S475/1
SUBSIDIARY
MATHEMATICS
Paper 1
July/August 2024
2⅔hours

UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

SUBSIDIARY MATHEMATICS

Paper 1

2 hours 40 minutes

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in Section A.*

*Answer only **four** questions from Section **B** with **at least one** question from each part.*

Any additional question(s) answered will not be marked.

*Each question in Section **A** carries **5** marks while each question in Section **B** carries **15** marks.*

*All necessary working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper..

Graph paper is provided.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be provided.

SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. Given that $\sqrt{6} = 2.4495$, $\sqrt{3} = 1.7321$ and $\sqrt{2} = 1.4142$, without using a calculator, evaluate $\frac{\sqrt{6}-\sqrt{3}}{\sqrt{3}+\sqrt{2}}$ to two significant figures; *(05 marks)*

2. Events M and N are independent such that $P(M) = \frac{1}{5}$ and $P(N) = \frac{2}{3}$; find:

   (i) $P(M \cap N)$; *(05 marks)*

   (ii) $P(M \cup N)$.

3. Evaluate $\int_1^3 \frac{2x^2+4}{x} dx$ *(05 marks)*

4. The data below was obtained in a survey to establish the age distribution of teachers in secondary schools in a district.

Age (years)	20-29	30-39	40-49	50-59	60-69
Number of teachers	20	61	35	29	5

   Draw the histogram. Hence estimate the mode *(05 marks)*

5. Given that the matrix $A = \begin{pmatrix} 3x & x-6 \\ -6 & x+2 \end{pmatrix}$ is singular, find the values of x. *(05 marks)*

6. A discrete random variable X has a probability distribution given below

x	1	2	3	4	5
$P(X = x)$	a	$\frac{3}{16}$	$\frac{1}{8}$	$3a$	$\frac{3}{2}a$

   (i) Find the value of a;

   (ii) Calculate $P(X < 3)$. *(05 marks)*

7. Find the values of θ in the equation: $3 - 3\sin\theta = 2\cos^2\theta$ for $0^0 \leq \theta \leq 360$. *(05 marks)*

8. The table below shows the animal production of milk in thousands of liters,from a dairy farm for the period 2010 to 2017.

Year	2010	2011	2012	2013	2014	2015	2016	2017
Animal production	180	195	160	170	165	155	145	160

   Construct the 4-point moving averages for the data. *(05 marks)*

SECTION B (60 MARKS)

*Answer only **four** questions from this section; with at least **one** question from each part. **All** questions carry **equal** marks.*

PART ONE: PURE MATHEMATICS

9. (a) The sum of the third and eighth terms of an arithmetic progression (AP) is 46 and the sum of the fifth and ninth terms is 58. Find the:
   (i) first term and common difference;
   (ii) sum of the first fourteen terms of the progression. *(08 marks)*

   (b) A woman makes baskets of different sizes, a basket of smaller size costs shs.1,000. The price difference between any two baskets of consecutive sizes is shs.500.
   (i) If she makes baskets whose sizes are in ascending order, how many baskets should she make to earn a total of shs.45,000?
   (ii) What would be the average price for all the baskets sold in b(i) above. *(07 marks)*

10. A manager of a certain hotel wishes to transport 870 crates of beer from Nile Breweries. He has to hire a lorry which can carry 150 crates at a time and a pick-up truck which can carry 60 crates at a time. The cost of each journey for a lorryisshs.250,000 and for the pick-up is shs.200,000. The pick-up makes more journeys than a lorry because it travels faster. The amount of money available for transportation of beer is shs.2,200,000.
   (a) Write down four inequalities representing the above information.
   (b) Plot the graph for the inequalities, shading out the unwanted region.
   (c) How many journeys should the lorry and pick-up make so as to keep the transport cost as low as possible. State how much money the manager saves by making these journeys *(15 marks)*

11. (a) The points P, Q, and R have position vectors $12\mathbf{i} + 16\mathbf{j}$, $3\mathbf{i} - 10\mathbf{j}$ and $5\mathbf{i} + \mathbf{j}$ respectively. Determine:
   (i) $|\overrightarrow{QR}|$
   (ii) $|\overrightarrow{PR}|$

(iii) the angle between $\overrightarrow{QR}$ and $\overrightarrow{PR}$ *(08 marks)*

(b) Given the points A (-4, -1), B(2, 5), and C(r, -7).Using vectors, find the value of r such that $\dot{AC}$ is perpendicular to $\dot{AB}$ *(07 marks)*

12. (a) Solve the differential equation $\frac{dy}{dx} = x(x^2 + 1)$ when $x = 2$ and $y = 8$. *(05 marks)*

(b) A certain culture of bacteria grows at a rate proportional to its size. If the size doubles in 4 days, find the time required for the culture to increase 10 times to its original size. *(10 marks)*

PART TWO: STATISTICS

13. A continuous random variable X has the probability density function (pdf) given by

$$f(x) = \begin{cases} kx^2(3-x); & 0 \leq x \leq 3 \\ 0 & ; \text{elsewhere} \end{cases}$$

where k is a constant.
Determine the:

(i) value of k; *(03 marks)*

(ii) $P(X \leq 3)$; *(03 marks)*

(iii) expected value of, E(X); *(04 marks)*

(iv) variance of X. *(05 marks)*

14. The points scored by eight schools 1 to 8, in Dance and Music competitions were:

School	1	2	3	4	5	6	7	8
Dance(x)	50	27	27	11	50	20	27	25
Music(y)	120	130	132	150	121	140	130	136

(a) (i) Draw a scatter diagram and use it to compare the school's performance in the Dance and Music.

(ii) Draw the line of best fit and use it to estimate the value of x when $y = 145$. *(07 marks)*

(b) (i) Calculate the rank correlation coefficient for the data.

(ii) Comment on your result. *(08 marks)*

15. (a) A binomial distribution $B(n, p)$ has a mean of 8 and variance 4. Find the values of n, p and q. *(08 marks)*

(b) In a bag containing balls, twenty percent of the balls are yellow. Find the probability that in a random sample of eight balls:

(i) exactly two are yellow;
(ii) at least two balls are yellow. *(07 marks)*

16. The prices (per kilogram) of Sugar, Soya, G'nuts and Rice in the years 2017 and 2018 as shown in the table below:

ITEM	PRICE		WEIGHT
	2017	2018	
Sugar	2500	3500	15
Soya	1000	1500	10
G'nuts	5000	7000	6
Rice	3000	5000	2

Taking 2017 as the base year, calculate the:

(i) price index for each item; *(04 marks)*

(ii) simple aggregate price index; *(03 marks)*

(iii) weighted average price index; *(04 marks)*

(iv) weight aggregate price index. *(04 marks)*

END