SANJOAC

S.5 END OF TERM II 2023

PURE MATHEMATICS

Time: 3 hours

SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. Solve the simultaneous equations

$$x - 2y - 2z = 0$$
$$2x + 3y + z = 1$$
$$3x - y - 3z = 3$$

2. In a geometric progression (G.P), the difference between the fifth and the second term 156. The difference between the seventh and the fourth term is 1404. Find the possible values of the common ratio.

3. Given that α and β are the roots of the equation $x^2 + px + q = 0$, express $(\alpha^2 - \beta^2)$ and $(\alpha^3 + \beta^3)$ in terms of $p \ and \ q$

4. If $x^2 + 3xy - y^2 = 0, find \ \frac{dy}{dx}$ at (1, 1) and find the equation of the normal at that point

5. Find the term independent of x in the expansion $\left(x + \frac{2}{x^2}\right)^{10}$

6. Express $4cosx + 3sinx$ in the form $Rcos(x - \alpha)$. Hence state the maximum value of the function $\frac{2}{4cosx+3sinx+10}$ and the smallest positive value of x within which it occurs

7. The length of a rectangular block is twice to its width, and the total surface area is $108cm^2$. Show that if the width of the block is x cm, the volume is $\frac{4}{3}x(27 - x^2)$. Find the dimensions of the block if the volume is maximum.

8. Find the remainder when $x^3 - 7x^2 + 6x + 1$ is divided by $x - 3$

SECTION B (60 MARKS)

*Answer any **five** questions from this section.*

9. a). Given that α *and* β are roots of the equation $ax^2 + bx + c = 0$, determine the equation whose roots are $\alpha + \beta$ *and* $\alpha^3 + \beta^3$.

   b). If the roots of the equation $ax^2 + bx + c = 0$ differ by 4, show that.

   $\frac{b^2}{4a} = 4a + c.$ ***(12 marks)***

10. (a) Peter is planning to have his graduation party at some points along the highway. He is to use a rectangular field of $5000m^2$ and is fenced off on the three sides not adjacent to the high way. What dimensions will give a least amount of fencing for the enclosure.

    (b) Sketch the curve $y = x^4 - 10x^2 + 9$. ***(12 marks)***

11. (a) Differentiate $(x + 1)^2$ from first principles

    (b) $x - 1$ and $x + 1$ are factors of $x^3 + ax^2 + bx + c$ and it leaves a remainder of 12 when divided by $x + 2$. Find *a, b* and *c.* ***(12 marks)***

12. (a) Prove that $sin^2\theta(1 + sec^2\theta) = sec^2\theta - cos^2\theta$

    (b) Solve the equation $4sec^2\theta = 3tan\theta + 5$ for $0^0 \leq \theta \leq 360^0$ ***(12 marks)***

13. (a) Find a number x which, when added to each of the numbers 21, 27, and 29 produces three numbers whose squares are in arithmetic progression.

    (b) A geometric progression (GP) and an arithmetic progression (AP) have the same first term. The sum of their first, second and third terms are 6, 10.5 and 18 respectively. Calculate the sum of their fifth terms. ***(12 marks)***

14. (a) Expand $(1-x)^{\frac{1}{3}}$ in ascending powers of *x* as far as the fourth term. By taking the first two terms of the expansion and substituting $x = \frac{1}{1000}$, find the value of $\sqrt[3]{37}$, correct to six significant figures. $[Hint\colon 27 \times 37 = 999]$ ***(06 marks)***

(a) Use Binomial theorem to find the value of $(1.01)^{10}$, correct to 3dps ***(06 marks)***

15. (a) A piece of wire of length *l* is cut into two portions. Each portion is then cut into twelve equal parts which are soldered together so as to form edges of a cube.

(i) Find an expression for the sum of the volumes of the two cubes formed.

(ii) What is the least value of the sum of the volumes? ***(06 marks)***

(b) An up turned cone with semi vertical angle of 45^0 is being filled with water at a constant rate of 30 cm^3 per second. When the depth of water is 60cm, find the rate at which the:

(i) depth of water is increasing

(ii) area of the water surface is increasing. ***(06 marks)***

16. (a) Express the following complex numbers in modulus – argument form

(i). $Z_1 = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

(ii). $Z_2 = -5i$

(b). (i) Express $Z = \frac{-1+2i}{1+3i}$ in the form $a + bi$

(ii) Find the values of *x* and *y* if $\frac{x}{1+i} + \frac{y}{2-i} = 2 + 4i$. ***(12 marks)***

END