P425/1

PURE MATHEMATICS

Paper 1

July/Aug 2024

3 hours

WENSSEC

Regional Mock Examination

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

Answer all the eight questions in section A and five questions from section B

Any additional question(s) answered will not be marked.

All working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Square paper is provided therefore graph work must be done in a graph paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

(WENSSEC 2024)

SECTION A (40 MARKS)

1. Differentiate with respect to x, $y = \frac{e^{2x}\sqrt{sinx}}{(2x+1)^3}$ (05 marks)

2. Given that $x = 3^y$, show that $x^3 - 3x^2 + 4 = 3^{3y} - 3^{2y+1} + 4$.hence solve the equation $3^{3y} - 3^{2y+1} + 4 = 0$ (05 marks)

3. If the line $2x - 3y + 26 = 0$ is a tangent to a circle with centre $(2, -3)$. Find the equation of the circle. (05 marks)

4. Solve the simultaneous equation below
$2^x = 4^y = 8^z$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = \frac{11}{16}$ (05 marks)

5. An arithmetic progression has first term 1000 and common difference -35. Calculate the value of the first negative term of the progression and the sun of all positive terms. (05 marks)

6. Solve the equation $\frac{1}{2}cos4x + \frac{1}{2}cos2x + sin^2x = 1$ for $0^0 < x < 360^0$
(05 marks)

7. A variable point M are on the positive $y - axis$ and two fixed points $P(1,0)$ and $Q(2,0)$ are on the$x - axis$. The perpendicular from the origin to MP meets MQ at$R(X, Y)$. Find the locus of R. (05 marks)

8. The region bounded by the curve $y = cosx$, the $y - axis$ and the $x - axis$ from $x = 0$ and $x = \frac{\pi}{2}$ is rotated about the$x - axis$. Find the volume of the solid formed. (05 marks)

SECTION B (60 MARKS)

9. (a) Evaluate $\int_1^{\frac{\pi}{2}} sin5xcos3x\,dx$ (05 marks)

(b) Express $\frac{x^4-2x^3-x^2-4x+4}{(x^2+1)(x-3)}$ into partial fraction, hence find $\int f(x)\,dx$. (07 marks)

10. (a) Given $x = 7\sqrt{t}+2$ and $y = (t+1)\sqrt{t}$. Show that $\frac{dy}{dx} = \frac{3(x-2)^2}{343} + \frac{1}{7}$ (05 marks)

(b) A farmer has shs 1,200,000 to spend in fencing her rectangular land. The land has already a natural fence on one straight side. She wants to use two types of wires costing shs 2,000 per metre and shs 3,000 per metre to fence opposite sides with the same type of wire. Determine the maximum area of the land that he can enclose to make sure that all the money is used. (07 marks)

11. Given the curve $y = \frac{x^2+3}{x-1}$

(i) Show that for real values of x, y cannot take any value between $-2$and 6.

(ii) Determine the turning points of the curve.

(iii) Find the equations of the asymptotes of the curve and sketch the curve. (12 marks)

12. (a) Solve the differential equation $xy\frac{dy}{dx} = y^2 + x^2e^{\frac{y}{x}}$. (06 marks)

(b) A body of mass $1kg$ falls under a constant acceleration of gms^{-2}in a medium in which the resistance is proportional to the velocity of the body. If the body was initially at rest, show that the velocity,vms^{-1}after time t is $\frac{g}{k}(1-e^{-kt})$ where k is a constant. (06 marks)

13. (a) Find the ratio in which the line joining points $A(1,2,-3) and B(-2,3,32)$ is divided externally by the point $C(7,0,11)$. (04 marks)

(b) (i) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{3-z}{-3}$ is parallel to the plane $4x - y - 3z = 4$. (04 marks)

(ii) Find perpendicular distance of the line from the plane in b(i) above. (04 marks)

14. (a) Use Maclaurines theorem to expand the function $f(x) = cosx$ in ascending powers of x upto and including the term in x^6. (06 marks)

(b) Expand $\sqrt{\frac{1+2x}{1-2x}}$ in ascending powers of x as far as the term in x^3. State the value of x for which the expansion is valid. (06 marks)

15. (a) $N(at^2, 2at)$ is the point on the parabola$y^2 = 4ax$. The normal at N meets the $x-$ axis of the parabola at P . Find the locus of the midpoint of NP. (06 marks)

(b) The line $y = mx + 5a$ cuts the parabola given by parametric equation$x = a(1 + t^2), y = 2a(1 + 2t)$ in the two distinct points $P$and Q. Show that the parameters of P and Q are the roots of the equation $mt^2 - 4t + (m + 3)$. Deduce the range of the possible values of m. (06 marks)

16. Show that $(x - 1)$ is a factor of $f(x) = 2x^3 - 7x^2 + 2x + 3$ and find the three roots of the equation $(x) = 0$. Hence or otherwise, solve the equation $2sin^3\theta - 7sin^2\theta + 2sin\theta + 3 = 0$, giving all solution in the interval $0^0 \leq \theta \leq 360^0$. (12 marks)

END

P425/2

APPLIED

MATHEMATICS

Paper 2

JULY/AUG 2024

3 Hours

WENSSEC

Regional Mock Examination

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

PAPER 2

3 Hours.

INSTRUCTION TO CANDIDATES

Attempt **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All necessary working **must** *be shown clearly.*

Begin each question on a fresh sheet of paper

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn over

SECTION A (40 MARKS)

Answer **all** *the questions in this section*

1. Given that $X \sim B(n, p)$ and that $P(X = 2) = P(X = 3)$, show that $E(X) = 3 - p$ (05 marks)
2. A particle is projected vertically upwards with a speed of$14ms^{-1}$. Determine the speed of the particle and its distance from the ground $0.5s$ after projection. (05 marks)
3. A five member committee is to be selected at random from a group consisting of 5 men and 3 women. Find the probability that the selected committee will contain;

   (a) Exactly 2 women. (02 marks)

   (b) No more than 2 women. (03 marks)
4. The table below shows grades scored by 8 students in Chemistry and Physics.

Chemistry	D_2	C_5	D_2	C_5	C_4	C_3	D_2	D_1
Physics	D_1	C_4	D_1	C_5	D_2	C_3	C_5	C_3

   Calculate the rank correlation coefficient and comment on your answer at 5% level of significance. (05 marks)
5. A man walks $4km$ due east, $3km$ due north and then, $3km$ on a bearing of $S60^{o}E$. Find the distance and bearing of the man's final position from his original position. (05 marks)
6. Given a function $y = \log_e(x - 3)$

   (a) Write the absolute error expression of y (01 mark)

   (b) Calculate the limits, correct to three decimal places, within which the exact value of y lie. If $x = 5.3 \pm 0.05$ (04 marks)
7. A particle is projected from O at time $t = 0$ and performs SHM with 0 as the center of oscillation. The motion is of amplitude $40cm$ and time period 8 seconds. Find:

   (a) The speed of projection (02 marks)

   (b) The speed of the projection when $t = 2.25$ seconds. (03 marks)
8. Given the table below

x	0.1	0.2	0.3	0.4
$\sqrt{x}$	0.3162	0.4472	0.5477	0.6325

   Use linear interpolation/extrapolation to find the value of:

   (a) $\sqrt{0.25}$

   (b) 0.75^2

   (05 marks)

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. (a) (i) On the same axes, draw graphs of $y = \tan x$, and $y = 1 + x$ for $0.1 \leq x \leq 1.5$ at intervals of 0.1

(ii) From your graph, obtain to one decimal place, an approximate root of the equation $\tan x - x - 1 = 0$. (05 marks)

(b) Using Newton Raphson method, find the root of the equation in $\tan x - x - 1 = 0$, taking the approximate root in (a) as an initial approximation, give your answer correct to three decimal places. (07 marks)

10. A particle of mass $4kg$ at rest at a point P(2,0,1)m is acted upon by a constant force of magnitude 24N acting in the direction $2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$

Determine the:

(a) Force acting on the particle. (02 marks)

(b) Displacement of the particle from P after 3 seconds. (07 marks)

(c) Work done by the force on the particle after 3 seconds. (03 marks)

11. (a) Use the trapezium rule with 4-subintervals to show that

$$\int_0^{\frac{1}{2}\pi} 3x \sin 2x \, dx \approx k\pi^2(\sqrt{2}+1)$$

Where k is a constant, Correct to three decimal places. (06 marks)

(b) Evaluate $\int_0^{\frac{1}{2}\pi} 3x \sin 2x \, dx$, correct to three decimal places. (03 marks)

(c) Calculate the error in your estimation and state how the error can be reduced. (03 marks)

12. The table below represents a cumulative frequency table for the times, taken by 80 customers to do their shopping in a supermarket.

Times(minutes)	Cumulative frequency
$0-<10$	7
$10-<20$	33
$20-<30$	57
$30-<40$	71
$40-<50$	78
$50-<60$	80

(a) Calculate the mean time for the customers to do shopping. (05 marks)

(b) Draw a cumulative frequency curve for the above data and use it to find the number of customers who used between 25 and 53 minutes to do shopping. (07 marks)

13. In a survey, 100 people were asked of the length of time,x they spent in the shower the last time they took. They following were the results. $\sum x = 1040.5$ and $\sum x^2 = 11102.11$

(a) Calculate the unbiased estimate for the population mean and variance (05 marks)

(b) Calculate the 99% confidence interval for the mean time spent.(07 marks)

14. A uniform ladder AB has length $16m$ and mass$30kg$. The ladder rests in equilibrium at 60^o to the horizontal with the end A on rough horizontal ground and the end B against a smooth vertical wall. A man of mass $70kg$ stands on the ladder at the point C, where AC= $2m$, and the ladder remains in equilibrium.

(a) Find the magnitude of the resultant force exerted on the ladder by the ground and the wall. (07 marks)

(b) The man climbs further up the ladder. When he is at the point D on the ladder, the ladder is about to slip. Given that the coefficient of friction between the ladder and the ground is 0.4. Find the distance AD, correct to four decimal places. (05 marks)

15. A box A contains 3 red, 5 green and 3 blue pens. Box B contains 4 red, 3 green and 4 blue pens. A balanced die is thrown and if the throw is a six, box A is chosen otherwise box B is chosen. Two pens are then picked at random one at a time without replacement from the chosen box.

(a) Find the probability that the pens are of the same colour.

(b) Given that the pens are of different colours, what is the probability that they are from box A. (12 marks)

16. A particle of mass 8 kg is acted upon by a force given by

$F = ((5t^3 - 4t^2)i - 4tj.)$ N.

The particle is initially at a point (0, 0, 0) and moving with a velocity

$(i + 2j - k)ms^{-1}$. Determine the:

(a) The magnitude of the acceleration the particle after $t = 4$ seconds. (04 marks)

(b) Displacement of the particle after $t = 4$ seconds. (08 marks)

END