

- (a) -1.492913 (3.5 marks)
 (b) 40.304846 (3.5 marks)
 (c) 130.931131 (3 marks)

2. (a) Required to prove: $\sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B$
 Considering right hand side (RHS)

$$\text{RHS} = \sinh A \cosh B + \cosh A \sinh B$$

By definition

$$\begin{aligned}\text{RHS} &= \frac{1}{2}(e^A - e^{-A})\frac{1}{2}(e^B + e^{-B}) + \frac{1}{2}(e^A + e^{-A})\frac{1}{2}(e^B - e^{-B}) \quad (6 \text{ marks}) \\ &= \frac{1}{4}\left[e^{A+B} + e^{A-B} - e^{B-A} - e^{-(A+B)}\right] \quad (1 \text{ mark}) \\ &= \frac{1}{4}\left[2e^{A+B} - 2e^{-(A+B)}\right] \\ &= \frac{1}{2}\left[e^{A+B} - e^{-(A+B)}\right] \\ &= \frac{1}{2}\left(e^{A+B} - e^{-(A+B)}\right) \\ &= \cosh(A+B) \quad \{\text{By definition}\} \\ &= \text{LHS proved.} \quad (0.5 \text{ mark})\end{aligned}$$

(ii) Required to prove: $\cosh^2 x \sinh^2 x - \cosh^2 x \sin^2 x - \sinh^2 x \cos^2 x = \frac{1}{2}[1 - \cosh 2x \cos 2x]$

Considering LHS

$$\begin{aligned}\text{LHS} &= \frac{[\cosh 2x + 1]}{2} \left[\frac{1 - \cos 2x}{2} \right] - \left[\frac{\cosh 2x - 1}{2} \right] \left[\frac{1 + \cos 2x}{2} \right] \quad (4 \text{ marks}) \\ &= \frac{1}{4} [\cosh 2x - \cosh 2x (\cosh 2x + 1) - \cosh 2x - \cosh 2x + \cosh 2x \cosh 2x + 1] \\ &\quad + \cos 2x \quad (0.5 \text{ marks})\end{aligned}$$

$$= \frac{1}{4} [2 - 2 \cos 2x \cosh 2x]$$

$$\begin{aligned}&= \frac{1}{2} [1 - \cos 2x \cosh 2x] \\ &\Rightarrow \text{RHS, proved} \quad (1/2 \text{ mark})\end{aligned}$$

(b) (i) Required to prove

$$\frac{d}{dx} \left[\tanh^{-1} \sqrt{\frac{\cosh x - 1}{\cosh x + 1}} \right] = \frac{1}{2}$$

2 (b) (i) Let $y = \tanh^{-1} \sqrt{\frac{\cosh x - 1}{\cosh x + 1}}$

$$= \tanh^{-1} \sqrt{\frac{\cosh^2 \frac{x}{2} + \sinh^2 \frac{x}{2} - 1}{\cosh^2 \frac{x}{2} + \sinh^2 \frac{x}{2} + 1}} \quad (1 \text{ mark})$$

$$= \tanh^{-1} \sqrt{\frac{(\cosh^2 \frac{x}{2} - 1) + \sinh^2 \frac{x}{2}}{\cosh^2 \frac{x}{2} + [1 + \sinh^2 \frac{x}{2}]}}$$

$$= \tanh^{-1} \sqrt{\frac{\sinh^2 \frac{x}{2} + \sinh^2 \frac{x}{2}}{\cosh^2 \frac{x}{2} + \cosh^2 \frac{x}{2}}}$$

$$= \tanh^{-1} \sqrt{\frac{2 \sinh^2 \frac{x}{2}}{2 \cosh^2 \frac{x}{2}}}$$

$$= \tanh^{-1} \sqrt{\tanh^2 \frac{x}{2}}$$

$$= \tanh^{-1} (\tanh \frac{x}{2})$$

$$= \frac{x}{2} \quad (0.5 \text{ mark})$$

$\therefore \frac{dy}{dx} = \frac{1}{2} \quad (\text{Proved})$

(ii) Required to prove:

$$\int_1^2 \frac{1}{(x+1) \sqrt{x^2-1}} dx = \frac{1}{2} \sqrt{3}$$

Let $I = \int_1^2 \frac{1}{(x+1) \sqrt{x^2-1}} dx$

Let $x = \cosh \theta$ so that $dx = \sinh \theta d\theta$ (0.5 mark)

$$= \int_a^b \frac{\sinh \theta}{(\cosh \theta + 1) \sqrt{\cosh^2 \theta - 1}} d\theta$$

$$= \int_a^b \frac{\sinh \theta}{(\cosh \theta + 1) \sinh \theta} d\theta$$

$$= \int_a^b \frac{1}{2 \cosh^2 \frac{\theta}{2}} d\theta \quad (1 \text{ mark})$$

$$\begin{aligned}
 2(b)(ii) &= \frac{1}{2} [2 \tanh \frac{\alpha}{2}]^b \\
 &= [\tanh \left[\frac{1}{2} \cosh^{-1} x \right]]_1^2 \\
 &= \tanh \left[\frac{1}{2} \ln \left| x + \sqrt{x^2 - 1} \right| \right]_1^2 \quad (0.5 \text{ mark}) \\
 &= \tanh \left[\frac{1}{2} \ln \left| 2 + \sqrt{4-1} \right| \right] - \tanh \left[\frac{1}{2} \ln \left| 1 + \sqrt{1-1} \right| \right] \\
 &\approx \tanh \left[\ln \sqrt{2+\sqrt{3}} \right] - \tanh(0) \\
 &= \frac{e^{\ln \sqrt{2+\sqrt{3}}} - e^{-\ln \sqrt{2+\sqrt{3}}}}{e^{\ln \sqrt{2+\sqrt{3}}} + e^{-\ln \sqrt{2+\sqrt{3}}}} \\
 &= \frac{\sqrt{2+\sqrt{3}} - \frac{1}{\sqrt{2+\sqrt{3}}}}{\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}}} \\
 &= \frac{(2+\sqrt{3}) - 1}{(2+\sqrt{3}) + 1} \\
 &= \frac{1+\sqrt{3}}{3+\sqrt{3}} \\
 &\approx \frac{1-\sqrt{3}}{2-\sqrt{3}+\sqrt{3}-2} \\
 &\approx \frac{-2}{2\sqrt{3}} \\
 &\approx \frac{1}{\sqrt{3}} \quad \text{proved (1 mark)}
 \end{aligned}$$

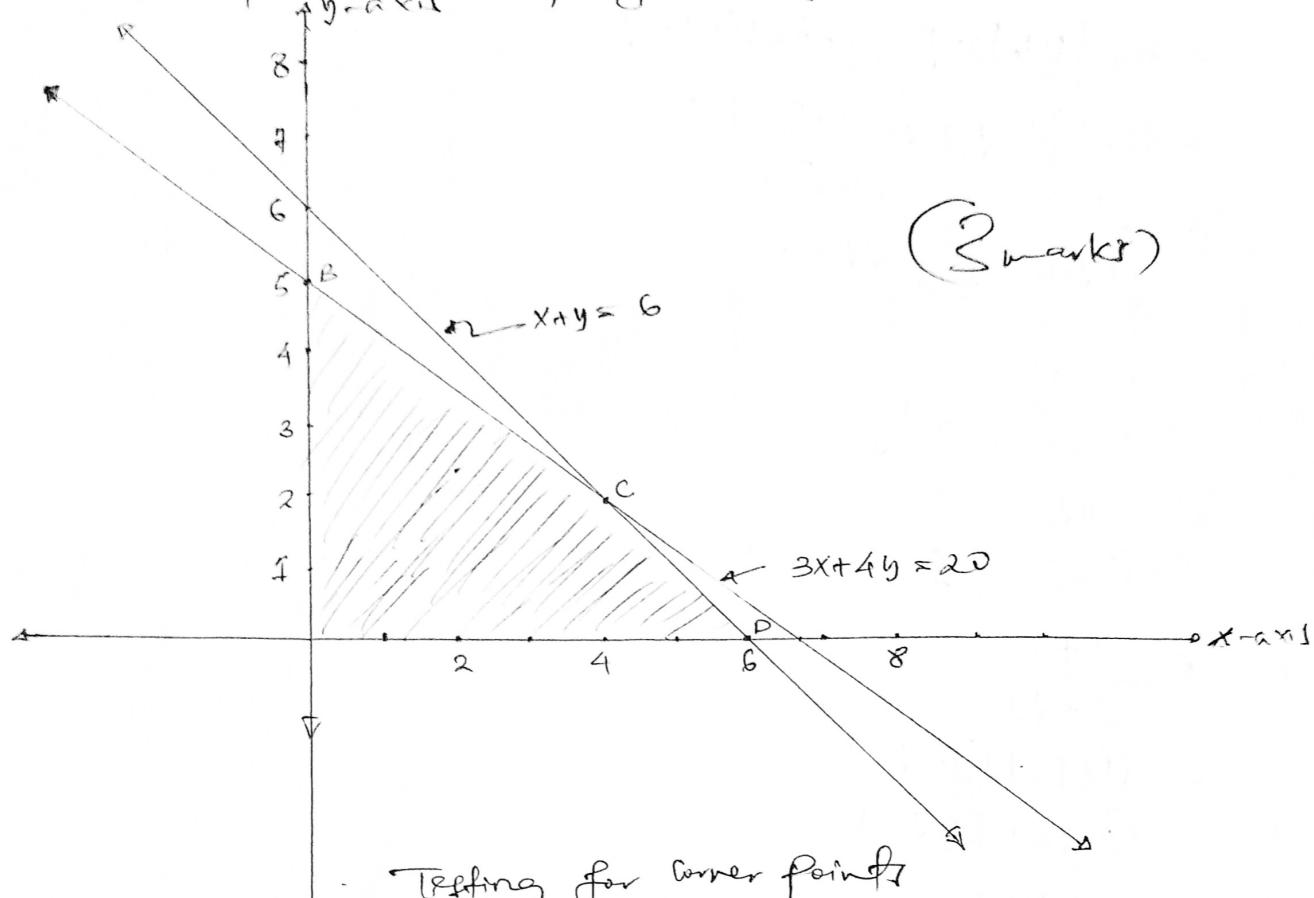
- 3 (a) Application of linear programming
- In transportation to minimize the cost of transportation
 - In manufacturing to maximize revenue
 - In agriculture to maximize profit
 - In management to minimize the cost of running an institution or company on paying salaries. (2 marks)

- (b) Let x be the number of type A answered (1 mark)
 y be the number of questions of type B answered
 objective function
 $f(x, y) = 15x + 20y \quad (0.5 \text{ mark})$

3 (b) Linear Constraints

$$2x+2by \leq 3(60) \text{ or } 3x+4y \leq 20$$
$$x+y \leq 6 \quad (\text{1.5 marks})$$
$$xy \geq 0$$

Graph of the linear programming problem.



Testing for corner points

corner point	Value of $f(x,y) = 15x+20y$
A (0,0)	0
B (0,5)	100
C (4,2)	100
D (6,0)	90

(1 mark)

To get the best score 4 questions of type A and two questions of type B must be answered. (1 mark)

4. The deviation of the assumed mean is given by

$$d_i^* = x_i - \bar{A}$$

$$f_i d_i^* = f_i (x_i - \bar{A})$$

Summing for all the data

$$\sum f_i d_i^* = \sum f_i (x_i - \bar{A}) \quad (0.5 \text{ mark})$$

$$= \sum f_i x_i - \bar{A} \sum f_i$$

Divide by $\sum f_i$ both sides

$$4(a) \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i - A}{\sum f_i} \text{ (1 mark)}$$

$$\frac{\sum f_i x_i}{\sum f_i} = A + \frac{\sum f_i^*}{\sum f_i}$$

$$\bar{x} = A + \frac{\sum f_i^*}{\sum f_i} \text{ (1 mark)}$$

Proved.

(b) (i) Evaluating the mean.

Score (%)	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
62.5 - 67.5	65	4	260
67.5 - 72.5	70	7	490
72.5 - 77.5	75	10	750
77.5 - 82.5	80	22	1760
82.5 - 87.5	85	22	1810
87.5 - 92.5	90	17	1530
92.5 - 97.5	95	9	855
97.5 - 102.5	100	5	500
TOTAL		100	8355

Mean is given by

(2 marks)

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{8355}{100} = 83.55 \text{ %} \quad (2 \text{ marks})$$

Median

$$\text{Median class} = 82.5 - 87.5$$

Lower class boundary, $L = 82.5$

$$\text{Median} = L + \left\{ \frac{\frac{1}{2} \sum f_i - f_b}{f_c} \right\} c \quad (0.5 \text{ mark})$$

$$\text{where } c = 87.5 - 82.5, f_b = 22 + 10 + 7 + 4 = 43, f_c = 26 \quad (0.5 \text{ mark})$$

$$f_c = 26$$

$$9. (b) (i) \text{ Median} = 82.5 + \frac{(50\% - 43)\times 5}{2.6} = 83.85\% \quad (\text{3 marks})$$

(ii) The i^{th} percentile is given by

$$P_i = L + \frac{(i \times n - f_c)}{f_w} c \quad (6 \text{ marks})$$

The 70th percentile class is 82.5 - 92.5

$$\therefore L = 82.5, i = 70, f_c = 4 + 7 + 10 + 24 + 26 = 69 \\ f_w = 12 \text{ and } c = 5 \quad (6 \text{ marks})$$

$$P_{70} = 82.5 + \left\{ \frac{\frac{70}{100} \times 100 - 69}{12} \right\} \times 5 = 87.79 \quad (4 \text{ marks})$$

5. (a) (i) Required to prove: $A \cap B' = \emptyset$

Given: $A \cap B$

Considering left hand side (LHS)

$$\text{LHS} = A \cap B'$$

But $A \cap B = A$ since $A \subseteq B$

$$\begin{aligned} \text{LHS} &= (A \cap B) \cap B' & A &= A \cap B \\ &\Rightarrow A \cap (B \cap B') && \text{Associative law} \\ &\Rightarrow A \cap \emptyset && \text{Complement law} \\ &= \emptyset && \text{Identity law.} \end{aligned}$$

(ii) Required to prove $[A \cup (A \cap B)] \cap [(A \cap B) \cup (A - B)] = A$

Considering left hand side (LHS)

Given $A \cup (A \cap B) \cap [(A \cap B) \cup (A - B)]$

$$\text{LHS} = [A \cup (A \cap B)] \cap [(A \cap B) \cup (A - B)] \quad \text{Definition}$$

$$= [A \cup (A \cap B)] \cap [(A \cap B) \cup (A \cap B')] \quad \text{Distributive law}$$

$$= [(A \cup A) \cap (A \cup B')] \cap [A \cap (B \cup B')] \quad \text{Complement law}$$

$$= [A \cap (A \cup B')] \cap [A \cap (B \cup B')] \quad \text{Identity law}$$

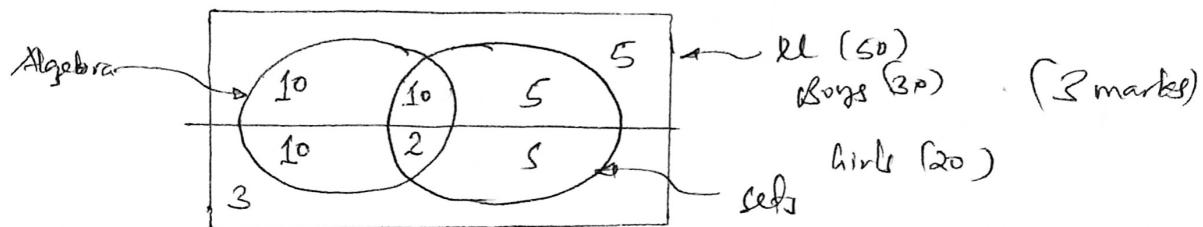
$$= A \cap (B \cup B') \quad \text{Distributive law}$$

$$= A \cap \emptyset \quad \text{Identity law}$$

$$= A$$

$\therefore \text{LHS proved} \quad (3 \text{ marks})$

5. (b) Summary of the set in Venn diagram.



- There are $10+5+5+2 = 22$ students who like sets (1 mark)
- 5 boys do not like any of the topics (1 mark)

6. (a)(i) Given $f(x) = 2x^2 + 1$ and $g(x) = 3x - 2$

$$\begin{aligned} f \circ g(x) &= f(3x-2) \\ &= 2[3x-2]^2 + 1 \\ &= 2[9x^2 - 12x + 4] + 1 \\ &= 18x^2 - 24x + 8 + 1 \end{aligned}$$

$$\therefore f \circ g(x) = 18x^2 - 24x + 9 \quad (1 \text{ mark})$$

$$\begin{aligned} g \circ f(x) &= g[f(x)] = g[2x^2 + 1] \\ &= 3(2x^2 + 1) - 2 \\ &= 6x^2 + 3 - 2 \end{aligned}$$

$$\therefore g \circ f(x) = 6x^2 + 1 \quad (1 \text{ mark})$$

$$(ii) \quad g \circ f(x) = 6x^2 + 1$$

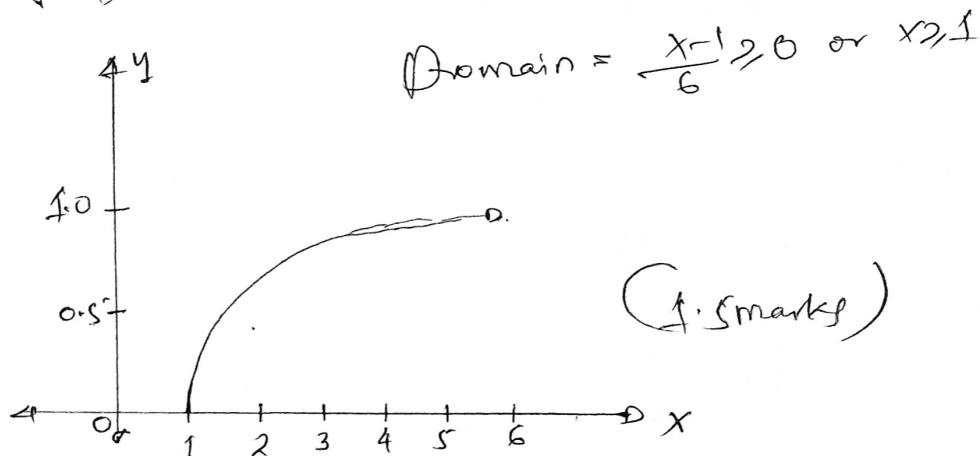
$$\text{Let } g \circ f(x) = y$$

$$y = 6x^2 + 1$$

$$x = \sqrt{\frac{y-1}{6}}$$

$$(g \circ f)^{-1}(x) = \sqrt{\frac{x-1}{6}} \quad (2 \text{ marks})$$

Graph of $(g \circ f)^{-1}(x)$



6 (b) Given $h(x) = \frac{5x+1}{x-3}$

x -intercept, put $y=0$, $x=-\frac{1}{5}$ (0.5 mark)

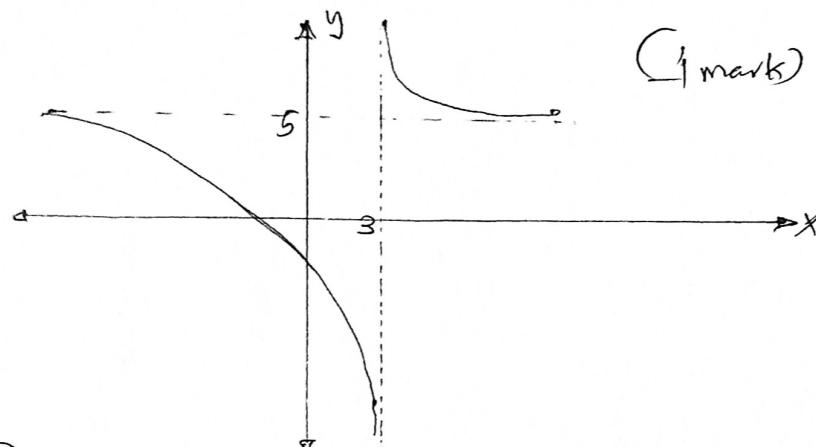
y -intercept, put $x=0$, $y=\frac{1}{3}$ (0.5 mark)

Vertical asymptote: $x=3$ (0.5 mark)

Horizontal asymptote:

$$y = \lim_{x \rightarrow \infty} \frac{5x+1}{x-3} = 5 \quad (0.5 \text{ mark})$$

Graph $h(x) = \frac{5x+1}{x-3}$



(1 mark)

Domain = $\{x: x \in \mathbb{R}, x \neq 3\}$ 0.5 mark

Range = $\{y: y \in \mathbb{R}, y \neq 5\}$ 0.5 mark

7. (a) Taylor's theorem state that: If a function $f(x)$ is continuous and has n continuous derivatives at $x=a$ then

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) \dots \quad (1 \text{ mark})$$

Derivation of Newton's-Raphson formula

Truncating higher terms, we get

$$f(x) \approx f(a) + \frac{(x-a)f'(a)}{1!}$$

If x is the root of a function, $f(x)=0$ (1 mark)

$$0 = f(a) + (x-a) f'(a)$$

$$x-a = \frac{-f(a)}{f'(a)}$$

$$x = a - \frac{f(a)}{f'(a)} \quad (1 \text{ mark})$$

where a is the value of x closer to the better approximation

7 (b) Given $f(x) = x^2 - 2x + 1 = 0$; $x_0 = 2$ and $x_1 = 3$

$$f(x_0) = 2^2 - 2(2) - 1 = -1$$

$$f(x_1) = 3^2 - 2(3) - 1 = 2$$

Since $f(x)$ has changed sign between $x=2$ and $x=3$, then there is a root in the interval.

The root of a function $f(x)$ by secant formula are given by

$$x_{n+2} = x_n - \left[\frac{x_{n+1} - x_n}{f(x_{n+1}) - f(x_n)} \right] f(x_n)$$

first iteration; $n=0$

$$\begin{aligned} x_2 &= x_0 - \left[\frac{x_1 - x_0}{f(x_1) - f(x_0)} \right] f(x_0) \\ &= 2 - \left[\frac{3-2}{2-(-1)} \right] (-1) \\ &= \frac{7}{3} \quad (\text{1mark}) \end{aligned}$$

Second iteration; $n=1$ and $f(x_2) = -\frac{2}{9}$

$$\begin{aligned} x_3 &= x_1 - \left[\frac{x_2 - x_1}{f(x_2) - f(x_1)} \right] f(x_1) \\ &= 3 - \left[\frac{\frac{7}{3} - 3}{-\frac{2}{9} - 2} \right] (2) \\ &= \frac{12}{5} \quad (\text{1mark}) \end{aligned}$$

Third iteration; $n=2$ and $f(x_3) = -0.04$

$$\begin{aligned} x_4 &= x_2 - \left[\frac{x_3 - x_2}{f(x_3) - f(x_2)} \right] f(x_2) \\ &= \frac{7}{3} - \left[\frac{\frac{12}{5} - \frac{7}{3}}{-0.04 - (-\frac{2}{9})} \right] (-\frac{2}{9}) \\ &= 2.41463 \end{aligned}$$

The root of the equation is 2.41463 to four decimal places.
 (1mark)

(c)

7 (e) Required to evaluate $\int_0^1 \sqrt{x} \cos x dx$

Compare with $\int_a^b f(x) dx$

$$a = 0, b = 1 \text{ and } f(x) = \sqrt{x} \cos x$$

Number of ordinates = 5 \therefore Number of strips = 4

$$\text{Width of intervals, } h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25 \quad (0.5 \text{ mark})$$

Table of values.

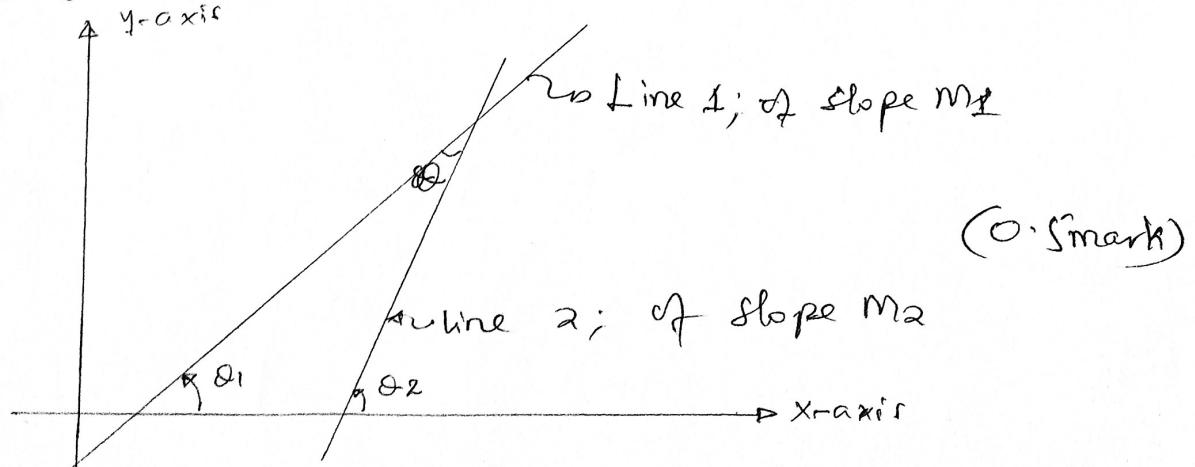
n	x_n	$f(x_n)$	$f(x_0) + f(x_4)$	even ordinate	odd ordinate
0	0	0	0		
1	0.25	0.48446			0.48446
2	0.50	0.62054		0.62054	
3	0.75	0.63366			0.63366
4	1.00	0.54030	0.54030		
TOTAL			0.54030		1.11812

The approximate value of the integral is given by (1 mark)

$$\begin{aligned} \int_0^1 \sqrt{x} \cos x dx &\approx \frac{1}{3} [(f(x_0) + f(x_4)) + 4 \sum \text{odd ordinate} + 2 \sum \text{even}] \quad (0.5 \text{ mark}) \\ &= \frac{0.25}{3} [0.54030 + (4 \times 1.11812) + (2 \times 0.62054)] \\ &= 0.521155 \end{aligned}$$

$$\therefore \int_0^1 \sqrt{x} \cos x dx = 0.5212 \text{ to four decimal places} \quad (1 \text{ mark})$$

8 (a) (i) Consider the sketch below



8. (a) (i) By angle properties of a triangle

$$\begin{aligned} \alpha + \alpha_1 &= \alpha_2 \\ \alpha &= \alpha_2 - \alpha_1 \end{aligned} \quad (1 \text{ mark})$$

Applying tangent half-angle, gives

$$\begin{aligned} \tan \alpha &= \tan(\alpha_2 - \alpha_1) \\ &= \frac{\tan \alpha_2 - \tan \alpha_1}{1 + \tan \alpha_1 \tan \alpha_2} \quad (0.5 \text{ mark}) \end{aligned}$$

where $\tan \alpha_1 = m_1$, $\tan \alpha_2 = m_2$

$$\tan \alpha = \frac{m_2 - m_1}{1 + m_1 m_2} \quad (0.5 \text{ mark})$$

(b) (i) Given line: $4y + 5x + 12 = 0$ and point $(x_1, y_1) = (6, 8)$
The perpendicular distance is given by

$$d = \left| \frac{4y_1 + 5x_1 + 12}{\sqrt{4^2 + 5^2}} \right| \quad (1 \text{ mark})$$

$$d = \left| \frac{(4 \times 8) + 5(6) + 12}{\sqrt{41}} \right| = \frac{50}{\sqrt{41}} \text{ units} \quad (1 \text{ mark})$$

(ii) Given: Circle equation $x^2 + y^2 - 2x + 6y - 3 = 0$

Tangent line: $3x - 4y + 7 = 0$

from $x^2 - 2x + y^2 + 6y - 3 = 0$

$$\begin{aligned} (x-2)^2 - 4 + (y+3)^2 - 9 - 3 &= 0 \\ (x-2)^2 + (y+3)^2 &= 16 \quad (0.5 \text{ mark}) \end{aligned}$$

Compare with: $(x-a)^2 + (y-b)^2 = r^2$

\therefore Centre of required circle $(a, b) = (2, -3)$ (0.5 mark)

Radius of the required circle is given by

$$r = \left| \frac{3x - 4y + 7}{\sqrt{3^2 + 4^2}} \right| \quad (x, y) = (2, -3)$$

$$r = \left| \frac{3(2) - 4(-3)}{\sqrt{25}} \right| = \frac{25}{5} = 5 \text{ units} \quad (1 \text{ mark})$$

8 (b) (ii) equation of required circle is given by

$$(x-a)^2 + (y-b)^2 = r^2$$

$$(x+2)^2 + (y+3)^2 = 5^2$$

$$\therefore x^2 + y^2 - 4x + 6y - 12 = 0 \quad (\text{1 mark})$$

8 (a) (ii) Given lines : $4x - 3y - 5 = 0$ and $2x + y - 1 = 0$

$$\text{from } 4x - 3y - 5 = 0; \quad 2x + y - 1 = 0$$

$$y = \frac{4x - 5}{3} \quad y = -2x + 1$$

Compare with $y = m_1 x + c$ $y = m_2 x + c$

$$\therefore m_1 = \frac{4}{3} \text{ and } m_2 = -2$$

Angle between the lines is $\tan \theta = \frac{-2 - \frac{4}{3}}{1 + (-2)(\frac{4}{3})}$

$$\tan \theta = \frac{-2 - \frac{4}{3}}{1 + (-2)(\frac{4}{3})}$$

$$\theta = -63^\circ 26'$$

\therefore The acute angle between the lines is $63^\circ 26'$

9. (a) Let $I = \int \frac{2x+1}{x^2 - 2x + 15} dx$

$$\text{Let } \frac{2x+1}{x^2 - 2x + 15} = \frac{2x+1}{(x-5)(x+3)} = \frac{A}{x-5} + \frac{B}{x+3}$$

$$2x+1 = A(x+3) + B(x-5)$$

Comparing Coefficients, we have

$$\begin{cases} A + B = 2 \\ 3A - 5B = 1 \end{cases}$$

$$\text{Hence } A = \frac{9}{2} \text{ and } B = -\frac{5}{2} \quad (\text{2 marks})$$

$$\begin{aligned} \therefore I &= \int \left[\frac{9}{2} \frac{1}{x-5} - \frac{5}{2} \frac{1}{x+3} \right] dx \\ &= \frac{9}{2} \ln(x-5) - \frac{5}{2} \ln(x+3) + C \quad (\text{2 marks}) \end{aligned}$$

(b)

9. (b) Given $\int_1^3 \frac{\ln x}{\sqrt{x}} dx$

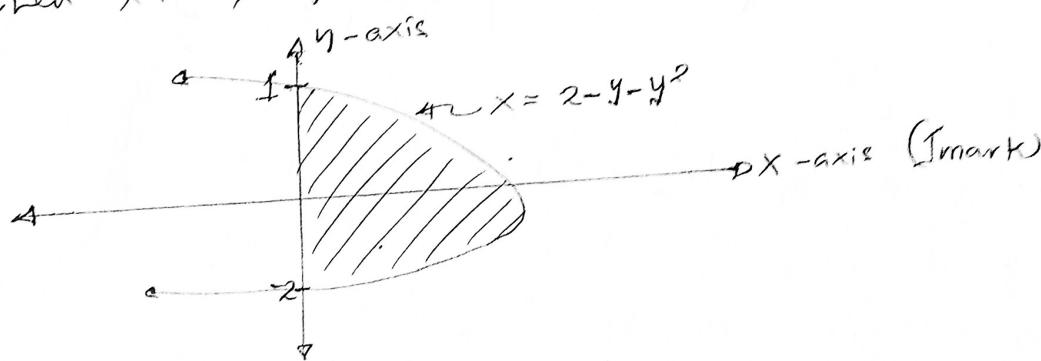
Integrating by parts

Let: $u = \ln x, dv = x^{-1/2} dx$

So that $du/dx = 1/x, du = 1/x dx$
 $v = 2\sqrt{x}$ (1 mark)

$$\begin{aligned}\int_1^3 \frac{\ln x}{\sqrt{x}} dx &= 2\sqrt{x} \ln x \Big|_1^3 - 2 \int_1^3 x^{1/2} \frac{1}{x} dx \\ &= 2\sqrt{x} \ln x - 4\sqrt{x} \Big|_1^3 \quad (\text{1 mark}) \\ &= [2\sqrt{3} \ln 3 - 4\sqrt{3}] - [2\sqrt{1} \ln 1 - 4\sqrt{1}] \\ &= 0.8775 \quad (\text{1 mark})\end{aligned}$$

10. (c) Given: $x = 2 - y - y^2$
 when $x = 0, y^2 + y - 2 = 0, \therefore y = 1 \text{ or } y = -2$



Required area is given by

$$\begin{aligned}A &= \int_{-2}^1 x dy \\ &= \int_{-2}^1 (2 - y - y^2) dy \quad (\text{1 mark}) \\ &= \left[2y - \frac{y^2}{2} - \frac{y^3}{3} \right] \Big|_{-2}^1 \\ &= \left[2 - \frac{1}{2} - \frac{1}{3} \right] - \left[-4 - 2 + \frac{8}{3} \right] \quad (\text{1 mark}) \\ &= \frac{9}{2} \text{ square units} \quad (\text{1 mark})\end{aligned}$$

10 (a) Given u and v are functions of x
 Let: $f(x) = u/v$ and $f(x+\Delta x) = \frac{u+\Delta u}{v+\Delta v}$

from first principle

$$\begin{aligned}
 \frac{d}{dx} \left(\frac{u}{v} \right) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \left\{ \frac{u+\Delta x - u}{v+\Delta v - v} \right\} \frac{1}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \left\{ \frac{v(u+\Delta u) - u(v+\Delta v)}{v(v+\Delta v)} \right\} \frac{1}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \left\{ \frac{uv + v\Delta u - uv - u\Delta v}{v^2 + v\Delta v} \right\} \frac{1}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \left\{ \frac{v \frac{\Delta u}{\Delta x} - u \frac{\Delta v}{\Delta x}}{v^2 + v\Delta v} \right\} \\
 \therefore \frac{d}{dx} \left(\frac{u}{v} \right) &= \left[\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \right] \quad (3 \text{ marks})
 \end{aligned}$$

(b) (i) Given $y = \log_3(x^2+2x+1)$ Required to find $\frac{dy}{dx}$

By law of logarithm $3^y = x^2+2x+1$

apply natural log both sides

$$\begin{aligned}
 \ln 3^y &= \ln(x^2+2x+1) \\
 y \ln 3 &= \ln(x^2+2x+1) \\
 y &= \frac{\ln(x^2+2x+1)}{\ln 3}
 \end{aligned}$$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}(x^2+2x+1)}{(x^2+2x+1)\ln 3}$$

$$\frac{dy}{dx} = \frac{2x+2}{(x^2+2x+1)\ln 3} \quad (2 \text{ marks})$$

10. (b) (ii) Given $y = \frac{\cos 3x + \cos x}{\sin 3x - \sin x}$

$$= \frac{2 \cos 2x \cos x}{2 \sin x \cos 2x} \quad [\text{By factor formula.}]$$

$$= \cos x \quad (\text{2 marks})$$

$\therefore \frac{dy}{dx} = -\operatorname{cosec}^2 x$

(c) Given $\sqrt{627} = \sqrt{625+2} = \sqrt{x+d}$

Let $y = \sqrt{x} = y = x^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$dy = \frac{1}{2\sqrt{x}} dx$$

$$= \frac{1}{2} \frac{x^{1/2}}{\sqrt{625}} = 0.04 \quad (1.5 \text{ mark})$$

Value of y $y = \sqrt{x} = \sqrt{625} = 25$

Now $\sqrt{627} = y+dy$

$$= 25 + 0.04$$

$$= 25.04$$

$\therefore \sqrt{627} = 25.04 \quad (1.5 \text{ mark})$