P425/1
PURE MATHEMATICS
AUGUST - 2024
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS – AUGUST, 2024

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all the eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) will **not** be marked.*

*All working **must** be shown clearly.*

Begin each question on a fresh sheet of paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn Over

SECTION A (40 MARKS)

Answer all questions in this section

1. Given that one root of the equation $4z^2 - 12z + p = 0$ is $k + 4i$, where k and p are real constants. Find the values of k and p. *(5 marks)*

2. Solve the equation ; $2\sin(60^0 - x) = \sqrt{2}\cos(135^0 + x) + 1$

   for $-180^0 \leq x \leq 180^0$. *(5 marks)*

3. If $y = \frac{\left(x-\frac{1}{3}\right)e^{2x}}{\cos x}$, find $\frac{dy}{dx}$. *(5 marks)*

4. A circle of radius 5 units has its centre in the second quadrant on the line $x + y = 4$ and passes through point (3, 2). Find the equation of the circle. *(5 marks)*

5. If $log_8 x - log_{16} x^2 + 3log_{32} x = 2.6$, find the value of x. *(5 marks)*

6. Show that $\int_0^3 x^2 log_2(3x)\, dx = \frac{9}{2ln2}[4\, ln3 - 1]$. *(5 marks)*

7. Find the acute angle between the lines $\frac{x-3}{5} = \frac{y+1}{-4} = \frac{z}{2}$ and

   $$y = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$ *(5 marks)*

8. A shell is formed by rotating the area bounded by the portion of the parabola $y^2 = 4x$ for which $0 \leq x \leq 1$ and $y \geq 0$, through two right angles about the x-axis. Find the volume of the solid generated.

   (5 marks)

SECTION B: (60 Marks)

Answer any five questions from this section. All questions carry equal marks.

9. (a.) When the quadratic expression $at^2 + bt + c$ is divided by $t + 1$, $t + 2$ and $t - 3$, the remainders are 6, 20 and 10 respectively. Find the values of a, b and c. *(7 marks)*

(b.) (i.) Simplify $\frac{2-\sqrt{3}+\sqrt{5}}{\sqrt{5}+\sqrt{3}}$.

(ii.) If $2x^2 + bx + 50 = 0$ has a repeated root, find the possible values of b.

(5 marks)

10. (a.) Prove that $\frac{\sin 2\theta - \cos 2\theta - 1}{2 - 2\sin 2\theta} = \frac{1}{\tan\theta - 1}$. *(3 marks)*

(b.) Solve $\sqrt{3}\cos 2\theta - \sin 2\theta + 1 = 0$, for $-180^0 \leq \theta \leq 180^0$. *(4 marks)*

(c.) Solve $\cos\theta - \cos 3\theta - \cos 7\theta + \cos 9\theta = 0$ for $0^0 \leq \theta \leq 90^0$. *(5 marks)*

11. (a.) Determine the equation of the normal to the curve $y = \frac{1}{x-2}$ at the point $\left(4, \frac{1}{2}\right)$.

And find the coordinates of the other point where the normal meets the curve again.

(7 marks)

(b.) Find the maximum and minimum values of $x^2 e^{-x}$. *(5 marks)*

12. (a.) Find the integrals;

(i.) $\int \ln\left(\frac{2}{x}\right) dx$.

(ii.) $\int (x \cos x)^2 \, dx$. *(7 marks)*

(b.) Evaluate $\int_{-1}^{0} \frac{x}{\sqrt{1-3x}} \, dx$. *(5 marks)*

13. (a.) Find the ratio of the term in x^6 to the term in x^9 in the expansion of $(2x + 3)^{18}$ to its simplest form. *(3 marks)*

(b.) If the first four terms in the expansion of $(1 - x)^n$ are

$1 - 6x + px^2 + qx^3$. Find the values of p and q. *(4 marks)*

(c.) The first term of an arithmetic progression is 5 and the common difference is 13. Find the sum to n terms of the progression. Hence find the least value of n for which the sum exceeds 1000.

(5 marks)

14. (a.) The line $y = x - c$ touches the ellipse $9x^2 + 16y^2 = 144$ .Find the possible values of c and the coordinates of the points of contact. *(7 marks)*

(b.) The point P lies on the ellipse $x^2 + 4y^2 = 1$ and N is the foot of the perpendicular from P to the line $x = 2$. Find the locus of the mid-point of PN as P moves on the ellipse. *(5 marks)*

15. (a.) Given $OT = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$ and $OS = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix}$ .Find the coordinates of R such that $TR : TS = 2 : 3$. *(5 marks)*

(b.) Show that the line $\frac{x-2}{4} = \frac{y-1}{9} = \frac{z+3}{5}$ is parallel to the plane $3x + 2y - 6z + 9 = 0$. Hence find the shortest distance between the line and the plane. *(7 marks)*

16. (a.) Solve the differential equation $(x^2 + 1)\frac{dy}{dx} - xy = x$. given that $y = 1$ when $x = 0$. *(4 marks)*

(b.) The rate of growth of the population of birds on an island is proportional to the number f birds present. If the birds doubled after 5 years. Form a differential equation for the rate of growth of the bird's population. Hence find after how long the birds will be 16 times the original number. *(8 marks)*