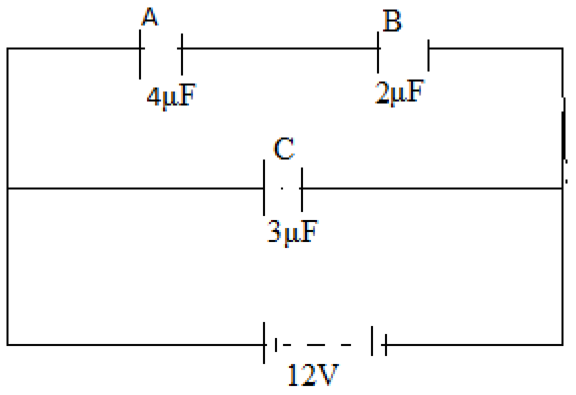
S.6 PHYSICS NOTES CONTINUATION OF CAPACITORS (all the streams should copy) BY MR.ALIA AMBROSE

(2) The figure below shows the network of the three capacitors A ,B and C



Calculate; (i) the charge on A, B and C

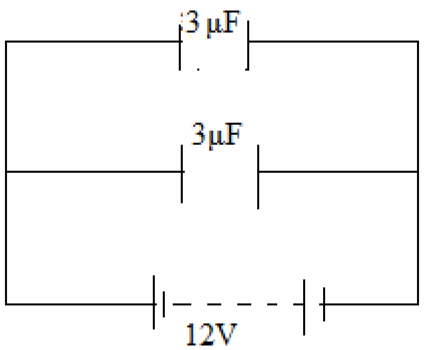
(ii) p.ds cross A,B and C

Soln.

A and B are in series ; C = $\frac{4\times10^{-6}\times12\times10^{-6}}{4\times10^{-6}+12\times10^{-6}} = 3\mu F$,

$3\mu F \; and \; 3\mu F \; are \; in \; parallel \;\; \Rightarrow C = 3\mu F + 3\mu F = 6\mu F$

$Q = CV = 6 \times 12^{-6} \times 12 = 7.2 \times 10^{-5}C$



Charge through A = charge through B = $\frac{3}{6} \times 10^{-5}C = 3.6 \times 10^{-5}C$

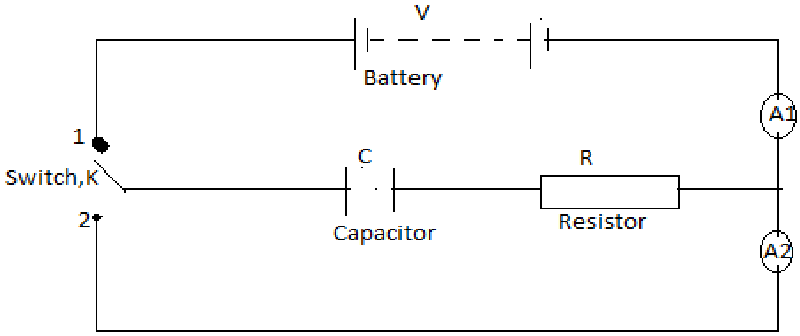
Charge through C = $\frac{3}{6} \times 7.2 \times 10^{-5}C = 3.6 \times 10^{-5}C$

(ii) p.d across A = $\frac{Q_A}{C_A} = \frac{3.6\times10^{-5}}{4\times10^{-6}} = 9V$

p.d across B = $\frac{Q_B}{C_B} = \frac{3.6\times10^{-5}}{12\times10^{-6}} = 3V$

P.d across C = $\frac{Q_C}{C_C} = \frac{3.6\times10^{-5}}{3.6\times10^{-6}} = 12V$

Charging and discharging a capacitor.



When switch, K is connected to position 1, the capacitor gets charged and the current flowing through the ammeter A_1 is initially very high but it decays with time to zero when the capacitor is fully charged.

The battery is then disconnected from the capacitor by removing the switch k from 1.

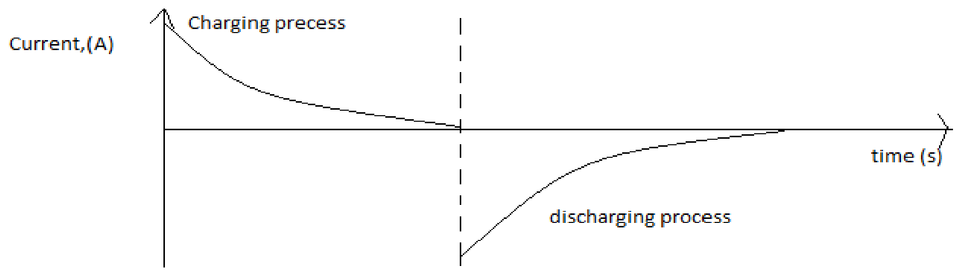
Switch k is then connected to 2 .The capacitor becomes discharged and initially current is maximum but eventually it decays to zero.

Flow of current shows that the capacitor stored charge when it was originally connected to the battery.

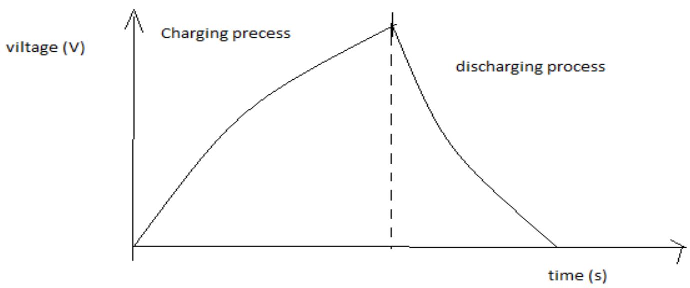
The energy changes which occur when a capacitor is being charged from a battery include;

Chemical energy from the battery being converted to heat energy in the connecting wires and electrical energy which is stored as electrostatic potential energy.

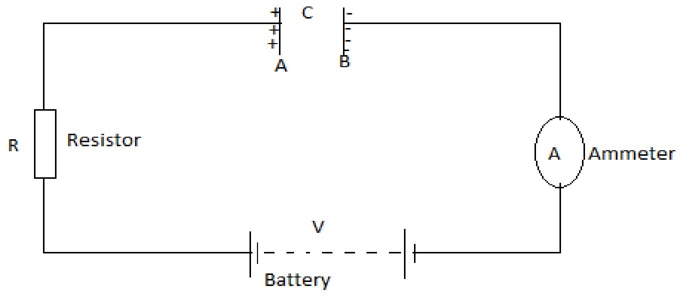
A graph of current with time during the charging and discharging processes.



A graph of voltage with time.



Explanation of the charging process.

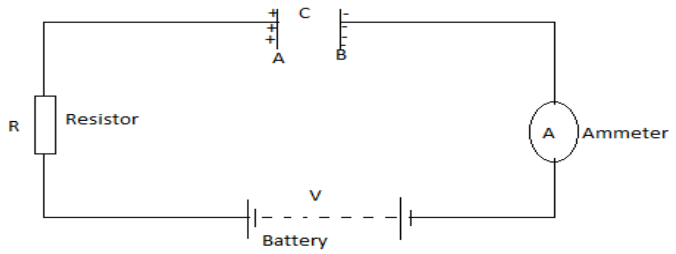


If a capacitor is connected to a battery, electrons flow from the negative terminal of the battery to plate B which leaves the plate negatively charged.

At the same time, electrons from plate A of the capacitor flow towards the positive terminal of the battery, which leaves plate A positively charged.

Positive and negative charges therefore appear on the plates of the capacitor and as these charges accumulate, p.d between the plates increases while the charging current decays to zero at that point when the p.d across the capacitor plates is equal to the battery voltage.

Explanation of the discharging process.

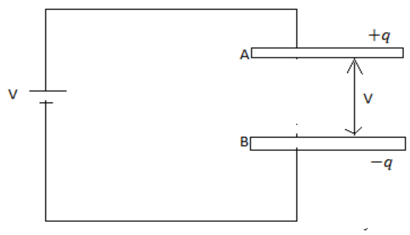


When the plates of the capacitor are connected together using connecting wires ,electrons flow from B to A until when all the charges on A are neutralised

Initially the current is maximum but due to the flow of electrons from B to A, current decays to zero when all the charges on A are neutralised and at the same time p.d between the plates are now neutral.

Energy stored in a charged capacitor.

When a capacitor is connected to a d.c supply ,it draws energy from the supply and stores in it's electric field.



When a small amount of charge δq is transferred from B and A

Total charge on A = $q + \delta q$ $and\ total\ p.d = V + \delta v$

Work done $\delta w = (v + \delta v)\delta q$ $\Rightarrow \delta w = v\delta q + \delta v\delta q$. But $\delta v\delta q$ is much smaller than q and can be neglected.

$\Rightarrow\delta w = v\delta q$ but $q = cv$ $\Rightarrow v = \frac{q}{C}$ so $\delta w = \frac{q}{C}\delta q$

To charge the capacitor plate A from $q = 0$ to $q = Q$, then the work done is given by

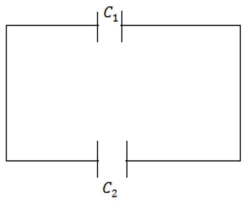
$\int dw = \int_0^Q \frac{q}{C} dq$, $\Rightarrow w = \frac{1}{C}\int_0^Q dq$, $w = \frac{1}{C}(\frac{q^2}{2})_0^Q$ $\Rightarrow W = \frac{Q^2}{2C}$

From Q = CV, w = $\frac{C^2V^2}{2C} = \frac{1}{2}CV^2$

Also from Q =CV $\Rightarrow$ V = $\frac{Q}{V}$ SO $W = \frac{Q^2}{2(\frac{Q}{V})}$, $\Rightarrow W = \frac{1}{2}QV$

Generally energy stored =$\frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

Joining two charged capacitors.



If a capacitor is charged and disconnected from the battery and then connected to another an uncharged capacitor, the following will be observed.

(i) There is no charge in total charge stored by the system.

(ii) The potential difference across the two capacitors becomes equal.

(iii) The two capacitors are considered to be in parallel therefore the effective capacitance $C = C_1 + C_2$

(iv) There is usually a loss in energy. This is because unless the p.ds across them are the same ,charge will flow to equalise the difference .

The flow of charge results in heating of the of the connecting wires and consequent loss of energy .

EXAMPLES

(1) A $2\mu F$ capacitor is fully charged by a 12V battery. It is disconnected from the battery and connected to an uncharged $5\mu F$ capacitor in parallel.

(a) find the total energy .

(i) before the two capacitors are connected.

(i) After they are connected.

(iii) Explain the difference in the two energy

(b) Calculate the p.d across the combination.

SOLUTION.

(i) Energy store before the capacitors are connected.

$E_0 = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6} 12^2 = 1.44 \times 10^{-4} J$

(ii) When the two capacitors are connected, total capacitance.

$C = C_1 + C_2$

$= (2+5)\ \mu F = 7\mu F$

$initial\ charge = C_1V_1 = 2 \times 10^{-6} \times 12 = 2.4 \times 10^{-5} C$

But initial charge =final charge $= 2.4 \times 10^{-5} C$

Energy, $E_1 = \frac{Q^2}{2C} = \frac{(2.4\times10^{-5})^2}{2x7\times10^{-6}} = 0.411 \times 10^{-4}$ J

Energy loss = 1.44×10^{-4} - 0.411×10^{-4} $= 1.03 \times 10^{-4} J$

(iii) the energy lost is dissipated as thermal (heat) energy in the connecting wires.

(b) let v be the p.d across the combination.

$Qo = C_1V + C_2V$

$2.4 \times 10^{-5} = (2 \times 10^{-6} + 5 \times 10^{-6})V$

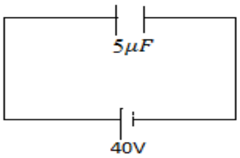
$V = \frac{2.4\times10^{-5}}{7.0\times10^{-6}} = 3.43$

2. A $5\mu F$ capacitor x is charged by a 40v supply and then disconnected from the supply source and connected across an charged capacitor of capacitance $20\mu F$. Calculate

(i) Final p.d across each capacitor

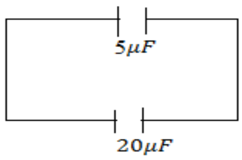
(ii)Final charge on each capacitor

(iii) initial and final energy stored by the capacitor
soln;



Initial charge $,Q_0 = CV = 5 \times 10^{-6} \times 40 = 2.0 \times 10^{-4} C$

After connection.



Initial charge Qo = final charge Q

$Q = CV_1 + C_2V \Rightarrow Q = (C_1 + C_2)V$, $2.0 \times 10^{-4} = (5 \times 10^{-6} + 2.010^{-6})V \Rightarrow Q = 8V$

(ii) from $Q = CV$

Charge on $5\mu F = 5 \times 10^{-6} \times 8 = 4.0 \times 10^{-5}C$

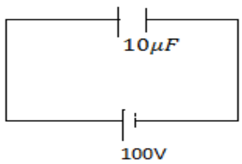
Charge on $20\mu F = 20 \times 10^{-6} \times 8 = 1.6 \times 10^{-4}$

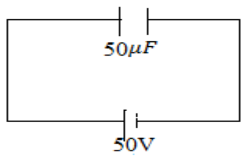
(iii) initial energy = $\frac{1}{2}CV^2 = \frac{1}{2}5 \times 10^{-6} \times 40^2 = 4.0 \times 10^{-3}J$

Energy stored after joining $= \frac{1}{2}CV^2 = \frac{1}{2}(25 \times 10^{-6}) \times 8^2 = 8 \times 10^{-4}J$

3. A capacitor of $10\mu F$ with a p.d of 100V across it is joined to another capacitor of $50\mu F$ with a p.d of 50V across it. Calculate the change in energy of the system.

Soln,





$E_1 = \frac{1}{2}CV^2 = \frac{1}{2} \times 10^{-6} \times 100^2 = 5 \times 10^{-2}J$ and $E_2 = \frac{1}{2}CV^2 = \frac{1}{2} \times 50 \times 10^{-6} \times 50^2 = 6.25x10^{-2}$

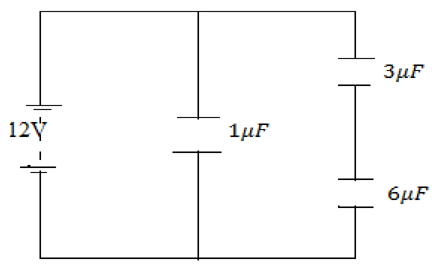
Initial total energy = $E_1 + E_2 = 50 \times 10^{-2} + 6.25 \times 10^{-2} = 11.25 \times 10^{-2}J$

Initial charge = total charge = $C_1V_1 + C_2V_2 = 50 \times 10^{-6} \times 50 + 10 \times 10^{-6} \times 100 = 3.5 \times 10^{-3}C$

Effective capacitance, $C = C_1 + C_2 50 \times 10^{-2} + 10 \times 10^{-6} = 60 \times 10^{-6}F$

Final energy $= \frac{Q^2}{2C} = \frac{(3.5 \times 10^{-3})^2}{2 \times 60 \times 10^{-6}} = 10.20 \times 10^{-6}J$

4. A $3\mu F$ capacitor is connected in series with a , $6\mu F$ capacitor. The combination is then connected in parallel with a $1\mu F$ capacitor to a 12V battery as shown in the figure below.



Calculate.

(i) The charge in each capacitor.
(ii) Energy stored in the $6\mu F$ capacitor.

Soln,

$3\mu F$ and $6\mu F$ are in series. $\Rightarrow C = \left(\frac{3\times6}{3+6}\right)\mu F = 2\mu F$

$2\mu F$ and $1\mu F$ are in parallel; $\Rightarrow C = (2+1)\mu F$; and

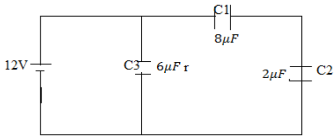
$Q = CV = 3 \times 10^{-6} \times 12 = 3.6 \times 10^{-5} C$

Charge through $2\mu F$ capacitor $= \frac{2}{3} \times 3.6 \times 10^{-5} = 2.4 \times 10^{-5} C$

Charge through $1\mu F$ capacitor $= \frac{1}{3} \times 3.6 \times 10^{-5} = 1.2 \times 10^{-5} C$

(ii) Energy stored by $6\mu F$ capacitor $= \frac{Q^2}{2C} = \frac{(2.4\times10^{-5})^2}{2x6\times10^{-6}} = 4.8 \times 10^{-5} J$

5. The diagram below shows an arrangement of three capacitors C_1, C_2 and C_3 of capacitance $8\mu F$,$2\mu F$ and $6\mu F$ respectively.



Calculate the total energy stored.

(i) in all the capacitors when fully charged.
(ii) when the space between the plate of C_2 is filled with a dielectric of dielectric constant 1.25.
Soln;

a) (i) $8\mu F$ and $2\mu F$ are in series; effective capacitance, $C = \left(\frac{2\times8}{2+8}\right)\mu F = 1.6\mu F$

$1.6\mu F$ and $6\mu F$ are in parallel so effective capacitance $7.6\mu F$

Energy, $E = \frac{1}{2}CV^2 = \frac{1}{2}x7 \times 10^{-6} \times 12^2 = 5.472 \times 10^{-4} J$

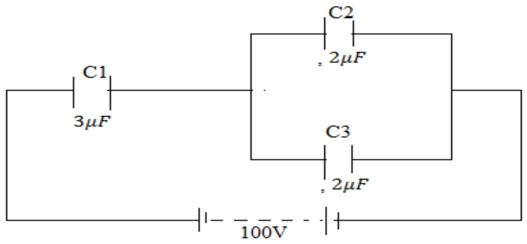
(iii) New capacitance of $C_2 = \varepsilon_r c_2 = 1.25 \times 2\mu F = 2.5\mu F$

Effective capacitance in series $= \left(\frac{2.5\times8}{2.5+8}\right)\mu F = 1.9\mu F$

Effective capacitance in parallel.$= (1.9+6)\mu F = 7.9\mu F$

New energy $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 7.9 \times 10^{-6} \times 12^2 = 5.69 \times 10^{-4} J$

6.



In the figure above C_1, C_2, and C_3 are capacitors of capacitance $3\mu F$, $2\mu F$ and $2\mu F$respectively, Connected to a battery of emf 100V. Calculate the energy stored in the system of capacitors if the space between the plates of C_1 is filled with an insulators of dielectic constant 3 and the capacitors are fully charged.

Soln,

$2\mu F$ and $2\mu F$ are in parallel, effective capacitance ,$= (2+2)\mu F = 4\mu F$

Let C_p be the capacitance when a dielectric is inserted.

$C_p = 3C_1 = 3 \times 3\mu F = 9\mu F$; so $9\mu F$ and $4\mu F$ are in series;

the effective capacitance, C = $\left(\frac{9\times4}{9+4}\right)\mu F = \frac{36}{13}\mu F$

$E = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{36}{13} \times 10^{-6} \times 100^2 = 0.0138J$

Calculating involving change of area of overlaps

When a capacitor is charge and the area of overlaps is reduced after disconnecting the capacitor from the source, the following will happen.

(i) the p.d between the plates is increased.
(ii) the charge ,Q remains constants.
(iii) the capacitance of a capacitor decreases.
(iv) the charge stored between the plates increases.

EXAMPLES

The capacitance of a variable radio capacitor can be changed continuously from 10PF to 900PF by turning the dial from 0^0 to 140^0. with the dial set aType equation here.t 140^0, the capacitor is connected to a 9V battery. After charging the capacitor is disconnected from the battery and the dial turned to 0^0. Calculate.

(i) charge on the capacitor.
(ii) energy stored on the capacitor with dial set at 140^0.
(iii) The work required to turn the dial from 140^0 to 0^0 if the friction is neglected.

Soln,

(i) charge stored in the capacitor

$Q = CV = 900 \times 10^{-12} \times 9 = 8.1 \times 10^{-9}C$

(ii) $E_1 = \frac{1}{2}CV^2 = \frac{1}{2} \times 900 \times 10^{-12} \times 9^2 = 3.65 \times 10^{-8}J$

(ii) energy stored after turning to 0^0; $E_2 = \frac{Q^2}{2C} = \frac{(8.1\times10^{-9})^2}{2\times100\times10^{-12}} = 3.3 \times 10^{-6}J$

$work\ done = (3.3 \times 10^{-6} - 0.0365 \times 10^{-6}) = 3.2635 \times 10^{-6}J$

Effect of increasing the plate separation d, on the energy stored.

When the plate separated ,d,is increased,the energy stored between the plates also increases. The electrostatic energy is given by $\frac{1}{2}CV^2$. The charge on the capacitor is given by CV and it remains constant but the capacitance C is inversely proportional to the distance between the plates. So as the separation increases, the capacitance decreases and V in the CV term must increase. therefore the stored energy increases, this is because work must be done to separate the plates against the electrostatic attraction.

NOTE; If the plate separation is doubled, the energy stored also doubles.

From $E = \frac{Q}{2C}$, $C = \frac{\varepsilon_0 A}{d}$ $\Rightarrow E = \frac{Q}{(\frac{\varepsilon_0}{d})}$ $\Rightarrow E = \frac{Qd}{\varepsilon_0 A}$,since Q, ε_0 and A are constant. $E \propto d$

Effects of increasing the plate separation on the potential between the plate.

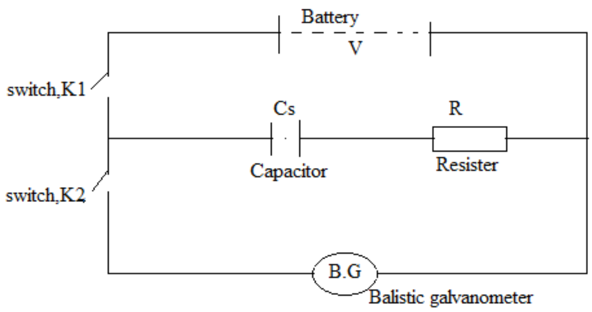
Since capacitance, $C = \frac{\varepsilon A}{d}$. where d is the separation between the plates, it implies that when d is increased, C reduces but from $Q = CV \Rightarrow V = \frac{Q}{C}$. Where, Q is the charge on either plate (constant) this implies that since C has reduced, then the p.d V increases hence the potential difference between the plate increases when the distance of separation is increased .

Qn, a capacitor of a capacitance c is charged by a battery and then later isolated. when the plates of the capacitor are taken apart, deduce what will happen to the potential difference between the plates.

Soln,

Refers to the above (effect of increasing d on the potential between the plates).

An experiment to measure capacitance of a capacitor using a ballistic galvanometer



The circuit is connected as shown above. The switch k_1 is closed leaving k_2 open, the standard capacitor of capacitance C_S is charge to a battery voltage v. Switch k_1 is open and k_2 closed to discharge the capacitor through the ballistic galvanometer.The maximum defection θ_S is noted. But $Q_S \propto \theta_S \Rightarrow Q_S = C_S V \Rightarrow C_S V \propto \theta_S \Rightarrow C_S V = K\theta_S$ ....(i)

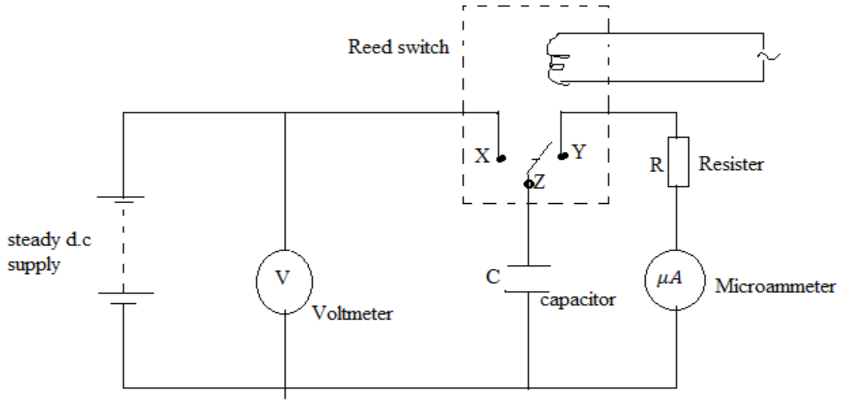
The standard capacitor is then replaced with a capacitor of unknown capacitance C. Switch K_1 is again closed leaving K_2 open. The capacitor of unknown capacitance, C is charge to a battery voltage v. K_1 is then open and switch k_2 closed and the capacitor discharges through the ballistic galvanomenter.

The maximum defection θ is noted.

That is $Q \propto \theta. but\ Q = CV \ \Rightarrow CV \propto \theta\ .Therefore\ CV = K\theta$ … … … … … (ii)

$(i) \div (ii)$ gives $\frac{C_S V}{CV} = \frac{K\theta_S}{K\theta} \Rightarrow \frac{C_S}{C} = \frac{\theta_S}{\theta} \quad \Rightarrow c = \frac{C_S\theta}{\theta_S}$

Experiment to measure capacitance of a capacitor using a vibrating reed switch.



The circuit is connected as shown above. The vibrating reed has two contact x and y and it is energized by a low a.c voltage from the mains.

During contact with x the capacitor is charged and when the contact is moved to y, the capacitor is discharged through the micrometre.

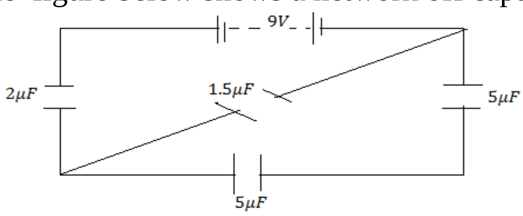
The micrometer receives a given amount of pulse of charge per second which causes a steady current I to flow through the microammeter.

If the charge on the capacitor is Q when the capacitor is fully charged and f is the frequency at which z is vibrating, then the current passing through the micrometer is given by

$I = Qf$, but $Q = CV \ \Rightarrow I = CVf \quad \Rightarrow C = \frac{I}{Vf}$.

<u>Revision questions</u>

(1) The figure below shows a network off capacitors connected a cross a 9V Supply .



Find (i)the energy stored in the system.

(iii) p.d across the $5\mu F$ capacitor.

Soln

$5\mu F\ and\ 5\mu F\ are\ in\ series$ so $C = \frac{5\times10^{-6} 5\times10^{-6}}{5\times10^{-6}+5\times10^{-6}} = 2.5\mu F$

$2.5\mu F\ and\ 1.5\mu F\ are\ in\ parallel\ \Rightarrow c = 2.5\mu F + 1.5\mu F = 4\mu F$

$4\mu F\ and\ 2\mu F\ are\ in\ series\ so\ C = \left(\frac{4\times2}{4+2}\right)\mu F = \frac{4}{3}\mu F$

Energy stored $= \frac{1}{2}CV^2 = \frac{1}{2}\times\frac{4}{3}\times 10^{-6}\times 9^2 = 5.4\times10^{-5}J$

(ii) p.d across the $5\mu F$ capacitor.

p.d across $2\mu F$; from $Q = CV \Rightarrow V = \frac{Q}{C} = \frac{\frac{4}{3}\times10^{-6}\times9}{2\times10^{-6}} = \frac{4\times9}{6} = 6V$

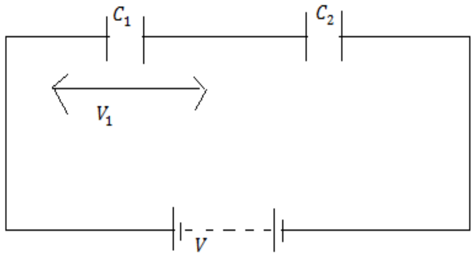
p.d across 1.5V = $6 - 9 = 3V$

2.(a) Define the following.

(i) capacitance

(ii) relative permittivity (dielectric constant)

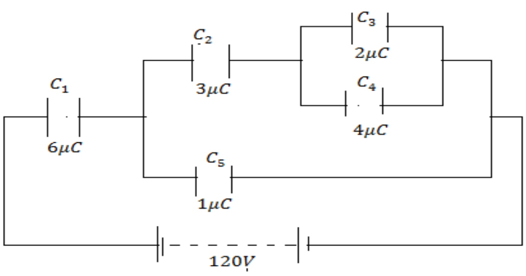
(b) two capacities of capacitance C_1 and C_2 are connected in sense with a battery of emf V as shown in the figure below.



If the p.d across a capacitor of capacitance c1 is v1 ,shows that $\frac{1}{V_1} = (\frac{1}{C_1} + \frac{1}{C_2})\frac{C_1}{V}$

c) Describe an experiment to determine the capacitance using a vibrating reed circuit .

d) A battery of emf 120 V is connected to a network of capacitors as shown in the figure below.



Calculate;

(i) the chare on C_1

(ii) Energy stored in C_5

(e) Describe how the effect of a dielectric material on the capacitance of a capacitor may be determined.

Soln

$C_1\ and\ C_2\ are\ in\ series\ so\ C = \frac{C_1C_2}{C_1+C_2}$

But $Q = CV\ \Rightarrow Q = \left(\frac{C_1C_2}{C_1+C_2}\right)V$

$V_1 = \frac{Q}{C_1} = \left(\frac{C_1C_2}{C_1+C_2}\right)V \div C_1 = \left(\frac{C_1C_2}{C_1+C_2}\right)\frac{V}{C_1} \quad \Rightarrow \frac{1}{V_1} = \left(\frac{C_1+C_2}{C_1C_2}\right)\frac{C_1}{V} \quad \Rightarrow \frac{1}{V_1} = (\frac{1}{C_1} + \frac{1}{C_2})\frac{C_1}{V}$

3.Define an expression for the energy stored in a capacitor of capacitance C charged to a voltage, V

b) A parallel plate capacitor with plates of area $2 \times 10^{-2}m^2$ separation $5.0 \times 10^{-3}m$ is connected to a $500V$ supply.

(i) calculate the energy stored in the capacitor .

(ii) if the space between the plates is completely filled with oil and the total charge in the capacitor becomes $4.42 \times 10^{-8}C$, find the dielectric constant of the oil.

(b) (i) $Q = CV.\ \ but\ C = \frac{\varepsilon_0 A}{d} \Rightarrow Q = \frac{\varepsilon_0 AV}{d} = \frac{8.85\times10^{-12}\times2\times10^{-2}\times500}{5\times10^{-3}} = 1.77 \times 10^{-8}\ C$

Energy $\frac{1}{2}QV = \frac{1}{2} \times 1.77 \times 10^{-8} \times 500 = 4.4 \times 10^{-6}J$

(ii). Capacitance $C' = \varepsilon_r c$ where ε_r = dielectric constant.

$Q = C'V \quad \Rightarrow Q = \frac{\varepsilon_r \epsilon_0 AV}{d}$

$$\varepsilon_r = \frac{Qd}{\varepsilon_0 AV} = \frac{4.42 \times 10^{-8} \times 5 \times 10^{-3}}{8.85 \times 10^{-12} \times 2 \times 10^{-2} \times 500} = 2.5$$

4. A capacitor of capacitance C_1 is charged by a battery of emf V_0. The charging battery is then removed and the capacitor is connected to an uncharged capacitor of capacitance C_2. Show that the loss in energy, E after connection is given by E = $\frac{1}{2}(\frac{C_1C_2}{C_1+C_2})V_0{}^2$