THE INTER SCHOOL VIRTUAL A LEVEL MATHEMATICS SEMINAR 2022.
An initiative of the Holistic elearning Platform(HeLP) Schools
Saturday 02nd July 2022 (9:00 a.m) and Sunday 03rd July 2022 (2:00p.m)

INSTRUCTIONS TO STUDENTS AND TEACHERS:

Dear students and teachers we would like to welcome you to participate in the forthcoming Mathematics seminar for senior six students. This is in preparation for the forthcoming final exams(UNEB) and the Mock Examinations. The seminar is organized by teachers under the Holistic eLearning programme. **This is a free seminar and no one should charge you any fees**.The process to be followed by both the teachers and students is suggested below:

1. Teachers share the Seminar questions with their students and ask for volunteers to discuss any of the questions.
2. Teachers talk to the school administrators to allow the children participate as presenters in the seminar on Saturday **02nd July from 09:00am - 2:00 pm.** Other students will just be participants.
3. There will be a repeat of this seminar on 03rd July 2022 starting at $2:00-5:00$pm to cater for the SDA community and others.
4. If your student is going to present then as the teacher(s) prepare her/him by looking through the calculations made by the student. Then encourage the student to write out the solution neatly in black pen including any graph. Then they scan or take a picture and send to the teacher . They can also type out the solution in a word or PowerPoint document and share with the teacher. The teacher or student will hand in the solutions to Kaziba Stephen (0787698238(Whats-App)) by monday **27th June 2022.** or upload the solution to the padlet https://padlet.com/holisticelearnplatform/Alevelmathematicsseminar
5. The teacher could now train the student on how to present in zoom as far as sharing a screen and using the whiteboard. Alternatively the students' presentation will be loaded on the computer screen and they explain to us their solution.

SEMINAR DETAILS

Holistic eLearning Platform is inviting you to a scheduled Zoom meeting.
Time: JULY 02, 2022 09:00 AM

This session will repeat on 03rd July 2022 at 2:00pm
Join Zoom Meeting
https://us02web.zoom.us/j/9133022418?pwd=cDlYOGNNV1hpdDU3SXBZUzdWMXorQT09
Meeting ID:9133022418
Passcode: HeLP
Kindly register your school using the survey link https://bit.ly/3sF2p7B .

P425/1	P425/2
1. Analysis (6 questions) (a) Differentiation (b) Intergration (c) Differential equations 2. Vectors (2 questions) (a) Vectors in 2-D (b) Vectors in 3-D (c) Ratio theorem (d) Line and their properties (e) Planes and their properties 3. Trigonometry (2 questions) 4. Geometry (2 questions) (a) Coordinate geometry of lines and triangles (b) Locus and circles (c) Parabola 5. Algebra (4 questions) (a) Surds,indices and logarithms (b) Quadratics (c) Polynomials (d) Simultaneous equations (e) Inequalities (f) Partial fractions (g) Complex numbers (h) Permutation and combinations	1. Mechanics (6 questions) (a) Calculus of vectors (b) General motion of the body (c) Relative motion (d) Projectiles (e) Newtonian mechanics (f) e.t.c 2. Numerical analysis (4 questions) (a) Location of the roots of an equation (b) Trapezium rule of numerical intergration (c) Newton raphson method (d) Errors (e) Flow charts 3. Statitics and probability(6 questions) (a) Mean ,node,median (b) Index numbers (c) Correlation coefficient (d) Scatter diagram (e) Discrete probability distributions (f) Continous probability distributions (g) Distributions i. Uniform distribution ii. Normal distribution iii. Binomial distribution iv. Normal approximation to binomial distribution (h) Estimations

"Yesterday's failures are today's seeds that must be diligently planted to be able to abundantly harvest tomorrow's success."

PURE MATHEMATICS (P425/1)

ALGEBRA

1. Solve the inequality $|2x+3| > 3|x+2|$

2. The polynomial $p(x)$ is defined by $p(x) = mx^3 + nx^2 - 17x - 6$, where m and n are constants. It is given that $(x+2)$ is a factor of $p(x)$ and that the remainder is 28 when $p(x)$ is divided by $(x-2)$.

   (a) Find the values of m and n .

   (b) Hence factorise $p(x)$ completely.

3. (a) Expand $\left(1+\frac{1}{4}x\right)^4$ up to the third term and use it to estimate $(1.025)^4$ correct to 3 decimal places.

   (b) Expand $(1+x)^{\frac{1}{3}}$ up to the term in x^2 . By using the substitution $x = \frac{1}{8}$, estimate the cube root of 9 correct to 3 decimal places.

   (c) Solve the equation ${}^nP_2 = 20$

4. (a) The coefficients of x^2 and x^3 in the expansion of $(3-2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$

   (b) i. Find the coefficient of x in the expansion of $\left(2x-\frac{1}{x}\right)^5$

   ii. Hence find the coefficient of x in the expansion of $(1+3x^2)\left(2x-\frac{1}{x}\right)^5$

5. (a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000.

   (b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression.

6. (a) Show that $1+i$ is a root of the equation $z^4 + 3z^2 - 6z + 10 = 0$.Hence find other roots

   (b) Given that the complex number z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z} - 2z + 2\bar{z} = 5 - 4i$.Find the possible values of z

7. (a) Solve the simultaneous equations

$$m^2 - 4mn + n^2 = 1$$
$$m^2 + n^2 - \frac{17m}{4} = 0$$

   (b) Find the range of values of x for which $\frac{2x+1}{x+2} > \frac{1}{2}$

8. Solve the simultaneous equations :

   (a)

$$\log(m+n) = 0$$
$$2\log m = \log(5+n)$$

(b)

$$2a - b + 4c = 26$$
$$3a - 2c - b = 0$$
$$a + 3b - c = 5$$

9. Express the following into partial fractions

   (a) $\frac{3x^3-x^2+2}{x(x^2-1)}$

   (b) $\frac{5x-12}{(x+2)(x^2-2x+3)}$

   (c) $\frac{2x^2+x-2}{x^3(x-1)}$

   (d) $\frac{x^3+5x^2+4x+5}{(x-1)(x^3-1)}$

VECTORS

10. (a) Determine the equation of the plane through the points A(1,1,2),B$(2,-1,3)$ and C$(-1,2,-2)$.

    (b) A line through the point D$(-13,1,2)$ and parallel to the vector $12i+6j+3k$ meets the plane in (a) at point E.Find :

    i. the coordinates of E

    ii. the angle between the line and the plane

11. (a) Find the cartesian equation of the line of intersection of the two planes $2x-3y-z=1$ and $4y+3x+2z=3$

    (b) Vectors **a,b** and **c** form the three sides of a triangle.Given that $|a|=5$,$|b|=12$ and **a.b**=$30\sqrt{3}$,find the area of the triangle

12. (a) The points A,B and C have position vectors **a**=$5i+3j+k$,**b**=$2i-j+3k$ and **c**=$7i-3j+10k$ respectively.Show that ABC is a triangle and find the area of triangle ABC

    (b) i. Find the coordinates of the point of intersection of the line $r=2i-k+\alpha(i+3j)$ and the plane $5x-y-7z=9$

    ii. Calculate the angle between a line and a plane in b(i) above

13. Two planes have equations $2x-y-3z=7$ and $x+2y+2z=0$

    (a) Find the acute angle between the planes

    (b) Find a vector equation for their line of intersection

TRIGONOMETRY

14. (a) Prove the identity

$$\left(\frac{1}{\cos\theta}-\tan\theta\right)\left(\sin^2+\frac{1}{\sin\theta}+\cos^2\theta\right)=\frac{1}{\tan\theta}$$

    (b) Hence solve the equation $\left(\frac{1}{\cos\theta}-\tan\theta\right)\left(\sin^2+\frac{1}{\sin\theta}+\cos^2\theta\right)=2\tan^2\theta$ for $0^0\leq\theta\leq 180^0$

15. (a) Prove that $\frac{\sin\beta}{1+\cos\beta} = \tan\frac{\beta}{2}$

    (b) Solve for θ in the equation $\cos 5\theta - \cos\theta = \sin 3\theta$ in the range $0^0 \leq \theta \leq 360^0$

16. If A,B ,C are angles of a triangle ,prove that

    (a) $\sin\frac{B}{2} = \cos\left(\frac{A+C}{2}\right)$

    (b) $\cos A + \cos B + \cos C - 1 = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

    (c) $\frac{\cos A+\cos B}{\sin A+\sin B} = \tan\frac{C}{2}$

    (d) $\cos 2A + \cos 2B + \cos 2C = 1 - \cos A\cos B\cos C$

17. Prove the following identities

    (a) $\frac{1-\cos 2B}{\sin 2B} = \tan B$

    (b) $\sec 2\theta + \tan 2\theta = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

    (c) $\frac{\sin 2A}{1+\cos 2A} = \tan A$

    (d) $\cos^4\beta - \sin^4\beta = \cos 2\beta$

    (e) $(\cos x + \text{cosec}x)^2 = \frac{1+\cos x}{1-\cos x}$,where $\cos x \neq -1$

18. (a) Solve $2\sin 2\theta = 3\cos\theta$ for $-180^0 \leq \theta \leq 180^0$

    (b) Solve $\sin\theta - \sin 4\theta = \sin 2\theta - \sin 3\theta$ for $-\pi \leq \theta \leq \theta$

19. (a) Solve $\cos\theta + \sqrt{3}\sin\theta = 2$ for $0^0 \leq \theta \leq 360^0$

    (b) Solve for β if $\cos\beta + \cos 5\beta + \cos 3\beta = 0$ for $0^0 \leq \beta \leq \pi$

    (c) If P,Q and R are angles of a triangle prove that :

$$\frac{1}{p}\cos^2\left(\frac{P}{2}\right) + \frac{1}{q}\cos^2\left(\frac{Q}{2}\right) + \frac{1}{r}\cos^2\left(\frac{R}{2}\right) = \frac{(p+q+r)^2}{4pqr}$$

GEOMETRY

20. The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$.The points A(1,1) and B$(2,-6)$ lie on the circle

    (a) Find the values of a and b and hence find the coordinates of the centre of the circle

    (b) Find the equation of the tangent to the circle at the point A,giving your answer in the form $px + qy = k$,where p,q and k are integers

21. The equation of a circle is $x^2 + y^2 - 8x + 4y + 4 = 0$.

    (a) Find the radius of the circle and the coordinates of its centre

    (b) Find the x- coordinates of the points where the circle crosses the x-axis giving your answers in exact form

    (c) Show that the point B$(6, 2\sqrt{3} - 2)$ lies on the circle

    (d) Show that the equation of the tangent to the circle at B is $\sqrt{3}x + 3y = 12\sqrt{3} - 6$

22. (a) $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are points on the parabola $y^2 = 4ax$.If the chord passes through the focus ,show that $pq = -1$.If M is the midpoint of PQ ,deduce that the locus of M is $y = 2a(x - a)$

    (b) Show that the equation $y^2 = 9(x + y)$ represents a parabola;hence determine its focus and directrix

23. (a) A tangent from the point $Q(q^2, 2q)$ touches the curve $y^2 = 4x$ .Find

        i. the equation of the tangent

        ii. the equation of line L parallel to the normal at $(q^2, 2q)$ and passes through $(1, 0)$

        iii. The point of intersection , P of the line L and the tangent

    (b) A point R(x,y) is equidistant from P and Q in (a) above .Show that the locus of R is $q^4 - 3q = 2(x + y)$

ANALYSIS

24. A curve has equation $y = 7 + 4In(2x + 5)$ Find the equation of the tangent to the curve at the point $(-2, 7)$, giving your answer in the form $y = mx + c$.

25. (a) Given that $y = \tan 2x$, show that $\frac{dy}{dx} = 2\tan x + 2\tan^3 x$

    (b) Show that the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\tan x + \tan^2 x + \tan^3 x)dx = \sqrt{3} - \frac{\pi}{2}$

26. The parametric equations of a curve are $x = 1 - \cos\beta$,$y = \cos\beta - \frac{1}{4}\cos 2\beta$.Show that

$$\frac{dy}{dx} = -2\sin^2\left(\frac{1}{2}\beta\right)$$

27. (a) Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x

    (b) The period ,T of a swing of a simple pendulum of length ,l is given by the equation $T^2 = \frac{4\pi^2 l}{g}$,where g is the acceleration due to gravity.An error of 2% is made in measuring the length ,l .Determine the resulting percentages error in the period ,T

    (c) If $y = 3x^2 - x$ .show that $y\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y + 1 = 6x$

28. (a) A curve is given by the equation $y = x^2 + 1$. Find the area bounded by the curve, the axes and the line$x = 2$.

    (b) Find the volume of the solid generated when this area is revolved about the ;

        i. x − axis.

        ii. y − axis.

29. (a) Find $\int(2\cos\theta - 3)(\cos\theta + 1)d\theta$

    (b) Prove the identity

$$\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = 2\cos^2\theta - 1$$

30. (a) Prove each of these identities

i. $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}$ ii. $\frac{\cos\theta}{\tan\theta(1+\sin\theta)}=\frac{1}{\sin\theta}-1$

(b) Hence solve the equation $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=1+3\sin\theta$ for $0^0\leq\theta\leq360^0$

31. (a) If $y=In(x^2-5)$,show that $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=2e^{-y}$

(b) If $y=e^x\sin x$,show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

32. (a)

$$\int xIn(x^2-25)dx$$

(b) Evaluate

$$\int_0^2\frac{dx}{x^2\sqrt{16-x^2}}$$

33. (a) Differentiate the following from first principles

i. $\sin^2 2x$

ii. $x10^{\sin x}$

(b) Given that $y=In\left(e^{2x}\left(\frac{x+3}{x-3}\right)\right)^{\frac{-2}{3}}$.Find $\frac{dy}{dx}$

34. (a) Solve the equations below

i.

$$(1+x^2)\frac{dy}{dx}-y(y+1)x=0,\text{given that y=1 when x=0}$$

ii.

$$\frac{dy}{dx}-y\tan x-\cos x=0,\text{given that y=0 at } x=\frac{\pi}{2}$$

(b) The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding .Initially the temperature of the body is 80^0C .After 10 minutes the temperature of the body is 60^0C .The temperature of the surrounding is 15^0C

i. Form a differential equation for the temperature of the body

ii. Determine the time it takes for the temperature of the body to reach 40^0C

35. Maize dwarf mosaic virus(MDMV) has infected a number of maize plants Mr Ronalds' garden .The growth in the number of maize plants infected is proportional to the number already infected .Initially 20 maize plants were infected

(a) Form a differential equation that models the growth in the number infected

(b) Thirty days after the initial number of infections ,60 maize plants were infected .After how many further days does the model predict that 200 maize plants will be infected?

36. Given that

(a) $y = \sin(log_e x)$,show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(b) $y = e^{2x}\cos 3x$,show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

37. (a) Sketch on the same diagram,the curves $y = x^2 - 5x$ and $y = 3 - x^2$,and find their points of intersection

(b) Find the area of the region bounded by the two curves

38. (a) Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$.

(b) Hence find the exact value of $\int_0^{\frac{1}{2}} \frac{8x^3+4x^2+2x+7}{4x^2+1}dx$

39. (a) Find $\int x^3 e^{x^4} dx$

(b) Use the substitution $t = \tan\theta$ to find $\int \frac{1}{1+\sin^2\theta}d\theta$

(c) Find the area enclosed by the line $y = x + 1$ and the curve $y = x^2 - 2x - 3$

40. (a) Express $\frac{8-6x}{(x+3)(x^2+4)}$ in partial fractions

(b) Hence show that $\int_1^2 \frac{8-6x}{(x+3)(x^2+4)}dx = In\left(\frac{125}{128}\right)$

41. At time t days ,the rate of decay of the mass of a radioactie material is proportional to the mass,m grams ,of the radioactive material that is present at that time .At $t = 0$,$m = 150g$ and $t = 5, m = 100g$.

(a) Show that $m = 150e^{-\frac{1}{5}In(\frac{3}{2})t}$

(b) Find the length of time that it takes for the mass of the radioactive material present to be halved

42. Intergrate with respect to x the functions.

(a) $\int \frac{6x-10}{(2x+1)^2}dx$

(b) $\int \frac{6x-2x^2-8}{(x-1)(x^4-1)}dx$

(c) $\int \sin^{-1} x\mathrm{d}$

(d) $\int \frac{\cos\theta}{\cos 2\theta}d\theta$

APPLIED MATHEMATICS (P425/2)

MECHANICS

43. A particle M is projected vertically upwards from horizontal ground with speed u ms $^{-1}$. M reaches a maximum height of 20m above the ground.

(a) Find the value of u .

(b) Find the total time for which M is at least 15 m above the ground.

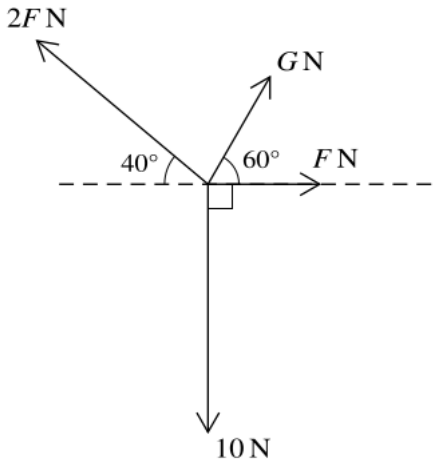
44. A cyclist starts from rest at a fixed point O and moves in a straight line, before coming to rest k seconds later. The acceleration of the cyclist at time t s after leaving O is a ms^{-2} , where a $= \frac{2}{\sqrt{t}} - \frac{3\sqrt{t}}{5}$ for $0 < t \leq k$

(a) Find the value of k .

(b) Find the maximum speed of the cyclist.

(c) Find an expression for the displacement from O in terms of t . Hence find the total distance travelled by the cyclist from the time at which she reaches her maximum speed until she comes to rest.

45. Four coplanar forces act at a point. The magnitudes of the forces are 10 N, F N, G N and 2 F N. The directions of the forces are as shown in the diagram.



(a) Given that the forces are in equilibrium, find the values of F and G .

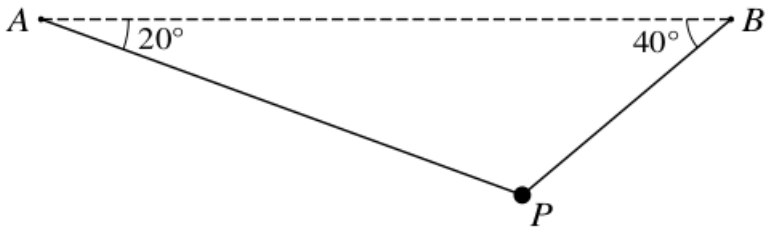
(b) Given instead that F = 3, find the value of G for which the resultant of the forces is perpendicular to the 10 N force.

46. A particle A is projected with speed 20 m s^{-1} at an angle of 60^0 below the horizontal, from a point O which is 30 m above horizontal ground.

(a) Calculate the time taken by A to reach the ground.

(b) Calculate the speed and direction of motion of A immediately before it reaches the ground.

47. A particle P of mass 1.6 kg is suspended in equilibrium by two light inextensible strings attached to points A and B . The strings make angles of 20^0 and 40^0 respectively with the horizontal . Find the tensions in the two strings.



48. A particle P moves in a straight line, starting from rest at a point O on the line.At time t s after leaving O the acceleration of P is $k(16 - t^2)$ ms^{-2} , where k is a positive constant, and the displacement from O is s m. The velocity of P is 8 ms^{-1} when t = 4.

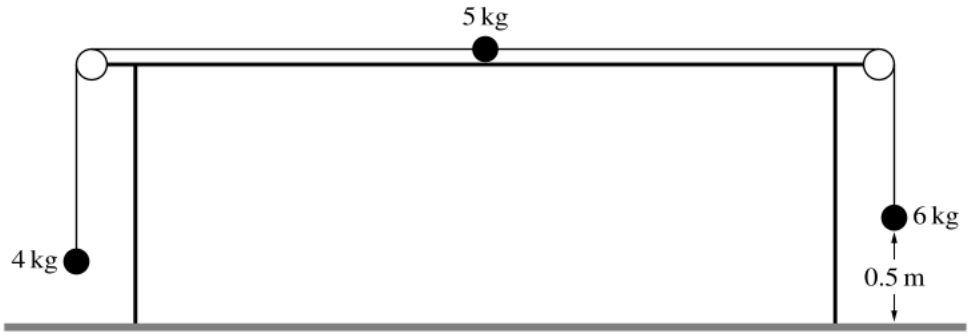
(a) Show that

$$s = \frac{1}{64}t^2(96 - t^2)$$

(b) Find the speed of P at the instant that it returns to O .

(c) Find the maximum displacement of the particle from O .

49. The diagram below shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.



(a) Find the coefficient of friction between the 5 kg particle and the table

(b) The 6 kg particle is now replaced by a particle of mass 8 kg and the system is released from rest. Find the acceleration of the 4 kg particle and the tensions in the strings.

50. Two uniform rods AB and BC of the same length and masses 8kg and 6kg respectively are smoothly joined at B.The rods A and C rest on a rough horizontal plane.The rods are inclined at 120^0 to each other when BC is on the point of slipping.Find the:

(a) Normal reactions at A and C

(b) Coefficient of friction between the ground and rod BC

(c) Magnitude of the reaction at B

51. At time $t = 0$,the position vector $\mathbf{r}$ and velocity $\mathbf{v}$ of two trains A and B are as follows.

Trains	Velocity vector	Position vector
A	$V_A = (-6i + k)ms^{-1}$	$r_A = (i + 2j + 3k)m$
B	$V_B = (-5i + j + 7k)ms^{-1}$	$r_B = (4i - 14j + k)m$

If the trains maintain these velocities,find the :

(a) Position of B relative to A at time t

(b) time that elapses before the trains are closest to each other

(c) least distance between the trains in the subsequent motion

52. A particle of mass $3kg$ is acted upon by a force $F = 6i - 3t^2j + 27tk$ newtons at time t.At time $t = 0$,the particle is at the point with a position vector $i - 5j - k$ and its velocity is $(3i + 3j)\text{ms}^{-1}$.Determine the:

(a) Position vector of the particle at time $t = 1$second.

(b) distance of the particle from the origin at time $t = 2$seconds

53. Given that f(0.9) = 0.2661, f(1.0) = 0.2420 and f(1.1.) = 0.2179, estimate:

    (i) f(0.94)

    (ii) $f^{-1}(0.2362)$ using linear interpolation or extrapolation

54. (a) Given that the equation $e^x = 10 - x$ has a root between 2 and 3.Show that the iterative formula based on Newton Raphson method is given as

$$x_{(n+1)} = \frac{10 + e_n^x(x_n - 1)}{e^{x_n} + 1}$$

    (b) Draw a flowchart that reads,prints the root x and the number of iterations.Carry a dry run of the flowchart and obtain a root with an error of less than 0.001,taking the initial approximation $x_0 = 2.45$

55. (a) i. On the same axes ,draw graphs of $y = e^{2x}$ and $y = 5x + 1$ for $-0.5 \leq x \leq 1$

    ii. From your graphs ,obtain to 1 significant figure ,an appropriate root of the equation $e^{2x} - 5x - 1 = 0$.

    (b) Using the Newton -Raphson method ,find the root of the equation $e^{2x} - 5x - 1 = 0$ taking the appropriate root in (a) as an initial approximation .Give your answer correc to 3 decimal places.

56. (a) Use the trapezium rule with 7 ordinates to estimate $\int_0^6 xe^{-x}dx$ correct to 3 significant figures

    (b) Find the percentage error made in your estimation ,giving your answer to 2 deciml places .Suggest how this error may be reduced.

57. Given the numbers $p = 30.75$ and $q = 6.125$ all measured to their nearest number of decimal places indicated

    (a) State the maximum possible errors in p and q

    (b) Find the absolute error in the quotient $\frac{p}{q}$

    (c) Find the limits within which the exact value of the quotient $\frac{p}{q}$ lies

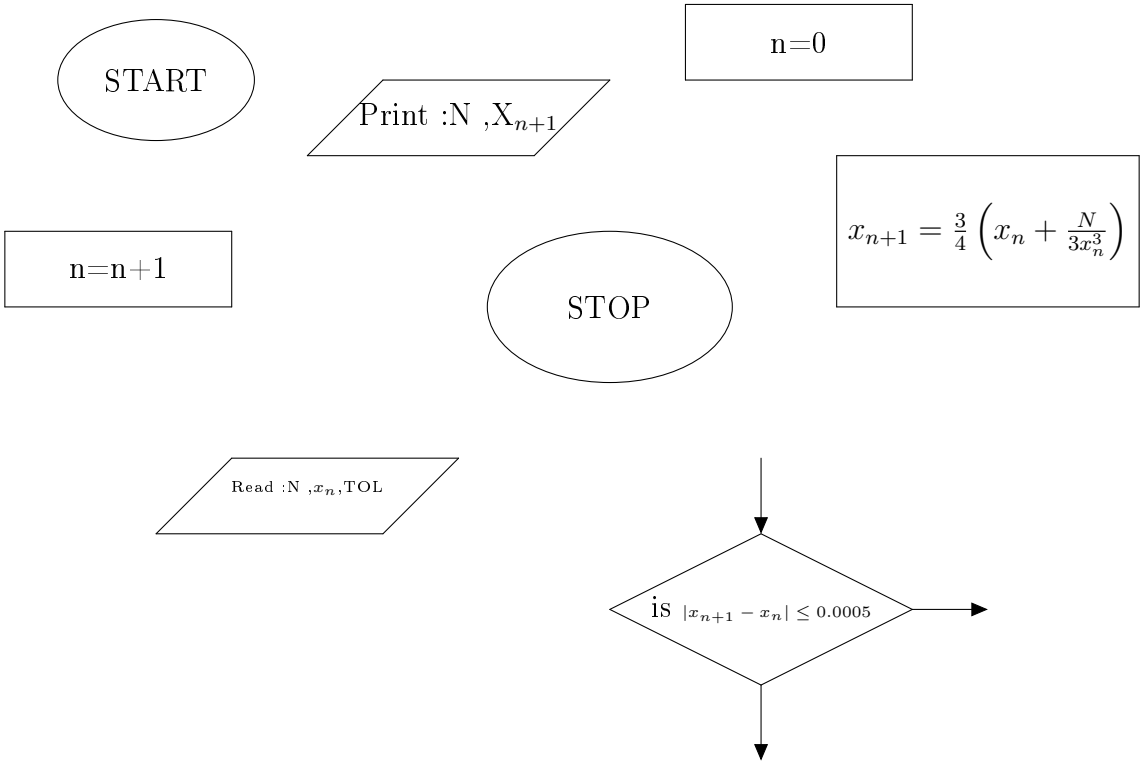
58. (a) The radius r and height h of a cylinder were measured with errors δr and δh respectively.Show that the maximum relative error in the volume of the cylinder is $2|\frac{\delta r}{r}| + |\frac{\delta h}{h}|$

    (b) The quantities x and y were measured with errors δx and δy respectively.Show that the maximum relative error in $\sqrt{xy}$ is $\frac{|y\delta x| + |x\delta y|}{2|xy|}$ hence find the limits within which $\sqrt{2.4 \times 4.86}$ is expected to lie

59. (a) Use the trapezium rule with three strips to estimate the integral $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1+\cos x}}$,correct to three decimal places

    (b) Calculate the percentage error in your estimation in (a) above

60. Given below are parts of a flow chart not arranged in order



(a) Re arrange them and draw a complete logical flow chart

(b) Using N=44 and $x_0 = 2$Perform a dry run of your arranged flow chart

(c) State the purpose of the flow chart

61. The table below shows the values of;

θ	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$
$\sin 10^0$	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822

Use linear interpolation to find,

(a) $\sin 10^0 16'$

(b) $\sin^{-1} 0.1747$

STATISTICS AND PROBABILITY

62. (a) Stephen keeps layer birds.Its assumed that the birds are likely to lay a yellow or white yolked egg.Find the probability that in the 5 eggs picked there will be more yellow yolked eggs than white yolked eggs

(b) Calculate an estimate of the mean height of 50 tomato plants , where there are sixteen 18cm height, twenty 19cm height and fourteen that are either 20 or 21 cm height.

63. Pack A consists of ten cards numbered 0, 0, 1, 1, 1, 1, 1, 3, 3, 3. Pack B consists of six cards numbered 0, 0, 2, 2, 2, 2. One card is chosen at random from each pack. The random variable X is defined as the sum of the two numbers on the cards.

(a) Show that $P(X = 2) = \frac{2}{15}$

(b) Draw up the probability distribution table for X .

(c) Given that $X = 3$, find the probability that the card chosen from pack A is a 1.

64. A random variable X has probability density function given by

$$f(x) = \begin{cases} 6x(1-x) & ;0 \leq x \leq 1 \\ 0 & ;\text{otherwise} \end{cases}$$

(a) Find the probability that X does not lie between 0.3 and 0.7.

(b) Sketch the graph of the probability density function and hence state the value of $E(X)$

(c) Find Var(X)

65. A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from $\{A, B, C, D, E\}$ and the digits are chosen from $\{1, 2, 3, 4, 5, 6, 7\}$. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed?

(b) Find the number of different codes that include the letter A or the digit 5 or both.

(c) A security code is formed at random. Find the probability that the code is DE followed by a number between 4500 and 5000.

66. The times taken, in minutes, by 360 employees at Mehta Sugar Factory in Lugazi to travel from home to work are summarised in the following table.

Time, t minutes	$0 \leq t < 5$	$5 \leq t < 10$	$10 \leq t < 20$	$20 \leq t < 30$	$0 \leq t < 50$
Frequency	23	102	135	76	24

(a) Calculate an estimate of the mean time taken by an employee to travel to work.

(b) Draw a histogram to represent this information.

67. The cost of making a well formulated feed for the layer birds on Mr Ronald's Poultry farm is calculated from the cost of Maize bran,Broken maize ,lime and concentrate .The table below gives the cost of these items in 2021 and 2022.

ITEMS	Price(UGX) in 2021	Price (UGX) in 2022	Weight
Maize bran/kg	500	735	12
Lime/kg	500	600	2
Broken maize/kg	800	1400	5
Concentrate/kg	196000	215600	1

Using 2021 as the base year

(a) Calculate the price relative for each item hence find the simple price index for the cost of making a complete feed

(b) Find the weighted aggregate price index for the cost of the feed

68. A bag contains 5 red and 4 blue marbles. Three marbles are selected at random from the bag,without replacement.

    (a) Show that the probability that exactly one of the marbles is red is $\frac{5}{14}$

    (b) The random variable X is the number of red marbles selected.Draw up the probability distribution table for X .

    (c) Find E(X)

69. The times taken, in minutes, by farmers to plant maize on a piece of land is normally distributed with mean 32.2 and standard deviation 9.6.

    (a) Find the probability that a randomly chosen farmers takes more than 28.6 minutes to complete the task.

    (b) 20% of the farmers take longer than t minutes to complete the task. Find the value of t .

    (c) Find the probability that the time taken to complete the task by a randomly chosen farmer differs from the mean by less than 15.0 minutes.

70. (a) M and N are two independent events such that $P(M) = \frac{2}{10}$ and $P(N) = \frac{3}{20}$,Find

        i. $P(M/N)$

        ii. Probability of M or N but not both

    (b) Two events M and N are such that $P(M) = \frac{7}{10}$,$P(M \cap N) = \frac{9}{20}$ and $P(\overline{M} \cap \overline{N}) = \frac{9}{50}$ .Find

        i. $P(\overline{N})$

        ii. P(M or N but not both M and N)

71. A continuous r.v X is uniformly distributed in the interval $[a, b]$.The lower quartile is 5 and the upper quartile is 9 .Find the

    (a) Values of **a** and **b** ,hence the probability density function

    (b) E(X) and Var(X)

    (c) $P(2 < X < 10)$

72. (a) The probability that a student passes Algebra is $\frac{3}{4}$ and the probability that she passes Vectors is $\frac{13}{20}$,the probability that she passes neither of the sections is $\frac{1}{5}$ .Find the probability that she passes Algebra but not Vectors.

    (b) A bag contains 4 red balls,3 white balls and 1 yellow ball.Two balls are picked in succession at random without replacement .Find the probability that :

        i. Both are of the same color

        ii. Atleast one black ball is picked

73. An experiment consists of removing 2 sweets one at a time without replacemnt from a box containing 3 red and 4 blue sweets.

(a) If M is the event that both sweets picked are of the same color ,find the probability that event M occured

(b) If the experiment is repeated 40 times ,find the probability that event M occured

i. between 20 and 35 times

ii. atleast 24 times

74. The continuous random variable X has probability density function $f(x)$ defined by

$$f(x) = \begin{cases} \frac{m}{x^4} & x < -1 \\ m(2 - x^2) & -1 \leq x \leq 1 \\ \frac{m}{x^4} & x > 1 \end{cases}$$

(a) Find the value of constant m

(b) Determine the expected value of X and its standard deviation

75. During the National Agriculture Education show ,two Agriculture experts Mr Brian and Mr Mathias were choosen to judge the tomatoes presented by 8 schools in a prize competition for the best school in Agriculture in Uganda.

Items	A	B	C	D	E	F	G	H
Mr Mathias	70	59	71	38	55	62	80	76
Mr Brian	67	42	85	51	39	97	81	70

Calculate the rank correlation coefficient between the scores and comment on your result at 5% level of significance

76. The continuous random variable X has probability distribution defined by

$$F(x) = \begin{cases} 0 & 0 \leq x \\ \frac{x^2}{4} & 0 \leq x \leq 1 \\ mx - \frac{1}{4} & 1 \leq x \leq 2 \\ n(5 - x)(x - 1) & 2 \leq x \leq 3 \\ 1 & x \geq 3 \end{cases}$$

(a) Find the values of m and n

(b) Determine the

i. p.d.f ,f(x) of x

ii. $P(1 \leq 2x \leq 4)$

iii. E(x)

77. The table below shows the scores obtained by a random sample of 80 learners who st for the mathematics contest

(a) Calculate the :

i. Mean

Scores	Number of learners
$20- < 25$	20
$25- < 30$	12
$30- < 35$	13
$35- < 45$	4
$45- < 55$	16
$55- < 75$	15

ii. Median

(b) Display the data on a histogram and use it to estimate the mode

78. The table below shows the distribution of weights of a certain type of fruit

Weight(grams)	Frequency
20–	4
30–	3
40–	21
50–	50
60–	14
70–	8

(a) Calculate the :

i. Mean weight

ii. Standard deviation

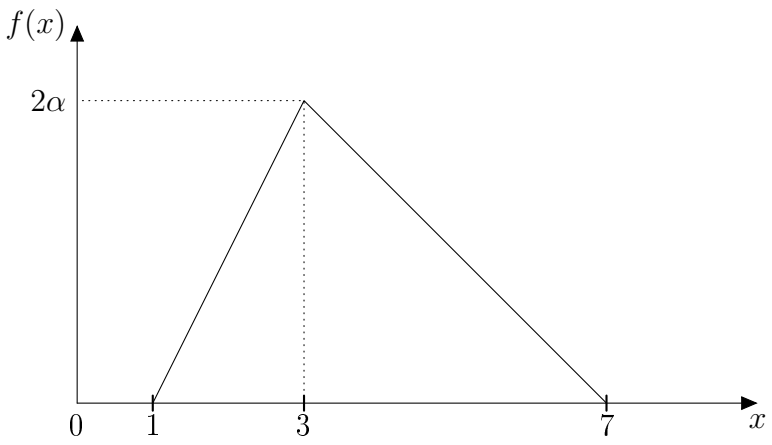
(b) Plot a cumulative frequency curve for the data and use it to estimate the

i. Median

ii. 80^{th} percentile

iii. quartile deviation

79. The p.d.f f(x) of a r.v X takes on the form shown in the sketch below



Find the

(a) Value of α

(b) Equations of the p.d.f hence the p.d.f

(c) Cumulative distribution F(x) and sketch it

(d) Median of X

80. Two locally developed antiviral drugs to cure COVID-19 were used to treat 12 symptomatic people in rotation of days and the results for the effectiveness of the herbal drugs was recorded in percentage as follows.

Covylice-1(x)	58	52	48	30	48	20	32	50	38	12	36	12
Covidex(y)	90	72	60	38	70	35	33	64	48	24	50	18

(a) Plot a scatter diagram for the data .Comment on the relationship between the two drugs

(b) Draw a line of best fit for the scatter diagram ,hence find x when $y = 68$

(c) Calculate the rank correlation coefficient for the effectiveness of the drugs .Comment on your result at 1% level of significance

END

PURE MATHEMATICS P425/1

1. Algebra

   (a) For the quadratic equation $ax^2 + bx + c = 0$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   (b) For an arithmetic series (A.P)

   $$u_n = a + (n-1)d$$
   $$S_n = \frac{1}{2}n\{2a + (n-1)d\}$$

   (c) For a geometric series (G.P)

   $$u_n = ar^{n-1}$$
   $$S_n = \frac{a(1-r^n)}{1-r} \qquad r \neq 1$$
   $$S_\infty = \frac{a}{1-r} \qquad |r| < 1$$

   (d) Binomial expansion

   $$(a+b)^n = a^n + \begin{pmatrix} n \\ 1 \end{pmatrix} a^{n-1}b + \begin{pmatrix} n \\ 2 \end{pmatrix} a^{n-2}b^2 + \begin{pmatrix} n \\ 3 \end{pmatrix} a^{n-3}b^3 + \cdots + b^n$$

   where n is a positive integer

   $$\begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!}$$
   $${}^nC_r = \frac{n!}{r!(n-r)!}$$
   $$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1(n-2)}{3!}x^3 \cdots$$

   Where n is rational and $|x| < 1$

   $${}^nP_r = \frac{n!}{(n-r)!}$$

   Where $r \leq n$

   (e) Summations

   $$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$
   $$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$
   $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

2. Trigonometry

No	Identity
1	$\tan\theta = \frac{\sin\theta}{\cos\theta}, \sec\theta = \frac{1}{\cos\theta}, co\sec\theta = \frac{1}{\sin\theta}$
2	$\cos^2\theta + \sin^2\theta = 1$
3	$1 + \tan^2\theta = \sec^2\theta$
4	$\cot^2\theta + 1 = co\sec^2\theta$
5	$\sin(A+B) = \sin A\cos B + \cos A\sin B$
6	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
7	$\cos(A+B) = \cos A\cos B - \sin A\sin B$
8	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
9	$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$
10	$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$
11	$\sin 2A = 2\sin A\cos A$
12	$\cos 2A = \cos^2 A - \sin^2 A$
13	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(a) For the t -formula

$$t = \tan\frac{1}{2}\theta$$
$$\sin\theta = \frac{2t}{1+t^2}$$
$$\cos\theta = \frac{1-t^2}{1+t^2}$$

(b) For any triangle with angles ,A,B and C and with sides a,b,and c .

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{Cosine rule}$$
$$s = \frac{a+b+c}{2}$$

3. Differentiation

No	y	$\frac{dy}{dx}$
1	x^n	nx^{n-1}
2	$\ln x$	$\frac{1}{x}$ for $x \neq 0$
3	e^x	e^x
4	$\sin x$	$\cos x$
5	$\cos x$	$-\sin x$
6	$\tan x$	$\sec^2 x$
7	uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
8	$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
9	$f(x)$	$\frac{f(x+\delta x)-f(x)}{\delta x}$
10	$\sec x$	$\sec x\tan x$
11	y=u,u=x	$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$
12	$e^{f(x}$	$f^1(x)e^{f(x)}$

4. Integration

No	$f(x)$	$\int f(x)dx$
1	x^n	$\frac{x^{n+1}}{n+1} + c$ for $n \neq -1$
2	$\frac{1}{x}$	$\ln \lvert x \rvert + c$ n
3	e^x	$e^x + c$
4	$\sin x$	$-\cos x + c$
5	$\cos x$	$\sin x + c$
6	$\sec^2 x$	$\tan x + c$
7	$\int u\frac{dv}{dx}dx$	$uv - \int v\frac{du}{dx}dx$
8	$\int \frac{f^1(x)}{f(x)}dx$	$\ln \lvert f(x) \rvert + c$
9	$co\sec x \cot x$	$-co\sec x + c$
10	$\sec x \tan x$	$\sec x + c$
11	$co\sec^2 x$	$-\cot x + c$
12	$\tan x$	$\ln \lvert \sec x \rvert + c$
13	$co\sec x$	$-\ln \lvert co\sec x + \cot x \rvert + c$
14	$\cot x$	$\ln \lvert \sin x \rvert + c$

(a)
$$\int \frac{1}{a^2 - b^2x^2}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right) + c$$

(b)
$$\int \frac{1}{a^2 + b^2x^2}dx = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right) + c$$

(c)
$$\int \frac{a}{p + qx}dx = \frac{a}{q}\ln|p + qx| + c$$

5. Vectors

(a) If $a = a_1i + a_2j + a_3k$ and $b = b_1i + b_2j + b_3k$ then
$$\begin{aligned} a.b &= a_1b_1 + a_2b_2 + a_3b_3 \\ &= |a||b|\cos\theta \end{aligned}$$

(b) $i.i = j.j = k.k = 1$ and $i.j = i.k = j.k = 0$

(c) $|a.a| = |a|^2$

(d) $a.(b + c) = a.b + a.c$

(e) $a.(kb) = (ka).b = k(a.b)$ where k is a constant

(f) $a.b = |a||b|\cos\theta$

(g) The cartesian equation of the line
$$\frac{x - a}{x_1} = \frac{y - b}{y_1} = \frac{z - c}{z_1}$$

APPLIED MATHEMATICS P425/2

1. Numerical Methods

   (a) Trapezium rule

$$\int_a^b f(x)dx \approx \frac{1}{2}h\{y_o + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n\}$$

   Where $h = \dfrac{b-a}{n}$

   (b) Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)} \qquad \text{Where } x = 0, 1, 2, \cdots$$

   (c) Ordinates and sub intervals

$$\text{The number of sub intervals} = \text{Number of ordinates} - 1$$

   (d) The maximum possible error made due to rounding off is given by

$$\text{Error} = 0.5 \times 10^{-n}$$

   Where **n** is the number of decimal places rounded off to

   (e) Error

$$\text{Error } = \text{Exact value} - \text{Approximate value.}$$

   (f) Absolute error This is the actual size of the error and is always positive .It is the magnitude of the error

$$\text{Error } = |\text{Exact value} - \text{Approximate value}|.$$

   (g) Relative error

$$\text{Relative Error } = \frac{\text{Absolute error}}{\text{Exact value}}$$

   The relative error must always be positive

$$\text{Relative Error } = \frac{|\text{Error}|}{\text{Exact value}}$$

   (h) Percentage error

$$\text{Percentage Error } = \frac{|\text{Error }|}{\text{Exact value}} \times 100$$

   (i) The interval or range with in which the exact value lies is given by Min value$\leq$ Exact value $\leq$ Max value or [Min,Max]

(j) Absolute error $= \frac{\text{Maximum value} - \text{Minimum value}}{2}$

2. Probability and Statistics

(a) The mean for ungrouped data is calculated using the formula

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of values in the data}}$$

$$\overline{X} = \frac{\sum x}{n}$$

(b) The mean for grouped data is calculated using the formula

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

Where **x** is the class mark and **f** is the frequency

(c) The mean for grouped data when given an assumed means is calculated using the formula

$$\text{Mean} = A + \frac{\sum fd}{\sum f}$$

Where **A** is the assumed mean or working mean

d is the deviation given as $d = x - A$

(d) For grouped data ,the median is calculated using

$$\text{Median } = L_1 + \left(\frac{\frac{\sum f}{2} - CF_b}{f_m} \right) \times C$$

Where

$L_1 =$ Lower class boundary of the median class

$CF_b =$ Cummulative frequency before the median class

$f_m =$ frequency within the median class

$C =$ Class width

$\sum f =$ Total frequency

(e) For grouped data with equal class width the mode is calculated using

$$\text{Mode} = L_1 + \left(\frac{d_1}{d_1 + d_2} \right) \times C$$

Where

$L_1 =$ Lower class boundary of the modal class
$d_1 =$ Modal frequency $-$Pre modal frequency
$d_2 =$ Modal frequency $-$Post modal frequency
$C =$ Class width

(f) For grouped data ,the lower quartile is calculated using

$$q_1 = L_1 + \left(\frac{\frac{\sum f}{4} - CF_b}{f_m} \right) \times C$$

Where

$L_1 =$ Lower class boundary of the q_1 class
$CF_b =$ Cummulative frequency before the q_1 class
$f_m =$ frequency within the q_1 class
$C =$ Class width
$\sum f =$ Total frequency

(g) For grouped data ,the upper quartile is calculated using

$$q_3 = L_1 + \left(\frac{\frac{3\sum f}{4} - CF_b}{f_m} \right) \times C$$

Where

$L_1 =$ Lower class boundary of the q_3 class
$CF_b =$ Cummulative frequency before the q_3 class
$f_m =$ frequency within the q_3 class
$C =$ Class width
$\sum f =$ Total frequency

(h) Inter quartile range $= q_3 - q_1$

(i) For grouped data ,the variance is calculated using

$$\text{Var(x)} = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$$

(j) Standard deviation $= \sqrt{\text{Var(x)}}$

3. Index numbers

(a)

$$\text{Price relative } = \frac{p_n}{p_0} \times 100$$

Where

p_n = Price of the commodity in the given year(new year)

p_0 = Price of the commodity in the base year(old year)

(b)

$$\text{Simple price index} = \frac{\text{Sum of the price relatives}}{\text{Number of items (N)}} \times 100$$
$$= \frac{\sum\left(\frac{p_n}{p_0}\right) \times 100}{N}$$

(c)

$$\text{Simple aggregate price index} = \frac{\text{Current year price total}}{\text{Base price total}} \times 100$$
$$= \frac{\sum p_n}{\sum p_0} \times 100$$

(d)

$$\text{Weighted price index} = \text{Price relatives} \times \text{weights}$$
$$= \frac{p_n}{p_o} \times w \times 100$$

(e)

$$\text{Weighted aggregate price index} = \frac{\sum p_n w}{\sum p_0 w} \times 100$$

(f)

$$\text{Weighted average price index} = \frac{\sum \frac{p_n}{p_0} \times 100 \times w}{\sum w}$$

(g)

$$\text{Value index} = \frac{\sum p_n q_n}{\sum p_o q_o} \times 100$$

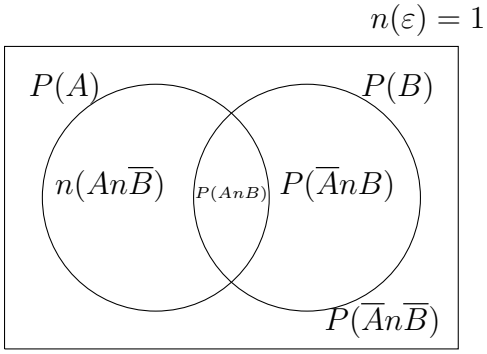
4. Spearman's rank correlation coefficient

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

Where n is the difference between the rankings of a given scores and n is the number of pairs

5. Probability theory

   (a) For any events A and B



$$P(A) = P(A \cap \overline{B}) + P(A \cap B)$$
$$P(\overline{A}) = P(\overline{A} \cap B) + P(A \cup B)^1$$
$$P(B) = P(\overline{A} \cap B) + P(A \cap B)$$
$$P(\overline{B}) = P(A \cap \overline{B}) + P(A \cup B)^1$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

   (b) $P(A) + P(\overline{A}) = 1$

   (c) $P(A \cup B)^1 = P(\overline{A} \cap \overline{B})$

   (d) $P(\overline{A} \cup \overline{B}) = P(A \cap B)^1$

   (e) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ for $P(B) \neq 0$

6. Mechanics

   (a) For projectile motion

$$y = x \tan\theta - \frac{gx^2}{2u^2} sec^2\theta$$

   (b) For calculus

Physical quantity	Formula	units	Formula	units	Formula
Force	F=ma	N	$a = \frac{dv}{dt}$	ms^{-2}	k.e=$\frac{1}{2}mv^2$
Power	P=F.v	W	$v = \frac{dr}{dt} or \frac{ds}{dt}$	ms^{-1}	Avg accel=$\frac{v(t_2)-v(t_1)}{t_2-t_1}$
Work done	W=F.s or F.r	j	W=$\int_{t_1}^{t_2} f.vdt$	j	speed=$\lvert v \rvert$
Impulse	I=F.t	Ns	$v = \int adt$	ms^{-1}	Avg vel=$\frac{r(t_2)-r(t_1)}{t_2-t_1}$
Momentum	momentum =m.v	$Kgms^{-1}$	$r = \int vdt$	m	distance=$\lvert r \rvert or \lvert s \rvert$

STATISTICAL TABLES

SIGNIFICANCE LEVELS FOR CORRELATION COEFFICEINTS

	Product-moment coefficient of correlation (r_{xy})		Spearman's rank Correlation coefficient (ρ)		Kendall's rank coefficient of correlation (τ)	
No. of pairs	Significance if $\lvert r_{xy} \rvert$ exceeds at 5%	at 1%	Significance if $\lvert \rho \rvert$ exceeds at 5%	at 1%	Significance if $\lvert \tau \rvert$ exceeds at 5%	at 1%
3	1.00	1.00				
4	0.95	0.99				
5	0.88	0.96	1.00			
6	0.81	0.92	0.89	1.00	0.87	1.00
7	0.75	0.88	0.75	0.89	0.71	0.81
8	0.71	0.83	0.71	0.86	0.64	0.79
9	0.67	0.80	0.68	0.83	0.56	0.72
10	0.63	0.77	0.65	0.79	0.51	0.64
11	0.60	0.74	0.60	0.74	0.49	0.60
12	0.58	0.71	0.58	0.71	0.45	0.58
13	0.55	0.68	0.55	0.68		
14	0.53	0.66	0.53	0.66		
15	0.51	0.64	0.51	0.64		
16	0.50	0.62	0.50	0.62		
17	0.48	0.61	0.48	0.61		
18	0.47	0.59	0.47	0.59		
19	0.46	0.58	0.46	0.58		
20	0.44	0.56	0.44	0.56	0.33	
30	0.35	0.45	0.35	0.45		
40	0.31	0.39	0.31	0.39		
50	0.27	0.35	0.27	0.35		
60	0.25	0.33	0.25	0.33		
70	0.23	0.31	0.23	0.31		
80	0.22	0.29	0.22	0.29		
90	0.21	0.27	0.21	0.27		
100	0.20	0.25	0.20	0.25		

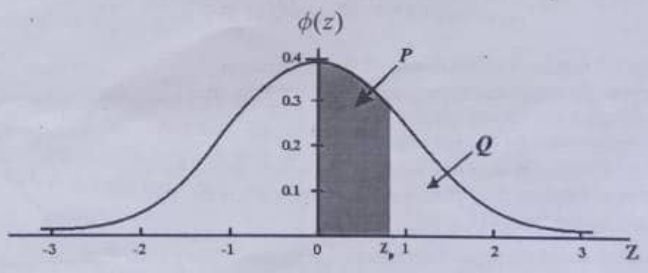
CUMULATIVE NORMAL DISTRIBUTION $P(z)$

Z	0	1	2	3	4	5	6	7	8	9	ADD 1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673	2704	2734	2764	2794	2823	2852	3	6	9	12	15	19	22	25	28
0.8	0.2881	2910	2939	2967	2995	3023	3051	3078	3106	3133	3	6	9	12	15	18	21	24	27
0.9	0.3159	3186	3212	3238	3264	3289	3315	3340	3365	3389	3	6	8	11	14	17	20	22	25
1.0	0.3413	3438	3461	3485	3508	3531	3554	3577	3599	3621	3	5	8	11	13	16	19	22	24
1.1	0.3643	3665	3686	3708	3729	3749	3770	3790	3810	3830	3	5	8	10	13	16	18	21	23
1.2	0.3849	3869	3888	3907	3925	3944	3962	3980	3997	4015	2	5	7	10	12	15	17	20	22
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	5	7	10	12	14	17	19	22
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	2	4	7	9	11	13	15	18	20
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	2	4	6	8	11	13	15	17	19
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	2	4	6	8	10	12	14	16	18
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	2	4	6	8	10	11	13	15	17
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	2	4	5	7	9	11	13	14	16
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	2	3	5	6	8	10	11	13	14
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	1	3	4	6	7	8	10	11	13
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	1	2	4	5	6	7	8	10	11
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	1	2	3	4	5	6	7	8	9
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	1	2	3	3	4	5	6	7	8
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	1	1	2	3	4	4	5	6	6
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952	1	1	2	2	3	4	4	5	5
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964	0	1	1	2	2	3	3	4	4
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974	0	1	1	2	2	2	3	3	4
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981	0	1	1	1	2	2	2	3	3
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986	0	0	1	1	1	2	2	2	2
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000	0	0	1	1	1	1	1	2	2

The table gives $P(z) = \int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P$(Shaded Area)
2. $P(Z > Z_p) = Q = ½ - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$



CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION) $\sum_{i \geq r} pi$

		x										
n	r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	1	0.0199	0975	1900	2775	3600	4375	5100	5775	6400	6975	7500
	2	0.0001	0025	0100	0225	0400	0625	0900	1225	1600	2025	2500
3	1	0.0297	1426	2710	3859	4880	5781	6570	7254	7840	8336	8750
	2	0.0003	0072	0280	0608	1040	1562	2160	2818	3520	4252	5000
	3		0001	0010	0034	0080	0156	0270	0429	0640	0911	1250
4	1	0.0394	1855	3439	4780	5904	6836	7599	8215	8704	9085	9375
	2	0.0006	0140	0523	1095	1808	2617	3483	4370	5248	6090	6875
	3		0005	0037	0120	0272	0508	0837	1265	1792	2415	3125
	4			0001	0005	0016	0039	0081	0150	0256	0410	0625
5	1	0.0490	2262	4095	5563	6723	7627	8319	8840	9222	9497	9688
	2	0.0010	0226	0815	1648	2627	3672	4718	5716	6630	7438	8125
	3		0012	0086	0266	0579	1035	1631	2352	3174	4069	5000
	4			0005	0022	0067	0156	0308	0540	0870	1312	1875
	5				0001	0003	0010	0024	0053	0102	0185	0312
6	1	0.0585	2649	4686	6229	7379	8220	8824	9246	9533	9723	9844
	2	0.0015	0328	1143	2235	3446	4661	5798	6809	7667	8364	8906
	3		0022	0158	0473	0989	1694	2557	3529	4557	5585	6562
	4		0001	0013	0059	0170	0376	0705	1174	1792	2553	3438
	5			0001	0004	0016	0046	0109	0223	0410	0692	1094
	6					0001	0002	0007	0018	0041	0083	0156
7	1	0.0679	3017	5217	6794	7903	8665	9176	9510	9720	9848	9922
	2	0.0020	0444	1497	2834	4233	5551	6706	7662	8414	8976	9375
	3		0038	0257	0738	1480	2436	3529	4677	5801	6836	7734
	4		0002	0027	0121	0333	0706	1260	1998	2898	3917	5000
	5			0002	0012	0047	0129	0288	0556	0963	1529	2266
	6				0001	0004	0013	0038	0090	0188	0357	0625
	7						0001	0002	0006	0016	0037	0078
8	1	0.0773	3366	5695	7275	8322	8999	9424	9681	9832	9916	9961
	2	0.0027	0572	1869	3428	4967	6329	7447	8309	8936	9368	9648
	3	0.0001	0058	0381	1052	2031	3215	4482	5722	6846	7799	8555
	4		0004	0050	0214	0563	1138	1941	2936	4059	5230	6367
	5			0004	0029	0104	0273	0580	1061	1737	2604	3633
	6				0002	0012	0042	0113	0253	0498	0885	1445
	7					0001	0004	0013	0036	0085	0181	0352
	8							0001	0002	0007	0017	0039
9	1	0.0865	3698	6126	7684	8658	9249	9596	9793	9899	9954	9980
	2	0.0034	0712	2252	4005	5638	6997	8040	8789	9295	9615	9805
	3	0.0001	0084	0530	1409	2618	3993	5372	6627	7682	8505	9102
	4		0006	0083	0339	0856	1657	2703	3911	5174	6386	7461
	5			0009	0056	0196	0489	0988	1717	2666	3786	5000
	6			0001	0006	0031	0100	0253	0536	0994	1658	2539
	7					0003	0013	0043	0112	0250	0498	0898
	8						0001	0004	0014	0038	0091	0195
	9								0001	0003	0008	0020
10	1	0.0956	4013	6513	8031	8926	9437	9718	9865	9940	9975	9990
	2	0.0043	0861	2639	4557	6242	7560	8507	9140	9536	9767	9893
	3	0.0001	0115	0702	1798	3222	4744	6172	7384	8327	9004	9453
	4		0010	0128	0500	1209	2241	3504	4862	6177	7340	8281
	5		0001	0016	0099	0328	0781	1503	2485	3669	4956	6230
	6			0001	0014	0064	0197	0473	0949	1662	2616	3770
	7				0001	0009	0035	0106	0260	0548	1020	1719
	8					0001	0004	0016	0048	0123	0274	0547
	9							0001	0005	0017	0045	0107
	10									0001	0003	0010
11	1	0.1047	4312	6862	8327	9141	9578	9802	9912	9964	9986	9995
	2	0.0052	1019	3026	5078	6779	8029	8870	9394	9698	9861	9941
	3	0.0002	0152	0896	2212	3826	5448	6873	7999	8811	9348	9673
	4		0016	0185	0694	1611	2867	4304	5744	7037	8089	8867
	5		0001	0028	0159	0504	1146	2103	3317	4672	6029	7256

CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION)

n	r	x 0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
11	6			0003	0027	0117	0343	0782	1487	2465	3669	5000
	7				0003	0020	0076	0216	0501	0994	1738	2744
	8					0002	0012	0043	0122	0293	0610	1133
	9						0001	0006	0020	0059	0148	0327
	10								0002	0007	0022	0059
	11										0002	0005
12	1	0.1136	4596	7176	8578	9313	9683	9862	9943	9978	9992	9998
	2	0.0062	1184	3410	5565	7251	8416	9150	9576	9804	9917	9968
	3	0.0002	0196	1109	2642	4417	6093	7472	8487	9166	9579	9807
	4		0022	0256	0922	2054	3512	5075	6533	7747	8655	9270
	5		0002	0043	0239	0726	1576	2763	4167	5618	6956	8062
	6			0005	0046	0194	0544	1178	2127	3348	4731	6128
	7			0001	0007	0039	0143	0386	0846	1582	2607	3872
	8				0001	0006	0028	0095	0255	0573	1117	1938
	9					0001	0004	0017	0056	0153	0356	0730
	10							0002	0008	0028	0079	0193
	11								0001	0003	0011	0032
	12										0001	0002
15	1	0.1399	5367	7941	9126	9648	9866	9953	9984	9995	9999	1.0000
	2	0.0096	1710	4510	6814	8329	9198	9647	9858	9948	9983	9995
	3	0.0004	0362	1841	3958	6020	7639	8732	9383	9729	9893	9963
	4		0055	0556	1773	3518	5387	7031	8273	9095	9576	9824
	5		0006	0127	0617	1642	3135	4845	6481	7827	8796	9408
	6		0001	0022	0168	0611	1484	2784	4357	5968	7392	8491
	7			0003	0036	0181	0566	1311	2452	3902	5478	6964
	8				0006	0042	0173	0500	1132	2131	3465	5000
	9				0001	0008	0042	0152	0422	0950	1818	3036
	10					0001	0008	0037	0124	0338	0769	1509
	11						0001	0007	0028	0093	0255	0592
	12							0001	0005	0019	0063	0176
	13								0001	0003	0011	0037
	14										0001	0005
20	1	0.1821	6415	8784	9612	9885	9968	9992	9998	1.0000	1.0000	1.0000
	2	0.0169	2642	6083	8244	9308	9757	9924	9979	9995	9999	1.0000
	3	0.0010	0755	3231	5951	7939	9087	9645	9879	9964	9991	9998
	4		0159	1330	3523	5886	7748	8929	9556	9840	9951	9987
	5		0026	0432	1702	3704	5852	7625	8818	9490	9811	9941
	6		0003	0113	0673	1958	3828	5836	7546	8744	9447	9793
	7			0024	0219	0867	2142	3920	5834	7500	8701	9423
	8			0004	0059	0321	1018	2277	3990	5841	7480	8684
	9			0001	0013	0100	0409	1133	2376	4044	5857	7483
	10				0002	0026	0139	0480	1218	2447	4086	5881
	11					0006	0039	0171	0532	1275	2493	4119
	12					0001	0009	0051	0196	0565	1308	2517
	13						0002	0013	0060	0210	0580	1316
	14							0003	0015	0065	0214	0577
	15								0003	0016	0064	0207
	16									0003	0015	0059
	17										0003	0013
	18											0002

To obtain $p(i \leq r)$ use: $p(i \leq r) = 1 - p(i \geq r+1)$

Where a space in the table is empty the probability is less than 0.00005.