CIRCLES TEST

TIME: 1HOIUR 30MINUTES

INSTRUCTIONS

Answer **ALL** Questions.

1. Show that the circles whose equations are $x^2 + y^2 - 4x - 5 = 0$ and $x^2 + y^2 - 8x + 2y + 1$ cut orthogonally. (5marks)

2. Find the equation of the circle whose centre lies on the line y = x which passes in points (2,5) and (4,-1). (5marks)

3. Find the equation of the circle whose centre lies on the line x + 3y = 8 and which touches the positive axes. (5marks)

4. A circle whose centre lies in the first quadrant touches the positive x-axis at +4 and touches the line 3y = 4x. Find the radius of the circle and state the coordinates of its centre. (5marks)

5. A circle whose centre is C(1,6) touches the line y = $\frac{3}{4}$x – 1 at point A. Find the;
   (a) equation of the circle (6marks)
   (b) coordinates of point A. (6marks)

6. A circle that passes through the points A(3,4) and B(6,1) and the equation of the tangent to this circle at A is the line 2y = x + 5. Find the,
   (i) coordinates of the centre of the circle (9marks)
   (ii) radius of the circle (2marks)
   (iii) equation of the circle. (1mark)

7. If y = mx is the tangent to a circle $x^2 + y^2 +$ 2fy+ c = 0, prove that c = $\frac{f^2m^2}{1+m^2}$. Hence find the equation of the tangents from origin to the circle $x^2 + y^2 - 10y + 20 = 0$ (6marks)

END

COMPLEX NUMBERS TEST I

TIME: 1HOUR 30MINUTES

INSTRUCTIONS;

Answer **ALL** the Question.

1. If Z is a complex number, find the locus described by $\left|\frac{z-1}{z+1}\right| = 2$. (5marks)

2. Given that x and y are real numbers such that:
   $xz + y\bar{z} = 7i - 2$ where $z = 2 + i$, find the modulus of $x + iy$. (5marks)

3. Given that $\frac{50}{(2+i)^2} = a + bi$, find the real numbers a and b. (5marks)

4. (a) Given that $z^3 = \frac{-(5+i)}{(2+3i)}$. find the three possible values of z. (5marks)
   (b) Given that z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z} + 2(z - i\bar{z}) = 3 + 2i$. Find the possible values of z. (5marks)

5. (a) Given that $z_1 = 2(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$ and $z_2 = 8(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$.
   Determine the following in the form x + iy
   (i) $\frac{z_1}{z_2}$ (ii) $\sqrt[3]{z_2}$ (5marks)
   (b) Find the locus of the complex numbers z represented by
   $\arg(\frac{z}{z-4+2i}) = \frac{\pi}{2}$ (5marks)

6. (a) Find the complex number z such that $9z\bar{z}+6i\bar{z}=33+10i$ where $\bar{z}$ is the conjugate of z. (5marks)
   (b) Illustrate the region represented by $|z+3i-2|<4$.
   where z is a complex number. (5marks)

7. (a) Given that $z=\frac{(2-i)^2(3i-1)}{(i+3)^3}$. Find
   (i) modulus of z
   (ii) argument of z. Hence express z in polar form. (5marks)
   (b) Show the region represented by $|z+i-2|\leq 1$ on an argand diagram and state the complex number of the centre of the wanted region. (5marks)

END

CONIC SECTION TEST

TIME: 1HOUR 30MINUTES

INSTRUCTIONS

Answer ALL Questions.

1. If y = mx + c is a tangent to the curve $4x^2 + 3y^2 = 12$, show that $c^2 = 4 + 3m^2$. (5marks)

2. Prove that x = $3t^2 + 1$ and 2y = 3t + 1 are parametric equations of a parabola. Find its vertex, focus and length of latus rectum. (5marks)

3. Show that $3x^2 + 2y^2 + 6x - 8y = 7$ is an ellipse and hence determine its centre and eccentricity. (5marks)

4. If $(y-2)^2 = 4(x-3)$ is a parabola, state its vertex, focus and equation of the directrix. (5marks)

5. The tangents to the parabola $y^2 = 4ax$ at the points P(ap^2,2ap) and Q(aq^2,2aq) meet at point R. Find the coordinates of R. (7marks)

6. If the line y = mx + c is the tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $c^2 = b^2 + a^2m^2$ hence determine; (4marks)

   (i) equations of four common tangents to the ellipses $\frac{x^2}{16} + \frac{y^2}{3} = 1$ and $\frac{x^2}{14} + \frac{y^2}{4} = 1$ (4marks)

   (ii) the equations of the tangents at the point (-3,3) to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. (4marks)

7. Show that the equation of the chord joining the points P(p^2,2p) and Q(q^2,2q) on the parabola is 2x$-(p+q)y + 2pq = 0$. (4marks)

   (b) If the chord in (a) above passes through the point R(4,0), show that pq = -4, hence

   (i) show that the chord PQ makes an angle at the origin (0,0)

   (ii) find the locus of the midpoint of PQ. (8marks)

END

EXPANSIONS TEST I

TIME: 1HOUR 30MINUTES

INSTRUCTIONS

Answer **ALL** Questions

1. Expand $\sqrt{1+4x}$ up to the third non-zero term and hence find $\sqrt{17}$, correct to two decimal places. (5marks)

2. Find the coefficients of x^2 in the expansion of $(2-3x+x^2)(1+2x)^4$. (5marks)

3. Expand $(\frac{1`+3x}{1-3x})^{\frac{1}{2}}$ as far as the term in x^3. By putting x= $\frac{1}{7}$ in your expansion, estimate $\sqrt{10}$, correct to two decimal places. (5marks)

4. Use maclaurin's theorem to expand $\ln(2+x)$, in ascending powers of x as far as the term in x^2. (5marks)

5. Expand $\sqrt{1-4x}$ up to the term in x^4. State the range of values of x within which the expansion is convergent. Hence evaluate $\sqrt{15}$ to 4dps. (7marks)

6. Find the first three terms of the binomial expansion of $\sqrt{(1+x)(1+x^2}$. (5marks)

7. Expand $(1+2x-3x^2)^6$ in ascending powers of x up to the term in x^3. (5marks)

8. Find the term independent of x in the expansion of $(2x+\frac{1}{x^2})^{12}$. (6marks)

9. (a) Expand $\sqrt{4-3x}$ upto the term in x^3, hence find the error made in using x = 1 in the expansion. (7marks)
   (b) Evaluate $\sqrt{61}$ to 4 decimal places. (5marks)

END

ROOTS AND REMAINDER THEOREM TEST

TIME: 1HOUR 30MINUTES

INSTRUCTIONS

Answer **ALL** Questions.

1. The polynomial $x^3 + ax^2 + bx - 7$ when divided by $x^2 - x - 2$ leaves a remainder of $8x - 1$. Find the values of a and b. (5marks)

2. The roots of a quadratic equation $2x^2 - 11x + 15 = 0$ are $\alpha\ and\ \beta$ where $\alpha > \beta$, form a quadratic equation whose roots are $\alpha\ and - \beta$. (5marks)

3. The polynomial f(x)= $3x^4 + ax^3 + 8x^2 + bx + 1$ is divisible by x-1 and has a remainder 9 when divided by x+2. Find the value of a and b (5marks)

4. If $\alpha\ and\ \beta$ are the roots of the equation $x^2 + px + q = 0$. Express
   (i) $\alpha^3 + \beta^3$
   (ii) $(\alpha - \beta^2)(\beta - \alpha^2)$ in terms of p and q.
   Deduce that the condition for one root of the equation to be the square of the other is $p^3 - 3pq + q^2 + q = 0$. (6marks)

5. When a polynomial p(x) is divided by $x - 1$ the remainder is 3 and when divided by $x - 2$ the remainder is 1. Prove that when divided by $x^2 - 3x + 2$ the remainder will be $5 - 2x$. (6marks)

6. (a) The roots of the equation $x^2 + px + (p + q) = 0$ differ by 3, find the possible values of p. (5marks)
   (b) Use the remainder theorem to find the remainder when the polynomial p $(x) = x^3 - 3x^2 + 2x - 5$ is divided by $(x - 2)^2$. (7marks)

7. (a) Given that f$x) = (x - \alpha)^2 g(x)$, show that $f^1(x)$ is divisible by $(x - \alpha)$ (4marks)
   (b) A polynomial p $(x) = x^3 + ax^2 + bx + 3$ is divisible by $(x - 1)^2$. Use the results in (a) to find the values of a and b. hence solve the equation P$(x) = 0$. (8marks)

END

SERIES TEST I

TIME: 1HOUR 30MINUTES

INSTRUCTIONS;

Answer **ALL** Questions.

1. A geometric progression has the sum of the sum of the first and second terms equal to -4, if the sum of the fourth and the fifth terms is 108. Calculate the;
   (a) first term
   (b) common ratio of the progression. (5marks)

2. How many terms of the G.P 2 + 2$\times$ $(1.1) + 2 \times (1.1)^2$+……… must be taken for the sum to exceed 100? (5marks)

3. The first, second and fourth terms of an arithmetic progression form a geometric progression. Find the common ratio of the G.P (5marks)

4. A man pays premium of 100 dollars at the beginning of every year to an insurance company on an understanding that at the end of 15years they can receive back the premium he had paid with 5% compound interest. what did he receive? (5marks)

5. .The n^{th} term of a series is 3^n+4n. Calculate the sum of the first 20 terms of the series. (5marks)

6. The first terms of the arithmetic progression (A.P) and geometric progression (G.P) are equal. The common ratio of a G.P is equal to the common difference of the A.P while the third term of the AP is also equal to the second term of the GP. If the fourth term of an AP is 10, find the two possible values of their first term. (7marks)

7. Mary operates an account with a bank which offers a compound interest of 5% per annum. She opened the account at the beginning of 2019 with shs 800,000 and continue to deposit the same amount at the beginning of every year. How much will she receive at the end of 2022 if she made no withdrawal within this period? (6marks)

8. a) The eighth term of an arithmetic progression is twice the third term and the sum of the first eight terms is 39. Find the first three terms of the progression and show that its sum to n terms is $\frac{3n}{8}$(n + 5). (6marks)
   b) Find how many terms of the series 1 + $\frac{1}{5} + \frac{1}{25} + \frac{1}{125}$ + …….. must be taken so that the sum will differ from the sum to infinity by less than 10^{-6}. (6marks)

END

APPLICATIONS OF DIFFERENTIATION TEST

TIME: 1HOUR 30MINUTES

INSTRUCTIONS

Answer **ALL** Questions.

1. Using small changes find the $\sqrt{627}$ to 4 significant figures. (5marks)

2. An inverted cone with vertcal angle 60^0 has water in it dripping out through a hole at the vertex at the rate of $9cm^3$ per minute. Find the rate at which its level will be decreasing at an instant when the volume of water left in the cone is $9\boldsymbol{\pi} cm^3$ (5marks)

3. A container in the shape of a hollow cone of semi vertical angle 30^0 is held with its vertex pointing downwards. Water is poured into the cone at the rate of $5cm^3s^{-1}$. Find the rate at which the depth of water in the cone is increasing when the depth is 10cm. (5marks)

4. The volume, v of a cone varies such that the height, h of the cylinder is twice its base radius .
   show that v= $\frac{\pi}{12}h^3$
   find the rate at which v changes with height, at the instant when h = 4cm

5. Use small changes to evaluate tan46^0 to 4dps. (5marks)

6. A conical vessel whose height is 10meters and the radius of the base 5m is being filled with water at a uniform rate of $1.5m^3min^{-1}$. Find the rate at which the level of the water in the vessel is rising when the depth is 4meters. (6marks)

7. An open box is to be made from a rectangular sheet measuring 16cm by 10cm by cutting squares of side xcm from each corner and turning up the edges. Calculate the value of x, so that the volume of the box is maximum. (7marks)

8. (a) If T= $2\boldsymbol{\pi}\sqrt{\frac{\boldsymbol{L}}{\mathbf{10}}}$. Find the approximate increase in T if l increases from 10.0m to 10.1m (5marks)
   (b) A circular cylinder open at the top is made so as to have a volume of $1cm^3$.if r is the radius of the base, prove that the total outside surface is $(\pi r^2 + \frac{2}{r})$. Hence prove that this surface area is minimum when h= $r = \frac{1}{\sqrt[3]{\pi}}$, where h is the height of the cylinder. (7marks)

END