P425/1
PURE MATHEMATICS
Paper 1
July/Aug, 2023
3 hours

PROVINCIAL - NAMIREMBE DIOCESE

COUHEIA SECONDARY

MOCK EXAMINATIONS 2023

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *questions in section* **A** *and only five questions from section* **B**.
- *All necessary calculations* **MUST** *be done on the same page as the rest of the answers.*
- *Any additional question(s) attempted in section* **B** *will not be marked.*
- *Begin each question on a fresh sheet of paper.*
- *All working must be shown clearly.*
- *Silent, non-programmable, scientific calculators and mathematical tables with a list of formulae may be used.*

TURN OVER

SECTION A (40 MARKS)

*Answer **all** the questions in this section.*

1. By reducing the appropriate matrix to echlon form, solve the simultaneous equations: *(05 marks)*

$$x - y + 2z = 1$$
$$2x + 3y + z = 3$$
$$3y - 2x - 4z = -3$$

2. The line L is concurrent to the lines $x + y = 7, 2x - y = 5$ and pependicular to the line $4x - y = 7$. Find the equation of the line L.

(05 marks)

3. Solve the inequality $\frac{3x^2-2x-11}{x^2-4x+3} \leq 3$ *(05 marks)*

4. Show from the first principles, that $\frac{d}{dx}(tanx + secx) = \frac{1}{1-sinx}$

(05 marks)

5. Given the points $P(3,4,2), Q(-2,1,-3)\ and\ R(5,-4,0)$, find the angle PQR using vectors. *(05 marks)*

6. Determine $\int x^2 lnx\, dx$ *(05 marks)*

7. Express $4cosx + 3sinx$ in the form $Rcos(x - \alpha)$. Hence state the maximum value of the function $\frac{2}{4cosx+3sinx+10}$ and the smallest positive value of x within which it occurs. *(05 marks)*

8. If $y = cos^2(x^2)$. prove that $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 16x^3y = 8x^3$. *(05 marks)*

SECTION B (60 MARKS)

*Answer any **five** questions from this section.*

9. (a) Given that the complex number z varies such that $|z-5|=3$, find the greatest and least values of $|z+2-4i|$. *(05 marks)*

(b) By De Moivres theorem, show that $tan3\theta = \frac{3t-t^3}{1-3t^2}$, where $t = tan\theta$ and hence solve the equation $1-3\,t^2 = 3t-t^3$, correct your answers to 3 significant figures. *(07 marks)*

10. (a) Prove that the line $\boldsymbol{r} = \boldsymbol{i} - 2\boldsymbol{j} + \lambda(\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k})$ is parallel to the intersection of the planes: $x+y-2z=2$ and $2x+y-z=0$. *(06 marks)*

(b) Find the perpendicular distance of the point $P(1,0,2)$ from the line:

$x-1=\frac{y-1}{-1}=z$ *(06 marks)*

11. Express $f(x)=\frac{2x^2+3x+5}{(x+1)(x^2+3)}$ into partial fractions and hence:

(a) show that $f^{|}(x) = -\frac{2}{3}$ when $x = 0$;

(b) evaluate $\int_0^{\sqrt{3}} f(x)dx$. *(12 marks)*

12. P and Q are the points $(ap^2, 2ap)$ *and* $(aq^2, 2aq)$ respectively on the parabola $y^2 = 4ax$ and M is the mid point of the chord PQ.

(a) Show that the area, A, enclosed by the curve and the chord PQ is given by $9A^2 = a^4(p-q)^6$. *(06 marks)*

(b) If $q=p-4$, give the cordinates of M in terms of p only and find the equation of the locus of M as the value of p varies continuously.

(06 marks)

13. Given the curve $y = \frac{x^2+x-2}{x^3-7x^2+14x-8}$

(a) Give the coordinates of the hole. *(02 marks)*

(b) Find the equations of the asymptotes. *(02 marks)*

(c) Determine the turning points and their nature. *(03 marks)*

(d) Find the intercepts and sketch the curve. *(05 marks)*

14. (a) A piece of wire of length l is cut into two portions. Each portion is then cut into twelve equal parts which are soldered together so as to form the edges of a cube.

(i) Find an expression for the sum of the volumes of the two cubes so formed.

(ii) What is the least value of the sum of the volumes? *(06 marks)*

(b) An up turned cone with semi vertical angle 45^o is being filled with water at a constant rate of $30cm^3$ per second. When the depth of water is 60cm, find the rate at which the:

(i) depth of water is increasing;

(ii) area of the water surface is increasing. *(06 marks)*

15. (a) If $cos\alpha - cos\beta = \frac{2}{5}$ and sin $\alpha - sin\beta = \frac{5}{6}$, find the value of:

(i) $sin\frac{1}{2}(\alpha + \beta)$

(ii) $cos\ (\alpha + \beta)$ *(06 marks)*

(b) In any triangle ABC, prove that
$sin^2A\ + sin^2B + sin^2C = 2 + 2cosAcosBcosC.$ *(06 marks)*

16. (a) Solve the differential equation $(x^2 + 1)\frac{dy}{dx}\ + 4xy = 12x^3$ for which $y = 1\ when\ x\ = 1.$ *(05 marks)*

(b) According to Newton's law, the rate of cooling of a body in air is proportional to the difference between the temperature, T, of the body and the temperature $, T_o$, of the air. If the air temperature is kept constant at 20^oC and the body cools from $100^oC\ \ to\ 60^oC$ in 20 minutes, in what further time will the body cool to $30^o\ C$?. *(07 marks)*

END.