P425/1
PURE MATHEMATICS
NOVEMBER 2023

UGANDA ADVANCED CERTIFICATE OF EDUCATION

END OF YEAR EXAMINATION 2023

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Answer all the eight questions in section A and only five questions in section B.
- Any additional question(s) will not be marked.
- All working must be shown clearly.
- Graph paper is provided.
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- Clearly indicate the questions you have attempted on the answer scripts as illustrated, DO NOT hand in the question paper.

Question		Mark
Section A		
Section B		
Total		

SECTION A

1. Differentiate from first principles $y = \frac{3}{x^2}$. (5 marks)

2. Solve the equation $\sin 2x + \cos 4x = 0$ for $0^o \leq x \leq 180^o$. (5 marks)

3. Find the equation of the tangent at $(2,\ 1)$ to the curve $y^2 + 3xy = 2x^2 - 1$.
(5 marks)

4. Show that $\tan\left(\frac{A+B}{2}\right) - \tan\left(\frac{A-B}{2}\right) = \frac{2\sin B}{\cos A + \cos B}$. (5 marks)

5. If the line $3x - 4y - 12 = 0$ is the tangent to the circle with a centre at $(1,\ 1)$. Find the equation of that circle. (5 marks)

6. An arithmetic progression has 12 terms, it's fifth term is 7 and common difference is 6. Find the first and last term and the the sum of the arithmetic progression. (5 marks)

7. The vertices of a triangle are $P(2,\ -1,\ 5)$, $Q(7,\ 1,\ 3)$, and $R(13,\ -2,\ 0)$. Find the area of the triangle. (5 marks)

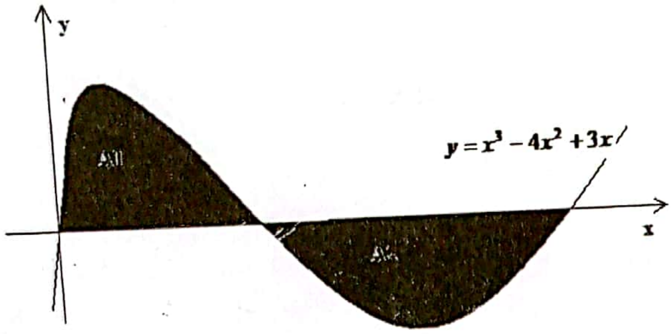
8. Given that $p = \log_2 3$ and that $q = \log_4 5$, show that $\log_{45} 2 = \frac{1}{2(p+q)}$.
(5 marks)

SECTION B(Attempt ONLY 5 questions)

9a) The curve $y = ax^3 + bx^2 + cx$ passes through the point $(-1,\ 16)$ and has a stationary point at $(1,\ -4)$. Find a, b, c. (6 marks)

b) A cylinder is inscribed in a right circular cone of height 12cm and radius 4cm, find the dimensions of the cylinder required to maximize the surface area. (6 marks)

10a) Shown is the sketch of the curve $y = x^3 - 4x^2 + 3x$. Find the area enclosed by the curve and the x-axis. (5 marks)



b) Sketch the curve $y = x(8 - x)$ and find the area enclosed by the curve $y = x(8 - x)$ and the line $y = 12$. (7 marks)

11a) A curve is given parametrically by $x = \frac{3t - 1}{t}$ and $y = \frac{t^2 + 4}{t}$, show that

$$\frac{d^2y}{dx^2} = 2t^3.$$ (6 marks)

b) Find and classify the nature of the turning points of the curve $x^2 + y^2 - 4x + 6y + 12 = 0$. (6 marks)

12. A, B, C are points whose position vectors are $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ respectively. L and M are the mid points of $\overrightarrow{AC}$ and $\overrightarrow{CB}$.

a) Show that $\overline{AB} = 2\overline{LM}$. (4 marks)

b) Find angle ACB. (5 marks)

c) The vector perpendicular to the plane containing the points A, B and C. (3 marks)

13a) Find the Cartesian equation of a curve whose parametric equations are $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$. (5 marks)

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b) A point $P(x,\ y)$ is such that its distance from the origin is five times its distance from the point $A(12,\ 0)$. Find the locus of P. (7 marks)

5PA

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PO = 5PA

14a) Prove that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. (6 marks)

b) Solve the equation: $\cos 6x + \cos 4x + \cos 2x = 0$ for $0^\circ \le x \le 180^\circ$. (6 marks)

15a) Solve the equation: $3\sec^2 x = \tan x + 5$ for $0^\circ \le x \le 360^\circ$. (5 marks)

b) Given that $A + B + C = 180^\circ$, prove that
$\sin A\cos(B - C) + \sin B\cos(C - A) + \sin C\cos(A - B) = 4\sin A\sin B\sin C$. (7 marks)

16a) If α and β are the roots of $x^2 - 5x + 4 = 0$, find an equation whose roots are $\frac{1}{\alpha - \beta}, \frac{1}{\alpha - \beta}$. (6 marks)

b) Solve the equation: $\sqrt{\dfrac{x-1}{3x+2}} + 2\sqrt{\dfrac{3x+2}{x-1}} = 3$. (6 marks)

END