OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

SUBSIDIARY MATHEMATICS SEMINAR QUESTIONS 2023

PART A: PURE MATHEMATICS

ALGEBRA

1. (a) If $(x+2)$ is a factor of $2x^3+6x+qx-5$,find the remainder when the expression is divided by $(2x-1)$.
   (b) If the roots of $px^2+qx+r=0$ differ by 3, show that $q^2=9p^2+4pr$.
   (c) If α *and* β are roots are roots of the quadratic equation $x^2-4x+2=0$, find the quadratic equation that has roots:$(\alpha+2)$ *and* $(\beta-2)$.

2. If the $\log_2 x + 2\log_4 y = 4$, show that $xy=16$. Hence solve for x *and* y in the simultaneous equation $\log_{10}(x+y)=1$ *and* $\log_2 x + 2\log_4 y = 4$,
3. Find the different values of $\ln m$ if $5^{2m+1}+4=21\times 5^m$ and the inverse function of $f\colon m\rightarrow \log_e m$ *is* function which maps $\log_e m$ on to m**.**

4. The gradient at any point on the curve is given by $6x-5$.if the curve passes through point $(0,2)$ find the discriminant of the equation of the curve y and hence state whether the equation will have two real distinct, no real or repeated roots at $y=0$.

SERIES AND APPLICATION

5. (a)The sum to infinity of a geometric progression (G. P) is $\frac{25}{4}$ and the first term is 5. find the Common ratio of the G.P and Sum of the first ten terms of the G.P.
   (b) A certain country registered 250 tourists at the beginning of February in 2003 if the tourists increased at a constant rate of 30 per month,
   Find:
   (i) the number of tourists at the end of October 2003.
   (ii) total number of tourists from end of February 2003 end of January 2004.
   (c) Find the sum of the multiples of 9 between 40 and 290.

TRIGONOMETRY

6. (a) Solve the following equations and find the values of the angles as instructed
   $(2\tan\beta)^2-\sec\beta=1.5\ for\ 0^0\leq\beta\leq 180^0$
   (b) By eliminating θ from the equation $x=asec\theta$ and $y=b+\mathrm{C}\cos\theta$
   show that $x(y-b)=ca$
   (c) Given that $tan\beta = -\frac{12}{5}$ and $90^0<\beta<180^0$ find without using tables or calculators or tables the value of $7sec\beta+12cosec\beta$.

VECTOR ANALYSIS

7. (a) Points O, A and B have coordinates (0,0), (6, -8) and (5,12) respectively, find angle AOB.

   (b) If $m = ai + 8j$ is parallel to $n = 2i + 4j$, find the value of a.

   (c)The points P, Q and R have position vectors $2i + 2j, i + 6j\ and - 7i + 4j\ respectively$. Show that triangle PQR is right-angled at Q.

   (d) $\overrightarrow{OM}$ and $\overrightarrow{ON}$ are two vectors such that $\overrightarrow{OM} = x - y$ and $\overrightarrow{ON} = 2x + 3y$. If $\overrightarrow{OM}$ is perpendicular to $\overrightarrow{ON}$ show that $x \cdot y = 3y^2 - 2x^2$.If x has magnitude 8 units and y has 6 units, find the angle between $x\ and\ y$ giving your answer to the nearest degree.

LINEAR PROGRAMMING

8. The manager of a cinema wishes to divide the seats available into two classes A and B. There are not more than 120 seats available. There must be at least twice as many A class as there are B class seats. Class A seats are priced at 1500 shs each and class B at 1000 shs each. At least 100,000/= should be collected at each show to meet the expenses. Taking x as the number of class A seats sold and y as the number of class B seats sold,

   (a) write down the inequalities representing the above information and plot them on a graph.

   (b) From the graph;

   (i) Find the number of seats of each kind which must be sold to give the maximum profit

   (ii) state the maximum profit.

   (iii) Find the least number of seats that must be sold in order to incur no loss.

MATRICES AND APPLICATIONS

9. (a) Given that matrices A= $\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$ and B= $\begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix}$.find Matrix C^2 and determinant of C such that *3A+2C +B*=**I, *where I*** is a 2x2 identity matrix and determinant of.

   (b) Given that matrix $M = \begin{pmatrix} cos^3\beta & cosec\beta \\ cos\beta & 2cosec^3\beta \end{pmatrix}$, $N = \begin{pmatrix} sec\beta & 0 \\ sin^3\beta & 1 \end{pmatrix}$ and

   $k = \begin{pmatrix} a & \mathrm{Y}cosec\beta \\ b & 4\mathrm{P}cosec^3\beta \end{pmatrix}$. find the values of a, b , Y and P if matrices $MN = K$

   (c) Find the value of m for which matrix equation $\begin{pmatrix} 3 & m \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ does not have a unique solution.

CALCULUS

10. (a) Determine the nature of the turning points of the curve $y + 9 = x^2$.hence sketch the curve

    (b) find the points of intersection of the curve $y + 9 = x^2\ and\ the\ line\ y = -5$

    (c) calculate the area bounded by the curve and the line in (b) above.

(d) Find the area enclosed between the curve $y = x^2 - 4x$ and the x –axis.

(e) Solve the differential equation $3x\frac{dy}{dx} - 4\frac{x^2}{y} = 0$ given that x=3 when y=4.

(f) Chemical H is converted into another chemical by a chemical reaction. The rate at which chemical H is being converted is directly proportional to the amount present at any time. initially 100g of chemical H as present after 5 minutes, 90g of H is present.

(i) Form a differential equation for the chemical reaction.

(ii) By solving the differential equation formed in (i), determine the

a) amount of chemical H present after 20 minutes.

b) time taken for the amount of chemical H to be reduced to 30g.

PART B. APPLIED MATHEMATICS

STATISTICS

11. (a) The table below shows prices in us dollars and weights of the five components of an engine, 1998 and 2005.

COMPONENT	A	B	C	D	E
PRICES 1998	35	70	43	180	480
PRICES 2005	60	135	105	290	800
WEIGHT	6	5	3	2	1

Taking 1998 as the base.

Calculate for 2005 the:

(i) Simple aggregate price index.
(ii) Price relative of each component
(iii) Weighted aggregate price index.
(iv) Estimate the cost of an engine in 1998 given that its cost in 2005 was 1600US dollars.

12. The sales of a computer company are given for the period of five years in the table below.

Year	First half	Second half
2007	230	810
2008	241	852
2009	259	902
2010	272	934

2011	288	966

Calculate a 2- moving average for the data above:

(i) Draw a graph of these sales and on it superimpose the 2 – point moving average.
(ii) Estimate from your graph the sales for the first half of 2012.

13. 100 students were tested to determine their intelligence quotient (IQ), and the results were as shown below. All IQs are given to the nearest integer).

IQ	45 –	55 –	65 –	75 –	85 –	95 –	105 –	115 –
Number of pupils	1	1	2	6	21	29	24	16

(a)Calculate the mean and standard deviation

(a) Draw a cumulative frequency curve and use it to estimate
(b) median
(c) Semi-interquartile range.

PROBABILITY

14. The people living in 3 houses are classified as Children(C), Parents (P) Or Grand Parents (G). The member living in each house are shown in the table below.

House Number 1	House Number 2	House Number 3
4C,1P,2G	*2C,2P,3G*	*1C,1G*

(i) All the people in all 3 houses meet for a party, if one person at the party is chosen at random, find the probability of choosing a Grandparent.
(ii) A house is chosen at random, then a person in that house is chosen at random. Calculate the probability that the person chosen is a Grandparent.

15. The events A and B are such that $P(A) = \frac{1}{2}$, $P(A/\overline{B}) = \frac{2}{3}$ and $P(A/B) = \frac{3}{7}$ where $\overline{B}$ is the event B does not occur.
(a) Find (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(B)$ (iv) $P(B/A)$.
(b) State with reason whether A and B are Independent or Mutually exclusive.
16. If the letters of the word PROBABILITY are arranged at random, find the probability that the two I"s are separated.

17. From 5 gentlemen and 6 ladies a committee of 5 is to be formed. In how many ways can this be done if;
    (i) The committee is to include at least one lady.
    (ii) There is no restriction about its formation.
    (iii) Not more than 3 gentlemen?

18. A certain examination had 10 statements that require a student to answer by filling in yes or no. A student failed if he or she scored at most seven questions wrong. Find the probability that a student gets 5 questions correct
(i) A student passes the test
(ii) Calculate the expected number and standard deviation of the correctly answered questions

19. The marks obtained by U.A.C.E candidates were found to be normally distributed with mean 50 and standard deviation 10.
a) Determine the percentage of candidates who obtained; -
    (i) more than 70 marks.
    (ii) between 40 and 60 marks.
b) assuming that the total number of candidates were 10,000. Find the number of candidates who scored;
    (i) more than 65.
    (ii) Less than 45.

20. The random variable X takes on values only and has Pdf given by

$$f(x) = \begin{cases} mx, & x = 1,2,3,4,5 \\ m(10-x), & x = 6,7,8,9 \end{cases}$$

Find:

(i) The value of the constant m.
(ii) $(a) P(X > 2/X \leq 6)$ $(b)\ E(3X-1)$

21. A random variable has a P.D.F given by $f(x) = \begin{cases} \frac{1}{4}x, & 0 \leq x \leq a \\ \frac{1}{4}(4-x), & a \leq x \leq b \\ 0, & elsewhere \end{cases}$

(i) Find the value of a and b
(ii) Sketch the graph of f (x)
(iii) Find the $P\left(\frac{1}{2} \leq x \leq 2\frac{1}{2}\right)$
(iv) Determine E(X) and Var (X).

END