P425/2
APPLIED MATHEMATICS
Paper 2
July /Aug. 2024
3 hours

UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in* ***section*** *A and any* **five** *from* ***section B.***

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section ***B*** *will* **not** *be marked.*

Turn Over

SECTION A (40 MARKS)

1. Given that $P(A) = 0.7$, $P(A' \cup B) = 0.6$, find
   (a) $P(A \cap B')$
   (b) $P(B/A)$ *(05 marks)*

2. A particle executing simple harmonic motion starts from rest and next comes to rest after 6 seconds, covering 10m in this time;
   Calculate the;
   (a) period of motion. *(02 marks)*
   (b) maximum speed of the particle. *(03 marks)*

3. Calculate the maximum error in the function $x\sin x$ at $x = 30^o \pm 0.5^o$ *(05 marks)*

4. Study the table below;

x	5	6	7	8	9	10
f	1	5	10	8	4	2

Using an assumed mean, m = 7, calculate the mean and variance of x. *(05 marks)*

5. The position vectors of two bodies A and B after t seconds of motion are; $r_A = \begin{pmatrix}8\\0\end{pmatrix} + t\begin{pmatrix}1\\8\end{pmatrix}$ and $r_B = \begin{pmatrix}0\\6\end{pmatrix} + t\begin{pmatrix}5\\5\end{pmatrix}$ metres. Calculate the shortest distance between the bodies, and state the value of t at which this occurs. *(05 marks)*

6. A box contains 3 red and 2 white balls of the same size. The balls are thoroughly mixed. Balls are picked one at a time with replacement. Ten balls are picked, find the probability that;
   (a) exactly 5,
   (b) over 8 balls are red. *(05 marks)*

7. The points in the table below were taken from the curve $y = x^2$

x	0.1	0.2
y	0.01	0.04

Calculate the absolute errors made in using linear interpolation or extrapolation to estimate;
   (a) y when $x = 0.18$
   (b) x when $y = 0.05$ *(05 marks)*

8. A particle is projected vertically upwards with a speed of $49ms^{-1}$. Calculate the time interval within which the particle is at least 78.4 metres above the ground. *(05 marks)*

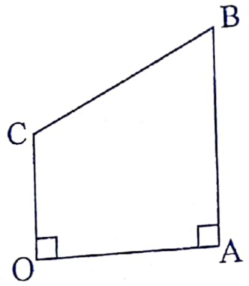
SECTION B (60 MARKS)

9. (a) Show that the equation $e^x = 2 - x$ has a root lying between 0 and 1. Hence use a graphical method to find the first approximate, x_o, to the equation. *(06 marks)*

   (b) Use the Newton – Raphson method to find the root correct to 4 decimal places. *(06 marks)*

10. (a) ABCD is a uniform rectangular tray of mass 1kg, at rest on a horizontal table. Masses of 2, 4, 5 and 8kg are placed at the corners A, B, C and D respectively. Find the position of the centre of gravity of the system, with respect to the sides AB and AD. *(04 marks)*

    (b)



OABC is a uniform lamina, in which $\overline{OA} = 6cm$, $\overline{AB} = 7cm$ and $\overline{OC} = 4cm$.

(i) Find the coordinates of the centre of gravity of the lamina, taking $\overline{OA}$ as the x – axis, and $\overline{OC}$ as the y – axis. *(04 marks)*

(ii) The lamina is suspended from point C, calculate the angle side OA makes with the vertical. *(04 marks)*

11. The table below shows the distribution of marks obtained by students in a paper 2 examination.

Marks (%)	0 - 10	10 - 20	20 – 40	40 – 45	45 – 60	60 - 100
Frequency density	0.8	1.0	1.5	4.4	2.8	0.4

(a) Construct a histogram for this data, and use it to calculate the modal mark. *(05 marks)*

(b) Plot a cumulative frequency curve, and use it to determine the;

(i) Number of student who scored at least a 50%. *(07 marks)*

(ii) Decile deviation.

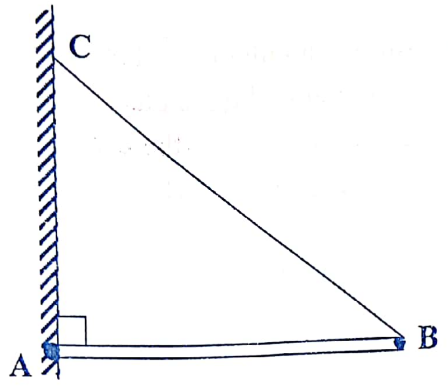
12. The heights of maize plants in a nursery bed are normally distributed with a mean of 16cm with a variance of $100cm^2$.

(a) Calculate the probability of obtaining a maize plant whose height is greater than 20cm. *(04 marks)*

(b) Twenty five plants are picked at random, find the probability that their mean height lies between 13cm and 19cm. *(04 marks)*

(c) How many plants should be picked at random before the probability that at least one of them has a height less than 16cm is greater than 0.9? *(04 marks)*

13.



AB is a uniform rod of mass 12kg, and of length 8m. The rod is smoothly hinged, to a vertical wall, at point A. A light inelastic string BC, of length 10m, keeps the rod at rest, in a horizontal position, as shown in the diagram above.

(a) Calculate the;

(i) Tension, **T** in the string. *(03 marks)*

(ii) Reaction at the hinge A. *(05 marks)*

(b) A boy of mass 40kg starts to walk along the rod from point A towards B. Given that the tension in the string cannot be exceed $\frac{3}{2}T$, calculate the distance the boy will walk before the string snaps. *(04 marks)*

14. (a) Use the trapezium rule with 6 intervals to evaluate; *(05 marks)*

$$\int_0^2 x\sin x \, dx, \qquad \textit{correct to 4 decimal places.}$$

(b) Given that $x = 1.60 \pm 0.005$ and $y = 4.8 \pm 0.05$; calculate the maximum error in computing y/x ; hence or otherwise state interval within which the exact value of y/x lies. *(07 marks)*

15. (a) A car of mass 1000 kg, travels a long a horizontal road against a constant resistance of 200 N. If the car develops a constant power of 30 kW, calculate the acceleration of the car at the instant when its speed is 30 ms^{-1}. *(05 marks)*

(b) A steamer is initially 60km north of an observatory, O and travelling at 40 kmh^{-1} due N60°E. At the same instant, a boat capable of a maximum speed of 64 kmh^{-1} and initially 80 km East of O, sets off to intercept the steamer.

Calculate the two possible courses the boat can take, and determine the shortest time of interception. *(07 marks)*

16. X is a continuous random variable whose probability density function is given as;

$$f(x) = \begin{cases} 2kx & ; \quad 0 \le x \le 2 \\ k(6-x) & ; \quad 4 \le x \le 6 \\ 0 & ; \quad elsewhere \end{cases}$$

(a) Sketch the graph of $f(x)$ and use it to determine the value of the constant k. *(04 marks)*

(b) Find, $F(x)$, the cumulative distribution function of X, hence compute;

(i) $P(X > 5)$ *(08 marks)*

(ii) the 80^{th} percentile of X.

END

UTEC – P425/2 MATHS PAPER 2

N	MARKING GUIDE	COMMENTS
1	(a) $P(A' \cup B) = P(A \cap B')' = 0.6$ M1 $\Rightarrow P(A \cap B') = 0.4$ A1 (b) $P(A) = P(A \cap B) + P(A \cap B')$ $\Rightarrow P(A \cap B) = 0.7 - 0.4$ M1 $= 0.3$ $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.3}{0.7}$ M1 $= 3/7$ A1	Accept any other method used correctly
2	(a) Period, $T = 2 \times 6$ M1 $= 12$ seconds. A1 (b) $V_{max} = \omega r$; $r = 5m$ A1, $\omega = \frac{2\pi}{T}$ $= \frac{2\pi}{12}$ B1 $= \frac{\pi}{6} \times 5$ M1 $= \frac{5\pi}{6} ms^{-1}$ or	$2.6179 ms^{-1}$

QN

3 Let $y = x\sin x$; $x = 30^\circ = \pi/6$; Δx

Alternative velocity

$$\Rightarrow \Delta y = (\sin x)\Delta x + (x\cos x)\Delta x \quad M_1$$

y_{max}

y_{min}

$$|\Delta y| = |\sin x||\Delta x| + |x||\cos x||\Delta x|$$

$$= \left\{|\sin 30^\circ| + \left|\frac{\pi}{6}\cos 30^\circ\right|\right\} \times \frac{\pi}{360} \quad M_1 B_1$$

$$\simeq 0.5079 \text{ (4 dps)} \quad A_1$$

4f

$d = x - 7$

x	d	f	fd	fd^2
5	−2	1	−2	4
6	−1	5	−5	~~25~~ 5
7	0	10	0	0
8	1	8	8	8
9	2	4	8	16
10	3	2	6	18 M_1
		$\Sigma f = 30$	$\Sigma fd = 15$	$\Sigma fd^2 = 51$

$$\text{Mean} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 7 + 15/30 \quad M_1$$

$$= 7.5 \quad A_1$$

$$Var(x) = \frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2$$

$$= \frac{51}{30} - \frac{225}{900} \quad M_1$$

$$= \cancel{2.1167} \; 1.45 \quad A_1$$

5 $\underset{\sim}{r}_A = \begin{pmatrix} 8+t \\ 8t \end{pmatrix}$; $\underset{\sim}{r}_B = \begin{pmatrix} 5t \\ 6+5t \end{pmatrix}$; $\underset{B\sim A}{r} = \underset{\sim}{r}_B - \underset{\sim}{r}_A$

$$= \begin{pmatrix} 4t-8 \\ 6-3t \end{pmatrix} \quad M_1$$

$$y = |\,{}_{B}\underset{\sim}{r}_{A}| = \sqrt{(4t-8)^2 + (6-3t)^2} \quad M_1$$

$$= 5(t-2) \quad B_1 \Rightarrow y_{min} = 0 \quad M_1 \text{ when } t = 2. \quad B_1$$

-2-

Alternative Method

velocity, $\underset{\sim}{v}_A = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$, $\underset{\sim}{v}_B = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

$\Rightarrow {}_B\underset{\sim}{v}_A = \underset{\sim}{v}_B - \underset{\sim}{v}_A$

$= \begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -3 \end{pmatrix} ms^{-1}$ (A1)

For shortest distance,

${}_B\underset{\sim}{v}_A \cdot {}_B\underset{\sim}{r}_A = 0$

$\Rightarrow \begin{pmatrix} 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 4t-8 \\ 6-3t \end{pmatrix} = 0$ (M1)

$16t - 32 - 18 + 9t = 0$

$25t = 50 \Rightarrow t = 2$ (A1)

Shortest dist. $= |{}_B\underset{\sim}{r}_A|$ (M1)

$= 0$ m. (i.e, planes collide) (A1)

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6 Let $X \sim$ no of red balls picked

$\sim B(n, p)$; $n = 10$, $p = 0.6$ (B1), $q = 0.4$

(a) $P(X=5) = {}^{10}C_5 (0.6)^5 (0.4)^5$ (M1)

$\simeq 0.2007$ (A1)

(b) $P(X=9) + P(X=10) = {}^{10}C_9 (0.6)^9 (0.4)^1 + {}^{10}C_{10} (0.6)^{10}$ (M1)

$\simeq 0.0403 + 0.0060$ (A1)

$\simeq 0.0463$

Note: Symmetry property can be used (then use tables)

7 (a)

x	0.1	0.18	0.2
y	0.01	y	0.04

$$\Rightarrow \frac{y-0.01}{0.04-0.01} = \frac{0.18-0.1}{0.2-0.1} \quad (M_1)$$

$$y = 0.01 + \frac{0.03 \times 0.08}{0.1}$$

$$\simeq 0.034 \quad (A_1)$$

but $y = 0.18^2$
$= 0.0324$

Absolute error $= |0.034 - 0.0324|$
$\simeq 0.0016$ (B_1)

(b)

x	0.1	0.2	x
y	0.01	0.04	0.05

$$\frac{x-0.2}{0.2-0.1} = \frac{0.05-0.04}{0.04-0.01}$$

$$x = 0.2 + \frac{0.1 \times 0.01}{0.03} \quad (M_1)$$

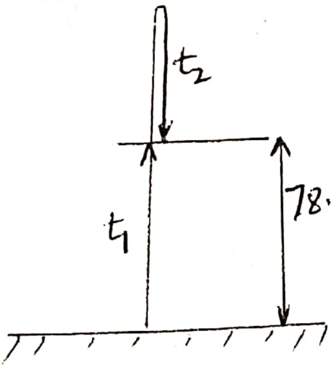
$$\simeq 0.2333 \quad (A_1)$$

$x^2 = 0.05$

$x \simeq 0.2236$

Absolute error $= |0.2333 - 0.2236|$
$\simeq 0.0097$ (A_1)

8



using $s = ut - \frac{1}{2}gt^2$

$78.4 = 49t - 4.9t^2$ (M_1)

$\Leftrightarrow t^2 - 10t + 16 = 0$

$(t-2)(t-8) = 0$ (M_1)

$\Rightarrow t_1 = 2$ and $t_2 = 8$ (A_1)

The required time $= t_2 - t_1$
$= 6$ seconds (M_1, A_1)

(2, 8) secon

~~Section B~~

(a) $f(x) = e^{x} + x - 2$

$f(0) = -1$; $f(1) = e - 2$
$= +0.72$

(M1, (B1. $y = e^{x}$

Since $f(0) \times f(1) < 0 \Rightarrow 0 < \text{root} < 1$ $y = x - 2$

x	y
0	-1
0.2	-0.8
0.4	-0.1
0.6	0.4
0.8	1.0 (B1
1	~~0.72~~ 1.72

6 2

From the graph, $x_0 \simeq 0.45$ (A1 ± 0.01

(b) $f(x) = e^{x} + x - 2$ | $x_{n+1} = x_n - \dfrac{(e^{x_n} + x_n - 2)}{e^{x_n} + 1}$

$f'(x) = e^{x} + 1$ (M1) | (M1

$$x_{n+1} = \frac{(x_n - 1)e^{x_n} + 2}{e^{x_n} + 1}; \; n = 0, 1, 2$$

$$x_0 = 0.45, \; x_1 = \frac{(0.45 - 1)e^{0.45} + 2}{e^{0.45} + 1}$$

$\simeq 0.44287$ (B1 ; $|x_1 - x_0| = 0.00713$
> 0.00005

$$x_2 = \frac{(0.44287 - 1)e^{0.44287} + 2}{e^{0.44287} + 1}$$

$\simeq 0.44285$ (B1 ; $|x_2 - x_1| = 0.00002$ (B1
< 0.00005

0.4429 (4dps)

Hence the root $\simeq$ 0.4429 (4 dps) (A1 -5-

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(To be fastened together with other answers to paper)

UACE

Candidate's Name ....

Signature ....

Subject Name .... Graph of $y = e^x + x - 2$ .... Paper code ....

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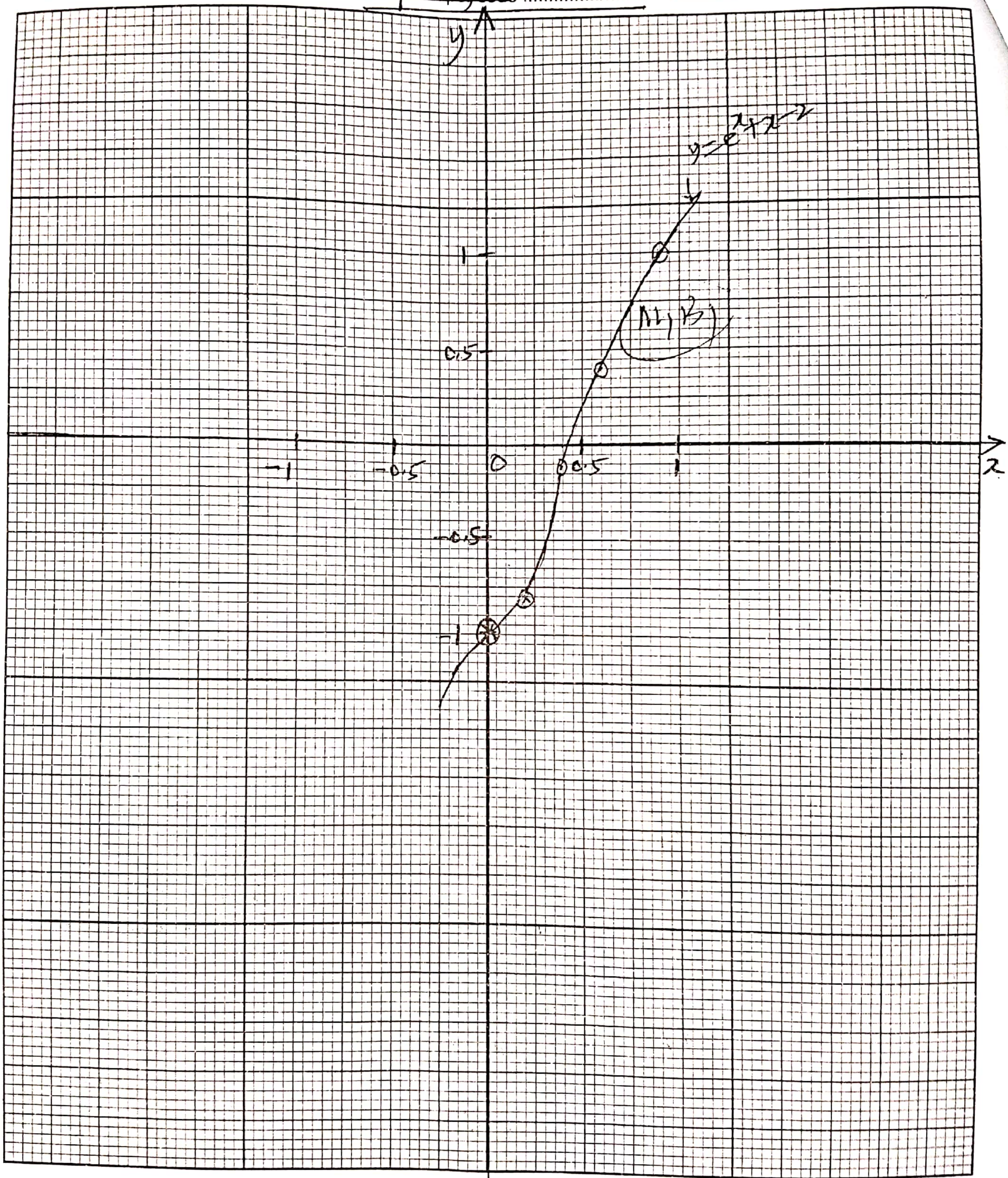
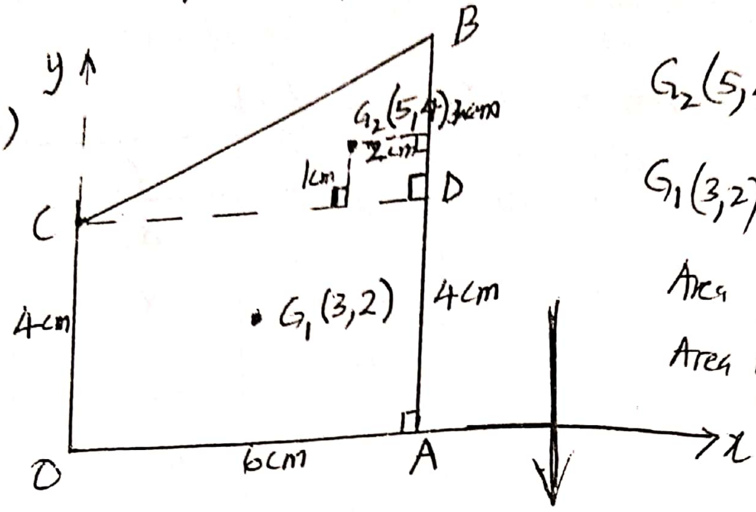


a) Marks shifted to part (b). Lengths AB, BC were not given.

(b)(i)



$G_2(5,4) \rightarrow$ C.O.G of ΔCBD

$G_1(3,2) \rightarrow$ C.O.G. of OADC.

Area of rectangle $= 24\,cm^2$

Area of triangle $= \frac{1}{2} \times 6 \times 3$

$= 9\,cm^2$

$\rho \equiv$ weight/area

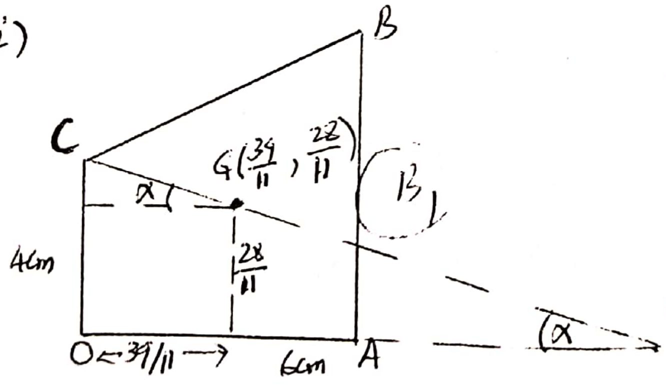
Portion	Weight	C.O.G
OADC	24ρ (B1)	$(3,2)$ (A1)
BCD	9ρ (B1)	$(5,4)$ (A1)
Whole Body	33ρ (B1)	$(\bar{x}, \bar{y})$ (A1)

$$\Rightarrow 24\rho\begin{pmatrix}3\\2\end{pmatrix} + 9\rho\begin{pmatrix}5\\4\end{pmatrix} = 33\rho\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$ (M1, A1) gives $\bar{x} = \frac{39}{11}$ cm

$\bar{y} = \frac{28}{11}$ cm

i.e, C.O.G $= \left(\frac{39}{11}, \frac{28}{11}\right)$ (A1)

(ii)



$\tan\alpha = \dfrac{4 - 28/11}{39/11}$ M1

$\tan\alpha = \dfrac{16}{39}$ (B1)

$\Rightarrow \alpha \simeq 23.31^\circ$ A1

(a) From the histogram, the mode $\simeq 43 \pm 0.5$ (A1)

(b)

Cumulative frequency table

Marks	freq. density	i	f	c.f.
0 – 10	0.8	10	8	8
10 – 20	1.0	10	10	18
20 – 40	1.5	20	30	48
40 – 45	4.4	5	22	70
45 – 60	2.8	15 (B1)	42 (B1)	112
60 – 100	0.4	40	16	128 (B1)
			$\Sigma f = 128$	

From the graph: (i) no that scored at least 50%

is $128 - 88 = 40 \pm 4$ (A1)

(ii) 1st decile, $D_1 = 17 \pm 0.5$ (A1)

9th decile, $D_9 = 56 \pm 0.5$ (A1)

Decile deviation $= D_9 - D_1$

$\simeq 56 - 17$ (M1)

$\simeq 39 \pm 1.0$ (A1)

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(To be fastened together with other answers to paper)

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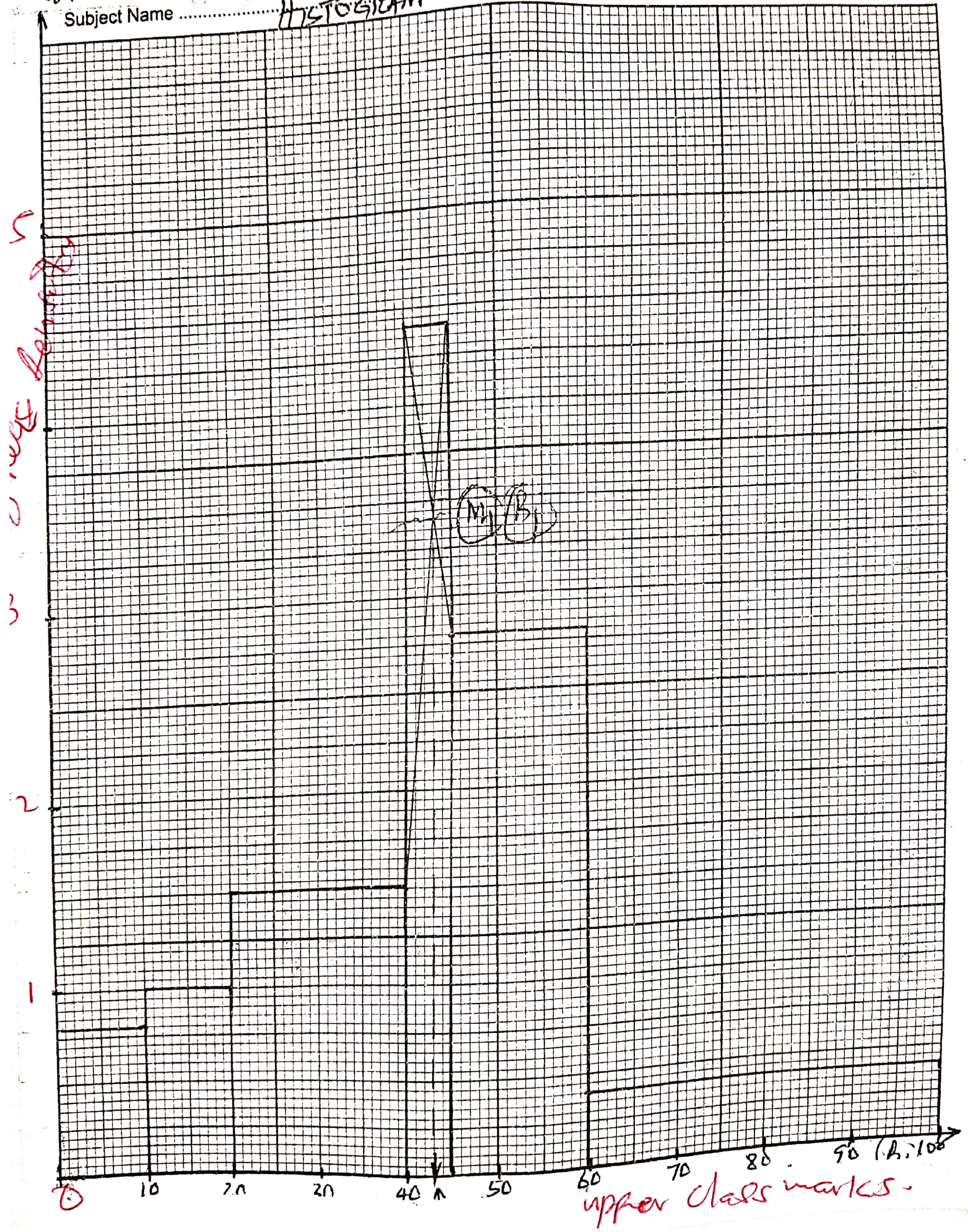
Candidate's Name .... No 11(a)

Signature .... Paper code ..../......

Subject Name .... HISTOGRAM



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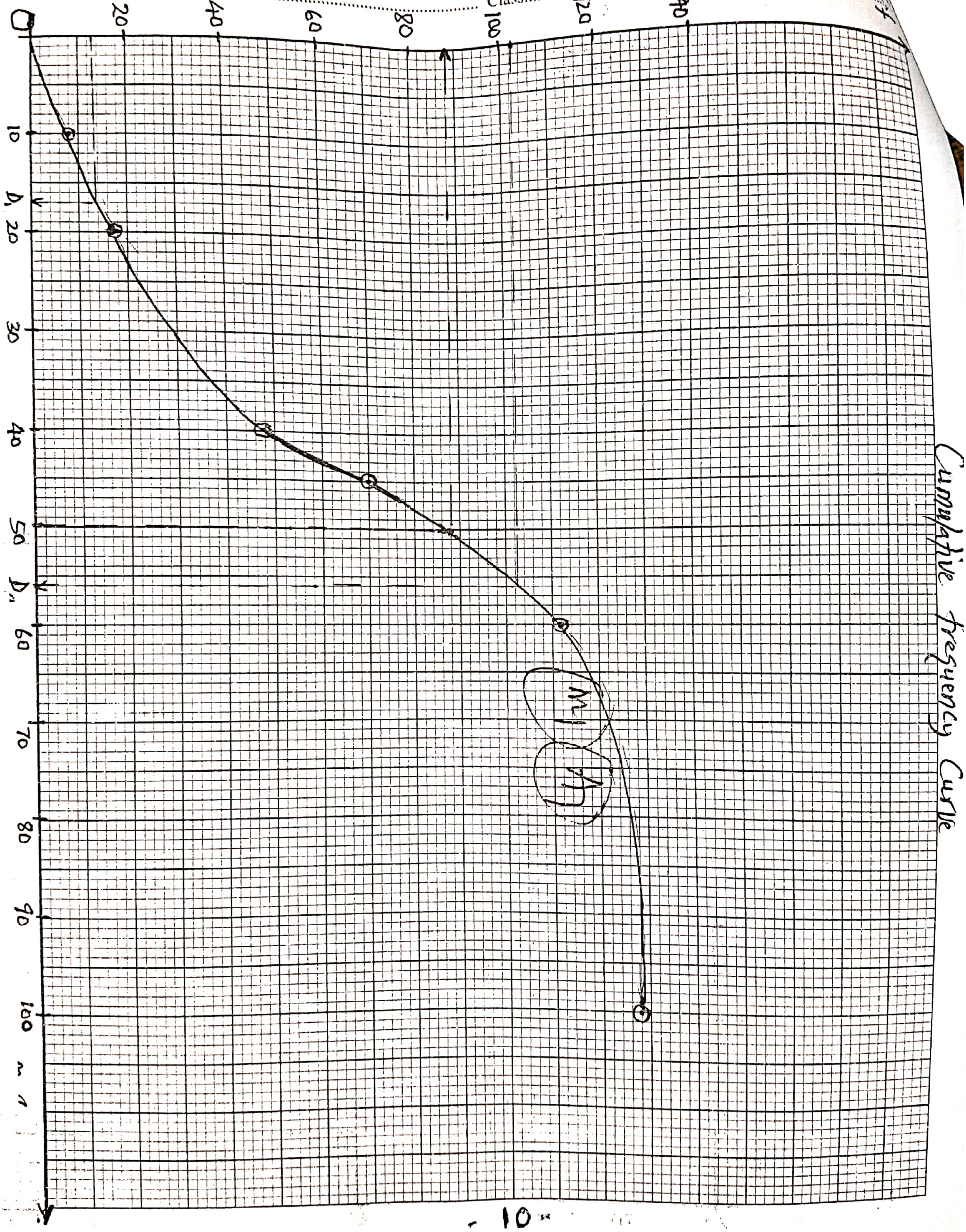
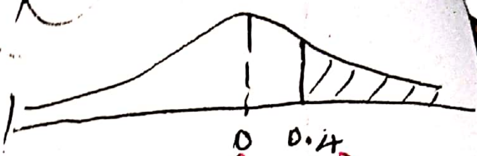


12) Let X the height of a maize plant.

$\Rightarrow X \sim N(\mu, \sigma^2)$; $\mu = 16$, $\sigma^2 = 100 \Rightarrow \sigma = 10$ (B1)

(a) $P(X > 20) \Leftrightarrow P\left(Z > \frac{20-16}{10}\right)$ (M1)

$= P(Z > 0.4)$ (B1)



$= 0.5 - \Phi(0.4)$ 0.1554 (cal)

$= 0.3446$ (A1)

(b) $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$; $\mu = 16$, standard dev. $= \frac{\sigma}{\sqrt{n}}$ (M1)

$= \frac{10}{5} = 2.$

$P(13 < \bar{X} < 19) = P\left(\frac{13-16}{2} < Z < \frac{19-16}{2}\right)$ (M1)

$= P(-1.5 < Z < 1.5)$

$= 2\Phi(1.5)$ (M1) (0.4330) (cal)

$= 0.8664$ (cal). Ans. (A1)

(c) $P(X \geq 16) = P\left(Z > \frac{16-16}{2}\right)$

$= P(Z > 0) = 0.5$ (B1)

Let n be the no of plants picked

$P(\text{at least 1 has a height} < 16) = 1 - 0.5^n$ (M1)

P.T.O

(c) Continued.

$\Rightarrow 1 - 0.5^n > 0.9$

$\Rightarrow 1 - 0.9 > 0.5^n$

$\Rightarrow 0.5^n < 0.1$

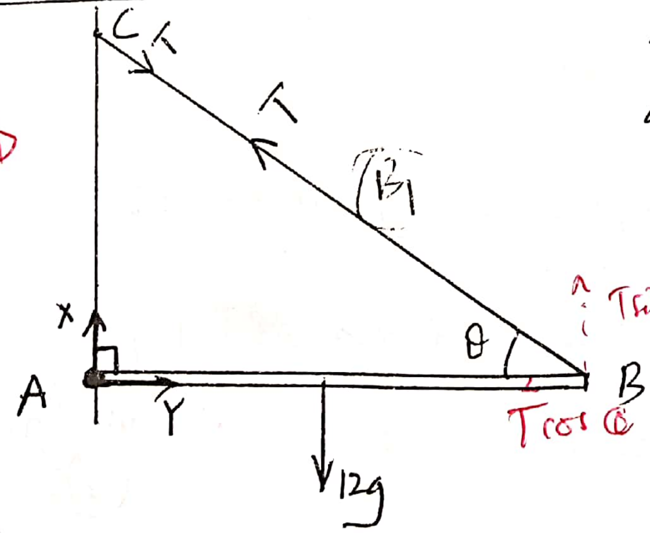
$\Rightarrow n \log_{10} 0.5 < \log_{10} 0.1$ (M1)

$n > \dfrac{\log_{10} 0.1}{\log_{10} 0.5}$

$n > \dfrac{\log 10}{\log 2}$ (B1)

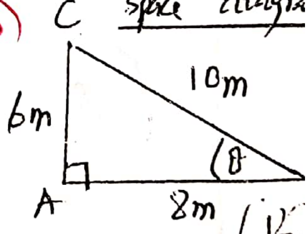
$\Rightarrow n > 3.32$, the least no, $n = 4$ (A1)

13 (a) F.D



X, Y are components of the rxn at A.

(S.D)



(B1)

$\cos\theta = 4/5$

$\sin\theta = 3/5$

(i) $\circlearrowleft$ A: $T\sin\theta \times 8 = 12g \times 4 \Rightarrow \frac{24}{5}T = 48g$ (M1)

$\Rightarrow$ Tension, $T = 10g$ N

$= 98$ N. (A1)

3 Cont'd.

(a)(ii) $(\rightarrow)$: $Y = T\cos\theta$

$= 98 \times 4/5$

$= 8g$ (B1)

$(\uparrow)$ $X + T\sin\theta = 12g$

$X = 12g - 98 \times 3/5$

$= 6g.$ (B1)

Magnitude of reaction

$R = \sqrt{X^2 + Y^2}$

$= 10g$

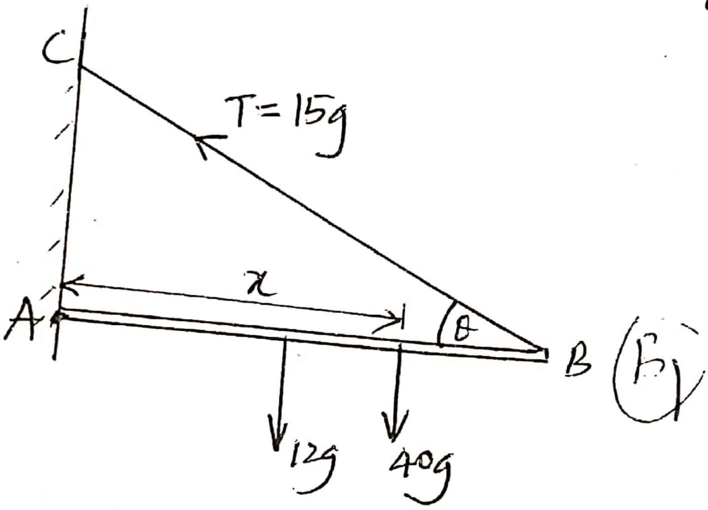
$= 98\ N$ (M1)

Direction of R

$\tan\alpha = \frac{Y}{X} = \frac{8g}{6g} = 4/3$ (A1)

$\alpha = 53.13^\circ$ with the horizontal

(b)



(B1)

$\frac{3}{2}T = \frac{3}{2} \times 10g$

$T = 15g\ N$

$\curvearrowleft A$: $40gx + 12g \times 4 = 15g\sin\theta \times 8$ (M1)

$40x + 48 = 15 \times 3/5 \times 8$ (B1)

$40x = 24 \Rightarrow x = 0.6\ m.$ (from A). (A1)

$h = \frac{2-0}{6} = \frac{1}{3}$

(a) $y = x\sin x$

x	y_0 , y_6	y_1, y_2, y_3, y_4, y_5
0	0	
$\frac{1}{3}$		0.10906
$\frac{2}{3}$		0.41225
1		0.84147
$\frac{4}{3}$		1.29592
$\frac{5}{3}$		1.65901
2	1.81859	
Sum	1.81859 (B)	4.31771 (B)

Note
→ Use Calculators in radians' Not in degrees

$\Rightarrow \int_0^2 x\sin x \, dx \simeq \frac{1}{2} \times \frac{1}{3}\{1.81859 + 2 \times 4.31771\}$ (M)

$\simeq 1.742335$ (A)

$\simeq 1.7423$ (4 dpl) (B)

(b) Maximum error in $\frac{y}{x} = \left|\frac{y}{x}\right| \left\{ \left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta x}{x}\right| \right\}$ (M)

$= \frac{4.8}{1.60} \left\{ \frac{0.05}{4.8} + \frac{0.005}{1.6} \right\}$ (B)

$\simeq 0.040625$ (A)

4dps as std.

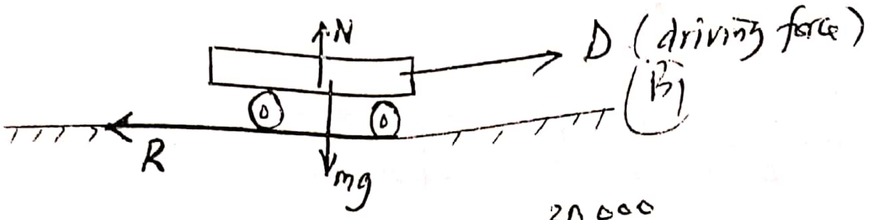
Approx. Value $\simeq \frac{4.8}{1.6} = 3$ (B)

Limits: $3 - 0.040625$ (M) and $3 + 0.040625$ (M)

$\Rightarrow$ Interval $= [2.9594, 3.0406]$ to 4 dps

Simple interval Arithmetic method Can be used.

15 (a)



Power $= DV \Rightarrow D = \frac{30\,000}{30}$ (M1)

$= 1000\text{N}$ (A1)

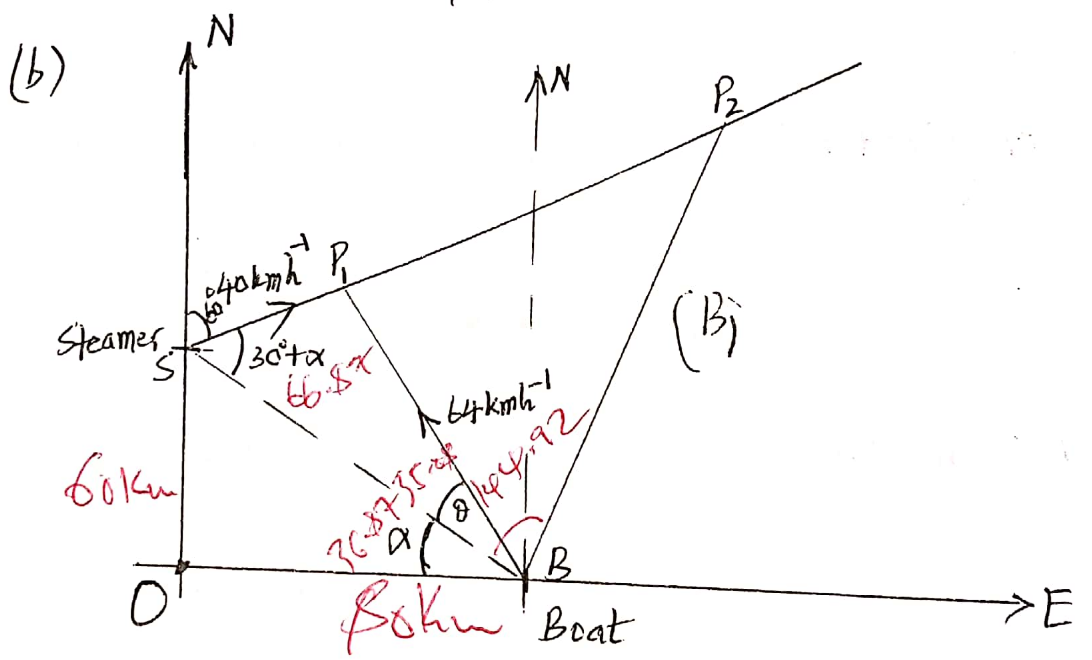
$R = 200\,\text{N}$ (A1)

Resultant force, $F = D - R$ (M1)

ie, $ma = 1000 - 200$

$1000a = 800 \quad \therefore$ Acc^n $a = 0.8\,ms^{-2}$. (A1)

(b)



$\tan\alpha = 3/4$

$\Rightarrow \sin\alpha = 3/5$ (B1)

$\cos\alpha = 4/5$

$\alpha \simeq 36.87°$

$\alpha + 30° = 66.87°$ (B1)

$\frac{\sin\theta}{40} = \frac{\sin 66.87°}{64}$

$\sin\theta = \frac{40 \sin 66.87°}{64}$ (M1)

$\theta \simeq 35.08°$ (A1) or $144.92°$ (Corresponding to BP_2.

Corresponding to BP_1 | The course BP_2 divergent.

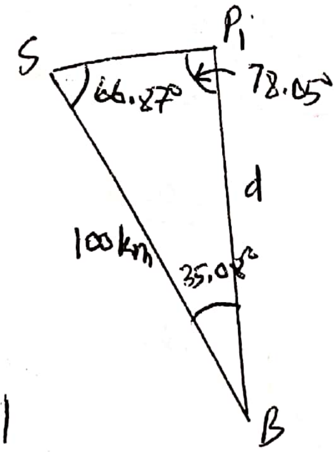
$\Rightarrow$ Course BP_1 is $N\,18.05°\,W$ (A1)

(b) Contd

Distance

5(b) Cont'd

Distance $\overline{BS} = 100km$



$$\frac{d}{\sin 66.87^\circ} = \frac{100}{\sin 78.05^\circ}$$

$$d = \frac{100 \sin 66.87^\circ}{\sin 78.05^\circ} \quad M1$$

$$\text{time taken} = \frac{d}{64} \text{ hrs}$$

$$\simeq 1.4687 \text{ hrs} \quad A1$$

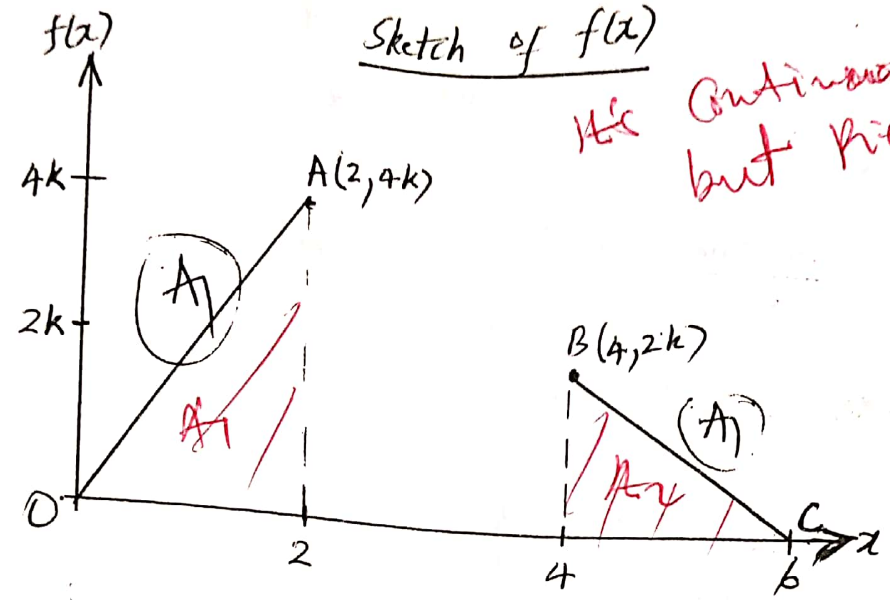
$$\simeq 1 \text{ hr } 28 \text{ mins}$$

PN 16 (a)

x	0	2	4	6
$f(x)$	0	$4k$	$2k$	0

Sketch of $f(x)$

It's continuous but piece wise.



Grad. of $OA = 2k$

A_1 f A_2 = 1

$\Rightarrow f(x) = 2kx$ for $0 \le x \le 2$.

Grad. of $BC = \frac{2k-0}{4-6} = -k$ | $y - 0 = -k(x-6)$

$\Rightarrow f(x) = +k(6-x)$

Area under the graph: $\frac{1}{2}(2)(4k) + \frac{1}{2}(2)(2k) = 1$ M1

$6k = 1 \therefore k = 1/6$ A1

$$\Rightarrow f(x) = \begin{cases} \frac{1}{3}x & ; 0 \le x \le 2 \\ \frac{1}{6}(6-x) & ; 4 \le x \le 6 \\ 0 & ; \text{elsewhere.} \end{cases}$$

b) For $x \le 0$, $F(x) = 0$

For $0 \le x \le 2$; $F(x) = \int_0^x \frac{1}{3} t \, dt$

$$= \frac{1}{6} x^2 \quad \text{M1}$$

$$F(2) = 2/3 \Rightarrow F(x) = \frac{2}{3} + \int_4^x \frac{1}{6}(6-t) dt$$

$$= 2/3 + \frac{1}{12} \int_4^x (12 - 2t) dt$$

$$= 2/3 + \frac{1}{12} \left[12t - t^2 \right]_4^x \quad \text{M1}$$

$$= 2/3 + \frac{1}{12} (12x - x^2 - 32)$$

$$\Rightarrow F(x) = \frac{1}{12} (12x - 24 - x^2) \quad \text{A1}$$

Check: $F(6) = \frac{1}{12}(72 - 24 - 36)$

$= 1$

Hence: $$F(x) = \begin{cases} 0 & ; x \le 0 \\ \frac{1}{6} x^2 & ; 0 \le x \le 2 \\ \frac{1}{12}(12x - x^2 - 24) & ; 4 \le x \le 6 \\ 1 & ; x \geq 6. \end{cases} \quad \text{A1}$$

6 (b)(i) $P(X>5) = 1 - F(5)$

$$= 1 - \frac{1}{12}(12\times5 - 5^2 - 24)$$

M1

$$= 1 - \frac{11}{12}$$

$$= 1/12$$

A

(ii) $F(2) = 2/3 < 4/5 \Rightarrow 4 < P_{80} < 6$

$$\Rightarrow \frac{1}{12}(12x - x^2 - 24) = \frac{4}{5}$$

Or $x^2 - 12x = -33.6$ M1

$$(x-6) = \pm\sqrt{2.4}$$

$x = 6 - \sqrt{2.4}$ (discard $+\sqrt{2.5}$, x is out of range)

Thus, 80th percentile $\simeq$ 4.451 A