

**INSTRUCTIONS:**

- Answer all the eight questions in Section A and only five questions in Section B. Each number in section B should start on a fresh page

**SECTION A (40 MARKS)**

Qn 1: Evaluate  $\frac{dy}{dx}$  at  $x = 2$ , given that  $y = \ln \left[ \frac{1+x^2}{1-x^2} \right]^{\frac{1}{2}}$  [5marks]

Qn 2: Solve the inequality:  $|x - 2| > |2x + 1|$  [5marks]

Qn 3: Differentiate  $y = 4x^2 + 6x$  from first principles. [5marks]

Qn 4: Solve the equation  $\sqrt{6x + 1} - \sqrt{2x - 4} = 3$  [5marks]

Qn 5: Solve for  $x$ ,  $\sin(x + 30^\circ) = \cos x$ , where  $0 \leq x \leq 2\pi$ . [5marks]

Qn 6: Show that  $\tan(\alpha + \beta) = 1$ , if  $\tan \alpha = \frac{a}{a+1}$  and  $\tan \beta = \frac{1}{2a+1}$  [5marks]

Qn 7: Find the equation of the line which passes through the point (3, 2) and the point of intersection of the lines  $3x - 4y - 6 = 0$  and  $2x + 3y - 1 = 0$ . [5marks]

Qn 8: Express the function  $f(x) = 1 - 6x - x^2$  in form  $f(x) = 1 - 6x - x^2$ , hence state the value of  $x$  at which it occurs [5marks]

**SECTION B (60 MARKS)**

**Question 9:**

(a). Solve the simultaneous equations

$$(x + 3)(y + 3) = 10 \text{ and } (x + 3)(x + y) = 2 \quad [05marks]$$

(b). Use a substitution  $y = x + \frac{2}{x}$  to solve,  $x^4 - 5x^3 + 10x^2 - 10x + 4 = 0$

[07marks]

**Question 10:**

- a) Express;  $\sqrt{5}\cos x + 2\sin x$  in the form  $R\cos(x - \alpha)$  where  $R > 0$  and  $0 < \alpha < 90^\circ$ .  
Hence state the maximum value and minimum value of  $\sqrt{5}\cos x + 2\sin x + 10$ .  
[06 marks]
- b) Given that  $a\cos^2 \theta + b\sin^2 \theta = c$ , prove that  $\tan^2 \theta = \frac{c-a}{b-a}$  hence solve for  $\theta$ , in the equation  $6\cos^2 \theta + 2\sin^2 \theta = 5$ , where  $\theta$  is acute. [06marks]

**Question 11:**

- (a). In an AP, the thirteenth term is 27, and the seventh term is three times the second term. Find the first term, common difference and the sum of the first ten terms.  
[06marks]
- (b). The first, second and third terms of geometric progression (G.P) are  $2k + 6$ ,  $2k$  and  $k + 2$  respectively, where  $k$  is a positive constant. Determine the;  
i) value of  $k$  and the common ratio ii) the sum to infinity of the progression.  
[06marks]

**Question 12:**

- (a) The polynomial  $f(x) = ax^3 + 3x^2 + bx - 3$  is exactly divisible by  $(2x + 3)$  and leaves a remainder  $-3$  when divided by  $(x + 2)$ . Find the values of  $a$  and  $b$ .  
[05marks]
- (b). The curve is given parametrically by the equations  $x = \frac{t^2}{1+t^2}$ ,  $y = \frac{t^3}{1+t^3}$ ,  
show that  $\frac{dy}{dx} = \frac{3t}{2-t^3}$  and that  $\frac{d^2y}{dx^2} = 48$  at a point  $(\frac{1}{2}, \frac{1}{2})$  [07marks]

**Question 13:**

- (a). Differentiate the following with respect to  $x$ .  
(i).  $(2x + 1)^3 \ln \sqrt{x - 3}$   
(ii).  $\frac{2x^2 - 3x}{(x+4)^2}$  [06marks]
- (b). Find the equation of the normal to the curve  $xy^3 - 2x^2y^2 + x^4 - 1 = 0$  at the point  $(1, 2)$  [06marks]

**Question 14.**

- a) Given that in the equation  $ax^2 + bx + c = 0$  one of the roots of the equation is 3 times the other. Show that  $3b^2 = 16ac$ . [04marks]
- b) Find the values of  $\beta$  for which the equation  $10x^2 + 4x + 1 = 2\beta x(2 - x)$  has equal roots [05marks]
- c) Use synthetic approach to obtain the remainder when  $(x + 4)$  divides the polynomial  $2x^4 + 6x^3 - 7x^2 + 9x + 11$  [03marks]

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