P425/1
PURE MATHEMATICS
PAPER 1
June 2023
3 hours

SAVIO SECONDARY SCHOOL - KAWEMPE
INTERNAL MOCK EXAMINATIONS 2023

Uganda Advanced Certificate of Education
PURE MATHEMATICS
Paper 1
3 Hours

INSTRUCTIONS TO CANDIDATES:

- Answer ***ALL*** the eight questions is Section A and five questions from Section B.
- Any additional question(s) answered will not be marked.
- All working must be shown clearly.
- Begin each answer on a fresh sheet of paper.
- Silent , non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A

1. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, determine the value of the integral

$$\int_0^{\frac{\pi}{2}} \frac{1}{2-\cos x}\,dx.$$

2. Write down all the values of θ that satisfy the equation $2-\cos\theta = 2\sin^2\theta$ where $-180^0 \le \theta \le 180^0$.

3. Solve for y when $2\sqrt{y-1} - \sqrt{y+4} = 1$.

4. Differentiate $\left(\frac{x^2-1}{x^2+1}\right)^{1/4}$ with respect to x.

5. Expand $(1-2x)^{\frac{1}{2}}$ in ascending powers of x upto the term in x^3. Taking $x = \frac{1}{9}$, find an approximation for $\sqrt{7}$ to four significant figures.

6. Find the equation of the tangent to the curve $x^3y - 3x^2y^2 + x^3 - 2x = 0$ at the point $P(2, 1)$.

7. Points A and B are $(-1, -2, 3)$ and $(2, 1, -3)$ respectively. If point P divides line AB externally in the ratio $1:4$, find the Cartesian equation of the plane containing P and perpendicular to the line AB.

8. Solve the differential equation $\frac{dy}{dx} + 3y = e^{2x}$ given that $y = \frac{6}{5}$ when $x = 0$.

SECTION B

9. (a) Determine the acute angle between the planes $2x + y - 3z = 10$ and $x + 2y - 2z = 10$.

(b) The line l_1 passes through the point $A(0, 6, 9)$ and the point $B(4, -6, -11)$.

The line l_2 has equation $r = \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$.

Show that the lines l_1 and l_2 intersect and find the coordinates of the point of intersection.

10. (a) Evaluate $\int_0^2 \frac{x^2}{4 + x^2} dx$.

(b) Differentiate $x^2 \cos x$ from first principles.

11. Given that $x = \cos\theta$, show that the equation $27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$ can be written in the form $16x^3 + 11x - 15 = 0$.
Hence solve the equation $27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$ for $0^\circ \le \theta \le 360^\circ$.

12. (a) Solve the differential equation $\frac{dy}{dx} = y^2 x \sin 3x$

given that $y = 1$ when $x = \frac{\pi}{6}$.

(b) A substance loses mass at a rate which is proportional to the amount M present at time.

(i) Form a differential equation connecting M, t and the constant of proportionality k.

(ii) If initially the mass of the substance is M_0, show that $M = M_0 e^{-kt}$

(iii) Given that half of the substance is lost in 1600 years, determine the number of years $15\ g$ of the substance would take to reduce to $13.6\ g$.

13. (a)(i) Express $\frac{5-8x}{(2+x)(1-3x)}$ in the form $\frac{A}{2+x} + \frac{B}{1-3x}$, where A and B are integers.

(ii) Hence show that $\int_{-1}^{0} \frac{5-8x}{(2+x)(1-3x)} dx = k \ln 2$, where k is a constant.

(b) Given that $\frac{9-18x-6x^2}{2-5x-3x^2}$ can be written as $C + \frac{5-8x}{2-5x-3x^2}$, find the value of C.

14. A circle with centre C has equation $x^2 + y^2 + 20x - 14y + 49 = 0$.

(a) Find the coordinates of the point B where the circle touches the $y-axis$.

(b) If the circle crosses the $x-axis$ at P and Q, find the coordinates of P and Q

(c) Given that the line $y = kx + 2$ is a tangent to the circle above, find the value of k.

15. (a) Given that $Z = 1 + i$ is a root of $Z^4 + 3Z^2 - 6Z + 10 = 0$, determine the remaining three roots of the polynomial.

(b) Sketch the locus of a number $Z = x + iy$ which moves such that $\left|\frac{Z-2}{Z-4}\right| \angle \frac{1}{2}$

16. Sketch the curve $y = \frac{x^2+3}{x-1}$.

END