P425/1
PURE MATHEMATICS
Paper 1
Jul./Aug. 2023
3 hours

WAKISO-KAMPALA TEACHERS' ASSOCIATION (WAKATA)

WAKATA MOCK EXAMINATIONS 2023

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All necessary working must be clearly shown.

Begin each answer on a fresh sheet of paper.

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

Neat work is a must!!

SECTION A (40 MARKS)

Answer **all** *questions in this section.*

1. Solve the equation: $(Z + 1 - 2i)^2 + 4i = 3$. *(05 marks)*

2. Given that $sin(\theta + \alpha) = a$ and $sin(\theta + \beta) = b$.
   Show that $cos2(\alpha - \beta) - 4abcos(\alpha - \beta) = 1 - 2a^2 - 2b^2$ *(05 marks)*

3. Find $\frac{dy}{dx}$ if $y = 3^{x^2+2x}$. *(05 marks)*

4. A straight line joining the points $(2, 1, 4)$ and $(a - 1,\ 4,\ -1)$ is parallel to the line joining points $(0,\ 2,\ b - 1)$ and $(5,\ 3, -2)$. Find the values of a and b. *(05 marks)*

5. P and Q are two points whose coordinates are $(at^2,\ 2at)$, $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ respectively and S is a point $(a, 0)$. Show that $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$. *(05 marks)*

6. Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{\sqrt{1 + cosx}}{(1 - cosx)^{\frac{5}{2}}} \right) dx$ *(05 marks)*

7. The quadratic equation $(P + 1)x^2 - 6(P + 1)x + 3(P + 9) = 0,\ P \neq 1$ has equal roots. Find the roots of the equation. *(05 marks)*

8. Solve the differential equation; $\frac{dy}{dx} + \frac{y}{x} = x^3$ *(05 marks)*

SECTION B (60 MARKS)

Answer any **five** *questions from this section. All questions carry equal marks.*

9. (a) The polymomial $f(x) = ax^3 + 3x^2 + bx - 3$ is exactly divisible by $(2x + 3)$ and leaves a remainder -3 when divided by $(x + 2)$. Find the values of a and b. *(07 marks)*

(b) Given that $(x - 2)$ and $(x - \frac{1}{2})$ are factors of $ax^2 + 5x + b$.

Show that $a = b$. *(05 marks)*

10. (a) Given that $y^3 - 3xy^2 = x^3 + 3x^2y$. Find $\frac{dy}{dx}$. *(06 marks)*

(b) The volume V of a liquid in a container is given by $V = (3h^2 + 4)^{\frac{3}{2}} - 8$; where h m is the depth of the liquid. The liquid is leaking from the container. It is observed that, when the depth of the liquid is 0.6m, the depth is decreasing at a rate of 0.015m per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is 0.6m. *(06 marks)*

11. (a) The lines $\frac{x-1}{3k} = \frac{y-2}{1} = \frac{z-3}{-5}$ and $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ are perpendicular.

Find the value of k. *(04 marks)*

(b) Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$. *(08 marks)*

12. Expand $\frac{2x^2 + 5x - 10}{(x + 1)(x + 2)}$ in ascending powers of x, as far as the term in x^2. *(12 marks)*

13. (a) Sketch the curve $y = x^2 + 2$. *(06 marks)*

(b) The area bounded by the curve and the line $y = x + 8$ is rotated about the x −axis through one revolution. Determine the volume of the solid generated. *(06 marks)*

14. (a) Prove that: $\frac{cosx}{1-sinx} = \frac{1+cosx+sinx}{1+cosx-sinx}$ *(08 marks)*

(b) Solve the equation $2sin\,x = \frac{4cosx-1}{tanx}$ for $0^\circ < x < 360^\circ$. *(04 marks)*

15. (a) Find the equation of the ellipse whose focus is $(1,-2)$, the directrix $3x-2y+5=0$ and accentricity equal to $\frac{1}{2}$. *(04 marks)*

(b) Points A and B are $10km$ a part and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is $6km$ closer to A than B. Show that the location of the explosion is restricted to a hyperbola whose equation is $\frac{x^2}{9}-\frac{y^2}{16}=1$. *(08 marks)*

16. In a culture, the bacteria count is 100,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 200,000, if the rate of growth of bacteria is proportional to the number present? *(12 marks)*

END