A LEVEL

P425/2

MATHEMATICS

Paper 1

March 1988

3 Hours.

1. Solve the equations

$$3x - y - 2z = 0$$
$$x + 3y - z = 5$$
$$2x - y + 4z = 26.$$

(b) Given that $\frac{\log_{10} 3}{\log_{10} 2} = \frac{8}{5}$, solve for x and y in the simultaneous equations .

$$3^{x} = 2^{3y+1}$$
$$4^{x-1} = 12^{2y+1}$$

2(a) If $p = x + \frac{1}{x}$ express $x^2 + \frac{1}{x^2}$, $x^2 + \frac{1}{x^3}$ and $x^4 + \frac{1}{x^4}$ in terms of p.

(b) Prove that (b - c) is a factor of
$a^3 (a - b) + b^3 (c - a) + a^3(a - b)$ and write down two other factors of the expression.

3.(a) Write down the first four terms of the expansion of $(p + q)^n$ in descending powers of p . Hence other wise find the value of $(1.01)^{10}$ to 5 significant figures.

(b) Using the substitution $x^2 - 4x = y$, or otherwise find the real roots of the equation

$$2x^4 - 16x^3 + 77x^2 - 180x + 63 = 0$$

4(a). Given that $z_1 = 2 - 3i$

$z_2 = 3 + 5i$

Find (i) $\left|\frac{z_1}{z_2}\right|$

(ii) $z_1z_2 + (z_1 + z_2)$

(b) Solve $x^2 + 4x + 13 = 0$

5. (a) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Hence solve the equation

$\sin 3\theta + \sin\theta = 0$ for $0^o \leq \theta \leq 360^o$

(b) Solve $3\sin\theta - \cos\theta = 3$ for

$0^o \leq \theta \leq 360^o$.

6.(a) Prove that in any triangle ABC

$a^2 = b^2 + c^2 - 2bc\cos A$.

(b) Prove that if $t = \tan\frac{1}{2}\theta$, then

$\sin\theta = \frac{2t}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$.

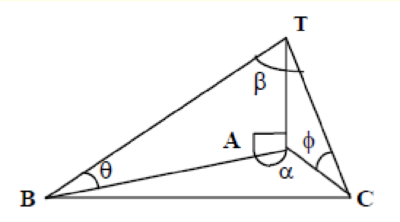
Hence prove that $\tan^2 22.5^o = 3 - 2\sqrt{2}$.

7. Find the equation of the normal at

$P(ap^2. 2ap)$ to the parabola $y^2 = 4ax$.

Show that the normal to the curve at L (a,2a) passes through the point B(5a, -2a). Prove that there is just one other point M on the curve at which the normal passes through B and determine the coordinates of M.

8.



In the figure A,B,C are points on a horizontal ground . AT is a vertical flag post subtending angles θ at B and φ at C . Line BC subtends angles α at A and β at T . Write down the two expressions equal to BC^2. Hence , or other wise prove that

$$\frac{\cos\theta\cos\varphi\cos\alpha - \cos\beta}{\sin\theta\sin\varphi} = 1.$$

9. The position vector of points P and Q are **2i - 3j +4k** and **3i -7j+ 12k** respectively. Determine the length of PQ.

PQ meets the plane 4x + 5y - 2z = 5 at point S. Find

(i) the coordinates of S

(ii) the angle between PQ and the plane.

10. Find and distinguish between the nature of the two turning points of the curve

$y = x^3 - x^2 - 5x + 6$ and sketch the curve .
Find the area enclosed between the curve and the lines $x = -2$, $x = 0$ and $y = 0$. Hence find the area enclosed between the curve and the line $y = x + 6$, where
$-2 \le x \le 0$.

11. Evaluate $\displaystyle\int_{a}^{2a} \frac{x^3}{x^4 + a^4}\,dx$

(ii) Using the substitution $x = \cos 2\theta$, or other wise prove that

$$\int_0^1 \sqrt{\left(\frac{1-x}{1+x}\right)}\,dx = \frac{1}{2}\pi - 1$$

12. Given that $y = \tan^{-1}(1-x)$ show that

(i) $(2-x)\dfrac{dy}{dx} + \dfrac{1}{2}(1-x)^{-1/2} = 0$

(ii) $(2-x)\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} + \dfrac{1}{4}(1-x)^{-3/2} = 0$

Hence or other wise determine MacLaurin's series expansion of y up to the x^3 term. Use your expansion to evaluate

$\tan^{-1}\sqrt{(1-\frac{1}{4}\pi)}$ to two decimal places.

13.(a) . Given that $y = e^{2x}\sin 3x$ prove that

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0.$$

(b) A shop keeper found that his net profit P when selling a number n of a certain type of shirt per week is given by

P = sh $(750n-0.1n^2 - 20{,}000)$.

He is selling 40 shirts per week ,show that if n increases from 40 to 41 profit will increase while an increase in n from 60 to 61 causes a decrease . Comment on your answer.

14. (a) . The quarterly cost , in shillings of water for a house hold over a period of 2 years is given in the table below.

	Quarters			
Year	1	2	3	4
1986	91	73	71	86
1987	99	80	64	95

Draw a graph to show these figures .

Calculate the four - quarterly moving averages and display these on the same axes.

(b) (i) Given n unlike objects , find the number of ways of dividing them into three unequal groups of sizes p, q and r where
$p + q + r = n$,
(ii) A committee of four is to be selected from six RC-1 and six RC -2 members. How many possible committees are there?
In how many ways will the members of RC -1 have a majority?
15. The table shows the number of people in millions in the different age groups in a country .

Age group	population in million
below 10	2
10 and under20	8
20 and under 30	10
30 and under 40	14
40 and under 50	10
50 and under 70	5

70 and under 90 1

Calculate

(i) the mean age

(ii) the mode

(iii) the standard deviation.

(iv) Draw a histogram to represent the above data.

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MATHEMATICS
Paper 2
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1. (a) Using the substitution y = xz or otherwise show that the solution of the equation

$$2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2} \quad \text{is given by} \quad \frac{(y-x)^2}{xy^2} = c$$

where c is a constant

(b) Solve the equation

$$\frac{dy}{dt} = -0.5x(t)y$$

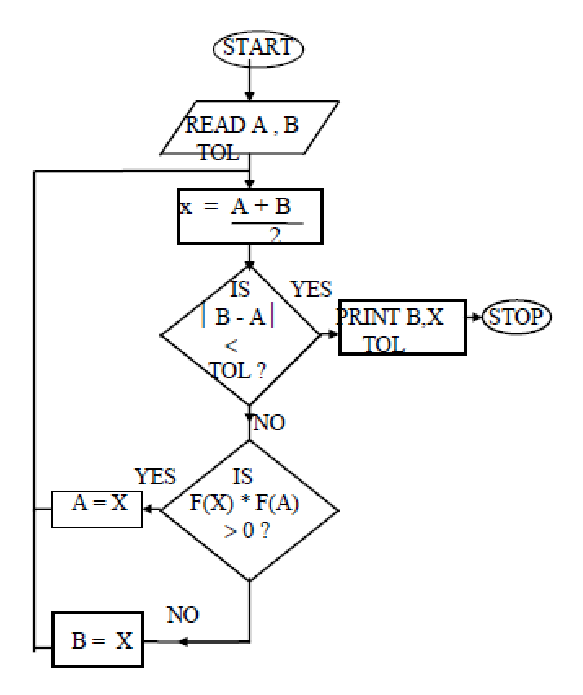
given that $x(t) = \dfrac{4}{(1+t)^2}$

and $y(0) = 10$

2. (a) Derive the simplest iterative formula based on Newton Raphson method for finding the root of the equation $e^{3x} - 3 = 0$

Use your formula with $x = \dfrac{1}{3}$ as the starting value to find the root correct to four significant figures. Hence find ln3 correct to 3 significant figures.

(b) An iterative method for approximating a root ot the equation $f(x) = 0$ is described in the flow chart below



Given that

A =1.6875, B =1.8750,TOL = 10^{-2} perform a dry run for the flow chart to determine $\sqrt[3]{5}$ tabulating the values of A,B and X at each stage.

3. (a) Define the term error and absolute error modulus.

obtain the range of values within which the exact value of 2.7654+3.8006-$\frac{15.78}{0.9876}$ lies .

(b) The numbers A and B are approximated by the numbers X and Y respectively such that A =X -a, B = y-b where a,b are small numbers compared to A and B .

Given that Y = f(x) and B = f(A) show that $|b| = |a| f^{1}(A)$.If f(A) = A^p where p is a constant deduce that $|b| = |a| pA^{p-1}$ and find the expression for the relative error.

4.(a) A particle projected from point O with an initial velocity **3i+4j** where **i** and **j** are vectors a long the x and y axis respectively. Find in vector form the velocity and position of the particle at any time.

(b) A particle P is projected from a point A with an initial velocity of 60ms^{-1} at an angle

30^o to the horizontal. At the same instant a particle Q is projected in opposite direction with initial speed of $50ms^{-1}$ from a point at the same level with A and 100m from A . Given that the particles collide find.

(i) the angle of projection of Q

(ii) the time when collision occurs.

5. (a) An object P passes through a point whose position vector is **3i - 2j** with constant velocity **i + j.** At the same instant an object Q moving with constant velocity **4i - 2j** passes through the point with position vector **i + 4j**.

Find :

(i) the displacement of P relative to Q after tseconds.

(ii)the time when P and Q are closest together and the closest distance at that time.

(b) To a cyclist riding due south at $20kmh^{-1}$ a steady wind appears to be blowing in the direction 240^o.

When he reduces his speed to15kmh^{-1} the wind appears to blow in the direction 210^0. Find the true velocity of the wind .

6. (a) Four forces acting on a particle are represented by $\mathbf{2i + 3j}$, $\mathbf{4i - 7j}$, $\mathbf{– 5i + 4j}$ and $\mathbf{i - j}$. Find the resultant force. A fifth force represented by p$\mathbf{i}$ + q$\mathbf{j}$ is added to the system which is then in equilibrium . Find the values of the constants p and q.

(b) Two uniform rods AB and BC of equal length but of masses M and 3M respectively are freely joined together at B. The rods stand in a vertical plane with the ends A and C on a rough horizontal ground. The coefficient of friction μ at the points of contact with the ground is the same and the rods are inclined at 60^0 to each other. Given that one of the rods is on point of slipping find μ . Find also the reaction at the hinge B when the rods are in this position.

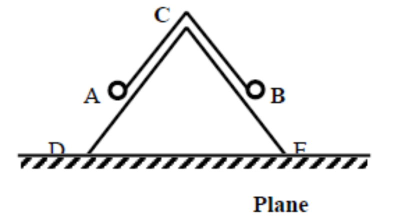
7. A particle moving with simple harmonic motion in a straight line passes through three points A,B,C in that order with velocities
0, $2ms^{-1}$ and -1 ms^{-1} respectively. Find the period and amplitude of the motion if
AB = 2 metres and AC = 8 metres.
(b) Two springs AB and BC are joined together end to end to form a long spring. The natural length of the separate springs are1.6metres and 1.4 metres and their moduli of elasticity are 20N and 28N respectively. The end A is fixed and the combined spring is stretched by one metre Find the tension in the spring.

8. (a) Two spheres A and B equal size have masses m and 2m respectively. Sphere A is at rest on horizontal plane and sphere B which is moving on that plane with speed 2u along the line of their centres collides directly with A. Given that the coefficient of restitution e is 1, find the loss in kinetic energy after impact.

(b) A gun of mass M fires a shell of mass m and recoils horizontally. Given that the shell travels along the barrel with speed v, find the speed with which the barrel begins to recoil if

(i) the barrel is horizontal

(ii) the barrel is inclined at an angle of 30^0 to the horizontal.



The diagram shows a wedge DCE of mass 10kg with the face DE in contact with asmooth horizontal plane. Two particles A and B of masses 3kg and 2kg respectively are connected by a light inextensible string passing over a smooth pulley fixed at the vertex C of the wedge . Given that the surface of the wedge is smooth and is moving freely, find

(i) the acceleration of the wedge

(ii) the acceleration of B.

10.(a) Define the independence of two events A and B

Given that A and B are independent events in a sample space such that $P(A) = \frac{2}{5}$ and $P(A \cup B) = \frac{4}{5}$.

Find

(i) P(B)

(ii) $P(\overline{A} \cup B)$.

(b) In a certain town the probability that a person owns a car is 0.25.Given that the probability that a person who owns a car is a University graduate is 0.2, find the probability that a person selected at random owns a car and is a University graduate.

11. (a) A random variable X has the following distribution

$P(X = 0) = P(X = 1) = 0.1$,

$P(X = 2) = 0.2$,

$P(X = 3) = P(X = 4) = 0.3$.

Find the mean and variance of X .

(b) A continuous random variable X has the distribution function

$$F(x) = \begin{cases} 3kx\left(1 - x^2/3\right) & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

Determine

(i) the value of k

(ii) the probability density of x

(iii) the mean of X

(iv) $P\left(X > 0.5 / 0.25 \le X \le 1\right)$

12.(a) In a certain clan the probability of a family having a boy is 0.6 . If there are 5 children in a family , determine

(i) the expected number of girls

(ii) the probability that there are at least three girls

(iii) the probability that they are all boys.

(b) The volume of soft drinks bottled by a certain company is approximately normally distributed with mean 300ml and standard deviation2mls. Determine the probability that in a sample of 10 bottles at least two bottles contain less than 297.4mls.

13. (a) The mean and standard deviation of a random sample of size 100 is 900 and 600 respectively. Given that the population is normally distributed find a 95 % confidence interval of the population mean.

(b) A random sample of size 9 drawn from a normally distributed population has the following values 297.5, 298.7, 596.5, 300, 297.4, 596.6, 297.5, 300.5, 300 .
Determine a 99% confidence interval for the population mean.

14.(a) The price of matoke is found to depend on the distance away from the nearest town. The table below gives the average price of matoke for markets around Kampala City.

Distance,d (km)	40	8	17	20	24	30	10	28	16	36
Price,p (sh)	120	160	140	130	135	125	150	130	145	125

(i) plot these data on a scatter diagram

(ii) Draw the line of best fit on your diagram.

(iii) Find the equation of your line in the form $p = \alpha + \beta d$ where α and β are constants.

Hence estimate the price of matoke when]

$d = 5$

(b) The following table gives the order in which six candidates were ranked in two tests X and Y.

X: 1 2 3 4 5 6

E C B F D A

Y: F A D E C C

1 0 3 4 6.5 5.5

Calculate the coefficient of rank of correlation and comment on your result.

15 (a) Simplify

(i) $p \vee (p \wedge) q$

(ii) $\sim(p \vee Q) \vee (\sim P \wedge Q)$

Hence deduce the relation between (I) and (ii)

(b) Given the compound statement

$\{p \Lambda Q\} v \left[\left(P^1 \Lambda Q^1\right) v Q\right] v P$

(i) Draw an electric circuit corresponding to the statement.

(ii) construct a truth table for the statement and state what it tells you about the circuit