P425/1
PURE MATHEMATICS
Paper 1
July/August
3 hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B.**
- *Any additional question(s) answered will* **not** *be marked.*
- *Show* **all** *necessary working clearly.*
- *Begin each answer on a fresh page of paper.*
- *Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Answer **all** questions in this section.

1. If the equation $x^2 + mx + n = 0$ and $x^2 + px + r = 0$ have common factors, prove that $(n-r)^2 = (m-p)(pn-mr)$. (5 marks)

2. From a class of 14 boys and 10 girls, 10 students are to be elected for a competition in which 5 boys and 5 girls or 2 girls and 8 boys are to go for it. In how many ways can they be selected? (5 marks)

3. Solve $10\sin^2 3x + 10\sin 3x\cos 3x - \cos^2 3x = 2$ for $0^\circ \le x \le 120^\circ$. (5 marks)

4. The area bounded by the curve $x^2 = 4ay$ and the y-axis and lines $y = 0$ and $y = 4b$ is rotated about y-axis through 2π to form a solid. Find the volume of the solid formed. (5 marks)

5. Given the points P(5,4,1) and Q($^-1$, $^-2$, 1). Find the position vector of the point R such that PR: PQ = 2:3. (5 marks)

6. Evaluate $\int x\sin^2 x\cos^2 x\, dx$. (5 marks)

7. Find the equation of the normal to curve $(x-1)^2 + (y+2) = 8$, at the point (3, $^-4$). (5 marks)

8. Q is a variable point given by the parametric equations; $x = \tan\theta - \sin\theta$ and $y = \sin\theta + \tan\theta$. Show that the locus of Q is $(y^2 - x^2)^2 = 16xy$. (5 marks)

SECTION B (60 marks)

Answer any **five** questions from his section.

9. The tangent to the parabola $y^2 = 4ax$ at T $(at^2, 2at)$ meets the x-axis at P. The straight line through T parallel to the axis of the parabola meets the directrix at Q. If S is the focus of the parabola. Prove that TPQS is a rhombus. (12 marks)

10. (a) Mary operates an account with a bank which offers a compound interest of 5% per annum. She opened the account at beginning of 2019 with Shs. 800,000 and continue to deposit the same amount at beginning of every year. How much will she receive at end of 2022 if she made no withdrawal within this period? (6 marks)

(b) Expand $\left(\frac{1+x}{1+3x}\right)^{\frac{1}{3}}$ in ascending powers of x up to the third term, hence by putting $x = \frac{1}{125}$ evaluate cube root of 63 correct to 4 decimal places. (6 marks)

11. Express $f(x) = \frac{3+2x+x^2}{x^3(x+2)}$ into partial fractions. Hence evaluate $\int_2^5 f(x)dx$. (12 marks)

12. (a) A, B and C are non-collinear points with position vectors **a**, **b** and **c** respectively. Point P and Q are on **BC** and **CA** such that **BP**: **PC** = 3:1 and **CQ**: **QA** = 2:3. If point R is on BA produced such that P, Q and R are collinear points. Find in terms of **a**, **b** and **c**, the position vectors of P, Q and R. (8 marks)

(b) Write down the equation of a line which passes through $(1, 0, {}^{-}2)$ in the direction of the vector $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ and find the coordinates of the point where the line intersects the plane $4x+3y+2z=25$. (4 marks)

13. (a) Given; $y = be^{-2t}\sin 3t$. Prove that, $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0$. (6 marks)

(b) A right circular cone is held with it's vertex down beneath a tap leaking at the rate of 2 cm^3s^{-1}. Find the rate of rise of water level when it's depth is 5 cm given that the height and radius of the cone are 15 cm and 5 cm respectively. (6 marks)

14. (a) Prove that $\frac{3\sin\theta+\sin2\theta}{1+3\cos\theta+\cos2\theta} = \tan\theta$, hence solve the equation;
$\frac{3\sin\theta + \sin2\theta}{1+3\cos\theta+\cos2\theta} = \frac{1}{\cos^2\theta} = 2$ for $0^0 \le \theta \le 360^0$. (6 marks)

(b) Express 7cos A + 24sinA in the form Rsin (A + β) where β is an acute angle and R is a constant. Find the range in which $\frac{2}{7\cos A + 24\sin A + 10}$ lies. (6 marks)

15. (a) Solve the equation; $2\log_4 x + \log_2(x+6) = 6\log_8(x+2)$. (5 marks)

Turn Over

(b) If $Z = \frac{(3-i)(5+12i)}{(1+3i)^2}$

Find the; (i) modulus of Z. (4 marks)

(ii) Argument of Z. (2 marks)

(iii) Polar form of Z. (1 marks)

16. In Focus High School Akiro of 1405 students, all students voted for the head prefect such that the rate of those who had voted is proportional to product of those who had voted and those had not yet voted. 20 students are to supervise the election and they voted before 7.00am, while the other students started to vote at 7:00am. If after 3 hours, 600 students had voted. Find the;

(i) number of students who had voted after 8:00am. (10 marks)

(ii) time when 800 students had voted. (02 marks)

END