P425/1
PURE MATHEMATICS
AUGUST - 2023
3 HOURS

Uganda Advanced Certificate of Education

MID TERM TWO EXAMINATIONS 2023
PURE MATHEMATICS
Paper 1
3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all the eight** questions in section **A** and any **five** from section **B**.*

*Any additional question(s) will **not** be marked.*

Begin each question on a fresh sheet of paper.

SECTION A

1. Find the sum of the roots of the equation $2\log x - \log(2x - 75) = 2$. *(5 marks)*
2. Reduce $\frac{(2+3\sqrt{-1})^2}{2+\sqrt{-1}}$ to the form $A + \mathrm{B}\sqrt{-1}$. State the values of A and B. *(5 marks)*
3. If α, β and the roots of the equation $mx^2 + bx + c = 0$, then find the value of

   $\frac{1}{(m\alpha+b)^2} + \frac{1}{(a\beta+b)^2}$ *(5 marks)*
4. When $P(z)$ is divided by $z + 1$ the remainder is -8 , and when divided by $z - 3$ the remainder is 4 . Find the remainder when $P(z)$ is divided by $(z - 3)(z + 1)$. *(5 marks)*
5. Solve the system of equations using Echelon form.

   $$2x - y - 4z = 3$$
   $$-x + 3y + z = -10$$
   $$3x + 2y - 2z = -2$$

   (5 marks)
6. Prove that in $\sin A \sin(60 - A) \cdot \sin(60 + A) = \frac{1}{4}\sin\ 3A$. *(5 marks)*
7. Solve the equation $5^{2x} - 5^{1+x} + 6 = 0$. *(5 marks)*
8. Show that the equation $\sqrt{5x - 11} - \sqrt{x - 3} = 4$ has only one real root and state the root. *(5 marks)*

SECTION B

9. (a) Solve for x in the range $0°$ to $360°$, $3\cos^2 2x - 3\sin 2x\cos 2x + 2\sin^2 2x = 1$. *(7 marks)*

(b) In a triangle ABC, prove that

$$\tan B\cot C = \frac{a^2+b^2-c^2}{a^2-b^2+c^2}$$ *(5 marks)*

10. (i) Express $\cos\theta + \sqrt{2}\sin\theta$ in the form $r\cos(\theta - \alpha)$, where $r > 0$ and $0° < \alpha < 90°$. *(5 marks)*

(ii) State the maximum and minimum values of $\frac{1}{3+\cos\theta+\sqrt{2}\sin\theta}$ and the smallest positive values of θ for which they occur. *(5 marks)*

11. (a) Show that, if the equations $x^2 + ax + 1 = 0$ and $x^2 + x + b = 0$ have a common root, then $(b-1)^2 = (a-1)(1-ab)$. *(6 marks)*

(b) If α, β are roots of the equation $x^2 - 5x + 6 = 0$ then find the equation whose roots are $\alpha + 3$ and $\beta + 3$. *(6 marks)*

12. (a) When the expression $x^5 + 4x^2 + ax + b$ is divided by $x^2 - 1$, the remainder is $2x + 3$. Find the values of a and b. *(7 marks)*

(b) Find the set of values of x for which

$$\frac{x^2-12}{x} > 1$$ *(5 marks)*

13. (a) If $\log 25 = m, \log 225 = n$, prove that

$$\log\left(\left(\frac{1}{9}\right)^2\right) + \log\left(\frac{1}{2250}\right) = 2m - 3n - 1$$ *(5 marks)*

(b) Find x, if $\log_2 x + \log_4 x + \log_8 x + \log_{16} x = \frac{25}{4}$ *(7 marks)*

14. (a) The angles α and β lie in the interval $0° < x < 180°$, and are such that

$$\tan\alpha = 2\tan\beta \text{ and } \tan(\alpha+\beta) = 3.$$

Find the possible values of α and β. *(5 marks)*

(b) (i) Show that the equation $\tan(30° + \theta) = 2\tan(60° - \theta)$ can be written in the form

$$\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0.$$

(ii) Hence, or otherwise, solve the equation $\tan(30° + \theta) = 2\tan(60° - \theta)$, for $0° \leqslant \theta \leqslant 180°$. *(7 marks)*

END