P425/1

PURE

MATHEMATICS

Paper 1

March, 2024

3 HOURS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

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3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in section **A** and any **five** from section **B.***
- *Any additional question (s) answered will not be marked*
- *All necessary working **must** be shown clearly*
- *Begin each answer on a fresh sheet of paper*
- *Graph paper is provided*
- *Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*
- *In numerical work, take g to be 9.8 ms^{-2}.*

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SECTION A (40MARKS)

1. Solve $\cos\theta + \sqrt{3}\sin\theta = 2$ for $0^0 \le \theta \le 360^0$ *(05 marks)*

2. Show that z = 1 is a root of the equation $z^3 - 5z^2 + 9z - 5 = 0$.Hence solve the equation for the other roots. *(05 marks)*

3. Differentiate $x10^{\sin x}$ with respect to x. *(05 marks)*

4. Find the points of intersection of the line $\frac{x}{5} = \frac{y+2}{2} = \frac{z-1}{4}$ with the plane $3x + 4y + 2z - 25 = 0$ *(05 marks)*

5. The population of a country increases by 2.75% per annum. How long will it take for the population to triple? *(05 marks)*

6. Solve the inequality $(0.6)^{-2x} < 3.6$, correct to two decimal places. *(05 marks)*

7. Solve the differential equation $\frac{dy}{dx} + 3y = e^{2x}$ given that when $x = 0$, $y = 1$. *(05 marks)*

8. Evaluate $\int_0^2 \frac{8x}{x^2 - 4x - 12}dx$ correct to **2** decimal places. *(05 marks)*

SECTION B (60MARKS)

9. The position vectors of points A and B are $\boldsymbol{OA} = 2\boldsymbol{i} - 4\boldsymbol{j} - \boldsymbol{k}$ and **OB** = 5**i** – 2**j** + 3**k** respectively. The line AB is produced to meet the plane $\boldsymbol{2x + 6y - 3z = -5}$ at a point C.

Find the;

(a) coordinates of C *(06 marks)*

(b) angle between AB and the plane. *(06 marks)*

10. (a) Solve $2\sin 2x = 3\cos x$, for $-180^0 \le x \le 180°$. *(04 marks)*

(b) Solve $\sin x - \sin 4x = \sin 2x - \sin 3x$ for $-\pi \le x \le \pi$ *(08 marks)*

11. (a) Using calculus of small increments or otherwise, find $\sqrt{98}$ correct to one decimal place. *(04 marks)*

(b) Use Maclaurines theorem to expand $\ln(1 + ax)$, where a is a constant. Hence or otherwise expand $\ln\left(\frac{(1+x)}{\sqrt{(1-2x)}}\right)$ up to the term in x^3.For what values of x is the expansion valid? *(08 marks)*

12. (a) Use De Moivres theorem to express tan 5θ in terms of tan θ *(07 marks)*

(b) Solve the equation $z^3 + 1 = 0$. *(05marks)*

13. Determine the nature of the turning points of the curve $y = \frac{x^2-6x+5}{(2x-1)}$. Sketch the graph of the curve for $x = -2$ to $x = 7$. State any asymptotes. *(12 marks)*

14. (a) A conic section is given by $x = 4 \cos \theta$; $y = 3 \sin \theta$. Show that the conic section is an ellipse and determine its eccentricity. *(04 marks)*

(b) Given that the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $c^2 = a^2m^2 + b^2$. Hence determine the equations of the tangents at the point (-3, 3) to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ *(08 marks)*

15. (a) Find $\int x^3 e^{x^4}\, dx.$ *(06 marks)*

(b) Use the substitution t = tan x to find $\int \frac{1}{1+\sin^2 x} dx$ *(06 marks)*

16. (a) Solve the differential equation $\frac{dR}{dt} = e^{2t} + t$, given that $R(0) = 3$ *(06 marks)*

(b) The acceleration of a particle after time t seconds is given by $a = 5 + \cos\frac{1}{2}t$. If initially the particle is moving at $1ms^{-1}$, find its velocity after 2π seconds and the distance it would have covered by then. *(06 marks)*

END