GSE & PSI - MoES

THE UGANDA INTER SCHOOL VIRTUAL A LEVEL MATHEMATICS SEMINAR 2024.

Saturday 06^{th} July 2024 (9:00 a.m)

INSTRUCTIONS TO STUDENTS AND TEACHERS:

Dear students and teachers we would like to welcome you to participate in the forthcoming Mathematics seminar for senior six students. This is in preparation for the forthcoming final exams(UNEB) and the Mock Examinations. **This is a free seminar and no one should charge you any fees.**The process to be followed by both the teachers and students is suggested below:

1. Teachers share the Seminar questions with their students and ask for volunteers to discuss any of the questions. Questions should be pinned up and learners write down all the questions in their books.
2. Teachers talk to the school administrators to allow the children participate as presenters in the seminar on Saturday **06th July from 09:00am - 2:00 pm.** Other students will just be participants.
3. The student together with the teachers select atleast two best done presentations and the students to represent the school.The solutions and pictures/videos should be uploaded on padlet.`https://bit.ly/S4MATHSEMINAR2023`
4. Hold a mock presentation where all your discussants present to the rest of the class.After that release the rest of the class and record your best presenter in a very quiet environment but with good light.Record each part of the question separately .
5. The teacher could now train the student on how to present on zoom as far as sharing a screen and using the whiteboard. Alternatively the students' presentation will be loaded on the computer screen and they explain to us their solution.

SEMINAR DETAILS

S.6 virtual Mathematics seminar 2024.

Time: 06 JULY 2024, 09:00 AM

Join Zoom Meeting

Meeting ID:99344801787

Passcode: HeLP2024

<table>
<tr><th>P425/1</th><th>P425/2</th></tr>
<tr><td>

1. Analysis (6 questions)
   (a) Differentiation
   (b) Intergration
   (c) Differential equations
2. Vectors (2 questions)
   (a) Vectors in 2-D
   (b) Vectors in 3-D
   (c) Ratio theorem
   (d) Line and their properties
   (e) Planes and their properties
3. Trigonometry (2 questions)
4. Geometry (2 questions)
   (a) Coordinate geometry of lines and triangles
   (b) Locus and circles
   (c) Parabola
5. Algebra (4 questions)
   (a) Surds,indices and logarithms
   (b) Quadratics
   (c) Polynomials
   (d) Simultaneous equations
   (e) Inequalities
   (f) Partial fractions
   (g) Complex numbers
   (h) Permutation and combinations

</td><td>

1. Mechanics (6 questions)
   (a) Calculus of vectors
   (b) General motion of the body
   (c) Relative motion
   (d) Projectiles
   (e) Newtonian mechanics
   (f) e.t.c
2. Numerical analysis (4 questions)
   (a) Location of the roots of an equation
   (b) Trapezium rule of numerical intergration
   (c) Newton raphson method
   (d) Errors
   (e) Flow charts
3. Statitics and probability(6 questions)
   (a) Mean ,node,median
   (b) Index numbers
   (c) Correlation coefficient
   (d) Scatter diagram
   (e) Discrete probability distributions
   (f) Continous probability distributions
   (g) Distributions
      i. Uniform distribution
      ii. Normal distribution
      iii. Binomial distribution
      iv. Normal approximation to binomial distribution
   (h) Estimations

</td></tr>
</table>

PURE MATHEMATICS (P425/1)

ALGEBRA

1. The first 3 terms, in ascending powers of x, in the binomial expansion of $(1+kx)^{10}$ are given by $1+15x+px^2$ where k and p are constants.
   (a) Find the value of k
   (b) Find the value of p
   (c) Given that, in the expansion of $(1+kx)^{10}$, the coefficient of x^4 is q, find the value of q
2. Solve the following simultaneous logarithmic equations.
$$\log_2(y-1) = 1+\log_2 x$$
$$2\log_3 y = 2+\log_3 x$$
3. (a) Find all the roots of the equation
$$x^2+5x = 42-\frac{216}{x^2+5x}$$
   (b) Use the substitution $= x+\frac{6}{x}$ to find all the roots of the equation
$$x^4-12x^3+47x^2-72x+36=0$$
4. Solve the equation $|3x-2y-11|+2\sqrt{31-8x+5y}=0$
5. The polynomial $p(x)$ is defined by $p(x) = mx^3+nx^2-17x-6$, where m and n are constants. It is given that $(x+2)$ is a factor of $p(x)$ and that the remainder is 28 when $p(x)$ is divided by $(x-2)$.
   (a) Find the values of m and n .
   (b) Hence factorise $p(x)$ completely.
6. (a) Expand $\left(1+\frac{1}{4}x\right)^4$ up to the third term and use it to estimate $(1.025)^4$ correct to 3 decimal places.
   (b) Expand $(1+x)^{\frac{1}{3}}$ up to the term in x^2 . By using the substitution $x=\frac{1}{8}$, estimate the cube root of 9 correct to 3 decimal places.
   (c) Solve the equation ${}^nP_2 = 20$
7. (a) The coefficients of x^2 and x^3 in the expansion of $(3-2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$
   (b) i. Find the coefficient of x in the expansion of $\left(2x-\frac{1}{x}\right)^5$
   ii. Hence find the coefficient of x in the expansion of $(1+3x^2)\left(2x-\frac{1}{x}\right)^5$
8. (a) The seventh term of an arithmetic progression is equal to twice the fifth term. The sum of the first seven terms is 84. Find the first term.

(b) The seventh term of a geometric progression is equal to twice the fifth term. The sum of the first seven terms is 254 and the terms are all positive. Find the first term, showing that it can be written in the form $p + q\sqrt{r}$ where p, q and r are integers.

9. (a) Use the factor theorem to express $x^3 + 2x^2 + x - 18$ as a product of two factors.

   (b) Find p and q for which $x^2 - 1$ is a factor of the polynomial $f(x) = 2x^3 + px^2 + qx + 6$, and hence solve the equation $f(x) = 0$

10. The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$,where k is a constant ,is 540.

   (a) Find the value of k

   (b) For this value of k ,find the coefficient of x^2 in the expansion

11. The polynomial $2x^3 - 5x^2 + ax + b$,where **a** and **b** are constants,is denoted by $f(x)$.It is given that when $f(x)$ is divided by $(x+2)$ the remainder is 8 and that when $f(x)$ is divided by $(x-1)$ the remainder is 50.

   (a) Find the value of a and the value of b.

   (b) When **a** and **b** have these values,find the quotient and remainder when $f(x)$ is divided by $x^2 - x + 2$.

12. Solve the simultaneous equations,giving your answers in exact form

   (a)

$$e^{3x+4y} = 2e^{2x-y}$$
$$e^{2x+y} = 8e^{x+6y}$$

   (b)

$$2\text{In}x + \text{In}y = 1 + \text{In}5$$
$$\text{In}10x - \text{In}y = 2 + \text{In}2$$

13. (a) Show that $1+i$ is a root of the equation $z^4 + 3z^2 - 6z + 10 = 0$.Hence find other roots

   (b) Given that the complex number z and its conjugate $\bar{z}$ satisfy the equation

$$z\bar{z} - 2z + 2\bar{z} = 5 - 4i$$

   Find the possible values of z

14. A geometric progression has first term a , where $a \neq 0$, and common ratio r , where $r \neq 1$. The difference between the fourth term and the first term is equal to four times the difference between the third term and the second term.

   (a) Show that $r^3 - 4r^2 + 4r - 1 = 0$.

   (b) Show that $r - 1$ is a factor of $r^3 - 4r^2 + 4r - 1 = 0$. Hence factorise $r^3 - 4r^2 + 4r - 1 = 0$.

   (c) Hence find the two possible values for the ratio of the geometric progression. Give your answers in an exact form

   (d) Prove that the sum to infinity is $\frac{1}{2}a(1 + \sqrt{5})$

15. Let

$$f(x) = \frac{3x}{(1+x)(1+2x^2)}$$

(a) Express $f(x)$ in partial fractions.

(b) Hence obtain the expression of $f(x)$ in ascending powers of x ,upto and including the term in x^3

16. The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement.

(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement.

17. (a) Use De - moivre's theory to show that

$$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$$

(b) Prove that $3i + 2$ is a root to the equation

$$Z^4 - 5Z^3 + 18Z^2 - 17Z + 13 = 0$$

and hence find all other roots to this equation.

18. The solutions of the equation $|4x - 1| = |x + 3|$ are $x = p$ and $x = q$, where $p < q$. Find the exact values of p and q , and hence determine the exact value of $|p - 2| = |q - 1|$.

19. (a) Solve the equation

i. $2 - 5e^{-x} + 5e^{-2x} = 0$

ii. $\sqrt{2-x} + \sqrt{3+x} = 3$

(b) Solve the simultaneous equation:

$$\frac{x-y}{4} = \frac{z-y}{3} = 2z - x$$
$$x + 3y + 2z = 4$$

20. (a) The function $f(x) = x^3 + px^2 - 5x + q$ has a factor $(x - 2)$ and has a value of 5 when $x = -3$.Find p and q

(b) The roots of the equation $ax^2 + bx + c = 0$ are α and β.Form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

(c) Simplify $\frac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form $p + q\sqrt{3}$ where p and q are rational numbers.

VECTORS

21. The line L_1 has vector equation

$$r = 3i + 2j + 5k + \lambda(4i + 2j + 3k)$$

The points A$(3, p, 5)$ and B$(q, 0, 2)$,where p and q are constants ,lie on the line L_1

(a) Find the value of p and the value of q

The line L_2 has vector equation

$$r = 3j + k + \mu(7i + j + 7k)$$

(b) Show that L_1 and L_2 intersect and find the position vector of the point of intersection.

(c) Find the acute angle between L_1 and L_2

22. (a) The vertices of a triangle are A$(1, 2, 3)$, B $(0, 1, 1)$ and C $(2, 1, 2)$. Determine the size of angle ABC .

(b) Determine the Cartesian equation of the plane containing triangle ABC .

(c) Find the angle between the plane in (b) above and the line

$$\frac{x}{5} = y = \frac{2 - z}{3}$$

23. (a) Find the cartesian equation of the line of intersection of the two planes $2x - 3y - z = 1$ and $4y + 3x + 2z = 3$

(b) Vectors **a,b** and **c** form the three sides of a triangle.Given that $|a| = 5$,$|b| = 12$ and $\mathbf{a.b}$=$30\sqrt{3}$,find the area of the triangle

24. The position vectors of points A and B are **OA**=$2i - 4j - k$ and **OB**=$5i - 2j + 3k$ respectively.The line AB is produced to meet the plane $2x + 6y - 3z = -5$ at a point C.Find the;

(a) Coordinates of C

(b) Angle between AB and the plane

25. Find the points of intersection of the line $\frac{x}{5} = \frac{y+2}{2} = \frac{z-1}{4}$ with the plane $3x+4y+2z-25 = 0$

26. (a) A,B and C are vertices of a triangle with position vectors $5i + 7j - 9k, 7i + 6j + 2k$ and $11i + 3j + k$ respectively.Using vectors prove that ABC is right angled and hence find its area.

(b) A perpendicular from a point Q$(3, -2, 10)$ meets the line

$$r = \begin{pmatrix} 8 \\ -1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} \text{ at N,find the}$$

i. coordinates of N

ii. perpendicular distance of Q from the line above

27. (a) The position vectors of the points P and Q are $4i - 3j + 5k$ and $i + 2j$ respectively.Find the coordinates of the point R such that $PQ : PR = 2 : 1$

(b) If the vector $5i - \alpha j + k$ is perpendicular to the line $r = i - 4j + t(2i + 3j - 4k)$.Find the value of α

28. (a) Find the vector equation of the line passing through the points $A(-2, 2, -4)$ and $\text{B}(0, 1, -2)$, and state,in coordinate form ,its x-intercept.

(b) Find the distance of the point $\text{P}(-2, 5, -7)$ from the line AB in (a) above.

29. Four points have coordinates P(3,4,k),Q(13,9,2),R(1,2,3) and S(10,8,6).The lines PQ and RS intersect at M .Determine the:

(a) Vector equation of lines PQ and RS

(b) Value of k

(c) Coordinates of M

TRIGONOMETRY

30. It is given that the angle θ satisfies the equation $\sin(2\theta + \frac{1}{4}\pi) = 3\cos(2\theta + \frac{1}{4}\pi)$.

(a) Show that $\tan 2\theta = \frac{1}{2}$.

(b) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle.

31. (a) Given that $8 + co\sec^2\theta = 6\cot\theta$,find the value of $\tan\theta$.

(b) Hence find the exact value of $\tan(\theta + 45^0)$

32. (a) Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$,where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.Give the exact values of R and α

(b) Hence prove that $\frac{1}{(\cos\theta+\sqrt{3}\sin\theta)^2} = \frac{1}{4}\sec^2\left(\theta - \frac{\pi}{3}\right)$

33. (a) Prove that

$$(\cot\theta + co\sec\theta)^2 = \frac{1 + \cos\theta}{1 - \cos\theta}$$

(b) Hence solve,for $0 < \theta < 2\pi$, $3(\cot\theta + co\sec\theta)^2 = 2\sec\theta$

34. (a) Prove the identity

$$\frac{1}{\sin(x + 30^0) + \cos(x + 60^0)} = \sec x$$

(b) Hence solve the equation $\frac{2}{\sin(x+30^0)+\cos(x+60^0)} = 7 - \tan^2 x$ for $0^0 < x < 360^0$

35. (a) Solve $2\sin 2\theta = 3\cos\theta$ for $-180^0 \leq \theta \leq 180^0$

(b) Solve $\sin\theta - \sin 4\theta = \sin 2\theta - \sin 3\theta$ for $-\pi \leq \theta \leq \theta$

36. (a) Solve the equation $\tan(\theta - 60^0) = 3\cot\theta$ for $-90^0 < \theta < 90^0$.

(b) Show that

$$\frac{\tan\beta}{\sin\beta} - \frac{\sin\beta}{\tan\beta} \equiv \tan\beta\sin\beta$$

37. (a) Simplify

$$\frac{\sin 105^0 - \sin(-15^0)}{\cos 105^0 + \cos(-15^0)}, \text{giving your answer in the form } A\sqrt{3}$$

(b) Given that $x = \tan\theta - \sin\theta$ and ,$y = \tan\theta + \sin\theta$.Prove that

$$(x^2 - y^2)^2 = 16xy$$

(c) If P,Q and R are angles of a traingle prove that ;

$$\frac{1}{p}\cos^2\left(\frac{P}{2}\right) + \frac{1}{q}\cos^2\left(\frac{Q}{2}\right) + \frac{1}{r}\cos^2\left(\frac{R}{2}\right) = \frac{(p+q+r)^2}{4pqr}$$

38. (a) Solve the simultaneous equations :

$$\cos x + 4\sin y = 1$$
$$4\sec x - 3\text{cosec } y = 5$$

(b) Determine the solution of the equation $\tan 2x + 2\sin x = 0$ for $0 \le x \le 90^0$.

39. Prove the following identities

(a) $2\text{cosec}2\theta = \text{cosec}\theta\sec\theta$

(b) $\frac{1+\tan^2\beta}{2-\tan^2\beta} = \sec 2\beta$

40. Prove that :

$$\tan 5\theta = \frac{5\tan\theta - \tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}$$

41. (a) Prove that

$$\cos(60^0 - x) + \cos(300^0 - x) = \cos x$$

(b) Hence

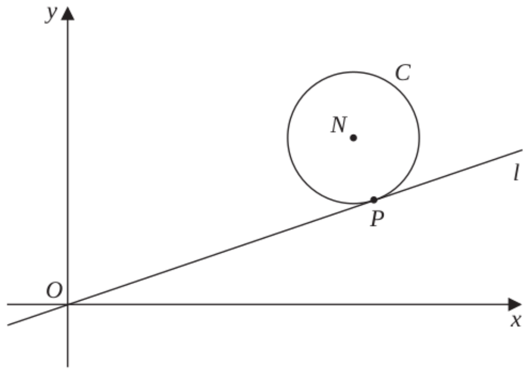
i. Find the exact value of $\cos 15^0 + \cos 255^0$

ii. Solve the equation

$$\cos(60^0 - x) + \cos(300^0 - x) = \frac{1}{4}\text{cosec}x \qquad \text{for} \qquad 0^0 < x < 180^0$$

GEOMETRY

42. The figure below shows a sketch of a circle C with centre N(7, 4) The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.



Find

(a) the equation of line PN in the form y = mx + c, where m and c are constants,

(b) an equation for C.

(c) The line with equation$\frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C. Find the value of k.

43. (a) $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are points on the parabola $y^2 = 4ax$.If the chord passes through the focus ,show that $pq = -1$.If M is the midpoint of PQ ,deduce that the locus of M is $y = 2a(x - a)$

(b) Show that the equation $y^2 = 9(x + y)$ represents a parabola;hence determine its focus and directrix

44. (a) A conic section is given by $x = 4\cos\theta$; $y = 3\sin\theta$.Show that the conic section is an ellipse and determine its eccentricity

(b) Given that the line $y = mx + c$ is atangent to th ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,show that $c^2 = a^2m^2 + b^2$.Hence determine the equations of the tangents at the point $(-3, 3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

45. The parametric coordinate of a curve is $(4\cos\theta, 3\sin\theta)$.

(a) Show that the curve represents an ellipse and hence determine its eccentricity.

(b) Find the equations of tangents to the ellipse in (a) above which passes through a point $(-3, 3)$

46. (a) Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$; and write down the equation of the tangent at the point $Q(9a, -6a)$.

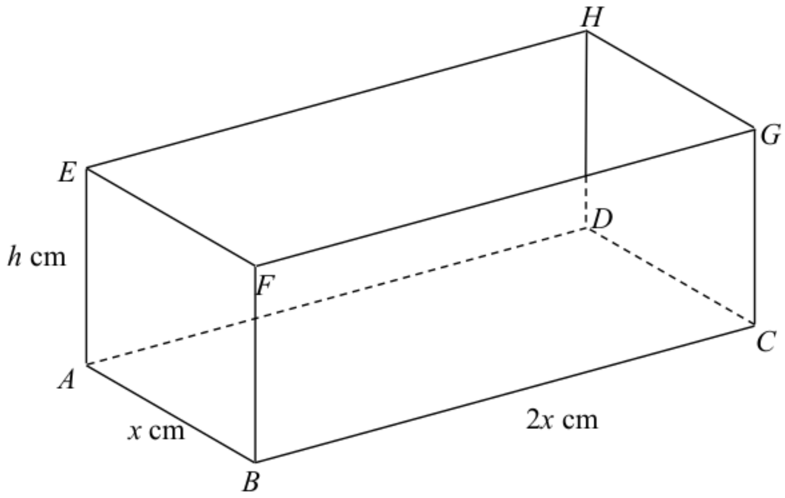
(b) Given the ellipse $\frac{x^2}{8} + \frac{y^2}{6} = 1$, find the

i. coordinates of the focii

ii. length of a latus rectum.

ANALYSIS

47. The figure below shows a solid cuboid ABCDEFGH used as a trough in feeding animals. AB = x cm, BC = 2x cm, AE = h cm The total surface area of the trough is $180cm^2$. The volume of the trough is $V cm^3$.



(a) Show that

$$V = 60x - \frac{4x^3}{3}$$

(b) Given that x can vary,use calculus to find, to 3 significant figures, the value of x for which V is a maximum.Justify that this value of x gives a maximum value of V.

(c) Find the maximum value of V, giving your answer to the nearest cm^3.

48. (a) Use the substitution $t = \tan\theta$ to solve

$$\int \frac{1}{4 + 5\cos 2\theta} d\theta$$

(b) Show that ;

$$\int_0^1 \tan^{-1} x dx = \frac{1}{4}(\pi - In4)$$

49. (a) Show that

$$\cos 3x = 4\cos^3 x - 3\cos x$$

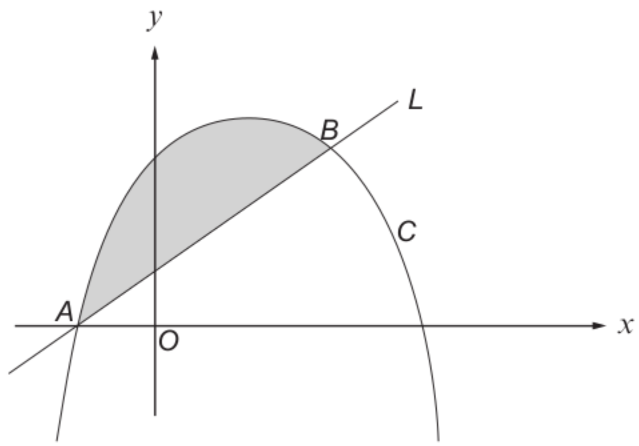
(b) Hence show that

$$\int_0^{\frac{\pi}{2}} \cos^3 x dx = \frac{2}{3}$$

50. (a) Express $8\sin\theta+6\cos\theta$ in the form $R\sin(\theta+\alpha)$,where $R>0$ and $0^0<\alpha<90^0$,giving the value of α correct to 2 decimal places

(b) Hence solve the equation $8\sin\theta+6\cos\theta=7$ giving all solutions in the interval $0^0<\theta<360^0$.

51. The sketch shows the curve C with equation $y=14+5x-x^2$ and line L with equation $y=x+2$. The line intersects the curve at the points A and B.



(a) Find the coordinates of A and B.

(b) Calculate the area enclosed by L and C.

52. (a) Given that $y=\cot^{-1}x$, show that $\frac{dy}{dx}=\frac{-1}{x^2+1}$

(b) Express $\frac{6x^2-10x-9}{(2x+3)(x^2+1)}$ in terms of partial fracions

(c) Hence find $\int\frac{6x^2-10x-9}{(2x+3)(x^2+1)}$

53. The parametric equations of a curve are $x=t+4\text{In}t$,$y=t+\frac{9}{t}$,for $t>0$.

(a) Show that

$$\frac{dy}{dx}=\frac{t^2-9}{t^2+4t}$$

(b) The curve has one stationary point.Find the y - coordinate of this point and determine whether it is a maximum or a minimum point.

54. (a) Show that $\cos 3x=4\cos^3x-3\cos x$

(b) Hence show that

$$\int_0^{\frac{1}{6}\pi}(4\cos^3x+2\cos x)dx=\frac{17}{6}$$

55. (a) Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x

(b) The period ,T of a swing of a simple pendulum of length ,l is given by the equation $T^2=\frac{4\pi^2 l}{g}$,where g is the acceleration due to gravity.An error of 2% is made in measuring the length ,l .Determine the resulting percentages error in the period ,T

(c) If $y = 3x^2 - x$ .show that $y\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y + 1 = 6x$

56. A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume V$m^3$of water in the tank is given by .

$$V = 243 - \frac{1}{3}(9 - x)^3$$

Water is being pumped into the tank at a constant rate of 3.6 m^3 per hour. Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute.

57. Determine the nature of the turning points of the curve

$$y = \frac{x^2 - 6x + 5}{(2x - 1)}$$

,Sketch the graph of the curve for $x = -2$ to $x = 7$.State any asymptotes.

58. (a) Solve the differential equation $\frac{dy}{dx} + 3y = e^{2x}$ given that when $x = 0, y = 1$

(b) The acceleration of a particle after time t seconds is given by $a = 5 + \cos\frac{1}{2}t$.If initially the particle is moving at $1ms^{-1}$,find its velocity after 20 seconds and the distance it would have covered by then.

59. (a) Find

$$\int x^3 e^{x^4} dx$$

(b) Use the substitution $t = \tan x$ to find

$$\int \frac{1}{1 + \sin^2 x} dx$$

60. (a) Express $f(x) = \frac{x^4 + 2x}{(x-1)(x^2+1)}$ into partial fractions

(b) Evaluate $\int_2^4 f(x)dx$

61. (a) Given that $y = e^{\tan^{-1} x}$,show that

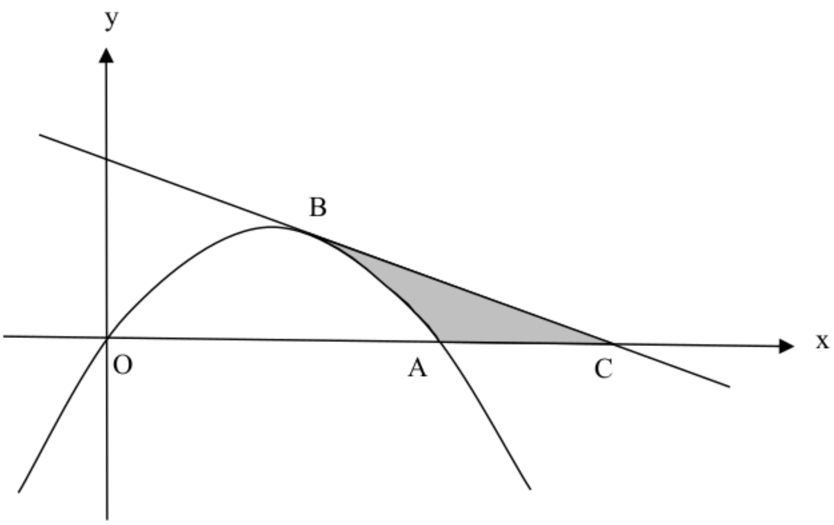
$$(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$$

(b) Differentiate x^{3x} with respect to x

62. (a) Solve the differential equation $(1 + x^2)\frac{dy}{dx} = 2 + y^2$

(b) In a culture of bacteria ,the rate of growth is proportional to the population present at time t.The population doubles every hour.Given that the initial population ,P_0 is three million,determine after how many hours the population will be 300 million

63. The diagram below shows a sketch of the curve $y = 3x - x^2$. The curve intersects the x -axis at the origin and at the point A. The tangent to the curve at the point B(2,2) intersects the x -axis at the point C.



(a) Find the equation of the tangent to the curve at B.

(b) Find the area of the shaded region.

64. (a) Differentiate e^{kx} from first principles

(b) Use small changes to show that $(16.02)^{\frac{1}{4}} = 2\frac{1}{1600}$

(c) Differentiate $\frac{e^{5x}\cos 2x}{\ln(1-x)}$ with respect to x

65. (a) Show that

$$\int_0^1 \frac{\ln(x+1)^2}{x+1}dx = (\ln 2)^2$$

(b) Express $5 + 4x - x^2$ in the form $a + b(x-2)^2$,hence evaluate;

$$\int_2^5 \frac{dx}{\sqrt{5+4x-x^2}}$$

66. (a) Given that $y = e^{\tan^{-1} x}$,show that

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

(b) Differentiate $\cos(x^2 e^x)$ with respect to x.

67. (a) Solve the equations below

i.

$$(1+x^2)\frac{dy}{dx} - y(y+1)x = 0, \text{given that y=1 when x=0}$$

ii.

$$\frac{dy}{dx} - y\tan x - \cos x = 0, \text{given that y=0 at } x = \frac{\pi}{2}$$

(b) The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding .Initially the temperature of the body is 80^0C .After 10 minutes the temperature of the body is 60^0C .The temperature of the surrounding is 15^0C

i. Form a differential equation for the temperature of the body

ii. Determine the time it takes for the temperature of the body to reach 40^0C

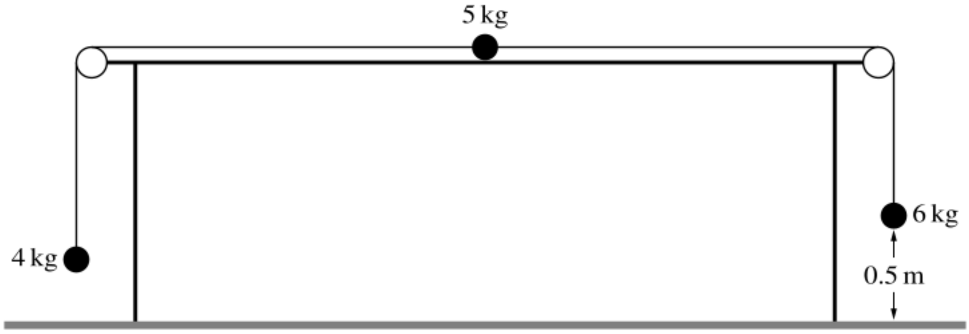
68. Maize dwarf mosaic virus(MDMV) has infected a number of maize plants in Mr Ronalds' garden .The growth in the number of maize plants infected is proportional to the number already infected .Initially 20 maize plants were infected

(a) Form a differential equation that models the growth in the number infected

(b) Thirty days after the initial number of infections ,60 maize plants were infected .After how many further days does the model predict that 200 maize plants will be infected?

MECHANICS

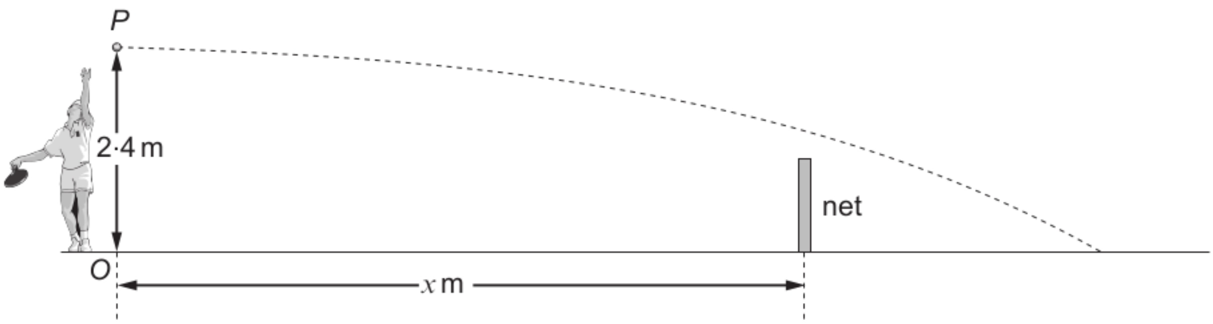
69. The diagram below shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.



(a) Find the coefficient of friction between the 5 kg particle and the table

(b) The 6 kg particle is now replaced by a particle of mass 8 kg and the system is released from rest. Find the acceleration of the 4 kg particle and the tensions in the strings.

70. A tennis ball is projected with velocity vector $(30i - 1.4j)$ ms^{-1} from a point P which is at a height of 2.4 m vertically above a horizontal tennis court. The ball then passes over a net of height 0.9 m, before hitting the ground after $\frac{4}{7}$s The origin O lies on the ground directly below the point P. The base of the net is x m from O.



(a) Find the speed of the ball when it first hits the ground, giving your answer correct to one decimal place.

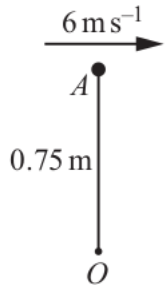
(b) After $\frac{2}{5}$s, the ball is directly above the net.

i. Find the position vector of the ball after $\frac{2}{5}$s

ii. Hence determine the value of x and show that the ball clears the net by approximately 16 cm.

(c) In fact, the ball clears the net by only 4 cm,explain why the observed value is different from the value calculated in (b)(ii).

71. One end of a light inextensible string of length $0.75m$ is attached to a particle A of mass 2.8 kg. The other end of the string is attached to a fixed point O. A is projected horizontally with speed 6 ms^{-1} from a point 0.75 m vertically above O (see Fig below). When OA makes an angle θ with the upward vertical the speed of A is vms^{-1} .



(a) Show that $v^2 = 50.7 - 14.7\cos\theta$.

(b) Given that the string breaks when the tension in it reaches 200 N, find the angle that OA turns through between the instant that A is projected and the instant that the string breaks.

72. A particle P of mass 0.5 kg moves on a horizontal plane such that its velocity vector v ms^{-1} at time t seconds is given by

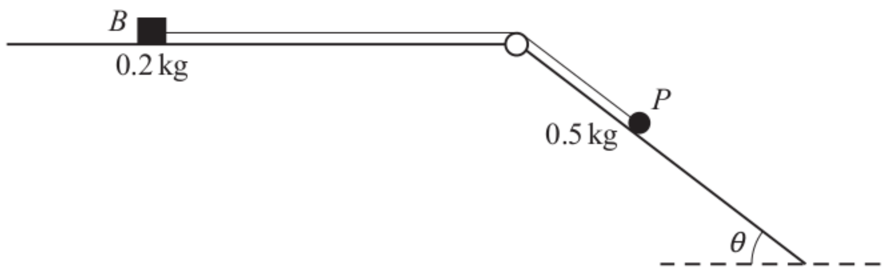
$$v = 12\cos(3t)i - 5\sin(2t)j$$

(a) Find an expression for the force acting on P at time t s.

(b) Given that when t = 0, P has position vector $(4i + 7j)$ m relative to the origin O, find an expression for the position vector of P at time t s.

(c) Hence determine the distance of P from O at time $t = \frac{\pi}{2}$

73. The diagram below shows a small block B, of mass 0.2 kg, and a particle P, of mass 0.5 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane.The block can move on the horizontal surface, which is rough. The particle can move on the inclined plane, which is smooth and which makes an angle of i with the horizontal where $\tan\theta = \frac{3}{4}$ .The system is released from rest. In the first 0.4 seconds of the motion P moves $0.3m$ down the plane and B does not reach the pulley.



(a) Find the tension in the string during the first 0.4 seconds of the motion.

(b) Calculate the coefficient of friction between B and the horizontal surface

74. An object of mass $4kg$ is initially at rest at a point whose position vector is $(-4i + 2j)m$.If its acted upon by a force $F = (14i + 21j + 24k)N$. Find

(i) acceleration of the object

(ii) its velocity and speed after $2 seconds$

(iii) its distance from the origin after 4seconds

75. A particle is projected from a point on level ground such that its initial velocity is $60ms^{-1}$ at an angle of elevation 30^0 and taking $g = 10ms^{-2}$,find

(a) the time taken for the particle to reach its maximum height

(b) the maximum height

(c) the time of flight

(d) the horizontal range of the particle

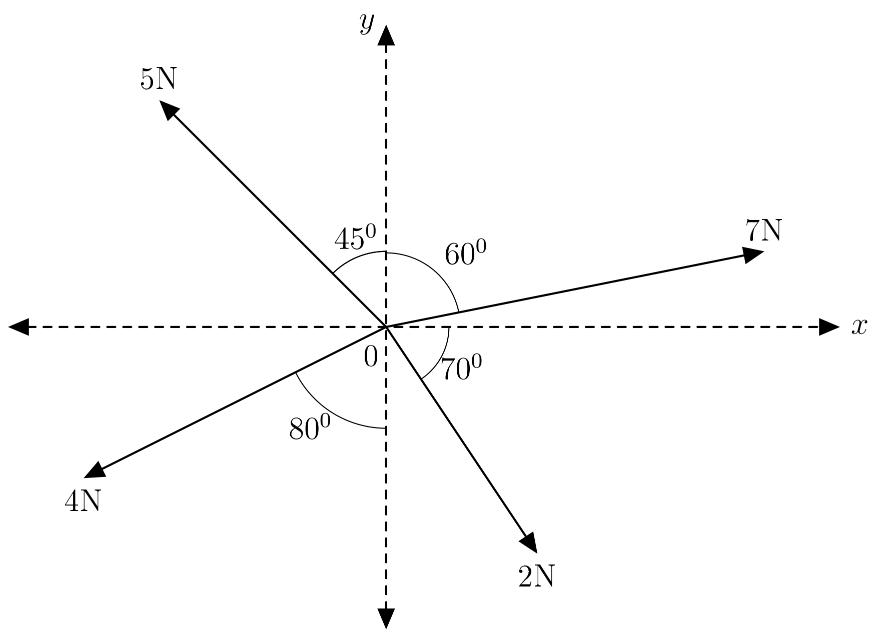
76. Two particles of masses 6kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;

(a) Acceleration of the particles

(b) The tension in the string

(c) The force on the pulley

77. The figure below shows a system of forces acting on a particle placed at O.Find the magnitude and direction of their resultant.



78. (a) A car of mass 1200kg pulls a trailer of mass 300kg up a slope of 1 in 100 against a constant resistance of 0.2N per kg .If the car moved at a constant speed of $1.5ms^{-1}$ for 5 minutes,calculate the ;

i. tension in the tow bar

ii. workdone by the car engine during this time

(b) A car of mass 800kg moved with a constant acceleration of $0.4ms^{-2}$ along a horizontal straight road against a resistance to motion of $250N$.Find the power developed at the instant when the car moved at $10ms^{-1}$

79. ABCD is arectangle in which AB=5M,BC=3M.forces of 2N,4N,3N and 11N act along AB,BC,CD and DA respectively,their directions being given by the order of the letters.Taking AB as the x-axis and AD as the y-axis .find the resultant force and the moment about A.

80. At time $t = 0$,the position vector **r** and velocity **v** of two trains A and B are as follows.

Trains	Velocity vector	Position vector
A	$V_A = (-6i + k)ms^{-1}$	$r_A = (i + 2j + 3k)m$
B	$V_B = (-5i + j + 7k)ms^{-1}$	$r_B = (4i - 14j + k)m$

If the trains maintain these velocities,find the :

(a) Position of B relative to A at time t

(b) time that elapses before the trains are closest to each other

(c) least distance between the trains in the subsequent motion

81. (a) An object with position vector 5i-8k moves with a constant speed of $5\sqrt{17}$ ms^{-1} in the direction 2i-2j+3k. Find its distance from the origin after 2 seconds. [5 Marks]

(b) A particle of mass 2kg is acted upon by a force $24t^2i + (36t - 6)j - 12tk$.Initially the particle is at a point $(3, -4, 4)$ and moving with velocity $16i + 15j - 8k$. Find the

(i) speed of the particle after one second .

(ii) distance covered by the particle in the first 2 seconds.

(iii) rate of doing work when t=2seconds.

NUMERICAL ANALYSIS

82. (a) The numbers $a = 26.23, b = 13.18$ and $c = 5.1$ are calculated with percentage errors $4, 3$ and 2 respectively.Find the errors in a, b and c hence the limits within which the exact value of the expression $ab - \frac{b}{c}$ lies correct to 3 decimal places

(b) Hence find the absolute error,relative error and percentage error in the above expression.

83. (a) Use the trapezium rule with 7 ordinates to estimate $\int_0^6 xe^{-x}dx$ correct to 3 significant figures

(b) Find the percentage error made in your estimation ,giving your answer to 2 deciml places .Suggest how this error may be reduced.

84. Jinja and Mukono are 66km apart.At 8:00am ,Gad starts cycling towards Mukono from Mbikko town which is between Jinja and Kampala 2km away from jinja.If at 8:00pm,he has only covered 36km,estimate ,

   (a) His distance from mukono at 10:00pm

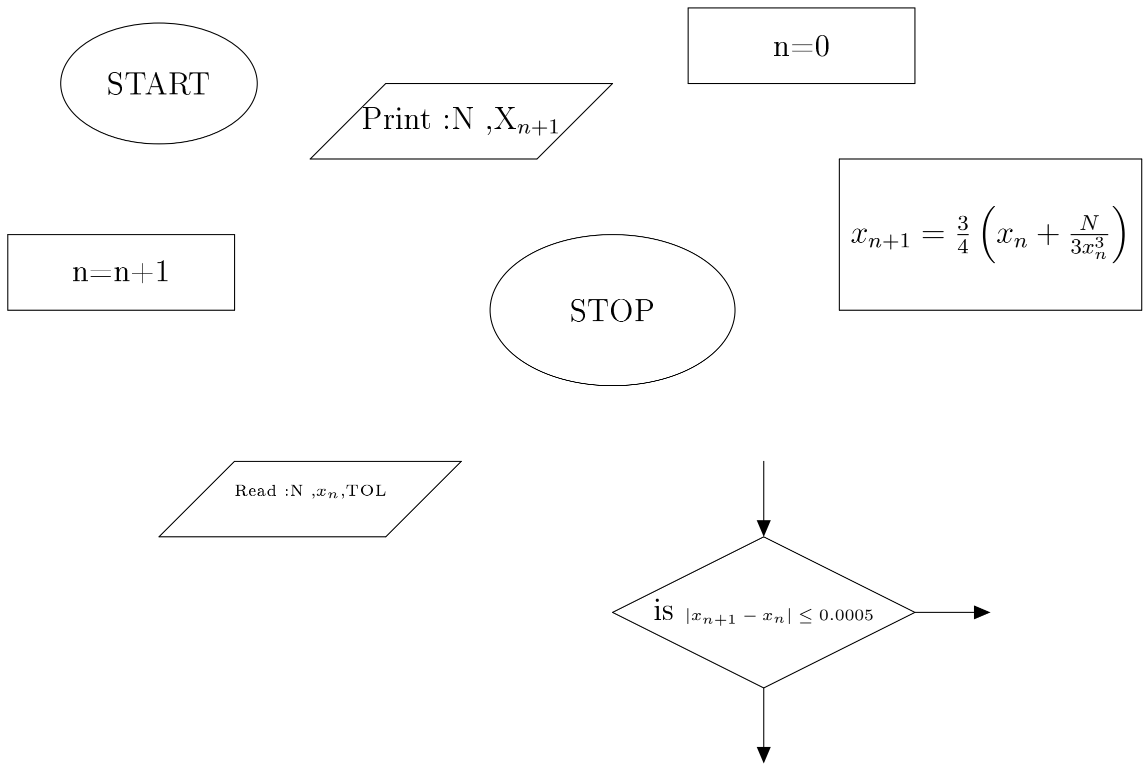
   (b) When he reaches mukono.

85. The distance between Buikwe and Lugazi town is 20km.Tenga, kibubu and makonge are 8km, 12km and 16km respectively from lugazi and the taxi charges are also respectively 500/=, 800/=, 1000/= and 1500/=. Namboowa is going to Visit her brother Kamyuka living 11km from lugazi

   (i) Find how much she will be charged in this taxi

   (ii) Suppose she had only 850/= and the taxi left her at a distance worth the money, find how far from Buikwe town the taxi leaves her

86. Given below are parts of a flow chart not arranged in order



   (a) Re arrange them and draw a complete logical flow chart

   (b) Using N=44 and $x_0 = 2$Perform a dry run of your arranged flow chart

   (c) State the purpose of the flow chart

87. (a) Derive the simplest iterative formulae based on Newton Raphson method for the equation $(x-2)\ln x = x-1$ and show that it is given by :

$$x_{n+1} = \frac{x_n(2\ln x_n + x_n - 3)}{x_n \ln x_n - 2}; n = 0, 1, 2, 3\cdots$$

(b) Construct a flow chart that ;

i. Reads initial approximations (x_0)

ii. Computes and limits the error to a root corrected to 3 decimal places.

iii. Prints the root

(c) Using $x_0 = 4$ as the first approximation ,perform a dry run for the flow chart above.

88. The table values of $\tan\Theta$ have been extracted from four figure tables

Θ	75	76	77	78	79
$\tan\Theta$	3.7321	4.0108	4.3315	4.7046	5.1446

Estimate

(i) $\tan^{-1}(4.6500)$

(ii) $\tan 79^0 36'$

89. The charges of sending parcels by JEFF distributing company depends on the weights of the parcels. For the parcels of weight 500g, 1kg, 1.5kg, and 2kg the charges are 1000/=, 2000/=, 3500/=, 4000/= respectively. Estimate

(i) What the distributor would charge for a parcel of weight 450g

(ii) What the distributor would charge for a parcel of weight 1.8kg

(ii) If the sender pays 6200/= what is the weight of his parcel

90. Given the numbers $x = 4, y = 6$ and $z = 8$ all measured to their nearest integers,find the minimum and maximum values

(a) $\frac{z-x}{y}$

(b) $\frac{x(y-z)}{z}$

91. The numbers $x = 4.6, y = 13.8$ and $z = 80.0$ are calculated with percentage errors of $0.5, 0.5$ and 0.05 respectively.Calculate the relative error in the expression $\frac{xy}{z}$

92. Given the numbers $x = 30.75$ and $y = 4.125$ all measured to their nearest number of decimal places indicated.

(a) state the maximum possible errors in x and y

(b) find the absolute error in the quotient $\frac{x}{y}$

(c) find the limits within which the exact value of the quotient $\frac{x}{y}$ lies

93. (a) Show graphically that the equation $e^{2x} + 4x = 5$ has one real root between 0 and 1

(b) Use the Newton- Raphson iterative method to find the root of the equation in (a) above giving your answer correct to 2 decimal places

94. (a) Show that the iterative formulabased on Newton Raphson's formula for finding the root of the equation $X = \sqrt[6]{N}$ is given by

$$x_{n+1} = \frac{5}{6}\left[x_n + \frac{N}{5x_n^5}\right]$$

(b) Draw a flow chart that:

(i) Reads N and the initial approximation x_0

(ii) Computes and prints the roots to 3 d.p

(iii) Print N and the root

(c) Taking N=26 and $x_0 = 1.5$ perform a dry run for the flow chart.

95. (a) Use the trapezium rule with 6 ordinates to estimate the area enclosed by the curve $y = xe^{-x^2}$,the x-axis and the lines $x = 1$ and $x = 3$.Give your answer to 4 d.p.

(b) Find the exact value of $\int_1^3 xe^{-x^2}dx$

(c) Find the percentage error in the estimation

96. Use the trapezium rule with 5 strips to estimate $\int_1^5 \frac{1}{3+x^2}dx$ to 3 d.p and find the percentage error made in the estimation.Hence,state ways of reducing such error

97. By drawing graphs of $y = e^{2x}$ and $y = 5x + 1$ on the same axes ,show that the equation $e^{2x} - 5x - 1 = 0$ has a root between 0 and 1.0 ,correct the root x_0 to 1 decimal place .Hence using x_0,the initial approximation and the newton Raphson method ,find the root correct it to 3 decimal places.

98. (a) Given that X = 4.52 and Y = 2.5 are rounded off to the given number of decimal places, Compute the minimum and maximum values of $\frac{X}{Y}$

(b) The floor of a room, $4.4m \times 6.5m$, is to be covered by tiles, $0.5m \times 0.45m$, assuming that the lengths given are rounded off to the given number of decimal points. Find the;

i. Minimum and maximum number of tiles required.

ii. Range of the total cost of laying the room with tiles if the cost of a tile is fixed at Ugx 4,000.

STATISTICS AND PROBABILITY

99. In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played. The random variable S, representing the player's score, has the following probability distribution where a, b and c are constants. The probability of scoring less than 2 points is twice the

s	0	1	2	3	4
$P(S = s)$	a	b	c	0.1	0.15

probability of scoring at least 2 points.Each game played is independent of previous games played. John plays the game twice and adds the two scores together to get a total. Calculate the probability that the total is 6 points.

100. For a set of ten data items

$$\sum(x - 20) = -140 \qquad \sum(x - 20)^2 = 2050$$

Find their mean and standard deviation

101. In an agricultural experiment ,320 plants were grown on a plot.The lengths of the stems were measured ,to the nearest centimetre,10 weeks after planting.The lengths were found to be distributed as in the following table.

Length,x(cm)	Number of plants.
$20.5 \leq x < 32.5$	30
$32.5 \leq x < 38.5$	80
$38.5 \leq x < 44.5$	90
$44.5 \leq x < 50.5$	60
$50.5 \leq x < 68.5$	60

(a) Calculate an estimate of the :

i. Mean of the stem lengths.

ii. Median of the stem lengths.

(b) Display the data on a histogram and use it to estimate the mode

102. Lyn buys electrical components from one of 3 suppliers A, B, C, in the ratio 2 : 1 : 7. The probability that the component is faulty is 0.33 for A, 0.45 for B and 0.05 for C. Lyn selects a component at random.

(a) Find the probability that the component works.

(b) Given that the component works, find the probability that Lyn bought the component from supplier B.

103. The cost of making a well formulated feed for the layer birds on Mr Ronald's Poultry farm is calculated from the cost of Maize bran,Broken maize ,lime and concentrate .The table below gives the cost of these items in 2023 and 2024.

ITEMS	Price(UGX) in 2023	Price (UGX) in 2024	Weight
Maize bran/kg	500	735	12
Lime/kg	500	600	2
Broken maize/kg	800	1400	5
Concentrate/kg	196000	215600	1

Using 2023 as the base year

(a) Calculate the price relative for each item hence find the simple price index for the cost of making a complete feed

(b) Find the weighted aggregate price index for the cost of the feed

104. Three events A,B and C are such that A and B are independent ,A and C are mutually exclusive.Given that $P(A) = 0.4, P(B) = 0.2, P(C) = 0.3$ and $P(CnB) = 0.1$.find $P(A/AUC)$

105. Events M and N are such that $3P(M \cap N) = 2P(\overline{M} \cap N) = P(\overline{M} \cap \overline{N}), P(M) = 0.6$.Find the probability that:

(a) Neither events occur

(b) Only one event occurs

106. The table below shows height in centimetres of 25 students in a certain school.

Height	< 10	< 20	< 25	< 30	< 40	< 55	< 60
Frequency	0	3	4	8	2	6	2

(a) Find the:

(i) Mean height

(ii) Variance

(iii) Middle 70% of the height.

(b) Represent the above information on a histogram and use it to estimate the modal height.

107. (a) The information below shows the grades scored by a group of students in biology and mathematics examinations

Student	1	2	3	4	5	6	7	8	9
Math	A	E	F	A	B	B	C	D	B
Biology	C	0	E	C	C	B	A	F	D

Compute the rank correlation coefficient for the perfomance between the two subjects .Comment on your result at 1% level of significance

(b) The table below shows the distribution of heights of 134 students in a Maths Class.

Heights	$20-<30$	$30-<35$	$35-<40$	$40-<55$	$55-<65$	$65-<80$	$80-<90$
No of students	9	12	27	13	25	18	30

(a) Calculate the mean mark

(b) Construct a cumulative frequency curve(ogive) and use it to find

(i) Median mark

(ii) Range between the 20^{th} and 60^{th} percentile

(iii) Range of the middle 40% of the mark

(iv) Probability that student selected at random scored below 60 marks.

108. A and B are events such that P(A)=$\frac{8}{15}$,P(B)=$\frac{1}{3}$ and P(A/B)=$\frac{1}{5}$.Find the probability that :

(i) neither A nor B occurs

(ii) Event B does not happen if event A has occured

(iii) Both events occur.

(iv) Only one of the two events occurs.

109. A school bus can arrive early,on time or late.The probability that it is late is 0.25. The probability that it is on time or late is $\frac{2}{3}$.Find the probability that the school bus is

(i) On time

(ii) early or on time

110. A bag contains 5 red balls and 3 blue balls .Three balls are selected in succession at random from it with replacement.Find the chance that

(a) they are of the same colour

(b) the first and last are of the same color

(c) At most one blue ball is drawn

111. A discrete r.v X has the following p.d.f

$$P(X=x)=\begin{cases} px & ;x=1,2,\cdots n \\ 0 & ;\text{otherwise} \end{cases}$$

Find

(i) values of p and n for which E(X)=7

(ii) P$(2 < X < 7/x \geq 4)$

112. A continuous r.v X has the following p.d.f.

$$f(x)=\begin{cases} \beta(3-x) & ;1 \leq x \leq 2 \\ \beta & ;2 \leq x \leq 3 \\ \beta(x-2) & ;3 \leq x \leq 4 \\ 0 & ;\text{otherwise} \end{cases}$$

(a) Sketch $f(x)$ hence deduce the mean and median of X

(b) Find

(i) the value of β

(ii) $P(|X-2| < 0.5)$

113. The distribution function of a continuous r.v X is as follows

$$F(x)=\begin{cases} 0 & ;x \leq 0 \\ \frac{1}{2}x^2 & ;0 \leq x \leq 1 \\ m+nx^3 & ;1 \leq x \leq 2 \\ 1 & ;x \geq 2 \end{cases}$$

Find

(a) the value of m and n

(b) $f(x)$

(c) Sketch the graph of f.Find

i. the mode

ii. the median

iii. the mean of X

114. It is given that

$$f(x)=\begin{cases} \beta(x+2) & ;-2 < x < 0 \\ \frac{1}{2}\beta(3-x) & ;0 < x < 2 \\ 0 & ;\text{otherwise} \end{cases}$$

Find

(a) the value of β

(b) $F(x)$

(c) $P(-1 < X < 1)$

(d) $P(1 < X < 3)$

115. The marks scored in an exam are normally distributed with mean 56 and standard deviation 14.2 .Find the probability that a candidate picked at random scored

(a) between 62 and 72 marks

(b) atleast 40 marks

116. Patience has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6.The other three coins are fair. Patience throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225.

(b) Complete the following probability distribution table for X .

x	0	1	2	3	4
$P(X = x)$	0.05	0.225			0.075

(c) Given that $E(X) = 2.1$, find the value of $Var(X)$.

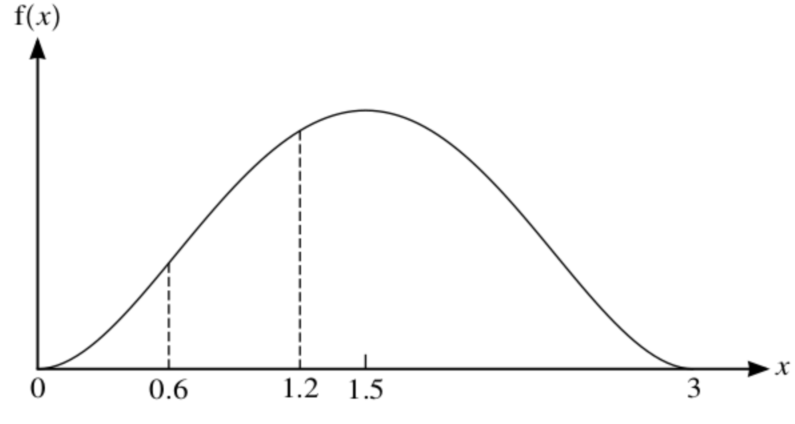
117. On a farm ,35%of the cows are infected by a tick disease.If a random sample of 400 cows is selected from the farm ,find the probability that:

(a) less than 30% of the cows are infected

(b) more than 155 cows are infected

(c) between 120 and 150 inclusive cows are infected

118. The diagram below shows the graph of the probability density function, f, of a random variable X that takes values between $x = 0$ and $x = 3$ only. The graph is symmetrical about the line $x = 1.5$.



(a) It is given that $P(X < 0.6) = a$ and $P(0.6 < X < 1.2) = b$. Find $P(0.6 < X < 1.8)$ in terms of a and b .

(b) It is now given that the equation of the probability density function of X is

$$f(x) = \begin{cases} kx^2(3-x)^2 & ; 0 \le x \le 3 \\ 0 & ; \text{otherwise} \end{cases}$$

i. Show that $k = \frac{10}{81}$

ii. Find Var (X.)

119. Three events A,B and C are such that A and B are independent ,A and C are mutually exclusive.Given that $P(A) = 0.4, P(B) = 0.2, P(C) = 0.3$ and $P(C \cap B) = 0.1$,.find

(a) $P(A \cup B^1)$

(b) $(A/A \cup C)$

120. The marks of 12 students in aptitude and stastics test were as follows

Aptitude	58	52	48	30	48	20	32	50	38	12	36	12
Stastics	90	72	60	38	70	35	33	64	48	24	50	18

(a) Plot a scatter diagram for the data.comment on the relationship between the two tests

(b) Draw a line of best fit for the scatter diagram,hence find x when y=68

(c) Calculate the rank correlation coefficient for the scores in the two tests .comment on your results at 1% level of significance

121. (a) A thrown biased dice is such that an even number is twice as likely to show up as an odd number.Find the probability of obtaining

i. A number less than 4

ii. An odd or prime number

(b) A box contains 3yellow ballotpapers,4 red ballot papers, and 1 green ballot paper.Two ballot papers are picked in succession at random without replacement .Find the probability that

i. there is no yellow ballot paper.

ii. Atleast one red ballot paper is picked.

122. The marks obtained in the UNEB Mathematics paper 2 by candidates of last year were normally distributed with a mean of 64 with a variance of 64.

(a) If the pass mark was 50, calculate the percentage of candidates that passed.

(b) Calculate the lowest mark for a distinction if 5% of the candidates scored distinctions.

(c) Given that a student obtained a distinction, calculate the probability that the student scored above 70.

END

PURE MATHEMATICS P425/1

1. Algebra

   (a) For the quadratic equation $ax^2 + bx + c = 0$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   (b) For an arithmetic series (A.P)

   $$u_n = a + (n-1)d$$
   $$S_n = \frac{1}{2}n\{2a + (n-1)d\}$$

   (c) For a geometric series (G.P)

   $$u_n = ar^{n-1}$$
   $$S_n = \frac{a(1-r^n)}{1-r} \qquad r \neq 1$$
   $$S_\infty = \frac{a}{1-r} \qquad |r| < 1$$

   (d) Binomial expansion

   $$(a+b)^n = a^n + \begin{pmatrix} n \\ 1 \end{pmatrix} a^{n-1}b + \begin{pmatrix} n \\ 2 \end{pmatrix} a^{n-2}b^2 + \begin{pmatrix} n \\ 3 \end{pmatrix} a^{n-3}b^3 + \cdots + b^n$$

   where n is a positive integer

   $$\begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!}$$
   $$^nC_r = \frac{n!}{r!(n-r)!}$$
   $$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1(n-2)}{3!}x^3 \cdots$$

   Where n is rational and $|x| < 1$

   $$^nP_r = \frac{n!}{(n-r)!}$$

   Where $r \leq n$

   (e) Summations

   $$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$
   $$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$
   $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

2. Trigonometry

No	**Identity**
1	$\tan\theta = \frac{\sin\theta}{\cos\theta}$,$\sec\theta = \frac{1}{\cos\theta}$,$co\sec\theta = \frac{1}{\sin\theta}$
2	$\cos^2\theta + \sin^2\theta = 1$
3	$1 + \tan^2\theta = \sec^2\theta$
4	$\cot^2\theta + 1 = co\sec^2\theta$
5	$\sin(A+B) = \sin A\cos B + \cos A\sin B$
6	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
7	$\cos(A+B) = \cos A\cos B - \sin A\sin B$
8	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
9	$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$
10	$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$
11	$\sin 2A = 2\sin A\cos A$
12	$\cos 2A = \cos^2 A - \sin^2 A$
13	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(a) For the t -formula

$$t = \tan\frac{1}{2}\theta$$
$$\sin\theta = \frac{2t}{1+t^2}$$
$$\cos\theta = \frac{1-t^2}{1+t^2}$$

(b) For any triangle with angles ,A,B and C and with sides a,b,and c .

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{Cosine rule}$$
$$s = \frac{a+b+c}{2}$$

3. Differentiation

No	y	$\frac{dy}{dx}$
1	x^n	nx^{n-1}
2	$\ln x$	$\frac{1}{x}$ for $x \neq 0$
3	e^x	e^x
4	$\sin x$	$\cos x$
5	$\cos x$	$-\sin x$
6	$\tan x$	$\sec^2 x$
7	uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
8	$\frac{u}{v}$	$\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
9	$f(x)$	$\frac{f(x+\delta x)-f(x)}{\delta x}$
10	$\sec x$	$\sec x\tan x$
11	y=u,u=x	$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$
12	$e^{f(x}$	$f^1(x)e^{f(x)}$

4. Integration

No	$f(x)$	$\int f(x)dx$
1	x^n	$\frac{x^{n+1}}{n+1}+c$ for $n\neq -1$
2	$\frac{1}{x}$	$\ln\lvert x\rvert+c$ n
3	e^x	e^x+c
4	$\sin x$	$-\cos x+c$
5	$\cos x$	$\sin x+c$
6	$\sec^2 x$	$\tan x+c$
7	$\int u\frac{dv}{dx}dx$	$uv-\int v\frac{du}{dx}dx$
8	$\int \frac{f^1(x)}{f(x)}dx$	$\ln\lvert f(x)\rvert+c$
9	$co\sec x\cot x$	$-co\sec x+c$
10	$\sec x\tan x$	$\sec x+c$
11	$co\sec^2 x$	$-\cot x+c$
12	$\tan x$	$\ln\lvert\sec x\rvert+c$
13	$co\sec x$	$-\ln\lvert co\sec x+\cot x\rvert+c$
14	$\cot x$	$\ln\lvert\sin x\rvert+c$

(a)

$$\int\frac{1}{a^2-b^2x^2}dx=\frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)+c$$

(b)

$$\int\frac{1}{a^2+b^2x^2}dx=\frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)+c$$

(c)

$$\int\frac{a}{p+qx}dx=\frac{a}{q}\ln|p+qx|+c$$

5. Vectors

(a) If $a=a_1i+a_2j+a_3k$ and $b=b_1i+b_2j+b_3k$ then

$$\begin{aligned}a.b&=a_1b_1+a_2b_2+a_3b_3\\&=|a||b|\cos\theta\end{aligned}$$

(b) $i.i=j.j=k.k=1$ and $i.j=i.k=j.k=0$

(c) $|a.a|=|a|^2$

(d) $a.(b+c)=a.b+a.c$

(e) $a.(kb)=(ka).b=k(a.b)$ where k is a constant

(f) $a.b=|a||b|\cos\theta$

(g) The cartesian equation of the line

$$\frac{x-a}{x_1}=\frac{y-b}{y_1}=\frac{z-c}{z_1}$$

APPLIED MATHEMATICS P425/2

1. Numerical Methods

   (a) Trapezium rule

   $$\int_a^b f(x)dx \approx \frac{1}{2}h\{y_o + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n\}$$

   Where $h = \dfrac{b-a}{n}$

   (b) Newton Raphson Method

   $$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)} \qquad \text{Where } x = 0, 1, 2, \cdots$$

   (c) Ordinates and sub intervals

   The number of sub intervals = Number of ordinates − 1

   (d) The maximum possible error made due to rounding off is given by

   $$\text{Error} = 0.5 \times 10^{-n}$$

   Where **n** is the number of decimal places rounded off to

   (e) Error

   $$\text{Error } = \text{Exact value} - \text{Approximate value.}$$

   (f) Absolute error This is the actual size of the error and is always positive .It is the magnitude of the error

   $$\text{Error } = |\text{Exact value} - \text{Approximate value}|.$$

   (g) Relative error

   $$\text{Relative Error } = \frac{\text{Absolute error}}{\text{Exact value}}$$

   The relative error must always be positive

   $$\text{Relative Error } = \frac{|\text{Error}|}{\text{Exact value}}$$

   (h) Percentage error

   $$\text{Percentage Error } = \frac{|\text{Error }|}{\text{Exact value}} \times 100$$

   (i) The interval or range with in which the exact value lies is given by Min value$\leq$ Exact value $\leq$ Max value or [Min,Max]

(j) Absolute error $=\frac{\text{Maximum value}-\text{Minimum value}}{2}$

2. Probability and Statistics

(a) The mean for ungrouped data is calculated using the formula

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of values in the data}}$$
$$\overline{X} = \frac{\sum x}{n}$$

(b) The mean for grouped data is calculated using the formula

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

Where **x** is the class mark and **f** is the frequency

(c) The mean for grouped data when given an assumed means is calculated using the formula

$$\text{Mean} = A + \frac{\sum fd}{\sum f}$$

Where **A** is the assumed mean or working mean

d is the deviation given as $d = x - A$

(d) For grouped data ,the median is calculated using

$$\text{Median } = L_1 + \left(\frac{\frac{\sum f}{2} - CF_b}{f_m}\right) \times C$$

Where

$$L_1 = \text{Lower class boundary of the median class}$$
$$CF_b = \text{Cummulative frequency before the median class}$$
$$f_m = \text{frequency within the median class}$$
$$C = \text{Class width}$$
$$\sum f = \text{Total frequency}$$

(e) For grouped data with equal class width the mode is calculated using

$$\text{Mode} = L_1 + \left(\frac{d_1}{d_1 + d_2}\right) \times C$$

Where

$$\begin{aligned} L_1 &= \text{Lower class boundary of the modal class} \\ d_1 &= \text{Modal frequency } - \text{Pre modal frequency} \\ d_2 &= \text{Modal frequency } - \text{Post modal frequency} \\ C &= \text{Class width} \end{aligned}$$

(f) For grouped data ,the lower quartile is calculated using

$$q_1 = L_1 + \left(\frac{\frac{\sum f}{4} - CF_b}{f_m} \right) \times C$$

Where

$$\begin{aligned} L_1 &= \text{Lower class boundary of the } q_1 \text{ class} \\ CF_b &= \text{Cummulative frequency before the } q_1 \text{ class} \\ f_m &= \text{frequency within the } q_1 \text{ class} \\ C &= \text{Class width} \\ \sum f &= \text{Total frequency} \end{aligned}$$

(g) For grouped data ,the upper quartile is calculated using

$$q_3 = L_1 + \left(\frac{\frac{3\sum f}{4} - CF_b}{f_m} \right) \times C$$

Where

$$\begin{aligned} L_1 &= \text{Lower class boundary of the } q_3 \text{ class} \\ CF_b &= \text{Cummulative frequency before the } q_3 \text{ class} \\ f_m &= \text{frequency within the } q_3 \text{ class} \\ C &= \text{Class width} \\ \sum f &= \text{Total frequency} \end{aligned}$$

(h) Inter quartile range $= q_3 - q_1$

(i) For grouped data ,the variance is calculated using

$$\text{Var(x)} = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$$

(j) Standard deviation $= \sqrt{\text{Var(x)}}$

3. Index numbers

(a)

$$\text{Price relative } = \frac{p_n}{p_0} \times 100$$

Where

$p_n =$ Price of the commodity in the given year(new year)

$p_0 =$ Price of the commodity in the base year(old year)

(b)

$$\text{Simple price index} = \frac{\text{Sum of the price relatives}}{\text{Number of items (N)}} \times 100$$
$$= \frac{\sum\left(\frac{p_n}{p_0}\right) \times 100}{N}$$

(c)

$$\text{Simple aggregate price index} = \frac{\text{Current year price total}}{\text{Base price total}} \times 100$$
$$= \frac{\sum p_n}{\sum p_0} \times 100$$

(d)

$$\text{Weighted price index} = \text{Price relatives} \times \text{weights}$$
$$= \frac{p_n}{p_o} \times w \times 100$$

(e)

$$\text{Weighted aggregate price index} = \frac{\sum p_n w}{\sum p_0 w} \times 100$$

(f)

$$\text{Weighted average price index} = \frac{\sum \frac{p_n}{p_0} \times 100 \times w}{\sum w}$$

(g)

$$\text{Value index} = \frac{\sum p_n q_n}{\sum p_o q_o} \times 100$$

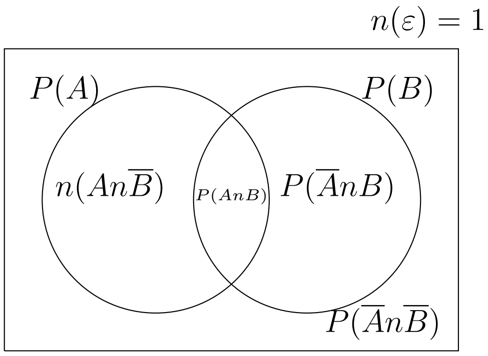
4. Spearman's rank correlation coefficient

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

Where n is the difference between the rankings of a given scores and n is the number of pairs

5. Probability theory

   (a) For any events A and B



$$P(A) = P(A \cap \overline{B}) + P(A \cap B)$$
$$P(\overline{A}) = P(\overline{A} \cap B) + P(A \cup B)^1$$
$$P(B) = P(\overline{A} \cap B) + P(A \cap B)$$
$$P(\overline{B}) = P(A \cap \overline{B}) + P(A \cup B)^1$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

   (b) $P(A) + P(\overline{A}) = 1$

   (c) $P(A \cup B)^1 = P(\overline{A} \cap \overline{B})$

   (d) $P(\overline{A} \cup \overline{B}) = P(A \cap B)^1$

   (e) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ for $P(B) \neq 0$

6. Mechanics

   (a) For projectile motion

$$y = x \tan\theta - \frac{gx^2}{2u^2} sec^2\theta$$

   (b) For calculus

Physical quantity	Formula	units	Formula	units	Formula
Force	F=ma	N	$a = \frac{dv}{dt}$	ms^{-2}	k.e=$\frac{1}{2}mv^2$
Power	P=F.v	W	$v = \frac{dr}{dt} or \frac{ds}{dt}$	ms^{-1}	Avg accel=$\frac{v(t_2)-v(t_1)}{t_2-t_1}$
Work done	W=F.s or F.r	j	W=$\int_{t_1}^{t_2} f.vdt$	j	speed=$\lvert v \rvert$
Impulse	I=F.t	Ns	$v = \int adt$	ms^{-1}	Avg vel=$\frac{r(t_2)-r(t_1)}{t_2-t_1}$
Momentum	momentum =m.v	$Kgms^{-1}$	$r = \int vdt$	m	distance=$\lvert r \rvert or \lvert s \rvert$

STATISTICAL TABLES

SIGNIFICANCE LEVELS FOR CORRELATION COEFFICEINTS

No. of pairs	Product-moment coefficient of correlation (r_{xy}) — Significance if $\lvert r_{xy} \rvert$ exceeds at 5%	at 1%	Spearman's rank Correlation coefficient (ρ) — Significance if $\lvert \rho \rvert$ exceeds at 5%	at 1%	Kendall's rank coefficient of correlation (τ) — Significance if $\lvert \tau \rvert$ exceeds at 5%	at 1%
3	1.00	1.00				
4	0.95	0.99				
5	0.88	0.96	1.00			
6	0.81	0.92	0.89	1.00	0.87	1.00
7	0.75	0.88	0.75	0.89	0.71	0.81
8	0.71	0.83	0.71	0.86	0.64	0.79
9	0.67	0.80	0.68	0.83	0.56	0.72
10	0.63	0.77	0.65	0.79	0.51	0.64
11	0.60	0.74	0.60	0.74	0.49	0.60
12	0.58	0.71	0.58	0.71	0.45	0.58
13	0.55	0.68	0.55	0.68		
14	0.53	0.66	0.53	0.66		
15	0.51	0.64	0.51	0.64		
16	0.50	0.62	0.50	0.62		
17	0.48	0.61	0.48	0.61		
18	0.47	0.59	0.47	0.59		
19	0.46	0.58	0.46	0.58		
20	0.44	0.56	0.44	0.56	0.33	
30	0.35	0.45	0.35	0.45		
40	0.31	0.39	0.31	0.39		
50	0.27	0.35	0.27	0.35		
60	0.25	0.33	0.25	0.33		
70	0.23	0.31	0.23	0.31		
80	0.22	0.29	0.22	0.29		
90	0.21	0.27	0.21	0.27		
100	0.20	0.25	0.20	0.25		

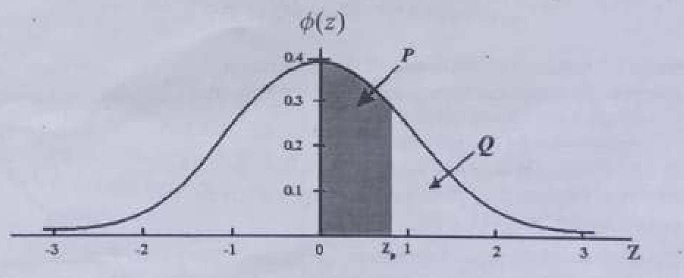
CUMULATIVE NORMAL DISTRIBUTION $P(z)$

											ADD								
Z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673							3	6	9	12	15	19	22	25	28
					2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023					3	6	8	11	14	17	20	22	25
							3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
0.9	0.3159	3186	3212	3238	3264	3289					3	5	8	10	13	16	18	21	23
							3315	3340	3365	3389	2	5	7	10	12	15	17	20	22
1.0	0.3413	3438	3461	3485	3508						2	5	7	10	12	14	17	19	22
						3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
1.1	0.3643	3665	3686	3708							2	4	6	8	11	13	15	17	19
					3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
1.2	0.3849	3869	3888	3907	3925						2	4	6	8	10	11	13	15	17
						3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives $P(z)=\int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P$(Shaded Area)
2. $P(Z > Z_p) = Q = ½ - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$



CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION) $\sum_{i \geq r} p_i$

n	r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	1	0.0199	0975	1900	2775	3600	4375	5100	5775	6400	6975	7500
	2	0.0001	0025	0100	0225	0400	0625	0900	1225	1600	2025	2500
3	1	0.0297	1426	2710	3859	4880	5781	6570	7254	7840	8336	8750
	2	0.0003	0072	0280	0608	1040	1562	2160	2818	3520	4252	5000
	3		0001	0010	0034	0080	0156	0270	0429	0640	0911	1250
4	1	0.0394	1855	3439	4780	5904	6836	7599	8215	8704	9085	9375
	2	0.0006	0140	0523	1095	1808	2617	3483	4370	5248	6090	6875
	3		0005	0037	0120	0272	0508	0837	1265	1792	2415	3125
	4			0001	0005	0016	0039	0081	0150	0256	0410	0625
5	1	0.0490	2262	4095	5563	6723	7627	8319	8840	9222	9497	9688
	2	0.0010	0226	0815	1648	2627	3672	4718	5716	6630	7438	8125
	3		0012	0086	0266	0579	1035	1631	2352	3174	4069	5000
	4			0005	0022	0067	0156	0308	0540	0870	1312	1875
	5				0001	0003	0010	0024	0053	0102	0185	0312
6	1	0.0585	2649	4686	6229	7379	8220	8824	9246	9533	9723	9844
	2	0.0015	0328	1143	2235	3446	4661	5798	6809	7667	8364	8906
	3		0022	0158	0473	0989	1694	2557	3529	4557	5585	6562
	4		0001	0013	0059	0170	0376	0705	1174	1792	2553	3438
	5			0001	0004	0016	0046	0109	0223	0410	0692	1094
	6					0001	0002	0007	0018	0041	0083	0156
7	1	0.0679	3017	5217	6794	7903	8665	9176	9510	9720	9848	9922
	2	0.0020	0444	1497	2834	4233	5551	6706	7662	8414	8976	9375
	3		0038	0257	0738	1480	2436	3529	4677	5801	6836	7734
	4		0002	0027	0121	0333	0706	1260	1998	2898	3917	5000
	5			0002	0012	0047	0129	0288	0556	0963	1529	2266
	6				0001	0004	0013	0038	0090	0188	0357	0625
	7						0001	0002	0006	0016	0037	0078
8	1	0.0773	3366	5695	7275	8322	8999	9424	9681	9832	9916	9961
	2	0.0027	0572	1869	3428	4967	6329	7447	8309	8936	9368	9648
	3	0.0001	0058	0381	1052	2031	3215	4482	5722	6846	7799	8555
	4		0004	0050	0214	0563	1138	1941	2936	4059	5230	6367
	5			0004	0029	0104	0273	0580	1061	1737	2604	3633
	6				0002	0012	0042	0113	0253	0498	0885	1445
	7					0001	0004	0013	0036	0085	0181	0352
	8							0001	0002	0007	0017	0039
9	1	0.0865	3698	6126	7684	8658	9249	9596	9793	9899	9954	9980
	2	0.0034	0712	2252	4005	5638	6997	8040	8789	9295	9615	9805
	3	0.0001	0084	0530	1409	2618	3993	5372	6627	7682	8505	9102
	4		0006	0083	0339	0856	1657	2703	3911	5174	6386	7461
	5			0009	0056	0196	0489	0988	1717	2666	3786	5000
	6			0001	0006	0031	0100	0253	0536	0994	1658	2539
	7					0003	0013	0043	0112	0250	0498	0898
	8						0001	0004	0014	0038	0091	0195
	9								0001	0003	0008	0020
10	1	0.0956	4013	6513	8031	8926	9437	9718	9865	9940	9975	9990
	2	0.0043	0861	2639	4557	6242	7560	8507	9140	9536	9767	9893
	3	0.0001	0115	0702	1798	3222	4744	6172	7384	8327	9004	9453
	4		0010	0128	0500	1209	2241	3504	4862	6177	7340	8281
	5		0001	0016	0099	0328	0781	1503	2485	3669	4956	6230
	6			0001	0014	0064	0197	0473	0949	1662	2616	3770
	7				0001	0009	0035	0106	0260	0548	1020	1719
	8					0001	0004	0016	0048	0123	0274	0547
	9							0001	0005	0017	0045	0107
	10									0001	0003	0010
11	1	0.1047	4312	6862	8327	9141	9578	9802	9912	9964	9986	9995
	2	0.0052	1019	3026	5078	6779	8029	8870	9394	9698	9861	9941
	3	0.0002	0152	0896	2212	3826	5448	6873	7999	8811	9348	9673
	4		0016	0185	0694	1611	2867	4304	5744	7037	8089	8867
	5		0001	0028	0159	0504	1146	2103	3317	4672	6029	7256

CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION)

		x										
n	r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
11	6			0003	0027	0117	0343	0782	1487	2465	3669	5000
	7				0003	0020	0076	0216	0501	0994	1738	2744
	8					0002	0012	0043	0122	0293	0610	1133
	9						0001	0006	0020	0059	0148	0327
	10								0002	0007	0022	0059
	11										0002	0005
12	1	0.1136	4596	7176	8578	9313	9683	9862	9943	9978	9992	9998
	2	0.0062	1184	3410	5565	7251	8416	9150	9576	9804	9917	9968
	3	0.0002	0196	1109	2642	4417	6093	7472	8487	9166	9579	9807
	4		0022	0256	0922	2054	3512	5075	6533	7747	8655	9270
	5		0002	0043	0239	0726	1576	2763	4167	5618	6956	8062
	6			0005	0046	0194	0544	1178	2127	3348	4731	6128
	7			0001	0007	0039	0143	0386	0846	1582	2607	3872
	8				0001	0006	0028	0095	0255	0573	1117	1938
	9					0001	0004	0017	0056	0153	0356	0730
	10							0002	0008	0028	0079	0193
	11								0001	0003	0011	0032
	12										0001	0002
15	1	0.1399	5367	7941	9126	9648	9866	9953	9984	9995	9999	1.0000
	2	0.0096	1710	4510	6814	8329	9198	9647	9858	9948	9983	9995
	3	0.0004	0362	1841	3958	6020	7639	8732	9383	9729	9893	9963
	4		0055	0556	1773	3518	5387	7031	8273	9095	9576	9824
	5		0006	0127	0617	1642	3135	4845	6481	7827	8796	9408
	6		0001	0022	0168	0611	1484	2784	4357	5968	7392	8491
	7			0003	0036	0181	0566	1311	2452	3902	5478	6964
	8				0006	0042	0173	0500	1132	2131	3465	5000
	9				0001	0008	0042	0152	0422	0950	1818	3036
	10					0001	0008	0037	0124	0338	0769	1509
	11						0001	0007	0028	0093	0255	0592
	12							0001	0005	0019	0063	0176
	13								0001	0003	0011	0037
	14										0001	0005
20	1	0.1821	6415	8784	9612	9885	9968	9992	9998	1.0000	1.0000	1.0000
	2	0.0169	2642	6083	8244	9308	9757	9924	9979	9995	9999	1.0000
	3	0.0010	0755	3231	5951	7939	9087	9645	9879	9964	9991	9998
	4		0159	1330	3523	5886	7748	8929	9556	9840	9951	9987
	5		0026	0432	1702	3704	5852	7625	8818	9490	9811	9941
	6		0003	0113	0673	1958	3828	5836	7546	8744	9447	9793
	7			0024	0219	0867	2142	3920	5834	7500	8701	9423
	8			0004	0059	0321	1018	2277	3990	5841	7480	8684
	9			0001	0013	0100	0409	1133	2376	4044	5857	7483
	10				0002	0026	0139	0480	1218	2447	4086	5881
	11					0006	0039	0171	0532	1275	2493	4119
	12					0001	0009	0051	0196	0565	1308	2517
	13						0002	0013	0060	0210	0580	1316
	14							0003	0015	0065	0214	0577
	15								0003	0016	0064	0207
	16									0003	0015	0059
	17										0003	0013
	18											0002

To obtain $p(i \le r)$ use: $p(i \le r) = 1 - p(i \ge r+1)$

Where a space in the table is empty the probability is less than 0.00005.