OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL APPLIED MATHEMATICS SEMINAR SOLUTIONS 2023

1 (a)

x	f	xf	x^2	x^2f
0	P	0	0	0
1	q	q	1	q
sum	P+q	q		q

(i) Mean= $\frac{q}{p+q}$

(ii) S.D= $\sqrt{\frac{q}{p+q} - (\frac{q}{p+q})^2} = \frac{\sqrt{pq}}{p+q}$

(b) (i)

X	f	Xf	X^2f
2	1	2	4
3	1	3	9
6	1	6	36
9	1	9	81
sum	4	20	130

Mean= $\frac{20}{4} = 5$

S.D= $\sqrt{\frac{130}{4} - 5^2} = 2.73861\ (4\ dps)$

(ii) old mean=5
new mean =6 (since the mean was increased by one)
old Var(x)=7.5
new Var(x)=10 (since it was increased by 2.5)

$$6 = \frac{20 + a + b}{6}$$
$$a + b = 16 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$$
$$10 = \frac{a^2 + b^2 + 130}{6} - 6^2$$
$$a^2 + b^2 = 146 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Solving $(i)\ and\ (ii)$

$$a = 16 - b$$
$$(16 - b)^2 + b^2 = 146$$
$$2b^2 - 23b - 110 = 0$$
$$b = \frac{32 \pm \sqrt{32^2 - 4x2x110}}{4}$$
$$b = 5\ or\ 11$$
$$\leftrightarrow a = 11\ or\ 5$$
$$\therefore the\ values\ of\ a\ and\ b\ are\ either\ 11\ or\ 5$$

(c)

R_H	R_m	d	d^2
1	2	-1	1
2	1	1	1
3	4	-1	1
4	3	1	1
5	7	-2	4
6	5	1	1
7	6	1	1

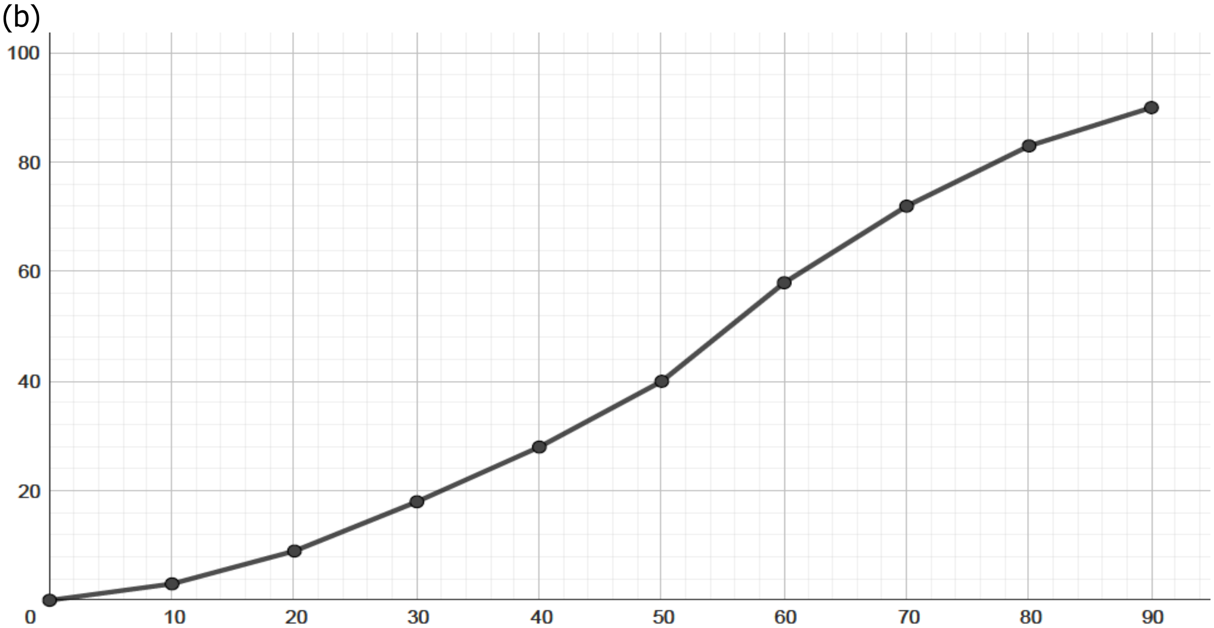
$$\rho = 1 - \frac{6x10}{7(7^2-1)} = 0.8214$$

2

Marks	Freq. density	Class interval	f	x	xf	x^2f	F
0-10	0.3	10	3	5	15	75	3
10-20	0.6	10	4	15	90	1350	9
20-30	0.9	10	9	25	225	5625	18
30-40	1.0	10	10	35	350	12250	28
40-50	1.2	10	12	45	540	24300	40
50-60	1.8	10	18	55	990	54450	58
60-70	1.4	10	14	65	910	59150	72
70-80	1.1	10	11	75	825	61875	83
80-90	0.7	10	7	85	595	50575	90
sum			90		4540	247780	

Mean $=\frac{4540}{90} = 50.4444$ standard deviation$=\sqrt{\frac{247780}{90} - \left(\frac{4540}{90}\right)^2} = 14.4385$

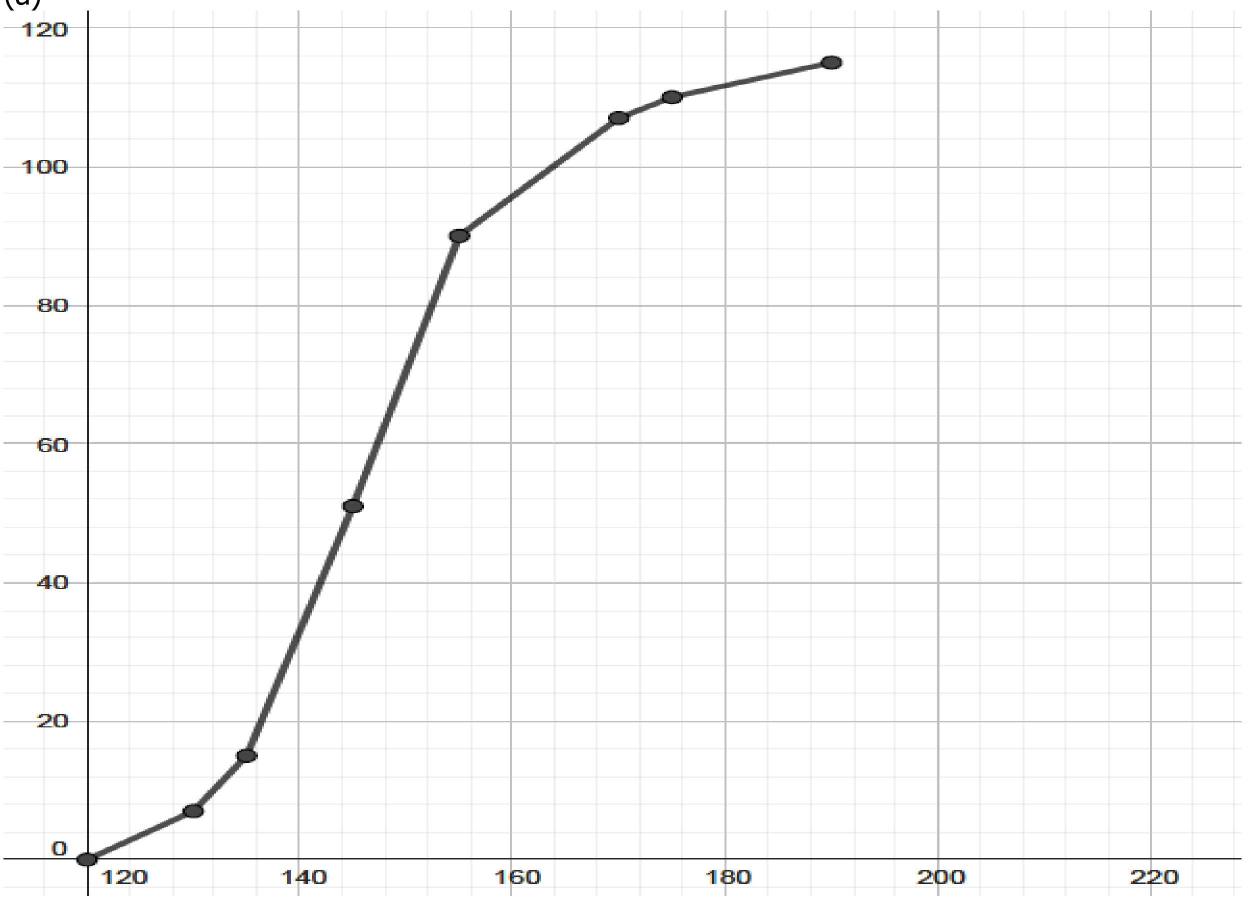
(b)



3

Height	f	x	xf	x^2f	F
120-130	7	125	875	109375	7
130-135	8	132.5	1060	140450	15
135-145	36	140	5040	705600	51
145-155	39	150	5850	877500	90
155-170	17	162.5	2762.5	448906.25	107
170-175	3	172.5	517.5	89268.75	110
175-190	5	182.5	912.5	166531.25	115
sum			17017.5	2537631.25	

(a)



(i) median$=135+\left(\frac{57.5-51}{36}\right)x10=136.80556$

(ii)

height	145	150	155
F	51	x	90

$$\frac{x-51}{150-145}=\frac{90-51}{155-145}$$
$$X=70.5\approx 71$$
$$\therefore The\ number\ less\ than\ the\ height\ of\ 150=71$$

(b) Mean $=\frac{17017.5}{115}=147.97826$

standard deviation$=\sqrt{\frac{2537631.25}{115}-\left(\frac{17017.5}{115}\right)^2}=12.9920$

4

Weight	f	X	Xf	X^2f	F	Interval	Freq.density
0-0.10	2	0.05	0.1	0.005	2	0.1	20
0.10-0.25	3	0.175	0.525	0.091875	5	0.15	20
0.25-035	5	0.3	1.5	0.45	10	0.1	50
0.35-0.50	9	0.425	3.825	1.625625	19	0.15	60
0.50-0.60	3	0.55	1.65	0.9075	22	0.1	30
0.60-0.65	2	0.625	1.25	0.78125	24	0.05	40
0.65-0.80	3	0.725	2.175	1.576875	27	0.15	20
sum			11.025	5.438125			

(a) i) S.D=$\sqrt{\frac{5.438125}{27}-\left(\frac{11.025}{27}\right)^2} = 0.1862\ (4\ dps)$

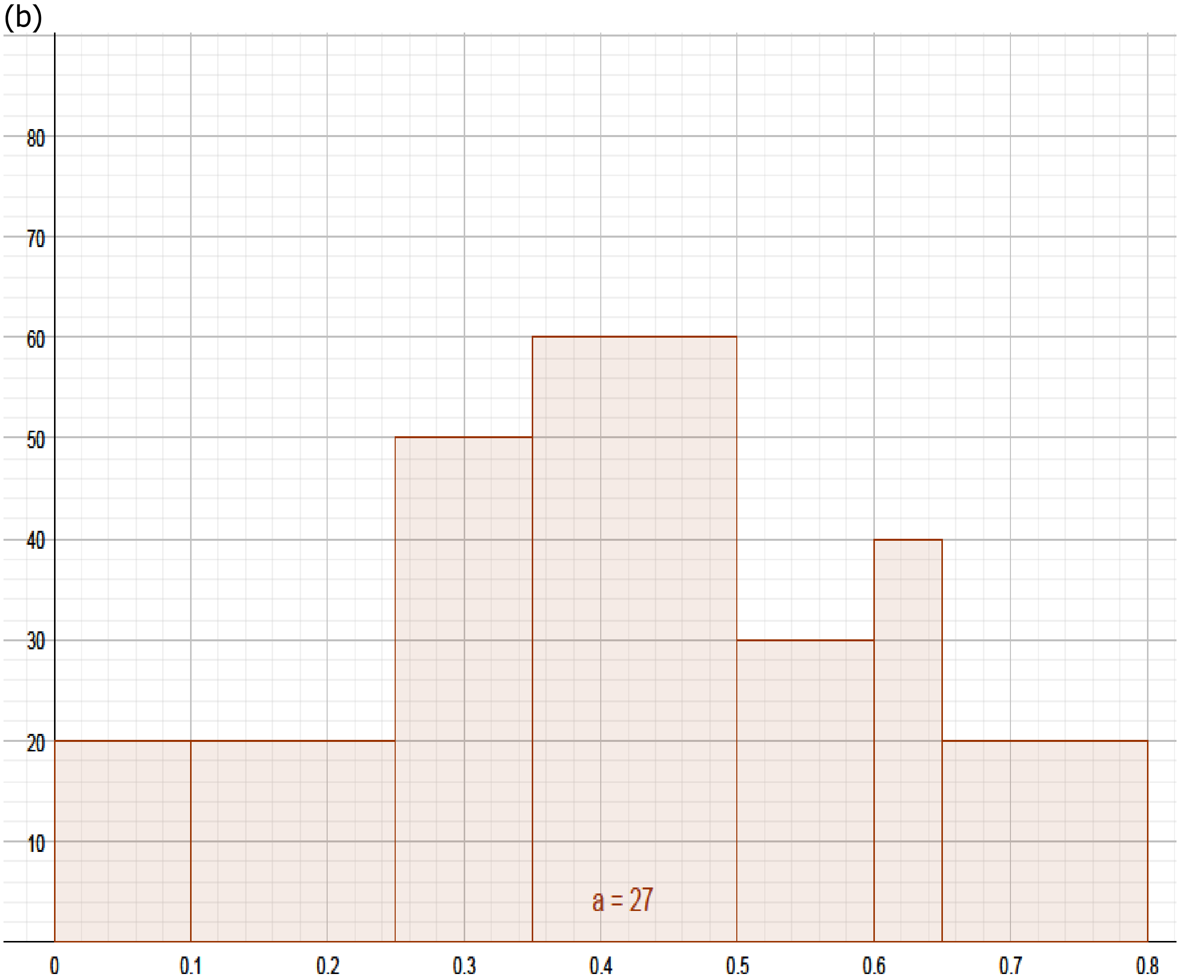
(ii)

Weight	0.50	0.57	0.60
F	19	x	22

$\frac{x-19}{0.57-0.50} = \frac{22-19}{0.6-0.5}$ $\qquad x = 21.1$

$\therefore Number\ of\ seedling\ that\ weigh\ more\ 0.57g = 27 - 21 = 6\ seedling$

(b)



Mode =0.39

5

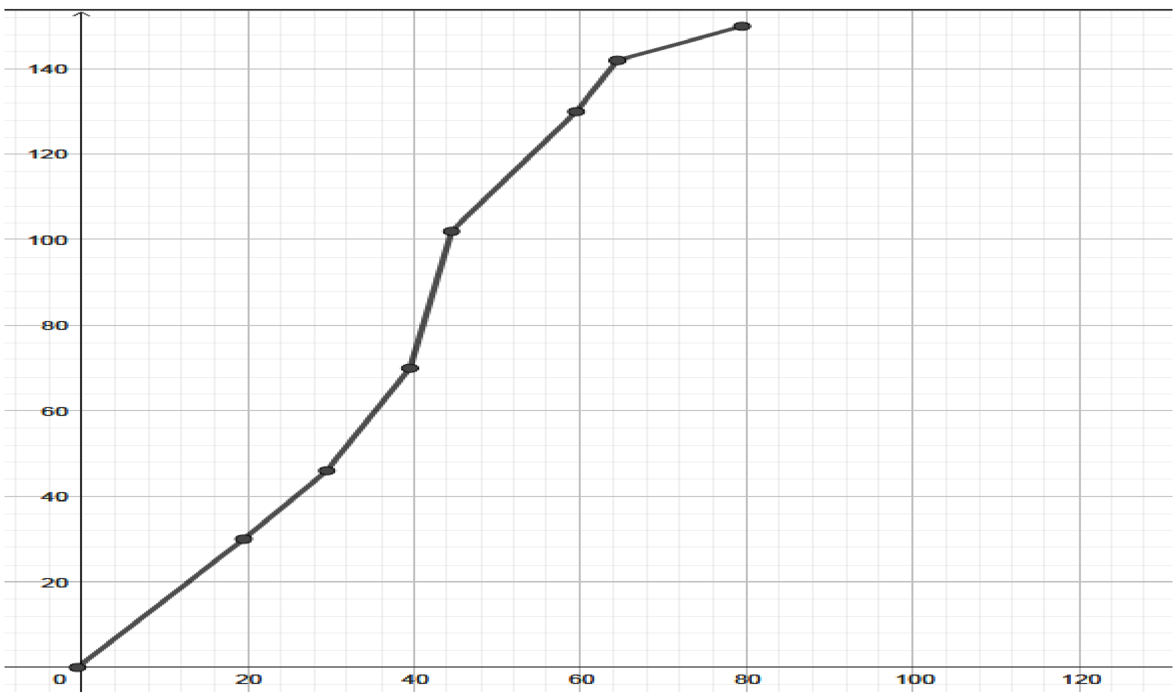
Weight	f	X	Xf	interval	Freq. density	F
0-19	30	9.5	285	20	1.5	30
20-29	16	24.5	392	10	1.6	46
30-39	24	34.5	828	10	2.4	70
40-44	32	42	1344	5	6.4	102
45-59	28	52	1456	15	1.867	130
60-64	12	62	744	5	2.4	142
65-79	8	72	576	15	0.5333	150

a)

$$Mean = \frac{5625}{150} = 37.5$$

$$Modal\ weight = 39.5 + \left(\frac{4}{4+4.533}\right)x5 = 41.8438$$

b)



6

(i) P(AUB)=q

P(A)+P(B)-P(AnB)=q

$$\frac{5}{12} + \mathrm{P(B)} - \frac{1}{6} = q$$

$$\mathrm{P(B)} = \frac{1}{4}(4q-1)$$

(ii) $P\left({}^{A}/_{B}\right) = \frac{P(AnB)}{P(B)} = \frac{{}^{1}/_{6}}{\frac{1}{4}(4q-1)} = \frac{2}{3(4q-1)}$

(iii) $P\left({}^{A}/_{B}\right) = P(A)$

$$\frac{2}{3(4q-1)} = \frac{5}{12}$$

$$q = \frac{13}{20}$$

(b) $P(3G, 2R) = \frac{\binom{8}{3}x\binom{4}{2}}{\binom{12}{5}} = \frac{14}{33} = 0.4242$

7

$$\int_1^3 \frac{k}{x(4-x)}dx \quad = 1$$

Let $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$

$$1 \equiv A(4-x) + B(x)$$
$$A = \frac{1}{4} \; and \; B = \frac{1}{4}$$

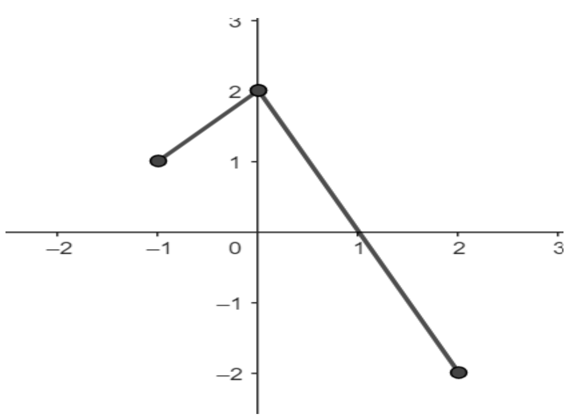
$\int_1^3 \frac{k}{x(4-x)}dx \quad = \frac{k}{4}\int_1^3 \frac{1}{x}dx + \frac{k}{4}\int_1^3 \frac{1}{4-x}dx = 1$

$$\frac{k}{4}[lnx - \ln(4-x)]_1^3 = 1$$
$$k = \frac{2}{ln3}$$
$$Var(x) = E(x^2) - E(x)^2$$
$$E(x) = \int_1^3 \frac{k}{(4-x)}dx \quad = -k[\ln(4-x)]_1^3 = \frac{-2[ln1 - ln3]}{ln3} = 2$$
$$E(x^2) = k\int_1^3 \frac{x}{(4-x)}dx$$
$$let\; u = 4-x$$
$$du = -dx$$
$$x = 4-u$$

x	u
1	3
3	1

$$E(x^2) = k\int_3^1 \frac{4-u}{u}(-du)$$
$$Var(x) = \frac{2}{ln3}[u - 4lnu]_3^1 - 4 = 0.3590$$

(b) (i)



$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

$$c\int_{-1}^{0}(x+2)dx + 2c\int_{0}^{1}(1-x)dx + -2c\int_{1}^{2}(1-x)dx = 1$$

$$c\left[\frac{x^2}{2}+2x\right]\begin{matrix}0\\-1\end{matrix} + 2c\left[x-\frac{x^2}{2}\right]\begin{matrix}1\\0\end{matrix} + -2c\left[x-\frac{x^2}{2}\right]\begin{matrix}2\\1\end{matrix} = 1$$

$$\frac{3c}{2} + 2cx\frac{1}{2} + 2cx\frac{1}{2} = 1$$

$$c = \frac{2}{7}$$

Or

Using the area under the curve

$$\frac{1}{2}x\ 1(c+2c) + \frac{1}{2}x1(2c) + \frac{1}{2}(2c) = 1$$

$$\frac{3c}{2} + 2c = 1$$

$$c = \frac{2}{7}$$

(ii) $P(|x-1| < 0.5) = P(-0.5 < x-1 < 0.5)$

$$= P(0.5 < x < 1.5)$$

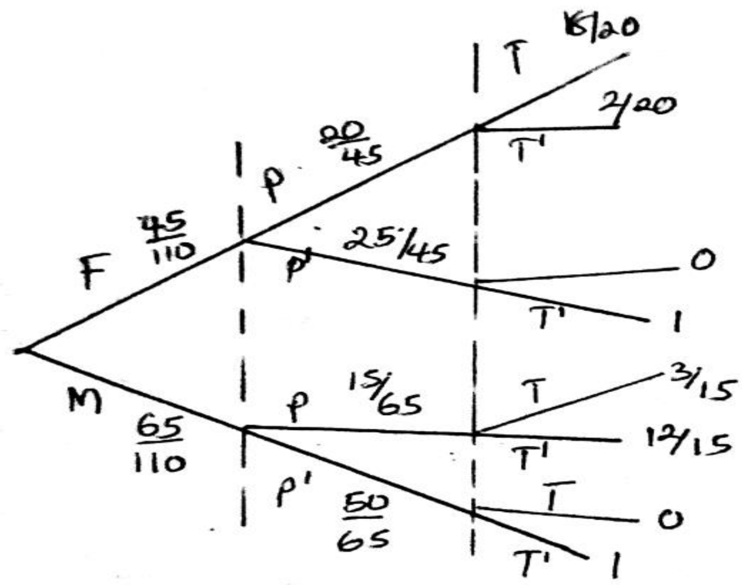
$$= 2c\int_{0.5}^{1}(1-x)dx + -2c\int_{1}^{1.5}(1-x)dx$$

$$= 2c\left[x-\frac{x^2}{2}\right]\begin{matrix}1\\0.5\end{matrix} + -2c\left[x-\frac{x^2}{2}\right]\begin{matrix}1.5\\1\end{matrix}$$

$$= 2cx\frac{1}{8} - 2c\left(-\frac{1}{8}\right)$$

$$= \frac{c}{2} = \frac{1}{7}$$

8 (i)



$$P(P) = P(FnP) + P(MnP) = \left(\frac{45}{11}x\frac{20}{45}\right) + \left(\frac{65}{110}x\frac{15}{65}\right) = \frac{7}{22}$$

OR

$$P(P) = \frac{n(E)}{n(S)} = \frac{20+15}{110} = \frac{7}{22}$$

	ii) $P(T) = P(FnPnT) + P(MnPnT)$ $= \frac{18}{110} + \frac{3}{110} = \frac{21}{110}$ iii) $P\left(\frac{M}{T}\right) = \frac{P(MnT)}{P(T)} = \frac{\frac{3}{110}}{\frac{21}{110}} = \frac{1}{7}$
9	$X \sim N(\mu, \delta)$ $P(X < 35) = 0.2$ $\frac{35 - \mu}{\delta} = -\emptyset^{-1}(0.3)$ $\mu - 0.842\delta = 35$…………………(1) $P(35 < X < 45) = 0.65$ $P(X < 45) = 0.2 + 0.65$ $\frac{45 - \mu}{\delta} = -\emptyset^{-1}(0.35)$ $\mu + 1.036\delta = 45$……………………(2) Solving (1) and (2) $\mu = 39.4835, \delta = 5.3248$ $Y \sim B(10, p)$ $p = P(x > 40)$ $= p\left(Z > \frac{40 - 39.4835}{5.3248}\right)$ $= P(z > 0.097)$ $= 0.5 - P(0 < z < 0.097))$ $= 0.5 - 0.0387$ $= 0.4613$ $\leftrightarrow q = 0.5387$ $P(Y \geq 1) = 1 - P(Y = 0)$ $= 1 - \binom{10}{0}(0.4613)^0(0.5387)^9)$ $= 0.9962$ (b) $X \sim B(200,0.45)$ $E(X) = np = 200X0.45 = 90$ $Var(x) = npq = 200x0.45x0.55 = 49.5$ $X \sim N(90,49.5)$ $P(x > 97) = P(x \geq 98) = P\left(Z > \frac{97.5 - 90}{\sqrt{49.5}}\right) = P(Z > 1.066)$ $= 0.5 - P(0 < Z < 1.066)$ $= 0.5 - 0.3568$ $= 0.1432$ (c) $X \sim N(\mu, \delta), n = 100$ $Z_{\frac{\propto}{2}} = \emptyset^{-1}\left(\frac{0.95}{2}\right) = 1.96$ $\bar{x} - Z_{\frac{\propto}{2}}\frac{\delta}{\sqrt{n}} = 177.22$ $\bar{x} - 0.196\delta = 177.22 \ldots\ldots(1)$

$$\bar{x} + Z_{\frac{\propto}{2}}\frac{\delta}{\sqrt{n}} = 179.18$$

$$\bar{x} + 0.196\delta = 179.18 \ldots.(2)$$

Solving (1) and (2)

$$\delta = 5 \text{ and } \bar{x} = 178.2 \text{ cm}$$

10

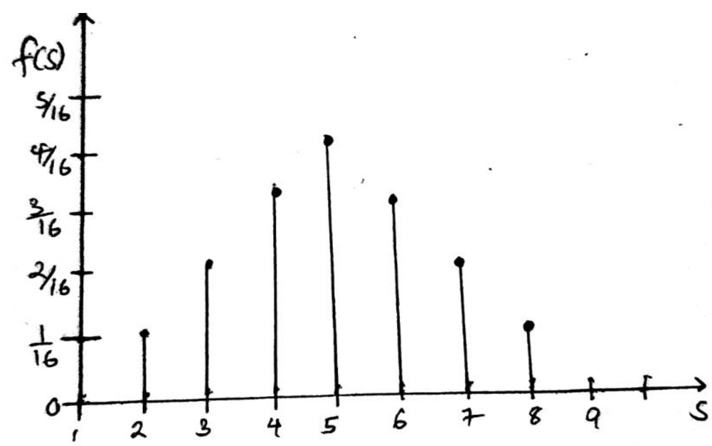
	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$$S \sim 2,3,4,5,6,7,8$$

S	2	3	4	5	6	7	8	
P(S=s)	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	
S P(S=s)	$\frac{2}{16}$	$\frac{6}{16}$	$\frac{11}{16}$	$\frac{20}{16}$	$\frac{18}{16}$	$\frac{14}{16}$	$\frac{8}{16}$	5

$$f(s) = P(S = s) = \begin{cases} \frac{s-1}{16}; 2,3,4 \\ \frac{9-s}{16}; 5,6,7,8 \\ 0; \quad elsewhere \end{cases}$$

(ii) sketch



exception $E(x) = \sum \mathrm{S\,P(S = s)} = 5$

b) $P(H) = \frac{2}{3}$ $\quad P(T) = \frac{1}{3}$ $\quad X \sim B(3, \frac{1}{3})$; $P(X = x) = \binom{n}{x} p^x q^{n-x}$)

$$P(X = 0) = \binom{3}{0}(\frac{1}{3})^0(\frac{2}{3})^3) = \frac{8}{27}$$

$$P(X = 1) = \binom{3}{1}(\frac{1}{3})^1(\frac{2}{3})^2) = \frac{4}{9}$$

$$P(X = 2) = \binom{3}{2}(\frac{1}{3})^2(\frac{2}{3})^1) = \frac{2}{9}$$

$$P(X = 3) = \binom{3}{3}(\frac{1}{3})^3(\frac{2}{3})^0) = \frac{1}{27}$$

x	0	1	2	3
P(X=x)	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$
s	1000	3000	6000	10000

$$\sum \mathrm{S\ P(X = s)} = \left(1000x\frac{8}{27}\right) + \left(3000x\frac{4}{9}\right) + \left(6000\ x\frac{2}{9}\right) + \left(10000x\frac{1}{27}\right) = 4391.5344$$
$$E(12s) = 4391.5344x12 = 52698.4128$$
$$Loss = (12x5000) - 52698.4128 = 7301.5872\ ugx$$

11

$$xy = 12y = \frac{12}{x}$$
$$with\ d = \frac{4-1}{6} = \frac{1}{2}$$

X	$y = \frac{12}{x}$	
1.0	12	
1.5		8
2.0		6
2.5		4.8
3.0		4
3.5		3.428571
4.0	3	
sum	**15**	**26.22857**

$$\int_1^4 \frac{12}{x}\,dx \approx 0.5x0.5\big(15 + 2(26.2287)\big)$$
$$\approx 16.86435$$

(b) the exact value of $\int_1^4 \frac{12}{x} dx = 12\ lnx|_1^4$ =12(ln4 – ln1) =16.63553

(c) the percentage error =$\frac{|16.63553-16.86435|}{16.63553} x100\%$ =1.3755%

The error can be reduced by increasing the number of ordinates

12

length	530	540	550
No. of components	15	x	39

Let x be the No. of components with life length of 540

$$\frac{x-15}{540-530} = \frac{39-15}{550-530}$$
$$x = 27$$

Let y be the No. of components with life length of 580

length	570	580	600
No. of components	72	y	93

Let y be the No. of components with life length of 580

$$\frac{y-72}{580-570}=\frac{93-72}{600-570} \quad ; \quad y=79$$

$$prob=\frac{79-27}{100}=0.52$$

(b)

duration	27	30	36
No. of calls	37	N	57

Let N be the number of calls exceeds 30 minutes

$$\frac{N-37}{30-27}=\frac{57-37}{36-27}$$
$$N=43.6666\approx 44$$
$$prob=\frac{60-44}{60}=\frac{4}{15}=0.2\bar{6}$$

13 (a)

value	error	Max value	Min value
A=3.3366	± 0.00005	3.33665	3.33655
B=0.559	± 0.0005	0.5595	0.5585

Since A is greater than B, then the bigger the negative the minimum it is and the smaller the negative the maximum it is.

$$\min value=\left(\frac{B-A}{AB}\right)_{max}=\frac{B_{max}-A_{min}}{A_{min}.B_{min}}=\frac{0.5595-3.33655}{3.33655x0.5585}=-1.49026$$
$$\max value=\left(\frac{B-A}{AB}\right)_{min}=\frac{B_{min}-A_{max}}{A_{max}.B_{max}}=\frac{0.5585-3.33665}{3.33665x0.5595}=-1.4881$$
$$Interval=[-1.490,-1.488]$$

(b) $T=2\pi\sqrt{\frac{l}{g}}$ let the error in T be $\partial T$and error in l be ∂l

$$T^2=4\pi^2\frac{l}{g}$$
$$(T+\partial T)^2=4\pi^2\frac{l+\partial l}{g}$$
$$T^2+2T\partial T+\partial T^2=4\pi^2\frac{l+\partial l}{g}$$
$$2T\partial T+\partial T^2=\frac{4\pi^2 l}{g}+\frac{4\pi^2\partial l}{g}-T^2$$

Assuming that $\partial T \ll T, \partial T^2\approx 0$

$$2T\partial T=\frac{4\pi^2\partial l}{g}$$

Divide through by T^2

$$\frac{\partial T}{T}=\frac{1}{2}\frac{\partial l}{l}$$

multiplying through by 100

$$\frac{\partial T}{T}x100 = \frac{1}{2}\frac{\partial l}{l}x100$$

$$\frac{\partial T}{T}x100 = \frac{1}{2}x4\%$$

=2%

14

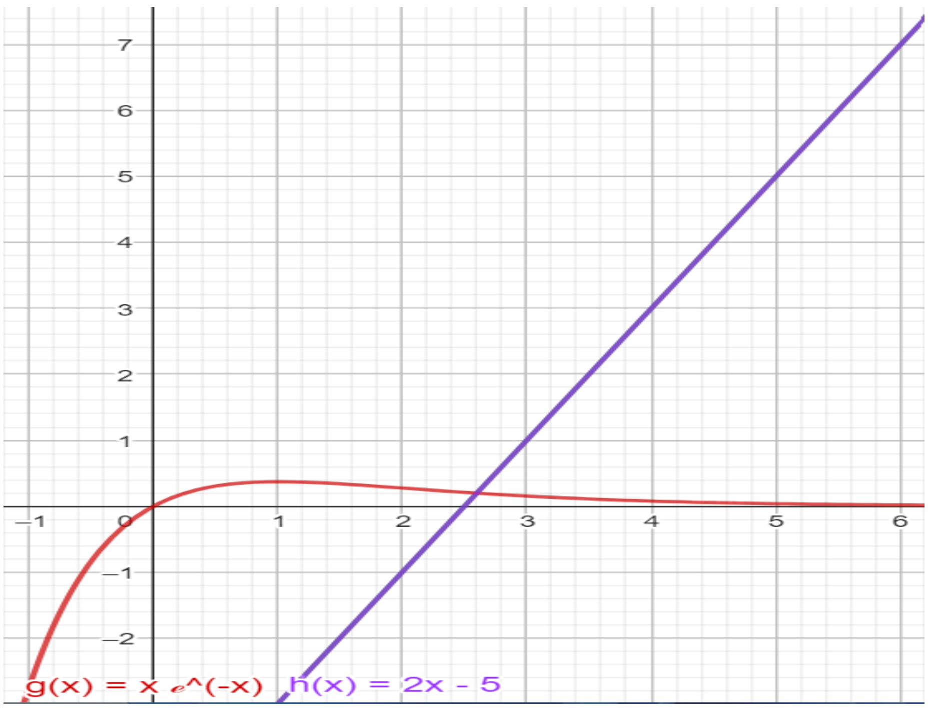
x	0	1	2	3	4
Y=2x-5	-5	-3	-1	1	3
$y = xe^{-x}$	0	0.368	0.271	0.149	0.07

Let $f(x) = xe^{-x} - 2x + 5$ $\quad f'(x) = e^{-x} - xe^{-x} - 2$

$f(x_n) = x_n e^{-x_n} - 2x_n + 5$ $\quad f'(x_n) = e^{-x_n} - x_n e^{-x_n} - 2 = e^{-x_n}(1 - x_n) - 2$

From $x_{n+1} = x_n - \frac{x_n e^{-x_n} - 2x_n + 5}{e^{-x_n}(1-x_n) - 2} = \frac{x_n[e^{-x}(1-x_n) - 2] - x_n e^{-x_n} - 2x_n + 5}{e^{-x_n}(1-x_n) - 2} = \frac{{x_n}^2 e^{-x_n} - 5}{e^{-x_n}(1-x_n) - 2}$

From the graph



Using $x_0 = 2.5$, tol=0.005

$$x_1 = \frac{2.5^2 e^{-2.5} - 5}{e^{-2.5}(1 - 2.5) - 2} = 2.1134$$

$$Error\ = |2.1134 - 2.5| = 0.3866 > 0.005$$

$$x_2 = \frac{2.1134^2 e^{-2.1134} - 5}{e^{-2.1134}(1 - 2.1134) - 2} = 2.0896$$

$$Error\ = |2.0896 - 2.1134| = 0.0238 > 0.005$$

$$x_2 = \frac{2.0896^2 e^{-2.0896} - 5}{e^{-2.0896}(1 - 2.0896) - 2} = 2.0890$$

$$Error\ = |2.0890 - 2.0896| = 0.0006 < 0.0006$$

Hence the root is 2.09 (2 dps)

15 a) Let length be $l = 1.25km$, and the error in l be ∂l

Width be $w = 0.44km$ and the error in w be ∂w

So $5 = \frac{\partial l}{1.25}x100$
$\partial l = 0.0625$
$4.2 = \frac{\partial w}{0.44}x100$
$\partial w = 0.01848$
The area of the plot $= lxw$
Lower limit $= l_{min}xw_{min} = (1.25 - 0.0625)(0.44 - 0.01848) = 0.500555 = 0.50(2\, sfgs)$
upper limit $= l_{max}xw_{max} = (1.25 + 0.0625)(0.44 + 0.01848) = 0.601755 = 0.60(2\, sfgs)$
(b) let volume of the cylinder be $v = \pi r^2 h$ and error in the volume be Δv
the exact height$= h + \Delta_1$ and the exact radius $= r + \Delta_2$

$$v + \Delta v = \pi(r + \Delta_2)^2(h + \Delta_1)$$
$$v + \Delta v = \pi(r^2 + 2r\Delta_2 + {\Delta_2}^2)(h + \Delta_1)$$

On expanding

$$\Delta v = 2\pi r\Delta_2 h + 2r\pi\Delta_2\Delta_1 + \pi h{\Delta_2}^2 + \pi{\Delta_2}^2\Delta_1 + \pi r^2\Delta_1$$

Assuming $|\Delta_1| \ll h$, $|\Delta_2| \ll r$ then ${\Delta_2}^2 \approx 0$, $\Delta_2\Delta_1 \approx 0$; $\Delta v = 2\pi r\Delta_2 h + \pi r^2\Delta_1$

$$|\Delta v| = |2\pi r\Delta_2 h + \pi r^2\Delta_1| \leq |2\pi r\Delta_2 h| + |\pi r^2\Delta_1|$$
$$\frac{|\Delta v|}{v} = \left|\frac{2\pi r\Delta_2 h}{\pi r^2 h}\right| + \left|\frac{\pi r^2\Delta_1}{\pi r^2 h}\right|$$

The maximum percentage error in volume$=2\left|\frac{\Delta_2}{r}\right| + \left|\frac{\Delta_1}{h}\right|$

16

$$r = \left(cost\tilde{\imath} + sint\tilde{\jmath} + \frac{1}{2}t^2\tilde{k}\right)$$

i) $K.E = \frac{1}{2}mv^2$

$$v = \frac{dr}{dt} = \frac{d}{dt}\left(cost\tilde{\imath} + sint\tilde{\jmath} + \frac{1}{2}t^2\tilde{k}\right) = \begin{pmatrix} -sint \\ cost \\ t \end{pmatrix}$$
$$v^2 = v.v = \begin{pmatrix} -sint \\ cost \\ t \end{pmatrix}.\begin{pmatrix} -sint \\ cost \\ t \end{pmatrix} = sin^2t + cos^2t + t^2 = 1 + t^2$$
$$K.E = 0.5x4(1 + t^2) = 2(1 + t^2)\, J$$

ii) $p = \underline{F}.\underline{v}$
but F=ma $a = \frac{dv}{dt} = \frac{d}{dt}\begin{pmatrix} -sint \\ cost \\ t \end{pmatrix} = \begin{pmatrix} -cost \\ -sint \\ 1 \end{pmatrix}$; $F = \begin{pmatrix} -4cost \\ -4sint \\ 4 \end{pmatrix}N$

$P = \begin{pmatrix} -4cost \\ -4sint \\ 4 \end{pmatrix}.\begin{pmatrix} -sint \\ cost \\ t \end{pmatrix} = 4costsint - 4sintcost + 4t = 4tW$

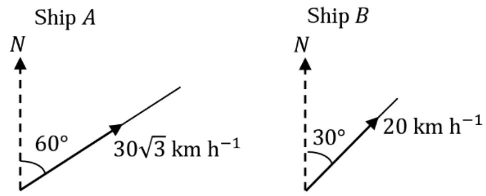
iii) $W = F.r\ or\ W = \int pdt = \begin{pmatrix} -4cost \\ -4sint \\ 4 \end{pmatrix}.\begin{pmatrix} cost \\ sint \\ \frac{1}{2}t^2 \end{pmatrix} = -4cos^2t - 4sin^2t + 2t^2 = -1 + 2t^2\, J$

$$at\ t = 4s\, , W_{t=4} = 31$$
$$at\ t = 0\, , W_{t=0} = -1$$

$Work\ done\ = 31 - -1 = 32J$

$$0r = \int_0^4 4t\, dt = 2t^2 \Big|_0^4 = 32J$$

17	Vector diagrams

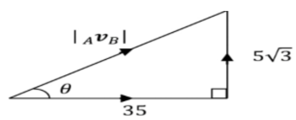


$$V_A = \begin{pmatrix} 30\sqrt{3}sin60 \\ 30\sqrt{3}cos60 \end{pmatrix} \qquad V_B = \begin{pmatrix} 20sin30 \\ 20cos30 \end{pmatrix}$$

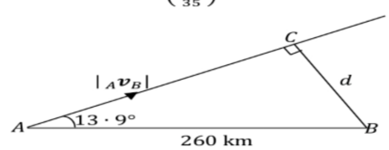
$$V_A = \begin{pmatrix} 45 \\ 15\sqrt{3} \end{pmatrix} \qquad V_B = \begin{pmatrix} 10 \\ 10\sqrt{3} \end{pmatrix}$$

${}_AV_B = \begin{pmatrix} 45 \\ 15\sqrt{3} \end{pmatrix} - \begin{pmatrix} 10 \\ 10\sqrt{3} \end{pmatrix} = \begin{pmatrix} 35 \\ 5\sqrt{3} \end{pmatrix} kmh^{-1}$

${}_AV_B = \sqrt{35^2 + (5\sqrt{3})^2} = 10\sqrt{13}\ kmh^{-1}$



$$\tan\theta = \left(\frac{5\sqrt{3}}{35}\right) \Rightarrow \theta = 13\cdot 9°$$



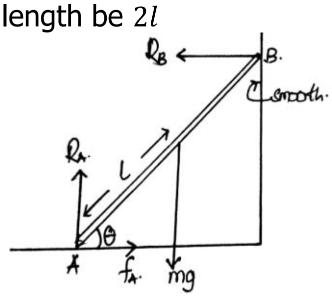
$$\sin 13.9 = \frac{d}{260}$$

$$d = 62.45km$$

$$d = 62km\ to\ nearst\ km$$

$$t = \frac{|\overline{AC}|}{|\overline{\mathrm{AVB}}|} = \frac{\sqrt{260^2 - 62^2}}{10\sqrt{13}} = 7.0\ hours$$

18 a) let m be the mass of the ladder and the length be $2l$



$$\uparrow;\ R_A = mg$$

$$\rightarrow;\ R_B = f_A = \mu R_A$$

Taking moment at A, $mglcos\theta = 2lR_Bsin\theta$

$$mgcos\theta = 2\mu mgsin\theta$$

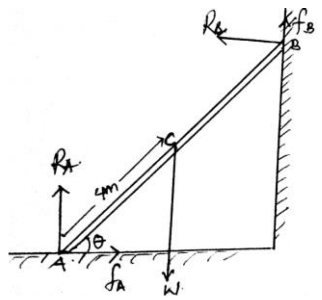
$$cos\theta = 2\mu sin\theta$$

$$\therefore 2\mu tan\theta = 1$$

b)

let W be the weight of ladder, f_A and f_B be friction force at A and B respectively

$$\mu_A = \frac{1}{3}, \mu_B = \frac{1}{4},$$



$$\uparrow;\ R_A + f_B = W \qquad , f_B = \mu_B R_B$$
$$\rightarrow;\ R_B = f_A = \mu_A R_A$$

$$R_A + f_B = W$$
$$R_A + \mu_B R_B = W$$
$$R_A + \mu_B \mu_A R_A = W$$
$$R_A(1 + \mu_B \mu_A) = W$$

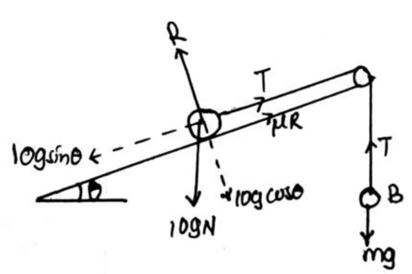
Taking moments at A

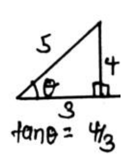
$$W x 4cos\theta = f_B x 6cos\theta + R_B x 6sin\theta$$

$$R_A(1 + \mu_B \mu_A) x 4cos\theta = \mu_B R_B x 6cos\theta + R_B x 6sin\theta$$
$$R_A(1 + \mu_B \mu_A) x 4cos\theta = \mu_B \mu_A R_A x 6cos\theta + \mu_A R_A x 6sin\theta$$

$$\left(1 + \frac{1}{3} x \frac{1}{4}\right) x 4cos\theta = \frac{1}{3} x \frac{1}{4}\ x 6cos\theta + \frac{1}{3} x 6sin\theta$$
$$\frac{13}{3} cos\theta = \frac{1}{2} cos\theta + 2sin\theta$$
$$\frac{23}{6} cos\theta = 2sin\theta$$
$$tan\theta = \frac{23}{12}\ as\ required$$

19 a) when m is minimum





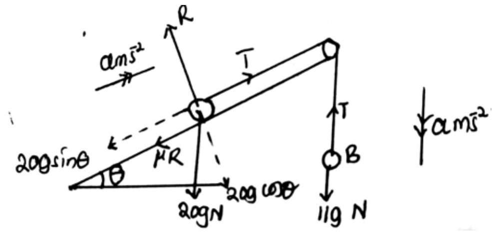
For B: $T = mg$.................(i)

For A: $T + \mu R = 10sin\theta$ ....(ii) $\quad \uparrow; R = 10gcos\theta$

Substituting for R&T in (ii)

$$\leftrightarrow \qquad mg + 0.5x10gx\frac{3}{5} = 10gx\frac{4}{5}$$
$$m = 5kg$$

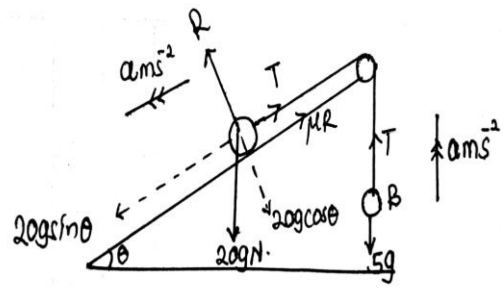
When m is maximum



For B: $T = mg$..................(i)
For A: $T = 10sin\theta + \mu R$ ....(ii) $\quad \uparrow; R = 10gcos\theta$
Substituting for R&T in (ii)

$$\leftrightarrow \qquad mg = 10gx\frac{4}{5} + 0.5x10gx\frac{3}{5}$$
$$m = 11kg$$

b) for m=20kg and m=5kg



For B: $T - 5g = 5a$..................(i)
For A:$20gsin\theta - (T + \mu R) = 20a$ ....(ii) $\quad \uparrow; R = 20gcos\theta$
Substituting for R&T in (ii)

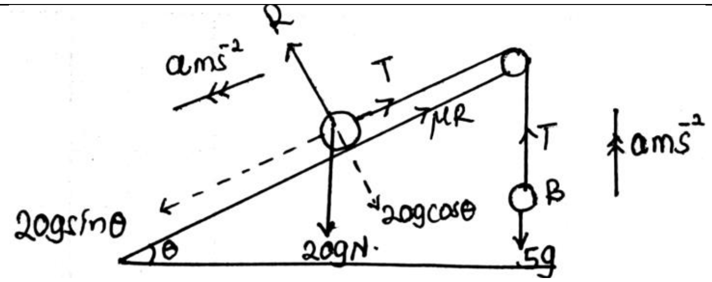
$$\leftrightarrow \qquad 20gx\frac{4}{5} - \left(T + 0.5x20gx\frac{3}{5}\right) = 20a$$

$10g - T = 20a$..............................(iii)
Adding (i) and (iii)

$$5g = 25a$$
$$5(9.8) = 25a$$

$$a = 1.96ms^{-2}$$

For m=11kg



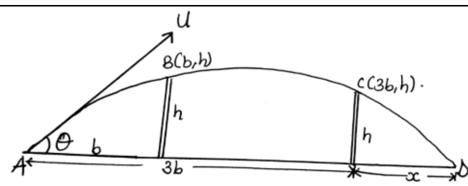
For B: $11g - T = 11a$..................(i)
For A:$T - (\mu R + 20gsin\theta) = 20a$ ....(ii) $\uparrow; R = 20gcos\theta$
Substituting for R&T in (ii)

$$\leftrightarrow \quad \left(T - 0.5x20gx\frac{3}{5} - 20gx\frac{4}{5}\right) = 20a$$
$$T - 6g - 16g = 20a$$

$T - 22g = 20a$.........................(iii)
Adding (i) and (iii)

$$-11g = 31a$$
$$-11(9.8) = 31a$$
$$a = -3.4774ms^{-2}$$
$$\therefore a = 3.4774ms^{-2} \text{ in opposite direction}$$

20



a) $At\ B(b,h)$ $\quad h = tan\theta - \frac{gb^2}{2u^2}sec^2\theta$ ....(i)

$At\ c(3b,h)$ $\quad h = 3btan\theta - \frac{9gb^2}{2u^2}sec^2\theta$ ....(ii)

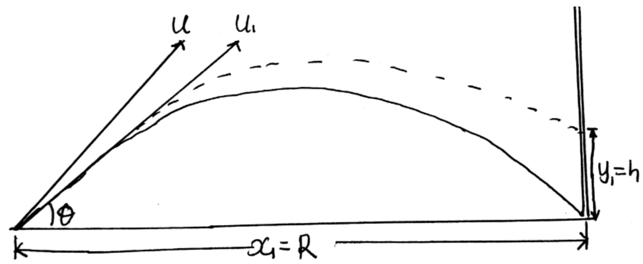
Equating (i) and (ii)

$$2btan\theta = \frac{4gb^2}{u^2}sec^2\theta$$
$$tan\theta = \frac{2gb}{u^2}sec^2\theta$$
$$\frac{sin\theta}{cos\theta}xcos^2\theta = \frac{2gb}{u^2}$$
$$sin\theta cos\theta = \frac{2gb}{u^2}$$
$$but\ sin\theta cos\theta = \frac{1}{2}sin2\theta$$
$$u^2 sin\theta cos\theta = 4gb$$

Horizontal range $R = \frac{u^2 sin2\theta}{g}$

$$= \frac{4gb}{g}$$
$$= 4b \text{ as required}$$

b)



$$R = \frac{u^2}{g} \quad \theta = 45^0$$

$$x_1 = u_1 cos\theta . t_1$$

$$\frac{u^2}{g} = u_1 cos45. t_1$$

$$\frac{u^2}{g} = \frac{u_1}{\sqrt{2}}. t_1$$

$$\frac{u^2\sqrt{2}}{gu_1} = t_1$$

$$y_1 = u_1 sin\theta . t - \frac{g{t^2}_1}{2}$$

$$h = u_1 sin45. t_1 - \frac{g{t^2}_1}{2}$$

$$h = \frac{u_1}{\sqrt{2}}\frac{u^2\sqrt{2}}{gu_1} - \frac{g}{2}(\frac{u^2\sqrt{2}}{gu_1})^2$$

$$h = \frac{u^2}{g} - \frac{u^2}{{u_1}^2 g}$$

$$h{u_1}^2 g = u^2 {u_1}^2 g - u^2$$

$$u^2 = {u_1}^2(u^2 - hg)$$

$${u_1}^2 = \frac{u^2}{(u^2 - hg)}$$

$${u_1}^2 = \frac{u}{\sqrt{(u^2 - hg)}}$$

END