Tr. Kabba

[illegible] EXAMINATIONS BOARD

MOCK EXAMINATIONS 2023

P425/1

P425/1 PURE MATHEMATICS

MARKING GUIDE

1. $a - 1^{st}\ term,$ $\qquad r - common\ ratio$

$a + ar = -4$ B1

$ar^3 + ar^4 = 108$ B1

$r^3(a + ar) = 108$

$-4r^3 = 108$ M1

$r^3 = -27$

$r = \sqrt[3]{-27}$

$= -3$

$a = \frac{-4}{a+r} = \frac{-4}{1+-3}$ A1

$a = 2$ A1

05mks

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2. $3x^2 + 2y^2 + 6x - 8y = 7$

$3(x^2 + 2x) + 2(y^2 - 4y) = 7$

$3(x^2 + 2x + 1^2 - 1^2) + 2(y^2 - 4y + 2^2 - 2^2) = 7$ M1

$3(x + 1)^2 - 3 + 2(y - 2)^2 - 8 = 7$

$3(x + 1)^2 + 2(y - 2)^2 = 18$

$\frac{(x+1)^2}{6} + \frac{(y-2)^2}{9} = 1$ B1

$centre \quad (-1,2)$ A1

$b^2 = a^2(1 - e^2)$

$a^2 = 9\ b^2 = 6$

$6 = 9\,(1 - e^2)$ M1

$e^2 = \frac{1}{3}$

$e = \frac{1}{\sqrt{3}}$ A1

05mks

3. $y^2 - 4xy = x^2 + 5$

$2y\frac{dy}{dx} - 4\left(x\frac{dy}{dx} + y\right) = 2x$

$(2y - 4x)\frac{dy}{dx} = 2x + 4y$

$\frac{dy}{dx} = \frac{2x+4y}{2y-4x} = \frac{x+2y}{y-2x}$ M1

$for\ horizontal\ tangent\ \frac{dy}{dx} = 0$

$x + 2y = 0$ B1

$x = -2y$

$y^2 - 4(-2y)y = (-2y)^2 + 5$ M1

$9y^2 = 4y^2 + 5$

$5y^2 = 5$

$y^2 = 1$

$y = \pm 1$

$y = 1 \quad x = -2 \quad ; \quad y = -1 \quad x = 2$

$(-2, 1) \quad (2, -1)$ A1 A1 05 mks

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4. $3\cos^2\theta - 4\cos\theta\sin\theta + \sin^2\theta = 2$

$3\cos^2\theta - 4\cos\theta\sin\theta + \sin^2\theta = 2$

$3\frac{1}{2}(1 + \cos2\theta) - 2\sin2\theta + \frac{1}{2}(1 - \cos2\theta) = 2$ M1

$3 - 4\tan\theta + \tan^2\theta = 2\sec^2\theta$

$3 + 3\cos2\theta - 4\sin2\theta + 1 - \cos2\theta = 4$

$2\cos2\theta = 4\sin2\theta$ B1

$3 - 4\tan\theta + \tan^2\theta = 2(1 + \tan^2\theta)$ M1

$\tan2\theta = \frac{1}{2}$

$\tan^2\theta + 4\tan\theta - 1 = 0$ B1

$2\theta = \tan^{-1}(0.5)$ M1

$\tan\theta = -4 \pm \sqrt{4^2 - 4\times1\times-1}$ M1

$2\theta = 26.57°, 206.57°, 336.57°, 566.57°$

$= 0.236, -4.236$ A1

$\theta = 13.29°, 103.229°, 193.29°, 283.29°$ A1 A1 05mks

$\theta = 13.3, 103.3°$ A1

5. Let $\frac{x-1}{k} = \frac{y}{-1} = \frac{z+3}{1} = \lambda \quad \Rightarrow x = 1 + k\lambda,\ y = -\lambda\ z = -3 + \lambda$

$\frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2} = \mu \quad \Rightarrow x = 4 + \mu, y = -3 + \mu, z = -3 + 2\mu$

For intersection

$1 + k\lambda = 4 + \mu$ ……………………….(i) M1

$-\lambda = -3 + \mu$ …………………………….(ii)

$-3 + \lambda = -3 + 2\mu$ ------ (iii)

$\lambda = 2\mu$

Substitute λ in (ii) M1

$-2\lambda = -3 + \mu$

$3\mu = 3$

$\mu = 1,\ \lambda = 2$ ~~B1~~ A1 for both

Using (i)

$1 + 2k = 4 + 1$

$K = 2$ A1

Point of intersection $(5, -2, -1)$ B1 05mks

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6. For $n = 1$

$LHS = \frac{1}{1(1+1)} = \frac{1}{2}$ ~~M1~~

$RHS = 1 - \frac{1}{2} = \frac{1}{2}$ M1

$Assume\ the\ result\ holds\ for\ n = k$

$\frac{1}{1\times2} + \frac{1}{2\times3} + - - - + \frac{1}{K(k+1)} = 1 - \frac{1}{k+1}$

For $n = k + 1$

LHS $\frac{1}{1\times2} + \frac{1}{2\times3} + - - - + \frac{1}{K(k+1)} + \frac{1}{(K+1)\ (k+2)}$ ~~M1~~ M1

$= 1 - \frac{1}{k+1} + \frac{1}{(k+1)(k+2)}$

$= 1 - \frac{1}{k+1}\left(1 - \frac{1}{k+2}\right) = 1 - \frac{1}{k+1} \cdot \left(\frac{k+1}{k+2}\right)$

$= 1 - \frac{1}{k+2}$ ~~B1~~

$RHS = 1 - \frac{1}{k+1+1}$ B1

$= 1 - \frac{1}{k+2}$ B1 A1

Hence $\sum_1^n \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$ *is true for* $n = 1,$ B1

$k, k+1$ *and all postive integral values* $n \geq 1$ 05mks

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7. $\int_2^6 \frac{\sqrt{x-2}}{x}\, dx$

x	u
6	2
2	0

$u = \sqrt{x-2}$

$u^2 = x - 2$

$2udu = dx$

$\int_0^2 \frac{u}{u^2+2} 2udu = \int_0^2 \frac{2u^2}{u^2+2} du$ M1

$$\begin{array}{r|l} & 2 \\ \hline u^2+2 & 2u^2 \\ - & 2u^2+4 \\ \hline & -4 \end{array}$$

$= \int_0^2 \left(2 - \frac{4}{2+u^2}\right) dx$ ~~B1~~ M1

$= \int_0^2 \left[2 - 4\left(\frac{1}{2+u^2}\right)\right] du$

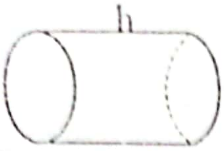
$= \left[2u - 4 . \frac{1}{\sqrt{2}} \tan^{-1} \frac{1}{\sqrt{2}} u\right]_0^2$ ~~M1~~ A1

$= (4 - 2\sqrt{2} \tan^{-1}\sqrt{2} - 0)$ M1

$= 1.2980$ (udp) A1

05 mks

8.



$2\pi r + h = 6$

$V = \pi r^2 h$

$V = \pi r^2(6 - 2\pi r)$ M1

$V = 6\pi r^2 - 2\pi^2 r^3$

$\frac{dv}{dr} = 12\pi r - 6\pi^2 r^2$

for the largest parcel $\frac{dv}{dr} = 0$ ~~M1~~

$6\pi r(2 - \pi r) = 0$ M1

$r = 0, \; r = \frac{2}{\pi}$ A1 ~~B1~~

$\frac{d^2v}{dr^2} = 12\pi - 12\pi^2 r$

$\frac{d^2r}{dr^2}\left(r = \frac{2}{\pi}\right) = 12\pi - 24\pi = -12\pi < 0$ M1

$r = \frac{2}{\pi}$ *Gives maximum volume*

$h = 6 - 2\pi \, . \frac{2}{\pi}$

$= 2$ A1

$r = \frac{2}{\pi}cm, \quad h = 2cm$ 05 mks

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9. (a)

$$\frac{Z_1}{Z_2} = \frac{2\left(cos\frac{\pi}{4} + i\,sin\frac{\pi}{4}\right)}{8\left(Cos\frac{\pi}{3} + \; i\;Sin\,\frac{\pi}{3}\right)}$$

$$= \frac{2(Cos\pi + i\;Sin\pi)^{\frac{1}{4}}}{8(Cos\pi + i\;Sin\pi)^{\frac{1}{3}}}$$ Demoivre's theorem M1

$= \frac{1}{4}\;(Cos\pi + i\;Sin\pi)^{\frac{-1}{12}}$

$= \frac{1}{4}\left(Cos\;\frac{1}{12}\pi - i\;Sin\,\frac{\pi}{12}\right)$ B1 M1

$= 0.2415 - i\;0.0647$ A1

(ii) $\sqrt[3]{z_2} = \left[8\left(Cos\frac{\pi}{3} + i\,Sin\frac{\pi}{3}\right)\right]^{\frac{1}{3}}$

$2\left[Cos\left(\frac{\pi}{3} \pm 2n\pi\right) + i\,Sin\left(\frac{\pi}{3} \pm 2n\pi\right)\right]^{\frac{1}{3}}$ M1 for the general angle.

$2\left[Cos\left(\frac{\pi}{9} + \frac{2n\pi}{3}\right) + i\,Sin\left(\frac{\pi}{9} + \frac{2n\pi}{3}\right)\right]$ $n = 0, 1, 2, --$

$n = 0$

$z^{1/n} = r^{\frac{1}{n}}\left(Cos\left(\frac{\theta + 2\pi k}{n}\right) \pm i\,Sin\,\frac{\theta + 2\pi k}{n}\right)$

$\sqrt[3]{z_2} = 2\left(Cos\frac{\pi}{9} + iSin\frac{\pi}{9}\right)$

$= 1.8794 + i\,0.6840$ A1

Accept degrees (answers)

$n = 1$

$2\left(Cos\frac{7\pi}{9} + i\,Sin\frac{7\pi}{9}\right)$

$= -1.5321 + i\,1.2856$ A1

$n = 2$

$= 2\left(Cos\,\frac{13\pi}{9} + i\,Sin\frac{13\pi}{9}\right)$

$= -0.3473 - i\,1.9696$ A1

9.(b) $arg\left(\frac{Z}{Z-4+2i}\right) = \frac{\pi}{2}$ $Z = x + iy$

For the locus, it must be x & y. No a+bi

$arg\ Z - \arg(Z - 4 + 2i) = \frac{\pi}{2}$

$arg(x + iy) - \arg\left[(x-4) + (y+2)i\right] = \frac{\pi}{2}$ B1

$tan^{-1}\frac{y}{x} - tan^{-1}\left(\frac{y+2}{x-4}\right) = \frac{\pi}{2}$ M1

Let $A = tan^{-1}\frac{y}{x}$ $B = tan^{-1}\frac{y+2}{x-4}$

$A - B = \frac{\pi}{2}$

$\tan(A - B) = tan\frac{\pi}{2}$

$\frac{tanA - tanB}{1 + tanA\ tanB} = tan\frac{\pi}{2}$ M1

$\frac{\frac{y}{x} - \frac{y+2}{x-4}}{1 + \frac{y}{x}\cdot\frac{y+2}{x-4}} = tan\ \frac{\pi}{2} = \frac{1}{0}$

$1 + \frac{y}{x}\cdot\frac{y+2}{x-4} = 0$ M1

$x(x-4) + y(y+2) = 0$

$x^2 + y^2 - 4x + 2y = 0$ A1 12mks

10. (a) $y = mx$

$x^2 + y^2 + 2\,y + c = 0 \quad ------(i)$

$x^2 + (mx)^2 + 2f(mx) + c = 0$

$(1+m^2)\,x^2 + 2fmx + c = 0$ M1

Compare with $Ax^2 + BX^2 + c = 0$

for equal roots

$B^2 = 4Ac$

$(2fm)^2 = 4\,.(1+m^2)\,.c$ M1

$4f^2m^2 = 4c\,(1+m^2)$

$c = \frac{f^2m^2}{1+m^2}$ B1

Hence

Compare $x^2 + y^2 - 10y + 20 = 0 \quad ----(ii)$with (i)

$2f = -10$

$f = -5$ B1 both f $ c

$c = 20$

Therefore

$20 = \frac{(-5)^2\,.m^2}{1+m^2}$ M1

$20 + 20m^2 = 25m^2$

$5m^2 = 20$

$m^2 = 4$ B1 (for M)

$m = \pm 2$ A1 (both correct)

The tangents are;

$y = 2x, \quad y = -2x$ A1 (both Corred) A1 A1

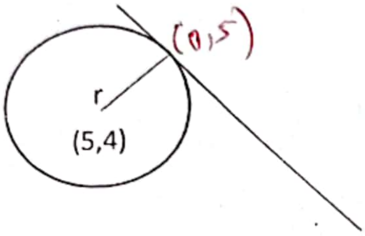
10 (b) Gradient of the tangent $= \frac{5-1}{0-4} = -1$

M1

Equation of the tangent $\frac{y-5}{x} = -1$

$y - 5 = -x$

$x + y - 5 = 0$ B1



$r = \left|\frac{5+4-5}{\sqrt{1^2 1^2}}\right| = \frac{4}{\sqrt{2}}$ B1

Equation of the circle

$(x-a)^2 + (y-b)^2 = r^2$

$(x-\cancel{6}5)^2 + (y-4)^2 = 8$ M1 A1

$x^2 + y^2 - 10x - 8y + 33 = 0$ ~~M1~~ ~~A1~~

12mks

11.(a) $y = \frac{12}{x^2-2x-3}$

$yx^2 - 2yx - 3y = 12$

$yx^2 - 2yx - \cancel{2yx} - 3y - 12 = 0$ B1

$for\ no\ real\ roots\ b^2 - 4ac < 0$

$(-2y)^2 - 4.y.(-3y-12) < 0$ M1

$y^2 + 3y^2 + 12y < 0$

$4y^2 + 12y < 0$ [illegible] ~~B1~~

$y(y+3) < 0$

$critical\ values\ y = 0, -3$ A1 ~~B1~~

	$y < -3$	$-3 < y < 0$	$y > 0$
$y(y+3)$	+	−	+

M1

There is no curve in the range $-3 < y \leq 0$ A1

$y = -3$

$-3x^2 + 6x + 9 - 12 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$X = 1$

$(1, -3)$ *is a maximaum point* A1

(b) from $-3 < y \leq 0$

$y = 0$ *is the horizontal asymptote* A1

Vertical a asymptotes

$x^2 - 2x - 3 = (x+1)\ (x-3) = 0$

$X = 3, \qquad X = -1$ A1 A1

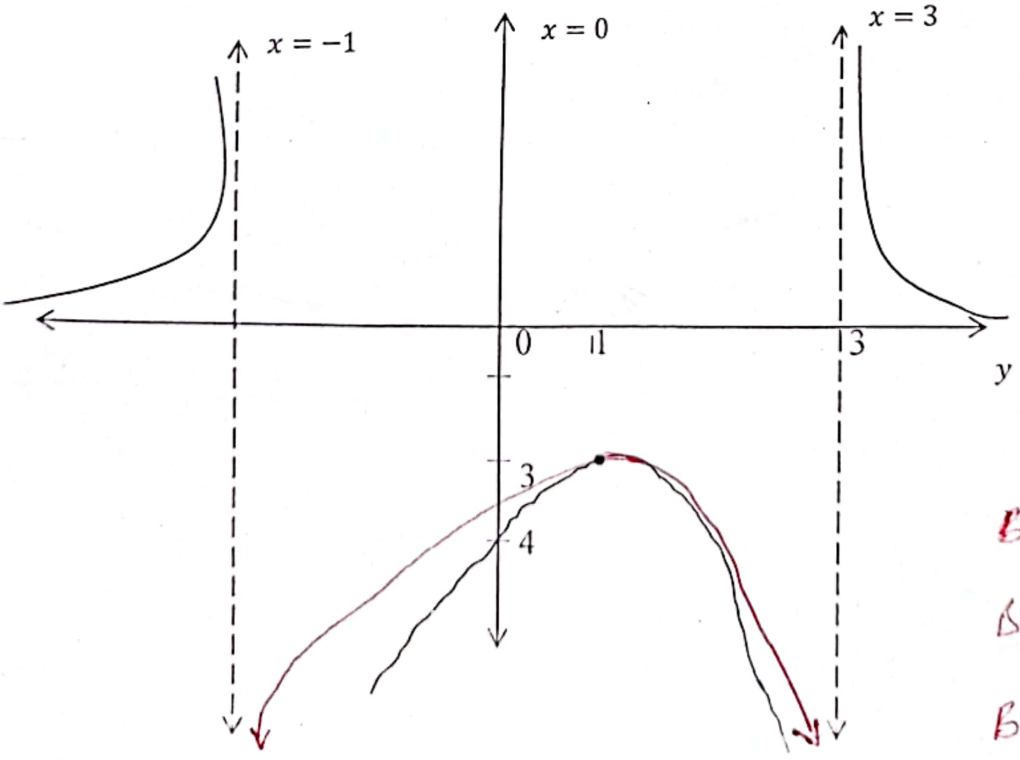
Intercepts $x = 0 \quad y = -4 \quad (0, -4)$

$y = 0 \quad$ *no x − Intercept*

Regions

	$x < -1$	$-1 < x < 3$	$x > 3$
y	+	−	+

B1



B1 B1

(correct curve only)

B1 - Correct Curve

B1 - Asymptotes

B1 - table

12. (i) $r = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

$r.\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = d$

For $\mu = 0$, $r = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

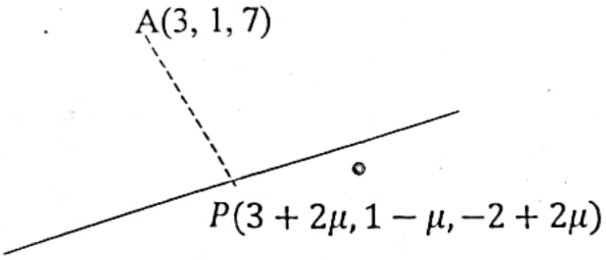
~~M1~~

$\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = d$ M1

$d = 3 - 2 + 4$

$d = 5$ A1 A1

(ii) $\overrightarrow{AP} = \begin{pmatrix} 3+2\mu \\ 1-\mu \\ -2+2\mu \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}$



$AP = \begin{pmatrix} 2\mu \\ -\mu \\ -9+2\mu \end{pmatrix}$ B1

$\begin{pmatrix} 2\mu \\ -\mu \\ -9+2\mu \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = 0$ M1

$4\mu + \mu - 18 + 4\mu = 0$

$9\mu = 18$ ~~A1~~

$\mu = 2$ A1

$\overrightarrow{AP} = \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix}$ B1

$|\overrightarrow{AP}| = \sqrt{4^2 + (-2)^2 + (-5)^2}$ M1

$= \sqrt{45}$

$= 3\sqrt{5}$ *units* A1

$= 6.70\ 2$

(b)

$$A(2,-5,3) \qquad B(7,0,-2)$$

$AC : CB \quad = 3 : -8$

$\frac{AC}{CB} = \frac{3}{-8}$ M1

$-8(OC - OA) = 3(OB - OC)$

$-8OC + 8OA = 3OB - 3OC$

$\overrightarrow{5OC} = \overrightarrow{8OA} - \overrightarrow{3OB}$

$\overrightarrow{5OC} = 8\begin{pmatrix}2\\-5\\3\end{pmatrix} - 3\begin{pmatrix}7\\0\\-2\end{pmatrix}$ M1

$\overrightarrow{5OC} = \begin{pmatrix}-5\\-40\\30\end{pmatrix}$ ~~B~~M1

$\overrightarrow{OC} = \begin{pmatrix}-1\\-8\\6\end{pmatrix}$ ~~B~~A1

$C(-1,-8,6)$ ~~A~~B1

12mks

13.(a) let $\log_{n^2}(ab) = k$

$n^{2k} = ab$

$log_n^{n^{2k}} = log_n ab$

$2k = \log_n a + \log_n b$ M1 (change of base n.

$k = \frac{1}{2}(log_n^a + log_n^b)$ B1 (PROOF).

$log_{n^2}ab = \frac{1}{2}(log_n^a + log_n^b)$

hence

$2log_9(xy) = 5 \Rightarrow 2log_{3^2}(xy) = 5$ M~~1~~2

$n = 3$

$2 \cdot \frac{1}{2}(log_3 x + log_3 y) = 5$ M1

$log_3 x + log_3 y = 5$

$\log_3 x \cdot \log_3 y + 6 = 0$

Let $\log_3 x = a, \quad \log_3 y = b$

$a + b = 5$ ~~M1~~ B1

$ab = -6 \quad \Rightarrow b = \frac{-6}{a}$

$a + \frac{-6}{a} = 5$

$a^2 - 5a - 6 = 0$ ~~B1~~ M1

$(a-6)\ (a+1) = 0$

$a = 6, -1$

$6 = \log_3 x \quad \Rightarrow x = 3^6$

$= 729$

$\log_3 x = -1 \quad \Rightarrow x = \frac{1}{3}$ A1

$when\ a = 6 \quad b = -1 \quad when\ a = -1 \quad b = 6$

$y = \frac{1}{3}, \ 729$ A1

13.(b) $(1+x)^n = 1 + n(x) + \frac{n(n-1)}{2!}x^2 + - - -$

$\sqrt{(1+x)(1+x^2)} = (1+x)^{\frac{1}{2}}(1+x^2)^{\frac{1}{2}}$ B1

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}x^2 + - - -$ B1

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + - - -$

$(1+x^2)^{\frac{1}{2}} = 1 + \frac{1}{2}x^2 + - - -$ B1

$(1+x)^{\frac{1}{2}}(1+x^2)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + - - - + \frac{1}{2}x^2 + - - -$ M1

$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + - - -$ A1

12mks

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14. (a) $\cos 3\theta = \cos(2\theta + \theta)$

$= \cos 2\theta \cos\theta - \sin 2\theta .\ \sin\theta$

$= (2\cos^2\theta - 1)cos\theta - 2sin\theta cos\theta . sin\theta$ M1 (for single angle)

$= cos^3\theta - cos\theta - 2sin^2\theta cos\theta$

$= 2cos^3\theta - cos\theta - 2(1 - cos^2\theta)cos\theta$ M1 (when he has cos

$= 2cos^3\theta - cos\theta - 2cos\theta + 2cos^3\theta$

$= 4cos^3\theta - 3cos\theta$ B1

$cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$

$cos3\theta = 4\cdot\left[\frac{1}{2}\left(a + \frac{1}{a}\right)\right]^3 - 3\cdot\frac{1}{2}\left(a + \frac{1}{2}\right)$ M1

$= \frac{1}{2}\left(a + \frac{1}{a}\right)\left[\left(a + \frac{1}{a}\right)^2 - 3\right]$

$= \frac{1}{2}\left(a + \frac{1}{a}\right)\left(a^2 + 2 + \frac{1}{a^2} - 3\right)$ M1

$= \frac{1}{2}\left(a + \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right)$

$= \frac{1}{2}\left(a^3 + \frac{1}{a} - a + a + \frac{1}{a^3} - \frac{1}{a}\right)$ ~~B1~~

$= \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ B1

14.(b) let

$K = sin^2\frac{1}{2}A + sin^2\frac{1}{2}B + sin^2\frac{1}{2}C$

$= \frac{1}{2}(1 - cosA) + \frac{1}{2}(1 - cosB) + \frac{1}{2}(1 - cosC)$ M1

$= \frac{3}{2} - \frac{1}{2}(cosA + cosB + cosC)$

$= \frac{3}{2} - \frac{1}{2}\left(2cos\frac{A+B}{2}\cos\frac{A-B}{2} + 1 - 2sin^2\frac{C}{2}\right)$ M1

$A + B + C = 180$

$\frac{A+B}{2} = 90 - \frac{C}{3}$

$$\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$$

$$K = \frac{3}{2} - \frac{1}{2}\left(2\sin\frac{C}{2}\cdot\cos\frac{A-B}{2} - 2\sin^2\frac{C}{2} + 1\right)$$ M1

$$= \frac{3}{2} - \frac{1}{2}\left[2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\frac{C}{2}\right) + 1\right]$$

$$= \frac{3}{2} - \frac{1}{2}\left[2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\frac{A+B}{2}\right) + 1\right]$$ B1

$$= \frac{3}{2} - \frac{1}{2}\left[2\sin\frac{C}{2}(-2)\sin\frac{A}{2}\sin\left(\frac{-B}{2}\right) + 1\right]$$ M1

$$\text{but } \sin\left(\frac{-B}{2}\right) = -\sin\frac{B}{2}$$

$$= \frac{3}{2} - \frac{1}{2} - \frac{1}{2}\cdot 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C.}{2}$$ B1

12mks

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15. (a) (i)

$$y = x^2\sin\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = \frac{vdu}{dx} + u\frac{dv}{dx}$$

$u = x^2 \qquad v = \sin\frac{1}{x}$ $\qquad \frac{du}{dx} = 2x, \quad \frac{dv}{dx} = \frac{-1}{x^2}\cos\frac{1}{x}$

$$\frac{dy}{dx} = \sin\frac{1}{x}\cdot 2x + x^2\cdot\cos\frac{1}{x}\cdot\left(\frac{-1}{x^2}\right)$$ M1 B1 B_1 fr deriv

$$= 2x\cdot\sin\frac{1}{x} - \cos\frac{1}{x}$$ A1

(ii) $y = xIn^3x \cdot = x\,(Inx)^3$

$u = x \qquad v = (Inx)^3$

$$\frac{dy}{dx} = (Inx)^3\cdot 1 + x\cdot 3(Inx)^2\cdot\frac{1}{x}$$ M1 B1 B_1 fr deriv

$$= In^3x + 3In^2x$$ A1

$$(In^2x)(Inx + 3)$$

(b) $y = \log_e \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$y = \log_e \frac{\left(1-\tan\frac{x}{2}\right)}{1+\tan\frac{x}{2}}$ M1

$= \log_e \cancel{\tan}\left(1 - \tan\frac{x}{2}\right) - \log_e\left(1 + \tan\frac{x}{2}\right)$ B1

$\frac{dy}{dx} = \frac{\frac{-1}{2}Sec^2\frac{x}{2}}{1-\tan\frac{x}{2}} - \cdot \frac{\frac{1}{2}sec^2\frac{x}{2}}{1+\tan\frac{x}{2}}$ M1

$= \frac{-\frac{1}{2}sec^2\frac{x}{2}\left(1+\tan\frac{x}{2}+1-\tan\frac{x}{2}\right)}{1-\tan^2\frac{x}{2}}$

$= \frac{-sec^2\frac{x}{2}}{2-sec^2\frac{x}{2}}$ B1

When you multiply by $\cos^2 x/2$

$= \frac{-1}{\cos^2\frac{x}{2}} \div \left(2 - \frac{1}{\cos^2\frac{x}{2}}\right)$ ~~M1~~

$= \frac{-1}{\cos^2\frac{x}{2}} \div \frac{2\cos^2\frac{x}{2}-1}{\cos^2\frac{x}{2}}$ ~~M1~~ ~~B1~~

$= \frac{-1}{2\cos^2\frac{x}{2}-1} = \frac{-1}{\cos x}$ M1 ~~B1~~

$= -\sec x$ B1

12mks

16. (a)

$$x\frac{dy}{dx} = 2x - y$$

$$x\frac{dy}{dx} + y = 2x$$ M1

$$\frac{d}{dx}(xy) = 2x$$ M1

$$\int \frac{d}{dx}(xy)dx = \int 2xdx$$ A1

$$xy = x^2 + c$$

Alt.

$$x\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx} + \frac{y}{x} = 2$$

$$If = e^{\int \frac{1}{x}dx}$$

$$= e^{\ln x} = x.$$

$$x\frac{dy}{dx} + y = 2x.$$

$$\frac{d}{dx}(xy) = 2x.$$

$$xy = \int 2x dx.$$

$$xy = x^2 + c.$$

16(b)

$$\frac{dx}{dt} \propto x(1-x)$$

$$\frac{dx}{dt} = Kx(1-x)$$ B1

$$\int \frac{dx}{x(1-x)} = \int Kdt$$

$$\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$$

$$A(1-x) + Bx = 1$$

$$x = 0 \quad A = 1$$ M1

$$x = 1 \quad B = 1$$

$$\int \left(\frac{1}{x} + \frac{1}{1-x}\right) dx = \int Kdt$$

$$Inx - In(1-x) = Kt + c$$

$$In\left(\frac{x}{1-x}\right) = Kt + c$$ A1

$$t = 0 \quad x = \frac{1}{2}$$

$$t = 6 \quad x = \frac{3}{4}$$

$$In\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = c$$ B1 (for c)

$c = 0$	
$In\left(\frac{x}{1-x}\right) = kt$	
$In\left(\frac{\frac{3}{4}}{1-\frac{3}{4}}\right) = 6k$	B1 (for k)
$K = \frac{1}{6}In3$	A1
$In\left(\frac{x}{1-x}\right) = \frac{1}{6}tIn3$	
$t = 12$	M1
$In\left(\frac{x}{1-x}\right) = In9$	
$\frac{x}{1-x} = 9 \Longrightarrow x = \frac{9}{10}$ A1	~~B1~~
Population destroyed = 90% B1	~~A1~~
	12mks

END

Qn 9 b)

$$\arg\left(\frac{z}{2-4+2i}\right)=\pi/2.$$

$$\arg\left(\frac{x+iy}{x+iy-4+2i}\right)=\pi/2 \quad B_1$$

$$\arg\left(\frac{x+iy}{(x-4)(2+yi)}\right)=\pi/2.$$

$$\arg\left(\frac{(x+iy)((x-4)-(2+y)i)}{((x-4)+(2+y)i)((x-4)-(2+y)i)}\right). \quad M_1$$

$$\arg\left(\frac{x^2-4x-2xi-2iy+xiy-4iy+2y+y^2}{(x-4)^2+(2+y)^2}\right)=\pi/2.$$

$$\tan^{-1}\left(\frac{xy-2x-6y}{x^2+y^2-4x+2y}\right)=\pi/2 \quad M_1$$

$$\therefore \quad \frac{xy-2x-6y}{x^2+y^2-4x+2y}=\frac{1}{0}. \quad M_1$$

$$x^2+y^2-4x+2y=0 \quad A_1.$$