P425/1
PURE MATHEMATICS
Paper 1
Jun. Jul. 2024
3 hours

DIOCESE OF KIGEZI

CHURCH OF UGANDA

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in section A and any* **five** *from* ***section B.***

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section ***B*** *will* **not** *be marked.*

SECTION A (40 MARKS)

1. Solve the equation: $sin4x\ cosx = cos3xsin2x\ \ for\ \ 0^o \leq x \leq 180^o$.
(05 marks)

2. The first, second and fourth terms of an arithmetic progression form a geometrical progression. Find the common ratio of the G.P. (05 marks)

3. The line through the point $C\ (2a, 8a)$ is a normal to the parabola $y^2 = 4ax$ at the point P. Find the coordinates of P.
(05 marks)

4. Show that $\frac{d}{dx}\ [\ln\ (1\ +\ x^x)]\ =\ \frac{1\ + \ln x}{1 +\ x^{-x}}$. (05 marks)

5. Given that $\frac{50}{(2\ +\ i)^2}\ =\ a\ +\ bi,$ find the real numbers $a\ \ and\ b.$
(05 marks)

6. Evaluate $\int_{o}^{1/2} \frac{4x}{4 -\ x^2}\ dx$ (05 marks)

7. Calculate the acute angle between the planes;
$x - z = 5\ \ and\ \ x + y + 4z = 10.$
(05 marks)

8. A curve is represented by the parametric equations;
$x = t^3 - 2t - 2\ \ and\ \ y = t^2 - 4t + 1.$ Find the coordinates of the stationary point of the curve.
(05 marks)

SECTION B (60 MARKS)

9. (a) Solve the equation: $3sin\theta\ -\ 4cos\theta = 4\ \ for\ \ 0^o \leq \theta \leq 360^o$
(06 marks)

(b) Prove that; $\frac{sin3A\ -\ sinA}{sin5A +\ sin3A}\ =\ \frac{1}{4}Sec^2 A$ (06 marks)

10. (a) Prove by induction that:

$$2^3 + 4^3 + 6^3 + \cdots + (2n)^3 = 2n^2(n+1)^2.$$ *(06 marks)*

(b) Obtain the Binomial expansion of $\frac{1+x}{\sqrt{1-2x}}$ up to the term in x^3. Use $x = {}^1/_8$ to evaluate $\sqrt{3}$ to 3 decimal places; and state the degree of accuracy to which this estimation is correct. *(06 marks)*

11. (a) Find the equation of the normal to the curve $y^2 = 4x$ at the point $A\,(1, 2)$. Deduce the y coordinate of point B where the normal meets the curve again. *(06 marks)*

(b) Calculate the volume generated when the area bounded by the curve $y^2 = 4x$ and the line AB (in (a) above) is rotated about the $y-$axis. *(06 marks)*

12. (a) Find the position vector of the point of intersection of the lines $r = \begin{pmatrix} 8 \\ -3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ *and* $r = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$; hence compute the acute angle between the lines. *(09 marks)*

(b) Write down the vector equation of the plane containing the two lines in (a) above. *(03 marks)*

13. (a) Use the substitution $\tan\theta = t$ to solve;

$$\int \frac{d\theta}{4 + 5\cos 2\theta}$$

(06 marks)

(b) Show that;

$$\int_0^1 tan^{-1} x\, dx = \frac{1}{4}(\pi - ln4).$$

(06 marks)

14. (a) Find the first three terms of the Maclaurin's series for $\ln(1 + x)$, and state the range of values of x within which the series is valid.

(06 marks)

(b) The letters of the word **COCACOLA** are to be arranged in a row. Find the number of arrangements of all the letters if;

(i) there is no restriction.

(ii) a **C** and an **A** begin and end an arrangement respectively.

(ii) the **O**'s are separated.

(06 marks)

15. A circle whose centre is $C\,(1, 6)$ touches the line $y = {}^{3}/_{4}\,x - 1$ at point A.

Find the; (a) equation of the circle. *(06 marks)*

(b) coordinates of point A. *(06 marks)*

16. (a) Solve: $Sin\, x\,\frac{dy}{dx} + y\cos x = \tan 3x$ *(05 marks)*

(b) The price P of a litre of petrol increases at a rate which is directly proportional to the price. If the price doubles every 10 days; find the percentage increase in the price after 20 days. *(07 marks)*

END