P425/1
PURE MATHEMATICS
Paper 1
July/August, 2024
3 hours

ACHOLI SECONDARY SCHOOLS EXAMINATIONS COMMITTEE

Uganda Advanced Certificate of Education

Joint Mock Examinations, 2024

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES:

- ✓ Answer all the **EIGHT** questions in section **A** and any **FIVE** questions from section **B**. Any additional question(s) will **NOT** be marked.
- ✓ **All** workings **MUST** be shown clearly.
- ✓ Graph paper is provided.
- ✓ Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- ✓ State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- ✓ Clearly indicate the questions you have attempted on the answer scripts as illustrated, **DO NOT** hand in the question paper.

Question		Mark
Section A		
Section B		
Total		

SECTION A: (40 MARKS)

*Answer **all** the questions in this section*

1. Solve the trigonometric equation $4sec^2\theta = 3tan\theta + 5$ for $0^0 \leq \theta \leq 360^0$. *(05 marks)*
2. Differentiate $\boldsymbol{x^{2x}tanx}$ with respect to x. *(05 marks)*
3. Find the equation of the line passing through $(2, 1, -1)$ and perpendicular to the plane $3x - 7y + 2z = 8$. Hence show that the point $(5, -6, 1)$ lies on the line. *(05 marks)*
4. Find the square roots of $-2 + 2\sqrt{3}\,i$ using De moivre's theorem. *(05 marks)*
5. In a geometric progression (GP) the third term is 18 and the 6th term is 486. Find the 10th term of the series. *(05 marks)*
6. Find $\int x^2 ln2x\ dx$. *(05 marks)*
7. A tangent is drawn from the point $R(5, 1)$ to touch the circle $x^2 + y^2 + 6x - 4y - 3 = 0$ at S. Find the length **RS** of the tangent. *(05 marks)*
8. The gradient of a curve satisfies $\frac{dy}{dx} = \frac{1}{3y^2(x-1)}$, x>1. Given the curve passes through $(2, -1)$. Determine the equation of the curve. *(05 marks)*

SECTION B (60 MARKS)

*Answer any **five** question from this section*

9. (a) Solve the equation $2(9^x) + 3^{x-1} + 4 = 2(3^{x+1})$. *(06 marks)*

   (b) If α and β are the roots of the equation $x^2 - x - 3 = 0$. Find a quadratic equation with integral coefficients whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. *(06 marks)*

10. (a) Evaluate $\int_0^{\frac{\pi}{4}} sin2xcos6x\ dx$. *(06 marks)*

    (b) Using the substitution $t = x^4$, find $\int \frac{x^3}{1+9x^8}\ dx$. *(06 marks)*

11. (a) A and B are respectively the points $(1, 0)$ and $(7, 8)$. A point P moves such that the angle APB is a right angle. Describe the locus of P.

(05 *marks*)

(b) Points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on the parabola $y^2 = 4ax$. Show that the locus of the mid- point of the chord PQ is
$y^2 = 2a(x + 2a^2)$, given that P and Q are those points for which $pq = 2a$.

(08 *marks*)

12. Two airplanes M and N were flown in the sky. Plane M described a path of $\boldsymbol{y = 20x - 2x^2}$ and N describe a path of $\boldsymbol{y = 4x + 14}$ where (x, y) is the grid reference of the planes in the sky.

(a) Using differentiation, sketch the path traced by the two planes.(05 *marks*).

(b) At what points were the two planes at the same level. (04 *marks*)

(c) Find the area enclosed by the path of the two planes. (03 *marks*)

13. (a) By expressing $3cos\theta - 4sin\theta$ in appropriate R-form,

(i) Solve the equation $3cos\theta - 4sin\theta = 5$.

(ii) Find the maximum value of $\frac{7}{18+3cos\theta-4sin\theta}$ and the smallest value of θ for which that occur. (08 *marks*)

(b) Show that $cos\theta + cos3\theta + cos5\theta + cos7\theta = 4cos\theta cos2\theta cos4\theta$.

(04 *marks*)

14. (a) The position vectors of points A and B are $\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$ and $7\boldsymbol{j} + 5\boldsymbol{j} - 2\boldsymbol{k}$ respectively. Find the position vector of a point C that divides AB in the ratio $3{:}-1$. (04 *marks*)

(b) If vectors $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = -\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ are parallel to the plane containing point $(1, -2, 3)$. Determine;

(i) The equation of the plane.

(ii) The angle the line $\frac{x-4}{4} = \frac{y}{3} = \frac{1-Z}{-2}$ makes with the plane in (i) above

(08*marks*)

15. (a) Prove by mathematical induction that $9^n - 5^n$is divisible by 4 for all positive integral values n. *(05 marks)*

(b) The first three terms in the expansion of $(\mathbf{1}+\boldsymbol{ax})^{\boldsymbol{n}}$ are $\mathbf{1}+\mathbf{12}\boldsymbol{x}+\mathbf{81}\boldsymbol{x}^{\mathbf{2}}$.

(i) Find the values of a and n

(ii) State the validity of the expansion. *(07 marks)*

16. The number, x of reported cases of an infectious disease, t months after it was reported is now dropping. The rate at which its dropping is proportional to the square of the reported cases. Initially there were 2500 reported cases and one month later they had dropped to1600 cases.

(a) Form a differential equation to model the information above. *(02 marks)*

(b) By solving the Differential equation, show that $x = \frac{40000}{9t+6}$ *(07 marks)*

(c) Find after how many months there will be 250 reported cases. *(03 marks)*

****THE END****