SECTION A (40 MARKS) 5 miles and 10 10

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Answer all the questions in this section

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Prove by induction that
$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)^r}{6}$$

If a line $y = mx + c$ is a tangent to the curve $4x^2 + 3y^2$

If a line
$$y = mx + c$$
 is a tangent to the curve $4x^2 + 3y^2 = 12$, show that $c^2 = 4 + 3m^2$. (05 marks)

3. Given that
$$y = e^x \cos 3x$$
, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$ (05 marks)

4. Find the angle between the line
$$\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix}$$
 and the plane $-x + 2y + 2z - 66 = 0$.

5.
$$\neq$$
 Solve the inequality $\frac{7-2x}{(x+1)(x-2)} \ge 0$. (05 marks)

C6. Evaluate
$$\int_0^{\pi/3} (1+\cos 3y)^2 dy. \qquad (05 \text{ marks})$$

7. Express
$$2\sin\theta + 3\cos\theta$$
 in the form $R\sin(\theta + \alpha)$. (05 marks)

Use Maclaurin's theorem to expand $\ln (2+x)$, in ascending powers of x as far as the term in x^2 .

(05 marks)

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

- 9. (a) Solve the equation $Z^3 7Z^2 + 19Z 13 = 0$. (06 marks)
 - (b) Find the fourth roots of $8(-\sqrt{3}+i)$. (06 marks)
- Express $f(x) = \frac{3x^3 + 2x^2 3x + 1}{x(1-x)}$ in partial fractions. -6 2.1 + 1Hence find $\int f(x) dx$. (12 marks)
 - A point E has coordinates (2, 0, -1). A line through E and parallel to the line whose equation is $\frac{x}{-2} = y = \frac{z+1}{2}$, meets a plane x + 2y 2z = 8 at a point B. A perpendicular line from E meets the plane at a point C. Determine the coordinates of;

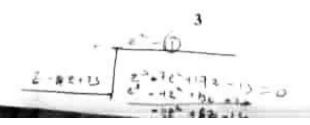
(a) B. (QZ marks)

- (b) C. (05 marks)
- 12. (a) Four different Mathematics books and six other different books are to be arranged on a shelf. In how many ways can the Mathematics books be arranged on the shelf? (02 marks)
- (b) On a certain day, Fatuma drunk 6 bottles of the 9 bottles of soda available. On the next day she drunk 5 bottles of the 7 bottles of soda available. In how many ways could she have chosen the bottles of soda to drink in the two days?

 (03 marks)
- 2+5 \mathcal{Q} +2-3 \mathcal{O} (c) Given that ${}^{20}C_r = {}^{20}C_{r-2}$, find the value of r. (07 marks)
- 13. (a) A curve is given by the parametric equations $x = t^2 3$, $y = t(t^2 3)$. Find the Cartesian equation of the curve. (04 marks)
 - A point P is such that its distance from the origin is five times its distance from (12, 0).
 - Show that the locus of P is a circle.

13

 $\dot{\xi} - w \dot{\xi} + \dot{\psi} = 0$ (ii) Determine the coordinates of the centre of the circle and its radius.
(08 marks)



Turn Over

14. Given the curve
$$y = \frac{1}{4x^2 - 1}$$
, determine the;

(a) coordinates of the turning points of the curve.

(03 marks)

(b) equation of the asymptotes. Hence sketch the curve.

(09 marks)

15. (a) Show that
$$\tan 3\theta = \frac{\tan \theta \left(3 - \tan^2 \theta\right)}{\left(1 - 3\tan^2 \theta\right)}$$
.

(05 marks

- (b) Solve the equation $\cos 4x + \cos 6x + \cos 2x = 0$ for $0^{\circ} \le x \le 180^{\circ}$.
- The rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. The body is placed in a room of temperature 25 °C. After 6 minutes the temperature of the body dropped from 90 °C to 60 °C.
 - (a) Form a differential equation for the rate of cooling of the body.
 (07 marks)
 - (b) Find the time it takes for the body to cool from 40 °C to 30 °C.
 (05 marks)

$$2^{5}-72^{2}+192-13=0$$
Try and wor

Idnon $2=+1$
 $+-7 (1)-7+19-13=0$