P425/1
PURE MATHEMATICS
Paper 1
Jul./Aug. 2024
3 hours

WAKISO-KAMPALA TEACHERS' ASSOCIATION (WAKATA)

WAKATA MOCK EXAMINATIONS 2024

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All necessary working must be clearly shown.

Begin each answer on a fresh sheet of paper.

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

Neat work is a must!!

Turn Over

SECTION A (40 MARKS)

*Answer **all** questions in this section.*

1. Find the square root of $15 + 8i$ *(05 marks)*

2. Solve: $3 + 2sin2\theta = 2sin\,\theta + 3cos^2\theta$ for $0° \leq \theta \leq 360°$. *(05 marks)*

3. Given that $x = a\,sec\theta, y = btan\theta$. Show that $\frac{d^2y}{dx^2} = \frac{-b}{a^2}\,cot^3\theta$ *(05 marks)*

4. A triangle ABC has position vectors $A(2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}), B(5\boldsymbol{i} + 4\boldsymbol{k})$ and $C(\boldsymbol{i} + 2\boldsymbol{j} + 12\boldsymbol{k})$. Find the area of the triangle. *(05 marks)*

5. A point $P(2at, at^2)$ lies on the parabola $4ay = x^2$. Given that the point S is $(0, a)$ and M is the midpoint of PS. Show that the equation of the locus of M is given by $x^2 + a^2 = 2ay$ *(05 marks)*

6. Using the substitution $x = tan\theta$. Evaluate $\int_0^1 \frac{dx}{(1+x^2)^2}$ *(05 marks)*

7. The sum of the first n terms of a certain progression is $\sum_{r=0}^{n-1} 3^r$. Find the least value of n such that the sum exceeds 10,000. *(05 marks)*

8. Solve the differential equation $x\frac{dy}{dx} = xy + e^x$, given that $y = 0$ when $x = 1$. *(05 marks)*

SECTION B (60 MARKS)

*Answer any **five** questions from this section. All questions carry equal marks.*

9. (a) Given that $Z_1 = 3 - i,\ Z_2 = 3 + i$, Find the modulus and argument of ${}^{Z_2}/_{Z_1}$ *(04 marks)*

   (b) Show that $Z_1 = 2$ and $Z_2 = {}^1/_2(-1 + i)$ are roots of the equation.

   $2Z^3 - 2Z^2 - 3Z - 2 = 0.$ *(04 marks)*

   (c) Use DeMoivre's theorem to solve $Z^4 + 1 = 0$. (*Leave your answer in surd form*) *(04 marks)*

10. (a) Given that $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, show that $\frac{d^2y}{dx^2} = \frac{-4x}{(1+x^2)^2}$ *(06 marks)*

(b) Show that $\frac{d}{dx}(\log_2 x + \log_x 2) = \frac{(\ln 2x)(\ln {}^{x}/_{2})}{(\ln 2^x)(\ln x)^2}$ *(06 marks)*

11. A and B are points whose position vectors are $a = i + k$ and $b = i - j + 3k$ respectively.

(a) Determine the position vector of the point P that divides AB in the ratio $-4 : 1$.

(b) Given that $a = i - 3j + 3k$ and $b = -i - 3j + 2k$, determine

(i) the equation of the plane containing a and b.

(ii) the angle the line $\frac{x-2}{4} = \frac{y}{3} = \frac{z-1}{3}$ makes with the plane in (i) above

(12 marks)

12. (a) Expand $(1-x)^{\frac{1}{3}}$ as far as the term in x^3. Use your expansion to deduce $(24)^{\frac{1}{3}}$ correct to 3sf *(05 marks)*

(b) Use Maclaurin's theorem to expand $In\left(\frac{1+\sin x}{1+x}\right)$ as far as the term x^3 *(07 marks)*

13. Sketch the curve: $y = \frac{(x+1)(x-6)}{(x+3)(x-2)}$ *(12 marks)*

14. (a) Show that $\frac{\sin 2\theta - 2\sin 4\theta + \sin 6\theta}{\sin 2\theta + 2\sin 4\theta + \sin 6\theta} = 1 - \sec^2\theta$ *(04 marks)*

(b) Prove that $\left(\frac{1+\sin 2\theta}{1-\sin 2\theta}\right)^{1/2} = \frac{1+\tan\theta}{1-\tan\theta}$

Hence or otherwise solve for θ, if $\left(\frac{1+\sin 2\theta}{1-\sin 2\theta}\right)^{1/2} + \sqrt{3} = 0$; for $0 \leq \theta \leq 90°$.

(08 marks)

15. (a) Given that $r = 3\tan\theta$ is the polar equation of the circle; find its Cartesian form.

(03 marks)

(b) Prove that the chord joining the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ on the parabola $y^2 = 4ax$ has the equation $(p+q)y = 2x + 2apq$. A variable chord PQ of the parabola is such that the lines OP and OQ are perpendicular, where O is the origin.

(i) Prove that the chord PQ cuts the $x-$ axis at a fixed point, and give the $x-$ coordinate of this point.

(ii) Find the equation of the locus of the mid-point of PQ.

(09marks)

16. The temperature of a sick student was measured by the school nurse at 4:00pm and was found to be 50^0C. The nurse noticed that the temperature of the sick bay at that instant was 25^0C. She again took the temperature of the student after one hour, when it showed 45^0C. Assuming that the rate of change of the student's temperature was directly proportional to the difference between student's temperature, T and that of the sick bay.

(a) (i) Write a differential equation to represent the rate of change of temperature of the student.

(ii) Using the conditions given, solve the differential equation.

(09 marks)

(b) At what time of the day did the temperature of the student reduce to $38°C$?

(03 marks)

END