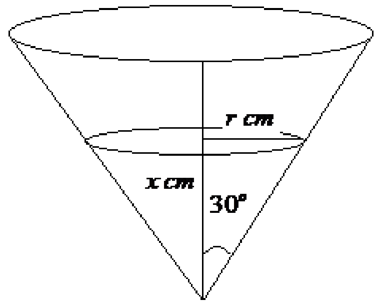
S 6 PURE MATHEAMTICS MARKING GUIDE

EXTERNAL MOCKS

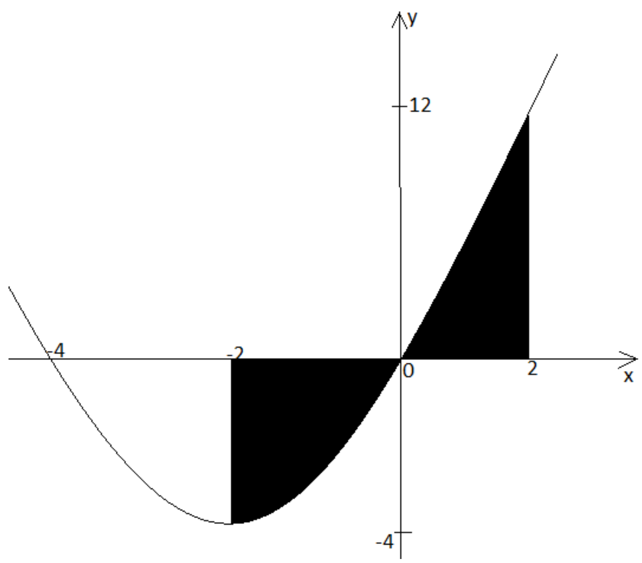
QN	**SOLUTION**	COMMENT
1.	$2+2\times(1.1)+2\times(1.1)^2+\ldots \quad a=2,\ d=1.1$ M1 $\frac{2(1.1^n-1)}{1.1-1}=1000$, M1 $1.1^n=51$, M1 $n=\frac{\log 51}{\log 1.1}=41.25$ M1 $\therefore \quad n=42$ A1	
2.	$3\cos\theta(2\tan\theta-1)+2(2\tan\theta-1)=0$ M1 $(3\cos\theta+2)(2\tan\theta-1)=0$, M1 $\cos\theta=-\frac{2}{3}, \quad \theta=\pm 131.81^o$ A1 $\tan\theta=\frac{1}{2}$ A1 $\quad \theta=26.57^o,\ -153.43^o$ A1	
3.	The point of intersection: $\begin{matrix} 2y+3x=5 & \times 2 \\ 3y-2x=14 & \times 3 \end{matrix}$, $\begin{matrix} 4y+6x=10 \\ 9y-6x=42 \end{matrix}$. M1 $13y=52$, $y=4$ and $x=-1$ so point is $(-1,\ 4)$ A1 For the line $y=3x-5$, $m=3$ B1 so equation is $\frac{y-4}{x--1}=3$, M1 $y-4=3x+3$, $3x-y+7=0$ A1	

4.	$y=(4x+5)^{\frac{1}{2}}$, $\frac{dy}{dx}=\frac{1}{2}(4x+5)^{-\frac{1}{2}}(4)=\frac{2}{(4x+5)^{\frac{1}{2}}}$, M1, A1 $\left.\frac{dy}{dx}\right\rvert_{x=1}=\frac{2}{3}$ M1 $\frac{y-3}{x-1}=\frac{2}{3}$, M1 $3y=2x+7$ A1	
5.	$\mathbf{OM}=\frac{\lambda\mathbf{q}+\mu\mathbf{p}}{\mu+\lambda}$ for $\mathbf{PM:MQ}=-1:2$ where $\lambda=-1$ and $\mu=2$ $\mathbf{OM}=\frac{-1\begin{pmatrix}1\\0\\4\end{pmatrix}+2\begin{pmatrix}4\\2\\6\end{pmatrix}}{-1+2}=7\mathbf{i}+4\mathbf{j}+8\mathbf{k}$, M1 M1 M1A1 thus the coordinates are $M(7,\ 4,\ 8)$ A1	
6.	$(2-3x+x^2)(1+2x)^4$ $(1+2x)^4=1+4(2x)+6(2x)^2+\ldots$ M1 $=1+8x+24x^2+\ldots$ A1 $(2-3x+x^2)(1+8x+24x^2)=48x^2-24x^2+x^2$ M1 M1 $=25x^2$ So the coefficient of x^2 is 25. A1	
7.	$y+\partial y=\cos 2(x+\partial x)$ $\partial y=\cos 2(x+\partial x)-\cos 2x$ M1 $\frac{\partial y}{dx}=\frac{-2\sin 2x\sin\partial x}{\partial x}$ M1 $\lim\limits_{\partial x\to 0}$, $\sin\partial x\approx\partial x(rads)$ M1 $=\frac{-2\sin 2x.\partial x}{\partial x}$ M1 $\therefore\ \frac{d}{dx}(\cos 2x)=-2\sin 2x$ A1	
8.	Let the depth of water be $x\ cm$ and radius be $r\ cm$	

	 $\tan 30^o = \frac{r}{x} \Rightarrow r = \frac{x}{\sqrt{3}}$. **M1** Volume of water in the cone is $V = \frac{1}{3}\pi r^2 x = \frac{1}{9}\pi x^3$, thus $\frac{dV}{dx} = \frac{1}{3}\pi x^2$, **M1** but $\frac{dV}{dt} = 5$ therefore $5 = \frac{1}{3}\pi x^2 \times \frac{dx}{dt}$, **M1** so $\frac{dx}{dt} = \frac{15}{\pi x^2}$ **M1** when $x = 10\,cm$, $\frac{dx}{dt} = \frac{15}{\pi(10)^2} = \frac{3}{20\pi}\,cm\,s^{-1}$ **A1** $\frac{dx}{dt} = 0.0477cm\,s^{-1}$	
9.	Midpoint of $AB = (4, 2)$, **M1** mid point $BC = (8, 4)$ **M1** Gradient of $AB = \frac{6 - -2}{7 - 1} = \frac{8}{6} = \frac{4}{3}$, **M1** Gradient of $BC = \frac{2-6}{9-7} = \frac{-4}{2} = -2$ **M1** Gradient of normal to $AB = \frac{-3}{4}$, **M1** Grad of normal to $BC = \frac{1}{2}$ **M1** Equation of normal through $(4, 2)$ is $\frac{y-2}{x-4} = \frac{-3}{4}$, $4y - 8 = -3x + 12$ to get $4y + 3x = 20$ ...(i) **A1** Equation of normal through $(8, 4)$ is $\frac{y-4}{x-8} = \frac{1}{2}$, $2y - 8 = x - 8$ to get $x = 2y$ ...(ii) **A1**	

	$4y+6y=20$, so $y=2$, $x=4$ **M1** so the point of intersection $(4,\ 2)$ **A1** $A(1,\ -2)$ $r=\sqrt{(4-1)^2+(2--2)^2}=\sqrt{9+16}=5$ **A1** So equation of circle is $(x-4)^2+(y-2)^2=5^2$, $x^2+y^2-8x-4y-5=0$ **A1**	
10a)	$\cos t+\cos 2t=0$, $2\cos^2 t+\cos t-1=0$, **M1** $2\cos^2 t+2\cos t-\cos t-1=0$ **M1** $2\cos t(\cos t+1)-1(\cos t+1)=0$, $(2\cos t-1)(\cos t+1)=0$ **M1** $\cos t=\frac{1}{2}$, $\cos t\neq -1$ **M1** $t=\frac{\pi}{3}s$ **A1**	
ii)	$v=\frac{ds}{dt}=-\sin t-2\sin 2t$, **M1** $v=-\sin\frac{\pi}{3}-2\sin\frac{2\pi}{3}$, $=-\frac{\sqrt{3}}{2}-\frac{2\sqrt{3}}{2}=-\frac{3\sqrt{3}}{2}\,m\,s^{-1}$ **M1 A1**	
iii)	$-\sin t-2\sin 2t=0$, $-\sin t-4\sin t\cos t=0$ $-\sin t(1+4\cos t)=0$, $\sin t\neq 0$, $\cos t=-\frac{1}{4}$ **M1** $a=\frac{dv}{dt}=-\cos t-4\cos 2t$, **M1** $a=-\left(-\frac{1}{4}\right)-4\left(2\left(-\frac{1}{4}\right)^2-1\right)$, **M1** $a=3\frac{3}{4}\,m\,s^{-2}$ **A1**	
11a)	$y=72x+3x^2-2x^3$ $\frac{dy}{dx}=72+6x-6x^2$, **M1** for max $\frac{dy}{dx}=0$	*Differentiating *Solving

	So, $72+6x-6x^2=0$, thus $x^2-x-12=0$ $(x-4)(x+3)=0$ **M1** so $x=4,\ x\neq-3$ **A1 for both** $\frac{d^2y}{dx^2}=6-12x$ $\left.\frac{d^2y}{dx^2}\right\|_{x=-3}=42>0$ discard $\left.\frac{d^2y}{dx^2}\right\|_{x=4}=-42<0$ so $x=4$ will give the max. **B1** Thus For $x=4$, $y=288+48-128=208$ **A1** is the maximum profit	
b)	Intercepts $x=0,\ y=0$ so $(0,\ 0)$ Turning points $\frac{dy}{dx}=2x+4$, so $x=-2$, $y=-4$ and $(-2,\ -4)$min **M1 for intercepts and t.p**	



	B1 curve $=\int_{-2}^{0}x^2+4x\,dx$ **M1** $=\left[\frac{x^3}{3}+2x^2\right]_{-2}^{0}=\left((0)-\left(-\frac{8}{3}+8\right)\right)=-\frac{16}{3}$ **A1** $=\int_{0}^{2}x^2+4x\,dx$ **M1** $=\left[\frac{x^3}{3}+2x^2\right]_{0}^{2}=\left(\left(\frac{8}{3}+8\right)-(0)\right)=\frac{32}{3}$ **A1** Total area is $\frac{16}{3}+\frac{32}{3}=16\,sq\,units$ **A1**	
12a)	$4\cot^2 2x-4\cot 2x+1=3(\cot^2 2x+1)-6$ **M1** $\cot^2 2x-4\cot 2x+4=0$, **M1**	

	$(\cot 2x - 2)(\cot 2x - 2) = 0$ $\cot 2x = 2$, $\tan 2x = \frac{1}{2}$ **A1** $2x = 26.6^o, 206.6^o, 386.6^o$ **M1** for all the angles $x = 13.3^o, 103.3^o, 193.3^o$ **A1 for all the angles**	
b)	$10\sin x\cos x + 12\cos 2x$ = $5\sin 2x + 12\cos 2x$ Let $5\sin 2x + 12\cos 2x \equiv R\sin 2x\cos\alpha + R\cos 2x\sin\alpha$ **B1** $\Rightarrow 5 = R\cos\alpha,\ 12 = R\sin\alpha$, thus $\tan\alpha = \frac{12}{5}$ **M1** $\therefore \alpha = 67.38^o$ **A1** $R = \sqrt{5^2 + 12^2} = 13$ **M1** $5\sin 2x + 12\cos 2x = 13\sin(2x + 67.38^o)$ **A1** $10\sin x\cos x + 12\cos 2x + 7 = 0$, $13\sin(2x + 67.38^o) = -7$ $2x + 67.38^o = 212.59^o,\ 327.41^o$, **M1** $2x = 145.21^o,\ 260.03^o$ Thus, $x = 72.61^o,\ 130.02^o$ **A1**	*identify the double angle
13a)	$\bar{\mathbf{AB}} = \bar{\mathbf{OB}} - \bar{\mathbf{OA}} = \begin{pmatrix} 3 \\ \alpha \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ 13 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ \alpha - 13 \\ 2 \end{pmatrix}$ M1 $\bar{\mathbf{AC}} = \bar{\mathbf{OC}} - \bar{\mathbf{OA}} = \begin{pmatrix} 6 \\ -7 \\ \beta \end{pmatrix} - \begin{pmatrix} 2 \\ 13 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ -20 \\ \beta - 5 \end{pmatrix}$ M1 $\bar{\mathbf{AB}} = \lambda\,\bar{\mathbf{AC}}$; $\begin{pmatrix} 1 \\ \alpha - 13 \\ 2 \end{pmatrix} = \lambda \begin{pmatrix} 4 \\ -20 \\ \beta - 5 \end{pmatrix}$ $1 = 4\lambda$, $\lambda = \frac{1}{4}$, M1 $\alpha - 13 = \frac{1}{4}(-20)$, M1 $\alpha = 8$, A1 $\frac{1}{4}(\beta - 5) = 2$, $\beta = 13$ A1	
b)	$\mathbf{OA}\,.\,\mathbf{OB} = \lvert\mathbf{OA}\rvert\lvert\mathbf{OB}\rvert\cos\theta$, $(4\mathbf{i} + 3\mathbf{j}).(\mathbf{i} + t\mathbf{j}) = \sqrt{4^2 + 3^2}\sqrt{1 + t^2}\left(\frac{2}{\sqrt{5}}\right)$ M1	

	$(4+3t)=5\left(\sqrt{1+t^2}\right)\left(\frac{2}{\sqrt{5}}\right)$, M1 $(4+3t)^2=25(1+t^2)\left(\frac{4}{5}\right)$ $16+24t+9t^2=20+20t^2$, $11t^2-24t+4=0$, M1 $11t^2-22t-2t+4=0$ M1 $11t(t-2)-2(t-2)=0$ $(11t-2)(t-2)=0$, $t=\frac{2}{11}$, $t=2$ A1 A1	
14a)	let $z=2-3i$, $(z-2)^2=(-3i)^2$, $z^2-4z+13=0$ **M1** $z^3+pz^2+qz+13\equiv(z^2-4z+13)(z+A)$ $z^3+pz^2+qz+13\equiv z^3+(A-4)z^2+(13-4A)z+13A$ **M1** $13A=13$, $A=1$ $p=A-4=1-4=-3$ **A1** $q=13-4A=13-4=9$ **A1** Hence the other roots are: -1, **A1** $2+3i$ **A1**	
b)	$(x+iy)(x-iy)-2(x+iy)+2(x-iy)=5-4i$ **M1** $x^2+y^2-4yi=5-4i$ **M1** $4y=4$, $y=1$ **A1** $x^2+1=5$, $x^2=4$, $x=\pm 2$ **M1, A1** $z=2+i$, $z=-2+i$ **A1**	
15a)	$x\log 10-x\log 5=\log 6-\log(1+2^x)$ $\log\frac{10^x}{5^x}=\log\frac{6}{1+2^x}$, **M1** $2^x=\frac{6}{1+2^x}$ **M1** $y=2^x$, $y=\frac{6}{1+y}$, $y^2+y-6=0$, **M1** $(y+3)(y-2)=0$, **M1** $y=-3$, $y=2$, $2^x=-3$ *Discard*, **B1** $2^x=2$, $x=1$ **A1**	
b)	Let $y=5^x$, $y^3-19y-30=0$	

	$P(5)=125-95-30=0$, **M1** therefore $y=5\ is\ a\ root$ $\begin{array}{cccc} 1 & 0 & -19 & -30 \\ 0 & 5 & 25 & 30 \\ \hline 1 & 5 & 6 & 0 \end{array}$, $(y-5)(y^2+5y+6)=0$ **M1** $(y-5)(y+2)(y+3)=0$, $y=5,\ y=-2,\ y=-3$ **A1, A1** $5^x=5,\ x=1$, **A1** $5^x\neq-2$, $5^x\neq-3$ discard **B1**	
16a)	$\frac{d^2y}{dx^2}=24x^2-2$, $\frac{dy}{dx}=\int 24x^2-2\,dx$, $\frac{dy}{dx}=8x^3-2x+c$, M1 $5=8-2+c$, $c=-1$ A1 $\frac{dy}{dx}=8x^3-2x-1$, $y=2x^4-x^2-x+k$, M1 $4=2-1-1+k$, $k=4$, A1 $y=2x^4-x^2-x+4$ A1	
b)	$x\frac{dy}{dx}=1-y^2$, by separating the variables, $\int\frac{dy}{1-y^2}=\int\frac{dx}{x}$ M1 Let $\frac{1}{(1+y)(1-y)}\equiv\frac{A}{1+y}+\frac{B}{1-y}$ M1 $\Rightarrow 1\equiv A(1-y)+B(1+y)$ Solving, $A=B=\frac{1}{2}$ A1 Thus $\frac{1}{2}\int\frac{1}{(1+y)}+\frac{1}{(1-y)}=\int\frac{dx}{x}$ $\frac{1}{2}In\left(\frac{1+y}{1-y}\right)=Inx+c$, M1 $x=2,\ y=0$, $c=-In2$ A1 $In\left(\frac{1+y}{1-y}\right)=In\frac{x^2}{4}$, $\left(\frac{1+y}{1-y}\right)=\frac{x^2}{4}$ M1 $4+4y=x^2-x^2y$, $y=\frac{x^2-4}{x^2+4}$ A1	