MATIGO EXAMINATIONS BOARD

S475/1
SUBSIDIARY MATHEMATICS
MARKING GUIDE 2023
PAPER 1

<table>
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$$\frac{\left(2m^{\frac{2}{3}}\right)^3}{\left(8m^{\frac{1}{6}}\right)^2} = \frac{2^3 \times m^{\frac{2}{3}\times 3}}{8^2 \times m^{\frac{1}{6}\times 2}} \quad M_1 \quad B_1$$

$$= \frac{2^3 \times m^2}{8^2 \times m^{\frac{2}{6}}} \quad M_1$$

$$= \frac{8}{81} \times m^{\left(2-\frac{1}{3}\right)} \quad M_1$$

$$= \frac{8}{81} m^{\frac{5}{3}} \quad A_1$$

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<tr><td>2</td><td>

speed	x	f	fx	cf
45 – 50	47.5	20	950	20
51 – 55	53	28	1484	48
56 – 60	58	16	928	64
61 – 65	63	13	819	77
66 – 70	68	3	204	80
		$\sum f = 80$	$\sum fx = 4385$	

B_1

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	(i) Modal speed= $$= L + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times \mathrm{I}$$ $$= 50.5 + \frac{(28-20)}{(28 \times 2 - 20 - 16)} \times 6 \quad M_1$$ $$= 52.9kmhr^{-1} \quad A_1$$	
	(ii) $Mean = \frac{\sum fx}{\sum f}$ $$= \frac{4385}{80} \quad M_1$$ $$= 54.8125kmhr^{-1} \quad A_1$$	
3	(i) $let\ the\ first\ number\ n$ $$second\ number = n + d$$ $$third\ number = n + 2d$$ $$n + n + d + n + 2d = 9$$ $$3n + 3d = 9$$ $$n + d = 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i) \quad B_1$$ $$n(n+d)(n+2d) = 24$$ $$n(3)(n+2d) = 24$$ $$n(3)(n+d+d) = 24$$ $$9n + 3nd = 24$$ $$3n + nd = 8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(ii) \quad B_1$$ $$d = 3 - n$$ $$3n + n(3-n) = 8$$ $$3n + 3n - n^2 = 8$$ $$n^2 - 6n + 8 = 0$$ $$(n-2)(n-4) = 0$$ $$Either, n = 2\ or\ n = 4 \quad M_1$$ $$when\ n = 2, d = 3 - 2, d = 1$$	

	$when\ n = 4, d = 3 - 4 = -1$ A 1 $first\ term = 2\ or\ 4$ $common\ difference = 1\ or - 1$ A 1	
4	$$W.A.P.I = \frac{\sum IW}{\sum W}$$	

Weight(W)	P_0	P_1	$I = \frac{P_1}{P_0} \times I$	IW
25	3500	4650	132.8671	3321.429
10	800	1200	150	1500
35	600	900	150	5250
20	2500	3000	120	2400
1	7500	10500	140	140
9	3000	3150	105	945 B 1
$\sum W = 100$			B 1	$\sum W = 13556.43$

$$W.A.P.I = \frac{\sum IW}{\sum W}$$

$$= \frac{13556.43}{100}$$ M 1

$$= 135.56$$ A 1

$Comment: the\ prices\ of\ items\ increased\ by\ 35.56\%$ B 1

5

$$x = \frac{\begin{vmatrix} 4 & -1 \\ -30 & 9 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 5 & 9 \end{vmatrix}},$$ M 1

$$= \frac{36 - 30}{27 - -5}$$ B 1

$$= \frac{6}{32}$$

$$= \frac{3}{16} \qquad A_1$$

$$y = \frac{\begin{vmatrix} 3 & 4 \\ 5 & -30 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 5 & 9 \end{vmatrix}} \qquad M_1$$

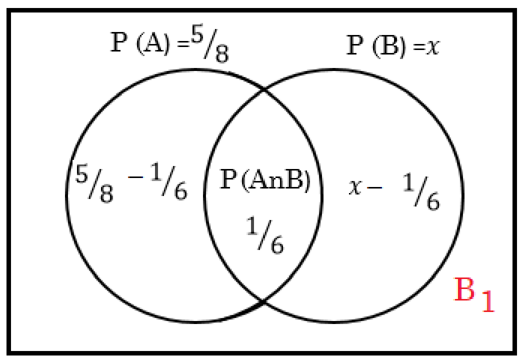
$$= \frac{-90 - 20}{27 - -5}$$

$$= \frac{-110}{32}$$

$$= \frac{-55}{16}$$

$$= -3\frac{7}{16} \qquad A_1$$

6

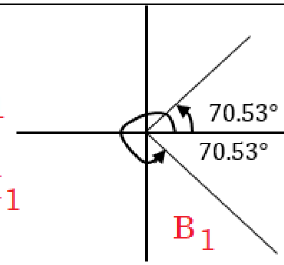
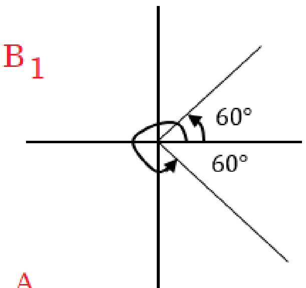


(a)

$$\frac{11}{24} + \frac{1}{6} + x - \frac{1}{6} = 1 \qquad M_1$$

$$x = \frac{13}{24} \qquad A_1$$

$$\therefore \text{P(B)} = \frac{13}{24}$$

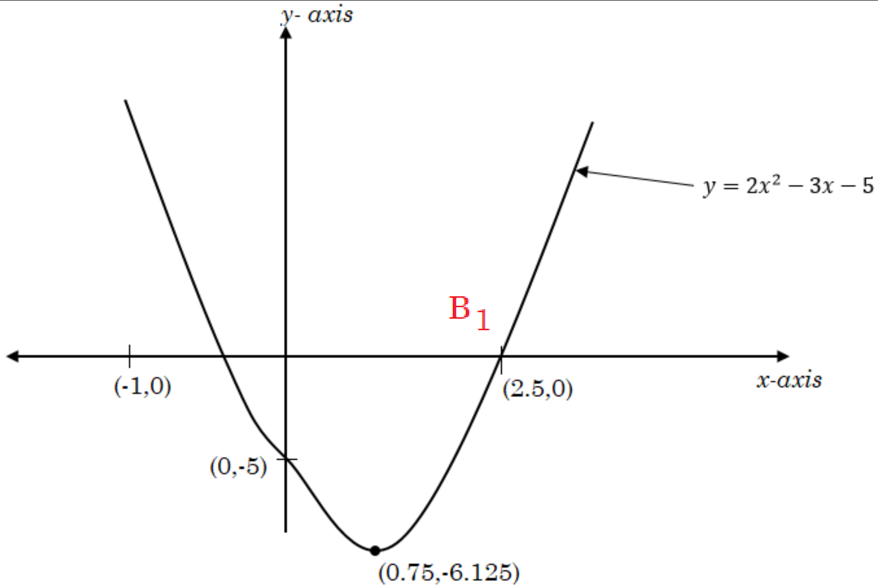
(b)	$P(^A/_B) = \frac{P(AnB)}{P(B)}$ $= \frac{^1/_6}{^{13}/_{24}}$ M1 $= {^1/_6} \times {^{24}/_{13}}$ A1 $= \frac{4}{13}$	
7	$6sin^2x + 5cosx = 7\ for\ 270° < x < 360°$ $6(1 - cos^2x) + 5cosx = 7$ $6cos^2x - 5cosx + 1 = 0$ M1 $cosx = --\frac{5 \pm \sqrt{(-5)^2 - 4(6)(1)}}{2 \times 6}$ M1 $Either\ cosx = \frac{1}{3}\ or\ \frac{1}{2}$ B1 $for\ cosx = \frac{1}{3}$ B1 $x = cos^{-1}\left(\frac{1}{3}\right)$ $x = 70.53°, 289.47°$ $for\ cosx = \frac{1}{2}$ $x = 60°, 300°$ A1 $\therefore x = 60°, 70.53°, 289.47°, 300°$   	
8(i)	$\sum_{allx} P(X = x) = 1$ M1 $0.1 + 0.3 + a + 0.2 + 0.15 = 1$ A1 $a = 0.25$	
(ii)	$P\left(x < \frac{3}{x} \geq 2\right) = \frac{P(x < 3\quad n \quad x \geq 2)}{P(x \geq 2)}$ M1 $= \frac{P(X = 2)}{1 - P(X = 1)}$ $= \frac{0.3}{1 - 0.1}$ B1 $= \frac{1}{3}$ A1	

9(a)(i)	$s = 3t^3 - 27t^2 + 72t - 50$ $\frac{ds}{dt} = 9t^2 - 54t + 72$ M1 $\frac{ds}{dt} = 0\ for\ v = 0$ $9t^2 - 54t + 72 = 0$ M1 $t^2 - 6t + 8 = 0$ $(t-4)(t-2) = 0$ M1 $Either\ t = 4s\ or\ t = 2s$ B1 A1 A1 $Vwlocity\ vanishes\ at\ t = 2s\ and\ at\ t = 4s$	
(ii)	$\frac{d^2s}{dt^2} = 18t - 54$ M1 $\frac{d^2s}{dt^2} = 0\ for\ a = 0$ M1 $18t - 54 = 0$ $t = \frac{54}{18}$ $t = 3s$ A1	
(b)	$at\ t = 2s$ $S = 3(2)^3 - 27(2)^2 + 72(2) - 50$ M1 B1 $s = 10$ A1 $at\ t = 4s$ $S = 3(4)^3 - 27(4)^2 + 72(4) - 50$ A1 M1 B1 $S = -2m$	
10(a)	$\frac{dy}{dx} = kx - 3$ $\int dy = \int (kx - 3)dx$ M1 $y = \frac{kx^2}{2} - 3x + c$ B1 $-6 = {}^1/_2\, k(1)^2 - 3(1) + c$ M1 $-6 = \frac{1}{2}k - 3 + c$ $-3 = \frac{k}{2} + c$ B1 $-6 = k + 2c$ $\frac{d^2y}{dx^2} = k = 4$ M1	

$$\therefore \textit{the turning point is a minimum}$$
$$-6 = 4 + 2c$$
$$2c = -10$$
$$c = -5 \quad \mathrm{B}_1$$

$$\therefore y = \frac{4}{2}x^2 - 3x + c$$
$$y = 2x^2 - 3x - 5 \quad \mathrm{A}_1 \quad \mathrm{M}_1$$

$$\frac{dy}{dx} = 4x - 3 = 0$$
$$4x - 3 = 0$$
$$x = {}^{3}/_{4}$$
$$y = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) - 5 \quad \mathrm{B}_1$$
$$y = -6.125$$
$$\left(\frac{3}{4}, -6.125\right) \textit{is a minimum turning point}$$
$$x - intercept$$
$$when\ y = 0$$
$$2x^2 - 3x - 5 = 0 \quad \mathrm{M}_1$$
$$x = -\frac{-3 \pm \sqrt{(-3)^2 - (2)(-5)}}{2 \times 2}$$
$$Either\ x = 1\ or\ x = 2.5$$
$$x\ intercept\ are\ (-1,0) and\ (2.5,0)$$
$$y - intercept$$
$$when\ x = 0$$
$$y = 2(0)^2 - 3(0) - 5$$
$$y = -5$$
$$(0, -5) is\ a\ y\ intercept$$

	 B1 $Area = \int_{-1}^{2.5} 2x^2 - 3x - 5dx$ M1 $= \left[\frac{2x^3}{3} - \frac{3x^2}{2} - 5x\right]_{-1}^{2.5}$ B1 $= \left[\left(\frac{2(2.5)^3}{3} - \frac{3(2.5)^2}{2} - 5(2.5)\right) - \left(\frac{2(-1)^3}{3} - \frac{3(-1)^2}{2} - 5(-1)\right)\right]$ A1 $=$	
11(a) (i)	$\tilde{a} + \tilde{b} - \tilde{c} = \begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}-3\\3\end{pmatrix} - \begin{pmatrix}12\\-5\end{pmatrix}$ $= \begin{pmatrix}-12\\10\end{pmatrix}$ M1 $\lvert\tilde{a} + \tilde{b} - \tilde{c}\rvert = \sqrt{(-12)^2 \ + 10^2}$ A1 $= 15.6205\ units$	
(ii)	$x\tilde{a} + y\tilde{b} = \tilde{c}$ $x\begin{pmatrix}3\\2\end{pmatrix} + y\begin{pmatrix}-3\\3\end{pmatrix} = \begin{pmatrix}12\\-5\end{pmatrix}$ M1 $\begin{pmatrix}3x - 3y\\2x + 3y\end{pmatrix} = \begin{pmatrix}12\\-5\end{pmatrix}$ M1 $3x - 3y = 12 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$ B1 $2x + 3y = -5 \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$ B1	

	$3x-3y=12$ $\underline{2x+3y=-5}$ $5x=7$ M1 $x=\frac{7}{5}$ A1 $3x-3y=12$ $x-y=4$ $y=x-4$ $y=\frac{7}{5}-4$ $y=-2.6$ A1	
(b)	$\lvert\tilde{a}\rvert=\sqrt{3^2+2^2}$ $=\sqrt{13}$ B1 $\lvert\tilde{c}\rvert=\sqrt{(12)^2+(-5)^2}$ $=\sqrt{169}$ $=13$ B1 $\tilde{a}.\tilde{c}=\lvert\tilde{a}\rvert\lvert\tilde{c}\rvert cos\theta$ $\begin{pmatrix}3\\2\end{pmatrix}.\begin{pmatrix}12\\-5\end{pmatrix}=\sqrt{13}.\sqrt{169}cos\theta$ M1 M1 $36+-10=13\sqrt{13}cos\theta$ B1 $cos\theta=\frac{26}{13\sqrt{13}}$ $\theta=cos^{-1}\left(\frac{26}{13\sqrt{13}}\right)$ $\theta=56.31°$ A1	
12(a)(i)	TROTTING $8 letters\ including\ 3T'd$ $Number\ of\ arrangments=\frac{8!}{3!}$ M1 M1 B1 $=6720$ A1	
(ii)	$number\ of\ arragment\ with\ R\ and\ O\ next=\frac{7!\times 2!}{3!}$ M1 M1 B1 $=1680$ A1	
(iii)	$Arrangment\ with\ R\ and\ O\ seperated=6720-1680$ M1 $=5040$ A1	

(b)	$Number\ of\ ways\ = C_2^5 \times C_3^7 + C_3^5 \times C_2^7 + C_4^5 \times C_1^7$ B1 M1 M1 M1 $= 350 + 210 + 35$ A1 $= 595$	
13(a)	(table below)	

Marks	*tally*	*f*	*c.f*	*Class boundaries*
20 − 30	///	3	3	20 − 30
30 − 40	~~////~~ //	7	10	30 − 40
40 − 50	~~////~~ ////	9	19	40 − 50
50 − 60	~~////~~	5	24	50 − 60
60 − 70	~~////~~	5	29	60 − 70
70 − 80	///	3	32	70 − 80
80 − 90	/	1 B1	33	80 − 90
	B1	$\sum f = 33$	B1	

13(b)

R_x	R_y	D	D^2
10	7.5	2.5	6.25
3	1	2	4
3	3	0	0
5.5	2	3.5	12.25
7.5	4.5	3	9
12	12	0	0
11	10.5	0.5	0.25
9	7.5	1.5	2.25
3	10.5	−7.5	56.25
7.5	7.5 B1	0	0
5.5	4.5	1.0	1
1	7.5	−6.5	42.25
			$\sum D^2 = 133.5$ B1

$$\rho = 1 - \frac{6\sum D^2}{n(n^2-1)}$$

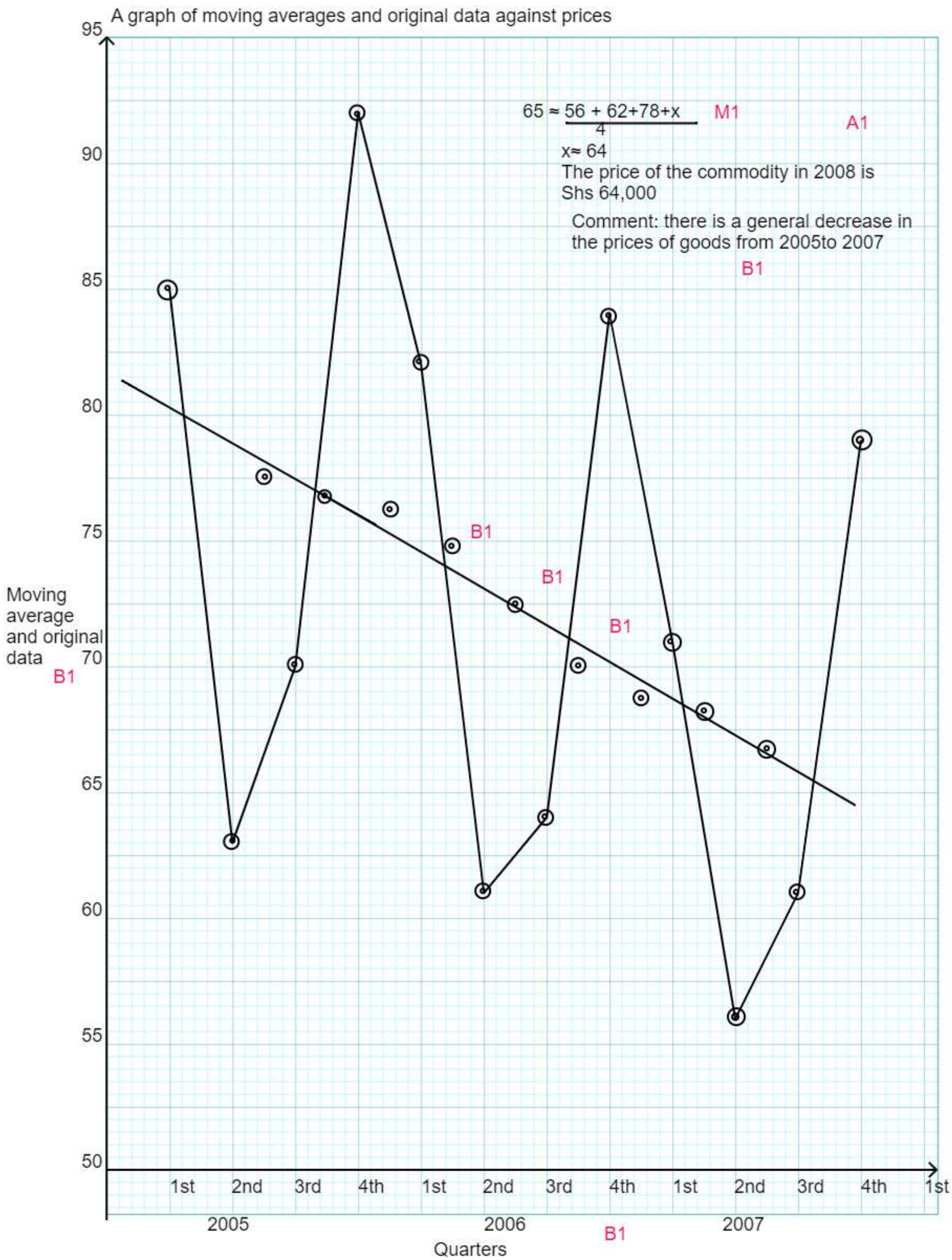
$$= 1 - \frac{6 \times 133.5}{12(12^2-1)} \qquad M_1 \quad B_1$$

$$= \frac{305}{572} \qquad A_1$$

$$= 0.53$$

There is a moderate postive correlation B_1

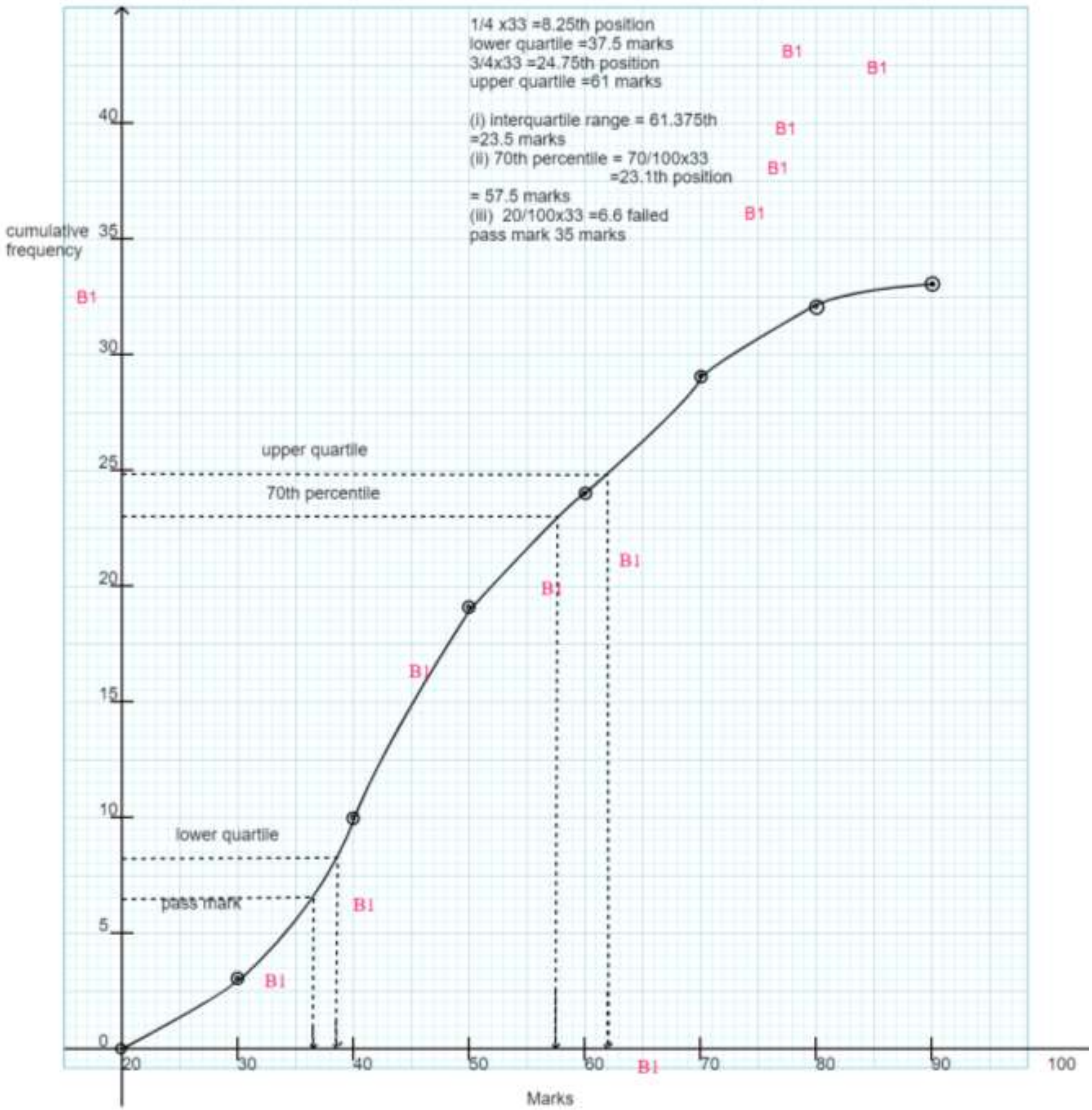
Year	Quarters	Prices (000")	Moving total	Moving average
2005	1^{st}	85		
	2^{nd}	63		
			310	77.5
	3^{rd}	70		
			30 B_1	76.75 B_1
	4^{th}	92		
			305	76.25
	1^{st}	82		
			299	74.75
	2^{nd}	61		
			291 B_1	72.75 B_1
	3^{rd}	64		
			280	70
	4^{th}	84		
			275	68.75
	1^{st}	71		
			273 B_1	68.25
	2^{nd}	56		
			267	66.75 B_1
	3^{rd}	62		
	4^{th}	78		



A graph of moving averages and original data against prices
$65 \approx \frac{56 + 62+78+x}{4}$ M1
A1
$x \approx 64$
The price of the commodity in 2008 is
Shs 64,000
Comment: there is a general decrease in
the prices of goods from 2005to 2007
B1
B1
B1
B1
Moving
average
and original
data
B1
95
90
85
80
75
70
65
60
55
50
1st
2nd
3rd
4th
1st
2nd
3rd
4th
1st
2nd
3rd
4th
1st
2005
2006
2007
B1
Quarters

15

A graph of cumulative frequency against marks



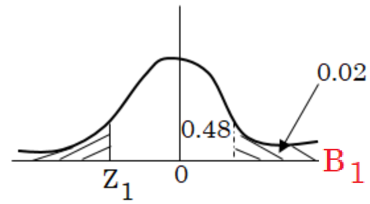
16(a)

$P(X < X_1) = 0.02$ B_1 M_1

$Z_1 = -2.054$

$$\frac{X_1 - 50}{2} = -2.054$$

$X_1 = 45.892Kg$ A_1 M_1



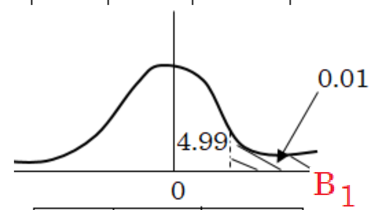
P	Q	Z
0.48	0.02	2.054

$P(X > X_2) = 0.01$ B_1

$$\frac{X_2 - 50}{2} = 3.090$$ B_1 A_1

$X_2 = 56.18Kg$

a bag should lie in the range $[45.892kg, 56.18kg]$ A_1



P	Q	Z
4.99	0.01	3.090

(b)

$$P(X < 47.5) = P\left(Z < \frac{47.5 - 50}{2}\right)$$ M_1 M_1 M_1

$= P(Z < -1.25)$ M_1

$= 0.3944$ A_1

END

(+256780413120)