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SECTIONA: MECHANICS

CHAPTER1: DIMENSIONS OF A PHYSICAL QUANTITY

1.1.0: Fundamental quantities

These are quantities which can't be expressed in terms of any other quantities by using any mathematical equation. E.g.

Mass - M Length - L Time- T

1.1.1: Derived quantities

These are quantities which can be expressed in terms of the fundamental quantities of mass, length, and time e.g.

i) Pressure

ii) Acceleration

iii) Momentum

iv) Density

1.1.2: DIMENSIONS OF A PHYSICAL QUANTITY

This refers to the way a derived quantity is related to the three fundamental quantities of length, mass and time.

Or It refers to the power to which fundamental quantities are raised.

Symbol of dimensions is [ ]

Examples

[Area] $= L^2$

[Volume]$= L^3$

$[Density] = \frac{[Mass]}{[Volume]} = \frac{M}{L^3}$ $= ML^{-3}$

[Velocity]$= \frac{[Displacement]}{[Time]} = \frac{L}{T} = LT^{-1}$

[Acceleration]$= \frac{[Change\ in\ Velocity]}{[Time]} = \frac{LT^{-1}}{T} = LT^{-2}$

[Momentum]=[Mass][Velocity]=MLT^{-1}

[Weight]=[Mass][Gravitational acceleration]=MLT^{-2}

[Force]=[Mass][Acceleration]= MLT^{-2}

[Pressure]$= \frac{[Force]}{[Area]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

NB. Dimension less quantity has no dimensions and is described by a number which is

independent of a unit of measurement chosen for the primary quantities

Examples of dimension less quantities

- ❖ Refractive index
- ❖ strain
- ❖ relative density
- ❖ all constants such as 2π, 2, π, 4 π,

.

They are always given a dimension of one, (1)

1.1.3: USES OF DIMENSIONS

1. Used to check the validity of the equation or check whether the equation is dimensionally consistent or correct.
2. Used to derive equations

a) Checking validity of equations (dimensional homogenity)

When the dimensions on the L-H-S of the equations are equal to the dimensions on the

R-H-S, then the equation is said to be dimensionally consistent.

Examples

1. The velocity V of a wave along a flat string is given by $V = \sqrt{\frac{TL}{M}}$

T - Tension in the string

L - Length of the string

M - Mass of the string

Show that the formula is dimensionally correct.

Solution

$$V = \sqrt{\frac{TL}{M}}$$

L.H.S $[V] = LT^{-1}$

R.H.S $\left[\sqrt{\frac{TL}{M}}\right]$ = $\left[\left(\frac{TL}{M}\right)^{\frac{1}{2}}\right]$= $\left(\frac{[T][L]}{[M]}\right)^{\frac{1}{2}}$

Tension (T) is a force therefore takes the dimensions of force.

$\left(\frac{MLT^{-2}L}{M}\right)^{\frac{1}{2}}$= $(L^2T^{-2})^{\frac{1}{2}}$

$= L^{2x\frac{1}{2}}T^{-2x\frac{1}{2}}$

$= LT^{-1}$

L.H.S = R.H.S

Since dimension on left are equal to dimensions on right then its correct

2. The period T, of a simple pendulum is given by T=$2\pi\sqrt{\frac{l}{g}}$ Show that the equation is dimensionally correct.

Where 2π = dimension less constant g = Acceleration due to gravity

l = length of pendulum

Solution

L.H.S [T] = T

R.HS = $\left[2\pi\sqrt{\frac{l}{g}}\right] = \left[2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}}\right] = [2\,\pi]\left(\frac{[l]}{[g]}\right)^{\frac{1}{2}} = \left(\frac{L}{LT^{-2}}\right)^{\frac{1}{2}} = (T^2)^{1/2}$ = T

Since the dimensions on the L.H.S are equal to the dimensions on the R.H.S then the equation is dimensionally consistent.

NB: Dimensions cannot be added or subtracted but for any equation to be added or subtracted

then they must have the same dimensions.

Example

Show that the equation $v^2 = u^2 + 2as$ is dimensionally correct.

Solution

L.H.S $[v^2] = (LT^{-1})^2 = L^2T^{-2}$

R.H.S = $[u^2]$ = [2as]

$= (LT^{-1})^2 = LT^{-2}$L

$= L^2T^{-2} = L^2T^{-2}$

Since dimensions on the L.H.S are equal to dimensions on the R.H.S then the equation is dimensionally correct.

Exercise

Show that the following equations are dimensionally consistent.

i) $S = ut + \frac{1}{2}at^2$

ii) $v = ut + at$

iii) $Ft = mv - mu$

b) Deriving equations (dimensional analysis)

The method of dimension analysis is used to obtain an equation which is relating to relevant variables

Example 1

Assume that the period (T) depend on the following

i) Mass (m) of the bob

ii) Length (l) of the pendulum

iii) Acceleration due to gravity (g)

Derive the relation between T, m, l, g

Solution

T $\propto m^x l^y g^z$

T = K $m^x l^y g^z$ ………x

Where K is a constant

If it's dimensionally consistent then

[T] = [K] $[m]^x [l]^y [g]^z$

T = $M^x L^y (LT^{-2})^z$

$M^0 L^0 T = M^x L^y L^Z T^{-2Z}$

$M^0 L^0 T = M^x L^{y+Z} T^{-2Z}$

Dimensions of M

$x = 0$……………1

Dimensions of L

$y + z = 0$…………2

Dimensions of T

$-2z$ = 1………..3

z = $\frac{-1}{2}$

Put into equation2

y + $\frac{-1}{2}$ =0

y = $\frac{1}{2}$

x = 0, y = $\frac{1}{2}$, z = $\frac{-1}{2}$

Since T = K $m^x l^y g^z$

T = K $m^0 l^{\frac{1}{2}} g^{\frac{-1}{2}}$

T = $K \frac{l^{\frac{1}{2}}}{g^{\frac{1}{2}}}$

T= $K\sqrt{\frac{l}{g}}$

Note:

Dimensional analysis does not give the value of a constant K. it can be determined using mathematical analysis.

Example ii

Use dimensional analysis to show how the velocity of transverse vibrations of a stretched string depends on its length (l) mass (m) and the tension force (F) in the string.

solution

V $\propto l^x m^y F^z$

V $= K\, l^x m^y F^z$

[V] $= [K\, l^x m^y F^z]$

[V] $= [K][l^x][m^y][F^z]$

$LT^{-1} = L^x M^y (MLT^{-2})^z$

since [K]=1

$M^0LT^{-1} = L^{x+z} M^{y+z} T^{-2z}$

For dimensions of M

$y + z = 0$…………(1)

For dimensions of L

$x + z = 1$…………(2)

For dimensions of T

$-2z = -1$………..(3)

$Z = \frac{1}{2}$

Put into equation(1)

$y + z = 0$

$y + \frac{1}{2} = 0$

$y = \frac{-1}{2}$

Also for equation(2)

$x + z = 1$

$x + \frac{1}{2} = 1 \therefore x = \frac{1}{2}$

but V $= K\, l^x m^y F^z$

V $= K\, l^{\frac{1}{2}} m^{\frac{-1}{2}} F^{\frac{1}{2}}$

V = K$\sqrt{\frac{l\,F}{m}}$

Example iii UNEB 1999 No 2 b(c)

The viscous force (F) on a small sphere of radius (a) falling through a liquid of coefficient of viscosity ŋ with a velocity V given by F=K$a^x ŋ^y V^z$

Use the method of dimensions to find the values of x, y, z (5marks)

Solution

[ŋ]=$\frac{[Force]}{[Area]x[\,vel\ gradient]}$

[F] = MLT^{-2} and [A] $=L^2$

[Velocity gradient] = $\frac{[V_2 - V_1]}{[l]}$

[Velocity gradient]= $\frac{LT^{-1}}{L}$ $=T^{-1}$

[ŋ] = $\frac{MLT^{-2}}{L^2\,T^{-1}}$

[ŋ] = M $L^{-1}\,T^{-1}$

[F]=[K][a^x][$ŋ^y$][V^z]

MLT^{-2}= L^x (M $L^{-1}T^{-1})^y$ $(LT^{-1})^z$

MLT^{-2} = $M^y L^{x+z-y}\ T^{-y-z}$

For M : $y = 1$……….(1)

For L: $x + z - y = 1$…….(2)

For T: $-y - z = -2$……..(3)

Put equation (1) into equation(3)

$-y - z = -2$

$-1 - z = -2$

$Z = 1$

Put into equation(2)

$x + z - y = 1$

$x + 1 - 1 = 1$

$x = 1$

F=K$a^x ŋ^y V^z$

F = K a ŋ V

Example iv UNEB 2005 No1 b

The equation for the volume V of a liquid flowing through a pipe in time t under a steady flow is given by $\frac{V}{t} = \frac{\pi r^4 P}{8 \eta l}$

Where r = radius of the pipe

l=length of the pipe

P=pressure difference between the 2 ends

ŋ = coefficient of viscosity of the liquid

Show that the equation is dimensionally consistent (3mks)

UNEB 2010 N0 4 (d)

The velocity V of a wave in a material of young modulus E and density ρ is given by

$V=\sqrt{\left(\frac{E}{\rho}\right)}$

Shows that the relationship is dimensionally correct (03 marks)

UNEB2009 No 3b

A cylindrical vessel of cross sectional area, A contains air of volume V, at pressure p trapped by frictionless air tight piston of mass, M. The piston is pushed down and released.

i) If the piston oscillates with simple harmonic motion, shows that its frequency f is given $f = \frac{A}{2\pi}\sqrt{\frac{\mathrm{p}}{\mathrm{MV}}}$

(06 marks)

ii) Show that the expression for f in b(i) is dimensionally correct (03 marks)

UNEB2003 No 1(a)

Distinguish between fundamental and derived physical quantities. Give two examples of each

(04marks)

UNEB2002 No1

a) i)What is meant by the dimension of a physical quantity (01mark)

ii)For a stream line flow of a non-viscous, in compressible fluid, the pressure P at a point is related to the width h and the velocity v by the equation.

(P -a) = ρg(h-b) + ½ ρ (v^2-d) where a, b and d are constant and ρ is the density of the fluid and g is the acceleration due to gravity. Given that the equation is dimensionally consistent, find the dimensions of a, b and d (03 marks)

Solution

NB: We only add and subtract quantities which have the same dimensions.

$(P - a) = \rho g(h - b) + \frac{1}{2}\rho(v^2 - d)$

LHS: [P] = [a]

$$[P] = \frac{[Force]}{[Area]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

[a] = $ML^{-1}T^{-2}$

On the RHS: $[\rho][g][h-b]$

$[\rho][g][h] = [\rho][g][b]$

[h] = [b]

[b] = L

$\frac{1}{2}\rho(v^2 - d)$

$\frac{1}{2}\rho v^2 - \frac{1}{2}\rho d$

$[\frac{1}{2}\rho v^2] = [\frac{1}{2}\rho d]$

$[v^2] = [d]$

$(LT^{-1})^2 = [d]$

$[d] = L^2T^{-2}$

UNEB 2001 No 2 b

The velocity V of sound travelling along a rod made of a material of young's modulus y and density ρ is given by $V=\sqrt{\frac{Y}{\rho}}$ Show that the formula is dimensionally consistent (03 mks)

UNEB 1997 No 1

a) i) What is meant by dimensions of a physical quantity (1mk)

ii)The centripetal force required to keep a body of mass $\boldsymbol{m}$ moving it a circular path of radius $\boldsymbol{r}$ is given by $F= \frac{m\,v^2}{r}$ show that the formula is dimensionally consistent. (04 marks)

CHAPTER2: MOTION

2.1.0: LINEAR MOTION

This is motion is a straight line

Distance

This is the length between 2 fixed point

Displacement

This is the distance covered in a specific direction

Speed

This is the rate of change of distance with time

OR It is the distance covered by an object per unit time.

The SI unit of speed is ms^{-1}

Velocity

It is the rate of change of displacement with time

OR It is the distance covered per unit time in a specific direction

The SI unit of velocity is ms^{-1}

Uniform velocity

Is the velocity of a body which covers equal displacement in equal time intervals.

Acceleration

It is the rate of change of velocity with time

It SI unit is ms^{-2}

Acceleration = $\frac{change\ in\ velocity}{time}$

2.1.1:UNIFORM ACCELERATION & EQUATIONS

Uniform acceleration

Its a motion in which the velocity of a body changes by equal magnitudes in equal time intervals no matter how small the intervals are.

OR Constant rate of change of velocity.

Equations of motion

1st equation

Suppose a body moving in a straight line with uniform acceleration a, increases its velocity from u to v in a time t, then from definition of acceleration

Acceleration = $\frac{change\ in\ velocity}{time}$

$$a = \frac{v-u}{t}$$

$$at = v - u$$

$$v = u + at$$

$\boxed{v = u + a\,t}$……………1

2nd equation

Suppose an object with velocity u moves with uniform acceleration for a time t and attains a velocity v, the distance s travelled by the object is given by

S = average velocity x time

$S = \left(\frac{v+u}{2}\right)t$

But $v = u + at$

$$S = \frac{(u + at + u)}{2}t$$

$$S = \frac{(2u + at)t}{2}$$

$$S = \frac{2ut + at^2}{2}$$

$\boxed{S = ut + \frac{1}{2}\,at^2}$……………………2

3rd equation

From $S = ut + \frac{1}{2}\,at^2$

Since $v = u + at$

$$t = \frac{v-u}{a}$$

S = $u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$

S = $\frac{vu-u^2}{a} + \frac{1}{2}a\,\frac{(v-u)^2}{a^2}$

S = $\frac{vu-u^2}{a} + \frac{1}{2}\,\frac{(v-u)^2}{a}$

S = $\frac{2vu-2u^2+(v-u)^2}{2a}$

S = $\frac{2vu-2u^2+v^2-2uv+u^2}{2a}$

2as = $v^2 - u^2$

$\boxed{v^2 = u^2 + 2as}$…………………3

Note

- The three equations apply only to uniformly accelerated motion
- When the object starts from rest then (u=0m/s) and when it comes to rest (v=0m/s)
- The acceleration can be positive or negative. When its negative, then it known as a retardation or deceleration

Numerical examples

1) A car moving with a velocity of $10ms^{-1}$ accelerates uniformly at $1ms^{-2}$ until it reaches a velocity of $15ms^{-1}$. Calculate,
   i) Time taken
   ii) Distance traveled during the acceleration
   iii) The velocity reached 100m from the place where acceleration began.

Solution

i) $v = u + at$

u=10m/s, a=1m/s, v=$15ms^{-1}$,

$15 = 10 + t$

$t = 5s$

ii) $v^2 = u^2 + 2as$

$15^2 = 10^2 + 2x1xs$

$225 = 100 + 2s$

$S = 62.5m$

iii) $S = 100m$, v=? u=$10ms^{-1}$a=1

$v^2 = u^2 + 2as$

$v^2 = 10^2 + 2x1x100$

$v = 17.32m/s$

2) A particle moving in a straight line with a constant acceleration of $2ms^{-2}$ is initially at rest;

   find the distance covered by the particle in the 3rd second of its motion.

Solution

Using $S = ut + \frac{1}{2}at^2$

u=0m/s, t=2s and t=3s a= $2ms^{-2}$

t=2 : $s = 0x2 + \frac{1}{2}x2x2^2 = 4m$

When t=3: a=$2ms^{-2}$ u=0m/s

$s = 0x3 + \frac{1}{2}x2x3^2 = 9m$

Distance in 3rd

Distance for 3s – distance for 2s

$= 9 - 4 = 5m$

Distance in 3rds in 5m

3)A Travelling car A at a constant velocity of 25m/s overtake a stationery car B. 2s later car B sets off in pursuit , accelerating at a uniform rate of $6ms^{-2}$. How far does B travel before catching up with A

Solution

For A: $S_A = ut + \frac{1}{2}at^2$

Since it moves with a constant velocity a=0

S_A = 25t---------------(1)

For B: $S_B = ut + \frac{1}{2}at^2$

If B is to catch up with A then it must travel faster i.e it will take a time of (t-2)s

$S_B = 0x(t-2)+ \frac{1}{2}x6(t-2)^2$

$S_B = 3t^2-12t+12$…………..(2)

For B to catch A then

$S_A=S_B$

$25t = 3t^2 - 12t + 12$

$3t^2 - 37t + 12 = 0$

$t = \frac{37 \pm \sqrt{37^2 - 4x12x3}}{2x3}$

t=12s or $t = \frac{1}{3}s$

Since the car leaves 2s later then time 12s is correct since it gives a positive value

S_B = 25x12

S_B = 300m

4) A train travelling at $72kmh^{-1}$under goes uniform deceleration of $2ms^{-2}$,when brakes are applied. Find the time taken to come to rest and the distance travelled from the place where brakes are applied.

Solution

$u = \frac{72x1000}{60x60} = 20ms^{-1}$

$a=-2ms^{-2}$, v=0 comes to rest

$v = u + at$

$0 = 20 - -2xt$

$t = 10s$

$s = ut + \frac{1}{2}at^2$

$s = 20x10 + 1/2\ x\text{-}2x10^2$

s= 100m

EXERCISE1

1. A car moving with a velocity of 54km/hr accelerates uniformly at a rate of $2ms^{-2}$. Calculate the distance travelled from the place where acceleration began, given that

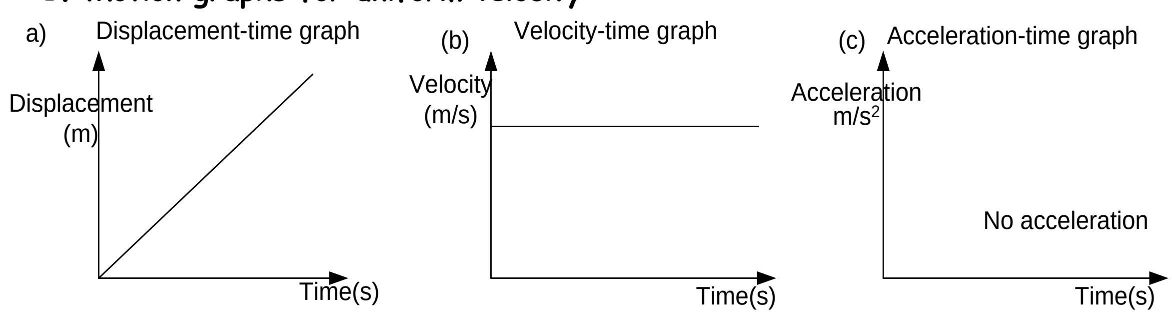
final velocity reached is 72km/hr and find the time taken to cover this distance. **An [$43\frac{3}{4}m$, 2.5s]**

2. A particle x travels with a constant velocity of 6m/s along straight line. It passes a particle y which is stationery. One second later y accelerates at 2m/s. how long after being passed does it take for y to draw level with x? [leave your answer in surd form] **An[t=(3+$\sqrt{15}$)]**

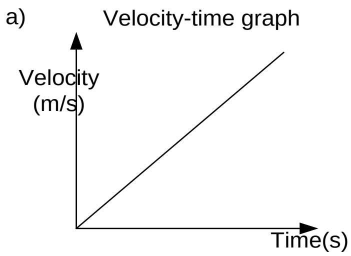
3. A body accelerates uniformly from rest at the rate of 6ms^{-2} for 15 seconds. Calculate
   i) velocity reached within 15 seconds
   ii) the distance covered within 15 seconds **An[90m/s, 675m]**

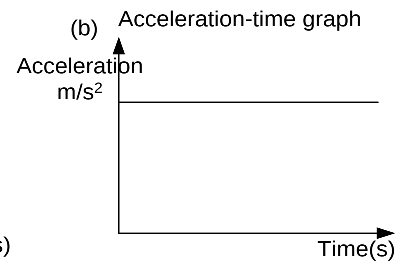
4. A particle moving on a straight line with constant acceleration has a velocity of 5ms^{-1} at one instant and 4s later it has a velocity of 15ms^{-1}. Find the acceleration and distance covered by particle. **An [a = 2.5ms^{-2}, s=40m]**

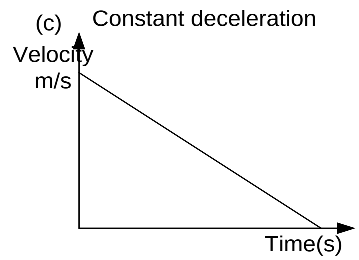
1. **Motion graphs for uniform velocity**



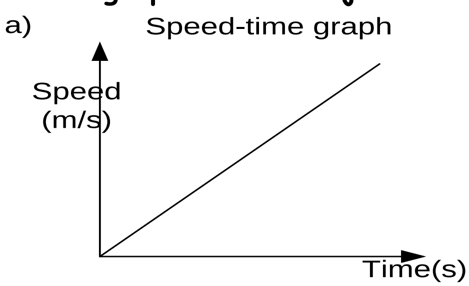
2. **Motion graph for uniform acceleration**

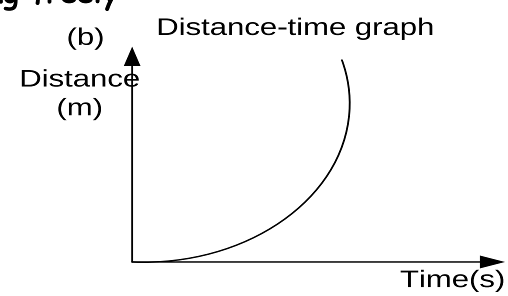




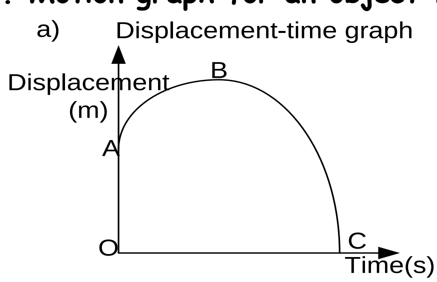


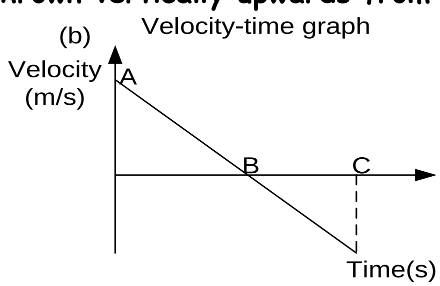
3. Scalar graphs for an object falling freely



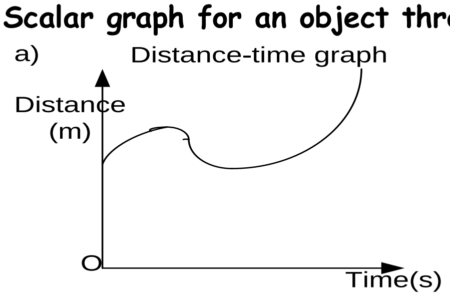


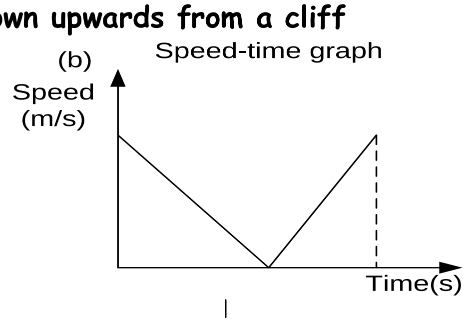
4. Motion graph for an object thrown vertically upwards from the top of a cliff





5. Scalar graph for an object thrown upwards from a cliff





Note

For a body thrown vertically downwards,

$v = u + at$ becomes $v = u + gt$

$S = ut + \frac{1}{2}gt^2$ becomes $S = ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2as$ becomes $v^2 = u^2 + 2gh$

For a body projected vertically upwards

$v = u + at$ becomes $v = u - gt$

$S = ut + \frac{1}{2}gt^2$ becomes $S = ut - \frac{1}{2}gt^2$

$v^2 = u^2 + 2as$ becomes $v^2 = u^2 - 2gh$

2.1.2: MOTION UNDER GRAVITY

1. Vertical motion

a) When a body is projected vertically upwards its acceleration (a) is given $a = -g$=9.81ms^{-2}

g= $-$9.81ms^{-2} means the velocity of a body decreases by a 9.81ms^{-1} after every second till the time when the velocity is zero (maximum height) and the body falls down again with a positive direction due to gravity.

b) An object freely falling vertically downwards has an acceleration of a=+g = +9.81ms^{-2}. This implies that its velocity increases by 9.81ms^{-1} after every second.

Definition

Acceleration due to gravity is rate of change of velocity with time for an object falling freely under gravity.

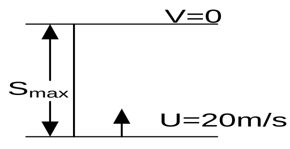
OR The force of attraction due to gravity exerted on a 1kg mass.

Numerical examples

1. A ball is thrown vertically upwards with an initial speed of 20ms^{-1}. Calculate.
    i) Time taken to return to the thrower
    ii) Maximum height reached

solution

u=20ms^{-1} $g = -9.81$ms^{-2} s projected upwards



From $v = u + gt$

At max height $v = 0$

$0 = 20 - 9.81t$

$t = 2.04s$

Time taken to reach maximum height $= 2.04s$

But the total time ta ken to return to the thrower = 2t

$= 2x2.04$

$= 4.08s$

$v^2 = u^2 + 2gs$

at max height v=0m/s, u=20m/s,

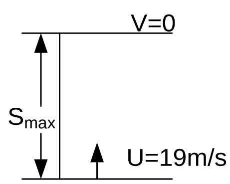
$g = -9.81$ms^{-2}

$0^2 = 20 - 2x9.81s_{max}$

$s_{max} = 20.39m$

2. A particle is projected vertically upwards with velocity of $19.6ms^{-1}$. Find
   i) The greatest height attained
   ii) Time taken by the particle to reach maximum height
   iii) Time of flight

Solution



At greatest height $v = 0m/s$

$g = -9.81ms^{-2}$

$v^2 = u^2 + 2gs$

$0^2 = 19.6^2 - 2x9.81\ s_{max}$

$s_{max} = \frac{19.6^2}{2x9.81}$

s_{max} =19.58m

ii) From $v = u + gt$

$u = 19.6, g\ -9.81ms^{-2}$ $v = 0$ at max height

$0 = 19.6 - 9.81t$

$t = 1.998s$

t ≈ 2.0s Time to maximum height = $2.0s$

iii) Time of flight

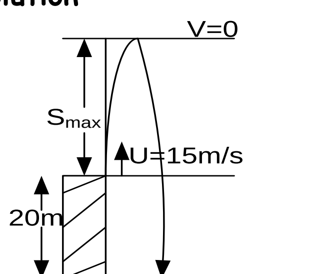
Time of flight = $2x$ time to max height

$= 2x2$

Time of flight $= 4.0s$

3. A man stands on the edge of a cliff and throws a stone vertically upwards at $15ms^{-1}$. After what time will the stone hit the ground 20m below the point of projection

Solution



u=15m/s, g=-9.81m/s

v=0m/s at max height, s_{max}=? t=?

Method I

$v = u + gt$

$0 = 15 - 9.81t$

$t = 1.53s$

Time to maximum height = 1.53s

$v^2 = u^2 + 2gs$

0 = 15^2–2x9.81s_{max}

$s_{max} = \frac{15^2}{2x9.81}$

s_{max}=11.47m

Maximum height = 11.47m

Total height = (11.47+20) = 31.47m

When the ball begins to return down from max height u=0m/s and a=g=+9.81m/s

$S = ut + \frac{1}{2}gt^2$

$31.47 = 0xt + \frac{1}{2} x9.81t^2$

$t = \sqrt{\frac{31.47x2}{9.81}}$

$t = 2.53s$

Total time = $(2.53 + 1.53) = 4.06s$

Time taken to hit the ground = $4.06s$

Method II

The height of the cliff = 20m which is below the point of project therefore

s=-2m g=-9.81m/s^2 u=15m/s

$S = ut - \frac{1}{2}gt^2$

$-20 = 15t - \frac{1}{2} x9.81t^2$

$-20 = 15t - 4.905t^2$

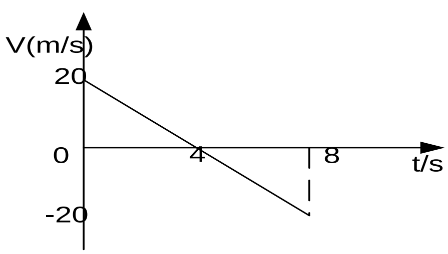
$t = 4.06s$

Time taken to hits the ground = 4.06s

4. A car decelerates uniformly from 20ms^{-1} to rest in 4s, then reverses with uniform acceleration back to it original starting point also in 4s
   a) Sketch the velocity-time graph for the motion, and use it to determine the displacement and average velocity
   b) Sketch the speed-time graph for the motion and use it to determine the total distance covered and the average speed.

Solution

a)Velocity-time graph



Displacement s = $\frac{1}{2} bh + \frac{1}{2} bh$

= $\frac{1}{2} x4x20$ + $\frac{1}{2} x4x(-20)$

$= 40 - 40$

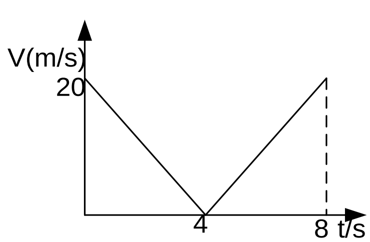
$s = 0m$

Average velocity = $\frac{total\ displacement}{total\ time}$

= $\frac{0}{8}$

= 0m/s

b) Speed-time graph



Total distance = $\frac{1}{2} x20x4 + \frac{1}{2} x20x4$

$= 40 + 40$

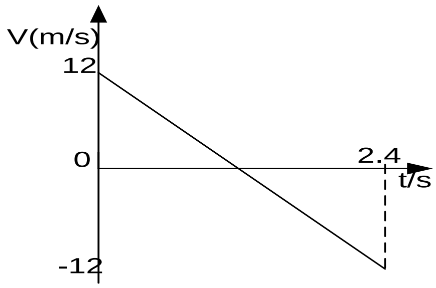
= 80m

Average speed = $\frac{total\ distance}{total\ time}$

= $\frac{80}{8}$

= 10ms^{-1}

5. The graph below shows a ball which is thrown up into the air and then caught



a) Describe the motion briefly

b) Calculate the acceleration

c) Find the height reached (above the point where the ball starts)

Solution

a) The ball decelerates uniformly from $12ms^{-1}$ to rest in 1.2s and then it reverses with uniform acceleration back to its original position in another 1.2s

b) $v=-12ms^{-1}$ $u=12ms^{-1}$ $t=2.4s$

$v = u + at$

$-12 = 12 + 2.4a$

$a = -10ms^{-2}$

$S = ut + \frac{1}{2}at^2$

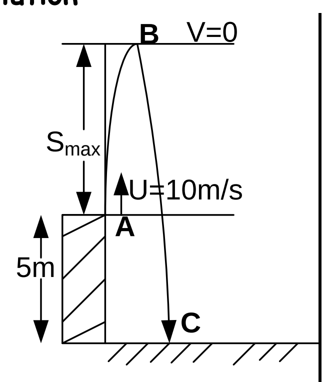
$S = 12x2.4 + \frac{1}{2}x - 10x2.4^2$

S = 0m

UNEB 2003 No 1 b(ii)

A ball is thrown vertically upwards with a velocity of $10ms^{-1}$ from a point 50m above the ground. Describe with the aid of a velocity - time graph, the subsequent motion of the ball. (10marks)

Solution



Time to reach max height

$v=0$, $u=10ms^{-1}$ $g=-9.81ms^{-2}$

$v = u + gt$

$0 = 10 - 9.81t$

$t = 1.02s$

Time to reach maximum height is 1.02s

At Max height v=0

$u=10ms^{-1}$ $g=-9.81ms^{-2}$

$v^2 = u^2 + 2gs$

$0 = 10^2 - 2x9.81Smax$

$S_{max} = 5.1m$

Total height = (5.1+5)

$= 10.1m$

Time taken to move from max height to the ground is

t=?, u=0m/s g=9.81ms^{-2}

$$S = ut + \frac{1}{2}gt^2$$

$$10.1 = 0xt + \frac{1}{2}x9.81t^2$$

$$t = \sqrt{\frac{20.2}{9.81}}$$

$$t = 1.43s$$

Final velocity when the ball hits the ground v=?

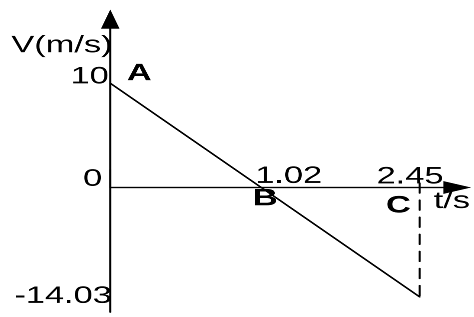
$$u = 0, t = 1.43s, g = 9.81ms^{-1}$$

$$v = ut + gt$$

$$v = 0 + 9.81x1.43$$

v=14.03m/s

Vel-time graph



- ✓ When the ball is thrown vertically upwards with a velocity of 10ms^{-1} it decelerates uniformly at -9.81ms^{-1} and its velocity reduces to zero at B(maximum height).
- ✓ The time taken to reach maximum height B is 1.02s and the maximum height is 5.1m
- ✓ After reaching the maximum height, the ball begins to fall downwards with a uniform acceleration of 9.81m/s^2 but the direction is now opposite and therefore the velocity is negative until it reaches a final velocity of 14.03m/s in a time of 2.45s from the time of projection.

UNEB 2014 No 1(c)

(i) State **Newton's laws of motion** (03marks)

(ii) Explain how a rocket is kept in motion (04marks)

(iii) Explain why passengers in a bus are thrown backwards when the bus suddenly starts moving. (03marks)

UNEB 2013 No 3(d)

(i) Define uniformly accelerated motion (03marks)

(ii) A train starts from rest at station **A** and accelerates at 1.25 m s^{-2} until it reaches a speed of 20 m s^{-1}. It then travels at this steady speed for a distance of 1.56km and then decelerates at 2 m s^{-2} to come to rest at station **B**. Find the distance from **A** and **B**

An (1 820m) (04marks)

UNEB 2011 No 1(a)

Define the following terms

(i) Uniform acceleration (01mark)

(ii) Angular velocity (01 mark)

UNEB 2010 No 1(d)

(i) Define uniform acceleration (01 mark)

(ii) With the aid of a vel-time graph, describe the motion of a body projected vertically upwards

(03 marks)

UNEB 2009 No 2

a) Define the following terms

(i) Velocity

(ii) Moment of a force (02marks)

b) i) A ball is projected vertically upwards with a speed of $50ms^{-1}$, on return it passes the point of projection and falls 78m below. Calculate the total time taken

An(11.57s)

(05 marks)

UNEB 2008 No 1(a)

i) Define the terms velocity and displacement (02 marks)

ii) Sketch a graph of velocity against time for an object thrown vertically upwards

(02

marks)

UNEB 2007 No 4(b)

(i) What is meant by acceleration due to gravity

UNEB 2006 No 1

a) i) What is meant by uniformly accelerated motion (01 mark)

ii) Sketch the speed against time graph for a uniformly accelerated body (01 mark)

b) (i) Derive the expression $S = ut + \frac{1}{2}at^2$

For the distance S moved by a body which is initially travelling with speed u and is uniformly accelerated for time t

(04 marks)

UNEB 1993 No 1

(a) Define the terms

(i) Displacement

(ii) Uniform acceleration

(b) i) A stone thrown vertically upwards from the top of a building with an initial velocity of 10m/s. the stone takes 2.5s to land on the ground.

ii) Calculate the height of the building

(iii) State the energy changes that occurred during the motion of the stone (03 marks)

EXERCISE2

1. A ball is dropped from a cliff top and takes 3s to reach the beach below. Calculate
   a) The height of the cliff

      An(44.1m)

b) Velocity acquired by the ball

An(29.4m/s)

2. With what velocity must a ball be thrown upwards to reach a height of 15m

An(17.1ms^{-1})

3. A body is thrown upwards with an initial velocity at *2m/s*. Calculate;

(i) The maximum height attained

(ii) Time taken to reach the maximum height.

4. An object is dropped from a helicopter. If the object hits the ground after *2s*. Calculate the height from which the object was dropped.

5. A body is thrown upwards with an initial velocity of *12m/s*.

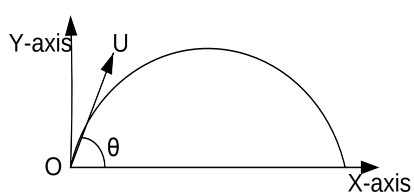
(i) Calculate the maximum height it attained.

(ii) Time taken by the body to return to the ground

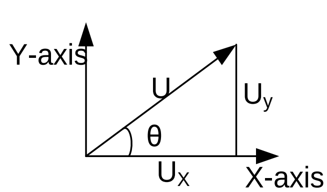
2. PROJECTILE MOTION

This is the motion of a body which after being given an initial velocity moves under the influence of gravity.

Consider a ball projected at O with an initial velocity um/s at an angle θ to the horizontal.



Resolution of velocity



From the figure: $sin\,\theta = \frac{u_y}{u}$

u_y = $usin\,\theta$ --------------------(1)

Also: $cos\,\theta$ = $\frac{u_x}{u}$

u_x = $ucos\,\theta$ --------------------(2)

Equation (1) is the initial vertical component of velocity

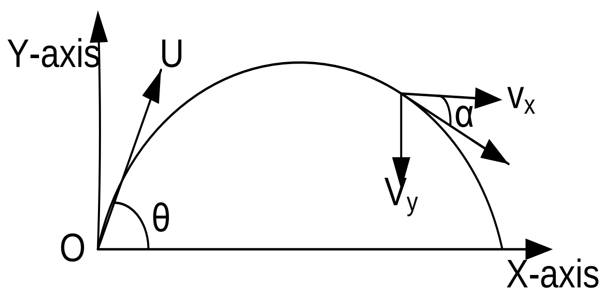
Equation (2) is the initial horizontal component of velocity

Note

The horizontal component of velocity [u_x = $ucos\ \theta$] is constant through the motion and therefore the acceleration is zero.

MATHEMATICAL FORMULAR IN PROJECTILES

All formulas in projectiles are derived from equations of linear motion



Finding velocity at any time t.

Horizontally: $v = u_x + at$

u_x = $ucos\ \theta$,

a=0 (constant velocity)

$\boxed{v_x = u\cos\theta}$

Vertically: $v = u_y + at$

u_y = $usin\ \theta$

a= $-g$

$\boxed{v_y = u\sin\theta - gt}$

Velocity at any time t

$\boxed{v = \sqrt{V_x^2 + V_y^2}}$

Direction of motion

$\boxed{\alpha = tan^{-1}\frac{V_y}{V_x}}$ to the horizontal

Finding distances at any time t

horizontally : $s_x = u_x t + \frac{1}{2}at^2$

$u_x = u\cos\theta$, a= 0

$\boxed{x = u\cos\theta\, t}$

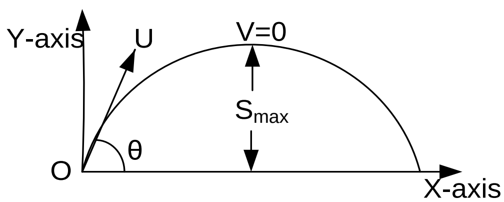
Vertically: $s_y = u_y t + \frac{1}{2}at^2$

$u_y = u\sin\theta,\ a = g$

$$\boxed{y = u\sin\theta t - \frac{1}{2}gt^2}$$

TERMS USED IN PROJECTILES

1. MAXIMUM HEIGHT [GREATEST HEIGHT] [S_{max}]



For vertical motion : at max height

v=0, $u_y = u\sin\theta,\ a = -g,$

s= S_{max}

$${v_y}^2 = {u_y}^2 + 2gs$$

$$0 = (u\sin\theta)^2 - 2gS_{max}$$

$$2gS_{max} = u^2\sin^2\theta$$

$$\boxed{S_{max} = \frac{u^2\sin^2\theta}{2g}}$$

Note : $\sin^2\theta = (\sin\theta)^2$ but $\sin^2\theta \neq \sin\theta^2$

2. TIME TO REACH MAX HEIGHT [t]

Vertically $v = u_y + at$ at max height

v=0

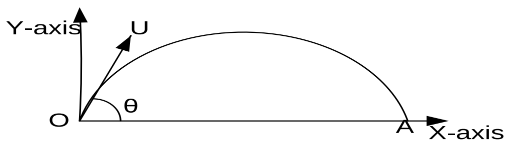
$u_y = u\sin\theta\ , a = g$

$$0 = u\sin\theta - gt$$

$$\boxed{t = \frac{u\sin\theta}{g}}$$

3. TIME OF FLIGHT [T]

It refers to the total time taken by the projectile to move from the point of projection to the point where it lands on the horizontal plane through the point of projection.



Vertically: $S_y = u_y t + \frac{1}{2} at^2$
at point A when the projectile return

to the plane S_y=0,

t=T(time of flight), $a = -g \quad u_y$

$= u \sin\theta$

$$0 = usin\theta T - \frac{gT^2}{2}$$

$T\left(usin\theta - \frac{gT}{2}\right) = 0$

Either T=0 or $\left(usin\theta - \frac{gT}{2}\right) = 0$

$\left(usin\theta - \frac{gT}{2}\right) = 0$

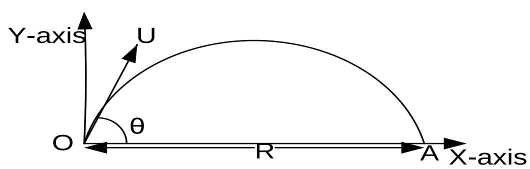
$$usin\theta = \frac{gT}{2}$$

$$\boxed{T = \frac{2\,u\,sin\theta}{g}}$$

Note: The time of flight is twice the time to maximum height

4. RANGE [R]

It refers to the horizontal distance from the point of projection to where the projectile lands along the horizontal plane through the point of projection.



Neglecting air resistance the horizontal component of velocity $\boldsymbol{ucos\theta}$ remains constant during the flight

Horizontally : $S_x = u_x t + \frac{1}{2} at^2$

$u_x = ucos\theta$, a=0 (constant velocity),

t=T

$$R = ucos\theta\, T + ½\, x 0 x T^2$$

$$R = ucos\theta T$$

But T = $\frac{2\,u\,sin\theta}{g}$

R = $\boldsymbol{\frac{u^2\, 2sin\theta\, cos\theta}{g}}$

But from trigonometry $2sin\theta cos\theta = sin2\theta$

$$\boxed{R = \frac{u^2\, Sin\, 2\theta}{g}}$$

5. MAXIMUM RANGE [R_{max}]

For maximum range sin2θ =1, R=R_{max}

2 θ= $sin^{-1}(1)$

2 θ = 90

$$R_{max} = \frac{u^2 sin90}{g}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

6. EQUATION OF A TRAJECTORY

A trajectory is a path described by a projectile.

A trajectory is expressed in terms of horizontal distance x and vertical distance y.

For horizontal motion at any time t

$x = ucos\,\theta t$

t = $\frac{x}{ucos\theta}$ -----------------[1]

For vertical motion at any time t

$y = usin\theta t - ½\, g t^2$ -------------[2]

Putting t into equation [2]

y = $usin\theta\ \frac{x}{ucos\theta} - \frac{1}{2} g \left(\frac{x}{ucos\theta}\right)^2$

$$\boxed{y = x\frac{u\sin\theta}{\cos\theta} - \frac{gx^2}{2u^2\cos^2\theta}}$$

Either $\boxed{y = xtan\theta - \frac{gx^2}{2u^2COS^2\theta}}$

Or

$$\boxed{y = xtan\theta - \frac{g\,x^2\,Sec^2\theta}{2\,u^2}}$$

Where $\frac{1}{cos^2\theta}$ = $sec^2θ$

Hint

If a particle is projected directly (vertically) upwards the motion it describes is purely kinematics (motion under gravity). But if it is projected at an angle to the horizontal, the motion is parabolic (projectile motion).

Numerical examples

A. Objects projected upwards from the ground at an angle to the horizontal

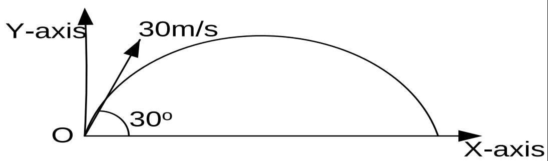
1. A Particle is projected with a velocity of 30ms^{-1} at an angle of elevation of 30^0. Find

   i) The greatest height reached

   ii) The time of flight

   iii) The velocity and direction of motion at a height of 4m on its way downwards

Solution



u = 30m/s , θ = 30^0,

g=9.81ms^{-1}

i) $Smax \ = \frac{u^2 \ sin^2\theta}{2g}$

$\boldsymbol{S}max$ =$\frac{30^2 sin^2 30}{2x9.81}$

$Smax$ = 11.47m

ii) **T** = $\frac{\boldsymbol{2\ u\ sin\theta}}{\boldsymbol{g}}$

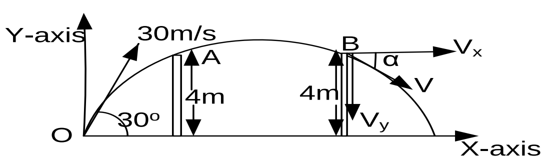
T =$\frac{2\ x\ 30\ sin30}{9.81}$

T= 3.06s

iii) R= $\frac{u^2\ sin2\theta}{g}$

R= $\frac{30^2\ sin2x30}{9.81}$

R= 79.45m



For vertical motion

$y \ = \ usin\theta t - \ ½ \ gt^2$

$4 \ = \ 30sin30t$ – ½ $9.81x \ t^2$

$4.905t^2 - 15t + 4 \ = 0$

$t = 2.76s$ or $t = 0.30s$

The value of t=0.30s is the correct time since it's the smaller value for which the body moves upwards.

$v_x = ucos\theta$

$v_x = \ 30cos \ 30$

$v_x \ = \ 25.98m/s$

$v_y = usin\theta - gt$

$v_y = \ 305x30 - 9.81x0.30$

v_y= 12.06m/s

$v = \ \sqrt{V_x^2 + \ V_y^2}$

$v = \sqrt{25.98^2 + 12.06^2}$

$v = \ 28.64m/s$

Direction : $\alpha = tan^{-1}\frac{V_y}{V_x}$

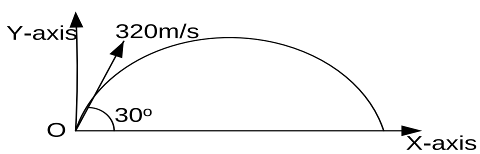
$\alpha = tan^{-1}\frac{12.06}{25.98}$

α =24.9^0 to the horizontal

Velocity is 28.64m/s at 24.9° to horizontal

2. A projectile is fired with a velocity of 320m/s at an angle of 30^0 to the horizontal. Find
   (i) time to reach the greatest height
   (ii) its horizontal range
   (iii) maximum range

Solution



i) At max height v=0,

$$v = usin\theta - gt$$

$$0 = 320sin30 - 9.81t$$

$$t = \frac{320sin30}{9.81}$$

$$t = 16.31s$$

ii) range R=$ucos\theta x$time of flight

Time of flight =twice time to max height

R= $320cos30x\ [2x16.31]$

R= 9039.92m

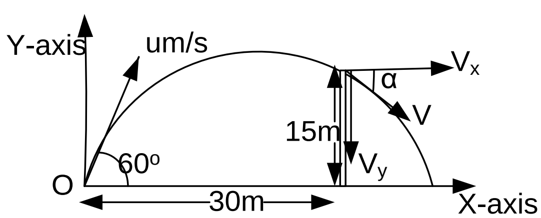
iii) max range

$$R_{max} = \frac{u^2}{g}$$

$$R_{max} = \frac{320^2}{9.81} = 10438.33\text{m}$$

3. A projectile fired at an angle of 60^0 above the horizontal strikes a building 30m away at a point 15m above the point of projection. Find
   (i) Speed of projection
   (ii) Velocity when it strikes a building

Solution



i) Horizontal distance at time t

$$x = ucos\theta t$$

$$30 = utcos60$$

$$t = \frac{60}{u}$$

Also vertical distance at any time t

$$y = usin\theta - \frac{1}{2}gt^2$$

$$15 = usin60x\frac{60}{u} - \frac{1}{2}x9.81\left(\frac{60}{u}\right)^2$$

15 = 51.96152423 $- \frac{4.905\ x\ 3600}{u^2}$

$u^2 = \frac{-4.905\ x\ 3600}{(15-51.96152423)}$

$u = \sqrt{477.7400383}$

u= 21.86m/s

ii)but since t =$\frac{60}{u}$

$$t = \frac{60}{21.86} = 2.75s$$

$v_x = ucos\theta$

$v_x = 21.86cos60$

$v_x = 10.93m/s$

$v_y = usin\theta - gt$

$v_y = 21.81sin60 - 9.81x2.75$

$V_y = -8.09m/s$

velocity at any time

$$v = \sqrt{V_x^2 + V_y^2}$$

$$v = \sqrt{10.93^2 + 8.09^2}$$

v =13.60m/s

$$\alpha = tan^{-1}\frac{V_y}{V_x}$$

$\alpha = tan^{-1}\ \frac{8.09}{10.9}$

α =36.58^0

The velocity is 13.60m/s at 36.58^0 to the horizontal

Alternatively

Using the equation of a trajectory

$$y = xtan\theta - \frac{gx^2}{2u^2COS^2\theta}$$

y=15m, x=30m, $\theta = 60°$, u=?

$$15 = 30tan60 - \frac{9.81x30^2}{2u^2Cos^260}$$

$$15 = 51.96152423 - \frac{17658}{u^2}$$

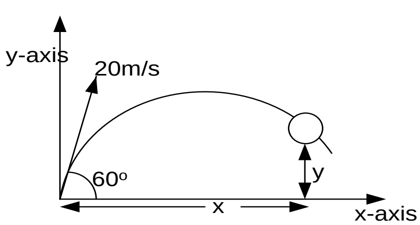
$$u^2 = \frac{17658}{(51.9652423 - 15)}$$

$u = \sqrt{477.7400383}$

u= 21.86m/s

4. A body is projected at an angle of 60^0 above horizontal and passes through a net after 10s. Find the horizontal and vertical distance moved by the body after it, was projected at a speed of 20m/s

Solution



Horizontal motion : $x = ucos\theta xt$

$x = 20cos60x10$

$x = 100m$

Vertical motion; $y = usin\theta t - ½\ gt^2$

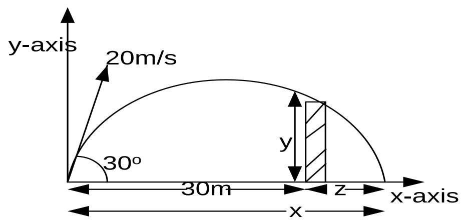
$y = 20(sin60)x10 - ½\ x9.81x10^2$

$y = -317.29m$

UNEB MARCH 1998 N0 1c

5. A ball is kicked from the spot 30m from the goal post with a velocity of 20m/s at 30^0 to the horizontal. The ball just clears the horizontal bar of a goal post. Find;
   (i) Height of the goal post
   (ii) How far behind the goal post does the ball land

Solution



horizontal motion : $x = ucos\theta xt$

$$30 = 20cos30xt$$

$$t = 1.732s$$

For vertical motion: $y = usin\theta t - ½ gt^2$

$$y = (20sin30)x1.732 - \frac{1}{2}x9.81x(1.732)^2$$

$$y = 2.61m$$

Height of the goal post = 2.61m

ii) Time of flight

$$T = \frac{2 u \sin\theta}{g} = \frac{2x20xsin30}{9.81}$$

T = 2.04s

iii) Horizontal distance: $x = ucos\theta xt$

$x = 20c\square s30x2.04$ =35.33m

but $x = 20 + z$

$$35.33 = 30 + z$$

$z = 35.33 - 30$

z = 5.33m The ball 5.33m behind the goal

EXERCISE 3

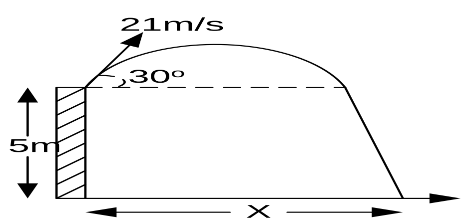
1. A particle is projected at an angle of 60° to the horizontal with a velocity of 20m/s. calculate the greatest height the particle attains **An[15.29m]**
2. A particle is projected from level ground towards a vertical pole, 4m high and 30m away from the point of projection. It just passes the pole in one second. Find
   i) Its initial speed and angle of projection **An [39.29m/s, 16.5^0]**
   ii) The distance beyond the pole where the particle will fall **An [24.42m]**
3. A stone thrown upwards at an angle θ to the horizontal with speed ums^{-1} just clears a vertical wall 4m high and 10m from the point of projection when travelling horizontally. Find the angle of projection **An[38.66^0]**

4. A particle is projected with a velocity of 30m/s at an angle of 40^0 above the horizontal plane. find ;
    a) The time for which the particle is in the air.
    b) The horizontal distance it travels **An [3.9s, 22.9m/s]**

B. **Objects projected upwards from a point above the ground at an angle to the horizontal**

1. A particle is projected at an angle of elevation of 30^0 with a speed of 21m/s. If the point of projection is 5m above the horizontal ground, find the horizontal distance that the particle travels before striking the ground

Solution



u= -5m since it's below the point of projection

For vertical motion: $y = usin\theta t - ½ gt^2$

$-5 = 21sin30t$ - $\frac{9.81t^2}{2}$

$4.905t^2 - 10.5t - 5 = 0$

t= $2.54s$ or t $= -0.40s$

Time of flight t=2.54s

For horizontal motion

$x = ucos\theta xt$

$x = 21(cos30)\, x2.54$

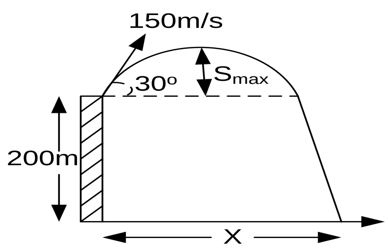
$x = 46.19m$

The horizontal distance = 46.19m

Alternatively use equation of trajectory but not y=-5m

2. A bullet is fired from a gun placed at a height of 200m with a velocity of $150ms^{-1}$ at an angle of 30^0 to the horizontal find
i) Maximum height attained
ii) Time taken for the bullet to hit the ground

Solution



i) $Smax = \frac{u^2 \, sin^2\theta}{2g}$

$$Smax = \frac{150^2 \, sin^2 30}{2x9.81}$$

$Smax$ = 286.70m

The max height attained is 286.70m from the point of projection

ii) Time taken for the bullet to hit the ground

Vertical motion : $y = usin\theta t - ½ \, gt^2$

$y = -200m$ since its below the point of projection

$-200 = 150sin30t - ½ \, x9.81t^2$

$-200 = 75t - 4.905t^2$

$t = 17.61s \; or \; t = -2.32s$

Time taken is 17.61s

EXERCISE 4

1. A particle is projected with a velocity of $10ms^{-1}$ at an angle of 45^0 to the horizontal, it hits the ground at a point which is 3m below its point of projection. Find the time fro which it is in the air and the horizontal distance covered by the particle in this time **An[1.76s, 12.42m]**

2. A pebble is thrown from the top of a cliff at a speed of 10m/s and at 30^0 above the horizontal. it hits the sea below the cliff 6.0s later , find;
   a) The height of the cliff . **An[150m, 52m]**
   b) The distance from the base of the cliff at which the pebble falls into the sea.

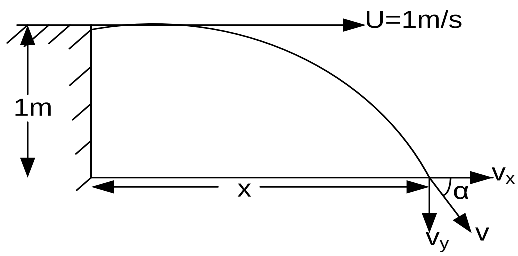
3. A pencil is accidentally knocked off the edge of a horizontal desktop. The height of the desk is 64.8cm and the pencil hits the floor a horizontal distance of 32.4cm from the edge of the desk, What was the speed of the pencil as it left the desk. **An[$0.9ms^{-1}$]**

C. An object projected horizontally from a height above the ground

Examples

1. A ball rolls off the edge of a table top 1m high above the floor with a horizontal velocity $1ms^{-1}$. Find;
   i) The time it takes to hit the floor
   ii) The horizontal distance it covered
   iii) The velocity when it hits the floor

Solution



u=$1ms^{-1}$ θ=0^0 y=-1m below the point of projection

vertical motion: $y = usin\theta t - ½ gt^2$

$-1 = 1xsin0xt - ½ x9.81t^2$

$-1 = -4.905t^2$

$t = 0.45s$

ii) $x = ucos\theta t$

$x = 1xcos0x0.45$

$x = 0.45m$

iii) velocity when it hits the ground

$v_x = ucos\theta$

$v_x = 1cos0$

$v_x = 1m/s$

$v_y = usin\theta - gt$

$v_y = 1sin0 - 9.81x0.45$

$v_y = -4.4m/s$

$\boldsymbol{v} = \sqrt{V_x^2 + V_y^2}$

$v = \sqrt{(1)^2 + (-4.4)^2}$

$v = 4.5m/s$

Direction: $\alpha = tan^{-1}\frac{V_y}{V_x}$

$\alpha = tan^{-1}\frac{4.4}{1}$

α =77.2^0

The velocity is 4.5m/s at 77.2° to the horizontal

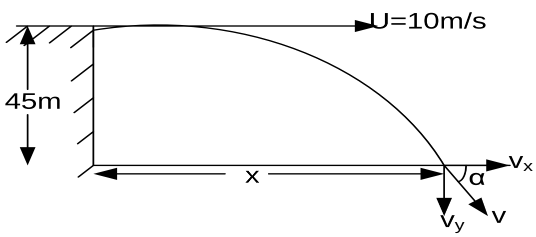
2. A ball is thrown forward horizontally from the top of a cliff with a velocity of 10m/s. the height of a cliff above the ground is 45m. calculate

i) Time to reach the ground

ii) Distance from the cliff where the ball hits the ground

iii) Direction of the ball just before it hits the ground

Solution



u=10ms^{-1} θ=0^0 y=-45m below the point of projection

For vertical motion

$$y = usin\theta t - ½\, gt^2$$

$$-45 = 10xsin0xt - ½\, x9.81t^2$$

$$-45 = -4.905t^2$$

$t = 3.03s$

ii) $x = ucos\theta t$

$$x = 10xcos0x3.03$$

$$x = 30.3m$$

iv) velocity when it hits the ground

$v_x = ucos\theta$

$v_x = 10cos0$

$v_x = 10m/s$

$v_y = usin\theta - gt$

$v_y = 10sin0 - 9.81x3.03$

$v_y = -4.4m/s$

$v_y = 29.72m/s$

$$v = \sqrt{V_x^2 + V_y^2}$$

$$v = \sqrt{(10)^2 + (29.72)^2}$$

v =$\sqrt{983.2784}$ m/s

$$\alpha = tan^{-1}\frac{V_y}{V_x}$$

$$\alpha = tan^{-1}\frac{29.72}{10}$$

α=71.4^0

The velocity is V =$\sqrt{983.2784}$ m/s at 71.4° to the horizontal

UNEB 2014 No1 (a)

(i) What is a **projectile motion** (01marks)

(ii) A bomb is dropped from an aero plane when it is directly above a target at a height of 1402.5m. the aero plane is moving horizontally with a speed of 500kmh^{-1}. Determine whether the bomb will hit the target. **An (misses target by 2347.2m)** (05marks)

UNEB 2012 No 3 (d)

(i) Derive an expression for maximum horizontal distance travelled by a projectile in terms of the initial speed u and the angle of projection θ to the horizontal [02 marks]

(ii) Sketch a graph to show the relationship between kinetic energy and height above the ground in a projectile.

UNEB 2010 No (d)

iii) Calculate the range of a projectile which is fired at an angle of 45^0 to the horizontal with a speed of 20m/s **An [40.77m]**

UNEB 2009 No 1 (d)

A stone is projected at an angle of 20^0 to the horizontal and just clears a wall which is 10m high and 30m form the point of projection. Find the;

i) Speed of projection (04marks)

ii) Angle which the stone makes with the horizontal as it clears the wall (03marks)

An[73.78m/s, 16.9^0]

UNEB 2006 No 1 (c)

A projectile is fired horizontally from the top of a cliff 250m high. The projectile lands 1.414×10^3m from the bottom of the cliff. Find the

i) Initial speed of the projectile (05 marks)

ii) Velocity of the projectile just before it hits the ground (05 marks)

An [198m/s, 210m/s at 19.5^0]

UNEB 2000 No 3 (b)

(i) Define the terms time of flight and range as applied to projectile motion (02 marks)

(ii) A projectile is fired in air with a speed um/s at an angle θ to the horizontal. Find the time of flight of the projectile (02marks)

MARCH UNEB 1995 No 1

a)i) write the equation of uniformly accelerated motion (03 marks)

ii) Derive the expression for the maximum horizontal distance travelled by a projectile in terms of the initial speed u and the angle of projectile θ to horizontal (04 marks)

b) A bullet is fired from a gun placed a height of 200m with a velocity of 150m/s at an angle of 30^0 to the horizontal. Find

i) The maximum height attained

ii) The time for the bullet to hit the ground (07marks)

CHAPTER 3: COMPOSITION AND RESOLUTION OF VECTORS

3.1.0: VECTOR QUANTITY

It is a physical quantity with both magnitude and direction.

Example; displacement, velocity, acceleration, force, weight and momentum

3.1.2: SCALAR QUANTITY

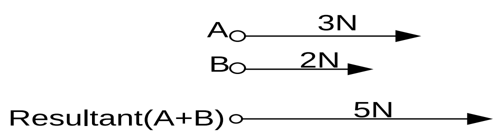
It is a physical quantity with only magnitude.

Example; distance, speed, time, temperature, mass and energy

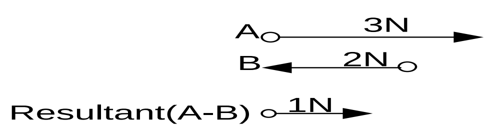
3.1.3: VECTOR ADDITION

A. Vectors acting in the same line

i) If vectors are acting in the same direction then resultant along that direction is just the sum of the two vectors.



ii) If they are moving in the opposite direction then, the resultant is difference of the vectors but along the direction of the bigger vector.



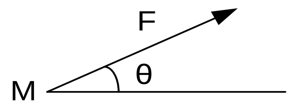
B . vectors acting at an angle

With vectors inclined at an angle to each other, a triangle of vectors is used to find the resultant. The resultant given by the line that completes the triangle.

Components of a vector

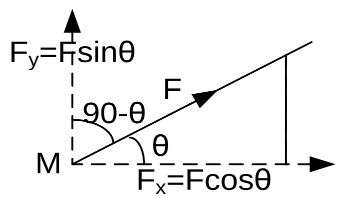
The component of a vector is the effective value of a vector a long a particular direction. The component along any direction is the magnitude of a vector multiplied by the **cosine of the angle** between its direction and the direction of the component.

Suppose a force F pulls a body of mass m along a truck at an angle θ to the horizontal as shown below;



The effective force that makes the body move along the horizontal is the component of F along the horizontal

$F_x = Fcos\theta$ ------------------------------ -[1]



$F_y = Fcos(90 - \theta)$

$F_y = F(cos90cos\theta + sin90sin\theta)$

$F_y = F(0xcos\theta + 1sin\theta)$

$F_y = Fsin\theta$ ------------------**[2]**

Alternatively

$Cos\theta = \frac{F_x}{F}$

$F_x = Fcos\theta$

$Sin\theta = \frac{F_y}{F}$

$F_y = Fsin\theta$

Hints

When a vector is inclined at an angle θ to the horizontal then;

- along the horizontal, the component of the vector is $cos\theta$
- along the vertical, the component of the vector is $sin\,\theta$

When a vector is inclined at θ to the vertical then;

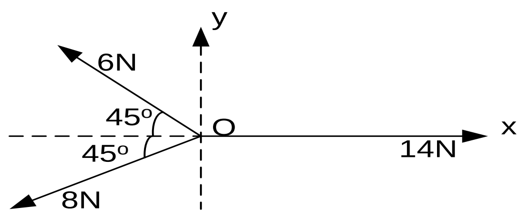
- along the horizontal, the component of the vector is $sin\,\theta$
- along the vertical, the component of the vector is $cos\theta$

$$\boxed{\text{Resultant vector } \boldsymbol{F_R} = \sqrt{\boldsymbol{F_x^2} + \boldsymbol{F_y^2}}}$$

$$\boxed{\text{Direction } \theta = \boldsymbol{tan^{-1}} \frac{\boldsymbol{F_y}}{\boldsymbol{F_x}}}$$

Examples

1. Three forces are applied to a point as shown below



a) The component in directions Ox and Oy respectively

b) Resultant force acting at O

Calculate

Solution

Components along Ox

F_x

$= 14 - 6cos45$

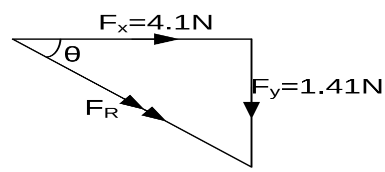
$- 8cos45$

$F_x = 4.10N$

Component along Oy

$F_y = 6sin45 - 8sin45$

$F_y = -1.41N$



$F_R = \sqrt{F_x^2 + F_y^2}$

$F_R = \sqrt{4.1^2 + 1.41^2}$

$F_R = 4.34N$

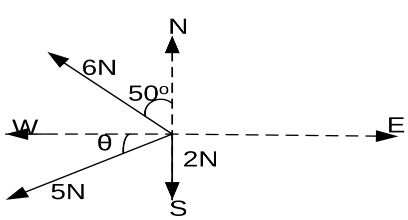
Direction $\theta = tan^{-1}\frac{F_y}{F_x}$

$\theta = tan^{-1}\frac{1.41}{4.1}$

$\theta = 19.0^0$ Below the horizontal

Resultant force is 4.34N at 19.0 below the horizontal

2. A particle is acted upon by three coplanar forces, the resultant force is in the direction due west



Calculate the value of θ to the nearest degree

Solution

Component along $E - W$

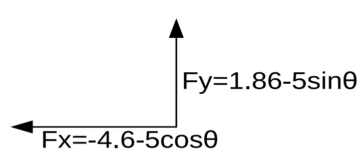
$F_x = -6sin50 - 5cos\theta$

$F_x = -4.6 - 5cos\theta$

Components along $N - S$

$F_y = 6cos50 - 2 - 5sin\theta$

$F_y = 1.86 - 5sin\theta$



Since the resultant force is due west, then the north – south component is zero

i.e. $F_y = 0$

$1.86 - 5sin\theta = 0$

$Sin\theta = \frac{1.86}{5}$

$\theta = sin^{-1}\left(\frac{1.86}{5}\right)$

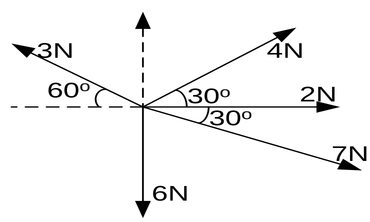
$\underline{\underline{\theta = 21.84^0}}$

3. Forces of 2N, 4N, 3N, 6N, and 7N act on a particle in the direction 0^0, 30^0, 120^0, 270^0 and 330^0 respectively. Find the magnitude and direction of a single force represented by the above forces.

Solution

Note.

The direction given has 1, 2 , 3digit values therefore, the direction is in form of angles and hence they should be read anticlockwise from the positive x-axis



Resultant component along x-axis

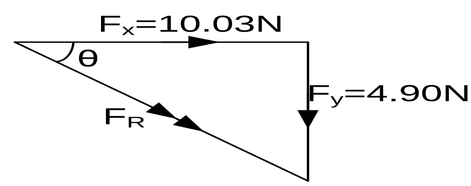
$$F_x = 2 + 4cos30 + 7cos30 - 3cos60$$

$$F_x = 10.03N$$

Resultant component along y-axis

$$F_y = 4sin30 + 3sin60 - 7sin30 - 6$$

$$F_y = -4.90N$$



$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$F_R = \sqrt{10.03^2 + 4.90^2}$$

F_R = 11.16N

$\theta = tan^{-1}\frac{F_y}{F_x}$ $\quad \theta = tan^{-1}\frac{4.90}{10.03}$

θ = 26.04^0

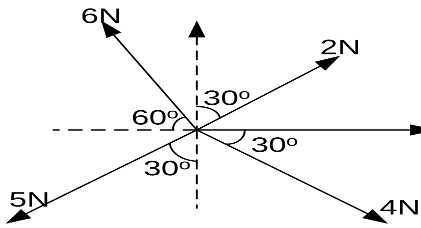
The resultant force is 11.16Nat 26.04^0 below the horizontal.

4. Forces of 2N, 3N, 4N, 5N, and 6N act on a particle in the direction 030^0, 090^0, 120^0, 210^0, and 330^0 respectively. Find the resultant force.

Solution

Note

The direction are given in 3-digit values, therefore they are bearings and must be measured from the north (positive y-axis) clock wise.



Resultant along the x-axis

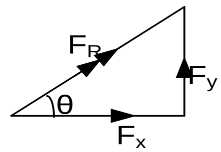
$$F_x = 3 + 2sin30 + 4cos30 - 5cos30 - 6cos60$$

$$F_x = 1.964N$$

Resultant along the y-axis

$$F_y = 6sin60 + 2cos30 - 5cos30 - 4sin30$$

$$F_y = 0.598N$$



$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$F_R = \sqrt{1.964^2 + 0.598^2}$$

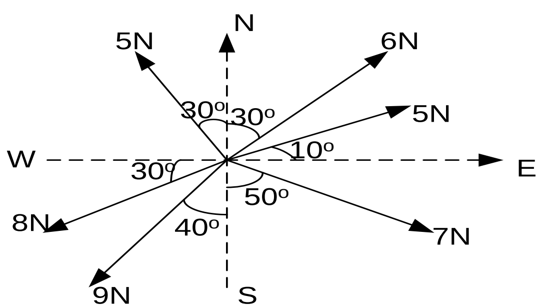
$$F_R = 2.053N$$

Direction $\theta = tan^{-1}\frac{F_y}{F_x}$ θ = 16.9^0

The resultant force is 2.053N at 16.9^0 above the horizontal

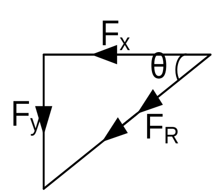
5. Forces of 6N, 5N, 7N, 8N, 5N, and 9N act pm a particle in the direction N30^{0}E, N30^{0}W, S50^{0}E, N60^{0}W, N80^{0}E and s40^{0}w, respectively. find the resultant force.

Solution



$$F_x = 5cos10 + 6sin30 + 7sin50 - 9sin40 - 8cos50 - 5sin30 = -1.927N$$

$$F_y = 5cos30 + 6cos30 + 5sin10 - 8sin30 - 9cos40 - 7cos50 = -4.999N$$



$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$F_R = \sqrt{1.927^2 + 4.999^2}$$

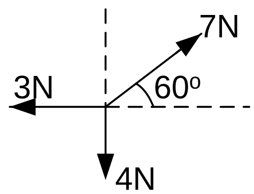
$$F_R = 5.36N$$

$$\theta = tan^{-1}\frac{F_y}{F_x}$$

$$\theta = tan^{-1}\frac{4.999}{1.927} \quad \theta = 68.9^0$$

Resultant force is 5.36N at 68.9^0 below horizontal

6. A particle at the origin 0 is acted upon by the three forces as shown below. Find the position of the particle after 2 seconds of its mass is 1kg.



Solution

Resultant along horizontal

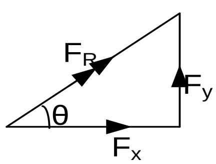
$$F_x = -3 + 7cos60$$

$$F_x = 0.5N$$

Resultant along vertical

$$F_y = 7sin60 - 4$$

$$F_y = 2.06N$$



$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$F_R = \sqrt{0.5^2 + 2.06^2}$$

$$F_R = 2.12N$$

But $F_R = ma$

$2.12 = 1a$

$a = 2.12ms^{-2}$

From $S = ut + ½\, at^2$

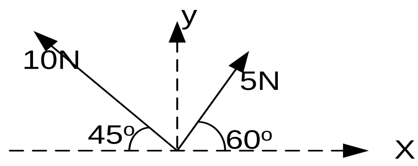
$u = 0 \;\; t = 2s \;\; a = 2.12ms^{-2}$

$S = 0x2 + ½\, x2.12x2^2$

$= 4.24m$ from the origin

EXERCISE 5

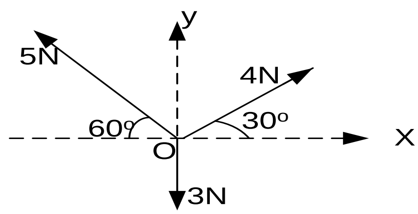
1. Two coplanar forces act on a point 0 as shown below



Calculate the resultant force

An[12.3N at 68.0^0 above the horizontal]

2. Three coplanar forces act at a point as shown below



Find the resultant force acting at 0

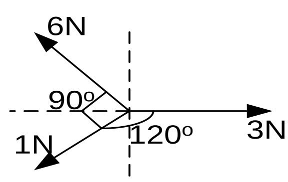
An[3.4N at 73.1^0 above the horizontal]

3. Forces of 2N, 1N, 3N and 4N act on a particle in the directions 0^0, 90^0, 270^0 and 330^0 respectively. Find the magnitude and direction of the resultant force.

   An[6.77N at 36.2^0 below the horizontal]

4. Forces of 7N, 2N, 2N, and 5N act on a particle in the direction 060^0, 160^0, 200^0 and 315^0 respectively. Find the resultant force. **An[4.14N at 52.36^0 below the horizontal]**

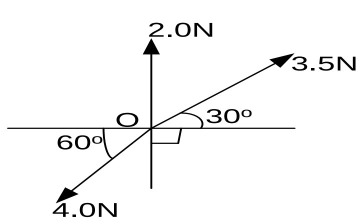
5. Three forces act on a body of mass 0.5kg as shown is the diagram. Find the position of the particle after 4 seconds.



An[3.44N, $6.88ms^{-2}$, 55.2m]

UNEB 2008 No1

b

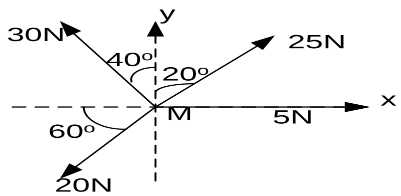


Three forces of 3.5N, 4.0N and 2.0N act at a point 0 as shown above. Find the resultant force. (4marks)

An[1.07N at 15.5^0 above the horizontal]

UNEB 2007 No 4

ii)



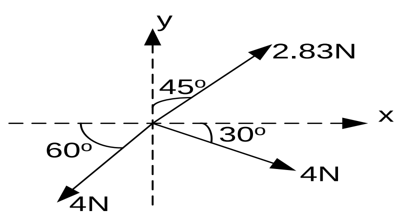
as shown above. Find the acceleration of m [05 marks]

An[5.5ms^{-2}]

A body m of mass 6kg is acted on by forces of 5N, 20N, 25N and 30N

UNEB NOV/DEC 1998 No1

c)



Forces of 2.83N, 4.00N and 6.00N act on a particle 0 as shown above. Find the resultant force on the particle [06marks]

3.2.0: RELATIVE MOTION

It comprises of;

1-Relative velocity

2-Relative path

3.2.1: Relative velocity

This is the velocity a body would have as seen by an observer on another body. Suppose A and B are two moving bodies, the velocity of A relative to B is the velocity of A as it appears to an observer on B.

It's denoted by $_A\mathbf{V}_B = \mathbf{V}_A - \mathbf{V}_B$

Note that $_AV_B \neq {_BV_A}$ since [$_B\mathbf{V}_A = \mathbf{V}_B - \mathbf{V}_A$]

Numerical calculations

There are two methods used in calculations

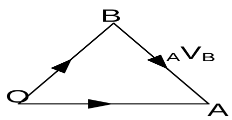
- Geometric method
- Vectorial method

1. Geometrical method

In this method make sure that the velocities of moving objects originates from a common point and their relative velocity closes to form a triangle of velocities.

Note

The direction of the relative velocity must be from the observer because it's the one observing where the other object is moving.



Apply either cosine formula or sine formula to obtain the unknown quantities

i.e $a^2 = b^2 + c^2 - 2bc\ CosA$ and $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

2. Vector method

Find component of velocity for each object separately

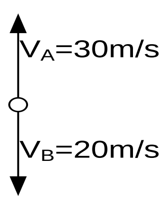
Therefore $_AV_B = V_A - V_B$

Example

1. Particle A is moving due to north at $30ms^{-1}$ and particle B is moving due south at 20m/s. find the velocity of A relative to B.

Solution

Method II



$V_A = \begin{pmatrix} 0 \\ 30 \end{pmatrix}$ $V_B = \begin{pmatrix} 0 \\ -20 \end{pmatrix}$

$_AV_B = V_A - V_B$

$_AV_B = \begin{pmatrix} 0 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ -20 \end{pmatrix}$

$_AV_B = \begin{pmatrix} 0 \\ 50 \end{pmatrix}$

$/_AV_B/ = \sqrt{0^2 + 50^2}$

$/_AV_B/$ = 50m/s due north

2. A cruiser is moving at 30km/hr due north and a battleship is moving at 20km/hr due north, find the velocity of the cruiser relative to the battleship.

Solution

Method II[vector]

$V_C = \begin{pmatrix} 0 \\ 30 \end{pmatrix}$ $V_B = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$

$_CV_B = V_C - V_B$

$_CV_B = \begin{pmatrix} 0 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ 20 \end{pmatrix}$

$_CV_B = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$ due north

$/_CV_B/ = \sqrt{0^2 + 10^2}$

$_CV_B$ =10km/h due north

3. A particle A has a velocity of 4i+6j-5k (m/s) while particle B has a velocity of -10i-2j+6k (m/s). find the velocity of A relative to B

Solution

${}_AV_B = V_A - V_B$

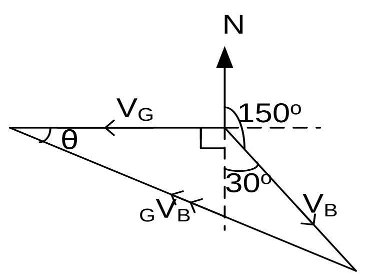
$${}_AV_B = \begin{pmatrix} 4 \\ 6 \\ -5 \end{pmatrix} - \begin{pmatrix} -10 \\ -2 \\ 6 \end{pmatrix}$$

$${}_AV_B = \begin{pmatrix} 14 \\ 8 \\ -11 \end{pmatrix}$$

${}_AV_B$ = 14i + 8j – 11k (m/s)

4. A boy runs at 5km/h due west and a girl runs 12km/hr at a bearing of 150^0. Find the velocity of the girl relative to the boy.

Method I [geometrical]



The arrow of relative velocity is from the boy since he is the observer

$({}_GV_B) = V_B^2 + V_G^2 - 2\,V_B\,V_G\cos 20^0$

$({}_GV_B)^2 = 5^2 + 12^2 - 2 \times 12 \times 5\cos 120$

$({}_GV_B)^2 = 229$

${}_GV_B = \sqrt{229}$

${}_GV_B = 15.13\text{kmhr}^{-1}$

Direction: using sine rule

$$\frac{V_B}{Sin\theta} = \frac{GVB}{Sin120}$$

$$\frac{12}{Sin\theta} = \frac{15.31}{Sin120}$$

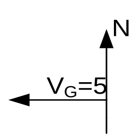
$$Sin\theta = \frac{12Sin120}{15.13}$$

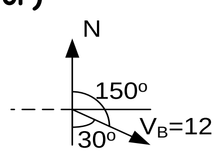
$$\theta = sin^{-1}\left(\frac{12sin120}{15.31}\right)$$

$\theta = 43.4^0$

The relative velocity is 15.13km/hr at 43.4^0 below the western direction or $S46.6^0W$

Method II (vector)





$V_G = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$ $V_B = \begin{pmatrix} 12Sin30 \\ -12Cos30 \end{pmatrix}$

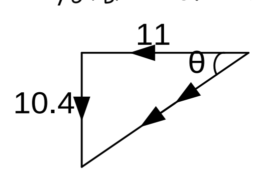
${}_GV_B = V_G - V_B$

${}_GV_B = \begin{pmatrix} -5 \\ 0 \end{pmatrix} - \begin{pmatrix} 12Sin30 \\ -12Cos30 \end{pmatrix}$

${}_GV_B = \begin{pmatrix} -11 \\ -10.4 \end{pmatrix}$

$/{}_GV_B/ = \sqrt{(-11)^2 + (-10.4)^2}$

$/{}_GV_B/$ = 15.14km/hr



$\theta = tan^{-1}\frac{10.4}{11}$ $\therefore \theta = 43.4^0$

below the western direction

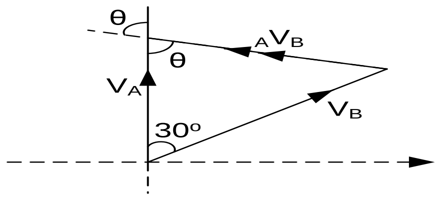
Relative velocity is 15.14km/hr at 43.4^0 below the horizontal.

5. Plane A is flying due north at 40km/hr while plane B is flying in the direction N30°E at 30km/hr. Find the velocity of A relative to B.

Solution

Method I (Geometric)

V_A=40km/hr V_B=30km/hr



$_AV_B^2 = V_B^2+V_A^2 - 2\,V_B\,V_A\cos30^0$

$= 30^2+40^2-2\text{x}30\text{x}40\cos30$

$_AV_B = \sqrt{421.539}$

$_AV_B$ = 20.53km/hr

Direction

$$\frac{V_B}{Sin\theta} = \frac{\text{AVB}}{Sin30}$$

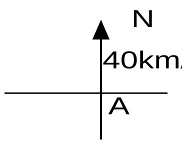
$$\frac{30}{Sin\theta} = \frac{20.53}{Sin30}$$

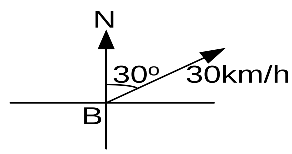
$\theta = \text{Sin}^{-1}(\frac{30\text{Sin}30}{20.53})$

$\theta = 46.9^0$

The relative velocity is 20.53 km/hr in the direction N46.9°W or S46.9°E

Method II vector





$V_A = \begin{pmatrix} 0 \\ 40 \end{pmatrix}$ $V_B = \begin{pmatrix} 30Sin30 \\ -30Cos30 \end{pmatrix}$

$_AV_B = V_A - V_B$

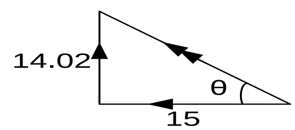
$_AV_B = \begin{pmatrix} 0 \\ 40 \end{pmatrix} - \begin{pmatrix} 30Sin30 \\ -30Cos30 \end{pmatrix}$

$_AV_B = \begin{pmatrix} -15 \\ 14.02 \end{pmatrix}$

$/_AV_B/ = \sqrt{(-15)^2 + (14.02)^2}$

$/_AV_B/$ = 20.53km/hr

Direction



$\theta = tan^{-1}\frac{14.02}{15}$

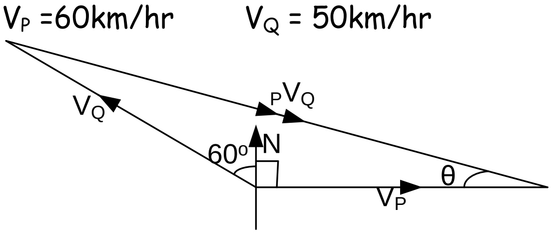
θ=43.07^0 above the Horizontal

The relative velocity is 20.53 at N46.93^{0}W

6. Ship P is steaming at 60km/hr due east while ship Q is steaming in the direction N60^{0}W at 50km/hr. Find the velocity of P relative to Q.

Method I (geometric)

V_P =60km/hr V_Q = 50km/hr



$_pV_q^2 = V_p^2+V_q^2-2V_pV_q\cos150$

$_pV_q^2 = 60^2+50^2-2\text{x}60\text{x}50\cos150$

$_pV_q$=106.3km/hr

$$\frac{pVq}{Sin150} = \frac{\text{Vq}}{Sin\theta}$$

$$\frac{106.3}{Sin150} = \frac{50}{Sin\theta}$$

$\theta = \text{Sin}^{-1}(\frac{50\ \text{Sin}150}{106.3})$

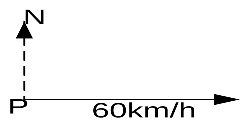
θ=13.6^0

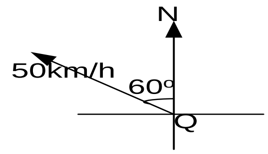
Direction is $S(90-13.6)°E$

S76.4^{0}E

Velocity of p relative to q is 106.3km/hr at S76.4^{0}E.

Method II vector





V$_p$ = $\begin{pmatrix}60\\0\end{pmatrix}$ V$_q$ =$\begin{pmatrix}-50Sin60\\50Cos60\end{pmatrix}$

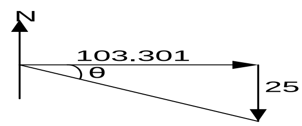
pVq=Vp-Vq

pVq =$\begin{pmatrix}60\\0\end{pmatrix}$ - $\begin{pmatrix}-50Sin60\\50Cos60\end{pmatrix}$

pVq= $\begin{pmatrix}103.301\\-25\end{pmatrix}$

$/pVq/ \;=\sqrt{(103.301)^2+\left(-25\right)^2}$

$pVq\;=$ 106.3km/hr



$\theta = tan^{-1}\dfrac{25}{103.301}$

Ɵ= 13.60^0

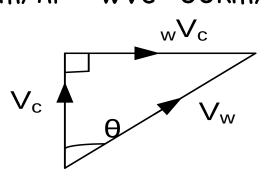
Direction $S(90-13.6)°E$

Relative velocity is 106.3km/hr at S76.4^{0}E

7. To a cyclist riding due north at 40km/hr, a steady wind appears to blow from west at 30km/hr. find the true velocity of the wind.

Method geometric

Vc =40km/hr wVc =30km/hr



$V_w^2 = wV_c^2 + V_c^2$ −2 $wVc\ x\ V_c$Cos90

$V_w = \sqrt{30^2+40^2}$

Vw= 50km/hr

$$\frac{wVc}{Sin\theta} = \frac{\text{Vw}}{Sin90}$$

$$\frac{30}{Sin\theta} = \frac{50}{Sin90}$$

θ= Sin^{-1}($\frac{30}{50}$)

Ɵ =36.9^0

True velocity is 50km/hr at N36.9^{0}E

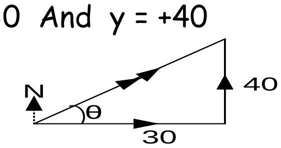
Method II vector

Vc = $\begin{pmatrix}0\\40\end{pmatrix}$ wVc= $\begin{pmatrix}30\\0\end{pmatrix}$ Vw= $\begin{pmatrix}x\\y\end{pmatrix}$

Vc =Vw−Vc

$\begin{pmatrix}30\\0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}0\\40\end{pmatrix}$

X=30 And y = +40



V w= $\begin{pmatrix}30\\40\end{pmatrix}$ Vw = $\sqrt{30^2+40^2}$

V w= 50km/hr

$\theta = tan^{-1}\frac{40}{30}$ Ɵ=53.13^0

Direction $N(90\ -\ 53.13)°E$

N36.87°E

Exercise 6

1. Car A is moving East wards at 20m/s and car B is moving Northwards at 10m/s. find the
   i) Velocity of A relative to B **An [10$\sqrt{5}$ m/s]**
   ii) Velocity of B relative to A **An [10$\sqrt{5}$ m/s]**

2. In EPL football match, a ball is moving at 5m/s in the direction of N45^{0}E and the player is running due north at 8m/s. Find the velocity of the ball relative to the player. **An[5.69m/s at S38.38^{0}E].**

3. A ship is sailing south East at 20km/hr and a second ship is sailing due west at 25km/hr. Find the magnitude and direction of the velocity of the first ship relative to the second. **An [41.62km/hr at S70.13^{0}E]**

4. On a particular day wind is blowing N30^{0}E at a velocity of 4m/s and a motorist is driving at 40m/s in the direction of S60^{0}E
   a) Find the velocity of the wind relative to motorist **An [40.2m/s at N54.28^{0}W]**
   b) If the motorist changes the direction maintaining his speed and the wind appears to blow due East. What is the new direction of the motorist? **An[N85.03^{0}W]**

3.2.2: RELATIVE PATH

Consider two bodies A and B moving with V_A and V_B from points with position vectors R_A and R_B respectively.

Position of A after time t is
R_{At}=OA + t x V_A
Position of B after time t is
R_{Bt}= OB + t x V_B

Relative path
${}_AR_B=R_{At}-R_{Bt}$
${}_AR_B=(OA+tV_A)-(OB+tV_B)$
${}_AR_B=(OA-OB)+t(V_A-V_B)$
${}_AR_B=(OA-OB)+t({}_AV_B)$

EXAMPLE

1. A car A and B are moving with their respective velocities $2i-j$ and $i+3j$, if their position vectors are $4i+j$ and $2i-3j$ respectively. Find the path of A relative to B

i) At any time t
ii) At t=2s

Solution

i) $V_A=\begin{pmatrix}2\\-1\end{pmatrix}$ $V_B=\begin{pmatrix}1\\3\end{pmatrix}$

$OA=\begin{pmatrix}4\\1\end{pmatrix}$ $OB=\begin{pmatrix}2\\-3\end{pmatrix}$

${}_AR_B$= (OA-OB) + t(${}_AV_B$)

${}_AR_B=\left[\begin{pmatrix}4\\1\end{pmatrix}-\begin{pmatrix}2\\-3\end{pmatrix}\right]+t\left[\begin{pmatrix}2\\-1\end{pmatrix}-\begin{pmatrix}1\\3\end{pmatrix}\right]$

${}_AR_B=\begin{pmatrix}2\\4\end{pmatrix}+t\begin{pmatrix}1\\-4\end{pmatrix}$

ii) When t=2 ${}_AR_B=\begin{pmatrix}2\\4\end{pmatrix}+2\begin{pmatrix}1\\-4\end{pmatrix}$

$=\begin{pmatrix}2\\4\end{pmatrix}+\begin{pmatrix}2\\-8\end{pmatrix}$

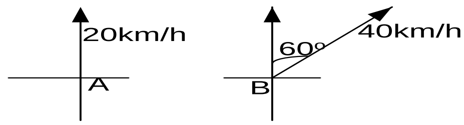
${}_AR_B=\begin{pmatrix}4\\-4\end{pmatrix}$

2. Two ships A and B move simultaneously with velocities 20km/hr and 40km/hr respectively. Ship A moves in the northern directions while ship B moves in N60^{0}E. Initially ship B is 10km due west of A. determine

a) The relative velocity of A to B
b) The relative path of A to B

Solution

a)



$V_A = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$ $V_B = \begin{pmatrix} 40Sin60 \\ 40Sin60 \end{pmatrix}$

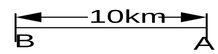
${}_AV_B = V_A - V_B$

${}_AV_B = \begin{pmatrix} 0 \\ 20 \end{pmatrix} - \begin{pmatrix} 40Sin60 \\ 40Sin60 \end{pmatrix}$

${}_AV_B = \begin{pmatrix} -34.64 \\ 0 \end{pmatrix}$

${}_AV_B = 34.64$km/hr

b)



${}_AR_B = (OA - OB) + t({}_AV_B)$

$OB = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $OA = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$

${}_AR_B = \left[\begin{pmatrix} 10 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] + t\begin{pmatrix} -34.64 \\ 0 \end{pmatrix}$

${}_AR_B = \begin{pmatrix} 10 \\ 0 \end{pmatrix} + t\begin{pmatrix} -34.64 \\ 0 \end{pmatrix}$

3.2.3: SHORTEST DISTANCE AND TIME TO SHORTEST DISTANCE [DISTANCE AND TIME OF CLOSEST APPROACH]

When two particles are moving simultaneously with specific velocities, time will come when they are closest to each other **without** colliding

Numerical calculations

There three methods used

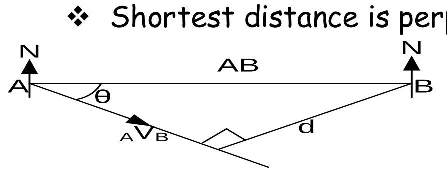
- Geometrical
- Vector
- Differential

1. Geometrical

It works when the information given is either a bearing (direction) or in two dimensions i.e. only **i** and **j** components

Procedure

- Draw a diagram showing the initial positions of the particles
- Consider the motion of one body relative to another i.e. A relative to B
- Represent the velocities of the bodies on the diagram with their directions specified.
- Superimpose the relative velocity to the observable body
- Shortest distance is perpendicular to the relative velocity as shown below;



Shortest distance

$$d = ABsin\theta$$

${}_AV_B$ is a velocity, therefore to express it as a distance, it becomes $/{}_AV_B/\ t$

But $cos\theta = \frac{/AVB/\ t}{AB}$ *Hence* $/{}_AV_B/\ t = ABcos\theta$

Time to shortest distance $t = \frac{AB\ Cos\theta}{/AVB/}$

2. Vector

Consider particles A and B moving with velocities V_A and V_B from point with positions vectors OA and OB respectively.

Then **shortest distance** **d**$=$ **$/_A R_B$ /**

For minimum distance to be attained then **$_A V_B$. $_A R_B$ = 0** This gives the time

Or time $= \frac{/AB\ .\ \mathbf{AVB}/}{/AVB/^2}$ Where **AB.** **$_A V_B$** is a dot product

3. Differential

The minimum distance is reached when $\frac{d}{dt}$**$/_A R_B$ /** $^2 =$ **0** This gives the time

Minimum distance **d** $=$ $/_A R_B$ **/**

EXAMPLE

1. A particle P starts from rest from a point with position vector $2j + 2k$ with a velocity $(j + k)$m/s. A second particle Q starts at the same time from a point whose position vector is $-11i - 2j - 7k$ with a velocity of $(2i + j + 2k)$m/s . Find;
   i) The shortest distance between the particles
   ii) The time when the particles are closest together
   iii) How far each has travelled by this time

Solution:

Method I vector

i) OP $= \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ Vp $= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} m/s$

$OQ = \begin{pmatrix} -11 \\ -2 \\ -7 \end{pmatrix}$ $V_Q = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$m/s

$_P V_Q = V_P - V_Q$

$_P V_Q = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$

$_P R_Q = (OP - OQ) + (_P V_Q)t$

$\text{PRQ} = \left[\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} -11 \\ -2 \\ -7 \end{pmatrix}\right] + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} t$

$_P R_Q = \begin{pmatrix} 11 \\ 4 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} t$

For minimum distance

$_P V_Q . _P R_Q = 0$

$\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} . \begin{pmatrix} 11 - 2t \\ 4 \\ 9 - t \end{pmatrix} = 0$

$-22 + 4t + 0 - 9 + t = 0$

$t = \frac{31}{5}$ $\therefore\ t = 6.2s$

ii) Shortest distance d=$/_P R_Q/$

$_P R_Q = \begin{pmatrix} 11 \\ 4 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} t$

t=6.2

$_P R_Q = \begin{pmatrix} 11 \\ 4 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} 6.2$

$_P R_Q = \begin{pmatrix} -1.4 \\ 4 \\ 2.8 \end{pmatrix}$

$/_P R_Q/ = \sqrt{(-1.4)^2 + 4^2 + 2.8^2}$

$/_P R_Q/ = 5.08$m

iii) How far each has travelled

$R_P = OP + V_P t$

$R_P = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} 6.2$

$R_P = \begin{pmatrix} 0 \\ 8.2 \\ 8.2 \end{pmatrix}$

$/R_P/ = \sqrt{0^2 + 8.2^2 + 8.2^2}$

$/R_P/ = 11.6$m

$R_Q = OQ + V_Q t$

$R_Q = \begin{pmatrix} -11 \\ -2 \\ -7 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} 6.2$

$R_Q = \begin{pmatrix} 1.4 \\ 4.2 \\ 5.4 \end{pmatrix}$

$/R_Q/ = \sqrt{1.4^2 + 4.2^2 + 5.2^2}$

$/R_Q/ = 6.8$m

Method II (differential)

$\frac{d}{dt}/_PR_Q/^2 = 0$

${}_PR_Q = \begin{pmatrix} 11 \\ 4 \\ 9 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} t$

${}_PR_Q = \begin{pmatrix} 11 - 2t \\ 4 \\ 9 - t \end{pmatrix}$

$/_PR_Q/^2 = (11 - 2t)^2 + (4)^2 + (9 - t)^2$

$/_PR_Q/^2 = 121 - 44t + 4t^2 + 16 + 81 - 18t + t^2$

$/_PR_Q/^2 = 218 - 62t + 5t^2$

$\frac{d}{dt}/_PR_Q/^2 = -62 + 10t$

$\frac{d}{dt}/_PR_Q/^2 = 0$

$-62 + 10t = 0$

$t = 6.2s$

Minimum Distance d = $/_PR_Q/$

$d = \begin{pmatrix} 11 - 2x6.2 \\ 4 \\ 9 - 6.2 \end{pmatrix}$

$d = \begin{pmatrix} -1.4 \\ 4 \\ 2.8 \end{pmatrix}$

$d = \sqrt{(-1.4)^2 + (4)^2 + (2.8)^2}$

d= 5.08m

2. Initially two ships A and B are 65km apart with B due East of A. A is moving due East at 10km/hr and B due south at 24km/hr. the two ships continue moving with these velocities. Find the least distance between the ships in the subsequent motion and the time taken to the nearest minute for such a situation to occur.

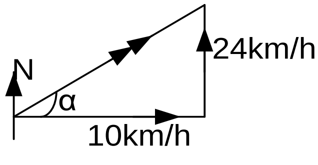
Solution Method I (Geometric)

First get the relative velocity

$V_A = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$ $V_B = \begin{pmatrix} 0 \\ -24 \end{pmatrix}$

${}_AV_B = V_A - V_B$

${}_AV_B = \begin{pmatrix} 10 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -24 \end{pmatrix} = \begin{pmatrix} 10 \\ 24 \end{pmatrix}$



$\alpha = \tan^{-1}\frac{24}{10}$ $\alpha = 67.38^0$

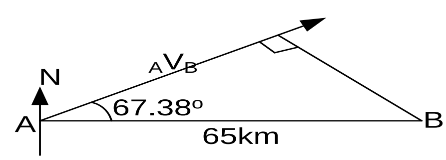
N22.61^0E

${}_AV_B = \sqrt{10^2 + 24^2}$

$/_AV_B/ = 26$km/h

Relative velocity is 26km/h at N22.62^{0}E

Sketch



Least distance $d = ABsin\theta$

= 65sin67.38

Least distance = **60.0km**

Time t= $\frac{A\,B\,Cos\theta}{/AVB/}$

${}_AV_B = \begin{pmatrix}10\\24\end{pmatrix}$

$/{}_AV_B/ = \sqrt{10^2 + 24^2}$

$/{}_AV_B/$=26km/h

Time **t=** $\frac{\mathbf{65Cos67.4}}{\mathbf{26}}$

Time = 0.96hr

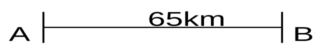
Time=0.96x60≈58minutes

Method II (vector)

least distance d = $/{}_AR_B/$

For least distance $({}_AV_B \,.\, {}_AR_B) = 0$

But $\;{}_AV_B = \begin{pmatrix}10\\24\end{pmatrix}$



$A = \begin{pmatrix}0\\0\end{pmatrix}$ $\qquad$ B= $\begin{pmatrix}65\\0\end{pmatrix}$

${}_AR_B = (OA - OB) + {}_AV_B\, t$

$= \left[\begin{pmatrix}0\\0\end{pmatrix} - \begin{pmatrix}65\\0\end{pmatrix}\right] + t\begin{pmatrix}10\\24\end{pmatrix}$

${}_AR_B = \begin{pmatrix}-65+10t\\24t\end{pmatrix}$

${}_AV_B \,.\, {}_AR_B = 0$

$\begin{pmatrix}10\\24\end{pmatrix}.\begin{pmatrix}-65+10t\\24t\end{pmatrix} = 0$

$-650 + 100t + 576t = 0$

$t = \frac{650}{676} \quad \therefore \; t = 0.96hrs$

Alternatively t can be obtained from

time $= \frac{/AB\ .\ \mathbf{AVB}/}{/AVB/^2}$ $\qquad$ AB = $\begin{pmatrix}65\\0\end{pmatrix}$

time $= \frac{/\begin{pmatrix}65\\0\end{pmatrix}.\begin{pmatrix}10\\24\end{pmatrix}/}{\left(\sqrt{10^2+24^2}\right)^2}$

t= $\frac{/(650+0)/}{676}$

t = $\frac{650}{676}$

t=0.96hr

least distance d = $/{}_AR_B/$

${}_AR_B$ = [(OA)-(OB)] + ${}_AV_B$ t

${}_AR_B = \begin{pmatrix}-65+[10x0.96]\\24x0.96\end{pmatrix}$

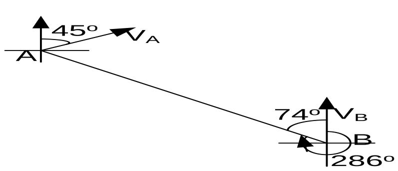
${}_AR_B = \begin{pmatrix}-55.4\\23.04\end{pmatrix}$

$/{}_AR_B/ = \sqrt{(-55.4)^2 + 23.04^2}$

$/\ {}_AR_B/ = 60km$

3. At noon a boat A is 30km from boat B and its direction from B is 286^0. Boat A is moving in the North east direction at 16km/hr and boat B is moving in the northern direction at 10km/hr. Determine when they are closest to each other. What is the distance between them?

Solution For relative velocity



$V_A = \begin{pmatrix}16sin45\\16cos45\end{pmatrix}$ $\qquad$ $V_B = \begin{pmatrix}0\\10\end{pmatrix}$

${}_AV_B = V_A - V_B$

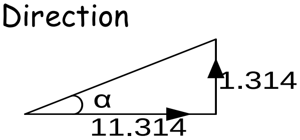
${}_AV_B = \begin{pmatrix}16sin45\\16cos45\end{pmatrix} - \begin{pmatrix}0\\10\end{pmatrix}$

${}_AV_B = \begin{pmatrix}11.314\\1.314\end{pmatrix}$

$/{}_AV_B/ = \sqrt{11.314^2 + 1.314^2}$

$/{}_AV_B/$ = 11.39km/hr

Direction

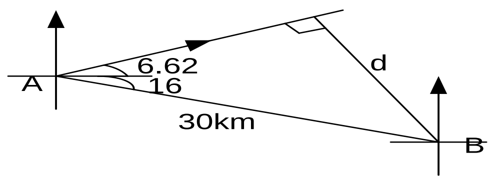


$\alpha = tan^{-1}\left(\frac{1.314}{11.314}\right)$

α = 6.62^0 $\quad N(90-6.62)°E$

Relative velocity is 11.39km/hr at N 83.38^{0}E

SKETCH



$$d = AB\sin\theta$$
$$d = 30\sin(6.62 + 16)$$
$$d = 30\sin(22.63)$$
$$d = 11.54km$$

Time t= $\frac{A\ B\ Cos\theta}{/\text{AVB}/}$

t= $\frac{30\ Cos22.62}{11.39}$

t = 2.43hrs

Time is 2.43hours from noon or 2 hours and 25.8 minutes

It occurs 2:26pm at a distance of 11.54km

Exercise7

1. A ship A is 8km due North of Ship B, ship A is moving at 150kmh^{-1} due west while B is moving at 200km/hr due N30^{0}W. After what time will they be nearest together and how far apart will they be. **An(2.22km, 0.043hrs)**

2. The point p is 50km west of q. Two air crafts A and B fly simultaneously from p and q velocities are 400km/hr N50^{0}E and 500km/hr N20^{0}W respectively. Find;
   (i) The closest distance between the air crafts
   (ii) The time of flight up to this point **An(20.35km, 5.24 minutes)**

3. Ship A steams North-west at 60km/hr whereas B steams southwards at 50km/hr, initially ship B was 80km due north of A. find;
   (i) The velocity of A relative to B
   (ii) The time taken for the shortest distance to be reached
   (iii)The shortest distance between A and B.
   An(101.675km/hr at N24.7^{0}W, 42.9minutes, 33.382km)

3.3.0: Motion of bodies with different frames of reference

It involves crossing the river and flying space

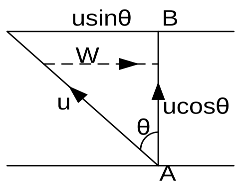
3.3.1: Crossing the river

There are three cases to consider when crossing a river

a. Case I (shortest route)

If the water is not still and the boat man wishes to cross **directly opposite** to the starting point.

In order to cross point A to another point B directly opposite A (perpendicularly), then the course set by the boat must be upstream of the river.

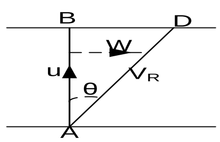


u is the speed of the boat in still water,
w is the speed of the running water
At point B: $usin\theta$ =w

$Sin\theta=\frac{w}{u}$
$\theta=Sin^{-1}\frac{w}{u}$
θ is the direction to the vertical but the direction to the bank is (90-θ)°
Time taken =$\frac{AB}{uCos\theta}$

b. Case II. The shortest time/as quickly as possible

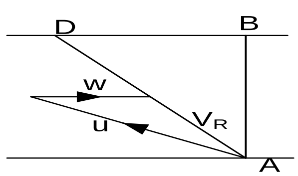
If the boat man wishes to cross the river as quickly as possible, then he should steer his boat directly from A to B as shown. The river pushes him down stream.



Time to cross the river **t** $=\frac{AB}{u}$
Distance covered downstream is = $\boldsymbol{wxt}$

Or distance downstream = $\boldsymbol{w\frac{AB}{u}}$
tan$\theta=\frac{w}{u}$ $\theta=tan^{-1}\frac{w}{u}$
The resultant velocity downstream V_R
$V_R^2 = W^2 + u^2$
$V_R = \sqrt{w^2+u^2}$

C. Case III



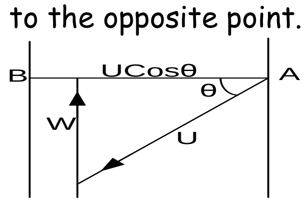
Resultant velocity $\overrightarrow{V_R}$ = $\vec{W}$ + $\vec{U}$

EXAMPLES

1. A river with straight parallel bank 400m apart flows due north at 4km/hr. Find the direction in which a boat travelling at 12km/hr must be steered in order to cross the river from East to West along the course perpendicular to the banks. Find also the time taken to cross the river.

Solution

Hint. Since the course is perpendicular to the bank, then it requires crossing directly to the opposite point.



W=4km/hr U=12km/hr
AB=400m=0.4km
Sinθ=$\frac{w}{u}$ $\theta=Sin^{-1}\frac{4}{12}$ θ=19.47^0

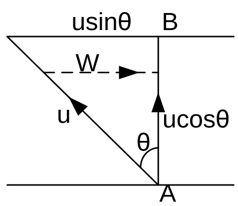
The direction is (90-19.47) to the bank.
Direction is 70.53^0 to the bank
Time taken =$\frac{AB}{uCos\theta}$
Time taken =$\frac{0.4}{12Cos19.47}$
Time = 0.035hrs
Time =2.1 minutes

2. A man who can swim at 6km/hr in still water would like to swim between two directly opposite points on the banks of the river 300m wide flowing at 3km/hr. Find the time he would take to do this.

Solution

U=6km/hr W= 3km/hr

AB=300m AB = 0.3km



Sinθ=$\frac{\mathrm{w}}{\mathrm{u}}$ θ=$Sin^{-1}\frac{3}{6}$ θ =30°

Time taken =$\frac{\mathrm{AB}}{\mathrm{uCos\theta}}$

Time taken =$\frac{0.3}{6\mathrm{Cos}30}$

Time = 0.058hrs

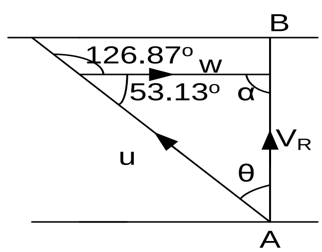
Time =3.46minute

He must swim at 30^0 to AB in order to cross directly and it will take 3.46minutes

3. A man who can swim at 8m/s in still water crosses a river by steering at an angle of 126.87^0 to the water current. If the river is 75m wide and flows at 5m/s, find;
(i) The velocity with which the person crosses the river
(ii) The time he takes to do this

Solution

u=8m/s w=5m/s AB=75m



α is not 90^0

Using cosine rule

$V_R^2 = 8^2 + 5^2 - 2\,u.wcos53.13$

$V_R = \sqrt{8^2 + 5^2 - 2\,x8x5cos53.13}$

V_R=6.4m/s

The person crosses with 6.4m/s.

ii) Time taken =$\frac{\mathrm{AB}}{\mathrm{uCos\theta}}$

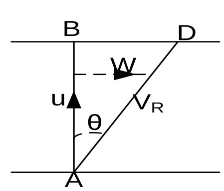
But V_R=$Ucos\theta$

Time = $\frac{75}{6.4}$ = 11.72 seconds

4. A man who can swim at 2m/s at in still water wishes to swim across a river 120m wide as quickly as possible. If the river flows at 0.5m/s, find the time the man takes to cross and how far down streams he travels.

Solution

U=2m/s w=0.5m/s AB=120m



t =$\frac{AB}{u}$

$t = \frac{120}{2}$ t=60s

i. Distance downstream = wt

= 0.5x60

Distance downstream =30m

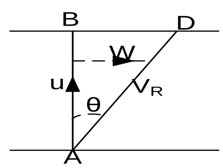
5. A boat can travel at 3.5m/s in still water. A river is 80m wide and the current flows at 2m/s, calculate

a) The shortest time to cross the river and the distance downstream that the boat is carried.

b) The course that must be set to apoint exactly opposite the starting point and the time taken for crossing

Solution

a) U=3.5m/s, w=2m/s AB=80m



Shortest time **t** =$\frac{AB}{u}$

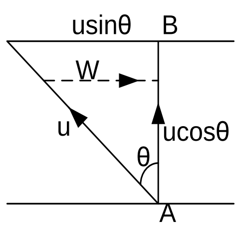
t =$\frac{80}{3.5}$ t= 22.95

Distance downstream BD=wt

=2x22.9

Distance downstream BD=45.8m

b. U=3.5m/s, w=2m/s, AB=80



Sinθ=$\frac{w}{u}$

θ=$Sin^{-1}\frac{2}{3.5}$

θ=34.8^0

The course must be 34.8^0 to AB.

Time for crossing **Time taken** =$\frac{AB}{uCos\theta}$

Time taken =$\frac{80}{3.5Cos34.8}$

t=27.8s

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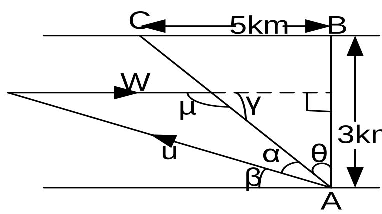
6. A boat crosses a river 3km wide flowing at 4m/s to reach a point on the opposite bank 5km upstream. The boat's speed in still water is 12m/s. Find the direction in which the boat must be headed. (04marks)

Solution

In order for a boat to cross to a point C upstream on the opposite bank then the course set must be such that the resultant velocity of the boat is along AC upstream.

U=12m/s, w=4m/s, AB=3km, AC=5km



$\tan\theta = \frac{5}{3}$ θ=59.04^0

But $\gamma + \theta = 90^0$

$\gamma = 90 - 59.04$

$\gamma = 30.96^0$

But $\mu + \gamma = 180^0$

$\mu + 30.96^0 = 180^0$

$\mu = 180^0 - 30.96^0$

$\mu = 149.04^0$

Also using sin rule $\frac{w}{\sin\alpha} = \frac{u}{\sin\mu}$

$$\frac{4}{\sin\alpha} = \frac{12}{\sin 149.04}$$

$\alpha = \sin^{-1}(\frac{4\sin 149.04}{12})$

$\alpha = 9.87^0$

But $\beta + \alpha + \theta = 90^0$

$\beta + 9.87 + 59.04 = 90^0$

$\beta = 21.09^0$

The boat must be headed at 21.09^0 to the river bank upstream

Exercise8

1. A man who an row at 0.9m/s in still water wishes to cross the river of width 1000m as quickly as possible. If the current flows at a rate of 0.3m/s. Find the time taken for the journey. Determine the direction in which he should point the boat and position of the boat where he lands **An [1111.11s, 71.57° to the bank, 333.33 downstream]**

2. A man swims at 5kmh^{-1} in still water. Find the time it takes the man to swim across the river 250m wide, flowing at 3kmh^{-1}, if he swims so as to cross the river;
   (i) By the shortest route **An [178.6s]**
   (ii) In the quickest time **An[217.4s]**

3. A boy can swim in still water at 1m/s, he swims across the river flowing at 0.6m/s which is 300m wide, find the time he takes;
   (i) If he travels the shortest possible distance
   (ii) If he travels as quickly as possible and the distance travelled downstream. **[375s,180m]**
   (iii)

4. A boy wishes to swim across a river 100m wide as quickly as possible. The river flows at 3km/hr and the boy can swim at 4km/h in still water. Find the time that

the boy takes to cross the river and how far downstream he travels. **An [90s,75m].**

CHAPTER 4: NEWTON'S LAWS OF MOTION

LAW I : Everybody continues in its state of rest or uniform motion in **a straight line** unless acted upon by an external force.

This is sometimes called the law of **inertia**

Definition

Inertia is the reluctance of a body to start moving once its at rest or to stop moving if its already in motion.

LAW II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

Definition

Linear momentum (p) is the product of the mass and the velocity of the body moving in a straight line.

Momentum is a vector quantity and its unit is **kgms^{-1}**

$$\boxed{\boldsymbol{momentum = mv}}$$

Consider a mass m moving with velocity u. If the mass is acted on by a force F and its velocity changes to v;

By Newton's law of motion

F $\alpha \ \frac{Change\ in\ momentum}{time}$

F $\alpha \ \frac{mv-mu}{t}$

F = $\frac{k(mv-mu)}{t}$

F = $km\frac{(v-u)}{t}$

But a = $\frac{v-u}{t}$

$F = kma$

Where k is a constant

The unit of force is **a newton**

Definition

A Newton is a force which gives a body of mass 1kg an acceleration of $1ms^{-2}$
F= 1N, m = 1kg, a = $1ms^{-2}$
But F = kma
1 = Kx1x1
K=1

$$\boxed{F = ma}$$

Note: F must be the resultant force

LAW III: To every action there is an equal but opposite reactions.
This law stresses that whenever two bodies interact with each other, the force exerted by the first body on the second body is equal and opposite to the force exerted by the second body on the first body

$$F_1 = -F_2$$

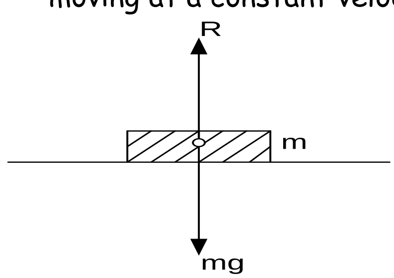
Example of 3rd law of motion

- A gun moves backwards on firing it.
- A ball bounces on hitting the ground.

NB: Though action and reaction forces are equal and opposite in direction, they do not cancel out because they act on different bodies.

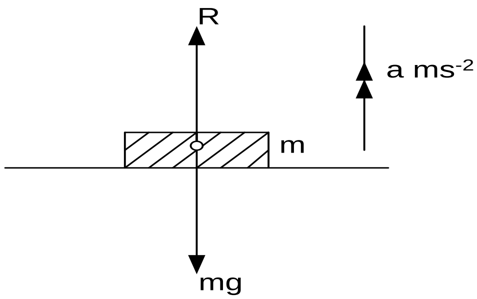
4.1.0: IDENTIFICATION OF FORCES AND THE APPLICATION OF NEWTON'S LAWS

1. Consider a body of mass **m** placed on either a stationary platform or a platform moving at a constant velocity



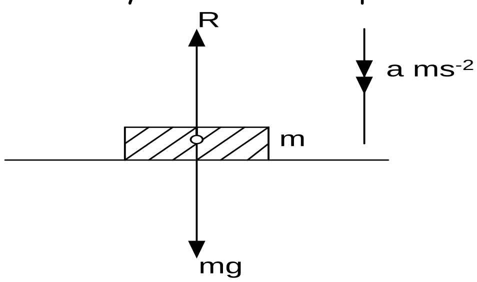
R= normal reaction
Mg= gravitational pull [weight]
R = mg since (a=0) constant velocity

2. A body of mass m placed on a platform moving vertical upward with an acceleration (a)



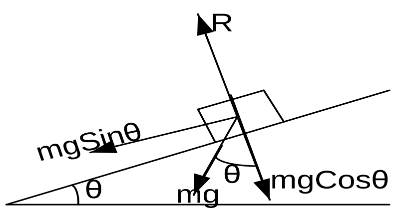
Resultant acts upwards in the direction of the acceleration.
Resultant force is $R - mg$
By Newton's second law R – mg = ma

3. A body of mass m on a platform having a acceleration a vertically downwards.



Resultant force downwards (mg - R)
From 2nd law mg - R = ma

4. Mass m placed on a smooth inclined plane of angle of inclination θ

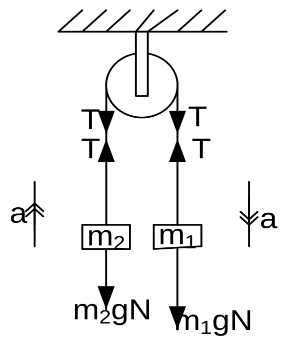


$$\boldsymbol{R = mgCos\theta}$$

NB:
- All objects placed on, or moving on an inclined plane experience a force $mgsin\theta$ **down**
  the plane. [It doesn't matter what direction the body is moving]
- If the plane is **rough** the body experiences a frictional force whose direction is opposite
  to the direction of motion.

5. Connected bodies of masses m_1 and m_2 over smooth a pulley by a light string ($M_1 > M_2$)

Since m_1 is greater than m_2, it Pulls m_2 down
T = tension in the string
a = common acceleration



Applying Newton's 2nd law separately to each mass

For m_2: $T - M_2g = M_2a$…………………(i)

For M_1: $M_1g - T = M_1a$…………………(ii)

Adding (i) and (ii)

$M_1g - M_2g = M_1a + M_2a$

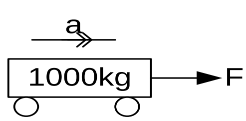
$$a = \frac{(M_1 - M_2g)}{M_2 + M_1}$$

Examples:

1. A car of mass 1000kg is accelerating at $2ms^{-2}$. What resultant force acts on the car? .

If the resistance to the motion is 1000N, what force is due to the engine?

Solution



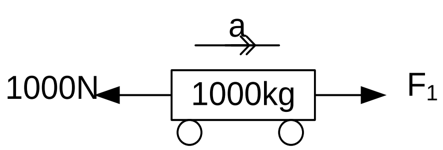
From 2nd law

$F = ma$

$F = 1000 \times 2$

$F = 2000N$

Resultant force is 2000N



The resistance force should act in opposite direction to the force due to the engine

Apply Newton's 2nd law

$F_1 - 1000 = ma$

$F_1 - 1000 = 1000 \times 2$

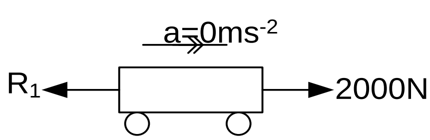
$F_1 - 1000 = 2000$

$F_1 = 3000N$

Force due to the engine is 3000N

2. A car moves along a level road at a constant velocity of 22m/s. If its engine is exerting a forward force of 2000N, what resistance is the car experiencing

Solution



Applying Newton's 2nd law

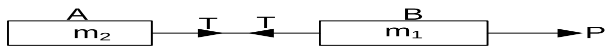
$2000 - R_1 = ma$

But a = 0 since it moves with constant velocity

$2000 - R_1 = 0$

$R_1 = 2000N$

3. Two blocks A and B connected as shown below on a horizontal friction less floor and pulled to the right with an acceleration of $2ms^{-2}$ by a force P, if $m_1 = 50kg$ and $m_2 = 10kg$. what are the values of T and P



Solution

Since the acceleration act to the right then

For m_1: $P-T=m_1a$…….[1]

$P-T=50x2$

$P-T=100$

For m_2: $T=m_2\,a$

$T=10x2$

$T=20N$

But into equation (1)

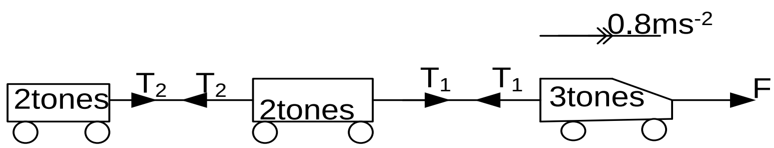
$P-T=100$

$P-20=100$

$P=120N$

4. A Lorry of 3 tones pulls 2 trailers each of mass 2 tones along a horizontal road, if the lorry is accelerating at $0.8ms^{-2}$, calculate
   a) Net force acting on the whole combination
   b) The tension in the coupling between the lorry and 1st trailer.
   c) The tension in the coupling between the 1st and 2nd trailer.

Solution



Let F be the net force acting on the combination

For the lorry: $F-T_1= 3000x0.8$

$F-T_1=2400$………..[1]

For the 1st trailer: $T_1-T_2= 2000x0.8$

$T_1-T_2=1600$…………[2]

For the 2ND trailer: $T_2= 2000x0.8$

$T_2=1600N$

Put into [2]: $T_1-T_2=1600$

$T_1-1600=1600$

$T_1=3200N$

Put into [1]

$F-T_1=2400$

$F-3200=2400$

$F=5600N$

$F=5600N, T_1=3200N\ T_2=1600N$

Exercise 10

1. A box of 50kg is pulled up from a ship with an acceleration of $1ms^{-2}$ by a vertical rope attached to it.
   i. Find the tension on the rope.
   ii. What is the tension in the rope when the box moves up with a uniform velocity of $1ms^{-1}$ ($g=9.8ms^{-2}$)

   An [540N, 490N]

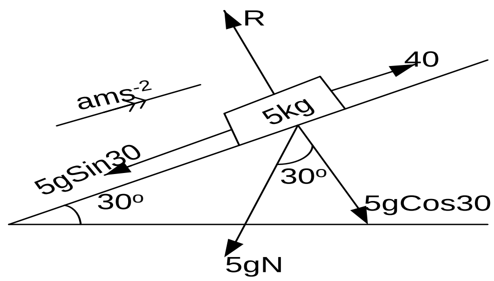
2. A lift moves up and down with an acceleration of $2ms^{-2}$. In each case, calculate the reaction of the floor on a man of mass 50kg standing in the lift. (take $g = 9.8ms^{-s}$)
   An[590N, 390N]

Motion on inclined planes

Example

1. A body of mass 5kg is pulled up a smooth plane inclined at 30^0 to the horizontal by a force of 40N acting parallel to the plane. Find
    a) Acceleration of the body
    b) Force exerted on the body by the plane

Solution



Since its moving up the plane, the resultant is upwards.

a) Resolving parallel to the plane

$$40 - 5gsin30 = ma$$

$$40 - 5x9.81sin30 = 5xa$$

1.5.475 =5a

a = 3.095m/s^2

a≈ 3.1 m/s^2

Acceleration ≈3.1 m/s^2

b) Force exerted on the body by the plane is the normal reaction

$$R = 5gcos30$$

$$R = 5x9.81cos30$$

R = 42.4N

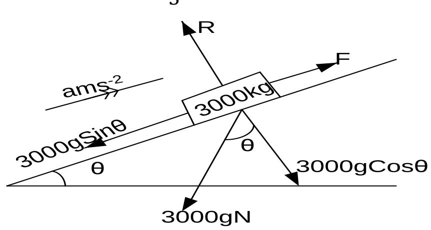
2. A lorry of mass 3 tones travelling at 90km/hr starts to climb an incline of 1 in 5. Assuming the tractive pull between its tyres and the road remains constant and that its velocity reduces to 54km/h in a distance of 500m. Find the tractive pull

Solution

U = 90km/h= $\frac{90x1000}{3600}$ =25ms^{-1}

V = 54km/h = $\frac{54x1000}{3600}$ =15ms^{-1}

S = 500m, Sin θ = $\frac{1}{5}$ $\therefore \theta$ = 11.54^0



Let F be the tractive pull

Resolving along the plane

$$F - 3000gsin\theta = 3000a$$

$$F - 3000x9.81x\frac{1}{5} = 3000a$$

$F - 5886 = 3000a$……………….(i)

But $v^2 = u^2+2as$

$15^2 = 25^2+2ax500$

a = - 0.4m/s^2

put into (i)

$F - 5886 = 3000a$

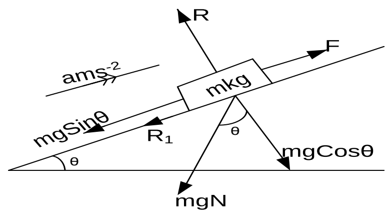
$F = -3000x0.4 + 5886$

F = 4686N

The tractive force is 4686N

3. A train travelling uniformly at 72km/h begins an ascent on 1 in 75. The tractive force which the engine exerts during the ascent is constant at 24.5kN, the resistance due to friction and air is also constant at 14.7kN, given the mass of the whole train is 225 tones. Find the distance a train moves up the plane before coming to rest.

Solution



1 in 75 means $\sin\theta = \frac{1}{75}$ ∴ $\theta = 0.76^0$

resistance force: R_1= 14.7kN

tractive force :F = 24.5kN

F – ($Mg\sin\theta$ +R_1) = ma

24500–($225000x9.81x\frac{1}{75}$ +14700) = 22500a

a = -0.087ms^{-2}

its deceleration = 0.087ms^{-2}

$v^2 = u^2 + 2as$ [$v = om/s$comes to rest]

u = 72km/h = $\frac{72x1000}{3600}$=20ms^{-1}

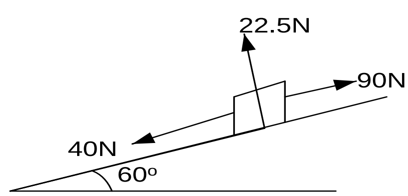
$0^2 = 20^2 + 2(-0.087)s$

$-400 = -0.174s$

S = 2298.85m

Exercise 11

1. The pull exerted by an engine is 1/80 of the weight of the whole train and the maximum brake force which can be exerted is 1/30 of the weight of the train. Find the time in which the train travels from the rest up a slope of 1 in 240 and 4800m along, if the brakes are applied when the engine is switched off . **An(379s).**
2. The resistance to the motion of the train due to friction is equal to 1/160 of the weight of the train, if the train is travelling on a level road at 72kmh^{-1} and comes to the foot of an incline of 1 in 150 and steam is then turned off, how far will the train go up the incline before it comes to rest.
   An(1579.99m)
3. 12m length of the slope. If the truck starts from the bottom of the slope with a speed of 18km/h, how far up will it travel before coming to rest
   An(71.43m).
4.



Three forces act on a block as shown, the block is placed on a smooth plane inclined at 60^0 calculate;

a) Acceleration of the block up the plane
b) Gain in kinetic energy in 5s after moving from rest **An(1.5ms^{-2}, 140.625J)**

4.1.1: MOTION OF CONNECTED PARTICLES

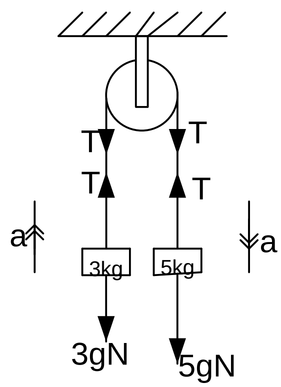
When two particles are connected by a light inextensible string passing over a smooth pully and allowed to move freely, then as long as the string is tight, the following must be observed.

- Acceleration of one body in general direction of motion is equal to the acceleration of the other
- The tension T in the string is constant.

Examples

1. Two particles of masses 5kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;
   (i) Acceleration of the particles
   (ii) The tension in the string
   (iii) The force on the pulley

Solution



For 5kg mass: $5g - T = ma$

$5g - T = 5a$……………………(i)

For 3kg mass: $T - 3g = ma$

$T - 3g = 3a$ ……………………(ii)

Adding (i) and (ii)

$2g = 8a$

$a = \frac{2x9.81}{8}$

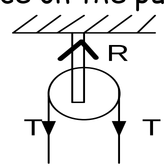
$a = 2.45ms^{-2}$

ii) $T - 3g = 3a$

$T = 3x2.45 + 3x9.81$

$T = 36.78N$

iii) Force on the pulley



$R = 2T$

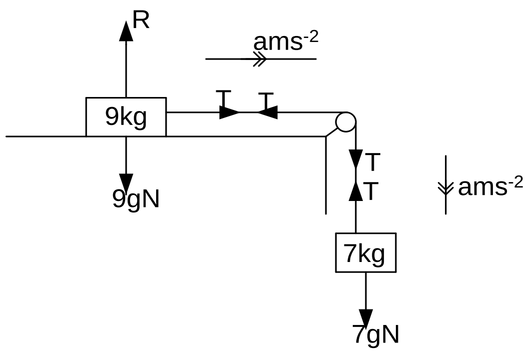
$R = 2x36.78$

$R = 73.56N$

Force on the pulley is 73.56N

2. A mass of 9kg resting on a smooth horizontal table is connected by a light string passing over a smooth pulley at the edge of the table, to the pulley is a 7kg mass hanging freely; find
   (i) Common acceleration
   (ii) The tension in the string
   (iii) The force on the pulley in the system if its allowed to move freely.

   Solution



For 7kg mass: $7g - T = 7a$..............(i)

For 9kg mass: T = 9a...........................(ii)

Put (ii) into (i)

$$7g - 9a = 7a$$

$$a = \frac{7g}{16}$$

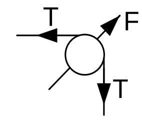
$$a = \frac{7x9.81}{16} \quad \therefore a = 4.292ms^{-2}$$

(ii) Tension : T = 9a

T = 9 x 4.292

T = 38.63N

(iii) The force on the pulley



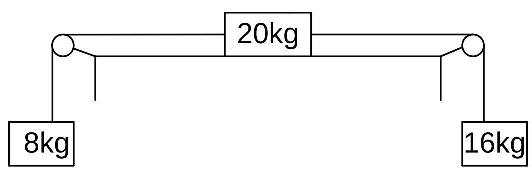
$$F^2 = \sqrt{T^2 + T^2}$$

$$F = T\sqrt{2}$$

F = 38.63$\sqrt{2}$

Force on the pulley = 54.63N

3.



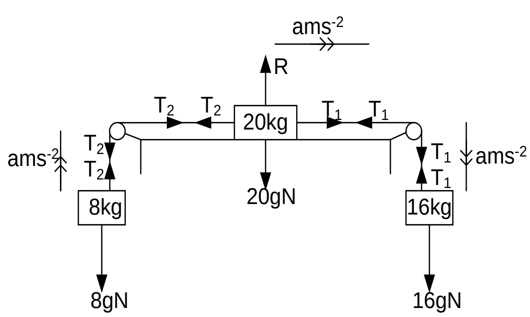
The figure shows a block of mass 20 kg resting on a smooth horizontal table. Its connected by strings which pass over pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang vertically. Calculate;

(i) Acceleration of 16kg mass

(ii) Tension in each string

(iii)Reaction on each pulley

Solution



For 16kg mass: $16g - T_1 = 16a$..........[1]

For 20kg mass: $T_1 - T_2 = 20a$............[2]

For 8kg mass: $T_2 - 8g = 8a$...............[3]

Adding 1 and 2

$16g - T_2 = 36a$.............[x]

And 3 and x

8g = 44a

$$a = \frac{8x\ 9.81}{44} \therefore a = 1.784ms^{-2}$$

ii) Tension in each string

$16g - T_1 = 16a$

$T_1 = 16x9.81 - 16x1.784$

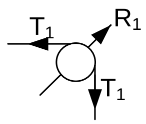
$T_1 = 128.416N$

$T_2 - 8g = 8a$

$T_2 = 8x1.784 + 8x9.81$

$T_2 = 92.752N$

iii) Reaction on each pulley

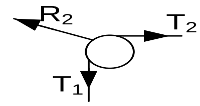


$R_1^2 = T_1^2 + T_1^2$

$R_1 = T_1\sqrt{2}$

$R_1 = 128.416x\sqrt{2}$

$R_1 = 181.61N$



$R_2 = T_2\sqrt{2}$

$R_2 = 92.752\sqrt{2}$

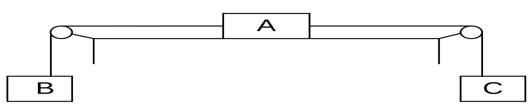
$R_2 = 131.171N$

Exercise12

1. Two particles of masses 20g and 30g are connected to a fine string passing over a smooth pulley, find;
   (i) Common acceleration **An[1.962ms^{-2}]**
   (ii) The tension in the string **An [0.235N]**
   (iii) The force on the pulley **An [0.471N]**

2. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate;
   a) The common acceleration of the masses **An[3.68m/s^2]**
   b) The tension in the string **An[18.4N]**
   c) The force acting on the pulley **An[26N]**

3.



The diagram shows a particle A of mass M = 2kg resting on a horizontal table. It is attached to particles B of m =5kg and C of m= 3kg by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string given that the surface of the table is rough and the coefficient of friction between the particle and the surface of the table is ½

An[0.98ms^{-2}, 32.37N, 44.15N]

[Hint: friction force = coefficient of friction x normal reaction]

4.1.2: LINEAR MOMENTUM AND IMPULSE

Momentum is the product of mass and velocity of the body moving in a straight line

Momentum (p) = mass x velocity

$\boxed{\vec{p} = m\vec{v}}$ Momentum is a vector quantity

IMPULSE

This is the product of the force and time for which the force acts on a body

i.e. Impulse (I) = Force(F) x time (t)

$$\vec{I} = \vec{F}\,t$$

The unit of impulse is Ns.

An impulse produces a change in momentum of a body. If a body of mass(m) has it velocity changed from u to v by a force F acting on it in time t, then from Newton's 2nd law.

$F = \frac{mv-mu}{t}$

Ft = mv-mu

$I = Ft$

I = mv-mu

Impulse = change in momentum

Example

1. A body of mass 5kg is initially moving with a constant velocity of $2ms^{-1}$, when it experiences a force of 10N is 2s, find
   (i) The impulse given to the body by the force
   (ii) The velocity of the body when the force stops acting

Solution

$I = ft$

Impulse = $10x2$ = 20Ns

$I = mv - mu$

20 = 5v-5x2

5v = 30

v = 6m/s

2. A body of mass 50kg jumps onto the ground from a height of 2m. Calculate the force which acts on him when he lands
   (i) As he bends his knees and stops within 0.2 seconds
   (ii) As he keeps his legs straight and stops within a shorter period of time of 0.05s

Solution

i) $v^2 = u^2 + 2gs$

$v^2 = 0^2 + 2x9.81x2$

$v = \sqrt{39.24}$

V = $6.3ms^{-1}$

Using $F = \frac{mv\text{-}mu}{t}$

$F = \frac{50(6.3\text{-}0)}{0.2}$

F = 1573N

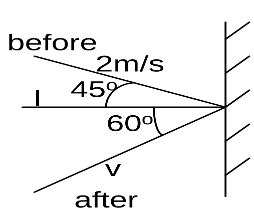
ii) ii) $F = \frac{mv-mu}{t}$

$F = \frac{50(6.3\text{-}0)}{0.05}$

F=6300N

3. A ball of mass 0.25kg moving in a straight line with a speed of $2ms^{-1}$ strikes a vertical wall at an angle of 45^0 to the normal. The wall gives it an impulse I in the direction of the normal and the ball rebounds at an angle of 60^0 to the normal. Calculate the magnitude of the impulse and the speed with which the ball rebounds.

Solution



Impulse I =mv-mu

$V = \begin{pmatrix} -vCos60 \\ -vSin60 \end{pmatrix}$ $V = \begin{pmatrix} \frac{-1}{2}v \\ \frac{-\sqrt{3}}{2}v \end{pmatrix}$

$U = \begin{pmatrix} 2cos45 \\ 2sin45 \end{pmatrix}$ $\therefore U = \begin{pmatrix} \sqrt{2} \\ -\sqrt{2} \end{pmatrix}$

$I = mv - mu$

$$I = 0.25\begin{pmatrix} \frac{-1}{2}v \\ \frac{-\sqrt{3}}{2}v \end{pmatrix} - 0.25\begin{pmatrix} \sqrt{2} \\ -\sqrt{2} \end{pmatrix}$$

$$I = \begin{pmatrix} -\frac{v}{8} - \frac{\sqrt{2}}{4} \\ -\frac{\sqrt{3}}{8}v + \frac{\sqrt{2}}{4} \end{pmatrix}$$

Since I is perpendicular to the wall then the vertical component is zero

$$-\frac{\sqrt{3}}{8}v + -\frac{\sqrt{3}}{8}v + \frac{\sqrt{2}}{4} = 0$$

$\frac{-\sqrt{3}}{8}v = \frac{-\sqrt{2}}{4}$

$V = \frac{2\sqrt{2}}{3}$m/s

$I = \frac{-v}{8} - \frac{\sqrt{2}}{4}$ Since vertical component is 0

$I = \frac{-2\sqrt{2}}{8\sqrt{3}} - \frac{\sqrt{2}}{4}$

I=2.23Ns

4.1.3: WHY LONG JUMPER BEND KNEES

The force F exerted on a long jumper on coming to rest is F= $\frac{change\ in\ momentum}{time\ taken}$

Since the change in momentum is constant. Therefore the knees are bent so as to increase the time taken to come to rest which reduces the rate of change of momentum, therefore the force on the jumpers legs is reduced thus less pain on the legs.

Question: Explain why, when catching a fast moving ball, the hands are drawn backwards while ball is being brought to rest.

Question: Explain why a long jumper must land on sand

4.1.4: LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that for a system of colliding bodies, their total momentum remains constant in a given direction provided no external force acts on them.

Suppose a body A of mass m, and velocity U_1, collides with another body B of mass m_2 and velocity U_2 moving in the same direction

By principle of conservation of momentum

$$\boxed{\underbrace{m_1u_1 + m_2u_2}_{\text{Total momentum before collision}} = \underbrace{m_1v_1 + m_2v_2}_{\text{Total momentum after collision}}}$$

4.1.5: Proof of the law of conservation of momentum using Newton's law

Let two bodies A and B with masses m_1 and m_2 moving with initial velocities u_1 and u_2 and let their velocities after collision be $v_1$and v_2 respectively for time t with (v_1<v_2)

By Newton's 2^{nd} law:

Force on m_1: $F_1 = \frac{m_1(v_1-u_1)}{t}$

Force on m_2: $F_2 = = \frac{m_2(v_2-u_2)}{t}$

By Newton's 3rd law: $\boldsymbol{F_1} = -\boldsymbol{F_2}$

$$\frac{m_1(v_1-u_1)}{t} = -\frac{m_2(v_2-u_2)}{t}$$

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$\therefore\ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Hence $m_1u_1 + m_2u_2 =$ **constant**

4.1.6: COLLISIONS

In an isolated system, momentum is always conserved but this is not always true of the kinetic energy of the colliding bodies.

In many collisions, some of the kinetic energy is converted into other forms of energy such as heat, light and sound.

Types of collisions

1. Elastic collisions

It is also perfectly elastic collision. This is a type of collision in which all kinetic energy is conserved. Eg collision between molecules, electrons.

2. Inelastic collision

This is a type of collision in which the kinetic energy is not conserved.

3. Completely inelastic collision

This is a type of collision in which the bodies stick together after impact and move with a common velocity. Eg a bullet embedded in a target

4. Explosive collision (super elastic)

This is one where there is an increase in K.E.

Summary

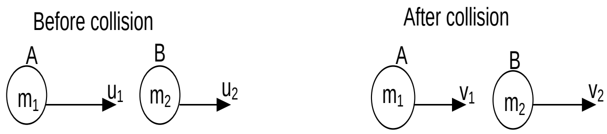
Elastic collision
- Linear momentum is conserved
- Kinetic energy is conserved
- Bodies separate after collision
- Coefficient of restitution (elasticity)=1 (e=1)

Inelastic collision
- Linear momentum is conserved
- Kinetic energy is not conserved
- Bodies separate after collision
- Coefficient of restitution is less than 1 (e<1)

Perfectly inelastic
- Linear momentum is conserved
- Kinetic energy is not conserved
- Bodies stick together and move with a common velocity
- e=0

4.1.7: Mathematic treatment of elastic collision

Consider an object of mass m, moving to the right with velocity u_1. If the object makes a head-on elastic collision with another body of mass m_2 moving with a velocity u_2 in the same direction. Let v_1 and v_2 be the velocities of the two bodies after collision.



By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$------------------- [1]

For elastic collision k.e is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2$

$m_1({u_1}^2 - {v_1}^2) = m_2({v_2}^2 - {u_2}^2)$ --------------- [2]

from equation 1 and 2 then

[1]÷ [2]

$$\frac{m_1(u_1 - v_1)}{m_1(u_1^2 - v_1^2)} = \frac{m_2(v_2 - u_2)}{m_2(v_2^2 - u_2^2)}$$

$$\frac{(u_1 - v_1)}{(u_1 + v_1)(u_1 - v_1)} = \frac{(v_2 - u_2)}{(v_2 + u_2)(v_2 - u_2)}$$

$$\frac{1}{(u_1 + v_1)} = \frac{1}{(v_2 + u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

$$v_2 - v_1 = -(u_2 - u_1)$$

Example

1. A particle P of mass m_1, travelling with a speed u_1 makes a head-on collision with a stationary particle Q of mass m_2. If the collision is elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Show that for $\beta = \frac{m_1}{m_2}$

   (i) $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ (ii) $\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

Solution

By law of conservation of momentum

$m_1u_1 = m_1v_1 + m_2v_2$--------------------[x]

$(u_1 - v_1) = \frac{m_2}{m_1}v_2$

Therefore $u_1 - v_1 = \frac{v_2}{\beta}$

$v_2 = \beta(u_1 - v_1)$----------------------------- [1]

for elastic collision k.e is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2 - 0)$

$\frac{m_1}{m_2}({u_1}^2 - {v_1}^2) = {v_2}^2$

$\beta(u_1^2 - v_1^2) = v_2^2$--------------------------[2]

equating [1] and [2]

$\beta(u_1^2 - v_1^2) = [\beta(u_1 - v_1)]^2$

$\beta(u_1^2 - v_1^2) = \beta^2(u_1 - v_1)(u_1 - v_1)$

$(u_1 - v_1)(u_1 + v_1) = \beta(u_1 - v_1)(u_1 - v_1)$

$(u_1 + v_1) = \beta(u_1 - v_1)$

$v_1 + \beta v_1 = \beta u_1 - u_1$

$v_1(1+\beta) = u_1(\beta - 1)$

$\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$

ii) From $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ ----------------------- [xx]

from equation[1]

$v_2 = \beta(u_1 - v_1)$

$v_2 = \beta u_1 - \beta v_1$

$u_1 = \frac{v_2 + \beta v_1}{\beta}$ put into (xx)

$$\frac{u_1}{v_1} = \frac{(1+\beta)}{(\beta-1)}$$

$$\frac{\left(\frac{v_2 + \beta v_1}{\beta}\right)}{v_1} = \frac{(1+\beta)}{(\beta-1)}$$

$(v_2 + \beta v_1)(\beta - 1) = (1+\beta)\beta v_1$

$\beta v_2 + \beta^2 v_1 - v_2 - \beta v_1 = \beta v_1 + \beta^2 v_1$

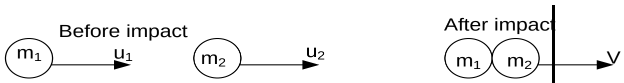
$\beta v_2 - v_2 = 2\beta v_1$

$v_2(\beta - 1) = 2\beta v_1$

$\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

4.1.8: Mathematical treatment of perfectly inelastic collision

Suppose a body of mass m, moving with velocity u_1 to the right makes a perfectly inelastic collision with a body of mass m_2 moving with velocity u_2 in the same direction



By law of conservation

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Total kinetic energy before collision

k.e$_i$ = $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

Total kinetic energy after collision

k.e$_f$= $\frac{1}{2}(m_1 + m_2)v^2$

Loss in k.e= k.e$_i$ – k.e$_f$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

Numerical examples

1. Ball P, Q and R of masses m_1, m_2 and m_3 lie on a smooth horizontal surface in a straight line. The balls are initially at rest. Ball P is projected with a velocity u_1 towards Q and makes an elastic collision with Q. if Q makes a perfectly in elastic collision with R, show that R moves with a velocity.

$$v_2 = \frac{2\,m_1m_2u_1}{(m_1 + m_2)(m_2 + m_3)}$$

Solution

Elastic collision of P and Q:

Conservation of momentum:

$m_1u_1 = m_1v_P + m_2v_Q$

$v_P = u_1 - \frac{m_2v_Q}{m_1}$.......(1)

Conservation of kinetic energy:

$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_P^2 + \frac{1}{2}m_2v_Q^2$.....(2)

Putting [1] into [2

$m_1u_1^2 = m_1\left(u_1 - \frac{m_2v_Q}{m_1}\right)^2 + m_2v_Q^2$

$v_Q = \frac{2\,m_1u_1}{m_1+m_2}$......(3)

In elastic collision of Q and R:

$m_2v_Q + m_3 0 = (m_2 + m_3)v_2$

$m_2\frac{2\,m_1u_1}{m_1 + m_2} = (m_2 + m_3)v_2$

$v_2 = \frac{2\,m_1m_2u_1}{(m_1 + m_2)(m_2 + m_3)}$

2. A 0.2kg block moves to the right at a speed of $1ms^{-1}$ and meets a 0.4kg block moving to the left with a speed of $0.8ms^{-1}$. Find the find velocity of each block if the collision is elastic.

Solution

By law of conservation

$M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$

$(0.2x1) + (0.4x - 0.8) = 0.2v_1 + 0.4v_2$

-0.8 because its moving to left

$0.2 - 0.32 = 0.2v_1 + 0.4v_2$

$v_1 + 2v_2 = -0.6$---------------------- [1]

for elastic collision K.E is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$0.2x1^2 + 0.4x(-0.8)^2 = 0.2v_1^2 + 0.4v_2^2$

$0.2 + 0.256 = 0.2v_1^2 + 0.4v_2^2$

$v_1^2 + 2v_2^2$ =2.28 ----------------------- [2]

But from [1] v_1=-0.6-2v_2 put into (2)

$v_1^2 + 2v_2^2$=2.28

$2v_2^2$+(0.6-2v_2)^2 = 2.28

$6v_2^2$+2.4 v_2-1.92=0

v_2 =0.4m/s , $v_2 = -0.8$m/s

v_2=0.4m/s is correct since m_2 is in front it supposed to move faster

Therefore from (1)

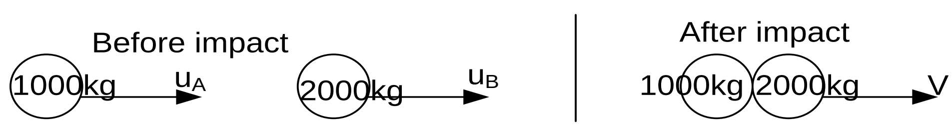
$v_1+2v_2=-0.6$

$v_1+2x0.4 = -0.6$

v_1=-1.4m/s

3. A truck of mass 1 tonne travelling at 4m/s collides with a truck of mass 2 tonnes moving at 3m/s in the same direction. If the collision is perfectly inelastic, calculate;
   (i) Common velocity
   (ii) Kinetic energy converted to other forms during collision

Solution



By law of conservation of momentum

$M_AU_A + M_BU_B = (M_A + M_B)V$

$(1000x4)$ + $(2000x3)$ = (1000 + 2000)v

10000 = 3000v

V =$\frac{10}{3}$ms^{-1}

Common velocity is$\frac{10}{3}$ms^{-1}

ii) Initial K.e = $\frac{1}{2}M_AU_A^2 + \frac{1}{2}M_BU_B^2$

= ½ x 1000x 4^2 + 1/2x2000x3^2

k.e.$_i$ = 17000J

Final k.e$_f$ = $\frac{1}{2}(M_A + M_B)V^2$

= ½(1000 + 2000)($\frac{10}{3}$)^2

= 16666.67J

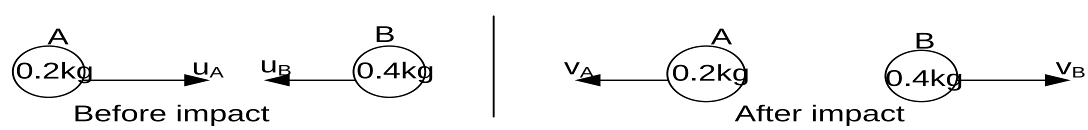
Kinetic energy converted = k.e$_i$ – k.e$_f$

= 17000 – 16666.67

= 333.33Joules

4. Two particles of masses 0.2kg and 0.4kg are approaching each other with velocities 4ms^{-1} and 3ms^{-1} respectively. On collision, the first particle reverses, its direction and moves with a velocity of 2.5ms^{-1}. find the;
   (i) velocity of the second particle after collision
   (ii) percentage loss in kinetic energy

Solution



By law of conservation of momentum

$M_AU_A + M_BU_B = M_AV_A + M_BV_B$

$0.2x4 + 0.4x - 3 = 0.2x2.5 + 0.4V_B$

-0.4 + 0.5 = 0.4V_B

V_B = 0.25m/s

The velocity of the second particle is 0.25m/s in opposite direction

ii) Initial k.e $_i$ = $\frac{1}{2}M_AU_A^2$ +$\frac{1}{2}M_BU_B^2$

= $\frac{1}{2}$ (0.2x4^2+0.4x$[-3]^2$)

= 3.4J

Final K.e$_f$ = $\frac{1}{2}M_AV_A^2$ +$\frac{1}{2}M_BV_B^2$

$= \frac{1}{2}x\ 0.2x2.5^2$ +$\frac{1}{2}x0.4x0.25^2$

=0.6475J

Loss in kinetic energy = k.e$_i$ – k.e$_f$

= 3.4 - 0.6375

= 2.7625J

% loss in k.e. $= \frac{loss\ of\ k.e}{\text{k.e}_i}x100\%$

$= \frac{2.7625}{3.4}x100\%$

% loss in k.e. =81.25%

5. A bullet of mass $1.5\ x\ 10^{-2}kg$ is fired from a rifle of mass $2.7x10^2kg$ with a muzzle velocity of 100km/h. Find the recoil velocity of the rifle.

Solution

V_b = $\frac{100\ x\ 1000}{60x60}$ V_b =27.78m/s

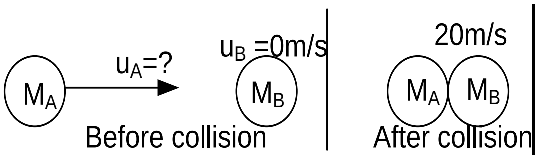
$M_gV_g\ =\ \text{M}_\text{b}V_b$

$3V_g\ =\ 1.5\ x\ 10^{-2}x27.78$

V_g =0.14m/s

6. A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of 20m/s. Calculate
   (i) The speed with which the bullet hits the wood
   (ii) The kinetic energy lost

Solution



By the principle of conservation of momentum

$M_AU_A\ + M_BU_B\ = (M_A\ + M_B)V$

$(0.02xU_A) + (0.4x0)$

$= (0.02 + 0.4)x20$

$u_A\ \ =\ \ 420m/s$

The original velocity of the bullet was *420m/s*

Initial K.e = $\frac{1}{2}M_AU_A^2$ +$\frac{1}{2}M_BU_B^2$

= $\frac{1}{2}\ x\ 0.02x\ 420^2 + \frac{1}{2}x0.4x0^2$

=1764J

Final K.e$_f$ = $\frac{1}{2}(M_A + M_B)V^2$

= $\frac{1}{2}x$(0.02 + 0.04)x(20)^2

= 84J

Loss in kinetic energy = k.e$_i$ – k.e$_f$

= 1764 - 84

= 1680J

Exercise13

1. A 2kg object moving with a velocity of 8m/s collides with a 3kg object moving with a velocity 6ms^{-1} along the same direction. If the collision is completely inelastic, calculate the decrease in kinetic energy collision. **An [2.4J]**

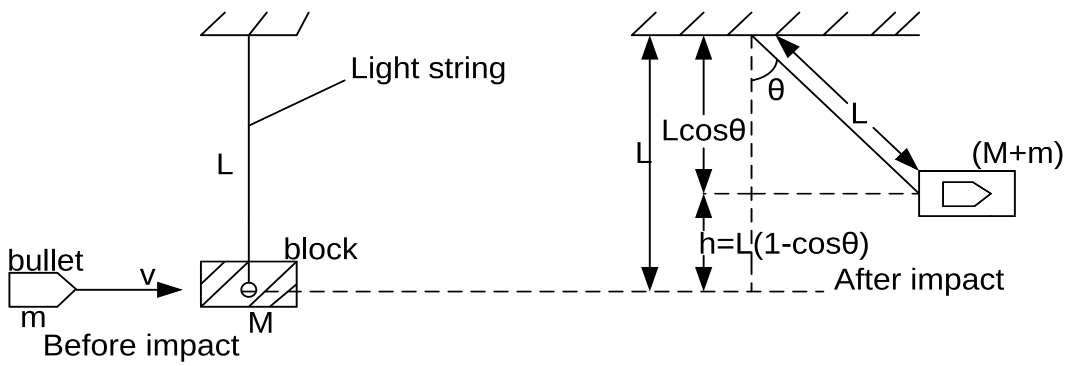
2. Two bodies A and B of mass 2kg and 4kg moving with velocities of 8m/s and 5m/s respectively collide and move on in the same direction. Object A's new velocity is 6m/s.

   (i) Find the velocity of B after collision
   (ii) Calculate the percentage loss in kinetic energy. **An(6m/s, 5.26%)**

3. A particle of mass 2kg moving with speed $10ms^{-1}$ collides with a stationary particle of mass 7kg. Immediately after impact the particles move with the same speeds but in opposite directions. Find the loss in kinetic energy during collision. **An(28J)**

4. A 2kg object moving with a velocity of $6ms^{-1}$ collides with a stationary object of mass 1kg. If the collision is perfectly elastic, calculate the velocity of each object after collision. **An[$2ms^{-1}$, $8ms^{-1}$]**

4.1.9: BALLISTIC PENDULUM



Resolving along the vertical gives $Lcos\theta$

But $L = Lcos\theta + h$

$h = L - Lcos\theta$

$h = L(1 - cos\theta)$

The device illustrates the laws of conservation of momentum and mechanical energy

a) **During impact**

- Mechanical energy is not conserved because of friction and other non conservative forces
- Linear momentum is conserved in the horizontal direction along which there is no external force

If V_c is the velocity of combined mass just after collision

$Mv + mx0 = (M + m)V_c$

$mv = (m + M)V_c$……………………………….(i)

The block was initially at rest.

b) **Swing after impact**

- ❖ Mechanical energy is conserved. The conserved gravitational force causes conversion of $k.e$ to $p.e$.
- ❖ Momentum is not conserved because an external resultant force (pull of the earth / weight) acts on the bullet-block system.

From (i) $k.e. = p.e.$

$$\frac{1}{2}(M+m)V_c^2 = (M+m)gh$$

$V_c^2 = 2gh$……………………..(x)

But $h = L(1-cos\theta)$

$$V_c^2 = 2gL(1-cos\theta)$$

$\boldsymbol{V_C} = \sqrt{2gL(1-cos\theta)}$ ………(2)

θ is the angle of swing

Factors on which angle of swing depends

- ➢ The speed of the bullet
- ➢ The length of the string

NB; the angle can be obtained from

$h = L(1-cos\theta)$

$\frac{h}{L} = (1-cos\theta)$

$Cos\theta = 1 - \frac{h}{L}$

$Cos\theta = \frac{L-h}{L}$

$\theta = Cos^{-1}\left(\frac{L-h}{L}\right)$

OR

$\boldsymbol{V_C} = \sqrt{2gL(1-cos\theta)}$

v_c^2 =2gL (1-cosθ)

$\frac{v_c^2}{2gL}$ =(1-cosθ)

$Cos\theta$ =1- $\frac{v_c^2}{2gL}$

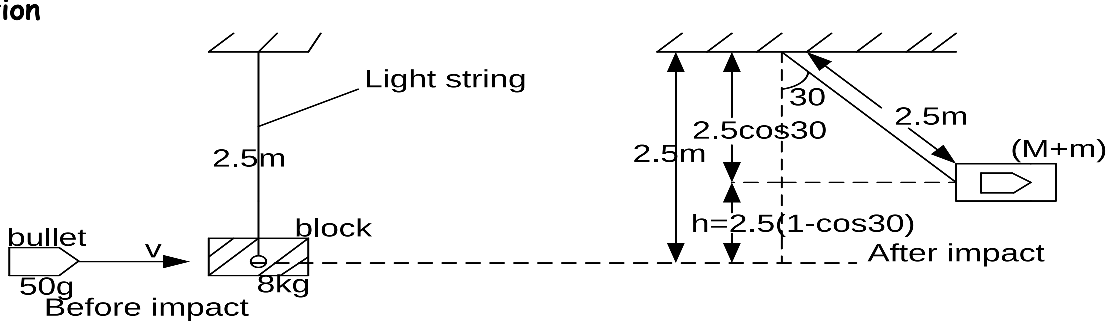
$Cos\theta = \left(\frac{2gL - v_c^2}{2gL}\right)$

θ =cos^{-1} $\left(\frac{2gL - v_c^2}{2gL}\right)$

Examples

1. A bullet of mass 50g is fired horizontally into a block of wood of mass 8kg which is suspended by a string of length 2.5m. after collision the block swing upwards through an angle 30^0. Calculate the velocity of the bullet assuming that it gets embedded in the block just after collision.

Solution



$$h = L\,(1-cos\theta)$$
$$h = 2.5(1-\cos 30)$$
$$h = 0.335m$$

Before impact (law of conservation of momentum)

$$mv + Mx0 = (M+m)\,V_C$$
$$\frac{50}{1000}v = \left(\frac{50}{1000}+8\right)V_C$$
$$0.05v = 8.05Vc$$
$$V_C = \frac{v}{161}$$

After impact (By conservation of mechanical energy)

$\frac{1}{2}(m+M)V_c^2 = (m+M)gh$

$$\frac{1}{2}(8+0.05)V_C^2 = (0.05+8)\,x9.81x0.335$$
$$V_C^2 = 6.5727$$
$$V_C = 2.564m/s$$

V_C is the velocity of bullet block system

But $V_C = \frac{v}{161}$

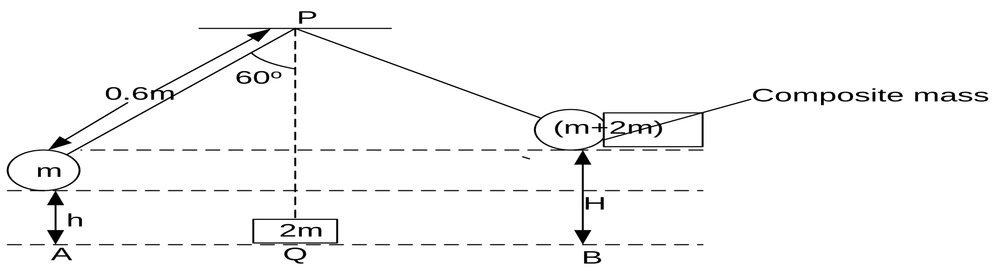
$V = 161V_C$

$V = 161x2.564$

$V = 412.804m/s$

The velocity of the bullet is 412.804ms^{-1}

2. A steel ball of mass m is attached to an inelastic string of length 0.6m. The string is fixed to a point P so that the steel ball and the string can move in a vertical plane through P. The string is held out at an angle of 60^0 to the vertical and then released. At Q vertically below P, the wall makes a perfectly inelastic collision with the lump of plasticine of mass 2m so that the two bodies move together after collision



Calculate

(i) The velocity of the composite just after collision

(ii) The position of the composite mass with respect to point Q when the mass first comes to rest.

(iii) The composite mass now oscillates about the point Q, state two possible reasons why the composite mass finally comes to rest.

Solution

$h = L(1 - cos\theta)$

$h = 0.6(1 - cos60)$

$h = 0.3m$

Applying the law of conservation of energy at A

$P.E = K.E$

mgh= $\frac{1}{2}mv^2$

$V = \sqrt{2gh}$

$V = \sqrt{2x9.81x0.3}$

$V = 2.43m/s$

The velocity of mass m just before collision is 2.43m/s

Applying law of conservation of momentum at Q where collision occurs

i) $mv + 2mx0 = (m + 2m)Vc$

$2.43m = 3mV_C$

Vc=0.81ms^{-1}

The velocity of the composite just after collision is 0.81ms^{-1}

ii) Principle of mechanical energy at B

$K.E = P.E$

$\frac{1}{2}M_cV_c^2 = M_c$gH but M_c=(m+2m)

H = $\frac{1}{2}\frac{V_c^2}{g}$

$H = \frac{1}{2}x\ \frac{0.81^2}{9.81}$

$H = 0.33m$

iii) -Frictional force

-Air resistance

Exercise 14

1. A bullet of mass 40g is fired horizontally into freely suspended block of wood of mass 1.96kg attached at the end of an inelastic string of length 1.8m. given that the bullet gets embedded in the block and the string is deflected through an angle of 60^0 to the vertical . Find:
   (i) The initial velocity of the bullet **An[210m/s]**
   (ii) The maximum velocity of the block **An[42m/s]**

2. A bullet of mass 20g travelling horizontally at $100ms^{-1}$ embedded itself in the centre of a block of wood of mass 1kg which is suspended by a light vertical string 1m in length. Calculate the maximum inclination of the string to the vertical .
   An(36.1^0)

UNEB 2013 No 3(a)

(i) State the law of conservation of linear momentum
(01mark)

(ii) A body explodes and produces two fragments of masses $\boldsymbol{m}$ and $\boldsymbol{M}$. If the velocities of the fragments are $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{\boldsymbol{E_1}}{\boldsymbol{E_2}} = \frac{\boldsymbol{M}}{\boldsymbol{m}}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M
(04marks)

UNEB 2011 NO.2

(a) State Newton's laws of motion
(04marks)

(b) Use Newton's laws of motion to show that when two bodies collide their momentum is conserved
(04marks)

(c) Two balls P and Q travelling in the same line in opposite directions with speeds of $6ms^{-1}$ and $15ms^{-1}$ respectively make a perfect inelastic collision. If the masses of P and Q are 8kg and 5kg respectively, find the
   (i) The velocity of P
   (04marks)
   (ii) Change in kinetic energy **An[v=2.08ms^{-1},278.38J]** (04marks)

(d) (i) what is an impulse of a force
(01marks)
   (ii)Explain why a long jumper should normally land on sand.
(04marks)

UNEB 2010 NO.1

a) i) State the law of conservation of linear momentum
(01mark)
ii)Use Newton's laws to derive the a(i)
(04marks)
b) Distinguish between elastic and inelastic collision
(01mark)
c) An object X of mass M, moving with a velocity $10ms^{-1}$ collides with a stationary object Y of equal mass. After collision X moves with speed U at an angle of 30^0 to its initial direction while Y moves with a speed of Y at an angle of 90^0 to the new direction.
(i) Calculate the speeds U and Y
(05marks)
(ii) Determine whether the collision is elastic or not.
(03marks)

Solution

i. Resolving the momentum along the horizontal

$mx10 = mucos30 + mvcos60$

$10m = mu\frac{\sqrt{3}}{2} + mv\frac{1}{2}$

$20 = u\sqrt{3} + v$

$v = 20 - u\sqrt{3} \ldots\ldots\ldots\ldots [1]$

Resolving the momentum along the vertical

$0 = musin30 - mvsin60$

$musin30 = mvsin60$

$\frac{u}{2} = v\frac{\sqrt{3}}{2}$

$u = v\sqrt{3}$..............[2]

Put into [1]

$v = 20 - \sqrt{3}\, v\sqrt{3}$

$4v = 20$

$v = 5ms^{-1}$

$u = v\sqrt{3}.$

$u = 5\sqrt{3}$

$u = 8.66ms^{-1}$

ii. Total K.E before collision

$K.e = \frac{1}{2}mx10^2 = 50mJ$

Total K.e after collision

$\frac{1}{2}m(5)^2 + \frac{1}{2}m(5\sqrt{3})^2$

$= 50mJ$

Since kinetic energy is conserved then the collision is elastic

UNEB 2009 NO.1

a) i) Define the term impulse (01mark)
ii) State Newton's laws of motion (03marks)
b) A bullet of mass 10g travelling horizontally at a speed of $100ms^{-1}$ strikes a block of wood of mass 900g suspended by a light vertical string and is embedded in the block which subsequently swings freely. Find the;

(i) Vertical height through which the block rises (04marks)
(ii) Kinetic energy lost by the bullet (03marks)

[Hint k.e. lost $=\frac{1}{2}m_b u_b^2-\frac{1}{2}m_b V_C^2$]

Where V_C is velocity of combined system.

m_b -is mass of the bullet

u_b is initial velocity of the bullet **An(6.2x10^{-2}m , 49.99J)**

UNEB 2008 NO 4

a) State

(i) Newton's laws of motion (03 marks)

(ii) The principle of conservation of momentum (01 mark)

b) A body A of mass M_1 moves with velocity U_1 and collides head on elasticity with another body B of mass M_2 which is at rest. If the velocities of A and B are V_1 and V_2 respectively and given that x= $\frac{m_1}{m_2}$ Show that;

i) $\frac{u_1}{v_1}=\frac{x+1}{x-1}$ (04 marks)

ii) $\frac{v_2}{v_1}=\frac{2x}{x-1}$ (03 marks)

c) Distinguish between conservative and non conservative forces (02 marks)

d) A bullet of mass 40g is fired from a gun at 200ms^{-1} and hits a block of wood of mass 2kg which is suspended by a light vertical string 2m long. If the bullet gets embedded in the wooden block

(i) Calculate the maximum angle the string makes with the vertical (06 marks)

(ii) State factors on which the angle of swing depends **An (53.4^0)** (01 mark)

UNEB 2006 No 2(c)

(i) State the work - energy theorem (01 mark)

(ii) A bullet of mass 0.1kg moving horizontally with a speed of 420ms^{-1} strikes a block of mass 2.0kg at rest on a smooth table becomes embedded in it. Find the kinetic energy lost if they move together. **An[8400J]** (04 marks)

UNEB 2005

C i) Define linear momentum (01 mark)

i) State the law of conservation of linear momentum (01 mark)

ii) Show that the law in c(ii) above follows Newton's law of motion (03 marks)

iii) Explain why, when catching a fast moving ball, hands are drawn back while the ball is being brought to rest. (02 marks)

d). A car of mass 1000kg travelling at uniform velocity of 20ms^{-1}, collides perfectly inelastically with a stationary car of mass 1500kg, calculate the loss in kinetic

energy of the car as a result of collision **An[1.68x10^{5}J)**
(04 marks)

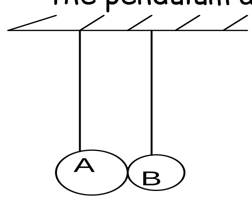
UNEB 2001 No 1

c)State the conditions under which the following will be conserved in a collision between two bodies.

i) Linear momentum [01mark]

ii) Kinetic energy [01mark]

d] Two pendula of equal length L have bobs A and B of masses 3m and m respectively the pendulum are lung with bobs in contact as shown.



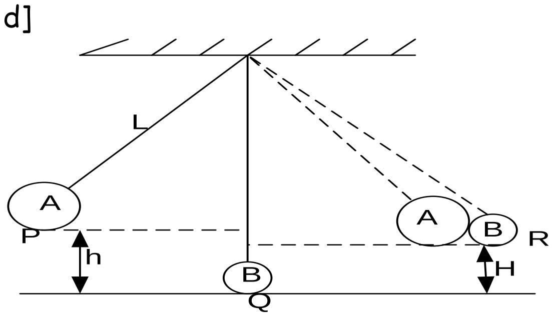
The bob A is displaced such hat the string makes an angle θ with the vertical and released. If A makes a perfectly inelastic collision with B, find the height to which B rises
[08marks]

Solution

i) Linear momentum is conserved if there is no external resultant acting on the colliding bodies.

ii) Total kinetic energy is conserved if the collision is perfectly elastic i.e the bodies separate after collision

d]



At P: $h = L(1 - cos\theta)$

$P.e = K.e$ by conservation of energy

$3mgh = \frac{1}{2}3mv^2$

Where v is the velocity with which A is released

$3mgh = \frac{1}{2}3mv^2$

$gh = \frac{v^2}{2}$

$$gL(1 - cos\theta) = \frac{v^2}{2}$$

$v^2 = 2gL(1 - cos\theta)$

$v = \sqrt{2gL(1 - Cos\theta)}$ ------------------------- [1]

At Q: Momentum is conserved

$3mv + mx0 = (3m + m)Vc$

Where Vc is the velocity of the combination

$3mv = 4mVc$

$3v = 4Vc$

$3\sqrt{2gL(1 - Cos\theta)} = 4Vc$

$Vc = \frac{3}{4}\sqrt{2gL(1 - Cos\theta)}$---------------------[2]

At R

mechanical energy is conserved

$½\,(3m + m)V_C^2 = (3m + m)gH$

H= $\frac{V_C^2}{2g}$

H= $\frac{\left(\frac{3}{4}\sqrt{2gL(1-Cos\theta)}\right)^2}{2g}$

$H= \frac{9}{16x2g}2gL(1-Cos\theta)$
$H= \frac{9gL(1-Cos\theta)}{16}$

B rises $\frac{9gL(1-Cos\theta)}{16}$

UNEB 2000 N0 1

a) i)State Newton's laws of motion [03marks]

ii) Define impulse and derive its relation to linear momentum of the body on which it acts

[03marks]

c) A ball of mass 0.5kg is allowed to drop from rest from a point at a distance of 5.0m above the horizontal concrete floor. When the ball first hits the floor, it rebounds to a height of 3.0m.

i) What is the speed of the ball just after the first collision with the floor

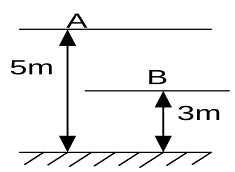
[04marks]

ii) if the collision last 0.01s, find the average force which the floor exerts on the ball

[05marks]

Solution

c)



i) By law of conservation of energy

k.e after collision =p.e at height of 3m

$½\, mv^2 \ = mgh$

$v^2 = 2gh$

$v = \sqrt{2x9.81x3}$

$v = \ 7.67m/s$

Where v is the velocity with which it rebounds from the floor .

ii) Force $= \frac{change\ in\ momentun}{time}$

k.e on hitting floor =p.e at height of 5m

mgh= $\frac{1}{2}$ mu^2

$u = \sqrt{2gh}$

$u = \sqrt{2x9.81x5}$

u =9.9ms^{-1}

Since velocity is a vector quantity

$v = -7.67$ since it rebounds (moves in opposite direction)

$F = \frac{mv-mu}{t}$

F= $\frac{(0.5x9.9)-(0.5x-7.67)}{0.01}$

F=878.5N

UNEB 1997 N0 2

a) Define the terms momentum [01marks]

b) A bullet of mass 300g travelling at a speed of 8ms^{-1} hits a body of mass 450g moving in the same direction as the bullet at 15ms^{-1}. The bullet and body move together after collision. Find the loss in kinetic energy [06marks]

c) i) State the work energy theorem [01mark]

ii)A ball of mass 500g travelling at a speed of 10ms^{-1} at 60^0 to the horizontal strikes a vertical wall and rebounds with the same speed at 120^0 from the original direction. If the ball is in contact with the wall for 8x10^{-3}, calculate the average force

exerted by the ball. **Ans[625N]**
[06marks]

4.2.0: SOLID FRICTION

Friction is the force that opposes relative motion of two surfaces in contact.

4.2.1: Types of friction

There are two types of friction i.e. static friction and kinetic friction.

1. Static friction

It's a force that opposes the tendency of a body to slide over another.

Note:

Limiting friction is the maximum frictional force between two surfaces in contact when relative motion is just starting.

2. Kinetic/sliding/dynamic friction

It's the force that opposes relative motion before two surfaces which are already in motion.

4.2.2: Laws of friction

1st law : Frictional forces between two surfaces in contact oppose their relative motion.

2nd law : Frictional forces are independent of the area of contact of the surfaces provided that normal reaction is constant.

3rd law : The limiting frictional force is directly proportional to the normal but independent of relative velocity of surfaces.

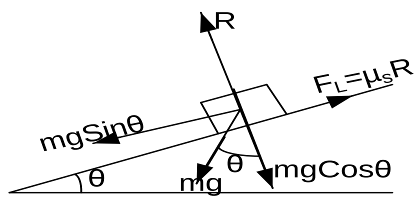
4.2.3: Molecular explanation for occurrence of friction

- Actual area of contact of two surfaces is very small, hence the pressure at points of contact is very high. Projections merge to produce welding and the force which opposes motion is obtained. This explains law 1

- The area of contact is small and the actual area of contact is the same. This explains law 2
- Increasing weight increases the pressure at the welds and hence a greater limiting frictional force . This explains law 3

4.2.4: MEASUREMENT OF COEFFICIENT OF STATIC FRICTION

Method I



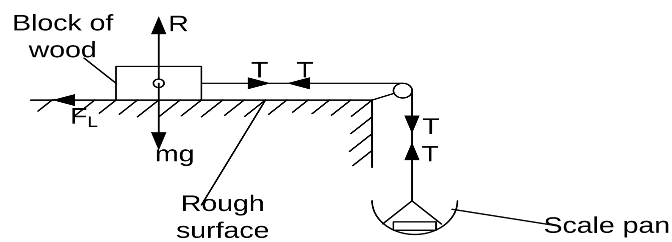
- Place a block on a horizontal plane.
- tilt the plane gently, until it **<u>just begins</u>** to slide.
- Measure and record the angle of tilt θ

  $\mu_s = tan\theta$

Coefficient of static friction $= tan\theta$

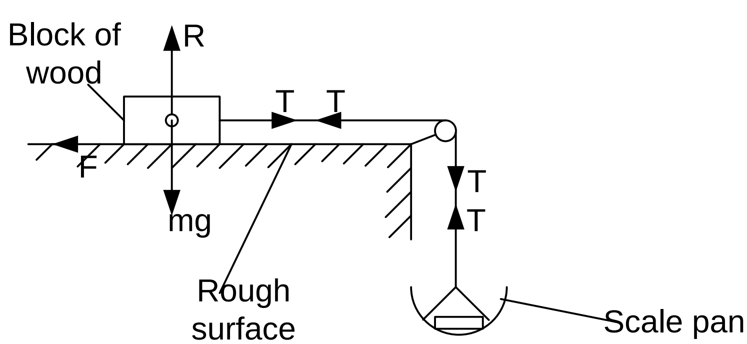
Where θ is the angle of friction

Method 2



- The mass m of the wooden block is determined and placed on a horizontal plane surface.
- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masse are added to the scale pan one at a time, till the block just slides
- The mass M of the scale pan and the masses added is obtained.
- Coefficient of static friction $\boldsymbol{\mu} = \frac{\boldsymbol{m}}{\boldsymbol{M}}$

4.2.5: Measurement of coefficient of kinetic friction



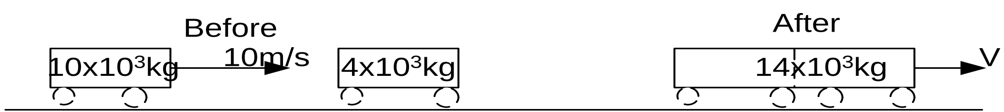
- The mass m of the wooden block is determined and placed on a horizontal plane surface.

- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masse are added to the scale pan one at a time, till the block moves with a uniform speed
- The mass M of the scale pan and the masses added is obtained.
- Coefficient of kinetic friction $\boldsymbol{\mu} = \frac{\boldsymbol{m}}{\boldsymbol{M}}$

EXAMPLES

1. A truck of mass 10 tones moving at 10ms^{-1} draws into a stationary truck of mass 4 tones. They stick together and skid to a stop a long a horizontal surface. Calculate the distance through which the trucks skid, if the coefficient of kinetic friction is 0.25.

Solution



By law of conservation of momentum

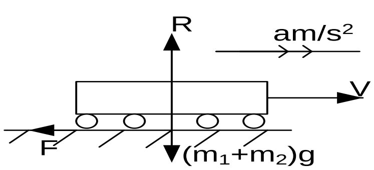
$m_1u_1+m_2u_2=(m_1+m_2)v$

$10^4x10+(4x10^3x0) =[10^4+4x10^3]v$

100000 = 14000v

V=7.143ms^{-1}

When they skid to a s top, they experience a friction force



$F = \boldsymbol{\mu}R$ but $R = (m_1 + m_2)g$

$F = \boldsymbol{\mu}(m_1 + m_2)g$

$F = 0.25(104 + 4x103)x9.81$

$Frictional\ force\ = 34335N$

Frictional force is the only resultant forces, therefore from Newton's 2nd law of motion

34335= $(m_1+m_2)a$

34335=$(10^4+4x10^3)a$

a ≈ 2.453ms^{-2}

The trucks come to a stop then

$a = -2.453$ms^{-2}(a deceleration)

To get the distance the trucks come to rest

u=7.143ms^{-1} $v = 0$m/s, a =2.453ms^{-2}

$V^2=u^2+2as$

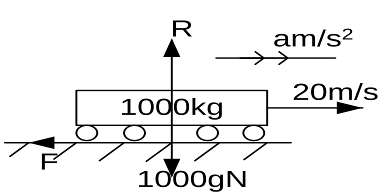
$0^2=7.143^2x2x-2.453s$

S =10.4m

2. A car of mass 1000kg moving along a straight road with a speed of 72kmh^{-1} is brought to rest by a speedy application of brakes in a distance of 50m. Find the coefficient of kinetic friction between the tyres and the road.

Solution

u=$\frac{72x1000}{3600}$=20m/s



$F = \mu R$ But

$R = mg$ =1000x9.81

R = 9810N

F= 9810 μ--------- [1]

$F = ma$ ----------------- [2]

To get the distance the car comes to rest

u=20m/s , v=0m/s, s=50m

$V^2=u^2+2as$

0=$20^2 + 2ax50$

$a = -4$ms^{-2}

$F = 1000x - 4$

$F = -4000N$

Frictional force =4000N

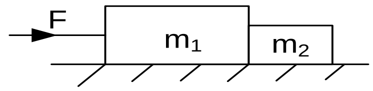
F=9810 μ

4000=9810 μ

μ =0.41

Coefficient of friction =0.41

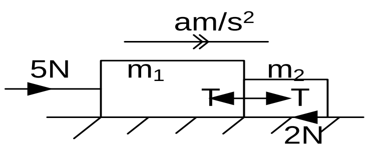
3. Two blocks of masses m_1=3kg and m_2=2kg are in contact on a horizontal table. A constant horizontal force F=5N is applied to the block of mass m_1 in the direction shown



There is a constant frictional force of 2N between the table and the block of mass m_2 but no frictional force between the table and the block of mass m_1. Find:

i) The acceleration of the two blocks
ii) The force of contact between the blocks

Solution



By Newton's 2nd law

For block m_1, $5 - T = m_1 a$

$5 - T = 3a$ ---------- [1]

For block m_2 : $T - 2 = m_2 a$

$T - 2 = 2a$----------- [2]

Adding 1 and 2

$3 = 5a$

$a = 0.6$ms^{-2}

but from 2

$T - 2 = 2a$

$T = 2x0.6 + 2$

$T = 3.2N$

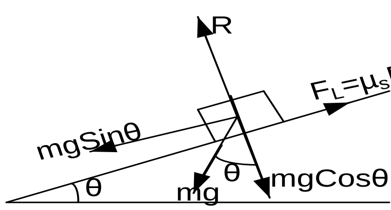
Acceleration of two blocks =0.6ms^{-2}

Force of contact=3.2N

4. A block of wood of mass 150g rests on an inclined plane. If the coefficient of static friction between the surface of contact is 0.3. find;
   a) The greatest angle to which the plane may be tilted without the block slipping
   b) The force parallel to the plane necessary to prevent slipping when the angle of the plane to the horizontal is 30^0.

Solution

a)



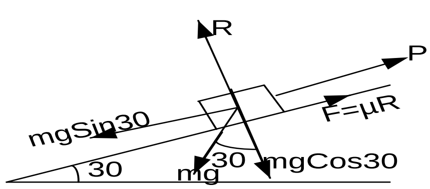
For the block not to slip then it experiences limiting friction

For limiting friction μ =tanθ

$\theta = tan^{-1}\mu$

$\theta = tan^{-1}0.3$ ∴θ=16.7^0

b)



Let P be the force parallel to the plane to prevent slipping

resultant force $= (P\ + \mu\,.R - mgsin30)$

Newton's 2nd law $P\ + \mu\,R - mgsin30 = ma$

$m = \frac{150}{1000}kg$ $(a = 0)$ no motion

but $R = mgcos30$

P+0.3$x\frac{150}{1000}x$9.81cos30= $\frac{150}{1000}x9.81sin30$

P= $(\frac{150}{1000}x9.81sin30-0.3x\frac{150}{1000}x9.81cos30)$

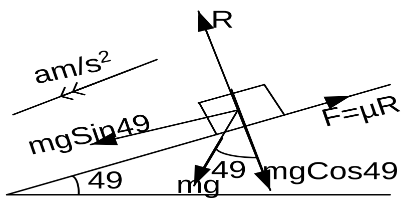
$P=0.353N$

5. A car of mass 500kg moves from rest with the engine switched off down a road which is inclined at an angle 49^0 to the horizontal

a)Calculate the normal reaction

b) If the coefficient of friction between the tyres and surface of the road is 0.32. Find the acceleration of the car

Solution



a)Resolving vertically R=mgcos49

R= $500x9.81cos49$

R=3217.97N

b) Resultant force $= mgsin49-\mu R$

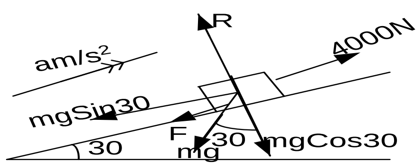
By 2nd law $mgsin49-\mu R\ = ma$

$500x9.81sin49-0.32x3217.97=500a$

a =5.34ms^{-2}

6. A car of mass 1000kg climbs a truck which is inclined at 30^0 to the horizontal. The speed of the car at the bottom of the incline is 36kmh^{-1}. If the coefficient of kinetic friction is 0.3 and engine exerts a force of 4000N how far up the incline does the car move in 5s?

Solution



u=36kmh^{-1}, u=$\frac{36x\ 1000}{3600}$ =10ms^{-2}

Resultant force $= 4000-(mgsin30+F)$

Where F= μ R

by Newton's 2nd law

$4000-(mgsin30+\mu R)=ma$

$4000-(1000x9.81sin30+0.3mgcos30)$=$1000a$

$a=\ -3.45$ms^{-2}

$S=ut\ +\ ½\ at^2$

$S=10x5+\frac{1}{2}x-3.45x5^2$

$S\ =\ 6.9m$

Exercise15

1. A particle of weight 4.9N resting on a rough inclined plane of angle equal to tan^{-1}(5/12) is acted upon by a horizontal force of 8N. If the particle is on the point of moving up the plane, find coefficient of friction between the particle and the plane. **An (μ = 0.72)**

2. A box of mass 2kg rests on a rough inclined plane of angle 25^0. The coefficient of friction between the box and the plane is 0.4.Find the least force applied parallel to the plane which would move the box up the plane.
An[15.39N]

3. A particle of mass 0.5kg is released from rest and slides down a rough plane inclined at 30^0 to the horizontal. It takes 6 seconds to go 3 meter.
    i. Find the coefficient of friction between the particle and the plane
    ii. What minimum horizontal force is needed to prevent the particle from moving?

    An[0.56, 0.086N]

4. A parcel of mass 2kg is placed on a rough plane inclined at 45^0 to the horizontal, the coefficient of friction between the parcel and the plane is 0.25. Find the force that must be applied to the plane so that the parcel is just.
    i. Prevented from sliding down the plane
    ii. On the point of moving up the plane.
    An[10.39N, 17.32N]

CHAPTER 5: WORK, ENERGY AND POWER

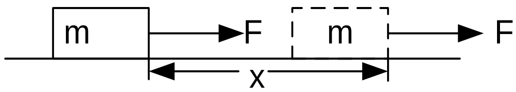
5.1.0: Work

5.1.1: Work done by a constant force

Work is said to be done when energy is transferred from one system to another

Case 1

When a block of mass m rests on a smooth horizontal



When a constant force F acts on the block and displaces it by x, then the work done by F is given by

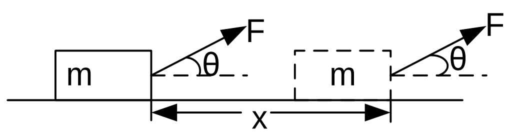
$$W = Fx$$

Definition

Work is defined as the product of force and distance moved in the direction of the force

Case II

If the force does not act in the direction in which motion occurs but at an angle to the it as shown below



$$W = (F\cos\theta)x$$

Definition

Work done is also defined as the product of the component of the force in the direction of motion and displacement in that direction

Note

1. Work done either can be positive or negative. If it is positive, then the force acts in the same direction of the displacement but negative if it acts oppositely.
   The work done by friction when it opposes one body sliding over it is negative.
2. Work and energy are scalar quantities and their S.I unit is Joules

Definition

A joule is the work done when a force of 1N causes a displacement of 1m in the direction of motion

Dimension of work

$W = Fx$

$[W] = [F]\,[x]$

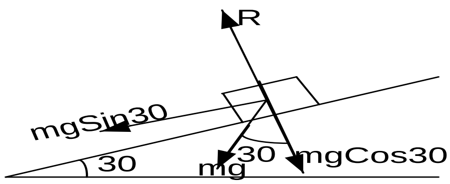
$= MLT^{-2}L$

$[W] = ML^2T^{-2}$

Examples

1. A block of mass 5kg is released from rest on a smooth plane inclined at an angle of 30^0 to the horizontal and slides through 10m. Find the work done by the gravitation force.

Solution



Work done by gravitational force

$W = mgsin30xd$

$W = \ 10x5x9.81sin30$

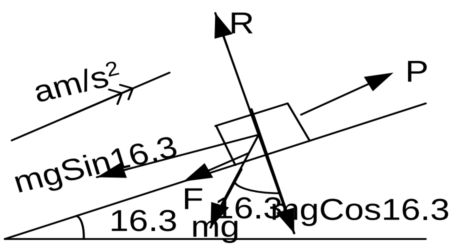
$W = 245.25J$

Note we use mgsin30 since it's the gravitational force along the direction of motion.

2. A rough surface is inclined at $\tan^{-1}\left(\frac{7}{24}\right)$ to the horizontal. A body of mass 5kg lies on the surface and is pulled at a uniform speed a distance of 75cm up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is $\frac{5}{12}$. Find;
   a) Work done against gravity
   b) Work done against friction

Solution

θ = $\tan^{-1}\left(\frac{7}{24}\right)$ ∴θ=16.3^0



a) Work done against friction

$W = Fd$ But $F = \mu R$

$R = mgcos\theta$

$W = \ \mu mgcos\theta d$

$= \frac{5}{12}x5x\frac{75}{100}x9.81cos16.3$

W=14.71J

b) Work done against gravity

W=$mgsin\,\theta xd$

W=$5x9.81sin16.3x\frac{75}{100}$

W=10.35J

5.2.0 : ENERGY

This is the ability to do work.

When an interchange of energy occurs between two bodies, we can take the work done as measuring the quantity of energy transferred between them.

5.2.1: KINETIC ENERGY

Kinetic energy is the energy possessed by a body due to its motion.

Formulae of kinetic energy

Consider a body of mass m accelerated from rest by a constant force, F so that in a distance, s it gains velocity, v

Then $v^2 = u^2 + 2as$ but $(u = 0)$

$a = \frac{v^2}{2s}$

resultant force $F = ma$

F= $\frac{mv^2}{2s}$

work done $= FxS$

$w = \frac{mv^2}{2s}$s

$w = \frac{mv^2}{2}$

by law of conservation of energy

$work\ done\ = k.e\ gained$

$$\boxed{k.e = \frac{1}{2}mv^2}$$

5.2.2: WORK-ENERGY THEOREM

The work energy theorem is the relation between work and energy.

Consider a body of mass m accelerated from u by a constant force F so that in a distance s it gains velocity v

$v^2 = u^2 + 2as$

$a = \frac{v^2-u^2}{2s}$ ------------------------ [1]

resultant $force\ F = ma$

$F = \frac{m(v^2-u^2)}{2s}$

But work done $= Fxs$

$W = \frac{m(v^2-u^2)}{2s}s$

W=$\frac{m(v^2-u^2)}{2}$

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

This is the work-energy theorem.

It states that:

The work done by the resultant external force on a body is equal to the change in the kinetic energy of the body.

Question: Explain why it is easier to walk on a straight road than an inclined road up hill.

Examples

1. A car mass 1000kg moving at 50ms^{-1} skid to rest under a constant retardation. Calculate the magnitude of the work done by the force of friction given that the car.
   a) Comes to a halt in 4s
   b) Skids through 150m

Solution

a) Using $v = u + at$

$0 = 50 + 4a$

$a = -12.5m/s^2$

Frictional force $= ma$

$F = 1000x - 12.5$

Frictional force =12500N

$S = ut + ½\ at^2$

$S = 50x4 + ½\ x - 12.5x4^2$

$S = 100m$

Work done $= FxS$

$= 12500x100$

Work done $= 1.25x10^6J$

Alternatively

Using work-energy theorem

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

W= $\frac{1}{2}x1000x50^2 - \frac{1}{2}\ x1000x0^2$

Work done $= 1.25x10^6J$

b) $v^2 = u^2 + 2as$

$0 = 50^2 + 2ax150$

$a = -8.33ms^{-2}$

$F = ma$

$F = 1000x - 8.33$

Frictional force= $8330N$

W= Fxs

$= 8330x150$

$\approx 1.25x10^6 J$

Alternatively

Using work energy theorem

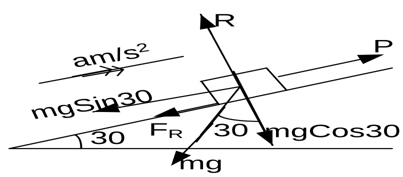
$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

W= ½ $x1000x50^2 - \frac{1}{2}\ x1000x0^2$

W=$1.25x10^6$J

2. A force FN is used to drag a body of mass 2kg up a rough plane of angle 30^0 and whose coefficient of friction is 0.25 at a steady speed. If the force drags the body through a distance of 5m. Calculate the amount of work done by the force F

Solution



F_R= μR but R=mgcos30

F_R= μmgcos30

F_R=0.25x9.81x2cos30

Using Newton's 2nd law of motion

$F - (mgsin30 + F_R) = ma$

$F - (2x9.91sin30 + 0.25x9.81x2cos30) = 2a$

a=0(steady speed)

$F - (2x9.91sin30 + 0.25x9.81x2cos30) = 0$

F=14.06N

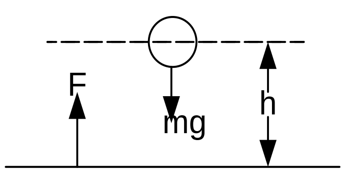
Work done =FxS

W= $14.06x5$

W= $73J$

5.2.3: GRAVITATIONAL POTENTIAL ENERGY

Potential energy is the energy that a body has due to its position in a gravitational field. Consider a body of mass m on the surface of the earth moved up a height h by a greater Force F.



Work done = $Fx\ S$

work done $= mgxh$

But work done =P.E gained at maximum height

$$\boxed{P.E = mgh}$$

Note

When a body is moving vertically upwards, it losses K.E but gains P.E and when moving downwards, it losses P.E and gains K.E

5.2.4: PRINCIPLES OF CONSERVATION OF MECHANICAL ENERGY

Mechanical energy includes kinetic energy, Gravitational potential energy and elastic potential energy.

Definition

Elastic potential energy is energy possessed by a stretched or compressed elastic material eg spring.

P.E (elastic)= ½ ke^2

Where k is the spring constant and e is the compression /extension

The principle of conservation of mechanical energy

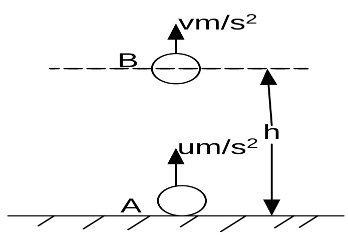
States that in a mechanical system the total mechanical energy is a constant provided that the no dissipative forces act on the system.

Examples of dissipative forces are;
Frictional force, air resistance, viscous drag

Examples of principle of conservation of M.E

i) A body thrown vertically upwards

Consider a body of mass m projected vertically upwards with speed u from a point on the ground. Suppose that it has a velocity v at a point B at a height h above the ground provided no dissipative forces act.



At A

Total mechanical energy $= \frac{1}{2} mu^2 + mgx0$

$= \frac{1}{2} mu^2$

Total mechanical energy $= \frac{1}{2} mv^2 + mgh$----- [1]

By second equation of motion

$v^2 = u^2 - 2gh$ ------------------------- [2]

Put 2 into 1

Total mechanical energy at B

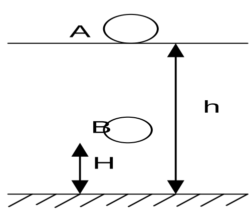
$= \frac{1}{2} m(u^2 - 2gh) + mgh$

$= \frac{1}{2} mu2 - mgh + mgh$

$= \frac{1}{2} mu^2$

∴Since total mechanical energy at A is equal to the total mechanical energy at B. Hence a stone thrown vertically upwards obeys the principles of conservation of mechanical energy.

ii) A body falling freely from a height above the ground



Suppose an object of mass m falls from a point A at a height h above the ground

At A:

Total mechanical energy $= mgh + \frac{1}{2} m0^2$

$= mgh$

At B :

Total mechanical energy $= mgH + \frac{1}{2} mv^2$-- [1]

But Newton's second law

$v^2 = 2g(h - H)$---------------------------- [2]

Total mechanical energy at B

$= mgH + \frac{1}{2} mx2g(h - H)$

$= mgH + mhg - mgH$

$= mgh$

Since the total mechanical energy at A=total mechanical energy at B than the mechanical energy of a freely falling object is conserved provided there is no dissipative force.

5.2.5: CONSERVATIVE AND NON CONSERVATIVE FORCES

1. **A conservative force** is one where the work done in moving around a closed path in the field of force is zero.

Examples of conservative forces

- Gravitational force
- Electrostatics force
- Magnetic force

2. **A non-conservative force** is one where the work done moving around a closed path in the field of force is not zero.

Examples of non- conservative force

- Frictional force
- Air resistance
- Viscous drag

Differences between conservative forces and non- conservative forces

Conservative forces	Non-conservative forces
Work done around a closed path is zero	Work done around a closed path is not zero
Work done to move a body from one point to another is independent on the path taken	Work done to move a body from one point to another is dependent on the path taken
Mechanical energy is conserved	Mechanical energy is not conserved

5.3.0: POWER

It's the rate of doing work.

Its units are watts(W) or joule per second [Js^{-1}]

Power $=\frac{\text{Work done}}{\text{time}}$

P$=\frac{\text{F x d}}{\text{t}}$

P$= Fx\ \frac{d}{t}$

$P = Fxv$

Dimensions of power

$[P] = [F]x[v]$

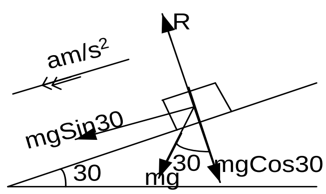
$[\mathrm{P}] = MLT^{-2}LT^{-1}$

$[\mathrm{P}] = ML^2T^{-3}$

Numerical examples

1. A 5kg mass is released from rest at the top of a smooth inclined plane. The angle of elevation of the incline is 30^0. What is the speed of the mass when it passes a point 20m down the incline.

Solution



From Newton's 2nd law

$$mgsin30 = ma$$
$$9.81sin30 = a$$
$$a = 4.905ms^{-2}$$

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2x4.905x10$$
$$v = \sqrt{2x4.905x10}$$
$$v = 9.9ms^{-1}$$

Alternatively

Loss in $p.e$ =gain in $k.e$

$$mgsin30x10 = ½\, mv^2$$
$$2x10x9.81sin30 = v^2$$
$$v = 9.9ms^{-1}$$

2. A body of mass 6kg initially moving with speed 12ms^{-1} experiences a constant retarding force of 10N for 3s. find the kinetic energy of the body at the end of this time .

Solution

$$W = ½\, mv^2 - ½mu^2$$
$$W = k.e_f - \frac{1}{2}\, x6x12^2$$
$$Fxd = k.e_f - \frac{1}{2}x6x144$$

But $d = ut$

$$Fxut = k.e_f - 3x144$$
$$-10x12x3 = k.e_f - 432$$

-10 is a retarding force

$$k.e_f = 432 - 360$$
$$k.e_f = 72J$$

Final kinetic energy = $72J$

3. A ball of mass of 0.1kg is thrown vertically up wards with an initial speed of 20m^{-1}. Calculate
   i) the time taken to return to the thrower
   ii) the maximum height
   iii) the kinetic and potential energy of the ball half way up.

Solution

i) Using $v = u + gt$

$$0 = 20 - 9.81t$$
$$t = 2.04s$$

This is the time to maximum height

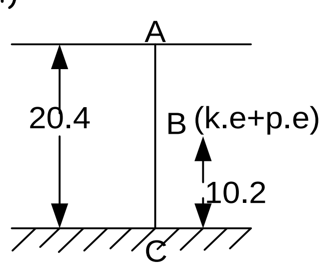
Time to return to the thrower= $2x2.04$

T= $4.08s$

ii) max height (v=om/s)

$$v^2 = u^2 - 2gs_{max}$$
$$0 = 20^2 - 2x9.81s_{max}$$
$$s_{max} = \frac{400}{2x9.81}$$
$$s_{max} = 20.39m$$

iii)



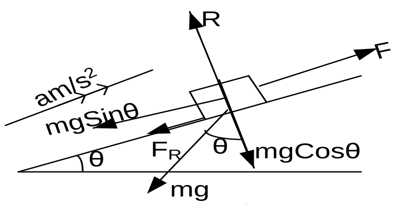
$k.e = \frac{1}{2}mv^2$ --------- (1)

But $v^2 = u^2 + 2gs$

$$v^2 = 20^2 + 2x9.81x10.2$$
$$v = 14.14m/s$$
$$k.e = \frac{1}{2}x0.1x14.14^2$$
$$k.e = 9.96J$$
$$p.e = mgh$$
$$p.e = 0.1x9.81x10.2$$
$$p.e = 10.01\ J$$

4. A train of mass $2x10^6$kg moves at a constant speed of $72kmh^{-1}$ up a straight incline against a frictional force of $1.28x10^4$N. The incline is such that the train rises vertically one meter for every 100m travelled along the incline. Calculates;
   i) Rate of increase of potential energy of the train.
   ii) The necessary power developed by the train.

Solution



$Sin\ \theta = \frac{1}{100}$

i) $P.e\ = mgh$

Rate of the potential energy change

$\left(\frac{P.e}{t}\right) = \frac{m\,g\,h}{t}$

$$s = ut + \frac{1}{2}at^2$$

$$100 = \frac{72x1000}{3600}t + \frac{1}{2}x0xt^2$$

(a=0) since it moves with constant speed

$$100 = 20t$$

$t = 5s$

$\frac{p.e}{t} = \frac{2x10^6x9.81x1}{5}$

$= 3.924x10^6Js^{-1}$

Alternatively

$\left(\frac{P.e}{t}\right) = \frac{mgSin\theta x100}{t}$

$= 2x10^6x9.81x\frac{1}{100}x100$

$\frac{P.e}{t} = 3.924x10^6Js^{-1}$

ii) We need to get the driving force (engine force)of the train.

P=driving force x velocity

By Newton's 2nd law

$$F - (mgSin\theta + F_R) = ma$$

$a = 0$ constant speed

$F - (2x166x9.81x\frac{1}{100} + 1.28x10) = 0$

$F = 2.09x10^5N$

Power= FxV

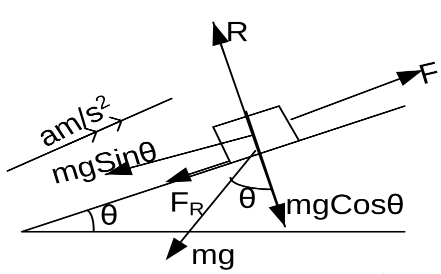
$=2.09x10^5x\frac{72X1000}{3600}$

Power=$4.18x10^6$W

5. The maximum power developed by the engine of a car of mass 200kg is 44kW. When the car is travelling at $20kmh^{-1}$ up an incline of 1 in 8 it will accelerate at $2ms^{-2}$. At what rate will it accelerate when travelling down an incline of 1 in 16 at $60kmh^{-1}$. If in both cases the engine is developing the maximum power and the resistance to motor is the same.

Solution

Case I : up the plane



But Sinθ = $\frac{1}{8}$

Power $= FxV$

$44x10^3 = Fx\frac{20x1000}{3600}$

$F = 7.92x10^3N$

By 2nd law of newton

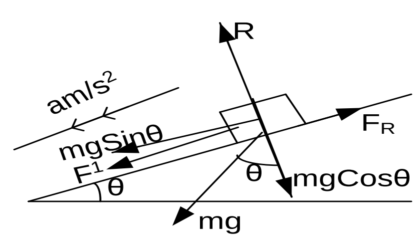
$$F - (mgsin\theta + F_R) = ma$$

$$7.92x10^3 - (200x9.81x\frac{1}{8} + F_R) = 200x2$$

$F_R = 7274.75N$

Retarding force=7275N

Case II : down the plane



But Sinθ $= \frac{1}{16}$

Power= F^1xV

$44x10^3 = F^1x\frac{20x1000}{3600}$

$F^1 = 2640N$

By newtons 2nd law

$Mgsin\theta + F^1 - FR = ma$

$200x9.81x\frac{1}{16} + 2640 - 7275 = 200a$

$2640 + 200x9.81x\frac{1}{16} - 7275 = 200a$

$a = -22.56$ms^{-2}

6. A bullet travelling at 150ms^{-1} will penetrate 8cm into a fixed block of wood before coming to rest. Find the velocity of the bullet when it has penetrated 4cm of the block.

Solution

Loss in k.e energy=work done against resistance

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = w$

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FxS$

$\frac{1}{2}mx0^2 - \frac{1}{2}mx150^2 = maxs$

$\frac{1}{2}mx150^2 = max\frac{8}{100}$

$a = -140625$ms^{-2}

Using $v^2 = u^2 + 2as$

$v^2 = 150^2 + 2x(-140625)x\frac{4}{100}$

$v = 106.06ms^{-1}$

PUMP RAISING AND EJECTING WATER.

Consider a pump which is used to raise water from a source and then eject it at a given speed.

The total work done is sum of potential energy in raising the water and kinetic energy given to the water. The work done per second gives the rate (power) at which the pump is working.

$$\boxed{\boldsymbol{work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second}}$$

Example

A pump draws 3.6$m^3$of water of density 1000kgm^{-3} from a well 5m below the ground in every minute, and issues it at ground level r a pipe of cross-sectional area 40cm^2. Find

(i) The speed with which water leaves the pipe
(ii) The rate at which the pump is working
(iii) If the pump is only 80% efficient, find the rate at which it must work
(iv) Find the power wasted

Solution

i) $volume\ per\ second = area\ x\ velocity$

$$\frac{3.6}{60} = 40x10^{-4}\ v$$

$$v = 15\ ms^{-1}$$

ii) Mass per second =volume per second $x\rho$=$\frac{3.6}{60}x1000 = 60kgs^{-1}$

$$work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second$$

$$work\ done\ per\ second = (mass\ per\ secondxgxh) + \left(\frac{1}{2}xmass\ per\ secondxv^2\right)$$

$$work\ done\ per\ second = (60x9.81x5) + \left(\frac{1}{2}x60x15^2\right)$$

$$Power = 9693W$$

iii) Efficiency $= \frac{power\ output}{power\ input} x100\%$

$$80\% = \frac{9693}{power\ input} x100\%$$

$$power\ input = 12116.25W$$

iv) Power wasted= power output – power input

Power wasted= $12116.25 - 9693 = 2423.25W$

EXERCISE16

1. A bullet of mass 50g travelling hormonally at 500ms^{-1} strikes as stationary block of wood and after travelling 10cm, it emerges from the block travelling at 100ms^{-1}. Calculate the average resistance of the block to the motion of the bullet.
   An[60000N]

2. A point A is vertically below the point B. A particle of mass 0.1kg is protected from A vertically up wards with a speed 21ms^{-1} and passes through point B with speed 7ms^{-1}. Find the distance from A to B
   An[20m]

3. The friction resistance to the motion of a car of mass 100kg is 30VN where V is the speed in ms^{1}. Find the steady speed at which the car a scends a hill of inclination sin^{-1}($\frac{1}{10}$). If the power exerted by the engine is 12.8kW.
   An[V=10m/s]

4. A load of 3Mg is being hauled by a rope up a slope which rises 1 in 140. There is a retardation force due to friction of 20gN per Mg at a certain instant when the speed is 16kmh^{-1} and the acceleration is 0.6ms$^{-2.}$ Find the pull in the rope and the power exerted at the instant. **An[2598N, 11.55kW]**

5. A car of mass 2 tones moves from rest down. A road of inclination sin^{-1} ($\frac{1}{20}$) to the horizontal. Given that the engine develops a power of 64.8kW when it is travelling at a speed of 54kmh^{-1} and the resistance to motion is 500N, find the acceleration.
   An[2.4m/s^2]

6. A car is driven at a uniform speed of 48kmh^{-1} up a smooth incline of 1 in 8. If the total mass of the car is 800kg and the resistance are neglected calculate the power at which the car is working. **An[1.31x10^{4}W]**

7. A train whose mass is 250Mg run up an incline of 1 in 200 at a uniform rate of 32km/h. The resistance due to friction is equal to the weight of 3Mg. at what power is the engine working?.

An[370.2kW]

8. A train of mass $1x10^5$kg acquires a uniform speed of $48kmh^{-1}$from rest in 400m. Assuming that the frictional resistance of 300gN. Find the tension in the coupling between the engine and the train. And the maximum power at which the engine is working during 400m run, the mass of the engine may be neglected.
An[25162N, 335.5kW]

9. A car of mass 2000kg travelling at $10ms^{-1}$ on a horizontal surface is brought to rest in a distance of 12.5m by the action of its brakes. Calculate the average retarding force. What power must the engine develop in order to take the vehicle up an incline of 1 in 10 at a constant speed of $10ms^{-1}$ if the frictional resistance is equal to 2000N.
An[8000N, 21600N]

10. A water pump must work at a constant rate of 900W and draws $0.3m^3$ of water from a deep well and issue it through a nozzle situated 10m above the level from which the water was drawn after every minute. If thee pump is 75% efficient, find;
    i) Velocity with which the water is ejected
    ii) The cross-sectional area of the nozzle

An (8.6ms^{-1}, $5.81cm^2$)

UNEB 2014 No3

(a) Define **work and energy**

(02marks)

(b) Explain whether a person carrying a bucket of water does any work on the bucket while walking on a level road

(03marks)

(c) A pump discharges water through a nozzle of diameter 4.5 cm with a speed of $62ms^{-1}$ into a tank 16 m above the intake.

(i) Calculate the work done per second by the pump in raising the water if the pump is ideal

(04marks)

(ii) Find the power wasted if the efficiency of the pump is 73%

(02marks)

(iii) Account for the power lost in (c) (ii)

(02marks)

An($2.05x10^5\, J\ s^{-1}$, $7.6x10^4 W$)

(d) (i) State the **work-energy theorem**

(01mark)

(ii) Prove the work-energy theorem for a body moving with constant acceleration.

(e) Explain briefly what is meant by internal energy of a substance

(03marks)

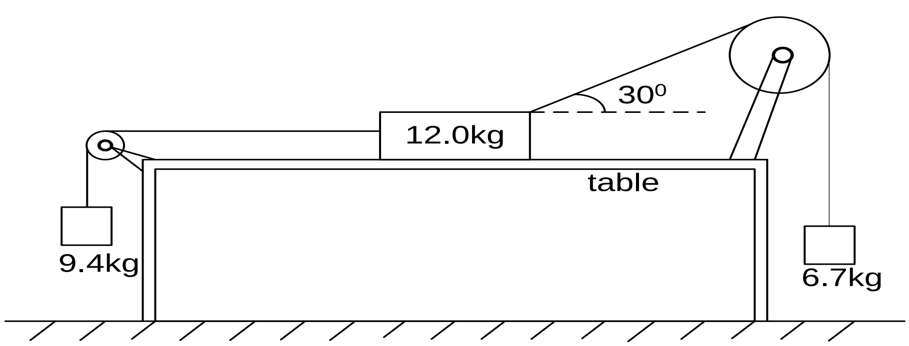
UNEB 2013 No1

(a) Using the molecular theory, explain the laws of friction between solid surface

(06marks)

(b) With the aid of a labeled diagram, describe how the coefficient of static friction for an interface between a rectangular block of wood and a plane surface can be determined. (06marks)

(c) The diagram below shows three masses connected by inextensible strings which pass over smooth pulleys. The coefficient of friction between the table and the 12.0 kg mass is 0.25

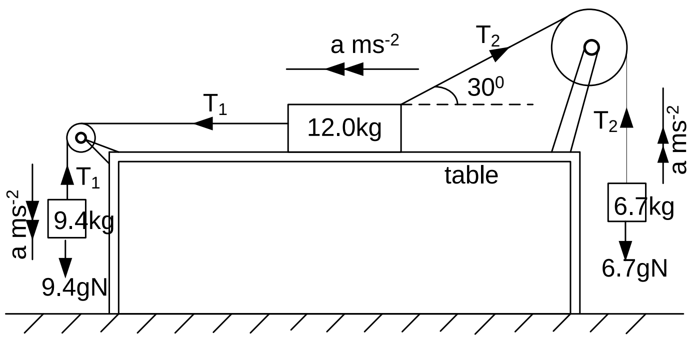


If the system is released from rest, determine the

(i) Acceleration of the 12.0kg mass (05marks)

(ii) Tension in each string (03marks)

Solution



(i) F=ma

9.4kg mass: $9.4gN \ -T_1 = 9.4a$

$T_1 = 9.4g - 9.4a$..............(1)

For 6.7kg mass: $T_2 \ -6.7gN = 6.7a$

$T_2 = 6.7a + 6.7g$...................(2)

For 12kg mass:

$T_1 - (T_2 \ Cos30° + 0.25\text{R}) = \ 12a$...........(3)

But $R + T_2 \ Sin30° = \ 12gN$

$\therefore R = 12g - T_2 \ Sin30°$

put into(3)

$T_1 - (T_2 \ Cos30° + 0.25[12g - T_2 \ Sin30])= 12a$

Put equation(1)

9.4g-9.4a-T_2 $Cos30°$-

0.25x12g+$0.25xT_2$ Sin30=12a

$$9.4g - 3g - T_2 \ \frac{\sqrt{3}}{2} + \frac{T_2}{8} = 12a + 9.4a$$

$$\frac{T_2(1 - 4\sqrt{3})}{8} = 21.4a - 6.4g$$

$T_2 = \frac{8(21.4a-6.4g)}{(1-4\sqrt{3})}$................(x)

equating equation (2) and (x)

$6.7a + 6.7g = \frac{8(21.4a-6.4g)}{(1-4\sqrt{3})}$

$50.97179677g = 210.8289616a$

$a \ = 2.372 \ m \ s^{-2}$

Acceleration of 12kg mass is $2.372 \ m$

s^{-2}

(ii)

Tension in each string

$T_1 = 9.4g - 9.4a$

$T_1 = 9.4x9.81 - 9.4x2.372$

$T_1 = 69.92N$

Also

$T_2 = 6.7a + 6.7g$

$T_2 = 6.7x2.372 + 6.7x9.81$

$T_2 = 81.62N$

UNEB2010No3

(c) i) State the laws of solid friction

[03marks]

ii) With the aid of a well labeled diagram describe an experiment to determine the co-efficient of kinetic friction between the two surfaces.

[05marks]

d) A body slides down a rough plane inclined at 30^0 to the horizontal. If the co-efficient of kinetic friction between the body and the plane is 0.4. Find the velocity after it has travelled 6m along the plane.

An[4.25m/s] [05marks]

UNEB2008 No2

a) i) state the laws of friction between solid surfaces

[03marks]

ii) Explain the origin of friction force between two solid surfaces it contact.

[03marks]

iii) Describe an experiment to measure the co-efficient of kinetic friction between two solid surfaces.

[03marks]

b) i) A car of mass 1000kg moves a long a straight surface with a speed of 20ms^{-1}. When brakes are applied steadily, the car comes to rest after travelling 50m. Calculate the co-efficient of friction between the surface and the types. **An[**$\boldsymbol{\mu = 0.\square 08}$**]**

[04marks]

c) ii) State the energy changes which occur from the time the brakes are applied to the time the car comes to rest. **An[kinetic energy**$\rightarrow \boldsymbol{heat + sound\ energy}$**]**

[02marks]

d) i)State two disadvantages of friction

[01marks]

e) ii) Give one method of reducing friction between solid surfaces.

[01mark]

UNEB2007No3

a) i)State the laws of solid friction

[03marks]

ii) Using the molecular theory, explain the laws stated in a i).

[03marks]

b) Describe an experiment to determine the co-efficient of static friction for an interface between a rectangular block of wood and plane surface.

[04marks]

c) i)State the different between conservative and non conservative forces, giving one example of each.

ii) State the work-energy theory.

[01marks]

iii) A block of mass 6.0 kg is protected with a velocity of $12ms^{-1}$ up a rough plane inclined at 45^0 to the horizontal if it travels 5.0m up the plane. Find the frictional force.

An[44.8N] [04marks]

UNEB2006No2

a) i)Define force and power

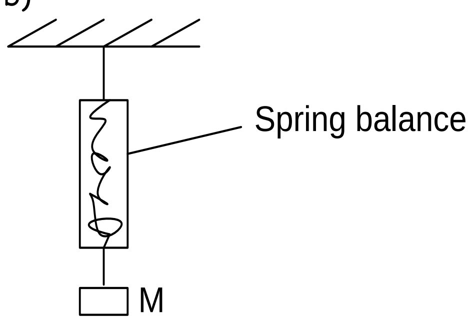
[02marks]

Force is anything that changes a body's state of rest or uniform motion in a straight line.

ii)Explain why more energy is required to push a wheelbarrow uphill than on a level ground.

[03marks]

b)



A mass M is suspended from a spring balance as shown above. Explain what happens to the reading on the spring balance when the set up is raised slowly to a very high height above the ground.

[02marks]

c) i)State the work-energy theorem

[01mark]

Solution

a)[ii] When pushing a wheelbarrow on a level ground, work is done only against the frictional force. While when pushing up hill, work is done against the frictional force plus the component of the weight of the wheelbarrow along the plane of the hill.

b)As the acceleration due to gravity reduces the weight of M decreases and it reading of the spring balance reduces proportionately.

UNEB2005No1

c) i)What is meant by conservation of energy? [01mark]

ii)Explain how conservation of energy applies to an object falling from rest in a vacuum. [02marks]

UNEB2004 N01

a) State the laws of friction [04marks]

b) A block of mass 5.0kg resting on the floor is given horizontal velocity of 5ms^{-1} and comes to rest in a distance of 7.0m. Find the co-efficient of kinetic friction between the block and the floor. **An[0.182]** [04marks**]**

c) i)State the laws of conservation of linear momentum [01mark]

ii)What is perfectly inelastic collision? [01mark]

d) A car of mass 1500kg rolls from rest down a round inclined to the horizontal at an angle of 35^0, through 50m. The car collides with another car of identical mass at the bottom of the incline. If the two vehicles interlock on collision and the co-efficient of kinetic friction is 0.20, final the common velocity of the vehicle. **An[20.05m/s]** [08marks]

[Hint loss of p.e at the top=gain in k.e at the bottom + work done against friction]

e) Discuss briefly the energy transformation which occur in(d) above. [01mark]

An[Potential energy$\rightarrow$ $kinetic\ energy + sound + heat$**]**

UNEB 2001 No1

a) i)State the principle of conservation of mechanical energy. [01mark]

ii)Show that a stone thrown vertically upwards obey, the principle in (c) throughout its upward motion. [04marks]

CHAPTER 6: STATICS

Is a subject which deals with equilibrium of forces $e.g$ the forces which act on a bridge.

Coplanar forces

Those are forces acting on the same point (plane).

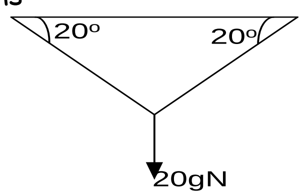
6.1.0: Conditions necessary for mechanical equilibrium

When forces act on a body then it will be in equilibrium when;

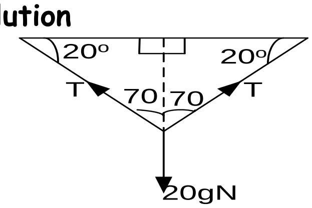
1. the algebraic sum of all forces on a body in any direction is zero
2. the algebraic sum of moments of all forces about any point is zero

Examples

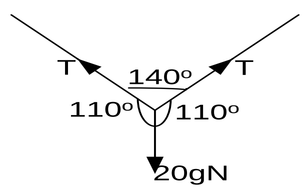
1. A mass of 20kg is hang from the midpoint P of a wire as shown below. Calculate the tension in the wire take g=9.8ms^{-1}



Solution



Method I
lami's theorem
(Apply to only three forces in equilibrium)

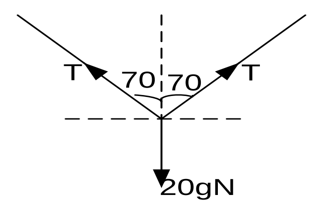


$\frac{20gN}{Sin140} = \frac{T}{Sin110}$

T=$\frac{20X9.81sin110}{Sin140}$

T= 286.83N

METHOD II using components



Resolving vertically

Tsin70 +Tcos70 = 20gN

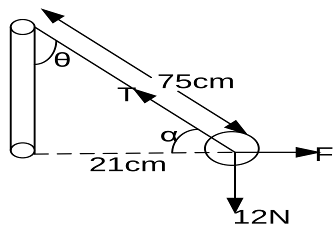
2Tcos70 = 20x9.81

T = $\frac{20x9.81}{2\ cos70}$

T = 286.83N

2. One end of a light in extensible string of length 75cm is fixed to a point on a vertical pole. A particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force. Find the magnitude of the force and the tenion in the string

Solution

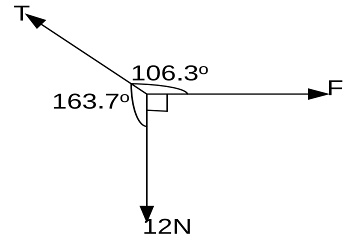


$\mathrm{Sin}\theta = \frac{21}{75}$ $\therefore \theta = 16.3°$

Also $\cos\alpha = \frac{21}{75}$

$\alpha = 73.7^0$

Using Lami's theorem



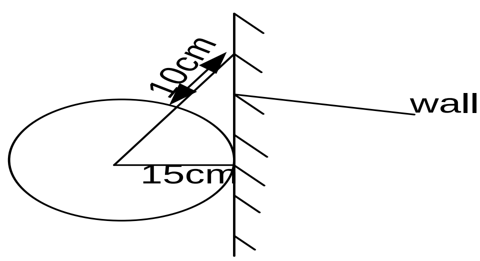
$$\frac{F}{Sin163.7} = \frac{12}{Sin106.3}$$

F = 3.51N

Also $\frac{T}{Sin90} = \frac{12}{Sin106.3}$

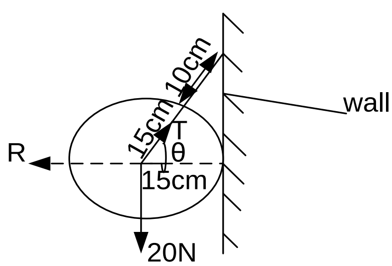
T = 12.5N

3. A sphere of weight 20N and radius 15cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown.



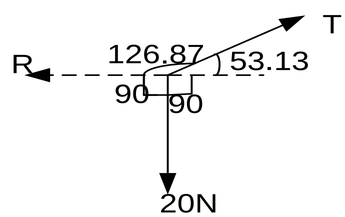
i) copy the diagram and show the forces acting on the sphere
ii) Calculate the reaction on the sphere due to the wall.
iii) Find the tension in the string

Solution



$$Cos\theta = \frac{15}{28} \quad \therefore \theta = 53.13°$$

Using Lami's theory



$$\frac{20}{Sin126.87} = \frac{T}{Sin90}$$

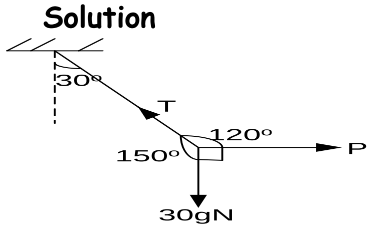
$$T = 25N$$

$$\frac{R}{Sin143.13} = \frac{20}{Sin126.87}$$

$$R = 15N$$

4. A mass of 30Kg hangs vertically at the end of a light string. If the mass is pulled aside by a horizontal force P so that the string makes an angle 30^0 with the vertical. Find the magnitude of the force P and the tension in the string.

Solution



$$\frac{30x9.81}{Sin120} = \frac{20}{Sin150}$$

P = 169.91N

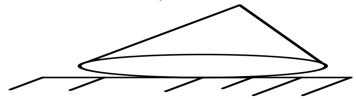
$$\frac{T}{Sin90} = \frac{30x9.81}{Sin120}$$

T= 339.83N

6.1.1: Types of equilibrium

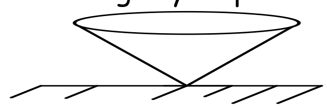
1. **Stable equilibrium**.

This when a body returns to its equilibrium position after it has been slightly displaced



2. **Unstable equilibrium**.

This is when a body does not return to its equilibrium position and does not remain in the displaced position after it has been slightly displaced



3. **Neutral equilibrium**.

This is when a body stays in the displaced position after it has been slightly displaced

6.2.0: Turning effect of forces

A force can produce a turning effect or moment about a pivot, this can be a clockwise or anti clockwise turning effect.

6.2.1: Moment of a force

This is the product of a force and the perpendicular distance of its line of action from the pivot.

The unit of a moment is Nm and it's a vector quantity.

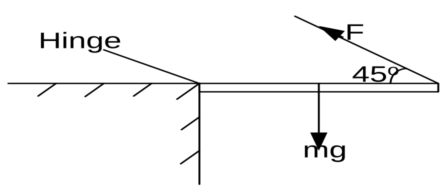
Moment of a force = Force x perpendicular distance of its line of action from pivot.

6.2.2: Principle `of moments

It states that when a body is in equilibrium, the sum of clockwise moments about a point is equal to the sum of anticlockwise moments about the same point.

Examples

1. A hinged trapped door of mass 15kg and length 1m is to be opened by applying a force F at an angle of 45^0 as shown below.

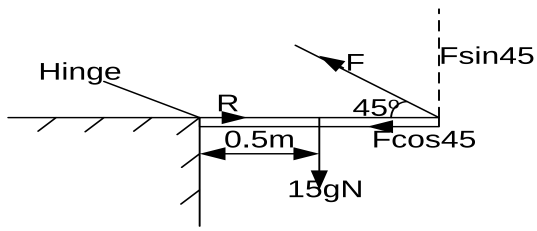


Calculate

i) The value of F

ii) The horizontal force on the hinge

Solution



Taking moments about the hinge
At equilibrium,
Anti clockwise moments = clockwise

$Fsin\, 45x1 \;=\; mg\; x\; 0.5$

$Fx\frac{1}{\sqrt{2}} \;=\; 15x9.81x0.5$

F = 104.1N

The horizontal force is the horizontal normal reaction

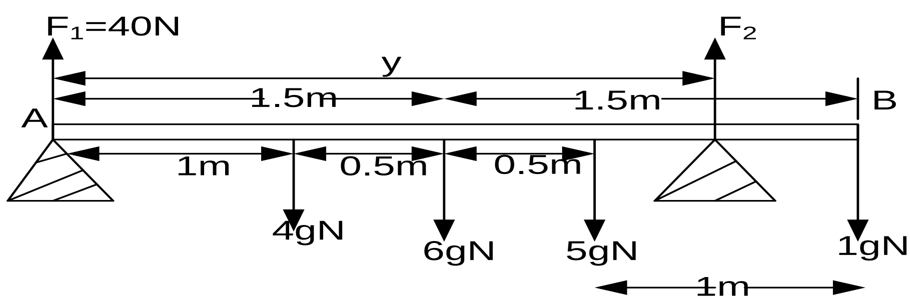
Resolving horizontally: $R = \;Fcos\; 45$

$R \;=\;\; 104.1cos45$

$R = \;73.61N$

2. A uniform bean AB, 3m long and of mass 6kg is supported at A and at another point. A load of 1 kg is suspended at B, loads 4kg and 5 kg at points 1m and 2m from A. If the pressure on the support at A is 40N. Where is the other support?

Solution



Note.
Uniform beam implies its weight acts at the centre of gravity (mid-point)
Resolving vertically:

$F_1 + F_2 = \;4gN \;+6gN \;+\;5gN\;+\;1gN$

40 +F_2 = 4x9.81 + 6x9.81+ 5x9.81 + 1x9.81

$F_2 \;=\;116.96N$

Taking moments about A at equilibrium
Clockwise moment = anti clockwise moment

4gNx1 +6gNx1.5 +5gNx2+1gNx3 =F_2 x y

$Y \;=\; \frac{4gN+9gN+10gN+3gN}{F_2}$

Y= $\frac{26gN}{F_2}$

Y = $\frac{26x9.81}{116.96N}$

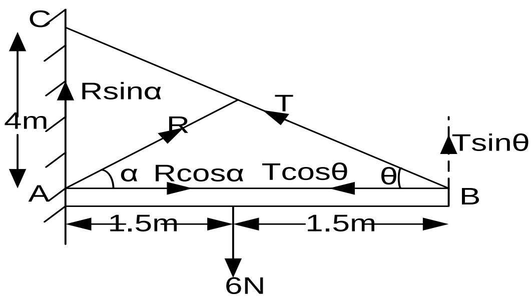
Y = 2.18m

The other support is 2.18 from A

6.2.3: Beams hinged against the wall

1. A Uniform beam AB, 3.0m long and of weight 6N is hinged at a wall at A and is held stationary by a rope attached to B and joined to a point C on the wall, 4.0m vertically above A. Find the tension T in the rope and the Reaction R of the hinge.

Solution



$\tan\theta = \frac{4}{3}$ $\theta = 53.13^0$

Taking moments about A at equilibrium

Clockwise moment = anticlockwise moment

$Tsin\theta \; x \; 3 = 6 \; x \; 1.5$

$(Tsin53.13) \; x \; 3 = 9$

$T = 3.75N$

Resolving vertically:

$Rsin\alpha + Tsin\theta = 6$

$Rsin\alpha = 6 - 3.75sin53.13$

$Rsin\alpha = 3$------------------------------i

Resolving horizontally:

$Rcos\alpha = Tcos\theta$

$Rcos\alpha = 3.75cos53.13$

$Rcos\alpha = 2.238$----------------------------ii

i/ii

$\frac{Rsin\alpha}{Rcos\alpha} = \frac{3}{2.238}$

$\tan\alpha = \frac{3}{2.238}$

$\alpha = 53.28^0$

Put into i

Rsin α = 3

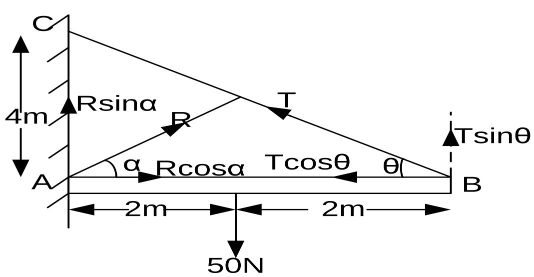
Rsin53.28 =3

R = 3.74N

The reaction at A is 3.74 at 53.28^0 to the beam

2. A uniform beam AB of length 4m and weight 50N is freely hinged at A to a vertical wall and is held horizontal in equilibrium by a string which has one end attached at B and the other end attached to appoint C on the wall, 4m above A. find the magnitude of the reaction at A.

Solution



$\tan\theta = \frac{4}{4}$ $\therefore \theta = 45°$

Taking moments about A

$Tsin \; \theta x4 = 50x2$

$Tsin45x4 = 50x2$

$T = 35.36N$

Resolving vertically: $Rsin \; \alpha + Tsin \; \theta = 50$

$Rsin \; \alpha + 35.36sin45 = 50$

$Rsin \; \alpha = 24.997$……………………….(i)

Resolving horizontally: $Rcos \; \alpha = Tcos \; \theta$

$Rcos \; \alpha = 35.36cos45$

$Rcos \; \alpha = 25$……………………….(ii)

(i)/(ii)

$\frac{Rsin\alpha}{Rcos\alpha} = \frac{24.997}{25}$

$\tan \alpha = \frac{24.997}{25} \therefore \alpha = 45^0$

Put into (ii)

$Rcos \; \alpha = 25$

$Rcos45 = 25$

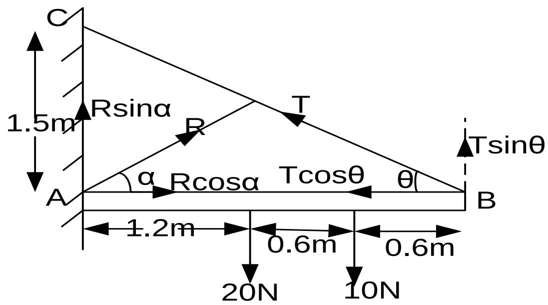
R = 35.36N at 45^0 to the beam

3. A uniform beam AB of mass 20kg and length 2.4m is hinged at a point A in a vertical wall and is maintained in a horizontal position by means of a chain attached to B and to point C in a wall 1.5m above. If the bar carries a load of 10kg at a point 1.8m from A. calculate.

i) The tension in the chain

ii) The magnitude and direction of the reaction between the bar and the wall

Solution



$\tan\theta = \frac{1.5}{2.4}$ $\therefore \theta = 32.01°$

Taking moments about A

$Tsin\ \theta\ x\ 2.4\ =\ 20gNx1.2\ +\ 10gN\ x\ 1.8$

$Tx2.4sin32.01 = 20x9.81x1.2\ +\ 10x9.81x1.8$

$T\ =\ 323.87N$

Tension in the chain = $323.87N$

(ii) Reaction at the wall

Revolving vertically

R sin α + Tsinθ = 20gN + 10gN

R sin α + 323.87sin32.01 = 30gN

R sin α = 122.63……………………(i)

Resolving horizontally; $R\ cos\ \alpha$ = Tcosθ

$R\ cos\ \alpha$ = 32.87cos32.01

$R\ cos\ \alpha$ = 274.63 ……………….(ii)

(i)/(ii)

$\frac{Rsin\alpha}{Rcos\alpha} = \frac{122.63}{274.67}$

tan α = 0.446528055

α = 24.1^0 Put α in eqn (ii)

R cos24.1 = 274.63

R = 300.85N

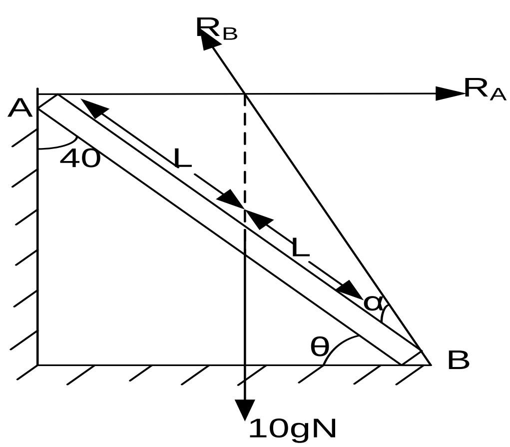
Reaction at A is 300.85 at 24.1^0 to the horizontal

6.2.4: Ladder problems

1. A uniform rod AB of mass 10kg is smoothly hinged at B and rests in a vertical plane with the end A against a smooth vertical wall. If the rod makes an angle of 40^0 with the wall, find the reaction on the wall and the magnitude of the reaction at B

Solution

Note: If there are three coplanar forces acting on a rigid body which is equilibrium, then their line of action meet at some specific point provided no force is parallel to each other.



Let the length of the ladder be 2L

θ+40^0+90^0 = 180^0

θ = 50^0

Taking moments about B

R_A x 2Lsin θ = 10gN x $Lcos\ \theta$

R_A x 2Lsin50 = $10x9.81cos50$

R_A = 41.16N

Resolving vertically; R_B sin(θ+ α) = 10gN

$RB\ sin(\theta\ +\ \alpha)\ =\ 10x9.81$………(i)

Resolving horizontally: R_B $cos(\theta$+ $\alpha)$ = R_A

$R_B\ cos(\theta\ +\ \alpha)\ =\ 41.16$……………(ii)

(i)/(ii)

$\frac{R_B sin(\alpha+\theta)}{R_B cos(\alpha+\theta)} = \frac{10x9.81}{41.16}$

$\theta + \alpha$ =67.24

α +50=67.24

α = 17.24^0

Put into equation (i)

R_B $(sin\theta + \alpha) = 10x9.81$…………(i)

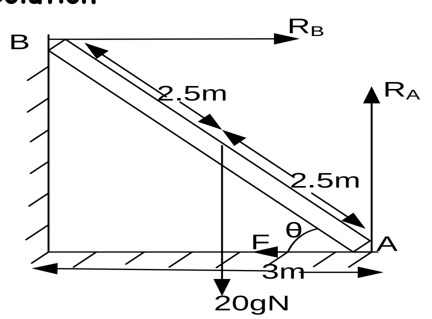
R_B $sin(17.24 + 50) = 10x9.81$

R_B = 106.38N

Reaction at B is 106.38N at 67.24^0 to the horizontal.

2. A uniform ladder which is 5m long and has a mass of 20kg leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The bottom of the ladder is 3m from the wall. Calculate the functional force between the ladder and the ground and the coefficient of friction

Solution



$Cos\theta = \frac{3}{5}$ ∴ θ = 53.13^0

Resolving vertically: R_A = 20gN

R_A= 20x9.81

R_A = 196.2N

Taking moments about A

R_B $x\ 5sin\theta = 20x9.81x2.5cos\theta$

R_B 5sin53.13=20x9.81x2.5cos53.13

R_B= 73.56N

Resolving horizontally: R_B= F

F = 73.56N

But F = μxR_A

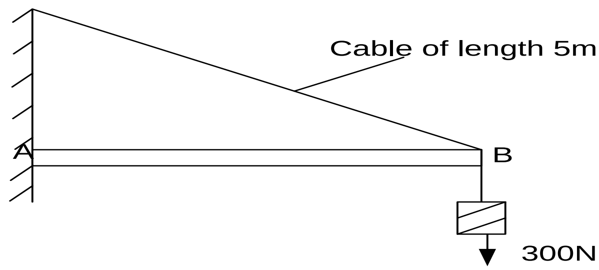
73.56 =$\mu x196.2$

μ = 0.37

Exercise17

1. One end of a uniform plank of length 4m and weight 100N is hinged to the vertical wall. An inelastic rope, tied to the other end of the plank is fixed at a point 4m above the hinge. Find
   i. The tension in the rope'
   ii. The reaction of the wall on the plank **An(388.9N, 302.1N at 24.4^0 to horizontal)**

2.



The figure shows a uniform rod AB of weight 200N and length 4m, the beam is hinged to the wall at A.

i. Find the tension in the cable
ii. The horizontal and vertical components of the force exerted on the beam by the wall
iii. The reaction of the wall on the beam at point A

An(666.7N, 533.3N, 99.98N, 542.59 at 10.6^0 to the horizontal)

3. A uniform beam AB of length 2L rests with end A in contact with a rough horizontal ground. A point C on the beam rests against a smooth support. AC is of length $\frac{3l}{2}$ with C higher than A and AC making an angle of 60^0 with the horizontal. If the beam is in limiting equilibrium, find the coefficient of friction between the beam and the ground.

4. A uniform ladder of mass 25kg rests in equilibrium with its base on a rough horizontal floor and its top against a smooth vertical wall. If the ladder makes an angle of 75^0 with the horizontal, find the magnitude of the normal reaction and of the frictional force at the floor and state the minimum possible value of the coefficient of friction μ between the ladder and the floor.

5. A ladder 12m long and weighing 200N is placed 60^0 to the horizontal with one end B leaning against the smooth wall and the other end A on the ground. Find;

a) reaction at the wall **An(57.7N)**
b) reaction at the ground **An(208.2N at 73.9^0 to the horizontal).**

6.3.0: Couples

A couple is a pair of **equal**, **parallel** and **opposite** forces with different lines of action acting on a body.

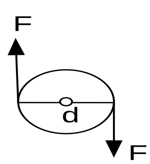
A true couple produces only a **rotation** but not a **translation**

Examples

- Forces in the driver's hands applied to a steering wheel
- Forces in the handles of a bike
- Forces in the peddles of a bike
- Forces experienced by two sides of a suspended rectangular coil carrying current in a magnetic field.

6.3.1: Moment of a couple (torque of a couple)

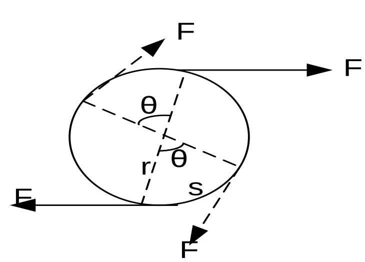
It is defined as the product of one of the forces and the far distance between the lines of action of the forces



Moment of a couple or torque of couple = F x d

6.3.2: Work done by a couple

Consider two opposite and equal forces acting tangentially on a wheel of radius r, suppose the wheel rotates through an angle θ radians as shown below.



Work done by each force $= FxS$

But $s = \frac{\theta}{360} x2\pi r$

$360^0 = 2\pi\, rads$

Work done by each force $= F\, x\, r\theta$

Total work done by the couple $= 2Fr\theta$

6.3.3: CENTRE OF MASS

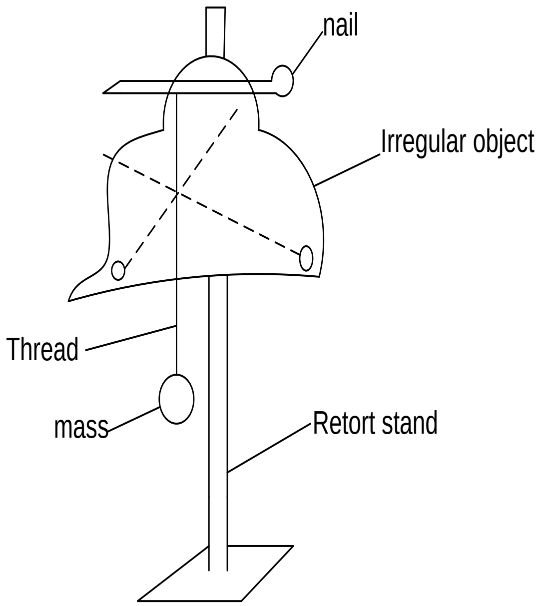
This is a point where an applied force produces a linear acceleration but not an angular acceleration (rotation)

6.3.4: CENTER OF GRAVITY

This point where the entire weight or resultant force of attraction of the body acts.

DETERMINATION OF CENTRE OF GRAVITY OF AN IRREGULAR LAMINA

Procedures:



- Clamp a nail on the stand so that the pointed end is free.
- Make holes at three points at the edge of the card board and hung the card board on the nail through one of the hole.
- Tie the thread on a mass to make a plumb line. Tie the plumbline on the nail allow it to rest freely with its thread tacking the card board
- Trace the thread using a pencils I.
- Repeat the procedure when the plumbline is suspended from the other holes.
- The point of intersection of the three lines is the centre of gravity of the board

UNEB 2009 No 2

a) Define the following terms

i) Velocity (2marks)

ii) Moment of a force

c)(i) State the condition necessary for mechanical equilibrium t be attained.(2 marks)

ii)A uniform ladder of mass 40kg and length 5m, rest with its upper end against a smooth vertical wall and with its lower end at 3m from the wall on a rough ground. Find the magnitude and direction of the force exerted at the bottom of the ladder (06 marks)

An[418.7N at an angle of 69.4° to the horizontal]

UNEB 2006 No 2

c) State the condition for equilibrium of a rigid body under the action of coplanar forces. (2mk)

d) A 3m long ladder at an angle 60^0 to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5kg and its centre of gravity is one third from the bottom of the ladder.

v) Draw a sketch diagram to show the forces acting on the ladder. (2mk)
vi) Find the reaction of the ground on the ladder. (4mk)

(Hint Reaction on the ladder =$\sqrt{R^2+F^2}$) An(49.95N at 79.11^0 to the horizontal)

UNEB 2006 No1

e) Describe an experiment to determine the centre of gravity of a plane sheet of material having an irregular shape. (4 marks)

UNEB 2005 No2

f) (i) Define centre of gravity (1 mark)

(ii) Describe an experiment to find the centre of gravity of a flat irregular piece of a card board. (3 marks)

UNEB 2002 No2

d) (i)Define moment of a force (1 mark)

(ii) A wheel of radius 0.6m is pivoted at its centre. A tangential force of 4.0N acts on the wheel so that the wheel rotates with uniform velocity find the work done by the force to turn the wheel through 10 resolutions.

Solution

Work done =force x distances

But distance= circumference x number of revolutions

=2π rx10

W=F x d

=4x2 π rx0.6x10

W =150.79J

UNEB 2000 No3

b) State the conditions for equilibrium of a rigid body under the action of coplanar forces.(2mk)

d) A mass of 5.0kg is suspended from the end A of a uniform beam of mass 1.0kg and length 1.0m. The end B of the beam is hinged in a wall. The beam is kept horizontal by a rope attached to A and to a point C in the wall at a height 0.75m above B

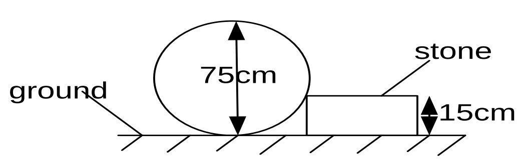
i. Draw a diagram to show the forces on the beam (2 marks)
ii. Calculate the tension in the rope (4 marks)
iii. What is the reaction exerted by the hinge on the beam (5 marks)

An (89.8N, 72.01N, at 3.95^0 to the beam)

UNEB 1998 No1

d) (i)Explain the term unstable equilibrium (3mk)

(ii) An oil drum of diameter 75cm and mass 90kg rests against a stone as shown

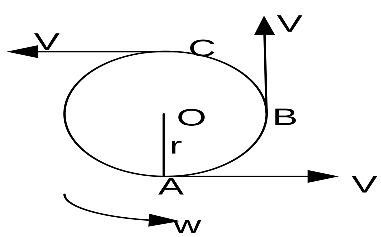


Find the least horizontal force applied through the centre of the drum, which will cause the drum to roll up the stone of height 15cm.

An(1177.2N) (5 marks)

CHAPTER 7: CIRCULAR MOTION

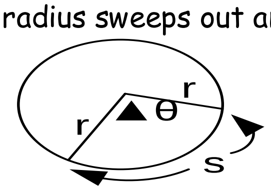
This is the motion of the body with a uniform speed around a circular path of fixed radius about a center.



Although the body moves round this circular path with a uniform speed, the direction of motion changes as it moves from point A to B, C and so on around the path. Since the direction of motion changes, then the velocity of the body changes as it moves round circular path. This change in velocity results into an acceleration because acceleration is the rate of change in velocity.

Terms used in circular motion

Consider a body of mass m initially at point A moving with a constant speed in a circle of radius r to point B in a time Δt, the radius sweeps out an angle $\Delta\theta$ at the centre



1. Angular velocity (ω)

This is the is the rate of change of the angle for a body moving in a circular path.

Mathematically $\boldsymbol{\omega} = \frac{\Delta\theta}{\Delta t}$

For large angles and big time intervals.

$\boldsymbol{\omega} = \frac{\theta}{t}$

Angular velocity is measured in radians per second ($rads^{-1}$)

2. Linear speed (v)

If the distance of the arc AB is, s and the speed is constant then velocity.

$v = \frac{Arc\ length}{time}$

$v = \frac{s}{\Delta t}$

But $v = \frac{\Delta\theta}{360}\ x2\pi r$

$360 = 2\pi$ rads

$s = \Delta\theta r$

But the equation

$v = \frac{\Delta\theta r}{\Delta t}$

$v = r\ \omega$

Where $\frac{\Delta\theta}{\Delta t} = \omega$

$\boxed{v = r\ \omega}$–units are ms^{-1}

Definition

Velocity is the rate of change of displacement for a body moving a round a circular path about a fixed point or centre.

3. **Period T**

This is the time taken for the body to describe one complete are revolution

T= $\frac{Circumference[distance\ around\ acircle]}{velocity}$

T= $\frac{2\pi r}{v}$

But $v = r\ \omega$

T= $\frac{2\pi r}{\omega r}$

$\boxed{T = \frac{2\pi}{\omega}}$ units seconds.

Period is independent of the radius and it's constant.

4. Acceleration

Centripetal acceleration is defined as the rate of change of velocity of a body moving in a circular path and is always directed towards the centre.

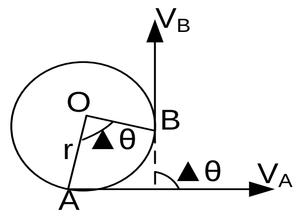
7.1.0: Derivation of $\boldsymbol{a} = \frac{V^2}{r}$

Question:

Show that the acceleration of a body moving round a circular path with speed v is given by $\frac{V^2}{r}$ where r is the radius of the path.

Solution

Consider a body of mass m moving around a circular path of radius r with uniform angular velocity ω and speed V. If initially the body is at point A moving with velocity V_A and after a small time internal Δt, the body is at point B where its velocity is V_B with the radius having moved an angle $\Delta\theta$



- Change in velocity parallel to OA= $V\sin\theta - 0 \approx V\theta$
- Change in velocity perpendicular to OA= $V\cos\theta - 0 \approx 0$
- If A is close to B so that for small angles $cos\theta \approx 1$ and $\sin\theta \approx \theta$

  Acceleration, a= $\frac{change\ in\ velocity}{time} = v\frac{\theta}{t}$

but $\frac{\theta}{t}$ =$\boldsymbol{\omega}$

$$\boxed{a = v\omega}$$

also v=r$\boldsymbol{\omega}$

$$\boxed{a = \omega^2 r}$$

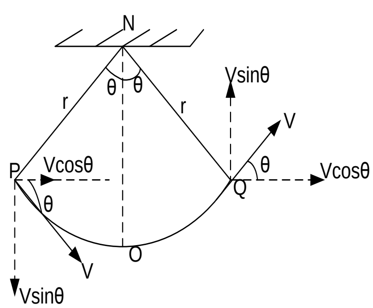
also $\boldsymbol{\omega}$= $\frac{\mathrm{v}}{\mathrm{r}}$

$a = v.\frac{v}{r}$

$$\boxed{a = \frac{\mathrm{v}^2}{\mathrm{r}}}$$

Question

A volume of mass m is oscillated from a fixed point by a string of length r with a constant speed V. Shows that the acceleration of the body is $\frac{\boldsymbol{V^2}}{\boldsymbol{r}}$ and directed towards the centre.



Acceleration

a= $\frac{change\ in\ velocity}{time}$

Horizontal component

$$a_X = \frac{vCos\theta - vCos\theta}{t}$$

$a_x = \frac{0}{t}$

a_x =0 ,the body does not accelerate horizontally.

Vertical component

$a_y = \frac{vSin\theta - vSin\theta}{t}$

$a_y = \frac{2vSin\theta}{t}$ ------------- (1)

but time (t) = $\frac{Arc\ length\ PQ}{Velocity}$

t = $\frac{2\theta}{360} x \frac{2\,\pi\, r}{v}$

$t = \frac{2\,\theta x\, 2\,\pi\, r}{2\,\pi\, v}$

where 360=2πrads

$t = \frac{2\,\theta\, r}{V}$

put into equation (1)

a_y= $\frac{2\ v\ sin\theta}{\left(\frac{2\,\theta\, r}{v}\right)}$

a_y= $\frac{v^2 Sin\ \theta}{r\ \theta}$

for small angle sinθ ≈θ

a_y= $\frac{v^2\theta}{r\ \theta}$

$$\boxed{a_y = \frac{v^2}{r}}$$

EXAMPLE

1. A particle moves along a circular path of radius 3.0m with an angular velocity of 20 rad s^{-1} calculate;

   a) The linear speed of the particle

b) Angular velocity in revolutions per second

c) Time for one revolution

d) The centripetal acceleration

Solution

r=3m $\quad \boldsymbol{\omega}$=20 rads^{-1}

a) Linear speed $v = r\boldsymbol{\omega}$

$v =$ 20x3

v =60ms^{-1}

b) Angular velocity in revolutions per second gives the frequency

$\boldsymbol{\omega} = 2\pi f \quad \therefore \; f=\frac{\omega}{2\pi}$

f=$\frac{20}{2\pi}$

f=3.18 revolutions per second

c) Time for one revolution (T)

T=$\frac{1}{f}$

T=$\frac{1}{3.18}$

T=0.31 second

d) Acceleration a = $\frac{v^2}{r}$

a = $\frac{60^2}{3}$

a =1200ms^{-2}

2. A body is fixed on the string and whirled in a circle of radius 10cm. If the period is 5s. find

i) The angular velocity

ii) The speed of the body in the circle

iii) The acceleration of the body

iv) The frequency

Solution

i) $\boldsymbol{\omega} = \frac{\theta}{t}$

its whirled in a circle (θ=360=2π)

$\boldsymbol{\omega} = \frac{2\pi}{t}$

$\boldsymbol{\omega} = \frac{2x\frac{22}{7}}{5}$

$\boldsymbol{\omega}$ =1.26rads^{-1}

ii) $v = \boldsymbol{\omega}$ r

$v = 1.26x\,\frac{10}{100}$

v=0.13ms^{-1}

iii) a = $\boldsymbol{\omega}^2$r

a= $(1.26)^2$x0.1

a = 0.169ms^{-2}

iv) f= $\frac{2\pi}{\omega}$

$f = \frac{2x\frac{22}{7}}{1.26}$

f=0.2Hz

EXERCISE 18

1. A particle of mass 0.2kg moves in a circular path with an angular velocity of 5 rads^{-1} under the action of a centripetal force of 4N. What is the radius of the particle. **An(0.8m).**

2. What force is required to cause a body of mass 3g to move in a circle of radius 2m at a constant rate of 4 revolutions per second. **An(3.8N)**

7.1.1: CENTRIPETAL AND CENTRIFUGAL FORCES

If a body is moving in a circle, it will experience an initial outward force called **centrifugal force**. These forces always act away from the center and are perpendicular to the direction of motion.

In order for the body to continue moving in a circle without falling off, there must be an equal and opposite force to the centrifugal force. This force which counter balances the centrifugal force is called the **centripetal force** and always acts towards the center of the motion.

Definition

Centripetal force is a force which keeps a body moving in a circular path and is directed towards the center of the circular path.
If the mass of the body is m then the centripetal force
F =ma
But a = $\frac{v^2}{r}$
$\boxed{F = \frac{m\,v^2}{r}}$ This is the expression for the centripetal force Or $\boxed{F = mr\,\omega^2}$

Question

Explain why there must be a force acting on a particle which is moving with uniform speed in a circular path. Write down an expression for its magnitude.

Solution

If a body is moving along circular path, there must be a force acting on it, for if there were not, it would move in a straight line in accordance with Newton's first law. Furthermore, since a body is moving with a constant speed, this force cannot at any stage have a component in direction of motion of a body. For it did, it would increase or decrease the speed of the body. The force on the body must therefore be perpendicular to direction of motion and directed towards the center.

7.1.2: Examples of centripetal forces

1. ***A* car moving around a circular truck**: For a car negotiating a corner or moving on a circular path, the frictional force between the wheels and the surface provides the necessary centripetal force required to keep it on the track.

2. ***A* car moving on banked track**:
   For a banked track, the centripetal force is provided by the frictional force and the horizontal components of the normal reaction.

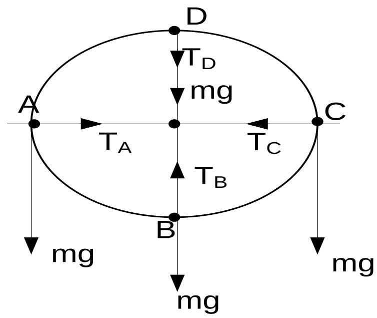
3. a) **Tension on the string keeping a whirling body in a vertical circle**.
   The tension force in the string provides the necessary centripetal force
   b) **For the conical pendulum, the horizontal component of the tension in the string** provides the necessary centripetal force

4. **Gravitational force on planets**
   For a planet orbiting round the sun or satellite revolving about the earth, the gravitational force between the two bodies provides the necessary centripetal force required to keep the satellite in the orbit.

5. **Electrostatics force on the electrons**
   For electrons moving round the nucleus, the electrostatics force provides the necessary centripetal force.

7.1.3: Motion in a vertical cycle

Consider a body of mass m attached to a string of length r and whirled in a vertical circle with a constant speed V. If there is no air resistance to the motion, then the net force towards the centre is the centripetal force.



At point A: $T_A = \frac{m\,v^2}{r}$ ------------------------ (2)

At point B: $T_B = \frac{m\,v^2}{r} + mg$ - ----------------(3)

At point C: $T_C = \frac{m\,v^2}{r}$------------------------- (4)

At point D: $T_D = \frac{m\,v^2}{r} - mg$ ------------------ (5)

The maximum tension in the vertical circle is experienced at B

$T_{max} = \frac{m\,v^2}{r} + mg$

The minimum tension is experienced on the top of the circle at point D

$T_{min} = \frac{m\,v^2}{r} - mg$

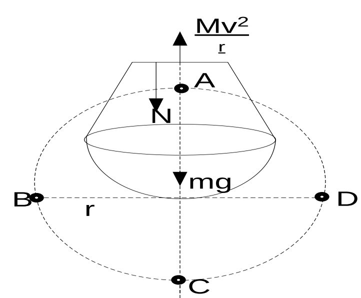
Note

If the speed of whirling is increased the string will most likely break at the bottom of the circle. Motion is tangential to the circle and when string breaks the mass will fly in a parabolic path.

Question

Explain why a bucket full of water can be swung round a vertical circle without spilling.

Solution

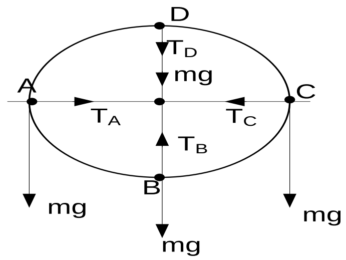


The water will only stay in the bucket if mg < $\frac{m\,v^2}{r}$. Consider a bucket at the top of the circle, if the weight, **mg** < $\frac{m\,v^2}{r}$, the normal reaction force N at the bottom of the bucket on the water provides the rest of the force on the bucket required to maintain the water in the circular path and therefore the water will not spill. However if the bucket is swung slowly then $Mg > \frac{m\,v^2}{r}$ and the un used part of the weight causes the water to leave the bucket

Examples

1. An object of mass 3kg is whirled in a vertical circle of radius 2m with a constant sped of $12ms^{-1}$, calculate the maximum and minimum tension in the string

Solution



Maximum tension is at B

T - mg = $\frac{m\,v^2}{r}$

T = $\frac{m\,v^2}{r}$ +mg

T = $\frac{3x12^2}{2}$ +3x9.81

T = 245.43N

Minimum tension is at D

T= $\frac{m\,v^2}{r} - mg$

= $\frac{3x12^2}{2}$ $- 3x9.81$

T =186.57N

2. A stone of mass 800g is attached to string of length 60cm which has a breaking tension of 20N. The string is whirled in a vertical circle the axis of rotation at a height of 100cm from the ground.
   i) What is the angular velocity where the string is most likely to break?
   ii) How long will it take before the stone hits the ground?

Solution

i) The string breaks when $T_{max} = \frac{m\,v^2}{r}$+ mg

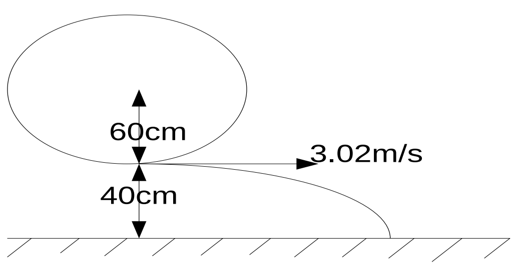
$$20 = 0.8(9.81 + \frac{v^2}{0.6})$$

$v = 3.02ms^{-1}$

But $v = r\,\omega$

$\boldsymbol{\omega}$ = $\frac{3.02}{0.6}$, $\therefore$ $\boldsymbol{\omega}$ = 5.03rads^{-1}

ii)



$y = utsin\theta - \frac{1}{2}gt^2$

$y = -40cm$ (below the point of projection)

$-0.4 = 3.02tsin\theta - \frac{1}{2}$ x9.81t^2

$-0.4 = 3.02tsin0 - \frac{1}{2}$ x9.81t^2

t = 0.286s

iii) Horizontal range

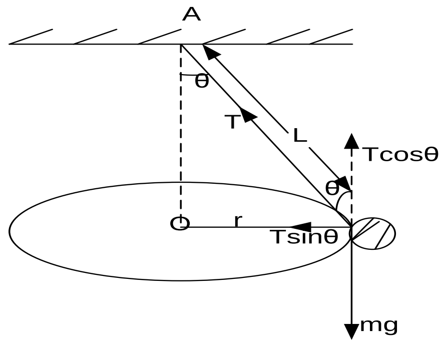
$x = utcos\theta$

$x = 3.02x0.285cos0$

$x = 0.86m$

7.1.4: MOTION IN A HORIZONTAL CIRCLE [CONICAL PENDULUM]

Consider a body of mass , m tied to a string of length L whirled in a horizontal circle of radius r at a constant speed V



If the string is fixed at A and the centre O of the circle is directly below O, the horizontal components of the tension provides the necessary centripetal force.

$Resolving\ horizontally$

$Tsin\theta = \frac{m\,v^2}{r}$ ----------- (1)

$Resolving\ vertically$

$Tcos\theta$ = mg ------------ (2)

(1) ÷ (2)

$\frac{\mathrm{Tsin\theta}}{\mathrm{Tcos\theta}} = \frac{m\,v^2}{r} \Big/ mg$

$tan\theta = \frac{v^2}{r\,g}$

$\boxed{v^2 = rgtan\theta}$--------- (3)

but also $sin\theta = \frac{r}{L}$

r= $Lsin\theta$

and v=r $\boldsymbol{\omega}$

put into equation (3)

(r $\boldsymbol{\omega}$)$^2 = rgtan\theta$

$r\omega^2 = \mathrm{g tan\theta}$

$\omega^2 = \frac{\mathrm{g\,tan\theta}}{\mathrm{r}}$

But $r = Lsin\theta$

$\omega^2 = \frac{\mathrm{g\,tan\theta}}{\mathrm{L\,sin\theta}}$

$\omega^2 = \frac{\mathrm{g}}{\mathrm{Lsin\theta}} sin\theta \Big/ cos\theta$

$\boldsymbol{\omega}^2 = \frac{\mathrm{g}}{\mathrm{Lcos\theta}}$

$\boxed{\omega = \sqrt{\frac{g}{Lcos\theta}}}$----------- (4)

Also

T =$\frac{2\,\pi}{\omega}$

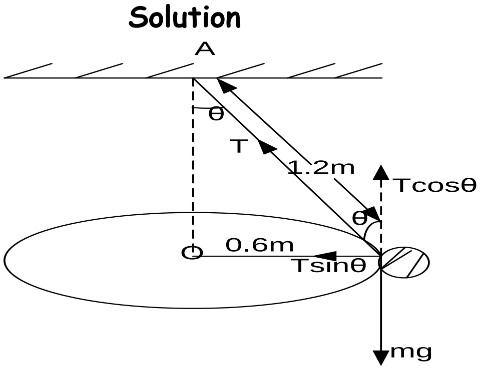
T =$\frac{2\,\pi}{\sqrt{\frac{g}{Lcos\theta}}}$

$\boxed{T = 2\pi\sqrt{\frac{Lcos\theta}{g}}}$

Example

1. A stone 0.5kg is tied to one end of a string 1.2m long and whirled in a horizontal circle of diameter 1.2m. Calculate;
   i) The length in the string
   ii) The angular velocity
   iii) The period of motion

Solution



i) Resolving vertically

$Tcos\theta = 0.5gN$ ------- (1)

But $sin\theta = \frac{0.6}{1.2}$ ∴ θ= 30^0

put into: (1) Tcos30=0.5x9.81

T =5.60N

ii) Angular velocity

$\omega = \sqrt{\frac{g}{Lcos\theta}}$

$\omega = \sqrt{\frac{9.81}{1.2cos30}}$

ω =3.07rads^{-1}

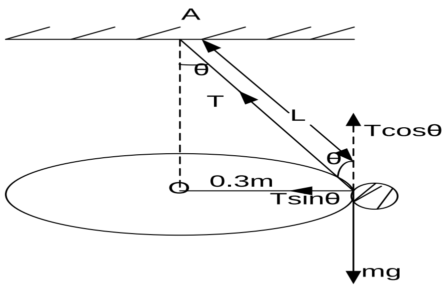
iii)Period , T =$\frac{2\,\pi}{\omega}$

T = $\frac{2x\frac{22}{7}}{3.07}$

T = 2.05s

2. A body of mass 4kg is moving with a uniform speed $5ms^{-1}$ in a horizontal circle of radius 0.3m, find:
    i) The angle the string makes with the vertical
    ii) The tension on the string

Solution



Resolving horizontally

$T sin\theta = \frac{m v^2}{r}$ ……….[1]

Resolving vertically

$T cos\theta = mg$ ……….. [2]

[1]÷[2]

$tan\theta = \frac{v^2}{r g}$

$\theta = tan^{-1} \frac{v^2}{r g}$

$\theta = tan^{-1} \frac{5^2}{0.3x9.81}$

$\theta = 83.3^0$

ii)Tension T

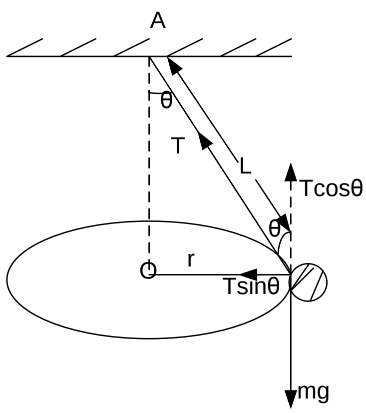
$T cos\theta = mg$

$T = \frac{4x9.81}{Cos\ 83.3}$

$T = 336.33N$

3. The period of oscillation of a conical pendulum is 2s. If the string makes an angle of 60^0 with the vertical at the point of suspension, Calculate;
    i) The length of the string
    ii) The velocity of the mass

Solution



$\theta = 60^0$ $\sin 60 = \frac{r}{l}$

$r = l\sin 60^0$------------- (1)

Also resolving vertically

$T cos\theta = mg$

Resolving horizontal

$T sin\theta = \frac{m v^2}{r}$

$Tan\theta = \frac{v^2}{rg}$

v^2 =rgtan60 ---------- (2)

Also T $= \frac{2\pi}{\omega}$

T $= \frac{2\pi}{2}$

T =3.14$rads^{-1}$

But v=r ω

V=3.14r

Put into equation (2)

V^2=rgtan60

$(3.14r)^2$=rgtan60

r$= \frac{g\ tan60}{314^2}$

put into equation (1)

r=Lsin60

$\frac{g\ tan60}{314^2}$ = $l$sin60

$L = \frac{g\ tan60}{3.14^2 sin60}$

L =1.986m

OR

$T = 2\pi\sqrt{\frac{Lcos\theta}{g}}$

$\frac{T^2}{4\pi^2} = \frac{Lcos\theta}{g}$

$L = \frac{T^2 g}{4\pi^2 cos\theta}$

$L = \frac{2^2 x9.81}{4\left(\frac{22}{7}\right)^2 cos60}$

L=1.986m

$v = r\,\omega$

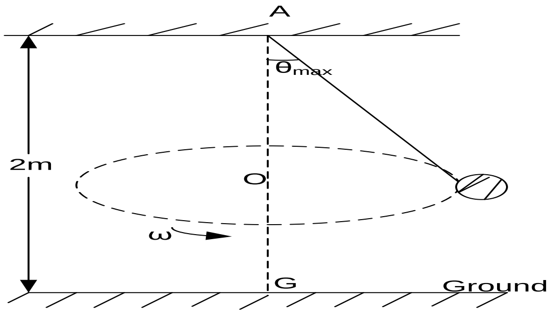
$v = \frac{2\pi}{T}$ r

$v = \frac{2\pi}{2}$xLsinθ

v =1.986xπsin60

v =5.4ms^{-1}

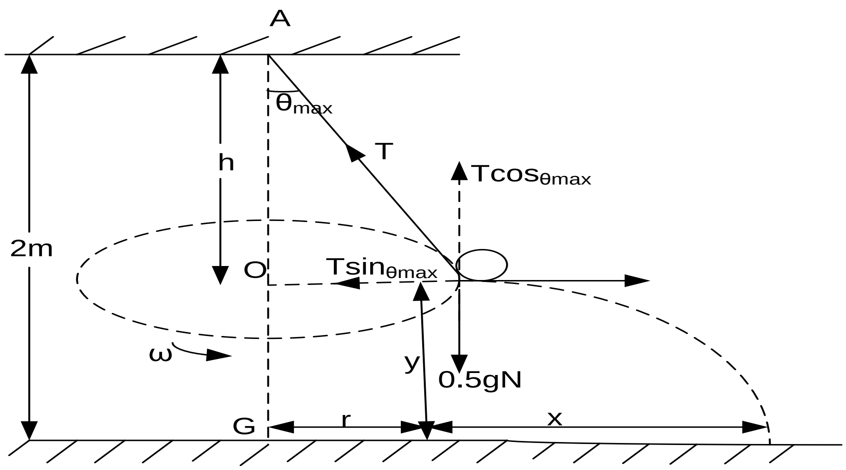
4. Stone of mass 0.5kg is tied to one end of the string 1m long. The point of suspension of the string is 2m above the ground. The stone is whirled in the horizontal circle with increasing angular velocity. The string will break when the tension in it is 12.5N and the angle θ is to the maximum (θ_{max})as shown in the figure below;



ii) Calculate the angular velocity of the stone when the string breaks
iii) How far from the point G on the ground will the stone hit the ground
iv) What will be the speed of the stone when it hits the ground

i) Calculate the angle θ_{max}

Solution



i) Resolving vertically

$T\cos\theta_{max} = 0.5gN$

$\cos\theta_{max} = \frac{0.5x9.81}{12.5}$

$\theta_{max} = 66.9^0$

ii) Resolving horizontally

$T\sin\theta = \frac{m\,v^2}{r}$

Also $\sin\theta = \frac{r}{l}$

$T\,\frac{r}{1} = \frac{m\,v^2}{r}$

$Tr^2 = mv^2$

But $v = r\,\omega$

$T = m\,\boldsymbol{\omega}^2$

$12.5 = 0.5\,\boldsymbol{\omega}^2$

$\boldsymbol{\omega}^2 = \frac{12.5}{0.5}$ rads^{-1}

$\boldsymbol{\omega}$ = 5rads^{-1}

$\cos\theta_{max} = \frac{h}{1}$

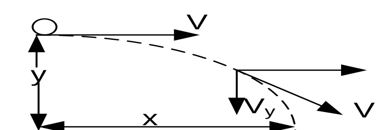
$h = cos66.9$

$h = 0.39m$

$y + h = 2$

$y = 2 - 0.39$

$y = 1.61m$



Using $y = ut\sin\theta - \frac{1}{2}gt^2$

$y = -1.61$ below the point of projection

$-1.61 = ut\ sin0 - ½\,x9.81t^2$

$-1.61 = -½\,x9.81t^2$

Horizontal distance

$x = v\cos\theta t$

$x = vcos0x0.57$

$x = 0.57v$

but $v = \omega r$

$x = 0.57\ \omega r$

$x = 0.57x5xsin66.9$

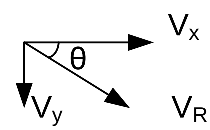
where $sin66.9 = \frac{r}{l}$

$x = 2.63m$

$\therefore G = r + x$

$G = 2.62 + sin66.9$

G=3.54m



v_x is constant

$v_x$ucosθt

$v_x = vcos0x0.57$

$v_x = 0.57v$

$v_x = 0.57\omega r$

v_x =0.57x5xsin66.9

v_x =4.599ms^{-1}

$v_y = usin\theta + gt$

$v_y = usin0 + 9.91x0.57$

$v_y = 5.592ms^{-1}$

$v_R = \sqrt{V_X^2 + V_y^2}$

$v_R = \sqrt{4.599^2 + 5.592^2}$

$V_R = 7.24ms^{-1}$

$\theta = \tan^{-1}\frac{5.592}{4.599}$

The speed as it hits the ground is	$7.24ms^{-1}$.

EXERCISE 19

1. A body of mass 20kg is whirled in a horizontal circle using an inelastic string which has a breaking force of 400N. If the breaking speed is at $9ms^{-1}$. Calculate the angle which the string makes with the horizontal at the point of breaking.
   An(θ=29.3^0).

2. A particle of mass 0.2kg is attached to one end of a light inextensible string of length 50cm. The particle moves in a horizontal circle with an angular velocity of $5.0rads^{-1}$ with the string inclined at θ to the vertical. Find the value of θ.
   An(37^0)

3. A particle of mass 0.25kg is attached to one end of a light in extensible string of length 3.0m. The particle moves in a horizontal circle and the string sweeps out the surface of a cone. The maximum tension that the string can sustain is 12N. Find the maximum angular velocity of the particle. **An[4rads^{-1}].**

4. A particle of mass 0.30kg moves with an angular velocity 0f $10rads^{-1}$ in a horizontal circle of radius 20cm inside a smooth hemispherical bowl. Find the reaction of the bowl on the particle and the radius of the bowl.
   An[6.7N, 22cm]

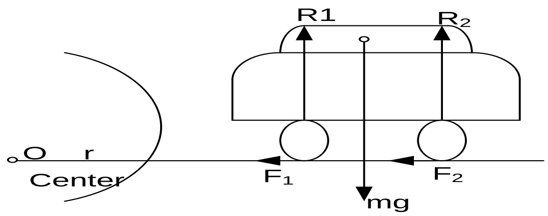
5. A child of mass 20kg sits on a stool tied to the end of an inextensible string 5m long, the other end of the string being tied to a fixed point. The child is whirled in a horizontal circle of radius 3m with a child not touching ground.
   i) Draw a diagram to show the forces acting on the child
   ii) Calculate the tension on the string
   iii) Calculate the speed of the child as it moves around the circle. **An[245.25N, 4.695ms^{-1}]**

7.1.5: MOTION OF A CAR ROUND A FLAT CIRCULAR TRACK [NEGOTIATING A BEND]

Consider a car of mass m moving round a circular horizontal arc of radius r with a speed v

A) Skidding of the car

Skidding is the failure of a vehicle to negotiate a curve as a result of having a centripetal force less than the centrifugal force and the car goes off the track or moves away from the centre of the circle.



Consider a car of mass m taking a flat curve of radius r at a speed v. F_1 and F_2 are the frictional forces due to the inner tyre and outer tyre respectively. R_1 and R_2 are the normal reactions due to inner and outer tyres respectively.

Resolving vertically :$R_1+R_2 = mg$---------(1)

Resolving Horizontally: $F_1+F_2 = \frac{m\,v^2}{r}$ -----(2)

The frictional forces F_1 and F_2 provide the necessary centripetal force

But $F_1 = \mu R_1$, $F_2 = \mu R_2$

$\mu R_1 + \mu R_2 = \frac{m\,v^2}{r}$

$\mu (R_1+R_2) = \frac{m\,v^2}{r}$ -------------------------(3)

Put equation (1) into equation (3)

$\mu\, mg = \frac{mv^2}{r}$

$\mu\, g = \frac{v^2}{r}$

$\mu = \frac{v^2}{r\,g}$ or $v^2 = rg\,\mu$

The maximum speed with which no skidding occurs is given by

$$\boxed{v_{max} = \sqrt{\mu rg}}$$

For no skidding

$\mu \geq \frac{v^2}{rg}$ Or $v^2 \leq \mu rg$

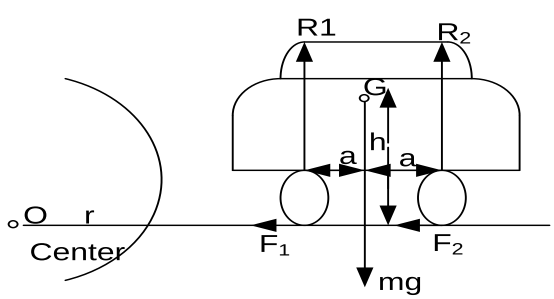
Conditions for no skidding/side slips

For a car to go round a bend successfully without skidding then:

1- The speed should not exceed $(\mu rg)^{\frac{1}{2}}$ or $[v \leq \sqrt{\mu rg}]$
2- The radius of the bend should be made big
3- Coefficient of friction should be increased
4- Centre of gravity should be low

B) Overturning/toppling of a car

Consider a car of mass m moving around a horizontal (flat)circular bend of radius r at speed v. let the height of the centre of gravity above the track be "h" and the distance between the wheels be "2a".



Resolving vertically : $R_1+R_2 = mg$-----(1)

Resolving Horizontally :$F_1+F_2 = \frac{m\,v^2}{r}$ -----(2)

Taking moments about G

Clockwise moments =anticlockwise moments

$F_1.h + F_2.h + R_1.a = R_2.a$

$(F_1 + F_2)h + R_1 a = R_2.a$ -----------------(3)

Put equation 2 into equation 3

$\frac{m\,v^2}{r}.h + R_1 a = R_2.a$

$\frac{m\,v^2}{r}.\frac{h}{a} = (R_2 - R_1)$----------------------[4]

Equation 1–Equation 4

$mg - \frac{m\,v^2}{r}.\frac{h}{a} = 2R_1$

R_1 =0, the car just about to overturn

$R_1 = \frac{mg}{2} - \frac{mv^2h}{2ra}$

$R_1 = \frac{m}{2}\left(g - \frac{v^2h}{ra}\right)$-------------------------- - (5)

Note

R_1 is the reaction of the inner tyre

- When $R_1>0$: The wheels in the inner side of the curve is in contact with the ground
- When $R_1=0$: The wheels in the inner side of the curve are at the point of loosing contact with the ground
- When $R_1<0$: The inner wheels have lost contact with the ground and the vehicle has over turned

For no toppling or no overturning

$R_1 \geq 0$

$\frac{m}{2}\left(g - \frac{v^2h}{ra}\right) \geq 0$

$g \geq \frac{v^2h}{ra}$ or $v^2 \leq \frac{rag}{h}$

$v_{max} = \sqrt{\frac{rag}{h}}$

Ways prevent toppling/overturning

i) Reduce the speed when negotiating a corner ($v^2 \leq \frac{rag}{h}$)

ii) Increase radius of a corner (r $>\frac{v^2h}{ra}$)

iii) The distance between the tyres should be made big (a $>\frac{v^2h}{ra}$)

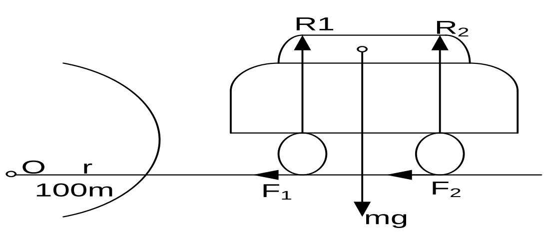
iv) Reduce distance from the ground to the centre of gravity (h) or C.O.G of the car should

be low (h$<\frac{rag}{v^2}$)

EXAMPLE

1. A car of mass 1000kg goes round a bend of radius 100m at a speed of 50km/hr without skidding. Determine the coefficient of friction between the tyres and the road surface

Solution



Resolving vertically : $R_1 + R_2 = mg$----- (1)

Resolving Horizontally : $F_1 + F_2 = \frac{m v^2}{r}$

$\mu(R_1 + R_2) = \frac{m v^2}{r}$------------------------- [2]

Put equation (1) and equation 2

μ mg= $\frac{m v^2}{r}$

$\mu = \frac{v^2}{rg}$

$$\mu = \frac{\left(\frac{50x1000}{3600}\right)^2}{100x9.81}$$

μ =0.1965

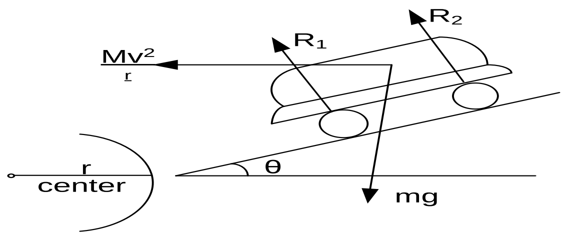
7.1.6: MOTION OF A CAR ON A BANKED TRACK

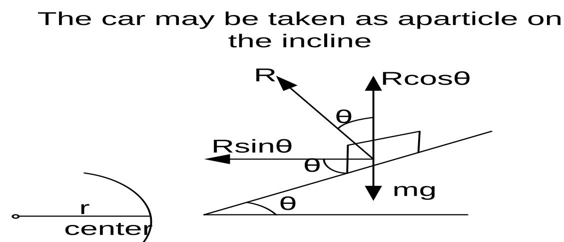
Definition

Banking a track is the raising of the outer part of the bend (corner) slightly above the inner part so as to ensure that only the horizontal component of normal reaction contributes towards the centripetal force.
Banking also enables the car to go round a bend at a higher speed for the same radius compared to a flat track.

A) NO SIDE SLIPP [No frictional force]

Consider a car of mass m negotiating a banked track at a speed v and radius of the bend is r.





Resolving vertically :$Rcos\ \theta = mg$ ------- (1)

Resolving horizontally :$Rsin\ \theta = \frac{\mathrm{m\ v^2}}{\mathrm{r}}$---- (2)

Equation2÷ $Equation1$

$\frac{R\mathrm{Sin}\theta}{R\mathrm{Cos}\theta} = \frac{\mathrm{m\ v^2}}{\mathrm{rmg}}$

tan θ = $\frac{\mathrm{v^2}}{\mathrm{rg}}$

$v^2 = rgtan\ \theta$

θ is the angle of banking and v is the designed speed of the banked track.

Examples

1. A racing car of mass 1000kg moves around a banked track at a constant speed of 108km/hr, the radius of the track is 100m. Calculate the angle of banking and the total reaction at the tyres.

Solution

$\theta = tan^{-1}\frac{v^2}{rg}$

$\theta = tan^{-1}\left[\frac{\left(\frac{108x1000}{3600}\right)^2}{100x9.81}\right]$ $\therefore \theta = 42.5^0$

Resolving vertically:$Rcos\theta = mg$

$R\ = \frac{1000x9.81}{cos42.5}$

R=13305N

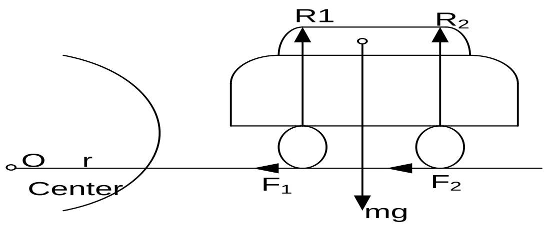
Exercise 20

A road banked at 10^0 goes round a bend of radius 70m. At what speed can a car travel round the bend without tending to side slip. **An[11ms^{-1}]**

Question:

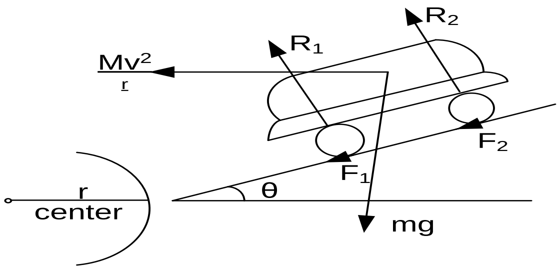
Explain why a car travels at a higher speed round a banked track without skidding unlike the flat tracks of the same radius.

Solution



Along a circular arc on a horizontal road the frictional force provides the centripetal force $F_{max} = \frac{\mathrm{m\,v^2}}{\mathrm{r}} = \mu$ R

At a higher speed, the frictional force is not sufficient enough to provide the necessary centripetal force and skidding would occur.



On a banked track the centripetal force is provided by both the horizontal component of normal reaction R and component of the frictional force. $F_c = Fcos\,\theta + Rsin\,\theta = \frac{\mathrm{m\,v^2}}{\mathrm{r}}$

For $0° < \theta < 90°$, $\mu cos\,\theta + sin\,\theta > \mu$

therefore $V_1 < V_2$

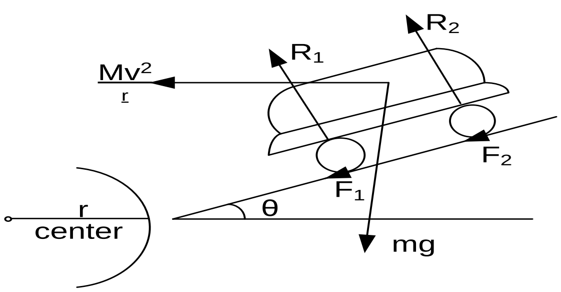
This is enough to keep the car on the track even at high speed.

B) FOR SKIDDING/SLIDE SLIPP

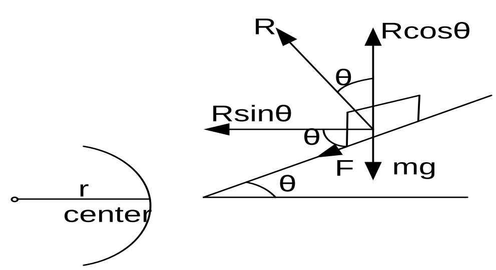
The frictional force must be there whose direction depends on the speed of the car.

i) MAXIMUM SPEED/GREATEST SPEED

If the car is moving at speed v, greater than the designed speed v, the force Rsinθ is enough to provide the necessary centripetal force. The car will tend to slid outwards from the circular path, the frictional force would therefore oppose their tendency up to the maximum value .



The car may be taken as aparticle on the incline



Resolving vertically : $Rcos\,\theta = mg + Fsin\,\theta$

R$cos\,\theta\; - \mu$Rsin θ$= mg$

R$cos\,\theta\; - \mu\, sin\,\theta) = mg$ ------------ (1)

Resolving horizontally: Rsin θ+$Fcos\theta = \frac{\mathrm{m\,v^2}}{\mathrm{r}}$

$Rsin\,\theta$+ μ R $cos\,\theta = \frac{\mathrm{m\,v^2}}{\mathrm{r}}$

R ($sin\theta$+ $\mu\;\; cos\,\theta) = \frac{\mathrm{m\,v^2}}{\mathrm{r}}$ --------- (2)

Divide equation 2 by 1

$\frac{\mathrm{R(sin\theta + \mu\;\, cos\,\theta)}}{\mathrm{R(cos\,\theta - \mu\, sin\,\theta)}} = \frac{m\,v^2}{rmg}$

$\frac{(Sin\theta + \mu Cos\theta)}{(Cos\theta - \mu Sin\theta)} = \frac{\mathrm{v^2}}{\mathrm{r}g}$

$$v_{max}^2 = rg\frac{(Sin\theta + \mu Cos\theta)}{(Cos\theta - \mu Sin\theta)}$$

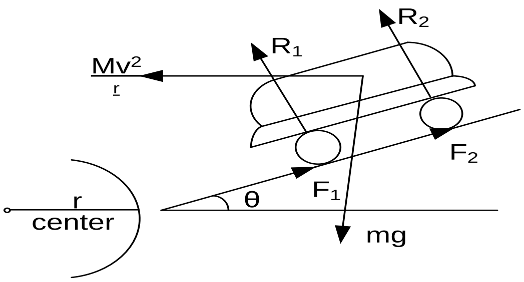
Or divide the right hand side by cosθ

$v_{max}^2 = rg\left[\frac{(tan\theta + \mu)}{(1 - \mu tan\theta)}\right]$

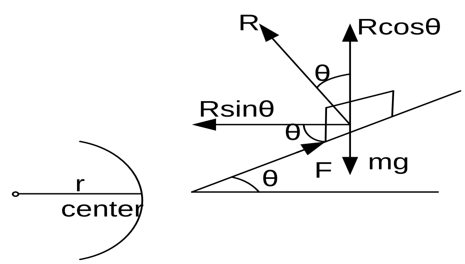
ii) MINIMUM SPEED/LEAST SPEED

If the speed v, is less than the designed speed v the component of the reaction $Rsin\,\theta$ produces an acceleration greater than the centripetal acceleration ($\frac{v^2}{r}$) which is required to keep the car on circular path.

The car tends to slip down the banked track and this tendency is opposed by the frictional force acting upwards.



The car may be taken as aparticle on the incline



Resolving vertically : $Rcos\,\theta + Fsin\,\theta = mg$

$Rcos\,\theta + \mu Rsin\,\theta = mg$

$R(\cos\theta + \mu \sin\theta) = mg$ ------------- (1)

Resolving horizontally: $Rsin\,\theta\ -\ Fsin\,\theta = \frac{m\,v^2}{r}$

$Rsin\,\theta - \ \mu Rcos\,\theta = \frac{m\,v^2}{r}$------------- (2)

Divide equation 2 by 1

$\frac{(Sin\theta - \mu Cos\theta)}{(Cos\theta + \mu Sin\theta)} = \frac{v_{min}^2}{rg}$

Divide the right hand side by cosθ

$v_{min}^2 = \ rg\ \left[\frac{(tan\theta - \mu)}{(1+\mu tan\theta)}\right]$

Example

1. A car travels round a bend which is banked at $22^{0.}$ If the radius of the curve is 62.5m and the coefficient of friction between the road surface and tyres of the car is 0.3, calculate the maximum and minimum speed at which the car can negotiate the bend without skidding.

Solution

$v_{max}^2 = \ rg\ \left[\frac{(tan\theta + \mu)}{(1-\mu tan\theta)}\right]$

$$v_{max} = \left[62.5x9.81\left(\frac{tan22 + 0.3}{1 - 0.3tan22}\right)\right]^{\frac{1}{2}}$$

v_{max} =22.15ms^{-1}

$v_{min}^2 = \ rg\ \left[\frac{(tan\theta - \mu)}{(1+\mu tan\theta)}\right]$

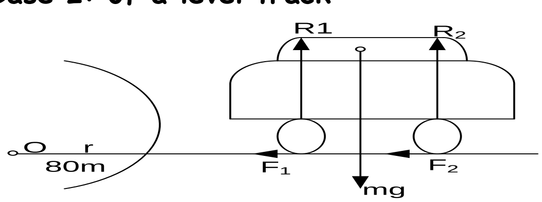
$$v_{min} = \left[62.5x9.81\left(\frac{tan22 - 0.3}{1 + 0.3tan22}\right)\right]^{\frac{1}{2}}$$

v_{min} =7.54ms^{-1}

2. On a level race track, a car just goes round a bend of radius 80m at a speed of 20ms^{-1} without skidding. At what angle must the track be banked so that a speed of 30ms^{-1} can just be reached without skidding, the coefficient of friction being the same in both cases.

Solution

Case I: of a level track



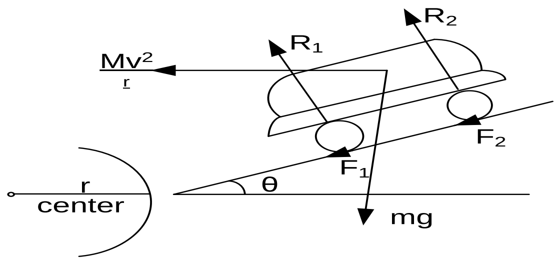
For no skidding $V_{max} = \sqrt{\mu rg}$

$V_{max} = \sqrt{\mu rg}$

$20^2 = \mu x80x9.81$

$\mu = \ 0.51$

Case II: on a banked track



$v_{max}^2 = rg\left[\frac{(tan\theta+\mu)}{(1-\mu tan\theta)}\right]$

$30^2 = 80x9.81\left[\frac{(tan\theta+\mu)}{(1-\mu tan\theta)}\right]$

$\frac{900}{80x9.81} = \frac{(tan\theta+0.51)}{(1-0.51tan\theta)}$

$1.1468-0.584868tan\theta = tan\theta + 0.51$

$tan\theta = \frac{0.6368}{1.58468}$

$\theta = 21.89°$

EXERCISE 21

1. A racing car of mass 2 tonnes is moving at a speed of 5ms^{-1} round a circular path. If the radius of the track is 100m. calculate;
   i) Angle of inclination of the track to the horizontal if the car does not tend to side slip
   ii) The reaction to the wheel if it's assumed to be normal to the track. **An[1.5°, 19606.7N]**

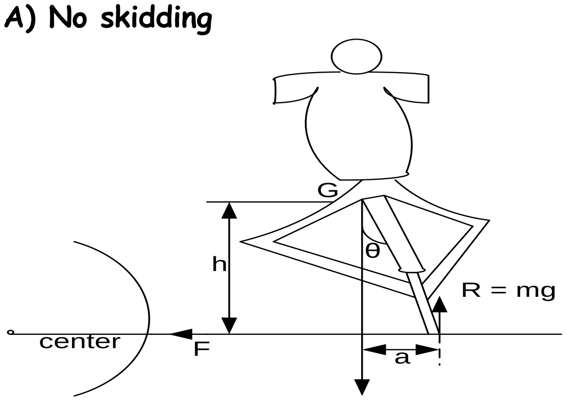
2. A car travels round a bend banked at an angle of 22.6^0. if the radius of curvature of the bend is 62.5m and the coefficient of friction between the tyres of the car and the road surface is 0.3. Calculate the maximum and minimum speed at which the car negotiates the bend without skidding. ***An*** **[22.38ms^{-1}, 7.96ms^{-1}]**

7.1.7: MOTION OF A CYCLIST ROUND A BEND

A cyclist must bend towards the centre while travelling round the bend to avoid toppling. When the cyclist bends, the weight creates a couple which opposes the turning effect of the centrifugal forces.

Consider the total mass of the cyclist and his bike to be m round the circle of radius r at a speed v.

A) No skidding



Resolving vertically: R =mg ---------- (1)

Resolving horizontal: F=$\frac{m\,v^2}{r}$

$\mu R = \frac{m\,v^2}{r}$ ------------------------------ (2)

Put 1 into 2

$\mu mg = \frac{m\,v^2}{r}$

$\mu = \frac{v^2}{rg}$

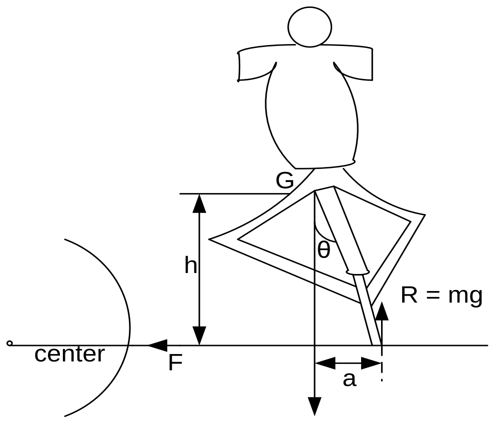
$\boldsymbol{V^2 = \mu rg}$

Vis the max speed at which a cyclist negotiates a bend of radius r without skidding

For no skidding | $V^2 \leq \mu rg$

B) No toppling/over turning

The force G has a moment about the centre of gravity G(F.h) which tends to turn the rider out.



Taking moment about G

$F.h = mg.a$

$\frac{a}{h} = \frac{F}{mg}$

But $\tan\theta = \frac{a}{h}$

$F = \frac{mv^2}{r}$

$\tan\theta = \frac{\frac{mv^2}{r}}{mg}$

$\boxed{v^2 = rg\tan\theta}$

v is the speed at which a cyclist can negotiate a corner without toppling

For no toppling

$v^2 \leq rg\tan\theta$

Why it is necessary for a bicycle rider moving round a circular path to lean towards a center of the path

When a rider moves round a circular path, the frictional force provides the centripetal force. The frictional force has a moment about the centre of gravity of the rider, the rider therefore tends to fall off from the centre of the path if this moment is not counter balanced. The rider therefore leans toward the center of the path so that his reaction provides a moment about the center of gravity , which counter balances the moment due to friction.

UNEB2014No1

(b) (i) Define angular velocity. (01mark)

(ii) satellite is revolving around the earth in a circular orbit at an altitude of $6\,x10^5 m$ where the acceleration due to gravity is $9.4ms^{-2}$. Assuming that the earth is spherical, calculate the period of the satellite. **An[$\mathbf{5.42x10^3 s}$]** (03marks)

UNEB2013No3

(b) Show that the centripetal acceleration of an object moving with constant speed, v, in a circle of radius, r, is $\frac{v^2}{r}$ (04marks)

(c) A car of mass 1000kg moves round a banked track at a constant speed of 108kmh^{-1}. Assuming the total reaction at the wheels is normal to the track, and the radius of curvature of the track

is 100m, calculate the;

(i) Angle of inclination of the track to the horizontal. **An[42.5°]** (04marks)

(ii) Reaction at the wheels **An[13305N]** (02marks)

UNEB 2012 No3

a) Explain what is meant by centripetal force (2mks)

b) i) Derive an expression for the centripetal force acting on a body of mass m moving in a circular path of radius r (6mrk)

ii) A body moving in a circular path of radius 0.5m makes 40 revolutions per second. Find the centripetal force if the mass is 1kg (3mk)

c) Explain the following;

i) a mass attached to a string rotating at a constant speed in a horizontal circle will fly off at a tangent if the string break (02mk)

ii) a cosmonaut in a satellite which is in a free circular orbit around the earth experiences the sensation of weightlessness even though there is influence of gravitation field of the earth.

Solution

b) ii

f=40revs^{-1} r=50m m=1kg	F=m ω^2r
$\omega = 2\pi f$	F=1(251.43)^{2}x50
$\omega = 2 \times \frac{22}{7} \times 40$	F= 3.161x$10^2 N$
ω =251.43rads^{-1}	

c)i) When a mass is whirled in a horizontal circle, the horizontal component of the tension ($T\sin\theta$) provides the necessary centripetal force which keeps the body moving in a circle without falling off.

When the string breaks, the only force acting in the centrifugal force which acts away from the centre and perpendicular to the direction of motion and therefore the mass will fly off at a tangent.

UNEB 2011 No1

a) Define the following terms

i) Uniform acceleration (1mk)

ii) Angular velocity (1mk)

b) i) what is meant by banking of a track

(ii) Derive an expression for the angle of banking θ for a car of mass, m moving at a speed, v around banked track of radius r. (4mk)

c) A bob of mass, m tied to an inelastic thread of length L and whirled with a constant speed in a vertical circle

i) With the aid of a sketch diagram, explain the variation of tension in the string along the circle
(5mk)

ii) If the string breaks at one point along the circle state the most likely position and explain the subsequent motion of the bob.
[2mk]

UNEB 2007 No1

d) Explain why the maximum speed of a car on a banked road is higher than that on an unbanked road.

e) A small bob of mass 0.20kg is suspended by an inextensible string of length 0.8m. The bob is then rotated in a horizontal circle of radius 0.4m. find the
   i) linear speed of the bob (3mk)
   ii) tension in the string (2mk)

UNEB 2005 No4

a) i) Define angular velocity (1mk)

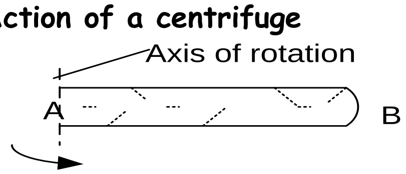
ii) Derive an expression for the force F on a particle of mass m, moving with angular velocity ω in a circle of radius r.

b) A stone of mass 0.5kg is attached to a string of length 0.5m which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of 1m above he ground. The angular speed is gradually increased until the string breaks.
   i) In what position is the string most likely to break? Explain.
   ii) At what angular speed will the string break **An [7.78rads^{-1}, 1.24m]**
   iii) Find the position where the stone hits the ground when the string breaks

c) Explain briefly the action of a centrifuge

Solution

Action of a centrifuge



A centrifuge is used to separate substances of different densities e.g. milk and fat by whirling in a horizontal circle at a high speed.

The mixture placed in a tube and the tube is rotated in a horizontal circle. The liquid pressure at the closed end B is more than that at the open end A. this sets up a pressure gradient along the tube. This pressure gradient creates a large centripetal force that causes matter of small density to move inwards while that of higher density to move away from the centre when rotation stops, the tube is placed in a vertical position and the less dense substance comes to the top which are then separated from the mixture.

UNEB 2004 No2

a) Define the term angular velocity (1mk)

b) A car of mass m, travels round a circular track of radius, r with a velocity v.

i) Sketch a diagram to show the forces acting on the car (2mks)

ii) Show that the car does not overturn if $v^2 < \frac{arg}{2h}$, where a is the distance between the wheel, h is the height of the C.O.G above the ground and g is the acceleration due to gravity (5mk)

c) A pendulum of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. the bob moves in a horizontal circle with the string inclined at 30^0 to the vertical . Calculate;

i) The tension in the string (2mk)

ii) The period of the motion **An[2.27N, 2.04s]** (4mk)

UNEB 2003 No2

a) Define the following terms

i) Angular velocity (1mk)

ii) Centripetal acceleration (1mk)

b) i) Explain why a racing car travels faster on a banked track than one which is flat of the same radius of curvature. (4mk)

ii) Derive an expression for the speed with which a car can negotiate a bend on a banked track without skidding (3mk)

UNEB 2002 No1

d) The period of oscillation of a conical pendulum is 2.0s. if the string makes an angel 60^0 to the vertical at the point of suspension, calculate the

i) Vertical height of the point of suspension above the circle (3mk)

ii) Length of the string (1mk)

iii) Velocity of the mass attached to the string (3mk)

An[0.995m, 1.99m, 5.41ms^{-1}]

UNEB 2002 No2

b i) Derive an expression for the speed of a body moving uniformly in a circular path (3mk)

ii) Explain why a force is necessary to maintain a body moving with a constant speed in a circular path.

c) A small mass attached to a string suspended from a fixed point moves in a circular path at a constant speed in a horizontal plane.

i) Draw a diagram showing the forces acting on the mass (1mk)

ii) Derive an equation showing how the angle of inclination of the string depend on the speed of the mass and the radius of the circular path (3mk).

CHAPTER 8: GRAVITATION

Gravitation deals with motion of planets in a gravitational field. This motion is governed by laws among which are Kepler's laws of gravitation and Newton's laws of gravitation.

8.1.0: KEPLER'S LAWS OF GRAVITATION

Law I: Planets describe elliptical orbit with the sun at one focus

Law II: The imaginary line joining the sun and a planet sweeps out equal areas in equal time intervals

Law III: The squares of the periods of revolution of a planet about the sun is directly proportional to the cube of the mean distance from the sun to the planet. ieT^2**ar^3**

8.1.1: NEWTON'S LAWS OF GRAVITATION

It states that: the force of attraction between two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

i.e. $F \propto m_1m_2$ -------------------------------- (1)

$F \propto \frac{1}{r^2}$ -- (2)

Combining 1 and 2

$F \propto \frac{m_1m_2}{r^2}$

$F = \frac{Gm_1m_2}{r^2}$

G is the gravitational constant

$G = 6.67\times10^{-11}Nm^2kg^{-2}$

This law is sometimes called the inverse square law of gravitation.

8.1.2: DIMENSION OF G AND ITS UNITS

From $F = \frac{Gm_1m_2}{r^2}$

$G = \frac{F\,r^2}{m_1m_2}$

$[G] = \frac{[F\,][r^2]}{[m_1][m_2]}$

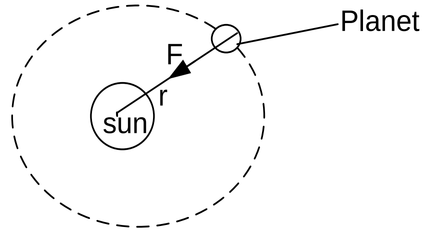
$[G] = \frac{M\,L\,T^{-2}L^2}{M^2}$

$[G] = M^{-1}L^3T^{-2}$

Units of $G = kg^{-1}m^3s^{-2}$ or Nm^2kg^{-2}

8.1.3: VERIFICATION OF KEPLER'S 3RD LAW

Consider a planet of mass m_p above the sun of m_s. If the distance separating the planet and the sun is r.



The force which keeps the planet moving around the sun is represented by;

$F = m_p\omega^2 r$

But $\omega = \frac{2\pi}{T}$

F= $m_p\left(\frac{2\pi}{T}\right)^2 r$---------------------------------(1)

From Newton's law of gravitation

F = $\frac{G\, m_p m_s}{r^2}$ ------------------------ (2)

For the planet moving round a sun, the centripetal force required to keep it in motion should be provided by the gravitational force of attraction between the planet and the sun.

$m_p r\frac{4\pi^2}{T^2} = \frac{G\, m_p m_s}{r^2}$

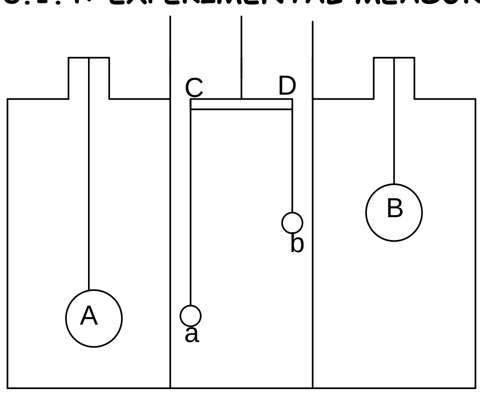
$$T^2 = \frac{4\pi^2 r^3}{Gm_s}$$

$$T^2 = \left(\frac{4\pi^2}{Gm_s}\right)r^3$$

Since $\frac{4\pi^2}{Gm_s}$ is a constant

$T^2 \alpha\, r^3$

8.1.4: EXPERIMENTAL MEASURE OF G



CD- highly polished bar

a, b -gold balls

A, B- lead spheres

- When A, B are brought near a, b respectively because of the attraction between the masses, two equal but opposite forces act on CD. The two forces form a couple and CD is deflected and the angle of deflection θ is. measured by a lamp and scale method by light reflected from CD.
- The distance d between a, A and b, B when the deflection is θ, is measured. Then masses m, M of a and A respectively are measured

$$G = \frac{k\,\theta\, d^2}{m\, M\, \overline{CD}}$$

Where k is torsional constant

Note

- Then torque of couple on CD = $\frac{G\,m\,M}{d^2}$x $\overline{CD}$

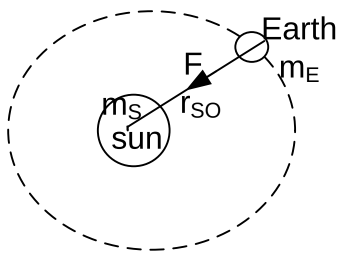
  But torque of couple = k θ

  k θ = $\frac{G\,m\,M}{d^2}$x $\overline{CD}$
- The high sensitivity of the quarts fibres enables the small deflection to be big enough to be measured accurately. The small size of the apparatus allowed it to be screened considerably from air convention currents.
- The constant k can be determined by allowing CD to oscillate through small angle and then observing its period of oscillation 'T' which was of the order of 3 minutes. If I is the known moment of inertia of the system about the torsion wire

  T=$2\pi\sqrt{\frac{I}{k}}$

8.1.5: MASS OF THE SUN

The mass of the sun can be estimated by considering the motion of the earth round the sun in an orbit of radius 1.5×10^{11}m.



Centripetal force: F=$m_E\,\omega^2 r_{so}$ ------------- (1)

Force of attraction: F = $\frac{G\,M_E M_S}{r_{so}^2}$ ---------- (2)

Force of attraction= Centripetal force

$\frac{G\,M_E M_S}{r_{so}^2}$ = $m_E\,\omega^2 r_{so}$

$m_s = \frac{\omega^2 r_{so}^3}{G}$

But $\omega = \frac{2\pi}{T}$

$m_s = \frac{4\pi^2 r_{so}^3}{GT^2}$

r_{so} is radius of the orbit of the earth around the sun

r_{so} =1.5×10^{11}m

G=6.67×10^{-11}Nm^{-2}kg^{-2}

T=1yr≈365days =365x24x60x60s

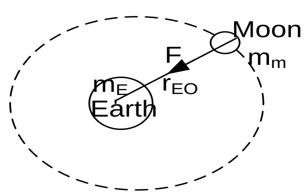
r_{so} = 1.5×10^{11}m

$m_s = \frac{4x\left(\frac{22}{7}\right)^2 x\left(1.5x10^{11}\right)^3}{6.67x10^{11}x(365x24x60x60)^2}$

m_s =$2.0x10^{30}kg$

8.1.6: MASS OF THE EARTH

The mass of the earth can be estimated by considerably the motion of the moon round the earth in an orbit of radius 4x10^8m



Centripetal force: F=$m_m\,\omega^2 r_{EO}$ ------------ (1)

Force of attraction: F = $\frac{G\,M_E M_m}{r_{EO}^2}$ ---------- (2)

Force of attraction= Centripetal force

$\frac{G\,M_E M_m}{r_{EO}^2}$= $m_m\,\omega^2 r_{EO}$

$m_E = \frac{\omega^2 r_{Eo}^3}{G}$ But $\omega = \frac{2\pi}{T}$

$m_E = \frac{4\pi^2 r_{Eo}^3}{GT^2}$

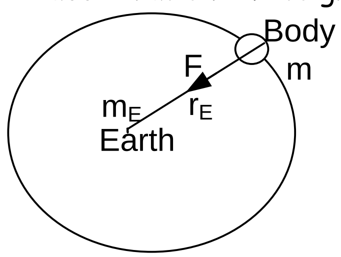
r_{Eo} is the radius of the orbit of the moon about the earth.
r_{Eo} = 4x10^{8}m
T=I month =30days =30x24x60x60
G=6.67x10^{-11}

$$m_E = \frac{4x\left(\frac{22}{7}\right)^2 x\left(4x10^8\right)^3}{6.67x10^{11}x(30x24x60x60)^2}$$

m_E=5.6x10^{24}
m_E ≈6.0x10^{24}kg

8.1.8: RELATION BETWEEN G AND g

Consider a body of mass m placed on the earth's surface of radius r_E where the acceleration due to gravity is g



Force of attraction F = $\frac{G\,M_E m}{{r_E}^2}$-------- (1)

If the body is on the earth's surface then it experiences a gravitational pull

F=mg ------------------------------------- (2)

Equating equation 1 and 2

$\frac{G\,M_E m}{{r_E}^2}$= mg

$$\boxed{G\,m_E = g\,{r_E}^2}$$

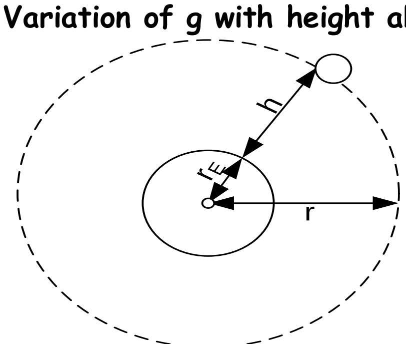
Where r_E is the radius of earth where r_E =6.4x10^{6}m

Differences between G and g

G	g
Units are $m^3kg^{-1}s^{-2}$ or Nm^2kg^{-2}	Units are ms^{-2}
Occurs due to forces of attraction between two bodies	Acts on only one body
Does not vary with attitude	Varies with attitude

8.1.9: VARIATION OF g OF A BODY DURING FREE FALL

1. Variation of g with height above the earth's surface



An object of mass m placed at a height h, above the surface of the earth where acceleration due to gravity at that height is g^1.

At a height h the gravitational force of attraction between the object and the earth is equal to the weight of the object.

$mg^1 = \frac{Gm_E\,m}{r^2}$---------------------- (1)

but $g{r_E}^2 = Gm_E$--------------------- (2)

(1)÷ (2)

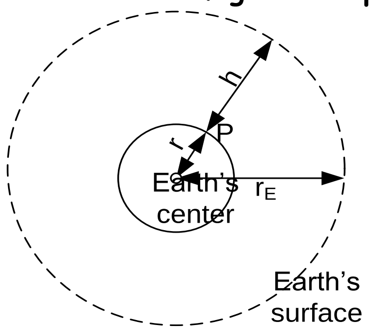
$$\frac{mg^1}{g{r_E}^2} = \frac{Gm_E m}{r^2 Gm_E}$$

$$g^1 = \frac{g{r_E}^2}{r^2}$$

Since ${r_E}^2$ and g are constant, $g^1 \propto \frac{1}{r^2}$

- Therefore for a point above the earth surface g varies inversely as the square of the distance r from the centre of the earth.

2. Variation of g with depth below the earth surface



Consider the earth to be a uniform sphere of uniform density. Suppose a body at a point h meters from the surface of the earth measured towards the centre of the earth.

When the object is on the surface of the earth .

$$mg = \frac{Gm_E m}{r_E{}^2}$$

$$M_E = \frac{r_E{}^2 g}{G} \quad \text{------------------ (1)}$$

at p $\quad m_E{}^1 g^1 = \frac{G\, m_E{}^1\, m}{r^2}$

$$m = \frac{r^2 g^1}{G} \quad \text{---------------------------- (2)}$$

Where $m_E{}^1$ is the effective mass of that part of the earth which has a radius of r

Equation 2 divided by 1

$$\frac{m}{M_E} = \frac{r^2 g^1}{G} \Big/ \frac{r_E{}^2 g}{G}$$

$$\frac{m}{M_E} = \frac{r^2 g^1}{r_E^2 g} \quad \text{------------------------- (3)}$$

For masses of uniform spheres are proportional to the cube of their radii

i.e. $m \propto r^3$ and $M_E \propto r_E^3$

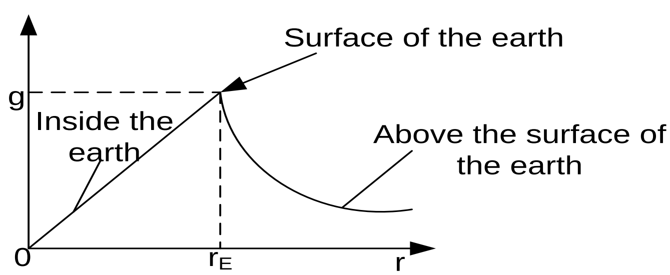
$$\frac{r^3}{r_E^3} = \frac{r^2 g^1}{r_E{}^2 g}$$

$$\frac{g^1}{g} = \frac{r}{r_E}$$

$$g^1 = g\frac{r}{r_E}$$

$\therefore g^1 \propto r$ for a point inside the earth

3. *Graph of variation of acceleration of free fall from the centre of the earth*



For points above the earth, the gravitational force obeys the inverse square law while for points inside the earth, g is proportional to the distance from the centre.

4. Variation of acceleration due to gravity with location on the surface of the earth

a) The earth is ellipsoidal with the equatorial radius being greater than the polar radius. At the equator body is less attracted towards the earth than at the poles : $g_{\text{equatorial}} < g_{\text{polar}}$

b) The earth rotates about its polar axis, so a body at the poles is practically stationary while towards the equator experiences a centripetal force mω^2r where r is the equatorial radius.

$$m\omega^2 r = mg_{polar} - mg_{equatorial}$$

$$g_{equatorial} = g_{polar} - \omega^2 r$$

Examples

1. A body has a weight of 10N on the earth. What will its weight be on the moon if the ratio of the moon's mass to the earth's mass is 1.2x10^{-2} and the ratio of the moon's radius to that of the earth is 0.27?

Solution

Consider the body on the earth's surface

$$mg = \frac{Gm_E m}{r_E{}^2}$$

$$g = \frac{Gm_E}{r_E{}^2} \text{ --------------- (1)}$$

Also on the moon's surface

$$mg^1 = \frac{Gm_m\, m}{r_m{}^2}$$

$$g^1 = \frac{Gm_m}{r_m{}^2} \text{ -------------- (2)}$$

eqn2 ÷ eqn 1

$$\frac{g^1}{g} = \frac{Gm_m}{r_m{}^2} \Big/ \frac{Gm_E}{r_E{}^2}$$

$$\frac{g^1}{g} = \frac{m_m}{m_E} x \frac{r_E{}^2}{r_m{}^2}$$

But $\frac{m_m}{m_e}$ = 1.2x10^{-2} and $\frac{r_m}{r_e}$ =0.27

$$\frac{g^1}{g} = 1.2x10^{-2}x\left(\frac{1}{0.27}\right)^2$$

$$g^1 = \frac{1.2x10^{-2}}{0.27^2}x9.81$$

g^1 = 1.6148ms^{-2}

but weight 10N

$$w = mg$$

10= mx9.81

$$m = \frac{10}{9.81}$$

m=1.0194kg

$w^1 = mg^1$

w^1 =1.0194x1.614

w^1 =1.646N

Alternatively

$$W_m = \frac{Gm_m\, m}{r_m{}^2} \text{ ----------- (1)}$$

$$W_E = \frac{Gm_E m}{r_E{}^2}$$

$$10 = \frac{Gm_E m}{r_E{}^2} \text{ ------------ (2)}$$

$$\frac{W_E}{10} = \frac{Gm_E m}{r_E{}^2} \Big/ \frac{Gm_m\, m}{r_m{}^2}$$

$$W_E = \frac{m_m}{m_E} x \frac{r_E{}^2}{r_m{}^2} x10$$

But $\frac{m_m}{m_e}$ = 1.2x10^{-2} and $\frac{r_m}{r_e}$ =0.27

$$W_m = 1.2x10^{-2}x\left(\frac{1}{0.27}\right)^2 x10$$

$$W_m = 1.65N$$

2. The acceleration due to gravity on the surface of mars is about 0.4times the acceleration due to gravity on the surface of the earth. How much wound a body weigh on the surface of mars if it weighs 800N on the earth's surface .

Solution

$W_m = mg^1$

But g^1=0.4g

W_m=mx0.4g

W_m=0.4mg

W_m=0.4x800 since mg=800N

W_m=320N

8.2.0: MOTION OF SATELLITE

A satellite is a small body that moves in space round a planet

- There are two types of satellite

1- Natural satellites eg moon
2- Artificial satellites eg Canal France International (CFI)

Artificial satellites are man made called space crafts/ space stations placed in orbits about the earth, moon or other planets for communication and scientific study purposes.

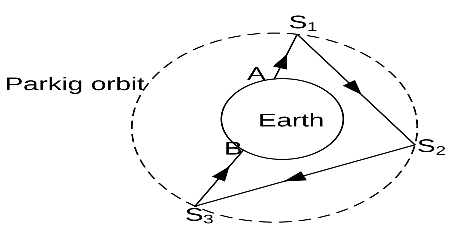
- Artificial satellites are grouped into
  i) Passive satellites, for these satellites, they simply reflect signals of the same strength from one point to another.
  ii) Active satellites, these satellites are able to amplify and retransmit signal that they pick from one point on the earth to another.
- Satellites are initially fired from the earth into space with the help of a rocket. When a rocket reaches that particular height, a second phase of the launch is initiated where the satellite is fired horizontally into the orbit to start its circular motion and the rest of the rocket burns in space.

8.2.1: GEOSTATIONARY/SYNCHRONOUS ORBIT

These are communication satellites with orbital period of 24hrs and stays at the same point above the earth surface provided it is above the equator and its moving in the same direction as the earth is rotating.

Satellites of this type are used to relay television signals and telephone messages from one point on the earth surface to another eg synchrom2, synchrom3 and the early bind.

8.2.2: HOW COMMUNICATION IS DONE USING A SATELLITE



- ❖ It is done by launching three or more geostationary communication satellites into space with help of a rocket.
- ❖ Microwave signals are transmitted from a large steerable horn antennae A to the geostationary satellites S_1. satellite S_1 has a number of aerials which receives them and amplify and retransmit them to satellite S_2, which in turn transmits then back to the earth and they are received by steerable dish aerial in an earth station.

8.2.3: PARKING ORBIT

It's a path in space of a satellite which makes it appear to be in same position with respect to a point on the earth.

Note:

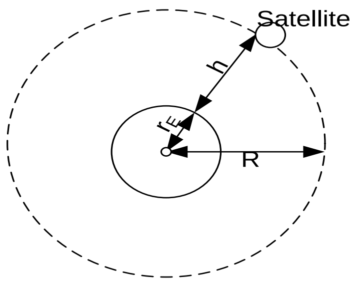
For an object (satellite) in parking orbit;

- It has a period of 24hrs

- Angular velocity relative to that of the earth is zero
- Direction of the object in the orbit is the same as the direction of rotation of the earth orbit about its axis.

8.2.4: PERIOD OF A SATELLITE

Consider a satellite of mass m moving in a circular orbit of radius h above the earth surface.



Attractive force=Centripetal force:

$m\omega^2 R = \frac{Gm_E m}{R^2}$

$\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{Gm_E}}$$

OR

Where $R = \mathrm{r_E} + \mathrm{h}$

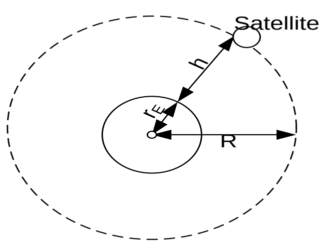
But also $Gm_E = g{r_E}^2$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{g{r_E}^2}}$$

Example

1. Find the period of revolution of a satellite moving in a circular orbit round the earth at a height of $3.6\text{x}10^6$m above the earth's surface.

Solution



Centripetal force $F = m\omega^2 R$

Attractive force $F = \frac{Gm_E m}{R^2}$

$m\omega^2 R = \frac{Gm_E m}{R^2}$

but $\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

$$T = \left(\frac{4\pi^2 R^3}{Gm_E}\right)^{\frac{1}{2}}$$

Where $R = \mathrm{r_E} + \mathrm{h}$

But also $Gm_E = g{r_E}^2$

$$T = \left(\frac{4\pi^2(\mathrm{r_E} + \mathrm{h})^3}{Gm_E}\right)^{\frac{1}{2}}$$

r_E is radius of earth $=6.4\text{X}10^6$m

m_E is mass of earth $=6\text{x}10^{24}$kg

$$T = \left(\frac{4\left(\frac{22}{7}\right)^2 (6.4\text{x}10^6 + 3.6\text{x}10^6)^3}{6.67x10^{-11}x6x10^{24}}\right)^{\frac{1}{2}}$$

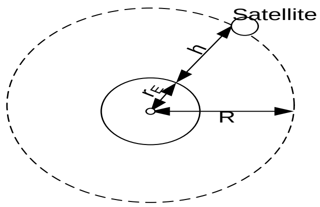
T=9932.10555s

T=2.759Hrs

2. An artificial satellite move round the earth in a circular orbit in the plane of the equator at height 30,000km above the earth's surface (mass of earth $=6.0\text{x}10^{24}$kg, radius of the earth= $6.4\text{x}10^6$m,)

i) Calculate its speed
ii) What is the time between successive appearances over a point on the equator
iii) What will be the additional distance of the satellite if it was to appear stationery

Solution



h=30,000km =$3x10^7$m, $r_E=6.4x10^6$m
G =$6.67x10^{-11}$$Nm^2kg^{-2}$ $M_E=6.0x10^{24}$kg

$$m\omega^2 R = \frac{Gm_E m}{R^2}$$

$$\omega^2 = \frac{Gm_E}{R^3}$$

$$\omega = \sqrt{\frac{6.67x10^{-11}x6.0x10^{24}}{(6.4x10^6+3x10^7)^3}}$$

ω =$9.1025x10^{-5}rads^{-1}$

$$v = r\omega$$

$$v = (6.4x10^6 + 3x10^7)x9.1025x10^{-5}$$

v =$3.313x10^3ms^{-1}$

(i) Its speed is $3.313x10^3ms^{-1}$
(ii) Time required is the period

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2x\frac{22}{7}}{9.1025x10^{-5}}$$

T= $6.903x10^4$s T=19.2h
(iii) From Kepler's third law

$$T^2 \propto R^3$$

$T^2 = kR^3$------------------------------------ (x)

$R=R_E+h$
$R=(6.4x10^6+3x10^7)$m
$R=36.4 x10^6$m
$19.2^2=k(36.4 x10^6)^3$------------------------ (1)

If the satellite is stationery, then the geostationary T^1=24hrs

$$(T^1)^2 \propto (R^1)^3$$

$(T^1)^2 = k(R^1)^3$------------------------------ (xx)

$(24)^2 = k(R^1)^3$---------------------------------(2)

Equation 2 $\div$equaiton 1

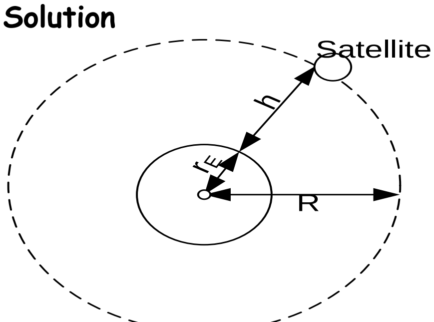
$$\frac{(24)^2}{(19.2)^2} = \frac{k\left(R^1\right)^3}{k(36.4\ x106)3}$$

$R^1=42.2x10^6$m
R^1=R+ extra distance
Extra distance = R^1-R
= $42.2x10^6 - 36.4\,x10^6$
=$5.8x10^6$m

3. **UNEB 2003 Qn 2(e)**

A communication satellite orbits the earth in synchronous orbits. Calculate the height of communication satellite above the earth.

Solution



$m\omega^2 R = \frac{Gm_E m}{R^2}$ but $\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

$$R = \left(\frac{T^2 Gm_E}{4\pi^2}\right)^{\frac{1}{3}}$$

G =$6.67x10^{-11}$$Nm^2kg^{-2}$
T=24hrs for synchronous orbits
$M_E=5.97x10^{24}$kg

$$R = \left(\frac{[24x60x60]^2 x6.67x10^{-11}x5.97x10^{24}}{4x\left[\frac{22}{7}\right]^2}\right)^{\frac{1}{3}}$$

$R=4.22x10^7$m
But $R=R_E+h$ $\therefore$ $R=6.4x10^6$+h

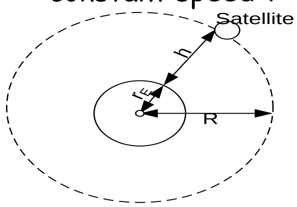
h=$4.22 \times 10^7 - 6.4 \times 10^6$

h=3.58×10^7m

8.2.5: ENERGY OF A SATELLITE

1. Kinetic energy

Consider a satellite of mass m moving in an orbit of radius R around the earth at a constant speed v



Centripetal force F = $\frac{mv^2}{R}$------------------ (1)

Force of attraction F = $\frac{Gm_E m}{R^2}$ -------------- (2)

Equating 1 and 2

$$\frac{mv^2}{R} = \frac{Gm_E m}{R^2}$$

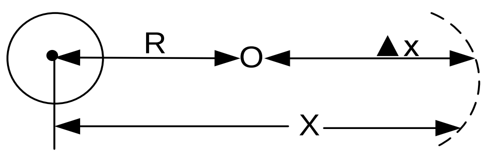
Introducing $\frac{1}{2}$ on both sides

$$\frac{1}{2}mv^2 = \frac{Gm_E m}{2R}$$

$$\boxed{\boldsymbol{K.E} = \frac{Gm_E\,m}{2R}}$$

2. Potential energy

Consider a satellite of mass m brought from infinity into the region of earth's gravitational force.



From Newton's law of gravitation

F = $\frac{Gm_E m}{x^2}$

Work done $\Delta w = F\Delta x$

Total work done

$$\int_0^W dw = \int_\infty^R F dx$$

$$[w]_0^W = \int_\infty^R \frac{Gm_E m}{x^2} dx$$

$$W = Gm_E m \int_\infty^R x^{-2} dx$$

$$W = Gm_E m \left[\frac{-1}{x}\right]_\infty^R$$

$$W = Gm_E m \left[\frac{-1}{R} - \frac{-1}{\infty}\right]$$

$$W = Gm_E m \left[\frac{-1}{R} - 0\right]$$

$$W = \frac{-Gm_E m}{R}$$

But work done = $p.e$

$$\boxed{P.E = \frac{-Gm_E m}{R}}$$

Definition

Gravitational potential energy P.E is the work done in bringing a unit mass from infinity to that point.

3. Mechanical energy/total energy

$M.E = K.E + P.E$

$$M.E = \frac{Gm_E\, m}{2R} + \frac{-Gm_E\, m}{R}$$

$$\boxed{M.E = \frac{-Gm_E\, m}{2R}}$$

Note:

i) Mechanical energy and kinetic energy only differ by the sign therefore their magnitude is the same

ii) If the radius of the orbit of the satellite decreases, the gravitational potential energy of the satellite becomes more negative implying that it has decreased.

- Decrease in radius however causes an increase in the kinetic energy, resulting in an increase in the speed of the satellite in its new orbit.
- Decrease in orbital radius also results into the mechanical energy becoming more negative hence it has decreased.

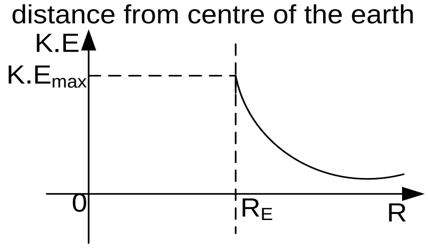
8.2.6: EFFECT OF FRICTION ON A SATELLITE

- If a satellite is located within the earth atmosphere as it moves in its orbit, the atmospheric gasses offer frictional resistance to its motion. The satellite thus would be expected to do work to overcome this resistance and is so doing, it falls to an orbit of lower radius.
- The decrease in the radius causes the total energy $\left(\frac{-Gm_E\, m}{2R}\right)$ to decrease while the kinetic energy of the satellite $\left(\frac{Gm_E\, m}{2R}\right)$ increases resulting into an increase in the speed of the satellite in its new orbit. Because of the increase of the speed the satellite becomes hotter and it may burnout.

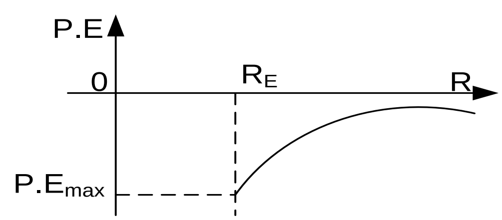
Question

Explain why any opposition to the forward motion of a satellite may cause it to burn.

A graph of K.E with variation of distance from centre of the earth



A graph of P.E with variation of distance from centre of the earth



Examples

1. A satellite of mass 100kg is in a circular orbit at a height 3.59×10^7m above the earth surface
   i) Calculate the kinetic energy, potential energy and the mechanical energy of the satellite in this orbit
   ii) State what happens when the mechanical energy of the satellite is reduced

Solution

i) $K.E = \frac{Gm_E\, m}{2R}$	$R = r_e + h$	

K.E. $=\frac{6.67x10^{-11}x6x10^{24}x100}{2x(6.4x10^6+3.59x10^7)}$

K.E. $= 4.7.5x10^8$J

$P.E. = -\frac{Gm_E m}{R}$

R=r_e+h

$P.E. = -\frac{6.67x10^{-11}x6x10^{24}x100}{(6.4x10^6+3.59x10^7)}$

P.E.=- 9.4992 x10^8J

P.E. $= -9.50\ x10^8$J

M.E=P.E +K.E

$= -9.4992\ x10^8 + 4.75x10^8$

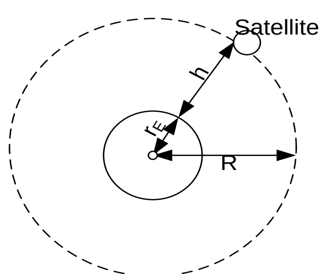
M.E= -4.75x10^8J

ii)

- ✓ Frictional force increases
- ✓ Satellite falls to orbit of small radius
- ✓ PE reduces
- ✓ K.E increases
- ✓ Satellite becomes hot and may burn

2. A 10^3 kg mass satellite is launched in a parking orbit about the earth
   i) Calculate the height of the satellite above the surface of the earth
   ii) Calculate the mechanical energy of the satellite [$R_E = 6.4x10^6 m, g = 9.81ms^{-2}, G = 6.67x10^{-11}Nm^2kg^{-2}$]

Solution



$m\omega^2 R = \frac{Gm_E m}{R^2}$ but $\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

$R = \left(\frac{T^2 G m_E}{4\pi^2}\right)^{\frac{1}{3}}$ But $G\,m_E = g\,r_E{}^2$

$$R = \left(\frac{T^2 g\,r_E{}^2}{4\pi^2}\right)^{\frac{1}{3}}$$

T=24hrs for parking orbits

M_E=5.97x10^{24}kg

$$R = \left(\frac{[24x60x60]^2 x9.81x(6.4x10^6)^2}{4x\left[\frac{22}{7}\right]^2}\right)^{\frac{1}{3}}$$

$R = 4.22x10^7 m$

But $R = R_E + h$

$R = 6.4x10^6 + h$

$h = R - 6.4x10^6$

$h = 4.24x10^7 - 6.4x10^6$

$h = 3.6x10^7 m$

$M.E = -\frac{Gm_E m}{2R}$

But $G\,m_E = g\,r_E{}^2$

$$= \frac{9.81x(6.4x10^6)^2}{2x4.22x10^7}$$

M.E= $-4.74x10^9$ J

EXERCISE 22

1. A satellite of mass 1000kg is launched on a circular orbit of radius 7.2x10^6m about the earth. Calculate the mechanical energy of the satellite [ME=6x10^{24}kg, $, G = 6.67x10^{-11}Nm^2kg^{-2}$]

   An [-2.78x10^{10}J]

2. An artificial satellite is launched at a height of 3.6×10^7mabove the earth's surface

   i) Determine the speed with which the satellite must be launched to maintain in the orbit.

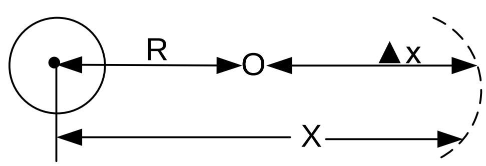
   An[**3.08×10^3**]ms^{-1}

   ii) Determine the period of the satellite. **An[24hrs]**

8.2.7: GRAVITATIONAL POTENTIAL [U]

Gravitational potential at a point in the gravitational field is defined as the work done to move a one kilogram mass (1kg) from infinity to that part. Ie $U = \frac{w}{m}$

Consider a body of mass 1kg moved from infinity to appoint O where the distance from the centre of the earth to O is R



From Newton's law of gravitation

F = $\frac{GMm}{x^2}$

$m = 1kg$

Work done $\Delta w = F\Delta x$

Total work done

$$\int_0^W dw = \int_\infty^R F dx$$

$$[w]_0^W = \int_\infty^{R_E} \frac{GM}{x^2} dx$$

$$W = GM \int_\infty^R x^{-2} dx$$

$$W = GM \left[\frac{-1}{x}\right]_\infty^R$$

$$W = GM \left[\frac{-1}{R} - \frac{-1}{\infty}\right]$$

$$W = GM \left[\frac{-1}{R} - 0\right]$$

$$W = \frac{-GM}{R}$$

But work done $=$ $potential\ U$

$$\boxed{U = \frac{-GM}{R}}$$

Generally On the earth surfaces

$$U = \frac{-G\,m}{R_E}$$

Note:

The amount of work done against the gravitational force of mas M to move the mass a distance $\boldsymbol{r_1}$ to position r_2 is given by

$$W = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Example

A body of mass 15kg is moved from the earth's surface to a point $2.8x10^6$m above the earth. If the radius of the earth is $6.4x10^6$m and its mass is $6.0x10^{24}$ kg calculate the work done in taking the body to that point

Solution

$$W = Gm_E m\left(\frac{1}{R} - \frac{1}{R_E}\right)$$

$R=R_e+h$

$$R = (6.4x10^6 + 2.8x10^6)$$

$R=9.2x10^6$

$$w = 6.67x10^{-11}x6.4x10^{24}x15\left(\frac{1}{9.2x10^6} - \frac{1}{6.4x10^6}\right)$$

work done in taking the body to that point

$W= 2.85x10^8$J

8.2.8: VELOCITY OF ESCAPE

This is the minimum vertical velocity with which a body is projected from the surface of the earth so that it escapes from the earth's gravitational pull.

Derivation of formulae

Suppose a rocket of mass is fired from the earth's surface so that it just escapes from the gravitational influence of the earth

K.E lost = P.E lost

$$\tfrac{1}{2}\, mV_{esc}{}^2 = 0 - \frac{-Gm_E m}{R_E}$$

$$V_{esc} = \sqrt{\frac{2\,Gm_E}{R_E}}$$

$$G\,m_E = g\,R_E{}^2$$

$$V_{esc} = \sqrt{2gR_E}$$

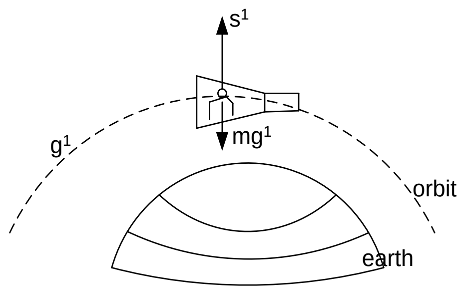
$$V_{esc} = \sqrt{2x9.81x6.4x10^6}$$

V_{esc} =11.2km/h

Note

- ❖ Air molecules at stp have an average speed of about $0.5ms^{-1}$ which is much less than the escape velocity and so the earth's gravitational field is able to maintain an atmosphere of air around the earth.
- ❖ Light gases like Neon, Argon, helium have mean thermal speed more than 3 time of air. This means that their speeds are higher than the mean speeds of air and this explain why they are rare in the earth atmosphere.
- ❖ The moon has no atmosphere because the gravitational field of the moon is not sufficiently strong to prevent gases from escaping.
- ❖ For other planets escape velocity is given by $V_{esc} = \sqrt{\frac{2\,Gm}{R}}$

8.2.9: WEIGHTLESSNESS



An astronaut in an orbiting space craft has a centripetal acceleration $a=g^1$ where $g^1$is the acceleration due to gravity at the height of the orbit.

If S^1 is the reaction of the surface of the space craft in contact with the astronaut then;

$F=mg^1-S^1$

$ma= mg^1-S^1$

but $a=g^1$

$mg^1= mg^1-S^1$

$S^1=0$

The astronaut therefore has no acceleration relative to his space craft and hence experiences zero reaction i.e. **weightless**.

Definition

Weightlessness is the condition of a body having zero reaction when a body moves with the acceleration as acceleration due to the gravity.

UNEB 2013No4(a)

(i) State Kepler's laws of planetary motion
(03marks)

(ii) Estimate the mass of the sun, if the orbit of the earth around the sun is circular
(04mks)

UNEB 2012 No3(c)

(ii) A cosmonaut in a satellite which is in a free circular orbit around the earth experience the sensation of weightlessness even though there is influence of the gravitational field of the earth Explain

(03marks)

UNEB 2011 No1(d)

A body of mass 15kg is moved from the earth's surface to a point 2.8×10^6m above the earth. If the radius of the earth is 6.4×10^6m and its mass is 6.0×10^{24} kg calculate the work done in taking the body to that point An **2.85×10^8J**

(06marks)

UNEB 2008 No3(c)

(i) with the aid of a diagram, describe an experiment to determine the universal gravitational constant G.

(06marks)

(ii) If the moon moves round the earth in a circular orbit of radius $=4.0 \times 10^8$m and takes exactly 27.3 days to go round once calculate the value of acceleration due to gravity g at the earth's surface.

(04marks)

$m\omega^2 R = \frac{Gm_E m}{R^2}$

but $\omega = \frac{2\pi}{T}$

$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$

$G\,m_E = g\,{r_E}^2$

$g = \frac{4\pi R^3}{T^2 {r_E}^2}$

g=$\frac{4x\left(\frac{22}{7}\right)^2 x\left(4.0x10^8\right)^3}{(27.3x24x60x60)^2 x(6.4x10^2)^2}$

g=11.09ms^{-2}

UNEB 2007 No2

a) State Kepler's laws of planetary motion (3mk)

b) i) A satellite moves in a circular orbit of radius R about a planet of mass m, with period T. show that $R^3 = \frac{G\,m\,T^2}{4\,\pi^2}$ where G is the universal gravitational constant

(04marks)

ii) The period of the moon round the earth is 27.3. if the distance of the moon from the earth is 3.88×10^8km. Calculate the acceleration due to gravity at the face of the earth.

An [$g=9.72ms^{-2}$] (04marks)

iii) Explain why any resistance to forward motion of an artificial satellite results into an increase in its speed.

(04marks)

c) i) what is meant by weightlessness (02marks)

ii) Why does acceleration due to gravity vary with location on the surface of the earth

(03marks)

UNEB 2004 No2

d) Explain and sketch the variation of acceleration due to gravity with distance from the centre of the earth.

(06marks)

UNEB 2003 No2

c) Show how to estimate the mass of the sun if the period and orbital radius of one of its planets are known.

d)The gravitational potential U at the surface of a planet of mass m and radius R is given by $U = -\frac{G\,m}{R}$ where G is the gravitational constant. Derive an expression for the lowest velocity, v which an object of mass m must have at the surface of the planet if it is the escape from the planet

(04marks)

e) Communication satellite orbits the earth in synchronous orbits. Calculate the height of a communication satellite above the earth **An [3.6×10^7m]**

(04marks)

UNEB 2000 No 4

a) State Keplers law's of gravitation (03marks)

b) I)Show that the period of a satellite in a circular orbit of radius r about the earth is given by

$T=\left(\frac{4\,\pi^2}{G\,M_E}\right)^{\frac{1}{2}} r^{\frac{3}{2}}$

Where the symbols have usual meanings

(05marks)

ii)Explain briefly how world wide, radius or television communication can be achieved with the help of satellites

(04marks)

c) A satellite of mass 100kg in a circular orbit at a height of 3.59×10^7m above the earth's surface

(i) Find the mechanical energy

(04marks)

(ii) Explain what would happened if the mechanical energy was decreased

(04marks)

CHAPTER 9: SIMPLE HARMONIC MOTION (S.H.M)

Definition

This is the periodic motion of a body whose acceleration is directly proportional to the displacement from a fixed point (equilibrium position) and is directed towards the fixed point.

$$a\,\alpha - x$$

$$a\ = -\omega^2 x$$

The negative signs means the acceleration and the displacement are always in opposite direction.

9.1.0: Characteristics of SHM

(1) It's a periodic motion (to and fro motion)

(2) Mechanical energy is always conserved

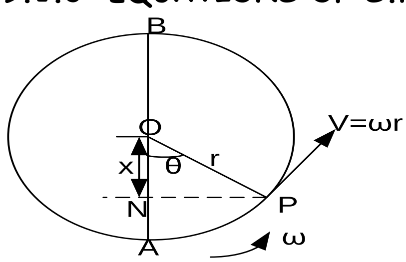
(3) The acceleration is directed towards a fixed point

(4) Acceleration is directly proportional to its displacement

9.1.1: PRACTICAL EXAMPLES OF S.H.M

- Pendulum clocks
- Pistons in a petrol engine
- Strings in music instruments
- Motor vehicle suspension springs
- Balance wheels of a watch

9.1.3: EQUATIONS OF S.H.M



Consider a particle, p of mass, m, moving round a circle of radius ,r, with uniform angular velocity, ω, . Let the particle be at point p, t seconds after starting motion from A such that the angle subtended at the centre is ,θ

a) Displacement x

This is the distance of N from O measured away from O

The displacement is obtained from triangle ONP

$$Cos\theta = \frac{ON}{OP}$$

$$Cos\theta = \frac{x}{r}$$

$x = rCos\theta$ But $\theta = \omega t$

$\boxed{x = rCos\omega t}$-- (1)

Note:

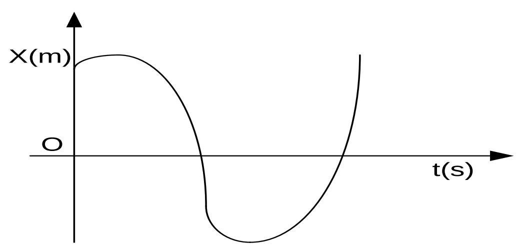
When the displacement x is maximum i.e. when N is at A or at B, then this displacement is known

as **amplitude**.

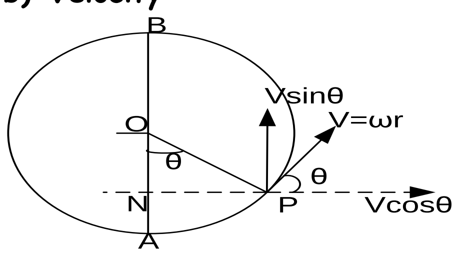
Definition

Amplitude is the maximum displacement of a body (a particle) from equilibrium position.

A GRAPH OF DISPLACEMENTAGAINST TIME



b) Velocity



Velocity of N as a result of the velocity P moving round circle. This is equal to the vertical component of velocity of p

$v_N = -vsin\theta$

But $\theta = \omega t$ and $v = \omega r$

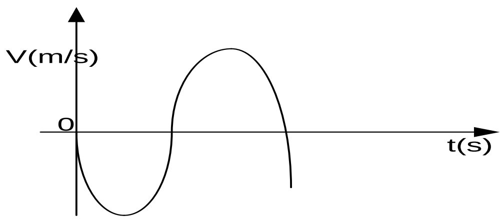
$\boxed{v_N = -\omega r sin\omega t}$--------------------------------- (2)

Note:

The velocity of N is negative because as p moves from A to B, N moves upwards and when it

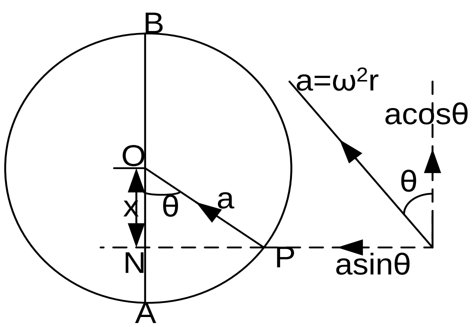
moves from B to A, N changes direction and moves downwards.

GRAPH OF VELOCITY AGAINST TIME



c) Acceleration $\ddot{x}$ or a

The acceleration of N is as a result of the acceleration of p. This is equal to the vertical component



a$_N = acos\theta$

but $a = \omega^2 r$ and $\theta = \omega t$

$a_N = \omega^2 r cos\omega t$

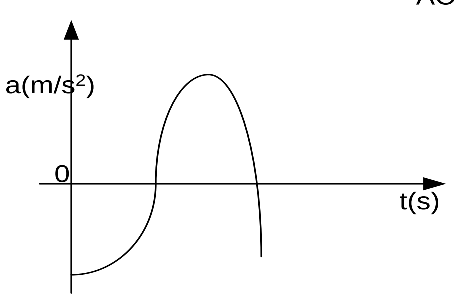
but fom equation 1 $x = rcos\omega t$

$a_N = \omega^2 x$

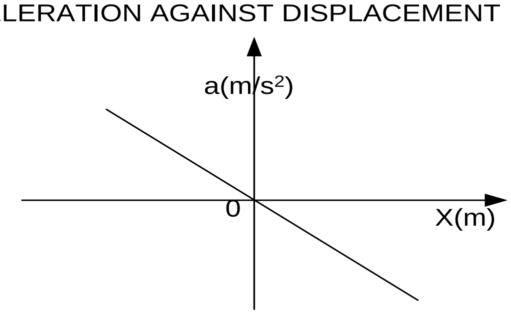
$\boxed{a_N = -\omega^2 x}$------------------------------ --- (3)

$a_{max} = -\omega^2 r$

ACCELERATION AGAINST TIME



ACCELERATION AGAINST DISPLACEMENT



d) Period T

This is the time taken for one complete oscillation. .i.e. N moving from A to B and back to A.

$T = \frac{distance}{speed}$

$T = \frac{2\pi}{v}$ but $v = r\omega$

$T = \frac{2\,\pi\, r}{\omega\, r}$

$T = \frac{2\,\pi}{\omega}$--------------------------- (4)

e) Frequency f

This is the number of complete oscillation made in one second

$f = \frac{1}{T}$

$f = \frac{\boldsymbol{\omega}}{\boldsymbol{2\pi}}$

f) Velocity in terms of displacement

Velocity of a body executing S.H.M can be expressed as a function of displacement x. this is obtained from the acceleration

a= $-\omega^2 x$

a= $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

but $\frac{dx}{dt} = v$

a = $v \cdot \frac{dv}{dx}$

$v \cdot \frac{dv}{dx} = -\,\omega^2 x$

$vdv = -\,\omega^2 x dx$

integrating both sides

$\int v\, dv = -\,\omega^2 \int x\, dx$

$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + C$ ....[1]

Where C is a constant of integration

When t = 0 v=0 and

$x = r$(amplitude)

$\frac{0^2}{2} = -\frac{\omega^2 r^2}{2} + C$

$C = \frac{\omega^2 r^2}{2}$

Put into [1]

$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 r^2}{2}$

$v^2 = \omega^2 r^2 - \omega^2 x^2$

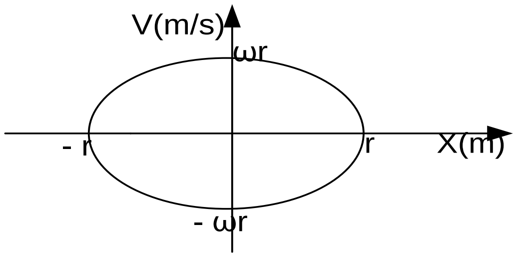
$\boxed{v^2 = \omega^2 (r^2 - x^2)}$

Velocity is maximum when x =0

$v^2 = \omega^2 r^2$

$\boxed{v_{max} = \boldsymbol{\omega r}}$

AGRAPH OF VELOCITY AGAINST DISPLACEMENT



From $\boldsymbol{v^2} = \omega^2 r^2 - \omega^2 x^2$

$\boldsymbol{v^2}$ **+** $\omega^2 x^2 = \omega^2 r^2$

$\frac{v^2}{\omega^2 r^2} + \frac{x^2}{r^2}$ ***= 1***

This an ellipse

Example

1. A particles moves in a straight line with S.H.M. Find the time of one complete oscillation when
   i) The acceleration at a distance of 1.2m is $2.4ms^{-2}$
   ii) The acceleration at a distance of 20cm is $3.2ms^{-2}$

Solution

i) From $a = -\omega^2 x$ Negative is ignored

$2.4 = \omega^2(1.2)$

$\omega^2 = \frac{2.4}{1.2}$

ω =1.4rads^{-1}

But $T = \frac{2\pi}{\omega}$

$T = \frac{2x\frac{22}{7}}{1.4} = 4.46s$

ii) $a = -\omega^2 x$

$3.2 = \omega^2(0.2)$

ω =4rads^{-1}

$T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{4}$ =1.57second

2. A Particle moving with S.H.M has velocities of $4ms^{-1}$ and $3ms^{-1}$ at distances of 3m and 4m respectively from equilibrium position. Find
   i) amplitude ,
   ii) period ,
   iii) frequency
   iv) velocity of the particle as it passes through equilibrium position

Solution

i) $v = 4ms^{-1}, x = 3m$ and

Using $v^2 = \omega^2(r^2 - x^2)$

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(r^2 - 9)$------- (1)

Also $v = 3ms^{-1}, x = 4m$

$3^2 = \omega^2(r^2 - 4^2)$

$9 = \omega^2(r^2 - 16)$------- (2)

Equation 1 divide by 2

$$\frac{16}{9} = \frac{\omega^2(r^2 - 9)}{\omega^2(r^2 - 16)}$$

$16(r^2 - 16) = 9(r^2 - 9)$

$r^2 = 25$

r=5m

Amplitude =5m

ii) period put r=5m into one of the equations

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(5^2 - 9)$

$\omega^2 = 1$

ω =1

But $T = \frac{2\pi}{T}$

T=$\frac{2x\frac{22}{7}}{1}$

T=6.28 seconds

iii) frequency = $\frac{1}{T}$

f = $\frac{1}{6.28}$ =0.16Hz

iv) velocity as it passes equilibrium position at equilibrium x=0

$v^2 = \omega^2(r^2 - x^2)$

$v^2 = 1^2(5^2 - 0^2)$

ω^2=25

$v = 5m/s$

3. A body of mass 200g s executing S.H.M with amplitude of 20mm. The maximum force which acts upon it is 0.064N. calculate
   a) its maximum velocity
   b) its period of oscillation

Solution

F= 0.064N

Mass m=200g = 0.2kg

Amplitude r =20mm = 0.02m

a) $v_{max} = \omega r$

But $F = ma_{max}$

0.064=0.2a_{max}

$a_{max} = 0.32m/s^2$

$a_{max} = -\omega^2 r$

0.32= $\omega^2 x$0.02

ω^2=16

ω =4rads^{-1}

$v_{max} = \omega r = 4x0.02$

v_{max} = 0.08ms^{-1}

b) T= $\frac{2\pi}{\omega}$

T= $\frac{2\pi}{4} = \frac{2x22}{4x7}$

T=1.57 seconds

4. A body of mass 0.30kg executes S.H.M with a period of 2.5s and amplitude of 4x10^{-2}m. determine

i) Maximum velocity of the body

ii) The maximum acceleration of the body

Solution

M=0.3kg, T=2.5s, r= $4x10^{-2}$m

vii) $v_{max} = \omega r$

$\omega = \frac{2\pi}{T}$

$$v_{max} = \frac{2\pi}{T}r$$

$$v_{max} = \frac{2x\frac{22}{7}x4x10^{-2}}{2.5}$$

$$v_{max} = 0.101m/s$$

viii) $a_{max} = \omega^2 r$

$$a_{max} = \left(\frac{2\pi}{T}\right)^2 r$$

$$a_{max} = \left(\frac{2x\frac{22}{7}}{2.5}\right)^2 x\ 4x10^{-2}$$

$$a_{max} = 0.25ms^{-2}$$

5. A particle moves with S.H.M in a straight line with amplitude 0.05m and period 12s. Find

   a) speed as it passes equilibrium position

   b) maximum acceleration of the particle

Solution

a)speed at equilibrium

$$v_{max} = \omega r$$

$$v_{max} = \frac{2\pi}{T}r$$

$$v_{max} = \frac{2x\frac{22}{7}x0.05}{12} = 0.026ms^{-1}$$

b) $a_{max} = \omega^2 r$

$$a_{max} = \left(\frac{2\pi}{T}\right)^2 r$$

$$a_{max} = \left(\frac{2x\frac{22}{7}}{12}\right)^2 x\ 0.05$$

$$a_{max} = 0.014ms^{-2}$$

9.2.0: TO SHOW A GIVEN MOTION IS SIMPLE HARMONIC

9.2.0: TO SHOW A GIVEN MOTION IS SIMPLE HARMONIC

This requires to show that a particular motion has an acceleration of the form [$a = -\omega^2 x$.] and find the period (T= $\frac{2\pi}{T}$)

Steps

i) Identify the forces acting on the body at equilibrium position

ii) Identify the forces acting on the body after displacement from the equilibrium position

iii) Obtain an expression for the restoring force after the displacement and equate this restoring

force to [ma] in accordance with Newton's second law of motion.

iv) Compare the expression got with basic standard expression for S.H.M. if it's comparable to

$(a = -\omega^2 x\ )$, then motion is simple harmonic.

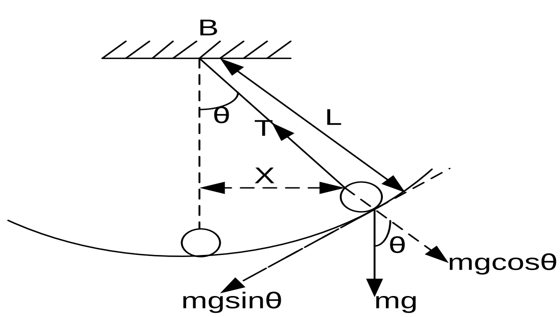
Examples of S.H.M

9.2.1: SIMPLE PENDULUM

Consider a mass m suspended by a light inelastic string of length L from a fixed point B.

At equilibrium the bob lies in a vertical plane with the tension in the string being balanced by the weight of the bob.

If the bob is given a small vertical displacement through an angel θ and released, we show that a bob moves with simple harmonic motion



Resolving tangentially, gives the restoring force

Restoring force $= -mgsin\theta$

By Newton's 2nd law $ma = -mgsin\theta$

$a \ = -gsin\theta$…………………………….........2

If the displacement is small, then θ is very small such that the arc length is equal to x.

$$Sin\theta \approx \theta \approx \frac{x}{l}$$

$$a = -g\theta$$

$$a = -g\frac{x}{l}$$

$$a = -\left(\frac{g}{l}\right)x$$

it is in the form $a = -\omega^2 x$ and hence performs S.H.M with period

$$\omega^2 = \frac{g}{l}$$

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

But $\boldsymbol{\omega} = \frac{2\pi}{T}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

9.2.1: Determination of acceleration due to gravity (g) using a simple pendulum

- Starting with a measured length L of the pendulum, the pendulum is clamped between 2 wood pieces from a retort stand.
- A bob is then given a small angular displacement from the vertical position and released.
- The time t for 20 oscillation is obtained, find period T and henceT^2
- Repeat the procedure for different values of length of the string.
- A graph of T^2 against L is then drawn and its slope S calculated.

Hence acceleration due to gravity is obtained from $g = \frac{4\pi^2}{s}$

Factors which affect the accuracy of g when using a simple pendulum.

1. The nature of the string. The string should be inelastic
2. Air resistance (dissipative force). In present of air the motion of a simple pendulum is highly damped such that the oscillation dies out quickly that affecting the period.

3. The displacement of the bob from the equilibrium position should be small such that the oscillation remain uniform.
4. The mass of the bob should be small to minimize the effect of dimension of the object.
5. In accuracies in the timing and measuring extensions.

Examples ;

A bob of a simple pendulum moves simple harmonically with amplitude 8.0cm and period 2.00s. its mass is 0.50kg, the motion of the bob is un damped. Calculate maximum values of;

a) The speed of the bob, and

b) The kinetic energy of the bob.

Solution

a) m=0.5kg , r=8cm=0.08m, T=2s

$v_{max} = \omega r$

$v_{max} = \frac{2\pi}{T} r = \frac{2x\frac{22}{7}}{2} x0.08$

$v_{max} = 0.25ms^{-1}$

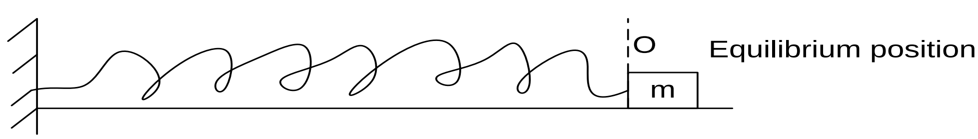
b) $K.E_{max} = ½\, m\, v_{max}^2$

$K.E_{max} = ½\, x0.5x(0.25)^2 = 1.563x10^{-2}J$

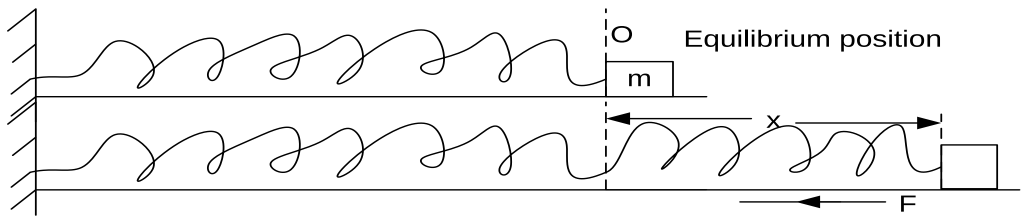
MASS ON A SRING

a) A horizontal spring attached to a mass

Consider a spring lying on a smooth horizontal surface in which one end of the spring is fixed and the other end attached to a particle of mass m



When the mass is slightly pulled a small distance x and the released. The mass executes S.H.M



The restoring force F is given by hooke's law

$F = -kx$-------------------------------------- (1)

By Newton's 2nd law

$F = ma$ -------------------------------------- (2)

$ma = -kx$

a= $-\left(\frac{k}{m}\right)x$-------------------------------- (3)

Where k is the spring constant

Equation (3) is in the form[$a = -\omega^2 x$], therefore the body performs S.H.M

$\therefore \omega^2 = \frac{k}{m}$

=> $\boxed{\omega = \sqrt{\frac{k}{m}}}$

But $\omega = \frac{2\pi}{T}$

$$\frac{4\,\pi^2}{T^2} = \frac{k}{m}$$

$T^2 = \frac{4\,\pi^2 m}{k}$

$$\boxed{T = 2\pi\sqrt{\frac{k}{m}}}$$

Also f = $\frac{1}{T}$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Example : UNEB 2011 No 4C

A horizontal spring of force constant 200 Nm^{-1} is fixed at one end and a mass of 2kg attached to the free end and resting on a smooth horizontal surface. The mass is pulled through a distance of 4.0cm and released. Calculate the;

i) Angular speed

ii) Maximum velocity attained by the vibrating body, acceleration when the body is half way towards the centre from its initial position.

Solution

i) From $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 10$$rads^{-1}$

ii) $v_{max} = \omega r$

$v_{max} = 10x\frac{4}{100}$ =0.4ms^{-1}

Note: the small distance pulled and released becomes the amplitude

$a = -\omega^2 x$

where its half towards the centre

$x = \frac{r}{2}$

$x = \frac{4x10^{-2}}{2}$

$a = -\omega^2 x = 10^2 x \frac{4x10^{-2}}{2} = 2ms^{-2}$

Alternatively

$F = ma$

$F = kx$

$k\frac{r}{2} = ma$

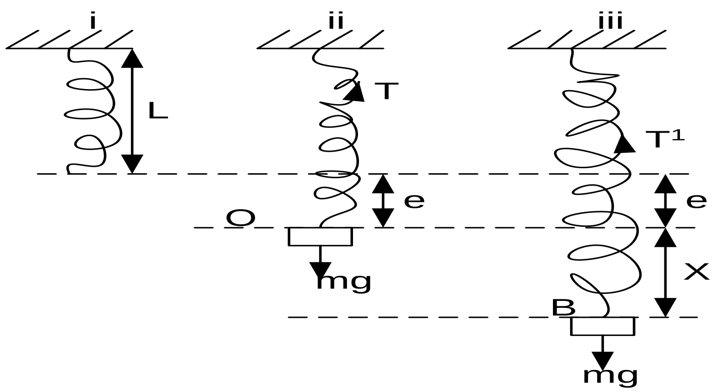
$a = \frac{200x4x10^{-2}}{2x2} = 2ms^{-2}$

b) Oscillation of mass suspended on a helical spring

Consider a helical spring or elastic string suspended from a fixed point.

When a mass is attached to the spring, it stretches by length, e and comes to equilibrium positions 0.

When the mass is pulled down a small distance, x and released the motion will be simple harmonic.



In position (ii) the mass is in equilibrium position

$T = mg$

And by hooke's law $T = ke$

$mg = ke$ -------------------------(1)

In position (iii) after displacement through x

The restoring fore is $= mg - T^1$

But by hooke's law $T^1 = k(e + x)$

Restoring force $= mg - k(e + x)$

By Newton's 2nd law $mg - k(e + x) = ma$

But from equation 1 $mg = ke$

$ke - k(e + x) = ma$

$ke - ke - kx = ma$

$-kx = ma$

$a = -\frac{k}{m}x$ -------------------------------- [3]

Equation 3 is in the form $a = -\omega^2 x$ and therefore performs S.H.M

$\omega^2 = \frac{k}{m}$

$\boxed{\omega = \sqrt{\frac{k}{m}}}$ ---------------------------------- - [4]

But $\omega = \frac{2\pi}{T}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Note:

From [1] $mg = ke$

$$\frac{k}{m} = \frac{g}{e}$$

$$\omega = \sqrt{\frac{g}{e}}$$

$$T = 2\pi\sqrt{\frac{e}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

9.2.2: Determination of acceleration due to gravity using a vertically loaded spring

- Clamp a spring with pointer vertically besides a meter rule, note and record the initial pointer reading;P_o
- Suspend a known mass on the spring and note and record the new reading P of the pointer on the meter rule
- The extension e of the spring is obtained from e= $P - P_0$
- Give the mas a small vertical displacement and obtain the time t for 20 oscillations. Find the period T and T^2
- Repeat the procedure for different values of the masses.
- Plot a graph of $\mathbf{T^2}$ against **e** and find the slope, s of the graph

Hence acceleration due to gravity is determined from $g = \frac{4\pi^2}{s}$

Examples

1. A 100g mass is suspended vertically from a light helical spring and the extension in equilibrium is found to be 10cm. The mass is now pulled down a further 0.5cm and it is released from rest.
    i) Show that the subsequent motion is simple harmonic
    ii) Find the period of oscillation
    iii) What is the maximum kinetic energy of the mass

Solution

$m = 100g \ = 0.1kg,$

$e = 10cm = 0.1m\,,$

$r = 0.5cm = 0.005m$

From $\omega = \sqrt{\frac{k}{m}}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

But also $mg = ke$

Therefore $\frac{k}{m} = \frac{g}{e}$

$$T = 2\pi\sqrt{\frac{e}{g}}$$

$$T = 2\pi\sqrt{\frac{0.1}{9.81}} = 0.63s$$

$$v_{max} = \omega r$$

$$v_{max} = \frac{2\pi}{T}r = \frac{2x\frac{22}{7}}{0.63}x0.05$$

$v_{max} = 0.0499ms^{-1}$

$K.E_{max} = ½\, m\, v_{max}^2$

$= ½\, x0.1x(0.0499)^2$

$K.E_{max} = 1.245x10^{-4}J$

2. A mass hangs from a light spring. The mss is pulled down 30mm from its equilibrium position and then released from rest. The frequency of oscillation is 0.5Hz. calculate
   a) The angular frequency, ω of the osculation
   b) The magnitude of the acceleration at the instant it is released from rest

Solution

Distance pulled down ward and released becomes the amplitude

$\therefore r = 30mm = 30x10^{-3}m$

$f = 0.5Hz$

a) Angular frequency ω

$\omega = 2\pi f$

$\omega = 2\pi x0.5$

$\omega = \ \pi\, rad\, s^{-1}$

$\omega = 3.14rad\ s^{-1}$

b) When it is released from rest the displacement is equals to amplitude and the acceleration is maximum.

$a_{max} = \omega^2 r$

$a_{max} = (3.14)^2 x30x10^{-3}$

$a_{max} = 0.296ms^{-2}$

Exercise:23

1. When a metal cylinder of mass 0.2kg is attached to the lower end of a light helical spring, the upper end of which is fixed, the spring extends by 0.16m. the metal cylinder is then pulled down a further 0.08m.
   i) Find the force that must be exerted to keep it there. **An [1.0N]**
   ii) The cylinder is then released. Find the period of vertical oscillation and the kinetic energy the cylinder posses when it passes through its mean position. **An[0.79s, 0.04J]**

2. A mass of 0.2kg is attached to the lower end of a helical spring and produces extension of 5.0cm. The mass is now pulled down at a further distance and released. Calculate
   a) the force constant of the spring
   b) The period of the subsequent motion
   c) The maximum value of the acceleration during the motion

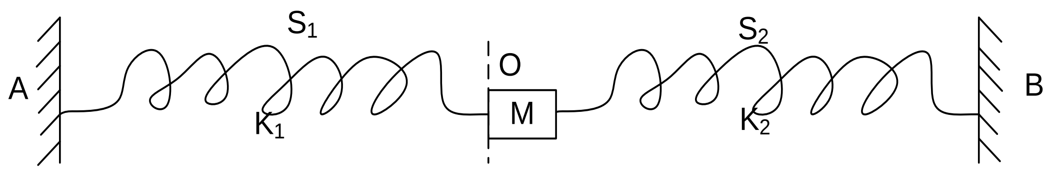
   An[39.24Nm^{-1}, 0.45s, 3.924ms^{-2}]

COMBINED SPRINGS

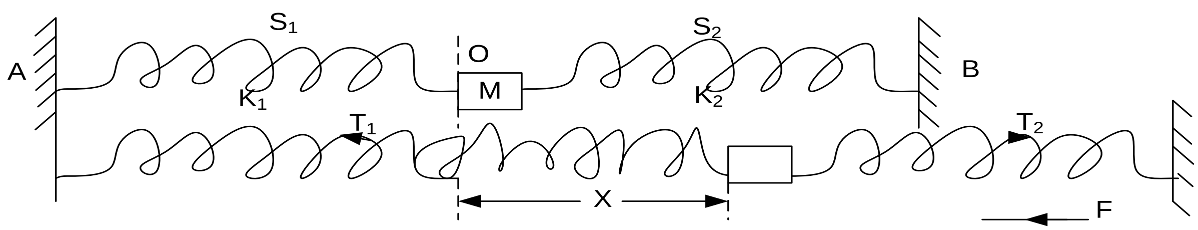
a) horizontal springs

9.2.3: TWO HORIZONTAL SPRINGS WITH A MASS BETWEEN THEM

Consider two springs with spring constants K_1, and K_2 attached to fixed points and mass attached between them.



Show that when the mass is displaced horizontally towards one side the resultant motion is S.H.M



Extension of $S_1 = x$

Compression of $S_2 = x$

Restoring force $F = -(T_1 + T_2)$

But by Hooke's law

$T_1 = k_1 x \text{ and } T_2 = k_2 x$

$F = -(k_1 x + k_2 x)$

$F = -(k_1 + k_2)x$ -------------------------- (1)

By Newton's 2nd law

$F = ma$ ------------------------------------- (2)

$ma = -(k_1 + k_2)x$

$a = -\left(\frac{k_1 + k_2}{m}\right)x$ ---------------------------

-- (3)

Equation 3 in the form $a = -\omega^2 x$ and therefore it performs S.H.M

$$\omega^2 = \left(\frac{k_1 + k_2}{m}\right)$$

$\boxed{\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$ ---------------------------

-- (4)

But $\omega = \frac{2\pi}{T}$

$$\frac{2\pi}{T} = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

$$T = \frac{2\pi}{\sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$

$$\boxed{T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}}$$

$$f = \frac{1}{T}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$

Note: when the springs are identical

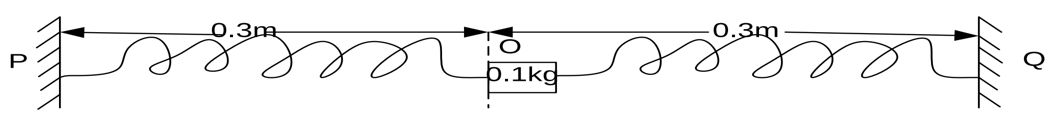
$k_1 = k_2 = k$

$$T = 2\pi\sqrt{\left(\frac{m}{2k}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{2k}{m}\right)}$$

Example

1. A mass of 0.1kg is placed on a frictionless horizontal surface and connected to two identical springs of negligible mass and a spring constant of 33.5Nm^{-1}. The springs are then attacked to fixed point p and Q on the surface as shown below.



The mass is given a small displacement along the line of the spring and released

i) Show that the system will execute S.H.M

ii) Calculate the period of oscillation

iii) If the amplitude of oscillation is 0.05m, calculate the maximum kinetic of the system.

Solution

ii) From $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

$$T = 2x\frac{22}{7}\sqrt{\left(\frac{0.1}{33.5 + 33.5}\right)}$$

$T = 0.243s$

iii) r=0.05m

$v_{max} = \omega r$

$v_{max} = \frac{2\pi}{T} r$

$= \frac{2x\frac{22}{7}}{0.243} x0.05$

$v_{max} = 1.293ms^{-1}$

$K.E_{max} = ½\, m\, v_{max}^2$

= ½ $x0.1x(1.293)^2$

$K.E_{max}$ =0.084J

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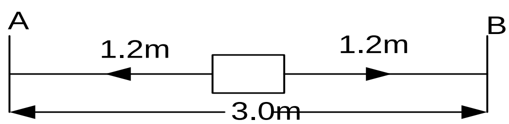
2. A body of mass 4kg rests on a smooth horizontal surface. Attached to the body are two pieces of light elastic strings each of length of 1.2m and force constant 6.25Nm^{-1}. The ends are fixed to two points A and B 3.0m apart as shown in the figure below. The body is then pulled through 0.1m towards B and then released.
   i) Show that the body executes S.H.M
   ii) Find the period of oscillation of the body
   iii) Calculate the speed of the body when it is 0.03m from the equilibrium position

Solution



ii) From $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

$$T = 2x\frac{22}{7}\sqrt{\left(\frac{4}{6.25 + 6.25}\right)}$$

T= 3.55s

iii) $v^2 = \omega^2(r^2 - x^2)$

Amplitude $r = 0.1\, m$

$x = 0.03m$

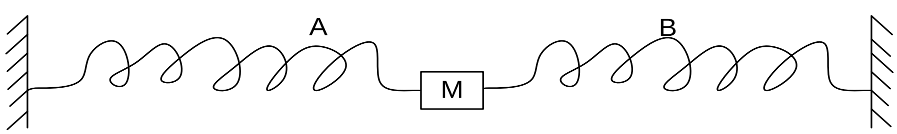
$\omega = \frac{2\pi}{T}$

$v^2 = \left(\frac{2\pi}{T}\right)^2 (r^2 - x^2)$

$v^2 = \frac{4x3.14^2}{3.55^2}(0.1^2 - 0.03^2)$

$v = 0.169m/s$

3. The figure below shows a mass of 200g resting on a smooth horizontal table, attached to two springs A and B of force constants k_1 and k_2 respectively



The block is pulled through a distance of 8cm to the right and then released.

(i) Show that the mass oscillates with simple harmonic motion and find the frequency of oscillation if $k_1 = 120Nm^{-1}$ and $k_2 = 200Nm^{-1}$

(ii) Find the new amplitude of oscillation when a mass of 120g is dropped vertically onto the block as the block passes the equilibrium position. Assume that the mass sticks to the block

Solution

i) From $f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$

$$f = {}^{1}\!\Big/\!\left(2x\frac{22}{7}\right)\sqrt{\left(\frac{60 + 100}{0.2}\right)}$$

$$f = 6.37Hz$$

ii) By conservation of momentum:

$$m_1 u = (m_1 + m_2)v_{max}$$

$$v_{max} = \frac{m_1 u}{(m_1 + m_2)}$$

$v_{max} = \omega^1 r^1$ and $u_{wax} = \omega r$

$$\omega^1 = \sqrt{\left(\frac{k_1 + k_2}{m}\right)} = \sqrt{\left(\frac{120+ 200}{0.32}\right)} = 10\sqrt{10}rads^{-1}$$

$$\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)} = \sqrt{\left(\frac{120+ 200}{0.2}\right)} = 40rads^{-1}$$

$$u_{wax} = \omega r = 40x0.08 = 3.2m/s$$

$$\therefore\ v_{max} = \frac{m_1 u}{(m_1 + m_2)}$$

$$10\sqrt{10}\,r^1 = \frac{0.2x3.2}{(0.2 + 0.12)}$$

$$r^1 = 0.632m$$

EXERCISE:24

A block of mass 0.1kg resting on a smooth horizontal surface and attached to two springs s_1 and s_2 of force constant 60Nm^{-1} and 100Nm^{-1}respectively. The block is pulled a distance of $4x10^{-2}$m to the right and the released.

i) Show that the mass executes S.H.M and fixed the frequency of oscillation
ii) Find the new amplitude of oscillation when the block is added a mass of 0.06kg on top as the

block passes the equilibrium position. **An (6.4 Hz, 0.032m)**

b) Vertical springs

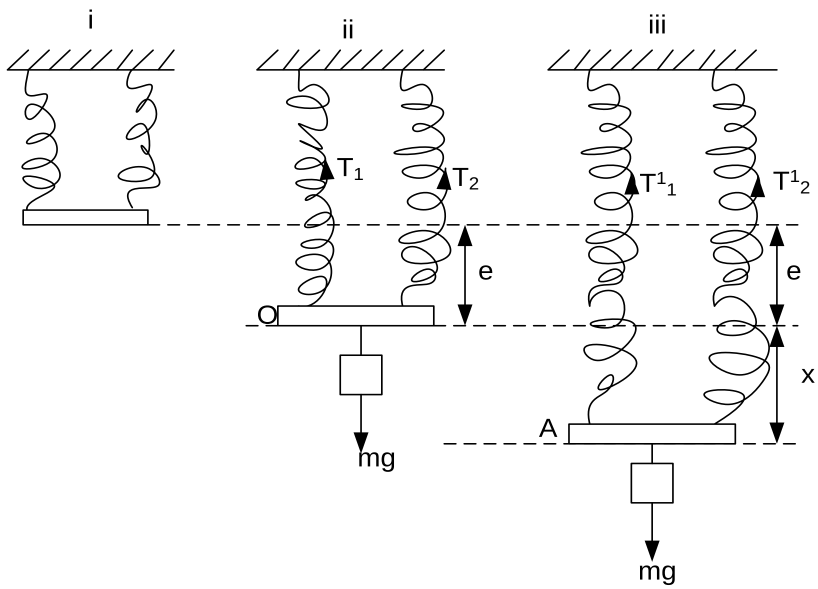
9.2.4: Vertically loaded springs in parallel

Consider two springs of force constants k_1 and k_2 suspended from the same rigid support side by side.

When a mass is attached to the mid point of a rod connected to the lower ends of the springs.

The system rests in equilibrium

When the mass is displaced a small distance vertically downwards and then released the system execute S.H.M



At equilibrium : $Mg \ = T_1 + T_2$

But by Hooked law $T_1 = k_1 e$ and$T_2 = k_2 e$

$mg = (\ k_1 + k_2)$ e------------------------

[1]

When the mass is displaced then:

Restoring force $= mg - (T_1{}^1 + T_2{}^1)$

But by Hooke's law

$T_1{}^1 = k_1(e + x)$ and $T_2{}^1 = k_2(e + x)$

Restoring force $= \ mg - [\ k_1(e + x) + \ \ k_2(e + x)]$

By Newton's second law Restoring force

$= ma$

$mg - [\ k_1(e + x) + \ \ k_2(e + x)] = ma$ --

(2)

But from equation 1 $mg = (\ k_1 + k_2)e$

$(k_1 + k_2)e - [\ k_1(e + x) + \ k_2(e + x)] = ma$

$-\ k_1 x - k_2 x = ma$

$-(\ k_1 x + k_2 x) \ = ma$

$a = \ -\left(\frac{k_1 + k_2}{m}\right)x$ ------------------------

(3)

Equation 3 is in the form $a = -\omega^2 x$ and

therefore performs S.H.M

$\omega^2 = \left(\frac{k_1 + \ k_2}{m}\right)$

$$\boxed{\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}} \text{------------------------- (4)}$$

T=$\frac{2\pi}{\omega}$

Period T=$2\pi\sqrt{\left(\frac{m}{k_1+k_2}\right)}$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

Note: From equation 1

$$mg = (k_1 + k_2)e$$

$$\frac{m}{(k_1+k_2)} = \frac{e}{g}$$

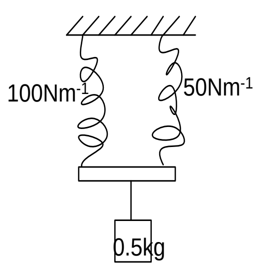
$$\omega = \sqrt{\frac{g}{e}}$$

T=$2\pi\sqrt{\left(\frac{e}{g}\right)}$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

Examples

1. A mass of 0.5kg is suspended from the free ends of two springs of force constant 100Nm^{-1} and 50Nm^{-1} respectively as shown in the figure below.



Calculate ;

i) The extension produced

ii) Tension in each string

iii) Energy stored in the string

iv) Frequency of small oscillations when he mass is given a small vertical displacement

Solution

i) At equilibrium $mg = (k_1 + k_2)$ e

$$e = \frac{mg}{k_1 + k_2} = \frac{0.5x9.81}{100 + 50} = 0.0327m$$

ii) Tension in each string

From Hooke's law $T_1 = k_1 e$

$$T_1 = 100x0.0327 = 3.27N$$

Also $T_2 = k_2 e = 50x0.0327 = 1.635N$

iii) Energy stored is always stored as elastic potential energy of the spring

$$P.E_{Elatic} = \frac{1}{2}ke^2$$

$E_1 = \frac{1}{2}k_1e^2 = \frac{1}{2}x\ 100\ x\ (0.0327)^2$ =0.0535J

$E_2 = \frac{1}{2}k_2e^2 =$

$\frac{1}{2}x\ 50\ x\ (0.0327)^2$ =0.0267J

$$P.E_{Elatic} = E_1 + E_2$$

$P.E_{Elatic}$ = 0.0535+ 0.026

$$P.E_{Elatic} = 0.0802J$$

iv) Frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{9.81}{0.0327}} = 2.757Hz$$

Alternatively

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

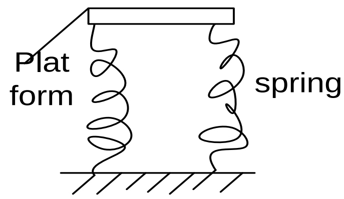
$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\left(\frac{100 + 50}{0.5}\right)} = 2.757Hz$$

2. A light platform is supported by tow identical springs each having spring constants

20Nm^{-1} as shown below.

2. A light platform is supported by tow identical springs each having spring constants

20Nm^{-1} as shown below.



a) Calculate the weight which must be placed on the centre of the platform in order to produce a displacement of 3.0cm.

b) The weight remains on the platform and the platform is depressed a further 1.0cm and then released

   i) What is the frequency of the oscillation

   ii) What is the maximum acceleration of the platform

Solution

a) Compression $e = 3.0cm \ = 0.03m$

At equilibrium

$mg = \ T_1 + T_2$

$mg = (\ k_1 + k_2)$ e

$mg = \ (20 + 20)x0.03$

$mg = 1.2N$

$weight \ = 1.2N$

b) Amplitude r=1.0cm=0.01m

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{9.81}{0.03}}$$

$f = 2.89Hz$

$a_{max} = \omega^2 r$

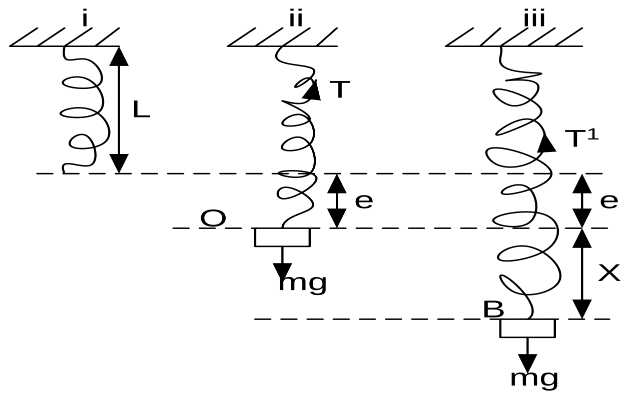
$a_{max} = (2\pi f)^2 r$

$$a_{max} = \left(2x\frac{22}{7}x2.89\right)^2 x0.01$$

$a_{max} = 3.297ms^{-2}$

9.2.5: Vertically loaded spring in series

Consider two springs of constants k_1 and k_2 suspended in series, mass m is then attached to the lower end of the last spring such that at equilibrium each spring extends by e_1 and e_2 respectively.



The springs are assumed to be light such that they have the same tension

Let e be extension in the combination

At equilibrium $mg = T$

$mg = ke$ ---------------------------------- (1)

Where k is the combined spring constant

$k = \frac{k_1 k_2}{k_1 + k_2}$ for series connection

After a small displacement, the restoring force

$mg - T^1 = ma$

$mg - k(e + x) = ma$

$mg - ke - kx = ma$ -------------------- [2]

but from equation[1] $mg = ke$

$ke - ke - kx = ma$

$-kx = ma$

$a = -\left(\frac{k}{m}\right)x$------------------------------- -[3]

it is in the form $a = -\omega^2 x$

$\therefore\ \omega^2 = \frac{k}{m}$

But $k = \frac{k_1 k_2}{k_1 + k_2}$

$$\omega^2 = \frac{k_1 k_2}{k_1 + k_2}\Big/ m$$

$\boxed{\omega = \sqrt{\frac{k_1 k_2}{k_1 + k_2}\Big/ m}}$------------------------ [4]

$$T = 2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1 k_2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{k_1 + k_2}\Big/ m}$$

Note

The tension is the same in both springs

$$mg = T$$
$$mg = ke$$
$\therefore mg = k_1e_1$ and $mg = k_2e_2$

$e_1 = \frac{mg}{k_1}$ and $e_2 = \frac{mg}{k_2}$

but $e = e_1 + e_2$

$$e = \frac{mg}{k_1} + \frac{mg}{k_2}$$
$$e = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$
$$e = mg\left(\frac{k_1 + k_2}{k_1k_2}\right)$$
$$e\left(\frac{k_1k_2}{k_1 + k_2}\right) = mg$$
$$\therefore ek = mg$$
$$mg = ke$$
$\therefore k = \frac{k_1k_2}{k_1 + k_2}$

Also

$$\omega = \sqrt{\frac{g}{e}}$$
$$T = 2\pi\sqrt{\left(\frac{e}{g}\right)}$$
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

Example

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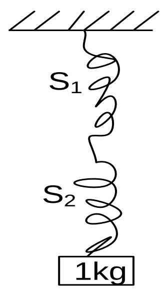
A mass of 1.0kg is hung from two springs S_1 and S_2 connected in series as shown

The force constant of the springs are 100Nm^{-1} and 200 Nm^{-1} respectively. Find

i) The extension produced in the combination

ii) The frequency of oscillation of the mass if it is pulled downwards and released



Solution

m=1kg , k_1=100Nm^{-1}, k_2=200 Nm^{-1}

At equilibrium $mg = ke$

$e = \frac{mg}{k}$ but $k = \frac{k_1k_2}{k_1 + k_2}$

$$e = mg\Big/\frac{k_1k_2}{k_1 + k_2}$$
$$e = 1x9.81\Big/\left(\frac{100x200}{100+200}\right)$$
$$e = 0.1472m$$

ii)

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{k_1 + k_2}\Big/m}$$
$$f = \frac{1}{2\pi}\sqrt{\left(\frac{100x200}{100 + 200}\right)\Big/1}$$

f=1.299Hz

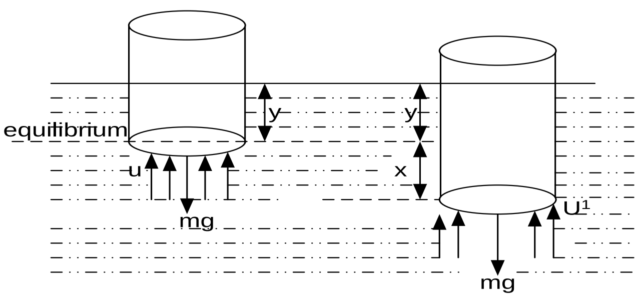
NB: For all S.H.M, the following assumptions hold

1- displacement from equilibrium position is small such that Hooke's law is obeyed throughout the motion

2- no dissipative forces act

9.2.6: S.H.M OF A FLOATING CYLINDER

Consider a uniform cylindrical rod of length L and cross sectional area A and density, ρ floating vertically in a liquid of density, δ. When the rod is given a small downward displacement x and released, the rod executes S.H.M.



At equilibrium, up thrust of the liquid on the rod is equal to the weight of the displaced fluid

U =weight of the liquid is displaced

U =mass of liquid displaced x g

U = volume of liquid displaced x density x g

$U = Ay\delta g$ -- [1]

When the rod is given a downward displaced x, the new up thrust is U^1

U^1 =weight of the liquid is displaced

=mass of liquid displaced x g

U^1 = volume of liquid displaced x density x g

$U^1 = A(y+x)\delta g$ ----------------------------- [2]

On release, the restoring force on the rod is $U - U^1 = ma$

$Ay\delta g - A(y+x)\delta g = ma$

$-A\delta g\, x = ma$

$a = -\left(\frac{A\delta g}{m}\right)x$

m is mass of cylinder =volume of cylinder x density of cylinder

$m = Al\rho$

$a = -\left(\frac{A\delta g}{A\,l\,\rho}\right)x$

$a = -\left(\frac{\delta g}{l\,\rho}\right)x$-------------------------- [3]

it is the form $a = -\omega^2 x$

$\omega^2 = \frac{\delta g}{l\,\rho}$

$\boxed{\omega = \sqrt{\frac{\delta g}{l\,\rho}}}$---------------------------- [4]

T=$2\pi\sqrt{\left(\frac{\rho l}{\delta g}\right)}$

$$f = \frac{1}{2\pi}\sqrt{\frac{\delta g}{l\,\rho}}$$

Examples : UNEB 2000 No2b

1. A Uniform cylindrical rod of length 8cm, cross sectional area 0.02m^2 and density 900kgm^{-3} floats vertically in a liquid of density 1000kgm^{-3}. The rod is depressed through a distance of 0.005m and then released.

i) Show that the rod performs S.H.M (5mk)

ii) Find the frequency of the resultant oscillation (4mk)

iii) Find the velocity of the rod when it is at a distance of 0.004m above the equilibrium position

Solution

ii) $f = \frac{1}{2\pi}\sqrt{\frac{\delta g}{l\,\rho}}$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{1000x9.81}{8x10^{-2}x900}}$$

$f = 1.858Hz$

iii) $v^2 = \omega^2(r^2 - x^2)$

r=0.005m , $x = 0.004m$, $\omega = 2\pi f$

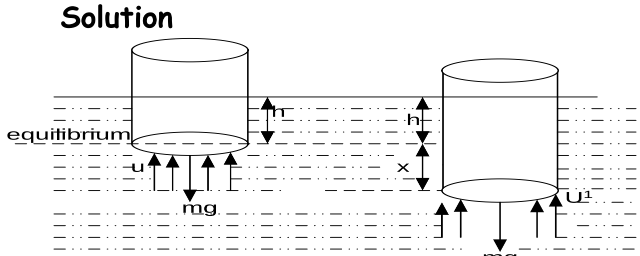
$v^2 = (2\pi f)^2(r^2 - x^2)$

$$v^2 = \left(2x\frac{22}{7}x1.858\right)^2(0.005^2 - 0.004^2)$$

$v = 3.5x10^{-2}ms^{-1}$

2. A wooden rod of uniform cross sectional area A floats with a height h immersed in a liquid of densityδ. The rod is given a slight downward displacement and released. Show that the resulting motion is S.H.M with a time period of $2\pi\sqrt{\frac{h}{g}}$

Solution



At equilibrium, upthrust of the liquid on the rod is equal to the weight of the displaced fluid

U =weight of the liquid is displaced

=mass of liquid displaced x g

U = volume of liquid displaced x density x g

$U = Ah\delta g$ ----------------------------------- [1]

When the rod is given a downward displaced x, the new up thrust is U^1

U^1 =weight of the liquid is displaced

=mass of liquid displaced x g

$U^1 =$ volume of liquid displaced x density x g

$U^1 = A(h+x)\delta g$ -------------------------- --- [2]

On release, the restoring force on the rod is $U - U^1 = ma$

$Ah\delta g - A(h+x)\delta g = ma$

$-A\delta g\, x = ma$

$a = -\left(\frac{A\delta g}{m}\right)x$

m is mass of cylinder =volume of cylinder x density of cylinder

$m = Al\delta$

$a = -\left(\frac{A\delta g}{A\,h\,\delta}\right)x$

$a = -\left(\frac{g}{h}\right)x$-------------------------- [3]

it is the form $a = -\omega^2 x$

$\omega^2 = \frac{g}{h}$

$\boxed{\omega = \sqrt{\frac{g}{h}}}$---------------------------- [4]

$T = \frac{2\pi}{\omega}$

T=$2\pi\sqrt{\left(\frac{h}{g}\right)}$

Example

A cylindrical test tube of thin wall and mass 1kg with a piece of lead of mass 1kg fixed at its inside bottom floats vertically in the liquid.

When the test tube is slightly depressed and released it oscillates vertically with a period of one second ($T = 1s$).

If some extra copper beads are put in the test tube, it floats vertically with a period of 1.5 seconds. Find the mass of the copper beads in the test tube.

Solution

T=$2\pi\sqrt{\left(\frac{h}{g}\right)}$

1=$2\pi\sqrt{\left(\frac{h_1}{9.81}\right)}$

$h_1 = \frac{1^2 x 9.81}{4\pi^2}$ =0.2485m

Also 1.5=$2\pi\sqrt{\left(\frac{h_2}{9.81}\right)}$

$h_2 = \frac{1.5^2 x 9.81}{4\pi^2}$ =0.5591m

at equilibrium U =weight of liquid displaced

$2g = A\, h_1 \delta\, g$

$2 = A\, h_1 \delta$ ------------------------------- (1)

Also when a mass m is added

$(2+m)g = A\, h_2 \delta\, g$

$(2+m) = A\, h_2 \delta$ --------------------- (2)

Equation 2 divided by equation 1

$$\frac{(2+m)}{2} = \frac{A\,h_2\delta}{A\,h_1\delta}$$

$$\frac{(2+m)}{2} = \frac{h_2}{h_1}$$

$$m = \frac{2\,x\,h_2}{h_1} - 2$$

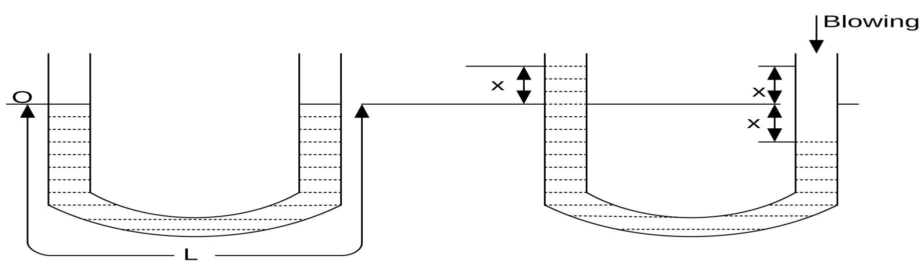
$$m = \frac{2\,x\,0.5591}{0.2485} - 2$$

$$m = 2.5kg$$

9.2.7: A LIQUID OSCILLATING IN A U-TUBE

Consider a column of liquid of density σ and total length l in a U-tube of uniform cross sectional area A.

Suppose the level of the liquid on the right side is depressed by blowing gently down that side, the levels of liquid will oscillate for a short time about their respective or equilibrium positions O.



When the meniscus is at a distance, x , from equilibrium position, a differential height of liquid of, $2x$, is produced

Excess pressure on liquid $= 2x\delta g$ from $[p = h\delta g]$

Force on liquid =pressure x Area $= 2x\delta gA$

Restoring force $= -2x\delta gA$ ----------------------------------[1]

Newton's 2nd law : $ma = -2x\delta gA$

$a = -\left(\frac{2\delta gA}{m}\right)x$ ---[2]

But mass of liquid in the tube =volume of liquid x $\delta = Al\delta$

$$a = -\left(\frac{2\delta gA}{A\,l\,\delta}\right)x$$

$a = -\left(\frac{2g}{l}\right)x$-- [3]

it is in the form $a = -\omega^2 x$

$$\omega^2 = \frac{2\,g}{l}$$

$$\boxed{\omega = \sqrt{\frac{2\,g}{l}}}$$

$T\ = \frac{2\pi}{\omega}$

$$\boxed{T = \ 2\pi\sqrt{\frac{l}{2\,g}}}$$

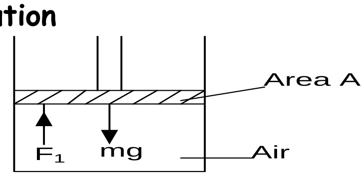
9.2.8:S.H.M IN A FRICTIONLESS AIR TIGHT PISTON

A volume v of air and pressure p is contained in a cylindrical vessel of cross section area A by frictionless air tight piston of mass m.

Show that on slight forcing down the piston and then releasing it, the piston will exert S.H.M given by

$$T = \frac{2\,\pi}{A}\sqrt{\frac{m\,v}{P}}$$

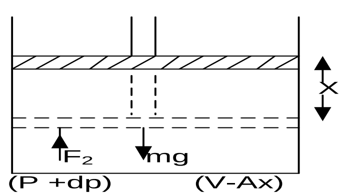
Solution



At Equilibrium

$F_1 = PA$

$PA = mg$ ------------------------------- [1]



When the piston is given a slight downward displaced x, the volume decrease to $[v - dv]$

where $dv = Ax$

$F_2 = \ (P + dp)A$

On releasing, the restoring force

$= (P + dp)A - mg$

But by Bewton's 2nd law

$ma = -[\ (P + dp)A - mg]$

$ma = -[\ PA + Adp - mg]$

from Equation 1 $PA = mg$

$ma = -[mg + Adp - mg]$

$ma = \ -Adp$ --------------------------- [2]

If the displacement x is small, the compression will be almost isothermal obeying Boyle's law. $[P_1V_1 = P_2V_2]$

$(P + dp)(v - Ax) = Pv$

$Pv - PAx + vdp - Axdp = Pv$

For small displacement $Axdp \approx 0$

$Pv - PAx + vdp - 0 = Pv$

$PAx = vdp$

$dp = \frac{PAx}{v}$

put into equation 2

$$ma = -A\left(\frac{PAx}{v}\right)$$

$$a = -\left(\frac{PA^2}{m\,v}.\right)x$$

it is in the form $a = -\omega^2 x$

$$\omega^2 = \frac{PA^2}{m\,v}.$$

$$\omega = \sqrt{\frac{PA^2}{m\,v}.}$$

$$\omega = A\sqrt{\frac{P}{m\,v}.}$$

But $T = \frac{2\pi}{\omega}$

T=$\frac{2\pi}{A}\sqrt{\left(\frac{mv}{p}\right)}$

$f = \frac{1}{T}$

$$f = \frac{\boldsymbol{A}}{\boldsymbol{2\pi}}\sqrt{\frac{\boldsymbol{P}}{\boldsymbol{m\,v}}}$$

Example

A piston in a car engine performs S.H.M. The piston has a mass of 0.50kg and its amplitude of vibration is 45mm. the revolution counter in the car reads 750 revolutions per minute. Calculate the maximum force on the piston.

Solution

$r = 45mm = 45x10^{-3}m, \; m = 0.5kg$

$$f = 750\,rev/min$$

$$f = \frac{750}{60} = 12.5rev/s$$

But $a_{max} = \omega^2 r$

$\omega = 2\pi f$

$$a_{max} = (2\pi f)^2 r$$

$$a_{max} = \left(2x\frac{22}{7}x12.5\right)^2 x12.5$$

a_{max} =277.583ms^{-2}

$$F_{max} = ma_{max}$$

$$F_{max} = 0.5x277.583$$

$$F_{max} = 138.792N$$

9.3.0: ENERGY CHANGES IN S.H.M

- In S.H.M there's always an energy exchange. At maximum displacement , all the energy is elastic potential energy while at equilibrium point all the energy is kinetic energy

a) Kinetic energy

It's the energy possessed by a body due to its motion

K.E = ½ mv^2

K.E = ½ m$\omega^2(r^2 - x^2)$

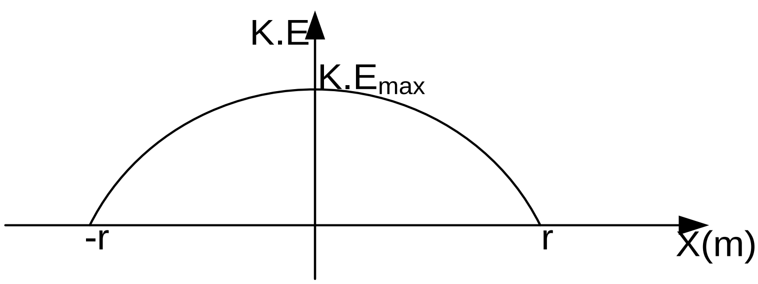
Note

i) The K.E is zero when the displacement x is equals to the amplitude

ii) K.E is maximum when the displacement x is zero

K. $E_{max} = \frac{1}{2}$ m$\omega^2 r^2$

9.3.1: A graph of K.E against displacement



b) Elastic potential energy

This is the energy possessed by a body due to the nature of its particle i.e. compressed or stretched.

Force is applied to make particles stretch or compress and therefore the force does work, which work is stored in the body.

$\Delta w = F\Delta x$

But $F = kx$

$\Delta w = k\Delta x$

Total work done $\int_0^w dw = \int_0^x kx\, dx$

$w = \left[\frac{kx^2}{2}\right]_0^x$

$w = \frac{kx^2}{2}$

Elastic potential energy $= \frac{1}{2}kx^2$

Or

$\Delta w = F\Delta x$

But F= mω^2x

$\Delta w = m\omega^2 x\Delta x$

$$\int_0^w dw = \int_0^x m\omega^2 x\, dx$$

$W = ½\, m\omega^2 x^2$

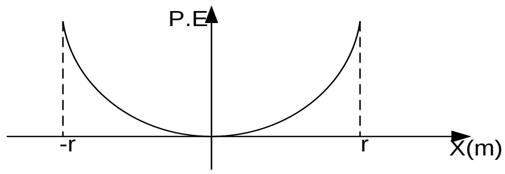
Elastic potential energy = ½ $m\omega^2 x^2$

Note :

i) Elastic potential energy is maximum when x is a maximum

ii) Elastic potential energy is zero when x=0 (equilibrium)

9.3.2: Graph of P.E against displacement



iii) Mechanical energy

This is the total energy possessed by a body due its motion and nature of its particles

$M.E = K.E + P.E$

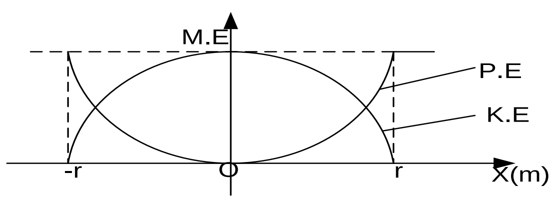
$= \frac{1}{2}\, m\omega^2(r^2 - x^2) + ½\, m\omega^2 x^2$

M.E = $½\, m\omega^2 r^2$

Note

Mechanical energy is constant

9.3.3: A graph of M.E against displacement



9.4.0: MECHANICAL OSCILLATION

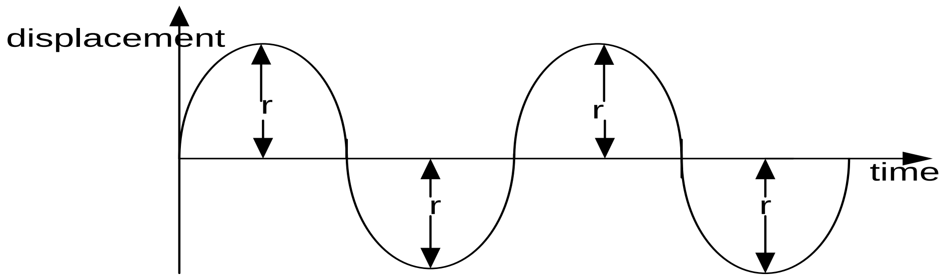
There are three types of oscillation i.e.

a) Free oscillation
b) Damped oscillation
c) Forced oscillation

a) Free oscillations

These are oscillations in which the amplitude remains constant and oscillating systems does not do work against dissipative force such as air friction, and viscous drag. Eg a pendulum bob in a vacuum

Displacement- time graph



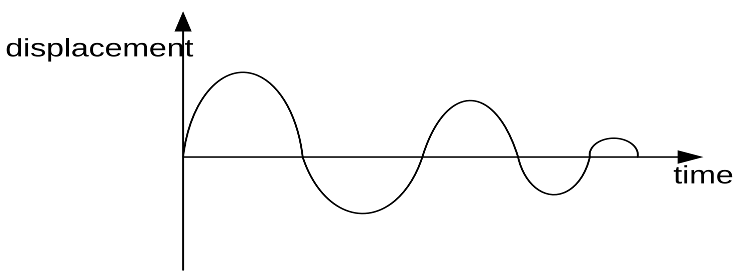
b) Damped oscillations

These are oscillations in which energy is lost and amplitude keeps on decreasing until it dies away due to dissipative forces.

Types of damped oscillations

i) Under damped/slightly damped/lightly damped oscillations

Is when energy is lost and amplitude gradually decreases until oscillation dies away.

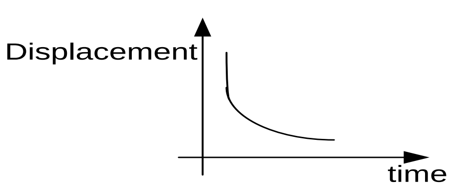


Examples

- ❖ Mass oscillating at the end of the spring oscillating in air
- ❖ Simple pendulum oscillating in air

ii)Over damped/highly damped/heavily damped

Is when a system does not oscillate when displaced but takes a very long time to return to equilibrium position.

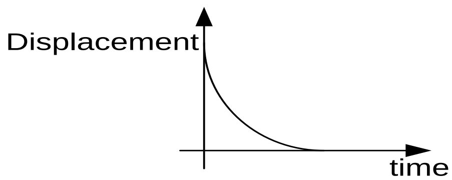


Example

- ❖ A horizontal spring with a mass on a rough surface

iii) Critically damped oscillations

Is when a system does not oscillate when displaced and returns to equilibrium position in a short time.



Example

❖ Shock absorber in a car

C) FORCED OSCILLATIONS

These are vibrations caused by an external force and the system oscillates at the same frequency as the vibrating force.

Example

❖ Oscillation of a guitar string

❖ Oscillation of a building during an earthquake

❖ Oscillation of air column in a musical pipe

UNEB 2013 No4

(b) Explain **Brownian motion** (03marks)

(c) Explain the energy changes which occur when a pendulum is set into motion (03marks)

An[p.e to k.e to p.e]

(d) A simple pendulum of length 1 m has a bob of mass 100g. It is displaced from its mean Position A to a position B so that the string makes an angle of 45° with the vertical. Calculate the ;

(i) Maximum potential energy of the bob (03marks)

(ii) Velocity of the bob when the string makes angle of 30° with the vertical. [Neglect air resistance]

(04marks)

Solution

i) $P.e\ = mgh$

$= mg(l - lCos\theta)$

$= 0.1x9.81(1 - 1cos45)$

$P.e\ = 0.287J$

ii) By law of conservation of energy

$K.e\ = P.e$

$\frac{1}{2}mv^2 = mgh$

$\frac{1}{2}mv^2 = mg(l - lCos\theta)$

$\boldsymbol{v} = \sqrt{2g(l - lCos\theta)}$

$\boldsymbol{v} = \sqrt{2x9.81(1 - 1Cos30)}$

$v = 0.164ms^{-1}$

UNEB 2012 No 2

a) Define the following terms as applied to oscillating motion

i) Amplitude [1mk]

ii) Period [1mk]

b) State four characteristics of simple harmonic motion [2mk]

c) A mass m, is suspended from a rigid support by a string of length, l. the mass is pulled a side so that the string makes an angle, θ with the vertical and then released.

i) Show that the mass executes simple harmonic motion with a period, $T = 2\pi\sqrt{\frac{l}{g}}$ [05mk]

ii) Explain why this mass comes to a stop [02mk]

d) A piston in a car engine performs simple harmonic motion of frequency 12.5Hz. If the mass of the piston is 0.50kg and its amplitude of vibration is 45mm, find the maximum force on the piston. **An[139N]** [03mk]

e) Describe an experiment to determine the acceleration due to gravity, g using a spiral spring of known force constant [06mk]

UNEB 2011 No 2

a) i) what is meant by simple harmonic motion [1mk]

ii) State two practical examples of simple harmonic motion

[1mk]

iii) Using graphical illustration distinguish between under damped and critically damped oscillation

[4mk]

b) i)describe an experiment to measure acceleration due to gravity using a spiral spring

[6mk]

ii) State two limitations to the accuracy of the value it b (i)

[02mk]

UNEB 2010 No 2

b) i) What is meant by a simple harmonic motion

[1mk]

ii) Distinguish between damped and forced oscillations

[2mk]

c) a cylinder of length l, cross sectional area A and density, δ, floats in a liquid of density, ρ, the cylinder is pushed down slightly and released.

i) Show that a performs simple harmonic oscillation [5mk]

ii) Derive the expression for the period of oscillation [2mk]

An($T=2\pi\sqrt{\left(\frac{\delta\, l}{\rho\, g}\right)}$ **)**

d) A spring of force constant $40Nm^{-1}$ is suspended vertically. A man of 0.1kg suspended from the spring is pulled down a distance of 5mm and released. Find the,

i) Period of oscillation **An[0.314s]** [2mk]

ii) Maximum oscillation of the mass **An[2ms-^{2}]** [2mk]

iii) Net force acting on the mass when it is 2mm below the centre of oscillation.

An[0.08N] [2mk]

UNEB 2009 No 3

(a) What is meant by simple harmonic motion

(01marks)

(b) A cylindrical vessel of cross-sectional area A, contains air of volume V, at a pressure P, trapped by frictionless air tight piston of mass M,. The piston is pushed down and released.

(i) If the piston oscillates with s.h.m, show that the frequency is given by $f = \frac{A}{2\pi}\sqrt{\frac{P}{m\,v}}$

(06marks)

(ii) Show that the expression for, f in b(i) is dimensionally correct

(02marks)

(c) Particle executing s.h.m vibrates in a straight line, given that the speeds of the particle are 4m$s^{-1}$and 2ms^{-1} when the particle is 3cm and 6cm respectively from equilibrium. calculate the;

(i) amplitude of oscillation **An(6.7x10^{-2}m)** (03marks)

(ii) frequency of the particle **An(10.68Hz)** (03marks)

(d) Give two examples of oscillatory motions which execute s.h.m and state the assumptions made in each case

UNEB 2008 No3

a) (i) Define simple harmonic motion

[01marks]

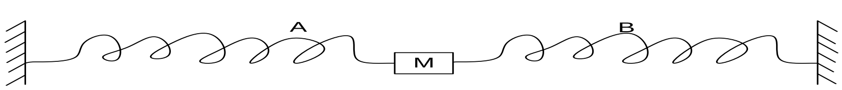
(ii) A particle of mass m executes simple harmonic between two point A and B about equilibrium

position O. Sketch a graph of the restoring force acting on the particle as a function of distance

r and moved by the particle

[02marks]

b)



Two springs A and B of spring constants K_A and K_B respectively are connected to a mass m as shown. The surface on which the mass slides is frictionless.

(i) Show that when the mass is displaced slightly, it oscillates with simple harmonic motion of frequency given by

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_A+k_B}{m}\right)}$$ [04marks]

(ii) If the two springs above are identical such that $k_A = k_B = 5\text{Nm}^{-1}$ and mass m=50g, calculate the period of oscillation **An[0.44s]** [03marks]

UNEB 2007 No 1

a) Define simple harmonic motion
 [01marks]

b) Sketch a graph of
 i) velocity against displacement
 [03marks]
 ii) acceleration against displacement for a body executing S.H.M

c) A glass U-tube containing a liquid is tilted slightly and then released
 i) Show that the liquid oscillates with S.H.M
 [04marks]
 ii) Explain why the oscillations ultimately come to rest
 [03marks]

UNEB 2007 No 4

b) i)What is meant by acceleration due to gravity
 [01mark]

ii)Describe how you would use a spiral string, a retort stand with a clamp, a pointer, seven 50g masses, meter rule and a stop clock to determine the acceleration due to gravity [6mk]

iii) State any two sources of errors in the experiment in bii) above.
 [01mark]

iv)A body of mass 1kg moving with simple harmonic motion has speed of $5ms^{-1}$ and $3ms^{-1}$ when it is at a distance of 0.1m and 0.2m respectively from the equilibrium point. Find the amplitude of motion

[04marsk]

CHAPTER 10: ELASTICITY

If a force is applied to a material in such a way as to deform it (change its shape or size), then the material is said to be stressed and there will be change in relative positions of the molecules within the body and the material become strained. Stress which results in increase in length is called tensile stress and one which results in decrease in length is called compressive stress.

Terms used

1. **Elasticity:** This is the ability of the material to regain its original shape and size when the deforming load has been removed.
2. **Elastic material:** This is a material which regains its original shape and size when the deforming load has been removed. E.g. Rubber band, spring.
3. **Elastic deformation:** This is when a material can recover its original length and shape when the deforming load has been removed.
4. **Elastic limit:** This is the maximum load which a material can experience and still regain its original size and shape once the load has been removed.

The elastic limit sometimes coincides with the limit of proportionality.

5. **Proportional limit:** This is the maximum load a material can experience for which the extension created on it is directly proportional to the load applied.
6. **Hooke's law:** it staes that; the extension of a wire or spring is proportional to the applied load provided the proportional limit is not exceeded.

The law shows that when the molecules of a material are slightly displaced from their mean positions, the restoring force is proportional to its displacement.

I.e. $F \alpha e$ $\boxed{F = ke}$ Where k is the constant of proportionality.

7. **Yield point:** this is a point at which there is a marked increase in extension when the stress or load is increased beyond the elastic limit.

   The internal structure of the material has changed and the crystal planes have effectively slid across each other. At yield poirt the material begin to show plastic behavior.

   Few materials exhibit yield point such as mild steel, brass and bronze.

8. **Plastic deformation:** this is when a material cannot recover its original shape and size when the deforming load has been removed.
9. **Breaking stress/ultimate tensile strength:** it is the maximum stress which can be applied to a material. Or it is the corresponding force per unit area of the narrowest cross section of the wire.
10. **Strength:** this is the ability of a material to with stand an applied force before breaking.

Or it is the maximum force which can be applied to a material without it breaking.

11. **Stiffness:** this is the ability of a material to resist changing its shape and size.
12. **Ductility:** it is the ability of the material to be permanently stretched. or it is the ability of the material to be stretched appreciably beyond elastic limit. It can be drawn into different shapes without breaking.
13. **Brittleness:** it is the ability of the material to break immediately it is stretched beyond to elastic limit.
14. **Toughness:** this is the ability of material to resist crack growth e.g. rubber
15. **Tensile stress:** it is force acting per unit area of cross-section of a material.

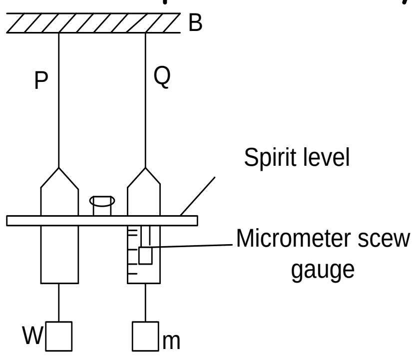
Stress = $\frac{F}{A}$

16. **Tensile strain**: it is the extension per unit original length of the material.

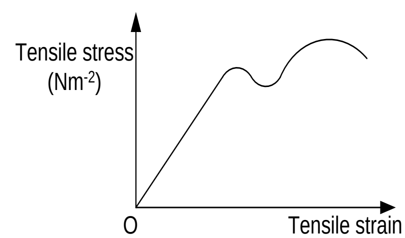
Strain = $\frac{e}{L}$

Strain has no units because it is a ratio of two similar units

10.1.0: Experiment to study elastic properties of steel



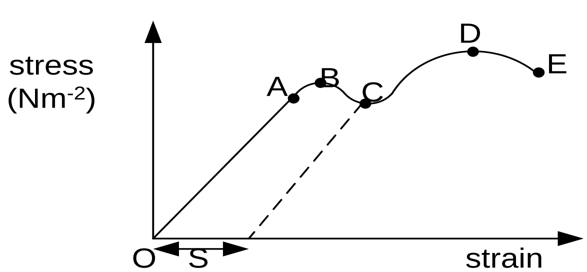
- Two <u>long, thin identical</u> steel wires are suspended besides each other from rigid support B
- The wire P is kept <u>taut</u> and free of <u>kinks</u> by weight A attached to its end
- The <u>original length l</u> of test wire Q is measured and recorded.
- The <u>mean diameter d</u> is determined and cross-sectional area $A = \frac{\pi d^2}{4}$ is found.
- Known weight, W is added to the free end of test wire Q and the corresponding extension e is read form the vernier scale.
- The procedure is repeated for different weights. however, vernier readings are also taken when the loads are removed. It ensures that the elastic limit is not exceeded.
- Results are tabulated including values of tensile stress $\left(\frac{W}{A}\right)$ and tensile strain $\left(\frac{e}{L}\right)$
- The graph of tensile stress versus tensile strain is plotted as below.



10.1.1: Stress-strain graphs

1. **Ductile material** e.g. copper, steel, iron

A ductile material is one which can be permanently stretched



A-Prportional limit

B-Elastic limit

C-Yield ponit

D-Breaking stress

E-Breaking point

Region OA: stress $\propto$ strain, all extensions are recovered when the load is removed. It is Hooke's law region and Young's Modulus can be defined only in this region.

Region AB: Hooke's law is not obeyed but extension is recovered when load is removed.

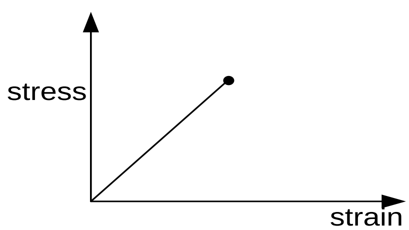
Region BC; Not all extensions are recovered when the load is removed

Region BC: Changes from elastic to plastic deformation

Point E: without any further increase in stress, the wire begins to under go physical changes, it **thins** out at some point and finally breaks

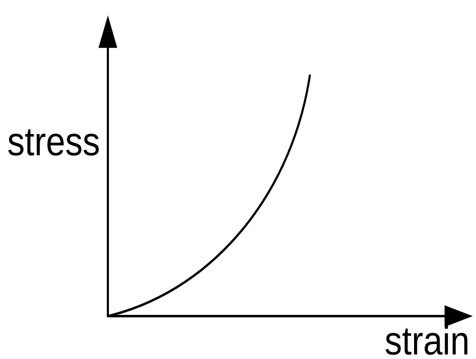
2. **Brittle materials** e.g. glass, chalk, rocks and cast iron

These are materials that can not be permanently stretched. It breaks as soon as the elastic limit has been reached



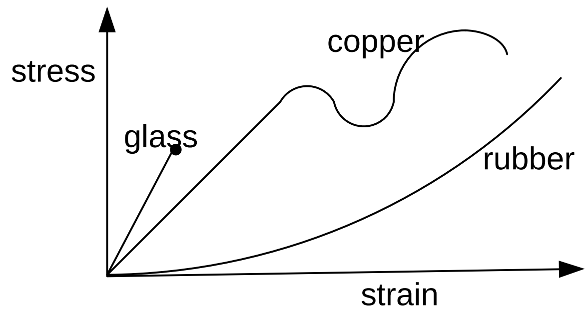
Brittle materials have only a small elastic region and do not under go plastic deformation. This behavior in glass is due to the existence of cracks in its surface. The high concentration of the stress at the crack makes the glass break.

3. **Rubber**



Rubber stretches very easily without breaking and has a greater range of elasticity. Rubber is much less stiff than metals and therefore its value of young's modulus is very much smaller than that of most metals. It does not undergo plastic deformation. Un stretched rubber has coiled molecules and when stretched they unwind and become straight and much harder

10.1.2: Stress-strain graph for glass, copper and rubber



10.1.3: Energy changes/physical process

1. Elastic deformation

Particles are slightly displaced from their equilibrium positions and work done by the force to displace the particles is stored as elastic potential energy. When the stretching force is removed, the potential energy of the particles changes to kinetic energy and moves them back to their equilibrium position.

2. Plastic deformation

It occurs when the material is stretched beyond the yield point. The crystal planes slide over each other and movement of dislocations takes place. When the stress is removed, original shape and size are not recovered due to energy loss in form of heat.

3. Work hardening

It is the process of increasing the resistance of a material to plastic deformation by plastically deforming it repeatedly.

During repeated plastic deformation, the metal dislocations move through the material and they entangle round each other and become immobile, it creates the resistance of the material to plastic deformation.

This explains why it is easier to break a copper wire by flexing it to and fro.

4. Annealing

It is a process by which a material restores its ductility.

Procedure

The metal is heated to high temperature above its melting point and maintained in this temperature for a period of time and relaxes the internal strains and hence the metal is re-crystallised and returns to the ductile state.

10.2.0: Young's modulus

It is also called the modulus of elasticity of a wire.

Young's modulus is the ratio of tensile stress to tensile strain of a material

Young modulus $= \frac{stress}{strain}$	$E = \frac{F}{A} \Big/ \frac{e}{L}$	$E = \frac{F\,L}{A\,e}$

A is area, L is original length, e is extension

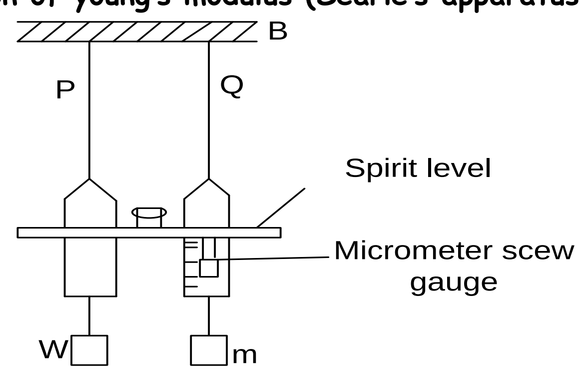
Dimensions of young's modulus

$[E] = \frac{[\,F\,][\,L\,]}{[\,A\,]\,[\,e\,]}$

[E] = $\frac{(M\,L\,T^{-2})(L)}{L^2\,L}$

[E]=$M\,L^{-1}T^{-2}$

10.2.1: Determination of young's modulus (Searle's apparatus)



- Two <u>long, thin identical</u> steel wires are suspended besides each other from rigid support B
- The wire P is kept <u>taut</u> and free of <u>kinks</u> by weight A attached to its end
- The <u>original length l</u> of test wire Q is measured and recorded.
- The <u>mean diameter d</u> is determined and cross-sectional area $A = \frac{\pi\,d^2}{4}$ is found.

- Known weight, W is added to the free end of test wire Q and the corresponding extension e is read form the vernier scale.
- The procedure is repeated for different weights. however, vernier readings are also taken when the loads are removed. It ensures that the elastic limit is not exceeded.
- A graph of weight W against extension e is plotted and its slope (s) obtained.
- Young's modulus is obtained from $E = \frac{S\,L}{A}$

Note

✓ Two identical wires are used to avoid errors due expansion as a result of temperature changes since they are affected equally.

✓ Wires are long as it is convenient, because a moderate load would produce a large tensile stress.

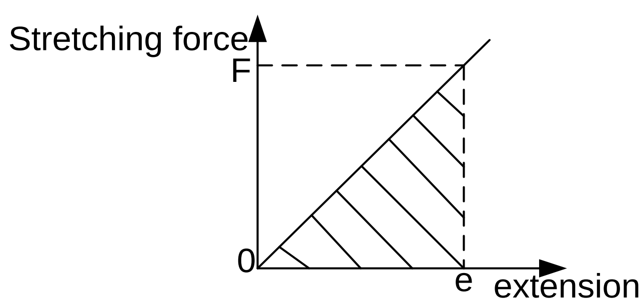
✓ Wires are thin so that a measurable extension is produced even with a small load. Otherwise if the wires were thick it requires a large load which would cause the support to yield.

✓ Micrometer/vernier readings are also taken when the load is removed to ensure that the elastic limit is not exceeded.

10.2.2: Energy stored in a stretched material [strain energy]

Consider a material of an elastic constant k, stretched by a force,$\boldsymbol{F}$ to extend by $\boldsymbol{e}$.

By Hooke's law, the extension is directly proportional to the applied force provided the elastic limit is not exceeded.



Work done = <u>area under the graph</u>

Work done =$\frac{1}{2}F\,e$

But $F = ke$

Work done = $\frac{1}{2}k\,e^2$

The work done to stretch the material is stored as elastic potential in the material

Energy stored = $\frac{1}{2}k\,e^2$

Or Energy stored = $\frac{1}{2}F\,e$

By calculus [integration]

If F is the force which gives an extension from O to e and $F = kx$ (from Hooke's law)

Work done $= \int_0^e F\,dx$

$= \int_0^e kx\,dx$

$= \left[\frac{k\,x^2}{2}\right]_0^e$

Work done $= \frac{1}{2}ke^2$

10.2.3: Energy stored per unit volume

Energy stored in the wire = $\frac{1}{2}F\,e$

If a wire is of cross sectional area A and natural length L, the volume = AL

Energy per unit volume = $\frac{Energy\ stored}{volume} = \frac{1}{2}F\,e \Big/ AL = \frac{Fe}{2AL}$

$$\boxed{\text{Energy per unit volume} = \frac{1}{2}\left(\frac{F}{A}\frac{e}{L}\right) \text{ or } \frac{1}{2}x\ stress\ x\ strain}$$

Numerical examples

1. A metal bar has a circular cross section of diameter 20mm. If the maximum permissible tensile stress is $8x10^{7}$Nm^{-2}, calculate the maximum force which the bar can withstand.

Solution

d = 20mm = 20x10^{-3}m

stress=$\frac{Force}{Area}$

Force = stress x area

$= 8x10^7 x\frac{\pi d^2}{4}$

$= 8x10^7 x\frac{\left[\frac{22}{7}x\left(20x10^{-3}\right)^2\right]}{4}$

Force = 2.513x10^{4}N

2. Find the maximum load which may be placed on steel of diameter 1mm if the permitted strain must not exceed $\frac{1}{1000}$ and young's modulus for steel is $2x10^{11}$Nm^{-2}

Solution

Young modulus $= \frac{stress}{strain}$

Stress = E x strain

$= 2x10^{11}x\frac{1}{1000}$

Stress = $2x10^8$Nm^{-2}

But stress = $\frac{F}{A}$

Force = stress x area

$= 2x10^8 x\frac{\pi d^2}{4}$

$= 2x10^8 x\frac{\left[\frac{22}{7}x(1x10^{-3})^2\right]}{4}$

Force = 1.571x10^{2}N

3. An elastic string of cross-sectional area 4mm^2 requires a force of 2.8N to increase its length by one tenth. Find young's modulus for the string if the original length of the string was 1m, find the energy stored in the string when it is extended.

Solution

A= 4mm^2= $4x\ 10^{-6}$m^2,

F = 2.8N,

L=1m, e= $\frac{1}{10}L$ e= 0.1

$$E = \frac{F\,L}{A\,e}$$

$$E = \frac{2.8\ x\ 1}{4x\ 10^{-6}\ x\ 0.1}$$

E = $7x10^6 Nm^{-2}$

Energy stored = ½ Fe

$$= \frac{1}{2}x2.8x0.1$$

= 0.14J

4. A rubber cord of a catapult has a cross-sectional area of 1.2mm^2 and original length 0.72m, and is stretched to 0.84m to fire a small stone of mass 15g at a bird. Calculate the initial velocity of the stone when it just leaves the catapult. Assume that Young's modulus for rubber is $6.2x10^8\ Nm^{-2}$

Solution

$e = 0.84 - 0.72 = 0.12m$

Stretching force, $F = \frac{EAeL}{l}$

$$F = \frac{6.2x10^8\ x1.2x10^{-6}x0.12}{0.72}$$

$$F = 124N$$

Energy stored in rubber= ½ Fe

$$\frac{1}{2}x124x0.12 = 7.44J$$

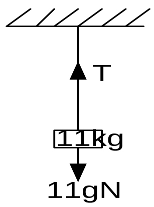
Kinetic energy of stone= $\frac{1}{2}mv^2$

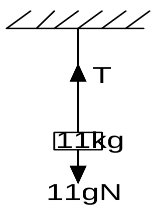
$$\frac{1}{2}x0.015xv^2 = 7.44$$

$$v = 31.5ms^{-1}$$

5. A mass of 11kg is suspended from the ceiling by an aluminum wire of length 2m and diameter 2mm, what is;
   a) The extension produced
   b) The elastic energy stored in the wire (young's modulus of aluminum is $7x10^{10}$Pa)

Solution





$L = 2m, d = 2mm = 2x10^{-3}m$

$T = 11gN$

$T = 11x9.81$

$T = 107.91N$

$$E = \frac{F\,L}{A\,e}$$

$$e = \frac{F\,L}{A\,E}$$

But $F = T = 107.91N$

$$A = \frac{\pi d^2}{4} = \frac{\left[\frac{22}{7}x\left(2x10^{-2}\right)^2\right]}{4}$$

$A = 3.14x10^{-6}m^2$

$$e = \frac{F\,L}{A\,E}$$

$$e = \frac{107.91x\ 2}{3.14x10^{-6}\ x7x10^{10}}$$

$e = 9.813x10^{-4}m$

Energy stored = ½ Te

$= ½\ x107.91x9.813x10^{-4}$

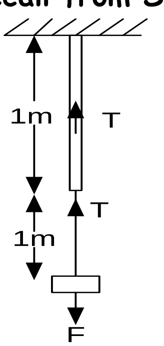
Energy stored =

$5.29x10^{-2}$J

6. A cylindrical copper wire and a cylindrical steel wire, each of length 1m and having equal diameter are joined at one end to form a composite wire 2m long. This composite wire is subjected to a tensile stress until its length becomes 2.002m. calculate the tensile stress applied to the wire (young modulus of copper =$1.2x10^{11}$Pa and Steel =$2x10^{11}$Pa)

Solution

[Recall from S.H.M wire in series experience the same tension and weight]



Total extension, $e =$

$2.002 - 2$

$e = 0.002m$

$e = e_1 + e_2$-------------- [1]

Note the two wires will experiences same stress

$0.002 = e_1 + e_2$

But E= $\frac{F\,L}{A\,e}$

$$e = \frac{F\,L}{A\,E}$$

$$0.002 = \frac{F\,L_1}{A\,E_1} + \frac{F\,L_2}{A\,E_2}$$

$$0.002 = \frac{F}{A}\left(\frac{L_1}{E_1} + \frac{L_2}{E_2}\right)$$

$$0.002 = \frac{F}{A}\left(\frac{1}{1.2x10^{11}} + \frac{1}{2x10^{11}}\right)$$

$$\frac{F}{A} = 1.5x10^{8}N$$

Stress $= 1.5x10^{8}N$

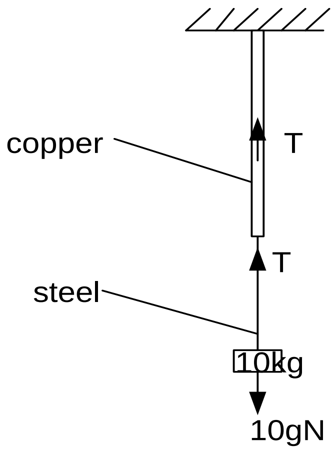
7. One end of a copper wire is welded to a steel wire of length 1.5m and diameter 1mm while the other end is fixed. The length of the copper wire is 0.8m while its diameter is 0.5mm. a bob 10kg is suspended from the free end of a steel wire. Find

i) Extension which results

ii) Energy stored in the compound wire

(Young's modulus for copper=$1x10^{11}$Nm^{-2} and steel =$2x10^{11}$Nm^{-2})

Solution



$E_1 = 1x10^{11}, l_1 = 0.8m$

$d_1 = 0.5mm = 0.5x10^{-3}m$

$E_2 = 2x10^{11}, l_2 = 1.6m$

$d_1 = 1mm = 1x10^{-3}m$

Recall from S.H.M for series wires

T=mg

But $e = \frac{F\,L}{A\,E}$

$$e_1 = \frac{F}{A_1}x\frac{L_1}{E_1}$$

$$e_1 = \frac{10x9.81x0.8}{\pi\frac{d^2}{4}x1x10^{11}}$$

$$e_1 = \frac{10x9.81x0.8}{\frac{22}{7}x\frac{(0.5x10^{-3})^2}{4}x1x10^{11}}$$

e_1=3.997x10^{-3}m

$$e_2 = \frac{F}{A_2}x\frac{L_2}{E_2}$$

$$e_2 = \frac{10x9.81x1.6}{\pi\frac{d^2}{4}x2x10^{11}}$$

$$e_2 = \frac{10x9.81x1.6}{\frac{22}{7}x\frac{(1x10^{-3})^2}{4}x2x10^{11}}$$

e_2=9.9924x10^{-4} m

$$e = e_1 + e_2$$

e=9.9924x10^{-4} + 3.997x10^{-3}

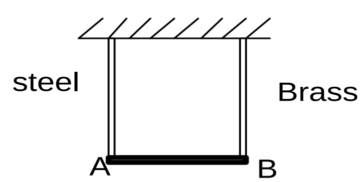
$$e = 1.039x10^{-3}m$$

iii) Energy stored in composite = $\frac{1}{2}$Fe

$$= \frac{1}{2}x(10x9.81)x1.039x10^{-3}$$

= 5.10x10^{-2}J

7.



A light rigid bar is suspended horizontally from two vertical wires, one of steel and one of brass as shown in the diagram. Each wire is 2.00m long. The diameter of the steel wire is 0.6mm and the length of the bar AB is 0.2m. when a mass of 10kg is suspended from the centre of AB the bar remains horizontal.

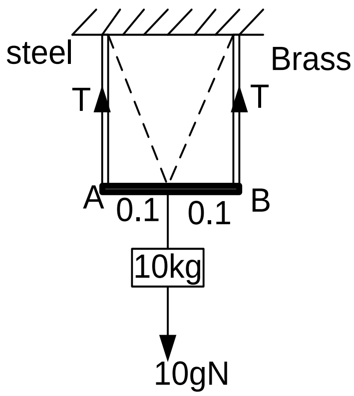
(i) What is the tension in each wire

(ii) Calculate the extension of the steel wire and the energy stored in it

(iii) Calculate the diameter of the brass wire

(iv) If the brass wires are replaced by another brass wire of diameter 1mm, where should the mass be suspended so that AB would remain horizontal.[young's modulus for steel = $2x10^{11}$Pa and brass= $1x10^{11}$Pa].

Solution



Assume that AB always remains horizontal

L_1 = 2m, d_1 =$0.6x10^{-3}$m, E_1 = $2x10^{11}$pa, e_1 = ?,

L_2 = 2m, d_2 = ?, E_2 = $1x10^{11}$pa, e_2 = ?

Taking moments about O: $0.1xT_1 = 0.1xT_2$

$$T_1 = T_2 \ldots\ldots..(i)$$

Also: $10gN = T_1 + T_2 \ldots.(\text{ii})$

$$2T_1 = 10x9.81$$

$$T_1 = 49.05N$$

Tension on each wire is $49.05N$

ii) for steel $e = \frac{F\,L}{A\,E}$

$$e_1 = \frac{T_1}{A_1}x\frac{L_1}{E_1}$$

$$e_1 = \frac{49.05x2}{\frac{22}{7}x\frac{(0.6x10^{-3})^2}{4}x2x10^{11}}$$

e_1 = $1.735x10^{-3}m$

Energy stored in steel $= \frac{1}{2}T_1e_1$

$=\frac{1}{2}x49.05x1.735x10^{-3}$

Energy stored in steel is $4.26x10^{-2}J$

iii) For the bar AB to remain horizontal e_1 = e_2

and $L_1 = L_2$

For brass : $A_2 = \frac{T_2}{e_2}x\frac{L_2}{E_2}$

$$A_2 = \frac{49.05}{1.735x10^{-3}}x\frac{2}{1x10^{11}}$$

$A_2 = 5.65x10^{-7}$m^2

$$A_2 = \frac{\pi\,d^2}{4}$$

$$d^2 = \frac{4x\,5.65x10^{-7}}{\frac{22}{7}}$$

d $= 8.485x10^{-4}m$

(v) Brass: d=1mm

$$A_2 = \frac{\pi\,(1x10^{-3})^2}{4}$$

A_2= $7.85x10^{-7}$m^2

$T_1 = \frac{e_1E_1A_1}{L_1}$ and $T_2 = \frac{e_2E_2A_2}{L_2}$

Taking moments about O

$$yxT_1 = (0.2 - y)xT_2$$

$y(2x10^{11}x2.825x10^{-7}) =(0.2-y)(1x10^{11}x7.85x10^{-7})$

$$y = 0.116m$$

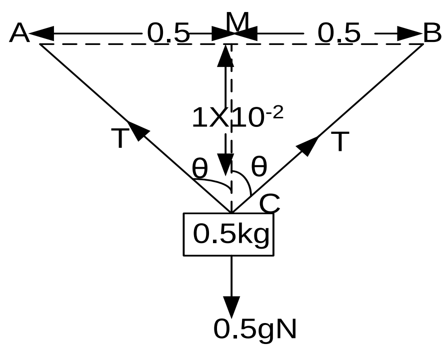
Mass should be placed 0.116m from the steel wire

8. The ends of a uniform wire of cross-sectional area $10^{-6}m^2$ and negligible mass are attached to fixed points A and B which are 1m apart in the same horizontal plane. The wire is initially straight and outstretched. A mass of 0.5kg is attached to the mid point of the wire and hangs in equilibrium with the mid point at a distance 10mm below AB. Calculate the value of young's modulus for the wire

8. The ends of a uniform wire of cross-sectional area $10^{-6}m^2$ and negligible mass are attached to fixed points A and B which are 1m apart in the same horizontal plane. The wire is initially straight and outstretched. A mass of 0.5kg is attached to the mid point of the wire and hangs in equilibrium with the mid point at a distance 10mm below AB. Calculate the value of young's modulus for the wire

Solution

A = 10^{-6}m^2, AB = 1m, e = 1m, m = 0.5kg,
Mc = $10x10^{-3}$m



Using Pythagoras theorem

$CB^2 = 0.5^2 + (1x10^{-2})^2$

$CB^2 = 0.2501$

$CB = 0.5001m$

$AC = CB = 0.5001m$

Length ACB = $0.5001 \; x \; 2$

$= 1.0002m$

Extension = 1.0002 – 1

$e = 2x10^{-4}m$

But $Tan\theta = \frac{0.5}{1x10^{-2}}$

θ = 88.9^0

Resolving vertically

$2Tcos\,\theta = 0.5g$

$2Tcos88.9 = 0.5x9.81$

$T = 127.75N$

$E = \frac{F\,L}{A\,e}$

But F = T (deforming force)

E = $\frac{127.75x1}{10^{-6}x2x10^{-4}}$

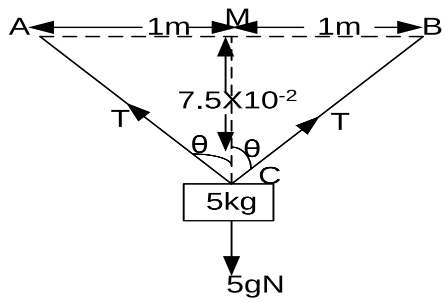
E = $6.39x10^{11}Nm^{-2}$

9. The ends of a uniform wire of length 2m are fixed to two points which are 2m apart in the same horizontal line. When a 5kg mass is attached to the mid point of the wire, the equilibrium position is 7.5cm below the line AB. Given that the young's modulus of the material of the wire is $2x10^{11}$pa. find the;
   i. Strain in the wire
   ii. Stress in the wire
   iii. Energy stored in the wire.

Solution

M = 5kg, AB = 2m, L = 2m, M_C = 7.5x10^{-2}m,

E = 2x10^{11}Pa



$CB^2 = MB^2 + MC^2$

$CB^2 = 1^2 + (7.5x10^{-2})^2$

CB = 1,003m

CB = AC = 1.003m

Stretched length ACB = 2x1.003

= 2.006

Extension = 2.006 – 2= 0.006m

Strain = $\frac{e}{l} = \frac{0.006}{2}$

Strain = 3x10^{-3}

Stress = $E\ x$ strain= 2x10^{11} x 3 x 10^{-3}

Stress = 6x10^{8}Nm^{-2}

Energy stored =$\frac{1}{2}Te$ …………….(i)

But resolving vertically

2Tcosθ = 5g …………………….(ii)

Also $tan\theta = \frac{1}{7.5x10^{-2}}$

θ= 85.7^0

$$2Tcos85.7 = 5x9.81$$

T = 327.92N

Energy stored = ½ x327.92 x 0.006

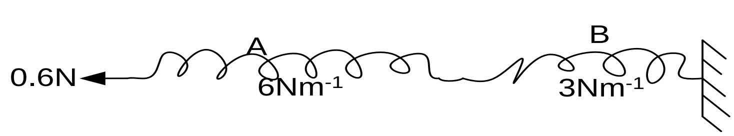
= 9.84 x 10^{-1}J

Exercise: 25 [use g = 10ms^{-2}]

1. A metal specimen has length of 0.5m. If the maximum permissible strain is not to exceed 10^{-3}, calculate its maximum extension **An (5x10^{-4}m)**

2. A metal bar of length 50mm and square cross-sectional side 20mm is extended by 0.015mm under a tensile load of 30kg, calculate
   a. Stress
   b. Strain in specimen
   c. Value of young's modulus for that metal. **An[7.25x10^{-3}Nm^{-2}, 3x10^{-4}, 24.5Nm^{-2}]**

3.



A spring A of force constant $6Nm^{-1}$ is connected in series with a spring B of force constant $3Nm^{-1}$ as shown below. One end of the combination is securely anchored and a force of 0.6N is applied to the other end

a. By how much does each spring extend
b. What is the force constant of the combination **An[0.1 (A), 0.2m(B), $2Nm^{-1}$]**

4. A copper wire and steel wire each of length 1.5 m and diameter 2mm are joined end to end to form a composite wire. The composite wire is loaded until its length becomes 3.003m. if young's modulus of steel is $2.0x10^{11}Pa$, and that of copper is $1.2x10^{11}Pa$

   (i) Find the strain in the copper and steel wires

   (ii) Calculate the force applied

   An[copper =0.0013, steel =$7.5x10^{-4}$, force=$4.7x10^{2}N$]

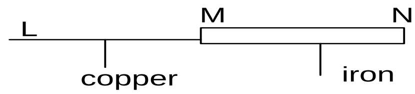
5. A thin steel wire initially 1.5m long and of diameter 0.50mm is suspended from a rigid support, calculate

   i. The final extension
   ii. Energy stored in a wire when a mass of 3kg is attached to the lower end. (young's modulus for steel = $2x10^{11}Nm^{-2}$) **An [1.1mm, $1.7x10^{-2}$J]**

6. Two wires of steel and phosphor bronze each of diameter 0.40cm and length 3.0m are joined end to end to form a composite wire of length 6.0m. calculate the tension in the wire needed to produce a total extension of 0.128cm in the composite wire. (Given that E of steel= $2.0x10^{11}Pa$ and E of bronze=$1.2x10^{11}Pa$)

   An[100.5N]

7. A copper wire LM is fused at one end M to an iron wire MN. The copper wire has length 0.9m and cross section $0.9x10^{-6}m^2$. The iron wire has length 1.4m and cross-section $1.3x10^{-6}m^2$. The compound wire is stretched and its total length increases by 0.01m



Calculate;

a) The ratio of the extension of the two wires

b) The extension of each wire

c) The tension applied to the compound wire (young's modulus for copper = $1.3x10^{11}Nm^{-2}$and $2.1x10^{11}$ pa) (Young's modulus for steel = $2x10^{11}Nm^{-2}$)

An (Cu:Fe 3:2, 0.6mm, 4.0mm,780N)

8. a) Define stress, strain and the young's modulus

b) i) describe an experiment to determine the young's modulus for a material in the form of a wire

ii) Which measurement require particular care, from the point of view of accuracy and why

c) i) derive an expression for the potential energy stored in a stretched wire

ii) A steel wire of diameter 1mm and length 1.m is stretched by a force of 50N, calculate the potential energy stored in the wire. ((young's modulus for steel = $2x10^{11}Nm^{-2}$)

An $1.2x10^{-2}$J

iii) The wire is further stretched to breaking where does the stored energy go

9. a) A heavy rigid bar is supported horizontally from a fixed support by two vertical wires A and B of the same initial length and which experience the same extension. If the ratio of the diameter of A and to that of B is 2 and the ratio of the young's modulus of A to that of B is 2, calculate the ratio of the tension in A to that in B. **An (8:1)**

b) if the distance between the wires is D, calculate the distance of wire A from the centre of

gravity of the bar. **(An**= $\frac{D}{9}$**)**

10. a) A rubber cord has a diameter of 5.0mm and on un stretched length of 1.0m. One end of the cord is attached to a fixed support A. When a mass of 1.0kg is attached to the other end of the cord so as to hang vertically below A, the cord is observed to elongate by 100mm, calculate the young's modulus of rubber.

b) If the 1kg mass is now pulled down a further short distance and then released, what is the period of the resulting oscillations**?** **An**

[$5.1\times10^{6}Nm^{-2}$, 0.63s]

11. A uniform steel wire of density $7800kgm^{-3}$ weighs 26g and 250cm long, it lengthens by 1.2mm,

when stretched by a force of 80N, calculate;

a) The value of young's modulus for steel

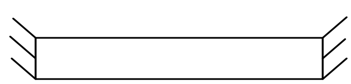
b) The energy stored in the wire

(**Hint volume** $=$ **Al** $= \frac{mass}{density}$ **)** **Ans ($2.03\times10^{11}Nm^{-2}$, 0.048J)**

10.2.4: FORCE ON A BAR DUE TO THERMAL EXPANSION OR CONTRACTION

When a bar is heated and then prevented from contracting as it cools, a force is exerted at the ends of a bar.

Consider a metal of young's modulus E, cross sectional Area A at a temperature $\theta_2{}^0c$ fixed between two rigid supports.



When the bar is cooled to a temperature θ_1°C, the bar can not contract hence there will be forces on the rigid support.

If α is the mean co-efficient of linear expansion then $L_\theta = L_0(1 + \alpha\theta)$

L_θ is length of the bar at temperature θ^0c

L_o is length of the bar at temperature 0^0c

$L_2 = L_0(1 + \alpha\theta_2)$ ………………….i

$L_1 = L_0(1 + \alpha\theta_1)$ …………………..ii

Subtracting

$L_2 - L_1 = L_0\ \alpha\ (\theta_2 - \theta_1)$

$L_2 - L_1 = L_0\ \alpha\theta$

$\alpha\ \theta = \frac{L_2 - L_1}{L_0}$

But strain $= \frac{L_2 - L_1}{L_0}$

$\boxed{Strain\ =\ \alpha\ \theta}$ where $\theta = \theta_2 - \theta_1$

From E = stress/strain

Stress = E x strain

$\frac{F}{A} = Ex\alpha\ \theta$

F = AE α θ

$\boxed{F\ = AE\ \alpha\ \theta}$

Coefficient of linear expansion α is defined as the fractional increase in length at0℃ for every degree rise in temperature.

Examples UNEB 2012 No1c (ii)

1. Two identical steel bars A and B of radius 2.0mm are suspended from the ceiling. A mass of 2.0kg is attached to the free end of bar A, calculate the temperature to which B should be raised so that the bars are again of equal length. (young's modulus of steel = $1.0x10^{11}Nm^{-2}$ and linear expansitivity of steel = $1.2x10^{-5}K^{-1}$)

Solution

For steel bar A, $r\ =\ 2x10^{-3}m$, m = 2kg,

$E\ =\ 1x10^{11}Nm^{-2}$, $\alpha\ =\ 1.2x10^{-5}K^{-1}$

But E = $\frac{stress}{strain}$ Strain = $\frac{stress}{E}$

Stress $= \frac{F}{A}$

Strain = $\frac{F}{A\,E}$

Strain = $\frac{2x9.81}{\pi(2x10^{-3})^2x1x10^{11}}$

Strain = $1.56x10^{-5}$

but strain = α θ

θ = $\frac{1.56x10^{-5}}{1.2x10^{-5}}$ θ = 1.3K

B should be raised by a temperature of 1.3K

2. A uniform metal bar of length 1m and diameter 2cm is fixed between two rigid supports at 25^0C. if the temperature of the bar is raised to 75^0C, find

2. A uniform metal bar of length 1m and diameter 2cm is fixed between two rigid supports at 25^0C. if the temperature of the bar is raised to 75^0C, find

(i) The force exerted on the support.

(ii) Energy stored in the bar at 75^0C. (young's modulus of metal = $2x10^{11}$Pa and coefficient of linear expansion = $1x10^{-5}K^{-1}$)

Solution

i) $\Theta_1 = 25^0C$, $\Theta_2 = 75^0C$, $E = 2x10^{11}$Pa,

L = 1m,

$d = 2x10^{-2}$m, $\alpha = 1x10^{-5}K^{-1}$

Force = EA $\alpha\ \theta$

$= 2x10^{11}\ x\ \frac{\pi\, d^2}{4} x1x10^{-5}(\theta_2 - \theta_1)$

$= 2x10^{11}\ x\ \frac{\frac{22}{7}x\,(2x10^{-2})^2}{4} x1x10^{-5}(75 - 25)$

$F = 3.14x10^4$N

ii) Energy stored = $\frac{1}{2}$Fe

but strain = $\alpha\ \theta$

and also strain = $\frac{e}{l}$

$\frac{e}{l} = \alpha\ \theta$

$e = l\ \alpha\ \theta$

Energy stored = $\frac{1}{2}F\ l\ \alpha\ \theta$

$= 3.14x10^4\ x1x10^{-5}x1x(75 - 25)$

= 7.85J

Exercise:26

1. A copper rod of length 0.8m and diameter 40mm is fixed between two rigid supports at a temperature of 20^0C. the temperature of the rod is raised to 70^0C, calculate;

   i. The force exerted on the rod at 70^0C

   ii. Energy stored per unit volume at 70^0C

   iii. Force exerted on the support if temperature was lowered to 45^0C

   [E for copper = $1.2x10^{11}Nm^{-2}$, α for copper between 20^0C to 70^0C is $1.7x10^{-5}K^{-1}$]

   Ans ($1.28x10^5$ N, 43.52J, $4.33x10^4Jm^{-3}$, $6.4x10^4$N)

2. Two identical cylindrical steel bars each of radius 3.00m and length 7m rest in a vertical position with their lower end on a rigid horizontal surface. A mass of 4.0kg is placed on the top of one bar. The temperature of the other bar is to be altered so that the two

bars are once again of equal length. Given that the coefficient of linear expansivity of steel is= $1.2x10^{-5}K^{-1}$

(i) By how much should the temperature be altered

(ii) Find the energy store in the bar due to the temperature change. **An[0.58K, $\mathbf{0.96}J$]**

UNEB 2014 No2

(a) **(i)** What is meant by **Young's modulus**

(01mark)

(ii) State **Hooke's law**

(01mark)

(iii) Derive an expression for the energy released in a unit volume of a stretched wire in terms of stress and strain

(04marks)

(b) A steel wire of length 0.6 m and cross- sectional area $1.5x10^{-6}m^2$ is attached at B to a copper wire BC of length 0.39 m and cross-sectional area $3.0x10^{-6}m^2$. The combination is suspended vertically from a fixed point at A and supports weight of 250 N at C. find the extension in each of the wires, given that Young's Modulus for steel is $2.0x10^{11}Pa$ and that of copper is $1.3x10^{11}Pa$. **An[$\boldsymbol{steel = 5.0x10^{-4}m, copper = 2.5x10^{-4}m}$]**

(05 marks)

(c) With the aid of a labeled diagram, describe an experiment to determine the Young's Modulus of steel wire

(07marks)

(d) Explain the term plastic deformation in metals

(02marks)

UNEB 2012 No1

a) State Hooke's law (I mark)

b) A copper wire is stretched until it breaks

i. Sketch a stress-strain graph for the wire and explain what happens to the energy used to stretch the wire at each stage. (4 marks)

ii. Derive the expression for the work done by a distance e (3 marks)

c) Define young's modulus (1 mark)

UNEB 2010

a) i) describe the terms tensile stress and tensile strain as applied to a stretched wire. (2 marks)

b) ii) Distinguish between elastic limit and proportional limit (2 marks)

c) With the aid of a labeled diagram, describe an experiment to investigate the relationship between tensile stress and tensile strain of a steel wire (4 marks)

d) i) A load of 60N is applied to a steel wire of length 2.5m and cross sectional area of $0.22mm^2$. if young's modulus for steel is 210GPa, find the expansion produced. (3 marks)

ii) If the temperature rise of 1k causes a fractional increase of 0.001%, find the change in the length of a steel wire of length 2.5mm when the temperature increases by 4K. (3 marks)

Solution

F = 60N, L = 2.5m, A = $0.22mm^2$ = $0.22x10^{-6}m^2$, E = 210Gpa or E = $210x10^9$Pa

Expansion required is the extension

$$E = \frac{F\,L}{A\,e}$$

e= $\frac{F\,L}{AE}$

$$e = \frac{60x2.5}{0.22x210x10^9x10^{-6}}$$

e = 3.247×10^{-3}m

ii) 1K gives 0.001%

%extension = $\frac{extension}{natural\ length}x100\%$

0.001% = $\frac{e}{2.5}x100\%$

e = $2.5x10^{-4}$m

1K = $2.5x10^{-4}$m

4K = $2.5x10^{-4}$x4

4K = $1x10^{-3}$m

UNEB 2006 No 3

a) i) Define stress and strain (2 marks)

ii) Determine the dimensions of young's modulus (3 marks)

b) Sketch a graph of stress versus strain for a ductile material and explain its features (6 marks)

c) A steel wire of cross-section area $1mm^2$ is cooled from a temperature of 60^0c to 15^0c, find the;

i. Strain (2marks)

ii. Force needed to prevent it from contracting young's modulus = $2x10^{11}Pa$, coefficient of linear expansion for steel = $1.1x10^{-5}K^{-1}$ (3 marks)

d) Explain the energy changes which occur during plastic deformation (4 marks)

Ans ($4.95x10^{-4}$, 99N)

UNEB 2005 No 2

a) Explain the terms

i. Ductility

ii. Stiffness

b) A copper wire and steel wire each of length 1.0m and diameter 1.0mm are joined and to end to form a composite wire 2.0m long, find the strain in each wire when the composite stretches by $2x10^{-3}m$ Young's modulus for copper and steel are $1.2\ x10^{11}$ and $2.0x10^{11}Pa$ respectively

Ans ($1.25x10^{-3}$, $7.5x10^{-4}$)

(7 marks)

UNEB 2003 No 3(d)

i) define the terms longitudinal stress and young's modulus of elasticity(2 marks)

ii) describe how to determine young's modulus for a steel wire. (07 marks)

UNEB 2001 No2

a) Define the following terms

i. Stress (1 mark)

ii. Strain (1 mark)

c) State the necessary measurements in the determination of young's modulus of a metal wire (2 marks)

d) Explain why the following precautions are taken during an experiment to determine young's

modulus of a metal wire.

i. Two long, thin wires of the same material are suspended from a common support (2 marks)

ii. The readings of the Vanier are also taken when the loads are gradually removed in steps

(1 mark)

CHAPTER 11: FLUID MECHANICS

Both liquids and gases are fluids because they 'flow' i.e. since their molecules are spaced and cause a change in shape without change in volume.

Fluid mechanics involves fluids at rest (hydro statistics) and fluids in motion (hydro dynamics/fluid flow)

11.1.0: Fluid flow

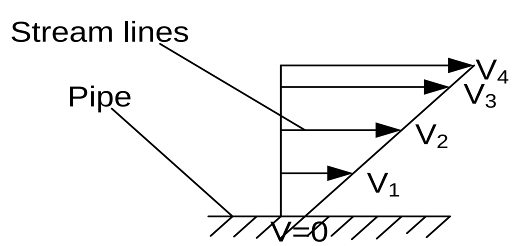
11.1.1: Viscosity

Definition

Viscosity is the frictional force between adjacent layers of a fluid moving at different velocities.

Fluid flow involves a shear. Fluids flow or move in form of layers, adjacent layers of a fluid are displaced over each other to form a shear. The different layers move at different speeds and therefore there will be a frictional force which opposes relative motion between the layers of the fluid. This frictional force is called **viscosity**. The greater the viscosity, the less easily it is for the liquid to flow and the stickier the liquid feels, the harder the liquid to flow.

Fluids stick to the solid surface so that when they flow the velocity must gradually decrease to zero as the wall of the pipe or vessel in approached.



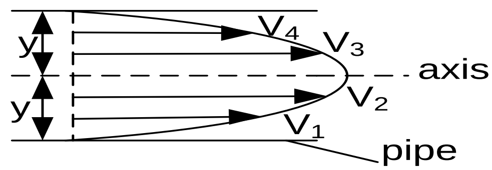
The arrows are known as stream lines and the length of the streamlines represents the magnitude of the velocity.

Definition: A streamline is a curve whose tangent at any point is along the direction of the velocity of the fluid particles at that point.

11.1.2: LAMINAR AND TURBULENT FLOW

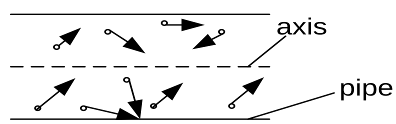
Laminar (steady/uniform) flow is the orderly flow of a liquid where lines of flow are parallel to the axis of the tube or pipe and liquid particles at the same distance from the axis have the same velocity.

Laminar flow occurs at low velocities below the critical velocity.

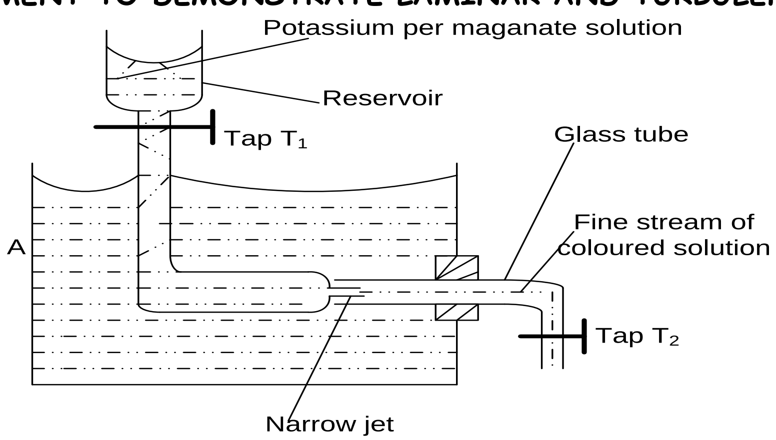


Turbulent flow is a disorderly flow where lines of flow are not parallel to the axis of the pipe and liquid particles at the same distance from the axis have different velocities (speeds and direction).

Turbulent flow occurs at high velocities, above the critical velocity.



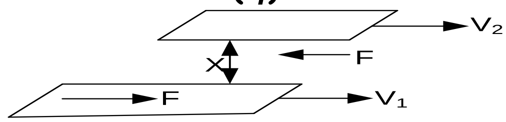
11.1.3: EXPERIMENT TO DEMONSTRATE LAMINAR AND TURBULENT FLOW



- ❖ The apparatus is set up as above. With taps T_1 andT_2 closed, potassium per magnate solution is poured into the reservoir and water poured into container A
- ❖ T_2 is then slightly opened to allow water to flow out of the glass tube and T_1 also opened slightly. A fine stream coloured solution is seen flowing along side the water to the glass tube and this illustrates laminar flow.

- Tap T_2 is then <u>widely opened</u> to allow more water to flow from the glass tube, a stage is reached when the coloured solution in the glass tube begins to spread out and fill the wall of the tube. The critical velocity has been exceeded and turbulence has begun.

11.1.4: COEFFICIENT OF VISCOSITY (η)



Consider two parallel layers of a liquid moving with velocities V_1 and V_2 and separated by a distance x with area of contact between the layers A

The slower lower layer exerts a tangential retarding force F on the faster upper layer the lower layer its self-experiences an equal and opposite tangential force F due to the upper layer.

Velocity gradient between the layers $= \frac{Velocity\ change}{distance\ apart} = \frac{V_2 - V_1}{x}$

Definition

Velocity gradient is the change in velocity between two layers (points) per unit length of separation of the points.

Frictional force F between adjacent layers depends on

Area of contact between the layers [F α A]

Velocity gradient between layers [F α velocity gradient]

Therefore $F\ \alpha\ A\ x\ velocity\ gradient$

$F\ =\ \eta x\ A\ x\ Velocity\ gradient$

$\boldsymbol{\eta} = \frac{F}{A\ x\ Velocity\ gradient}$

Definition

Coefficient of viscosity is the frictional force acting on a unit area of a fluid when it is in a region of unit velocity gradient

OR

Coefficient of viscosity is the tangential stress which one layer of a fluid exerts on another layer in contact with it when the velocity gradient between the layers is $1s^{-1}$.

Dimensions of η

$$\eta = \frac{F}{A\,x\,Velocity\ gradient}$$

$$[\eta] = \frac{[F]}{[A]x\,[Velocity\ gradient]}$$

$$[\eta] = \frac{M\,L\,T^{-1}}{L^2\left(\frac{LT^{-1}}{L}\right)}$$

$$[\eta] = M\,L^{-1}\ T^{-1}$$

Units of η = Nsm^{-2}

11.1.5: Effects of temperature on viscosity

- In liquids, viscosity is due to intermolecular forces of attraction between layers moving at different speeds. Increase in temperature reduces(weakens) intermolecular forces which increases molecular separation and speed, consequently viscosity in liquids decreases rapidly with increase in temperature
- In gases, viscosity is due to transfer of momentum . Molecules are further apart and have negligible intermolecular forces , molecules move randomly colliding with one another and continuously transferring momentum to the neighboring layers. Increasing the temperature of the gas increases the average speed (increases K.E) of the gas molecules hence increasing the transfer of momentum which results into increase in viscosity of the gas.

Differences between viscosity and solid friction

Solid friction	**Viscosity**
Independent of area of contact	Depends on area of contact
Independent of relative velocity between layers in contact	Directly proportional to velocity gradient
Independent of temperature but dependent on normal reaction	Depends on temperature

11.1.6: Steady flow of a liquid through a pipe (poiseuille's formula)

Poiseuille derived an expression for the volume of a liquid flowing out of a pipe per second. He assumes that the flow was steady/laminar.

The volume of liquid flowing out of a pipe per unit time (V/t) dependents on;

i. The coefficient of viscosity η of the liquid
ii. The radius of the pipe r
iii. The pressure gradient P/L causing the flow

$$\frac{V}{t} \propto \eta^x r^y \left(\frac{P}{l}\right)^z$$

$$\frac{V}{t} = K\eta^x r^y \left(\frac{P}{l}\right)^z \text{..........................x}$$

$$\frac{[V]}{[t]} = [K][\eta]^x [r]^y \left(\frac{[P]}{[l]}\right)^z$$

K is a dimensionless constant

$$L^3T^{-1} = (M L^{-1}T^{-1})^x \ L^y \ (M L^{-2}T^{-2})^z$$

$$L^3T^{-1} = M^{x+z} L^{y-x-2z} \ T^{-x-2z}$$

For M, $0 = x + z$..........................1

For L, $3 = y - x - 2z$.........................2

For T, $-1 = -x - 2z$........................3

From equation 1: $0 = x + z$

$x = -z$

Put into equation 3: $-1 = -(-z) - 2z$

$-1 = -z$

$z = 1$

$x = -1$

Put into equation 2

$3 = y - -1 - 2$

$3 = y - 1$

$y = 4$

$x = -1, y = 4, z = 1$

But from x

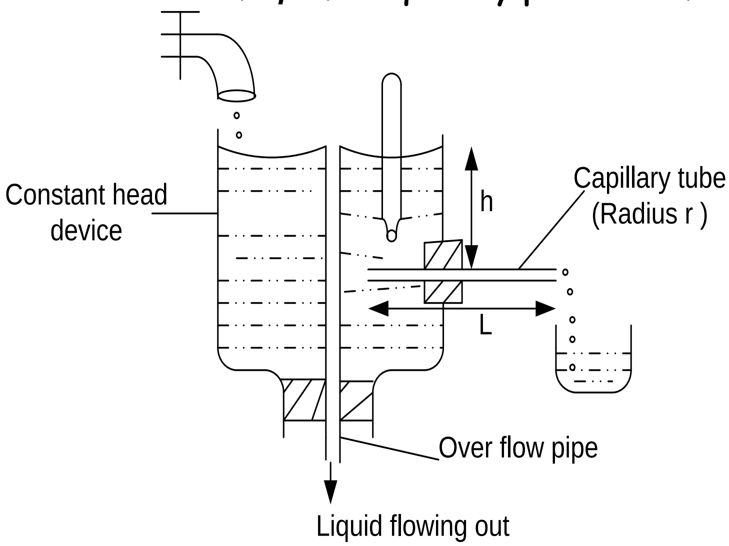
$$\frac{V}{t} = K\eta^x r^y \left(\frac{P}{l}\right)^z$$

$$\frac{V}{t} = \frac{K r^4 P}{\eta l}$$

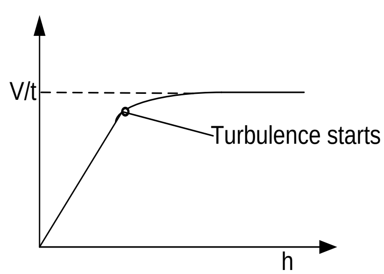
By experiment K = $\frac{\pi}{8}$

$\boxed{\frac{V}{t} = \frac{\pi r^4 P}{8 \eta l}}$ - Poiseuille's formula

a: Measurement of η of a liquid by poiseuille's formula



- The liquid under test flows steadily through the capillary tube from a constant head device and the volume V of liquid which emerges in a known time t is collected in a beaked and measured.
- The radius r of the tube is measured using a travelling microscope and a meter rule is used to measure its length L and height h of the liquid above the tube.
- The procedure is repeated by lowering or raising the overflow pipe to obtain several values of $\left(\frac{V}{t}\right)$ for each h.
- A graph of $\left(\frac{V}{t}\right)$ against h is plotted and its slope is determined from the straight part of laminar flow.



$$From\ \frac{V}{t} = \frac{\pi\, r^4 P}{8\,\eta\, l}$$

But P = hρg where $\rho$is the density of the liquid

$$\frac{V}{t} = \left(\frac{\pi\, r^4 \rho\, g}{8\eta l}\right)h$$

Comparing with y = mx +c

Slope S = $\left(\frac{\pi\, r^4 \rho\, g}{8\eta l}\right)$

$\boldsymbol{\eta}$ = $\frac{\pi\, r^4 \rho\, g}{8\ l\, s}$

Note:

- The experiment must be carried out at a constant temperature to avoid changes in $\boldsymbol{\eta}$

- ❖ Constant head apparatus is used to ensure that the rate of liquid flowing through the capillary tube is uniform. Since Poisseulle's formular holds for only laminar flow
- ❖ Great care is needed when measuring r because it appears in the calculation of η as r^4. This makes the % error in η due to an error in r four times the % error in r
- ❖ A capillary tube is used because r needs to be small so that h is large enough to be measured accurately

11.2.0: STOKE'S LAW AND TERMINAL VELOCITY

11.2.1: Derivation of Stoke's law

Stoke's suggested that any particle moving through a fluid experiences a retarding force called **viscous drag** due to the viscosity of the fluid. This force depends on the speed of the body V and acts in opposite direction to its motion

Note:

Viscosity of a fluid is the frictional force opposing relative motion between adjacent layers while **viscous drag** is the frictional force experienced by a body moving in a fluid due to its viscosity.

The viscous drag F on a spherical body depends

- ✓ On the radius (r) of the sphere
- ✓ Velocity V of the sphere
- ✓ Coefficient of viscosity η

F $\propto \eta^x V^y r^z$

F $= K\eta^x V^y r^z$……….(x)

[F] $= [K][\eta]^x [V]^y [r^z]$

K is a dimensionless constant

$MLT^{-2} = (ML^{-1}T^{-1})^x (LT^{-1})^y (L)^z$

$MLT^{-2} = M^x L^{y-x+z} T^{-x-y}$

For M

$x = 1$………………..(1)

For L,

$1 = y - x + z$

$y + z = 2$……………….(2)

For T,

$-2 = -x - y$

$-2 = -1 - y$

$y = 1$

Put into eqn 2

$y + z = 2$

$z = 1$

$x = 1, y = 1, z = 1$

From equation x

$F = K\eta^x V^y r^z$

F = k $\boldsymbol{\eta}$ Vr

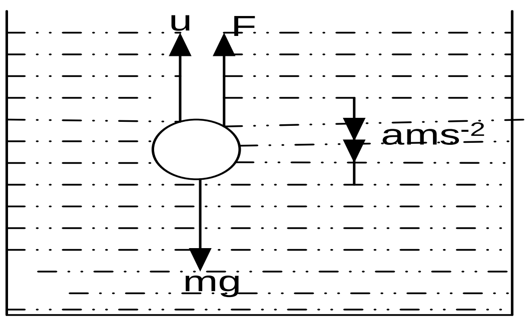
Experiment showed that K = 6π

$\boxed{\boldsymbol{F = 6\pi\eta\ rV}}$ **-Stoke's law**

11.2.2: TERMINAL VELOCITY

Consider a sphere of radius,r falling from rest through a viscous fluid.

- The forces acting on the sphere are its weight downwards , up thrust upwards due to the displaced fluid and the viscous drag, F upwards due to viscosity of the fluid.
- As the body accelerate downwards, its velocity increase and from $F = \pi\ \boldsymbol{\eta}\ r\ V_o$ so does the viscous drag increase .When the body is completely immersed in the fluid, up thrust remains constant since no more fluid is being displaced.
- A point is reached when $Mg = U + F$. This implies that the net force acting on the body is zero and body continues to move down with a maximum constant velocity called ***terminal velocity***.



If σ and ρ re the densities of the fluid and sphere respectively, the;

At the terminal velocity: $Mg = U + F$……(1)

$\frac{4}{3}\pi r^3\rho\ g = \frac{4}{3}\pi r^3\sigma\ g + 6\pi\ \boldsymbol{\eta}\ rV_o$

$6\pi\ \boldsymbol{\eta}\ rV_o = \frac{4}{3}\pi r^3\ g(\rho - \sigma)$

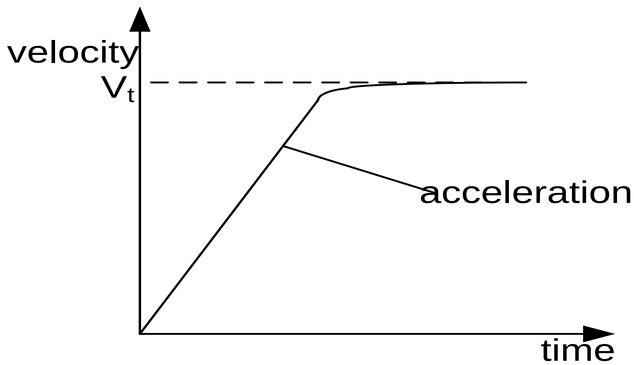
$V_o = \frac{4\ \pi\ r^3\ g\ (\rho-\sigma)}{3\ x 6\pi\eta r}$

$V_o = \frac{2\ r^2\ g\ (\rho-\sigma)}{9\eta}$

Definition

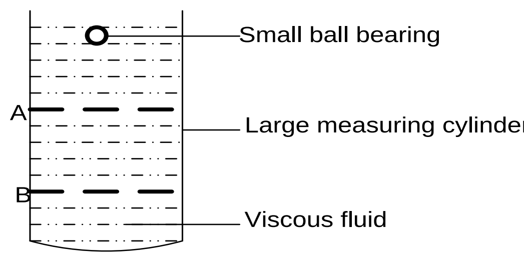
Terminal velocity is the maximum constant velocity attained by a body falling through aviscous fluid.

A graph of velocity against time for an object falling in a fluid



b: Measurement of η liquid by Stoke's law

The method is suitable for liquids of high viscosity such as glycerin and treacle



- Densities of the ball bearing and liquid ρ and σ respectively are obtained.
- Three reference marks A , B and C at equal distances are made on the sides of a tall transparent tube filled with the liquid.
- The ball is allowed to fall centrally through the liquid. The times t_1 and t_2 taken for the ball to fall from A to B and from B to C respectively are measured and noted .

When $t_1 = t_1 = t$, terminal velocity is obtained from

$V_o = \frac{AB}{t} = \frac{BC}{t} = \frac{AC}{2\,t}$......................[1]

- The and diameter d and hence radius r of the ball bearing is measured using a micrometer screw gauge.

Coefficient of viscosity is then calculated from Stoke's using

$V_o = \frac{2\,r^2\,g\,(\rho-\sigma)}{9\eta}$

$\boldsymbol{\eta} = \frac{2\, r^2\, g\, (\rho - \sigma)}{9\, V_t}$.....................................[2]

Note:

i) A measuring cylinder which is wide compared with the diameter of the ball bearing.

ii) Point C should be far away from the top of the tube so that the temperature remains constant.

iii) using a highly viscous liquid and a small ball bearing makes t large enough to be measured

Question

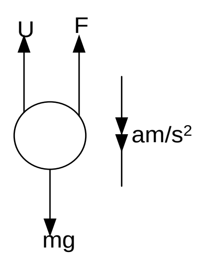
Describe how you can determine terminal velocity of a sphere falling in a viscous fluid.

An [The whole experiment of Stoke's law is the answer but only end in equation 1]

Numerical examples

1. A spherical raindrop of radius $2.0 x 10^{-4}$m, falls vertically in air at 20^0c, if the densities of air and water are $1.3 kgm^{-3}$ and $1x10^3 kgm^{-3}$ respectively and the viscosity of air at 20^0c is $1.8x10^{-5}$pa. Find the terminal velocity of the drop

Solution



At terminal velocity : Mg = U + F

$\frac{4}{3}\pi r^3 \rho_s\, g = \frac{4}{3}\pi r^3 \rho_f\, g + 6\pi\, \boldsymbol{\eta}\, rV_o$

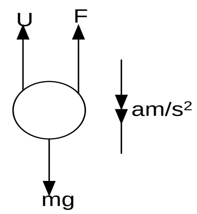
$V_o = \frac{2\, r^2\, g\, (\rho_f - \rho_s)}{9\, \eta}$

$V_o = \frac{2x\,(2x10^{-4})^2 x9.81x\,(1x10^3 - 1.2)}{9x1.8x10^{-5}}$

$V_o = 4.84 ms^{-1}$

2. A spherical oil drop of density $900 kgm^{-3}$ and radius $2.5x10^{-6}$m has a charge of $1.6x10^{-19}$C. the drop falls under gravity between two plates
   i. Calculate the terminal velocity attained by the drop
   ii. What electric field intensity must be applied between the plates in order to keep the drop stationary (density air = $1 kgm^{-3}$, coefficient of viscosity of air = $1.8x10^{-3} Nm^{-2}s^{-1}$)

Solution



At terminal velocity: Mg = U + F

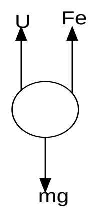
$\frac{4}{3}\pi r^3\rho_s\, g = \frac{4}{3}\pi r^3\rho_f\, g + 6\pi\boldsymbol{\eta}\, rV_o$

$V_o = \frac{2\, r^2\, g\,(\rho_f - \rho_s)}{9\eta}$

$V_o = \frac{2x\,(2.5x10^{-6})^2 x9.81x\,(900-1)}{9x1.85x10^{-5}}$

$V_o = 6.62x10^{-6} m/s$

Since the sphere is moving down, the electric field must be applied upwards to keep it stationary and there will be no vicious drag



When it is stationary Mg = U + F_e

$\frac{4}{3}\pi r^3\rho_s\, g = \frac{4}{3}\pi r^3\rho_f\, g + EQ$

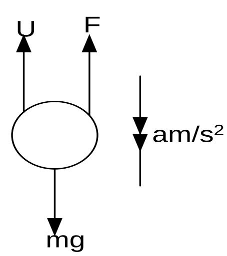
$$E = \frac{4\pi r^3 g(\rho_f - \rho_s)}{3xQ}$$

E = $\frac{4x\frac{22}{7}x(2.5x10^{-6})^3 x9.81x(900-1)}{3x1.6x10^{-19}}$

E = $3.60\times10^{6}Vm^{-1}$

3. Find the terminal velocity of an oil drop of radius 2.5×10^{-6}m which falls through air. Neglecting the density of air.(Viscosity of air = $1.8\times10^{-5}Nm^{-2}$, density of oil = $900kgm^{-3}$**)**

Solution



At terminal velocity: Mg = U + F

$\frac{4}{3}\pi r^3\rho_s\, g = \frac{4}{3}\pi r^3\rho_f\, g + 6\pi\, \boldsymbol{\eta}\, rV_o$

$V_o = \frac{2\, r^2\, g\,(\rho_f - \rho_s)}{9\,\eta}$

But ρ_f= $0kgm^{-3}$

$V_o = \frac{2x\,(2.5x10^{-6})^2 x9.81x\,(900-0)}{9x1.8x10^{-5}}$

$V_o = 6.81x10^{-4} m/s$

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4. A metal ball of diameter 10mm is timed as it falls through oil at a steady speed, it takes 0.5s to fall through a vertical distance of 0.03m. Assuming that density of the metal is $7500kgm^{-3}$ and that of oil is $900kgm^{-3}$, find

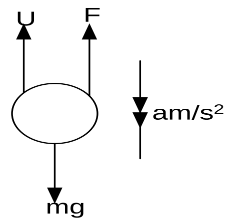
   i. The weight of the ball (2 marks)

   ii. The up thrust on the ball

   iii. The coefficient of viscosity of oil (03 marks

(Assume the viscous force = $6\pi\,\boldsymbol{\eta}\, r\, V_0$ where $\boldsymbol{\eta}$ is the coefficient of viscosity, r is radius of *the ball and V_0 is terminal velocity)*

Solution



i) Weight = mg

$= \frac{4}{3}\pi r^3 \rho_s\, g$

$= \frac{4}{3} x \frac{22}{7} x (10x10^{-3})^3 x 7500 x 9.81$

Weight = 0.31N

ii) Up thrust U $= \frac{4}{3}\pi r^3 \rho_f\, g$

$= \frac{4}{3} x \frac{22}{7} x (10x10^{-3})^3 x 900 x 9.81$

U = 0.037N

iii) At terminal velocity Mg = U +F

0.31 = 0.037 + 6π η r V_0

$\eta = \frac{0.31\text{-}0.037}{6\pi\, r\, V_o}$

but $V_o = \frac{0.3}{0.5}$

$V_o = 0.6 m/s$

$\eta = \frac{0.31\text{-}0.037}{6x\frac{22}{7}x10x10^{-3}x0.6}$

η = 2.414Nsm^{-2}

Exercise:27

1. A small oil drop falls with terminal velocity of 4x10^{-4}ms^{-1} through air. Calculate the radius of the drop. What is the terminal velocity of oil drop if its radius is halved.

(viscosity of air = 1.8x10^{-5}Nm^{-2}s, density of oil = 900kgm^{-3}, neglect density of air)

An [1.92x10^{-6}m, 1.0x10^{-4}ms^{-1}]

2. A spherical rain drop of radius $2.0x10^{-4}$m falls vertically in air at 20℃. If the densities of air and water are 1.2kgm^{-3} and 1000kgm^{-3} respectively and that the coefficient of viscosity of air at 20℃ is $1.8x10^{-5} Pa$ s, calculate the terminal velocity of the drop. **An[4.484 ms^{-1}].**

3. A metal sphere of radius 2.0x10^{-3}m and mass 3.0x10^{-4}kg falls under gravity, central down a wide tube filled with a liquid at 35^{0}c, the density of the liquid is 700kgm^{-3}, the sphere attains a terminal velocity of magnitude 40x10^{-2}ms^{-1}. The tube is emptied and filled with another liquid at the same temperature and of density 900kgm^{-3}. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude 25x10^{-2}ms^{-1}. Determine at 35^{0}C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[an 1.640]**

4. In an experiment to determine the coefficient of viscosity of motor oil, the following measurements were made

Mass of glass of sphere = 1.2×10^{-4}kg

Diameter of sphere = 4.0×10^{-3},

Terminal velocity of sphere = $5.4 \times 10^{-2}ms^{-1}$

Density of oil = $860kgm^{-3}$

Calculate the coefficient of viscosity of the oil [**an 0.45Nsm^{-2}**]

5. A metal sphere of radius 3.0×10^{-3}m and mass 4.0×10^{-4}kg falls under gravity, central down a wide tube filled with a liquid at 25°C, the density of the liquid is $800kgm^{-3}$, the sphere attains a terminal velocity of magnitude $45cms^{-1}$. The tube is emptied and filled with another liquid at the same temperature and of density $100kgm^{-3}$. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude $20cms^{-1}$. Determine at 25^0C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[An 2.09]**

6. A steel sphere of diameter 3.0×10^{-3}m falls through a cylinder containing a liquid x .When the sphere has attained a terminal velocity, it takes 1.08 s to travel between two fixed marks on the cylinder. When the experiment is repeated using another steel sphere of diameter 5.0×10^{-3}m with the cylinder containing liquid y, the time of fall between two fixed points is 4.8 s. if the density of liquid x is $1.26x10^3kgm^{-3}$, that of liquid y is$0.92x10^3kgm^{-3}$ and that of the steel ball is $7.8x10^3kgm^{-3}$, determine the ratio of the coefficient of viscosity of the liquid x to that of the liquid y, if the temperature remains constant throughout. **[An 0.77]**

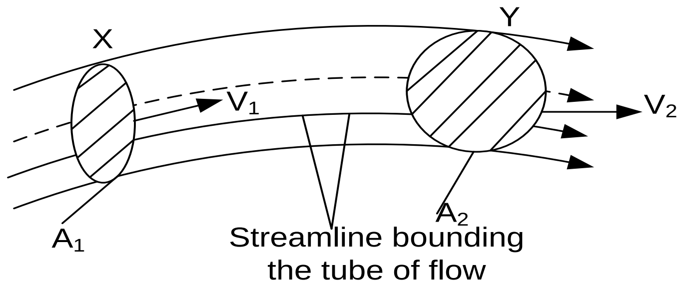
11.3.0: Equation of continuity

Consider a fluid undergoing steady flow and consider a section XY of a tube of flow with the fluid.

Let A_1 and A_2 be the cross-section areas of the tube of flow at X and Y respectively

ρ_1 and ρ_2 be the densities of the fluid at X and Y respectively

V_1 and V_2 be the velocities of the fluid particles at X and Y respectively.



In a time interval Δt the fluid at X will move forward a distance $V_1\Delta t$. therefore, a volume $A_1V_1\Delta t$ will enter the tube at X. the mass of fluid entering at X in time Δt will be there be

$$\rho_1 A_1V_1\Delta t$$

Similarly the mass leaving at Y in the same time is $\rho_2 A_2V_2\Delta t$

Since the mass entering at X is equal to mass leaving at Y

$$\rho_1A_1V_1\Delta t = \rho_2 A_2V_2\Delta t$$

For an incompressible fluid $\rho_1 = \rho_2$

$$\boxed{A_1V_1 = A_2V_2} \text{.......................}1$$

Equation 1 is an equation of continuity for an incompressible fluid

Definition

An incompressible fluid is a fluid in which changes in pressure produce no change in the density of

the fluid

11.3.1: WHY LIQUIDS FLOW FASTER IN CONSTRICTIONS

Volume flow per second is constant, so by the equation of continuity: $\mathbf{A_1V_1 = A_2V_2}$

$V_2 = \frac{A_1}{A_2}V_1$ It implies that $A_2 \propto \frac{1}{V_2}$ if $A_1 > A_2$ then $V_2 > V_1$

Hence the velocity at the wider part is less than that at the constructed part

11.3.2: Bernoulli's equation

It states that for an incompressible non viscous fluid undergoing steady flow, the pressure plus the potential energy per unit volume plus kinetic energy per unit volume is constant at all points on a stream line.

$$\text{i.e. } \boxed{P + \tfrac{1}{2}\rho v^2 + \rho g h = a\ constant}$$

P is the pressure with in the fluid

ρ is the density of the fluid

v is the velocity of the fluid

g is the acceleration due to gravity

h is height of the fluid (above reference line)

11.3.3: Derivation of Bernoulli's equation

Consider a tube of flow with in a non-viscous incompressible fluid undergoing steady flow

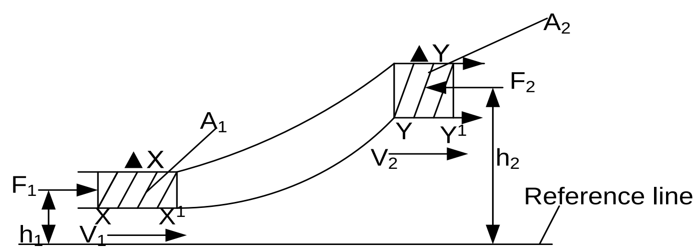
Let

ρ_1 and ρ_2 = pressure at X and Y

V_1 and V_2 = velocities at X and Y

A_1 and A_2 = area of cross section at X and Y

h_1 and h_2 = Average heights at X and Y



Let X^1 be close to X so that each of the parameters above has the same value at X^1 and at X. Let Y^1 be close to Y with similar consequences

Since the fluid is incompressible, the density will be the same at all points. Let this be ρ.

Consider the section of fluid which is between X and Y moving to occupy the region between X^1 and Y^1. The fluid moves in this direction because the force F_1 is greater than the force F_2. The force F moves a distance Δx and the fluid moves a distance Δy against the force F_2

Network done on the fluid is therefore given by

$\boldsymbol{W = F_1\,\Delta x - F_2\,\Delta y}$……………………………………1

Since the fluid is undergoing ste4ady flow, the mass of fluid that was originally between X and X^1 is equal to the mass which is now between Y and Y^1. Let this mass be M, thus a mass M which originally had velocityV_1 and average height H_1 has been replaced by an equal mass with velocity V_2 and average height h_2, therefore,

Gain in kinetic energy $= \frac{1}{2}m\,{v_2}^2 - \frac{1}{2}m\,{v_1}^2$………………….2

Gain in potential energy $= mgh_2 - mgh_1$…………………….3

Name of the work done on the fluid has been used to overcome internal friction since the fluid is non-viscous and therefore by the principle of conservation of energy.

Work done= Gain in kinetic energy + Gain in potential energy

Work done$= \frac{1}{2}m\,{v_2}^2 - \frac{1}{2}m\,{v_1}^2 + mgh_2 - mgh_1$

$F_1\,\Delta x - F_2\,\Delta y = \frac{1}{2}m\,{v_2}^2 - \frac{1}{2}m\,{v_1}^2 + mgh_2 - mgh_1$

$P_1A_1V_1\Delta t - P_2\ A_2V_2\Delta t = \frac{1}{2}m\,{v_2}^2 - \frac{1}{2}m\,{v_1}^2 + mgh_2 - mgh_1$

But $A_1\,\Delta x$ (volume between X and X_1) = $\frac{m}{\rho}$

And similarly volume between Y and Y^1 ($A_2\,\Delta y$) = $\frac{m}{\rho}$

$\rho_1\frac{m}{\rho} - \rho_2\frac{m}{\rho} = \frac{1}{2}m\,{v_2}^2 - \frac{1}{2}m\,{v_1}^2 + mgh_2 - mgh_1$

$P_1 - P_2 = \frac{1}{2}\rho\,{v_2}^2 - \frac{1}{2}\rho\,{v_1}^2 + \rho gh_2 - \rho gh_1$

$P_2 + \frac{1}{2}\rho\,{v_2}^2 + \rho gh_2 = P_1 + \frac{1}{2}\rho\,{v_1}^2 + \rho gh_1$

$P + ½\,\rho v^2 + \rho gh =$aconstant

Assumptions

- ✓ The flow is laminar
- ✓ The fluid is incompressible and non viscous
- ✓ The pressure and velocity are uniform at any cross section of the tube

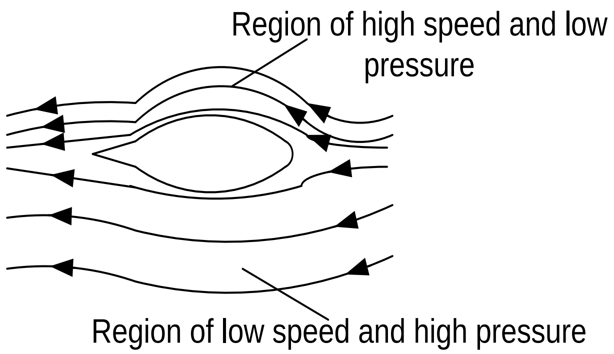
Note

In accordance with equation of continuity, fluids speed up at constrictions and therefore there is a decrease in pressure at constrictions. This effect is made use of in such devices are filter pumps, Bunsen burners and carburetors

11.3.4: Application of Bernoulli's principle

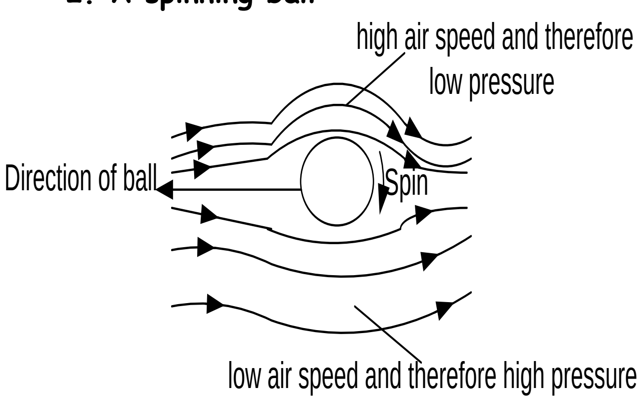
It follows from Bernoulli's equation that whenever a flowing fluid speeds up, there is a corresponding decrease in the pressure and for the potential energy of the fluids. If the flow is horizontal, the whole of the velocity increase is accounted for by a decrease in pressure.

1. **Aero foil lift**



- ❖ An aero foil e.g. an air craft wing is shaped so that air flows faster along the top of the wings than below the wings.
- ❖ By Bernoulli's principle pressure below becomes greater than that above the wings.
- ❖ This pressure difference produces the resultant force called lift upwards force. It is this force which provides a force that lifts the plane off the ground at take off

2. **A spinning ball**

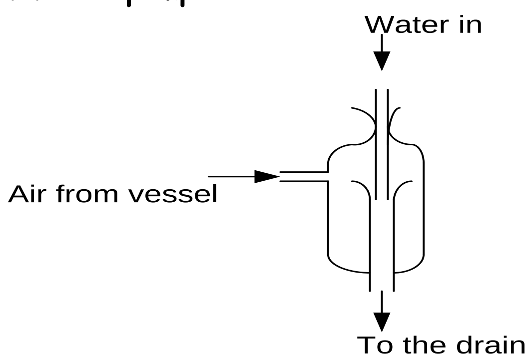


A ball such as a football, tennis or golf ball that is projected to travel through air experiences a sideways force which makes it curve in flight. This is because the spin drags air around with the ball such that air moves faster on one side of the ball than the other. The pressure difference causes a resultant force which makes the ball the curve as it spins.

3. **Sanction effect**

This is experienced by a person standing close to the platform at the station when a fast moving train passes. The fast moving air between the person and the train produces a decrease in pressure and the excess pressure on the other side pushes the person towards the train

4. Filter pump



This pump has a narrow cross section in the middle so that the jet of water from the top flows faster here. This reduces the pressure around it and thus air flows in from the side of a tube connected to a vessel. The air and water are expelled together through the bottom of the pump

5. Bunsen burner

The gas passes the narrow jet at high speed creating a low pressure region. Atmospheric pressure then pushes air in through the hole and the mixture flows up the tube to burn at the top

6. Carburetor

The air passage through a carburetor is partially constructed at the point where petrol and air are mixed. This increases the speed of air but lowers its pressure and permits more rapid evaporation of the petrol.

Examples

1. Water flows along a horizontal pipe of cross section area 30cm^2. The speed of water is 4ms^{-1} but this rises to 7.5m/s in constriction pipe. What is the area of this narrow part of the tube.

Solution

From the equation of continuity

$$A_1V_1 = A_2V_2$$

$$30x10^{-4}x4 = A_2x7.5$$

$$A_2 = 1.6x10^{-3}m^2$$

Area of the narrow part is 16cm^2

2. Water leaves the jet of a horizontal horse at 10m/s. If the velocity of the water with in the horse is 0.4m/s. Calculate the pressure P with in the horse (density of water $1000kgm^{-3}$) and atmospheric pressure 10^5Nm^{-2}

Solution

V_1 = 0.4m/s, P_1 = ?, $1000kg/m^3$,

V_2 =10m/s, P = 10^5

P + ½ ρv^2 + ρgh = a constant

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ for horizontal flow

$P_1 + \frac{1}{2} x1000x0.4^2 = 10^5 + \frac{1}{2} x1000x10^2$

$P_1 = 1.5x10^5Pa$

3. A fluid of density $1000kgm^{-3}$ flows in a horizontal tube. If the pressure the entry of the tube is 10^5Pa and at the exit is 10^3Pa, given that the velocity of the fluid at the entry is $8ms^{-1}$, calculate the velocity of the liquid at the exit.

Solution

P + ½ ρv^2 + ρgh = a constant

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ for horizontal flow

$$10^5 + ½\, x1000x8^8 = 10^3 + ½\, x1000xV_2^2$$

$$V_2 = 45.2ms^{-1}$$

4. An air craft design requires a dynamic lift of $2.4x10^4N$ on each square meter of the wing when the speed of the air craft through the air is $80ms^{-1}$. Assuming that the air flows past the wing with streamline line flow and that the flow past the lower surface is equal to the speed of the air craft, what is required speed of the air over the upper surface of the wing if the density of the air is $1.29kgm^{-3}$.

Solution

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ for horizontal flow

$$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

$$P_2 - P_1 = \frac{1}{2}x1.29x(V_1^2 - 80^2)$$

$$\text{lift force}, F = (P_2 - P_1)A$$

$$24000 = \left[\frac{1}{2}x1.29x(V_1^2 - 80^2)\right]x1$$

$$V_1 = 208.8ms^{-1}$$

5. Air flows over the upper surface of the wings of an aero plane at a speed of $81ms^{-1}$ and past the lower surfaces of the wings at $57ms^{-1}$. Calculate the lift force on the aero plane if it has a total wing area of $3.2m^2$. (density of air = $1.3kgm^{-3}$)

Solution

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ for horizontal flow

$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$

$P_2 - P_1 = \frac{1}{2}x1.3x(81^2 - 57^2)$

lift force, $F = (P_2 - P_1)A$

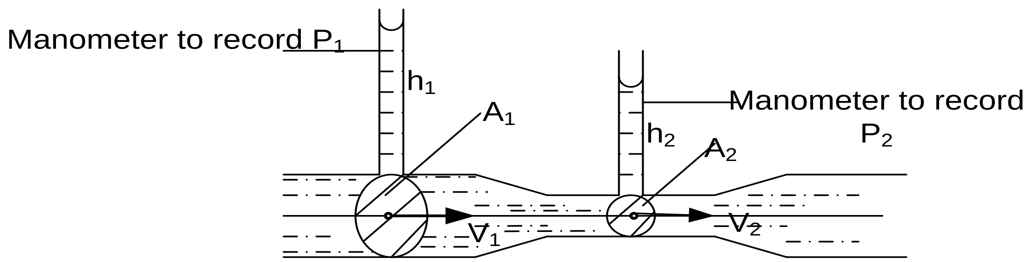
$F = \left[\frac{1}{2}x1.3x(81^2 - 57^2)\right]x3.2$

$F = 6.9x10^3 N$

11.3.5: Measurement of fluid velocity

5. Venture meter

This is a device which introduces a constriction into a pipe carrying a fluid in order that the velocity of the fluid can be measured by measuring the resulting drop in pressure.



Consider the fluid to be non viscous, incompressible and of density ρ in a horizontal steady flow let the pressure and velocity be P_1 and V_1 at the main pipe and P_2 and V_2 at the constricted pipe along the same stream line

Applying Bernoulli's equation

$\mathbf{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2}$ ……………….(1) (horizontal flow)

If the cross sectional areas at main and constructed equation of continuity.

$A_1V_1 = A_2V_2$

$V_2 = \frac{A_1V_1}{A_2}$

Put into equation 1

$\mathbf{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho\left(\frac{A_1V_1}{A_2}\right)^2}$

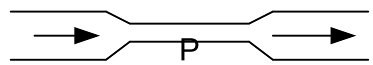
$$\boxed{\mathbf{P_1 - P_2} = \frac{1}{2}\rho\, v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$$

Thus by measuring pressures $\mathbf{P_1}$ and $\mathbf{P_2}$ and knowing ρ, A_1, and A_2 it is possible to find the velocity of V_1 of the fluid in the un constricted (main) section of the pipe.

Note: $P_1 = \rho h_1 g$ and $P_2 = \rho h_2 g$

Examples

1.a)



a horizontal pipe of a diameter 36.0cm tapers to a diameter of 18.0cm at P. An ideal gas at a

pressure of 2×10^5Pa is moving along the wider part of the pipe at a speed of $30ms^{-1}$, the pressure of the gas at P is 1.8×10^5Pa. Assuming the temperature of the gas remain constant

calculate the speed of the gas at P.

b) For the gas in (a) recalculate the speed at P on the assumption that it can be treated as an incompressible fluid, and use Bernoulli's equation to calculate corresponding value for the pressure at P. Assume that in the wider part of the pipe the gas speed is still $30.0ms^{-1}$, the

pressure is still 2.00×10^5 Pa and at this pressure the density of the gas is $2.60kgm^{-3}$.

Solution

a) $P_1 = 2 \times 10^5$Pa $d_1 = 36 \times 10^{-2}$m, $v_1 = 30ms^{-1}$

$P_2 = 1.8 \times 10^5$Pa $d_2 = 18 \times 10^{-2}$m $v_2 = ?$

An ideal gas at constant temperature obeys Boyle's law.

$P_1 V_1{}^1 = P_2 V_2{}^1$ ----------------------------- [1]

volume $V_1{}^1 = A_1 L_1$ and volume $V_2{}^1 = A_2 L_2$

But L_1 = speed $V_1 \times t$ and L_2= speed $V_2 \times t$

Put into equation 1 : $P_1 A_1 L_1 t = P_2 A_2 L_2 t$

$$P_1 \frac{\pi d_1{}^2}{4} L_1 t = P_2 \frac{\pi d_2{}^2}{4} L_2 t$$

$$2 \times 10^5 x \frac{\frac{22}{7} x (36 \times 10^{-2})^2}{4} x 30 =$$

$$2 \times 10^5 x \frac{\frac{22}{7} x (18 \times 10^{-2})^2}{4} x V_2$$

$$V_2 = 133.33 m/s$$

b) For an incompressible fluid

$A_1 V_1 = A_2 V_2$----------------- [2]

$P_1 = 2 \times 10^5$Pa $d_1 = 36 \times 10^{-2}$m $v_1 = 30ms^{-1}$

$P_2 = ?$ $d_2 = 18 \times 10^{-2}$m, $v_2 = ?$

$$\frac{\pi d_1{}^2}{4} V_1 = \frac{\pi d_2{}^2}{4} V_2$$

$$\frac{22}{7} x \frac{(36 \times 10^{-2})^2}{4} x 30 = \frac{22}{7} x \frac{(18 \times 10^{-2})^2}{4} x V_2$$

$$V_2 = 120 m/s$$

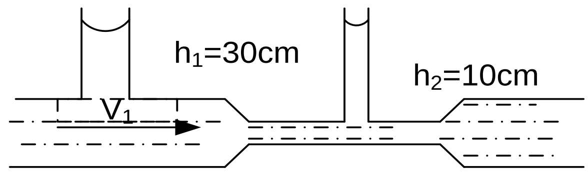
Using Bernoulli's equation

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ for horizontal flow

$2x10^5 + \frac{1}{2} x2.6x30^2 = P_2 + \frac{1}{2} x2.6x120^2$

$P_2 = 1.825x10^5 Pa$

6. A venturimeter consists of a horizontal tube with a constriction tube which replaces part of the piping system as shown below



If the cross-section area of the main pipe is $5.8x10^{-3}m^2$ and that of the constriction is $2.58x10^{-3}m^2$. Find the velocity V_1 of the liquid in the main pipe

Solution

$h_1 = 30x10^{-2}$, $h_2 = 10x10^{-2}m$, $\rho_1 = ?$ $\rho_2 = ?$,

$A_1 = 5.81x10\text{-}3m^3$, $A_2 = 2.58x10\text{-}3m^3$

$P_1 = h_1\rho g$ and $P_2 = h_2\rho g$

$P + \frac{1}{2}\rho v^2 + \rho gh$ = a constant

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ for horizontal flow

$$\frac{1}{2}\rho\, v_1{}^2 + \rho g h_1 = \frac{1}{2}\rho\, v_2{}^2 + \rho g h_2$$

From equation of continuity

$A_1V_1 = A_2V_2$ $\therefore V_2 = \frac{A_1V_1}{A_2}$

$$\frac{1}{2}\rho\, v_1{}^2 + \rho g h_1 = \frac{1}{2}\rho\left(\frac{A_1V_1}{A_2}\right)^2 + \rho g h_2$$

$$30x10^{-2}x9.81 + \frac{1}{2}xV_1^2 = 10x10^{-2}x9.81 + \frac{1}{2}x\left(\frac{5.81x10^{-3}xV_1}{2.58x10^{-3}}\right)^2$$

$2.943 - 0.981 = 2.035612343V_1^2$

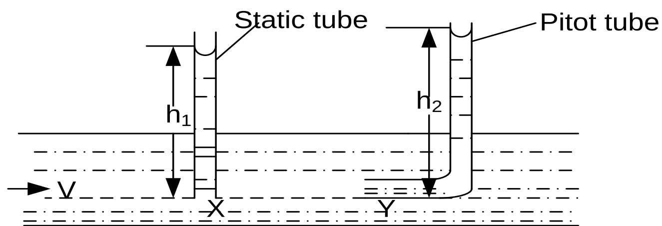
$V_1^2 = 0.963837739$

$V_1 = 0.982m/s$

3. **Pitot-static tubes**

3. **Pitot-static tubes**

The Pitot - static tube is a device used to measure the velocity of a moving fluid. It consists of two manometer tubes, the pitot tube and the static tube. The pitot tube has its opening facing the fluid flow, the static tube has its opening at right angles to this



The total pressure exerted by a flowing liquid has two components ie the static pressure and dynamic pressure. Static tube measures the static pressure while pitot tube measures total pressure.

P + $\frac{1}{2}$ ρv^2 + ρgh = a constant

Static pressure= P + ρgh_1 and Dynamic pressure = $\frac{1}{2}\rho V^2$

$Total pressure = static pressure + dynamic pressure$

D$ynamic pressure = total pressure - static\ pressure$

$\frac{1}{2}\rho V^2 = total pressure - static\ pressure$

V= $\sqrt{\frac{2(total pressure - static\ pressure)}{\rho}}$

- **Static pressure**

Static pressure at a point is the pressure that the fluid would have if it were at rest.

- **Dynamic pressure**

It is the pressure of a fluid due to its velocity

- **Total pressure**

It is the sum of the dynamic and static pressure.

Example

1. The static pressure in a horizontal pipe line is $4.3x10^4$Pa, the total pressure is $4.7x10^4$Pa and the area of cross-section is $20cm^2$. The fluid may be considered to be incompressible and non viscous and has a density of $10kgm^{-3}$. Calculate
   i. The flow velocity in the pipeline
   ii. The volume flow rate in the pipeline

Solution

Static pressure $=4.3x10^4$Pa

Total pressure = $4.7x10^4$Pa

A = $20 \times 10^{-4}m^2$, P = $10kgm^{-3}$

Dynamic pressure=total pressure–static pressure

Dynamic pressure $=4.7x10^4 - 4.3x10^4$

Dynamic pressure $=0.4 \times 10^4$Pa

Dynamic pressure = $\frac{1}{2}\rho V^2$

$0.4 \times 10^4 = \frac{1}{2}x10^3 V^2$

V = 2.83m/s

ii) Volume flow rate = $\frac{volume}{time}$

$\frac{volume}{time} = \frac{A\,L}{time} \quad = \frac{A\,v\,t}{t}$

$= 20x10^{-4}x\ 2.83$

$\frac{volume}{time} = 5.66x10^{-3}m^3s^{-1}$

2. Water flows steadily along a uniform flow tube of cross-sectional area $30cm^2$. The static pressure is $1.20x10^5Pa$ and the total pressure is $1.28x10^5Pa$. assuming that the density of water is $1000kgm^{-3}$, calculate the;
   (i) Flow velocity
   (ii) Volume flux
   (iii) Mass of water passing through a section of the tube per second

Solution

(i) $V= \sqrt{\frac{2(totalpressure - static\ pressure)}{\rho}}$

$V= \sqrt{\frac{2(1.28x10^5 - 1.20x10^5)}{1000}} = 4ms^{-1}$

(ii) volume per second = area x velocity

$=30x10^{-4}x4$

Volume flux = $0.012m^3s^{-1}$

(iii) Mass per second= volume per second x ρ

$= 0.012x1000$

Mass per second= $12kgs^{-1}$

3. A pitot – static tube fitted with a pressure gauge is used to measure the speed of a boat at sea. Given that the speed of the boat does not exceed 10m/s and the density of sea water is 1050kgm^{-3}, calculate the maximum pressure on the gauge

Solution

Maximum pressure is the dynamic pressure

Dynamic pressure = $\frac{1}{2}\rho V^2$

= $\frac{1}{2}x1050x10^2$

Dynamic pressure =5.25x10^{4}Pa

Exercise: 28

6. Water flows speedily along a horizontal tube of cross-sectional area 25cm^2. The static pressure with in the pipe is 1.3x10^{5}pa and the total pressure 1.4x10^{5}pa. Calculate the velocity of the water flow and the mass of the water flow past a point in a tube per second. [**an 4.47m/s, 11.175kg/s]**

7. A lawn sprinkler has 20 holes each of cross sectional area 2x10^{-2}cm^2 and its connected to a horse pipe of cross sectional area 2.4cm^2, if the speed of the water in the horse pipe is 1.5m/s, estimate the speed of the water as it emerges from the holes. [**an9m/s]**

8. Water flows speedily along a uniform flow tube of cross section 30cm^2. The static pressure is 1.2x10^{5}Pa and the total pressure is 1.28x10^{5}Pa. Calculate the flow velocity and the mass of water per second flowing past a section of the tube. (Density of water is 1000kgm^{-3}. **)[an 4m/s, 12kg/s]**

9. Air flows over the upper surface of the wings of an aero plane at a speed of 120ms^{-1} and past the lower surfaces of the wings at 110ms^{-1}. Calculate the lift force on the aero plane if it has a total wing area of 20m^2. (density of air $= 1.29kgm^{-3}$) **[an= 2.97x10^{4}N]**

11.4.0: FLUIDS AT REST

11.4.1: DENSITY AND RELATIVE DENSITY

Density of a substance is defined as the mass per unit volume of a substance.

$$\rho = \frac{m}{v}$$

S.I unit's kgm^{-3}

Relative density

Definition

It is the ratio of the density of a substance to density of an equal volume of water at 4^{0}c

It is at 4℃ because water has maximum density of 1000kgm^{-3} at that temperature

$$R.D = \frac{density\ of\ asubstance}{density\ of\ water\ at\ 4℃}$$

$$R.D = \frac{m_s/v_s}{m_w/v_s}$$

$$R.D = \frac{m_s}{m_w}$$

It can also be defined as the ratio of the mass of a substance to mass of an equal volume of water

$R.D = \frac{m_s}{m_w}$ for W = mg $\qquad \frac{w_s/g}{w_w/g} \qquad$ R.D = $\frac{w_s}{w_w}$

It can also be defined as the ratio of weight of a substance to weight of an equal volume of water.

Note: Relative density has no units.

11.4.2: ARCHIMEDE'S PRINCIPLE

It states that when a body is wholly or partially immersed in a fluid, it experiences an up thrust equals to the weight of the fluid displaced.

I.e. Up thrust = weight of fluid = apparent loss of weight of the object in a fluid.

Definition

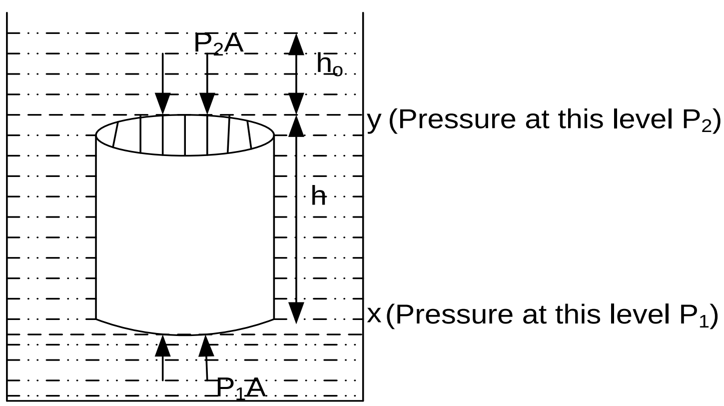
Up thrust is the apparent loss of weight of an object immersed in a fluid

Or

It is the resultant upward force on the body due to the fluid.

11.4.3: Verification of Archimedes' principle using a cylindrical rod

Consider a cylindrical rod of cross-sectional area A and height h immersed in a large quantity of a fluid of density ρ_f such that its top is at level Y, h_o meters below the surface of the fluid while its bottom is at level X shown below



Volume of fluid displaced = volume of cylinder

= Ah

Mass of fluid displaced = Ahρ_f

Weight of fluid displaced = Ahρ_f g………………………….(i)

The fluid exerts forces of P_1A and P_2A on the bottom and top faces of the cylinder.

The up thrust (resultant upward force due to the fluid is therefore given by

$Upthrust = P_2A - P_1A$

$Upthrust = (h + h_o)\rho_f\ gA - h_o\rho_f\ gA$

$Up\ thrust = Ah\rho_f\ g$………………………………………….(ii)

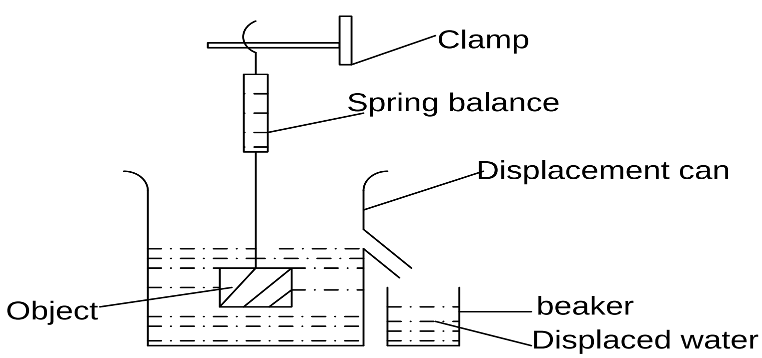
From equation (i) and equation (ii), therefore;

$Upthrust$ = weight of fluid displaced

Question: Show that the weight of fluid displaced by an object is equal to up thrust on the object

11.4.4: Verification of Archimedes' principle using a spring balance.

- Fill the displacement can with water till water flows through the spout and wait until the water stops dripping.
- Weigh a solid object in air using a spring balance and record its weight W_a
- Place a beaker of known weight beneath the spout of the can.
- With the help of the spring balance, the solid object is carefully lowered into the water in the displacement can and wait until water stops dripping when it is completely immersed, its weight (apparent weight) is then read and recorded from the spring balance as W_w.
- Re weigh the beaker and the displaced water and record the weight as $W_{(b+w)}$



Results

Let the weight of the empty beaker be W_b

Weight of water displaced = weight of (beaker +water) – weight of beaker

Weight of water displaced = $W_{(b+w)} - W_b$…………………………..1

Apparent loss of weight of object = weight of object in air – weight of object in water

Apparent loss of weight of the object = W_a - W_w

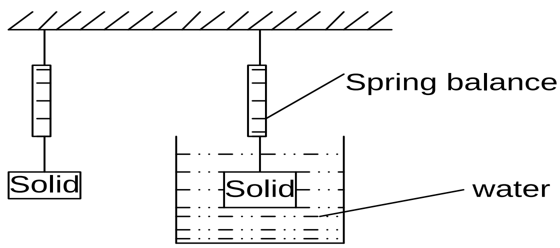
If (W_a - W_w) = ($W_{(b+w)} - W_b$), then Archimedes's principle is verified

11.4.5: Application of Archimedes' principle

It can be used to determine density and relative density of a solid and a liquid.

a) **Determination of density and relative density of a solid**

- Weigh the solid in air and its weight (W_a) recorded using a spring balance.
- Immerse the solid wholly in water and record its apparent weight from balance (W_w)



- Up thrust in water = W_a - W_w
- $R.D = \frac{Weight\ of\ a\ substance}{Up\ thrust\ in\ water}$
- $R.D = \frac{W_a}{W_a - W_w}$
- Density of solid = RD of solid x density of water

Example

1. An object suspended from the spring balance is found to have a weight of 4.92N in air and 3.87N when immersed in water. Calculate the density of the material from which the object is made of the density of water is 1000kgm^{-3}

Solution

W_a = 4.92, W_w = 3.87N

$$R.D = \frac{W_a}{W_a - W_w}$$

$$R.D = \frac{4.92}{4.92 - 3.87}$$

RD = 4.686

Density of substance = RD x ρ of water

= 4.686 x 1000

= 4686kgm^{-3}

Exercise : 29

1. A piece of glass weighs 0.5N in air and 0.30N in water. Find the density of the glass.
   An[2500kgm^{-3}]

2. A spherical stone has a mass of 1.546kg, if its radius is 20cm. find the density of the stone in
   (i) $g\ cm^{-3}$ (ii) $kg\ m^{-3}$
   An [46.848 $g\ cm^{-3}$, 4.6848 $kg\ m^{-3}$]

3. What is the mass of the sphere of diameter 20cm if its relative density is 14.1
   An[59.22kg]

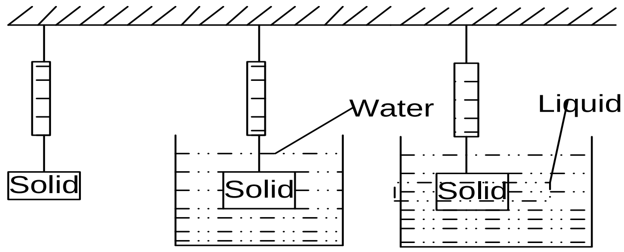
4. A glass block weighs 25N in air. When wholly immersed in water, the block weighs 15N. calculate
   i. The up thrust on the block
   ii. The density of the glass in $kg\ m^{-3}$ **An[10N, 2500 $kg\ m^{-3}$]**

b) Density of a floating object (eg cork)

The same experiment as above except the solid and the sinker are fixed together so that both become totally immersed in water.

c) Determination of density and relative density of a liquid

- Weigh a solid (sinker) in air and record its weight W_a using a spring balance.
- Immerse the solid (sinker) wholly in water and record the apparent weight W_w
- Wipe the surface of the solid (sinker) with a piece of dry cloth and immerse it wholly in the liquid whose relative density is to be measured, read and record its apparent weight in the liquid W_L



Weight of water displaced (up thrust in water) = W_a - W_w

Weight of liquid displaced (up thrust in liquid) = W_a - W_L

Relative density = $\frac{upthrust\ in\ Liquid}{upthrus\ in\ water}$

- $R.D\ of\ the\ liquid\ = \frac{W_a - W_L}{W_a - W_w}$

Density of liquid = R.D of liquid x density of water

Example

1. A solid has a weight of 160N in air and 120N when wholly immersed in a liquid of relative density 0.8, determine the density of a solid

Solution

R.D of Liquid =

$\frac{Weight\ of\ solid\ in\ liquid}{Weight\ of\ equal\ volume\ of\ water}$

0.8 = $\frac{160-120}{Weight\ of\ solid\ in\ water}$

Weight of solid in water = $\frac{40}{0.8}$ = 50N

$R.D\ of\ solid = \frac{weight\ of\ solid\ in\ air}{weight\ of\ solid\ in\ water}$

$R.D\ of\ solid = \frac{160}{50}$

R.D of solid= 3.2

Density of a solid = RD of solid x $\rho$of water

= 3.2 x 1000

Density of a solid = 3200 kgm^{-3}

2. A piece of iron weighs 555N in air when completely immersed in water, it weighs 530N and weighs 535N when completely immersed in alcohol. Calculate the relative density of alcohol and the density of alcohol.

Solution

$W_a = 555$N $W_w = 530$N

W_L=535N

R.D of alcohol $= \frac{W_a - W_L}{W_a - W_w} = \frac{555-535}{555-530}$

R.D of alcohol =0.8

Density of alcohol = R.D of alcohol x ρ of H_2O

$= 0.8\ x1000$

Density of alcohol = $800kgm^{-3}$

3. A string supports a solid iron of mass 0.18kg totally immersed in a liquid of density 800 $kg\ m^{-3}$. Calculate the tension in the string if the density of iron is 800 $kg\ m^{-3}$

Solution

Weight of iron $= mg = 0.18x9.8 = 1.77N$

Volume of iron= $\frac{mas}{density} = \frac{0.18}{8000} = 2.25x10^{-5}m^3$

Mass of liquid displaced=2.25x$10^{-5}x$8000

$= 0.018kg$

Weight of the liquid displaced = $0.018x9.81$

Weight of the liquid displaced = 0.177N

At equilibrium ; mg = T+U

$1.77 = T + 0.1777$

$T = 1.593N$

4. A specimen of an alloy of silver and gold whose densities are 10.5gcm^{-3} and 18.9cm^{-3} respectively, weigh 3.2g in air and 33.13 g in water. Find the composition by mass of the alloy assuming that there has been no volume change in the process of producing he alloy. Assume that the density of water is 1 g cm^{-3}

Solution

$m_s + m_g = 325.2$……………………………1

R.D of alloy $= \frac{35.2}{35.2-33.13}$ =17

Density of alloy= R.D x density of water

Density of alloy= $17x1$=17 g cm^{-3}

Volume of alloy= $\frac{m}{\rho} = \frac{35.2}{17} = 2.07cm^3$

Volume of alloy= $V_s + V_g$

Volume of alloy= $\frac{m_s}{\rho_s} + \frac{m_g}{\rho_g}$

2.07= $\frac{m_s}{10.5} + \frac{m_g}{18.9}$…………………2

Solving 1 and 2 simultaneously

$m_s = 30.3g\ and\ m_g = 4.9g$

Exercise: 30

1. A block of mass 0.1kg is suspended from a spring balance when the block is immersed in water of density 1000kgm^{-3}, the spring balance reads 0.63N. When the block is immersed in a liquid of unknown density the spring balance reads 0.7N, find
   i) Density of the solid
   ii) Density of the liquid **An [2800kgm^{-3}, 800kgm^{-3}]**

2. An alloy contains two metals X and Y of densities $3.0x10^3 kgm^{-3}$ and $5.0x10^3 kgm^{-3}$ respectively. Calculate the density of the alloy if,

   (i) The volume of X is twice that of Y

   (ii) The mass of X is twice that of Y

   An[(i)=$3.7x10^3 kgm^{-3}$(ii)= $3.5x10^3 kgm^{-3}$]

3. An alloy contains two metals A and B, has a volume of $5.0x10^{-4} m^3$ and a density of $5.6x10^3 kgm^{-3}$. The densities of A and B are $8.0x10^3 kgm^{-3}$ and $4.0x10^3 kgm^{-3}$ respectively. Calculate the mass of A and mass of B. **An [A= 1.6kg, B=1.2kg]**

4. A piece of glass has a mass 62 kg in air. It has a mass of 32kg when completely immersed in water and a mass of 6kg when completely immersed in an acid.

   (a) The glass (b) The acid in $kg\ m^{-3}$

   An[(a)=1550 $kg\ m^{-3}$ (b)= 1400 $kg\ m^{-3}$]

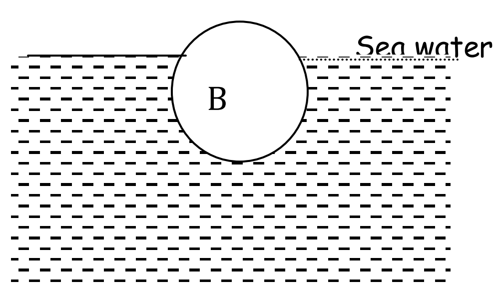
5. A body of mass 0.1kg and relative density 2 is suspended by a thread and completely immersed in a liquid of density 920kgm^{-3}.

   i) Find the tension in the thread. **An[0.53N]**

   ii) If the thread breaks, what will be the initial acceleration? **An [5.3ms^{-2}]**

6. A body of weight 0.52N in air weighs only 0.32N when totally immersed in water while its weight when immersed in another liquid is 0.36N. What is the density of the second liquid if the density of water is 1000kgm^{-3}?

7. The figure below shows a buoy B, of volume 40 litres and mass 10kg. It is held in position in sea water of density 1.04g/cm^3 by a light cable fixed to the bottom so that $\frac{3}{4}$ of the volume of the buoy is below the surface of the sea water.



(i) Name the three forces that keep the buoy in equilibrium and state the direction in which each acts
(ii) Determine the tension in the cable

8. A tank contains a liquid of density 1200kgm^{-3}. A body of volume 5x10^{-3}m^3 and density 900kgm^{-3} is totally immersed in the liquid and attached to by a thread to the bottom of the tank. Find the tension in the thread. **An [14.72N]**

9. A block of metal weighs 50N in air and 25N in water
   (a) Determine the density of the metal in $kg\ m^{-3}$
   (b) Find the weight of the metal in paraffin whose relative density is 0.8

An[2000 $kg\ m^{-3}$, 30N]

11.5.0: FLOATATION

A body floats in a liquid if its density is less than the density of the liquid.

11.5.1: Law of floatation

It states that a floating body displaces its own weight in the fluid in which its floating.

Experiment to verify the law of floatation

(Same as verification of Archimedes' principle)

Note:

1. For a floating body
   - The weight of floating body =weight of fluid displaced
   - The weight of fluid =Up thrust
   - The weight of floating body = Up thrust

- The mass of the floating body = the mass of the fluid displaced
- A floating body sinks deeper in liquids of less density than in liquids of higher densities.

2. Density of a floating body = fraction submersed x density of liquid
3. Volume of displaced liquid = fraction submerged x volume of floating body.

Example

1. A solid weighs 237.5g in air and 12.5g when totally immersed in a fluid of density 0.9g/cm^3. Calculate
   a) Density of the solid.
   b) The density of the liquid in which the solid would float with 1/5 of its volume exposed above the liquid surface.

Solution

a) W_a =237.5g W$_L$ =12.5g

Up thrust in liquid $= W_a - W_L = 237.5 - 12.5$

Up thrust in liquid (mass of liquid displaced)

=225g

Volume of liquid displaced $= \frac{m}{\rho} = \frac{225}{0.9}$

Volume of liquid displaced =250cm^3

Volume of solid =250cm^3

Density of solid = $\frac{Mass\ of\ solid}{volume\ of\ solid} = \frac{237.5}{250}$

=0.95g/cm^3

b) If $\frac{1}{5}$ of its volume is exposed, then $\frac{4}{5}$ of its volume is submerged.

Volume of liquid = fraction x volume of the solid submerged

$= \frac{4}{5} x25$ =200cm^3

Mass of solid =237.5

Density of liquid = $\frac{237.5}{200}$ =1.19g/cm^3

OR

ρ of floating body = fraction submerged x ρ liquid

0.95 = $\frac{4}{5}$ x density of liquid

Density of liquid= $\frac{0.95x5}{4}$=1.19gcm^{-3}

2. A solid of volume 10^{-4}m^3 floats in water of density 10^{3}kgm^{-3} with $\frac{3}{5}$ of its volume submerged
   i) Find the mass of the solid

ii) If the solid floats in another liquid with $\frac{4}{5}$ of its volume submerged. What is the density of the liquid?

Solution

a) V=10^{-4} ρ_ω =1000kgm^{-3}

Volume submerged $= \frac{3}{5}$

Volume of water displaced $= \frac{3}{5} x$ volume of solid = $\frac{3}{5} x$ 10^{-4} $= 6x10^{-5}\, m^3$

mass of displaced water = volume of water displaced x density of water = $6\text{x}10^{-5} x\ 1000$

$= 6x10^{-2}\, kg$

By law of floatation, mass of water displaced is equals to the mass of the solid

∴ Mass of solid = $6x10^{-2}\, kg$

b) Fraction submerged $= \frac{4}{5}$

Density of solid = $\frac{mass\ of\ solid}{volume\ of\ solid} = \frac{6\ x\ 10^{-2}}{10^{-4}}$ = 600kgm^{-3}

Density of solid = fraction submerged x density of liquid

600 $= \frac{4}{5}\, x$ density of liquid

Density of liquid = 750kgm^{-3}

Exercise: 31

1. A Ball with a volume of 32cm^3 floats on water with exactly half of the ball below the surface. What is the mass of the ball (density of water =1.0x10^{3}kgm^{-3}) **An [1kg]**

2. An object floats in a liquid of density 1.2x10^{3}kgm^{-3} with one quarter of its volume above the liquid surface. What is the density of the object. **An[900kgm^{-3}]**

3. A solid weighs 237.g in air and 212.5g when totally immersed in a liquid of density0.9gcm^{-3}. Calculate the;

   (i) Density of the solid

(ii) Density of a liquid in which the solid would float with $1/5$ of tis volume exposed above the liquid surface. **An[9500 kgm^{-3}. 1190 kgm^{-3}].**

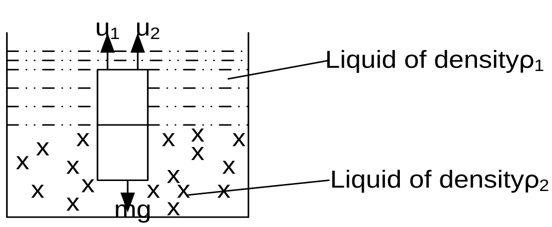
4. Object with a volume of $1.0x10^{-5}m^3$ and density $4.0x10kgm^{-3}$ floats on water in a tank of cross sectional area $1.0x10^{-3}m^2$
    a) By how much does the water level drop when the object is removed
    b) Show that this decrease in water level reduces the force on the base of the tank by an equal amount to the weight of the (density of water = $1.0x10^3kgm^{-3}$) **An [$4x10^3kgm^{-3}$]**

5. A block of wood floats in water of density 1000 kgm^{-3} with $2/3$ of its volume submerged . in oil it has $9/10$ of its volume submerged . find the densities of wood and oil
An[740.74 kgm^{-3}, 666.67 kgm^{-3}].

6. A piece of metal of mass 2.60g and density 8.4 g cm^{-3} is attached to a block of wax of 1.0 g and density 0.92 g cm^{-3}. When the system is placed in a liquid, it floats with wax just submerged. Find the density of the liquid. **An [$1.13x10^{-6}$ g cm^{-3}]**

11.5.2: RELATION BETWEEN DENSITIES AND VOLUME FOR AN OBJECT FLOATING IN TWO LIQUIDS

Consider an object which floats at the interface between two immiscible liquids of density ρ_1 and ρ_2 with the objects having a density of $\boldsymbol{\rho}$



V_1 volume submerged to liquid of density ρ_1

V_2 e volume submerged to liquid of density ρ_2

U_1 the upthrust in liquid of density ρ_1

U_2 the upthrust in liquid of density ρ_2

Weight of object = Total upthrust

$$mg = U_1 + U_2$$

$$\rho(V_1 + V_2)g = \rho_1 V_1 g + \rho_2 V_2 g$$

$$\rho V_1 g + \rho V_1 g = \rho_1 V_1 g + \rho_2 V_2 g$$
$$\rho V_1 g - \rho_1 V_1 g = \rho_2 V_2 g - \rho V_2 g$$
$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

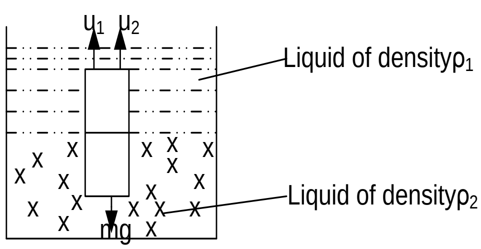
Note

When the object is replaced by a denser one, then the new object will sink much deeper and therefore V_1 decreases and V_2 increases by the same magnitude.

EXAMPLE: UNEB 2006 Q.4 (iii)

A block of wood floats at an interface between water and oil with 0.25 of its volume submerged in the oil. If the density of the wood is 7.3x10^{2}kgm^{-3} . Find the density of the oil.

Solution



$V_1 = 0.75$ $V_2 = 0.25$

ρ =730kgm^{-3} ρ_1 =1000kgm^{-3} ρ_2 = ?

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$
$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$
$$\frac{0.75}{0.25} = \frac{\rho_2 - 730}{730 - 1000}$$
$$3x(730 - 1000) = \rho_2 - 730$$
$$\rho_2 = -810 + 730$$
$\rho_2 = -80$kgm^{-3}

ρ_2 =80kgm^{-3}

Applications of law of floatation

1- Balloons 2- Ships 3- Submarines

- **Balloons**

A balloon filled with a light gas such as hydrogen gas rises up because the weight of the displaced air is greater than the weight of the balloon plus its content. It's the net upward force (up thrust) which pushes the balloon upwards and the balloon continues rising until the up thrust acting on it becomes equal to the weight of the balloon plus its content then it begins floating.

$\therefore$ U $= W_b + W_h + W_L$

$V_a\, \rho_a$g $= M_b$g+ $V_h \rho_h$g+ M_Lg

U=Up thrust

W_h =weight of hydrogen

W_b =weight of balloon

W_L =weight of load

M_b=mass of balloon M_L=mass of load V_a =volume of air	V_h=volume of hydrogen ρ_a =density of air	ρ_h =density of hydrogen

Note

Volume of air displaced = volume of balloon

$$V_a = V_b$$

EXAMPLES

1. A balloon has a capacity of 10m^3 and is filled with hydrogen. The balloon's fabric and the container have a mass of 1.25kg. Calculate the maximum mass the balloon can lift .

   [ρ=0.089kgm-^{3}, ρ of air=1.29 kgm-^{3}]

   Solution

   V_b=10m^3 ρ_h =0.089, ρ_a =1.29 kgm-^{3}.

   M_b=1.25 V_a=10m^3 V_b=10m^3

   But up thrust = weight of balloon + weight of hydrogen +load

   U= $W_b + W_h + W_L$

   $V_a \rho_a g = M_b g + V_h \rho_h g + M_L g$

   10x1.29g=1.25g+10x0.089+ $M_L g$

   M_L =10.76kg

2. A hot air balloon has a volume of 500m^3. The balloon moves upwards at a constant speed in air of density 1.2kgm^{-3} when the density of the hot air inside it is 0.80kgm^{-3}.
   a) What is the combined mass of the balloon and the air inside it.
   b) What is the upward acceleration of the balloon when the temperature of the air inside it has been increased so that its density is 0.7kgm^{-3}.

   Solution

V_b=500m^3 V_h=500m^3 V_a=500m^3 ρ_a =1.2kgm^{-3} ρ_h =0.8kgm^{-3}	U=W_b+W_h+W_L $V_a \rho_a g$ = $M_b g$+ $V_h \rho_h g$+ $M_L g$

$V_a\,\rho_a g = (M_b + M_L)g + V_h\rho_h g$

$500x1.2g = (M_b + M_L)g + M_h x g$

$500x1.2 = (M_b + M_L + M_h)$

$600 = (M_b + M_L + M_h)$

Combined mass = 600kg

b) V_b=500m^3 V_h=500m^3 V_a=500m^3

ρ_a =1.2kgm^{-3}, ρ_h =0.8kgm^{-3}

At equilibrium : $U = W_b + W_h + W_L$

$V_a\,\rho_a g = M_b g + V_h\rho_h g + M_L g$

$500x1.2x9.81 = (M_b + M_L)x9.81 + 500x0.8x9.81$

$(M_b + M_L)$=200kg

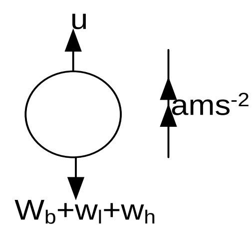
when ρ_h =0.7kg/m^3, V_h=500

$W_h = V_h\rho_h g = 500x0.7x9.81 = 3433.5N$

$(W_b + W_L) = (M_b + M_L)x9.81$

$W_b + W_L = 200x9.81 = 1962N$

U= $V_a\,\rho_a g = 500x1.2x9.81 = 5886\text{N}$



$U - (W_b + W_h + W_L) = ma$

$5886 - (1962 + 3433.5) = 600xa$

$a = 0.82ms^{-2}$

11.6.0: PRESSURE

The pressure acting on a surface is defined as the force per unit area acting normally on the surface

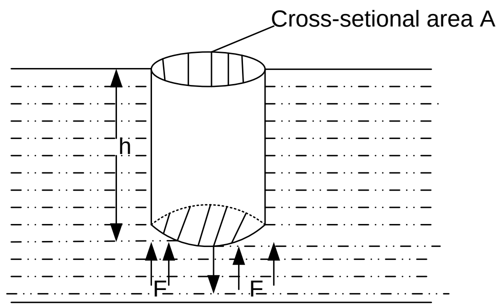
$$P = \frac{F}{A}$$

PRESSURE IN FLUIDS

The pressure in a fluid increased with depth, and all points at the same depth in the fluid are at the same pressure.

11.6.1: RELATION OF PRESSURE P WITH DEPTH h

Consider a cylindrical region of cross sectional area A and height h in a fluid of density ρ



The top of the cylinder is at the surface of the fluid and the vertical forces acting on it are its weight (mg) and an upward force F due to press p at the bottom of the cylinder.

The cylinder is in equilibrium and therefore

$F = mg$-------------------------[1]

But: $m = v\rho$ and $v = Ah$

F= Ahρg----------------------------- [4]

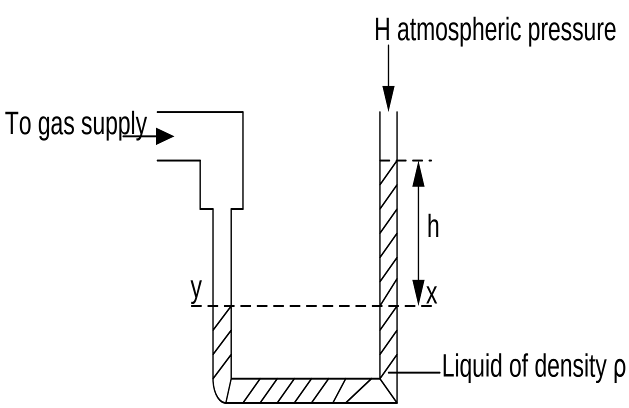
But P= $\frac{F}{A} = \frac{A\,h\,\rho\,g}{A}$

$P = h\rho g$

11.6.2: PRESSURE OF A GAS [U-TUBE MANOMETER]

This consists of a U-shaped tube containing a liquid. It is used to measure pressure.

The pressure to be measured (i.e. that of a gas) is applied to one arm of the manometer and the other arm is open to the atmosphere.



The gas pressure p is the same at the pressure at y

But pressure at y = pressure at x

$P = H + h\rho g$

Where $H = 1.01x10^5 Pa$

Or H=760mmHg

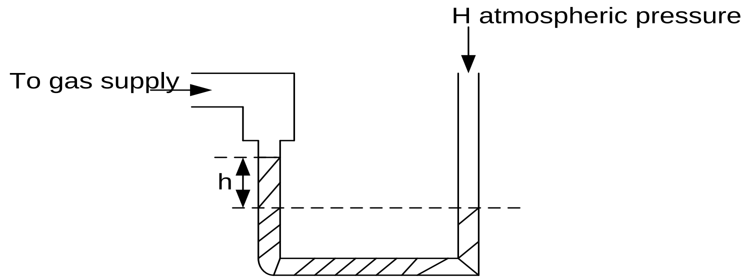
Or H=76cmHg

Note

The pressure recorded by the manometer (hρg) is known as gauge pressure. The actual pressure

($H +\ h\rho g$) is called absolute pressure.

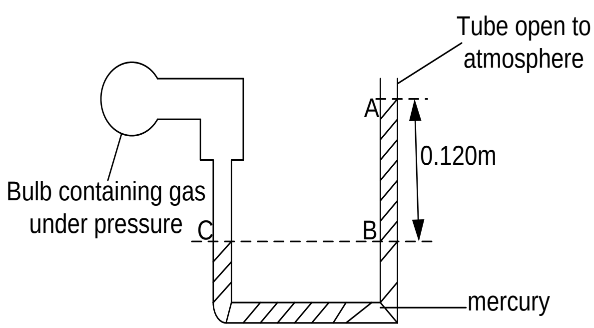
Suppose the level of the liquid in open limb of the manometer is lower than the level of the other side connected to a gas.



Pressure of gas $P = H - h\rho g$

Examples

1. Calculate the pressure of the gas in the bulb [Atmospheric pressure = 1.01x10^5Pa] density of mercury =1.30x10^4kgm^{-3} g=9.81ms^{-2}] Given the figure below;



Solution

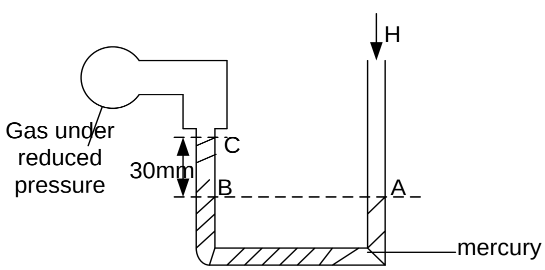
Pressure at C = pressure at B

Pressure at C =$H + h\rho g$

=1.01x10^5+$(0.12x1.36x10^4x9.81)$

Pressure of gas = 1.17x10^5Pa

2. Using the diagram below, calculate the pressure of the gas in the bulb. (atmospheric pressure =760mmHg)



Pressure at B = pressure at A

=760mmHg

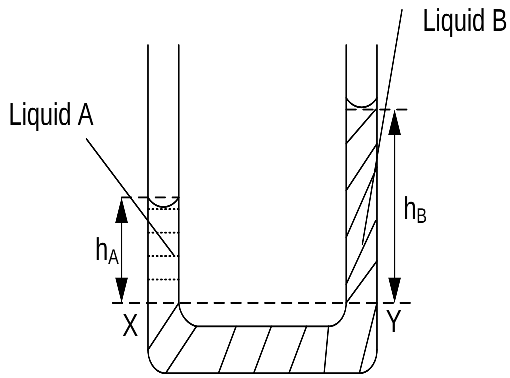
Pressure at C = (H-h)

Pressure at C $= 760 - 30 = 730mmHg$

Gas pressure $= 730mmHg$

11.6.3: DENSITY OF A LIQUID [U-TUBE MANOMETER]

It uses two immiscible liquids



The pressure P_x at X is equal to atmospheric pressure H plus the pressure exerted by the height h_A of liquid A i.e.

P_x= H+ $h_A \rho_A g$

Where ρ_A is the density of liquid A

Similarly at Y

P_y = H+ $h_B \rho_B g$

Where ρ_B is the density of liquid B

Since x and y are at the same level

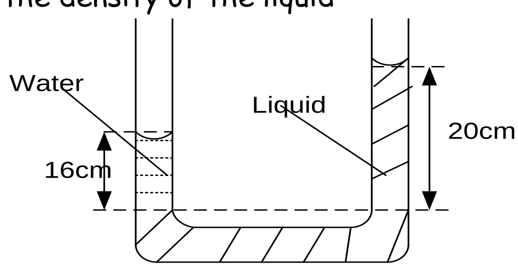
$P_x = P_y$

H+ $h_A \rho_A g$ = H+ $h_B \rho_B g$

$$\boxed{h_A\rho_A = h_B \rho_B}$$

Example

Find the density of the liquid



Solution

$$h_w\rho_w = h_L\rho_L$$

$$\frac{16}{100}x1000 = \frac{20}{100}x\rho_L$$

ρ_L = 800kgm^{-3}

UNEB 2014Q.4

(a)Define coefficient of viscosity and state its units

(02marks)

(b) Explain the origin of viscosity in air and account for the effect of temperature on it (05marks)

(c)Describe, stating the necessary precautions an experiment to measure the coefficient of viscosity of a liquid using Stoke's law

(07marks)

(d) A steel ball bearing of diameter 8.0mm falls steadily through oil and covers a vertical height of 20.0cm in 0.56 s. if the density of the steel is 7800kgm^{-3} and that of oil is 900 kgm^{-3}. Calculate:

(i) Up thrust on the ball **An** $2.37x10^{-3}N$ (03 marks)

(ii) Viscosity of oil **An** $0.674Nsm^{-2}$ (03 marks)

UNEB 2013 Q.2

(a)Define terminal velocity. (01mark)

(b) Explain laminar flow and turbulent flow. (03marks)

(c)Describe an experiment to measure the coefficient of viscosity of water using Poiseulle's formula. (07marks)

(d) (i) State Bernoulli's principle. (01marks)

(ii) Explain why a person standing near a railway line is sucked towards the railway line when a fast moving train passes. (03marks)

(e)A horizontal pipe of cross-sectional area 0.4 m^2, tapers to a cross-sectional area of 0.2 m^2. The pressure at the large section of the pipe is 8.0 x $10^4 N\,m^{-2}$ and the velocity of water through the pipe is 1.2 m s^{-1}. If atmospheric pressure is 1.01 x$10^5 N\,m^{-2}$, find the pressure at the small section of the pipe. **An[9.884x$10^4 N\,m^{-2}$,]** (05marks)

UNEB 2012 Q 4

a) i) What is meant by the following terms steady flow and viscosity. (02marks)

ii) Explain the effect of increase in temperature on the viscosity of a liquid. (03marks)

b) i) Show that the pressure p exerted at a depth h below the free surface of a liquid of density ρ

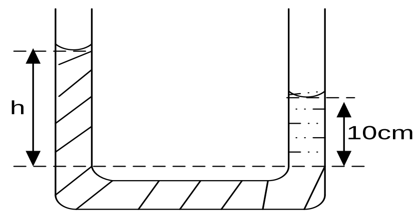
is given by $P = h\rho g$

(03marks)

ii) Define relative density

(01mark)

iii) A U-tube whose ends are open to the atmosphere, contains water and oil as shown below.



Given that the density of oil is 800kgm^{-3}, find the value of h. **An [12.5cm]**

UNEB 2011 Q 3

a) i) What is meant by viscosity.

(01mark)

ii) Explain the effect of temperature on the viscosity of a liquid.

(03marks)

b) Derive an expression for the terminal velocity of a sphere of radius a, falling in liquid of viscosity η (4mks)

c) Explain why velocity of a liquid at a wide part of tube is less than that at a narrow part. (2mks)

UNEB 2010 Q 3

a) Define viscosity of a fluid (01mark)

b) i) Derive an expression for the terminal velocity attained by a sphere of density δ, and radius a, falling through a fluid of density ρ and viscosity η

(05marks)

ii) Explain the variation of the viscosity of a liquid with temperature.

(02marks)

UNEB 2009 Q 4

a) i) State Archimedes principle
(01mark)

ii) A tube of uniform cross sectional area of $4x10^{-3}m^2$ and mass 0.2kg is separately floated vertical in water of density $1000kgm^{-3}$ and in oil of density $800kgm^{-3}$. Calculate the difference in the
lengths immersed. **An [$1.25x10^{-2}$m]**
(04marks)

UNEB 2006 Q 4

a) i)State Archimedes principle
(01mark)

ii) Describe an experiment to determine relative density of an irregular solid which floats in water.

UNEB 2005 Q 3

a) What is meant by the following terms
   i) Velocity gradient (01mark)
   ii) Coefficient of viscosity
   (01mark)

b) Derive an expression for the terminal velocity of a steel-ball bearing of radius r and density ρ falling through a liquid of density σ and coefficient of viscosity η.
(05marks)

d) Explain with the aid of a diagram why air flow cover the wings of an air craft at take-off causes a lift.
(03marks)

UNEB 2003 Q 3

a) State the law of floatation . (01mark)

b) With the aid of a diagram describe how to measure the relative density of a liquid using Archimedes principle and the principle of moments. **An [refer to Abbot Pg 133]** (06marks)

c) A cross sectional area of a ferry at its water line is $720m^2$. If sixteen cars of average mass 1100kg are placed on board, to what extra depth will the boat sink in the water. **An[$2.4x10^{-2}$m]** (04marks)

UNEB 2002 Q 3

a) i) Show that the weight of fluid displaced by an object is equal to the up thrust on the object. (5mks)

ii) A piece of metal of mass $2.60x10^{-3}$kg and density $8.4x10^{3}kgm^{-3}$ is attached to a block of wax of mass $1.0x10^{-2}$ kg and density $9.2x10^{2}kgm^{-3}$. When the system is placed in a liquid, it floats with wax just submerged. Find the density of liquid. (04marks)

b) Explain the

i) Terms laminar flow and turbulent flow (04marks)

ii) Effects of temperature on the viscosity of liquids and gases (03marks)

c) i) Distinguish between static pressure and dynamic pressure (02marks)

Solution

a) ii) By law of floatation, a floating body displaces its own weight

Mass of liquid displaced = ($2.60x10^{-3}$+ $1.0x10^{-2}$)

= $1.26x10^{-2}$kg

Volume of liquid displaced = $\frac{2.6x10^{-3}}{8.4x10^{3}} + \frac{1x10^{-2}}{9.2x10^{2}}$.

$= 1.12x10^{5}m^{3}$

ρ of liquid = $\frac{mass\ of\ liquid\ displaced}{volume\ of\ liquid\ displaced}$

$=\frac{1.26\ x\ 10^{-2}}{1.12\ x\ 10^{-5}}$

ρ of liquid = $1.13x10^{3}kg\ m^{-3}$

UNEB 1998 Q 4

a) i) Distinguish between laminar and turbulent flow

(02marks)

ii) What are the origins of viscosity in liquid

(02marks)

iii) Explain the temperature dependence of viscosity of a liquid.

(02marks)

b) i) State Bernoulli's principle

ii) Account for the variation of pressure and velocity of a liquid flowing in a horizontal pipe of

varying diameter.

(04marks)

CHAPTER 12: SURFACE TENSION

The surface of a liquid behaves like an elastic skin in a state of tension.

It is responsible for the following observations;

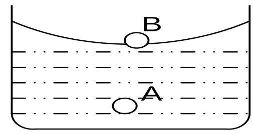
1- A needle floating on an undisturbed water surface though made of material which is denser than water

2- Some insects walk on water surface without sinking

3- Drops of water remaining suspended and becoming nearly spherical when falling from a tap

4- Mercury gathering into small droplets when spilt

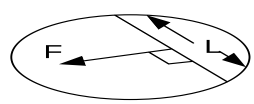
12.1.0: Molecular explanation for existence of surface tension



- Liquid molecules attract each other. In the bulk of the liquid the resultant force on any molecule such as A is zero.
- A surface molecule such as B is subjected to intermolecular forces of attraction below therefore potential energy of surface molecules exceeds that of the interior. Average separation of the surface molecules is greater than that of molecules in the interior. At any point on a liquid surface there is a net force away from that point and this makes the surface behave like an elastic skin in a state of tension. This accounts for surface tension.

Definition

Surface tension coefficient $\gamma$of a liquid is defined as the force per unit length acting in the surface and perpendicular to one side of an imaging line drawn in the surface.



$\gamma = \frac{F}{L}$

Units of γ are Nm^{-1}

Dimensions of γ

$\gamma = \frac{F}{L}$

$[\gamma] = \frac{[F]}{[L]} = \frac{M\,L\,T^{-2}}{L}$

$[\gamma] = M\,T^{-2}$

Other units of γ are kgs^{-2}

12.1.2: Factors affecting surface tension

i) Temperature

When the temperature of a liquid is increased, the liquid molecules gain kinetic energy and the molecules become more free to move and rush to the surface. The number of molecules in the surface increase, potential energy of the surface molecules is lowered and the separation of molecules decreases leading to a reduction in the intermolecular attraction, this reduces tension energy of molecules and hence surface energy tension is also reduced.

ii) Impurities

Impurities detergents and soap get between the molecules of the liquid reducing the intermolecular forces between the liquids and hence reducing surface tension

iii) Nature of the liquid

Different liquids have different surface tension

12.1.3: SHAPES OF LIQUID SURFACE

The surface of a liquid must be at right angles to the resultant force acting on it otherwise there would be component of this force parallel to the surface which would cause motion. Normally a liquid surface is horizontal i.e. at right angles with the force of gravity but where it's in contact with the solid it's usually curved.

The particular form that this curvature takes is determined by the strengths of what are called the **cohesive** and **adhesive** forces.

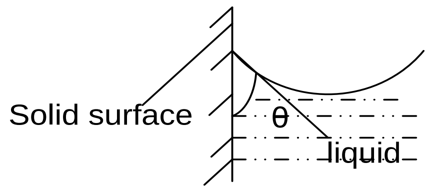
Definition

Cohesive force is the attractive force exerted on a liquid molecules by the neighboring liquid molecules.

Adhesive forces is the attractive force exerted on a liquid molecule by the molecules in the surface of the solid.

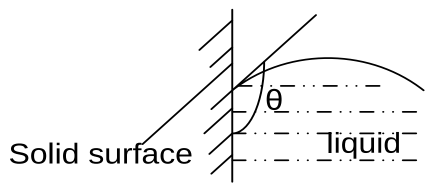
Consider a liquid in a container with vertical sides

- If the adhesive force is large comparative with the cohesive force, the liquid tends to stick to the wall and so has a concave meniscus (curves upwards).



e.g water and glass

- If the cohesive force is large compared with adhesive , the liquid surface pulls away from the wall and the meniscus is convex(curves downwards)



e.g mercury and glass

12.14: ANGLE OF CONTACT θ

This is the angle between the solid surface and the tangent plane to the liquid surface at the point where it touches the solid measured through the liquid.

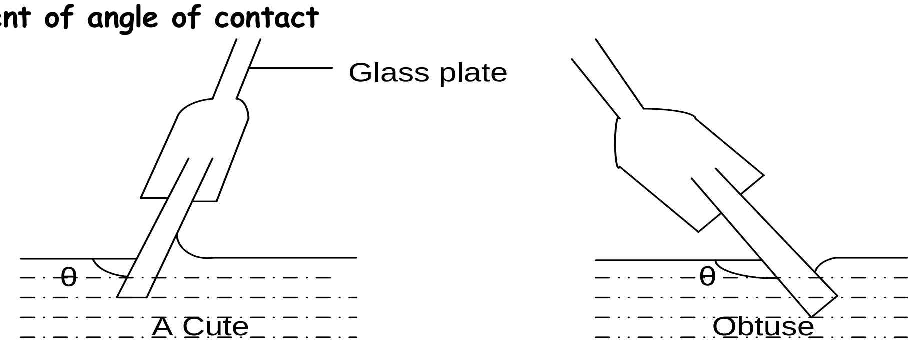
From the diagrams above, the meniscus is concave when θ is less than 90^0 and is convex when θ is greater than 90^0.

A liquid is said to wet a surface with which its angle of contact is less than 90^0.

The angle of contact of water and clean glass is **zero**, and that between mercury and clean glass is **137**°. Thus water wets clean glass, mercury does not.

Addition of a detergent to a liquid lowers its surface tension and reduces the contact angle.

Measurement of angle of contact

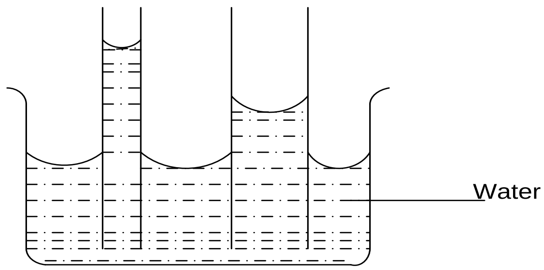


A clean glass plate is placed at varying angles to a liquid until the surface on one side of the plate remains horizontal. The angle θ made between the horizontal surface and the plate is the angle of contact.

12.3.0: CAPILLARITY

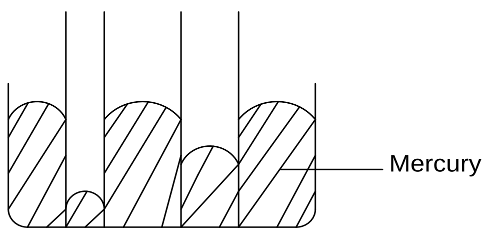
When a capillary tube is immersed in water and the plane vertical with one end of water. Water rises to a height above the surface of water in the container. This is due to the fact that adhesive forces are greater than the cohesive forces.

The narrower the tube, the greater is the height to which water rises.



If the capillary tube is dipped inside mercury liquid is depressed below the outside level. This is because the cohesion of mercury is greater than the adhesion of mercury and glass.

The depression of the tube increases with decreases the diameter of the tube



Definition

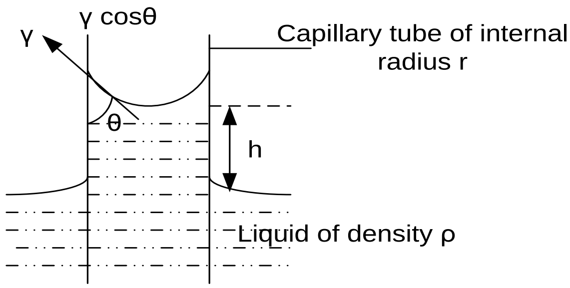
Capillarity: Is the rise or fall of a liquid in a capillary tube

12.3.1: Capillary rise

Around the boundary where the liquid surface meets the tube, surface tension forces exert a downward pull on the tube since they not balanced by any other surface tension forces.

The tube therefore exerts an equal but upwards force on the liquid which forces it to rise.

The liquid stops rising when the weight of the raised column acting downwards equals to vertical component of the upward force exerted by the tube in the liquid.



Force acting upwards F = γcos θ x L

But L=2πr

F = γ cos θx2πr ------------[1]

Weight W= mg =Vρg

W =Ahρg

W= πr^{2}hρg ----------------- [2]

At equilibrium

Weight = vertical component of surface tension

W=F

πr^{2}hρg= πr

h= $\frac{2\,\gamma\,cos\theta}{r\,\rho\,g}$

12.3.2: Capillary depression

Consider mercury inside a tube and the angle of contact θ

$$P_2 - P_1 = \frac{2\,\gamma\,cos\theta}{r}$$

But P_1 = H(atmospheric)

$P_2 - H = \frac{2\,\gamma\,cos\theta}{r}$

$P_2 = \frac{2\,\gamma\,cos\theta}{r} + H$ ---------------------------

-- [1]

Also: $P_2 = H + h\rho g$-----------------------

-- [2]

Equating

$H + h\rho g = \frac{2\,\gamma\,\cos\theta}{r} + H$

hρg = $\frac{2\,\gamma\,cos\theta}{r}$

$$\boxed{h = \frac{2\,\gamma\,cos\theta}{r\,\rho\,g}}$$

Example

1. A clean glass capillary tube of internal diameter 0.04cm is held with its lower end dipped in water contained in a beaker and with 12cm of the tube above the surface of water.
   i) To what height will water rise in the tube.
   ii) What will happen if the tube is now depressed until only 4cm of its length is above the surface.

(γ of water =7.0x10^{-2}Nm^{-1}, ρ of water =1000kgm^{-3})

Solution

i) Using $h = \frac{2\,\gamma\cos\theta}{r\,\rho\,g}$

But for a clean glass of water θ =0

$$h= \frac{2\ x\ 7x\ 10^{-2}\ cos0}{\left(\frac{0.04\ x10^{-2}}{2}\right) x\ 1000\ x9.81}$$

h=0.071m

ii) If only 4cm of the tube is left above the water surface, this length is less than h in part (i) above so water must change its angle of contact so that it can fit the 4cm

$$h= \frac{2\,\gamma\cos\theta}{r\,\rho\,g}$$

$$4x10^{2} = \frac{2\ x\ 7x\ 10^{-2}\ \cos 0}{\left(\frac{0.04\ x10^{-2}}{2}\right) x\ 1000\ x9.81}$$

$\theta=55.9^{0}$

water forms a new surface with an angle of contact 56^{0}

2. A U-tube is made with an internal diameter of one arm 2.0cm and the other 4mm and mercury is poured in the two tubes. If the angle of contact of mercury with glass after exposure to air is 160^{0}. What will be the difference in level of surface in the tubes, take surface tension of mercury as $0.0472Nm^{-1}$

Solution

$r_1=2x10^{-3}m$ $r_2=1x10^{-2}m$ ρ =$13600kgm^{-3}$(density of mercury)

γ=0.0472 $\theta=180-160^{0}$ $\theta=20^{0}$(we subtracted to obtain a positive value of the $cos\theta$)

Note: we only subtract for angles greater than 90^{0}

$$h_1= \frac{2\,\gamma\cos\theta}{r_1\,\rho\,g} = \frac{2x0.047Cos20}{2x10^{-3}x13600x9.81} = 3.32x10^{-4}m$$

$$h_2= \frac{2\,\gamma\cos\theta}{r_2\,\rho\,g} = \frac{2x0.047Cos20}{1x10^{-3}x13600x9.81} = 6.65x10^{-5}m$$

Difference $=h_1- h_2$

$$=3.32x10^{-4} - 6.65x10^{-5}$$

$$= 2.655x10^{-4}m$$

Exercise: 32

1. A liquid of density 1000k$gm^{-3}$and surface tension $7.26x10^{-2}Nm^{-1}$, dipped in it is a capillary tube with a bore radius of 0.5mm. If the angle of contact is 0^{0} determine,
   i) the height of the column of the liquid rise

ii) if the tube is pushed until its 2cm above the level of the liquid, explain in what happen **An[$2.96x10^{-2}$m, 47.5^0]**

2. The two vertical arms of manometer containing water, have different internal radii of 10^{-3}m and $2x10^{-3}$m respectively. Determine the difference in height of the two liquids levels when the arms are open to the atmosphere. (surface tension and density of water are $7.2x10^{-2}Nm^{-11}$ and 10^3kgm^{-3} respectively)
An[$7.14x10^{-3}$m]

3. The end of a clean glass capillary tube having internal diameter 0.6mm is dipped into a beaker containing water, which rises up the tube to a vertical height of 5.0cm above the water surface in the beaker. Calculate the surface tension of water (Density of water =1000kgm^{-3}, g=10ms^{-2}).
What would be the difference if the tube were not perfectly clean so that the water did not wet it, but had an angle of contact of 30^0 with the tube surface.
An[7.5x10Nm^{-1}, the water would rise to only 4.3cm]

4. A capillary tube which is clean is immersed in water of surface tension $7.2x10^{-2}Nm^{-1}$ and water rises 6.2cm in the capillary tube. What will be the difference in the mercury level, if the same capillary tube is immersed in the mercury (surface tension of mercury = 0.84 Nm^{-1}, angle of contact between mercury and glass =140^0, ρ of mercury =$1.36x10^4kgm^{-3}$, ρ of water =10^3kgm^{-33}) **An[h=4.2cm]**

5. Mercury is poured into glass U-tube with vertical limbs of diameters 2.0mm and 12.0mm respectively. If the angle of contact between mercury and the glass is 140° and the surface tension of mercury is 0.52Nm^{-1}, calculate the difference in the levels of mercury. (density of mercury is $1.36x10^4kgm^{-3}$) **An($4.9x10^{-3}m$)**

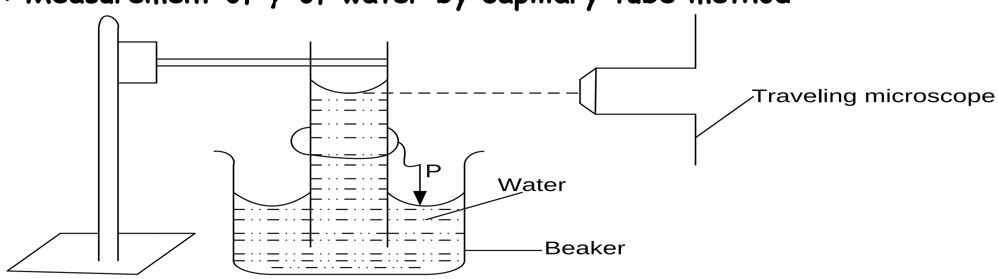
6. A U-tube with limbs of diameter 7mm and 4mmm contains water of surface tension $7x10^{-2}Nm^{-1}$, angle of contact $0°$ and density 1000kgm^{-3}. Find the difference in the levels.
   ***An* $\mathbf{3.1mm}$**

7. A glass U-tube is such that the diameter of one limb 4.0mm while that of the other is 8.0mm. the tube is inverted vertically with the open ends below the surface of water in a beaker. Given that surface tension of water is $7.2x10^{-2}Nm^{-1}$, angle of contact between water and glass is zero, and that density of water is 1000kgm^{-3}. What is the difference between the heights to which water rises in the two limbs.
   ***An* $\mathbf{7.34mm}$**

8. Calculate the height to which the liquid rises in the capillary tube of diameter 0.4mm placed vertically inside
   (i) A liquid of density $800kgm^{-3}$ and surface tension $5x10^{-2}Nm^{-1}$ and angle of contact 30°
   (ii) Mercury of angle of contact 139° and surface tension 0.52Nm^{-1}

   An[0.032m, 0.0294m]

12.4.0: Measurement of γ of water by capillary tube method



- A clean capillary tube is dipped in water as shown and a wire p which is bent is tied along the capillary tube with a rubber band.
- When the tube is dipped into water, the wire p is adjusted so that its top just touches the surface of the water.
- A travelling microscope is focused on the water meniscus in the capillary tube and the reading noted, say h_1.

- ❖ The beaker is then removes and the travelling microscope is focused on the tip of the wire p and scale reading h_2 is noted.
- ❖ The height of the water rise $h=/h_1-h_2/$.
- ❖ The capillary tube is removed and its diameter and hence radius , r is determined by using a travelling microscope. The surface tension can be obtained from ;

$$h=\frac{2\,\gamma\cos\theta}{r\,\rho\,g}$$

$$\gamma=\frac{h\,r\,\rho\,g}{2\cos\theta}$$

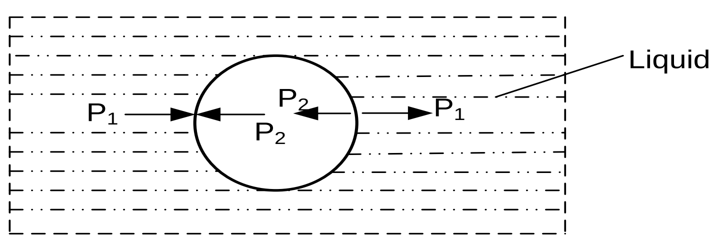
$\gamma=\frac{h\,r\,\rho\,g}{2}$ for clean glass of water $\theta=0^0$

12.2.0: PRESSURE DIFFERENCE ACROSS A SPHERICAL INTERFACE

The pressure inside a soap bubble is greater than the pressure of the air outside the bubble. If this were not so, the combined effect of the external pressure and the surface tension forces in the soap film would cause the bubble to collapse, similarly the pressure inside an air bubble in a liquid exceeds the pressure in the liquid and the pressure inside a mercury drop is greater than that outside it.

12.2.1: Pressure difference across an air bubble

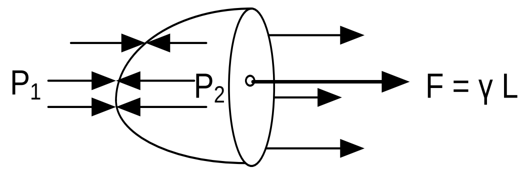
Consider an air bubble of radius r which is spherical and formed in a liquid of surface tension γ



P_1 = External pressure on the bulb due to the liquid

P_2 = internal pressure of air in the bubble

Considering half of the bubble. The remaining half experiences surface tension force due to the other half and this force acts towards the right.



For the bubble to maintain its shape the, internal pressure should be bigger than the external pressure.

At equilibrium; Force due to P_2 = force due to P_1 + surface tension

$$AP_2 = AP_1 + \gamma L$$

$$\pi r^2 P_2 = \pi r^2 P_1 + 2\pi r \gamma$$

$$\pi r^2 (P_2 - P_1) = 2\pi r \gamma$$

$$\boxed{P_2 - P_1 = \frac{2\gamma}{r}}$$

OR Excess pressure $= \frac{2\gamma}{r}$

Note:

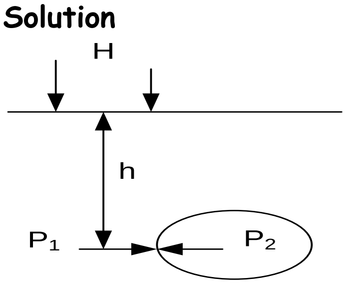
The pressure inside an air bubble is greater than that outside, otherwise the combined effect of the external pressure and the surface tension forces in the air bubble to collapse.

The same case can be extended to a soap bubble.

Example

Calculate the pressure inside a spherical air bubble of diameter 0.1cm blown at depth of 20cm below the surface of a liquid of density $1.26x10^3 kgm^{-3}$ and surface tension $0.064Nm^{-1}$.(height of mercury barometer is 0.76m, and density of mercury is $13.6x10^3 kgm^{-3}$).

Solution



$P_1 = H + h\rho g$

$P_1 = 0.76x13.6x10^3x9.81 + \frac{20}{100}x1.26x10^3x9.81$

P_1=101643Pa

Excess pressure of air bubble $= \frac{2\gamma}{r}$

$P_2 - P_1 = \frac{2\gamma}{r}$

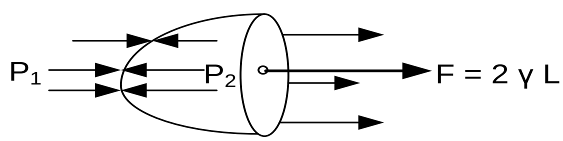
$P_2 - 101643 = \frac{2\ x\ 0.064}{0.05\ x\ 10^{-2}}$

$P_2 = 1.02x10^5$Pa

12.2.2: Excess pressure (pressure difference) for a soap bubble

For a soap bubble of radius r, there are two surfaces of liquid in contact with air (the air inside the bubble and air outside the bubble). Therefore the total length of surface in contact with air is 2L such that surface tension force.

$$F=2L\gamma$$



At equilibrium : Inside force due to P_2 = external force due to P_1 +surface tension force

$$AP_2=AP_1+2\gamma L$$

$$\pi r^2P_2=\pi r^2P_1+4\pi r\gamma$$

$$\pi r^2(P_2-P_1)=4\pi r\gamma$$

$$\boxed{P_2-P_1=\frac{4\gamma}{r}}$$

$$\text{Excess pressure} = \frac{4\gamma}{r}$$

Example

A soap bubble has a diameter of 4mm. calculate the pressure inside it, if the atmospheric pressure is $10^5 Nm^{-2}$, and that the surface tension of soap solution is $2.8x10^{-2}Nm^{-1}$

Solution

$P_2-P_1=\frac{4\gamma}{r}$

$P_2-10^5=\frac{4\,x2.8x10^{-2}}{2\,x\,10^{-3}}$

$P_2=1.0006x10^5Pa$

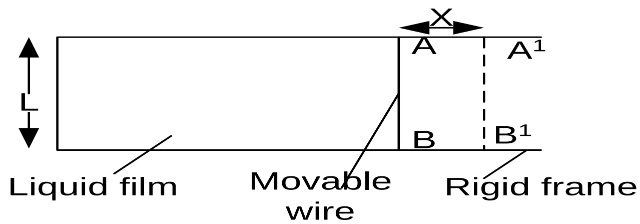
12.1.1: FREE SURFACE ENERGY (σ)

It is defined as the work done in creating a unit area of a new surface under isothermal conditions .

Units of σ are Jm^{-2} or Nm^{-1}

Consider stretching a thin film of a liquid on a horizontal frame as shown below. Since the film has both an upper and lower surface, the force F on AB due to surface tension is given by

$F=2L\gamma$-------------------[1]



If AB is moved a distance x to A^1B^1, then work has to be done against this force

Work done = force x distance

$$W = Fx\, X = 2L\, \gamma\, X \text{---------------------------[2]}$$

The increases in surface area is 2LX (upper and lower surface).

Therefore the work done per unit area increases the surface energy (σ) is given by;

$$\sigma. = \frac{W}{2\, L\, X} = \frac{2\, L\, \gamma\, X}{2\, L\, X}$$

$$\sigma = \gamma$$

∴ free surface energy =surface tension

Alternative definition of γ

Is the work done per unit area in increasing the surface area of a liquid under isothermal conditions.

Example

1. Calculate the work done against surface tension force on blowing a soap bubble of diameter 15mm , if the surface tension of the soap solution is $3.0x10^{-2}Nm^{-1}$.

Solution

$$\gamma = \frac{work\ done}{increase\ in\ S.A}$$

Work done $= \gamma\ x$ increase in surface area $=\ \gamma x(2x4\pi r^2)$

$$=3.0x10^{-2}x2x4x\frac{22}{7}x\left(\frac{15x10^{-3}}{2}\right)^2$$

Work done $= 4.241x10^{-5}$J

Increases in surface area is multiplied by 2 for both the upper and lower surface of a soap bubble.

2. Calculate the change in surface energy of a soap bubble when its radius decreases from 5cm to 1cm, given that the surface tension of soap solution is $2x10^{-2}Nm^{-1}$

Solution

$\gamma = \frac{work\ done}{increase\ in\ S.A}$

Work done = $\gamma\ x$ increase in surface area = $\gamma x(2x4\pi r^2)$

5cm bubble: Work done $=2.0x10^{-2}x2x4x\frac{22}{7}x(5x10^{-2})^2 = 1.257x10^{-3}$J

1cm bubble: Work done $=2.0x10^{-2}x2x4x\frac{22}{7}x(1x10^{-2})^2 = 5.027x10^{-5}$J

Change in surface energy $=1.257x10^{-3} - 5.027x10^{-5} = 1.207x10^{-3}$J

3. A liquid drop of diameter 0.5cm breaks up into 27 tiny droplets all of the same size. If the surface tension of the liquid is 0.07Nm^{-1} calculate the resulting change in energy.

   Solution

   Diameter of big drop, D= 0.5cm $\therefore R = 0.25cm = 2.5x10^{-3}m$

   Volume of big drop= $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(2.5x10^{-3})^3$

   Volume of 27 tiny droplets= $27x\frac{4}{3}\pi r^3$

   $$27x\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.5x10^{-3})^3$$

   $\boldsymbol{r} = 8.3x10^{-4}m$

Work done = $\gamma\ x$ increase in surface area= $\gamma x(4\pi r^2)$

Big drop: Work done$=0.07x4x\frac{22}{7}x(2.5x10^{-3})^2 = 5.5x10^{-6}$J

27 drop lets: Work done= $27x0.07x4x\frac{22}{7}x(8.3x10^{-4})^2 = 1.637x10^{-5}$J

Change in surface energy $=1.637x10^{-5} - 5.5x10^{-6} = 1.087x10^{-5}$J

4. Calculate the work done in breaking up a drop of water of radius 0.5cm in to tiny droplets of water each of radius 1mm assuming isothermal conditions, given that surface tension of water is $7x10^{-2}$ Nm^{-1}.

   Solution

   Radius of big drop, R= 0.5cm $= 5x10^{-3}m$ and Radius of n tiny droplets, r= 1mm $= 1x10^{-3}m$

   Volume of big drop= $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(5x10^{-3})^3$

   Volume of n tiny droplets= $nx\frac{4}{3}\pi r^3 = nx\frac{4}{3}\pi(1x10^{-3})^3$

$$nx\frac{4}{3}\pi(1x10^{-3})^3 = \frac{4}{3}\pi(5x10^{-3})^3$$

$$\boldsymbol{n} = 125\ droplets$$

Work done = $\gamma\ x$ increase in surface area= $\gamma x(4\pi r^2)$

Big drop: Work done= $7x10^{-2}\ x4x\frac{22}{7}x(5x10^{-3})^2 = 2.2x10^{-5}$J

125 drop lets: Work done= $125x7x10^{-2}\ x4x\frac{22}{7}x(1x10^{-3})^2 = 1.1x10^{-4}$J

Change in surface energy =$1.1x10^{-4} - 2.2x10^{-5} = x1.09x10^{-4}$J

EXERCISE: 33

1. A spherical drop of mercury of radius 2mm falls to the ground and breaks into 10smaller drops of equal size. Calculate the amount of work that has to be done. (Surface tension of mercury = 4.72×10^{-1} Nm^{-1}) **An[$2.74x10^{-5}$J]**

Relationship between surface area and shape of a drop

The area of a liquid has the least number of molecules in it under surface tensional forces. Surface are of a given volume of a liquid is there minimum when its is spherical and this explains why the meniscus and small droplets of mercury and rain are spherical in shape.

Why small mercury droplets are spherical and larger one flatten out

A small drop takes on a spherical shape to minimize the surface energy which tends to be greater than the gravitational potential energy. Therefore, the gravitational potential force cannot distort the spherical shape due to the very small mass of tiny droplets.

A large drop flattens out in order to minimize the gravitational potential energy, which tends to exceed the surface energy. Due to its large weight, gravitational force distorts the spherical shape of large drops. The shape of the drop must agree with the principle that the sum of gravitational potential energy and surface energy must be a minimum

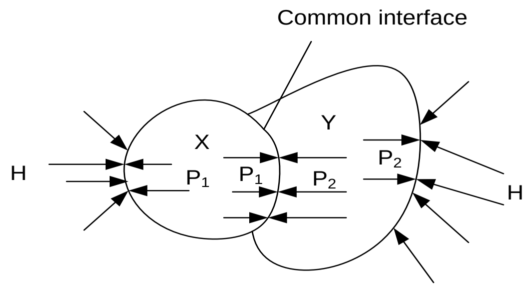
COMBINED BUBBLES

CASE 1

A soap bubble x of radius r_1, and another bubble y of radius r_2, are brought together so that the combined bubble has a common interface of radius R. show that

$R = \frac{r_1 \ r_2}{r_2 - r_1}$

Solution



Excess pressure on x

$P_1 – H = \frac{4\gamma}{r_1}$------------------------------ [1]

Excess pressure on y

$P_2 – H = \frac{4\gamma}{r_2}$ ------------------------------ [2]

Equation –equation 2 gives

$P_1 – P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$

$P_1 – P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$ ----------------------- [3]

Excess pressure at the interface

$P_1 - P_2 = \frac{4\gamma}{R}$ ----------------------------- [4]

Equating equation 3 and equation 4

$\frac{4\gamma}{R} = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$

$\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2}$

$\frac{1}{R} = \frac{r_1 - r_2}{r_1 r_2}$

$R = \frac{r_1 \ r_2}{r_2 - r_1}$

Example

1. A soap bubble x of radius 0.03m and another bubble y on radius,0.04m are brought together so that the combined bubble has a common interface of radius r. calculate r

Solution

Using $r = \frac{r_1 \ r_2}{r_2 - r_1} = \frac{0.03x\ 0.04}{0.04 - 0.03}$ =0.12m

2. Two soaps bubble A and B of radii 6cm and 10cm respectively coalesce so that the combined bubble has a common interface . calculate the radius of curvature of this common surface and hence the pressure difference. Given that surface tension of soap is $2.5 10^{-2} Nm^{-1}$

   Solution

   Using $r = \frac{r_1 \ r_2}{r_2 - r_1} = \frac{0.06x\ 0.1}{0.1 - 0.06}$ =0.15m pressure difference $= \frac{4\,\gamma}{r} = \frac{4x\ 2.5x10^{-2}}{0.15} = 0.667Pa$

CASE 2

Two bubbles of a soap solution of radii r_1 and r_2 of surface tension γ and pressure P coalence under isothermal conditions to form one bubble. Find the expression for the radius of the bubble formed.

Solution

Let R be the radius of the new bubble

A_1 be the surface area of bubble with radius r_1

A_2 be the surface area of bubble with radius r_2

A be the surface area of bubble with radius R

Under isothermal conditions, work done in enlarging the surface area of a bubble is given by

$$2\gamma A = 2\gamma A_1 + 2\gamma A_2$$

$$2\gamma 4\pi R^2 = 2\gamma 4\pi\, r_1{}^2. + 2\gamma 4\pi\, r_2{}^2$$

$$R^2 = r_1{}^2 + r_2{}^2$$

$$R = \sqrt{r_1{}^2 + r_2{}^2}$$

Examples

1. Two soap bubbles have radii of 3cm and 4cm, the bubbles are in a vacuum and they combine to form a single larger bubble. Calculate the radius of this bubble

   Solution

$$R = \sqrt{r_1{}^2 + r_2{}^2} = \sqrt{3^2 + 4^2} = 5cm$$

2. Two soap bubbles of radii 2cm and 4cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5x10^{-2}$

   Solution

$$R = \sqrt{r_1{}^2 + r_2{}^2} = \sqrt{(2x10^{-2})^2 + (4x10^{-2})^2} = \sqrt{20x10^{-4}}m$$

pressure difference $= \frac{4\,\gamma}{r} = \frac{4x\ 2x10^{-2}}{\sqrt{20x10^{-4}}} = 1.789Pa$

EXERCISE: 34

1. A soap bubble whose radius is 12mm becomes attracted to one of radius 20mm. Calculate the radius of curvature of the common interface. **An[30mm]**

2. Two soap bubbles of radii 2.0cm and 4.0cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5x10^{-2}Nm^{-1}$. Calculate the excess pressure inside the resulting soap bubble. **An[2.36Pa]**

UNEB 2009 Q.4

C) i)Define surface tension in terms of work

(1mk)

ii)Use the molecular theory to account for the surface tension of liquid

(4mk)

iii)Explain the effect of increasing temperature of a liquid on its surface tension

(4mk)

iv) Calculate the excess pressure inside a soap bubble of diameter 3.0cm if the surface tension of the soap solution is $2.5x10^{-2}Nm^{-1}$. **An[6.67Pa]**

(2mk)

UNEB 2008 Q.3

c) i) Define surface tension

(01mark)

ii) Explain the origin of surface tension

(03marks)

iii) Describe an experiment to measure the surface tension of a liquid by the capillary method

(06marks)

UNEB 2002 Q.4

a) Define the term surface tension in terms of surface energy

(01mark)

b) i) Calculate the work done against surface tension is blowing a soap bubble of diameter 15mm, if the

surface tension of the soap solution is $3.0\times10^{-2}Nm^{-1}$ **An** [$4.24\times10^{-5}J$]

(03marks)

ii) A soap bubble of a radius r_1 is attached to another bubble of radius r_2. If r_1 is less than r_2. Show

that the radius of curvature of the common interface is $\frac{r_1\ r_2}{r_2 - r_1}$

(05marks)

UNEB 2001 Q.3

a) Define surface tension and derive its dimension (3mk)

b) Explain using the molecular theory the occurrence of surface tension (4mk)

c) Describe an experiment to measure surface tension of a liquid by the capillary tube method (6mk)

d) i)Show that the excess pressure in a soap bubble is given by $P = \frac{4\gamma}{r}$

ii) Calculate the total pressure within a bubble of air of radius 0.1mm in water, if the bubble is formed 10cm below the water surface and surface tension of water is $7.27\times10^{-2}Nm^{-1}$. [Atmospheric pressure $=1.01\times10^{5}Pa$] **An 1.03×10^{5}Pa**

END