**P425/1**
**PURE MATHEMATICS**
**Paper 1**
**March 2024**
**3 Hours**

# Uganda Advanced Certificate of Education

PURE MATHEMATICS

**PAPER 1**

**3 Hours**

## INSTRUCTIONS TO CANDIDATES:

*Attempt all the* ***eight*** *questions in section* ***A*** *and any* ***five*** *from section* ***B****.*

*All working must be clearly shown.*

*Mathematical tables with a list of formulae and squared paper are provided.*

*Silent, simple non programmable scientific calculators and a list of formulae may be used.*

*State the degree of accuracy at the end of each answer using* ***CAL*** *for calculator and* ***TAB*** *for tables.*

## SECTION A (40 Marks)

**Attempt ALL questions in this section.**

**1.** Solve the inequality $\frac{1+x}{4+x} \geq \frac{5-2x}{x}$ *(05 marks)*

**2.** Using the substitution $t = \tan x$, find $\int \frac{1}{1+\sin 2x} dx$ *(05 marks)*

**3.** Solve the equation $2tan\theta + sin2\theta sec\theta = 1 + sec\theta$ for $0 \leq \theta \leq 2\pi.$ *(05 marks)*

**4.** Find the locus of a point $P$ which moves such that its distance from the point $A(1, 2)$ is equidistant to the point $B(0.1)$. *(05 marks)*

**5.** Expand $(25 - 2x)^{\frac{1}{2}}$ in ascending powers of $x$ up to the term in $x^3$. Hence by taking $x = 1$, obtain the value of $\sqrt{23}$ correct to four significant figures. *(05 marks)*

**6.** If $y = e^{2x} sin2x$, show that $\frac{d^2y}{dx^2} = 8(2e^{2x} cos^2 x - 1).$ *(05 marks)*

**7.** Find the area bounded by the curve $x = y^2 - 4$ and the y – axis. *(05 marks)*

**8.** A line $r = \begin{pmatrix} 5 \\ 5 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$ intersects the plane $2x + y + 3z = 16$ at $A$. Find the coordinates of $A$. *(05 marks)*

## SECTION B (60 marks)

*Answer **any five** in this section*

**9.** (a) Express the complex numbers $z_1 = 1 - 4i$ and $z_2 = 2 + i$ in the polar form Hence find $z_1(z_2)^2$. *(04 marks)*

(b) Given that $z = 1 + i$ is a root of $z^4 + 3z^2 - 6z + 10 = 0$. Determine the remaining three roots of the polynomial. *(04 marks)*

(c) Simplify to the form $\boldsymbol{a} + \boldsymbol{bi}$ if $\boldsymbol{P} = \dfrac{\left(\boldsymbol{cos\frac{2\pi}{5} + isin\frac{2\pi}{5}}\right)^{8}}{\left(\boldsymbol{cos\frac{3\pi}{5} - isin\frac{3\pi}{5}}\right)^{3}}$ *(04 marks)*

**10.** (a) The polynomial $\boldsymbol{f(x)}$ leaves a remainder of $\boldsymbol{3}$ when divided by $\boldsymbol{x+3}$ and a remainder of $\boldsymbol{18}$ when divided by $\boldsymbol{x-2}$. Find the remainder when $\boldsymbol{f(x)}$ is divided by $\boldsymbol{x^2 + x - 6.}$ *(06 marks)*

(b) The roots of the equation $\boldsymbol{25x^2 + x + 1 = 0}$ are $\boldsymbol{\alpha^2}$ and $\boldsymbol{\beta^2}$ . Find the equation with integral coefficients whose roots are $\frac{1}{\alpha}$ *and* $\frac{1}{\beta}$ .

*(06 marks)*

**11.** (a) Show that $\boldsymbol{\dfrac{cos6\theta + cos10\theta}{sin10\theta - sin6\theta} = cot2\theta}$ Hence solve for $\boldsymbol{\theta}$ if $\boldsymbol{\dfrac{cos6\theta + cos10\theta}{sin10\theta - sin6\theta}} = \dfrac{1}{\sqrt{3}}$ for $\boldsymbol{-180^o \leq \theta \leq 360^o}$ *(07 marks)*

(b) Solve the equation $\boldsymbol{5sin\theta cos\theta - 6cos2\theta = 2}$ for $\boldsymbol{0^o \leq \theta \leq 360^o}$ *(05 marks)*

**12.** (a) Show that the curve $\boldsymbol{y = \dfrac{2x-3}{x^2+2x-3}}$ does not exist in the range $\boldsymbol{\frac{1}{4} < y < 1}$ *(04 marks)*

(b) Sketch the above curve by stating the turning points and asymptotes as well. *(08 marks)*

**13.** (a) The first term of an arithmetic progression is $\boldsymbol{73}$ and the ninth is $\boldsymbol{25}$. Find:

(i) common difference

(ii) the number of terms that must be added to give the sum of 96. *(06 marks)*

(b) A geometrical progression has first term as $\mathbf{15}$ and sum to infinity as $\mathbf{225}$.

Find the: (i) the common ratio

(ii) sum of the first ten terms. *(06 marks)*

**14.** (a) Differentiate with respect to x.

(i) $\boldsymbol{3x^x}$ (ii) $\cos^2 3x$ *(07 marks)*

(b) Find the equation of the normal and tangent to the curve.

$\boldsymbol{xy}^3 - \boldsymbol{2x}^2\boldsymbol{y}^2 + \boldsymbol{x}^4 - 1 = 0$ at a point $\boldsymbol{P(1,2)}$. *(05 marks)*

**15.** (a) Evaluate:

(i) $\int_0^4 \frac{dx}{x+\sqrt{x}}$. *(04 marks)*

(ii) $\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) dx.$ *(03 marks)*

(b) Given that $\boldsymbol{x} = \frac{3t-1}{t}$ and $\boldsymbol{y} = \frac{t^2+4}{t}$, show that $\frac{d^2y}{dx^2} = \mathbf{2}\boldsymbol{t}^3$ *(05 marks)*

**16.** Express $\boldsymbol{f(x)} = \frac{x^4-2x^3-x^2-4x+4}{(x-3)(x^2+1)}$ into partial fractions. Hence find $\int \boldsymbol{f(x)}\ \boldsymbol{dx}.$

*(12 marks)*

**END**