

**MATIGO MOCKS 2024
UACE MARKING GUIDE 2024
PURE MATHEMATICS P425/1**

NO	SOLUTION	REMARKS	MKS
NO.1	$4^x - 2^{x+3} + 15 = 0$ $2^{2x} - 2^{3+3} + 15 = 0$ $(2^x)^2 - 2^3 - 2^x + 15 = 0$ <p>let $2^x = m$</p> $m^2 - 8m + 15 = 0$ $m = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(15)}}{2 \times 1}$ <p>either $m = 5$ or $m = 3$</p> <p>for $m = 5$</p> $2^x = 5$ $x \log 2 = \log 5$ $x = \frac{\log 5}{\log 2}$ $= \underline{2.3219}$ <p>for $m = 3$</p> $2^x = 3$ $x \log 2 = \log 3$ $x = \frac{\log 3}{\log 2}$ $= \underline{1.5850}$		<p>M1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>A1</p> <p>05</p>

No.2	$\frac{\tan \theta}{\sin \theta} - \frac{\sin \theta}{\tan \theta} = \frac{\sin \theta}{\cos \theta \sin \theta} - \frac{\sin \theta}{\frac{\sin \theta}{\cos \theta}}$ $= \frac{1}{\cos \theta} - \sin \theta \cdot \frac{\cos \theta}{\sin \theta}$ $= \frac{1}{\cos \theta} - \frac{\cos \theta}{1}$ $= \frac{1 - \cos^2 \theta}{\cos \theta}$ $\frac{\tan \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta}$ $= \tan \theta \cdot \sin \theta$		M1 M1 M1 M1 A1 05
NO.3	$\vec{OP} = \frac{\mu \vec{OA} + \lambda \vec{OB}}{\lambda + \mu}$ $\vec{OM} = \frac{3(3\hat{i} + 2\hat{j} - 5\hat{k}) + 4(\hat{i} + 3\hat{j} + 2\hat{k})}{2 + 4}$ $= \frac{13\hat{i} + 18\hat{j} - 7\hat{k}}{7}$ $= \frac{13}{7}\hat{i} + \frac{18}{7}\hat{j} - \hat{k}$		M1 M1 M1 M1 A1 05
No.4	$3x - 4y + 14 = 0$ $x = \left(\frac{4y - 14}{3}\right)$ $\text{from } x^2 + y^2 + 4x + 6y - 3 = 0$		M1 M1

	$\left(\frac{4y-14}{3}\right)^2 + y^2 + 4\left(\frac{4y-14}{3}\right) + 6y - 3 = 0$ $\frac{16y^2 - 112y + 169}{9} + y^2 + \frac{16y - 56}{3} + 6y - 3 = 0$ $16y^2 - 112y + 196 + 9y^2 + 48y - 168 + 54y - 12 = 0$ $25y^2 - 10y + 1 = 0$ $A = 25, B = 10, \quad C = 1$ $B^2 = (-10)^2 = 100$ $4AC = 4 \times 25 \times 1 = 100$ <p>Since $B^2 = 4AC$, the line is a tangent to circle.</p>	<p>Accept $B^2 - 4AC = 0$ $B^2 = 4AC$</p> <p>Give marks to a student who solves the equation and gets one point of intersection.</p>	<p>M1</p> <p>M1</p> <p>B1</p>																
NO.5	$ x - 2 < 3x - 4$ $(x - 2)^2 < (3x - 4)^2$ $x^2 - 4x + 4 < 9x^2 - 24x + 16$ $-8x^2 + 20x - 12 < 0$ $2x^2 - 5x + 3 > 0$ $(2x - 3)(x - 1) > 0$ <p>Critical points are $x=1$ and $x = 1.5$</p> <table border="1"> <tr> <td>x</td><td>$x < 1$</td><td>$1 < x < 1.5$</td><td>$x > 1.5$</td></tr> <tr> <td>$2x - 3$</td><td>-</td><td>-</td><td>+</td></tr> <tr> <td>$x - 1$</td><td>-</td><td>+</td><td>+</td></tr> <tr> <td>$(2x - 3)(x - 1)$</td><td>+</td><td>-</td><td>+</td></tr> </table> <p>$x < 1$ and $x > 1.5$</p> <p>testing</p>	x	$x < 1$	$1 < x < 1.5$	$x > 1.5$	$2x - 3$	-	-	+	$x - 1$	-	+	+	$(2x - 3)(x - 1)$	+	-	+		<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p>
x	$x < 1$	$1 < x < 1.5$	$x > 1.5$																
$2x - 3$	-	-	+																
$x - 1$	-	+	+																
$(2x - 3)(x - 1)$	+	-	+																

	$\text{if } m_1 = \frac{1}{3}$ $\frac{y-3}{x-2} = \frac{1}{3}$ $3y - 9 = x - 2$ $3y - x - 7 = 0$		M1 A1 05
No.8	$\frac{dy}{dx} = \frac{114y^2}{e^2}$ $\frac{dy}{1+4y^2} = \frac{dx}{e^x}$ $\int \frac{1}{1+4y^2} dy = \int e^{-x} dx$ $\frac{1}{1 \times 2} \tan^{-1}\left(\frac{2y}{1}\right) = e^{-x} + c$ $\frac{1}{2} \tan^{-1}(2y) + e^{-x} = c$		M1 M1 M1 M1 A1 05
No.9	<p>SECTION B:</p> <p>(i) $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -2 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$</p> $= \begin{pmatrix} 1 \\ -2 \\ 13 \end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix} -1 \\ 0 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}$ $ \overrightarrow{AB} = \sqrt{1^2 + (-2)^2 + (-1)^2} = \sqrt{14}$ $ \overrightarrow{CD} = \sqrt{(-2)^2 + (-1)^2 + (-4)^2} = \sqrt{21}$		M1 M1

	$\overrightarrow{AB} \cdot \overrightarrow{CD} = \overrightarrow{AB} \cdot \overrightarrow{CD} \cos \theta$ $\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix} = \sqrt{14} \cdot \sqrt{21} \cos \theta$ $-2 + 2 + 12 = \sqrt{294} \cos \theta$ $12 = \sqrt{294} \cos \theta$ $\cos \theta = \frac{12}{\sqrt{294}}$ $\theta = \cos^{-1} \left(\frac{12}{7\sqrt{6}} \right)$ $\theta = 45.58^\circ$		M1
			M1
			A1
	$(ii) \text{ } \underbrace{r}_{\text{r}} AB = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}$ <p>For intersection:</p> $\underbrace{r}_{\text{r}} AB = \underbrace{r}_{\text{r}} CD$ $\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}$ $\begin{pmatrix} 4 + \lambda \\ -2\lambda \\ 1 - 3\lambda \end{pmatrix} = \begin{pmatrix} 1 - 2\mu \\ 1 - \mu \\ -4\mu \end{pmatrix}$ $4 + \lambda = 1 + 2\mu$ $\lambda + 2\mu = -3 \dots (i)$ $-2\lambda + \mu = 1 \dots (ii)$ $-3\lambda + 4\mu = -1 \dots (iii)$ $2\lambda + 4\mu = -6$		M1
			M1

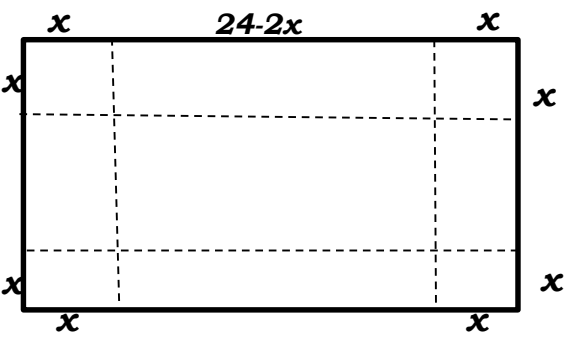
	$\underline{-2\lambda + \mu = 1}$	M1
	$5\mu = -5$	
	$\mu = -1$	
	From equation (ii) $-2\lambda - 1 = 1$	
	$-2\lambda = 2$	
	$\lambda = -1$	
	Subtracting μ and λ in equation (iii)	
	$-3(-1) + 4(-1) = -1$ $3 - 4 = -1$ $-1 = -1$	
	Since R.H.S. = L.H.S. the lines intersect.	A1
(iii)	$\overrightarrow{OM} = \begin{pmatrix} 4 + \lambda \\ -2\lambda \\ 1 - 3\lambda \end{pmatrix} \overrightarrow{AB}$ $\rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}$ $\overrightarrow{PM} = \begin{pmatrix} 4 + \lambda \\ -2\lambda \\ 1 - 3\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}$ $\overrightarrow{PM} = \begin{pmatrix} 3 + \lambda \\ -2\lambda - 5 \\ -5 - 3\lambda \end{pmatrix}$ $\overrightarrow{PM}.d = 0$ $\begin{pmatrix} 3 + \lambda \\ -5 - 2\lambda \\ -5 - 3\lambda \end{pmatrix} . \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} = 0$	M1
		M1

	$3 + \lambda + 10 + 4\lambda + 15 + 9\lambda = 0$ $14\lambda + 28 = 0$ $\lambda = \frac{-28}{14}$ $\lambda = -2$ $\overrightarrow{PM} = \begin{pmatrix} 3 - 2 \\ -5 + 4 \\ -5 + 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $ \overrightarrow{PM} = 1^2 + (-1)^2 + 1^2$ $= \sqrt{3}$		A1 12
NO.10	$\text{a) } \frac{\cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} = \frac{4}{\sin \theta \tan \theta}$ $L.H.C = \frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta}$ $= \frac{(1 + \cos \theta)^2 - (1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}$ $= \frac{1 + 2 \cos \theta + \cos^2 \theta - 1 + 2 \cos \theta - \cos^2 \theta}{1 - \cos^2 \theta}$ $= \frac{4 \cos \theta}{\sin^2 \theta}$ $= \frac{4 \cos \theta}{\sin \theta \cdot \frac{\sin \theta}{\cos \theta}}$ $= \frac{4}{\sin \theta \tan \theta}$ $\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} = 3$		M1 M1 M1 M1

	$\frac{4}{\sin \theta \tan \theta} = 3$ $\frac{4}{\sin \theta \cdot \frac{\sin \theta}{\cos \theta}} = 3$ $\frac{4 \cos \theta}{1 - \cos^2 \theta} - 3 = 0$ $\frac{4 \cos \theta - 3(1 - \cos^2 \theta)}{1 - \cos^2 \theta} = 0$ $3 \cos^2 \theta + 4 \cos \theta - 3$ $\cos \theta = \frac{-4 \pm \sqrt{4^2 - 2(3)(-3)}}{2 \times 3}$ $\text{either } \cos \theta = \frac{-2 + \sqrt{13}}{3} \text{ or } \cos \theta = \frac{-2 - \sqrt{13}}{3}$ $\text{for } \cos \theta = \frac{-2 + \sqrt{13}}{3} \quad \text{for } \cos \theta = \frac{-2 - \sqrt{13}}{3}$ $\theta = 57.64^\circ, \quad 302.36^\circ$		M1
			M1
			M1
	<p>b) Let $\tan^{-1}(1/2) = \alpha, \tan^{-1}(1/3) = \beta$</p> $\tan \alpha = 1/2; \tan \beta = 1/3$ $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ $\alpha + \beta = \tan^{-1} \left[\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} \right]$ $\alpha + \beta = \tan^{-1}(1)$ $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \pi/4$		A1

No.11	$a + ar = -4 \dots (i)$ $ar^2 = 4ar^2 \dots (ii)$ $ar^2 \cdot r^2 = 4a \cdot r^2$ $r^2 = 4$ $r = \pm 2.$ <p>from equation (i) $a(1 + r) = -4$</p> $a(1 + 2) = -4$ $a = -4/3$ <p>when $r = -2$</p> $a(1 - 2) = -4$ $a = -4/-1$ $a = 1$ <p>when $r = 2, a = \frac{4}{3}$</p> <p>when $r = -2, a = 1$</p> <p>the possible G. P. S are;</p> $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \frac{-32}{3} + \dots$ $1 - 2 + 4 - 8 + 16 - 32 + \dots$ <p>b)</p> $(i) \quad \alpha + \beta = 7k$ $\alpha \beta = k^2$ $(\alpha - \beta) = \sqrt{\alpha^2 - 2 \alpha \beta + \beta^2}$ $(\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4 \alpha \beta}$ $= \sqrt{(7k)^2 - 4k^2}$	<p>M1</p> <p>M1</p> <p>B1</p> <p>B1</p> <p>A1</p> <p>M1</p>
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	$= \sqrt{49k^2 - 4k^2}$ $= \sqrt{45k^2}$ $= \sqrt{9k^2 \times 5}$ $3k\sqrt{5}$ <p>(ii) sum of roots = $\alpha + 1 + \beta - 1$ $= \alpha + \beta$ $= 7k$</p> <p>product of roots = $(\alpha + 1)(\beta - 1)$ $= \alpha\beta - \alpha + \beta - 1$ $= \alpha\beta - 1 - (\alpha - \beta)$</p> <p>product of roots = $k^2 - 1 - 3k\sqrt{5}$ $= k^2 - 3k\sqrt{5} - 1$</p> $x^2 + 7kx + k^2 - 3k\sqrt{5} - 1 = 0$		<p>A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>
No.12 (a)	$\frac{dy}{dx} = \sqrt{1 + 2x}$ $\frac{dy}{dx} = (1 + 2x)^{\frac{1}{2}}$ $\int dy = \int (1 + 2x)^{\frac{1}{2}} dx$ $y = \frac{(1 + 2x)^{\frac{3}{2}}}{\frac{3}{2}(2)} + c$ $y = \frac{1}{3}(1 + 2x)^{\frac{3}{2}} + c$ $y(4) = 11$ $11 = \frac{1}{3}(1 + 2(4))^{\frac{3}{2}} + c$		<p>M1</p> <p>M1</p>

(b)	$11 = \frac{1}{3}(9)^{\frac{3}{2}} + c$ $c = 2$ $\therefore y = \frac{1}{3}(1 + 2x)^{\frac{3}{2}} + 2$  $v = L \times w \times h$ $v = (24 - 2x)(9 - 2x)x$ $v = (216 - 48x - 18x + 4x^2)x$ $v = (216 - 66x + 4x^2)x$ $v = 216x - 66x^2 + 4x^3$ $\frac{dy}{dx} = 216 - 132x + 12x^2$ <p>for maximum volume</p> $\frac{dy}{dx} = 0$ $216 - 132x + 12x^2 = 0$ $12x^2 - 132x + 216 = 0.$ $x = \frac{132 \pm \sqrt{(-132)^2 - 4(12)(216)}}{2(12)}$ <p><i>Either $x = 9$ or $x = 2$.</i></p>		<p>A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p>
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[illegible]

Dimension for maximum volume are 20cm, 5cm, and 2cm.

No.13

$$\frac{dy}{dx} = \tan^2 x$$

$$v = \int \tan^2 x dx$$

$$v = \tan x - x$$

$$\begin{aligned}\int x \tan^2 x dx &= x(\tan x - x) - \int \tan x - x dx \\ &= x \tan x - x^2 + -\ln|\sec x| + \frac{x^2}{2} + c\end{aligned}$$

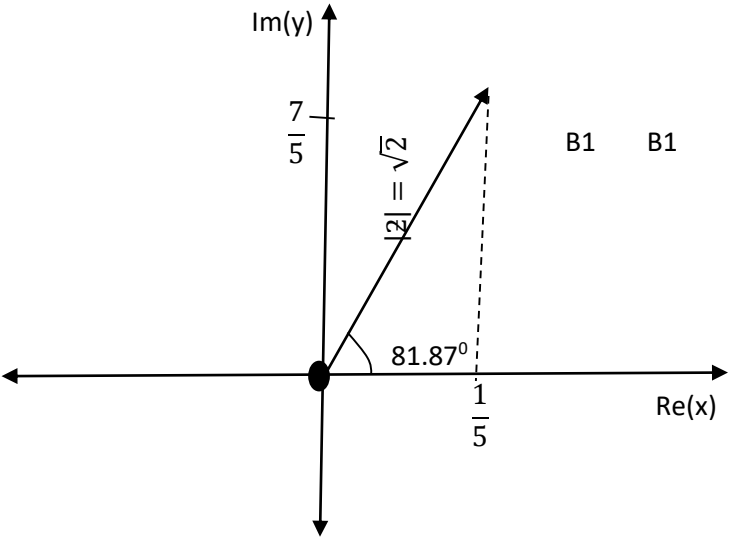
$$= x \tan x - x^2 + -i n | \sec x | + \frac{x^2}{2} + c$$

$$\int \sqrt{1-x^2} \, dx = \frac{1}{4} \cdot 2 \sin u \cos u + \frac{u}{2} + k$$

$$= \frac{1}{2} x \sqrt{1-x^2} + \frac{\sin^{-1} x}{2} + k$$

$$\int x \sin^{-1} x dx = \frac{x^2}{2} \sin^{-1} x + \frac{-1}{2} \sin^{-1} x + \frac{1}{2} \left(\frac{x}{2} \sqrt{1-x^2} + \frac{\sin x^{-1}}{2} + c \right)$$

	$= \frac{x^2}{2} \sin^{-1} x + \frac{1}{4} x \sqrt{1-x^2} - \frac{1}{4} \sin^{-1} x + c$ $= \frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{1}{4} x \sqrt{1-x^2} + c.$		M1 A1
14(a)	$\sqrt{-3+4i} = \pm(a+bi)$ $-3+4i = a^2 + 2bi \pm b^2$ $-3 = a^2 - b^2 \dots (i)$ $2b = 4 \dots (ii)$ $b = 2$ from (i) $-3 = a^2 - 2^2$ $a^2 = 1$ $a = 1$ $\sqrt{-3+4i} = \pm(1+2i)$ $= 1+2i \text{ and } -1-2i$		M1 M1 M1 M1 A1A1
(b)(i)	$= \frac{-1+3i}{2+i}$ $= \frac{(-1+3i)(2-i)}{(2+i)(2-i)}$ $= \frac{-2+i+6i+3}{2^2+1}$ $z = \frac{1+7i}{5} \quad M1$ $z = \frac{1}{5} + \frac{7}{5}i \quad A1$		M1 M1

			
15(a)	<p>$y^2 = 4ax$ let the point $A(at^1, 2at)$ lie on the parabola.</p> $2y \frac{dy}{dx} = 4a$ $\frac{dy}{dx} = \frac{4a}{2y}$ $\frac{dy}{dx} = \frac{4a}{4at}$ $\frac{dy}{dx} = 1/t$ <p>equation of a tangent is;</p> $(y - 2at) = 1/t (x - at^2)$ $ty - 2at^2 = x - at^2$ $ty = x + at^2 \dots (i)$ $y = \frac{x + at^2}{t}$ <p>from $y^2 = 8ax \dots (iii)$</p>		<p>M1</p> <p>M1</p> <p>M1</p>

	$\left(\frac{x + at^2}{t}\right)^2 = 8ax$ $x^2 + 2axt^2 + a^2 + 4 = 8axt^2$ $x^2 - 6axt^2 + a^2 + 4 = 0$ <p>let the mid – point of LM be (X, Y)</p> $X = 3at^2 \dots\dots (iii)$ <p>from equation (i) and (ii)</p> $x = ty - at^2$ $: y^2 = 8a(ty - at^2)$ $y^2 = 8aty - 8a^2t^2$ $y^2 - 8aty + 8a^2t^2 = 0$ $Y = 4at \dots (iv)$ $: t = Y/4a.$ <p>from eqn (iii)</p> $X = 3a \frac{Y^2}{(4a)^2}$ $X = \frac{3y^2}{16a}$ $16ax = 3y^2$ $16ax = 3y^2$		M1
			M1
			M1
			A1
(b)	<p>Equation of chord of contact is</p> $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ <p>when $x = 0$</p> $y = \frac{b^2}{y_1}$ <p>when $y = 0$</p> $x = \frac{a^2}{x_1}$		M1
			M1

	$L \text{ is } \left(0, \frac{b^2}{y_1}\right) \text{ and } M \text{ is } \left(\frac{a^2}{x_1}, 0\right)$ $\text{mid point is } \left(\frac{a^2}{2x_1}, \frac{b^2}{2y_1}\right)$ $\left(\frac{a^2}{2x_1}\right)^2 + \left(\frac{b^2}{2y_1}\right)^2 = 1$ $\frac{a^4}{4x_1^2} + \frac{b^4}{4y_1^2} = 1$ $\therefore \text{The locus of } (x_1, y_1) \text{ is } \frac{a^4}{4x_1^2} + \frac{b^4}{4y_1^2} = 1$		<p>M1</p> <p>M1</p> <p>A1 12</p>
16.	$\ln y + \frac{x}{y} \frac{dy}{dx} = \sec x \tan x ; y(0) = \frac{\pi}{2}$ $\int \frac{d}{dx} (x \ln y) dx = \int \sec x \tan x dx$ $x \ln y = \sec x + c$ $(0) \ln \left(\frac{\pi}{2}\right) = \sec(0) + c$ $0 = 1 + c$ $c = -1$ $\therefore x \ln y = \sec x - 1$		<p>M1</p> <p>M1</p> <p>M1</p> <p>B1</p> <p>A1</p>
(b)	$\text{Rate in} = 0$ $\text{Rate out} = \frac{y}{1000} (kg/L) \times 20(L/min)$ $= \frac{y}{50} (kg/min)$ $\frac{dy}{dt} = 0 - \frac{y}{50}$		M1

	$\frac{dy}{dt} = \frac{-y}{50}$ $\int \frac{dy}{y} = \int \frac{-1}{50} dt$ $\ln y = \frac{-1}{50}(0) + c.$ $c = \ln 10.$ $\ln y = \frac{-1}{50}t + \ln 10$ $\ln y - \ln 10 = \frac{-t}{50}$ $e^{\frac{-t}{50}} = \frac{y}{10}$ $y = 10 e^{\frac{-t}{50}}$		M1
(i)	$\text{at } t = 5, y = ?$ $y = 10 e^{\frac{-5}{50}}$ $y = 9.0484 \text{ kg.}$		M1 A1
(ii)	$y = 5 \text{ kg}, t = ?$ $5 = 10 \cdot e^{\frac{-t}{50}}$ $\ln 0.5 = \frac{-t}{50}$ $t = -50 \ln(0.5)$ $t = 34.6574 \text{ minutes}$		M1 A1 12