P425/1
PURE
MATHEMATICS
Paper 1
31 July 2024
3 hours

**ENTEBBE JOINT EXAMINATION BUREAU**

**Uganda Advanced Certificate of Education**

**MATHEMATICS**

Paper 1

**3 hours**

**INSTRUCTIONS TO CANDIDATES:**

*Attempt* **ALL** *the eight questions in* Section A *and any* **five** *from* Section **B.**

*Begin every answer on a fresh page.*

*Any additional questions answered will* **not** *be marked.*

*Mathematical tables and squared paper shall be provided*

*Silent, non – programmable calculators may be used.*

*State the degree of accuracy at the end of each answer attempted using a calculator or table and indicate* **cal** *for calculator or* **tab** *for mathematical table.*

## SECTION A: 40 MARKS

*Attempt all questions in this Section.*

1. If $\alpha$ and $\beta$ are roots of the equation $x^2 - x - 2 = 0$.
   Find a quadratic equation whose roots are $\beta - \frac{1}{\alpha^2}$ and $\alpha - \frac{1}{\beta^2}$ *(05 marks)*

2. $A$, $B$ and $C$ are angles of a triangle $Cos\ A = \frac{3}{5}$ $Cos\ B = \frac{5}{13}$
   Without using tables or a calculator, show that $Cos\ C = \frac{33}{65}$ *(05 marks)*

3. Use Maclaurin's theorem to expand $ln\sqrt{1-2x}$ up to the term in $x^3$. *(05 marks)*

4. Solve for $x$: $e^x = 1 + 6e^{-x}$ *(05 marks)*

5. Find the perpendicular distance from the point $P\ (1,\ -1,\ 4)$ to the line $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda\ (2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ *(05 marks)*

6. Evaluate $\int_0^{\pi/12} x\ sin^2 3x\ dx$ *(05 marks)*

7. A line with a variable gradient is passing through the point $A\ (2,\ 3)$ and cuts the y – axis and x – axis at $P$ and $Q$ respectively. Find the locus of midpoint of $PQ$. *(05 marks)*

8. Find the volume of the solid generated when the region bounded by the curve $y = sin2x$ and the x – axis from $x = 0$ to $x = \frac{\pi}{2}$ is rotated about the x – axis. *(05 marks)*

## SECTION B

9. (a) Show that $z = -1 + i$ is a root of the equation $z^4 - 2z^3 - z^2 + 2z + 10 = 0$.
   Find the remaining roots. *(06 marks)*

   (b) If $z_1 = 4\left(\text{Cos}\ \frac{13}{24}\pi + i\ \text{Sin}\ \frac{13}{24}\pi\right)$ and $z_2 = 2\left(\text{Cos}\ \frac{5}{24}\pi + i\ \text{Sin}\ \frac{5}{24}\pi\right)$

   Find $z_1z_2$ and $\frac{z_1}{z_2}$ in the form $a + ib$

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10. By substituting $u = e^x$, show that

$$\int_0^{\ln 3} \frac{e^{2x}}{e^{2x} + 3e^x + 2}\, dx = \ln\left(\frac{8}{5}\right)$$ (12 marks)

11. (a) Express $\sqrt{6}\, Cos\, \theta + \sqrt{10}\, Sin\, \theta$ in the form $R\, Cos\,(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < 90^0$. Hence solve the equation $\sqrt{6}\, Cos\, \theta + \sqrt{10}\, Sin\theta = 3$ for $0 \leq \theta \leq 180^0$. (06 marks)

(b) If $t = tan\ \frac{\theta}{2}$ ; state expressions for $sec\ \theta$ and $tan\ \theta$ in terms of $t$.
Hence show that: $\sec\theta + \tan\theta = \tan\left(45^0 + \frac{\theta}{2}\right)$ (06 marks)

12. The line $L_1$ passes through the points $A\ (8, -1, 3)$ and $B\ (4, 0, 3)$ and line $L_2$ has vector equation $r = -2i + 8j - k + \mu\ (i + 3j + ak)$ and plane $M$ has equation $4x - 2y - z + 5 = 0$.

(a) Find in Cartesian form the equation of the line $L_1$. (05 marks)

(b) Find the point of intersection of line $L_1$ and the plane $M$. (04 marks)

(c) Given that line $L_2$ and plane $M$ are parallel, find the value of $a$. (03 marks)

13. Show that the curve $y = \dfrac{12x}{x^2 + 2x + 4}$ entirely lies in the range $-6 \leq y \leq 2$.

Hence, find the turning points and their nature. Sketch the curve. (12 marks)

14. (a) Solve the simultaneous equations $7x + 2y - 3z = 8$ and $\dfrac{3x - y}{3} = \dfrac{4x - z}{4} = 3y - 2z$ (06 marks)

(b) Find the ranges of values of $k$ for which the equation $2x^2 + 3x = kx - k - 3$ has two distinct roots. (06 marks)

(a) $ABCD$ is a square inscribed in a circle $x^2 + y^2 - 6x - 4y - 12 = 0$. Find the area of the square. (05 marks)

(b) Show that the curve $16x^2 + 9y^2 - 64x - 54y + 1 = 0$ represents an ellipse. Find the foci and equations of directrices. (07 marks)



Turn Over

16. (a) Solve $(x^2 + 4)\dfrac{dy}{dx} = 6xy$ given that $y(0) = 32$. *(04 marks)*

(b) Mr. Lubega starts to sip a bottle of soda of 1000 $cm^3$ at a rate of 10 $cm^3$ per minute. Given that the rate of consumption is inversely proportional to that of the volume of soda remaining at anytime. $t$. Find the time he takes to empty the bottle.

*(08 marks)*

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