P425/1

PURE MATHEMATICS

Paper 1

Oct/Nov. 2022

3 hours

PRE-UNEB SET 2

Uganda Advanced Certificate of Education

PURE MATHEMATICS

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3 hours

INSTRUCTIONS TO CANDIDATES:

Answer ***all*** *the* ***eight*** *questions in section A and any* ***five*** *from section* ***B****.*

Any additional question(s) answered will ***not*** *be marked.*

All *necessary working* ***must*** *be shown clearly.*

Begin each answer on a fresh page.

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Solve the inequality: $\frac{6}{x-4} \leq x+1$. *(05 marks)*

2. Evaluate: $\int_2^3 \frac{2}{x^2+5x+6}dx$ *(05 marks)*

3. Find the equation of the normal to the curve $x^2 tanx - xy - 2y^2 + 2 = 0$ at the point (0, 1). *(05 marks)*

4. Show that $\frac{sin8\theta cos\theta - sin6\theta cos3\theta}{cos2\theta cos\theta - sin3\theta sin4\theta} = tan2\theta$ *(05 marks)*

5. Show that the lines $x-2 = \frac{y+1}{3} = \frac{z-1}{3}$ and $\mathbf{r} = \mathbf{i} - 3\mathbf{j} + \lambda(2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$ intersect. Hence state the position vector of the point of intersection. *(05 marks)*

6. Peter deposits sh.80,000 per year month in a bank that offers a compound interest rate of 2% per month. Find the interest he will earn after saving for 2 years. *(05 marks)*

7. A point P moves such that its distance from the point (3, 1) is equal to its perpendicular distance from the line x+3=0. Find the equation of locus. Hence sketch the locus. *(05 marks)*

8. Find the volume generated by rotating about the x-axis the area bounded by part of the curve $y = x^2 + 2$ for which x is positive, the y-axis and the line y=3. *(05 marks)*

SECTION B: (60 MARKS)

*Attempt **only five** questions in this section.*

9. (a) Given that $z = 2 + 3i$ is a root of the equation $z^4 + az^3 + 6z^2 + bz + 65 = 0$, find the values of a and b. Hence state the remaining roots of the equation. *(07 marks)*

   (b) Show on the Argand diagram the locus of the points given by values of z satisfying $|z - 2 - 4i| = 4$. Sketch the locus on an Argand diagram.

   (05 marks)

10. The plane π_1 has equation $3x - 4y + 2z = 5$ and plane π_2 has equation

$$r = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$ where λ and μ are constants.

   (a) Find the Cartesian equation of plane π_2. *(04 marks)*

   (b) Obtain the acute angle between the planes. *(04 marks)*

   (c) Find the vector equation of line of intersection of the two planes.

   (04 marks)

11. (a) Solve the equation: $4 sinx cos2x sin3x = 1$ for $0^0 \leq x \leq 180^0$

   (05 marks)

   (b) Solve the equation $5\,tan\,2\,\theta + sec\,2\,\theta + 5 = 0$ for $0^0 \leq \theta \leq 240^0$

   (06 marks)

12. (a) Express $\dfrac{x-5}{(x+1)(x^2-1)}$ in partial fractions. Hence $\int_2^3 \frac{x-5}{(x+1)(x^2-1)}\,dx$

   (08 marks)

   (b) Find $\int (secx + tanx)^2\,dx$ *(04 marks)*

13. (a) Solve for n if $nP_4 = 60(nC_2)$ *(05 marks)*

   (b) Find the values of a and b if $f(x) = x^3 + 4ax^2 + bx + 3a$ is divisible by $(x-1)^2$. Hence factorise the polynomial. *(07 marks)*

14. (a) Find the value of $\cos^2 58^0$ using small changes correct to 4s.f

(05 marks)

(b) A piece of wire 128cm is cut into two parts of un equal length. The former is bent into the shape of a square and the other into a rectangle of whose length is double the base. Find the dimensions of the rectangle for which the sum of the areas is a minimum. *(07 marks)*

15. The tangents to the parabola at the point $P(p^2, 2p)$ and $Q(q^2, 2q)$ intersect at T.

(a) Find the equation of tangent at P and deduce the one at Q.

(05 marks)

(b) Find the coordinates of T. *(03 marks)*

(c) If the angle PTQ is 45^0, find the equation locus of T.

(04 marks)

16. (a) Given that $y = Ae^x + Be^{2x}$, find a differential equation connecting x and y which is independent of the arbitrary constants A and B.

(04 marks)

(b) According to Newton's law of cooling, the rate at which a hot object cools is directly proportional to the difference between the temperature of the object and the temperature of air (assumed to be constant equal to 25^0C). If an object cools from 200^0C to 160^0C in 20 minutes, find the time taken to cool from 160^0 to 100^0C. *(08 marks)*

GOOD LUCK