P425/1
PURE MATHEMATICS
Paper 1
Jul./Aug. 2022
3 hours

WAKISO-KAMPALA TEACHERS' ASSOCIATION (WAKATA)

WAKATA MOCK EXAMINATIONS 2022

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All necessary working must be clearly shown.

Begin each answer on a fresh sheet of paper.

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

Neat work is a must!!

SECTION A (40 MARKS)

Answer **all** *questions in this section.*

1. Given that $p(x) = 8x^3 + ax^2 + bx - 1$ has a remainder 1 when divided by $(2x + 1)$ and it is exactly divisible by $(x + 1)$. Factorize $p(x)$ completely. *(05marks)*

2. The angles θ and ϕ lie between 0^0 and 180^0, and are such that $tan(\theta - \phi) = 3$ and $tan\theta + tan\phi = 1$. Find the possible values of θ and ϕ. *(05marks)*

3. A curve has equation $y = \frac{3x+1}{x-5}$. Find the coordinates of the points on the curve at which the gradient is -4. *(05marks)*

4. The points A, B and C have position vectors $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$.

   The plane M is perpendicular to AB and contains the point C. The line through A and B intersect the plane M at point N. Find the position vector of N. *(05marks)*

5. The complex number $Z = 3 - i$ has a complex conjugate Z^*.
   (a) On an argand diagram with origin O, show the points A, B and C representing the complex numbers Z, Z^* and $Z^* - Z$ respectively and name the quadrilateral $OABC$. *(03marks)*
   (b) Express $\frac{Z^*}{Z}$ in the form $x + iy$ where x and y are real. *(02marks)*

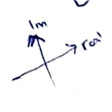
6. Show that the equation of the tangent to the parabola $y^2 = 4ax$ at (x_1, y_1) is $yy_1 = 2a(x + x_1)$. *(05marks)*

7. Evaluate $\int_0^1 x\,e^x dx$ *(05marks)*

8. Solve the differential equation $\frac{dx}{d\theta} = (x + 2)sin^2 2\theta$, given that $x = 0$ when $\theta = 0$ *(05marks)*

$(x-5)(x-5)$
$x^2-5x-5x+25$
$x^2-10x+25$

$\cos^2A-\sin^2A = (1-\cos^2 ... = 1-2\sin^2A$
$\cos 2A = 1-2\sin^2A$
$3+i-(3-i)$
$2i$
$\cos^2\theta+\sin^2\theta=1$



$\tan(\theta-\phi)$
3
$\tan(\theta+\phi) =$
$\tan(\theta+\phi)$
$\tan(\theta+\phi)$
$3-3\tan$

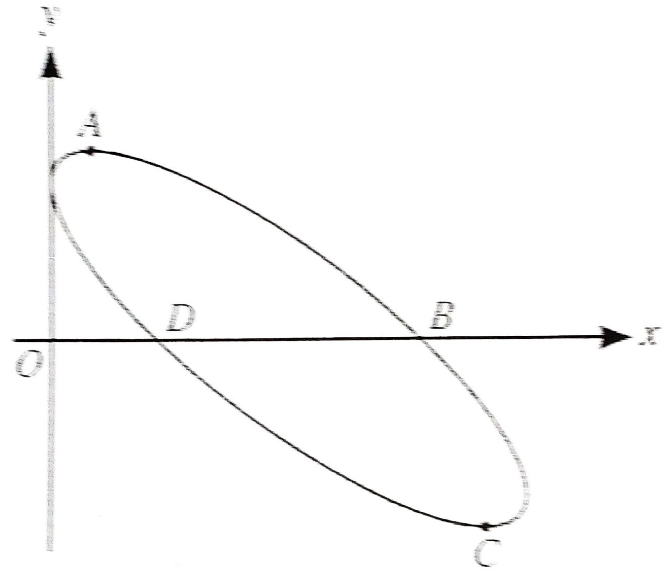
SECTION B (60 MARKS)

Answer any **five** *questions from this section. All questions carry equal marks.*

9. (a) Prove that $1\times 4 + 2\times 9 + 3\times 16 + \dots\dots + n(n+1)^2 = \frac{n(n+1)(n+2)(3n+5)}{12}$ *(07marks)*

(b) Expand $\frac{(1+2x)^2}{(2-x)}$ in ascending powers of x up to and including the term in x^3 and state the values of x for which the expansion is valid. *(05marks)*

10. The diagram below shows a curve with parametric equations $x = 6\sin^2 t$, $y = 2\sin 2t + 3\cos 2t$, for $0 \le t < \pi$. The curve crosses the $x-$axis at points B and D and the stationary points are A and C.



(a) Show that $\frac{dy}{dx} = \frac{2}{3}(\cot 2t - 1)$ *(05marks)*

(b) Find the;

(i) values of t at A and C

(ii) gradient of the curve at B *(07marks)*

11. Ressolve $\frac{16x}{(x^4-16)}$ into partial fractions. Hence evaluate $\int_0^{2} \frac{16x}{(x^4-16)}\,dx$ correct to 3 significant figures *(12marks)*

$\sin 2t = 2\sin t$
$= 2\sin t\cos t$

12. (a) Show that $\frac{\cos 3\theta}{\cos\theta} - \frac{\cos 6\theta}{\cos 2\theta} = 2(\cos 2\theta - \cos 4\theta)$. *(05marks)*

(b) Solve the equation $\sin 5x - \sin x + \sqrt{3}\cos 3x = 0$, for $-180^0 \le x \le 180^0$ *(07marks)*

$\frac{\tan\theta - \tan\alpha}{1+\tan\theta\tan\alpha}$

$\frac{\tan\theta + \tan\alpha}{1-\tan\theta\tan\alpha}$

$3-48$
$3-48 = -4$
$(x^2-4)(x^2+4)$

Turn Over

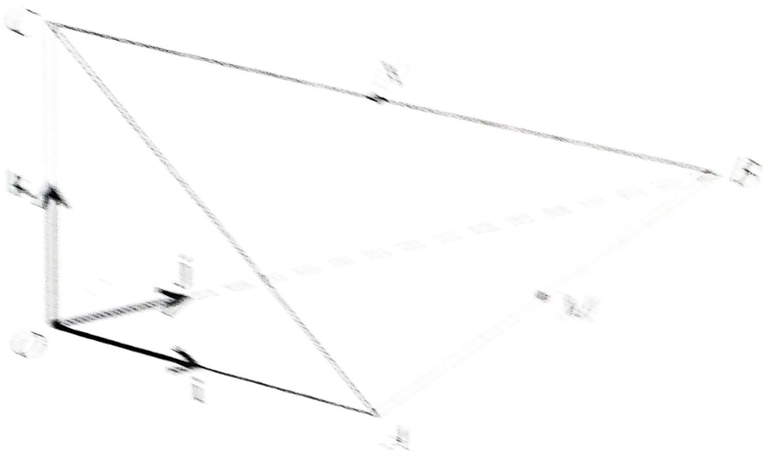
13. (a) By row reducing the appropriate matrix to echelon form, solve the system of equations

$$2x - y + z - 3 = 0$$
$$x - 3y + 2z - 1 = 0$$
$$2x + y + 4z + 5 = 0$$

(05 marks)

(b) Find the solution set for the inequality $\frac{x+4}{x-1} < \frac{x-2}{x-4}$ (07 marks)

14. In the diagram below, $OABC$ is a pyramid in which $\overline{OA} = 2$ units, $\overline{OB} = 4$ units and $\overline{OC} = 2$ units. The edge $\overline{OC}$ is vertical, the base OAB is horizontal and angle $AOB = 90^0$. Unit vectors i, j and k are parallel to $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$ respectively. The mid points of AB and BC are M and N respectively.



(a) Express the vectors $\overrightarrow{ON}$ and $\overrightarrow{CM}$ in terms of i, j and k hence calculate the angle between directions of $\overrightarrow{ON}$ and $\overrightarrow{CM}$.

(07 marks)

(b) Show that the length of the perpendicular from M to ON is $\frac{3}{5}\sqrt{5}$ units. (05 marks)

15. (a) Show that at $(a\sec\theta, b\tan\theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the equation of a tangent is $bx - ay\sin\theta - ab\cos\theta = 0$

(07 marks)

(b) The line $y = mx + c$ is also a tangent to the hyperbola in (a) above.
Show that $c = \pm\sqrt{a^2m^2 - b^2}$.

(05 marks)

16. In a chemical reaction, a compound X is formed from two compounds Y and Z. The masses in grams of X, Y and Z present at time, t seconds after the start of reaction are x, $10 - x$ and $20 - x$ respectively. At any time the rate of formation of X is proportional to the product of the masses of Y and Z present at the time. When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 2$.

(a) Show that x and t satisfy the differential equation $\frac{dx}{dt} = 0.01(10 - x)(20 - x)$.

(04 marks)

(b) Solve the differential equation and state what happens to the value of x when t becomes large.

(08 marks)

END