

P425/1

Pure mathematic

Paper 1

3hours

Uganda Advanced Certificate of Education

END OF TERM 1 EXAMINATIONS – 2016

PURE MATHEMATICS

P425/1

3 Hours

INSTRUCTIONS TO CANDIDATES

Answer **all** the eight questions in section **A** and any **five** from section **B**

Any addition question(s) answered will **not** be marked

All necessary working **must** be clearly shown

Begin each answer on a fresh sheet of paper

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

1. Find x if $\log_x 27 - \log_{x^2} 81 = 1$. (05 marks)
2. The roots of the equation $3x^2 + 2x - 5 = 0$ are α and β . Find the value of $\alpha^4 + \beta^4$. (05 marks)
3. Prove that $\tan^{-1} \frac{q}{p+q} + \tan^{-1} \frac{p}{p+2q} = \frac{\pi}{4}$. (05 marks)
4. Differentiate $(\sin 3x^2)^x$ with respect to x . (05 marks)
5. Find the possible values of t given that:
$$\int_0^t (8x^3 - 27x^2 + 26x - 6) dx = 0$$
 (05 marks)
6. Find the equation of the normal to the curve $x^2 - 2xy - 2y^2 + x = 2$ at the point $(-4, 1)$. (05 marks)
7. Find a vector perpendicular to the vector $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$. (05 marks)
8. A is the point $(0, 4)$, P is a variable point such that its distance from A is twice its distance from the line $3x = 4y$. Find the locus of P.

SECTION B (60 MARKS)

9. (a) Express $\frac{2x^4 - 3x^3 + 7x^2 - 8x + 5}{(x-1)^2(x^2+2)}$ into partial fractions. (07 marks)
(b) Hence find $\int \frac{2x^4 - 3x^3 + 7x^2 - 8x + 5}{(x-1)^2(x^2+2)} dx$. (05 marks)
10. (a) In how many ways can the letters in the word **OMWANAWOMWANAWOMUNTU** be arranged in a row? (05 marks)
(b) Prove by induction that: $\sum_{r=1}^n (r+1)(r+5) = \frac{n}{6}(n+7)(2n+7)$.
Hence calculate the value of: $\sum_{r=1}^{40} (r+1)(r+5)$ (07 marks)

11. (a) The complex number Z has modulus 1 and argument 120° . Find the fourth root of Z . (05 marks)
- (b) (i) If $Z_1 = 1 + i\sqrt{3}$ and $Z_2 = \sqrt{3} + i$ represent $\frac{Z_1}{Z_2}$ on an argand diagram. (03 marks)
- (ii) Given that $Z = 1 + i$ is a root of $Z^4 - 4Z^3 + 3Z^2 + 2Z - 6 = 0$. Find the other roots. (04 marks)
12. (a) Find the position vector of the point of intersection of the line $x - 2 = 2y + 1 = 3 - z$ and the plane $x + 2y + z = 3$. (04 marks)
- (b) Show that the points with position vectors $\mathbf{OA} = 4\mathbf{i} - 8\mathbf{j} - 13\mathbf{k}$, $\mathbf{OB} = 3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{OC} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ are vertices of a triangle ABC. (04 marks)
- (c) Find the equation of the plane through the origin parallel to the lines $r_1 = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k} + m(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ and $r_2 = 4\mathbf{i} - 5\mathbf{j} - 8\mathbf{k} + t(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ where m and t are scalars. (04 marks)
13. (a) Prove that:

$$\frac{\sin(A+B)}{\cos(A+B)} + 1 = \frac{(1+\cot A)(1+\tan B)}{\cot A + \tan B}$$
 (04 marks)
- (b) A point P lies on the line AC of a triangle ABC such that BCP is an equilateral triangle. Show that $AP^2 = a^2 + c^2 - ac \cos B - \sqrt{3}ac \sin B$. Deduce that $AP^2 = \frac{1}{2}(a^2 + b^2 + c^2) - 2\sqrt{3} \Delta$ where Δ is the area of a triangle ABC . (08 marks)
14. (a) $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are points on the parabola $y^2 = 4ax$. If the chord passes through the focus, show that $pq = -1$. If M is the midpoint of PQ , deduce that the locus of M is $y^2 = 2a(x - a)$. (06 marks)
- (b) The normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point $R(a \cos \theta, b \sin \theta)$ cuts the x and y axes at point A and B respectively. Find the area of the triangle AOB . (06 marks)

15. A curve is given by $y = \frac{(x+1)(x-3)}{x(x-2)}$.

- (i) Show that for real x , y cannot be between 1 and 4. (04 marks)
- (ii) Hence determine the turning points and distinguish them. (02 marks)
- (iii) State the asymptotes and the intercepts of the curve. (03 marks)
- (iv) Hence sketch the curve. (03 marks)

16. (a) Solve the differential equation:

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2. \quad (04 \text{ marks})$$

- (b) A police patrol on Jinja road found a dead body lying in the middle of the road at Banda at 7:00 am and its body temperature was 30°C . Ten minutes later, the police surgeon measured the body temperature and found it to be 28.5°C , the air temperature was 20°C . The body temperature loses heat at a rate proportional to the difference between the body temperature T and the surrounding temperature T_0 . If the normal body temperature is 37°C . Estimate the time when the man was killed. (08 marks)

End