

P425/1
PURE
MATHEMATICS
Paper 1
July/Aug. 2024
3 hours.



ACEITEKA JOINT MOCK EXAMINATIONS 2024

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

Time: 3 Hours

INSTRUCTIONS TO CANDIDATES:

- Answer all the eight questions in section A and only five questions in section B.
- Indicate the five questions attempted in section B in the table aside.
- Additional question(s) answered will not be marked.
- All working must be shown clearly.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Question		Mark
Section A		
Section B		
Total		

Section A (40 Marks)

Answer all the questions in this section.

Qn 1: Solve for x : $64x^{\frac{2}{3}} + x^{\frac{-2}{3}} = 20$. [5 Marks]

Qn 2: Evaluate $\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{\frac{3}{2}}} dx$ by using a suitable substitution. [5 Marks]

Qn 3: Show that: $-\sqrt{5} \leq \cos x + 2 \sin x \leq \sqrt{5}$. [5 Marks]

Qn 4: Given that w and z are two complex numbers, solve the simultaneous equations:

$$\begin{aligned} 3z + w &= 9 + 11i \\ iw - z &= -8 - 2i \end{aligned}$$

[5 Marks]

Qn 5: Using Maclaurin's theorem, expand $(x+1) \sin^{-1} x$ up to the term in x^2 . [5 Marks]

Qn 6: The equation of a curve is given by $3y^2 - 18y + 2x + 37 = 0$.

(a). Show that the curve is a parabola. [3 Marks]

(b). Find the equation of the directrix. [2 Marks]

Qn 7: Given that points $A(1, 3, 2)$, $B(2, -1, 1)$, $C(-1, 2, 3)$ and $D(-2, 6, 4)$ are vertices of a parallelogram, find the area of the parallelogram $ABCD$. [5 Marks]

Qn 8: The area bounded by the curves $y^2 = 32x$ and $y = x^3$ is rotated about the x -axis through one revolution. Show that the volume of the solid so formed is $\frac{320\pi}{7}$ cubic units. [5 Marks]

4.9710

Section B (60 Marks)

Answer any **five** questions from this section.
All questions carry equal marks.

Question 9:

- (a). Find the ratio of the coefficient of x^7 to that of x^8 in the expansion of $\left(3x + \frac{2}{3}\right)^{17}$.
[5 Marks]
- (b). (i). Expand $(1 + x)^{-2}$ in descending powers of x including the term x^{-4} .
(ii). If $x = 9$, find the % error in using the first two terms of the expansion in (b)(i) above.
[7 Marks]

Question 10:

- (a). Express $\frac{x^3 + 9x^2 + 28x + 28}{(x+3)^2}$ into partial fractions.
- (b). Hence or otherwise show that:
$$\int_0^1 \frac{x^3 + 9x^2 + 28x + 28}{(x+3)^2} dx = \frac{1}{3} \left(10 + \ln \frac{4}{3} \right)$$

[12 Marks]

Question 11:

- (a). Determine the equation of the plane equidistant from the points $A(1, 3, 5)$ and $B(2, -4, 4)$.
[4 Marks]
- (b). (i). Find the coordinates of the point, P , in which the plane $4x + 5y + 6z = 87$ intersects the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z+4}{2}$.
(ii). Calculate the angle between the line and the plane in (b)(i) above.
[8 Marks]

Question 12:

- (a). Given that $\left(\frac{x}{2x-1}\right) + \left(\frac{x}{2x-1}\right)^2 + \left(\frac{x}{2x-1}\right)^3 + \dots = \frac{3}{4}$,
find the value of x .
[3 Marks]
- (b). Given that the factors $(x - 1)$ and $(x + 1)$ are factors of the polynomial, $f(x) = ax^4 + 7x^3 + x^2 + bx - 3$, find the values of the constants a and b . Hence, find the set for real values of x for which $f(x) > 0$.
[9 Marks]

Question 13:

- (a). Show that the equation of the circle passing through the points $(-2, -4)$, $(3, 1)$ and $(-2, 0)$ is $(x - 1)^2 + (y + 2)^2 = 13$.
[7 Marks]
- (b). With reference to the circle in (a) above, show that the tangent at point $(3, 1)$ is parallel to the diameter that passes through the point $(-2, 0)$.
[5 Marks]

Question 14:

(a). Using calculus of small changes, show that $\cos 44.6^\circ = \frac{\sqrt{2}}{2} \left(\frac{900+2\pi}{900} \right)$

[5 Marks]

(b). The pressure in an engine cylinder is given by:

$$P = 8000[1 - \sin(2\pi t - 3)] \text{ N m}^{-2}$$

At what time does this reach a maximum and what is the maximum pressure?

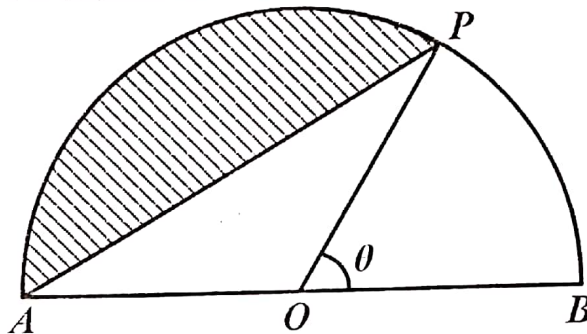
[7 Marks]

Question 15:

(a). Solve the equation: $\frac{4 \sin^2 \theta}{\operatorname{cosec} \theta} + \frac{3}{\operatorname{cosec}^2 \theta \sec \theta} = \sin^2 \theta$,
for $0^\circ \leq \theta \leq 360^\circ$.

[6 Marks]

(b). The diagram shows a semicircle APB on AB as diameter.



The midpoint of AB is O . The point, P , on the semicircle is such that the area of the sector POB is equal to twice the area of the shaded segment. Given that angle POB is θ radians,

show that: $3\theta = 2(\pi - \sin \theta)$.

[6 Marks]

Question 16:

(a). Solve the differential equation $(x+2)^2 \frac{dy}{dx} + 3y^2 = 0$ given that $y(0) = -2$.

[4 Marks]

(b). A girl returning from a milling point is carrying mealie-meal in a cylindrical container. The container has a hole at its base and the mealie-meal trickles out through this hole. It is estimated that the rate of reduction of mealie-meal is proportional to the mass, M , of mealie-meal remaining in the container after a time, t .

(i). Form a differential equation connecting M , t and the constant of proportionality, k .

(ii). Find the general solution of the differential equation in (b)(i) above and show that it reduces to $M = M_0 e^{-kt}$; where M_0 is the initial mass of mealie-meal.

(iii). The girl takes 2 hours to walk from the milling point to her home. Given that after one hour, ten percent of the mealie-meal is lost, calculate the percentage of the mealie-meal in the container when she arrives home.

[8 Marks]

END