P425/1

PUREMATHEMATICS

PAPER 1

JUNE 2023

3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

RESOURCEFUL EXAMINATION 2023

PURE MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae provided.
- Silent, non-programmable calculators should be used.
- Clearly indicate the questions you have attempted in a grid on your answer scripts.

SECTION A

1. Solve the inequality: $\frac{6}{1-x} \geq x+4$

(**5** ***marks***)

2. Evaluate: $\int_1^2 \frac{1}{x^2+6x+5} dx$

(**5** ***marks***)

3. Solve the equation; $tan4\beta + tan2\beta = 0$ for $0^0 \leq \beta \leq 360^0$

(**5** ***marks***)

4. Using small changes Approximation $sec^2 44.6^0$

(**5** ***marks***)

5. Show that the equation $4y^2 + 4y + 16x - 15 = 0$ represents a parabola. Hence determine the latus rectum, directrix and focus.

(**5** ***marks***)

6. M is a point which divides line AB externally in the ratio of 4:3. A is (1,4,1) and B is $(-1,-2,3)$. Find the Cartesian equation of the line through M and $N(2,1,0)$.

(**5** ***marks***)

7. Evaluate $\int_0^{\pi/2} x^2 cos2x dx$ (**5** ***marks***)

8. How many ways can the word *SUCCEEDED* be arranged when the vowels are not together. (**5** ***marks***)

SECTION B

9. (a) Show that $arg\left(\frac{Z+1}{Z-1}\right) = \frac{\pi}{4}$ represents a circle. Hence state the coordinates of the Centre and radius.

(6 marks)

(b) Given the equation $|Z + 2 + 3i| = 3$. Find the minimum and maximum of $|Z - 1 - i|$

(6 marks)

10. (a) Show that the parametric equations $x = 9cos\theta$ and $y = 16sin\theta$ represents an ellipse. Hence determine foci and diretrices.

(5 marks)

(b) If the line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
Express c in terms of a, b and m. Hence show that $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$ is point of contact.

(7 marks)

11. (a) Solve the equation $5tanx + secx + 5 = 0$ for $0^0 \leq x \leq 360^0$

(6 marks)

(b) A and B are acute angles such that $cosA = \frac{2}{3}$ and $cosecB = 5$.
Find the value of $\tan(A - B)$. Leave your answer in surd form.

(6 marks)

12. The plane L_1 has equation $3x - 4y + 2z = 5$ and plane L_2 has equation $r = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

(a) Find the Cartesian equation of plane L_2 **(4 marks)**

(b) Obtain the cute and between the planes. **(4 marks)**

(c) Find the vector equation of the line of intersection of the two planes. **(4 marks)**

13. Express $\frac{6x+4}{(x^2-4)(x+2)}$ into partial fractions. Hence evaluate

(i) $\int_0^1 \frac{6x+4}{(x^2-4)(x+2)} dx$

(ii) $\frac{d}{dx}\left(\frac{6x+4}{(x^2-4)(x+2)}\right)$ **(12 marks)**

14. (a) Find the values of m and n if $f(x) = x^3 + 4mx^2 + nx + 3m$ is divisible by $(x-1)^2$ **(4 *marks*)**

(b) Timothy deposits shs.100, 000 per month in a bank that offers a compound interest of 5% per month. Find the interest he will earn after saving for one year. **(4 *marks*)**

(c) Find the square root of $21 - 6\sqrt{6}$. Leave your answer in surd form. **(4 *marks*)**

15. Given the curve $y = \frac{8}{(x+3)(x-1)}$

(a) Find the range of values of y for real x. hence determines the turning point and the nature.

(b) State all the asymptotes and intercepts.

(c) Sketch the curve.

(12 *marks*)

16. (a) Solve the differential equation $\frac{dy}{dx} + y\tan x = 1$, given that $y = 2$ and $x = 0$ **(6 *marks*)**

(b) Find the area bounded by the curve $y = 11 - x^2$, lines $y = 2$ and $y = 7$ **(6 *marks*)**

END