Work, energy and power

Work done by constant force

Work is said to be done when energy is transferred from one system to another

When a block of mass m (kg) rests on a smooth horizontal surface. When a constant force F (N) acts on the block and displace it by a distance s (m), then the work done by F is given by

W = Fs

Example 1

Find the work done against gravity when a body of mass 5kg is moved through a vertical distance of 2m.

Solution

W = Fs = mgs	W = 5 x 9.8 x 2 = 98J

Example 2

A man building a wall lifts 50 bricks through a vertical distance of 3m. If each brick has a mass 4 kg. how much does the man do against gravity

Solution

W = Fs = mgs	W = 50 x 4 x 9.8 x 3 = 5880J

Example 3

A body of mass 2kg is moved vertically upwards at constant speed of $5ms^{-1}$. Find how much work is done against gravity in each second.

Solution

W = Fs = mgs	W = 2 x 9.8 x 5 x 1 = 98J

Example 4

A horizontal force pulls a body of mass 5kg a distance of 8m across a rough horizontal surface, coefficient of friction 0.25. The body moves with uniform velocity, find the work done against friction.

Solution

W = Fs = mgs

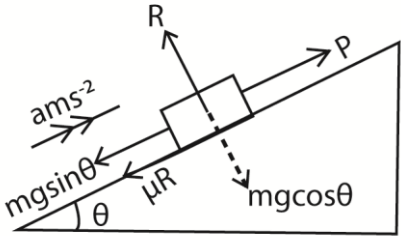
W = 0.25 x 5 x 9.8 x 8 = 98J

Example 5

A rough surface is inclined at $\tan^{-1}\left(\frac{7}{24}\right)$to the horizontal. A body of mass 5kg lies on the surface and is pulled at uniform speed of 75cms^{-1} up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is$\frac{5}{15}$. Find

(a) work done against gravity (b) work done against friction

Solution



θ = $\tan^{-1}\left(\frac{7}{24}\right)$= 16.3^0

(a) work against gravity

W = mgsinθd

= 5 x 9.8sin16.3^0 x $\frac{75}{100}$ = 10.35J

(b) work against friction

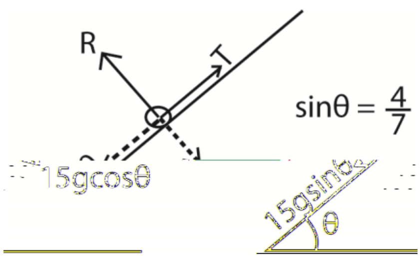
W = μRd but R = mgcosθ

=$\frac{5}{12}$ x 5 x 9.8cos16.3^0 x $\frac{75}{100}$ = 14.71J

Example 6

A particle of mass 15kg is pulled up a smooth slope by a light inextensible string parallel to the slope. The slope is 10.5m long and inclined at sin$^{-1}\left(\frac{4}{7}\right)$ to the horizontal. The acceleration of the particle is 0.98ms^{-2}. Determine the

(a) Tension in the string (03marks)



sinθ = $\frac{4}{7}$

T – 15gsinθ = 15a

T – 15 x 9.8 x $\frac{4}{7}$ = 15 x 0.98

T- 84 = 14.7

T = 98.7N

(b) Work done against gravity when the particle reached the end of the slope. (02marks)

Work = force x distance

= 15gdsinθ

= 15 x 9.8 x 10.5 x $\frac{4}{7}$

= 882J

Revision exercise

1. Find the work done against gravity when a body of mass 1kg is raised through a vertical distance of 3m. [29.4J]
2. Find the work done against gravity when a person of mass 80kg climbs a vertical distance of 25m. [19600J]
3. A body of mass 200g is moved vertically upwards at a constant speed of $2ms^{-1}$. Find how much work is done against gravity in each second. [3.92J]
4. A body of mass 10kg is pulled a distance of 20m across a horizontal surface against resistance totalling 40N. If the body moves with a uniform velocity, find the work done against the resistance. [800J]
5. A horizontal force pulls a body of mass 3kg a distance of 20m across a rough horizontal surface, coefficient of friction$\frac{2}{7}$. The body moves with a uniform velocity and the only resistance is that due to friction. Find the work done.[168J]
6. A horizontal force drags a body of mass 4kg a distance of 10m across a rough horizontal floor at a constant speed. The work done against friction is 49J. Find the coefficient of friction between the body and the surface. [0.125]
7. A block of mass 15kg rests on a smooth plane inclined at an angle of 30^0 to the horizontal. The block is pulled at a uniform speed a distance of 10m up the line of greatest slope. Find the work done against gravitation force. [735J]
8. A surface inclined at $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. A body of mass 50kg lies on the surface and pulled at uniform speed a distance of 5m to the line of greatest slope against a resistance to totalling 50N. Find
   (a) work done against gravity [1470J]
   (b) work done against friction [250J]
9. A rough surface inclined at $\tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal. A body of mass 130kg lies on the surface and is pulled at uniform speed a distance of 50m up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface if $\frac{2}{7}$. Find
   (a) the frictional force acting [336N]
   (b) work done against friction[16800J]
   (c) work done against gravity [24500J]
10. A rough surface is inclined at 30^0 to the horizontal. A body of mass 100kg lies on the surface and is pulled at a uniform speed a distance of 20m up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is 0.1. Find
    (a) work done against friction [1700J]
    (b) work done against gravity. [9800J]
11. A rough surface is inclined at $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. A body of mass 50kg lies on the surface and is pulled at a uniform speed a distance of 15m up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is $\frac{1}{3}$. Find the total work done on the body. [6370J]
12. A rough surface is inclined at an angle θ to the horizontal. A body of mass mkg lie on the surface and is pulled at a uniform speed by a force acting along a lines of greatest slope. The coefficient of friction between the body and the surface is μ. show that the total work done on the body is mgx(sinθ + μcosθ)

Work – energy theorem

It state that the work done by the **net force** acting on a body is equal to the change in its energy. Consider a body of mass m accelerated from velocity, u by a constant force, F so that in a distance, s it gains velocity, v.

a = $\frac{v^2-u^2}{2s}$

resultant force = ma = $\frac{m(v^2-u^2)}{2s}$

But work done = F x s

W = $\frac{m(v^2-u^2)}{2s} x\ s$ = $\frac{m(v^2-u^2)}{2}$

$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$ work – energy theorem

Example 7

A constant force pushes a mass of 4kg in a straight line across a smooth horizontal surface. The body passes a point A with speed of 5ms^{-1} and then through a point B with a speed of 8ms^{-1}. B is 6m from A. Find the magnitude of force acting on the mass.

a = $\frac{v^2-u^2}{2s}$ = $\frac{8^2-5^2}{2\ x\ 6s}$ = 3.25ms^{-2}

F = ma = 4 x 3.25 = 13N

Alternatively

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

F x 6 = $\frac{1}{2}\ x\ 4\ (8^2 - 5^2)$

F = 13N

Example 8

A car of mass 1000kg moving at 50ms^{-1} skids to rest in 4s under a constant retardation. Calculate the magnitude of the work done by the force of friction

Using v = u + at

0 = 50 + 4a

a = -12.5ms^{-2}

Frictional force = ma

= 1000 x -12.5 = 1250N

s = ut + ½ at2

s = 50 x 4 + ½ x -12.5 x 4^2

= 100m

W = F x s = 1250 x 100

= 125 000J

Alternatively

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

W = $\frac{1}{2}\ x\ 1000\ (50^2 - 0^2)$

Work done =125 000J

Example 9

A body of mass 4kg is moving with an initial velocity of 4ms-1 on a plane. The kinetic energy of the body is reduced by 16J in a distance of 40m. Find the deceleration of the body

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

16 = $\frac{1}{2}\ x\ 4(5^2 - v^2)$

v^2 = 17

a = $\frac{v^2-u^2}{2s}$ = $\frac{17-5^2}{2\ x\ 40}$ = -0.1ms^{-2}

Example 10

A body of mass 5kg moves in a straight line across a horizontal surface against a constant resistance of magnitude 10N. The body passes through point A and then comes to rest at point B, 9m from A. Find the speed of the body when it is at A.

F = ma

-10 = 5a

a = $-2ms^{-2}$

$v^2 = u^2 + 2a$

$0^2 = u^2 + 2 \times -2 \times 9$

u = $6ms^{-2}$

Or

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

10 x 9 = $\frac{1}{2}\ x\ 5(u^2 - 0^2)$

u = $6ms^{-2}$

Example 11

A body of mass 5kg slides over a rough horizontal surface. in sliding 5m, the speed of the body decreases from 8ms-1 to 6ms-1, find

(a) Frictional force

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

F x 5 = $\frac{1}{2}\ x\ 5(8^2 - 6^2)$

F = 14N

F = μR

μ = $\frac{14}{5\ x\ 9.8}$ = 0.286

Alternatively; $v^2 = u^2 + 2as$

a = $\frac{v^2-u^2}{2s} = \frac{6^2-8^2}{2\ x\ 5}$

= $-2.8ms^{-2}$

F= ma = 5 x 2.8 = 14N

Example 12

A bullet of mass 15g is fired towards a fixed wooden block and enters the block when travelling horizontally at $400ms^{-1}$. It comes to rest after penetrating a distance of 25cm. Find the

(i) work done against resistance of the wood

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}\ x0.015(400^2 - 0^2)$ = 1200J

(ii) magnitude of resistance

W = F x s

1200 = F x 0.25

F = 4800N

Example 13

A particle of mass 5kg falls vertically against a constant resistance. The particle passes through two point A and B 2.5m apart with A above B. Its speed is $2ms^{-1}$ when passing through A and $6ms^{-1}$ when passing through B. Find the magnitude of the resistance.

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

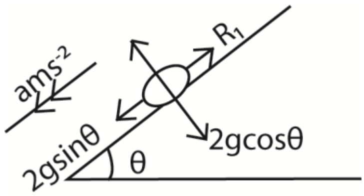
(5g – R) x 2.5 = $\frac{1}{2}\ x\ 5\ (6^2 - 2^2)$

R = 17N

Inclined Planes

Example 14

A rough slope of length 5m is inclined at angle 30^0 to the horizontal. A body of mass 2kg is released from rest at the top of the slope and travels down the slope against a constant resistance. The body reaches the bottom of the slope with speed of $2ms^{-1}$, find the magnitude of the resistance.



W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$(2g\sin 30 - R_1) \times 5 = \frac{1}{2}\ x\ 2(2^2 - 0^2)$

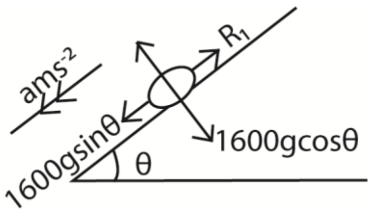
R_1 = 9N

Example 15

A car of mass 1600kg slides down a hill of slope 1 in 25. When the car descends 200m along the hill its speed increases from $3ms^{-1}$ to $10ms^{-1}$. Calculate

$\Delta K.E = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}\ x\ 1600(10^2 - 3^2)$ = 72800J



$a = \frac{v^2-u^2}{2\ x\ s} = \frac{10^2-3^2}{2\ x\ 200}$ = $0.228ms^{-2}$

F = ma

$1600g\sin\theta - R_1 = 1600a$

R1 = 1600 x 9.8 x $\frac{1}{25}$ – 1600 x 0.228

= 262.4N

OR; W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$(1600g\sin\theta - R_1) \times 200 = \frac{1}{2}\ x\ 1600(10^2 - 3^2)$

R = 262.4N

Revision exercise 2

1. A carton of mass 0.4kg is thrown across a table with velocity of 25ms-1. The resistance of the table to its motion is 50N. How far will it travel before coming to rest? What must the resistance be if it travels only 2m? [2.5m, 62.5N]
2. A and B are two points 4m apart on a smooth horizontal surface. A body of mass 2kg is initially at rest at A and is pushed by a force of constant magnitude acting in direction from a to B. The body reaches B with speed of $4ms^{-1}$. Find the magnitude of force [4N]
3. A and B are two points 3m apart on a smooth horizontal surface. A body of mass 6kg is initially at rest at A and is pushed towards B with a constant force of 9N. Find the speed of the body when it reaches B. [$3ms^{-1}$]
4. A constant force of magnitude 8N pushes a body of mass 4kg in a straight line across a smooth horizontal surface. The body passes through a point a with speed of $4ms^{-1}$ and then through appoint B 5m from A. Find the speed of the body at B. [$6ms^{-2}$]
5. A particle of mass 100g moves in a straight line across a horizontal surface against of constant magnitude. The particle passes through a point A with a speed of $7ms^{-1}$ and then through B with speed of $3ms^{-1}$, B being 2m from A. Find the magnitude of resistance. [1N]

6. A and B are two points 15m apart in the same vertical line, with A above B. A body of mass 5kg is released from rest at A and falls vertically against constant resistance of 25N. Find the speed of the body when it passes B. [$12ms^{-1}$]
7. A particle of mass 6kg is released from rest and falls freely under gravity. Find its speed when it has fallen a distance of 90m. [42ms-1]
8. A particle of mass 6kg is released from rest and falls freely under gravity. Find the distance it has fallen when its speed is $7ms^{-1}$. [2.5m]
9. A body of mass 3kg is projected vertically upwards from a point A with speed $4ms^{-1}$. The body passes through a point B 5m below A. Find the speed at B. [$10.7ms^{-1}$]
10. A particle of mass 2kg falls vertically against a constant resistance of 14N. The particle passes through two points A and B with a speed of $3ms^{-1}$ and $10ms^{-1}$ respectively. The distance AB. [16.25m]
11. A bullet of mass 8g is fired towards a fixed wooden block and enters the block when travelling horizontally at $300ms^{-1}$. How far does the bullet penetrate if the wood provide a constant resistance of 1800N. [20cm]
12. A bullet of mass 50g travelling horizontally at $100ms^{-1}$ strikes a stationary block of wood and coming to rest through a distance of 5m. Calculate the average resistance of the block to the motion of the bullet. [50N]
13. A bullet of mass 50g travelling horizontally at $500ms^{-1}$ strikes a stationary block of wood and after travelling 10cm, it emerges from the block travelling at $100ms^{-1}$. Calculate the average resistance of the block to the motion of the bullet. [60000N]
14. A bullet of mass 20g travelling horizontally at $210ms^{-1}$ strikes a stationary block of wood of thickness 0.1m and emerges from the block travelling at $50ms^{-1}$. Calculate the average resistance of the block to the motion of the bullet.[4160N]
15. A smooth slope is inclined at $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. A block of mass 4kg is released from rest at the top of the slope and travels down the slope, reaching the bottom of the slope with speed of $7ms^{-1}$, find the length of the slope. [4.17m]
16. A point A is situated at the bottom of a smooth slope inclined at an angle of $\tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal. A body is projected from A with a speed of 14ms-1 along and up a line of greatest slope and the body comes to rest at a point B. Find distance AB. [26cm]
17. A rough slope of length 10m is inclined at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. A block of mass 50kgis released from rest at the top of the slope and travels down the slope, reaching the bottom of the slope with speed of $8ms^{-1}$, find the
    (i) magnitude of the friction force. [134N]
    (ii) work done by friction force. [1340J]
    (iii) coefficient of friction [0.342]

18. Point A is situated at the bottom of a rough slope of length 10m is inclined at angle of $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. A body is projected from A with a speed of 14ms-1 along and up a line of greatest slope. The coefficient of friction between the body and the plane is 0.25. The body first comes to rest at point B. Find the distance. [12.5m]

Power

Power is the rate of doing work

Power = $\frac{work\ done}{time}$

P = $\frac{F\ x\ d}{t}$

P = $F\ x\ \frac{d}{t}$

P = F x v

F = $\frac{P}{v}$

Example 16

What is the average rate at which work must be done in lifting a mass of 100kg a vertical distance of 5m in 7s.

P = $\frac{F\ x\ d}{t} = \frac{100\ x\ 9.8\ x\ 5}{5}$ = 700W

Example 17

What is the rate at which work must be done lifting mass of 500kg at constant speed of 3ms^{-1}?

P = F x v = 500 x 9.8 x 3 = 14700W

Motion of cars

Consider a car being driven along a road, the forward or tractive force F_T moves the car is supplied by the engine working at constant rate of P watts.

Power = $\frac{work\ done}{time}$

P = $\frac{F_T\ x\ d}{t}$

P = $F_T\ x\ \frac{d}{t}$

P = F_T x v

F_T = $\frac{P}{v}$

Example 18

A cyclist travels along a road at constant speed of 8ms^{-1}. If the resistance to motion is 50N, find the rate at which the cyclist is working.

Solution

F =ma at constant speed a = 0ms-2

F – R = m x 0

$$\frac{P}{8} - 50 = 0$$

P = 400W

Example 19

A car of mass 800kg is driven along a level road against a constant resistance to motion of 200N. With the engine working at a steady rate of 14kW, find

(i) acceleration of the car when its speed is 10ms-1

(ii) maximum speed at which the car can move

Solution

(i) F = ma

F_T – R = m x a

$$\frac{14000}{10} - 200 = 800a$$

a = 1.5ms^{-1}

(ii) F = ma at maximum speed a = 0

$F_T - R = m \times a$

$\frac{14000}{v} - 200 = 800 \times 0$

$v = 70ms^{-1}$

Example 20

A car of mass 500kghas an engine of maximum power 2.5kW

(a) Calculate the force resisting the motion of the car when it is travelling at its maximum speed of $72kmh^{-1}$ on a level road.

$72kmh^{-1} = \frac{72 \times 1000}{3600} = 20ms^{-1}$

F = ma

$F_T - R = m \times a$

at maximum speed a = 0

$\frac{2500}{20} - R = 500 \times 0$

R = 125N

(b) If the resistance remains unaltered, find the acceleration of the car when travelling at $36kmh^{-1}$ on a level road with the engine working at the same rate.

$F_T - R = m \times a$

$\frac{2500}{10} - 125 = 500 \times a$

$a = 0.25ms^{-2}$

Example 21

A force on a particle of mass 15kg moves it along a straight line with velocity of 10ms-1. The rate at which work is done by the force is 50W. If the particle starts from rest, determine the time it takes to move a distance of 100m.

F = ma

$\frac{50}{10} = 15a$

$a = 0.33ms^{-2}$

$s = ut + \frac{1}{2}at^2$

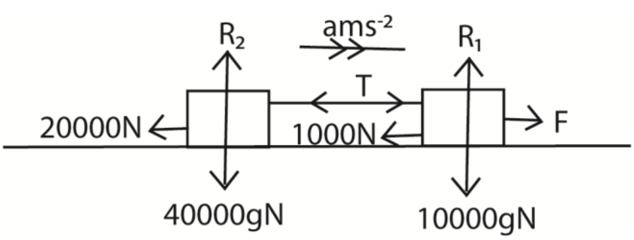
$100 = 0 \times t + \frac{1}{2} \times 0.33 \times t^2$

t = 24.51s

Example 22

A car of mass 10 tonnes tows a trailer of mass 40 tonnes along a level road. The car exerts experience a resistance of 100N and the trailer a resistance of 2000N, if the car engine working at constant rate of 4,000kW. Find the acceleration produced and the tension in the two bar at the instant the speed is $72kmh^{-1}$.

Solution



$72kmh^{-1} = \frac{72,000}{3600} = 20ms^{-1}$

$F = \frac{P}{v} = \frac{4,000,000}{20} = 20,000N$

10,000kg: 20000- (T+100) = 10000a

19,900 – T = 10000a ……… (i)

40,000kg; T – 2000 = 40000a …… (ii)

(i) + (ii): $a = 0.358ms^{-2}$

Alternatively

50000kg: 20,000-200-100 = 5000a

17,900 = 50000a

a = 0.358ms^{-2}

T- 2000 = 40,000a

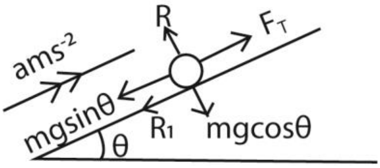
T =40000 x 0.358 + 2000

= 16,320N

Inclined planes

Example 23

A train of mass 20,000kg moves at constant speed of 72kmh^{-1} up a straight inclined plane against a frictional force of 128N. The incline is such that the train rise vertically 1m for every 100m travelled along the incline. Calculate the necessary power developed by the train



72kmh^{-1} = $\frac{72\ x\ 1000}{3600}$ =20ms^{-1}

sinθ = $\frac{1}{100}$

at constant speed, a = 0

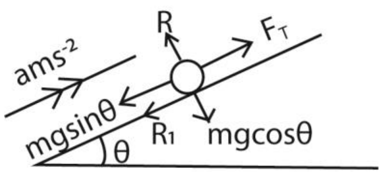
Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) =$ ma

$\frac{P}{20} - \left(20000x\ 9.8\ x\ \frac{1}{100} + 128\right)$ = 20000 x 0

P= 41760W

Example 24

A car of mass 1.5metric tonnes moves at constant speed of 6ms^{-1} up a slope inclined at $\sin^{-1}\frac{1}{7}$. Given that the engine of the car is working at constant rate of 18kW. Find the resistance to the motion.



sinθ = $\frac{1}{7}$

at constant speed, a = 0

Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) =$ ma

$\frac{P}{6} - \left(1500x\ 9.8\ x\ \frac{1}{7} + R_1\right)$ = 1500 x 0

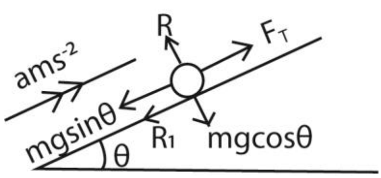
R_1 =900N

Example 25

A car of mass 800kg with engine working at constant rate of 15kW climbs a hill of inclination 1 in 98 against a constant resistance to motion of 420N. Find the

(i) the acceleration when the speed is 10ms^{-1}

(ii) maximum speed of the car up the hill



Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) =$ ma

$\frac{P}{10} - \left(800x\ 9.8\ x\ \frac{1}{98} + 420\right)$ = 800 x a

a =1.25ms^{-2}

(ii) at constant speed, a = 0

$\frac{15000}{v} - \left(8000x\ 9.8\ x\ \frac{1}{98} + 420\right)$ = 800 x 0 v = 30ms^{-1}

Example 26

A car of mass 1000kg has a maximum speed of 40ms^{-1} on a level road and the engine s working at 32kW against constant resistance

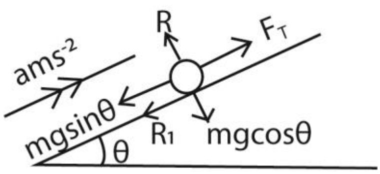
(i) find the resistance to motion

$F_T - \mathrm{R}_1 = \mathrm{ma}$; at constant speed a = 0

$\frac{32000}{40} - R_1 = 1000\ x\ 0$

$R_1 = 800N$

(ii) Given that the resistance in both cases varies as the speed, find the rate at which the engine must work for the car to ascend a slope of 1 in 98 at constant speed of 20ms^{-1}.



R_1 = kv

k = $\frac{800}{40} = 20$

Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) = \mathrm{ma}$

at constant speed a =0

$\frac{P}{20} - \left(1000x\ 9.8\ x\ \frac{1}{98} + 20\ x\ 20\right)$ = 1000 x 0

P = 10,000W

Example 27

A car of mass 1000kg has maximum speed of 150kmh^{-1} on a level road and working at 60kW

(i) find the coefficient of friction between the car and the road if all resistance is due to friction

150kmh-1 = $\frac{150000}{3600}$ = 41.67ms^{-1}

F = ma

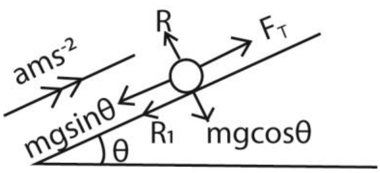
$F_T - \mu R$ = ma

At constant speed, a = 0

$\frac{60000}{41.67} - \mu(1000\ x\ 9.8) = 1000\ x\ 0$

μ = 0.147

(ii) Given that the tractive force remains unaltered and the non-gravitation resistance in both cases varies as square of the speed, find the greatest slope on which a speed of 120kmh^{-1} could be maintained.



$\mu R = kv^2$

k = $\frac{0.147\ x\ 1000\ x\ 9.8}{(41.67)^2}$ = 0.8297

120kmh^{-1} = $\frac{120\ x1000}{3600}$ = 33.33ms^{-1}

Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) = \mathrm{ma}$

at constant speed a =0

$\frac{60000}{41.67} - (1000x9.8sin\theta + 0.8297(33.33)^2$

$= 1000\ x0$

θ = 3.04^0

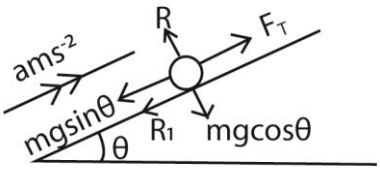
Example 28

A car of mass 900kg with the engine working at a constant rate of 7.35kW climbs a hill of inclination 1 in 63 against constant resistance to motion. Find the

(i) resistance to motion when the car is travelling with a constant speed of $15ms^{-1}$

(ii) maximum speed of the car when travelling down the same slope with the engine working at the same rate as before and the resistance to motion unchanged.

Solution



$F_T = \frac{P}{v} - (mgsin\theta + R_1) = \text{ ma}$

$\frac{7350}{15} - \left(900\ x\ 9.8\ x\ \frac{1}{63} + R_1\right) = 900\ x\ 0$

R_1 = 350N

(ii) $F_T = \frac{P}{v} + (mgsin\theta - R_1) = \text{ ma}$

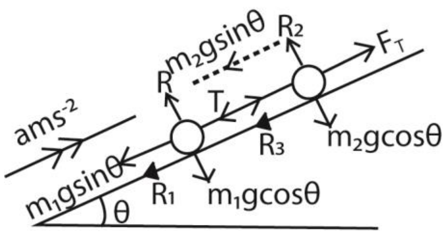
$\frac{7350}{v} - \left(900\ x\ 9.8\ x\ \frac{1}{63} - 350\right) = 900\ x\ 0$

v = $35ms^{-1}$

Example 28

A car of mass 1200kg is pulling a trailer of mass 300kg up a slope 1 in 10, the resistance to motion for car and trailer is $0.2Nkg^{-1}$. Given that the car moved at a constant speed of $1.5s^{-1}$ for 5 minutes, find the

(i) Tension in coupling between the car and the trailer
(ii) work done by the engine of the car during this time
(iii) total resistance, if the engine develops a power of 15kW at maximum speed of $120kmh^{-1}$ on level road.



At constant speed, a = 0; $\sin\theta = \frac{1}{10}$

300kg: T –($m_1gsin\theta + 0.2m_1$) = m_1 x 0

T = 300 x 9.8 x $\frac{1}{10}$ + 0.2 x 300 = 354N

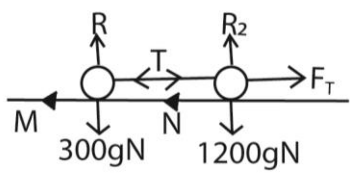
1200kg: F_T – (T +$m_2gsin\theta$ + 0.2m_2) =1200 x 0

FT = (354 + 1200 x 9.8 x $\frac{1}{10}$ + 0.2 x 1200)

= 1770N

W = FT x d = 1770 x 1.5 x 5 x 60 =796,500J

(iii)



$120kmh^{-1} = \frac{120\ x\ 1000}{3600} = 33.33ms^{-1}$

$F_T = \frac{P}{v} = \frac{15000}{33.33}$ = 450N

1500kg: 450 –(M + N) = 1500 x 0

(M + N) = 450

Example 30

A car of mass mkg has an engine which works at a constant rate of 2H kW. The car has constant speed of V ms^{-1} a long a horizontal road.

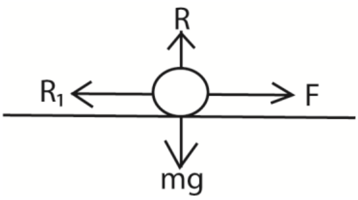
(a) Find in terms of m, H, V and θ the acceleration of the car when travelling
  (i) up a road of inclination θ with a speed of $\frac{3}{4}V\ ms^{-1}$.
  (ii) down the same road with a speed of $\frac{3}{5}V\ ms^{-1}$, the resistance to the motion of the car apart from gravitational force, being constant

(b) If an acceleration in (a)(ii) above is 3 times that of (a)(i) above, find the angle of inclination θ of the road.

(c) If the car continues directly up the road, in case (a)(i) above, show that its maximum speed is $\frac{12}{13}V\ ms^{-1}$

Solution

(a) When the car moves along a horizontal road



With constant speed, F = R_1

But power, P = Fv = R_1v

⇨ 2000H = Fv = R_1v

R_1v= 2000H

$R_1 = \frac{2000H}{v}$

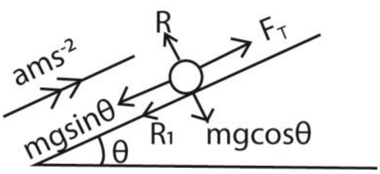
(i) When ascending with velocity $\frac{3v}{4}ms^{-1}$

Power = Fv

2000H = $\frac{3v}{4}F$

F = $\frac{8000H}{3v}$

Let a = acceleration



Net accelerating force = F – (mgsinθ + R_1)

ma = $\frac{8000H}{3v}$ - (mgsinθ + $\frac{2000H}{v}$)

ma = $\frac{2000H}{3v} - \text{mgsin}\theta$

a = $\frac{2000H-3mvgsin\theta}{3mv}$

(ii) When descending with velocity $\frac{3v}{5}ms^{-1}$

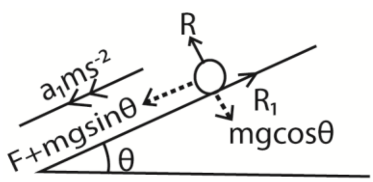
Power = Fv

2000H = $\frac{3v}{5}F$

F = $\frac{10000H}{3v}$

Let a_1 = acceleration

Net accelerating force = F +mgsinθ - R_1) = ma_1



ma_1 = $\frac{10000H}{3v} + mgsin\theta - \frac{2000H}{v}$

a_1 =$\frac{4000H+3mvgsin\theta}{3mv}$

(b) Given $a_1 = 3a$

$$\frac{4000H+3mvgsin\theta}{3mv} = 3\left(\frac{2000H-3mvgsin\theta}{3mv}\right)$$

$$\frac{4000H+3mvgsin\theta}{3mv} = \frac{2000H-3mvgsin\theta}{mv}$$

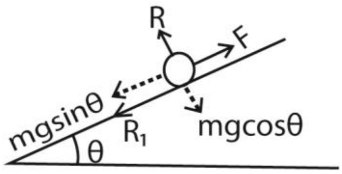
$$4000H + 3mvgsin\theta = 6000H - 9mvgsin\theta$$

2000H = $12mvgsin\theta$

$$sin\theta = \frac{2000H}{12mvg}$$

$$\theta = \sin^{-1}\left(\frac{2000H}{12mvg}\right)$$

(c) With maximum speed, a = 0



Net accelerating force = F – (mgsinθ + R_1)

m x 0 = F – (mgsinθ + R_1)

F = R_1 + mgsinθ

Let v1 be maximum velocity

Power, P = Fv_1

2000H = Fv_1

2000H = (R_1 + mgsinθ)v_1

Substituting for R and sinθ

2000H $=\left(\frac{2000H}{v} + mg\left[\frac{2000H}{12mvg}\right]\right)v_1$

2000H $=\left(\frac{2000H}{v} + \frac{2000H}{12v}\right)v_1$

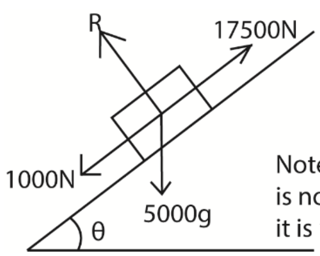
1 $=\left(\frac{1}{v} + \frac{1}{12v}\right)v_1$

1 = $\left(\frac{13}{12v}\right)v_1$

$$v_1 = \frac{12v}{13}$$

Example 31

The engine of a lorry of mass 5,000kg is working at a steady rate of 350 kW against a constant resistance force of 1,000N. The lorry ascends a slope of inclination θ to the horizontal. If the maximum speed of the lorry is 20ms^{-1}, find the value of θ (05marks)



Note resistance of 1000N is not on the surface because it is not friction alone

Resultant force = 17500 – (1000 + 5000sinθ)

5000a = 17500 – (1000 + 5000sinθ)

At maximum speed a = 0

⇨ 0 = 17500 – (1000 + 5000sinθ)
16500 = 5000sinθ
θ = 19.7^0

Example 32

A car is working at 5kW and is travelling at constant speed of 72kmh^{-1}. Find the resistance to motion

Power = 5kW = 5000w

V = 72kmh^{-1} = $\frac{72 \, x \, 1000}{3600} = 20ms^{-1}$

At constant speed, F = R

power = Fv= Rv

R = $\frac{5000}{20} = 250N$

Revision exercise 3

1. A car is drive along a level road against a constant resistance to motion of 400N. Find the maximum speed at which the car can move when its engine works at steady rate of 8.8kW. [22ms^{-1}]
2. What is the maximum speed which a car can travel along road when its engine is developing 24kW and there is a resistance to motion of 800N. [30ms^{-1}]
3. A car is driven along a level road against a constant resistance to motion of 400N. Find the maximum speed at which the car can move when its engine works at a steady rate of 4kW. [10ms^{-1}]
4. A car is working at 14kW and is travelling at constant speed of 75kms^{-1} along a level road. Find the resistance to motion. [187N]
5. A car of mass 1000kg is driven along a level road against constant resistance to motion of 200N. With the engine of car when working at steady rate of 8kW, find
   (i) acceleration of the car when its speed is 5ms^{-1} [1.4ms^{-2}]
   (ii) the maximum speed of the car. [40ms^{-1}]
6. A car of mass 900kg is driven along a level road against a constant resistance to motion of 300N. With the engine working at steady rate of 12kW, find
   (i) acceleration of the car when its speed is 10ms^{-1} [1ms^{-2}]
   (ii) the maximum speed of the car. [40ms^{-1}]

7. A car of mass 1600kg is freewheeling down a hill of slope 1 in 25. When the car descends 200m along a hill, its speed increases from 3ms^{-1} to 10ms^{-1}, find the average resistance to motion [263.2N]
8. A train of mass 100 tonnes has an engine of maximum power 60kW.
   (i) Calculate the force resisting the motion of the car when it is travelling at its maximum speed of 108kmh^{-1} on a level road. [2000N]
   (ii) If the resistance remains unaltered, find the acceleration of the car when travelling at 54kmh^{-1} on a level road with the engine working at the same rate.[0.02ms^{-2}}
9. A cyclist of mass 75kg moves on a level road and working at a rate of 210W against a constant resistance of 21N
   (i) find the maximum speed that a cyclist can attain. [10ms^{-1}]
   (ii) with the resistance and the rate of working unchanged, find the maximum speed can ascend a slope of inclination 1 in 15. [3ms^{-1}]
10. A car of mass 900kg moves on a level road at maximum speed of 48ms^{-1} against a constant resistance of 350N.
    (i) find the rate at which the engine is working. [16.8kw]
    (ii) with the resistance and rate of working unchanged, find the maximum speed the car ascends a slope of inclination 1 in 18 [20ms-1]
11. A car of mass 900kg working at a rate of 19.2kW moves on a level road at maximum speed of 20ms^{-1} against a constant resistance of (160 + av^2)N, where a is a constant and v is the speed of the car in ms^{-1}.
    (i) find the value of a [a = 2]

(ii) with the resistance and the rate of working unchanged, find the maximum speed of the car can ascend a slope of inclination of 1 in 30. [1.40667ms^{-1}]

12. A car of mass 100 metric tonnes moves with a constant speed of 654kmh^{-1} up a slope of inclination$\sin^{-1}\left(\frac{1}{59}\right)$. Given that the engine of the car is working at constant rate of 369kW. Find the resistance to motion. [5000N]
13. A man of mass 70kg rides a bicycle of mass 15kg at steady speed of 4.0ms-1 up a road which rises 1.0m for every 20m of its length. What is his power if resistance to motion is 20N. [250N]
14. A lorry of mass 2000kg moving at 10ms^{-1} on horizontal surface is brought to rest in a distance of 12.5m by the brakes being applied.
    (i) calculate the average retarding force. [8000N]
    (ii) what power must the engine produce if the constant speed of 10ms-1, frictional resistance being 200N. [22kW]
15. A lorry of mass 2.4 metric tonnes carrying goods of 9.8 metric tonnes ascends a hill of 1 in 8 with an acceleration of 0.45ms^{-2}.
    (i) calculate the tractive force of the engine. [20,435N]
    (ii) If at a speed of 36kmh^{-1}, the engine develops a power of 235kW, calculate the resistance of the lorry. 3,065N]
16. A car of mass 2 tonnes moves from rest down a road of inclination $\sin^{-1}\left(\frac{1}{20}\right)$to the horizontal. Given that the engine develops a power of 64.8kW when travelling at a speed of 54kmh^{-1} and the resistance to motion is 500N, find the acceleration. [2.4ms^{-2}]
17. The force opposing the motion of a car is (a + bv^2)N, where a and b are constants and v is the speed of the car in ms^{-1}. the power required to maintain a steady speed of 20ms^{-1} and at 30ms^{-1} is 15.3kW.
    (a) Find the values of a and b. [a = 150, b = 0.4]
    (b) power developed for a steady speed of 40m^{-1} [31.6kW]
18. A car is driven at uniform speed of 48kmh^{-1} up a smooth incline of 1 in 8. If the total mass of the car is 800kg and the resistance are neglected, calculate the power at which the car is working. [13076W]
19. A truck of mass 2000kg moving at constant speed of 10ms^{-1} up a hill which is inclined at an angle α, where sinα = 0.25. There is a constant force of 400N opposing the motion.
    (i) calculate the power used[53kW]
    (ii) If the power is instantaneously increased to 75kW, calculate the rate at which the truck begins to accelerate. [1.1ms^{-2}]
20. With its engine working at a constant rate of 9.8kW, a car of mass 800kg can descend a slope of 1 in 56 at twice the steady speed that it can ascend the same slope, the resistance to motion remaining the same throughout. Find the magnitude of the resistance and the speed of ascent. [420N, 17.5ms^{-1}]
21. The frictional resistance to motion is (kv)N where k is a constant and v is the speed of a car in ms-1. The car ascends a hill of inclination 1 in 10 at a steady speed of 8ms^{-1}. The power exerted by the engine is 9.76kW.
    (i) find the value of k [k = 30]
    (ii) Find the steady speed at which the car ascends the hill if the power exerted by the engine is 12.8kW. When the car is travelling at this speed, the power exerted by the engine is increased by 2kW, find the immediate acceleration of the car. [10ms^{-1}, 0.2ms^{-1}]

Pump raising and ejecting water

Consider a pump which is used to raise water from the source and then eject it at a given speed. The work done per second gives the rate (power) at which the pump is working.

Power of the pump = P.E given to water per second + K.E given to water per second

Example 32

A pump raises water through a height of 3.0m at a rate of 300kg per minute and delivers it with a velocity of 8.0ms^{-1}. Calculate the power output of the pump.

solution

Power of the pump = P.E given to water per second + K.E given to water per second

work done per second = (mass per second x g x h) + ($\frac{1}{2}$ $x\ mass\ per\ second\ x\ v^2$)

$$= \frac{300}{60}\ x\ 9.8\ x\ 3 + \frac{1}{2}\ x\ \frac{300}{60}\ x\ 8^2 = 310W$$

Example 33

A pump draws 6m3 of water of density 1000kgm^{-3} from a well 9m below the ground in every minute and issues it at a speed of 12ms^{-1}. Find the rate at which the pump is working.

Solution

Power of the pump = P.E given to water per second + K.E given to water per second

work done per second = (mass per second x g x h) + ($\frac{1}{2}$ $x\ mass\ per\ second\ x\ v^2$)

$$= \frac{6\ x\ 1000}{60}\ x\ 9.8\ x\ 9 + \frac{1}{2}\ x\ \frac{6\ x\ 1000}{60}\ x\ 12^2 = 16020W$$

Example 34

A pump raises water through a vertical distance of 10m in one and half minutes, and discharges it at a speed of 25ms-1. Show that the power developed is approximately 2.25kW.

Solution

Power of the pump = P.E given to water per second + K.E given to water per second

work done per second = (mass per second x g x h) + ($\frac{1}{2}$ $x\ mass\ per\ second\ x\ v^2$)

$$= \frac{2\ x\ 1000}{90}\ x\ 9.8\ x\ 10 + \frac{1}{2}\ x\ \frac{2\ x\ 1000}{90}\ x\ 25^2 = 2247.22W \approx 2.25kW$$

Example 35

A pump draws 3.6m^3 of water of density 1000kgm^{-3} from a well 5m below the ground in every minute and issues it at ground level through a pipe of cross-section area 40cm^2. Find

(i) the speed with which water leaves the pipe.
(ii) the rate at which the pump is working
(iii) if the pump is only 80% efficient, find the rate at which it must work
(iv) find the power wasted.

Solution

(i) volume per second = area x velocity

$\frac{3.6}{60} = \frac{40}{10000}$v

$v = 15ms^{-1}$

(ii) Power of the pump = P.E given to water per second + K.E given to water per second

work done per second = (mass per second x g x h) + ($\frac{1}{2}$ x $mass$ per $second$ x v^2)

$$= \frac{3.6 \, x \, 1000}{60} \, x \, 9.81 \, x \, 5 + \frac{1}{2} \, x \, \frac{3.6 \, x \, 1000}{60} \, x \, 15^2 = 9693W$$

(iii) efficiency = $\frac{power\ output}{power\ input}$ x 100%

80% = $\frac{9693}{power\ input}$ x 100%

power input = 12116.25W

(iv) Power wasted = power input – power output

= 12116.25 – 9693 = 2423.25W

Revision exercise 4

1. A pump with power output of 600W raises water from a lake a height of 3.0m and delivers it with a velocity of 6.0ms^{-1}. What is the mass of water removed from the lake in one minute? [7500kg]
2. In every minute a machine pumps 300kg of water along a horizontal hose from rest at one end to eject at a speed of 4ms^{-1} at the other. Find the average rate at which the machine is working? [40W]
3. In every minute a pump draws 6m^3 of water from a well and issues it at a speed of 5ms^{-1} from a nozzle situated 4m above the level from which the water is drawn. Find the average rate at which the pump is working. [5.17W]
4. In every minute a pump draws 5m^3 of water from a well and issues it at a speed of 6ms^{-1} from a nozzle situated 6m above the level from which water was drawn. Find the average rate at which the pump is working. [6.4kW]
5. A pump draws water from a tank and issues it at a speed of 8ms^{-1} from the end of a pipe of cross-section area of 0.01m^2 situated at 10m above the level from which the water is drawn. find
   (i) the mass of water issued from the pipe in each second. [80kg]
   (ii) the rate at which the pump is working. [10.4kW]
6. A pump draws water from a tank and issues it at a speed of 10ms^{-1} from the end of the pipe of cross-sectional area 5cm^2 situated at 4m above the level from which the water is drawn. Find the rate at which the pump is working. [446W]
7. In every minute a pump draws 2.4m^3 of water from a well 5m below the ground and issues it at ground level through a pipe of cross-sectional area 50cm^2. Find the
   (i) speed with which the water leaves the pipe. [8ms^{-1}]
   (ii) rate at which the pump is working if its only 75% efficient. [3.24kW]
8. In every minute a pump working at 3.48kW raises 15m3 of water from underground tank and issues it from the end of a pipe situated at ground level. The water leaves the pipe with a speed of 10ms-1 and the pump is 50% efficient. Find
   (i) the cross-sectional area of the pipe. [25cm^2]

(ii) the depth below the ground level from which the water is drawn. [2m]

9. In each minute a pump working at 825W draws 0.3m^3 of water from a well and issues from the nozzle situated 5m above the level from which the water was drawn. If the pump is 60% efficient, find
   (i) the velocity with which water is ejected. [10ms-1]
   (ii) the area of a cross-section of the nozzle. [5cm^2]
10. A pump raises 75kg of water a vertical distance of 20m in 14s. Find the average rate at which the pump is working. [1.05kW]