P425/1
**PURE**
**MATHEMATICS**
Paper 1
**July /Aug. 2022**
3 hours

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# UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

## PURE MATHEMATICS

Paper 1

**3 hours**

### INSTRUCTIONS TO CANDIDATES:

*Answer* **all** *questions in* **section** *A and any* **five** *from* **section B.**

*All necessary working must be shown clearly.*

*Silent non – programmable scientific calculators and mathematical tables may be used.*

*Any extra question(s) attempted in section* ***B*** *will* **not** *be marked.*

## SECTION A (40 MARKS)

**Answer ALL questions in this section**

1. Solve the equation: $1 - cos2\theta = sin\theta \; for \; 90^o \leq \theta \leq 180^o$. *(05 marks)*

2. Given that $\frac{50}{(2+i)^2} = a + bi$, find the real numbers $a$ and $b$.

   *(05 marks)*

3. Show that the stationary points of the curve $y^2 = x^2 + 2xy + 8$ lie on the line $y = -x$. State the $x -$ coordinates of the points. *(05 marks)*

4. The acute angle between the lines $2y - x - 3. = 0$ and $y = px + 3$ is $45^o$. Find the possible values of **P**. *(05 marks)*

5. Calculate the distance of the origin O $(0, 0, 0)$ from each of the planes $3x - 4y + 12Z + 13 = 0$ and $3x - 4y + 12Z - 39 = 0$; hence deduce the distance between the planes. *(05 marks)*

6. Evaluate $\int_0^{1/2} \frac{4x}{4 - x^2} \, dx$ *(05 marks)*

7. The first, second and fourth terms of an arithmetic progression form a geometrical progression. Find the common ratio of the G.P. *(05 marks)*

8. The volume **V** of a cone varies such that the height , $h$, of the cylinder is twice its base radius.

   (a) Show that $V = \frac{\pi}{12} h^3$

   (b) Find the rate at which **V** changes with height, at the instant when $h = 4cm$. *(05 marks)*

## SECTION B (60 MARKS)

9. A curve is represented by the parametric equations ;

$x = t^2 - 2t - 3$

$y = t^2 + 2t - 3$ where $t$ is the parameter.

Find: (a) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of $t$, hence find and determine the nature of the stationary points of the curve. *(08 marks)*

(b) the equation of the tangent to the curve at the point where the curve cuts the positive $y - axis$. *(04 marks)*

10. A circle whose centre is $C\,(1, 6)$ touches the line $y = {}^3/_4\, x - 1$ at point **A**.

Find the; (a) equation of the circle. *(06 marks)*

(b) coordinates of point A. *(06 marks)*

11. (a) Solve the equation: $3\sin\theta - 4\cos\theta = 4$ for $0^o \leq \theta \leq 360^o$ **(06 marks)**

(b) Prove that; $\frac{\sin 3A - \sin A}{\sin 5A + \sin 3A} = \frac{1}{4}Sec^2 A$ **(06 marks)**

12. (a) Expand $\sqrt{4 - 3x}$ up to the term in $x^3$, hence find the error made in using $x = 1$ in the expansion. **(07 marks)**

(b) Evaluate $\sqrt{61}$ to 4 decimal places. **(05 marks)**

13. The tangent to the curve $y = x^3$ at the point $A\,(-1, -1)$ meets the curve again at point **B**.

(a) Find the;

(i) equation of the tangent at **A**

(ii) coordinates of point **B**. **(07 marks)**

(b) Calculate the area bounded by the line **AB** and the curve. **(05 marks)**

14. The line $r = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ meets the plane $3x + 2y + Z = 29$ in point A. Find the:

(a) coordinates of point **A** *(07 marks)*

(b) acute angle between the line and the plane *(05 marks)*

15. (a) Given $Z_1 = -1 - i\sqrt{3}$, $Z_2 = -1 + i$, $Z_3 = 4i$; find the principal argument of $\frac{Z_1^3 Z_2^2}{Z_3}$. *(06 marks)*

(b) Find, in Cartesian form, the cube roots of $-8$. *(06 marks)*

16. (a) Solve: $Sin\, x \frac{dy}{dx} + y\cos x = \tan 3x$ *(05 marks)*

(b) The price $P$ of a litre of petrol increases at a rate which is directly proportional to the price. If the price doubles every 10 days; find the percentage increase in the price after 20 days. *(07 marks)*

**END**