PROPOSED
MARKING GUIDE
UTEC P425/2
APPLIED MATHEMATICS 2023

NO	SOLUTION	Mks	Comments
1	(a) From $\mathrm{P}(A' \cup B) = 1 - \mathrm{P}(A \cap B')$ $\frac{1}{2} = 1 - \frac{5}{8}\mathrm{P}(B')$ $\frac{5}{8}\mathrm{P}(B') = \frac{1}{2}$ $\quad \therefore \mathrm{P}(B') = \frac{4}{5}$ ⇨ $\mathrm{P}(A \cup B') = 1 - \mathrm{P}(A' \cap B)$ $= 1 - \left(\frac{3}{8} \times \frac{1}{5}\right)$ $= \frac{37}{40}$ (b) $\mathrm{P}(A' \cup B') = \mathrm{P}(A \cap B)^1$ $= 1 - \mathrm{P}(A \cap B)$ $= 1 - \left(\frac{5}{8} \times \frac{1}{5}\right)$ $= \frac{7}{8}$		
		05	
2	i) $\begin{array}{\|c\|c\|c\|} 0.5 & 0.8 & 1.2 \\ \hline A & -0.24 & 0.18 \end{array}$ $\frac{A+0.24}{0.5-0.8} = \frac{-0.24-0.18}{0.8-1.2}$ $A = -0.555$ ii) $\begin{array}{\|c\|c\|c\|} 0.8 & B & 1.2 \\ \hline -0.24 & -0.12 & 0.18 \end{array}$ $\frac{B-0.8}{-0.12+0.24} = \frac{1.2-0.8}{0.18+0.24}$ $B = 0.9143$		
		05	

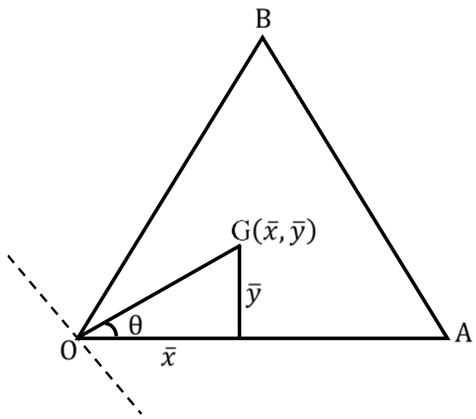
3

(a) $G(\bar{x}, \bar{y}) = G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= G\left(\frac{0+9+6}{3}, \frac{0+0+6}{3}\right)$

$= G(5, 2)$

(b)



$\tan\theta = \frac{2}{5}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta = 21.80^0$

05

4

R_H	R_M	d	d^2
1	2	-1	1
2	1	1	1
3	4	-1	1
4	3	1	1
5	7	-2	4
6	5	1	1
7	6	1	1
			$\sum d^2 = 10$

$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$

$\rho = 1 - \frac{6\times 10}{7(7^2-1)}$

$\rho = 0.8214$

Very high positive correlation

Or

Significant at 5%

Or

Not significant at 1%

05

5

Velocity at $t = 1$s;

$\boldsymbol{v}_{(t=1)} = 12\boldsymbol{i} + (8+23)\boldsymbol{j} = \mathbf{12}\boldsymbol{i} + 31\boldsymbol{j}\ ms^{-1}$

	Speed at $t = 1$s; $v = \sqrt{12^2 + 31^2} = \sqrt{1105}\, ms^{-1}$ Velocity at $t = 3$ s $\boldsymbol{v}_{(t=3)} = 12(3)^2\boldsymbol{i} + (8 \times 3 + 23)\boldsymbol{j} = 108\,\boldsymbol{i} + 47\boldsymbol{j}\, ms^{-1}$ Speed at $t = 3$s; $v = \sqrt{108^2 + 47^2} = \sqrt{13873}\, ms^{-1}$ ⇨ Average speed = $\frac{\sqrt{1105}+\sqrt{13873}}{2} = 75.5126\, ms^{-1}$		
		05	
6	$e_x = 0.005, e_y = 0.0005$ Max value = $(xy)_{max}$ $= (1.25 + 0.005) \times (1.600 + 0.0005)$ $= 2.0086$ Min value = $(xy)_{min}$ $= (1.25 - 0.005) \times (1.600 - 0.0005)$ $= 1.9914$ Interval = $1.9914 \leq xy \leq 2.0086$ Or $= [1.9914, 2.0086]$ Maximum error = $\frac{1}{2}(2.0086 - 1.9914)$ $= 0.0086$		
		05	
7	$\mathrm{P}(H) = 3\,\mathrm{P}(T)$ $\mathrm{P}(H) + \mathrm{P}(T) = 1$ $3P(T) + P(T) = 1$ $4\mathrm{P}(T) = 1$ $\quad\therefore \mathrm{P}(T) = \frac{1}{2} = 0.25, \mathrm{P}(H) = 0.75$ Let X = Number of heads that occurs $X \sim B(15, 0.75)$ $\mathrm{P}(X \geq 7) = \mathrm{P}(X' \leq 8)$ $= 1 - P(X' \geq 9)$		

	$= 1 - 0.0042$ $= 0.9958$		
		05	
8	From $v = u + at$ $0 = 12 + 5a$ $a = -2.4\ ms^{-2}$ $s = s_{(t=5)} - s_{(t=4)}$ $s = \left(12 \times 5 - \frac{1}{2} \times 2.4 \times 5^2\right) - \left(12 \times 4 - \frac{1}{2} \times 2.4 \times 4^2\right)$ $s = 30 - 28.8$ $s = 1.2\ m$		
		05	
9	(see below)		

$c.b$	f	x	fx	c	$f.d$	$c.f$
0 – 10	8	5	40	10	0.8	8
10 – 15	10	12.5	125	5	2	18
15 – 25	25	20	500	10	2.5	43
25 – 40	15	32.5	487.5	15	1	58
40 – 50	4	45	180	10	0.4	62
50 – 60	2	55	110	10	0.2	64
$\sum$	64		1442.5			

(a) (i) mean $= \frac{\sum fx}{\sum f}$

$= \frac{1442.5}{64}$

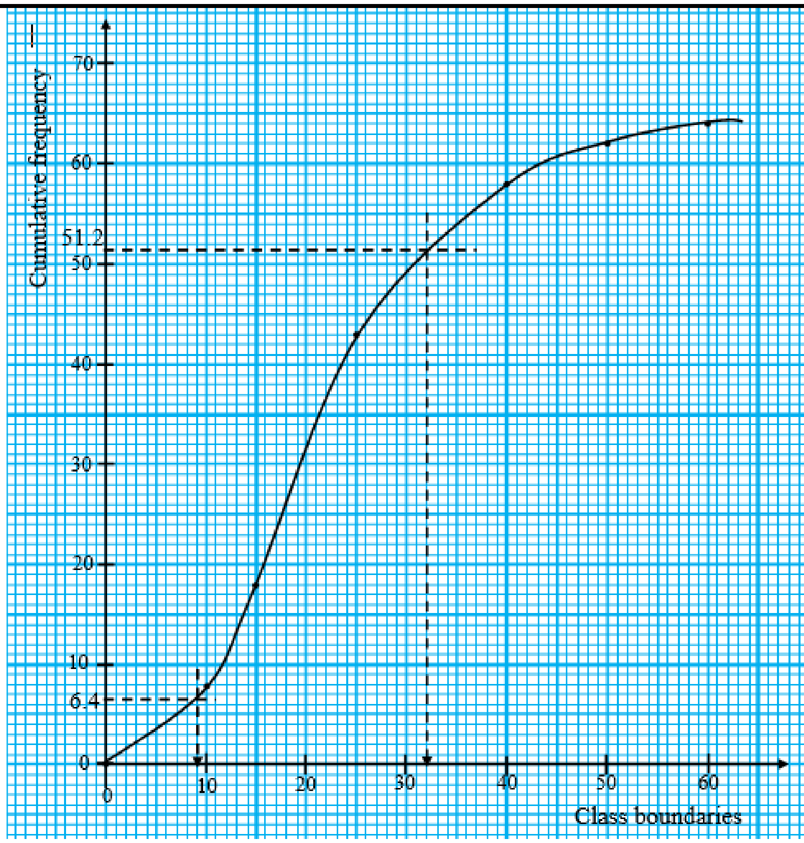
$= 22.5391$

(ii) mode $= l_1 + \left(\frac{d_1}{d_1+d_2}\right) \times c$

$= 15 + \left(\frac{0.5}{0.5+1.5}\right) \times 25$

$= 21.25$

(b)



Percentile deviation $= P_{80} - P_{10}$

$$= \left(\frac{80}{100} \times 64\right)^{th} - \left(\frac{10}{100} \times 64\right)^{th}$$

$$= 51.2^{th} - 6.4^{th}$$

$$= 52 - 9$$

$$= 43$$

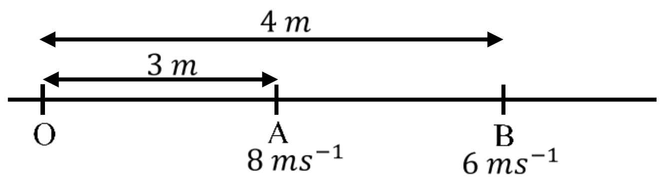
12

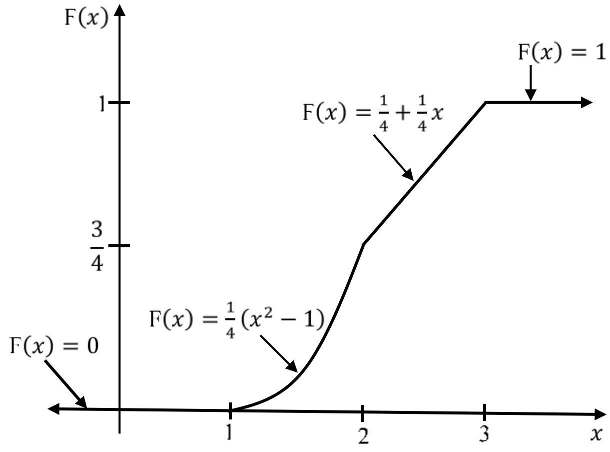
10

(a) Let $y = 2x + \cos x$, $h = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$

x	y	
0	1.00000	
$\pi/12$		1.48952
$\pi/6$		1.91322
$\pi/4$		2.27790
$\pi/3$		2.59440
$5\pi/12$		2.87681
$\pi/2$	3.14159	
Total	**4.14159**	**11.15185**

	$\int_0^{\pi/2}(2x+\cos x)dx \approx \frac{1}{2}\times\frac{\pi}{12}[4.14159+2(11.15185)]$ ≈ 3.461680366 ≈ 3.4617 (4dps) (b) Exact $= [x^2+\sin x]_0^{\frac{\pi}{2}}$ $= \left(\frac{\pi^2}{4}+\sin\left(\frac{\pi}{2}\right)\right)-0$ $= 3.4674011$ ≈ 3.4674 (4dps) %age error $= \frac{\lvert 3.4674-3.4617\rvert}{3.4674}\times 100$ $= 16.4388$ % or 16.44 % It can be minimized by increasing the number of ordinates		
		12	
11	(a) From $v^2=\omega^2(a^2-x^2)$ When $x=3m, v=8\ ms^{-1}$ $64=\omega^2(a^2-9)$ …………………(i) When $x=4\ m, v=6\ ms^{-1}$ $36=\omega^2(a^2-16)$ ………………..(ii) $(i)\div(ii)$; $\frac{64}{36}=\frac{a^2-9}{a^2-16}$ $16(a^2-16)=9(a^2-9)$ $16a^2-256=9a^2-81$ $7a^2=175$ $a^2=25 \qquad \therefore a=5\ m$ From (i); $64=\omega^2(25-9)$ $\omega^2=4 \qquad \therefore \omega = 2$ rads^{-1} From T $=\frac{2\pi}{\omega}$ $T=\frac{2\pi}{2}$ $T=\pi=3.1416$ s		

	(b)  From $x = a\sin\omega t$ Time at point A from centre, O $3 = 5\sin(2t_1)$ $t_1 = \frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ Time at point B from centre, O $4 = 5\sin(2t_2)$ $t_2 = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$ Time from A to B, $t = t_2 - t_1$ $t = \frac{1}{2}\left[\sin^{-1}\left(\frac{4}{5}\right) - \sin^{-1}\left(\frac{3}{5}\right)\right]$ $t = 8.1301$ s		
		12	
12	(a) F(2); $3a = a + 2b$ $2a = 2b$ $a = b$ F(3) $= 1$; $a + 3b = 1$ $b + 3b = 1$ $4b = 1 \qquad \therefore b = \frac{1}{4}, a = \frac{1}{4}$		



(b) $P(X < 2.5/X < 1.5) = \frac{P(X<2.5\cap X>1.5)}{P(X>1.5)}$

$= \frac{P(1.5<X<2.5)}{P(X>1.5)}$

$= \frac{F(2.5)-F(1.5)}{1-F(1.5)}$

$= \frac{\left(\frac{1}{4}+\frac{1}{4}(2.5)\right)-\left(\frac{1}{4}(1.5^2-1)\right)}{1-\frac{1}{4}(1.5^2-1)}$

$= \left(\frac{7}{8} - \frac{5}{16}\right) \div \left(1 - \frac{5}{16}\right)$

$= \frac{9}{16} \times \frac{16}{11}$

$= \frac{9}{11}$

(c) For $1 \leq x \leq 2, f(x) = \frac{d}{dx}\left[\frac{1}{4}(x^2 - 2)\right] = \frac{x}{2}$

For $2 \leq x \leq 3, f(x) = \frac{d}{dx}\left[\frac{1}{4} + \frac{1}{4}x\right] = \frac{1}{4}$

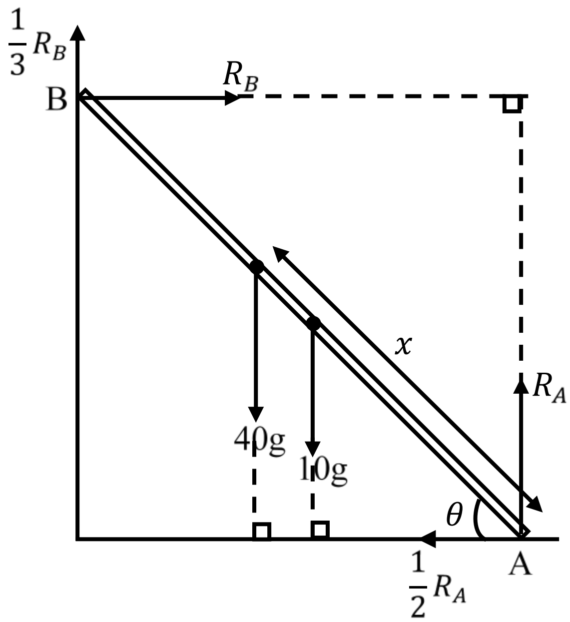
For $x \geq 1, f(x) = \frac{d}{dx}(1) = 0$

$$f(x) = \begin{cases} \frac{x}{2}, 1 \leq x \leq 2 \\ \frac{1}{4}, 2 \leq x \leq 3 \\ 0, elsewhere \end{cases}$$

$E(x) = \int_1^2 \frac{1}{4}x^2\, dx + \int_2^3 \frac{1}{4}x\, dx$

$E(x) = \left[\frac{x^3}{12}\right]_1^2 + \left[\frac{x^2}{8}\right]_2^3$

$E(x) = \frac{1}{12}(8 - 1) + \frac{1}{8}(9 - 4)$

<table>
<tr><td></td><td>$\text{E}(x) = \frac{29}{24}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>12</td><td></td></tr>
<tr><td>13</td><td>(a) Let $2l$ =length of the ladder, $x =$ distance the man climbs before the ladder slides.
Let $\theta = \tan^{-1}\frac{3}{4}$; $\tan\theta = \frac{3}{4}$


$(\rightarrow)$; $R_B = \frac{1}{2}R_A$………….(i)
$(\uparrow)$; $R_A + \frac{1}{3}R_B = 40\text{g} + 10\text{g}$
$R_A + \frac{1}{3}R_B = 50\text{g}$
$2R_B + \frac{1}{3}R_B = 50g$
$\frac{7}{3}R_B = 50g$
$R_B = \frac{3\times50\times9.8}{7} = 210\text{N}$
From (i);
$R_A = 2\times210 = 420\text{N}$
Taking moments about point A;
$40g\times x\cos\theta + 10g\times l\cos\theta = R_B\times 2l\sin\theta + \frac{1}{3}R_B\times 2l\cos\theta$
Dividing through by $\cos\theta$;</td><td></td><td></td></tr>
</table>

$40gx + 10gl = 2lR_B \tan\theta + \frac{2}{3}lR_B$

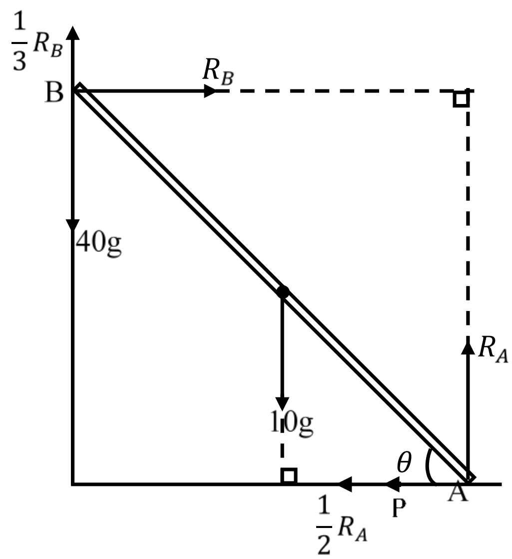
$40gx = \frac{2}{3} \times l \times 210 + 2 \times l \times 210 \times \frac{3}{4} - 10 \times 9.8 \times l$

$40gx = 140l + 315l - 98l$

$40gx = 357l$

$x = \frac{357}{392}l = \frac{51}{56}l$ m or $= 0.9107l$ m from A

(b) Let P be the minimum horizontal force



$(\uparrow)$; $\frac{1}{3}R_B + R_A = 50g$

$R_B = 150g - 3R_A$

$R_B = 150 \times 9.8 - 3R_A$

$R_B = 1470 - 3R_A$……………….(i)

$(\rightarrow)$; $R_B = P + \frac{1}{2}R_A$……………….(ii)

$(i) = (ii)$;

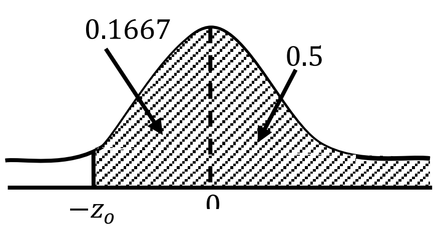
$1470 - 3R_A = P + \frac{1}{2}R_A$

$\frac{7}{2}R_A = 1470 - P$

$R_A = 420 - \frac{2}{7}P$

Taking moments about point B;

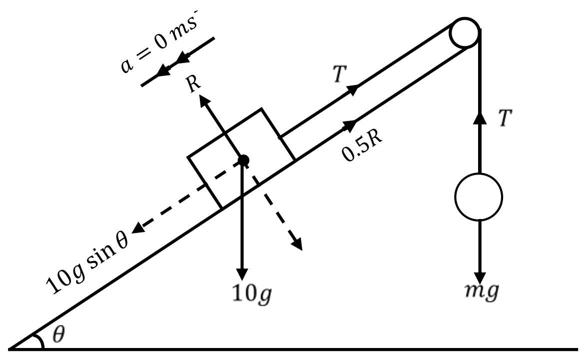
$10g \times l\cos\theta + \frac{1}{2}R_A \times 2l\sin\theta + P \times 2l\sin\theta = R_A \times 2l\cos\theta$

	$10 \times 9.8 + R_A \tan\theta + 2P \tan\theta = 2R_A$ $98 + \frac{3}{4}R_A + 2P \times \frac{3}{4} = 2R_A$ $98 + \frac{3}{2}P = \frac{5}{4}R_A$ $392 + 6P = 5R_A$ But $R_A = 420 - \frac{2}{7}P$ ⇨ $392 + 6P = 5\left(420 - \frac{2}{7}P\right)$ $392 + 6P = 2100 - \frac{10}{7}P$ $\frac{52}{7}P = 1708$ $\therefore P = \frac{2989}{13}N$ or 229.9231N		
		12	
14	(a) Let X = weights of the goats sold $\mu = 16$ kg, $\delta = ?$ $\mathrm{P}(X > 20) = \frac{8}{12} = 0.6667$ $\mathrm{P}\left(z > \frac{20-16}{\delta}\right) = 0.6667$ Let $\frac{20-16}{\delta} = z_0$ $\mathrm{P}(z > z_0) = 0.6667$  $P(0 < z < z_0) = 0.1667$ $z_0 = -0.431$ $\frac{20-16}{\delta} = -0.431$ $-0.431\delta = 4$ $\therefore \delta = -9.28074 \approx -9$ ***Helo members check on that number in my opinion I think there is a mistake somewhere***		
		12	

15 (a) From $\theta = \tan^{-1}\frac{4}{3}$, $\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}$

For minimum:

This is when the particle is just at the point of moving down the plane



At equilibrium;

$T = mg$ ……….(i)

Along the plane;

$T + o.5R = 10g\sin\theta$

But R $= 10g\cos\theta$

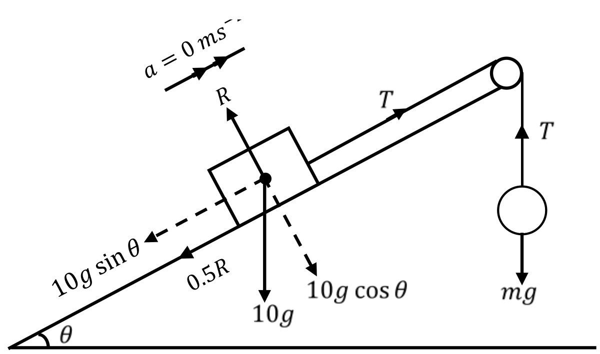
$mg + 0.5 \times 10g\cos\theta = 10g\sin\theta$

$m = 10 \times \frac{4}{5} - 5 \times \frac{3}{5}$

$m = 8 - 3 = 5$ kg

For maximum;

This is when the particle is just at the point of moving up the plane



At equilibrium;

T $= mg$

Along the plane;

$T = 0.5R + 10g\sin\theta$

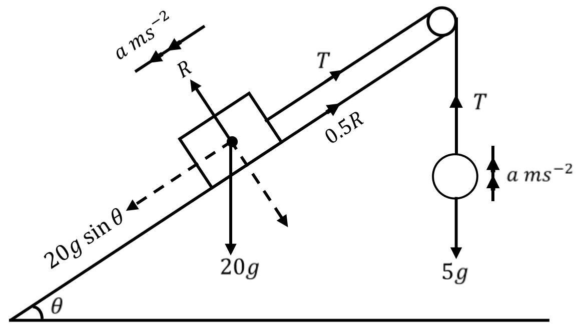
But $R = 10g\cos\theta$

$mg = 0.5\times 10g\times\cos\theta + 10g\times\sin\theta$

$m = 5\times\frac{3}{5} + 10\times\frac{4}{5}$

$m = 3 + 8 = 11$ kg

(b) When the mass of B is 5kg



For 5 kg mass;

$T - 5g = 5a$

$T = 5a + 5g$ ……………………(i)

For 20 kg mass;

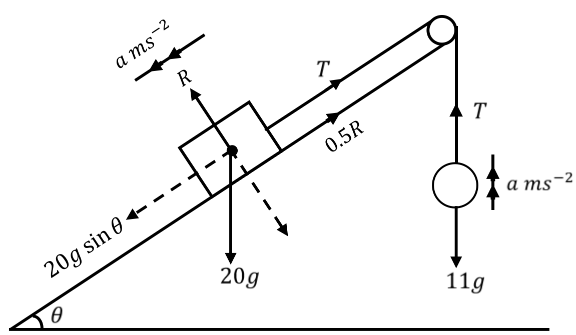
$20g\sin\theta - 0.5R - T = 20a$

But R= $20g\cos\theta$

$20\times 9.8\times\frac{4}{5} - 0.5\times 20\times 9.8\times\frac{3}{5} - 5a - 5\times 9.8 = 20a$

$25a = 49$ $\therefore a = 1.96\,ms^{-2}$

When he mass of B is 11 kg;



For 11 kg mass;

	$T - 11g = 11a$ $T = 11a + 11g$ For 20 kg mass; $20g\sin\theta - 0.5R - T = 20a$ But R= $20g\cos\theta$ $20 \times 9.8 \times \frac{4}{5} - 0.5 \times 20 \times 9.8 \times \frac{3}{5} - 11a - 11 \times 9.8 = 20a$ $31a = -9.8;$ $a = -\frac{49}{155}\ ms^{-2}$ $\therefore a = \frac{49}{155}\ ms^{-2}$ or $0.3161\ ms^{-2}$		
		12	
16	(a) Let $f(x) = x\sin x - 1$ $f(1) = 1\sin(1) - 1 = -0.15853$ $f(1.5) = 1.5\sin(1.5) - 1 = 0.49624$ $\therefore$ Since $f(1) \cdot f(1.5) < 0$, $1 <$ root < 1.5 $\begin{array}{\|c\|c\|c\|} 1 & x_0 & 1.5 \\ \hline -0.15853 & 0 & 0.49624 \end{array}$ $\frac{x_0 - 1}{0 + 0.15853} = \frac{1.5 - 1}{0.49624 + 0.15853}$ $x_0 = 1.121057776$ $x_0 \approx 1.12106$ (b) $f'(x) = x\cos x + \sin x$ $x_{n+1} = x_n - \left(\frac{x_n\sin x_n - 1}{x_n\cos x_n + \sin x_n}\right)$ Taking $x_0 = 1.12106$ $x_1 = 1.12106 - \left(\frac{1.12106\sin(1.12106) - 1}{1.12106\cos(1.12106) + \sin(1.12106)}\right)$ $= 1.11415$ $x_2 = 1.11415 - \left[\frac{1.11415\sin(1.11415) - 1}{1.11415\cos(1.11415) + \sin(1.11415)}\right]$ $= 1.11416$ Since $\|x_2 - x_1\| = 0.00001 < 0.00005$, then the root is 1.1142		
		12	

NB:

1. *The solutions in this Guide were according to my opinion.*
2. *I accept to own any mistakes detected*
3. *Try out the numbers and we compare the solutions thanx.*