P 425/2

APPLIED MATHEMATICS

PAPER 2

JULY/AUGUST 2023

3 HOURS

KAMMSA MOCK EXAMINATIONS 2023

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

GUIDE

3 HOURS

INSTRUCTIONS:

- Answer **all** the **eight** questions in **section A** and **only five** questions from **Section B.**
- Any additional question(s) answered will not be marked.
- All necessary working must be shown clearly.
- Begin each answer on a fresh sheet of paper.
- Graph papers are provided.
- Silent, non-programmable scientific calculators and mathematical tables may be used.
- In numerical work, take acceleration due to gravity g, to be 9.8 ms^{-2}

SECTION A (40 MARKS)

1. Forces $\begin{pmatrix}-3\\-1\end{pmatrix}$ N, $\begin{pmatrix}2\\9\end{pmatrix}$ N and $\begin{pmatrix}4\\-6\end{pmatrix}$N act on a body of mass 2kg. Find the magnitude of the acceleration. (05 marks)

Resultant force = $\begin{pmatrix}-3\\-1\end{pmatrix}+\begin{pmatrix}2\\9\end{pmatrix}+\begin{pmatrix}4\\-6\end{pmatrix}$

$= \begin{pmatrix}-3+2+4\\-1+9-6\end{pmatrix}$

$= \begin{pmatrix}3\\2\end{pmatrix} N$

Magnitude of the resultant = $\sqrt{3^2+2^2}$

$= \sqrt{13}$

= 3.6056N

From F = ma

$a = \frac{F}{m}$

$a = \frac{3.6056}{2}$

a = 1.8028ms^{-2}

2. Two events A and B are such that $P(A/B) = 2/7$, $P(A) = 1/2$ and $P(B) = 3/8$.
Find

(i) $P(A\ n\ B)$ (02 marks)

$P\left(\frac{A}{B}\right) = \frac{7}{2}$ $P(A) = \frac{1}{2}$ $P(B) = \frac{3}{8}$

$P\left(\frac{A}{B}\right) = P\frac{(AnB)}{P(B)}$

$P(AnB) = P\left(\frac{A}{B}\right).P(B)$

$= \frac{2}{7} x \frac{3}{8}$

$= \frac{6}{56}$

$P(AnB) = \frac{3}{28}$

(ii) $P\left(B/A'\right)$ (03 marks)

$P\left(\frac{B}{A^{-1}}\right) = P\frac{(A^1nB)}{P(A^1)}$

But P(B) = P(AnB) + P(A^1nB)

P(A^1nB) = P(B) – P(AnB)

P(A^1)+P(A) = 1

P(A^1) = 1-P(A)

$P\left(\frac{B}{A^1}\right) = \frac{3}{8} - \frac{3}{28}$

$= 1-\frac{1}{2}$

$= \left(\frac{15}{56}\right) \div \frac{1}{2}$

$=\frac{15}{56}x2$

$P\left(\frac{B}{A^1}\right) = \frac{15}{28}$

3. The following data relates to the percentage of unemployment and percentage change in wages for ten years.

% Unemployment	1.6	2.2	2.3	1.7	1.6	2.1	2.6	1.7	1.5	1.6
% Change in wages	5.0	3.2	2.7	2.1	4.1	2.7	2.9	4.6	3.5	4.4

Calculate the rank correlation coefficient between the percentage employment and percentage change in wages and comment on your result a 5% level of significance.

(05 marks)

Let the percentage of unemployment be x and percentage change in wages be y

x	y	Rx	Ry	d	d^z
1.6	5.0	8	1	7	49
2.2	3.2	3	6	-3	9
2.3	2.7	2	8.5	-6.5	42.25
1.7	2.1	5.5	10	-4.5	20.25
1.6	4.1	8	4	4	16
2.1	2.7	4	8.5	-4.5	20.25
2.6	2.9	1	7	-6	36
1.7	4.6	5.5	2	3.5	12.25
1.5	3.5	10	5	5	25
1.6	4.4	8	3	5	25
				£d² = 255	

$P = 1 - \frac{6£d^2}{n(n^2-1)}$

$P = 1 - \frac{6x255}{10(10^2-1)}$

$P = 1 - \frac{1530}{990}$

P = -0.5455

4. P, Q and R are three points in that order, on a straight road with $PQ = 40km$ and $QR = 90km$. If Maria travels from P to Q at 10kmh^{-1}. And then from Q to R at 15kmh^{-1}. Calculate her average speed for the journey from P to R. (05 marks)

P 40Km Q 90Km R

S=10Kmh^{-1} S=15Kmh^{-1}

Time taken $= \frac{D_1}{S_1} + \frac{D_2}{S_2}$

$= \frac{40}{10} + \frac{90}{15}$

= 4+6

= 10hrs

Average speed = $\frac{\textit{Total distance}}{\textit{Total time}}$

$= \frac{130}{10}$

Average speed = 13Kmh^{-1}

5. (i) Use the trapezium rule with equal strips of width $\pi/6$ to find an approximation for the $\int_0^{\pi}$ x sinx. Give your answer to 4 significant figures. (04 marks)

Let y=x sinx

x	**y**	
0	**0**	
$\frac{\pi}{6}$		**0.2618**
$\frac{\pi}{3}$		**0.9069**
$\frac{\pi}{2}$		**1.5708**
$\frac{2\pi}{3}$		**1.8138**
$\frac{5\pi}{6}$		**2.6180**
π	**0**	
	0	**7.1713**

$\int_0^{\pi} x\ sinx\ dx \approx \frac{h}{2}(0+0+2)$

$\approx \frac{\frac{\pi}{6}}{2}(0+2(7.1713))$

$\pi \approx \frac{\pi}{12}(14.3426)$

$\int_0^{\pi} x\ sinx\ dx \approx 3.755$

(ii)Comment on how you could obtain a better approximation to the value of the integral using trapezium rule. (01 mark)

By in-creasing the number of sub intervals
Reducing the width of the strips
Increasing the number of decimal places

6. The table below shows the values of x and f(x)

x	75.01	75.22	75.40	75.60
$f(x)$	1.8751	1.8762	1.8774	1.8785

Use linear interpolation or linear extrapolation to find

(i) $f(75.70)$ (02 marks)

(ii) x	**75.40**	**75.60**	**75.70**
F(x)	**1.8774**	**1.8785**	**F(x)**

A B C

Grad AB = Grad AC

$$\frac{1.8785-1.8774}{75.60-75.40} = \frac{f_{(x)}-1.8774}{75.70-75.40}$$

$$\frac{0.0011}{0.2} = \frac{f_{(x)}-1.8774}{0.3}$$

$$\frac{0.0011x0.3}{0.2} + 1.8774 = f_{(x)}$$

$f_{(x)}$ =1.87905

(iii) x when $f(x) = 1.8768$ (03 marks)

x	**75.22**	**x**	**75.40**
$f_{(x)}$	**1.8762**	**1.8768**	**1.8774**

D E F

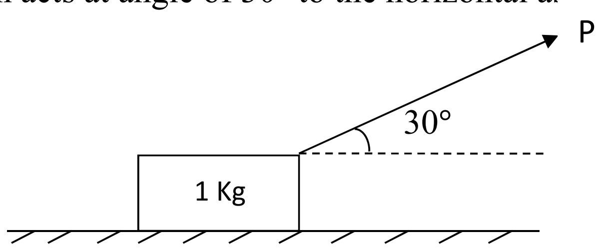
Grad DE = Grad DF

$$\frac{1.8768-1.8762}{x-75.22} = \frac{1.8774-1.8762}{75.40-75.22}$$

$$\frac{0.0006}{x-75.22} = \frac{0.0012}{0.18}$$

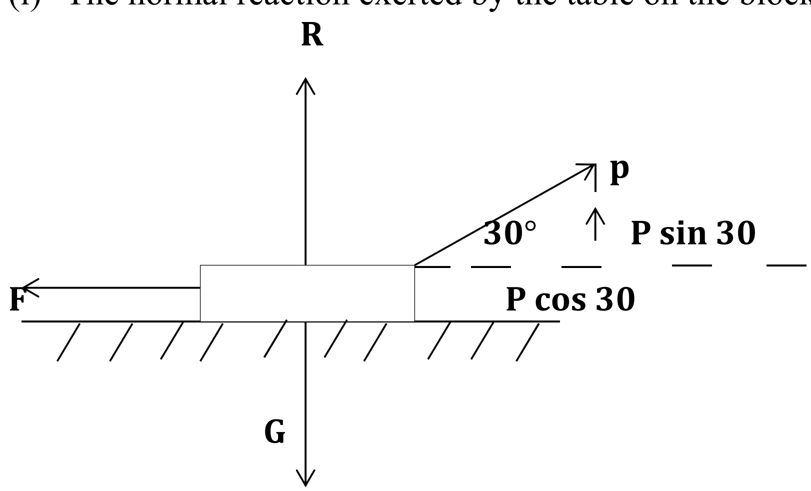
X = 75.31

7. A block of mass 1 kg rests in equilibrium on a rough horizontal table under the action of a force P which acts at angle of 30° to the horizontal as shown in the diagram below.



Given that the magnitude of P is 2.53N. Calculate

(i) The normal reaction exerted by the table on the block. (02 marks)



Resolving vertically
R + Psin 30 = 1x9.8
R + 2.53 sin 30 = 9.8
R = 9.8 – (2.53 sin 30)
Normal reaction R = 8.54N

(ii) The frictional force on the block (03 marks)
Resolving Horizonically
F = P cos 30
F = 2.53 cos 30
F = 2.19N Is the frictional force

8. A discrete random variable X has p.d.f $P(X = x) for\ x = 1, 2, 3$

$\boldsymbol{x}$	1	2	3
$\boldsymbol{P(X=x)}$	0.2	0.3	0.5

Find
(i) $E\ (5x\ + 3)$ (02 marks)

(ii) x	**1**	**2**	**3**
P(x=x)	**0.2**	**0.3**	**0.5**
xp(x=x)	**0.2**	**0.6**	**1.5**
x^2p(x=x)	**0.2**	**1.2**	**4.5**

E(x) = £xP(x=x) = 0.2+0.6+1.5=2.3
E(5x+3) =E(5x) + E(3)
= 5E(x) + 3
= (5x2.3) + 3
= 14.5

(iii) $Var\ (5x\ + 3)$ (03 marks)
Var(x) = E(x^2)-($E_{(x)}$)^2
= (0.2 + 1.2+4.5)-(2-3)^2
= 0.61
Var(5x+3) = Var(5x) + Var(3)
= 5^2Var(x)+0
= 25x061
= 15.25

SECTION B (60 MARKS)

Answer <u>any five</u> questions from this section.

9. A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 4x - 4x^3, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

Find
a) The mode (03 marks)

$F_{(x)}$ = $\sum_{,}$ $4x - 4x^3, 0 \le x \le 1$
0, otherwise
Mofe is the value of x for which $f^1_{(x)}$ = 0
$F_{(x)}$ = 4x-4x^2
$F^1_{(x)}$ = 4-12x^2=0

4-12x^2=0

$\frac{4}{12} = \frac{12x^2}{12}$

$x^2 = \frac{1}{3}$

$x \frac{1}{\pm\sqrt{3}}$

Either $x = \frac{1}{\sqrt{3}}$ *or* $x = -\frac{1}{\sqrt{3}}$

But x=$\frac{1}{\sqrt{3}}$ **dosenot lie in the range** $0 \leq x \leq 1$

Therefore the mxe is $\frac{1}{\sqrt{3}}$ **or** $\frac{\sqrt{3}}{3}$ **or 0.5774**

b) The cumulative distribution function and use it to find

(i) $P(0.1 < x < 0.6)$

For $x < 0, F_{(x)} = 0$

For $0 \leq x \leq 1, F_{(x)} = \int_0^x 4t - 4t^3 dt$

$= \frac{4t^2}{2} - \frac{4t}{4}^4 \, {}^x_0$

$= 2x^2 - x^4$

$F_{(1)} = 2_{(1)}{}^2 - {}_{(1)}{}^4 = (2-1) = 1$

For $x > 1, F_{(x)} = 1$

$f_{(x)} = \sum_1^0 2x^2 - x^4, 0 \leq x < 1$

$x > 1$

(i)

P(01<x<0.6) = F(0.6)-F(0.1)

$=2(0.6)^2-(0.6)^3-[2(0.1)^2 - (0.1)^3]$

=0.504-0.019

=0.4050

(ii) The median of x (09 marks)

Let the M be the median

$$\text{From } F_{(x)} = \sum_1^0 2x^2 - x, \quad \begin{array}{l} x < 0 \\ 0 \leq x \leq 1 \\ x > 1 \end{array}$$

$2m^2 - m^4 = \frac{1}{2}$

$4m^2 - 2m^4 = 1$

Either $X = \frac{2+\sqrt{2}}{2}$ **or** $x = \frac{2-\sqrt{2}}{2}$

$\therefore m = \sqrt{\frac{2+\sqrt{2}}{2}}$

M = 1.3066

Or $m = \sqrt{\frac{2-\sqrt{2}}{2}}$

Therefore, the median is 1.3066

$2m^4 - 4m^2 - 1 = 0$

Let $x = m^2$

$2x^2 - 4x - 1 = 0$

$X = \frac{+4 \pm \sqrt{4^2 - (4x2x1)}}{2x2}$

$X = \frac{4 \pm \sqrt[2]{2}}{4}$

= 0.54119

Testing m = 1.3066

$2(1.3066)^2 - (1.3066)^4 = 0.5$

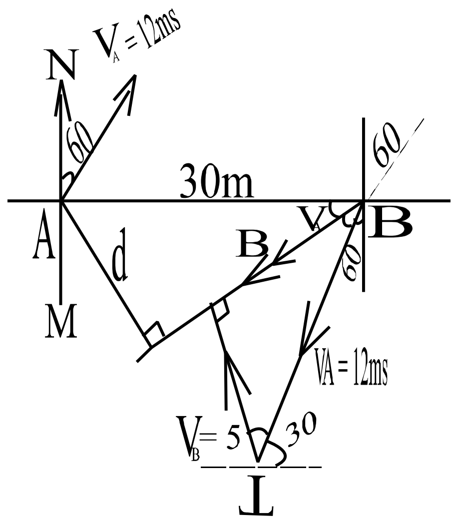
Testing m =0.54119

$2(0.54119)^2 - (0.54119)^4$

10. A particle at point **A** travels on a bearing of 060° at 12ms^{-1}. A second particle starts at point **B**, which is 30m due East of point **A** and has a maximum speed of 5ms^{-1}.
Find the

(i). The course the second particle B must set to get as close as possible to the first particle. (04 marks)

NO. 10



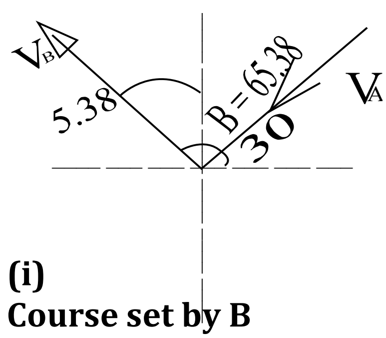
From ΔBNL,

$\sin \propto = \frac{5}{12}$

$\propto = 24.62°$

$\beta = 90 - 24.62$

= 65.38°



(i)

Course set by B

Is 360 ° = 354.38.6– 52°

(ii). The closest distance between the particles and the time at which this will occur. (08 marks)

Shortest distance, d is calculated from m angle ABM

$\theta = 90 - (60 + 2462)$.

= 5.38°

$\sin \theta = \frac{d}{30}$

d = 30 sinθ

= 30sin 5.38

= 2.8128m

Time take = $\frac{distance\ BM}{BVA}$

BVA =$\sqrt{12^2 - 5^2}$

= 10.9087ms^{-1}

Cosθ = $\frac{distance\ BM}{30}$

Distance BM = 30 Cosθ

= 30 Cos 5.38

= 29.8678m

Time take = $\frac{29.8678}{10.9087}$

= 2.7380 seconds

11(a). Given the equation $3x^3 + x - 5 = 0$.

(i). Show that the equation has a root between $x = 1$ and $x = 1.5$. (03 marks)

$F_{(x)} = 3x^3 + x - 5$

$F_{(1)} = 3_{(1)}{}^3 + 1 - 5 = {}^-1$

$F_{(1.5)} = 3(1.5)^3 + 1.5 - 5 = 6.625$

Since there is a sing change, the root lies between x=1 and x=1.5

(ii). Hence use linear interpolation to obtain approximation of root. (03 marks)

x	1	x	1.5
$f_{(x)}$	-1	0	6.625

Using linear interpolation

$$\frac{6.625 - {}^-1}{1.5 - 1} = \frac{0 - {}^-1}{x - 1}$$

$$\frac{7.625}{0.5} = \frac{1}{x - 1}$$

$x = 1.0656$ (4dp)

(b). Use Newton Raphson's formula to find the root of the equation by performing two iterations correct to two decimal places. (06 marks)

$f_{(x)} = 3x^3 + x - 5$

$f^1{}_{(x)} = 9x^2 + 1$

$X_{nti} = X_n - \frac{f(Xn)}{f^1(Xn)}$

$Xnti = Xn - \frac{(3X^3n + Xn - 5)}{9X^2n + 1}$

$Xnti = \frac{6X^3n + 5}{9X^2n + 1}$

Taking $X_0 = 1.0656$

1.0927 = 0.0006<0.005

Hence the root is 1.09 (2dp)

$X_1 = \frac{6(1.0656)^3 + 5}{9(1.0656)^2 + 1}$

$X_1 = 1.0927$

$X_2 = \frac{6(1.0927)^2 + 5}{9(1.0927)^2 + 1}$

$= 1.0921$

e = 11.0921-

12. The table below shows the marks obtained by students in a certain school

Class	30−<40	40−< 50	50−< 60	60−< 70	70−< 80	80−< 90	90−< 100
Frequency	4	6	8	12	10	7	3

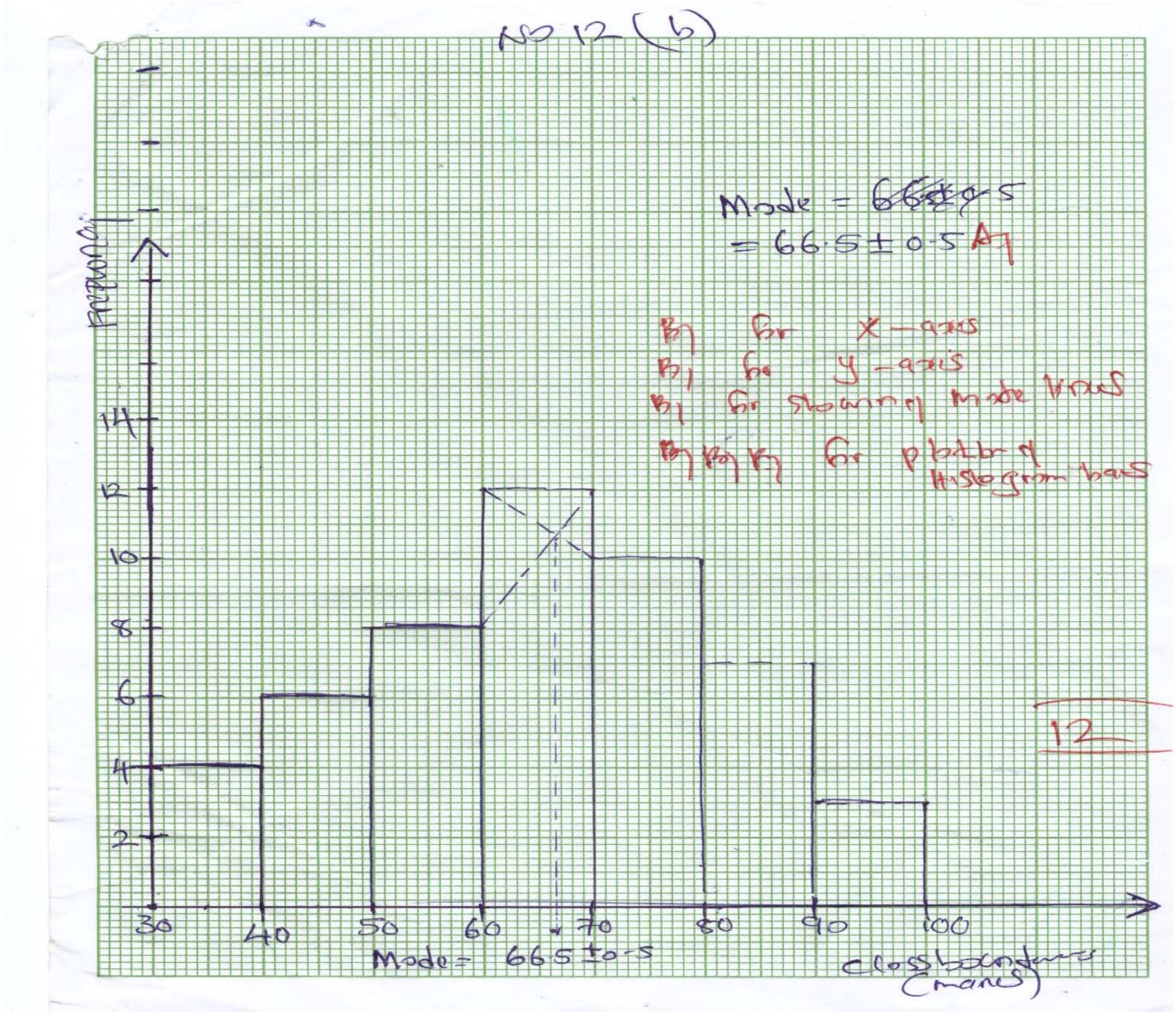
a) Calculate the average marks of the students (06 marks)

b) Class	**f**	**x**	**F(x)**
30-<40	**4**	**35**	**140**
40-<50	**6**	**45**	**270**
50-<60	**8**	**55**	**440**
60-<70	**12**	**65**	**780**
70-<80	**10**	**75**	**750**
80-<90	**7**	**85**	**595**
90-<100	**3**	**95**	**285**
	£f =50		**£fx = 3260**

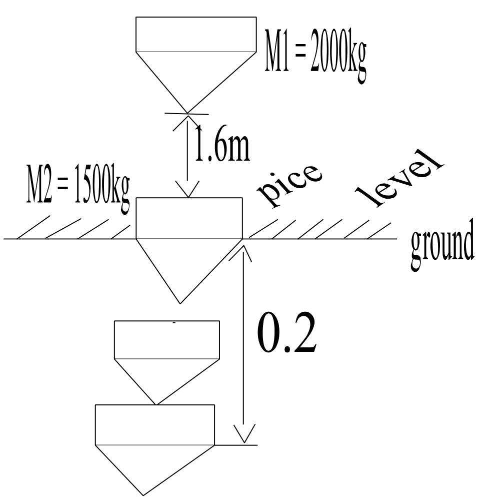
Average mark = $\frac{3260}{50}$

= 65.2marks

c) Draw a Histogram and use it to estimate the mode. (06 marks)



13(a). A pile driver of mass 2000kg falls from a height of 1.6m onto a pile of mass 1500kg without rebounding and the pile is driven 0.2m into the ground. Find the average resistance to the ground. (04 marks)



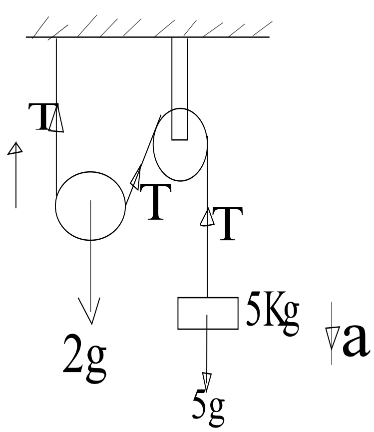
From $V^2 = u^2 + 2as$,
When fauing through distance
$V^2 = 0^2 + 2 \times 9.8 \times 1.6$
$V = \sqrt{31.36} = 5.6ms^{-1}$

On couision,
$M_1V + M_2 \times 0 = (M_1+M_2) V$
Where V = final velocity after couision
$2000 \times 5.6 = (2000+1500).V$
$V = 3.2ms^{-1}$

On moving into the ground
(Kinetic energy lost) = total work done by resistance force
$\frac{1}{2}(\boldsymbol{m^1} + \boldsymbol{m_2})(\boldsymbol{V})^2 = \boldsymbol{Fp\ x\ d}$
$\frac{1}{2}(\mathbf{2000} + \mathbf{1500})(\mathbf{3.2})^2 = \boldsymbol{Fp\ x}\ \mathbf{0.2}$
Fp = 89,600N

(b). A light inextensible string has one end attached to a ceiling. The string passes under a smooth movable pulley of mass 2kg and then over a smooth fixed pulley. A particle of mass 5kg is attached at the free end of the string. The sections of the string not in contact with the pulley are vertical. If the system is released from rest and moves in a vertical plane. **Find the**:-

(i). Acceleration of the system (03 marks)



$\underline{\boldsymbol{for\ 5kg\ mass}}$
$\boldsymbol{5g - T = 5a}$
$\underline{\boldsymbol{for\ moveable\ puues}}$
$\boldsymbol{2T - 2g = 2\left(\frac{1}{2}a\right)}$
$T - g = \frac{1}{2}a$
From (1) + (2),
$4g = 5a + \frac{1}{2}a^2$
$4g = \frac{11}{2}a$

(i)

a = $\frac{8g}{11} = \frac{8\,x\,9.8}{11}$

= 7.1273ms^{-2}

(ii). Tension in the string (02 marks)

T = $\frac{1}{2}$**a + g**

= $\frac{1}{2}\left(\frac{8g}{11}\right) + g$

= $\frac{15}{11}$**g = 13.3636N**

(iii).Distance moved by the moveable pulley in 1.5 seconds. (03 marks)

S = at +$\frac{1}{2}$**at²**

S = 0+ $\frac{1}{2}\left(\frac{1}{2}(7.1273)\right) x\ 1.5$

= 2.6727m

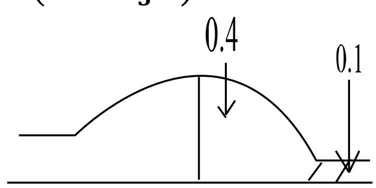
14(a). The marks in an examination were normally distributed with a mean μ and standard deviation δ. 10% of the candidates scored more than 75 marks and 20% scored less than 40 marks.

(i). Calculate the values of μ and σ (08 marks)

P(x>75) = 0.1

P(x<40) =0.2

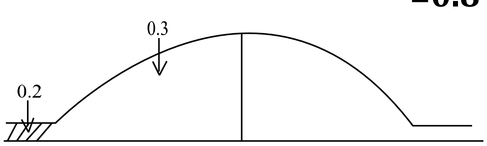
P$\left(z > \frac{75-m}{5}\right) = 0.1$



$\frac{1.30+1.29}{2} = 1.295$

P$\left(z < \frac{40-m}{8}\right) = 0.2$

=0.845



$\frac{75-m}{8} = 1.295$

1.2958+m = 75 ———— **(i)**

$\frac{40-m}{8} = -0.845$

-0.8458+m =40 ———— **(ii)**

Solving (i) on (ii)

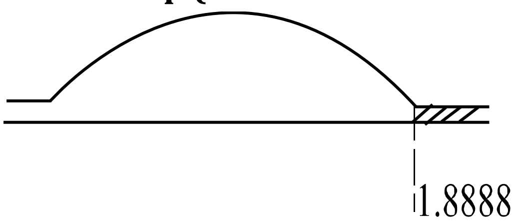
8 = 16.36

M = 53.82

(ii). If a sample of 25 candidates is selected at random from those who sat for the examination, find the probability that their average marks exceed 60. (04 marks)

P(X>60) = P$\left(Z > \frac{60-m}{\frac{16.36}{\sqrt{25}}}\right)$

= p ($Z > 1.8888$



P($Z > 60$**) = 0.5 – p(0<**Z**< 1.8888)**

= 0.5 – 0.4705

P(Z**> 60) = 0.0295**

15(a). Particles of mass $2kg, 3kg, 5kg$ and $6kg$ are located in the plane at points with position vectors $2i + 3j$,$4i - 3j$, $3i + 5j$ and $5i + 3j$ respectively. Find the coordinates of the centre of gravity of the system of particles. (05 marks)

$(m^1 + m^2 + Mn)\ell\binom{x}{y} = my\binom{x_1}{y_1}$+$\left(x^2 + \frac{\sqrt{3}}{4}x^2\right)\ell$. x = $x^2\ell.\frac{1}{2}x+\frac{1}{4}\sqrt{3}x^2\ell\left(x+\frac{\sqrt{3}x}{6}\right)$

Mg$\binom{x^2}{y^2}$+$\left(\frac{4+\sqrt{3}}{4}\right)\bar{x}$ = $\frac{1}{2}x + \frac{1}{4}\sqrt{3}\left(\frac{6+\sqrt{3}}{6}\right)x$

$\cdots + Mn\ell\binom{x}{y}(4+\sqrt{3})$x = $2x + \sqrt{3}\,\frac{(6+\sqrt{3})}{6}x$

(2+3+5+6g$\binom{\bar{x}}{\bar{y}}$ = 2$\binom{2}{3}$+2g$\binom{4}{-3}$+$(4+\sqrt{3})\,\bar{x} = \frac{(12+6\sqrt{3}+3)x}{6}$

5g$\binom{3}{5} + 6g\binom{5}{3}\ \bar{x} = \frac{1}{6}\frac{(15+6\sqrt{3})}{4+\sqrt{3}}x$

16g$\binom{\bar{x}}{\bar{y}} = \binom{61g}{40g} = \frac{1}{6}\frac{(15+6\sqrt{3})(4-\sqrt{3})x}{(4)^2=(\sqrt{3})^2}$

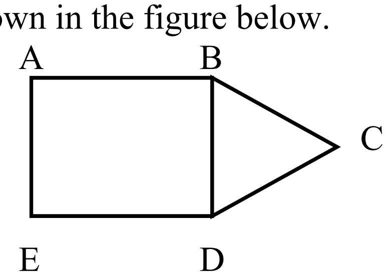
$\binom{\bar{x}}{\bar{y}} = \binom{3.8125}{2.5}m = \frac{1}{6}\frac{(60-15\sqrt{2}+24\sqrt{3}-18)x}{13}$

C$(\bar{x}, \bar{y}) = (3.8125, 2.5)m = \frac{1}{6}\left(\frac{(42+9\sqrt{3})}{13}\right)x$

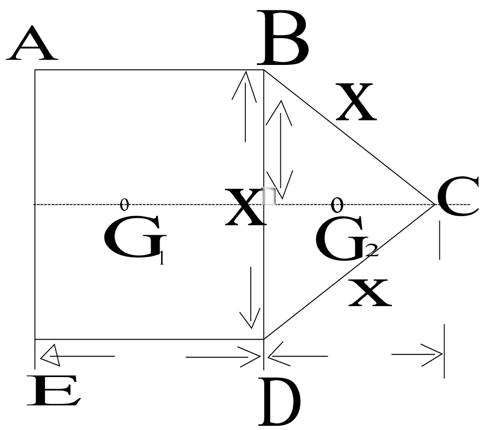
$= \frac{1}{6}.\frac{3(14+3\sqrt{3})x}{13}$

$= \frac{1}{26}(14+3\sqrt{3})x$

(b). A uniform laminar ABCDE is made from a square ABDE and an equilateral triangle BCD as shown in the figure below.



Show that the position of the centre of gravity from the side AE is given by $^1/_{26}\,(14 + 3\sqrt{3})$ (07 marks)



h = $\sqrt{x^2 - \left(\frac{1}{2}x\right)^2}$

$= \frac{\sqrt{3}}{2}x$

Let ℓ = weight per unit area

Figure	**Weight**	**C.O.G from A.E**
ABDE	$x^2\ell$	$\frac{1}{2}x$
BCD	$\frac{1}{2}$ x $x.\frac{\sqrt{2}}{2}$ x ℓ $=\frac{1}{4}\sqrt{3}x^2\ell$	$x+\frac{1}{3}.\frac{\sqrt{3}}{2}x$ $=x+\sqrt{3}x/6$
wde	$\left(x^2 + \frac{\sqrt{3}}{4}x^2\right)\ell$	$\bar{x}$

Using moment

16(a). A triangle base and height had dimensions 24.6cm and 15.4cm respectively.

(i). State the maximum possible error in the dimensions. (02 marks)

x=24.6, y=15.4

Ex = 0.5x10^{-1} = 0.05

Ey = 0.5x10^{-1} = 0.05

Exy = $|x\Delta y| + |y\Delta x| = \boldsymbol{Absolvte\ error}$

$= (24.6x0.05) + (15.4x0.05)$

= 1.23+0.77

Exy = 2

(ii). Find the range within which the area of the triangle lies. (03 marks)

Working value = L x W or x x y

= 24.6 x 15.4

= 378.84

Range at values = 378.84$\pm$2 = (376.84, 380.4)

= (376.84, 380.04)

(b). The numbers $P = 37.15$, $Q = 13.72$ and $r = 8.4$ are calculated with percentage errors of 6, 4 and 3 respectively. Find the limits to two decimal places within which the exact value of the expression

$\frac{P}{r} - Qr$ lies. (07 marks)

P = 37.15, Ep = 2.229

Q = 13.72, Eq = 0.5488

r = 8.4, Er = 0.252

Maximum value

(p/r-Qr) max

(37.15+2.229)/(8.4-0.253)-(13.72-0.5488)(8.40.252)

4.84845 - (107.3189)

= -102.4784

(p/r-Qr)Min = 4.0361766 – (123.45)

= 119.34825

(p/r-Qr)max=^- 102.4784

(p/r-Qr) min 〚=^- 119.34825〛

Limit = (min,max)

Limit =(^- 119.34825,⁻102.4784)

Limit =(⁻119.35,⁻102.48)

END