P425/1
PURE MATHEMATICS
Paper 1
Nov./Dec. 2023
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn Over

SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

1. Prove by induction that $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$. *(05 marks)*

2. If a line $y = mx + c$ is a tangent to the curve $4x^2 + 3y^2 = 12$, show that $c^2 = 4 + 3m^2$. *(05 marks)*

3. Given that $y = e^x \cos 3x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$. *(05 marks)*

4. Find the angle between the line $r = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix}$ and the plane $-x + 2y + 2z - 66 = 0$. *(05 marks)*

5. Solve the inequality $\frac{7-2x}{(x+1)(x-2)} > 0$. *(05 marks)*

6. Evaluate $\int_0^{\pi/3} (1+\cos 3y)^2 \, dy$. *(05 marks)*

7. Express $2\sin\theta + 3\cos\theta$ in the form $R \sin(\theta + \alpha)$. *(05 marks)*

8. Use Maclaurin's theorem to expand $\ln(2+x)$, in ascending powers of x as far as the term in x^2. *(05 marks)*

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. (a) Solve the equation $Z^3 - 7Z^2 + 19Z - 13 = 0$. *(06 marks)*

   (b) Find the fourth roots of $8(-\sqrt{3}+i)$. *(06 marks)*

10. Express $f(x) = \dfrac{3x^3 + 2x^2 - 3x + 1}{x(1-x)}$ in partial fractions.

    Hence find $\int f(x)\,dx$. *(12 marks)*

11. A point E has coordinates $(2, 0, -1)$. A line through E and parallel to the line whose equation is $\dfrac{x}{-2} = y = \dfrac{z+1}{2}$, meets a plane $x + 2y - 2z = 8$ at a point B. A perpendicular line from E meets the plane at a point C.

    Determine the coordinates of;

    (a) B. *(07 marks)*

    (b) C. *(05 marks)*

12. (a) Four different Mathematics books and six other different books are to be arranged on a shelf. In how many ways can the Mathematics books be arranged on the shelf? *(02 marks)*

    (b) On a certain day, Fatuma drunk 6 bottles of the 9 bottles of soda available. On the next day she drunk 5 bottles of the 7 bottles of soda available. In how many ways could she have chosen the bottles of soda to drink in the two days? *(03 marks)*

    (c) Given that ${}^{20}C_r = {}^{20}C_{r-2}$, find the value of r. *(07 marks)*

13. (a) A curve is given by the parametric equations $x = t^2 - 3,\ y = t\,(t^2 - 3)$.

    Find the Cartesian equation of the curve. *(04 marks)*

    (b) A point P is such that its distance from the origin is five times its distance from $(12, 0)$.

    (i) Show that the locus of P is a circle.

    (ii) Determine the coordinates of the centre of the circle and its radius.

    (08 marks)

14. Given the curve $y = \frac{1}{4x^2 - 1}$, determine the;

(a) coordinates of the turning points of the curve. *(03 marks)*

(b) equation of the asymptotes.
Hence sketch the curve. *(09 marks)*

15. (a) Show that $\tan 3\theta = \frac{\tan\theta\left(3 - \tan^2\theta\right)}{\left(1 - 3\tan^2\theta\right)}$. *(05 marks)*

(b) Solve the equation $\cos 4x + \cos 6x + \cos 2x = 0$ for $0^\circ \leq x \leq 180^\circ$.
(07 marks)

16. The rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. The body is placed in a room of temperature 25 °C. After 6 minutes the temperature of the body dropped from 90 °C to 60 °C.

(a) Form a differential equation for the rate of cooling of the body.
(07 marks)

(b) Find the time it takes for the body to cool from 40 °C to 30 °C.
(05 marks)