P425/1

PURE MATHEMATICS

PAPER 1

April, 2024

3 hours

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eigh**t questions in section **A** and **five** questions from section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** working **must** be show clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn Over

SECTION A: (40 MARKS)

*Answer **all** the questions in this section.*

1. Solve the equation $5\sin 2x + 4 = 10\sin^2 x$ for $-\pi \le x \le \pi$. *(05 marks)*

2. The second and third terms of a geometric progression are 24 and $12(\alpha + 1)$ respectively. Find α if the sum of the first three terms of the progression is 76.

(05 marks)

3. Points A and B have position vectors $2\boldsymbol{i} - 5\boldsymbol{j} + 3\boldsymbol{k}$ and $7\boldsymbol{i} - 2\boldsymbol{k}$ respectively. Find the coordinates of the point C which divides AB internally in the ratio 2:3 and point D which divides AB externally in the ratio 3:8. *(05 marks)*

4. Given that $y = \frac{\sin x}{x}$, show that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$. *(05 marks)*

5. Find the coordinates of the points of intersection of the curve with parametric equations $x = 2t^2 - 1,\ y = 3(t+1)$ and the line $3x - 4y = 3$. *(05 marks)*

6. Find $\int \frac{\cos x}{4 + \sin^2 x} dx$ *(05 marks)*

7. Solve the equation $\log_x 32 - \log_{256} x = 1$ *(05 marks)*

8. A spherical water container of internal radius 10cm has water to a maximum depth of 18cm. Find the volume of the water in the container. *(05 marks)*

SECTION B: (60 MARKS)

*Answer any **five** questions from this section.*

9. (a) Use binomial theorem to obtain the first four terms of the expansion $(1-6x)^{1/4}$. Hence find $\sqrt[4]{39}$ correct to 4s.f. *(06 marks)*

(b) Use maclaurin's theorem to find the expansion of $e^{x}\sin x$ in ascending powers of x as far as the term in x^{3}. *(06 marks)*

10. By splitting the numerator , find

(a) $\int \frac{2x-1}{4x^{2}+3}dx$ (b) $\int \frac{\cos\theta-2\sin\theta}{3\cos\theta+4\sin\theta}d\theta$ *(12 marks)*

11. (a) If θ is acute and $\cot\theta=\frac{x^{2}-y^{2}}{2xy}; x>0, y>0,$ find the value of $\sec\theta$ in the simplest form. *(04 marks)*

(b) Prove that $\frac{1+\cos x+\sin x}{1-\cos x+\sin x}=\cot x$. *(04 marks)*

(c) Solve for x if $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{2}$ *(04 marks)*

12. (a) Find the Cartesian equation of the plane which passes through the point (1,2,3) and which is parallel to the vectors $2\boldsymbol{i}+4\boldsymbol{j}-10\boldsymbol{k}$ and $6\boldsymbol{i}-4\boldsymbol{j}+2\boldsymbol{k}$. *(05 marks)*

(b) Find the equation of line of intersection of the planes $4x+3y+z=10$ and $x+y+z=6.$ Find the angle between the planes above.

(07 marks)

13. (a) Show that $z=2+3i$ is a root of the equation $z^{4}-5z^{3}+18z^{2}-17z+13=0,$ hence find other roots. *(06 marks)*

(b) If $z=1+\cos 2\theta+i\sin 2\theta,$ prove that $|z|=2\cos\theta$ and $\arg z=\theta.$

(06 marks)

14. Sketch the curve $y = x^3 - 3x^2 + 2x$ and find the area enclosed by the curve and the x-axis between $x = 0$ and $x = 4$. If this area is now rotated about the $x - axis$ through 2π radians , determine the volume of the solid generated, correct to three significant figures. *(12 marks)*

15. (a) $P(ap^2, 2ap)$ is any point on the parabola $y^2 = 4ax$ and the chord from P passing through the focal point meets the parabola again at $Q(aq^2, 2aq)$. If $pq = c$, find the value of c. *(04 marks)*

(b) Find the equation of the normal at $R(a\cos\theta, b\sin\theta)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the normal at R to the ellipse meets the x-axis at N and the y-axis at S. Find the equation of locus of the midpoint of **NS.** Find the area of triangle ***NOS*** where O is the centre of the ellipse.

(08 marks)

16. (a) Find the particular solution of the equation $\frac{dy}{dx} = x - \frac{2y}{x}$ given $y(2) = 4$. *(05 marks)*

(b) The rate of increase of the population ,P, of baboons in Busitema forest reserve is proportional to the number present in the forest at any time , t years. On first June 2010, there were 300 baboons in the forest and a year later, they were found to be 380.

(i) Form a differential equation involving P and t where t is time.

(01 mark)

(ii) Predict the population of baboons by first June, 2018.

(06 marks)

END