JINJA JOINT EXAMINATIONS BOARD

MOCK EXAMINATIONS 2022

P425/1 MATHEMATICS

MARKING GUIDE

1. $$(1+3x)^{\frac{1}{3}} = 1 + \frac{1}{3}(3x) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)(3x)^2}{2!} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)(3x)^3}{3!}$$ M1

$$= 1 + x - x^2 + \frac{5}{3}x^3 + ---$$ A1

$$x = \frac{1}{125}$$

$$\therefore \left(1+\frac{3}{125}\right)^{\frac{1}{3}} = 1 + \frac{1}{125} - \left(\frac{1}{125}\right)^2 + \frac{5}{3}\left(\frac{1}{125}\right)^3 ---$$ M1

$$\left(\frac{64\times 2}{5^3}\right)^{\frac{1}{3}} = 1.007936853$$

$$\sqrt[3]{2} = \frac{5}{4}(1.007936853)$$ M1

$180° +$

$$= 1.25992$$

$$= 1.26$$ A1

05

2. $$\cot\theta + \tan\theta = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$ ~~B1~~

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$\therefore \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin^2\theta}$$ M1

$\sin^2\theta - 2\sin\theta\cos\theta = 0$

$\sin\theta(\sin\theta - 2\cos\theta) = 0$

~~$\sin\theta(\sin\theta - 2\cos\theta) = 0$~~ $\frac{1}{\sin\theta}\left(\frac{1}{\cos\theta} - \frac{2}{\sin\theta}\right) = 0$ ~~B1~~ M1

~~$\sin\theta = 0$ and $\sin\theta - 2\cos\theta = 0$~~ $\frac{1}{\sin\theta} \neq 0$ B1 M1

$\tan\theta = 2$ and $\tan\theta = 2$ M1

Accept $63.4°$ and $243.4°$ A1 (1 dp)

1

$\cot\theta + \tan\theta = 2\,\text{cosec}^2\theta$

$\cot\theta + \frac{1}{\cot\theta} = 2(1+\cot^2\theta)$ M1

$\cot^2\theta + 1 = 2\cot\theta(1+\cot^2\theta)$

$(\cot^2\theta + 1)(1 - 2\cot\theta) = 0$ M1

M1 $\tan\theta = 2$ and $\cot^2\theta \neq -1$ B1 $\quad \theta = 0^0, 180^0, 360^0$, and $63.43^0, 243.43^0$ A1

$\theta = 63.43^\circ, 243.43^\circ \quad \theta = 0^0, 63.43^0, 180^0, 243.43^0, 360^0$ A1

A1

05

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3. $\int \sin(\sqrt{x})dx$

Let $t = \sqrt{x}$

$\frac{dt}{dx} = \frac{1}{2}x^{\frac{-1}{2}}$ B1

$\therefore \int \sin(\sqrt{x})\,dx = \int \sin t \,.\, 2\,t\,dt$ M1

Let $u = t \qquad , \qquad 2 = \int \sin t\ dt$

$\frac{du}{ut} = 1 \qquad\qquad 2 = -\cos t$ ~~B1~~ B1

$\therefore 2\int t\ \sin t\,dt = [t\ \cos t + \int \cos t\,dt] \times 2$ M1

$= -2t\ \cos t + 2\sin t + C.$ M1

$\Rightarrow \int \sin(\sqrt{x})dx = -2(\sqrt{x})\cos(\sqrt{x}) + 2\sin(\sqrt{x}) + C$ A1

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4. From $4x - 3y + 5 = 0$; $\quad y = \frac{4}{3}x + \frac{5}{3}$

; $\quad M_1 = \frac{4}{3}$ B1

Using $\tan\theta = \frac{M_1 - M_2}{1 + M_1 M_2}$

$-1\left(\frac{3+4m_2}{3}\right) = \frac{4-3m_2}{3}$ $\quad \tan 135^0 = \frac{\frac{4}{3} - m_2}{1 + \frac{4}{3}(m_2)}$ M1

$-3 - 4m_2 = 4 - 3m_2$, $\quad -1\left(1 + \frac{4}{3}\ m_2\right) = \frac{4}{3} - m_2$

$-m_2 = 7$

$m_2 = -7$ $\qquad m_2 = -7$

At point (2, 3) A1

$y - 3 = -7x + 14$ $\qquad \frac{y-3}{x-2} = -7$ M1

$7x + y - 17 = 0$ $\qquad$ ~~$y = -3x + 17$~~ 2

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$ ..   From eqn; $m_2 = 4/3$.

$\tan 135 = \dfrac{m_1 - 4/3}{1 + 4/3 m_1}$ M1 $m_1 = \frac{1}{7}$ A1

$-1(1 + 4/3 m_1) = m_1 - 4/3$ $\dfrac{y-3}{x-2} = \frac{1}{7}$ M1

~~$y = 3x + 17$~~   A1 $x - 7y + 19 = 0$ A1

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5. $n = \begin{pmatrix}2\\6\\5\end{pmatrix}$ and the vectors on the plane are   If not corrected

$$b = \begin{pmatrix}2\\1\\-2\end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix}1\\-2\\2\end{pmatrix}$$ ~~B1~~

$r = \begin{pmatrix}1\\1\\0\end{pmatrix} + s\begin{pmatrix}2\\1\\-2\end{pmatrix} + t\begin{pmatrix}1\\-2\\2\end{pmatrix}$

If n is normal to plane, then

$n.b = 0$ and $n.c = 0$

$n = 2\underline{i} + b\underline{j} + 5\underline{k}$

$\therefore n.b = \begin{pmatrix}2\\6\\5\end{pmatrix} \cdot \begin{pmatrix}2\\1\\-2\end{pmatrix} = 4 + 6 - 10 = 0$ M1 ~~A1~~

$\underline{n}\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix} = \underline{n}\cdot\begin{pmatrix}1\\-2\\2\end{pmatrix} = 0$

$n.c = \begin{pmatrix}2\\6\\5\end{pmatrix} \cdot \begin{pmatrix}1\\-2\\2\end{pmatrix} = 2 - 12 + 10 = 0$ ~~M1~~ A1

$\begin{pmatrix}2\\b\\5\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix} = \begin{pmatrix}2\\b\\5\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\2\end{pmatrix} = 0$ M1

$\therefore n$ is a normal vector the plane. B1

$4 + b - 10 = 2 - 2b + 10 = 0$

$b = 6$ A1

Equation of the plane is $2x + 6y + 5z + \cancel{d} = 0$ $\begin{pmatrix}2\\6\\5\end{pmatrix}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}$ M1

$\Rightarrow 2x + 6y + 5z = 8$ A1

It is perpendicular and normal to the plane B1

Or $r.\begin{pmatrix}2\\6\\5\end{pmatrix} = 8$

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6   $\log_2 x + \log_2 y = \log_2 xy = 3$

$\therefore xy = 8$ ------ (i) B1

$\log_2 x + \log_2 y = 3$ $\log_4 x - \log_4 y = \log_4\left(\frac{x}{y}\right) = \frac{-1}{2}$

$\log_4 x - \log_4 y = -\frac{1}{2}$ $\frac{x}{y} = 4^{\frac{-1}{2}}$ B1

$\frac{1}{2}\log_2 x - \frac{1}{2}\log_2 y = -\frac{1}{2}$ $y = 2x$ -------- (ii)

$\therefore 2x^2 = 8$ ~~B1~~ M1

let $p = \log_2 x$, $q = \log_2 y$

$p + q = 3$ B1

$-p - q = -1$ B1

$2p = 2$ M1

$p = 1$

$q = 2$

$x = 2^1 = 2$ A1

$y = 2^2 = 4$ A1

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$x = \pm 2\,;\quad x = -2$ discard

$\therefore x = 2$ A1

And $2y = 8$

$y = 4$ A1

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7. $x^2 + 4xy + 3y^2 = 5$

Differentiating the equation with respect to x,

$2x + 4y + 4x\dfrac{dy}{dx} + 6y\dfrac{dy}{dx} = 0$ M1

$(4x + 4y)\dfrac{dy}{dx} = (2x + 4y)$

$\therefore \dfrac{dy}{dx} = -\dfrac{x+2y}{2x+3y}$ B1

$\dfrac{d^2y}{dx^2} = \dfrac{-(2x+3y)\left(1+2\frac{dy}{dx}\right) - (x+2y)\left(2+3\frac{dy}{dx}\right)}{(2x+3y)^2}$ M1

$= -\dfrac{-y+x\frac{dy}{dx}}{(2x+3y)^2}$

$= \dfrac{y-x\left(-\frac{x+2y}{2x+3y}\right)}{(2x+3y)^2}$ M1

$= \dfrac{x^2+4xy+3y^2}{(2x+3y)^2}$

$= \dfrac{5}{(2x+3y)^3}$ B1

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8. Gradient of line $x - 2y - 4 = 0$ is $\frac{1}{2}$. B1

For $(y-2)^2 = x\,;$

$2\,(y-2)\dfrac{dy}{dx} = 1$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{2\,(y-2)}$ B1

Let $A\,(x_1, y_1)$ be the point contact of the required tangent.

$\Rightarrow \dfrac{1}{2\,(y_1-2)} = \dfrac{1}{2}$ M1

$y_1 = 3$

$\therefore (3-2)^2 = x_1$

$x_1 = 1$

Hence equation of tangent is

$y - 3 = \frac{1}{2}(x - 1)$ M1

$x - 2y + 5 = 0$ A1

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SECTION B

9. (a) let $a + ib$ be the square root of $15 + 8i$ ~~M1~~

$\Rightarrow (a + ib)^2 = 15 + 8i$

$(a^2 - b^2) + 2abi = 15 + 8i$ M1

Reals:

$\therefore\ a^2 - b^2 = 15$ -------(1) B1

$Im : 2ab = 8$ ---------(2) ~~B1~~

$a = \frac{4}{b}$

$\therefore \frac{16}{b^2} - b^2 = 15$ M1

$b^4 + 15b^2 - 16 = 0$

$(b^2 + 16)(b^2 - 1) = 0$ M1

$\therefore b^2 = -16$ or $b^2 = 1$

but $b^2 \neq -16$ since $b \neq R$

so $b^2 = 1 \Rightarrow b = \pm 1$

and $a = \pm 4$

A1 A1

$\therefore$ The square roots are $4 + i$ and $-4 - i$

$z^2 = 15 + 8i$

$|z^2| = \sqrt{15^2 + 8^2}$ B1

$= \sqrt{289} = 17$

$\arg(z^2) = \tan^{-1}\left(\frac{8}{15}\right)$

$= \underline{28.07^\circ} + 360k$ B1

$z^2 = r\{\cos(\theta + 2\pi) + i\sin(\theta + 2\pi)\}$

$z^2 = 17\{\cos(28.07^\circ + 360k) + i\sin(28.07 + 360k)\}$ M1

$z = \sqrt{17}\{\cos(14.04 + 180k) + i\sin(14.04 + 180k)\}$ M1

where $k = 0, 1, 2, \ldots$

when $k = 0$

$z = \sqrt{17}\{\cos 14.04 + i\sin 14.04\}$

$z = \underline{4 + i}$ A1

when $k = 1$

$z = \sqrt{17}\{\cos 194.04 + i\sin 194.04\}$

$= \underline{-4 - i}$ A1

(b) If $Z = x + iy$

$|z - 3| = |(x - 3) + iy| = 3$

$= \sqrt{(x - 3)^2 + y^2} = 3$ M1

$: (x - 3)^2 + y^2 = 9$ A1

Locus of C1 is a circle centre $(3, 0)$ and radius 3. B1

And $\text{Arg}(z - 1) = \frac{\pi}{4}$, is a half line starting at point $(1, 0)$ on the real

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$\arg(z-1) = \pi/4$.

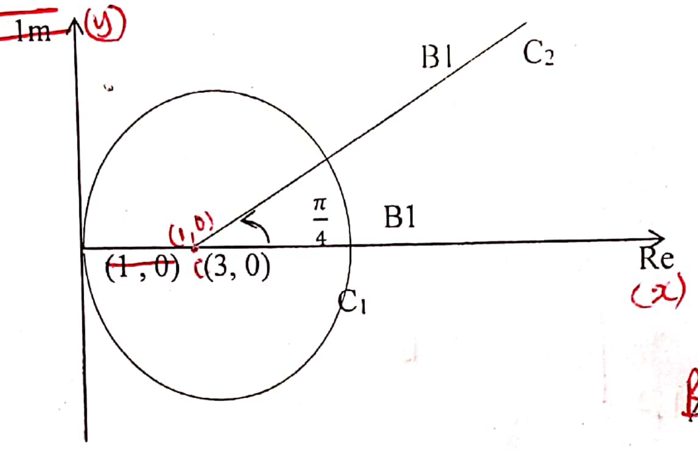
$\arg(x+iy-1) = \pi/4$.

$\arg(x-1+iy) = \pi/4$.

$\frac{y}{x-1} = \tan(\pi/4)$ axis making an angle of $\frac{\pi}{4}$ **B1**

$\frac{y}{x-1} = 1$

$y = x-1$



~~A1~~ B1 – Axes.

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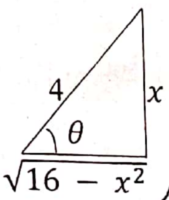
10. (a)

Let $x = 4\sin\theta$ B1

$dx = 4\cos\theta\, d\theta$



M1

$$\therefore \int \frac{\sqrt{16-x^2}}{x^2}\, dx = \int \frac{(16-16\sin^2\theta)^{\frac{1}{2}}}{16\sin^2\theta} . \cancel{4100\theta}\ 4\cos\theta\, d\theta$$ **M1**

$$= \int \cot^2\theta\, d\theta$$

$$= \int (\operatorname{cosec}^2\theta - 1)\, d\theta$$ **B1**

$$= -\cot\theta - \theta + c$$ **M1**

$$= -\sqrt{\frac{16-x^2}{x}} - \sin^{-1}\left(\frac{x}{4}\right) + c \qquad \text{A1}$$

(b)

$x = 2\sin t$

$dx = 2\cos t\, dt$

x	t
1	$\frac{\pi}{6}$
$\sqrt{3}$	$\frac{\pi}{3}$

B1 M1

$$\therefore \int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\sin t + 3}{\sqrt{4-4\sin^2 t}} \cdot 2\cos t\, dt \qquad \text{M1}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (2\sin t + 3)\, dt$$

$$= [-2\cos t + 3t]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \qquad \text{A1}$$

$$= \left[\left(-2\cos\frac{\pi}{3} + 3\left(\frac{\pi}{3}\right)\right)\right] - \left[-2\cos\frac{\pi}{6} + 3\left(\frac{\pi}{6}\right)\right] \qquad \text{M1}$$

$$= \left[\left(-2\times\frac{1}{2} + \pi\right) + \sqrt{3} - \frac{\pi}{2}\right]$$

$$= \frac{\pi}{2} + \sqrt{3} - 1 \qquad \text{B1}$$

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11. $\tan(\theta + 60^0)\ \tan(\theta - 60^0) = \underset{M1}{\left(\frac{\tan\theta + \tan 60^0}{1 - \tan\theta\tan 60^0}\right)} \underset{M1}{\left(\frac{\tan\theta - \tan 60^0}{1 + \tan\theta\tan 60^0}\right)}$

$$= \frac{tan^2\theta - (\sqrt{3})^2}{(1)^2 - (\sqrt{3}\ \tan\theta)^2} \quad \text{M1}$$

$$= \frac{tan^2\theta\ - 3}{1 - 3\ tan^2\theta} \quad \text{B1}$$

For $\tan(\theta + 60^0)\ \tan(\theta - 60^0) = 4sec^2\theta - 3$.

$$\frac{tan^2\theta - 3}{1 - 3\ tan^2\theta} = 4sec^2\theta\ - 3 \quad \text{M1}$$

$\frac{\tan^2\theta - 3}{1 - 3\tan^2\theta} = 4\sec^2\theta - 3$ M1

$$\frac{sec^2\theta - 4}{4 - sec^2\theta} = 4sec^2\theta\ - 3 \quad \text{M1}$$

$$3sec^4\theta\ - 6sec^2\theta\ + 2\ = 0 \quad \text{B1}$$

$\frac{\tan^2\theta - 3}{1 - 3\tan^2\theta} = 4(1 + \tan^2\theta) - 3$

$$sec^2\theta\ = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3} \quad \text{M1}$$

$\frac{\tan^2\theta - 3}{1 - 3\tan^2\theta} = 4 - 3 + 4\tan^2\theta$ M1

$$= 1.5774\ \text{ or } 0.4226$$

$$\therefore sec\theta\ = \ \pm 1.2559\ \text{ or } sec\theta\ = \ \pm 0.6501$$

$12\tan^4\theta - 4 = 0$

$\cos\theta\ = \ \pm 0.7962$ **B1** or $\cos\theta = \ \pm 1.5382$ B1
Discard

$\tan^4\theta - 1 = 0$ B1

$\tan^4\theta = \frac{1}{3}$ B1

$\theta\ = 37.~~32~~23^0,\ 142.~~73~~77^0,\ 217.23^0,\ 322.~~74~~77^0$ M1 A1

$\theta = \tan^{-1}\left(\sqrt[4]{\frac{1}{3}}\right) = 37.23^0$ M1 (correct to 1dp) ✓

$\theta = \{37.23^0, 142.77^0, 217.23^0, 322.77^0\}$ A1

12. (a) direction vector for L_1, $d_1 = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$ ~~B1~~

Direction vector of x-axis $d_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ B1 For both

$$\therefore \cos\theta = \frac{\begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\sqrt{2^2 + 2^2 + (-3)^2} \circ \sqrt{1^2 + 0^2 + 0^2}}$$ M1 ~~M1~~

$\frac{2}{\sqrt{17} \times \sqrt{1}}$ ~~Cosθ~~ $= \frac{2}{\sqrt{17}}$ M1 $\theta = \cos^{-1}(0.48507127)$

$$\theta = 60.98^0$$ A1

(b) (i) for L_2 : $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & + & as \\ 6 & + & 4s \\ 2 & + & 9s \end{pmatrix}$

At A : $\begin{pmatrix} 2 \\ -2 \\ b \end{pmatrix} = \begin{pmatrix} 4 & + & as \\ 6 & + & 4s \\ 2 & + & 9s \end{pmatrix}$ M1

Taking coefficient of **j**;

$-2 = 6 + 4s$

$s = -2$ ~~B1~~

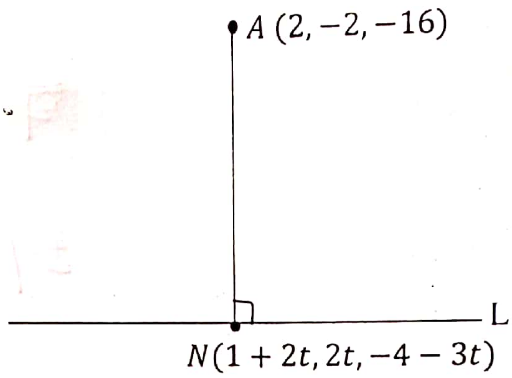
For $\boldsymbol{l}$: $2 = 4 + a\,(-2)$

$a = +1$ A1

For $\boldsymbol{k}$; $\quad b = 2 + 9(-2)$

$\qquad = -16$ **A1**

(b) (ii)



$d = \dfrac{|AB \times b|}{|b|}$

$$\overrightarrow{b} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$$

$$\mathbf{AN} = \begin{pmatrix} 1+2t \\ 2t \\ -4-3t \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ -16 \end{pmatrix} = \begin{pmatrix} -1 & + & 2t \\ 2 & + & 2t \\ 12 & - & 3t \end{pmatrix} \qquad \textbf{B1}$$

AN . b = 0

$$\begin{pmatrix} -1 & + & 2t \\ 2 & + & 2t \\ 12 & - & 3t \end{pmatrix} . \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} = 0 \qquad \textbf{M1}$$

$$17t = 34$$

$$t = 2 \qquad \textbf{B1}$$

$$\therefore AN = \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix}$$

$$|AN| = \sqrt{(3)^2 + 6^2 + 6^2} \qquad \textbf{M1}$$

$$d = \frac{|\overline{AB} \times \underline{b}|}{|\underline{b}|} \qquad \underline{b} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \qquad \overline{AB} \times \underline{b} = \begin{vmatrix} i & j & k \\ -1 & 2 & 12 \\ 2 & 2 & -3 \end{vmatrix}$$

$\overline{AB} = \cancel{(1-2)}\ \overline{OB} - \overline{OA}$ $\qquad A = (2, -2, -16)$

$= \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ -16 \end{pmatrix}$ M1 $\qquad B = (1, 0, -4)$

$= \begin{pmatrix} -1 \\ 2 \\ 12 \end{pmatrix}$

M1 $= \dfrac{|-30i + 21j + 6k|}{\sqrt{(2)^2 + (2)^2 + (-3)^2}}$

= 9 units A1

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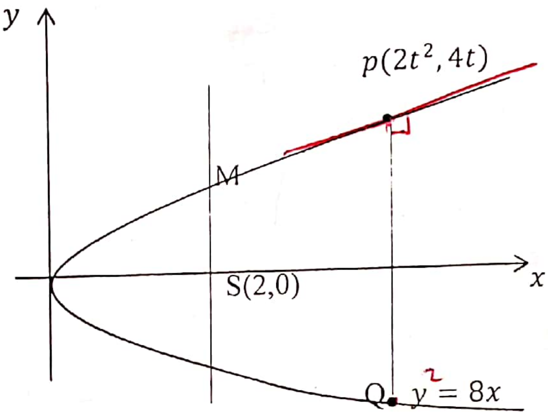
$\dfrac{\sqrt{1377}}{\sqrt{17}}$ M1

$\sqrt{81} = 9$ A1

13.



(a) solving simultaneously equation of normal and curve.

$y + t\left(\frac{y^2}{8}\right) = 4t + 2t^3$ M1

$\Rightarrow ty^2 + 8y - 32t - 16t^3 = 0$ B1

Let y_1 and y_2 be the roots of the quadratic equation;

$\Rightarrow y_1 + y_2 = \frac{-8}{t}$

$\therefore 4t + y_2 = \frac{-8}{t}$ M1

$y_2 = \frac{-8}{t} - 4t$

From $y^2 = 8x$

$x = \dfrac{\left(\frac{-8}{t} - 4t\right)^2}{8}$

$= \dfrac{(t^2+2)^2}{t^2}$

$\therefore Q\left[\dfrac{(t^2+2)^2}{t^2}, -\left(\dfrac{8+4t^2}{t}\right)\right]$ B1

$$\therefore \overline{PQ}^2 = \left[2t^2 - \frac{(t^2+2)^2}{t^2}\right]^2 + \left[4t + \frac{8+4t^2}{t}\right]^2$$ M1

$$= \left[\frac{(8+8t^2)^2}{t^4}\right] + \left[\frac{(8+8t^2)^2}{t^2}\right]$$

$$= (8+8t^2)^2 \left[\frac{1}{t^4} + \frac{1}{t^2}\right]$$

$$= (8+8t^2)^2 \cdot t^2 \frac{(1+t^2)}{t^4 \cdot t^2}$$

$$\overline{PQ} = \frac{8(1+t^2)\cdot(1+t^2)^{\frac{1}{2}}}{t^2}$$ M1

$$\therefore \overline{PQ} = \frac{8\,(1+t^2)^{\frac{3}{2}}}{t^2}$$ B1

(b) grad of normal is $-t$

$\therefore$ equation of line SM;

$$\frac{y-0}{x-2} = -t$$

$$y = -xt + 2t \text{ ------}(1)$$ B1 ~~B1~~ ~~B1~~

But equation of tangent at P is

$$y = \frac{1}{t}x + 2t \text{ --------}(2)$$

At M, 1 = 2

$$-xt - \frac{1}{t}x = 0$$

$$\therefore x\left(t + \frac{1}{t}\right) = 0$$

$$\therefore x = 0 \text{ since } \left(t + \frac{1}{t}\right) \neq 0$$

$$\Rightarrow y = 2t$$

$\therefore M\,(0, 2t)$ B1

$$\overline{SM} = \sqrt{(0-2)^2 + (2t - 0)}$$

$$= 2\,(1 \times t^2)^{\frac{1}{2}}$$ B1

But $5\,\overline{SM} = \overline{PQ}$

$$\Rightarrow 5 \times 2 (1 + t^2)^{\frac{1}{2}} = \frac{8(1+t^2)^{\frac{3}{2}}}{t^2}$$ M1

$$t^2 = 4$$ B1

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4. (a) $\left.\begin{array}{l} \propto + \beta = -p \\ \propto \beta = q \end{array}\right\}$ B1

(i) $\propto^3 + \beta^3 = (\propto + \beta)(\propto^2 - \propto p + \beta^2)$ ~~M1~~

$= (\propto + \beta)[(\propto + \beta)^2 - 3 \propto \beta]$

$= -p[(-p)^2 - 3q]$ M1

$= 3pq - p^3$ A1

(ii) $(\propto - \beta^2)(\beta - \propto^2) = \propto \beta - \propto^3 + \propto^2 \beta^2$

$= q - [3pq - p^3] + q^2$ M1

$= q^2 + q + p^3 - 3pq$ A1

(iii) If $\propto = \beta^2$

$\Rightarrow (\beta^2 - \beta^2)(\beta - \beta^4) = q^2 + q + p^3 - 3pq$

$\therefore p^3 - 3pq + q^2 + q = 0$ B1

(b) $\propto + \propto r + \propto r^2 = 8400$; $\propto r^2 = 4800$ B1

$\frac{\propto(1 + r + r^2)}{\propto r^2} = \frac{8400}{4800}$ dividing the equation M1

$4(1 + r + r^2) = 7r^2$

$3r^2 - 4r - 4 = 0$

$(3r + 2)(r - 2) = 0$ M1

$\therefore r = \frac{-2}{3}$ or $2.$

But $r > 0$;

$\therefore r = 2$ B1

From $\propto r^2 = 4800$

$\propto = \frac{4800}{4}$

$= 1200$

$\therefore \propto r = 2 \times 1200$ A1

$= 2400$

$\therefore$ the prices of other items are 1200/= and shs 2400/= A1

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15 (a) Given the equation $x^3 + y^3 = 3xy$

Differentiating both sides with respect to x,

$3x^2 + 3y^2 \frac{dy}{dx} = 3x\frac{dy}{dx} + 3y\ (1)$ M1 ~~M1~~

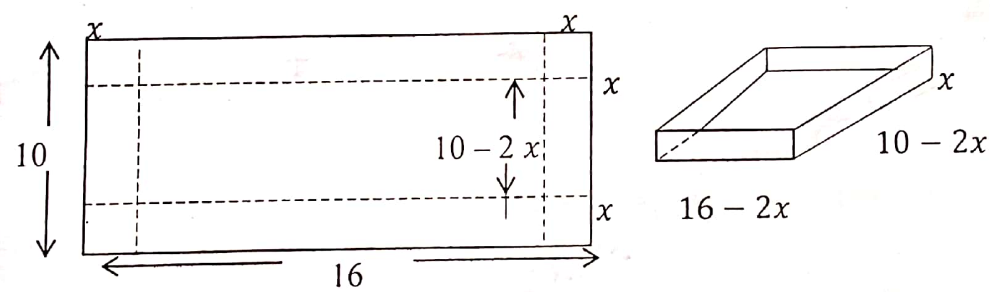
$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$ M1 A1

At point $\left(\frac{3}{2}, \frac{3}{2}\right)$,

Gradient, $\frac{dy}{dx} = \frac{\frac{3}{2} - \left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2 - \frac{3}{2}}$ M1

$= -1$ A1

(b)



$\therefore$ volume, $v = x(10 - 2x)(16 - 2x)$ M1

$= 160x - 52x^2 + 4x^3$

$\frac{dv}{dx} = 160 - 104x + 12x^2$ M1

$\therefore 160 - 104x + 12x^2 = 0$ M1

$(3x - 20)\ (x - 2) = 0$ M1

$: x = \frac{20}{3}$ or $x = 2$ A1

The length of the side of the square must be less than half the side of the rectangle

$\therefore x = 6\frac{2}{3}$ is not a possible solution.

$\Rightarrow x = 2$ B1

For maximum value. $\frac{d^2v}{dx^2} < 0$

$$\frac{d^2v}{dx^2} = -104 + 24x$$

$$= -104 + 24x$$

$$= -56 < 0$$ ~~B1~~

$$\therefore x = 2$$ ~~A1~~

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16. (a) $\frac{dy}{dx} - \frac{2}{x}y = x^2 Inx, x > 0$

$R = e^{\int \frac{2}{x}dx} = e^{-2Inx} = x^{-2}$ B1

$\therefore x^{-2}\frac{dy}{dx} - 2x^{-3}y = Inx$

$x^{-2}y = \int Inx\, dx$

Let $u = Inx$ $\quad \frac{dv}{dx} = \int Idx$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = x$

$\int Inx\, dx = x\, Inx - \int x.\frac{1}{x}dx$ M1

$= xInx - x + c$

$\therefore x^{-2}y = x\,(Inx - 1) + c$ A1

When $x = 1$ and $y = 2$

$\Rightarrow 2 = -1 + c$ B1

$c = 3$

$\therefore y = x^3(Inx - 1) + 3x^2$ A1

(b) (i) $\frac{dT}{dt}\ \alpha\ (T - \theta)$

$\frac{dT}{dt} = -K(T - \theta)$ B1

(ii) $\therefore \int \frac{1}{T-\theta}\, dT = -K\int dt$

$In\,(T - \theta) = -Kt + c$ A1

$$T - \theta = Ae^{-kt}$$

$$T = \theta e^{-kt}$$ B1

At $t = 0s$, $T = 100^0C$; $t = 600s, T = 84^0C, \theta = 25^0C$

$$\therefore 100 = 21 + Ae^{-k(10)}$$

$$A = 79$$ B1

And $84 = 21 + 79\, e^{-600k}$

$$K = 0.00038$$ B1

$$\therefore T = 21 + 79e^{-0.00038t}$$ A1

When $t = 21$ minutes $= 1260sec.$

(iii) $\Rightarrow T = 21 + 79e^{-0.00038(1260)}$ M1

$$\therefore T = 70^0C$$ A1

	12

END

P425/1
PURE MATHEMATICS
AUGUST - 2022
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS – AUGUST, 2022

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all the eight** questions in section A and any **five from** section **B**.*

*Any additional question(s) will **not** be marked.*

*All working **must** be shown clearly.*

Begin each question on a fresh sheet of paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. By using the Binomial theorem, expand $(1+3x)^{\frac{1}{3}}$ up to the fourth term. Hence by substituting $x = \frac{1}{125}$, evaluate $\sqrt[3]{2}$ to 3 significant figures. (05 marks)

2. Solve the equation $\cot\theta + \tan\theta = 2\cos ec^2\theta$ for $0^0 \leq \theta \leq 360^0$. (05 marks)

3. Find $\int \sin(\sqrt{x})\, dx$ (05 marks)

4. Find the equation of a line through point (2 , 3) which makes an angle of 135^0 with the line $4x - 3y + 5 = 0$. (05 marks)

5. Show that the vector $2\boldsymbol{i} + b\boldsymbol{j} + 5\boldsymbol{k}$ is normal to the plane
$$\boldsymbol{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + s\begin{pmatrix}2\\1\\-2\end{pmatrix} + t\begin{pmatrix}1\\-2\\2\end{pmatrix}$$
Hence determine the equation of the plane in the form $\boldsymbol{r}.\boldsymbol{n} = \boldsymbol{d}$. (05 marks)

6. Solve the simultaneous equations
$$\log_2 x + \log_2 y = 3$$
$$\log_4 x - \log_4 y = -\frac{1}{2}$$
(05 marks)

7. Given that $x^2 + 4xy + 3y^2 = 5$, show that $\frac{d^2y}{dx^2} = \frac{5}{(2x+3y)^3}$ (05 marks)

8. Find the equation of the tangent to the curve $(y-2)^2 = x$ which is parallel to the line $x - 2y - 4 = 0$. (05 marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks

9. (a) Calculate the square roots of $15 + 8i$ (06 marks)

   (b) The loci of C_1 and C_2 are given by $|z-3| = 3$ and $\text{Arg}\ (z-1) = \frac{\pi}{4}$ sketch on the same argand diagram the loci of C_1 and C_2. (06 marks)

10. (a) Integrate with respect to x
$$\frac{\sqrt{16-x^2}}{x^2}$$
(06 marks)

    (b) By using substitution $x = 2sint$, show that
$$\int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}}\, dx = \frac{\pi}{2} + \sqrt{3} - 1$$
(06 marks)

11. Prove that $\tan(\theta + 60^0)\ \tan(\theta - 60^0) = \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta}$. Hence solve the

equation $\tan(\theta + 60^0)\ \tan(\theta - 60^0) = 4\sec^2\theta - 3$, for $0^0 < \theta < 360^0$ (12 marks)

12. The lines L_1 and L_2 have equations.

$L_1: \boldsymbol{r} = (1 + 2t)\boldsymbol{i} + 2t\boldsymbol{j} - (4 + 3t)\boldsymbol{k}$

$L_2: \boldsymbol{r} = (4 + as)\boldsymbol{i} + (6 + 4s)\boldsymbol{j} + (2 + 9s)\boldsymbol{k}$

Respectively, where a is a constant

(a) Find the acute angle between L_1 and the $x - axis$. (05 marks)

(b) Given that point A has position vector $2\boldsymbol{i} - 2\boldsymbol{j} + b\boldsymbol{k}$ and that the line L_2 passes through point A, determine the

(i) values of a and b

(ii) perpendicular distance of A from the line L_1. (07 marks)

13. (a) The equation of the normal to the parabola $y^2 = 8x$ at the point $P(2t^2, 4t)$, $(t \neq 0)$ is given by $y + xt = 4t + 2t^3$.

Given that this normal meets the curve again at Q, show that the length of

$$\overline{PQ} = \frac{8\left(1 + t^2\right)^{\frac{3}{2}}}{t^2}$$ (07 marks)

(b) The line through the point of focus, S (2, 0) parallel to PQ meets the tangent at P to the parabola at point M. Given that 5SM = PQ, prove that $t^2 = 4$. (05 marks)

14. (a) If α and β are the roots of the equation $x^2 + Px + q = 0$; Express

(i) $\alpha^3 + \beta^3$ and

(ii) $(\alpha - \beta^2)(\beta - \alpha^2)$, in terms of p and q. Deduce that the condition for one root of the equation to be the square of the other is

$p^3 - 3pq + q^2 + q = 0$ (06 marks)

(b) The prices of three items are in a Geometric progression (G.P). If the total prices for these three items is shs 8400 and the most expensive item priced at shs 4800, find the prices of the other two items. (06 marks)

15. (a) The equation of the curve is given by $x^3 + y^3 = 3xy$

Find the gradient of the tangent to the curve at $\left(\frac{3}{2}, \frac{3}{2}\right)$ (05 marks)

(b) An open box is to be made from a rectangular sheet measuring 16cm by 10cm by cutting squares of side $x$cm from each corner and turning up the edges. Calculate the value of x, so that the volume of the box is maximum. (07 marks)

16. (a) Solve the differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \, Inx \;, \;\; x > 0 \text{ given that } y = 2 \text{ and } x = 1$$ (05 marks)

(b) At 2.23pm, the temperature of water in a kettle boiled at 100^0C and that of the surrounding 21^0C. At 2.33pm the temperature of water in the kettle had dropped to 84^0C. If the rate of cooling of the water was directly proportional to the difference between its temperature θ and that of the surroundings,

(i) write a differential equation to represent the rate of cooling of water in the kettle. (01 mark)

(ii) solve the differential equation using the given conditions. (04 marks)

(iii) find the temperature of the water at 2.44pm (02 marks)