

$$\sqrt{\frac{x-1}{3x+2}} + 2\sqrt{\frac{3x+2}{x-1}} = 3$$

separate the roots.

$$\frac{\sqrt{x-1}}{\sqrt{3x+2}} + \frac{2\sqrt{3x+2}}{\sqrt{x-1}} = 3$$

get LCM.

$$\frac{(\sqrt{x-1})^2 + 2(\sqrt{3x+2})^2}{\sqrt{(3x+2)(x-1)}} = 3$$

$$\frac{(3x+2)(x-1)}{3x^2 - 3x + 2x - 2}$$

$$3x^2 - x - 2$$

$$x-1 + 2(3x+2) = 3\sqrt{3x^2-x-2}$$

$$x-1 + 6x+4 = 3\sqrt{3x^2-x-2}$$

$$7x+3 = 3\sqrt{3x^2-x-2}$$

apply squares on both sides.

$$49x^2 + 42x + 9 = 9(3x^2 - x - 2)$$

$$49x^2 + 42x + 9 = 27x^2 - 9x - 18$$

$$49x^2 - 27x^2 + 42x + 9x + 9 + 18 = 0$$

$$22x^2 + 51x + 27 = 0$$

$$x_1 = -0.8181 \text{ or } -\frac{9}{11}$$

$$x_2 = -1.5 \text{ or } -\frac{3}{2}$$