**P425/1**

**PURE MATHEMATICS**

**PAPER 1**

**JULY/AUGUST**

**3 HOURS**

KYENJOJO JOINT EXAMINATIONS BOARD

*"Together We Serve"*

**ASSHU - KJEB**

**ASSHU-KYENJOJO DISTRICT JOINT MOCK EXAMINATIONS 2023**

UGANDA ADVANCED CERTIFICATE OF EDUCATION

PURE MATHEMATICS

PAPER 1

3 HOURS

**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** the **eight** questions in section **A** and any **five** from section **B**
- Any additional question(s) answered will not be marked
- **All** necessary working **must** be shown clearly.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used

## SECTION A (40 MARKS)

Answer **all** questions in this section.

1. Solve for $x$: $log_5(2^x - 1) - log_5 2 = log_5\left(\frac{17}{2} + 2^{x-1}\right) - log_5(2^x - 3)$ (5 marks)

2. Solve the equation: $8\sec x - 4\tan x = 7$ for $0 \leq x \leq \frac{\pi}{2}$ (5 marks)

3. Evaluate: $\int_{\pi}^{\frac{4\pi}{3}} \operatorname{cosec}\left(\frac{x}{2}\right) dx$ (5 marks)

4. Find the equation of the tangent to the curve $y^2 = 20x$ which makes an angle of $45^0$ with the $x$-axis. (5 marks)

5. A group of 9 members is to be selected from 10 boys and 9 girls. It can consist of either 5 boys and 4 girls or 4 boys and 5 girls. How many different groups can be chosen? (5 marks)

6. Given that $y^2 - 2y\sqrt{1+x^2} + x^2 = 0$, show that: $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{1+x^2}}$ (5 marks)

7. Find the shortest distance of a point $P(1,\ 6,\ 3)$ from the line $\boldsymbol{r} = \boldsymbol{i} + \mathbf{j} + \boldsymbol{k} + \beta(-\boldsymbol{i} + \mathbf{j} + 2\boldsymbol{k})$ (5 marks)

8. Solve the differential equation: $\cos x \dfrac{dy}{dx} = 1 - y\sin x$. (5 marks)

**SECTION B (60 MARKS)**

Answer any **five** questions from this section.

All questions carry equal marks

9. (a) Given that one root of the equation $Z^2 + kZ + p = 0$ where $k$ and $p$ are real constants is $2 + 5i$. Find the values of $k$ and $p$. (5 marks)

   (b) Show that $2 + i$ is a root of the equation $2Z^3 - 9Z^2 + 14Z = 0$. Hence find the other roots. (7 marks)

10. (a) Differentiate $3x^{2x}\sin^3 2x$ with respect to $x$. (6 marks)

    (b) Given the curve $y = \sqrt{1 - cos^2 x^2}$, find the gradient of the curve at the point where $x = \dfrac{\sqrt{\pi}}{2}$ (6 marks)

11. (a) An arithmetic progression contains $n$ terms. The first term is 2 and its common difference is $\frac{2}{3}$. If the sum of the last four terms is 72 more than the sum of the first four terms, find the value of $n$. (6 marks)

    (b) The sum of $n$ terms of the series is $2n + 3n^2$. Find the $n^{th}$ term. Hence find the sum of the terms from the $10^{th}$ to the $20^{th}$. (6 marks)

12. (a) Find the Cartesian equation of the plane given by $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and hence find the Cartesian equation of the line passing through the point $P(1, \ 4, \ -2)$ and perpendicular to the plane. (8 marks)

(b) Show that the point $P(2 \quad 5 \quad -5)$ lies on the line in (a) above. (4 marks)

13. (a) Evaluate $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ (6 marks)

(b) Find the volume of a solid generated by rotating the portion of the curve $y^2 = 4x$ from $x = 0$ to $x = 1$ through two right angles about the *x*-axis. (6 marks)

14. (a) Solve for *x*; $sinx - sin2x = sin4x - sin3x$ for $0 \le x \le 2\pi$ (7 marks)

(b) Show that: $tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$ (5 marks)

15. (a) Find the equation of the normal to the curve $xy = c^2$ at the point $P\left(ct, \frac{c}{t}\right)$. (5 marks)

(b) Given that the normal in (a) above meets the curve $xy = c^2$ again at $Q$, find the co-ordinates of $Q$ and hence the equation of the tangent to the curve at $Q$. (7 marks)

16. (a) Given that $ye^{2x} = Acos3x + Bsin3x$ where $A$ and $B$ are constants, show that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$. (5 marks)

(b) A student walks to school at a speed proportional to the square root of the distance he still has to cover. If the student covered 900*m* in 100 minutes and the school is 2500*m* from home, find how long he takes to get to school. (7 marks)

**END**