

**EASTER RECESS PAPER 2024**  
**S6 PURE MATHEMATICS**

**SECTION A**

1. Find  $x$  if  $\log_x 8 - \log_{x^3} 64 = 1$
2. Given that  $A = \tan^{-1} \left( \frac{1}{5} \right)$ ,  $B = \tan^{-1} \left( \frac{1}{3} \right)$ .  
Prove that  $2A + B = \tan^{-1} \left( \frac{27}{31} \right)$
3. Differentiate  $\frac{\sqrt[3]{2x-1}}{\sqrt{x^2-2}}$  with respect to  $x$
4. Find a vector that is perpendicular to the two vectors  $\mathbf{a} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$  and  $\mathbf{b} = -\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ .
5. A polynomial has remainders 3 and  $-5$  when divided by  $x - 2$  and  $x + 3$  respectively. Find the remainder when the polynomial is divided by  $x^2 + x - 6$ .
6. A spherical balloon is being filled with air at a rate of  $54\pi \text{ cm}^3 \text{ s}^{-1}$ . Find the rate at which the radius of the balloon increases when the volume is  $36\pi \text{ cm}^3$ .
7. The tangent to the parabola  $y^2 = 8x$  at a point  $M(2t^2, 4t)$  is perpendicular to the line whose equation is  $3y + x - 6 = 0$ . Determine the coordinates of  $M$ .
8. Evaluate  $\int_0^{0.5} 3\sin\theta \cos 3\theta \, d\theta$

**SECTION B**

9. A curve is given by the equation  $y = \frac{4x-3}{x^2-4x+3}$ . Find the  
(a) (i) intercepts of the curve (ii) asymptotes of the curve  
(iii) range of values of  $y$  within which the curve lies and hence find the turning points  
(b) Sketch the curve
10. (a) Find the binomial expansion of  $\frac{1}{\sqrt[4]{1-3x}}$  in ascending powers of  $x$  up to the term in  $x^3$ . Hence evaluate  $\frac{1}{\sqrt[4]{13}}$  correct to 4 decimal places.  
  
(b) Using the substitution  $p = x + \frac{1}{x}$   
solve the equation  $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$
11. The points  $A(1, 2, -3)$ ,  $B(-1, 8, 0)$ ,  $C(-17, 15, 8)$  are in the same plane.  
a. Prove that  $A, B, C$  are vertices of a triangle  
  
b. If point  $P$  is the foot of the perpendicular from  $C$  to  $AB$  produced.  
Find the coordinates of  $P$ .

12. (a) Find the area bounded by the curve  $y = (x - 2)^3$  and the lines

- (i)  $y = 1$ ,  $y = 8$  and  $x = 0$
- (ii)  $y = 0$ ,  $x = 4$  and the  $y$ -axis

(b) If the area bounded by the curve in (a) and the lines  $x = 2$ ,  $x = 4$  and  $y = 0$  is rotated through one revolution about the  $x$ -axis, find the volume of the solid generated.

13. Given that  $y = \tan^{-1}\sqrt{1-x}$

a. Show that

(i)  $(2-x)\frac{dy}{dx} + \frac{1}{2}(1-x)^{-\frac{1}{2}} = 0$

(ii)  $(2-x)\frac{d^2y}{dx^2} - \frac{dy}{dx} + \frac{1}{4}(1-x)^{-\frac{3}{2}} = 0$

b. Hence determine the Maclaurin's expansion of  $y$  up to the term in  $x^3$

14. The points  $A(1, 2, -3)$ ,  $B(-1, 8, 0)$ ,  $C(-17, 15, 8)$  are in the same plane.

a. Prove that  $A$ ,  $B$ ,  $C$  are vertices of a triangle

b. If point  $P$  is the foot of the perpendicular from  $C$  to  $AB$  produced. Find the coordinates of  $P$ .

**END**