Formula:

The orbital speed of the satellite around the earth is given by

$$v=\sqrt{\frac{GM}{r}}$$

Where 'r' is the total distance from the centre of earth.

$$r=R+r_e$$

$$r=6400+900=7300km$$

$$r=7300\times10^3 m=7.3\times10^6$$

Substituting all the values we get

$$v=\sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{7.3\times10^6}}$$

$$v=\sqrt{\frac{4.002\times10^{13}}{7.3\times10^6}}$$

$$v=\sqrt{54.8\times10^6}$$

$$v=7.4\times10^3 m/s$$

$$\boxed{v=7.4\,km/s}$$

Unit 06

FLUID DYNAMICS

Q.1 Define the terms fluid dynamics, fluid and viscosity.

Ans. Fluid Dynamics: 11106001

"Fluid dynamics is the study of the behaviour of the fluids when they are moving".

Upto moderate velocities of the fluid, the fluid can be imagined in the form of thin layers. When fluid is flowing then it means its one layer is sliding over its other layer.

Fluids:

"The substances having the property to flow are known as fluids." The fluids are classified into: i. Gases ii. Liquids

Viscosity:

"Viscosity is a measure of the force needed to slide one layer of the fluid over its other layer".

Substances that do not flow easily like honey and thick tar have greater viscosity, and the substances that flow easily like water; petrol and air have smaller viscosity.

Q.2 Define Drag Force. What are the factors of its dependence? 11106002

Ans. Drag Force or Viscous Drag:

The opposing force offered by the fluid to the motion of the body moving in that fluid is known as drag force or viscous drag.

Factors of Dependence of Drag Force:

Drag force depends on the following factors:

1. **Viscosity of the fluid (η):**
   Greater is the viscosity; greater will be the drag force.
2. **Velocity of the body (v):**
   Greater is the velocity of the body in the fluid, greater is the drag force.
3. **Size of the body:**
   Greater size implies greater drag force and vice-versa.

Q.3 State the Stoke's law. 11106003

Ans. Stoke's Law:

"For a spherical body of radius 'r' moving slowly with velocity 'v' in a fluid of viscosity 'η' the drag force on the spherical body is given by the following formula

$$F=6\pi\eta rv$$

This equation is known as Stoke's Law because it was formulated by Stoke."

It may be remembered that at high speeds the drag force is not proportional to speed of the body.

Q.4 Define Terminal Velocity of body and show that terminal velocity is d proportional to the square of radius of body.

Ans. Terminal Velocity:

Definition:

"When an object is moving in a fluid then its maximum and constant velocity in that known as terminal velocity of the object". An object achieves terminal velocity when its becomes equal to the drag force of the viscous medium in which it is moving."

Explanation:

Consider a spherical water droplet like that of fog slowly falling vertically down. Tw will be acting on it:

1. Weight force of droplet = mg   2. Drag force of air = $6\pi\eta rv$

The net force on the droplet in the vertically downward direction is:

Net force = Weight – Drag force

$\Rightarrow F = mg - 6\pi\eta rv$ .... (1)

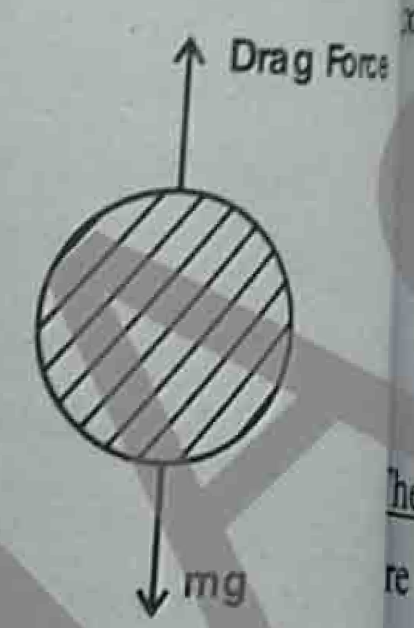
Because of its weight the downward velocity of the droplet increases which impl increase in the drag force. Ultimately a stage comes when drag force becomes equal to the weig this state the net force on the droplet becomes zero and hence no further increase in veloc droplet occurs. So this velocity is the maximum and constant velocity known as **terminal** ve 'v_t' of the droplet.

Thus above equation (1) becomes as written below:

$$0 = mg - 6\pi\eta rv_t$$

$$\Rightarrow 6\pi\eta rv_t = mg$$

$$\Rightarrow \boxed{v_t = \frac{mg}{6\pi\eta r}} \text{ --------------- (2)}$$



If $\frac{g}{6\pi\eta r}$ = constant then v_t = constant × m

$$\therefore \quad v_t \propto m$$

This shows terminal velocity is directly proportional to mass of the spherical droplet.

This formula can also be written in an other form as below:

Since $\quad m = \text{density} \times \text{volume}$

$$\Rightarrow \quad m = \rho \times \frac{4}{3}\pi r^3$$

Here ρ is the density of the droplet.

$$\therefore \quad m = \frac{4\rho\pi r^3}{3}$$

Substituting this value in above equation (2) we get

$$v_t = \frac{4\rho\pi r^3}{3} \times \frac{g}{6\pi\eta r}$$

$$\Rightarrow \quad \boxed{v_t = \frac{2\rho g r^2}{9\eta}} \text{ --------------- (3)}$$

Here $\frac{2\rho g}{9\eta}$ = constant

$$\boxed{v_t = \text{constant} \times r^2}$$

$$\therefore \quad v_t \propto r^2$$

This shows that terminal velocity is directly proportional to the square of the radius of the pherical droplet. Equations (2) and (3) are the formulas for the terminal velocity of a spherical bject should the particle be falling in vacuum, the terminal velocity will be infinite because for acuum $\eta = 0$.

Q5 Differentiate between stream line flow and turbulent flow. 11106005

Ans. Types of fluid flow:

Motion of fluid is of two types:-

Streamline flow or steady flow or laminar flow or regular flow.

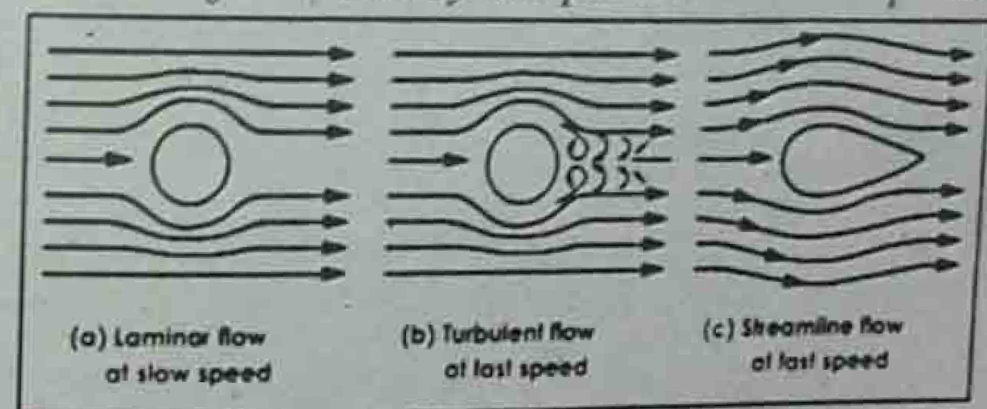
Turbulent flow or irregular flow.

Streamline Flow:

Definition:

"The flow is said to be streamline if every proceeding particle follows the motion of the ucceeding particle". OR

"The flow is set to be streamline flow if every particle that passes a particular point moves xactly along the same path as was followed by the particles which passed that point earlier".



he smooth path along which the fluid particles move is called streamline. Few of the streamlines re shown in figure above.

bove a certain velocity called "critical velocity", the streamline flow switches to turbulent flow. he value of critical velocity is different for different fluids.

haracteristics of Streamlines:

ollowing are the characteristics of streamlines:

- No two different streamlines cross each other.
- The direction of tangent at any point of the streamline is the direction of motion of the fluid at that point.
- Streamlines are closer where the fluid speed is high, and streamlines are farther apart where the fluid speed is low.

urbulent Flow:

"The irregular and unsteady flow of the fluid is called Turbulent flow."

Above certain velocity of the fluid the particles do not follow the motion of the other rticles. This type of flow is known as irregular flow or turbulent flow.

Q.6 What is an ideal fluid? State and prove equation of continuity. (Board 2010)

Ans. Characteristics of Turbulent Flow:

Ideal Fluid:

Definition:

"The fluid which is non-viscous and incompressible is known as ideal fluid wh… definition non-viscous fluid has zero viscosity and incompressible fluid has constant density".

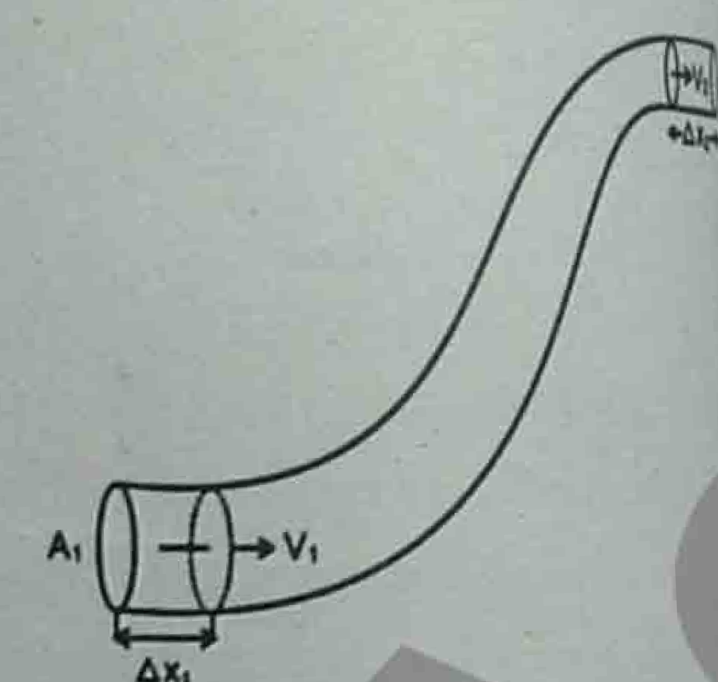
Equation of Continuity: **(Board 2010)**

Statement:

The product of cross sectional area of the pipe and the fluid speed at any point along th… is constant. This constant equals the volume flow per second of the fluid or simply the flow rat…

Explanation:

Let A_1, V_1, and ρ_1 are area of cross-section, uniform velocity and density of the fluid at the lower end respectively. Similarly let A_2, V_2 and ρ_2 are area of cross-section, uniform velocity and density of the fluid at the upper end respectively of the pipe as shown in figure. The pipe is of non-uniform area of cross section.



Suppose distances covered by the fluid in the lower and upper ends of the pipe are Δ… Δx_2 during Δt time.

Where by using the formula (s = vt) we have

$$\Delta x_1 = v_1.\Delta t$$
$$\Delta x_2 = v_2.\Delta t$$

Volume of the fluid entered through the lower end of the pipe is the volume of the pipe shown… bottom.

$$\therefore \quad V_1 = A_1\Delta x_1$$
$$V_1 = A_1 v_1 \Delta t$$

Now Δm_1 = mass entered in time Δt

$$\Delta m_1 = \text{density} \times \text{volume}$$
$$\Delta m_1 = \rho_1 A_1 v_1 \Delta t \quad \text{-----------------------} (1)$$

Similarly volume of the fluid flown out of the upper end of the pipe is again volume of th… shown at the top.

$$\therefore \; V_2 = A_2 \Delta x_2$$
$$V_2 = A_2 v_2 \Delta t$$

Thus Δm_2 = mass flown out in time Δt

$$\Delta m_2 = \text{density} \times \text{volume}$$
$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t \quad \text{-----------------------} (2)$$

Using law of conservation of mass for streamline flow we have

$$\Delta m_1 = \Delta m_2$$

Put values from equations (1) and (2)

$$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$
$$\Rightarrow \boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

This equation is called equation of continuity.

During streamline flow the fluid is incompressible then its density remains constant. So we can put $\rho_1 = \rho_2 = \rho$ in above equation.

$$\therefore \; \rho A_1 v_1 = \rho A_2 v_2$$
$$\boxed{A_1 v_1 = A_2 v_2} \quad \text{-------------------} (3)$$

OR

$$AV = \text{constant}$$

This equation is also called the equation of continuity applicable for steady flow of an ideal flow. In words this equation can be stated as below:

The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is constant. This constant equals the volume flow per second of the fluid or simply the flow rate. In other words we can say that "the rate of flow of an ideal fluid during streamline flow remains constant".

Conclusion:

From equation (3) we can write

$$\frac{v_1}{v_2} = \frac{A_2}{A_1}$$

Therefore the velocity of fluid and area of the cross section of pipe are inversely proportional to each other. So speed of fluid is high where its area of cross-section is small and vice-versa.

11106007

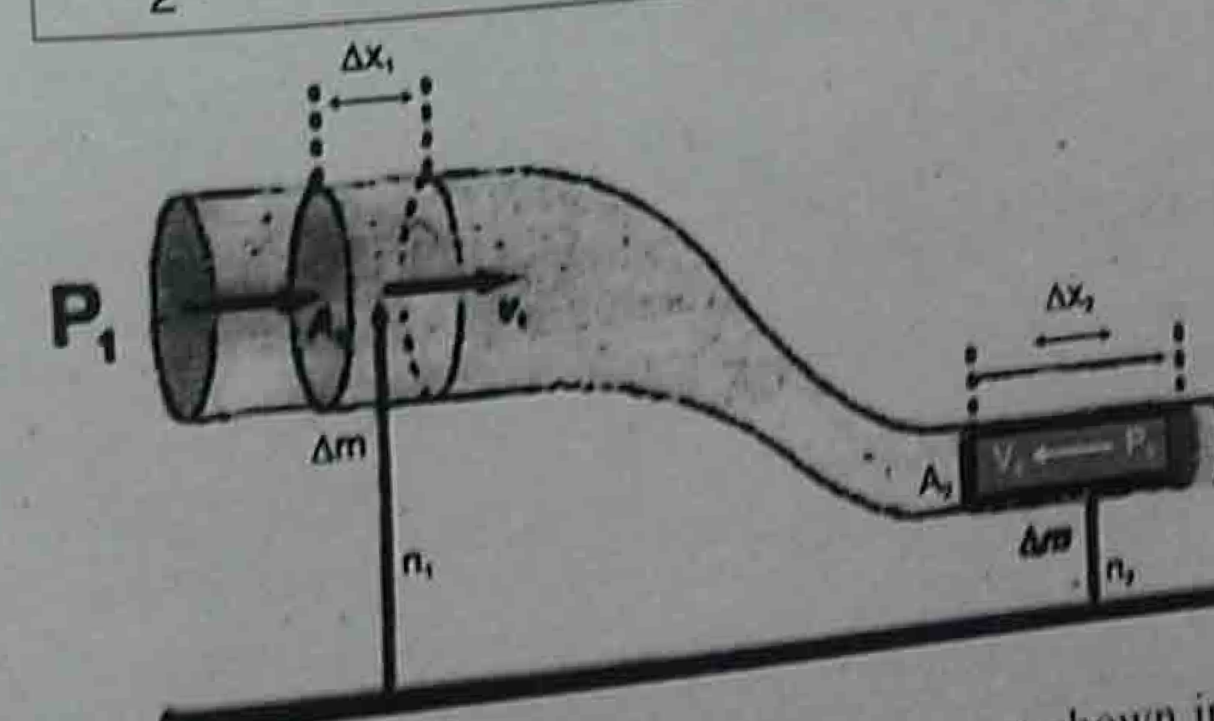
Q.7 State and prove Bernoulli's Equation.

Ans. Bernoulli's Equation:

Statement:

"The sum of pressure, K.E per unit volume and P.E per unit volume of an ideal fluid during streamline flow remains constant."

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$



Explanation:

Consider an ideal fluid in a streamline flow in the pipe as shown in fig. Let A_1 is the area of cross-section of the top end of the pipe whose height is h_1 from the reference surface. The uniform fluid speed at the top end is v_1. Suppose the pressure P_1 pushes the fluid inside through a dis Δx_1. Therefore, work done on the fluid in pushing it into the pipe through distance Δx_1 is

$$W_1 = \vec{F}_1 . \Delta \vec{x}_1$$

$$= F_1 \Delta x_1 \cos 0^\circ$$

Since $\cos 0^\circ = 1$

$\therefore \quad W_1 = F_1 \Delta x$

Where $\Delta x_1 = v_1 t$ and $F_1 = P_1 A_1$

Therefore $W_1 = P_1 A_1 v_1 t$ ---------------- (1)

Now suppose A_2 is the area of cross-section of the lower end where speed of the fluid is The fluid already present at the lower end exerts pressure P_2 on the fluid coming from the upside the fluid coming from the upside travels a distance of Δx_2 in a time t in the presence of the inw pressure P_2. So work done by the fluid at the lower end on the fluid coming from the upside is

$$W_2 = \vec{F}_2 . \Delta \vec{x}_2$$

$\Rightarrow \quad W_2 = F_2 \Delta x_2 \cos 180^\circ$

Where $\cos 180^\circ = -1$, $\Delta x_2 = v_2 t$ and $F_2 = P_2 A_2$

Therefore, $W_2 = -P_2 A_2 v_2 t$ ---------------- (2)

By law of conservation of mechanical energy, the total work done on the fluid as it pa from top to the bottom is equal to the change in K.E plus change in P.E of the fluid.

Therefore $W = \Delta(K.E) + \Delta(P.E)$

$$\Rightarrow \quad W_1 + W_2 = \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + (mgh_2 - mgh_1)$$

Put values from equations (1) and (2) in left hand side we get

$$P_1 A_1 v_1 t - P_2 A_2 v_2 t = \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + (mgh_2 - mgh_1)$$

Where

$A_1 v_1 t = A_2 v_2 t =$ Volume of fluid flown in time $t = V$

$$P_1 v - P_2 v = \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + (mgh_2 - mgh_1)$$

$$v(P_1 - P_2) = \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + (mgh_2 - mgh_1)$$

Dividing both sides by 'V' (volume):

$$P_1 - P_2 = \frac{1}{2}\frac{m}{V}v_2^2 - \frac{1}{2}\frac{m}{V}v_1^2 + \frac{m}{V}gh_2 - \frac{m}{V}gh_1$$

Here by definition $\frac{m}{V} = \rho$

Therefore $P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2} \quad \text{(3)}$$

OR it can be written as:

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}} \quad \text{(4)}$$

The equation (3) in the box is known as Bernoulli's equation in fluid dynamics. Eq. (4) is its in general form.

Q.8 Describe the following applications of Bernoulli's Equation. 11106008

(i) Torricelli's Theorem (ii) Relation between speed and pressure

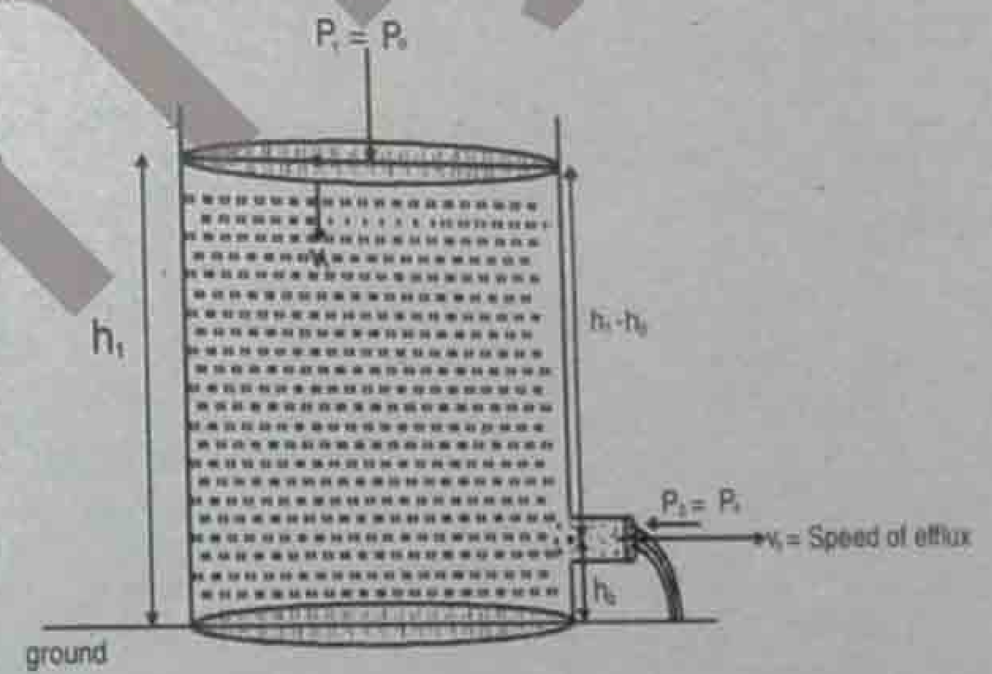
(iii) Venturi Relation

Ans. Application of Bernoulli's Equation:

(i) Torricelli's theorem:

Statement:

"The speed of efflux of the fluid is equal to the velocity gained by it as it falls through the height 'h' under the action of gravity".



$$V_{eff} = \sqrt{2gh}$$

Where $h = h_1 - h_2$

$$V_{eff} = \sqrt{2g(h_1 - h_2)}$$

Proof:

Consider a fluid in a tank with small hole in its wall as shown in figure. Let A_1 is the area of Cross-section of the top face of tank and A_2 is the area of cross-section of hole or orifice. Using Bernoulli's equation we have:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

In our situation we have $P_1 = P_2 = P_o$

Therefore $P_o + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_o + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$$\Rightarrow \quad \frac{1}{2}\rho v_1^2 + \rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Rightarrow \quad \rho g h_1 - \rho g h_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\Rightarrow \quad \rho g(h_1 - h_2) = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

As $A_1 >> A_2$ which implies $v_1 << v_2$, so we can neglect v_1^2 in the right hand side of the above equation.

Therefore $g(h_1 - h_2) = \frac{1}{2}v_2^2$

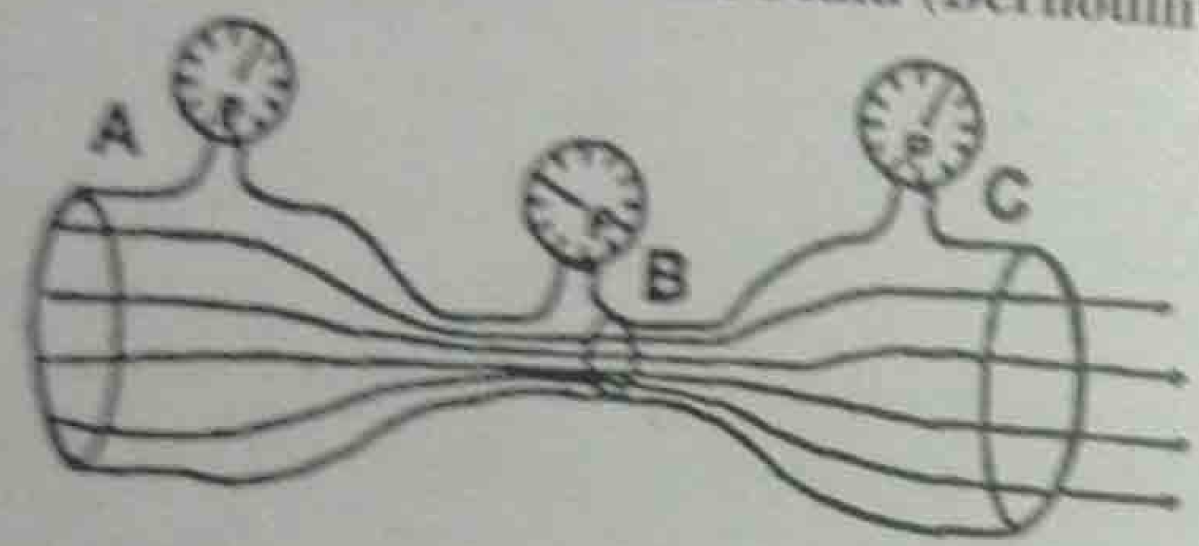
$$\Rightarrow v_2^2 = 2g(h_1 - h_2)$$

Take square root on both sides we get

$$\boxed{v_2 = \sqrt{2g(h_1 - h_2)}}$$

This is the proof of Torricelli's theorem.

2. Relation Between Speed and Pressure of the Fluid (Bernoulli's Effect):



Consider a horizontal pipe as shown in figure. The pipe is constricted at the middle. The velocities of the fluid are v_A and v_B where the areas of cross-section of the pipe are A_1 and respectively.

Using Bernoulli's equation and note that the average P.E. is the same at both places, we have

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_1 = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_2$$

As $\rho g h_1 = \rho g h_2$

$$\therefore \quad P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

Let $v_A = 0.20\ ms^{-1}$; $v_B = 2.0\ ms^{-1}$ and $\rho = 1000\ Kgm^{-3}$

We get $\quad P_A - P_B = 1980\ Nm^{-2}$

This shows that the pressure in the narrow pipe where streamlines are closer together is smaller than in the wider pipe. Thus, where the speed is high, the pressure will be low.

Examples of Bernoulli's Effect are:

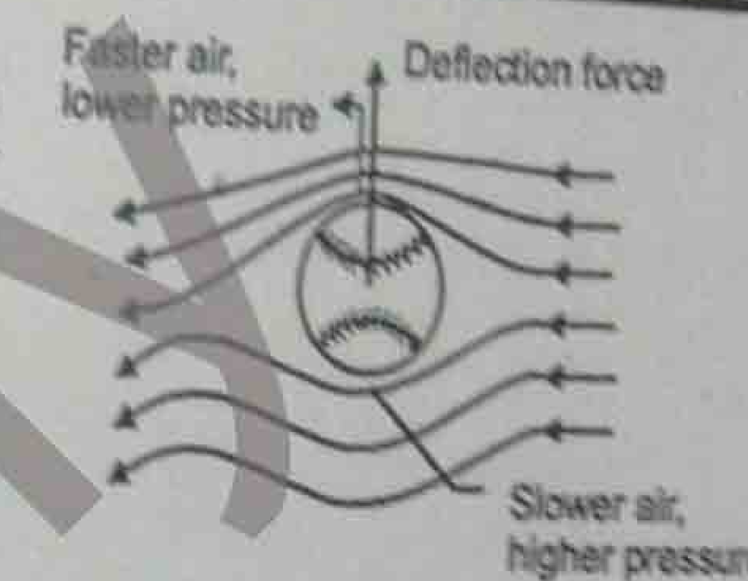
Example 1:

The aerodynamic lift on an aeroplane is due to Bernoulli's effect. The flow of air around aeroplane wing is shown in fig. The wing is designed such that velocity of air at the top is greater than at the bottom of wing.

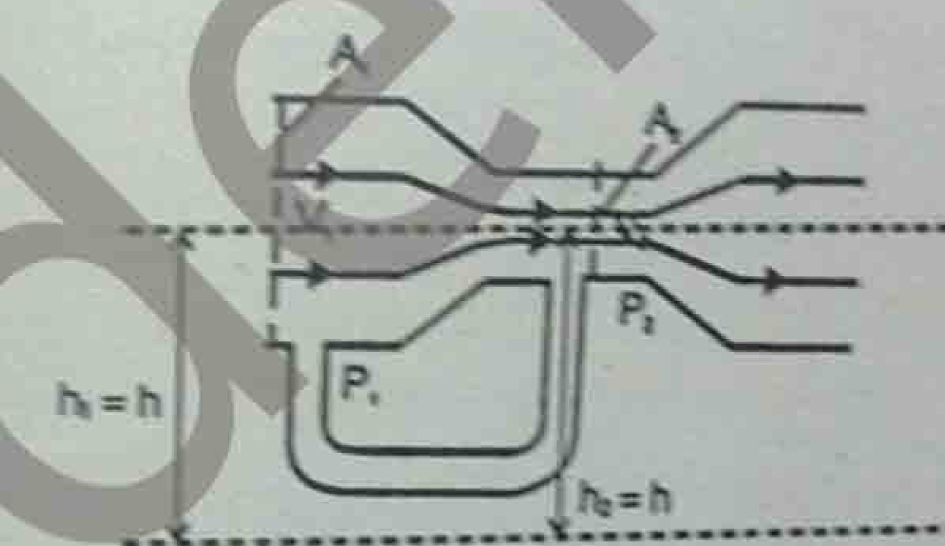
Therefore, the pressure will be lower at the top of the wing than at the bottom. So wing will be forced upward and hence the aeroplane.

Example 2:

When a Tennis ball is struck by a racket in such a way that it spins as well as moves forward, the velocity of the air on one side of the ball increases than on its other side due to spin. This will make the pressure of air to increase on the side of ball where speed of air is low. Thus the ball along with forward motion will deflect from higher to the side of lower pressure.



3. Venturi relation:



Consider the pipe of smaller area of cross section connected with a pipe of bigger cross-section as shown in above figure. This arrangement can be used for the measurement of the speed of the fluid and is known as venturi meter.

Using Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

As $\quad h_1 = h_2 = h$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h$$

$$\therefore \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2)$$

As $\quad A_1 >> A_2$

$\therefore \quad v_1 << v_2$

So we can neglect v_1^2 in the above equation.

Therefore $\boxed{P_1 - P_2 = \frac{1}{2}\rho\ v_2^2}$

This equation is known as Venturi's Relation, which may also be written as below:

$$v_2^2 = \frac{2(P_1 - P_2)}{\rho}$$

Taking square root on both sides we get

$$\boxed{v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}}}$$

Since density of an ideal fluid is constant, so

$$v_2 = \text{constant} \times \sqrt{(P_1 - P_2)}$$

Therefore, the velocity of the fluid through the venturi duct is directly proportional square root of pressure difference. The pressure difference (P_1 - P_2) is measured by the ma and hence the velocity of the fluid can be determined by the above equation in last box.

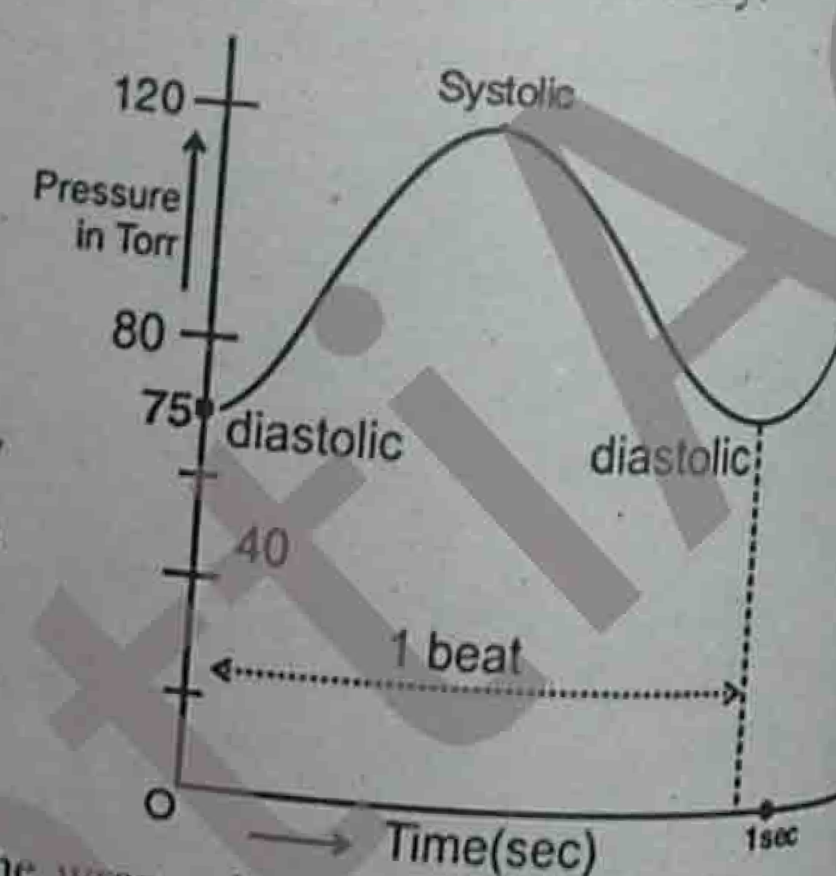
Q.9 Explain the blood flow in the human body. Describe working of sphygmoman used for measuring upper and lower limits of blood pressure.

Ans. Blood Flow and its Measurement:

Density of human blood is nearly equal to the density of water but high concentratio 50% of red cells makes its viscosity 3 to 5 times greater than that of water. Arteries or vessels b like stretch rubber pipes. Pressure of the blood in human body for its circulation is always g than atmospheric pressure. There are two limits of pressure. Lower limit is known as di pressure. It varies for normal human body from 75 torr to 80 torr. Upper limit of the press known as systolic pressure and its value is 120 torr for normal and healthy human body.

Where $\boxed{1 \text{ torr} = 133.3 \text{ N m}^{-2}}$

- At systolic pressure the blood flow is turbulent.
- At diastolic pressure the flow of blood is streamline.



"The blood pressure measurement can be made by using an apparatus known "**sphygmomanometer**" as explain below:

Sphygmomanometer consists of inflatable bag, which is wrapped around the arm of patient. The external pressure on the arm is increased by inflating the wrapped bag through the air bulb att with the bag. For this purpose the air bulb is compressed and released for a few times. When ex pressure exceeds the systolic pressure the blood vessels constrict cutting off the flow of blood release valve provided with the air bulb is opened and external pressure decreases gradually.

The stethoscope detects the instant when external pressure becomes equal to sy pressure. At this point the surge of blood rush through the narrow stricture in the blood v produces a sound due to turbulent flow. Finally the external pressure becomes equal to dia pressure and flow become steady and sound in the stethoscope disappears. This is an indicati record diastolic pressure.

Short Questions

6.1: Explain what do you understand by the term viscosity? **(Board 2008,10,14) 11106010**

Ans. Property of the fluid due to which it internally resists its flow is known as viscosity. Viscosity measures how much force is required to slide one layer of the liquid over its other layer.

Viscosity depends upon the nature of fluid and on its temperature. Increase in temperature of the liquids decreases its viscosity. Its S.I units are Nsm^{-2}. Increase in temperature increases the viscosity of gases.

6.2: What is meant by the drag force, what are the factors upon which drag force acting upon a small sphere of radius r, moving down through a liquid depend? **11106011**

Ans. Definition:

"The opposing force on the body when it is moving through the fluid is known as drag force".

For a small sphere, it depends upon:

(1) Its velocity in the fluid
(2) Viscosity of the fluid
(3) Radius of the sphere

By Stoke's law value of the drag force acting on a sphere is:

$$\boxed{F = 6\pi\eta r v}$$

6.3: Why fog droplets appear to be suspended in air? **(Board 2010,14) 11106012**

Ans. Fog droplets are particles of very small mass and very small radius. So they have very, very small terminal velocity v_t because $v_t \propto m$ and $v_t \propto r^2$. Hence due to their small terminal velocity the fog droplets appear to be suspended in air.

6.4: Explain the difference between laminar flow and turbulent flow? **(B. 2008, 2009) 11106013**

Ans. A flow of fluid is said to be laminar when every particle that passes a particular point moves exactly along the same path, followed by particles which passed that point earlier. And the irregular or unsteady flow of the fluid is called turbulent flow. The fluid particles do not move on definite path in this type of flow.

Also there is a definite flow pattern in a laminar flow while in turbulent flow there is a constantly changing flow pattern.

6.5: State Bernoulli's relation for a liquid in motion and describe some of its application? **(B. 2009) 11106014**

Ans. Its states that "sum of pressure, K.E. per unit volume, and P.E. per unit volume of an ideal fluid is constant". i.e.

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Its applications are

(1) Swing of cricket ball in the air.
(2) Aerodynamic lift on an aeroplane.
(3) Chimney

6.6: A person is standing near a fast moving train. Is there any danger that he fall towards it? **(B. 2008) 11106015**

Ans. Yes a person standing near a fast moving train has the danger of falling towards it. The speed of air between man and train is very high as compare to the speed of air behind the man. So according to Bernoulli's affect, the pressure of air between man and train is low as compared to the pressure of air behind the man. Hence greater pressure of air on the man from his rear side may push him towards the train.

6.7: Identify the correct answer according to Bernoulli's effect. **11106016**

Ans.

(i) Where the speed of the fluid is high the pressure will be low. **(True)**
(ii) Where the speed of the fluid is high the pressure is also high. **(False)**
(iii) This theorem is valid only for turbulent flow of the liquid. **(False)**

6.8: Two row boats moving parallel in the same direction are pulled towards each other. Explain. (Board 2010) 11106017

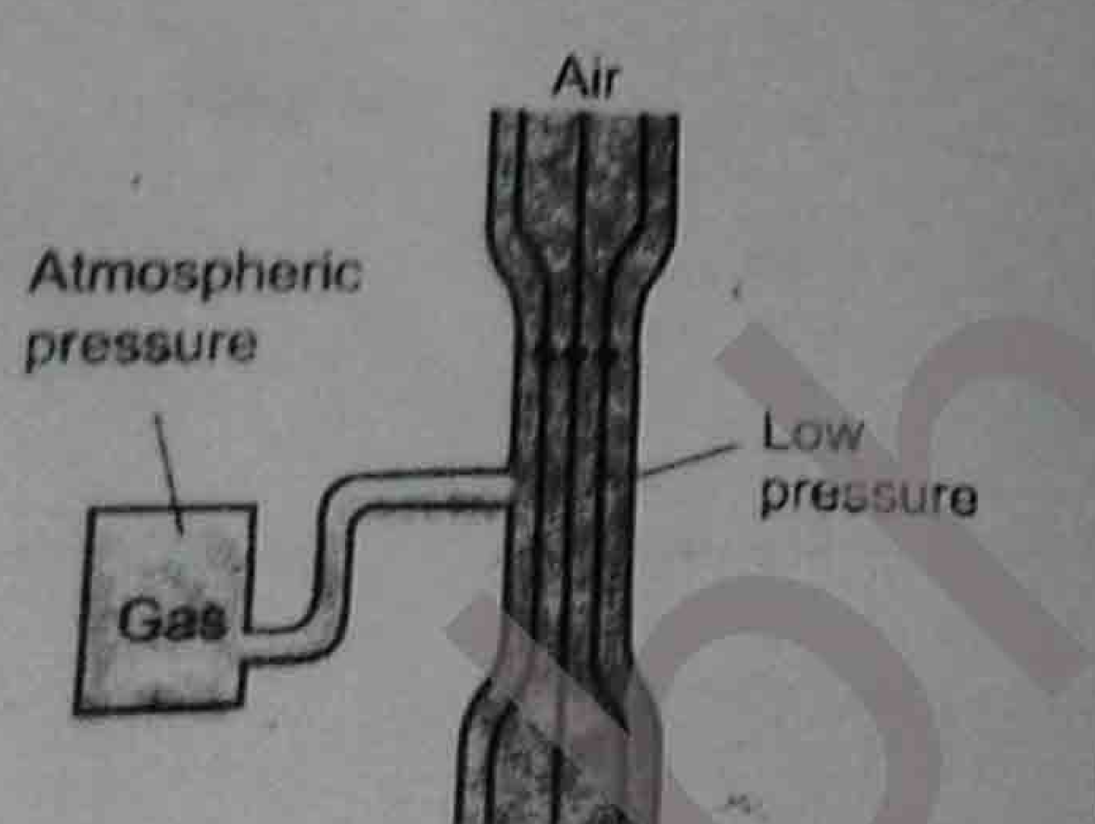
Ans. The water between the two boats has higher velocity than the velocity of water on their outer sides. Therefore according to Bernoulli's effect, the pressure of water present between the boats will be smaller than its pressure on the outer sides of the boats. Hence greater pressure of water on the outer sides of the boats pushes the boats towards each other, that is, the boats moving parallel in the same direction are pulled towards each other.

6.9: Explain how the swing is produced in a fast moving cricket ball. (Board 2009) 11106018

Ans. If the fast moving ball moves in the forward direction along with its spin motion then the speed of the stream lines of the air on the sides of the ball is different due to spin motion of the ball. Therefore, the pressure of air on one side of the ball will become greater than on its other side. This pressure difference will deviate the ball from straight path into a curved path known as swing in the cricket ball.

6.10: Explain the working of a carburetor of a motor car using Bernoulli's principle. 11106019

Ans. Carburetor of the car consists of a metallic pipe connected with a metallic pipe of narrow cross-section known as venturi duct. It is connected with a fuel tank. The pressure of the surface of the fuel in the tank is maintained equal to the atmospheric pressure. When air rushes into venturi duct the pressure will decrease in the duct. This will make fuel to flow from tank into the venturi duct in the form of vapours which mix with air. The mixture of petrol and air is known as carbureted air which enters into combustion chamber for its ignition.



6.11: For which position will the maximum blood pressure in the body has the smallest value: (a) standing up right (b) sitting (c) lying horizontally (d) standing on one's head? 11106020

Ans. (c) Lying horizontally, because each part of the body is at equal distance from the surface of earth and approximately at the level of heart.

6.12: In an orbiting space station would the blood pressure in major arteries in the leg ever be greater than the blood pressure in major arteries in the neck? 11106021

Ans. No, due to weightlessness, all parts will have same blood pressure. But if the artificial gravity is provided, then the blood pressure in major arteries of the legs can be greater than the blood pressure in major arteries in the neck. It happens when the spin frequency of the space station is such that $a_c > g$.

Solved Examples

Example 1: A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal velocity. Given that η for air $= 19 \times 10^{-6}$ kg m^{-1} s^{-1} and density of water $\rho = 1000$ kgm^{-3}. 11106022

Solution:

$$r = 1.0 \times 10^{-4}\ \text{m}\ ,\ \rho = 1000\ \text{kgm}^{-3}$$
$$\eta = 19 \times 10^{-6}\ \text{kg m}^{-1}\ \text{s}^{-1}$$

Putting the above values in Eq. $v_t = \dfrac{2gr^2\rho}{9\eta}$

$$v_t = \frac{2 \times 9.8\ \text{ms}^{-2} \times (1 \times 10^{-4}\ \text{m})^2 \times 1000\ \text{kgm}^{-3}}{9 \times 19 \times 10^{-6}\ \text{kgm}^{-1}\text{s}^{-1}}$$

We get Terminal velocity $= 1.1$ m s^{-1}

Example 2: A water hose with an internal diameter of 20 mm at the outlet discharges 30 kg of water in 60s. Calculate the water speed at the outlet. Assume the density of water is 1000 kgm^{-3} and its flow is steady. 11106023

Solution:

Mass flow per second $= \dfrac{30\text{kg}}{60\text{s}} = 0.5$ kgs

Cross sectional area $A = \pi r^2$

The mass of water discharging per second through area A is

$$\rho A v = \frac{\text{mass}}{\text{second}}$$

or

$$v = \frac{\text{mass}/\text{second}}{\rho A}$$

$$= \frac{0.5\ \text{kgs}^{-1}}{1000\ \text{kgm}^{-3} \times 3.14 \times (10 \times 10^{-3}\text{m})^2}$$

$$= 1.6\ \text{ms}^{-1}$$

Example 3: Water flows down hill through a closed vertical funnel. The flow speed at the top is 12.0 cms^{-1}. The flow speed at the bottom is twice the speed at the top. If the funnel is 40.0 cm long and the pressure at the top is 1.013×10^5 Nm^{-2}, what is the pressure at the bottom? 11106024

Solution: Using Bernoulli's equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Or

$$P_2 = P_1 + \rho g h + \frac{1}{2}\rho (v_1^2 - v_2^2)$$

Where $h = h_1 - h_2 =$ the length of the funnel

$$P_2 = (1.013 \times 10^5\ \text{Nm}^{-2}) + (1000\ \text{kgm}^{-3} \times 9.8\ \text{ms}^{-2} \times 0.4\text{m})$$
$$+ [\frac{1}{2}(1000\ \text{kgm}^{-3}) \times \{(0.12\ \text{ms}^{-1})^2 - (0.24\ \text{ms}^{-1})^2\}]$$
$$= 1.05 \times 10^5\ \text{N m}^{-2}$$

Numerical Problems

6.1: Certain globular protein particle has a density of 1246 Kgm^{-3} It falls through pure water ($\eta = 8.0 \times 10^{-4}\ N\ m^{-2}\ s$) with a terminal speed of 3.0 cm h^{-1}. Find the radius of the particle. 11106025

Data:

$\rho = 1246\ Kg^{-3}$

$\eta = 8.0 \times 10^{-4}\ N\ m^{-2}\ s$

$v_t = 3.0\ cm/h$

$= \dfrac{3.0 \times 10^{-2}}{3600}$

$v_t = 8.33 \times 10^{-6}\ m/s$

$r = ?$

Formula:

As we know that terminal velocity is

Since $v_t = \dfrac{2\rho g r^2}{9\eta}$

$\therefore \quad r^2 = \dfrac{9\eta v_t}{2\rho g}$

By taking square root on both sides:

$r = \sqrt{\dfrac{9\eta v_t}{2\rho g}}$

$r = \sqrt{\dfrac{9 \times 8 \times 10^{-4} \times 8.33 \times 10^{-6}}{2 \times 1246 \times 9.8}}$

Using calculator we get

$r = 1.56 \times 10^{-6}\ m$

Or $\boxed{r = 1.6 \times 10^{-6}\ m = 1.6\mu m}$

6.2: Water flows through a hose, whose internal diameter is 1 cm, at a speed of $1ms^{-1}$. What should be the diameter of the nozzle if the water is to emerge at $21ms^{-1}$? 11106026

Data:

$D_1 = 1\ cm = \dfrac{1}{100}\ m = 0.01\ m.$

$v_1 = 1\ m\ s^{-1}$

$D_2 = ?$

$v_2 = 21\ m\ s^{-1}$

Solution:

Formula:

We know that

$A_1 v_1 = A_2 v_2$

Since $A = \pi r^2 = \pi \left(\dfrac{D}{2}\right)^2$

$\therefore \quad \pi \left(\dfrac{D_1}{2}\right)^2 v_1 = \pi \left(\dfrac{D_2}{2}\right)^2 v_2$

$\Rightarrow \quad D_1^2 v_1 = D_2^2 v_2$

$D_2^2 = \dfrac{D_1^2 v_1}{v_2}$

Take square root on both sides.

$\sqrt{D_2^2} = D_1 \sqrt{\dfrac{v_1}{v_2}}$

$D_2 = D_1 \sqrt{\dfrac{v_1}{v_2}}$

$= 0.01 \times \sqrt{\dfrac{1}{21}}$

$D_2 = 0.002\ m$

$D_2 = 0.002 \times 100\ cm$

$\boxed{D_2 = 0.2\ cm}$

6.3: The pipe near the lower end of a la[rge] water storage tank develops a small le[ak] and a stream of water shoots from it. T[he] top of water in the tank is 15m above [the] point of leak. 11106027

(a) With what speed does the water ru[sh] from the hole?

(b) If the hole has an area of 0.060 cm^2 h[ow] much water flows out in one second?

Data:

$h = 15\ m$

$A = 0.060\ cm^2$

$= 0.060 \times 10^{-4}\ m^2$

$A = 6.0 \times 10^{-6}\ m^2$

$v = ?$

Rate of flow = ?

Solution (a):

By Torricelli's theorem we have

$v = \sqrt{2gh}$

$= \sqrt{2 \times 9.8 \times 15}$

$= \sqrt{294}$

$= 17.14\ m\ s^{-1}$

OR $\boxed{v = 17\ m\ s^{-1}}$

(b): We know that:

Rate of flow = Av

$= 6.0 \times 10^{-6} \times 17$

$= 1.02 \times 10^{-4}\ m^3\ s^{-1}$

$= 1.02 \times 10^{-4} \times 10^{6}\ cm^3\ s^{-1}$

$= 102\ cm^3\ s^{-1}$

So volume of water flown in one second is equal to 102 cm^3.

6.4: Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3m s^{-1} while at another point 3m higher, the speed is 4.0 m s^{-1}. If pressure is 80 kPa at the lower point, what is pressure at the upper end? 11106028

Data:

$v_1 = 3\ m\ s^{-1}$

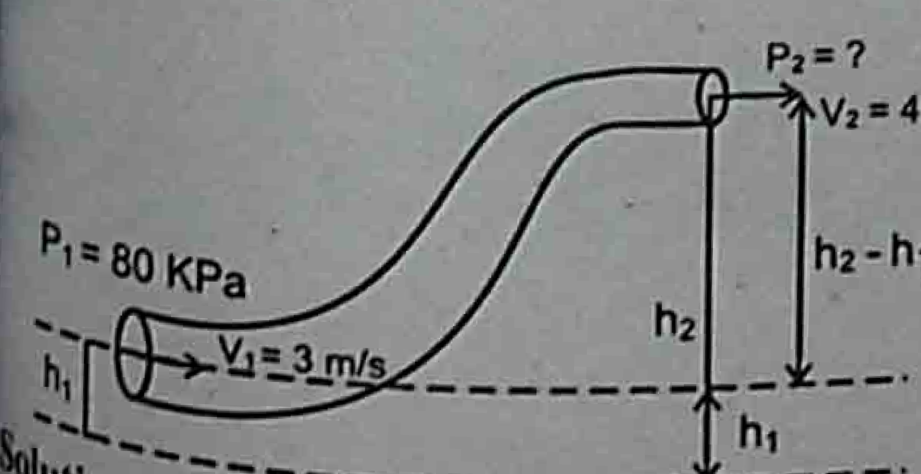
$P_1 = 80\ kPa = 80{,}000\ Pa$

$v_2 = 4\ m\ s^{-1}$

$h_2 - h_1 = 3m$

$\rho = 1000\ kg\ m^{-3}$

$P_2 = ?$



Solution:

Using Bernoulli's Equation

$P_1 + \dfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \dfrac{1}{2}\rho v_1^2 + \rho g h_2$

$\therefore \quad P_1 + \dfrac{1}{2}\rho v_1^2 + \rho g h_1 - \dfrac{1}{2}\rho v_2^2 - \rho g h_2 = P_2$

$P_1 + \dfrac{1}{2}\rho\left(v_1^2 - v_2^2\right) - \rho g\,(h_2 - h_1) = P_2$

$\Rightarrow 80{,}000 + \dfrac{1}{2} \times 1000(3^2 - 4^2) - 1000 \times 9.8 \times 3 = P_2$

Using calculator:

$80{,}000 - 3500 - 29400 = P_2$

$80{,}000 - 32900 = P_2$

$47100 = P_2$

$\boxed{P_2 = 47100 Pa}$

$\dfrac{47100}{1000}\ k\,Pa = P_2$

$P_2 = 47.1\ kPa$

Or $\boxed{P_2 = 47\ kPa}$

6.5: An air plane wing is designed so that when the speed of air across the top of the wing is 450 m s^{-1}, the speed of air below the wing is 410 m s^{-1}. What is the pressure difference between the top and bottom of the wing? (density of air = 1.29 kg m^{-3}). 11106029

Data:

$v_1 = 410\ m\ s^{-1}$

$v_2 = 450\ m\ s^{-1}$

$\rho = 1.29\ kgm^{-3}$

$P_1 - P_2 = ?$

Solution:

Using Bernoulli's effect we have

$P_1 + \dfrac{1}{2}\rho v_1^2 = P_2 + \dfrac{1}{2}\rho v_2^2$

$P_1 - P_2 = \dfrac{1}{2}\rho v_2^2 - \dfrac{1}{2}\rho v_1^2$

$= \dfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

$= \dfrac{1}{2} \times 1.29\ [(450)^2 - (410)^2]$

$= \dfrac{1}{2} \times 1.29\ [202500 - 168100]$

$= \dfrac{1}{2} \times 1.29 \times 34400$

$\boxed{P_1 - P_2 = 22188\ Pa}$

$= \dfrac{22188}{1000}\ kPa$

$= 22.188\ kPa$

$\boxed{P_1 - P_2 = 22\ kPa}$

6.6: The radius of the aorta is about 1.0 cm and the blood flowing through it has a speed of about 30 cms^{-1}. Calculate the average speed of the blood in the capillaries, using the fact that although each capillary has a diameter of about 8×10^{-4} cm, these are literally millions of them so that the total cross section is about 2000 cm^2. 11106030

Data:

$r_1 = 1\text{ cm} = 0.01\text{ m}$

$v_1 = 30\text{ cm s}^{-1} = \dfrac{30}{100}\text{ m s}^{-1} = 0.30\text{ m s}^{-1}$

$A_2 = 2000\text{ cm}^2 = 2000 \times 10^{-4}\text{ m}^2$

$A_2 = 0.2\text{ m}^2$

$v_2 = ?$

Solution: Using equation of continuity we have

$A_1v_1 = A_2v_2$

But $A_1 = \pi r_1^2$

$\therefore \quad \pi r_1^2 v_1 = A_2 v_2$

$\Rightarrow \quad v_2 = \dfrac{\pi r_1^2 v_1}{A_2}$

$= \dfrac{3.14 \times (0.01)^2}{2000 \times 10^{-4}} \times 0.30$

$= 4.7 \times 10^{-8} \times 10^4$

$\boxed{v_2 = 5.0 \times 10^{-4}\text{ m s}^{-1}}$

6.7: How large must a heating duct be if air moving 3.0 ms^{-1} along it can replenish the air in a room of 300 m^3 volumes every 15min? Assume the air's density remains constant. 11106031

Data:

$t = 15\text{min} = 15 \times 60\text{ s} = 900\text{ s}$

Volume of air $= V = 300\text{ m}^3$

$v = 3\text{ m s}^{-1}$

$r = ?$

Solution:

Formula:

We know that volume of fluid flown in t is:

$V = Avt$

$\Rightarrow \quad V = \pi r^2 vt$

$\Rightarrow \quad \dfrac{V}{\pi vt} = r^2$

Taking square root of both sides

$r = \sqrt{\dfrac{V}{\pi vt}}$

$= \sqrt{\dfrac{300}{3.14 \times 3 \times 900}}$

$= 0.19\text{ m}$

$= 0.19 \times 100\text{ cm}$

$\boxed{r = 19\text{ cm}}$

6.8: An airplane design calls for a "l... due to the net force of the moving air on ... wing of about 1000N m^{-2} of wing a... Assume that air flows past the wing of ... air craft with streamline flow. If the sp... of flow past the lower wing surface ... $160ms^{-1}$, what is the required speed over ... upper surface to give a "lift" of 1000Nm... The density of air is 1.29 kgm^{-3} and assu... maximum thickness of wing to be ... metre. 111060...

Data:

$P_1 - P_2 = 1000\text{ N m}^{-2}$

$V_1 = 160\text{ m s}^{-1}$

$V_2 = ?$

$\rho = 1.29\text{ kg m}^{-3}$

$h_2 - h_1 = 1\text{ m}$

Solution:

Formula:

Using Bernoulli's equation

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$P_1 - P_2 + \rho g h_1 - \rho g h_2 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$

$(P_1 - P_2) - \rho g(h_2 - h_1) + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$

Put values in left hand side.

$1000 - 1.29 \times 9.8 \times 1 + \frac{1}{2} \times 1.29 \times (160)^2$

$= \frac{1}{2} \times 1.29 v_2^2$

$1000 - 12.642 + 16512 = 0.645 v_2^2$

$17499.358 = 0.645 v_2^2$

$v_2^2 = \dfrac{17499.358}{0.645}$

$v_2^2 = 27130.8$

$v_2 = 164.7$

or $\boxed{v_2 = 165\text{ m s}^{-1}}$

6.9: What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15 m? 11106033

Data:

$h = 15\text{ m}$

$g = 9.8\text{ m s}^{-2}$

$\rho = 1000\text{ kg m}^{-3}$

Gauge pressure $= P_1 - P_2 = ?$

Solution:

Formula:

As we know that

By Torricelli Equation

$v_2 = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 15}$

$v_2 = \sqrt{294}$

$v_2 = 17.1\text{ m s}^{-1}$

Using venturi relation

$P_1 - P_2 = \frac{1}{2}\rho v_2^2$

$= \frac{1}{2} \times 1000\,(17.1)^2$

$= \frac{1}{2} \times 1000 \times 294$

$= 147000\text{ Pa}$

$\boxed{P_1 - P_2 = 147\text{ kPa}}$