## MHS Senior 6 Pure Maths Revision Questions 2020

1. If $\alpha + \beta = 3$ and $\alpha^2 + \beta^2 = \dfrac{5}{2}$, form a quadratic equation with roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

2. Given that $V = \sqrt{8 - 0.1Q}$ and $R = -\dfrac{\mathrm{d}V}{\mathrm{d}Q} \cdot \dfrac{Q}{V}$, find $R$ when $Q = 60$.

3. If $x$ is real, determine the range of the possible values of $y = \dfrac{x}{1+x^2}$.

4. The area of a triangle $PQR$ is $3\,\mathrm{m}^2$. If $p = 5\,\mathrm{m}$ and $r = 1.5\,\mathrm{m}$, find $\cos Q$.

5. When $\left(1 + \dfrac{x}{2}\right)^n$ is expanded in increasing powers of $x$, the coefficients of the first three terms of the expansion form an arithmetic progression. Find $n$.

6. Given the points $A(7, 1, 2)$, $B(3, -1, 4)$ and $C(4, -2, 5)$, find angle $ABC$.

7. Show that $\dfrac{\log 3^{3/2} + \frac{1}{2}\log 8 - \log\sqrt{125}}{\log 6 - \log 5} = \dfrac{3}{2}$.

8. Express $f(x) = 32 + 12x - 3x^2$ in the form $a + b(c + x)^2$. Hence find the maximum value of $f(x)$.

9. Differentiate $\log_5\left(\dfrac{\mathrm{e}^{\cot x}}{\operatorname{cosec}^2 x}\right)$ with respect to $x$.

10. Solve the inequality $\dfrac{3x^2 - 1}{x + 2} \geq 2$.

11. Find the equation of the normal to $y^3 + y^2 - x^4 = 1$ at the point $(1, 1)$.

12. The sum of the first $n$ terms of a progression is $6n^2 - n$. If the $k^{\text{th}}$ term of the progression is 221, find $k$.

13. Prove that: $1 - 4\sin\theta\sin 3\theta = \dfrac{\cos 5\theta}{\cos\theta}$.

14. Find $\displaystyle\int \cot^{-1}\theta \;\, \mathrm{d}\theta$.

15. By solving $5^x = \dfrac{10}{3} - 5^{-x}$, show that $x = \pm\log_5 3$.

16. The plane $x + 3y + 2z = 8$ intersects with the line $\dfrac{x}{2} = \dfrac{y+1}{-1} = \dfrac{z-1}{0}$ at point $Q$. Find the coordinates of $Q$.

17. Show that: $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\sin^{-1}(2x - x^2)\right\} = \dfrac{2}{\sqrt{1 + 2x - x^2}}$.

18. The line $3x + 4y = 27$ is a tangent to a circle with centre $(2, -1)$. Find the equation of the circle.

19. Solve the differential equation: $\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{y}{x} = \sec\left(\dfrac{y}{x}\right)$.

20. Find the values of $p$ for which $4x^2 + 2x + 1 = px(1 + px)$ has no real roots.

21. When a polynomial $p(x)$ is divided by $x + 1$, the remainder is 6. If $x - 2$ is a factor of $p(x)$, find the remainder when $p(x)$ is divided by $x^2 - x - 2$.

22. Find the centre and the radius of the circle $r = 2(4\cos\theta - 3\sin\theta)$.

23. Show that $\tan\left(\dfrac{\pi}{4} + \theta\right) = \tan 2\theta + \sec 2\theta$. Hence, find $\tan\left(\dfrac{5\pi}{12}\right)$ precisely.

24. Solve the differential equation $\sin x \dfrac{\mathrm{d}y}{\mathrm{d}x} - y\cos x = \mathrm{e}^x \sin^2 x$ if $y\left(\dfrac{\pi}{2}\right) = \mathrm{e}^{\frac{\pi}{2}}$.

25. The angle between the line $x = \dfrac{y+1}{2} = \dfrac{z-1}{-2}$ and the plane $2x + \lambda y + z = 8$ is $\sin^{-1}\dfrac{4}{9}$. Find the possible values of $\lambda$.

26. If $x = 3(1 - 2\cos 2\theta)$ and $y = 4\cos^3\theta$, prove that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{1}{48}\sec\theta$.

27. A spherical retort of internal radius $r$ contains a liquefied gas up to a depth $d$. Show that the volume of the gas is $\dfrac{1}{3}\pi d^2(3r - d)$.

28. Solve the equation $x^2 + 2x + \dfrac{12}{x^2 + 2x} = 7$.

29. Evaluate $\displaystyle\int_0^1 \frac{\mathrm{d}x}{(x^2+1)^2}$.

30. Show that the planes $2x - 3y - z + 1 = 0$ and $6x - 9y - 3z = 5$ are on opposite sides of the origin.

31. The letters of the word **INSIPIDITY** are to be arranged in a row. In how many different ways can this be done:
   (i) without any restrictions.
   (ii) if the letters **I** are not all together.

32. Solve the equation $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \dfrac{1}{2}\tan^{-1} x$.

33. Find the equation of the circle which passes through the origin and cuts each of the circles $x^2 + y^2 - 2x - 2y = 7$ and $x^2 + y^2 - 6x + 8 = 0$ orthogonally.

34. Differentiate $\dfrac{(x+1)^2(x+2)}{(x+3)^3}$ and simplify as far as possible.

35. Sand pours out at a rate $\dfrac{\pi}{100}$ $\mathrm{cm}^{-3}$ from the vertex of a conical funnel of height 10 cm and top radius 2 cm. Find the rate at which the height of the sand is decreasing when the funnel is half way full.