P425/2
APPLIED MATHEMATICS
Paper 2
Nov./Dec. 2023
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g, to be 9.8 ms^{-2}.

SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

1. A coin is biased such that when it is tossed the head is twice as likely to occur as the tail. Find the probability that in seven tosses, there will be exactly two tails. *(05 marks)*

2. Two bodies A and B of masses 6 kg and 2 kg moving along a straight line with velocities 4 ms^{-1} and 2 ms^{-1} respectively, collide head on. After collision, A moves with a velocity of 2.6 ms^{-1} in the same direction.

   Calculate the;

   (a) velocity of B after collision. *(02 marks)*

   (b) loss in kinetic energy. *(03 marks)*

3. The values of a function $f(x)$ are given in the table below.

x	0	½	1	1½	2	2½	3
$f(x)$	0.1003	0.0391	0.0801	0.0602	0.0649	0.0380	0.0327

   Use the trapezium rule to estimate the value of

$$\int_0^3 f(x)\,dx$$

   correct to **three** decimal places. *(05 marks)*

4. A ball of mass 1 kg rolls from rest down a rough plane inclined at 30° to a horizontal ground. The ball rolls for 4 m before it reaches the ground. The coefficient of friction between the ball and the plane is ¼. Find the velocity with which the ball reaches the ground. *(05 marks)*

5. The table below shows the age distribution of a population of a certain town in a census.

AGE (years)	NUMBER ('000)
Under 10	15
10 and under 20	19
20 and under 30	16
30 and under 40	18
40 and under 60	30
60 and under 80	6
80 and under 90	1

   (a) Draw a histogram for the data. *(03 marks)*

(b) Use the histogram to estimate the modal age of the population.

(02 marks)

6. The numbers $x = 6.45$, $y = 0.00215$ and $z = 2.7$ are each rounded off to the given number of decimal places.

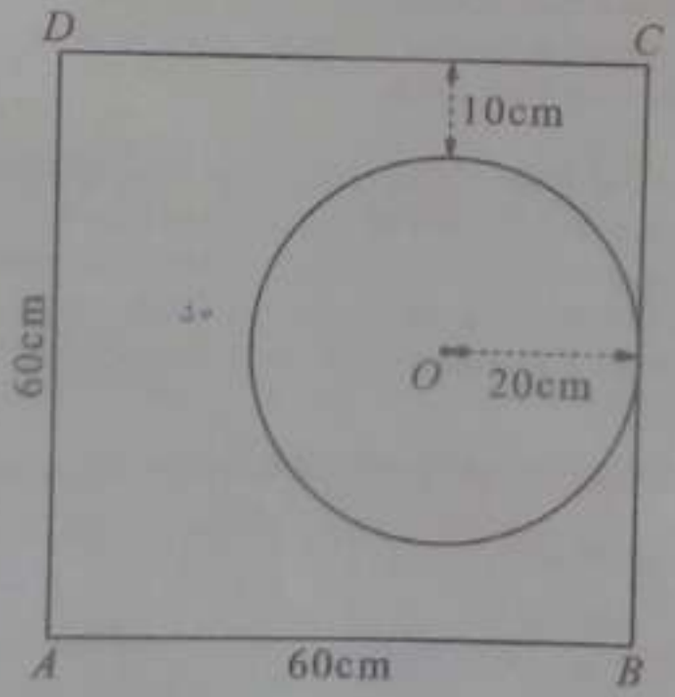
Determine the interval in which $w = \frac{x + z^3}{\sqrt{y}}$ lies. *(05 marks)*

7. Two independent events R and S are such that $P(R) = 3/4$ and $P(S) = P(S' \cap R')$. Find;

(a) $P(S)$. *(03 marks)*

(b) $P(S' \cap R)$. *(02 marks)*

8. A uniform lamina in form of a square with side 60 cm has a circular hole of radius 20 cm made in it as shown in the diagram below.



Find the position of the centre of gravity of the lamina from side AD.

(05 marks)

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. The table below shows the scores of 10 candidates in Biology and Economics.

CANDIDATE	A	B	C	D	E	F	G	H	I	J
BIOLOGY	45	63	56	61	75	83	73	50	77	70
ECONOMICS	90	64	76	70	55	53	62	85	53	62

(a) (i) Plot a scatter diagram for the data.

(ii) Draw a line of best fit on the scatter diagram.

(iii) Use your line of best fit to estimate the Biology mark for a candidate who scored 57 in Economics.

(06 marks)

(b) Calculate a rank correlation coefficient between the candidates' performance in the two subjects.
Comment on your result. *(06 marks)*

10. Two points A and B are 526 m apart along a straight road. A car moving along the road passes point A with a constant speed of 25 ms^{-1}. The car maintains this speed for 10 seconds and then decelerates uniformly for 8 seconds until it attains a speed of V ms^{-1}. The car maintains this speed until it passes point B. The total time taken by the car to move from point A to B is 30 seconds.

(a) Sketch a Velocity – Time graph for the motion of the car. *(04 marks)*

(b) Determine the;

(i) value of V. *(05 marks)*
(ii) deceleration of the car. *(03 marks)*

11. Given that $f(x) = xe^x + 5x - 10$;

(a) (i) Evaluate $f(1)$ and $f(2)$, correct to **four** decimal places.
(ii) Deduce that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$. *(04 marks)*

(b) Use linear interpolation **twice** to obtain the root of the equation

$$xe^x + 5x - 10 = 0,$$

correct to **three** decimal places. *(08 marks)*

12. A continuous random variable X has a cumulative distribution function given by

$$F(x)=\begin{cases}\frac{1+x}{6}, & -1\le x\le 0\\ \frac{1+2x}{6}, & 0\le x\le 2\\ \frac{4+3x}{12}, & 2\le x\le \frac{8}{3}\\ 1, & x\ge \frac{8}{3}\end{cases}$$

Find;

(a) the median. *(03 marks)*

(b) the probability density function $f(x)$. *(03 marks)*

(c) $P(1\le X\le 2.5)$. *(03 marks)*

(d) the mean of X. *(03 marks)*

13. Three forces $F_1=(2i-3j)$ N, $F_2=(5i+2j)$ N and $F_3=(-2i-11j)$ N act at points (2, 3), (−2, 3) and (3, −2) respectively.

Determine the;

(a) magnitude of their resultant force. *(03 marks)*

(b) equation of the line of action of the resultant force. *(05 marks)*

(c) distance from the origin at which the resultant cuts the x-axis. *(02 marks)*

(d) force that should be added to form a couple. *(02 marks)*

14. (a) Show that the formula based on Newton Raphson method for approximating the k^{th} root of a number N is given by

$$x_{n+1}=\frac{(k-1)x_n^{\,k}+N}{kx_n^{\,k-1}} \qquad n=0,1,2,3,\ldots$$

(04 marks)

(b) Construct a flow chart that,

(i) reads in the initial approximation x_0, k and N,

(ii) computes and prints N and its k^{th} root correct to **three** decimal places. *(05 marks)*

(c) Perform a dry run for your flow chart when $N=13$, $x_0=1.6$ and $k=4$. *(03 marks)*

15. (a) A woman travelling to work by a car goes through three police check points A, B and C. The probabilities that she is delayed at A, at B and at C are 0.3, 0.5 and 0.7 respectively.

Determine the probability that she is delayed at;

(i) only one check point. *(03 marks)*

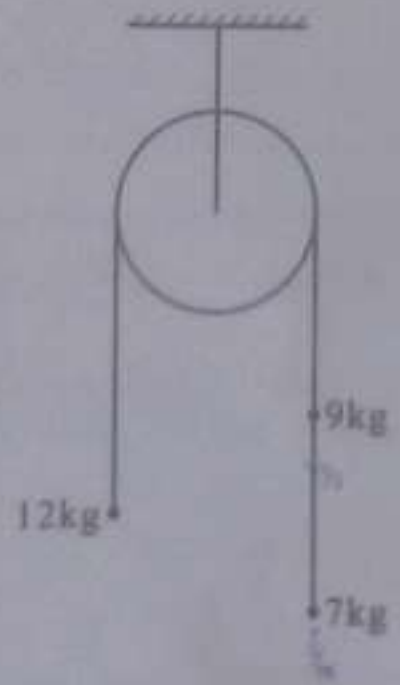
(ii) two or more check points. *(03 marks)*

(b) A man goes to work by route P or route Q. The probability that he takes route P is 0.6. The probability that he is late given that he goes through P is $^2/_3$ and through Q is $^1/_3$.

(i) Find the probability that he is late for work on a certain day. *(03 marks)*

(ii) Given that he is not late, determine the probability that he went through P. *(03 marks)*

16. The diagram below shows three masses of 12 kg, 9 kg and 7 kg connected by light inelastic strings. The string connecting the 12 kg and 9 kg masses passes over a smooth fixed pulley. The other string connects the 9 kg and 7 kg masses.



The system is released from rest and the 12 kg mass accelerates upwards.

(a) Calculate the;

(i) acceleration of the system.

(ii) tensions in the strings. *(10 marks)*

(b) Determine the velocity of the 12 kg mass after 1.5 seconds. *(02 marks)*

END

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100%

(a)

H	T
2x	x

$2x + x = 1$

$3x = 1$

$x = \frac{1}{3}$

$P(x=2) = {}^{7}C_{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{5} = 0.30727$

2. (a) $MAUA + M_B(-U_B) = MAV_A + M_BV_B$

$6 \times 4 + 2x - 2 = 6 \times 2.6 + 2V_B$

$2V_B = 4.4$

$V_B = 2.2\ ms^{-1}$

(b) Loss in kinetic energy $= \left(\frac{1}{2} \times 6 \times 4^2 + \frac{1}{2} \times 2 \times 2^2\right) - \left(\frac{1}{2} \times 6 \times 2.6^2 + \frac{1}{2} \times 2 \times 2.2^2\right)$

$= 52J - 25.12$

$= 26.88J$

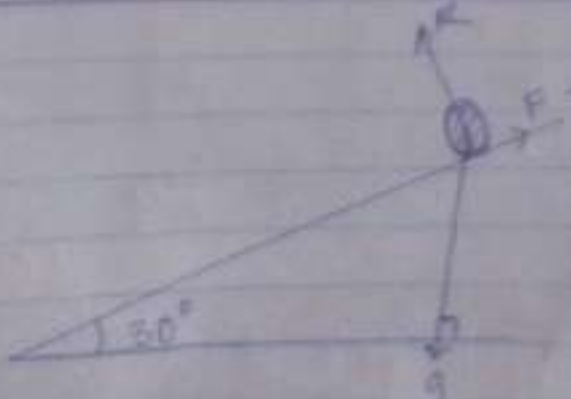
3.

x	y_0, y_6	$y_1 - y_5$
0	0.1003	
0.5		0.0791
1		0.0601
1.5		0.0402
2		0.0249
2.5		0.0080
3	0.0327	
Total	0.133	0.2823

$\therefore \int_0^3 f(x)\,dx = \frac{1}{2} \times \frac{1}{2}\left(0.133 + 2(0.2823)\right)$

$= 0.17764$

$= 0.174$



Resolving $\perp$ to the line of greatest slope

$R = g\frac{\sqrt{3}}{2} N = 8.48705 N$ B1

Parallel to the line of the greatest slope.

$\frac{g}{2} - \frac{1}{4}(R) = a$ B1

$a = 2.778238 \, ms^{-2}$ A1

$v^2 = 0^2 + 2 \times 2.778238 \times 4$ M1

$V = 4.7144355 \, ms^{-1}$. A1 (05)

5.

Age	f ('000)	c.w	f d
0-10	15	10	1.5
10-20	19	10	1.9
20-30	16	10	1.6
30-40	18	10	1.8
40-60	20	20	1.5
60-80	6	20	03
80-90	1	10	0.1

B1

6. $W_{max} = \frac{6.455 + 2.75^3}{\sqrt{0.002145}} = 588.41365$ M1 A1

$W_{min} = \frac{6.445 + 2.65^3}{\sqrt{0.002155}} = 539.71473$ M1 A1

The interval $= [539.71473, 588.41365]$ B1 (05)

7. (a) $P(S) = (1 - P(S))\,P(R')$

$P(S) = \frac{1}{4}(1 - P(S))$ M1

$4P(S) + P(S) = 1$ M1

$P(S) = \frac{1}{5}$. A1

(b) $P(S' \cap R) = P(S') \times P(R)$ M1

$= \frac{4}{5} \times \frac{3}{4}$ A1 (05)

$= \frac{3}{5}$.

Signature

Subject Name

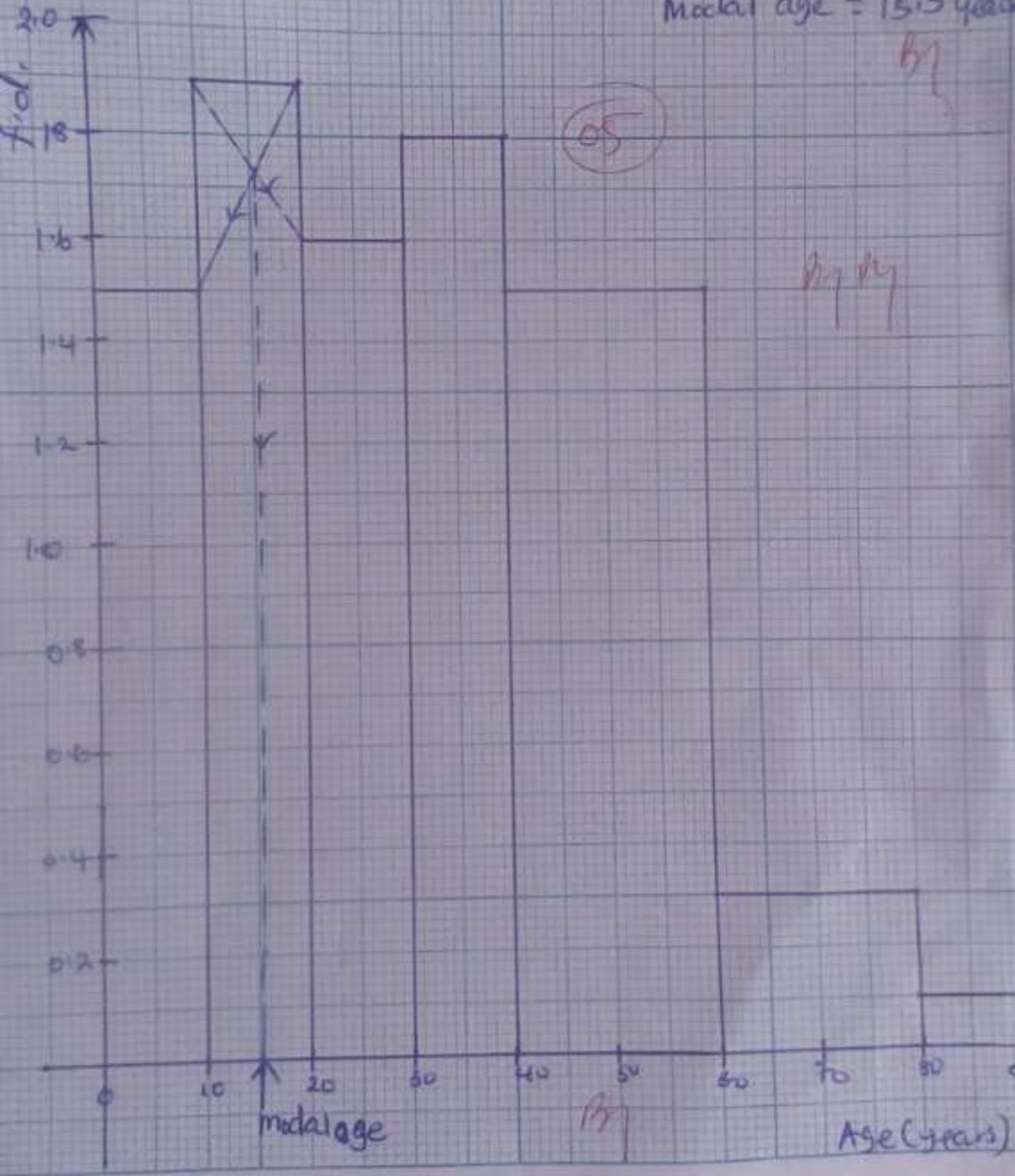
Paper code /

Random No

Personal Number

A Histogram:

Modal age = 15.5 years



Subject Name ..... Paper code ..... / Personal Number

(11) $\bar{x} = \frac{45+63+56+61+75+83+73+50+77+70}{10}$

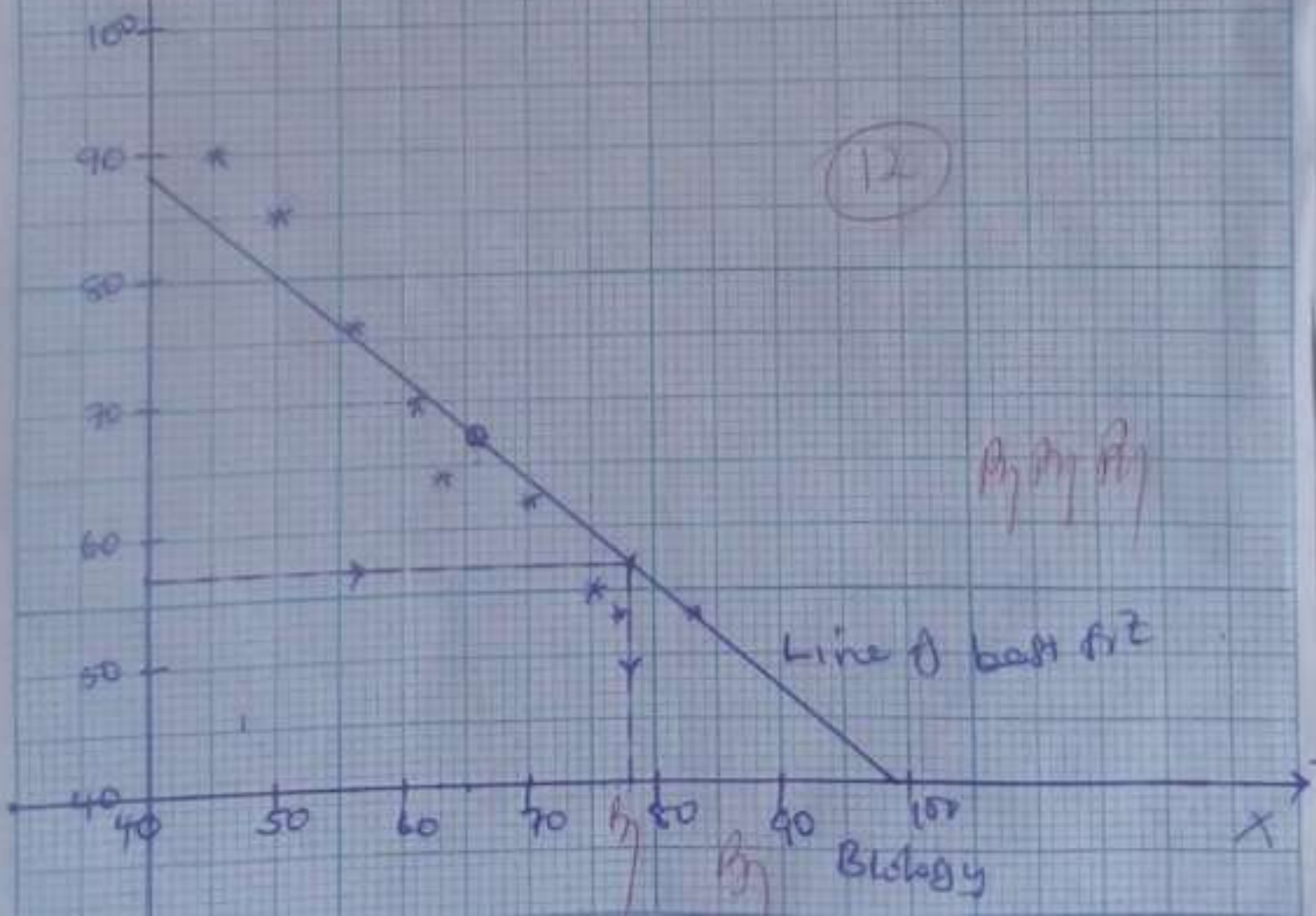
$= 65.3$

$\bar{y} = \frac{90+64+76+70+55+53+62+85+53+62}{10}$

$= 67$

$(\bar{x}, \bar{y}) = (65.3, 67)$

(a) Biology mark is 78 B1



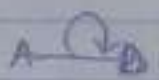
(12)

B1 B1 B1

B1 B1

8. Let W be the weight per unit area.

Part	Weight Per unit area	C.O.G from AD
ABCD	$3600W\,cm^2$	30 A1
Circle	$400\pi W$	40 A1
Remaining lamina	$(3600W - 400\pi W)\,cm^2$	$\bar{x}$ A1



$3600W\,cm^2 \times 30 - 400\pi W \times 40 = (3600W - 400\pi W)\bar{x}$ M1

$$\bar{x} = \frac{3600 \times 30 - 16000\pi}{3600 - 400\pi}$$

$\bar{x} = 24.6375$ cm from AD A1 (05)

9.

RB	RE	d	d^2
10	1	9	81
6	5	1	1
8	3	5	25
7	4	3	9
3	8	5	25
1	9.5	8.5	72.25
4	6.5	2.5	6.25
9	2	7	49
2	9.5	7.5	56.25
5	6.5	1.5	2.25
			$\Sigma d^2 =$
			327

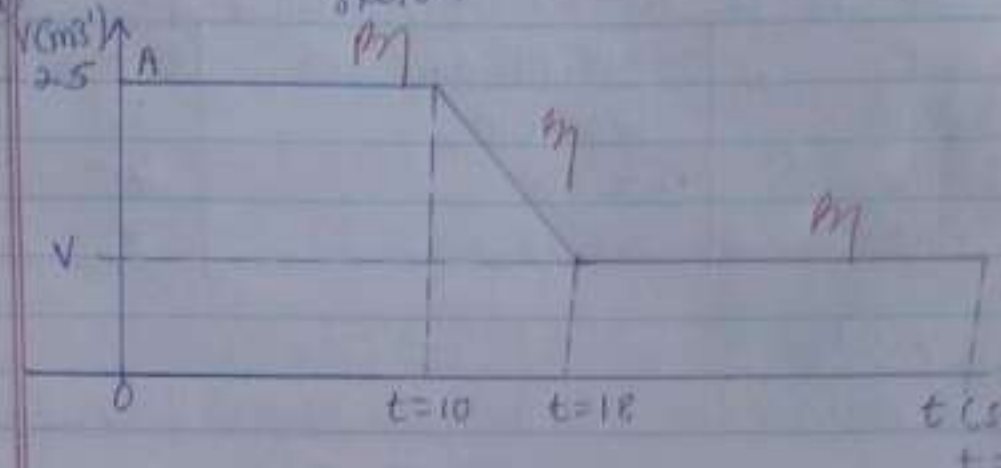
A1 A1 A1

$$\rho = 1 - \frac{6(327)}{10(10^2 - 1)}$$ M1

$= -0.98182$ A1

It is significant at 5% level. A1 (05)

Qn 10 (a) Sketch.



B1 B1 B1 B1

(b) (i) $10 \times 25 + \frac{1}{2}(25+V) \times 8 + V \times 12 = 526$ M1 M1

$250 + 4(25+V) + 12V = 526$

$100 + 4V + 12V = 276$ M1 M1

$16V = 176$

$V = 11 \text{ ms}^{-1}$ A1

(ii) $V = u + at$.

$11 = 25 + 8a$ M1

$8a = -14$

$a = -1.75 \text{ ms}^{-2}$ M1

The deceleration of the car is 1.75 ms^{-2} A1

12

Qn 11 (a) (i) $f(1) = e^1 + 5(1) - 10 = -2.28172 \approx -2.2817$ M1 B1

$f(2) = 2e^2 + 10 - 10 = 14.77811 \approx 14.7781$ M1 A1

Since $f(1) \times f(2) < 0$ then the root lies between 1 and 2. B1

(ii)

1	x_0	2
-2.2817	0	14.7781

B1

$\frac{x_0 - 1}{2.2817} = \frac{2-1}{14.7781}$ M1

$x_0 = 1.13375$ A1

$f(1.13375) = 1.13375\, e^{1.13375} + 5(1.13375) - 10 = -0.080836$ B1

1.13375	x_0	2
-0.080836	0	14.7781

M1

$\frac{x_0 - 1.13375}{0.080836} = \frac{2 - 1.13375}{14.77814 \; 0.080836}$ M1

$x_0 = 1.178676$

$x_0 \approx 1.179$ A1

12

12 (a) $\int_0^m \frac{1+2x}{6}\,dx = \frac{1}{2}$ M1

$\left[\frac{1+2x}{6}\right]_0^m = \frac{1}{2}$

$\frac{1+2m}{6} = \frac{1}{2}$ M1

$\frac{m}{3} + \frac{m}{6} = \frac{1}{2}$

$2m = 3-1$ B1

$\frac{m}{2} = \frac{1}{6}$

$m = 1$

median; $m = \frac{1}{2}$.

(b) for $-1 \le x \le 0$, $f(x) = \frac{1}{6}$.

for $0 \le x \le 2$; $f(x) = \frac{1}{3}$

for $2 \le x \le 8/3$; $f(x) = \frac{1}{4}$

for $x > 8/3$; $f(x) = 0$.

$$\therefore f(x) = \begin{cases} \frac{1}{6}; & -1 \le x \le 0 \\ \frac{1}{3}; & 0 \le x \le 2 \\ \frac{1}{4}; & 2 \le x \le 8/3 \\ 0; & \text{otherwise} \end{cases}$$ B1 B1 B1

(c) $F(2.5) - F(1)$

$= \frac{4+3(2.5)}{12} - \frac{1+2\times1}{6}$ M1

$= \frac{23}{24} - \frac{1}{2}$ M1

$= \frac{11}{24}$ A1

(d) Mean of $X = \int_{-1}^{0} \frac{x}{6}\,dx + \int_0^2 \frac{x}{3}\,dx + \int_2^{8/3} \frac{x}{4}\,dx$

$= \frac{x^2}{12}\Big|_{-1}^{0} + \frac{x^2}{6}\Big|_0^2 + \frac{x^2}{8}\Big|_2^{8/3}$ M1

$= -\frac{1}{12} + \frac{4}{6} + \frac{8}{9} - \frac{1}{2}$. M1

$= \frac{35}{36}$ A1 (09)

12

13(a) Resultant force $= \underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3}$

$= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -11 \end{pmatrix}$ B1

$= \begin{pmatrix} 5 \\ -12 \end{pmatrix} N$

Magnitude of the Resultant force $= \sqrt{(5)^2 + (-12)^2}$ M1

$= 13N.$ A1

(b) Taking moment about $(0,0)$ in clockwise direction

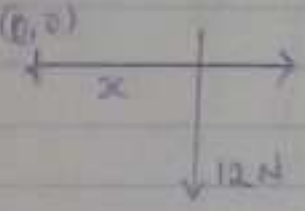
$G = \begin{vmatrix} 2 & -3 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 5 & 2 \\ -2 & 3 \end{vmatrix} + \begin{vmatrix} -2 & -11 \\ 3 & -2 \end{vmatrix}$ M1

$= 12 + 19 + 37$

$= 68$ clockwise A1

But $\Sigma x = 5 \quad \Sigma y = -12.$

$m = \tan\theta = -\frac{12}{5}$



$68 = 12x$ M1

$x = \frac{17}{3}.$ A1

The line cuts the x-axis at $(\frac{17}{3}, 0)$ B1

$y = -\frac{12}{5}x + c$

$0 = -\frac{12}{5} \times \frac{17}{3} + c$ B1

$c = \frac{68}{5}$

$\therefore y = -\frac{12}{5}x + \frac{68}{5}$ B1

$\Rightarrow 5y + 12x - 68 = 0.$

(c) distance from origin $= \frac{17}{3}$ units.

(d) The force to be added $F = 13N$ in the direction N 22.6° W at the origin M1 A1

14. (a) $N^{\frac{1}{K}} = x$

$N = x^K$ M1

$f(x) = x^K - N$

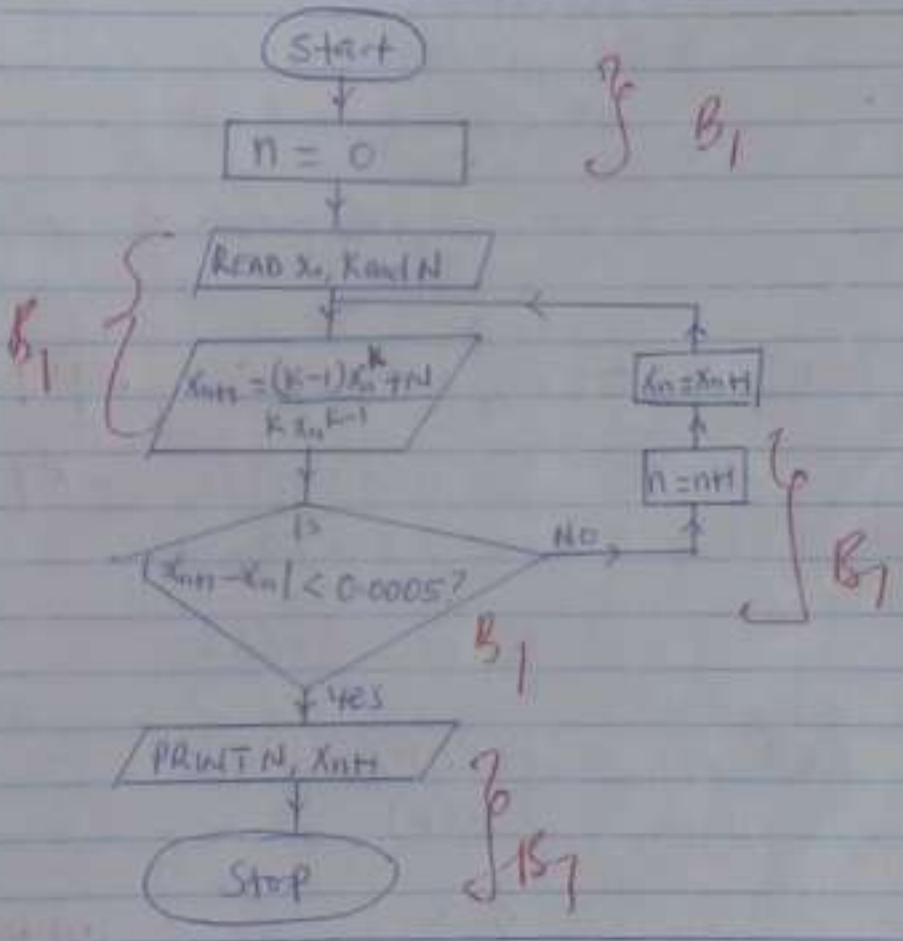
$f'(x) = K x^{K-1}$ B1

$x_{n+1} = x_n - \frac{x_n^K - N}{K x_n^{K-1}}$

$= \frac{K x_n^K - x_n^K + N}{K x_n^{K-1}}$ M1

$= \frac{x_n^K (K-1) + N}{K x_n^{K-1}}$ $= \frac{-(K-1)x_n^K + N}{K x_n^{K-1}}$ for $n = 0, 1, 2$ A1

(b)



B1 B1 B1 B1 B1

$K = 4$

(c)

N	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
13	1·6	1·9935	0·3935
13	1·9935	1·9054	0·0881
13	1·9054	1·8989	0·0065
13	1·8989	1·8988	0·0001

B1 B1

The 4th root of 13 is 1·8988

≈ 1·900. B1

12

(i) = 0·3 x 0·5 x 0·3 + 0·7 x 0·5 x 0·3 + 0·7 x 0·5 x 0·7 any one #1

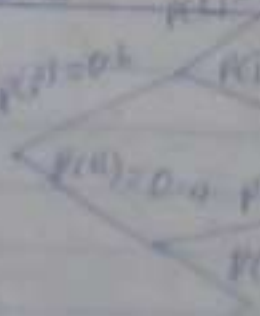
= 0·295

(ii) = 0·3 x 0·5 x 0·3 + 0·3 x 0·5 x 0·7 + 0·7 x 0·5 x 0·7 + any two #1

0·7 x 0·3 x 0·5

= 0·5 #1

(b)



(i) P(L) = 0·6 x 2/3 + 0·4 x 1/3 #1 #1

= 8/15 or 0·53333 #1

(ii) P(L') = 0·6 x 1/3 + 0·4 x 2/3

= 7/15 or 0·46667

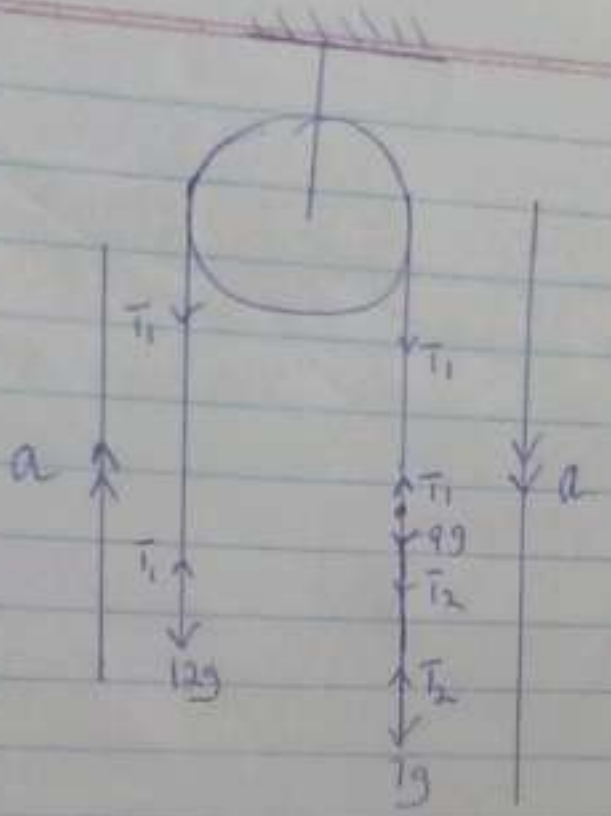
$P(\bar{L}/P) = \frac{P(\bar{L} \cap P)}{P(P)}$

$= \frac{0·2}{0·46667}$ #1 #1

= 3/7 or 0·42857 #1

12

(a)



B7

$T_1 - 12g = 12a$ — ① A1

$T_2 + 9g - T_1 = 9a$ — ② A1

$7g - T_2 = 7a$ — ③ A1

Adding ① + ② + ③

$16g - 12g = 28a$ M1

$39.2 = 28a$ M1

$a = 1.4\ ms^{-2}$ A1

$T_1 - 12g = 12 \times 1.4$ M1

$T_1 = 134.4\ N$ A1

from ②

$T_2 = 7g - 7(1.4)$ M1

$T_2 = 58.8\ N$ A1

Therefore the tensions on the strings are $T_1 = 134.4\ N$ and $T_2 = 58.8\ N$.

(b) $V = u + at$ M1

$V = 0 + 1.4 \times 1.5$ A1

$= 2.1\ ms^{-1}$ A1

12