MATIGO EXAMINATIONS BOARD

P425/1
PURE MATHEMATICS
MARKING GUIDE 2023
PAPER 1

Qn	Answer	marks
	SECTION A	
1	$nC_3 = 35$	
	$nC_3 = \frac{n!}{(n-3)!\,3!} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)! \times 6}$	**B1**
	$= \frac{n(n-1)(n-2)}{6}$	
	$\frac{n(n-1)(n-2)}{6} = 35$	**M1**
	$n(n-1)(n-2) = 210$	
	$n(n^2-2n-n+2) = 210$	
	$n(n^2-3n+2) = 210$	**B1**
	$n^3-3n^2+2n-210 = 0$	
	$F_{120} = \{\pm1, \pm2, \pm3, \pm5, \pm7, \pm30, \pm42, \pm70, \pm105, \pm210\}$	
	$n = 7, \qquad \Rightarrow n-7$	
	$n^2+4n+30$	
	$n-7\sqrt{n^3-3n^2+2n-210}$	
	$\underline{n^3-7n^2}$	
	$4n^2+2n-210$	
	$\underline{4n^2-28n+0}$	
	$30n-210$	
	$\underline{30n-210}$	**M1**
	00	

	$n^2+4n+80=0$ $b^2-4ac<0,$ $\therefore n=7$	A1
2	$\sqrt{3}cosec20°-sec20°=4$ $\sqrt{3}\times\frac{1}{sin20°}-\frac{1}{cos20°}$ $\frac{\sqrt{3}cos20°-sin20°}{cos20°sin20°}$ $let\ \sqrt{3}cos20°-sin20°=Rcos(20+\alpha)$ $\sqrt{3}cos20°-sin20°=Rcos20cos\alpha-Rsin20sin\alpha$ $Rcos\alpha=\sqrt{3}$ $Rsin\alpha=1$ $tan\alpha=\frac{1}{\sqrt{3}}\Rightarrow\alpha=tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=30°$ $R^2cos^2\alpha+R^2sin^2\alpha=\left(\sqrt{3}\right)^2+(1)^2$ $R^2=4$ $R=2$ $\sqrt{3}cos20°-sin20°=2\cos(30°+20°)$ $=2cos50°$ $\frac{\sqrt{3}cos20°-sin20°}{cos20°sin20°}=\frac{2cos50}{2sin40}$ $sin40°=cos50°$ $\frac{4cos50}{cos50}=4$	B1 M1 B1 M1 A1
		05
3	$\int_2^4\frac{x+2}{x^2+4x-7}dx$ $let\ t=x^2+4x-7$ $dx=\frac{dt}{2x+4}$ x | t: 2 | 5; 4 | 25	B1 M1 M1

x	t
2	5
4	25

	$\int \frac{x+2}{t} \times \frac{dt}{2(x+2)}$ $\frac{1}{2}\int_{5}^{25} \frac{1}{t} dt = \frac{1}{2}[lnt]_5^{25}$ $= \frac{1}{2}[ln25 - ln5]$ $= \frac{1}{2}[\ln\left(\frac{25}{5}\right)]$ $= \frac{1}{2}ln5$ $= 0.80472$ ≈ 0.805	**B1** **A1**
4	$3ln2 + ln5 - \frac{1}{2}ln10000 = lnP$ $ln2^3 + ln5^2 - \ln(10{,}000)^{\frac{1}{2}} = lnP$ $ln8 + ln25 - ln100 = lnP$ $\ln(8 \times 25) - ln100 = lnP$ $ln200 - ln100 = lnP$ $ln\left(\frac{200}{100}\right) = lnP$ $ln2 = lnP$ $P = 2$	**M1** **M1** **M1** **M1** **A1**
		05
5	$\frac{x-1}{3} = \frac{y+2}{2} = \frac{z-1}{5}$ $and\ 2x - y + 4z = 0$ $let\ \lambda = \frac{x-1}{3} = \frac{y+2}{2} = \frac{z-1}{5}$ $x = 3\lambda + 1, y = 2\lambda - 2, z = 5\lambda + 1$ $2(3\lambda + 1) - (2\lambda - 2) + 4(5\lambda + 1) = 6$ $6\lambda + 2 - 2\lambda + 4 + 20\lambda + 4 = 6$ $24\lambda = -4$ $\lambda = -\frac{1}{6}$ $x = 3 \times -\frac{1}{6} + 1 = -\frac{1}{2} += \frac{1}{2}$	**B1** **A1** **M1** **B1**

	$y = 2 \times -\frac{1}{6} - 2 = -\frac{1}{3} - 2 = \frac{-7}{3}$ $z = 5 \times -\frac{1}{6} + 1 = -\frac{5}{6} + 1 = \frac{1}{6}$ $\left(\frac{1}{2}, \quad -\frac{7}{3}, \quad \frac{1}{6}\right)$	**A1**
		05
6	$x + 2 = 0$ $x = -2$ $(-2)^2 + b(-2) + c = 0$ $4 - 2b + c = 0$ $-2b + c = -4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$ $(-2)^2 - 2d + c = 0$ $-2d + e = -4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$ $solving\ (i) and\ (ii) simultaneously$ $-2b + 2b + c - e = 0$ $2(d - b) = e - c$	**M1** **M1** **M1** **M1** **A1**
		05

<table>
<tr><td>7</td><td>

$$\frac{x^2+4x+5}{x+3} \le 1$$

$$\frac{x^2+4x+5}{x+3} - 1 \le 0$$

$$\frac{x^2+4x+5-x-3}{x+3} \le 0$$

$$\frac{x^2+3x+2}{x+3} \le 0$$

$$\frac{x(x+1)+2(x+1)}{(x+3)} \le 0$$

$$\frac{(x+2)(x+1)}{x+3} \le 0$$

$$x=-1, x=-2, x=-3$$

	$x<-3$	$-3<x<-2$	$-2<x<-1$	$x>-1$
$(x+2)$	$-$	$-$	$+$	$+$
$(x+1)$	$-$	$+$	$-$	$+$
$(x+3)$	$-$	$-$	$+$	$+$
$\frac{(x+2)(x+1)}{(x+3)}$	$-$	$+$	$-$	$+$

$$x<-3 \; and -2 \le x \le -1$$

</td><td>M1
M1
M1
B1
A1
05</td></tr>
<tr><td>8</td><td>

$$3e^{4x} - \frac{dv}{dx} = 2$$

$$\frac{dv}{dx} = 3e^{4x} - 2$$

$$\int dV = \int (3e^{4x}-2)dx$$

$$V = \frac{3}{4}e^{4x} - 2x + C$$

$$x=0, \quad V=-4$$

$$-4 = \frac{3}{4} - 0 + C$$

$$C = -\frac{4}{1} - \frac{3}{4} = -\frac{19}{4}$$

</td><td>M1
B1
M1
M1</td></tr>
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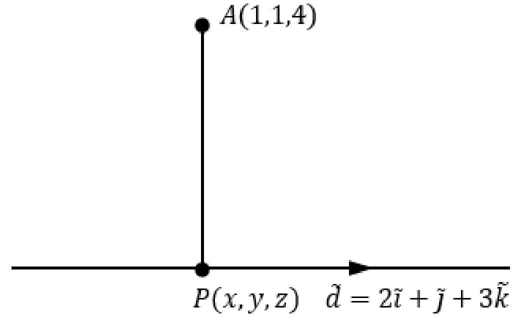
	$V=\frac{3}{4}e^{4x}-2x-\frac{19}{4}$ $V=\frac{1}{4}(3e^{4x}-8x-19)$	**A1** **05**
	SECTION B	
9(a)	$P(x)=\frac{x^2+7x+2}{(1+x^2)(2-x)}\equiv\frac{Ax+B}{1+x^2}+\frac{C}{2-x}$ $x^2+7x+2\equiv(2-x)(Ax+B)+C(1+x^2)$ $when\ x=2$ $20=5C$ $C=4$ $when\ x=0$ $2=2(B)+C(1)$ $2=2B+4$ $2B=-2$ $B=-1$ $when\ x=1$ $10=A+B+2C$ $10=A-1+8$ $A=3$ $\frac{x^2+7x+2}{(1+x^2)(2-x)}=\frac{3x-1}{1+x^2}+\frac{4}{2-x}$	**M1** **A1** **A1** **A1** **A1**
9(b)	$\int\frac{x^2+7x+2}{(1+x^2)(2-x)}dx=\int\left(\frac{3x}{1+x^2}-\frac{1}{1+x^2}+\frac{4}{2-x}\right)dx$ $let\ t=1+x^2$ $dx=2xdx$ $dx=\frac{dt}{2x}$ <table><tr><td>x</td><td>t</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>2</td></tr></table>	**M1** **B1**

	$$\int \frac{3x}{t} \times \frac{dt}{2x}$$ $$= \frac{3}{2}[lnt]_1^2$$ $$= \frac{3}{2}[ln2 - ln1]$$ $$= \frac{3}{2}ln2$$ $$\int_0^1 \frac{1}{1+x^2}dx = \tan^{-1}x$$ $$= \tan^{-1}(1) - \tan^{-1}(0)$$ $$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$ $$\int_0^1 \frac{4}{2-x}dx = -4[\ln(2-x)]_0^1$$ $$= -4[ln2 - ln2]$$ $$= -4 \times -ln2 = 4ln2$$ $$\int_0^1 \frac{x^2+7x+2}{(1+x^2)(2-x)}dx = \frac{3}{2}ln2 - \frac{\pi}{4} + 4ln2$$ $$= \left(\frac{3}{2}+4\right)ln2 - \frac{\pi}{4} \Rightarrow \frac{11}{2}ln2 - \frac{\pi}{4}$$	A1 M1 A1 M1 A1 A1
		12
10(a)(i)	$$let\ y = \sin^{-1}\left(tan\frac{x}{2}\right)$$ $$siny = tan\left(\frac{x}{2}\right)$$ $$cosydy = \frac{1}{2}sec^2\left(\frac{x}{2}\right)dx$$ $$\frac{dy}{dx} = \frac{1}{2}\frac{sec^2\left(\frac{x}{2}\right)}{cosy}$$ $$cos^2y + sin^2y = 1$$ $$cosy = \sqrt{1-sin^2y} = \sqrt{1-tan^2\left(\frac{x}{2}\right)}$$	M1 M1 M1

	$\frac{dy}{dx}=\frac{1}{2}\frac{sec^2\left(\frac{x}{2}\right)}{\sqrt{1-tan^2\left(\frac{x}{2}\right)}}$ $=\frac{sec^2\left(\frac{x}{2}\right)}{2\sqrt{1-(1-sec^2\left(\frac{x}{2}\right)}}$ $=\frac{sec^2\left(\frac{x}{2}\right)}{\sqrt{sec^2\left(\frac{x}{2}\right)}}$ $=\frac{sec^2\left(\frac{x}{2}\right)}{sec\left(\frac{x}{2}\right)}$ $=sec\left(\frac{x}{2}\right)$	A1
10(a)(ii)	$let\ y=x^{lnx}$ $lny=lnx^{lnx}$ $lny=lnxlnx$ $let\ t=lnx$ $lny=t^2$ $\frac{1}{y}\frac{dy}{dt}=2t$ $\frac{dy}{dt}=2ty=2tx^{lnx}$ $\frac{dt}{dx}=\frac{1}{x}$ $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ $=2tx^{lnx}\times\frac{1}{x}$ $\frac{dy}{dx}=\frac{2lnx(x^{lnx})}{x}$	M1 M1 M1 A1
10(b)	$y=e^x sinx$ $\frac{dy}{dx}=e^x cosx+\ e^x sinx$	M1

	$$\frac{dy}{dx} = e^x cosx + y$$ $$\frac{d}{dx}\left(\frac{dy}{dx}\right) = -e^x sinx + e^x cosx + \frac{dy}{dx}$$ $$\frac{d^2y}{dx^2} = -y + \left(\frac{dy}{dx} - y\right) + \frac{dy}{dx}$$ $$\frac{d^2y}{dx^2} = -y + \frac{dy}{dx} - y + \frac{dy}{dx}$$ $$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y$$ $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$	M1 M1 A1
	OR	
	$$y = e^x sinx$$ $$\frac{dy}{dx} = e^x cosx + e^x sinx$$ $$\frac{d}{dx}\left(\frac{dy}{dx}\right) = -e^x sinx + e^x cosx + e^x cosx + e^x sinx$$ $$= 2e^x cosx$$ $$\frac{d^2y}{dx^2} = 2e^x cosx$$ $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$$ $$= 2e^x cosx - 2(e^x cosx + e^x sinx\ ) + 2e^x sinx$$ $$= 2e^x cosx - 2e^x cosx - 2e^x sinsx + 2e^x sinx$$ $$= 0$$	M1 M1 M1 A1
		12
11(a)	$$A(3, -4, 2) \qquad B(-5, 2, -8)$$ $$\overrightarrow{AB} = \overrightarrow{0B} - \overrightarrow{OA}$$ $$= \begin{pmatrix} -5 \\ 2 \\ -8 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -8 \\ 6 \\ -10 \end{pmatrix}$$ $$\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{(-8^2) + (6^2) + (-10^2)}$$ $$= \sqrt{64 + 36 + 100}$$	M1M1 M1 M1

	$=\sqrt{200}$ $10\sqrt{2}$	**A1**
11(b)	 $\overrightarrow{AP}=\overrightarrow{0P}-\overrightarrow{OA}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}-\begin{pmatrix}1\\1\\4\end{pmatrix}=\begin{pmatrix}x-1\\y-1\\z-4\end{pmatrix}$ $\overrightarrow{AP}.\tilde{d}=0$ $\begin{pmatrix}x-1\\y-1\\z-4\end{pmatrix}.\begin{pmatrix}2\\1\\3\end{pmatrix}=0$ $2(x-1)+1(y-1)+3(z-4)=0$ $2x-2+y-1+3z-12=0$ $2x+y+3z-15=0$ $Let\ ,\lambda=\dfrac{x-1}{2}=\dfrac{y}{1}=\dfrac{z+1}{3}$ $x=2\,\lambda+1,\ y=\ \lambda,\ \ z=3\,\lambda-1$ $2(2\,\lambda+1)+\ \lambda+3(3\,\lambda-1)-15=0$ $4\,\lambda+2+\ \lambda+9\,\lambda-3-\ \lambda=0$ $14\,\lambda=16$ $\lambda=\dfrac{16}{14}=\dfrac{8}{7}$ $x=2\dfrac{8}{7}+1=\ \dfrac{16}{7}+1=\dfrac{23}{7}$	**M1** **M1** **A1** **M1** **A1** **M1**

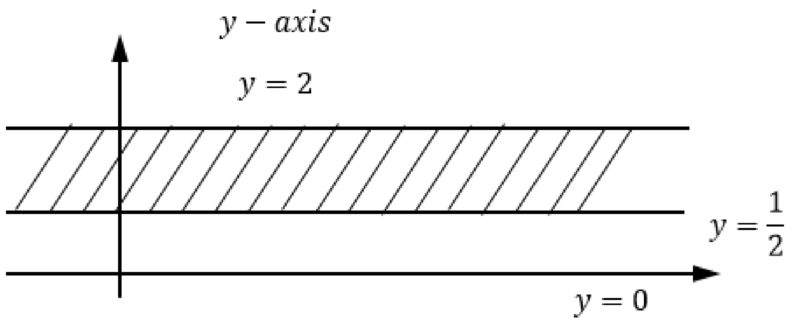
	$y=\frac{8}{7}$ $z=3\times\frac{8}{7}-1=\frac{17}{7}$ $\lvert\overrightarrow{AP}\rvert=\sqrt{\left(\frac{23}{7}-1\right)^2+\left(\frac{8}{7}-1\right)^2+\left(\frac{17}{7}-4\right)^2}$	A1
		12
12(i)	$y=\frac{4x-10}{x^2-4}$ $y(x^2-4)=4x-10$ $yx^2-4y=4x-10$ $yx^2-4x+10-4y=0$ $b^2-4ac<0$ $(-4)^2-4y(10-4y)<0$ $16-40y+16y^2<0$ $8y^2-20y+8<0$ $2y^2-4y-y+2<0$ $2y(y-2)-1(y-2)<0$ $(2y-1)(y-2)<0$ $y=\frac{1}{2}, y=2$	M1 M1
	(sign table below)	M1 M1
	$\frac{1}{2}<y<2$	
	$when\ y=2$ $\frac{4x-10}{x^2-4}=\frac{2}{1}$ $4x-10=2(x^2-4)$	

	$y<\frac{1}{2}$	$\frac{1}{2}<y<2$	$y>2$
$2y-1$	$-$	$+$	$+$
$y-2$	$-$	$-$	$+$
$(2y-1)(y-2)$	$+$	$-$	$+$

$$4x - 10 = 2x^2 - 8$$
$$x^2 - 2x + 1 = 0$$
$$x^2 - x - x + 1 = 0$$
$$x(x-1) - 1(x-1) = 0$$
$$(x-1)(x-1) = 0$$
$$x = 1$$

M1

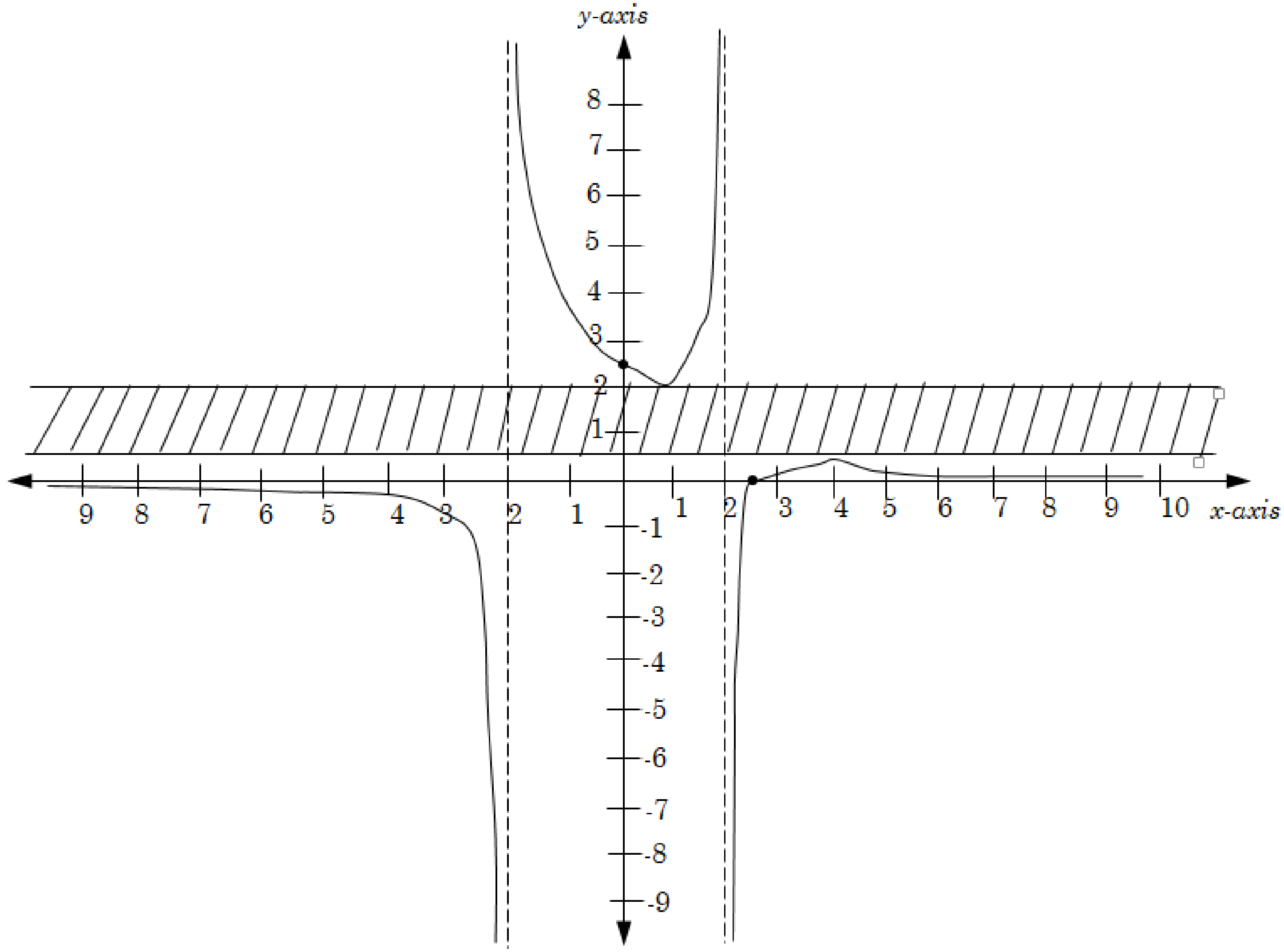
$$when\ y = \frac{1}{2}$$
$$\frac{4x-10}{x^2-4} = \frac{1}{2}$$
$$2(4x-10) = x^2 - 4$$
$$8x - 20 = x^2 - 4$$
$$x^2 - 8x + 16 = 0$$
$$x^2 - 4x - 4x + 16 = 0$$
$$x(x-4) - 4(x-4) = 0$$
$$(x-4)(x-4) = 0$$
$$x = 4$$



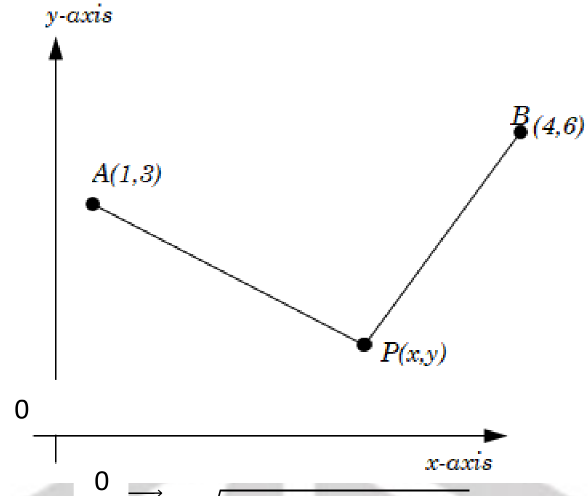
$$(1,2)\ is\ a\ minimum\ point$$
$$\left(4, \frac{1}{2}\right)\ is\ a\ maximum\ point$$

A1

12(ii)	$x^2-4=0$ $x^2=4$ $x=\pm 2$ $x=-2 \; and \; x=2$ $y=\dfrac{\frac{4x}{x^2}-\frac{10}{x^2}}{1-\frac{4}{x^2}}$ $As\; x\to\pm\infty, y\to 0$ $y=0$	**M1** **A1** **A1**
(iii)	$intercepts$ $when\; x=0, y=\frac{10}{4}=\frac{5}{2}$ $\left(0,\frac{5}{2}\right)$ $when\; y=0, x=\frac{5}{2}$ $\left(\frac{5}{2},0\right)$	**B1** **B1**

		**B1**
		12

13(a)



M1

$$\overrightarrow{AP} = \sqrt{(x-1)^2 + (y-3)^2}$$

$$\overrightarrow{AP} = \sqrt{(x-4)^2 + (y-6)^2}$$

$$2\overrightarrow{AP} = \overrightarrow{BP}$$

$$\left(2\sqrt{(x-1)^2 + (y-3)^2}\right)^2 = \left(\sqrt{(x-4)^2 + (y-6)^2}\right)^2$$ **M1**

$$4[(x-1)^2 + (y-3)^2] = [(x-4)^2 + (y-6)^2]$$

$$4(x^2 - 2x + 1 + y^2 - 6y + 9) = x^2 - 8x + 16 + y^2 - 12y + 36$$

$$4x^2 - 8x + 4 + 4y^2 - 24y + 36 = x^2 - 8x + 16 + y^2 - 12y + 36$$ **A1**

$$3x^2 + 3y^2 - 24y + 12y + 40 = 52$$

$$3x^2 + 3y^2 - 12y = 12$$

$$3x^2 + 3y^2 - 12y - 12 = 0$$

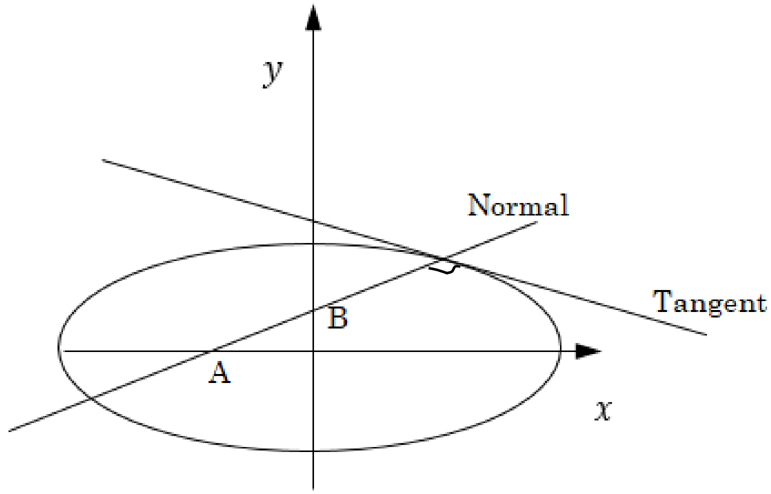
$$x^2 + y^2 - 4y - 4 = 0$$ **M1**

$$(x-0)^2 + (y-2)^2 - 4 - 4 = 0$$

$$(x-0)^2 + (y-2)^2 = 8$$

$$(x-0)^2 + (y-2)^2 = \left(\sqrt{8}\right)^2$$ **A1**

$$(0,2)\ radius = \sqrt{8}$$

13(b)	 $P(3Cos\beta, 4sin\beta)$ $x = 3cos\beta, y = 4sin\beta$ $\frac{dx}{d\beta} = -3sin\beta, \frac{dy}{d\beta} = 4cos\beta$ $\frac{dy}{dx} = \frac{dy}{d\beta} \times \frac{d\beta}{dx}$ $= 4cos\beta \times \frac{1}{-3sin\beta}$ $= \frac{4cos\beta}{3sin\beta}$ $M_N \times M_T = -1$ $M_N \times -\frac{4cos\beta}{3sin\beta} = -1$ $M_N = \frac{3sin\beta}{4cos\beta}$ $Equation\ of\ normal$ $\frac{y - 4sin\beta}{x - 3cos\beta} = \frac{3sin\beta}{4cos\beta}$ $4ycos\beta - 16sin\beta cos\beta = 3xsin\beta - 9cos\beta sin\beta$ $4ycos\beta = 3xsin\beta + 7cos\beta sin\beta$ $x = 0, y = \frac{7}{4}sin\beta$	**M1** **M1** **A1** **M1**

	$B\left(0, \frac{7}{4}sin\beta\right)$ $y = 0$ $3xsin\beta = -7cos\beta sin\beta$ $3x = -7cos\beta$ $x = \frac{-7}{3}cos\beta$ $A\left(\frac{-7}{3}cos\beta, 0\right)$ $mid\ point\ of\ AB,$ $x = 0 \pm \frac{7}{3}cos\beta = -\frac{7}{8}cos\beta$ $y = \frac{{}^{7}/_{4}\,sin\beta}{2} = \frac{7}{8}sin\beta$ $-\frac{6x}{7} = cos\beta, \frac{8y}{7} = sin\beta$ $\frac{36x^2}{49} + \frac{64y^2}{49} = 1$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{36x^2}{49} = \frac{x^2}{a^2}$ $38a^2 = 49$ $a^2 = \frac{49}{36}$ $b^2 = \frac{49}{64}$ $\frac{49}{64} = \frac{49}{36}(1 - e^2)$ $\frac{36}{64} = 1 - e^2$ $e^2 = 1 - \frac{36}{64}$ $e^2 = \frac{28}{64} = \frac{7}{16}$	M1 A1
		12
14(a)	$Z = -3 + 2i, \overline{Z = -3 - 2i}$ $Z + \bar{Z} = -6$ $Z\bar{Z} = (-3)^2 - (2i)^2$ $= 9 - 4(-1)$	M1

	$= 9 + 4$ $= 13$ $Z^2 - (-6)Z + 13 = 0$ $Z^2 + 6Z + 13 = 0$ OR $Z = -3 + 2i$ $Z + 3 = 2i$ $(Z + 3)^2 = (2i)^2$ $Z^2 + 6Z + 9 = 4i^2$ $Z^2 + 6Z + 9 = -4$ $Z^2 + 6Z + 13 = 0$	**A1** **M1** **M1** **A1**
(b)	$\lvert Z + 2i\rvert = \lvert Z - 1\rvert$ $Let\ Z = x + iy$ $\lvert x + (2 + y)i\rvert = \lvert(x - 1) + iy\rvert$ $\sqrt{x^2 + (2 + y)^2} = \sqrt{(x - 1)^2 + y^2}$ $\left(\sqrt{x^2 + (2 + y)^2}\right)^2 = \left(\sqrt{(x - 1)^2 + y^2}\right)^2$ $x^2 + 4 + 4y + y^2 = x^2 - 2x + 1 + y^2$ $4y = -2x + 1 - 4$ $4y = -2x - 3$	**M1** **M1** **A1**
(c)	$(2 + 5i)(Z + 2i) = -7 - 32i$ $2Z + 4i + 5Zi + 10i^2 = -7 - 32i$ $2Z + 5Zi = -7 - 32i + 10 - 4i$ $Z(2 + 5i) = 3 - 36i$ $Z = \dfrac{3 - 36i}{2 + 5i}$ $Z = \dfrac{(3 - 36i)(2 - 5i)}{(2 + 5i)(2 - 5i)}$ $= \dfrac{6 - 15i - 72i \pm 180}{(2)^2 - (5i)^2}$ $= \dfrac{-174 - 87i}{4 + 25}$ $= \dfrac{-174 - 87i}{29}$ $= \dfrac{-174}{29} - \dfrac{87}{29}i$	**M1** **M1** **A1** **M1** **M1** **A1**

		12
15(a)	$Given, 10^{th} term\ of\ an\ AP = 5$ $a + (10 - 5)d = 5$ $a + 9d = 5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$ $and\ 18^{th}\ term = 77$ $a + 18 - 1)d = 77$ $a + 17d = 77 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots.(ii)$ $(ii) - (i),$ $8d = 72$ $\therefore d = 9$	M1 M1 M1 M1 M1 A1
15(b)	$x + y - 4 = 0$ $x^2 - 4x - 3y = 0$ $y = 4 - x$ $x^2 - 4x - 3(4 - x) = 0$ $x^2 - 4x - 12 + 3x = 0$ $x^2 - x - 12 = 0$ $x = 4, x = -3$ $4 + y = 4$ $y = 0$ $-3 + y = 4$ $y = 7$ $\therefore x = 4, y = 0, x = -3, x = 7$	M1 M1 M1 M1 A1 A1
16(a)	$let\ V\ be\ the\ volume\ of\ balool\ at\ any\ time\ t.$ $\frac{dV}{dt} \propto \frac{1}{V^2}$ $\frac{dV}{dt} = \frac{K}{V^2}$	M1 A1
16(b)	$V^2 dV = K dt$ $\int V^2 dv = \int K dt$ $\frac{V^3}{3}\ Kt + C$ $t = 0, C = 0$	M1 M1

	$\frac{V^3}{3} = Kt$ $when\ t = 400, V = 600$ $(600)^3 = 3K \times 400$ $600^3 = 1200K$ $K = 180{,}000$ $\frac{V^3}{3} = 180{,}000t$ $V^3 = 540{,}000t$	**M1** **M1** **M1** **A1**
(c)	$(1250)^3 = 540{,}000t$ $t = 3{,}616.8981$ $t = 3617s$ $1hour = 3600s$ $t = 1.0047hours$ $12:00:00$ $\underline{-1:00:47}$ $\underline{10:59:13}$am	**M1** **A1** **A1** **A1**

END

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