A LEVEL

MATH

PAPER 1

1986.

SECTION A.

1. The first term of a geometric progression is A and the sum of the first 3 terms is $\frac{7}{4}A$

(i) Show that there are two possible progressions.

(ii) Given that A = 4 , find the next two terms of each progression.

(b) Expand the expression

$$\frac{1}{(x+2)(3x+1)}$$

in ascending powers of x as far as the term in x^4.

2. (a) Solve the inequality .

$$\frac{x^2 - 2x + 3}{x - 1} < 3.$$

(b) Given that the system of inequalities

$|x + 2| \leq 4$

$\frac{1}{2}x + y \leq 4$

$y \geq \frac{1}{2}x + 2$

Find the equation

$z = y + 2x.$

(c) Find by a graphical method or otherwise the set of integral values (x, y) that satisfy the system of inequalities. Hence determine the maximum value of Z.

3.(a) By row reducing the appropriate matrix to an echelon form solve the system of equations

$$\begin{aligned} x + 3y - z &= 4 \\ 2x + 4y + z &= 8 \\ 3x + 6y + 2z &= 10. \end{aligned}$$

(b) Find the values of p and q which make

$x^4 + 6x^3 + 13x^2 + px + q$

a perfect square.

Note: The above question was not part of this paper(b); the real question had some mistake!!

4.(a) Differentiate

(i) $x^{\sin x}$

(ii) $\log_{10}(1+\cos 2x)$

(b) A curve is represented parametrically by the equations

$x = 3t^2$, $y = 4t^3$.

Find the equation of the curve at any point.

5.(a) By dividing the interval [2,4] into five equal subintervals , use the trapezium rule to estimate the area under the curve.

$y = \dfrac{5}{x}$ between $x = 2$, $x = 4$

(b) Show that the coordinates of the centre of mass of a solid formed by rotating the curve $y^2 = 4x$ between $x =$ a and $x = b$ about the x - axis are given by

$$\left[\frac{2}{3}\frac{(b^2 + a^2 + ab)}{a + b}, 0\right]$$

6.(a) Find $\int x^2 \log(1-x)dx$

(b) By using the substitution $t = \tan x$, evaluate

$$\int_0^{\pi/4} \frac{2\cos^2 + \sin^2 x}{1+\cos^2 x}dx$$

7. Sketch the curve

$$\frac{x+1}{(x-1)(2x+1)}$$

showing clearly the nature of the turning points.

SECTION III :TRIGONOMETRY AND GEOMETRY.

8.(a) Prove that

$$\sin 4\theta = \frac{4\tan\theta(1-\tan^2\theta)}{(1+\tan^2\theta)^2}$$

(b) Show that in any triangle ABC

$\sin A + \sin B = 2\cos \tfrac{1}{2}C\sin(A + \tfrac{1}{2}C)$.

(c) Find the general solution of the equation

$\cos 4\theta + \sin 2\theta = 0$.

9. Show that the angle two lines with gradients λ_1

and λ_2 is given by

$$\theta = \tan^{-1}\left(\frac{\lambda_1 - \lambda_2}{1+\lambda_1\lambda_2}\right)$$

Hence find the angle between the lines.

$$y + 3x - 6 = 0$$
$$3y - x - 2 = 0$$

Show that the locus of point P which moves that the sum of the squares of the distance from these lines is 2 is a circle . Find the centre and radius of the circle.

10. Find the equation of the tangent to the ellipse.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

at the point (a $\cos\theta$, b $\sin\theta$) Hence show that the line $y = mx + c$ is a tangent to the allipse if $c^2 = a^2 m^2 + b^2$ and find the equation of the tangents from the point

(-3,3) to the ellipse.

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

11. The tangent to the parabola $y^2 = 4ax$ with vertex O(0,0) at the point P(at^2, 2at) meets the matrix at Q . Show that SP and SQ are perpendicular where S is

the focus of the parabola. A perpendicular from the vertex meets the tangent at R . Find the locus of the midpoint of OR.

SECTION IV STATISTICS:

12.(a) The table shows the consumer price index and the average wages in shillings per hour of wages in a certain company for the period 1980-1984.

Year	1980	1981	1982	1983	1984
Price index	100	102	110	115	120
Wage per hour	120	130	144	160	180

Using 1980 as the base year calculate :

(i) the wage index

(ii) the real wages per hour

(iii) the purchasing power of the shilling for the given period .

(b) In a group of seven children there are three girls . Find the number of ways they can sit on a bench given that

(i) the girls sit together.

(ii) no two girls sit together.

(iii) Each girl has to sit in the middle of two boys.

13. (a) The table shows the distribution of marks obtained by a class of 30 students.

Marks	Frequency.
15-19	10
20-24	6
25-29	5
30-34	4
35-39	5

Calculate the mean , median and the mode for the above data.

(b) There are an equal number of boys and girls in a class of 2n students . In a certain tent the earn mark standard deviation of the boys are X_1 and δ_1 and of the girls X_2 and δ_2. respectively show that the variance of the marks of all the students is given by

$$\delta_1 = \tfrac{1}{2}\left(\delta_1^2 + \delta_2^2\right) + \tfrac{1}{4}\left(\delta_1 - \delta_2\right)^2$$

. **SECTION V. VECTORS.**

14.(a) The vertices of a triangle ABC are represented by the position vectors o ,.a,b respectively where 0 is the zero vector . show that the position vector of any point on BC is given by the position vector

$p = k\mathbf{a} + (1 - k)\mathbf{b}$ for a suitable real number k.

(b) Find a vector r perpendicular to the vectors **s = 5i +3j + k** and **- i + 3j + 2k**. Hence the equation of a plane passing through the point (5,-1,-2) and parallel to s and t , find the angle between the plane and the line. $x - 2 = \frac{y-2}{2} = \frac{z-2}{3}$

15.(a) Given that

$$z = \frac{(1+2i)^3}{(1+i)(1-3i)}$$

Find (i) $|z|$

(ii)Arg(z)

(b) Show that 2 + 4i is a root of

$z^4 - 4z^3 + 21z^2 - 4z + 20 = 0$. Hence , find the other roots .

(c) Find the square root of $-1 + i\sqrt{3}$ giving your answer in the form x + iy.

PAPER 2
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SECTION A.

1. Given the equation $ax^2 + bx + c = 0$, show that the Newton Raphson method leads to t iterative formula.

$$X_{n+1} = \frac{aX_n^2 - c}{2aX_n + b}$$

Hence , construct a flow chart without subscripted variables to

(i) read the values of a, b,c and the first approximation A.

(ii)calculate the root.

(iii)test whether the difference between successive approximations to the root is less than the error limit ε_1.

(iv) print the equation , the root and number of iterations.

Use your flow chart to calculate the positive square root of 20 correct to 3 significant figures.

2.(a) The table shows the variables of a function f(x) at a set of points.

x	0.9	1.0	1.1	1.2
f(x)	0.266	0.242	0.218	0.192

Use linear interpolation to find

(i) the value of f(1.04)

(ii) the value of x corresponding to f(x) = 0.25.

(b) Given that Y_1 and Y_2 are approximations to X_1 and X_2 with error E_1 and E_2 respectively show that the maximum possible relative error in X_1 / X_2 is

$$\left|\frac{E_1}{Y_1}\right| + \left|\frac{E_1}{Y_2}\right|$$

Given that the error in measuring an angle is up to 0.5^o , find the maximum possible percentage error in

$$\frac{\sin x}{\cos x}$$

VECTORS AND MECHANICS.

3.(a) A particle of mass 5kg at rest at a point (1,4,4) is acted upon by the three forces

F_1 = 3i + 3j ,F_2 = 2j + 4k , F_3 = 2i + 6k

Find

(i) the position and momentum of the particle after 4 seconds.

(ii) the work done by the forces in the seconds.

(b)) A particle of mass marks is projected with an initial speed u at angle θ to the horizontal. Given that the force due to air resistance is equal to mkv , show that the velocity at any time is given by

$$V = (u\cos\theta\mathbf{i} + u\sin\theta\mathbf{j})e^{-kt} - \frac{g}{k}\left(1 - e^{-kt}\right)j$$

where k is constant and **i** and **j** are orthogonal unit vectors.

4. (a) A ship Y appears to an observer in ship Y at 10 o'clock to be travelling at a speed 20kmh^{-1} due North. After 30 minutes shipX which is travelling at a speed of 60kmh^{-1} N 60^o collides with ship Y.

Find (i) the actual velocity of Y

(ii) the distance and bearing of ship Y at 10 o'clock.

5. (a) A car of mass 2 tonnes moves from rest down a road of inclination

$\sin^{-1}\left(\frac{1}{20}\right)$ to horizontal . Given that the engine

develops a power of 64.8kw when it is travelling at a speed of kmh^{-1} and the resistance to motion is 500N . Find the acceleration of the car?

(b) A bullet of mass 40g is fired horizontally into a freely suspended block of wood of mass 1.96kg attached at the end of an inelastic string of length 1.8cm . Given that the bullet gets imbedded in the block and the string is deflected through an angle of 60^o to the vertical , find

(i) the initial velocity of the block.

(ii) the maximum velocity of the block.

6.(a) A particle of mass m is placed on a rough plane inclined at an angle 30^o to the horizontal. Given that the angle of friction

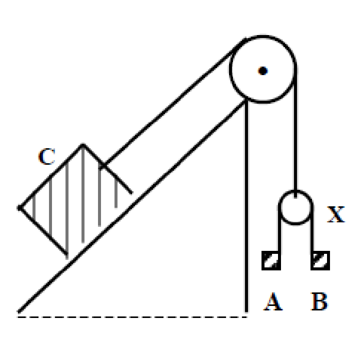
$\lambda > 30^o$ show that the minimum force required to move the body to the plane is given by

$$\frac{1}{2}mg\left(\cos\lambda + \sqrt{3}\sin\lambda\right)$$

If this force is three times the least force that would cause the body to move down the plane show that

$$\lambda = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

7. The diagram shows two masses A and B of 0.5kg and 1kg respectively connected by a light inextensible string passing over a smooth pulley X of mass 0.5kg . Pulley X is connected to a mass C of 2kg lying on a smooth plane inclined at an angle 45^o to the horizontal by a light inextensible string passing over a fixed pulley.



Find

(i) the acceleration of the masses B and C.

(ii) the tension in the string when the system is released.

8. Find the centre of gravity of a semicircular lamina of radius r with the diameter as the base. A semicircle lamina of radius r and base OA is cut from a larger semicircle lamina of radius 2r and base AOB and the remainder is hung from A . Find the inclination of AOB to the vertical .

DIFFERENTIAL EQUATIONS.

9.(a) The gradient of the tangent at any point (x,y) of a curve is $x - \frac{2y}{x}$. Given that the curve passes through the point (2,4) , find the equation of the curve .

(b) Use the substitution $y = ux$ to solve the differential equation.

$$x^2 \frac{dy}{dx} = x^2 + y^2 + xy$$

given that $y = 0$ when $x = \frac{1}{2}\pi$

STATISTICS .

10.(a) Given that A and B are two events such that $P(A) = 0.5$, $P(B) = 0.7$ and $P(A\cup B) = 0.8$ find.

(i) $P(A\cap B)$

(ii) $P(A\cap \overline{B})$.

(b) A bag contains 3 black and 5 white balls . Two balls are drawn at random one at a time without replacement.

Find .

(i) the probability that the second ball is white.

(ii) the probability that the first ball is white given the second is white.

(c) The probability that a student X can solve a certain problem is $\frac{2}{3}$ and that student Y can solve it is $\frac{1}{2}$. Find the probability that the problem will be solved if both X and Y try to solve it independently.

11. A discrete random variable X has a probability function

$$P(X = x) = \begin{cases} x / k & x = 1,2\ldots\ldots n \\ 0 & \text{otherwise} \end{cases}$$

Where k is a constant.

Given that the expectation of x is 3 , find

(i) the value of n and the constant k

(ii) the median and variance of X.

(iii) $P[X = 2 / x \geq 2]$

12.(a) A continuous variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0<x<1 \\ \frac{1}{8} & 2<x<8 \\ 0 & \text{elsewhere} \end{cases}$$

Find

(i) the distribution function and expectation of X.

(ii) $P[\frac{1}{2} < x < 3]$.

(b) A normal population has mean 150 and variance 25. Find the probability that in a random sample size 5 taken from the population at least 1 will have a value less than 146.

13. (a) A pair of dice is tossed 144 times and the sum of the outcomes recorded . Find the probability that a sum of 7 occurs at least 26 times.

(b) Three people play a game in which each person tosses a coin . The game is success if one of the players gets an outcome different from the others. Determine the probability that

(i) a success will occur at the first trial.

(ii) in two trials at least one success will occur.

14. (a)The table shows the distribution of weight of a random sample of 16 tins taken from large consignment.

Weight(gm)	97	98	99	100	101	102
frequency	2	1	2	3	6	2

Assuming the weights are normally distributed, determine a 95% confidence interval for the mean weight of all the tins.

(b) The life period of a certain machine approximately follows a normal distribution with 5

years and standard deviation 1 year. Given that the manufacturer of this machine replaces the machine that fall under guarantee, determine the length of the guarantee required so that not more an 2 % of the machines that fail are replaced.
Determine the proportion of the machines that would be replaced if the guarantee period was years.

15. (a) Prove that if a,b,c are elements of a group (G.o) then

(i) aob = aoc $\Rightarrow$ b = c

(ii) $(aob)^{-1} = b^{-1}$ o a^{-1}

Given the set S =

$$\left\{W_1 = 1, w_2 = \tfrac{1}{2} + \tfrac{1}{2}\sqrt{3i}\, w_3 = -\tfrac{1}{2} - \tfrac{1}{2}\sqrt{3i}\right\}$$

determine whether (S,o) is a group , where o is the ordinary multiplication .