NABISUNSA GIRL'S SCHOOL.

A – LEVEL MATHEMATICS SEMINAR 2024.

P425/1 PURE MATHEMATICS.

PAPER STRUCTURE

	SECTION A	SECTION B
TRIGONOMETRY	1	1
VECTORS	1	1
GEOMETRY	1	1
ALGEBRA	2	2
ANALYSIS (CALCULUS)	3	3

GEOMETRY	ALGEBRA	ANALYSIS
• Coordinate geometry • Circles and locus • Parabolas • Ellipse • Hyperbola	• Permutations and combination • QEs and polynomials. • logarithms and indices. • Partial fractions • Series • complex numbers. • Inequalities • equations • Ratio theorem. • binomial and pascals triangle.	• Integration • Differentiation • Curve sketching, I and II • Differential equations • Inequalities • Rates of change • Small changes • Maclaurin's theorem

TRIGONOMETRY.

1. Solve the equation $\left[cos\left(\frac{x}{2}\right) + sin\frac{x}{2}\right]^2 = \frac{3}{2}$ for $0^0 \leq x \leq 360^0$.
2. Solve the equation $sin2x + sin3x + sin4x = 0$ for $-90 \leq x \leq 90$.
3. Solve the equation: $5sin2x - 10sin^2x + 4 = 0$ for $-\pi \leq x \leq \pi$.
4. Solve; $4sin^2x - 12sin2x + 35cos^2x = 0$ for $0 \leq x \leq 90$.
5. Solve the equation. $5cos^2 3x = 3(1 + sin3x)$ fo $0 \leq x \leq 90$.
6. Solve $8cos^4x - 10cos^2x = 0$ for $0 \leq x \leq 2\pi$.
7. solve the equation.$tan^2x - sin^2x = 1$ for $0^0 \leq x \leq 360$
8. solve the equation: $tan^{-1}(2x + 1) + tan^{-1}(2x - 1) = tan^{-1}12$
9. *Find the maximum and minimum values* of $\frac{1}{6+4sinx-3cosx}$
10. *Express* $5sin^2x - 3cosxsinx + cos^2x$. in form of $a + bcos(2x - B)$ *and hence find the minimum and maximum values of the expression.*
11. *Given that* $cosA = \frac{3}{5}, cosB = \frac{12}{13}$ *where A is obtuse and B is acute, find the exact values* of $\tan(A + B), cosec(A - B)$ *without using calculators* or *tables.*
12. *Express* $\sqrt{\frac{1-sin2x}{1+sin2x}}$ *in terms of tanx.*
13. If $tan^{-1}a + tan^{-1}b + tan^{-1}c = \pi$, *show that* $a + b + c = abc$.
14. If $sin3x = p$, $sin^2x = \frac{3}{4} - q$, *prove that* $p^2 + 16q^3 = 12q^2$.
15. *Eliminate* θ *between the equations* $x = acos^2\theta + bcos^2\theta$ and $y = (a - b)sin\theta cos\theta$.
16. *In triangle,* $s - a = 3cm$, $s - b = 4cm$, $s - c = 5cm$ *where S is semi perimeter, find the area of triangle.*

Prove the following identities.

17. $4cos3xcosx + 1 = \frac{sin5x}{sinx}$.
18. $\frac{sinx+sin3x+sin5x}{cosx+cos3x+cos5x} = tan3x$.
19. $\frac{sinx+2sin2x+sin3x}{sinx-2sin2x+sin3x} = cot^2\left(\frac{x}{2}\right)$.
20. $Cos2A + cos2B + cos2C = -1 - 4cosAcosBcosC$.
21. $\frac{\sin(A+B)}{\cos(A-B)} + 1 = \frac{(1+cotA)(1+tanB)}{cotA+tanB}$.

VECTORS.

22. Vectors $\boldsymbol{a} = 2i - 2j - 2k$ and $\boldsymbol{b} = i - 3j + 2k$ form two sides of the triangle, find its area.

23. Show that $A(4,10,6)B(6,8,-2)$ and $C(1,10,3)$ are vertices of a right angled triangle.

24. Show that A (4, -8, -13), B (3, -2, -3), C (3,1, -2) are vertices of the triangle.

25. The point R divides the line AB externallyin the ratio of $3:1$ with $A(4,-3)$ and $B(-6,a)$. find coordinates of R and state the ratio in which point B divides AR.

26. Given the points $A(3,-2,5), B(9,1,-1)$. Find the coordinates of C such that C divides AB in the ratio of $5:3$. (i) externally (ii) internally.

27. The line passes through the points $A(4,6,3)$ and $B(1,3,3)$.
(a) find the vector equation of the line containing such points and hence show that a point $C(2,4,3)$ lies on the line.

28. Given that $a = i + 2j + 3k$ and $b = 4i - j + 2k$ are vectors, find a vector which is perpendicular to both a and b.

29. find the equation of the line through $A(1,-2,3)$ perpendicular to line $\frac{x-5}{2} = y - 2 = \frac{z-1}{3}$.

30. The points $A(2,3,-4), B(5,-1,2), C(11,\alpha,14)$.(
(i) find the unit vector parallel to AB.
(ii) find the position vector of D such that $ABCD$ is a parallelogram.

31. Given that $P(4,-3,5), Q(1,0,2)$. Find the coordinates of R such that $PR:PQ = 1:2$ and P, Q, R are collinear.

32. find the equation of a plane containing the line whose equation $r = (t-1)i + (t+2)j + (2t-4)k$ which is parallel to the direction vector $2i + 3j + k$, hence state the distance from the origin to this plane.

33. Given that the vectors $r = \begin{pmatrix} 5 \\ 3 \\ -5 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $r = \frac{x-5}{-1} = \frac{2-y}{3} = \frac{z+4}{4}$ intersect,
(i) find theposition vector of the point of intersection.
(ii) Cartesian equation of the line passingthrough the point of intersection of the lines above and parallel to the line $x = \frac{y-2}{2} = \frac{z-2}{3}$.

34. Show that the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{2-y}{-1} = \frac{2-z}{-1}$ don't intersect, hence find the shortest distance between them.

35. Plane passes through the point $(1,2,3)$ and is perpendicular to the vector $i - 5j + 4k$. The plane meets $z-$plane in P and the $y-$plane in Q, find the equation of the plane and the distance PQ.

36. *A line through the point* $D(-13,1,2)$ *and parallel to the vector* $12i + 6j + 3k$ meets *the plane containing the lines* $r = (-2i + 5j - 11k) + \beta(3i + j + 2k)$ and $r = 8i + 9j + \alpha(4i + 2j + 5k)$ at E.
find *the coordinates of E and the angle between the line and the plane.*

37. *Determine the equation of the plane passing through the point* $P(1,2,3)$ *and parallel to the lines* $r = 3i + 3j - k + \alpha(i - j - k)$ and $r = 4i - 5j - 8k + \beta(3i + j - 2k)$.

38. *Find the line of intersection of the planes* $2x + 3y + 4z = 1$ and $x + y + 3z = 0$.

39. *Find the orthocentre of the triangle* $A(-2,1), B(3,-4)\ C(-6,-1)$. (b) *a point* $C(a,4,5)$ *divide the line joining* $A(1,2,3)$ *and* $B(6,7,8)$ *in the ratio of* $\alpha:3$. *Find the values of* a *and* α.

40. *A plane contains points* $A(4,-6,5)$ *and* $B(2,0,1)$.
A perpendicular to the plane from P$(0,4,-7)$ *intersect the plane at C.*
find the Cartesian equation of the line PC.

41. *The point* $(6,-9,5)$ *lies on the line* $\frac{x-a}{3} = \frac{y-5}{b} = \frac{z-c}{-4}$
which is parallel to the plane $3x + y + 4z = 3$.
Find the values of a, b, c *and the shortest distance between the line and the plan*

42. In *the triangle OAB has* $OA = a, OB = b$, C *is a point on OA such that* $OC = 2/3a$. *D is a midpoint of AB, when CD is produced it meets OB produced at E such that* $DE = nCD$ and $BE = kb$.
Express DE in terms of (I) n, a, b *(ii)* k, a, b *and hence find the values of* n *and* k

43. *Given the equation of two lines* $y = m_1x + c_1$ and $y = m_2x + c_2$,
show that the vector equations are $\begin{pmatrix} 0 \\ c_1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ m_1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ c_2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ m_2 \end{pmatrix}$.
Hence *show that the angle between the lines is* $tan^{-1}\left(\frac{m_1-m_2}{1+m_1m_2}\right)$.

GEOMETRY.

44.$A(-3,0)$ and $B(3,0)$ are fixed points. Show that the locus of $P(x, y)$ which moves such that $PB = 2PA$ is a circle and find its radius and circle.

45.Given that $r = 3\cos\theta$ is an equation of a circle. Find its Cartesian form.

46.The point $A(x, 1)$and $B(-6, -5)$ are equidistant from the point $C(3, -2)$. Find the value of x.

47.The line $y = mx$ intersects the curve $y = 2x^2 - x$ at the points A &B. Find the equation of locus of point P which divides AB in the ratio $2:5$.

48.Find the equation of the circumscribing circle which passés through the points $(1,2), (2,5)$ and $(-3,4)$.

49.Find the parametric equation of the circles $(x + 1)^2 + (y - 2)^2 = 9$

50.Find the length of the tangent to the circle from the point $(5,7)$ to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$.

51.Show that $y = mx + c$ is a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$.

52.Show that the line $y = x + 1$ touches the circle $x^2 + y^2 - 8x - 2y + 9 = 0$ hence find the point(s) of intersection.

53.Prove that the circle $x^2 + y^2 - 2x - 6y + 1 = 0$ and $x^2 + y^2 - 8x - 8y + 31 = 0$ in two
distinct places and find the equation of the common chord.

54.Show that the tangents of $x^2 + y^2 + 4x - 2y - 11 = 0$ and $x^2 + y^2 - 4x - 8y + 11 = 0$ are intersecting at right angles.

55.Find the equation of a circle whose center lies on the line $y = 3x - 1$ and passes through the points $(1, 1)$ and $(2, -1)$.

56.Sketch the parabola $y^2 + 8y - 4x + 12 = 0$ showing clearly the focus and the directrix.

57.Show that the parametric equations $x = 3t^2 - 2, y = -6t$ represent the parabola. Find the focus and the directrix and hence sketch it.

58.If the normal at $P(ap^2, 2ap)$ to the parabola $y^2 = 4ax$ meets the curve again at $Q(aq^2, 2aq)$. Prove that $p^2 + pq + 2 = 0$.

59.Show that the that the tangent drawn from the end points of the focal chord joining the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ intersect at 90^0 at the directrix.

60.Prove that the chord $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ on the parabola $y^2 = 4ax$ has the equation $(p + q)y = 2x + 2apq$.
(b) a variable chord PQ of the parabola is
such that the line OP and OQ are perpendicular, where O is the origin.

(i)*prove that the chord PQ cuts the* $x-$ *axis* at *the fixed points, give the* $x-$ *coordinate* of the point. (ii) *find the equation of the locus of the midpoint PQ.*

61.*If the line* $y = mx + c$ *is a tangent to the ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when $c \pm \sqrt{a^2m^2 + b^2}$. *Hence find the equation of tangents to the ellipse* $\frac{x^2}{4} + \frac{y^2}{1} = 1$ *from the point* $(0, \sqrt{5})$.

62.*Given that* $y = mx + c$ *is a tangent to the hyperbola* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ *show that* $c^2 = a^2m^2 - b^2$.

63.*Show that* $25x^2 + 9y^2 - 100x - 54y = 44$ *represents an ellipse*, state the *coordinates of the center and the eccentricity.*

64.*The normal to the parabola* $y^2 = 4ax$ at the point $P(at^2, 2at)$ meets the axis of *the parabola at G and GP is produced beyond P to Q so* that $GP = PQ$, show *that the locus of Q is given by* $y^2 = 16a(x + 2a)$.

65.*Prove that the equation of the chord joining the points* $P(cp, c/p)$ and $Q(cq, c/q)$ *on the rectangular hyperbola* $xy = c^2$ is $x + pqy = c(p + q)$. (ii) *if this chord is also normal at P*, Show that $p^3q + 1 = 0$. *If in this case the normal* at *Q cuts the hyperbola again at R. prove that* PR *has the equation* $x + py = cp(1 + p)$.

ALGEBRA.

66.*Use the substitution* $x^2 - 4x = y$ to solve $2x^4 - 16x^3 + 77x^2 - 180x + 63 = 0$.

67.*Find the square root* of $6 + 14\sqrt{5}$.

68.*Solve the equation* $\sqrt{(2x + 3)} - \sqrt{(x + 1)} = \sqrt{x - 2}$.

69.*Solve the equation:* $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

70.If $a^2 + b^2 = 7ab$, show that $\log\frac{1}{3}(a + b) = \frac{1}{2}(\log a + \log b)$

71.*Solve the equation:* $x^2 + 2x + \frac{12}{x^2+2x} = 7$

72.*Solve the equation:* $x^{\frac{4}{3}} + 16x^{\frac{-4}{3}} = 17$.

73.*Solve:* $(0.4)^{-3x} < 3.6$.

74.*Solve the inequality:* $\frac{x+1}{2x-1} \leq \frac{1}{x-3}$.

75.*Prove by induction that* $2^{4n} - 1$ *is a multiple of* 15.

76.*Prove by induction that* $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n + 1)^2$ *and deduce that* $(n + 1)^3 + (n + 2)^3 + \cdots + (2n)^3 = \frac{1}{4}n^2(3n + 1)(5n + 3)$.

77. The sum of the first $n-$ terms of a certain progression is n^2+5n for all integral value of n. find the first three terms and prove that the progression is an AP.

78. The second, fourth and eighth term of An AP are in GP. If the sum of the third and the fifth term is 20. Find the sum of the first 4 terms of the progression.

79. Expand $\sqrt{\frac{1+5x}{1-5x}}$ as far as the term including x^3. Taking the first three terms, evaluate $\sqrt{14}$ to 3sf.

80. Given that $Z=1$ and $Z=1+i$ are roots of the equation $Z^3+aZ^2+bZ+c=0$. Find the values of a, b and c.

81. Given that the complex number Z varies such that $|Z-7|=3$. Find the greatest and least value of $|Z-i|$.

82. Find the locus defined by $|Z-2+3i|\geq 2$ if Z is a complex number.

83. Show that the locus of $\arg\left(\frac{Z-1}{Z-i}\right)=\frac{\pi}{6}$ is a circle, find its centre and radius.

84. Using Demoivre's theorem, prove that $16\sin^5\theta=\sin 5\theta-5\sin 3\theta+10\sin\theta$.

85. Given the complex number Z, $Z=\frac{(3i+1)(i-2)^2}{i-3}$, determine the modulus and argument of Z.

86. Prove that if $\frac{Z-6i}{Z+8}$ is real, then the locus of the point representing the complex number Z is a straight line.

87. If Z_1 and Z_2 are complex number, solve the simultaneous equations. $4Z_1+3Z_2=23$ and $Z_1+iZ_2=6+i8$ in $x+iy$.

88. Simplify. $\frac{(\cos\frac{2\pi}{7}-i\sin\frac{2\pi}{7})^3}{(\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7})^4}$

89. Show that if the equation $x^2+ax+1=0$ and $x^2+x+b=0$ have common roots, then $(b-1)^2=(a-1)(1-ab)$.

90. When the polynomial $3x^3+ax^2-bx+1$ is divided by $(x-2)^2$ the remainder is $39x-51$. Find the values of a and b.

91. Given that the equation $y^3-2y+4=0$ and the $y^2+y+c=0$ have a common root, show that $c^3+4c^2+14c+20=0$.

92. If α^2 and β^2 are roots of the equation $x^2-21x+4=0$. Form an equation with roots α and β.

93. A polynomial $p(x)$ is a multiple of $x-3$ and the remainder when divided by $x+3$ is 12. Find the remainder when the polynomial is divided by x^2-9.

ANALYSIS

94. $find\ the\ area\ bounded\ by\ the\ curve\ y = (5-x)(x+1)\ the\ y-axis\ and\ the\ line\ y = 5.$

95. $Prove\ that\ the\ area\ enclosed\ by\ the\ two\ parabolas\ y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. $If\ this\ area\ is\ rotated\ through\ four\ right\ angles\ about\ the\ x-axis, show\ that\ the\ volume\ generated$ is $\frac{96}{5}\pi a^3$.

96. $Show\ that\ \int_e^{e^2} \frac{dx}{xlnx} = ln2.$

$Integrate\ the\ following.$

97. $\int \frac{1-(log10^x)^{-1}}{xln10} dx.$

98. $\int \frac{x^3 dx}{1+x^8}$

99. $\int \frac{dx}{3sin^2x+cos^2x}.$

100. $\int \frac{dx}{1+sinx+cosx}.$

101. $\int sin4xcos2xdx.$

102. $\int \sqrt{\frac{3+x}{3-x}} dx$, using $x = 3sin\theta$

103. $Show\ that\ \int_0^1 \frac{8x+6}{(x^2+1)(x+2)} dx = \pi + ln\frac{8}{9}.$

104. $Differentiate\ cos2x\ from\ first\ principles.$

105. $If\ y = 4^x sinx, find\ the\ value$ of $\frac{dy}{dx}$

106. $Given\ that\ y = ae^{-2x}sin3x, prove\ that\ \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0.$

107. If $y = Ae^{3x} + Be^{-2x}. show\ that\ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$

108. $Differentiate: sin(x^x), 2x^x\ and\ 2x^{cosx}.$

109. $If\ x = \frac{3t-1}{t}\ and\ y = \frac{t^2+4}{t}, show\ that\ \frac{d^2y}{dx^2} = 2t^3.$

110. $If\ y = \sqrt{\frac{1+sinx}{1-sinx}}, show\ that\ \frac{dy}{dx} = \frac{1}{1-sinx}$

111. $Differentiate: cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).$

112. $Given\ that\ y = \tan xy, show\ that\ \frac{dy}{dx} = \frac{y}{\cos^2 xy - x}$

113. $Differentiate: y = ln\sqrt{\frac{1+x}{1-x}}.$

114. $Use\ small\ changes\ to\ evaluate\ sin48^0$

115. *Use small changes to evaluate* $\sqrt[3]{30}$.

116. *A cylinder of radius r and height 8r. The radius increases from* 4cm *to* 4.1cm. *find the approximate increase in volume.*

117. *The length of a rectangular block is three times its width. The total surface area of the block is* $180cm^2$. *Find its maximum value.*

118. *A piece of wire of length L is cut into two portions of length x* and $l - x$.
Each piece is the cut into twelve equal parts soldered together so as to form the edges of the cube. Show that the volume is given by $V = \frac{l^3 - 3l^2x + 3lx^2}{1728}$ *and that the minimum volume is* $\frac{l^3}{6912}$

119. *The base radius of a circular cone increases and the volume changes* by 2%. *If the height of the cone remains constant, find the percentage increase in the circumference of the base.*

120. *A capsule consists of a cylinder and two identical hemispheres as shown.*
Show that the surface area $S = \frac{4\pi}{3}(r^2 + \frac{18}{\pi r})$.
Given that the volume of the capsule is $12mm^3$,
determine the minimum value of S

121. *A container in shape of an inverted right circular cone, the height is* 12*cm and in a vertical angle* 60^0.
A tap delivers water in the container at a *rate which is proportional to the depth of water collected in the container* at *any time*. (i) *show that the rate at which the water level is riding* is *proportional to the depth.* (ii) *if it takes* 1 *minute to collect a depth* of 6cm of *water, calculate the time it takes to fill the whole container.*

Solve the following differential equations..

122. $\frac{dy}{dx} - y = x^3 e^{x^2}$.

123. $\frac{dy}{dx} + y\tan x = \cos x$.

124. $\frac{dy}{dx} = xy^2 - x$ given $y(0) = 2$

125. $(1-x^2)\ \frac{dy}{dx} - xy^2 = 0$, given $y(0) = 1$

126. $x^2\ \frac{dy}{dx} = x^2 + xy + y^2$.

127. *A student walks to school at a speed proportional to the square root of the distance he still has to cover.*
If the student covered 900*m* in 100

minutes and the school is 2500m from home, find how long he takes to get to school

128. *The population of a certain type of fish in a reserved part* of a *lake is allowed to change at rate* $\frac{dx}{dt} = 10 - 2t$, *where X is a population at* time *t years.* (a)*If the population is 2000 birds initially, show that* $x = 2000 + 10t - t^2$. (b) *find how long the population takes to grow to its maximum population.*

(c)*calculate the population of birds at the instant* when it's *decreasing at 14birds per day.*

129. *Given that* $y = \frac{x(x-3)}{(x-1)(x-4)}$. (i) *show that the curve doesn't have* the *turning points.* (ii)*find the equation of asymptotes and hence sketch* the *curve.*

130. *The curve with the equation* $y = \frac{ax+b}{x(x+2)}$ *where a and b are constants has* a *turning point at* $(1, -2)$. *Find values of a and b,* (ii)find *the equation of the asymptotes and hence sketch the curve.*

131. *Sketch the curve* $f(x) = x^2(x + 2)$ and *hence sketch the curve* of $\frac{1}{f(x)}$.

132. *Given that* $y = \frac{9}{3+x^2}$, *find the intercepts, asymptotes and hence sketch the curve.*

NGS SEMINAR QUESTIONS

P425/2: APPLIED MATHEMATICS.

STATISTICS AND PROBABILITY

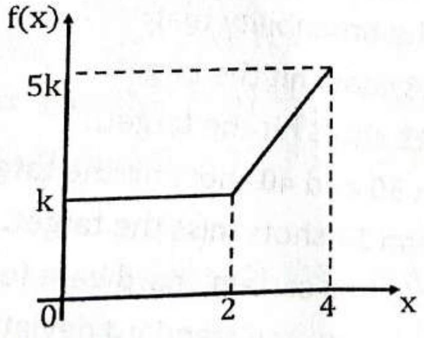
1) A continuous random variable X is uniformly distributed in the interval $[\alpha, \beta]$. The lower quartile is 5 and the upper quartile is 9. Find;
   a) Values of α and β, hence the p.d.f.
   b) E(X) and Var (x).
   c) P (4< X < 8)

2) A random variable y has a Cumulative distribution function, $F(y)$ given below.

$$F(y) = \begin{cases} 0 & : \quad y \leq -1 \\ k(y+1) & : \ -1 \leq y \leq 0 \\ k(2y+1) & : 0 \leq y \leq 1 \\ 3k & : y \geq 1 \end{cases}$$

Determine the;
   (i) value of k
   (ii) p. d. f of y.
   (iii) mean, μ.
   (iv) $P(|y - \mu| < \frac{1}{3})$.

3) The p.d.f. of a continuous random variable X is distributed as follows:



Find:
   (i) the value of k
   (ii) the equations of the p.d.f.

(iii) $P(1 < X < 3)$

(iv) the cumulative distribution function $F(x)$ and sketch it.

4) (a) Two events A and B are said to be independent. Show that A' and B' are also independent.

(b) It is assumed that pupils of a certain school hate their teachers due to their constant abuses, over testing and due to over caning with corresponding probabilities of 30%, 50% and 20%. Given the corresponding probabilities that the student would quit the school as 2%, 5% and 3%.

(i) Find the probability that the pupil quits the school.

(ii) Given that the pupil quits the school, find the probability that it was due to over caning

5) Independent observations are taken from a normal distribution with a mean of 30 and a variance of 5. Find the;

(a) probability that the average of 40 observations exceed 30.5.

(b) value of n such that the probability that the average of n observation exceeds 30.5 is less than 1%.

6) A soldier is thrice as likely to hit the target as missing it. If 48 shots are fired, find the probability that:

a) exactly **28** shots hit the target.

b) at least **29** shots hit the target.

c) between **30** and **40** shots hit the target.

d) fewer than **17** shots miss the target.

6. The length of rods sold in a certain hardware follow a normal distribution with a mean of 17.2 meters and standard deviation of 3.6 meters.

a) Find the 90% central limits of the length of the rods.

b) If 25 rods are chosen at random find the probability that the mean length of the rods will lie between 16m and 18m.

c) Find the probability that at least three among the five rods picked at random will have a length of more than 20 meters.

7. Two soldiers A and B in that order take turns shooting a bullet at a target. The first one to hit the target wins the game. If their chances of hitting the target on each occasion they shoot are $\frac{1}{3}$ and $\frac{1}{4}$ respectively, find the chance that:
   a) **A** wins the game on his third shot.
   b) **A** wins the game.
8. A task in mathematics is given to three students whose chances of solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$. Find the probability that:
   a) the task is solved.
   b) only one student solves it.
   c) at least two of them solved it.

9. The p.d.f. of a discrete random variable, X is as follows:

$$P(X = x) = \begin{cases} \beta x\ , & x = 1,2,3 \\ \beta(x+1)\ , & x = 4,5 \\ 0\ , & \text{otherwise} \end{cases}$$

   Find;
   a) the value of **k**
   b) $P(2 \leq X < 5)$
   c) mode and median of **X**
   d) E(X)
   e) Var(Y) if $Y = 2x - 1$
10. Mutually exclusive events A and B are such that $P(A \cup B) = 0.75$ and P(A) = 0.27, find:
   (i) $P(A' \cup B)$ (ii) $P(A' \cap B')$ (iii) $P(A \cap B)'$
11. Events A and B are such that $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{17}{24}$. Find:
   (i) $\mathbf{P(A \cap B)}$ (ii) $\mathbf{P(A' \cap B)}$ (iii) $\mathbf{P(A' \cap B)}$ (iv) $\mathbf{P(A' \cup B')}$

12. Exhaustive events A and B are such that 5P(A) = 4P(B) and $P(A \cap B) = \frac{1}{5}$ .Find: (i) P(A) (ii) P(A / B)
13. The mark, X, scored by candidates in a test is normally distributed with mean, μ and standard deviation, σ. Given that 80% of the candidates scored above 30 marks and 20% were awarded a distinction with at least 70 marks. Find the;

a) Value of μ and σ.

b) Number of candidates who passed out of the 500 who sat for the test if the pass mark was 50 marks.

14.Three pens are drawn without replacement from a bag containing **5** red and **3** blue pens. Find the:

a) probability distribution for the number of red pens drawn.

b) expected number of red pens drawn.

15.probability of obtaining at least **2** red pens

16.Bag **X** contains **5** red and **4** white beads, bag **Y** contains **7** red and **5** white beads, while bag **Z** contains **3** red and **5** white beads. A bag is selected at random and two beads are picked from it without replacement. Find the probability that:

a) they are of different colours.

b) bag **Y** is selected given that the beads drawn are of the same colour.

17.a) A random variable has a distribution of the form,

$f(x) = c\left(\frac{4}{5}\right)^x, x = 0,1,2,, \ldots.,$ Find c and P $(x \geq 2/_{x \leq 6})$.

b) Okello played 15 chess games. The probability that he wins a game is 0.6.

(i) Find the probability that he won between 6 to 10 games.

(ii) Calculate the most likely number of games he won.

c) Mary and Peter play a game in which they each throw a die in turn until someone throws a six. The person who throws a six wins the game, Peter starts the game. find the probability that she wins.

18. A random sample of 100 observations from a normal distribution with mean, μ gave the following data. $\sum x = 8200$. And $\sum x^2$= 686800. Calculate the;

(i) 99% confidence limits for μ.

(ii) 96 % confidence limits for μ.

19. The table below show the frequency distribution of marks obtained by a group of students in a paper two mathematics examination.

Marks (%)	10 -	20 -	35 -	45 -	65 -	80 -	90 -
Frequency density	1.8	2.4	5.8	3.3	1.2	0.4	0

a) Calculate the

(i) Modal mark

(ii) mean mark,

(iii) standard deviation,

(iv) number of students who scored above 54%.

b) Draw a cumulative frequency curve and use it to estimate the;

i) P_{10}-P_{60} range,

ii) number of students who scored below 40%,

iii) least mark if 20% of the students scored a distinction.

20. The table below shows the consumer prices per unit of commodities A, B, C and D in 2015 and 2017, with corresponding weights.

	Price in Shs.		
Commodity	2015	2017	Weights
A	9000	11000	9
B	6000	7000	6
C	x	y	2
D	6000	8000	3

Given that the simple aggregate index and the cost-of-living index were 124 and 122.5 respectively in 2017 basing on 2015, find the values of x and y. (05 marks)

(b) The table below show the items and quantities bought by a household in 2020 and 2022.

Item	Price 2020=100	Price 2022	Weight
Rice	12500	17500	12
Bread	4000	5500	20
Milk	2000	2500	15
Vegetables	1200	1800	30
Fruits	6000	7200	25

Calculate the

(i) simple aggregate price index

(ii) composite index

If the household spent shs 250,000/= in 2020, how much did it spend in 2022

21. The marks of **8** students in **GP** and **ICT** were as follows:

GP (x)	72	80	50	64	72	56	50	60
ICT (y)	78	79	65	60	85	67	54	65

(a) Plot a scatter diagram for the data. Comment on the relationship between the two tests.

(b) Draw a line of best fit for the scatter diagram and use it to find:

(i) x when y = 70. (ii) y when x = 55.

(c) Calculate the rank correlation coefficient for the scores in the two tests. Comment on your result based on 1% level.

22. The table below shows the weights in kg of 100 babies:

Weights	2.0	2.5	4.5	6.0	7.0	8.0
No of babies	35	20	20	10	5	5

a) Calculate the mean, variance and median for the above data.

Assuming this was a sample taken from a normal population, find the 97.5% confidence interval for the mean weight of all babies

23. A random variable **X** is normally distributed such that $P(X < 81) = 0.8849$ and $P(X > 66) = 0.9641$. Find:

a) the mean and standard deviation of **X**.

b) $P(69 < X < 83)$.

c) the interval which contains the middle 95% of distribution.

24.A random sample of 36 items drawn from a normal population is such that the 95% confidence interval for the mean of all the items is [67.9, 77.7] Find the 90% confidence limits for the mean of all the items.

NUMERICAL METHODS

25.The numbers x = 14.8, y = 8.75 and z = 12.4 were calculated with errors of 4%, 3% and 5% respectively. Find the range within which $\frac{y}{x-z}$ lies

26.The numbers x and y were estimated with errors Δx and Δy respectively.

a) Show that the maximum relative error in the product xy and quotient $\frac{x}{y}$ is the same.

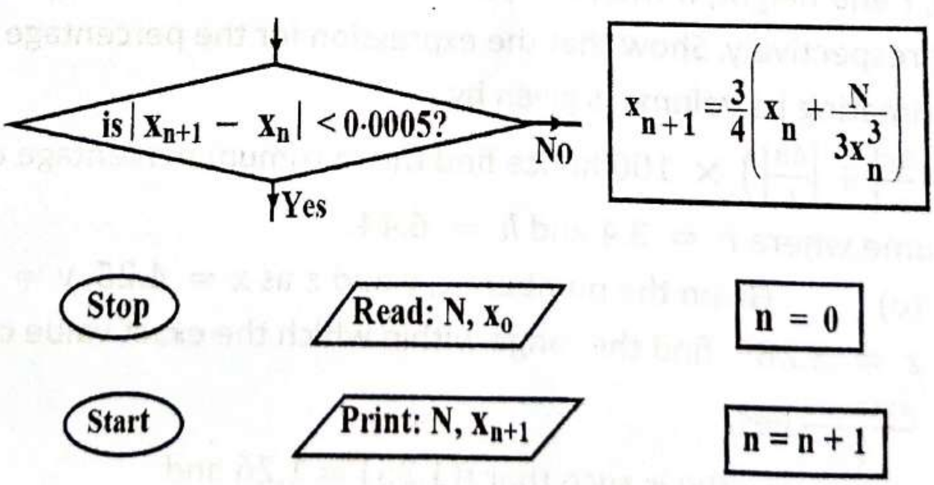
b) If $x = 4.96$ and $y = 2.243$ were rounded off to the given number of decimal places, calculate the:

(i) absolute error in $\frac{x}{y}$

(ii)limits within which $\frac{x}{y}$ is expected to lie, correct to 3 decimal places

27. By drawing graphs of $y = e^{2x}$ and $y = 5x + 1$, on the same axes, show that the equation $e^{2x} - 5x - 1 = 0$ has a root between 0 and 1. Use Newton Raphson Method to determine the root correct to 3 decimal places.

28.The iterative method for solving an equation is described by the following parts of the flowchart:



(i) By rearranging the given parts, draw a flow chart that shows the algorithm for the described method.

(ii) Using N = 38 and x_0 = 2, perform a dry run for the flow chart and state its purpose.

29. Draw a flow chart that computes and prints the product of the first **7** natural numbers. Hence perform a dry run of your flow chart.

30. a) Given the two iterative formulae $x_{n+1} = \frac{2{x_n}^3+1}{3{x_n}^2-5}$ and $x_{n+1} = \sqrt{5+\frac{1}{x_n}}$ using $x_0 = 2.3$, deduce with a reason the more suitable formula, hence give the root to 2dps

b) Use the trapezium rule with six ordinates to estimate $\int_1^2 6^{-2x}\,dx$ correct to three decimal places.

(i) Find the exact value of $\int_1^2 6^{-2x}\,dx$.

(ii) Calculate the percentage error made in your estimation in (a).

31. (a) The volume V of a cone can be calculated from the measurement of radius, r and height, h where Δv, Δr and Δh are errors in volume, radius and height respectively. Show that the expression for the percentage error in approximating its volume is given by

$\left(2\left|\frac{\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right|\right) \times 100$ hence find the maximum percentage error in the volume where $r = 3.4$ and $h = 6.44$.

(c) Given the numbers x, y and z as $x = 4.25, y = 8.9$ and $z = 3.289$, find the range within which the exact value of $\frac{xz(z-x)}{y}$ lies.

32. A certain function f(x) is such that f(1.25) = 1.26 and $f^{-1}(1.19) = 1.35$. Use linear interpolation or extrapolation to find the value of:

(i) $f^{-1}(1.22)$ (ii) $f(1.15)$

33. The bus charges over distances of 4km and 10km from town P are UGX 2800 and UGX 5800 respectively. Estimate the:

(i) charge that is worth 7km from P.

(ii) distance from P worth UGX 5300.

(iii) travel refund when a ticket of 8km is cancelled to 5km.

34. (a) The table below is an extract from the table of $Cos\ x°$

$x = 80°$	0	10'	20'	30'	40'
$Cos\ x$	0. 1736	0.1708	0.1679	0.1650	0.1622

Find the;

(i) $Cos\ 80°\ 36'$

(ii) $Cos^{-1}(0.1685)$

(b) Show that the equation $In\ x + x - 2 = 0$ has a root between $x = 1.5$ and $x = 1.6$. Use linear interpolation thrice to determine the root of the equation correct to 3 dps.

35. (i) Show that the iterative formula for finding the reciprocal of a number A is given by $x_{n+1} = x_n(2 - Ax_n)$.

(ii) Draw a flow chart that reads A and the initial approximation x_o, computes and prints A and its reciprocal after 2 iterations and gives it with an error of less than 0·0001.

(iii) Perform a dry run for your flow chart using A = $7/16$ and $x_o = 2.$

MECHANICS

36. (a) A mass of 5kg is pulled up at a Constant speed along a rough surface inclined at 20° to the horizontal. The angle of friction is 15° and the force pulling the mass is 3ON. Find the angle between the 30N and the line of greatest slope.

(b) A body of mass 30kg is fixed at the lower end of a light inextensible string whose other side is fixed on a ceiling. A horizontal force, P and a vertical force of 2.8 N keep the body in equilibrium with the string inclined at 60 ° to the vertical. Find P and the tension in the string.

37. ABCDEF is a regular hexagon. Forces of 3.5N, 4N, 6N, 1.5N and 2N act along the sides BC, AC, DA, AE and DE respectively with the direction of the forces being indicated by the direction of letters. Find the;

a) magnitude and direction of the resultant force.
b) Equation of line of action of the resultant force and hence or otherwise find where it cuts AB.

38. (a) A force $\boldsymbol{F}$ of magnitude 28N acts in the direction $-\boldsymbol{i}+\sqrt{3}\,\boldsymbol{j}$. Show that $\boldsymbol{F}=\left(-14\boldsymbol{i}+14\sqrt{3}\boldsymbol{j}\right)$.

(b) Two forces A and B have magnitudes $\boldsymbol{u}$ and $\boldsymbol{v}$ and act in directions $\boldsymbol{i}-\boldsymbol{j}$ and $\boldsymbol{i}+\boldsymbol{j}$ respectively. Given that the resultant of A and B is equal to $\boldsymbol{F}$ in (a) above, show that $\frac{u+v}{v-u}=\frac{1}{\sqrt{3}}$.

39. A non uniform rod AB of mass 12 kg has its Centre of gravity at a distance of $\frac{3}{4}$ AB. The rod is Smoothly hinged at A. It is maintained in equilibrium at 60° above the horizontal by a light inextensible string fixed at B and at 70° to AB. Calculate the magnitude and direction of the reaction at A.

40. A particle of mass 2 kg is acted upon by a force $\boldsymbol{F}=54t^3\,\boldsymbol{i}+24t^3\,\boldsymbol{j}-18t\,\boldsymbol{k}$ where t is time. Initially, the particle is located at a point with position vector $(1,0,0)\,m$ and moving with velocity $(1,0,1)ms^{-1}$.

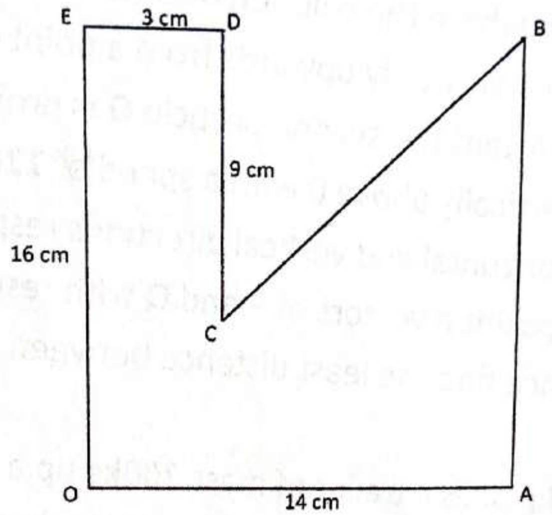
(a) Determine its;

(i) distance from the origin after 2s.

(ii) velocity at t = 1s.

(b) Show that the work done in the time interval t = 1s to t = 2s is equal to the change in the Kinetic energy of the particle.

41. (a) Find the centre of gravity of the lamina shown below.



(b) If the lamina is suspended from 0, find the angle that OE makes with the vertical.

42.a) An aircraft, at a height of 180m above horizontal ground and flying horizontally with a speed of $70ms^{-1}$, releases emergency supplies. If these supplies are to land at a specific point, at what horizontal distance from this point must the aircraft release them?

b) A particle is projected vertically upwards with velocity ums^{-1} and after t seconds another particle is projected vertically upwards from the same point with the same initial velocity. Prove that they will meet at a height of $\frac{4u^2 - g^2t^2}{8g}$

c) A car approaching a town covers two successive half-kilometers in 16 and 20 seconds respectively. Assuming the retardation is uniform. Find the further distance the car runs before stopping.

43.At 4:00 pm, a battle ship is at a place with position vectors $-6i + 12j\ km$ and it is moving with a velocity$16i - 4jkm/h$. At 4:30 pm, a cruiser is at a place with position vector $12i - 15jkm$ and is moving with a velocity $8i +$

$16jkm/h$.Show that if these velocities are maintained, the ships will collide. Find when and where the collision occurs.

44.A particle **P** is projected vertically upwards from a point **0** with a speed of **16ms^{-1}**. At the same instant the second particle **Q** is projected horizontally from point **A .25m** vertically above 0 with a speed of **12ms^{-1}**. using **i** and **j** as unit vectors in the horizontal and vertical directions respectively. find expressions for the position vectors of **P** and **Q** with respect to 0 at time t after projection, hence find the least distance between the particles (take g =10ms^{-2})

45.A car of mass **1200kg** pulls a trailer of mass **300kg** up a slope of **1 in 100** against a constant resistance of **0.2N** per kg. Given that the car moved at a constant speed of **1.5ms^{-1} for 5 minutes**, calculate the;

(i) tension in the tow bar.

(ii) work done by the car engine during this time

(iii) a car has an engine that can develop **15kw**. If the maximum speed of the car on a level road is **120kmh^{-1}**, calculate the total resistance at this speed.

END