

P425/1
PURE MATHEMATICS
Paper 1
3 hours



MMM JOINT MOCK EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section **A** and **five** questions from section **B***

*Any additional question(s) answered will **not** be marked.*

***All** working **must be** shown clearly.*

Begin each answer on a fresh sheet of paper

Graph paper is provided.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A

Answer **all** questions in this section.

1. Find the values of x for which the function $\frac{\sqrt{(2x-1)}+\sqrt{(x+1)}-9}{e^{3x}\cos 3x}$ is equal to zero. (5 marks)
2. Prove that if A, B and C are angles of a triangle, then $\frac{b-c}{b+c} \cot \frac{A}{2} = \tan \left(\frac{B-C}{2} \right)$ (5 marks)
3. Solve the equation $5 \tan \theta + \sec \theta + 5 = 2 \sec \theta$ ($0^\circ \leq \theta \leq 360^\circ$) (5 marks)
4. Use the substitution $U = \sqrt{x}$ to evaluate $\int_1^4 \frac{dx}{x+\sqrt{x}}$ (5 marks)
5. A and B are points (1,0) and (-1, 0) respectively. P is a variable point such that $AP + PB = 7$. Show that the locus of P is an ellipse. (5 marks)
6. If $y = \frac{\sin x}{x^2}$, Show that $\frac{d^2y}{dx^2} + \frac{4}{x} \left(\frac{dy}{dx} \right) + \left(1 + \frac{2}{x^2} \right) y = 0$ (5 marks)
7. Show that the solution of the differential equation $2x^2 \frac{dy}{dx} - xy = y^2$ is given by $\frac{(y-x)^2}{xy^2} = B$ where B is a constant. (5 marks)
8. Use the maclaurin's theorem to expand $e^{-x} \sin x$ up to and including the term in x^3 (5 marks)

SECTION B

Answer any **five** questions from this section. **All** questions carry equal marks.

9. a) Given that $Z = -1 - i$, express Z in modulus - argument form. (3 marks)
b) Find the locus of a complex number represented by $\arg (Z + 2) = \frac{-2\pi}{3}$ (3 marks)
c) Given that $\arg \left(\frac{Z-3}{Z-1} \right) = \frac{\pi}{4}$ Show that the locus of Z is a circle. Determine the centre and radius of the circle (6 marks)

10. a) Given that $x = a(2\theta + \sin 2\theta)$ and $y = a(1 + \cos 2\theta)$

Find $\frac{d^2y}{dx^2}$ (6 marks)

b) Given that $y = e^{-x} \cos x$. Show that $\frac{dy}{dx} = -\sqrt{2} e^{-x} \cos\left(x - \frac{\pi}{4}\right)$ (6 marks)

11. a) Show that the equation $3x^2 + 4y^2 + 18x - 16y - 11 = 0$ is an ellipse and determine its centre, foci and equations of the directrices. (6 marks)

b) The curve $b^2x^2 + a^2y^2 = a^2b^2$ intersects the positive X – axis at A and the positive Y – axis at B . Determine the equation of the perpendicular bisector of AB. Given that this line intersects the X- axis at P. Find the coordinates of P. (6 marks)

12. a) The roots of the equation $2x^2 - 4x - 6 = 0$ are α and β .

Find the values of $\frac{\alpha - \beta}{(x + \alpha)^2(2 + \beta)^2}$ (4 marks)

b) Find the range of values of K for which the equation $2x^2 + (3 - k)x + (k + 3) = 0$ have two real distinct roots. (4 marks)

c) Solve the equation $3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$ (4 marks)

13. a) Find the Cartesian equation of the line passing through A(2, - 3, 4) and parallel to another line $\underline{r} = 4\underline{i} + 5\underline{j} - 2\underline{k}$ (3 marks)

b) Find the angle between the plane $4x + 3y - 3z + 1 = 0$ and the line obtained in (a) above. (4 marks)

c) The point P, Q and R have position vectors $\underline{P} = 5\underline{i} + 4\underline{j} + \underline{k}$, $\underline{Q} = -\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{R} = \underline{i} + 2\underline{j} - \underline{k}$ respectively. Show whether or not P, Q and R are vertices of a triangle. (5 marks)

14. a) Use the substitution $U = x^{1/3}$, evaluate $\int_0^1 (2 - x) x^{\frac{1}{3}} dx$ (5 marks)

b) Express $\frac{4x^2+4x+25}{x(4x^2+25)}$ into partial fractions. Hence integrate $\int \frac{4x^2+4x+25}{x(4x^2+25)} dx$
(7 marks)

15. A hunter killed a lion and recorded the temperature of the body of the lion. Where he noticed that the body originally at 38°C was cooling in accordance with Newton's law of cooling. After 2hrs the temperature of the body was 34°C and the temperature of the surrounding air was constant 20°C .

a) Find the temperature, θ , of the body as a function of t , the time in hours since the lion was killed. (6 marks)

b) If at 5:00pm, the temperature of the body was 30°C , find the time when the lion was killed. (6 marks)

END