P425/2
APPLIED
MATHEMATICS
PAPER 2
June/July. 2023
3 hours

UGANDA ADVANCED CERTIFICATE OF EDUCATION
MOCK EXAMINATIONS 2023
Applied Mathematics
Paper 2
Time: 3 Hours

NAME: ....INDEX No:.............

INSTRUCTIONS TO CANDIDATES:

Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.

Indicate the five questions attempted in section B in the table aside.

Additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Graph paper is provided.

Where necessary, take acceleration due to gravity, $g = 9.8$ m s^{-2}.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

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SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

Qn 1: The discrete random variable, x, has the following probability distribution, where θ is an unknown parameter belonging to the interval $\left[0, \dfrac{1}{3}\right]$

Value of x	1	3	5
Probability	θ	$1-3\theta$	2θ

Obtain the expression for $E(X)$ in terms of θ and show that $Var(X) = 40(3-\theta)$ [5 Marks]

Qn 2: At time $t = 0$, two particles A and B have position vectors $(2i + 3j - 4k)m$ and $(8i + 6k)m$ and respectively.

Particle A moves with constant velocity $(-i + 3j - 5k)ms^{-1}$ and B with constant velocity, Vms^{-1}. Given that when $t = 5$ seconds, B passes through the point that A passed through one second earlier, find V.   [5 Marks]

Qn 3: The table below is an extract from the table of a certain function $f(x)$.

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	0.0998	0.1987	0.2955	0.3894	0.4794

Use linear interpolation to find:

(i). $f(0.15)$ (ii). $f^{-1}(0.35)$ [5 Marks]

Qn 4: A spinner can land on red or blue. When the spinner is spun, there is a probability of $\dfrac{1}{3}$ that it lands on blue. The spinner is spun repeatedly. Given that the random variable, X, represents the number of the spin when the spinner first lands on blue, find $p(X \le 4)$.

[5 Marks]

Qn 5: Three boys are pulling a heavy trolley by means of three ropes.
The boy in the middle is exerting a pull of 100 N. The other two boys, whose ropes both make an angle of $30°$ with the centre rope, are pulling with forces of 80 N and 140 N. Determine the magnitude of the resultant pull on the trolley.   [5 Marks]

Qn 6: Use the trapezium rule with six ordinates to estimate $\displaystyle\int_{2}^{5} xe^{-x}dx$, correct to 3 decimal places.

[5 Marks]

Qn 7: A particle is describing simple harmonic motion in a straight line directed towards a fixed point, O. When its distance from O is 3m, its velocity is $27ms^{-1}$ and its acceleration is $81ms^{-2}$. Determine the amplitude of oscillation.   [5 Marks]

Qn 8: Show that the variance of n one's, 6 two's and 7 threes is a factor of the reciprocal of $(n + 13)$

[5 Marks]

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

The awards to 8 schools by Judges A and B in a quiz contest were:

Judge A (x)	60	56	50	56	60	52	56	54
Judge B (y)	52	60	75	66	54	70	60	68

(a).   (i).   Plot a scatter diagram for the given data. Comment on your result.

         (ii).   Draw a line of best fit on the scatter diagram.

         (ii).   Estimate the marks awarded by Judge A if Judge B awarded 55.   **[7 Marks]**

(b).   Calculate the rank correlation coefficient between the two judges. Comment on your result.

         [5 Marks]

Question 10:

(a).   The numbers x and y are approximated by X and Y with error Δx and Δy respectively. Show that the maximum relative error in $\dfrac{x}{y}$ is given by: $\left|\dfrac{\Delta x}{x}\right| + \left|\dfrac{\Delta y}{y}\right| \cdot \left|\dfrac{x}{y}\right|$ **[6 Marks]**

(b).   Given that $x = 2.45$ and $y = 5.250$ are rounded off to the given number of decimal places. Determine the interval within which the exact value of $\dfrac{y-x}{y+x}$ lies. Give your answer to 4 decimal places.   **[6 Marks]**

Question 11:

A particle A, of mass, m kg, has position vector $(11i + 6j)$ metres and a velocity $(2i + 7j)ms^{-1}$. At the same moment, a second particle B, of mass, 2m kg, has position vector $(7i + 10j)$ metres and a velocity $(5i + 4j)ms^{-1}$.

(a).   If the particles continue to move with these velocities, prove that the particles will collide.   **[4 Marks]**

(b).   Given that the particles coalesce after collision, find the common velocity of the particles after collision.   **[4 Marks]**

(c).   Calculate the loss of kinetic energy caused by the collision.   **[4 Marks]**
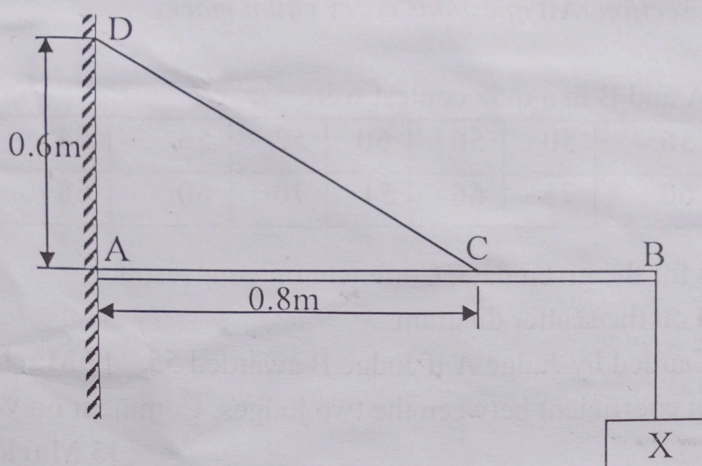
Question 12:

Calculate the probability of arranging the letters of the word "**PARALLELOGRAM**" in a row such that:

(i).   the A's are separated.   **[6 Marks]**

(ii).   each word begins and ends with "**R**".   **[3 Marks]**

(iii).   "**P**" and "**E**" are always next to each other?   **[3 Marks]**

Question 13:

The diagram below shows a uniform rod, AB of weight 10N, hinged to a vertical wall at A. The rod is held in a horizontal position by means of a light inextensible string. One end of the string is attached to a point C on the rod and the other end is attached to a point D on the wall. The point D is 0.6 m vertically above A and the length of AC is 0.8 m. A particle X, of weight 25N is attached to the rod at B and the tension in the string is 75N.

(a). Find the length of the rod AB.

(b). Calculate the magnitude and direction of the reaction at the hinge at A. **[12 Marks]**

Question 14:

(i). By plotting graphs of $y = \sin x$ and $y = \ln x$ on the same axes.

(ii). Show that the equation $\sin x = \ln x$ has a root between 2 and 3. Hence use Newton Raphson method to find the root, correct to three decimal places. **[12 Marks]**

Question 15:

The heights of the students at a university are assumed to follow a normal distribution. 1% of the students are over 200 cm tall and 76% are between 165 cm and 200 cm tall. Find:

(a). the mean and standard deviation of the distribution.

(b). the percentage of the students who are under 158 cm tall. **[12 Marks]**

Question 16:

(a). Village B is in a direction $N12^0 W$ from village A. When a man cycles from A to B at $12kmh^{-1}$, the wind appears to be coming from $S50^0 W$. When he returns from B to A at the same speed, the wind appears to be from due south. Assuming that the velocity of the wind is the same throughout, find its true velocity. **[8 Marks]**

(b). Two points A and B on the banks of a river are directly opposite.

A boy capable of swimming at $1\frac{7}{18}ms^{-1}$ in still water wishes to swim directly from A to B.

Given that the river is flowing at a rate of $\frac{5}{6}ms^{-1}$, determine:

(i). the boy's speed along AB,

(ii). the width of the river if it takes 2 minutes to cross the river. **[4 Marks]**

END