UTEC - P425/2 MATHS PAPER 2

N	MARKING GUIDE	COMMENTS
1	(a) $P(A' \cup B) = P(A \cap B')' = 0.6$ (M1 $\Rightarrow P(A \cap B') = 0.4$ (A1 (b) $P(A) = P(A \cap B) + P(A \cap B')$ $\Rightarrow P(A \cap B) = 0.7 - 0.4$ (M1 $= 0.3$ $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.3}{0.7}$ (M1 $= 3/7$ (A1	Accept other methods used correctly
2	(a) Period, $T = 2 \times 6$ (M1 $= 12$ seconds. (A1 (b) $V_{max} = \omega r$; $r = 5m$ (A) $\omega = \frac{2\pi}{T}$ $= \frac{2\pi}{12}$ (B1 $= \frac{\pi}{6} \times 5$ (M1 $= \frac{5\pi}{6}\ ms^{-1}$ or 2.6179 ms^{-1}	

1

3 Let $y = x\sin x$; $x = 30° = \pi/6$; Δx

$\Rightarrow \Delta y = (\sin x)\Delta x + (x\cos x)\Delta x$ (M1

accept use of error bound

$|\Delta y| = |\sin x||\Delta x| + |x||\cos x||\Delta x|$

$= \left\{|\sin 30°| + \left|\frac{\pi}{6}\cos 30°\right|\right\} \times \frac{\pi}{360}$ (M1B1

$\simeq 0.5079$

(A1

7 $d = x - 7$

x	d	f	fd	fd^2
5	−2	1	−2	4
6	−1	5	−5	5
7	0	10	0	0
8	1	8	8	8
9	2	4	8	16
10	3	2	6	18 (M1
		$\Sigma f = 30$	$\Sigma fd = 15$	$\Sigma fd^2 = 71$

$\Sigma fd^2 = 51$

$Mean = A + \frac{\Sigma fd}{\Sigma f}$

$= 7 + \frac{15}{30}$ (M1

$= 7.5$ (A1

$Var(x) = \frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2$

(M1

$= \frac{51}{30} - \frac{225}{900}$

1.45 $= \cancel{2.1167}$ (A1

5 $r_A = \begin{pmatrix} 8+t \\ 8t \end{pmatrix}$; $r_B = \begin{pmatrix} 5t \\ 6+5t \end{pmatrix}$; $r_{B\sim A} = r_B - r_A$

$= \begin{pmatrix} 4t-8 \\ 6-3t \end{pmatrix}$ (M1

$y = |r_{B\sim A}| = \sqrt{(4t-8)^2 + (6-3t)^2}$ (M1

$= 5(t-2)$ (B1 $\Rightarrow y_{min} = 0$ (M1 when $t = 2$. (B1

Alternative Method

Velocity, $\underset{\sim}{V}_A = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$, $\underset{\sim}{V}_B = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

$\Rightarrow {}_B\underset{\sim}{V}_A = \underset{\sim}{V}_B - \underset{\sim}{V}_A$

$= \begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -3 \end{pmatrix} ms^{-1}$ (A1)

For shortest distance,

${}_B\underset{\sim}{V}_A \cdot {}_B\underset{\sim}{r}_A = 0$

$\Rightarrow \begin{pmatrix} 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 4t-8 \\ 6-3t \end{pmatrix} = 0$ (M1)

$16t - 32 - 18 + 9t = 0$

$25t = 50 \Rightarrow t = 2$ (A1)

Shortest dist. $= |{}_B\underset{\sim}{r}_A|$ (M1)

$= 0$ m. (i.e, poles collide) (A1)

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6 Let $X \sim$ no of red balls picked

$\sim B(n, p)$; $n = 10$, $p = 0.6$ (B1), $q = 0.4$

(a) $P(X=5) = {}^{10}C_5 (0.6)^5 (0.4)^5$ (M1)

$\simeq 0.2007$ (A1)

(b) $P(X=9) + P(X=10) = {}^{10}C_9 (0.6)^9 (0.4)^1 + {}^{10}C_{10} (0.6)^{10}$ (M1)

$\simeq 0.0403 + 0.0060$

$\simeq 0.0463$ (A1)

Note: Symmetry property can be used (then use tables)

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7 (a)

x	0.1	0.18	0.2
y	0.01	y	0.04

$$\Rightarrow \frac{y-0.01}{0.04-0.01} = \frac{0.18-0.1}{0.2-0.1} \quad (M_1)$$

$$y = 0.01 + \frac{0.03 \times 0.08}{0.1}$$

$$\simeq 0.034 \quad (A_1)$$

but $y = 0.18^2$
$= 0.0324$

Absolute error $= |0.034 - 0.0324|$
$\simeq 0.0016$ (B_1)

(b)

x	0.1	0.2	x
y	0.01	0.04	0.05

$$\frac{x-0.2}{0.2-0.1} = \frac{0.05-0.04}{0.04-0.01}$$

$$x = 0.2 + \frac{0.1 \times 0.\cancel{01}}{0.03}$$

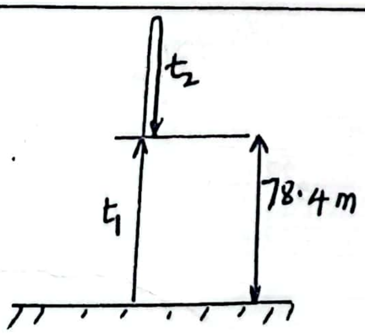
$$\simeq 0.2333 \quad (A_1)$$

$x^2 = 0.05$

$x \simeq 0.2236$

Absolute error $= |0.2333 - 0.2236|$
$\simeq 0.0097$ (A_1)

8



using $s = ut - \frac{1}{2}gt^2$

$78.4 = 49t - 4.9t^2$ (M_1)

$\Leftrightarrow t^2 - 10t + 16 = 0$

$(t-2)(t-8) = 0$ (M_1)

$\Rightarrow t_1 = 2$ and $t_2 = 8$ (A_1)

The required time $= t_2 - t_1$
$= 6$ seconds (M_1, A_1)

(2, 8) seconds

4

1) $f(x) = e^x + x - 2$

$f(0) = -1$; $f(1) = e - 2$
$\quad\quad\quad\quad\quad\quad = +0.72$ (1M) (1M)

Since $f(0) \times f(1) < 0 \Rightarrow 0 < \text{root} < 1$

x	y
0	-1
0.2	-0.8
0.4	-0.1
0.6	0.4
0.8	1.0 (B1)
1	~~0.72~~ 1.72

From the graph, $x_0 \simeq 0.45$ (A1) $x_0 \simeq \pm 0.45$

(b) $f(x) = e^x + x - 2$

$f'(x) = e^x + 1$ (1M)

$$x_{n+1} = x_n - \frac{(e^{x_n} + x_n - 2)}{e^{x_n} + 1}$$

$$x_{n+1} = \frac{(x_n - 1)e^{x_n} + 2}{e^{x_n} + 1} \text{ (1M)} ; n = 0, 1, 2, \ldots$$

$$x_0 = 0.45, \quad x_1 = \frac{(0.45 - 1)e^{0.45} + 2}{e^{0.45} + 1}$$

$\simeq 0.44287$ (B1) ; $|x_1 - x_0| = 0.00713$
> 0.00005

$$x_2 = \frac{(0.44287 - 1)e^{0.44287} + 2}{e^{0.44287} + 1}$$

0.4429 $\simeq 0.44285$ (B1) ; $|x_2 - x_1| = 0.00002$ (B1)
< 0.00005

Hence the root $= 0.4429$ (4 d.p.) (A1)

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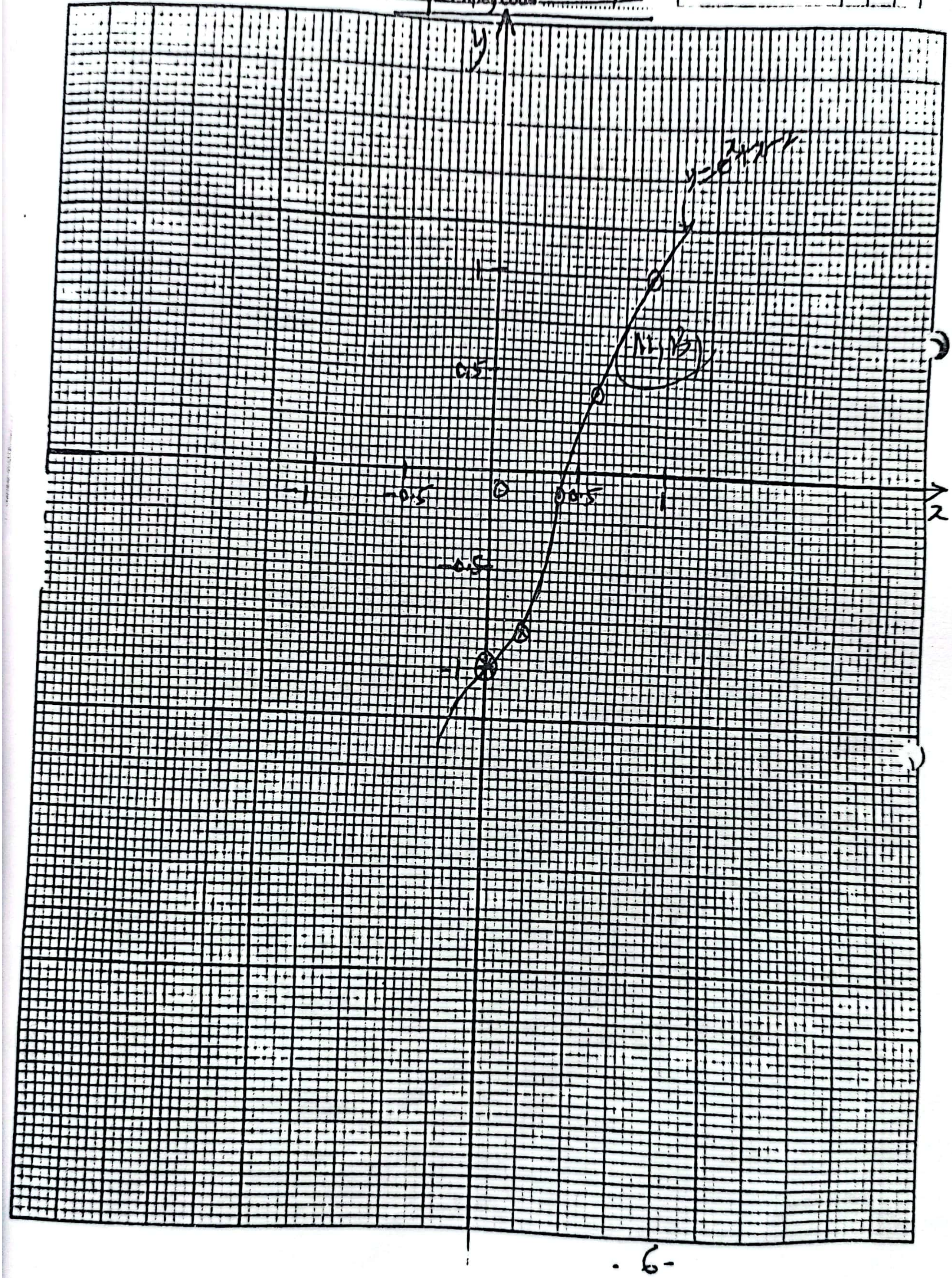
Signature ....

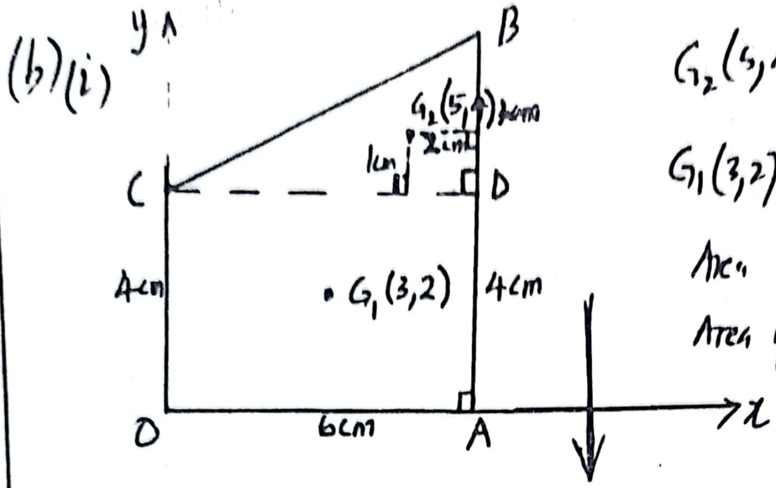
Subject Name .... Graph of $y = e^x + x - 2$ Paper code ....

Personal Number



(a) Marks shifted to part (b). Lengths AB, BC were not given.

(b)(i)



$G_2(5,4) \rightarrow$ c.o.g of ΔCBD

$G_1(3,2) \rightarrow$ c.o.g of OADC.

Area of rectangle $= 24\,cm^2$

Area of triangle $= \frac{1}{2} \times 6 \times 3$

$= 9\,cm^2$

$\rho \equiv$ weight/area

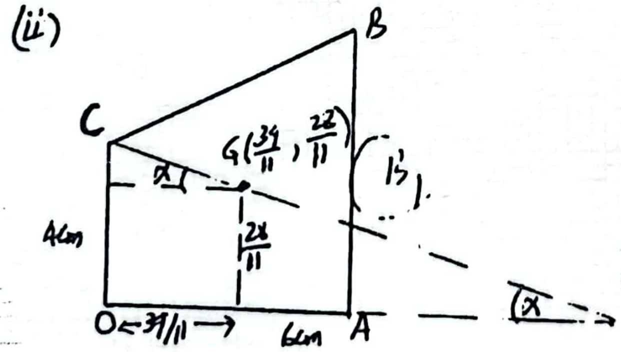
Portion	Weight	C.O.G
OADC	24ρ (B1)	$(3,2)$ (A1)
BCD	9ρ (B1)	$(5,4)$ (A1)
Whole Body	33ρ (B1)	$(\bar{x}, \bar{y})$ (A1)

$$\Rightarrow \quad 24\rho\begin{pmatrix}3\\2\end{pmatrix} + 9\rho\begin{pmatrix}5\\4\end{pmatrix} = 33\rho\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} \text{ (M1, A1) gives } \bar{x} = \frac{39}{11}\,cm$$

$$\bar{y} = \frac{22}{11}\,cm$$

i.e, C.O.G $= \left(\frac{39}{11}, \frac{22}{11}\right)$ (A1)

(ii)



$$\tan\alpha = \frac{4 - 22/11}{39/11} \text{ M1}$$

$$\tan\alpha = \frac{16}{39} \text{ (B1)}$$

$$\Rightarrow \alpha \simeq 23.31^\circ \text{ A1}$$

(a) From the histogram, the mode $\simeq 43 \pm 0.5$ (A1)

(b)

Cumulative frequency table

Marks	freq. density	i	f	c.f.
0 – 10	0.8	10	8	8
10 – 20	1.0	10	10	18
20 – 40	1.5	20	30	48
40 – 45	4.4	5	22	70
45 – 60	2.8	15 (B1)	42	112
60 – 100	0.4	40	16 (B1)	128 (B1)
			$\Sigma f = 128$	

From the graph: (i) no that scored at least 50%

is $128 - 88 = 40 \pm 4$ (A1)

(ii) 1st decile, $D_1 = 17 \pm 0.5$ (A1)

9th decile, $D_9 = 56 \pm 0.5$ (A1)

Decile deviation $= D_9 - D_1$

$\simeq 56 - 17$ (M1)

$\simeq 39 \pm 1.0$ (A1)

Candidate's Name .... No 11(a)

Signature ....

Subject Name .... HISTOGRAM Paper code 1

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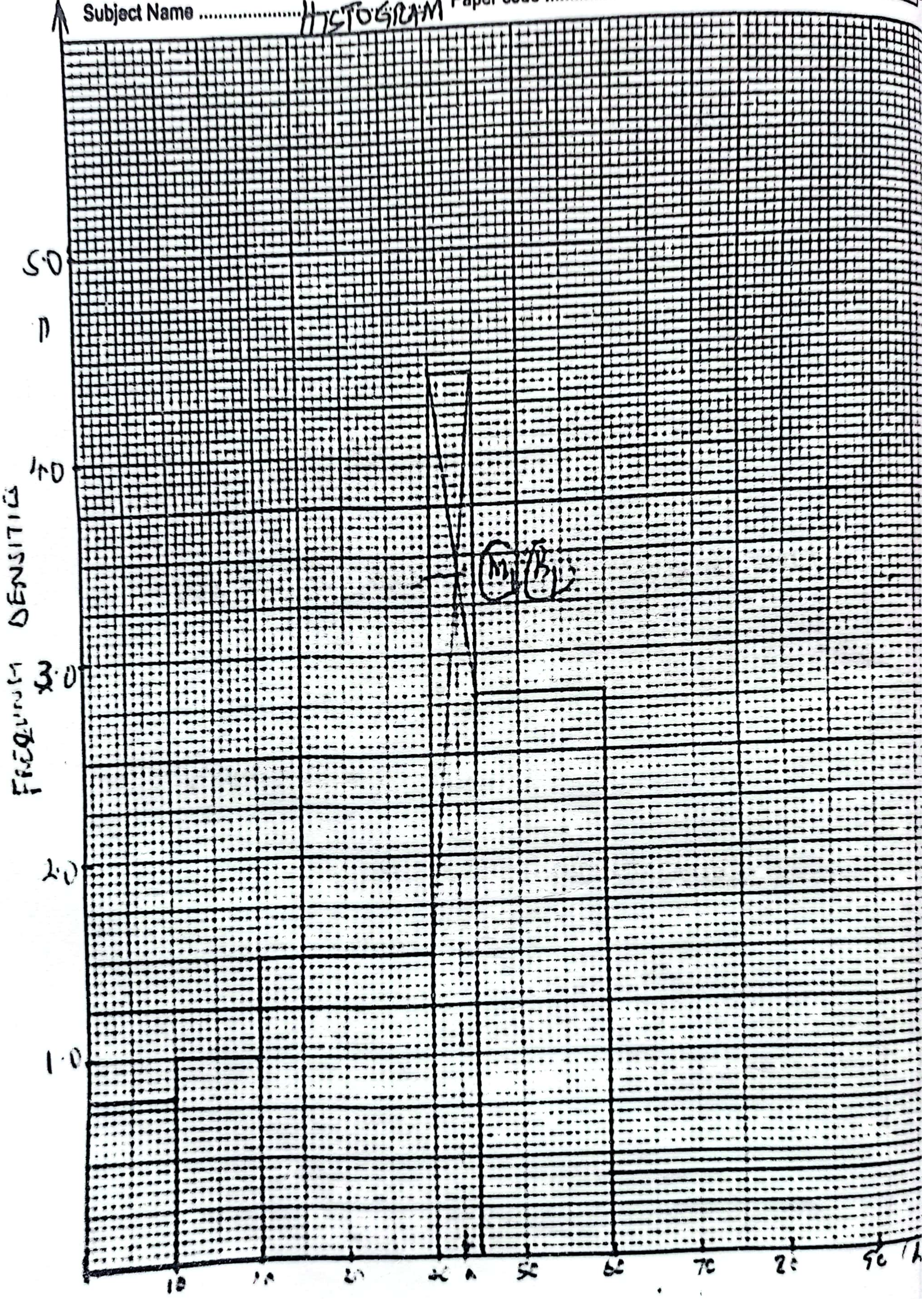

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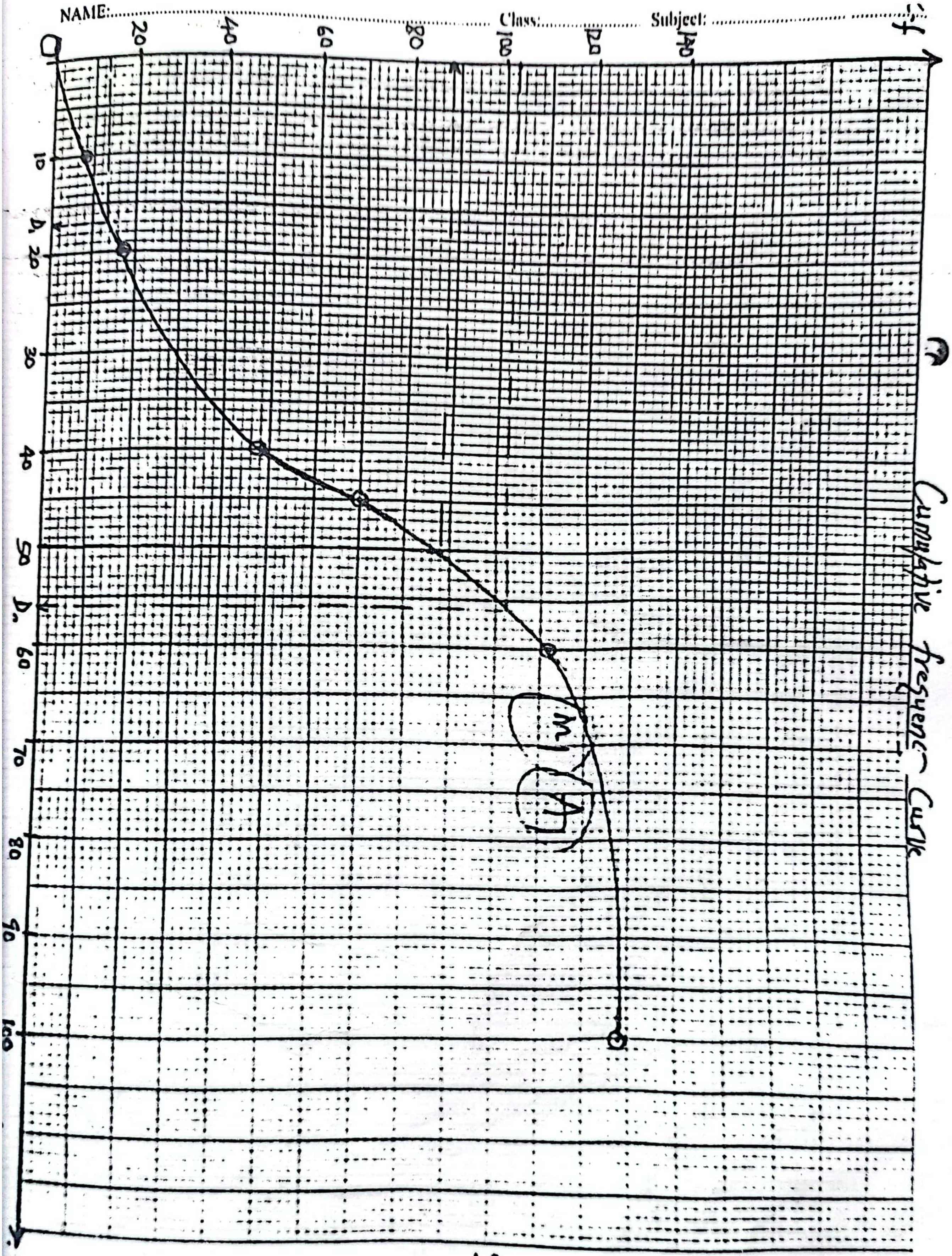


(Q11 1)

2) Let X the height of a maize plant.

$\Rightarrow X \sim N(\mu, \sigma^2)$; $\mu = 16$, $\sigma^2 = 100 \Rightarrow \sigma = 10$ (B1)

(a) $P(X > 20) \Leftrightarrow P\left(Z > \frac{20-16}{10}\right)$ (M1)

$= P(Z > 0.4)$ (B1)

$= 0.5 - \Phi(0.4)$ ~~0.336~~ ~~0.8664~~ 0.1564 (Cal)

$= \underline{0.3346}$ Ans (A1)

(b) $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$; $\mu = 16$, standard dev. $= \frac{\sigma}{\sqrt{n}}$ (M1)

$= \frac{10}{5} = 2.$

$P(13 < \bar{X} < 19) = P\left(\frac{13-16}{2} < Z < \frac{19-16}{2}\right)$ (M1)

$= P(-1.5 < Z < 1.5)$

$= 2\Phi(1.5)$ (M1) 0.4330 (Cal)

$= 0.8664$ (Cal) Ans A1

(c) $P(X \geq 16) = P\left(Z > \frac{16-16}{2}\right)$

$= P(Z > 0) = 0.5$ (B1)

Let n be the no of plants picked

$P(\text{at least 1 has a height} < 16) = 1 - 0.5^n$

(M1)

P.T.O

(?) Continued.

$\Rightarrow 1 - 0.5^n > 0.9$

$\Rightarrow 1 - 0.9 > 0.5^n$

$\Rightarrow 0.5^n < 0.1$

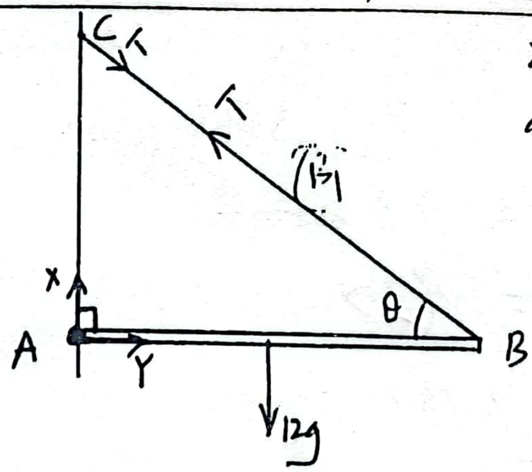
$\Rightarrow n \log_{10} 0.5 < \log_{10} 0.1$ (M1)

$n > \dfrac{\log_{10} 0.1}{\log_{10} 0.5}$

$n > \dfrac{\log 10}{\log 2}$ (B1)

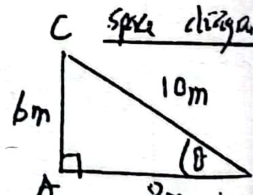
$\Rightarrow n > 3.32$, the least no, $n = 4$ (A1)

13 (a)



X, Y are components of the rxn at A.

C space diagram



(B1)

$\cos\theta = 4/5$

$\sin\theta = 3/5$

(i) $\circlearrowleft A$: $T\sin\theta \times 8 = 12g \times 4 \Rightarrow \frac{24}{5} T = 48g$ (M1)

$\Rightarrow$ Tension, $T = 10g$ N

$= 98$ N. (A1)

- 13 -

3 contd.

(a)(ii) $(\rightarrow)$: $Y = T\cos\theta$

$= 98 \times 4/5$

$= 8g$ (B1)

$(\uparrow)$ $X + T\sin\theta = 12g$

$X = 12g - 98 \times 3/5$

$= 6g.$ (B1)

<u>Magnitude of reaction</u>

$R = \sqrt{X^2 + Y^2}$

$= 10g$

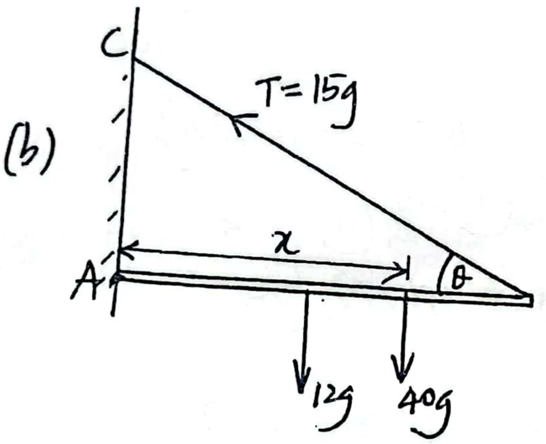
$= 98\ N$ (M1)

<u>Direction of R</u>

$\tan\alpha = \frac{Y}{X} = \frac{8g}{6g} = 4/3$ (A1)

$\alpha = 53.13^\circ$ with the horizontal

(b)



(B1)

$\frac{3}{2}T = \frac{3}{2} \times 10g$

$= 15g\ N$

$\curvearrowleft A$: $40gx + 12g \times 4 = 15g\sin\theta \times 8$ (M1)

$40x + 48 = 15 \times 3/5 \times 8$ (B1)

$40x = 24 \Rightarrow x = 0.6\ m.$ (from A). (A1)

$h = \frac{2-0}{6} = \frac{1}{3}$

(a)

$y = x\sin x$

x	y_0, y_6	y_1, y_2, y_3, y_4, y_5
0	0	
1/3		0.10906
2/3		0.41225
1		0.84147
4/3		1.29592
5/3		1.65901
2	1.81859	
Sum	1.81859 (B1)	4.31771 (B1)

We calculate in radians

$$\Rightarrow \int_0^2 x\sin x \, dx \simeq \frac{1}{2} \times \frac{1}{3} \left\{ 1.81859 + 2 \times 4.31771 \right\} \text{(M1)}$$

$$\simeq 1.742335 \text{ (A1)}$$

minimum of 5dp

$$\simeq 1.7423 \text{ (4 dpl) (B1)}$$

(b) Maximum error in $\frac{y}{x} = \left|\frac{y}{x}\right| \left\{ \left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta x}{x}\right| \right\}$ (M1)

$$= \frac{4.8}{1.60} \left\{ \frac{0.05}{4.8} + \frac{0.005}{1.6} \right\} \text{(B1)}$$

$$\simeq 0.040625 \text{ (A1)}$$

Not below 4dps

Approx. value $\simeq \frac{4.8}{1.6} = 3$ (B1)

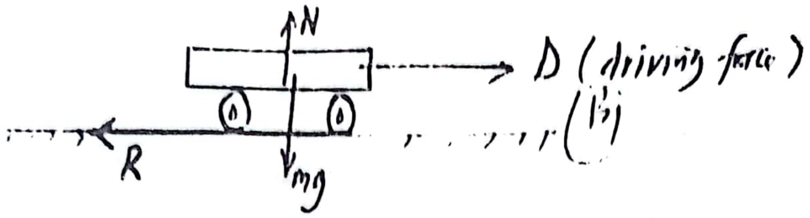
or Limits: $3 - 0.040625$ (M1) and $3 + 0.040625$ (M1)

simple interval can be used

$\frac{1}{2}(\text{Max} - \text{min}) \Rightarrow$ Interval $= [2.9594, 3.0406]$ to 4dps (A1)

$L \leq E \leq U$

15 (a)



$$\text{Power} = DV \Rightarrow D = \frac{30\,000}{30} \quad (M1)$$

$$= 1000\text{N} \quad (A1)$$

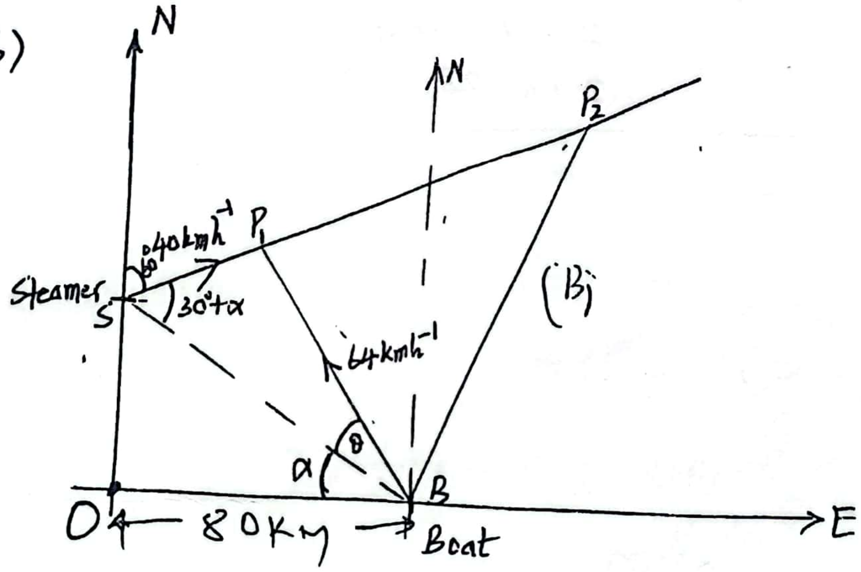
$$R = 200\text{N} \quad (A1)$$

Resultant force, $F = D - R$

ie, $ma = 1000 - 200$ (M1)

$1000a = 800 \quad \therefore$ Accl^n $a = 0.8\,\text{ms}^{-2}$ (A1)

(b)



$\tan\alpha = \frac{3}{4}$

$\Rightarrow \sin\alpha = \frac{3}{5}$ (B1)

$\cos\alpha = \frac{4}{5}$

$\alpha \simeq 36.87°$

$\alpha + 30° = 66.87°$ (B1)

$$\frac{\sin\theta}{40} = \frac{\sin 66.87°}{64}$$

$$\sin\theta = \frac{40\sin 66.87°}{64} \quad (M1)$$

$\theta \simeq 35.08°$ (A1) or $144.92°$ (corresponding to BP_2)

↓

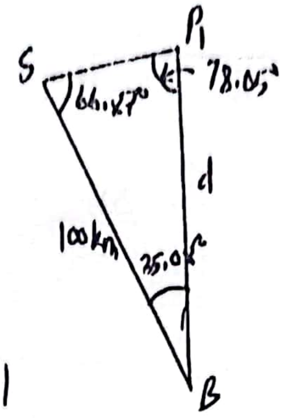
Corresponding to BP_1

$\Rightarrow$ Course BP_1 is N$18.05°$W (A1)

The course BP_2 is divergent.

5(b) Cont'd

Distance $\overline{BS} = 100km$



$$\frac{d}{\sin 66.27°} = \frac{100}{\sin 78.05°}$$

$$d = \frac{100 \sin 66.27°}{\sin 78.05°}$$ M1

time taken $= \frac{d}{64}$ hrs

$\simeq 1.4687$ hrs A1

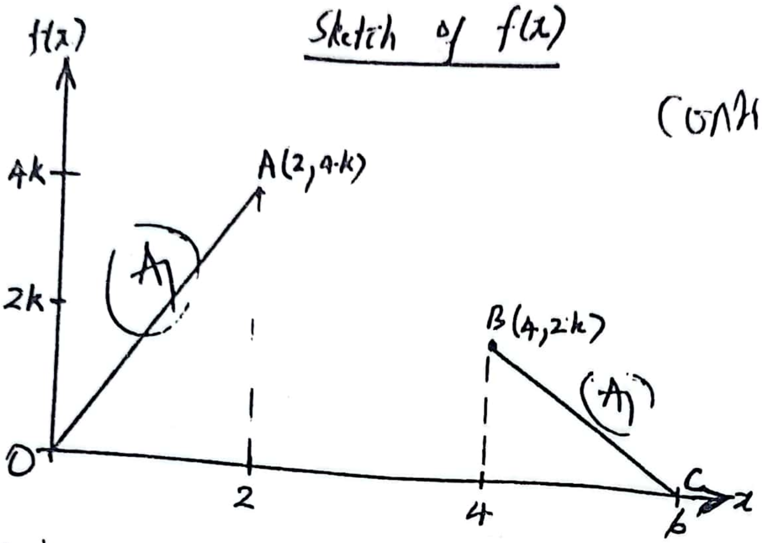
$\simeq 1$ hr 28 mins

16 (A)

x	0	2	4	6
$f(x)$	0	$4k$	$2k$	0

Sketch of $f(x)$

continuous pdf



Grad. of $OA = 2k$

$\Rightarrow f(x) = 2kx$ for $0 \le x \le 2$.

Grad. of $BC = \dfrac{2k - 0}{4 - 6} = -k$ | $y - 0 = -k(x - 6)$

$\Rightarrow f(x) = +k(6 - x)$

Area under the graph: $\frac{1}{2}(2)(4k) + \frac{1}{2}(2)(2k) = 1$ M1

$6k = 1 \therefore k = \frac{1}{6}$ A1

$$\Rightarrow f(x) = \begin{cases} \frac{1}{3}x & ; \ 0 \le x \le 2 \\ \frac{1}{6}(6 - x) & ; \ 4 \le x \le 6 \\ 0 & ; \ \text{elsewhere.} \end{cases}$$

(ii) For $x \le 0$, $F(x) = 0$

For $0 \le x \le 2$; $F(x) = \int_0^x \frac{1}{3}t \, dt$

$$= \frac{1}{6}x^2 \quad (M1)$$

$$F(2) = \frac{2}{3} \Rightarrow F(x) = \frac{2}{3} + \int_4^x \frac{1}{6}(6-t)\,dt$$

$$= \frac{2}{3} + \frac{1}{12}\int_4^x (12-2t)\,dt$$

$$= \frac{2}{3} + \frac{1}{12}\left[12t - t^2\right]_4^x \quad (M1)$$

$$= \frac{2}{3} + \frac{1}{12}(12x - x^2 - 32)$$

$$\Rightarrow F(x) = \frac{1}{12}(12x - 24 - x^2) \quad (A1)$$

Check: $F(6) = \frac{1}{12}(72 - 24 - 36)$

$$= 1$$

Hence $$F(x) = \begin{cases} 0 & ; x \le 0 \\ \frac{1}{6}x^2 & ; 0 \le x \le 2 \\ \frac{1}{12}(12x - x^2 - 24) & ; 4 \le x \le 6 \\ 1 & ; x \geq 6. \end{cases} \quad (A1)$$

6 (b)(i) $P(X>5) = 1 - F(5)$

$$= 1 - \frac{1}{12}(12\times5 - 5^2 - 24)$$ (M1)

$$= 1 - \frac{11}{12}$$

$$= 1/12$$ (A1)

(ii) $F(2) = 2/3 < 4/5 \Rightarrow 4 < P_{80} < 6$

$$\Rightarrow \frac{1}{12}(12x - x^2 - 24) = \frac{4}{5}$$

or $x^2 - 12x = -33.6$ (M1)

$$(x-6)^2 = \pm\sqrt{2.4}$$

$$x = 6 - \sqrt{2.4}$$ (discard $+\sqrt{2.4}$, x is out of range)

Thus, 80^{th} percentile $\simeq 4.451$ (A1)