

OKWANG SECONDARY SCHOOL

P.O BOX 741 – LIRA (U)

Uganda Advanced Certificate of Education.

S.5 END OF TERM II EXAMINATIONS, 2024.

PURE MATHEMATICS

P425/1

Time 3 Hours

Instructions

- Answer all eight questions in section A and only five questions from section B,
- Any additional question(s) will not be marked
- All working must be shown clearly.

Section A (40 marks): 72 Minutes

1. If the equation $x^2 + mx + n = 0$ and $x^2 + px + r = 0$ have common factors, prove that $(n - r)^2 = (m - p)(pn - mr)$. (5 marks)
2. Show that $\cos \frac{A-B}{2} = \frac{a+b}{c} \sin \frac{C}{2}$. (5 marks)
3. Solve for x in the equation $\log_{16}(21x - 5) = \log_4 2x$ (5 marks)
4. A spherical balloon is being inflated by gas being pumped at the constants rate of 200cm^3 per second. What is the rate of increase of the surface area of the balloon when its radius is 100cm? (5marks)
5. The curve $y = f(x)$ is such that $f(1) = 2$ and $f(x+h) = f(x) + 3x^2h - 6h + xh^2$, where h is a small increment in x. find the equation of the curve. (5 marks)
6. Use $\tan \frac{\theta}{2} = t$, to solve $3 \sin \theta - \cos \theta = 3$, for $0^\circ \leq \theta \leq 360^\circ$ (5 marks)
7. Sketch the curve $y = x^3 - x^2 - 5x + 6$. (5 marks)
8. Solve the equation $3^{(2x+1)} - 11(3^x) + 4 = 0$ (5 marks)

Section b

9. A curve is defined by the paranemic equations

$$x = t^2 - t$$

$$y = 3t + 4$$

Find the equation of the tangent to the curve at (2, 10)

(5 marks)

10. (a) If $f(x) = x + \frac{1}{x}$. Find $f^1(x)$ by first principles.

(4 marks)

(b) Differentiate the following with respect to x

(i) $\left(x^3 + \frac{1}{x^3}\right)^2$

(4 marks)

(ii) $\frac{(x+1)^4}{2x^2}$

(4 marks)

11. (a) Find the equations of the tangent and normal at the point P (4, 1) on the curve

$$y = \frac{4}{x}$$

(6 marks)

- (b) The tangent at P intersects the x – axis at X and the y – axis at Y. the normal at P intersects this line $y = x$ at L and the line $y = x$ at M. find;

- (i) The coordinates of the points X, Y, L and M.

- (ii) Prove that LYMX is a square.

(6 marks)

12. (a) By row reduction to echelon form solve the system of simultaneous equations.

$$5x - 3y + 2z = -5$$

$$2x + 2y - z = 4$$

$$4x - y + z = -1$$

(6 marks)

- (b) Given that α^2 and β^2 are the roots of the equation $12x^2 + 12x + 75 = 0$, find equation(s) whose roots are α^3 and β^3

(6 marks)

13. (a) Prove that if one of the differences between the roots of the equation

$$px^2 + qx + r = 0 \text{ is } k, \text{ then } k^2p^2 = q^2 - 4pr$$

(7 marks)

- (b) Express $-3x^2 + 12x - 4$ in the form $a(x + b)^2 + c$. state the values of a, b and c. hence determine the maximum value of $y = -3x^2 + 12x - 4$.

(5 marks)

*** The End ***