P425/2
APPLIED MATHEMATICS
Paper 2
16th October 2024

$Target\ (400^{+}\ A's)$

Road To Uneb Excellence Practice Questions

SECTION A TYPE QUESTIONS (Maximum total 40 marks-Target 30)

MECHANICS

1. A body of weight $20N$ rests on a rough horizontal ground whose coefficient of friction between the body and the ground is μ. A force P acting upwards, pulls the body at an angle 60^0 above the horizontal and is just sufficient to move the body. Show that:

$$P = \frac{40\mu}{\mu + \sqrt{3}}$$

2. Forces of $\begin{pmatrix}-2\\-3\end{pmatrix}N$, $\begin{pmatrix}7\\4\end{pmatrix}N$, $\begin{pmatrix}p\\2\end{pmatrix}N$ and $\begin{pmatrix}1\\-q\end{pmatrix}N$ are in equilibrium. Find the values of p and q.

3. The resultant of two forces YN and $3N$ acting at an angle θ to each other is $7N$. When the $3N$ is reversed, the resultant becomes $\sqrt{19}N$. Find the value of Y and θ.

4. The forces $\begin{pmatrix}-4\\-2\end{pmatrix}$N, $\begin{pmatrix}5\\-1\end{pmatrix}$N, $\begin{pmatrix}-2\\-1\end{pmatrix}$N and $\begin{pmatrix}1\\4\end{pmatrix}$N act on a lamina at points $(1,5)$m, $(2,-9)$m, $(2,-2)$m and $(7,\ 1)$m respectively. Show that the forces reduce to a couple

5. The initial velocity of a particle moving with a constant acceleration is $(3i-5j)ms^{-1}$. If after 3 seconds the velocity of the particle is $25ms^{-1}$ parallel to $(3i+4j)$. Find the acceleration of the particle.

6. Two particles A and B are moving a straight path. B is ahead of A by 14m, the speed of B is $16ms^{-1}$ and that of A is $25ms^{-1}$. Given that A and B have constant retardations of $6ms^{-2}$ and $4ms^{-2}$. Find the two possible times of overtake.

7. A stone is projected vertically upwards from the ground, with a velocity of $49ms^{-1}$. Find
(i) Time to reach maximum height (ii) Maximum height attained by the stone

8. A car of mass 20 metric tonnes travel up a plane of inclination 1 in 50 against a constant resistance of 0.1N per kilogram. Determine the;
(i) Tractive force required to produce an acceleration of $0.15ms^{-2}$.
(ii) Power developed by the car while at a speed of $36kmh^{-1}$.

9. Two trucks of mass 500kg and 400kg are travelling in the same direction at speed $2.4ms^{-1}$ and $1.05ms^{-1}$ respectively. If they collide and coalesce, find the loss in kinetic energy due to the collision.

10. A uniform rod AB of mass m, which is smoothly hinged at A is maintained in equilibrium by a horizontal force p acting at B. Given that the rod is inclined at 30^0 to the horizontal with B below A, find;
(i) an expression for p (ii) the magnitude and direction of the reaction at the hinge.

11. A particle moving in a straight line covers 12m in 4seconds and 30m in the next 5 seconds of its motion. Determine the
(i) acceleration. (ii) initial velocity.

12. A uniform beam AB of mass 8kg and length 4.8m rests horizontally with ends A and B supported by forces P and Q respectively. Masses of 6 and 4kg are hang at distances 160cm and 420cm from A. Calculate the reaction at each support.

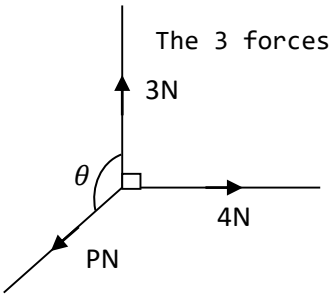
13. A particle, accelerating uniformly, moves with an average velocity of $8ms^{-1}$ for 4 seconds. If it's final velocity is $12ms^{-1}$. Calculate the;

(i) distance covered (ii) acceleration of the particle.

14.A particle is projected from a point O on a horizontal ground with a speed of $20ms^{-1}$ at an elevation $tan^{-1}\frac{4}{3}$. Calculate the;

(i) time of flight, (ii) height risen in a $\frac{1}{3}$ of the time of flight.

15.



The 3 forces aside are in equilibrium, find the values of **P** and **Q**

16. A projected particle achieves the greatest range of 120m. Find the;

(i) speed of projection, (ii) greatest height attained.

17.ABCD is a square of side 2a. Forces of magnitude 9N,5N and $3\sqrt{2}N$ act along $\vec{AB}$, $\vec{BC}$, and $\vec{BD}$ respectively. Find the equation of the line of action of the resultant force.

18. A ball projected from level ground with a speed of $25\sqrt{2}ms^{-1}$at an elevation of 45^{o} passes just above the top of two vertical posts each of height 30m. Find the distance between these posts.

19. A train is timed between successive stage A,B and C each 2km apart. If it takes 100s to travel from A to B and 150s from B to C. Calculate the:

(i) Retardation and initial velocity

(ii) Distance beyond C the train travels.

20. A non-uniform rod AB of mass 20kg and length 4m is suspended horizontally from the ends of the strings AC and BD of 60^{0} and 45^{0} respectively with the vertical. Calculate the;

(i) tension in the string BD

(ii) Distance from A where the weight of the rod acts.

21. A uniform rod AB $3.5m$ long and weight $5kg$ rests horizontally on two supports A and B. A load $24N$ is attached at a point C such that $AC = 1.5m$. Find the reactions at the supports A and B. (Take $g = 10ms^{-2}$)

22. A horizontal force X is just sufficient to prevent a body of mass M from sliding down a rough plane of inclination θ. A horizontal force $4x$ applied to the same mass on the same rough plane, causes the mass to be on the a point of moving up the plane, show that

$$5\mu\tan^2\theta - 3(\mu^2 + 1)\tan\theta + 5\mu = 0$$

SECTION B TYPE QUESTIONS

1. A bullet is fired from a point O which is at the top of a hill $50m$ above the ground. The speed with which the bullet is fired is $140ms^{-1}$ and it hits the ground at a point A which is at a horizontal distance of $200m$ from the foot of the hill. Find the:
   (i) two possible values of the angle of projection
   (ii) two possible times of flight
   (iii) direction of the bullet after travelling for 1 second.

2. (a) Show that for uniform motion , $s = \text{ut} + \frac{1}{2}at^2$.
   (b) A cyclist starts from rest and accelerates uniformly to a speed of Vms^{-1} in 9s. He maintains this speed for another 50s and then decelerates uniformly to rest. If his deceleration is numerically three times his previous acceleration,

   (i) Sketch a velocity – time graph of the motion of the cyclist,
   (ii) Calculate the time during which the cyclist was decelerating,
   (iii) Determine the value of V, given that the total distance travelled is 840m,
   (iv) Calculate the acceleration of the cyclist.

3. A truck of mass 3 metric tonnes moves with 3ms^{-1} on straight horizontal rails. It collides with truck B of mass 1 tonne, which is moving on the same rails at 4ms^{-1}, and the truck are moving towards each other. If the trucks couple to form a single body C, which continues to move on the rails,
   Find:
   (i) the speed and direction of C after the collision.
   (ii) the magnitude of the impulse exerted by B on A in the collision.
   (i) a constant braking force 250N is applied to C and it comes to rest in a distance d metres. Calculate the values d.

4. A body of mass mkg lies on a rough plane inclined at θ^0 to the horizontal. When a force of $\frac{mg}{2}N$ parallel to and up the plane is applied to the body, it is just about to move up the plane. When a force of $\frac{mg}{4}N$ parallel to and down the plane is applied to the body, it is just about to move down the plane. Calculate correct to two decimal places the value of:
   (i) θ
   (ii) The coefficient of friction between the body and the plane.

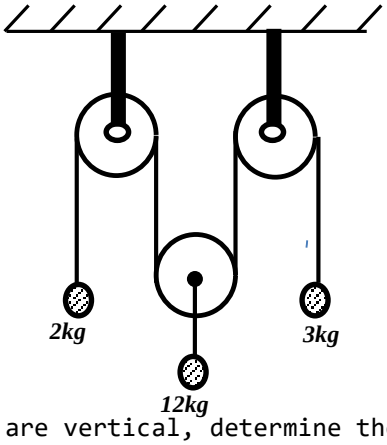
5. ABCDEF is a regular hexagon of side 4m forces of magnitude 2N, 6N, 5N, 8N and 3N act along the sides AB, BC, CD, ED and EF respectively. Taking AB and AE as the reference positive x and y axes respectively, determine the;
   (a) Magnitude of the resultant force and its direction.
   (b) Line of action by taking moments about A and point where it crosses the x – axis.

6. A sport car driver whose mass combined with the car is 250kg drives up a road of inclination 1 in 10 at a maximum power of 12kW.
   When the speed is $25ms^{-1}$, the car accelerates at $0.05ms^{-2}$.
   (a) Find the resistance to motion. (take $g = 10ms^{-2}$)
   (b) At the top of the road, he picks up a passenger of mass 70kg and drives on a horizontal road. If the resistance is increased by 10%, find;
   (i) the greatest speed that can be achieved when the engine is working at 80% of the maximum power.
   (ii) the immediate acceleration produced if the maximum power is suddenly engaged.

7. PQRST is a regular pentagon of side 6m. Forces of magnitude 3N, 2N, 1N, 4N, 5N and 6N act along the sides PQ, QR, SR, ST, TR and PT respectively. Taking PQ as the reference positive x axis, determine the;
   (a) Magnitude of the resultant force and its direction.
   (b) Line of action of the resultant and point where it crosses PQ.

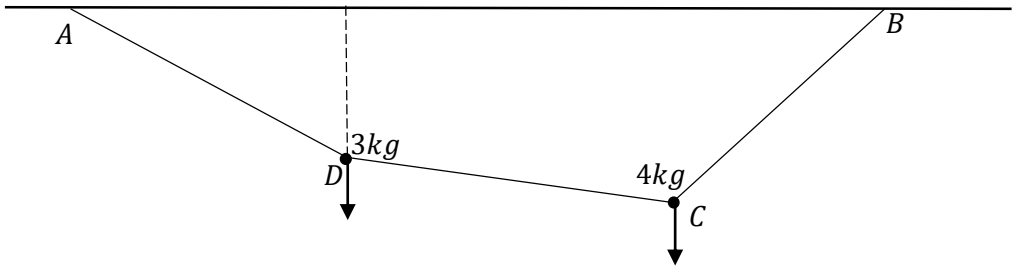
8. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:
   (i) acceleration of the system.
   (ii) tension in the string.
   (iii) distance moved by the moveable pulley in 1.5 s.

9. Three particles of masses 2kg,12kg and 3kg are connected by means of a light inextensible string passing over smooth light pulleys as indicated below



Assuming the strings are vertical, determine the
   (a) accelerations of 2kg,12kg and 3kg masses
   (b) tensions in the strings.

10. The figure below shows masses of $3kg$ and $4kg$ connected by a string and are at rest.



If AD is inclined at an angle 45^0 to the horizontal and $\angle ADC = 150^0$, find
(i) Tension in each portion of the string.
(ii) Angle string CB makes with the horizontal.

11. A uniform rod AB has a mass M and length $2a$. A particle of mass $2M$ is attached to the rod at a point C, where $AC = 1.5a$.The rod rests with end A on a rough horizontal ground, held in equilibrium at an angle θ to the ground by a light inextensible string attached perpendicular to the rod at B.
   (a) Show that $T = 2Mg\cos\theta$.
   (b) Given that $\cos\theta = 0.6$, show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$.
   (c) The coefficient of friction between the rod and the ground is μ. Given that the rod is in limiting equilibrium, show that $\mu = \frac{8}{19}$.

PROBABILITIES

1. Two events X and Y are such that $P(X) = \frac{2}{5}$, $P(X/Y) = \frac{1}{2}$ and $P(Y/X) = \frac{2}{3}$. Find

   (a) P(XnY) (b) $P(X'/Y')$

2. A discrete random variable Y has the probability distribution function given by

y	5	8	9	11	12
$P(Y = y)$	a	0.1	a	0.4	0.1

Find the; (i) value of a. (ii) $E(5Y - 7)$ (iii) $Var(x)$ (iv) median of Y

3. A random variable T is defined by; $f(t) = \begin{cases} \beta\left(\frac{1}{2}\right)^t ; & t = 1,2,3,4,\ldots\ldots \\ 0 ; & else\ where \end{cases}$

Where β is a constant. Find

(i) Value of β.

(ii) $P\left(T \geq 2/T \leq 6\right)$

4. Given that A and B are two events such that $P(A') = 0.3$, $P(B) = 0.1$ and $P(A/B) = 0.2$. find (i) $P(A \cup B)$ (ii) $P(A/B')$

5. A box *P* contains 3 red and 5 black balls, while another box *Q* contains 6 reds and 4 black balls. A box is chosen at random and from it a ball is picked and put into another box. A ball is then randomly drawn from the later. Find the probability that;

   (i) Both balls are red. (ii) First ball drawn is black.

6. A batch of 7 nails is drawn from machine output which is 40% defective. Find the;

(i) Probability that the batch contains at most 3 defective nails.

(ii) Most likely number of defective nails.

7. Given that $X \sim B(12,0.3)$. Find

   (a) $P(X < 6)$ (b) $Var(X)$ hence the standard deviation

8. (a) Given that events A and B are independent events show that A^1 and B are also independent

   (b) Two events A and B are independent such that their chance of occurring together is 0.2 and the chance of either A or B occurring is 0.875. find the values of $P(A)$ and $P(B)$.

9. The chances of three soldiers A, B and C hitting a bullet at a target are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. If all of them shoot once, find the probability of hitting the target by;

   (i) Only one of them (ii) At least two of them

10. A certain manager travels to work by routes A and B. the probability that the manager chooses route A is 0.25 and the probability that the manager is late for work if he uses route A is 0.6, and the corresponding probability via route B is 0.3. Determine the probability that he;

(a) is late for work.

(b) went via route B given that he is late for work.

11. (a) At a bus park, 60% of the buses are of Teso coaches, 25% are Kakise buses and the rest are Y.Y buses. Of the Teso coaches 50% have TVs, while for the Kakise and Y.Y buses only 5% and 1% have TVs respectively. If a bus is selected at random form the park, determine the probability that

(i) It has a TV (ii) Kakise bus is selected given that it has a TV

(b) On a certain day, fresh fish from lakes, Kyoga, Victoria, Albert and George were supplied to a market in ratio a 30%, 40%, 20% and 10% respectively. Each lake had an estimated ratios of poisoned fish of 2%, 3%, 3% and 1% respectively. If a health inspector picked a fish at random.

(i) What is the probability that the fish was poisoned?

(ii) Given that the fish was poisoned, what is the probability that it was from lake Albert.

12. Two bags A and B contains Red and Green fruits. Bag A has 4 red and 3 green fruits while bag B contains 2 Red and 5green fruits. A bag is selected at random and two fruits are randomly picked from it without replacement.

Find (i) Both fruits are green (ii) The fruits are of different colours,

(iii) The second fruit is green given that the first was also green

13. A Committee of three students had to be chosen from 8 girls and 10 boys. Find the probability that the Committee will consist of at least one girl.

14. Eight letters of the word **ELEPHANT** are written and arranged at random. Find the probability of the eight letter arrangements in which;

(i) The first letter is a "P".

(ii) "N and T" are next to each other

15. A continuous random variable X has a probability density function $f(x)$ given by;

$$f(x)=\begin{cases}\dfrac{k}{2} & ;0<x<2\\ \dfrac{k}{2}(3-x) & ;2<x<3\\ 0 & ;\ elsewhere\end{cases}$$

(a) Sketch $f(x)$, state the mode hence find the constant k.

(b) Find the; (i) Mean, $E(x)$

(ii) $P\left(X\leq 1.5/X>1\right)$

16. A continuous random variable X has a probability density function defined by

$$f(x)=\begin{cases}\alpha x & 0\leq x<1\\ \alpha & 2\leq x<2\\ \alpha(3-x) & 2\leq x<3\end{cases}$$

(a) Sketch $f(x)$ hence find the value of $\propto$

(b) Deduce the cumulative distribution function $F(x)$ hence find $P(\mathbf{1.5}<\boldsymbol{x}<\mathbf{2.5})$

17. A random variable T has a pdf $f(t)$ such that:

$$f(t) = \begin{cases} k(3+t) & -3 \le t \le 0 \\ k(3-t) & 0 \le t \le 3 \\ 0 & otherwise \end{cases}$$

Construct a cumulative distribution function $F(t)$ hence sketch it.

18. (a) Two events A and B are such that $P(A/B) = \frac{5}{11}$, $P(A \cup B) = \frac{9}{10}$, and $P(B) = x$.

(i) Show that $P(A) = \frac{9}{10} - \frac{6x}{11}$

(ii) If $P(A \cap B) = 2P(A \cap B')$, find the value of x.

(b) In a survey conducted in a S.6 mathematics class, 35% of the students watched football and not cricket, 10% watched cricket but not football, and 40% did not watch either game. If a student is chosen at random from those in the survey, find the probability that he watches;

(i) football given that he watches cricket

(ii) football given that he does not watch cricket

19. The events A and B satisfy $P(A) = x$, $P(B) = y$, $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 0.6$, $\mathrm{P}(B/A) = 0.2$

(a) Show that $4x + 5y = 3$

The events B and C are mutually exclusive such that $\mathrm{P}(\mathrm{B} \cup \mathrm{C}) = 0.9$ and $\mathrm{P}(\mathrm{C}) = x + y$,

(b) Find the value of x and y

(c) Determine, showing all the relevant workings, whether *A* and *B* are statistically independent.

20. The events *A* and *B* are such that $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cup B) = 0.6$.

Determine;

(a) $P(A \cap B)$ (b) $P(A \cap B')$ (c) $P(A' \cup B)$

21. The events *A* and *B* satisfy $P(A) = 0.6$, $P(B) = 0.52$ and $P(A \cup B) = 0.88$.

(a) Find the value of $P(A \cap B)$ and hence illustrate this probability information in a fully completed Venn diagram.

(b) Determine;

i. $P(B/A)$

ii. $P\left(A'/B'\right)$

(c) State, giving a reason, whether A and B are …

i. independent.

ii. mutually exclusive.

22. Given that $X \sim R[a,b]$, find (i) pdf of X. (ii) $E(X)$ (iii) $Var(X)$

23. A random variable $\boldsymbol{X}$ has a uniform distribution over an interval $[e,f]$.

(a) Show that; $E(X) = \frac{(e+f)}{2}$

(b) Given that $E(X) = 1$ and $P(X \le 0.25) = 0.25$, find the;

(i) Values of e and f (ii) Variance hence standard deviation of $\boldsymbol{X}$.

24. The table below shows the number of apples put in boxes A, B and C.

Apples	Boxes		
	A	B	C
Green	4	7	3
Red	7	5	11

A box is randomly selected and two apples are randomly selected from it without replacement. Box A is twice as likely to be picked as B. While A and C have the same chance of being picked.

(a) Determine the probability that both apples are:
   (i) Of the same colour
   (ii) From box B given that they are of the same colour.

(b) If X is the number of green apples taken, draw a probability distribution table for X. hence calculate the mean and standard deviation of X.

STATISTICS

1. The data below shows marks obtained in a math test.

Marks	$10-<20$	$20-<25$	$25-<35$	$35-<55$	$55-<70$	$70-<80$
frequency	10	20	15	40	10	5

(a) Calculate the;
   (i) Mean (ii) Mode

(b) Draw an Ogive and use it to find the;
   (i) Median (ii) Semi-interquartile range (iii) Pass mark if 50 students passed

2. The table below shows the times taken (in minutes) of a group of senior six students to think of the right solution to an examination question.

Time(minutes)	$0 \le x \le 2$	$2 \le x \le 4$	$4 \le x \le 6$	$6 \le x \le 10$	$10 \le x \le 14$	$14 \le x \le 18$
Frequency	2	12	20	14	8	4

(a) Calculate the;
   (i) Mean time taken by the student
   (ii) Probability that a student selected at random used a time between 6 and 15 minutes.

(b) Draw an ogive and use it to estimate the middle 60^{th} percentile range of the times used by the students while thinking.

3. The table below shows the prices of commodities sold in the market in the months of July and October of 2024.

Commodity	A	B	C	D	E
July	16000	4000	3000	3800	8000
October	15000	5200	3600	5500	6500
Weight	1	2	4	3	5

Take July=100

(a) Calculate the (i) Simple aggregate price index of October hence comment
   (ii) Price relative of for each commodity (iii) cost of living index

(b) If the cost of a commodity in October is 500000, calculate its cost in July using the answer in a (iii) above.

4. The table below shows the prices of items for the years 2016 and 2017

Item	PRICE (£)		Weights
	IN 2016	IN 2017	
A	25	28	5
B	x	y	3
C	30	36	2

Given that the simple aggregate price index and weighted mean price index for 2017 based on 2016 are 120 and 119 respectively, find the values of x and y.

5. In 1994, the price index of a commodity was 135 taking 1991 as the base year. The price of the commodity in 1994 was Shs 5400 and in 1992 was Shs 4600.
Find (a) The price of the commodity in 1991
(b) The price index of the commodity in 1992 taking 1991 as the base year.

6. A class of 60 students performed an experiment to estimate the diameter of a circular object and scored the following marks.

Marks	10	20	30	40	50	60	70
No. of students	2	10	x	12	9	6	4

Find the (i) Value of x
(ii) Mean mark
(iii) Median mark
(iv) Variance using a working mean value of 40

8. The table below shows the cumulative frequency of ages of 240 students admitted to a certain university.

Age (years)	< 18	< 19	< 20	< 24	< 26	< 30	< 32
Cumulative frequency	0	24	94	170	< 218	< 234	< 240

Calculate the
(i) $D_{7^{th}}$
(ii) P_{60}
(iii) Mean age
(iv) Draw an Ogive hence use it to find the median age.

9. The table below gives the data about the masses of form six students in a certain school in 2024.

Mass (Kg)	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64	65 – 69	70 – 74
Frequency	6	4	8	10	12	7	3

(i) Calculate the mean mass
(ii) Determine the number of students whose mass exceeded the mean mass.
(iii) Find the median mass
(iv) Draw a cumulative frequency curve and estimate the 10 – 90 percentile range.

10. The table below shows the distribution of heights of 60 students in a Math Class.

Heights (cm)	$50 - < 55$	$55 - < 75$	$75 - < 85$	$85 - < 90$	$90 - < 100$	$100 - < 120$
No. of students	4	20	15	12	7	2

(i) Calculate the modal height.

(ii) Draw a histogram.

11. The table below shows marks given by two judges to ten teams in a music contest.

Judge P	75	78	74	72	75	73	76	71	77	70
Judge Q	79	73	73	70	76	75	77	71	78	71

(i) Calculate the spearman's rank correlation coefficient.

(ii) Comment on your solution.

12. The table below shows Grades in maths and ICT obtained by 8 students in a mock examination.

Math	A	O	E	B	C	B	F	D
ICT	D_2	F_9	C_4	D_1	D_1	C_3	P_8	C_6

Calculate the rank correlation. Comment on results.

END

APPLIED MATHEMATICS P425/2 EXAMINATION FORMAT

Consists of 16 questions out of which 8 in Section A are compulsory and from the 8 in section B a student attempts 5 making a total of 13 questions attempted. Each section contains 3 mechanics questions, 2 numerical questions and 3 questions from Statistics and Probability.

Numerical Methods questions (4 questions) are set from topics: Linear interpolation/extrapolation, the Trapezium rule, Error analysis, Roots of equations (location, approximation and finding of accurate roots by linear interpolation and newton Raphson method) and Flow charts.

Statistics questions (2 questions) are set from topics: Descriptive Statistics, Measurement of relationship (Correlation and scatter diagrams) and index numbers.

Probability questions (4 questions) are set from topics: probability theory, probability density functions (p.d.f's), uniform/rectangular distributions, binomial distributions, normal distribution, normal to binomial, normal approximation to binomial (n, large), estimations/sampling (sample mean ($\bar{x}$) and confidence interval for μ).

Mechanics questions (6 questions). This is divided into three branches:

- ✓ ***Dynamics*** (1A and 1B= 2 questions)
  Newton's laws of motion e.g. connected particles, linear momentum, work, power and energy, elasticity, SHM and circular motion.
- ✓ ***Kinematics*** (1A and 1B= 2 questions)
  Linear motion, projectile motion, resultant velocity, relative motion, vector mechanics.
- ✓ ***Statics*** (1A and 1B= 2 questions)
  Composition and resolution of coplanar forces, friction (statics of a particle), moments, equilibrium of rigid bodies (suspended, pivoted, limiting equilibrium, hinged and jointed rods) and C.O.G.

WISHING YOU THE BEST PREPARATIONS