P425/2
APPLIED MATHEMATICS
PAPER 2
July/August 2024
3 hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Attempt* **all** *questions in section* **A** *and any* **five** *questions from section* **B**.
- *Any additional question(s) answered will* ***not*** *be marked.*
- **All** *working must be shown clearly.*
- *Begin each answer on a fresh sheet of paper.*
- *Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.*
- *In numerical work, take* ***g*** *to be* $9.8ms^{-2}$.
- *State the degree of accuracy at the end of the answer to each question attempted using a calculator or table and indicate* ***Cal*** *for calculator, or* ***Tab*** *for mathematical tables.*

Turn Over

SECTION A (40 MARKS)

Answer **all** questions in this section.

1. Two events A and B are such that P (A) = $\frac{1}{2}$, P (A or B but not both A and B) = $\frac{1}{3}$ and P (B) = $\frac{1}{4}$. Calculate;

   (a) P (A $\cap$ B) (03 marks)

   (b) P (B′/A) (02 marks)

2. In an examination, scaling is done such that a student K whose original mark is 35% gets 50% and student N whose original mark is 40% becomes 65%
   Determine the;

   (a) original mark for student M whose new mark is 80%

   (b) new mark for a student whose mark is 37%

   (05 marks)

3. A particle moving in a straight line covers distances of 90 meters and 240 meters in successive times of 2 seconds and 4 seconds respectively.
   Find the;

   (a) acceleration of the particle

   (b) initial velocity of the particle.

   (05 marks)

4. A discrete random variable y has a probability density function (p.d.f) given as;

$$P(y = y) = \begin{cases} Ky^2 ; & y = 1, 2, 3 \\ K(y+1); & y = 4, 5 \\ 0 , & \text{otherwise} \end{cases}$$

   Determine the;

   (a) value of constant K

   (b) mean of y

   (c) P ($2 < y \leq 4$)

   (05 marks)

5. The pressure (P), absolute temperature (T) and volume (V) of a fixed mass of a gas is given by the following relationship: $P = \frac{KT}{V}$, where K is the constant of proportionality. If e_1 and e_2 are the errors made in measuring the absolute temperature and volume respectively, show that the maximum possible relative error made in pressure is $\left|\frac{e_1}{T}\right| + \left|\frac{e_2}{V}\right|$

   (05 marks)

6. A football striker took a free kick with a velocity of 21 ms^{-1} at 30^0 above the horizontal. The defender intercepted it with a header at a height of 3 m above the ground.
   Calculate the speed of the ball at impact. (05 marks)

7. The price relatives for the years 2000 and 2004 and their weights of a house hold expenditure on essential goods are given in the table below.

Item	Price Relatives		Weight
	2000	2004	
Food	100	125	35
Water	100	121	11
Rent	100	112	8
Electricity	100	108	6
Transport	100	118	22

Calculate the;

(a) cost of living index for 2004. Comment on your results. (03 marks)

(b) cost of a cloth in 2004 that cost UGX 11,200 in 2000. (02 marks)

8. A particle executing simple harmonic motion about a point O has speeds of $3\sqrt{3}\ ms^{-1}$ and $3\ ms^{-1}$ when covering distances 1m and 0.268 m respectively from the end point. Find the amplitude of the motion. (05 marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section.

All questions carry equal marks.

9. At 7:30a.m daily, a bus leaves Kampala for Jinja. The times (minutes) taken to cover the journey were recorded over a certain period of time shown in the table below;

Time(min)	80-84	85-89	90-94	95-99	100-104	105-109	110-114	115-119
Frequency	10	15	35	40	28	15	4	3

(a) Calculate the;

(i) Mean time of travel from Kampala to Jinja by the bus.

(ii) Median time of travel

(07 marks)

(b) (i) Draw a cumulative frequency curve. (Ogive) for the data.

(ii) Use the Ogive to estimate the number of times the bus arrived in Jinja between 9:00a.m to 9:25a.m (05 marks)

10. (a) A square lamina ABCD of side 2M is made of uniform thin material. When a semi- circular piece with AB as its diameter is removed from square, show that the centre of mass of the remaining lamina is at a distance $\frac{20M}{3(8-\pi)}$ from the line AB. (07 marks)

(b) The remainder of the lamina is suspended from a light string attached to C and hangs in equilibrium. Show that CD is inclined to the down ward vertical at angle α where; $\tan\alpha = \frac{2(14-3\pi)}{3(8-\pi)}$ (05 marks)

11. (a) Use the trapezium rule with 6 ordinates to estimate the values of $\int_1^2 \left(\frac{x}{7x^2-3}\right)dx$, correct to four significant figures. (05 marks)

(b) (i) find the exact value of $\int_1^2 \left(\frac{x}{7x^2-3}\right)dx$, correct four significant figures.

(ii) Calculate the percentage error in your calculation in (a) above.

(iii) Suggest how the percentage error may be reduced.

(07 marks)

Turn Over

12. Box X contains 4 red battons and 3 green battons. Box Y contains 5 red and 6 green battons. Box X is twice as likely to be picked as box Y.
A box is chosen at random and two battons are removed from it at a time without replacement.

(a) Find the probability that the two battons removed are of the same colour. (05 marks)

(b) Construct the probability distribution table for the number of red battons picked. (04 marks)

(c) Calculate the mean number of red battons. (03 marks)

13. A particle moving with an acceleration given by:

$\mathbf{a} = (4e^{-3t}\,\boldsymbol{i} + 12\sin t\,\boldsymbol{j} - 7\cos t\,\boldsymbol{k})\ ms^{-2}$ is initially located at the point (5, -6, 2) and velocity $\mathbf{V} = (11\,\boldsymbol{i} - 8\,\boldsymbol{j} + 3\boldsymbol{k})\ ms^{-1}$

Find the;

(a) magnitude of the acceleration when t = 0. (02 marks)

(b) velocity at any time t seconds. (04 marks)

(c) displacement at time t = 1 second. (06 marks)

14. (a) Use a graphical method to show that the equation $e^x + x - 4 = 0$ has only one real root. Hence obtain to one decimal place, an approximate root of the equation. (06 marks)

(b) (i) Show that the Newton Raphson formula for estimating the root of the equation $e^x + x - 4 = 0$ is given by $x_{n+1} = \dfrac{e^{xn}(x_n - 1) + 4}{e^{xn} + 1}$, n = 0, 1, 2, 3, ...

(ii) Using equation in b (i) and taking the approximate root in (a) as an initial approximation, obtain the root of the equation. Give your answer correct to three significant figures. (06 marks)

15. A certain company came up with a program of rearing goats. They visited a certain village to find out the modern ways of rearing goats and discovered that the number of goats owned by the residents were normally distributed 15% of the residents had less than 60 goats and 5 % of the residents had more than 90 goats.

(a) Determine the mean and standard deviation of the number of goats. (08 marks)

(b) If the village had 300 residents rearing goats, find how many residents had more than 80 goats. (04 marks)

16. Forces, $(-2\underset{\sim}{i} + 3\underset{\sim}{j})$N, $(-\underset{\sim}{i} + 2\underset{\sim}{j})$N, $(4\underset{\sim}{i} - 2\underset{\sim}{j})$N and $(-\underset{\sim}{i} - 3\underset{\sim}{j})$N act at points A(-2, 3), B(3, 1), C(-1, -3) and D(3, 1) respectively.

(a) Show that the system of forces forms a couple. (05 marks)

(b) If the force acting at points D (3, 1) is replaced by the force $(2\underset{\sim}{i} + \underset{\sim}{j})$N, determine the Cartesian equation of the line of action of the resultant force and where it crosses the x – axis. (07 marks)

END