P425/1

PURE MATHEMATICS

Paper 1

Oct/Nov. 2022

3 hours

PRE-UNEB SET 1

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer ***all*** *the* ***eight*** *questions in section A and any* ***five*** *from section* ***B****.*

Any additional question(s) answered will ***not*** *be marked.*

All *necessary working* ***must*** *be shown clearly.*

Begin each answer on a fresh page.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Solve the equation $4^{x+4} - 2^{x+5} + 1 = 0.$ *(05 marks)*

2. P is a point which divides line AB externally in the ratio 3:2. A is (1,-2,1) and B is (0,-1,2). Find the Cartesian equation of line through P and Q(1,0,3). *(05 marks)*

3. Evaluate : $\int_0^2 \frac{x^2}{(x^3+1)^{1/2}} dx$ *(05 marks)*

4. Express $sinx - 2cosx$ in the form $Rsin(x - \alpha)$ where α is an acute angle. Hence solve $sinx - 2cosx = 1.5$ for $-180^0 < x < 180^0$.

   (05 marks)

5. Given that the expression $ax^3 + 8x^2 + bx + 6$ is exactly divisible by $x^2 - 2x - 3,$ find the values of a and b. *(05 marks)*

6. ABCD is a square where A(1,-1) and C(6,2) form the diagonal. Find the equation of line AB if it has a positive gradient. *(05 marks)*

7. A cylinder of height h and volume **V** fits exactly into a sphere of radius **a**. Show that its volume is given by $\boldsymbol{V} = \frac{1}{4}\pi h(4a^2 - h^2)$. Hence find the height of the cylinder in terms of **a** that gives maximum volume of the cylinder. *(05 marks)*

8. Find the volume generated by rotating about the x-axis the area bounded by the curve $y = 4 - x^2$, and the x-axis from $x = 0$ to $x = 1$ through 360^0. *(05 marks)*

SECTION B: (60 MARKS)

*Attempt only **five** questions from this section.*

9. (a) Express $Z = \frac{1+2i}{i^3(1-2i)} - \frac{(2+i)^3}{(3-i)^2}$ in polar form. *(06 marks)*

(b)Solve the equation: $\left(\frac{Z+1}{Z-1}\right)^2 = i$ where $Z = x + yi$

(06 marks)

10.(a) The second and third terms of a G.P are 24 and (b+1) respectively. The sum of the first three terms of the progression is 76, find the value of b for which the series is convergent. *(06 marks)*

(b) Assuming that x is so small that terms in x^3 and higher powers may be neglected, find a quadratic approximation to $\sqrt{\frac{1+2x}{1-x}}$. Hence state the range of values of x for which the expansion is valid. *(06 marks)*

11. (a) For $y = \frac{sinx}{x^2}$, show that $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2+2)y = 0$

(05 marks)

(b) Evaluate $\int_0^{\pi/3} x^2 sin3xdx$ *(07 marks)*

12.(a) If A, B and C are angles of a triangle, show that

$sinA - sinB + sinC = 4sin\frac{A}{2}cos\frac{B}{2}sin\frac{C}{2}$. *(05 marks)*

(b)Solve : $sin2\theta + cos2\theta = sin\theta - cos\theta + 1$ for $-180^0 \leq \theta \leq 180^0$

(07 marks)

13.(a) Show that the lines l_1 and l_2 with vector equations $\boldsymbol{r} = \boldsymbol{k} + \lambda(\boldsymbol{i} - \boldsymbol{j} - 3\boldsymbol{k})$ and $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{j} + \mu(3\boldsymbol{j} + 5\boldsymbol{k})$ respectively intersect. Find the angle between the two lines. *(07 marks)*

(b)Find the cartesian equation of the plane through the points A(1,0,-2) and B(3,-1,1) and parallel to the line $x = y = \frac{z-1}{2}$. *(05marks)*

14.(a) Differentiate the following with respect to x:

(i) $e^{-4x}x^3\tan(1-2x)$ *(03 marks)*

(ii) $log2x^2 + (sinx)^x$ *(03 marks)*

(b) Find the turning points on the curve $y = x^2e^x$ and distinguish between them. *(06 marks)*

15. (a) Find the equation of the circle which has its centre at point (2, -1) and touches the line $3x + y = 0$. *(04 marks)*

(b)The normal to the parabola $y^2 = 8x$ at the point $P(2t^2, 4t)$ meets the x-axis of the parabola at G and GP is produced, beyond P to Q so that $GP = PQ$. Show that the equation of locus of Q is another parabola. State its focus and vertex. *(08 marks)*

16.(a) Use the substitution $y = vx$, where v is a function of x, to solve the differential equation $x\frac{dy}{dx} = x + y$, given that $y = -1$ when $x = 1$.

(05 marks)

(b) A radioactive material decays so that the rate of decrease of mass at any time is proportional to the mass present at that time. Denoting by x the mass remaining at time t,

(i) Write down a differential equation satisfied by x. Hence show that $x = x_0e^{-kt}$ where x_0 is the initial mass and k is the decay constant.

(03 marks)

(ii) The mass is reduced to $4/5$ of its initial value in 30 days. Calculate to the nearest day, the time required for the mass to be reduced to $1/3$ its initial value. *(04 marks)*

GOOD LUCK