PURE MATHEMATICS
P425/1
DURATION: 3 HOURS: 00 MINUTES.

Instructions:

- Answer all the eight questions in section A and any five questions from section B.
- Any additional question(s) answered will not be marked.
- All working must be shown clearly.
- Silent, non –programmable scientific calculators and mathematical tables with a list of formulae may be used.
- Dont write wrong answers; Be a serious student (Tonswaaza)

SECTION A [40marks]

1. Solve for x, given that : $\log_a(x+3)+\frac{1}{\log_x a}-\log_a 4=0$. [05marks]

2. The first term of an A.P is equal to the first term of a G.P whose common ratio is $1/3$ and sum to infinity is 9. If the common difference of the A.P is 2, find the sum of the first ten terms of the A.P. [05marks]

3. In a triangle PQR all the angles are acute. Angle $PQR=50^0$, $p=10cm$ and $q=9cm$. Solve for the values of $r, <RPQ \ and \ <PRQ$. [05marks]

4. $ABCD$ is a square inscribed in a circle $x^2+y^2-6y=36$. Find the length of the diagonals and the area of the square.

5. The gradient function of a curve $y=ax^2+bx+C \ is \ 4x+2$. The function has a minimum value of 1. Find the values of $a, b \ and \ c$.

6. If $\propto^2 \ and \ \beta^2$ are the roots of the equation $x^2-21x+4=0$; form an equation whose roots are $\propto and \ \beta$; where $\propto and \ \beta$ are positive integers.

7. Solve the equation $3\cos\theta-4\sin\theta=5$, for $0\le\theta\le360^0$.

8. Find the locus of a point $p(x,y)$which is equidistant from the line $x=4$ and the centre of the circle $x^2+y^2=4$. Illustrate this with a Sketch.

SECTION B [60MARKS]

9. (i). Find the term independent of x in the expansion of $\left(x^2-1/3x\right)^9$ [04marks]

(ii). Expand $\left(\frac{1+3x}{1-3x}\right)^{1/2}$ as far as the term in x^3 and use this expansion to evaluate$\left(\frac{1.3}{0.7}\right)^{1/2}$ to 4 decimal places and hence deduce $\sqrt{91}$. [08marks]

10. (a). Express each of the following complex number $Z_1 = (1-i)(1+2i), Z_2 = \frac{2+6i}{3-i}$ *and* $Z_3 = \frac{-4i}{1-i}$ in form of $a + bi$. [06marks]

(b). Find the modulus and arguments of Z_1, Z_2, Z_3 above. [06marks]

11. (a). Mr.Mabirizi has five cows. Each cow is assigned a number. The sum of the five numbers in arithmetic progression is 25 and the sum of the squares of the number is 165.
Find the numbers. [07marks]

(b). Prof.Biraaro deposited shs.820,000/= in a certain bank at the beginning of 2012. If the bank gives compound interest of 10% per year, find how much money he will receive at the end of 2016 if he continues depositing the same amount at the beginning of every year and makes no withdrawal. [05marks]

12. (a). Solve the equation $\sin 4\theta - \sin\theta = \sin 3\theta - \sin 2\theta$ for $0^0 \leq \theta \leq 180^0$. [05marks]

(b). If A, B *and* C are angles of a triangle, show that;
$\text{Sin}(B + C - A) + \sin(A + B - C) + \sin(C + A - B) = 4\sin A \sin B \sin C$ [07marks]

13. (a). $f(x) = 6x^2 + px^2 + qx + r$, where p, q *and* r are constants. When $f(x)$ is divided by $x^2 - 4$ the remainder is $23x - 26$ and when divided by $(x + 3)$ the remainder is -220

(i). Find the values of p, q *and* r. [06marks]

(ii). If $(x - 1)$ is a factor of $f(x)$, factorize $f(x)$ completely.

(b). P is the equation of the tangent to the curve $y = x^2 + 6x - 4$ $(1,3)$ *and* q is the equation of the normal to the curve $y = x^2 - 6x + 18$ $at (4,10)$. Find the coordinates of the point of Intersection of P *and* Q. [06marks]

14. (a). Differentiate $\frac{1}{\sqrt{x^2+1}}$ from first principles. [04marks]

(b). Given that $y = \frac{\sin x}{x}$, show that
$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$ [04marks]]

(c). Given that $y = e^{3x}\sin 2x$, show that
$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$ [04marks]

15. (a). Show that the tangent to the curve $y = 4 - 2x - 2x^2$ at points $(-1,4)$ *and* $\left({}^1/_2, {}^5/_2\right)$ respectively, pass through the point $\left(-{}^1/_4, {}^{11}/_2\right)$, calculate the area of the region enclosed between the curve and the $x - axis$. [07marks]

(b). If $\propto$ *and* β are the roots of the equation $x^2 - px + q = 0$, Find the equation whose roots are $\frac{\propto^3-1}{\propto}$ *and* $\frac{\beta^3-1}{\beta}$. [05marks]

16. (a) Given that $y = \sqrt[3]{\frac{x-1}{(x^2-1)^2}}$ show that $\frac{dy}{dx} = \frac{1-3x}{3(x+1)^{5/3}(x-1)^{4/3}}$

(6 marks)

(b) A curve is given parametrically by the parametric equations x = 3t, y = $\frac{4}{t^2+1}$.

Find the equation of the tangent to the curve at (3, 2).

(6 marks)

END