Connected particles and Projectiles

Simple connections

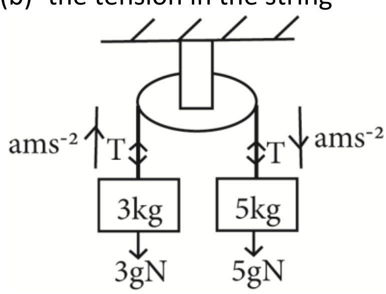
When two particles are connected by a light inextensible string passing over a smooth pulley and allowed to move freely, then as long as the string is tout, the following must be observed.

- acceleration of the particles is the same
- tension in the uninterrupted string is constant
- tensions in interrupted strings are different.

Example 1

Two particles of masses 5kg and 3kg are connected by a light inextensible string passing over a smooth fixed pulley. Find

(a) acceleration of the particle
(b) the tension in the string



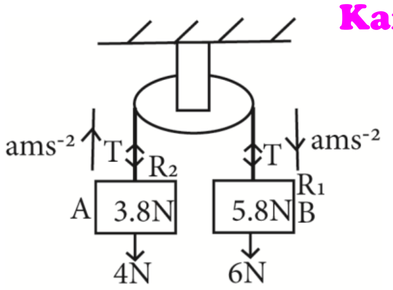
For 5kg mass: 5g – T = 5a …. (i)
For 3kg mass: T – 3g = 3a …. (ii)

(i) and (ii)
2g = 8a
$a = \frac{2 \times 9.8}{8} = 2.45ms^{-2}$
(ii) tension in the string
T -3g = 3a
T = 3 x 2.45 + 3 x 9.8 = 36.78N
(iii) Force on the pulley
R = 2T = 3 x 36.78 = 73.56N

Example 2

An inextensible string attached to two scale A and B each of weight 20g passes over a smooth fixed pulley. Particles of weight 3.8N and 5.8N are placed in pans A and B respectively. If the system s released from rest (take g = $10ms^{-2}$). Find the

(a) Tension in the string
(b) Reaction of the scale pan holding the 3.8N weight



Weight of the scale pan = $\frac{20}{1000} x\ 10 = 0.2N$

Total weight of A = 3.8 + 0.2 = 4N

Total weight of B = 5.8 + 0.2 = 6N

For 6N: 6 – T = 0.6a …….. (i)

For 4N: T – 4 = 0.4a …….. (i)

Adding (i) and (ii)

a = $2ms^{-2}$

T = 4 + 0.4 x 2 = 4.8N

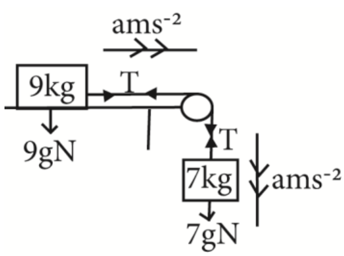
For scale pan A $R_2 - 3.8 = 0.38a$

R_2 = 3.8 + 2 x 0.38 x 2 = 4.56N

Example 3

A mass of 9kg resting on a smooth horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table to the pulley is a 7kg mass hanging freely 1.5m above the ground. Find

(a) common acceleration
(b) tension in the string
(c) force on the pulley when the system is allowed to move freely
(d) time taken for the 7kg mass to hit the ground



F = ma

For 7kg mass: 7g – T = 7a

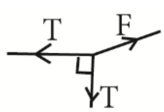
For 9kg mass: T = 9a

(i) + (ii): 7g = 16a

a = $\frac{7\ x\ 9.8}{16} = 4.29ms^{-2}$

(b) Tension: T = 9a = 9 x 4.29 = 38.61N

(c) The force on the pulley, F:



F =$\sqrt{T^2 + T^2} = T\sqrt{2}$ = 38.61$\sqrt{2}$ = 54.603N

(d) s = ut + $\frac{1}{2}at^2$

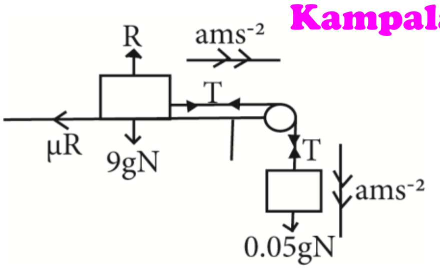
1.5 = 0 x t + $\frac{1}{2} x\ 4.29\ x\ t^2$

t = 0.84s

Example 4

A mass of 90g resting on a rough horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table attached to a 50g mass hanging freely. The coefficient of friction between the 90g mass and the table is $\frac{1}{3}$ and the system is released from rest, find

(a) common acceleration (b) the tension in the string



For 50g mass: 0.05g – T = 0.05a ….(i)

For 90g mass: T – μR = 0.09a

$T - \frac{1}{3} x\ 0.09\ x\ 9.8 = 0.09a$ …. (ii)

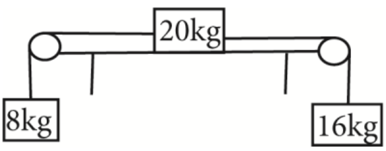
(i) + (ii): $0.05g - \frac{1}{3} x\ 0.09\ x\ 9.8 = 0.14a$

$a = \frac{0.02g}{0.14} = 1.4ms^{-2}$

(b) 0.05g – T = 0.05a

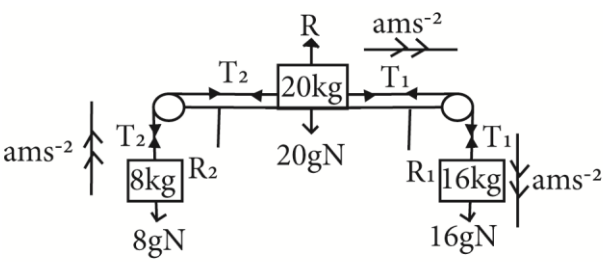
T = 0.05 x 9.8 – 0.05 x 1.4 = 0.42N

Example 5



The figure shows a block of mass 20kg resting on a smooth horizontal table. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) Acceleration of 16kg mass
(b) tension in the string
(c) reaction on each pulley



For 16kg mass: $16g - T_1 = 16a$ …… (i)

For 20kg mass: $T_1 - T_2 = 20a$ ……. (ii)

For 8kg mass: $T_2 - 8g = 8a$ ……… (iii)

(i) + (ii) + (iii): 8g = 44a

$a = \frac{8\ x\ 9.8}{44} = 1.782ms^{-2}$

(b) Tension in the string

$16g - T_1 = 16a$

$T_1 = 16 \times 9.8 - 16 \times 1.782 = 128.288N$

$T_2 - 8g = 8a$

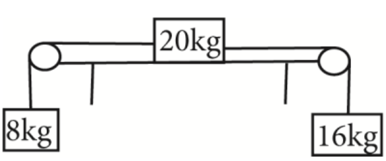
$T_2 = 8 \times 9.8 + 8 \times 1.782 = 92.656N$

(c) Reaction on each pulley

$R_1 = \sqrt{T_1^{\ 2} + T_1^{\ 2}} = T_1\sqrt{2} = \sqrt{2} \times 128.288 = 181.427N$

$R_2 = \sqrt{T_2^{\ 2} + T_2^{\ 2}} = T_2\sqrt{2} = \sqrt{2} \times 92.626 = 131\ N$

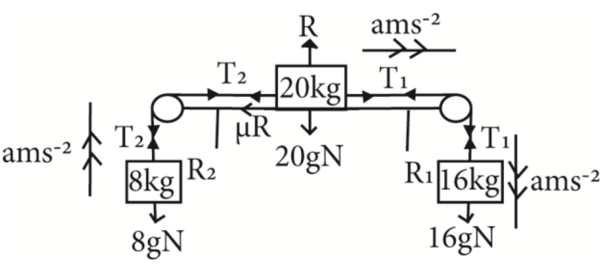
Example 6



The figure shows a block of mass 20kg resting on a rough horizontal table of coefficient of friction 0.21. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) acceleration of the 16kg mass
(b) Tension in each string
(c) reaction on each pulley

Solution



For 16kg mass: $16g - T_1 = 16a$ …… (i)

For 20kg mass: $T_1 - T_2 - 20g\mu = 20a$ ……. (ii)

For 8kg mass: $T_2 - 8g = 8a$ ……… (iii)

(i) + (ii) + (iii): $8g - 20g\mu = 44a$

$$a = \frac{8 \times 9.8 - 20 \times 9.8 \times 0.21}{44} = 0.846ms^{-2}$$

(b) Tension in the string

$16g - T_1 = 16a$

$T_1 = 16 \times 9.8 - 16 \times 0.846 = 143.264N$

$T_2 - 8g = 8a$

$T_2 = 8 \times 9.8 + 8 \times 0.846 = 85.168N$

(c) Reaction on each pulley

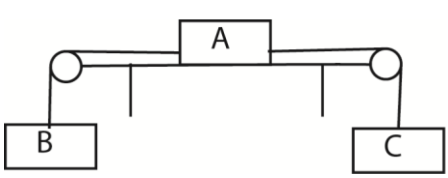
$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 2 \times 128.291 = 202.606N$

$R_2 = \sqrt{T_2^2 + T_2^2} = T_2\sqrt{2} = \sqrt{2} \times 85.168 = 120.446N$

Revision exercise 1

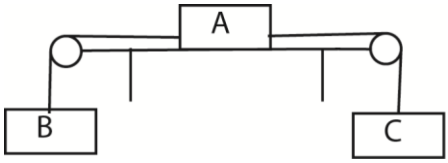
1. Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find
   (a) acceleration of the particles [$3.92ms^{-2}$]
   (b) the tension in the string [41.16N]
   (c) the force on the pulley [82.32N]
2. Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find
   (a) acceleration of the particles [$4.9ms^{-2}$]
   (b) the tension in the string [29.4N]
   (c) distance moved by the 6kg mass in the first 2s of motion [9.8m]
3. A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a rope which passes over a frictionless pulley so that they hang vertically downwards.
   (a) what is the tension in the section of the rope supporting the man [807.06N]
   (b) what is the acceleration of the bucket [$1.73ms^{-2}$]
4. Two particles of masses 200g and 300g are connected to a light inelastic string passing over a smooth pulley; when released freely find
   (i) common acceleration [1.96ms-2]
   (ii) the tension in the string [2.352N]
   (iii) the force on the pulley [4.704N]

5. The diagram below shows a particles of mass 8kg connected to a light scale pan by a light inextensible string which passes over a smooth fixed pulley. The scale pan holds two blocks A and B of mass 3kg and 4kg, with B resting on top of A. If the system is released from rest find
   (a) acceleration of the system [$0.653ms^{-2}$]
   (b) the reaction between A and B[41.813N]
6. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate
   (a) the common acceleration of the masses [$3.675ms^{-2}$]
   (b) the tension in the string [18.375N]
   (c) the force acting on the pulley[26N]
7. A mass of 3kg on a smooth horizontal table is attached by a light inextensible sting passing over a smooth pulley at the edge of the table, to another mass of 2kg hanging freely 2.1m above the ground; find
   (a) common acceleration[$3.92ms^{-2}$]
   (b) the tension in the string [11.76N]
   (c) The force on the pulley in the system if it's allowed to move freely. [16.63N]
   (d) the velocity with which the 2kg mass hits the ground [$4.06ms^{-1}$]
8. A mass of 5kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a 3kg mass hanging freely. the coefficient of friction between the 5kg mass and the table is 0.25 and the system is released from rest find
   (a) common acceleration[$2.144ms^{-2}$]
   (b) tension in the string [22.97N]
9. A mass of 11kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to 500g mass hanging freely. The coefficient of friction between the 1kg mass and the table is 0.1 and the system is released from rest find
   (a) common acceleration [$2.61ms^{-2}$]
   (b) the tension in the string [3.593N]
10. The objects of mass 3kg and 5kg are attached to ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3kg mass touching the floor and the 5kg mass at 4m above the floor and then release, what is
    (a) the acceleration of the system[$2.45ms^{-2}$]
    (b) tension in the chord[36.75N]
    (c) the time that will elapse before the 5kg object hits the floor [1.81s]
11.



The diagram shows a particle A of mass 2kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 5kg and C of mass 3kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the
   (a) common acceleration [$0.98ms^{-2}$]
   (b) the tension of each string [12.37N, 44.15N]

12.

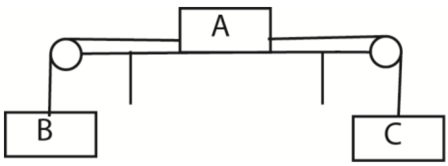


The diagram shows a particle A of mass 3kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 4kg and C of mass 6kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

(a) common acceleration [$0.75ms^{-2}$]

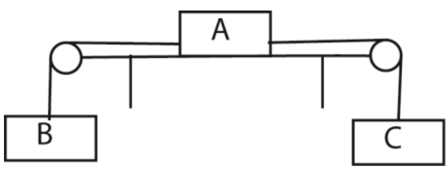
(b) the tension of each string [54.277N, 31.662N]

13.



The diagram shows a particle A of mass 5kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 3kg and C of mass 2kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest body B descend with an acceleration of 0.28ms-2, find the coefficient of friction between the body A and the surface of the table. [0.143]

14.



The diagram shows a particle A of mass 10kg resting on a smooth horizontal table. It is attached to the particle B of mass 4kg and C of mass 7kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

(c) common acceleration [$1.4ms^{-2}$]

(d) the tension of each string [44.8N, 58.8N]

Connected particles on inclined planes

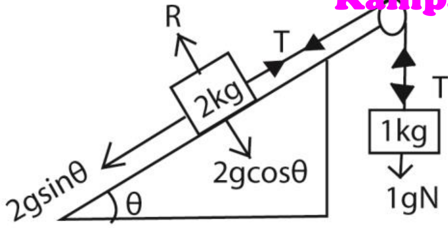
Example 7

A mass of 2kg lies on a smooth plane of inclination 1 in3. One end of a light inextensible sting is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its end. If the system is released from rest , find the

(i) acceleration of the masses

(ii) tension in the string

(iii) distance each particle travels in the first 2s.



$\sin\theta = \frac{1}{3}$ F =ma

For 2kg mass: T – 2gsinθ = 2a….. (i)

For 1kg mass: 1g – T = 1a ……….. (ii)

(ii) + (i): = 1g – 2gsinθ = 3a

$a = \frac{9.8 - 2\ x\ 9.8\ x\ \frac{1}{3}}{3} = 1.089ms^{-2}$

(ii) Tension: 1g – T = 1a

T = 9.8 – 1.089 = 8.71N

(iii) $s = ut + \frac{1}{2}at^2$

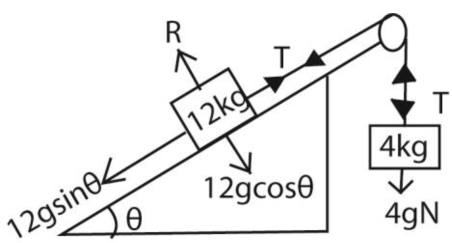
$s = 0 \times 2 + \frac{1}{2} x\ 1.089\ x\ 2^2 = 2.178m$

Example 8

A mass of 12kg lies over a smooth incline plane 6m long and 1m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane to freely suspended mass of 4kg at its other end. If the system is released from rest, find the

(a) acceleration of the system
(b) velocity with which the 4kg mass hits the ground
(c) time the 4kg mass takes to hit the ground.

Solution



$\sin\theta = \frac{1}{6}$ F =ma

For 12kg mass: T – 12gsinθ = 2a….. (i)

For 4kg mass: 4g – T = 4a ……….. (ii)

(ii) + (i): = 4g – 12gsinθ = 16a

$a = \frac{4\ x 9.8 - 12\ x\ 9.8\ x\ \frac{1}{6}}{16} = 1.225ms^{-2}$

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 1.225 = 34.3N

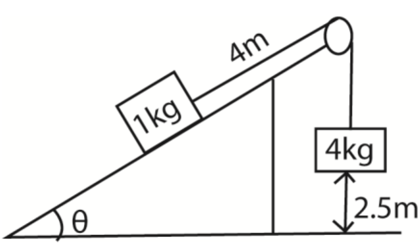
(iii) $s = ut + \frac{1}{2}at^2$

$1 = 0 \times t + \frac{1}{2} x\ 1.225\ x\ t^2$

t = 1.28s

Example 9

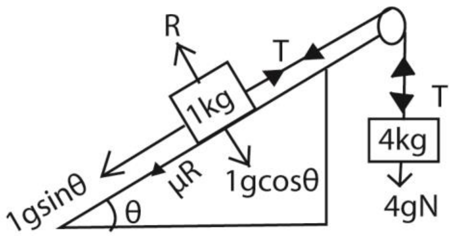
A mass of 1kg lies on a rough plane with coefficient of friction 0.25. One end of a light inextensible string is attached to 1kgmass and passes up the line of greatest slope over a smooth fixed pulley at the top of the plane and the other end of a string is tied to a mass of 4kg hanging freely.



The plane makes an angle θ with the horizontal where sinθ = $\frac{3}{5}$. When the system is released from rest, find:

(i) the acceleration of the system
(ii) tension in the string
(iii) velocity with which the 4kg mass hits the floor
(iv) velocity with which the 1kg mass hits the pulley

Solution



sinθ = $\frac{3}{5}$; cosθ = $\frac{4}{5}$ F =ma

For 12kg mass: T – 1gsinθ- 0.25R = 1a….. (i)

For 4kg mass: 4g – T = 4a ……….. (ii)

(ii) + (i): = 4g – 1gsinθ - 0.25R = 5a

a = $\frac{4\ x 9.8 - 1\ x\ 9.8\ x\frac{3}{5} - 0.25\ x\ 1\ x\ 9.8\ \ x\frac{4}{5}}{5}$ =6.272ms^{-2}

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 6.272 = 14 .112N

(iii) $v^2 = u^2 + 2as$ but u = 0

v = $\sqrt{2\ x\ 6.272\ x\ 2.5}$ = 5.6ms^{-1}

(iv) When a 4kg mass hits the floor, the 1kg mass has still to move 4 – 2.5 = 1.5m before hitting the pulley It will experience a retarding force due to gravity and friction

F = 1a = 1gsinθ + 0.25R

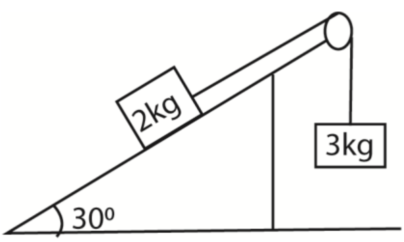
=$(1\ x\ 9.8\ x\ \frac{3}{5} + 0.25\ x\ 1\ x\ 9.8\ x\frac{4}{5})$

a = - 7.84ms^{-2}

$v^2 = u^2 + 2as$

v = $\sqrt{5.6^2 - 2\ x\ 7.84\ x\ 1.5}$ =2.8ms^{-1}

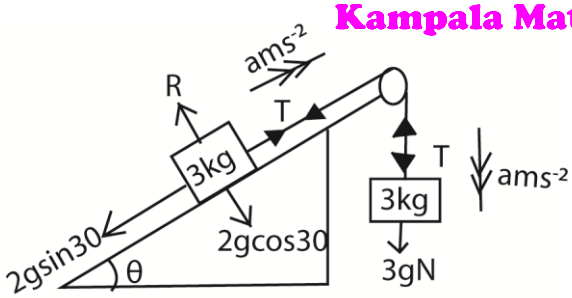
Example 10



A particle of mass 2kg on a rough plane inclined at 300 to the horizontal is attached by means of light inextensible string passing over a smooth pulley at the top edge of the plane to a particle of mass 3kg which hangs freely. If the system is released from rest with above parts of the strings taut, the 3kg mass travels a distance of 0.75m before attains a speed of 2.25ms^{-1}. Calculate

(a) acceleration
(b) coefficient of friction between the plane and 2kgmass
(c) reaction of the pulley on the string

Solution



(i) $v^2 = u^2 + 2as$

$a = \frac{2.25^2 - 0^2}{2 \times 0.75} = 3.375ms^{-2}$

(ii) F = ma

For 2kg mass: T – 2gsinθ- μR = 2x a ….. (i)

For 4kg mass: 3g – T = 3 a ………. (ii)

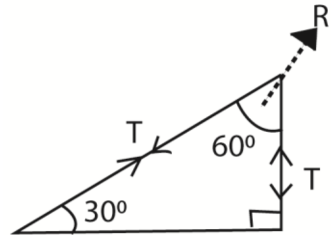
(ii) + (i): 3g – 2gsinθ – μ(2gcosθ) = 5a

$$\mu = \frac{(3 \times 9.8) - (2 \times 9.8 sin30 + 5 \times 3.375)}{2 \times 9.8 \times cos30} = 0.161$$

(iii)

Tension: 3g – T = 3a

T = 3 x 9.8 – 3 x 3.375 = 19.275N



Using parallelogram law of force

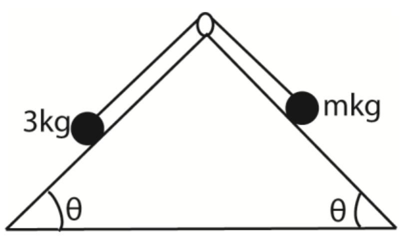
$R^2 = T^2 + T^2 + 2 \times T \cos 60 = 3T^2$

$R = 19.275\sqrt{3} = 33.4N$

Double inclined plane

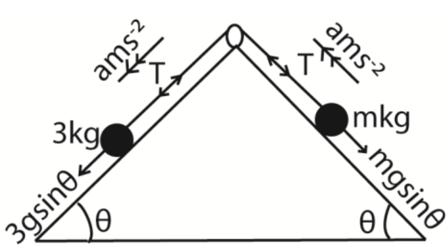
Example 11

The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where sinθ = 0.6. Two particles of mass 3kg and mkg, where m< 3kg are connected by a light inextensible string passing over a smooth fixed pulley.



The particles are released from rest with a string taut. After travelling a distance of 1.08m, the speed of the particle is 1.8ms-1. Calculate

(i) acceleration
(ii) tension in the string
(iii) value of m



(i) $v^2 = u^2 + 2as$

$1.8^2 = 0^2 + 2 \times a \times 1.08$

$a = 1.5ms^{-2}$

(ii) F = ma

For 3kg mass: 3gsinθ – T = 3a

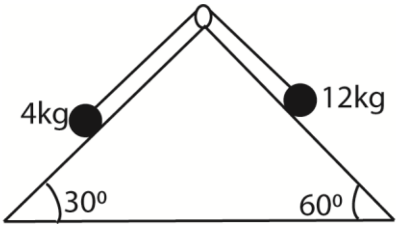
T = 3 x 9.8x 0.6 – 3 x 1.5 = 13.14N

(iii) For mkg mass; T – mgsinθ = ma

13.14 = m(9.8 + 1.5); m = 1.78kg

Example 12

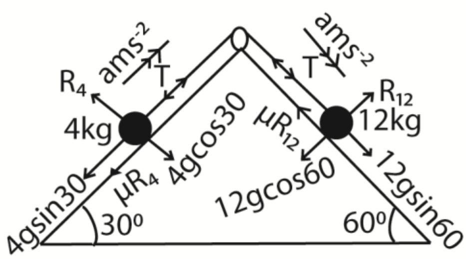
Two rough planes inclined at 300 and 600 to the horizontal and of the same height are placed back to back. Masses of 4kg and 12 kg are placed on the faces and connected by a light string passing over smooth pulley on the top of the planes.



If the coefficient of friction is 0.5 on both faces, find

(a) acceleration
(b) Tension in the strings

Solution



For 4kg mass: T -4gsin30 – 0.5 x 4gcos30 = 4a ……………. (i)

For 12kg mass: 12gsin60 –T – 0.5 x 12gcos 60 = 12a ….. (ii)

(i) + (ii)

12gsin60 – 4gsin30 - 0.5(4gcos30 + 12gcos 60) = 16a

a = 2.25ms^{-2}

(b) For 4kgmass

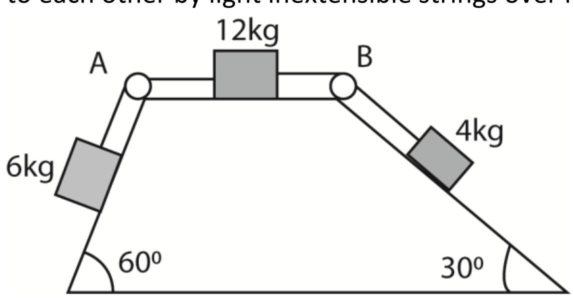
T -4gsin30 – 0.5 x 4gcos30 = 4a

T = 4gsin30 + 0.5 x 4gcos30 + 4x2.25

T = 45.54

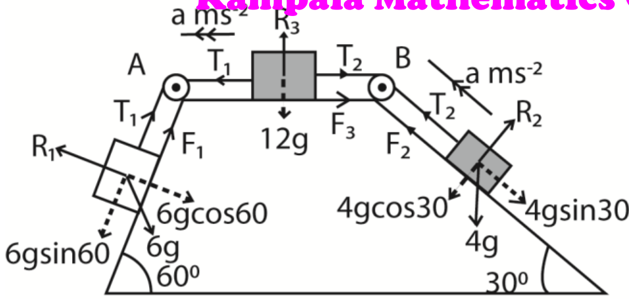
Example 13

1. The diagram below shows a 12kg mass on a horizontal rough plane. The 6kg and 4kg masses are on rough planes inclined at angles 600 and 300 respectively. The masses are connected to each other by light inextensible strings over light smooth pulleys A and B.



The planes are equally rough with coefficient of friction $\frac{1}{12}$. If the system is released from rest find the;

(a) Acceleration of the system (08marks)



For 6kg mass

6gsin60-(T_1 + $\frac{1}{12}\ x\ 6gcos60) = 6a$

6gsin60 – T_1 - $\frac{1}{2}cos60 = 6a$ ………………………… (i)

For 4kg mass

T_2 – ($\frac{1}{12}\ x\ 4gcos30 + 4gsin30) = 4a$

T_2 - $\frac{1}{3}gcos30 - 4gsin30$ = 4a ………………….. (ii)

For 12kg mass

T_1 – (T_2 +$\frac{1}{12}\ R_3$) = 12a

T_1 – (T_2 +$\frac{1}{12}\ x\ 12g$) = 12a

T_1 –T_2 - g = 12a……………………………………. (iii)

Eqn. (i) + Eqn. (ii) + Eqn. (iii)

6gsin60 - $\frac{1}{2}gcos60 - \frac{1}{3}gcos30 - 4gsin\ 30 - g = 22a$

16.24327742 = 22a

a = $\frac{16.24327742}{22} = 0.73833ms^{-2}$

(b) Tensions in the strings. (04marks)

From equation (i)

$T_1 = 6gsin60 - \frac{1}{2}cos60 - 6a$

= $6gsin60 - \frac{1}{2}cos60 - 6\ x\ 0.73833$

= 44.0423N

From eqn. (ii)

T_2 = $\frac{1}{3}gcos30 + 4gsin30 + 4a$

= $\frac{1}{3}gcos30 + 4gsin30 + 4\ x\ 0.73833$

= 25.3823N

Revision exercise 2

1. A mass of 2kg lies on a smooth inclined plane 9m long and 3m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope

over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its other end. If the system is released from rest, find

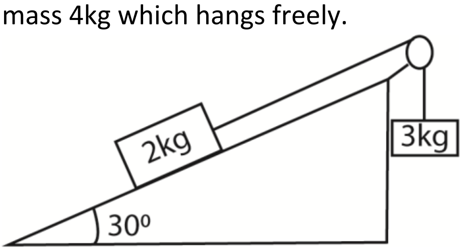
(i) acceleration of the system[$1.089ms^{-2}$]

(ii) tension in the string. [8.711N]

(iii) velocity with which the 1kg mass will hit the ground[$2.556ms^{-1}$]

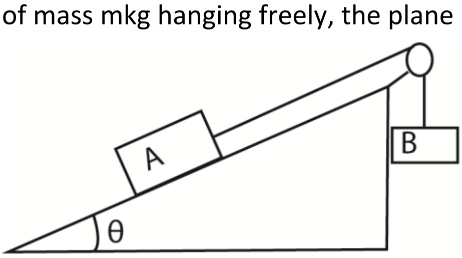
(iv) time the 1kg mass will hit the ground[2.347s]

2. A mass of 15kg lies on a smooth plane of inclination in 49. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 10kg at its other end. If the system is released from rest, find the acceleration of the masses and the distance each travel in the first 2s. [$3.8ms^{-2}$, 7.6m]
3. A mass of 2kg lies on a rough plane which is inclined at 300 to the horizontal. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 5kg at its other end. The system is released from rest as the 2kg mass accelerates up the slope, it experiences a constant resistance to motion of 14N down the slope due to friction. Find the tension of the string. [31N]
4. A mass of 10kg lies on a smooth plane which is inclined at θ to the horizontal. The mass is 5m from the top, measured along the plane. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 15kg at its other end. The 15kg mass is 4m above the floor. The system is released from rest and the string first goes slack $1\frac{3}{7}s$ latter. Find the value of θ. [30^0]
5. One of two identical masses lies on a smooth plane, which is inclines at $\sin^{-1}\left(\frac{1}{4}\right)$ to the horizontal and is 2m from the top. A light inextensible string attached to this mass passes along the line of greatest slope over a smooth pulley fixed at the top of the incline, the other end carries the other mass hanging freely 1m above the floor. If the system is released from rest, find the time taken for the hanging mass to reach the floor. [0.663s]
6. A particle of mass 2kg on a smooth plane inclined at 300 to the horizontal is attached by means of a light inextensible string passing over a smooth pulley at the edge of the plane to a particle of mass 4kg which hangs freely.



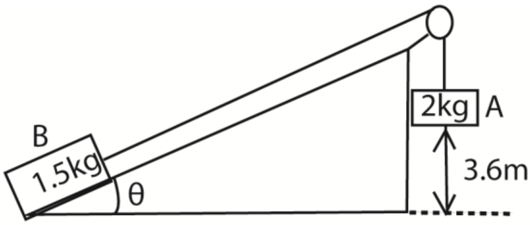
If the system is released from rest with above parts of the string taut, find the speed acquired by the particles when both have moved a distance of 1m [$2.8ms^{-1}$}

7. A body A of mass 13kg lying on a rough inclined plane, coefficient of friction, μ. From A, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley to a body B of mass mkg hanging freely, the plane makes an angle θ with the horizontal where sinθ = $\frac{5}{13}$.



When m = 1kgand the system is released from rest, B has upward acceleration of a ms^{-2}. When m = 11kg and the system released from rest, B has downward acceleration of ams^{-2}. Find a and μ. [1.96ms^{-2}, 0.1]

8. A particle A of mass 2kg and B of mass 1.5kg are connected by light inextensible string passing over a smooth pulley. The system is released from rest with A at height of 3.6m above the horizontal ground and B at the foot of a smooth slope inclined at an angle θ to the horizontal where $\sin\theta = \frac{1}{6}$. Take g = 10ms^{-2}.



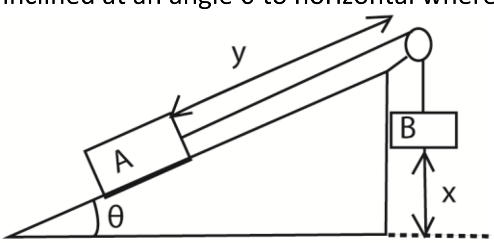
Calculate

(i) the magnitude of the acceleration of particles [6ms^{-1}]

(ii) the speed with which A reaches the ground[5ms^{-2}]

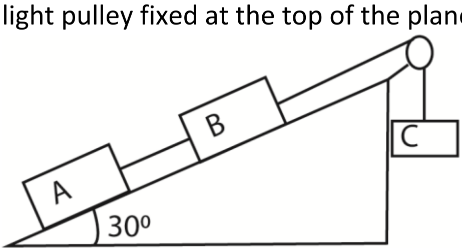
(iii) the distance B moves up the slope before coming to instantaneous rest. [14.4m]

9. A mass A of 4kg and a mass B of 3kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest and mass accelerates up along a smooth slope inclined at an angle θ to horizontal where θ = 30^0.



If y = 3m and x = 2.8m, calculate the velocity with which A hits the pulley [2.42ms^{-1}]

10. The diagram below shows particles A, B and C of masses 10kg, 8kg and 2kg respectively connected by a light inextensible strings. The string connecting B and C passes over a smooth light pulley fixed at the top of the plane.

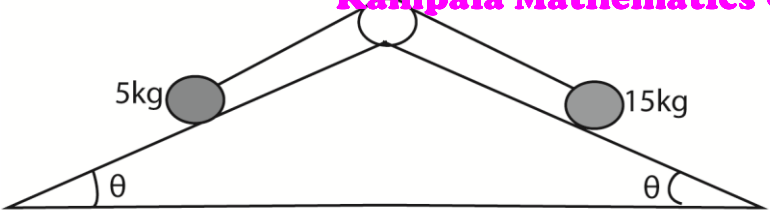


If the coefficient of friction between the plane and particles A and B are 0.22 and 0.25 respectively, calculate

(i) acceleration of the system [1.6477ms^{-2}]

(ii) tension in the strings[22.89N, 13.851N]

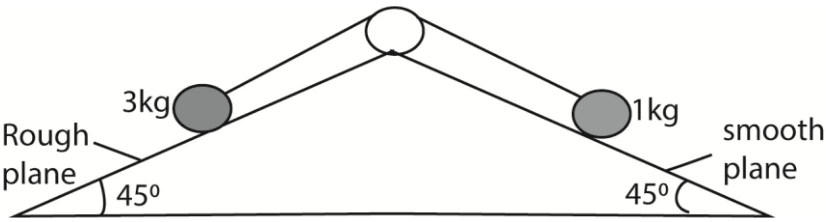
11. The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where $\sin\theta = \frac{3}{5}$. Two particles of mass 5kg and 15kg are connected by a light inextensible string over a smooth fixed pulley.



The particles are releases from rest with a string taut calculate

(i) Acceleration of the particles
(ii) Tension in the string

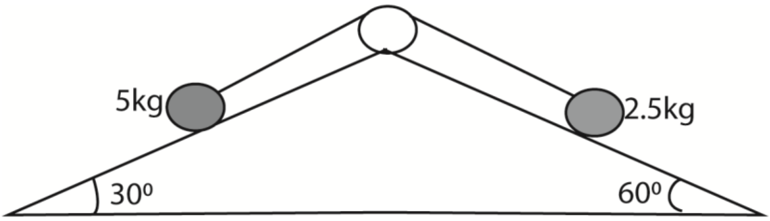
12. The diagram below shows two smooth plane and a rough plane both inclined at 450 to the horizontal. Two particles of mass of mass 1kg and 3kg are connected by light inextensible string passing over a smooth fixed pulley.



The particle are released from rest with a string taut. Calculate

(i) acceleration of the particle [1.4ms^{-2}]
(ii) tension in the string [.48N
(iii) coefficient of friction [0.4]

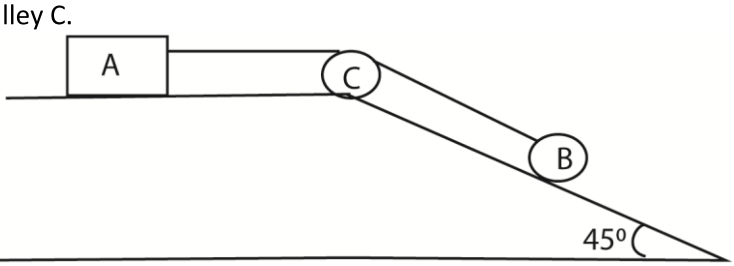
13. Two equally rough planes inclined at 30^0 and 60^0 to the horizontal and of the same height are placed back to back. Masses of 5kg and 2.5kg are placed on the faces and connected by a light string passing over a smooth pulley at the top of the planes.



If the string is taut and 5kg is just about to slip downwards find the

(i) coefficient of friction[0.06]
(ii) tension in the string [21.9538N]

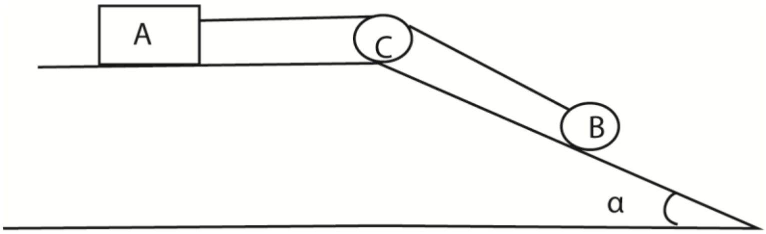
14. In the diagram, particle A and particle B are masses of 10kg and 8kg respectively and rest on planes as shown below. They are connected by a light inextensible string passing over a smooth pulley C.



Find the acceleration in the system and the tension in the string if

(i) the particles are in contact with smooth planes [3.08ms^{-2}, 30.N]
(ii) the particles are in contact with rough planes with coefficient of friction 0.25. [0.95ms^{-2}, 33.98N]

15. In the diagram particles A and B are of masses mkg and 5mkg respectively and rest on the planes as shown below. They are connected by a light inextensible string passing over a smooth fixed pulley at C

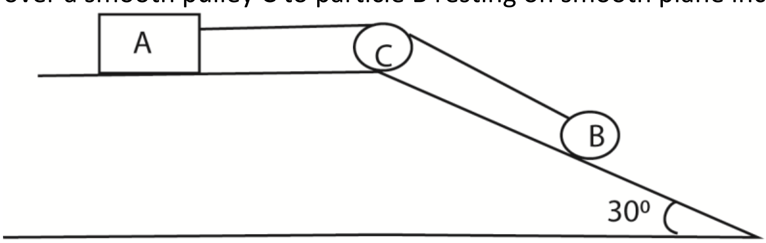


Find the acceleration of the system and the tension in the string if sinα = $\frac{4}{5}$ when;

(i) the particles are in contact with smooth plane[6.533ms-2, 6.533N]

(ii) the particles are in contact with rough plane of coefficient of friction $\frac{1}{3}$. [4.356ms^{-2}, 7.622N]

16. In the diagram particles A and B of masses 2.4kg and 3.6kg respectively. A rests on a rough horizontal plane (coefficient of friction 0.5), it is connected by a light inextensible string passing over a smooth pulley C to particle B resting on smooth plane inclined at 30^0 to the horizontal.



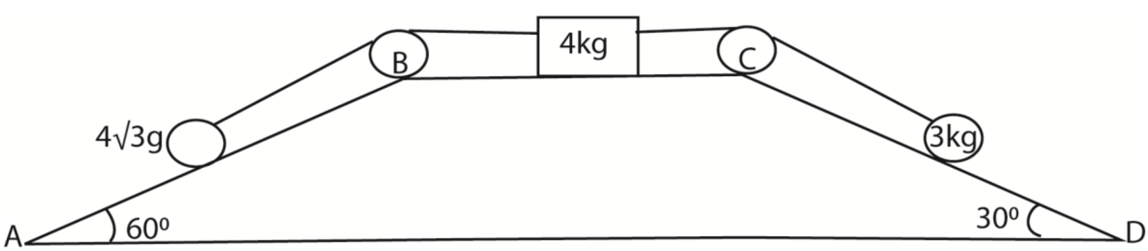
When the system is released from rest find

(i) acceleration of the system and tension in the string [0.98ms^{-2}, 14.112N]

(ii) the force on the pulley C [7.3049N]

(iii) the velocity of A mass after 2 seconds[1.96ms^{-2}]

17. The diagram below shows a 4kg mass on a horizontal rough plane with coefficient of friction 0.25. The $4\sqrt{3}kg$ mass rests on a smooth plane inclined at angle 600 to the horizontal while the 3kg mass rests on a rough plane inclined at an angle 300 to the horizontal and coefficient of friction $\frac{1}{\sqrt{3}}$. the masses are connected to each other by a light inextensible strings over light smooth fixed pulleys B and C.



Find the

(i) acceleration of the system[1.407ms-2]

(ii) tension in the string [49.051N, 33.622N]

(iii) work done against frictional force when the particles each moved 0.5m [12.25J]

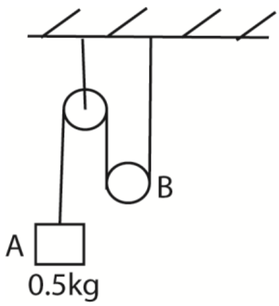
Multiple connections

- Acceleration of a particle moving between to portions of the string is equal to half the net acceleration of the particle (s) attached to the end of the string
- The tension in uninterrupted string is constant
- The tensions in interrupted strings are different.

Case I: A pulley moving between two portions of a string

Example 15

The diagram below shows particle A of mass 0.5kg attached to one end of alight inextensible string passing over a fixed pulley and under a movable light pulley B. The other end of the string is fixed as shown

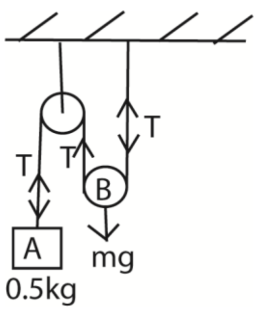


(i) What mass should be attached at B for the system to be in equilibrium
(ii) If B is 0.8kg what are the accelerations of particles A and pulley B?
(iii) find the tension in the string in (ii)

Solution

(i) Let T = tension in string

m = mass at B



For the system to be in equilibrium upward forces are equal to downward force. by resolving vertically

For mass A: T= 0.5g ……… (i)

For pulley B: 2T = mg

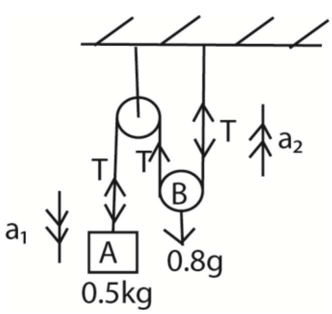
$T = \frac{mg}{2}$ ……… (i)

equating (i) to (ii)

$\frac{mg}{2} = 0.5g$; m = 1kg

(ii) Let a_1 = acceleration of A

a_2 = acceleration of B



For mass A: $0.5g - T = 0.5a_1$……… (i)

For pulley B: $2T - 0.8g = 0.8a_2$ but $a_2 = ½\, a_1$

⇨ $2T - 0.8g = 0.4a_1$

$T - 0.4g = 0.2a_1$ …….. (i)

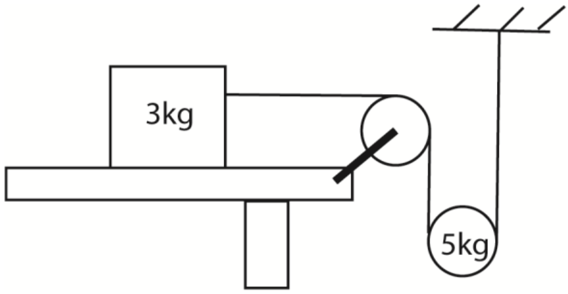
(i) + (ii) $a1 = \frac{9.9}{7} = 1.4ms^{-2}$ and $a2 = 0.7\ ms^{-2}$

From eqn. (i)

T = 0.5 x 9.8 - 0.5 x 1.4 = 4.2N

Example 16

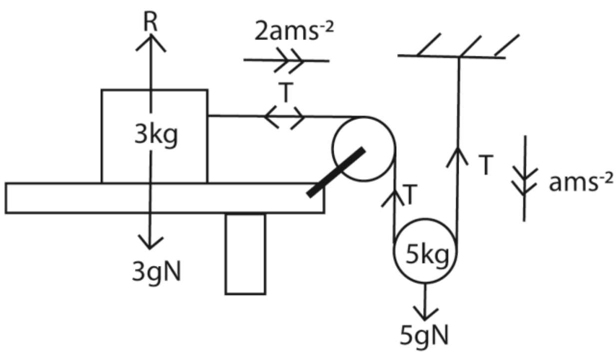
A particle of mass 3kg on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass 5kg, its other end being fixed so that the string beyond the table are vertical.



Find

(i) acceleration of 3kg and 5g
(ii) Tension in the string

Solution



F = ma

For 3kg: T = 3 x 2a ……… (i)

For 5kg: 5g – 2T = 5a …… (ii)

(ii) + 2 x (i)

5x 9.8 = 17a

a = $\frac{49}{17}$ = 2.8824ms^{-2}

Acceleration of 5kg: = 2.8824ms^{-2}

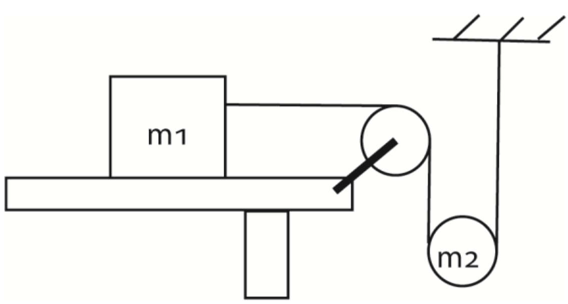
Acceleration of 3kg: = 2.8824 x 2ms^{-2}

= 5.7648ms^{-2}

T = 6a = 2.8824 x 6 = 17.2944N

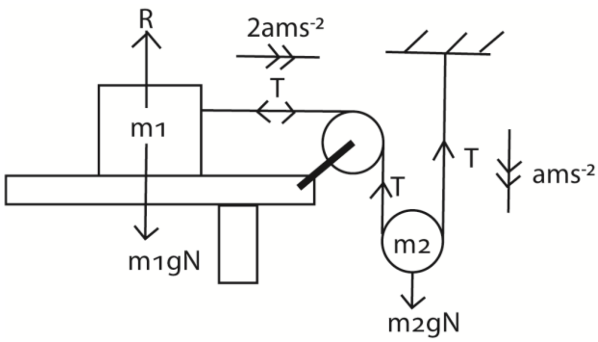
Example 17

A particle of mass m_1 on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass m_2, its other end being fixed so that the parts of the string beyond the table is vertical.



Show that m_2 descends with acceleration $\frac{m_2 g}{4m_1+m_2}$

Solution



F = ma

For m_1 kg mass: $T = m_1 \times 2a$ …… (i)

For m_2 kg mass: $m_2g - 2T = m_2a$ … (ii)

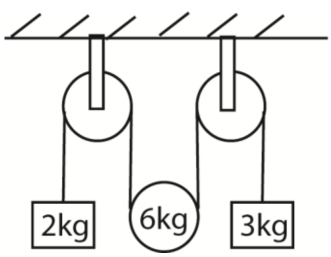
(ii) + 2 x (i)

$m_2g = 4m_1a + m_2a$

$a = \frac{m_2g}{4m_1+m_2}$

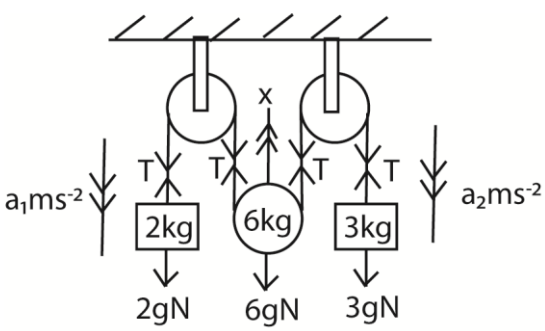
Example 18

A string has a load of mass 2kg attached at one end. The string passes over a smooth fixed pulley then under a movable pulley of mass 6kg and over another fixed pulley and has a load of mass 3kg attached to its end.



Find the acceleration of the movable pulley and the tension in the string

Solution



$x = \frac{a_1 + a_2}{2}$

For 2kg mass: $2g - T = 2a_1$ …………………….. (i)

For 3kg mass: $3g - T = 3a_2$ …………………….. (ii)

For 6kg mass: $2T - 6g = 6 \times \frac{1}{2}(a_1 + a_2)$…. (iii)

(ii) – (i): $g = (3a_2 - 2a_1)$ ……………………………. (iv)

2 x (ii) + (iii): $0 = 9a_2 + 3a_1$ ……………………. (v)

3 x (iv)- (v): $3g = -9a_1$

$a1 = \frac{-g}{3} = -3.267ms^{-2}$

From (v): $0 = 9a_2 + 3a_1$

$0 = 9a_2 + 3(-3.267)$

$a_2 = 1.089ms^{-2}$

Acceleration of pulley $= \frac{1}{2}(a_1 + a_2)$

$= \frac{1}{2}(-3.267 + 1.089)$

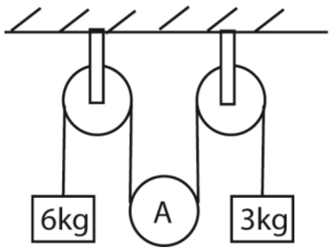
$= -1.089ms^{-2}$

Tension: $T = 2g - 2a_1$

$T = 2 \times 9.8 - 2 \times -3.267 = 26.134N$

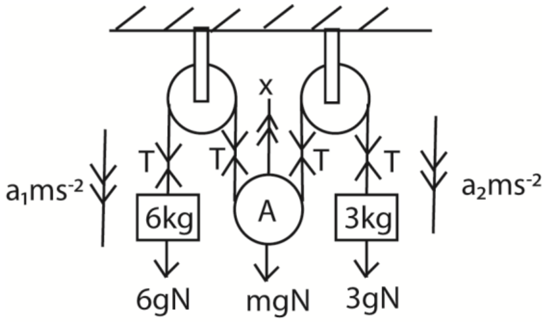
Example 19

In the pulley system below, A is a heavy pulley which is free to move



Find the mass of pulley A if it does not move upwards of downwards when the system is released from rest.

Solution



$$x = \frac{a_1 + a_2}{2}$$

For 2kg mass: $6g - T = 6a_1$ …………………..…. (i)

For 3kg mass: $3g - T = 3a_2$ ……………………. (ii)

For mkg mass: $2T - mg = 0$…………………….. (iii)

$a_1 = -a_2$

$6g - T = -6a_2$ ………. (iv)

$3g - T = 3a_2$ ………… (v)

(iv) – (v)

$3g = 9a_2$

$a_2 = \frac{-g}{3} = -3.267ms^{-2}$

$3g - T = 3a2$

$T = 3 \times 9.8 - 3(-3.267) = 39.201$

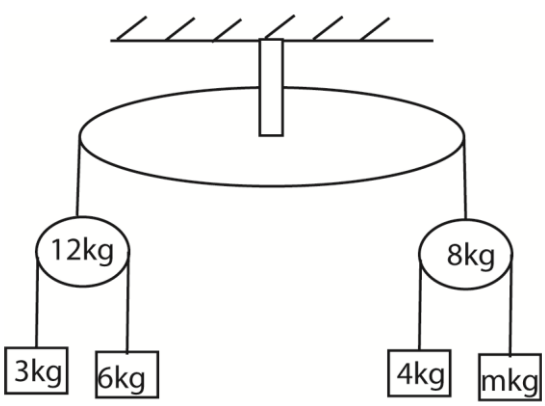
$2T - mg = 0$

$m = \frac{2 \times 39.201}{9.8} = 8kg$

Case 2: A pulley moving on one portion of a string

Example 20

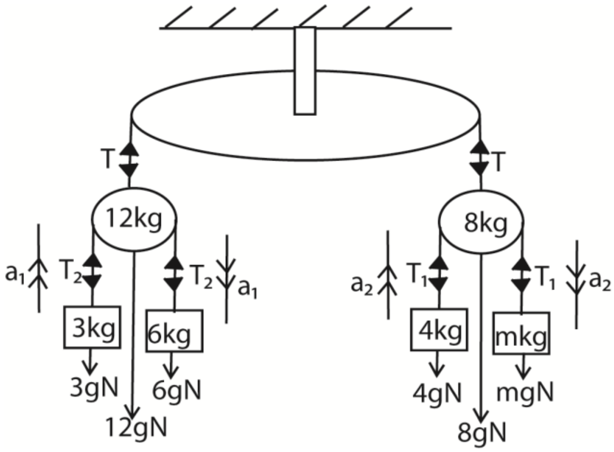
The diagram below shows two pulley to pulleys of masses 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.



The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the

(i) tensions in the string
(ii) value of m

Solution.



For 3kg mass: $T_2 - 3g = 3a_2$ .... (i)

For 6kg mass: $6g - T_2 = 6a_2$ .... (ii)

For 4kg mass: $T_1 - 4g = 4a_1$ .... (iii)

For mkg mass: $mg - T_1 = ma_1$ ....(iv)

For 8kg mass: $2T_1 + 8g = T$ .... (v)

For 12kg mass: $2T_2 + 12g = T$ .... (vi)

eqn. (i) + eqn. (ii): $3g = 3a_2$

$a_2 = \frac{3 \times 9.8}{9} = 3.2667\text{ms}^{-2}$

eqn. (i): $T_2 - 3g = 3a_2$

$T_2 = 3 \times 9.8 + 3 \times 3.2667 = 39.2001\text{N}$

eqn. (vi): $2T_2 + 12g = T$

$T = 2 \times 39.2001 + 12 \times 9.8 = 196.0002\text{N}$

eqn. (v): $2T_1 + 8g = T$

$T_1 = \frac{196.0002 - 8 \times 9.8}{2} = 58.8001\text{N}$

eqn. (iii): $T_1 - 4g = 4a_1$

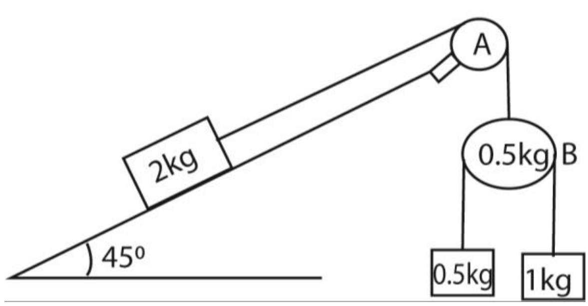
$a_1 = \frac{58.8001 - 4 \times 9.8}{4} = 4.9\text{ms}^{-2}$

eqn. $mg - T_1 = ma_1$

$m = \frac{58.8001}{9.8 - 4.9} = 12\text{kg}$

Example 21

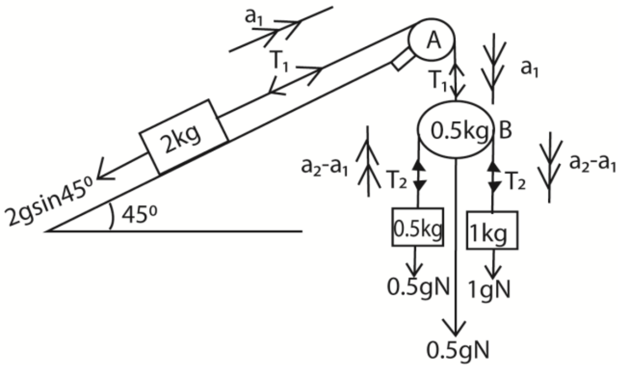
The diagram shows a particle of mass 2kg on a smooth plane inclined at 450 to the horizontal and attached by means of a light inextensible string over a smooth pulley, A at the top of the plane to pulley B of mass 0.5kg which hangs freely. Pulley B carries to particles of mass 0.5kg and 1kg on either side



Find

(a) acceleration of 2kg, 0.5kg and 1kg mass
(b) the tension in the strings

Solution



For 2kg mass: $T_1 - 2g\sin45 = 2a_1$ ……….. (i)

For 0.5kg mass: $T_2 - 0.5g = (a_2 - a_1)$ …… (ii)

For 1kg mass: $gN - T_2 = 1(a_1 + a_2)$ ……… (iii)

For pulley B: $2T_2 + 0.5g - T_1 = 0.5a_1$ ……. (iv)

eqn. (ii) + eqn (iii): $0.5g = 1.5a2 + 0.5a1$

$9.8 = 3a_2 + a_1$ ……….. (v)

eqn. (i) + eqn. (iv): $2T_2 - 2g\sin45 + 0.5g = 2.5a_1$

$2T_2 - 8.9593 = 2.5a_1$ ………… (vi)

2 x eqn. (iii) + eqn. (vi): $10.6407 = 4.5a_1 + 2a_2$

$5.3204 = 2.5a_1 + a2$……………. (vii)

eqn. (vii) – eqn. (v): $5.75a_1 = 6.1612$

$a_1 = \frac{6.1612}{5.75} = 1.0715\text{ms}^{-2}$

from eqn. (v): $9.8 = 3a_2 + a_1$

$a_2 = \frac{9.8-1.0715}{3} = 2.9095\text{ms}^{-2}$

Acceleration of 2kg mass = 1.0715ms^{-2}

Acceleration of 0.5kg mass = 2.9095ms^{-2}

Acceleration of 1kg = 2.9095 + 1.0715

$= 3.981\text{ms}^{-2}$

From eqn. (i): $T_1 - 2g\sin45 = 2a_1$

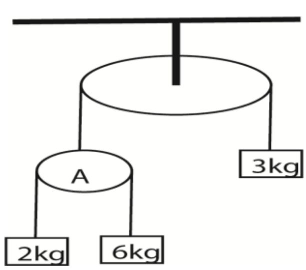
$T1 = 2 \times 1.0715 + 2 \times 9.8\sin45 = 16.0023\text{N}$

from eqn. (iv): $2T_2 + 0.5g - T_1 = 0.5a_1$

$T_2 = \frac{0.5 \times 1.0715 + 16.0023 - 4.9}{2} = 5.8190\text{N}$

Example 22

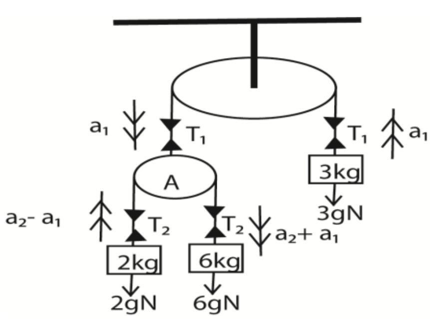
The diagram shows a fixed pulley carrying a string which has a mass of 3kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 6kg at one and a mass of 2kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 2kg, 6kg and 3kg masses
(c) tension in the string

solution



For 3kg mass: $T_1 - 3g = 3a_1$ ……………….. (i)

For 6kg mass: $6g - T_2 = 6(a_2 + a_1)$ ……… (ii)

For 2kg mass: $T_2 - 2g = 2(a_2 - a_1)$ ……… (iii)

For pulley A: $2T_2 - T_1 = 0 \times a_1$ …………… (iv)

eqn. (ii) and eqn. (iii): $4g = 8a_2 + 4a_1$ ….. (v) eqn. (i) + eqn. (iv): $2T_2 - 3g = 3a_1$...... (vi)

2 x eqn. (iii) - eqn. (vi)

$-g = 4a_2 - 7a_1$ .... (vii)

2eqn. (vii) – eqn. (v)

$-18a_1 = -6g$

$a_1 = \frac{6 \times 9.8}{18} = 3.27ms^{-2}$

$4g = 8a_2 + 4a_1$

$a_2 = \frac{9.8-3.27}{2} = 3.27ms^{-2}$

Acceleration of pulley A $=3.27ms^{-2}$

Acceleration of 2kg $= 3.27ms^{-2} - 3.27ms^{-2} = 0$

Acceleration of 6kg $= 3.27ms^{-2} + 3.27ms^{-2}$

$= 6.54ms^{-2}$

Acceleration of 3kg $=3.27ms^{-2}$

$T_1 - 3g = 3a_1$

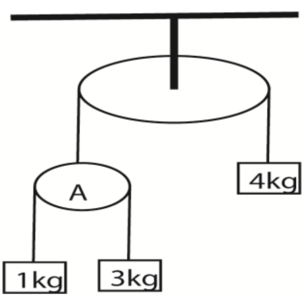
$T_1 = 3 \times 3.27 + 3 \times 9.8 = 39.21N$

$2T_2 - T_1 = 0 \times a_1$

$T_2 = \frac{39.21}{2} = 19.61N$

Example 23

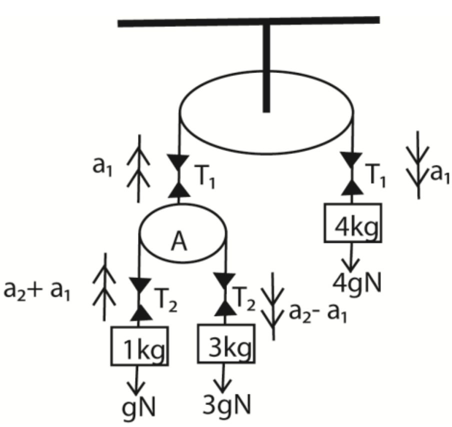
The diagram shows a fixed pulley carrying a string which has mass of 4kg attached at one end and a light pulley A at the other end. Another string passes over pulley A and carries a mass of 3kg at one end and a mass of 1kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 1kg, 3kg and 4kg masses
(c) tension in the string

Solution



For 4kg mass: $4g - T_1 = 4a_1$ .... (i)

For 3kg mass: $3g - T_2 = 3(a_2 - a_1)$ .... (ii)

For 1kg mass: $T_2 - g = (a_2 + a_1)$ .... (iii)

For pulley A: $T_1 - 2T_2 = 0 \times a_1$ .... (iv)

eqn. (ii) and eqn. (iii): $g = 2a_2 - a_1$ ..... (v)

eqn. (i) + eqn. (iv): $4g - 2T_2 = 4a1$ .... (vi)

2 x eqn.(iii) + eqn. (v): $2g = 2a_2 + 6a_1$ ..... (vii)

eqn. (vii) – eqn. (i): $7a_1 = g$

$a_1 = \frac{9.8}{7} = 1.4ms^{-2}$

$g = 2a_2 - a_1$

$a_2 = \frac{9.8+1.4}{2} = 5.6ms^{-2}$

Acceleration of pulley A $= 1.4ms^{-2}$

Acceleration of1kg mass $= 5.6 + 1.4 = 7ms^{-2}$

Acceleration of 3kg mass $= 5.6 - 1.4 = 4.2ms^{-2}$

Acceleration of 4kg mass $= 1.4ms^{-2}$

$4g - T_1 = 4a_1$

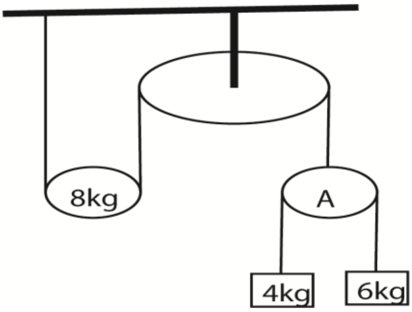
$T_1 = 4 \times 9.8 - 4 \times 1.4 = 33.6N$

$T_1 - 2T_2 = 0 \times a_1$

$T_2 = \frac{33.6}{2} = 16.8N$

Example 24

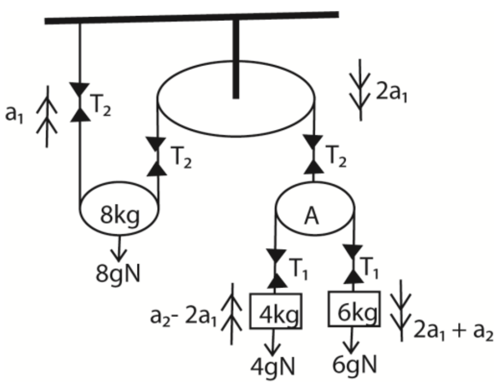
The diagram below shows a fixed pulley carrying a movable pulley of mass 8kg at one end and a light pulley A attached at the other end. A string passes over pulley A and carries a mass of 4kg at one end and a mass of 6kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 8kg, 6kg and 4kg masses
(c) tension in the string

Solution



For 8kg mass: $2T_2 - 8g = 8a_1$ ………………. (i)

For 4kg mass: $T_1 - 4g = 4(a_2 - 2a_1)$ ……… (ii)

For 6kg mass: $6g - T_1 = 6(2a_1 + a_2)$ ……… (iii)

For pulley A: $2T_1 - T_2 = 0 \times a_1$ …………… (iv)

eqn. (ii) and eqn. (iii): $2g = 10a_2 + 4a_1$

$4.9 = 2.5a_2 + a_1$ ….. (v)

eqn. (i) + 2 x eqn. (iv): $4T_1 - 8g = 8a_1$…. (vi)

4 x eqn. (iii) + eqn. (vi): $16g = 56a_1 + 24a_2$

$2g = 7a_1 + 3a2$ ………………………………. (vii)

7 x eqn. (v) – eqn. (vii): $14.5a_2 = 14.7$

$a_2 = \frac{14.7}{14.5} = 1.0138ms^{-2}$

eqn. (v); $4.9 = 2.5a_2 + a_1$

$a_1 = 4.9 - 2.5 \times 1.0138 = 2.3655ms^{-2}$

Acceleration of pulley = $2a_1 = 2 \times 2.3655$

$= 4.731ms^{-2}$

Acceleration of 6kg = 4.731 + 1.0138

$= 5.7448ms^{-2}$

Acceleration of 3kg = $a_2 - 2a_1$

$1.0138 - 4.731 = -3.7172ms^{-2}$

From eqn. (i): $2T_2 - 8g = 8a_1$

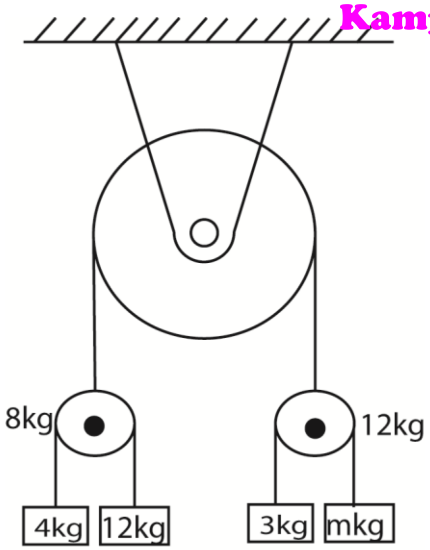
$T_2 = \frac{8 \times 4.731 + 8 \times 9.8}{2} = 58.124N$

From eqn. (iv)

$T_1 = \frac{T_2}{2} = \frac{58.124}{2} = 29.062N$

Example 25

The diagram below shows two pulleys of mass 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.



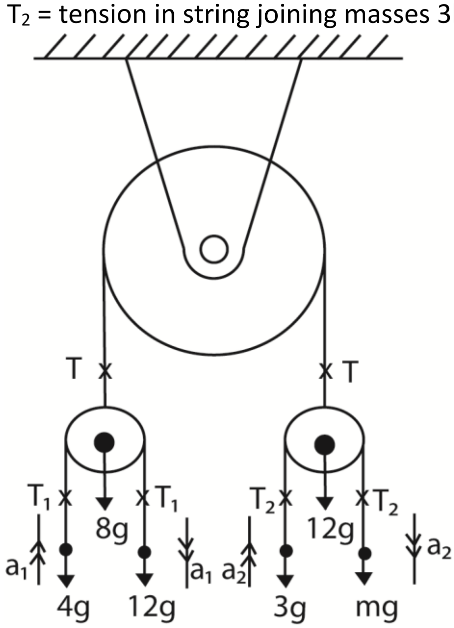
The acceleration of 4kg and 12kg masses are $\frac{g}{2}$ upward and $\frac{g}{2}$ downward respectively. The acceleration of the 3kg and m masses are $\frac{g}{3}$ upwards and $\frac{g}{3}$ downwards respectively. The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the

(a) tensions in the strings (09marks)

Let T = tension n the string joining masses 8kg and 12kg

T_1 = tension in the string joining masses 4kg and 12kg

T_2 = tension in string joining masses 3kg and mkg



Since the string of the fixed pulley remains stationary, this means the pulleys of the 8kg and 12kg are stationary or fixed

Resolving vertically

2T = 8g + 12g

T = 10g = 10 x 9.8 = 98N

For 4kg mass; resultant force = T_1- 4g

$4a_1 = T_1 - 4g$

$T_1 = 4g + 4a_1 = 4g + 4 \times \frac{g}{2} = 6g$

= 6 x 9.8 = 58.8N

For 3kg mass; resultant force = T_2- 3g

⇨ $3a_2 = T_2 - 3g$

$T_2 = 3g + 3a_2 = 3g + 3 \times \frac{g}{3} = 4g$

= 4 x 9.8 = 39.2N

Hence the tensions in the strings are 98N, 58.8N and 39.2N

(b) value of m. (03marks)

For the m kg mass

Resultant force = mg – T_2

$ma_2 = mg - T_2$

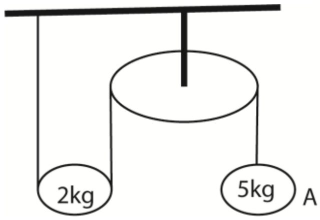
$m(\frac{g}{3}) = mg - 4g$

$\frac{2}{3}mg = 4g$

$m = \frac{12}{2} = 6kg$

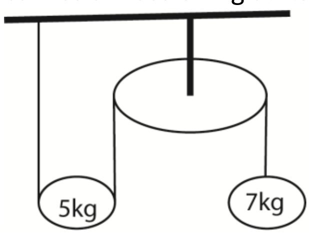
Revision exercise 3

1. A string with one end fixed passes under a movable pulley of mass 2kg, over a fixed pulley and carries a 5kg mass at its end



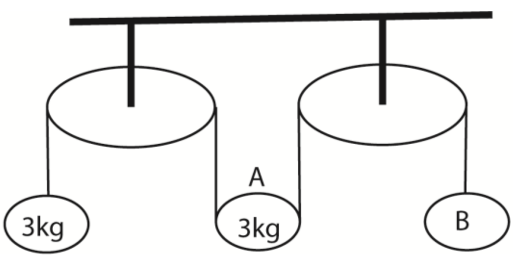
Find the acceleration of the movable pulley and the tension in the string. [$3.56ms^{-2}$, 13.36N]

2. a string with one end fixed passes under a movable pulley of mass 5kg, over a fixed pulley and carries a mass of7kg at its other end.



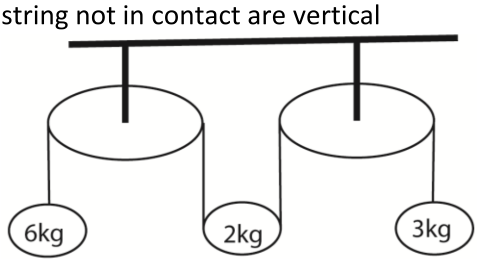
Find the acceleration of the movable pulley and the particle [$2.673ms^{-2}$, $5.146ms^{-2}$]

3. In the pulley system below, A is a heavy pulley which is free to move.



Find the mass B, if it does not move upwards or downwards when the system is released from rest. [1kg]

4. Two particles of mass 3kg and 6kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 2kg, the portions of the string not in contact are vertical

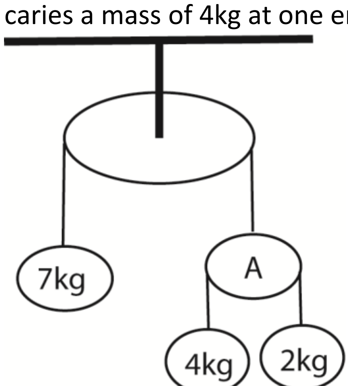


If the system is released from rest, find

(a) acceleration of movable pulley [$5.88ms^{-2}$]

(b) tension in the string [15.6N]

5. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over the pulley A and caries a mass of 4kg at one end and a mass of 2kg at the other end.

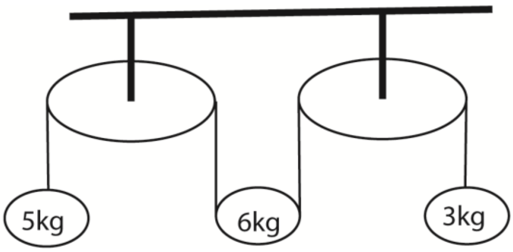


If the system is released from rest, find

(a) acceleration of 4kg mass [$2.38ms^{-2}$]

(b) tension in the strings [59.33N, 29.66N]

6. Two particles of mass 3kg and 5kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 6kg, the portions of the string not in contact are vertical

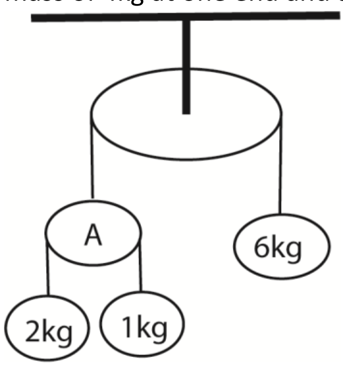


If the system is released from rest, find

(a) acceleration of movable pulley [$1.089ms^{-2}$]

(b) tension in the string [32.667N]

7. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 4kg at one end and a mass of 2kg at the other end.

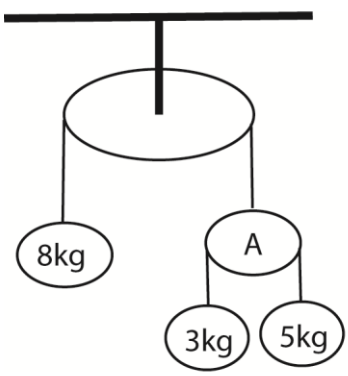


If the system is released from rest, find

(a) acceleration of 1kg mass [$8.2923ms^{-2}$]

(b) tension in the strings
[18.0923N, 36.1846N]

8. The diagram shows a system of masses and pulleys.

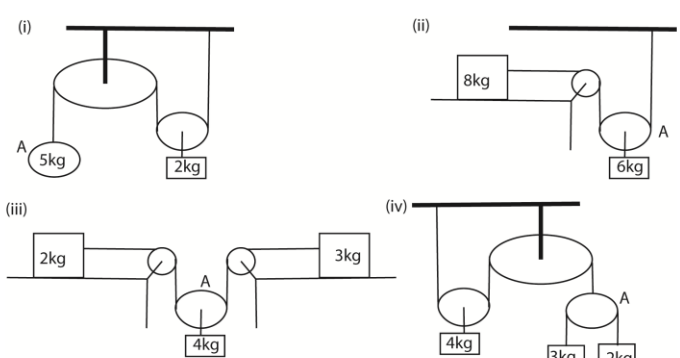


If the system is released from rest, find

(a) acceleration of 5kg mass [$2.8451ms^{-2}$]

(b) tension in the strings
[75.8712N, 37.9356N]

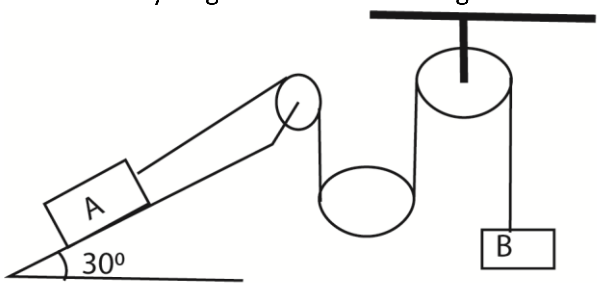
9. For each of the systems below: all the strings are light and inextensible, all pulleys are light and smooth and all surface are smooth. In each case find the acceleration of A and the tension in the string.



(i) [7.127ms^{-2}, 13.364N] (ii) [1.547ms^{-2}, 24.758N]

(iii) [3.564ms^{-2}, 10.691N] (iv) [4.731ms^{-2}, 12.166N, 24.331N]

10. Two particles A and B of mass 4kg and 2kg respectively and a movable pulley c of mass 6kg are connected by a light inextensible string as shown below



Given that the coefficient of friction between A and the plane is 0.2 and the system is released from rest, find the acceleration of A, B, C and the tension in the string.
[A = -0.25ms^{-2}, B = 2.9ms^{-2}, C = 1.325ms^{-2}]

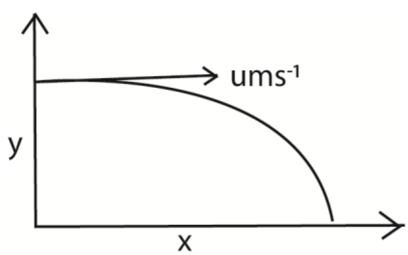
Projectile motion

This is the motion of a body which after being given an initial velocity moves freely under the influence of gravity.

Terms used in projectiles

1. ***Angle of projection:*** is the angle the initial velocity makes with the horizontal
2. ***Maximum height/ greatest height*** is the greatest height reached by projectile
3. ***Time of flight (T)*** is the time taken for projectile to complete motion. Note: the time of flight is twice the time to maximum height.
4. ***Range, R:*** is the horizontal distance covered by projectile
5. ***Maximum range (Rmax)*** is the greatest horizontal distance covered
6. ***Trajectory;*** is the path described by a projectile.

A. An object projected horizontally from a height above the ground.



Horizontal motion: Ux = u, a = 0

$x = ut$

Vertical motion: Uy = 0, a = $-9.8ms^{-2}$

$s = ut + ½ at^2$

$-y =- ½ gt^2$

$V = u + at$

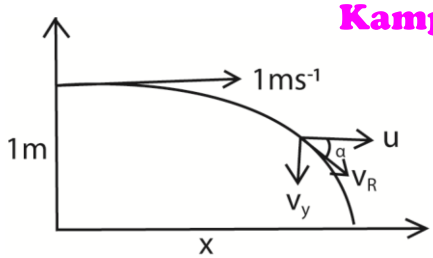
$V_y = -gt$

Example 1

A ball rolls of the edge of a table top 1m high above the floor with horizontal velocity 1ms-1. find:

(i) time taken to hit the floor
(ii) horizontal distance covered
(iii) the velocity when it hits the floor.

Solution



(i) vertical motion: u = 1ms^{-1}, θ = 0

y = 1m below the point of projection

y = ½ gt^2

-1 = $\frac{1}{2}\ x - 9.8\ t^2$

t = 0.4518s

(ii) x = ut = 1 x 0.4518 = 0.4518m

(iii) v_x = 1ms^{-1}

v_y = -gt = -9.8 x 0.4518 = -4.428ms^{-1}

$v_R = \sqrt{1^2 + (-4.428)^2}$ = 4.54ms^{-1}

$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4.428}{1}\right)$ = 77.3^0

Example 2

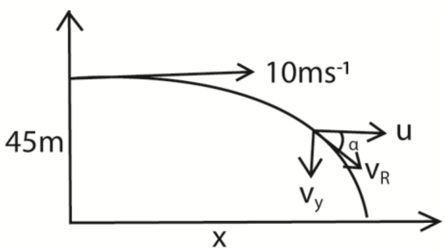
A ball is thrown forward horizontally from the top of a cliff with a velocity of 10ms^{-1}. The height of a cliff above the ground is 45m. Calculate

(i) time to reach the ground

(ii) distance from the cliff where the ball hits the ground

(iii) velocity and direction of the ball just before it hits the ground

Solution



(ii) vertical motion: u = 10ms^{-1}, θ = 0

y = 45m below the point of projection

y = ½ gt^2

-45 = $\frac{1}{2}\ x - 9.8\ t^2$

t = 3.03s

(ii) x = ut = 10 x 3.03 = 30.3m

(iii) v_x = 10ms^{-1}

v_y = -gt = -9.8 x 3.03 = -29.694ms^{-1}

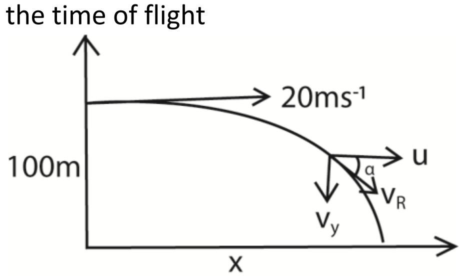
$v_R = \sqrt{10^2 + (-29.694)^2}$ = 31.33ms^{-1}

$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.694}{10}\right)$ = 71.4^0

Example 3

An object is projected horizontally at a speed of 20ms^{-1} from a height of 100m. Find

(i) the time of flight



y = ½ gt^2

-100 = $\frac{1}{2}\ x - 9.8\ t^2$

t = 4.52ms^{-1}

(ii) the horizontal range

x = ut = 20 x 4.52 = 90.4m

(iii) its velocity on reaching the ground

$v_x = 20ms^{-1}$

$v_y = -gt = -9.8 \times 4.52 = -32.7ms^{-1}$

$v_R = \sqrt{20^2 + (-32.7)^2} = 38.33ms^{-1}$

$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{32.7}{20}\right) = 58.5^0$

Example 4

At time t = 0, a particle is projected with a velocity of 3ms-1 from a point with position vector (5i + 25j)m. Find the

(i) speed and direction of the particle when t = 2s

$v_x = u = 3ms^{-1}$

$v_y = gt = -9.8 \times 2 = -19.6ms^{-1}$

$v_R = \sqrt{3^2 + (-19.6)^2} = 19.83ms^{-1}$

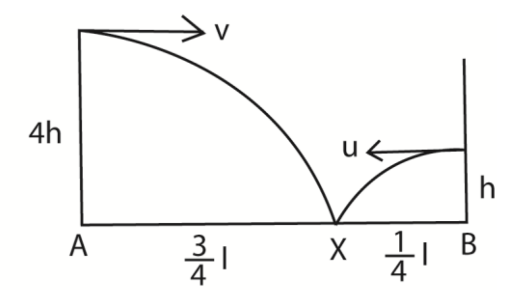
$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{19.6}{3}\right) = 81.3^0$

(ii) Position vector of the particle when t = 2s

$$P_{(t=t)} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 25 \end{pmatrix} + \begin{pmatrix} ut \\ -\frac{1}{2}gt^2 \end{pmatrix} = \begin{pmatrix} 5 \\ 25 \end{pmatrix} + \begin{pmatrix} 3 \ x \ 2 \\ -\frac{1}{2}x9.8 \ x \ 2^2 \end{pmatrix} = \begin{pmatrix} 11 \\ 5.4 \end{pmatrix} m$$

Example 5

A and B are two points on a level ground. A vertical tower of height 4h has its base at A and vertical tower of height h has its bas at B. When a stone is thrown horizontally with speed v from the top the taller tower towards the smaller tower, it lands at point X where AX = $\frac{3}{4}AB$. When a stone is thrown horizontally with speed u from the top of the smaller tower towards the taller tower, it also lands at the point A. Show that 3u = 2V



For A Vertical motion: y = $\frac{1}{2}gt^2$

$$4h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{8h}{g}}$$

Horizontal motion: x = vt

$$\frac{3}{4}l = vt$$

$$l = \frac{4}{3}vt = \frac{4}{3}v\sqrt{\frac{8h}{g}} \ldots.(i)$$

For B Vertical motion: y = $\frac{1}{2}gt^2$

$$h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

Horizontal motion: x = ut

$$\frac{1}{4}l = ut$$

$$l = \frac{4}{3}ut = \frac{4}{3}v\sqrt{\frac{2h}{g}} \ldots.(ii)$$

Eqn (i) and (ii)

$$\frac{4}{3}v\sqrt{\frac{8h}{g}} = \frac{4}{3}u\sqrt{\frac{2h}{g}};$$

v = $\frac{3}{2}u$

2v = 3u

Revision exercise 1

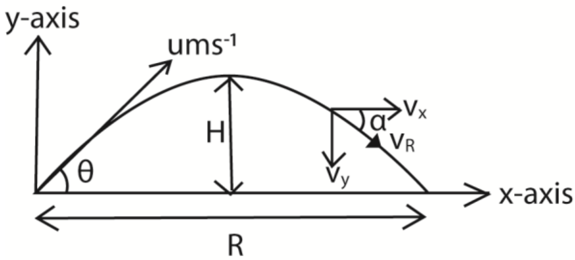
1. A pencil is accidentally knocked off the edge of a horizontal desktop. The height of the desk is 64.8cm and the pencil hits the floor a horizontal distance of 32.4cm from the edge of the desk. What was the speed of the pencil as it left the desk? [$0.9ms^{-1}$]
2. A particle is projected horizontally at $20ms^{-1}$ from a point 78.4m above a horizontal surface. Find the time taken for the particle to reach the surface and the horizontal distance travelled in that time. [4s, 80m]
3. A particle is projected horizontally with a speed of $2ms^{-1}$
   (i) Find the horizontal and vertical displacements of the particle from the point of projection, $2\frac{6}{7}$s after projection [60m, 40m below]
   (ii) Find how far the particle is then from the point of projection. [72.1m]
4. A particle is projected horizontally from a point 2.5m above the horizontal surface. The partial hits the surface at a point which is horizontally 10m from the point of projection. Find the initial speed of the particle.[$14ms^{-1}$]
5. At time t= 0, a particle is projected with a velocity of $2ims^{-1}$ from a point with position vector (10i + 150j)m. Find the
   (i) speed and direction of the particle when t = 5s [$49.04ms^{-1}$, at 87.6^0]
   (ii) position vector of the particle when t = 1s $\left[\begin{pmatrix}20\\27.5\end{pmatrix}m\right]$
6. At time t = 0, a particle is projected with a velocity of 5ims-1 from a point with position vector (20j)m. find the position vector of the particle when t = 2s. $\left[\begin{pmatrix}10\\0.4\end{pmatrix}m\right]$.
7. A batsman strikes a ball horizontally when it is 1m above the ground. The ball is caught 10cm above the grounder by a fielder standing 12m from the batsman. Find the speed with which the batsman hits the ball. [$28ms^{-1}$]
8. A darts player throws a dart horizontally with a speed of $14ms^{-1}$. The dart hits the board at a point 10cm below the level at which it is released. Find the horizontal distance travelled by the dart. [2m]
9. A tennis ball is served horizontally with initial speed of $21ms^{-1}$ from a height of 2.8m, by what distance does the ball clears a net 1m high situated 12m horizontally from the server? [20cm]
10. A fielder retrieves cricket ball and throws it horizontally with a speed of $28ms^{-1}$ to a wicket-keeper standing 12m away. If the fielder releases the ball at a height of 2m above level ground, find the height of the ball when it reaches the wicket-keeper. [110cm]
11. Initially a particle is at an origin and is projected with a velocity of ai ms^{-1}. After t second, the particle is at the point with position vector (30i – 21j)m. Find the value of t and a. [$1\frac{3}{7}$, 21]
12. Two vertical towers stand a horizontal ground level and are of height 40m and 30m. A ball is thrown horizontally from the top of the higher tower with a speed of $24.5ms^{-1}$ and just clears the short tower. Find the distance
   (i) between the two towers [35m]
   (ii) between the short and the point on the ground where the ball first lands. [35m]
13. The top of a vertical tower is 20m above ground level. When a ball is thrown horizontally from the top of this tower. By how much does the ball clears a vertical wall of height 13m situated 12m from the tower.[2m]
14. A stone is thrown horizontally with speed u from the edge of a vertical cliff of height h. The stone hits the ground at a point which is a distance d horizontally from the base of the cliff. Show that $2hu^2 = gd^2$.
15. A vertical tower stands with its base on a horizontal ground. Two particles A and B are both projected horizontally and in the same direction from the top of the tower with initial velocities

of $14ms^{-1}$ and $17.5ms^{-1}$ respectively. If A and B hit the ground at two points 10cm apart, find the height of the tower[40m]

16. O, A and B are three points with O on level ground and A and B respectively 3.6 and 40m vertically above O. A particle is projected horizontally from B with a speed of $21ms^{-1}$ and 2seconds later, a particle is projected horizontally from A with a speed of $70ms^{-1}$. Show that the two particles reach the ground at the same distance from O, find this distance.
17. An aeroplane moving horizontally at 150ms-1 releases a bomb at height of 500m. The aeroplane hits the intended target. What was the horizontal distance of aeroplane from the target when the bomb was released? [1500m]
18. A projectile is fired horizontally from the top of a cliff 250m high. The projectile lands 144m from the bottom of the cliff. Find the
    (i) initial speed of the projectile [198ms-1]
    (ii) velocity of the projectile just before it hits the ground [$210ms^{-1}$ at 19.5^0]

b. Object projected upwards from the ground at an angle to the horizontal

Suppose an object is project with velocity u at an angle θ from a horizontal ground. H and R are the maximum height reached and range respectively.



Horizontally; $u_x = u\cos\theta$, a = 0

v = u + at

$v_x = u\cos\theta$

$x = u\cos\theta t$

$v_y = s = ut + \frac{1}{2}at^2$

$x = u\cos\theta t$

Vertically; $u_y = u\sin\theta$, $a = -9.8ms^{-2}$

v = u + at

$v_y = u\sin\theta - gt$

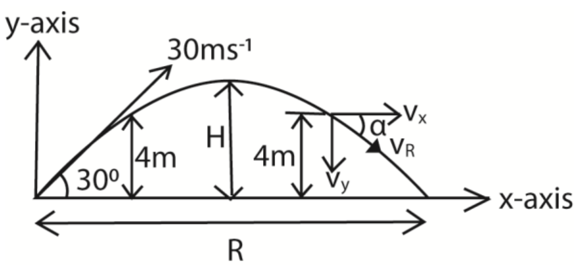
$s = ut + \frac{1}{2}at^2$

$y = u\sin\theta t - \frac{1}{2}gt^2$

Example 6

A particle is projected with a velocity of $30ms^{-1}$ at an angle of elevation of 30^0. Find

(i) the greatest height reached
(ii) the time of flight
(iii) the velocity and direction of motion at a height of 4m on its way upwards.



(i) (↑) $v_y = u\sin\theta$, $a = -9.8ms^{-2}$

At maximum height, $v_y = 0$

From $v^2 = u^2 + 2as$

$H = \frac{(30\sin 30)^2}{2\ x\ 9.8} = 11,47m$

(ii) at time of flight, s = 0

$s = ut + \frac{1}{2}at^2$

$0 = 30\sin 30T - \frac{1}{2}x\ 9.8\ xT^2$

$0 = (30\sin 30 - \frac{1}{2}x\ 9.8\ T)T$

Either T = 0

or $(30\sin 30 - \frac{1}{2}x\ 9.8\ T) = 0$

T = 3.0612s

(→): u_x = ucosθ, a = 0

s = ut + $\frac{1}{2}at^2$

R = ucosθT

R = (30cos 30) x 3.0612 = 79.5329m

(↑) y = usinθt - $\frac{1}{2}gt^2$

4 = 30sin30t - $\frac{1}{2}x\ 9.8\ x\ t^2$

t = 0.30s or t = 2.76s

t = 0.30s is the correct time since it is smaller indicating that the body is moving upwards

u_x = ucosθ = 30cos30 = 25.98ms^{-1}

v_y = usinθ – gt

= 30sin30 – 9.8 x 0.30 = 12.06ms^{-1}

v = $\sqrt{(v_x)^2 + (v_y)^2}$

= $\sqrt{25.98^2 + 12.06^2}$ = 28.64ms^{-1}

direction α = $\tan^{-1}\left(\frac{12.06}{25.98}\right)$= 24.9^0

Example 7

A particle is projected from the origin at a velocity of (10i + 20j)ms^{-1}. Find the position and velocity vectors of the particle 3s after projection. (Take g = 10ms^{-2})

Solution

$$P_{t=t} = \binom{x_0}{y_0} + \binom{x}{y}$$

$$P_{(t=3)} = \binom{0}{0} + \begin{pmatrix} ucos\theta t \\ usin\theta t - \frac{1}{2}gt^2 \end{pmatrix}$$

$$P_{(t=3)} = \begin{pmatrix} 10\ x\ 3 \\ 20\ x\ 3 - \frac{1}{2}\ x 9.8\ x\ 3^2 \end{pmatrix} = \binom{30}{15} m$$

$$v_{(t=t)} = \binom{v_x}{v_y} = \begin{pmatrix} ucos\theta \\ usin\theta - gt \end{pmatrix}$$

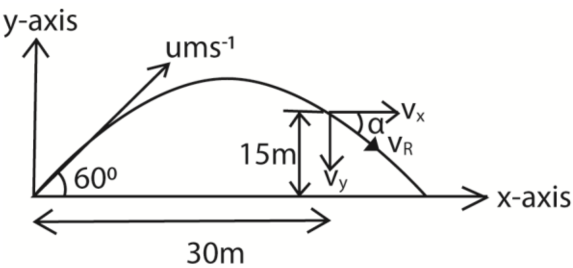
$$v_{(t=3)} = \begin{pmatrix} 10 \\ 20 - 10x\ 3 \end{pmatrix}$$

$$= \binom{10}{-10} ms^{-1}$$

Example 8

A projectile fired at an angle of 60^0 above the horizontal strikes a building 30m away at a point 15m above the point of projection. Find

(i) speed of projection (i) velocity when it strikes a building



(→) x = ucosθt

30 = utcos 60

t = $\frac{60}{u}$

(↑) y = $usin\theta t - \frac{1}{2}gt^2$

15 = usin60 x $\frac{60}{u}$ - $\frac{1}{2}\ x\ 9.8\ x\ \left(\frac{60}{u}\right)^2$

u = 21.86ms^{-1}

(ii) t = $\frac{60}{u} = \frac{60}{21.86}$ = 2.75s

u_x = ucosθ

= 21.86x cos 60 = 10.93ms^{-1}

u_y = usinθ – gt

= 21.86x sin 60 – 9.8 x 2.75

= -8.09ms^{-1}

v = $\sqrt{(v_x)^2 + (v_y)^2}$

= $\sqrt{10.93^2 + (-8.93)^2}$ = 13.60ms^{-1}

direction α = $\tan^{-1}\left(\frac{8.09}{10.93}\right)$= 36.58^0

Example 9

A football player projects a ball at a speed of $8ms^{-1}$ at an angle of 30^0 with the ground. The ball strikes the ground at a point which is level with the point of projection. After impact with the ground, the ball bounces and the horizontal component of the velocity of the ball remains the same but the vertical component is reversed in direction and halved in magnitude. The player running after the ball. Kicks it again at appoint which is at a horizontal distance 1.0m from the point where it bounced, so that the ball continues in the same direction. Find the

(a) horizontal distance between the point of projection and the point at which the ball first strikes the ground. (Take $g = 10ms^{-2}$)
(b) (i) the time interval between the ball striking the ground and the player kicking it again.
(ii) the height of the ball above the ground when it is kicked again (take $g = 10ms^{-2}$)

Solution

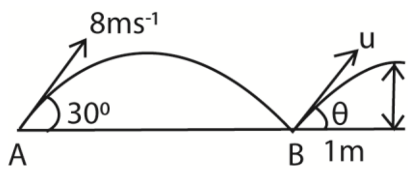
$y = u\sin\theta t - \frac{1}{2}gt^2$

At AB:

$0 = 8\sin 30 \times t - \frac{1}{2} x\ 10\ t^2$

t = 0.8s

$x = \cos\theta t = 8\cos 30 \times 0.8 = 5.543m$



(b)(i) $x = u\cos\theta t$

$1 = 8\cos(30)t$

t = 0.1443s

(ii) $y = u\sin\theta t - \frac{1}{2}gt^2$

$h = 4\sin 30 \times 0.1443 - \frac{1}{2}\ x\ 10\ x\ (0.1443)^2$

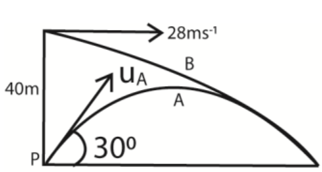
h = 0.185m

Example 10

Two objects A and B are projected simultaneously from different points. A is projected from the top of vertical cliff and A from the base. Particle B is projected horizontally with a speed $28ms^{-1}$ and A is projected at an angle θ above the horizontal. The height of the cliff is 40m and the particles hit the same point on the ground, find;

(a) time taken and the distance from P to where they hit
(b) speed and angle of projection of A

Solution



(a) For B: $y = \frac{1}{2}gt^2$

$-40 = \frac{1}{2}\ x - 9.8t^2$

$t = \frac{20}{7}s$

$x = ut = 28 \times \frac{20}{7} = 80m$

For A: $x = u\cos\theta t$

$80 = u_A\cos\theta \times \frac{20}{7}$

$u_A\cos\theta = 28$ …….. (i)

$y = u\sin\theta t - \frac{1}{2}gt^2$

$0 = u_A\sin\theta \times \frac{20}{7} - \frac{1}{2}\ x - 9.8\ x\ \left(\frac{20}{7}\right)^2$

$u_A\sin\theta = 14$………… (ii)

(i) ÷ (ii)

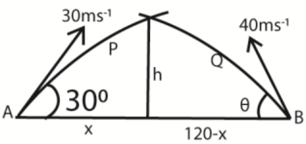
$\theta = \tan^{-1}\frac{14}{28} = 26.6^0$

$u_A\sin 26.6 = 14$

$u_A = 31.3ms^{-1}$

Example 11

A particle P is projected a point A with initial velocity 30ms-1at an angle of elevation 300 to the horizontal. At the same instant a particle Q is projected in the opposite direction with initial speed of 40ms-1 from a point at the same level with a and 120m from A. Given that the particles collide. Find (i) angle of projection of Q (ii) time when collision occur.



y = usinθt - $\frac{1}{2}gt^2$

For P

h = 30sin30 x t - $\frac{1}{2}$ $x9.8t^2$.... (i)

For Q

h = 40sinθ x t - $\frac{1}{2}$ $x9.8t^2$..... (ii)

(i) and (ii)

30sin30 x t = 40sinθ x t

θ = 24.5^0

For P

x – 30cos30t .... (iii)

For Q

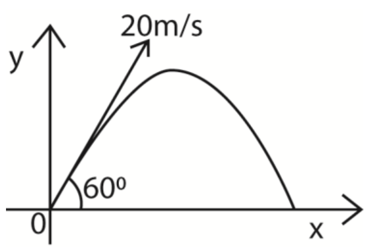
120 –x = 40cos24.5t ... (iv)

(iii)and (iv)

t = 1.9s

Example 12

A particle is projected from a point O with speed20m/s at an angle 60^0 to the horizontal. Express in vector form its velocity v and its displacement, r, from O at any time t seconds . (05marks)



At time t = 0

V = 20cos60^{0}i + 20sin60^{0}j = 10i + 10$\sqrt{3}$j

But A = -gj

At any time t,

V = $\int adt$ = -gj$\int dt$

= -gtj + c

At t = 0

10i + 10$\sqrt{3}$j = 0 + cn

⇨ At time t

V= 10i + (10$\sqrt{3} - gt$)j = 10i + (10$\sqrt{3} - 9.8t$)j

Or

V = $\begin{pmatrix} 10 \\ 10\sqrt{3} - 9.8t \end{pmatrix}$

r = $\int vdt = \int \begin{pmatrix} 10 \\ 10\sqrt{3} - 9.8t \end{pmatrix} dt$ = $\begin{pmatrix} 10t \\ 10\sqrt{3}t - 4.9t^2 \end{pmatrix}$+ c

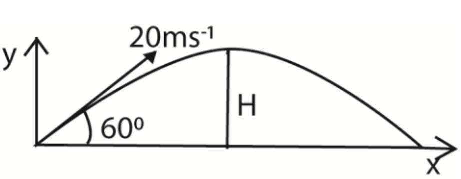
at t = 0, r = $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

⇨ $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + c$, c = $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Hence at time t, r = $\begin{pmatrix} 10t \\ 10\sqrt{3}t - 4.9t^2 \end{pmatrix}$

Example 13

A particle is projected at an angle 600 to the horizontal with velocity of 20ms^{-1}. Calculate the greatest height the particle attains. [Use g = 10ms^{-2}]



Use v^2 = u^2 + 2as; at maximum height, v = 0

0 = $(20 \sin 60)^2 - 2gH$

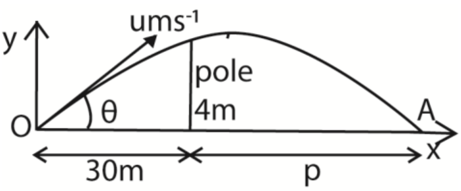
H = 15m

Examples 14

A particle is projected from ground level towards a vertical pole 4m high and 30m away from the point of projection. It just passes the pole in one second. Find

(i) its initial speed and angle of projection
(ii) the distance beyond the pole where the particle falls

Solution



(→) $x = u_x t$ but t = 1

$30 = u\cos\theta \times 1$

$30 = u\cos\theta$ .... (i)

(↑) $y = u_y t - \frac{1}{2}gt^2$

$4 = u\sin\theta \times 1 - \frac{1}{2} \times 9.8 \times 1^2$

$4 = u\sin\theta - 4.9$

$8.9 = u\sin\theta$ .... (ii)

(ii) ÷ (i)

$\theta = \tan^{-1}\left(\frac{8.9}{30}\right) = 16.5^0$

$30 = u\cos 16.5$

$u = 31.29ms^{-1}$

(b) At point O and A, y=0

$y = u\sin\theta t - \frac{1}{2}gt^2$

$0 = 31.29\sin 16.5T - \frac{1}{2} \times 9.8 \times T^2$

T = 1.8136s

Range, $R = u\cos\theta T$

R = 31.29cos 16.6 x 1.8136 = 54.36m

p = 54.36 – 30 = 24.36m

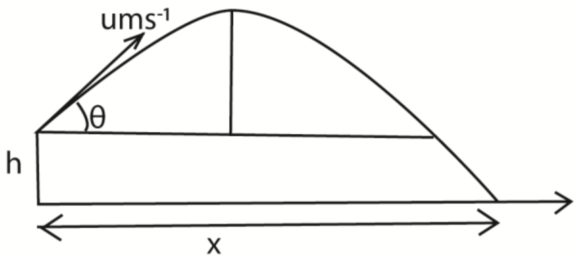
Revision exercise 2

1. A particle projected at an angle of 30^0 to the horizontal with velocity of $60ms^{-1}$. Calculate
   (a) time taken for the particle to reach maximum height [3s]
   (b) maximum height [45m]
   (c) horizontal range of the particle [312m]
2. A particle is projected from an origin O and has an initial velocity of $30\sqrt{2}$ ms^{-1} at angle 450 above the horizontal. Find the horizontal and vertical components of displacement 2s after projection. [60m, 40.4m] Hence find the distance of motion of the particle at that time [72.3m]
3. A particle projected from a point on the level ground has horizontal range of 240m and time of flight of 6s. find the magnitude and direction of velocity of projection [$50ms^{-1}$, 36.9^0]
4. A particle is projected with a velocity of $30ms^{-1}$ at an angle of 40^0 above the horizontal plane. Find
   (i) the time for which the particle is in air [3.9s]
   (ii) the horizontal distance it travels [22.9m]
5. A body is projected with a velocity of 200ms-1 at an angle of 300 above the horizontal. Calculate
   (i) time taken to reach the maximum height[10.2s]
   (ii) its velocity after 16s [$183ms^{-1}$ at 19.1^0]
6. A football is kicked from O on a level ground. 2s later the football just clears a vertical wall of height 2.4m. If O is 22m from the wall, find the velocity with which the ball is kicked. [$15.6ms^{-1}$ at 45^0 above the horizontal]

7. A particle is projected from a level ground in such away that its horizontal and vertical components of velocity are $20ms^{-1}$ and $10ms^{-1}$ respectively. find
   (a) maximum height of the particle [5.0m]
   (b) its horizontal distance from the point of projection when it returns to the ground. [40m]
   (c) the magnitude and direction of the velocity on landing [$22.4ms^{-1}$ at 26.6^0 below the horizontal]
8. A particle is projected with a speed of 25ms-1 at 300 above the horizontal. Find
   (a) time taken to reach the height point of the trajectory. [1.5s]
   (b) the magnitude and direction of velocity after 2.0s.[$22.9ms^{-1}$ at 19.1^0 below the horizontal]
9. A particle is projected from the origin at a velocity $(4i + 13j)ms^{-1}$. Find the position vector and distance of the particle in 2s after projection (take $g = 10ms^{-2}$) [(8i + 6j)m, 10m]
10. A particle is projected from the origin at a velocity of $(4i + 11j)ms^{-1}$. and passes a point P which has a position vector (8i + xj)m. Find the time taken for the particle to reach P from O and the value of x. [2s, 2.4m]
11. A Particle is projected from the origin at a velocity of $(7i + 5j)ms^{-1}$ and passes a point P which has position vector (xi – 30j)m. Find the time taken for the particle to reach P from O and the value of x [3s, 21]
12. A particle is projected from the origin at velocity of $(4i + 2j)ms^{-1}$. Find
   (a) the direction in which it is moving after 1s[63.43^0]
   (b) Two second later after launch of the first particle, a second particle is projected from the same point with a velocity $(8i – 26j)ms^{-1}$. Show that the two particles collide and find the time and position at which this occurs [t =4s, (16i – 72j)m]
13. A particle is projected at $84ms^{-1}$ to hit a point 360m away and on the same horizontal level at the point as the point of projection. Find the two possible angles of projection. [15^0, 75^0]
14. A golfer hits a golf ball at $30ms^{-1}$ and wishes it to land at a point 45m away, on the same horizontal level as the starting point. Find the two possible angles of projection. [4.7^0, 75.3^0]
15. A particle is projected from a horizontal ground and has an initial speed of $35ms^{-1}$. When the ball is travelling horizontally, it strikes a vertical wall. If the wall is 25m from the point of projection, find the two possible angles of projection. [11.8^0, 8.2^0]
16. A particle is projected from a point O and passes through a point A when the particle is travelling horizontally. If A is 10m horizontally and 8m vertically from, find the magnitude and direction of the velocity of projection. [$14.8ms^{-1}$, 58^0 above the horizontal]
17. A stone is projected at an angle of 20^0 to the horizontal and just clears a wall which is 10m high and 30m from the point of projection. Find the
   (a) speed of projection [$73.78ms^{-1}$]
   (b) angle which the stone makes with the horizontal as it clears the wall [16.9^0]
18. A particle is projected from a point on a horizontal plane and has an initial speed of 28ms-1. If the particle passes through a point above the plane, 40m horizontally and 20m vertically from the point of projection, find the possible angles of projection. [45^0, 71.6^0]
19. Two objects A and B are projected simultaneously from different points. A is projected from the top of a vertical cliff and B from the base. Particle A is projected horizontally with a speed $3u\ ms^{-1}$ and B is projected at an angle θ above the horizontal with speed $5u\ ms^{-1}$. The height of the cliff is 56m and the particles collide after 2s, find
   (a) horizontal and vertical distances from the point of collision to the base of the cliff. [42m, 36.4m]
   (b) value of angle u and θ. [$7ms^{-1}$, 53.1^0]

c. Objects projected upwards from a point above the ground at an angle to the horizontal

Suppose an object is projected with velocity u at an angle θ from a height h.



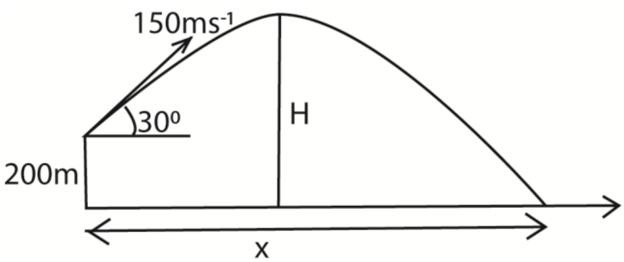
Horizontal: $u_x = u\cos\theta$, a = 0

Vertical: $u_y = u\sin\theta$, a = -g = -9.8

Example 15

A bullet is fired from a gun at a height of 200m with velocity150ms-1 at an angle of 300 to the horizontal. Find

(i) maximum height attained.



$v^2 = u^2 + 2as$

at maximum height, H, v =0

$0^2 = (150\sin 30)^2 - 2 \times 9.8H$

H = 86.70m

(ii) Time taken for the bullet to hit the ground

$y = u\sin\theta t - \frac{1}{2}gt^2$

y = -200m since it's below the point of projection

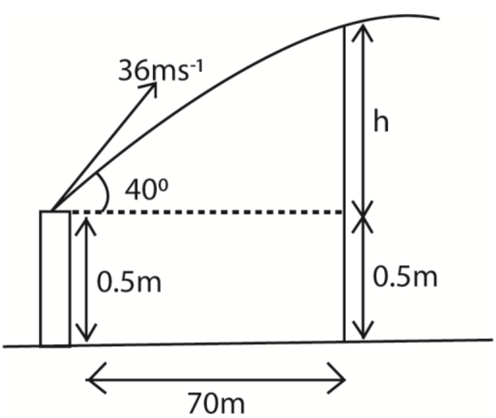
$-200 = 150\sin 30t - \frac{1}{2}x\ 9.8t^2$

t = 17.61s, or t = -2.32s

time taken = 17.61

Example 16

A particle I projected with a speed of $36ms^{-1}$ at an angle of 40^0 to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70 metres on the horizontal plane from a point of projection. Find the;



$Vx = 36\cos 40^0$
$Vy = 36\sin 40^0$

(a) (i) time taken for the particle to reach the wall.

Time take to clear the wall = time taken to cover a horizontal distance of 70m

Usin X= Vxt

$t = \frac{70}{36\sin 40^0} = 2.5384s$

(ii) height of the wall (08marks)

Using h = usinθt - $\frac{1}{2}gt^2$

=36sin40^0 x 2.5384 - $\frac{1}{2}$ x 9.8 x $(2.5384)^2$

= 27.1664 + 0.5 = 27.6664m

(a) Maximum height reached by the particle from the point of projection. (04marks)

From $v^2 = u^2 + 2as$

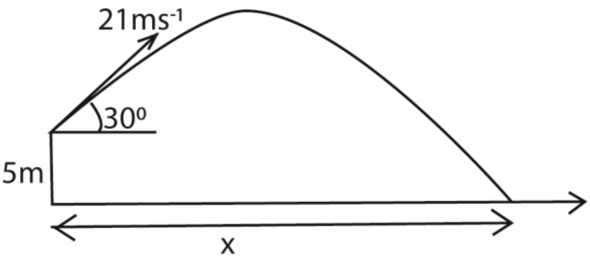
At maximum height vertical component of velocity is zero

⇨ 0 = $(36 \sin 40^2)^2 - 2\ x\ 9.8H$

H = $\frac{(36 \sin 40^2)^2}{2\ x\ 9.8}$ =27.32m

Example 17

A particle is projected at an angle of 30^0 with speed of 21ms^{-1}. If the point of projection is 5m above the horizontal ground, find the horizontal distance that the particle travels before hitting the ground



y = usinθt - $\frac{1}{2}gt^2$

y = -5m since it's below the point of projection

-5 = 21sin30T - $\frac{1}{2}x\ 9.8T^2$

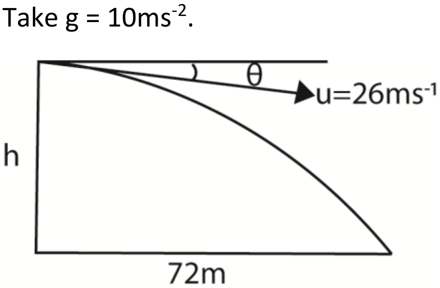
T = 2.54s

(→): x = ucosθ x T

= 21cos30 x 2.54 = 46.19m

Example 18

A stone is thrown from the edge of a vertical cliff and has initial velocity of 26ms^{-1} at an angle of $\tan^{-1}\left(\frac{5}{12}\right)$ below the horizontal. The stone hits the sea at a point level with the base of the cliff and 72m from it. Find the height of the cliff and the time for which the stone is in the air.
Take g = 10ms^{-2}.



y = xtanθ - $\frac{gx^2(1+tan^2\theta)}{2u^2}$

h = 72 x$\left(-\frac{5}{12}\right)$ = $\frac{10\ x\ 72^2\left[1+\left(\left(-\frac{5}{12}\right)\right)^2\right]}{2\ x\ 26^2}$

= -75m

h= 75m below the point of projection

x = ucosθt

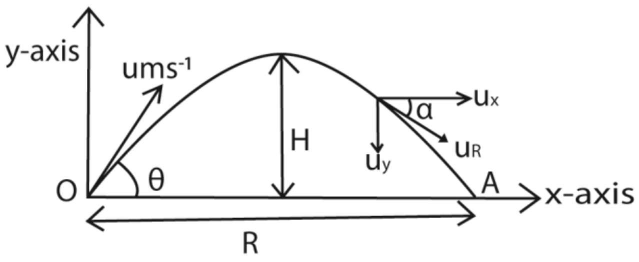
72 = 26$\left[cos\left(-\tan^{-1}\frac{5}{12}\right)\right]t$; t =3s

Revision exercise 3

1. A stone is thrown from the edge of a vertical cliff with velocity of 50ms^{-1} at an angle of $\tan^{-1}\frac{7}{24}$ above the horizontal. The stone strikes the sea at a point 240m from the foot of the cliff. Find the time for which the stone is in air and the height of the cliff. [5s, 52.5m]
2. A particle is projected with a velocity of 10ms-1 at an angle of 45^0 to the horizontal; it hits the ground at a point which is 3m below the point of projection. Find the time for which it is in the air and the horizontal distance covered the particle in this time. [1.76s, 12.42m]
3. A batsman hits a ball with velocity of 17ms^{-1} at an angle $\tan^{-1}\frac{3}{4}$ above the horizontal, the ball initially being60cm above the level ground. The ball is caught by a fielder standing 28m from the batsman. Find the time taken for the ball to reach the fielder and the height above the ground at which he takes the catch. [2s, 2m]
4. A stone is thrown from the edge of a vertical cliff 70m high at an angle of 300 below the horizontal. the stone hits the sea at a point level with the base of the cliff and 20m from it. Find the initial speed of the stone and the direction it is moving when it hits the sea. [6.69mms^{-1}, 81.2^0]
5. A vertical tower stands on a level ground. A stone is thrown from the top of the tower and has an initial velocity of 24.5ms^{-1} at an angle of $\tan^{-1}\frac{4}{3}$ above the horizontal. The stone strikes the ground at a point 73.5m from the foot of the tower. Find the time taken for the stone to reach the ground and the height of the tower. [5s, 24.5m]
6. A stone is thrown from the top of a vertical cliff, 100mabove sea level. The initial velocity of the stone is 13ms-1 at an angle of elevation of$\tan^{-1}\frac{5}{12}$. Find the time taken for the stone to reach the sea and its horizontal distance from the cliff at that time. Take g = 10ms^{-2}. [5s. 60m]
7. A golfer hits a golf ball with a velocity of 30ms^{-1} at an angle of $\tan^{-1}\frac{4}{3}$ above the horizontal. The ball lands on green 5m below the level from which it was struck. Find the horizontal distance travelled by the ball. Take g = 10ms^{-2}. [90m]
8. A pebble is thrown from the top of a cliff at a speed of 10ms^{-1} and at 30^0 above the horizontal. It hits the sea below the cliff 6s later. Find
   (a) the height of the cliff [150m]
   (b) the distance from the base of the cliff at which the pebble falls into the sea. [52m]
9. An arrow is fired from a point at a height 1.5m above the horizontal. It has a velocity of 12ms^{-1} at an angle 30^0 above the horizontal. The arrow hits the target at a height of 1m above the horizontal ground, find
   (i) time taken for the arrow to hit the target [1.3s]
   (ii) horizontal distance between where the arrow is fired and the target. [13.51m]
   (iii) speed of the arrow when the arrow hit the target [12.39ms^{-1}]

Standard equations of the projectile

Suppose an object is projected with velocity u at an angle θ from a horizontal ground



(a) Maximum height (greatest height), H

For vertical motion, at maximum height v = 0,

a = -g = -9,8ms^{-2}

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$

$H = \frac{(u\sin\theta)^2}{2g}$

(b) Time to reach maximum height

(↑): $v = u_y + at$, at maximum height, v = 0

0 = usinμ – gt

$t = \frac{u\sin\theta}{g}$

(c) Time of flight, T

(↑): $s_y = u_y t - \frac{1}{2}gt^2$

At A $s_y = 0$

$0 = u\sin\theta T - \frac{1}{2}gT^2$

$T = \frac{2u\sin\theta}{g}$

(d) Range

(→) x = ucosθt

R = ucosθT

$= u\cos\theta . \frac{2u\sin\theta}{g}$

$= \frac{2u^2\cos\theta\sin\theta}{g}$

$R = \frac{2u^2\sin2\theta}{g}$

(e) Maximum range

for maximum range sin2θ = 1

$2\theta = \sin^{-1} 1$

2θ =90

$R_{max} = \frac{2u^2\sin90}{g} = \frac{u^2}{g}$

(f) Equation of trajectory

A trajectory is expresses zin terms of horizontal distance x and vertical distance y

(→): x = ucosθt

$t = \frac{x}{u\cos\theta}$ .... (i)

(↑): $y = u\sin\theta t - \frac{1}{2}gt^2$ .... (ii)

putting (i) into (ii)

$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$= x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$

$y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$

Example 19

A ball is projected from the horizontal ground and has an initial velocity of 20ms^{-1} at an angle of elevation$\tan^{-1}\frac{7}{24}$. When the ball is travelling horizontally it strike a vertical wall. How high above the ground does the impact occur.

Projectile travel horizontally at maximum height

H = $\frac{(u\sin\theta)^2}{2g} = \frac{20^2\sin^1\left(\tan^{-1}\frac{7}{24}\right)}{2\ x\ 9.8}$ = 1.6m

Example 20

A particle is projected from a point on a horizontal ground at a speed of 84ms^{-1}. If the particle hits a point 300m away and on the same horizontal plane as the projection, find the

(i) angle of projection

R =$\frac{2u^2\sin 2\theta}{g}$

360 = $\frac{2x\ 84^2\ x\ \sin 2\theta}{9.8}$; θ = 15^0 0r θ = 75^0

(ii) maximum height

H$_1$ = $\frac{(u\sin\theta)^2}{2g} = \frac{84^2\sin^2(15)}{2\ x\ 9.8}$ = 24.1m

H$_2$ = $\frac{(u\sin\theta)^2}{2g} = \frac{84^2\sin^2(75)}{2\ x\ 9.8}$ = 335.9m

(iii) times of flight

$T_1 = \frac{2u\sin\theta}{g} = \frac{2x\ 84\ x\ \sin 15}{9.8}$ = 4.44s

$T_2 = \frac{2u\sin\theta}{g} = \frac{2x\ 84\ x\ \sin 75}{9.8}$ = 16.56s

Example 21

A gun has its barrel set at an angle of elevation of 15^0. The gun fires a shell with initial speed of 210ms^{-1}. Find the

(a) horizontal range of the shell

R =$\frac{2u^2\sin 2\theta}{g} = \frac{2x\ 84^2\ x\ \sin 30}{9.8}$ = 2250m

(b) maximum range

R =$\frac{2u^2}{g}$ = R =$\frac{2\ x\ 84^2}{g}$= 4500m

Example 22

A stone thrown upwards at an angle θ to the horizontal with speed ums-1 just clears a vertical wall4m high and 10m from the point of projection when travelling horizontally. Find the angle of projection.

Solution

Projectile travel horizontally

at maximum height

H = $\frac{(u\sin\theta)^2}{2g}$

4 = $\frac{(u\sin\theta)^2}{2g}$

8g = u^{2}sin^{2}θ =……… (i)

Also, x = ucosθt and t = $\frac{u\cos\theta}{g}$

x = ucosθ$\left(\frac{u\cos\theta}{g}\right)$

10g = u^{2}cosθsinθ ……(ii)

(i)÷(ii)

θ = $\tan^{-1}\frac{8}{10}$ = 38.7^0

Example 23

If the horizontal range of a particle with velocity u is r, show that the greatest height H is satisfied by the equation $16gH^2 - 8Hu^2 + gR^2 = 0$

$H = \frac{(usin\theta)^2}{2g}$

$\sin^2\theta = \frac{2gH}{u^2}$……… (i)

$R = \frac{2u^2 sin2\theta}{g}$

$gR = u^2(2\cos\theta\sin\theta)$

$\cos\theta = \frac{gR}{2u^2 sin\theta}$

$\cos^2\theta = \frac{(gR)^2}{4u^4 sin^2\theta}$ …….. (ii)

(i) and (ii)

$\cos^2\theta = \frac{(gR)^2}{4u^4\left(\frac{2gH}{u^2}\right)}$

$\cos^2\theta = \frac{gR^2}{8u^2H}$

$\cos^2\theta + \sin^2\theta = 1$

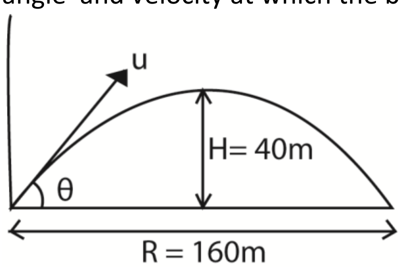
$\frac{2gH}{u^2} + \frac{gR^2}{8u^2H} = 1$

$16gH^2 - 8Hu^2 + gR^2 = 0$

Example 24

A ball is projected from point A and falls at point B which is in level with A at a distance of 160m from A. The greatest height of the ball attained is 40m. find the;

(a) angle and velocity at which the ball is projected (10marks)



Using $v^2 = u^2 + 2as$

$v^2 = u^2\sin^2\theta - 2gh$

At maximum height $v_y = 0$

$0 = u^2\sin^2\theta - 2gH$

$u^2 = \frac{2gH}{sin^2\theta}$ ……………………………………………… (i)

Range, $R = \frac{2u^2 sin\theta cos\theta}{g}$ ……………………………… (ii)

Eqn. (i) and eqn. (ii)

$R = 2\ x\ \frac{2gH}{sin^2\theta}.\frac{sin\theta cos\theta}{g}$ = 4Hcotθ ……………. (iii)

Substituting for R and H in eqn. (iii)

160 = 4 x 40cotθ

tan θ = 1

$\theta = \tan^{-1} 1 = 45^0$

Substituting for θ

$u^2 = \frac{2gH}{sin^2\theta} = \frac{2\ x\ 40\ x\ 9.8}{sin^2\ 45^0}$

$u = \sqrt{\frac{2\ x\ 40\ x\ 9.8}{sin^2\ 45^0}}$ = $39.60ms^{-1}$

(b) time taken for the ball to attain the greatest height (02marks)

$t = \frac{usin\ \theta}{g} = \frac{39.60\ x\ \sin 45^0}{9.8} = 2.8573s$

Example 25

A boy throws a ball at an initial speed of $40ms^{-1}$ at an angle of elevation θ. Taking $g = 10ms^{-2}$, show that the times of flight corresponding to a horizontal range of 80m are positive roots of the equation $T^4 - 64T^2 + 256$.

$R = \frac{2u^2 cos\theta sin\theta}{g}$

$80 = \frac{2\ x\ 40^2 cos\theta sin\theta}{10}$

sinθcosθ =0.25

$\sin\theta = \frac{1}{4cos\theta}$ ……. (i)

x = ucosθt

80 = 40cosθT

$\cos\theta = \frac{2}{T}$ ………. (ii)

but $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{4x\frac{2}{T}}\right)^2 + \left(\frac{2}{T}\right)^2 = 1$

$\frac{T^2}{64} + \frac{4}{T^2} = 1$

$T^4 - 64T^2 + 256$

Example 26

A particle projected from a point O on a horizontal ground moves freely under gravity and it hits the ground again at A. Taking O as the origin, the equation of the path of the particle is 60y = $20\sqrt{3}$x -x^2 where x and y are measured in meters. Determine the

(a) initial speed and angle of projection

60y = $20\sqrt{3}$x -x^2

y = $\frac{\sqrt{3}}{3}$x - $\frac{x^2}{60}$

Comparing with

y = xtanθ -$\frac{gx^2(1+tan^2\theta)}{2u^2}$

θ = $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$= 30^0

$\frac{gx^2(1+tan^2\theta)}{2u^2} = \frac{x^2}{60}$

$\frac{g(1+tan^2\theta)}{2u^2} = \frac{1}{60}$

$\frac{9.8\left(1+\frac{3}{9}\right)}{2u^2} = \frac{1}{60}$; u = 19.8ms^{-1}

(b) Distance OA

At A, y = 0

0= $20\sqrt{3}$x -x^2

0= ($20\sqrt{3}$ – x)x

x =$20\sqrt{3}$m

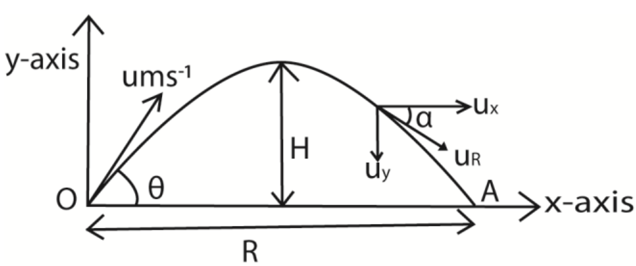
Example 27

A particle is projected from a point O on a level ground with initial speed 30ms^{-1} to pass through a point which is a horizontal distance 40m from O and a distance 10 vertically above the level of O.

(a) Show that there are two possible angles of projection

(b) If these angles are α and β, prove that tan (α+ β) = -4, take g = 10ms^{-2}

Solution



y = xtanθ -$\frac{gx^2(1+tan^2\theta)}{2u^2}$

10 = 40tanθ -$\frac{10\ x\ 40(1+tan^2\theta)}{2\ x\ 30^2}$

8tan$_2$θ – 36tanθ + 17 = 0

since it's quadratic equation in tanθ;

it has two roots and hence two values of θ < 90

(b) tanα + tanβ = $\frac{36}{8}$

tanα tanβ = $\frac{17}{8}$

tan(α + β) = $\frac{\tan\alpha+tan\beta}{1-\tan\alpha tan\beta}$

= $\frac{\frac{36}{8}}{1-\frac{17}{8}}$

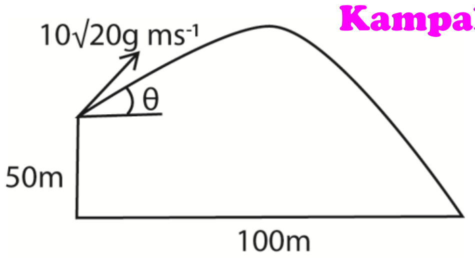
= $\frac{\frac{36}{8}}{\frac{8-17}{8}}$

=$\frac{36}{8}\ x\ \frac{8}{-9}$

tan(α + β) = -4

Example 28

A particle is projected with a speed $10\sqrt{2g}$ms-1from the top of a cliff 50m high. The particle hits the sea at a distance of 100m from the vertical through the point of projection. Show that there two possible directions of projection which are perpendicular. Determine the time taken from the point of projection in each case.



y = xtanθ - $\frac{gx^2(1+tan^2\theta)}{2u^2}$

-50 = 100tanθ - $\frac{g\ x\ 100^2(1+tan^2\theta)}{2x\ 100\ x\ 2g}$

$\tan^2\theta - 4\tan\theta - 1 = 0$

$\tan\theta_1 = 2+\sqrt{5}$ and $\tan\theta_2 = 2-\sqrt{5}$

$\tan\theta_1 \tan\theta_2 = (2+\sqrt{5})(2-\sqrt{5}) = -1$

Hence they are perpendicular

For horizontal motion

x = ucosθt

$\theta_1 = \tan^{-1}(2+\sqrt{5})$= 76.72^0

$t_1 = \frac{100}{10\sqrt{2g}\cos(76.72^0)}$ = 9.83s

$\theta_2 = \tan^{-1}(2-\sqrt{5})$= -13.36^0

$t_2 = \frac{100}{10\sqrt{2g}\cos(-13.38^0)}$ = 2.32s

Revision exercise 4

1. A golfer hits a ball with a velocity of 44.1ms-1 at an angle of $\sin^{-1}\frac{3}{5}$ above the horizontal. The ball lands on the green at a point which is level with the point of projection. Find the time for which the golf ball was in air. [5.4s]
2. a tennis ball is served horizontally from a point which is 2.5m vertically above a point A. The ball first strikes the horizontal ground through A at a distance 20m from A.
   (i) show that the ball is served with speed 28ms-1
   (ii) During its flight the ball passes over a net which is horizontal distance 12m from A. Find the vertical distance of the ball above the horizontal ground at the instant when it passes over the net. [1.6m]
3. An aircraft, at a height of 180m above horizontal ground and flying horizontally with speed of 70ms^{-1} releases emergency supplies. If these supplies are to land at a specific point, at what horizontal distance from this point must the aircraft release them? {take g = 10ms^{-2}) {420m]
4. A stone is projected from top of a vertical cliff of height of h and the stone attains a maximum height (h+ b) above the ground. The stone strikes a sea at a distance, a from the foot of the cliff. Prove that the angle of elevation θ of the stone is given by $a^2\tan^2\theta - 4ab\tan\theta - 4bh = 0$
5. At time t = 0 a particle is projected from a point O on a horizontal plane with speed 14ms^{-1} in a direction inclined at an angle $\tan^{-1}\frac{3}{4}$ above the horizontal. The particle just clears the top of a vertical wall, the base of which is 8m from O. Find
   (a) time at which the particle passes over the wall [$\frac{5}{7}s$]
   (b) height of the wall [3.5m]
6. A particle P is projected from a point O with a speed of 60ms-1 at an angle $\cos^{-1}\frac{4}{5}$ above the horizontal. Find
   (a) time the particle takes to reach the point Q whose horizontal displacement from O is 96m. [2s]
   (b) height of Q above O [52.4m]
   (c) speed of the particle 2s after projection [50.7ms^{-1}]
7. A particle P is projected from a point O with a speed50ms^{-1} at an angle $\sin^{-1}\frac{7}{25}$ above the horizontal. Find
   (a) height of P at the point where its horizontal displacement from O is 120m[4.375m]

(b) speed of P 2s after projection. [48.3ms^{-1}]

(c) times after projection at which P is moving at an angle of $\tan^{-1}\frac{1}{4}$ to the ground [0.204s, 2.65s]

8. A child throws a small ball from a height of 1.5m above level ground, aiming at a small target. The target is on top of a vertical pole of height 2m from the ground and horizontal displacement of the child from the pole is 6m. The initial velocity of the ball has magnitude ums^{-1} at an angle of elevation 40^0. The ball moves freely under gravity. (Take g = 10ms^{-2})
   (a) For u = 10, find the greatest height above the ground reached by the ball. [3.6m]
   (b) Calculate the value of u for which the ball hits the target[8.2]
9. A girl thrown a stone from a height of 1.5m above the ground with speed of 10ms^{-1} and hits a bottle standing on a wall 4m high and 5m from her. Take g = 10ms^{-1}.
   (a) Show that if α is the angle of projection of the stone as it leaves her hand then $1.25\tan^2\alpha - 5\tan\alpha + 3.75 = 0$
   (b) the horizontal component of the stone's velocity has to be6ms^{-1} for the bottle to be knocked off. By solving the above equation or otherwise, show that α has to be 45^0 for the bottle to be knocked off.
10. A basketball is released from player's hands with a speed of 8ms^{-1} at inclination of α^0 above the horizontal so as t land in the centre of the basket, which is 4m horizontally from the point of release and a vertical height of 0.5m above it. Take g =10ms^{-2}.
   (a) show that α satisfies the quadratic equation; $5\tan^2\alpha - 16\tan\alpha + 7 = 0$
   (b) Given that the player throws the ball at a large angle of projection find α for the ball to land in the basket. [70^0, 1.43s]