

P425/1
PURE
MATHEMATICS
Paper 1
AUGUST 2022
3 HOURS



(MEPSA) RESOURCEFULL ASSESSMENT

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS set.2

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Answer **all** the **eight** questions in section **A** and any **five** from section **B**.
- Any additional question (s) answered will not be marked
- All necessary working **must** be shown clearly
- Begin each answer on a fresh sheet of paper
- Graph paper is provided
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** questions in this section.

1. Solve the simultaneous equations;

$$x + y = 4$$

$$x^2 + y^2 - 3xy = 76 \quad (05 \text{ marks})$$

2. Solve the equation; $\sqrt{3} \sin \theta - \cos \theta + 2 = 0$ for $0 < \theta < 2\pi$. (05 marks)

3. Find the equations of the lines which pass through the point A(3, -2) and makes an angle θ with the line $2x - 3y - 4 = 0$, where $\tan \theta = 2$. (06 marks)

4. Show that $\frac{(\sqrt{3} - i)^5}{\sqrt{3} + i} = -16$ (05 marks)

5. If $y = A x^K$, where A and K are non – zero constants , find the values of K such that; $x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 2y = 0$ (05 marks)

6. Using the substitution $x = e^t$, evaluate the $\int_1^e \frac{3 - 1nx}{x^2} dx$. (05 marks)

7. Given that A and B are points whose position vectors are $\mathbf{a} = 2\mathbf{i} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ respectively.
Determine the position vector of the point that divides AB in the ratio -4 : 1 (04 marks)

8. Find the area bounded by the three curves $y = x^2$, $y = \frac{1}{4}x^2$ and $y = \frac{1}{x^2}$ in the first quadrant. (05 marks)

SECTION B: (60 MARKS)

Answer any **five** questions from this section. **All** questions carry equal marks.

9. (a) Find $\int \frac{1}{x^3 \sqrt{x^2 - 4}} dx$ (06 marks)

- (b) Evaluate $\int_3^4 \frac{x^3}{x^2 - x - 2} dx$ (06 marks)

10. (a) The eighth term of an arithmetic progression is twice the fourth term, and the sum of the eight terms is 30. Find the
- (i) first four terms, (06 marks)
 - (ii) sum of the first 12 terms, of the progression (02 marks)
- (b) Find the number of ways in which the letters of the word STATISTICS can be arranged in a straight line so that,
- (i) the last two letters are both Ts. (02 marks)
 - (ii) all the three Ss must be together (02 marks)
11. (i) Given that the roots of the equation $ax^2 + bx + c = 0$ are α and β . Show that $a^2 = b^2 - 4ac$ if $\alpha - \beta = 1$. (06 marks)
- (ii) Find a quadratic equation whose roots are $(\alpha + \alpha\beta)$ and $(\beta + \beta\alpha)$ in terms of a, b and c. (06 marks)
12. (a) Differentiate with respect to x,
- (i) $2^{\cos x^2}$ (03 marks)
 - (ii) $\log_e \left(\frac{(1+x)e^{-2x}}{1-x} \right)^{1/2}$ (03 marks)
- (b) (i) Determine the equation of the normal to the curve $y = \frac{1}{x}$ at the point $x = 2$. (03 marks)
- (ii) Find the coordinates if the other point where the normal meets the curve again (03 marks)
13. (a) Given the points A (3, 1, 2) and B (2, -2, 4), find the sine of the angle BOC. Hence determine the area of triangle AOB. Where O is the origin. (06 marks)
- (b) Show that the line $\frac{x-2}{2} = \frac{2-y}{1} = \frac{3-z}{-3}$ is parallel to the plane $r \cdot (4\mathbf{i} - \mathbf{j} - 3\mathbf{k}) = 4$. Hence find the perpendicular distance between the line and the plane. (06 marks)

14.(a) $\sin(2 \sin^{-1} x + \cos^{-1} x) = \sqrt{1 - x^2}$. (05 marks)

(b) Prove that $\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, hence solve the equation

$$\tan (x - 45^\circ) = 6 \tan x, \text{ where } -180^\circ \leq x \leq 180^\circ \quad (07 \text{ marks})$$

15. (a) Find the equation and radius of a circle passing through the points A (0,1), B (0, 4) and C (2, 5). (05 marks)

(b) A circle passes through the point P(1, -4) and is tangent to the y-axis. If its radius is 5 units, find its equation (07 marks)

16. (a) Given that $y = 0$ when $x = 0$, solve the equation $\frac{dy}{dx} = 2y + 3$,
Expressing y as a function of x . (05 marks)

(b) When a uniform rod is heated it expands in such a way that the rate of increase of its length, l , with respect to the temperature, $\theta^\circ \text{C}$, is proportional to the length. When the temperature is 0°C the length of the rod is L . Given that the length of the rod has increased by 1% when the temperature is 20°C , find the value of θ at which the length of the rod has increased by 5%. (07 marks)

END

