PURE MATHS SOLUTIONS

SECTION A			
	WORKING	**Marks**	**Comments**
1	Let $t = \tan\frac{\theta}{2}$ $\cos\theta = \frac{1-t^2}{1+t^2}$ $\sin\theta = \frac{1-t^2}{1+t^2}$ From $\cos\theta + \sqrt{3}\sin\theta = 2$ $\Rightarrow \frac{1-t^2}{1+t^2} + \sqrt{3}\left(\frac{2t}{1+t^2}\right) = 2$ $1-t^2+2\sqrt{3}t = 2(1+t^2)$ $1 - t^2 + 2\sqrt{3}t = 2 + 2t^2$ $3t^2 - 2\sqrt{3}t + 1 = 0$ $t = \frac{2\sqrt{3} \pm \sqrt{\left(-2\sqrt{3}\right)^2 - 4\times3\times1}}{6}$ $= \frac{2\sqrt{3} \pm \sqrt{12-12}}{6}$ $t = \frac{\sqrt{3}}{3}$ But $t = \tan\frac{\theta}{2}$ $\tan\frac{\theta}{2} = \frac{\sqrt{3}}{3}$ $\frac{\theta}{2} = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $\frac{\theta}{2} = 30°, 210°$ $\theta = 60^0, 420^0$ ***Alternatively***, $\cos\theta + \sqrt{3}\sin\theta = R\cos(\theta - \alpha)$ $\cos\theta + \sqrt{3}\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ Comparing L.H.S and R.H.S $R\cos\alpha = 1$............................. (i) $R\sin\alpha = \sqrt{3}$ .... (ii) $[\text{Eqn (i)}]^2 + [\text{Eqn (ii)}]^2$ $R^2\cos^2\alpha + R^2\sin^2\alpha = 1 + 3$ $R^2(\cos^2\alpha + \sin^2\alpha) = 4$ $R^2 = 4$ $R = 2$ Eqn (ii) ÷ Eqn (i) $\tan\theta = \sqrt{3}$ $\theta = \tan^{-1}(\sqrt{3})$ $\theta = 60^0$		

	$\Longrightarrow$ $2\cos(\theta - 60^0) = 2$ $\cos(\theta - 60^0) = 1$ $\theta - 60^0 = \cos^{-1}(1)$ $\theta - 60^0 = 0^0$ $\theta = 0^0 + 60^0$ $\theta = 60^0$		
2.	Taking $y = z^3 - 5z^2 + 9z - 5$ …………… (i) Substituting for z = 1 into Eqn (i) above, $y = 1^3 - 5(1)^2 + 9(1) - 5$ $= 1 - 5 + 9 - 5$ $= 0$ Hence $z = 1$ is a root ***Alternatively***; By using long division method, $\begin{array}{r} z^2 - 4z + 5 \\ z-1\overline{)\,z^3 - 5z^2 + 9z - 5} \\ \underline{z^3 - z^2} \\ -4z^2 + 9z - 5 \\ \underline{-4z^2 + 4z} \\ 5z - 5 \\ \underline{5z - 5} \\ 0 \quad 0 \end{array}$ Now when solving for other roots, we have $(z - 1)(z^2 - 4z + 5) = 0$ Taking $(z^2 - 4z + 5) = 0$ $Z = \dfrac{4 \pm \sqrt{4^2 - 4\times1\times5}}{2\times1}$ $= \dfrac{4 \pm \sqrt{16-20}}{2}$ $= \dfrac{4 \pm \sqrt{-4}}{2}$ $Z = \dfrac{4 \pm 2i}{2}$ $= 2 \pm i$ Hence the roots are $2 - \boldsymbol{i}$ and $2 + \boldsymbol{i}$		Note: By use of the remainder theorem, the remainder of the equation must be zero for z = 1 to be a root
3.	Taking $y = x\,10^{\sin x}$ Introducing $\log_e$ to both sides, $\log_e y = \log_e[x10^{\sin x}]$ $\log_e y = \log_e x + \sin x \log_e 10$ Differentiating with respect to x,		

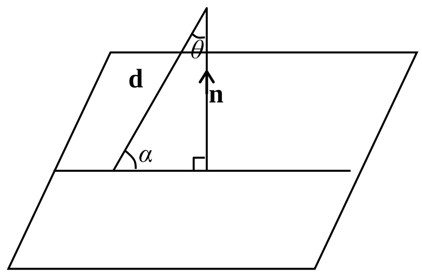
	$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\cos x\log_e 10$ $\frac{dy}{dx}=\left(\frac{1}{x}+\cos x\log_e 10\right)y$ $=x10^{\sin x}\left(\frac{1}{x}+\cos x\log_e 10\right)$		
4.	Let $\frac{x}{5}=\frac{y+2}{2}=\frac{z-1}{4}=\lambda$ Making x, y and z the subject; $x = 5\lambda$ $y = 2\lambda - 2$ $z = 4\lambda + 1$ Substituting for x, y and z in $3x + 4y + 2z - 25 = 25$, $3(5\lambda) + 4(2\lambda - 2) + 2(4\lambda + 1) - 25 = 0$ $15\lambda + 8\lambda - 8 + 8\lambda + 2 - 25 = 0$ $31\lambda = 31$ $\lambda = 1$ By substitution, $x = 5$ $y = 2 - 2 = 0$ $z = 4 + 1 = 5$ Hence the coordinates of the point of intersection are (5, 0, 5)		
5.	Let P = current population, n = number of years it will take the population to triple $\Rightarrow 3P=P\left(1+\frac{r}{100}\right)^n$ $3=\left(1+\frac{r}{100}\right)^n$ $3=(1+0.0275)^n$ $3=(1.0275)^n$ Introducing log to base ten on both sides, $\log 3 = n\log 1.0275$ $n=\frac{\log 3}{\log 1.0275}=\frac{0.4771212547}{0.0117818305}$ $= 40.5$ (I d.p.) Hence it will take 40.5 years for the population to triple.		
6.	$0.6^{-2x}<3.5$ Introducing $\log_e$ to both sides		

	$\ln 0.6^{-2x} < \ln 3.5$ $-2x \ln 0.6 < \ln 3.6$ $-2x(-0.222) < 0.558$ $0.444x < 0.556$ $x < \frac{0.556}{0.444}$ $x < 1.252$ (3 d p)		
7.	$\frac{dy}{dx} + 3y = e^{2x}$ .... (i) I.F. = $e^{\int 3dx} = e^{3x}$ Multiplying Eqn (i) by the I.F. $e^{3x}\frac{dy}{dx} + 3y\left(e^{3x}\right) = e^{2x}\left(e^{3x}\right)$ $e^{3x}\frac{dy}{dx} + 3y\left(e^{3x}\right) = e^{5x}$ $\Rightarrow \frac{d}{dx}y\left(e^{3x}\right) = e^{5x}$ $\int \frac{d}{dx}y\left(e^{3x}\right)dx = \int e^{5x}dx$ $y\left(e^{3x}\right) = \frac{1}{5}e^{5x} + C$ $y = \frac{1}{5}e^{2x} + C\left(e^{-3x}\right)$ .... (ii) Substituting for x = 0 and y = 1 into Eqn (ii) $1 = \frac{1}{5}e(^{0}) + C(e^{0})$ $C = \frac{4}{5}$ Substituting for C into Eqn (ii) $y = \frac{1}{5}e^{2x} + \frac{4}{5}e^{-3x}$ $y = \frac{1}{5}\left(e^{2x} + 4e^{-3x}\right)$		
8.	$x^2 - 4x - 12 = x^2 - 6x + 2x - 12$ $= x(x-6) + 2(x-6)$ $= (x-6)(x+2)$ $\Rightarrow \frac{8x}{x^2-4x-12} = \frac{8x}{(x-6)(x+2)}$ Changing to partial fractions $\frac{8x}{(x-6)(x+2)} \equiv \frac{A}{x-6} + \frac{B}{x+2}$ $8x \equiv A(x+2) + B(x-6)$ Putting x = -2 -16 = -8B B = 2 Putting x = 6 48 = 8A		

	$A = 6$ $\Rightarrow \int_0^2 \frac{8x}{x^2-4x-12}dx = \int_0^2 \left(\frac{6}{x-6}+\frac{2}{x+2}\right)dx$ $=\left[6\ln(x-6)+2\ln(x+2)\right]_0^2$ $=\left[\ln(x-6)^6+\ln(x+2)^2\right]_0^2$ $=\left(\ln(-4)^6\right)-\left(\ln(-6)^6\right)+\left(\ln 4^2\right)-\left(\ln 2^2\right)$ $=\ln(4096)-\ln(46656)+\ln 16-\ln 4$ $=\ln\left(\frac{4096\times 4}{46656}\right)=\ln\left(\frac{16384}{46656}\right)$ $=-1.05 \quad (2\,\text{dp})$		

SECTION B

		working	Marks	Comments
9.	(a)	let P(x, y, z) lie on the line AB $AP = \lambda AB$ $\mathbf{OP}-\mathbf{OA}=\lambda[\mathbf{OB}-\mathbf{OA}]$ $\mathbf{OP}=\mathbf{OA}+\lambda[\mathbf{OB}-\mathbf{OA}]$ $\mathbf{p}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\-4\\-1\end{pmatrix}+\lambda\left[\begin{pmatrix}5\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\-4\\-1\end{pmatrix}\right]...(i)$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\-4\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\2\\4\end{pmatrix}$ Substituting for (x, y, z) into the equation of the plane $2(2 + 3\lambda) + 6(-4 + 2\lambda) - 3(-1+ 4x) = -5$ $4 + 6\lambda - 24 + 12\lambda + 3 - 12\lambda = -5$ $6\lambda = 12$ $\lambda = 2$ Substituting for λ into Eqn (i) $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\-4\\-1\end{pmatrix}+2\begin{pmatrix}3\\2\\4\end{pmatrix}=\begin{pmatrix}8\\0\\7\end{pmatrix}$ Hence the coordinates of C are (8,0,7)		
	(b)	The vector parallel to the line is $\mathbf{d} = 3\mathbf{i}+2\mathbf{j}+4\mathbf{k}$ and the normal to the plane is $\mathbf{n} = 2\mathbf{i}+6\mathbf{j}-3\mathbf{k}$		

		$\mathbf{d}\cdot\mathbf{n}=\lvert\mathbf{d}\rvert\lvert\mathbf{n}\rvert\cos\theta$ $(3\mathbf{i}+2\mathbf{j}+4\mathbf{k}).(2\mathbf{i}+6\mathbf{j}-3\mathbf{k})$ $=\sqrt{3^2+2^2+4^2}.\sqrt{2^2+6^2+(-3)^2}$ $6+12-12=\sqrt{9+4+16}.\sqrt{4+36+9}\cos\theta$ $6=\sqrt{29\times 49}\cos\theta$ $\cos\theta=\frac{6}{\sqrt{1421}}$ $\theta=80.8414^0$  The angle between the line and the plane is $\alpha=90^0-\theta$ $=90^0-80.84^0$ $=9.16^0$		
10.	(a)	2 sin 2x = 3 cos x 2 sin 2x - 3 cos x = 0 4 sin x cos x – cos x = 0 cos x (4sin x – 3) = 0 cos x = 0 x = $\cos^{-1}(0)$ x = 90^0, -90 4 sin x – 3 = 0 $\sin x = \frac{3}{4}$ $x = \sin^{-1}\left(\frac{3}{4}\right)$ $x=48.59^0, 131.41$ Hence x = ($^-$90°, 48.6°, 90°, 131.4°) ***Alternatively***: By using t = tan $\frac{1}{2}x$ $\cos x=\frac{1-t^2}{1+t^2}$ $\sin x=\frac{2t}{1+t^2}$ By substitution $4\left[\frac{2t}{1+t^2}.\frac{1-t^2}{1+t^2}\right]-3\left[\frac{1-t^2}{1+t^2}\right]=0$		

	(b)	$\frac{8t(1-t^2)}{1+t^2}-3(1-t^2)=0$ $8t(1-t^2)-3(1-t^2)(1+t^2)=0$ $(1-t^2)\left[\left(8t-(3+3t^2)\right)\right]=0$ Taking $1-t^2 = 0$ $1 = t^2$ $t = \pm 1$ When $t = 1$ $\tan\frac{1}{2}x=1$ $\frac{1}{2}x=\tan^{-1}$ (1) = 45, 135, -135 $x = 90^0$ When $t = -1$ $\tan\frac{1}{2}x=-1$ $\frac{1}{2}x=\tan^{-1}(-1)=-45,$ $x = -90^0$ Taking $8t-3-3t^2=0$ $3t^2-8t+3=0$ $t=\frac{8\pm\sqrt{64-36}}{6}$ $t=\frac{8\pm5.29}{6}$ If $t=\frac{8+5.29}{6}=2.215$ $\tan\frac{1}{2}x=2.215$ $\frac{1}{2}x=\tan^{-1}(2.225)$ $\frac{1}{2}x=65.7,\ 245.7$ $x=131.4^0$ If $t=\frac{8-5.29}{6}=0.4517$ $\frac{1}{2}x=\tan^{-1}(0.4517)$ = 24.31°, 204.3° x = 48.6° For $^-180<\theta<180°$ x = ⁻90°, 48.6°, 90°, 132.4°) $\sin x-\sin 4x=\sin 2x-\sin 3x$ $\sin 3x+\sin x=\sin 4x+\sin 2x$ $2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)=2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)$ $2\sin(2x)\cos x=2\sin 3x\cos x$ $\sin 2x\cos x-\sin 3x\cos x=0$ $\cos x(\sin 2x-\sin 3x)=0$		

		Taking cos = 0 $x = \cos^{-1}(0)$ $x = \frac{-\pi}{2}, \frac{\pi}{2}$ Taking sin 2x – sin 3x = 0 sin 3x – sin 2x = 0 $2\cos\left(\frac{3x+2x}{2}\right)\sin\left(\frac{3x-2x}{2}\right) = 0$ $\cos\left(\frac{5}{2}x\right)\sin\left(\frac{1}{2}x\right) = 0$ Either $\cos\left(\frac{5}{2}x\right) = 0$ $\frac{5}{2}x = \cos^{-1}(0)$ $\frac{5}{2}x = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi$ $x = \pm\frac{\pi}{5}, \pm\frac{3\pi}{5}$ **Or** $\text{Sin}\left(\frac{1}{2}x\right) = 0$ $\frac{1}{2}x = \sin^{-1}(0) = 0, \pm\pi$ $x = 0^0$ Hence $x = -\frac{\pi}{5}^0, -\frac{\pi}{2}^0, -\frac{3\pi}{5}^0, 0^0, \frac{\pi}{5}^0, \frac{\pi}{2}^0, \frac{\pi}{5}^0, \frac{3\pi}{5}^0$		
11.	(a) (b)	Let $y = \sqrt{x}$ Suppose x changes by δx, then y also changes by δy $\Rightarrow y - \delta y = \sqrt{x - \delta x}$ But $\sqrt{98} = \sqrt{100-2}$ Where $y = \sqrt{100} = 10$, $x = 100$, $\delta x = 2$ As $\delta x \to 0$, $\frac{\delta y}{\delta x} \longrightarrow \frac{dy}{dx}$ $\Longrightarrow$ When x is very small, $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ $\delta y = \frac{dy}{dx} \cdot \delta x$ $= \frac{1}{2\sqrt{x}} \cdot \delta x$ $= \frac{1}{2\sqrt{100}} \cdot 2$ $= \frac{1}{10} = 0.1$ $\Rightarrow \sqrt{98} = y - \delta y$ = 10 – 0.1 = 9.9 Let $f(x) = \ln(1 + ax)$ By Maclaurin's theorem;		

		$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + ...$ Now $f(0) = \ln 1 = 0$ From $f(x) = \ln(1 + ax)$ $f'(x) = \frac{a}{1+ax}, \quad f'(0) = a$ $f''(x) = \frac{-a^2}{(1+ax)^2}, \quad f''(0) = -a^2$ $f'''(x) = \frac{2a^3}{(1+ax)^3}, \quad f'''(0) = 2a^3$ By substitution, we have; $f(x) = 0 + \frac{a}{1}x + \frac{-a^2}{2}x^2 + \frac{2a^3}{6}x^3 + ...$ $= ax - \frac{a^2x^2}{2} + \frac{a^3x^3}{3} + ...$ $\ln\left(\frac{1+x}{\sqrt{1-2x}}\right) = \ln(1+x) - \ln(1-2x)^{\frac{1}{2}}$ $= \ln(1+x) - \frac{1}{2}\ln(1-2x)$ By comparing with $\ln(1 + ax)$, For $\ln(1 + x)$, $a = 1$ For $\ln(1 - 2x)$, $a = -2$ => $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + ...$ $\ln(1 - 2x) = -2x - 2x^2 - \frac{8x^3}{3} - ....$ By substitution, $\ln\left(\frac{1+x}{\sqrt{1-2x}}\right) = \left[x - \frac{x^2}{2} + \frac{x^3}{3} + .\right] - \frac{1}{2}\left[-2x - 2x^2 - \frac{8x^3}{3}...\right]$ $= x - \frac{x^2}{2} + \frac{x^3}{3} + x + x^2 + \frac{4x^3}{3} + ...$ $= 2x + \frac{x^2}{2} + \frac{5x^3}{3} + ...$ Validity of the expression For $\ln(1 + x)$ to be valid, $-1 < x < 1$ For $\ln(1 - 2x)$ to be valid, $-1 < -2x < 1$ Where $-½ < x < ½$ Hence the expansion for $\ln\left(\frac{1+x}{\sqrt{1-2x}}\right)$ is valid for $-½ < x < ½$ or $\lvert x\rvert < ½$		
12.	(a)	Note: $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$ => $(\cos\theta + i\sin\theta)^5 = (\cos 5\theta + i\sin 5\theta)$ By expanding $(\cos\theta + i\sin\theta)^5$ $(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$		

(b)

$\Longrightarrow (\cos 5\theta + i\sin 5\theta) = (\cos^5\theta - 10\cos^3\theta\sin^2 + (5\cos\theta\sin^4\theta) +$
$i(5\cos^4\theta\sin\theta + \sin^5\theta - 10\cos^2\theta\sin^3\theta)$

Equating real parts;

$\cos 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$........................ (i)

Equating imaginary parts

$\sin 5\theta = (5\cos^4\theta\sin\theta + \sin^5\theta - 10\cos^2\theta\sin^3\theta)$ ....(ii)

Eqn (ii) ÷ Eqn (i)

$$\tan 5\theta = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$$

Dividing the numerator & denominator on the R.H.S. by $\cos^5\theta$

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$$

Let $y = z^3 + 1$

Taking $z = -1$

$y = -1^3 + 1 = 0$

Hence $z + 1$ is a factor

Using long division to obtain the remaining equation

$$\begin{array}{r|l} & z^2 - z + 1 \\ \hline z+1 & z^3 + 1 \\ & \underline{z^3 - z^2} \\ & \quad -z^2 + 1 \\ & \underline{\quad -z^2 - 2} \\ & \qquad z + 1 \\ & \qquad \underline{z + 1} \\ & \qquad - \ - \end{array}$$

=> $z^3 + 1 = (z + 1)(z^2 - z + 1) = 0$

Taking $z^2 - z + 1 = 0$

$$z = \frac{1 \pm \sqrt{1^2 - 4\times 1}}{2}$$
$$= \frac{1 \pm \sqrt{1-4}}{2}$$
$$= \frac{1 \pm i\sqrt{3}}{2}$$

$Z = -1$, $z = \frac{1}{2} + \frac{i\sqrt{3}}{2}$ and $z = \frac{1 - i\sqrt{3}}{2}$

Alternatively:

Let $z = x + iy$

Substituting for z in $z^3 + 1 = 0$

$(x + iy)^3 + 1 = 0$

$(x + iy)^3 = -1$

$x^3 + 3ix^2y - 3xy^2 - iy^3 = -1$

$(x^3 - 3xy) + i(3x^2y - y^3) = -1$

Equating real and imaginary parts;

$x^3 - 3xy^2 = -1$ ....(i)

$3x^2y - y^3 = 0$.............................. (ii)

From Eqn (ii);

$y(3x^2 - y^2) = 0$

Either $y = 0$ or $3x^2 - y^2 = 0$

$y^2 = 3x^2$

Substituting for $y = 0$ in Eqn (i)

$x^3 = -1;\ x = -1$

Substituting for $y^2 = 3x^2$ in Eqn (i)

$x^3 - 3x(3x^2) = -1$

$x^3 - 9x^3 = -1$

$-8x^3 = -1$

$x^3 = 1/8; \quad x = 1/2$

$$y^2 = 3\left(\frac{1}{4}\right)$$

$$y^2 = \frac{3}{4}$$

$$y = \pm\frac{\sqrt{3}}{2}$$

Hence $z = -1$, and $z = \dfrac{1 \pm i\sqrt{3}}{2}$

13.

At turning points, $\dfrac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{(2x-1)(2x-6)-(x^2-6x+5)(2)}{(2x-1)^2} = 0$$

$(2x - 1)(2x - 6) - (x^2 - 6x + 5)(2) = 0$

By opening brackets and simplifying,

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x - 2) + 1(x - 2) = 0$

$(x + 1)(x - 2) = 0$

Either $x + 1 = 0$

$x = -1$

Or $x - 2 = 0$

$x = -2$

Substituting for $x = -1$

$$y = \frac{(-1)^2 + 6(-1) + 5}{2(-1) - 1} = -1$$

Investigating the nature of the turning points

For (-1, -4)

x	-2	-1	0
$\frac{dy}{dx}$	+	0	−

Maximum

For (2, -1)

x	1	2	3
$\frac{dy}{dx}$	−	0	+

Minimum

Finding the intercepts

For x – intercept, y = 0

=> $x^2 - 6x + 5 = 0$

$x^2 - x - 5x + 5 = 0$

$x(x - 1) - 5(x - 1) = 0$

$(x - 5)(x - 1) = 0$

Either $x - 1 = 0$

$x = 1$

Or $x - 5 = 0$

$x = 5$

For y-intercept, $x = 0$

$\Rightarrow$ $y = {}^{5}\!/_{-1} = -5$

Finding asymptotes

Vertically: $2x - 1 = 0$

$x = {}^{1}\!/_{2}$

Horizontally:

$(2x - 1)y = x^2 - 6x + 5$

$x^2 - 6x + 5 - 2xy + \ = 0$

$x^2 - (6 + 2y)x + 5 + y = 0$

For real values of x

$(6 + 2y)^2 \geq 4(5 + y)$

$36 + 24y + 4y^2 \geq 20 + 4y$

$4y^2 + 20y + 16 \geq 0$

$(y + 1)(y + 4) = 0$

Either $y + 1 = 0$

$y = -1$

Or $y + 4 = 0$

$y = -4$

For slanting asymptote:

$$
\begin{array}{r l}
 & \frac{x}{2} - \frac{11}{4} \\
(2x-1) & \overline{)\, x^2 + 6x + 5} \\
 & \underline{x^2 - \frac{x}{2}} \\
 & \quad -\frac{11}{2}x + 5 \\
 & \quad -\frac{11}{2}x + \frac{11}{4}
\end{array}
$$

Hence the slanting asymptote is y = $\frac{x}{2} - \frac{11}{4}$

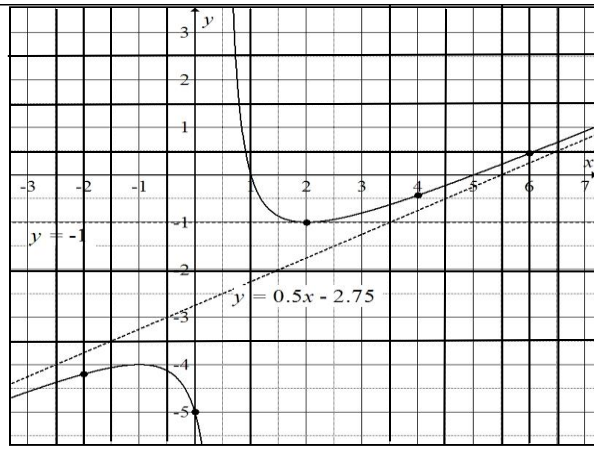
Investigating the range of x where the curve lies

x	$x < 1/2$	$½ < x < 1$	$1 < x < 5$	$x > 5$
x-1	−	−	+	+
x-5	−	−	−	+
$(x-1)(x-5)$	+	+	−	+
$2x - 1$	−	+	+	+
y	−	+	−	+

Investigating the range of y

	$y < -4$	$-4 < y < -1$	$y > -1$
$(y+1)(y+4)$	+	−	+

Graph of $y = \frac{x^2 - 6x + 5}{2x - 1}$

		 **Note:** The graph must be drawn from x = -2 to x = 7 (This is not an open graph)		
14.	(a) (b)	Given $x = 4\cos\theta$ $\frac{x}{4} = \cos\theta$ $y = 3\sin\theta$ $\frac{y}{3} = \sin\theta$ $\sin^2\theta + \cos^2\theta = \frac{y^2}{9} + \frac{x^2}{16}$ $\frac{y^2}{9} + \frac{x^2}{16} = 1$ Which is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and this is an ellipse $\Rightarrow a = 4$ and $b = 3$ From $b^2 = a^2(1 - e^2)$ $\frac{b^2}{a^2} = 1 - e^2$ $e^2 = 1 - \frac{b^2}{a^2}$ Where $b^2 < a^2$ $= 1 - \frac{9}{16} = \frac{7}{16}$ $e = \frac{\sqrt{7}}{4}$ At the point of contact, y and x are the same for the two equations. Substituting for $y = mx + c$ in equation $b^2x^2 + a^2y^2 = a^2b^2$ $b^2x^2 + a^2(mx + c)^2 = a^2b^2$ $b^2x^2 + a^2m^2x^2 + 2a^2mxc + a^2c^2 = a^2b^2$ $(b^2 + a^2m^2)x^2 + (2a^2mc)x + a^2(c^2 - b^2) = 0$ For tangency, the roots must be equal		

		$(2a^2mc)^2 = 4a^2(b^2 + a^2m^2)(c^2 - b^2)$ $4a^4m^2c^2 = 4a^2b^2c^2 - 4a^2b^4 + 4a4m^2c^2 - 4a^4m^2b^2$ $a^2m^2c^2 = b^2c^2 - b^4 + a^2m^2c^2 - a^2m^2b^2$ $b^2c^2 - b^4 - a^2m^2b^2 = 0$ $c^2 = b^2 + a^2m^2$ For $y = mx + c$, at point (-3, 3) $\Rightarrow 3 = -3m + c$ $c = 3(1 + m)$ Substituting for c in $c^2 = b^2 + a^2m^2$ $9(1 + m)^2 = b^2 + a^2m^2$ $9(1 + 2m + m^2) = b^2 + a^2m^2$ Since $a = 4$ and $b = 3$ $\Rightarrow 9(1 + 2m + m^2) = 9 + 16m^2$ $9 + 18m + 9m^2 = 9 + 16m^2$ $7m^2 - 18m = 0$ $m(7m - 18) = 0$ Either $m = 0$ Or $7m = 18$ $m = 18/7$ When $m = 0$, $c = 3(1 + 0) = 3$ When $m = 18/7$, $c = 3\left(1 + 18/7\right) = 75/7$ Hence the equation of the tangents are $y = 3$ and $y = \frac{18}{7}x + 75/7$		
15.	(a) (b)	$\int x^3 e^{x^4}\,dx$ Let $t = x^4$ $dt = 4x^3dx$ $dx = \frac{dt}{4x^3}$ $\int x^3 e^{x^4}\,dx = \int x^3 e^t \cdot \frac{dt}{4x^3}$ $= \frac{1}{4}\int e^t\,dt$ $= \frac{1}{4}e^t + c$ $= \frac{1}{4}e^{x^4} + c$ $t = \tan x$, $\frac{dt}{dx} = \sec^2 x = 1 + t^2$, $dx = \frac{dt}{1+t^2}$ Dividing through by $\cos^2 x$		

		$\int \frac{1}{1+\sin^2 x}dx = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}}dx$ $= \int \frac{\sec^2 x}{\sec^2 x+\tan^2 x}dx$ $= \int \frac{1+\tan^2 x}{\left(1+\tan^2 x\right)+\tan^2 x}dx$ $= \int \frac{1+\tan^2 x}{\left(1+2\tan^2 x\right)}dx$ $= \int \frac{1+t^2}{\left(1+2t^2\right)} \bullet \frac{dt}{1+t^2}$ $= \int \frac{dt}{1+2t^2}$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\ t\right)+C$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+C$ Hence $\int \frac{1}{1+\sin^2 x}dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+C$		
16.	(a) (b)	$\frac{dR}{dt} = e^{2t}+t$ $dR = (e^{2t}+t)dt$ $R = \int (e^{2t}+t)dt$ $R = \frac{1}{2}e^{2t}+\frac{1}{2}t^2+c$ Now when $t = 0$, $R = 3$ $\Rightarrow 3 = \frac{1}{2}e^{2(0)}+\frac{1}{2}(0)^2+c$ $3 = \frac{1}{2}(1)+c$ $c = 5/2$ $\Rightarrow R = \frac{1}{2}e^{2t}+\frac{1}{2}t^2+5/2$ $\frac{dV}{dt} = a$ $\Rightarrow \frac{dV}{dt} = 5+\cos(\frac{1}{2}t)$ $dV = 5+\cos(\frac{1}{2}t)dt$ $V = \int 5+\cos(\frac{1}{2}t)dt$ $= 5t+2\sin(\frac{1}{2}t)+c$ At time $t = 0$, $V = 1\ \text{ms}^{-1}$		

		$\Rightarrow$ $1 = 5(0) + 2\sin\frac{1}{2}(0) + c$ $c = 1$ $\Rightarrow$ $V = 5t + 2\sin(\frac{1}{2}t) + 1$ At $t = 2\pi$, $V = 10\pi + 2\sin(\pi) + 1$ $V = 10\pi + 1$ Hence its velocity is $10\pi + 1$ Also $\frac{ds}{dt} = V$ $\Rightarrow$ $\frac{ds}{dt} = [5t + 2\sin(\frac{1}{2}t) + 1]dt$ $S = \int\left(5t + 2\sin(\frac{1}{2}t) + 1\right)dt$ $S = \frac{5}{2}t^2 - 4\cos(\frac{t}{2}) + t$ $at \quad t = 2\pi,$ $S = \frac{5}{2}(2\pi)^2 - 4\cos(\pi) + 2\pi$ $= 10\pi^2 - 4(-1) + 2\pi$ $S = 10\pi^2 + 2\pi + 4$ Hence its distance covered is $10\pi^2 + 2\pi + 4$		