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MARKING GUIDE, P425/2, APPLIED MATHEMATICS

(1)

$P(T) = \frac{2}{3}$		$P(\bar{T}) = \frac{1}{3}$	
$P(G) = 0.9$	$P(\bar{G}) = 0.1$	$P(G) = 0.6$	$P(\bar{G}) = 0.4$

(a) $P(G) = \frac{2}{3} \times 0.9 + \frac{1}{3} \times 0.6 = 0.8 = \frac{4}{5}$ (B1)(M1)(A1)

(b) $P(T|G) = \frac{P(T \cap G)}{P(G)} = \frac{\frac{2}{3} \times 0.9}{0.8} = 0.75 = \frac{3}{4}$ (M1)(A1)

(2)

Horizontal motion	Vertical motion
$u_x = 5ms^{-1}$	$u_y = 0ms^{-1}$
$a_x = 0ms^{-2}$	$a_y = 10ms^{-2}$
$s_x = xm$	$s_y = 80m$
$t_x = ts$	$t_y = ts$

(a) ↑ $s = ut + \frac{1}{2}at^2$, $80 = 0 \times t + \frac{1}{2} \times 10 \times t^2$ (B1)

$5t^2 = 80$, $t^2 = 16$ $\therefore t = 4s$. (M1)(A1)

(b) → $x = 5 \times 4 + \frac{1}{2} \times 0 \times (4)^2 = 20m$ (M1)(A1)

(3) let $P = \frac{xy}{z} = \frac{4 \times 16.2}{2.53} = 25.6126$

$P_{max} = \frac{x_{max} \cdot y_{max}}{z_{min}} = \frac{4.5 \times 16.26}{2.523} = 29.0012$ (M1)(B1)

$P_{min} = \frac{x_{min} \cdot y_{min}}{z_{max}} = \frac{3.5 \times 16.14}{2.537} = 22.2665$ (B1)

$\Delta P = \frac{1}{2}(P_{max} - P_{min}) = \frac{1}{2}(29.0012 - 22.2665)$ (M1)

$\Delta P = 3.367345$ (A1)

(4) H : T

2 : 1 = 3

$P(H) = \frac{2}{3} : P(T) = \frac{1}{3}$ (B1), $n = 6$

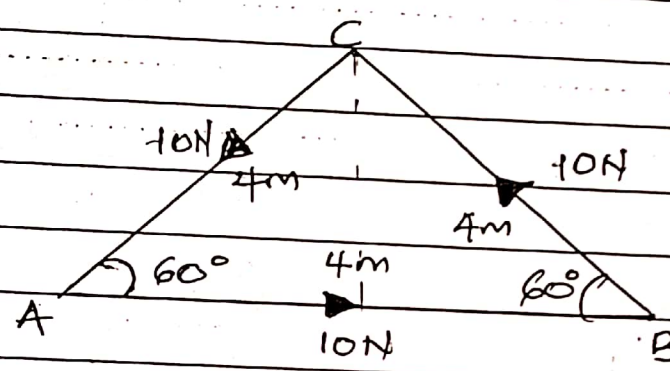
(a) $P(X_H \geq 5) = P(X_H = 5) + P(X_H = 6)$

$= {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0 = \frac{256}{729} = 0.3512$ (M1)

(b) $P(X_T \leq 1) = P(X_T = 1) + P(X_T = 0)$

$= {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 + {}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 = \frac{256}{729} = 0.3512$ (M1)

(5)



(B1)

→ $R_x = 10 - 10\cos 60° - 10\cos 60° = 0$ (M1) $\quad \underline{R} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} N$

↑ $R_y = 10\sin 60° - 10\sin 60° = 0$ (M1)

(A) A.C.M $= 4 \times 10\sin 60° = 20\sqrt{3}$ Nm. (B1)

Since $\underline{R} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} N$ and A.C.M $= 20\sqrt{3}$ Nm $\therefore$ HS a couple. (B1)

(6) (a)

	60	c	40
	150	170	200

(B1)

$\frac{c - 60}{170 - 150} = \frac{40 - 60}{200 - 150}$ (M1)

$c = 52$ (A1)

(b)

	100	90	75
	S	50	100

$\frac{S - 50}{100 - 90} = \frac{50 - 100}{90 - 75}$ (M1)

$S = 16.6667 = \frac{50}{3}$ (A1)

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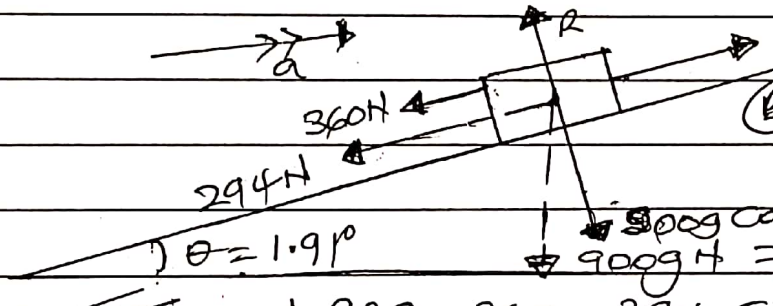
(7)

	x	F(x)	P(X=x)	x P(X=x)	x^2 P(X=x)
	3	0.01	0.01	0.03	0.09
	4	0.23	0.22	0.88	3.52
	5	0.64	0.41	2.05	10.25
	6	0.85	0.21	1.26	7.56
	7	1	0.15	1.05 (M1)	7.35

(B1) ..... E(X) = 5.27 (A1) $E(X^2)$ = 28.77 (B1)

$Var(X) = E(X^2) - (E(X))^2 = 28.77 - (5.27)^2 = 0.997$ (B1)

(8)



$F = \frac{P}{v} = \frac{19.2 \times 10^3}{10}$ (B1)

$= 1,920N$

$1,920 - 360 - 294 = 900a$ (M1) (B1)

$900a = 1,266$ (B1)

$a = \frac{211}{150} ms^{-2} = 1.4067 ms^{-2}$ (A1)

(9) (a) $\int_a^{15} \frac{1}{a(b-a)}\,dx = \frac{1}{4}$

$\left|\frac{x}{b-a}\right|_a^5 = \frac{1}{4}$ (M1), $\frac{5-a}{b-a} = \frac{1}{4}$ (B1), $20 - 4a = b - a$

$3a + b = 20$, (*)

$\int_a^{19} \frac{1}{a(b-a)}\,dx = \frac{3}{4}$

$\left|\frac{x}{b-a}\right|_a^9 = \frac{3}{4}$ (M1), $\frac{9-a}{b-a} = \frac{3}{4}$ (B1), $36 - 4a = 3b - 3a$

$a + 3b = 36$, (**) $\times 3$ $\quad 3a + 9b = 108$, (+)

$3a + b = 20$, (*) $\quad 3a + b = 20$, (+), (-)-(+)

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$$\overrightarrow{AB}(t=t) \cdot {}_A\underset{B}{\gamma} = 0$$

$$\begin{pmatrix} 7.0096t \\ 30-7.5t \end{pmatrix} \cdot \begin{pmatrix} 7.0096 \\ -7.5 \end{pmatrix} = 0 \quad (M_1)$$

$$105.3846t - 225 = 0 \quad (B_1)$$

$t = 2.135\text{hr} \approx 2.14\text{hr}$ (B1) $= 2\text{hrs } 0.14 \times 60\text{min}$, $\therefore t = 14:08\text{hrs}$ (A1)

(b) $\overrightarrow{AB}(t=2.14\text{hr}) = \begin{pmatrix} 7.0096 \times 2.14 \\ 30 - 7.5 \times 2.14 \end{pmatrix} = \begin{pmatrix} 15.0005 \\ 13.95 \end{pmatrix}$ km (B1)

$|\overrightarrow{AB}(t=2.14\text{hr})| = \sqrt{(15.0005)^2 + (13.95)^2} = 20.48\text{ km}$ (M1) (A1)

(11) (a) $h = \frac{6-1}{6} = \frac{5}{6}$ (B1)

x	$f(x) = x\ln x$	$f(x) = x\ln x$
1	0.0000	
$\frac{11}{6}$		1.1112
$\frac{8}{3}$		2.6156
$\frac{7}{2}$		4.3847
$\frac{13}{3}$		6.3544
$\frac{31}{6}$		8.4848
6	10.7506	
Sum (B1)	10.7506 (B1)	22.9504 (B1)

$$\int_1^6 x\ln x\,dx \approx \frac{5}{12}\left(10.7506 + 2\times 22.9504\right) \approx 23.6036 \quad (M_1)$$

≈ 23.6045 (3d.p.s) (A1)

(b) $E.V = \int_1^6 x\ln x\,dx$, $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$

$\frac{dv}{dx} = x$, $v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln x - \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx = \frac{x^2}{4}$

$= \left|\frac{x^2}{2}\ln x - \frac{x^2}{4}\right|_1^6$ (M1) $= (18\ln 6 - 9) - (\frac{1}{2}\ln 1 - \frac{1}{4})$ (M1)

$= 23.5017 \approx 23.502$ (3d.p.s) (A1)

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$A \cdot E = |E \cdot V - A \cdot V| = |23.502 - 23.605| = 0.103$ (B1)

$P \cdot E = \frac{A \cdot E}{E \cdot V} \times 100 = \frac{0.103}{23.502} \times 100 = 0.438$ (B1)

Increase number of strips or Sub-Intervals or ordinates. (B1)

(12)(b)

R_x	R_y	d^2
4	3.5	0.25
9.5	8	2.25
7	3.5	12.25
6	7	1
2.5	5	6.25
9.5	9	0.25
1	1	0
2.5	2	0.25
8	10	4
5	6	1
(B1) Sum	(B1) $\Sigma d^2 = 27.5$	(B1)

$\rho_c = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 27.5}{10 \times 99}$ (M1) $= 0.8333$ (A1)

$\rho_c = 0.8333 > \rho_T = 0.65$, ∴ Significant at 5%. (B1)

(13)(a)

$$\underset{\sim}{v} = \begin{pmatrix} 3t \\ -4t \\ t^2 \end{pmatrix} ms^{-1}$$

$$\underset{\sim}{r}_{(t=2)} = \int \underset{\sim}{v}\, dt = \int \begin{pmatrix} 3t \\ -4t \\ t^2 \end{pmatrix} dt = \begin{pmatrix} \frac{3t^2}{2} \\ -2t^2 \\ \frac{t^3}{3} \end{pmatrix} + \underset{\sim}{c}$$ (M1)

(To be fastened together with other answers to paper)

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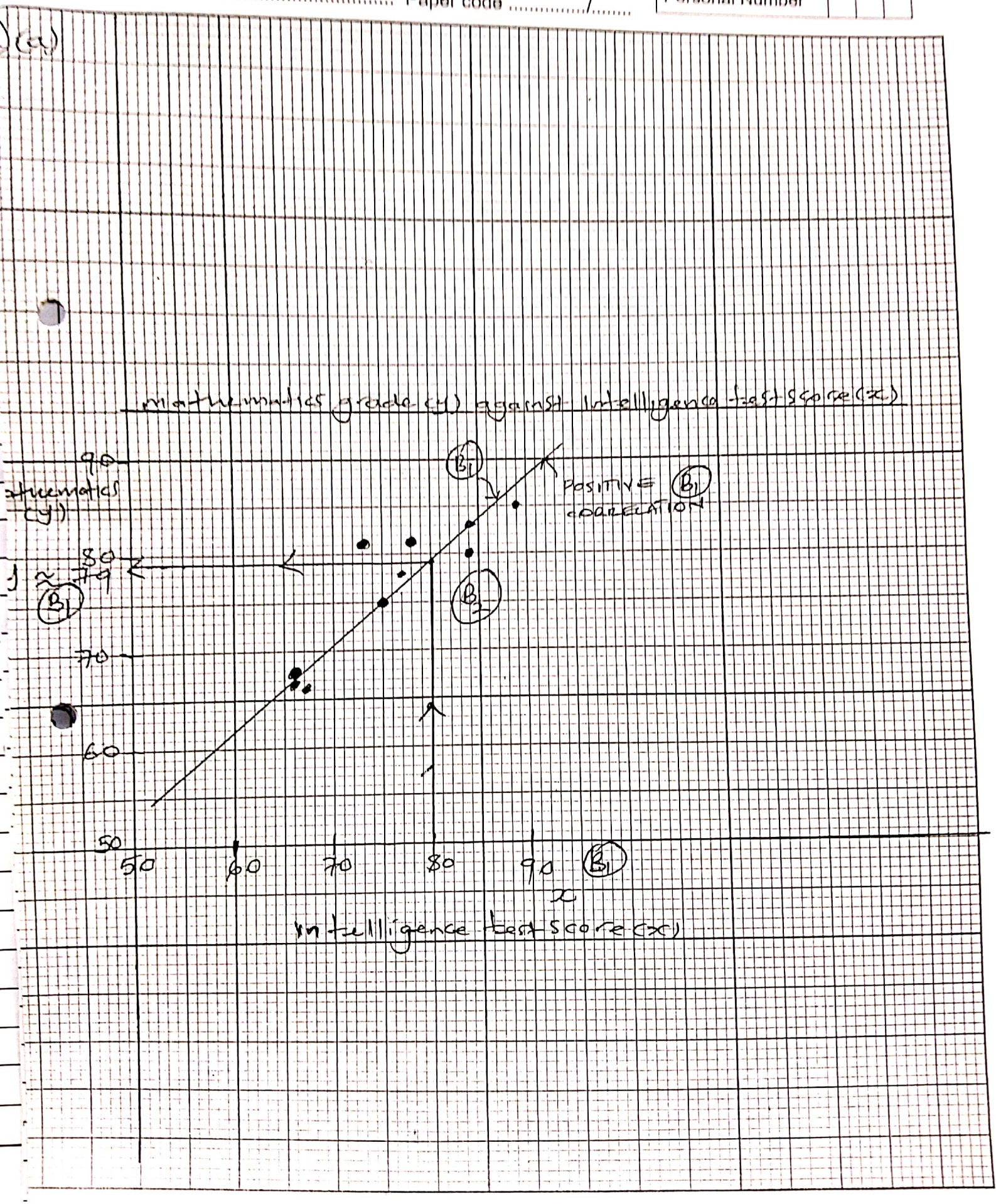
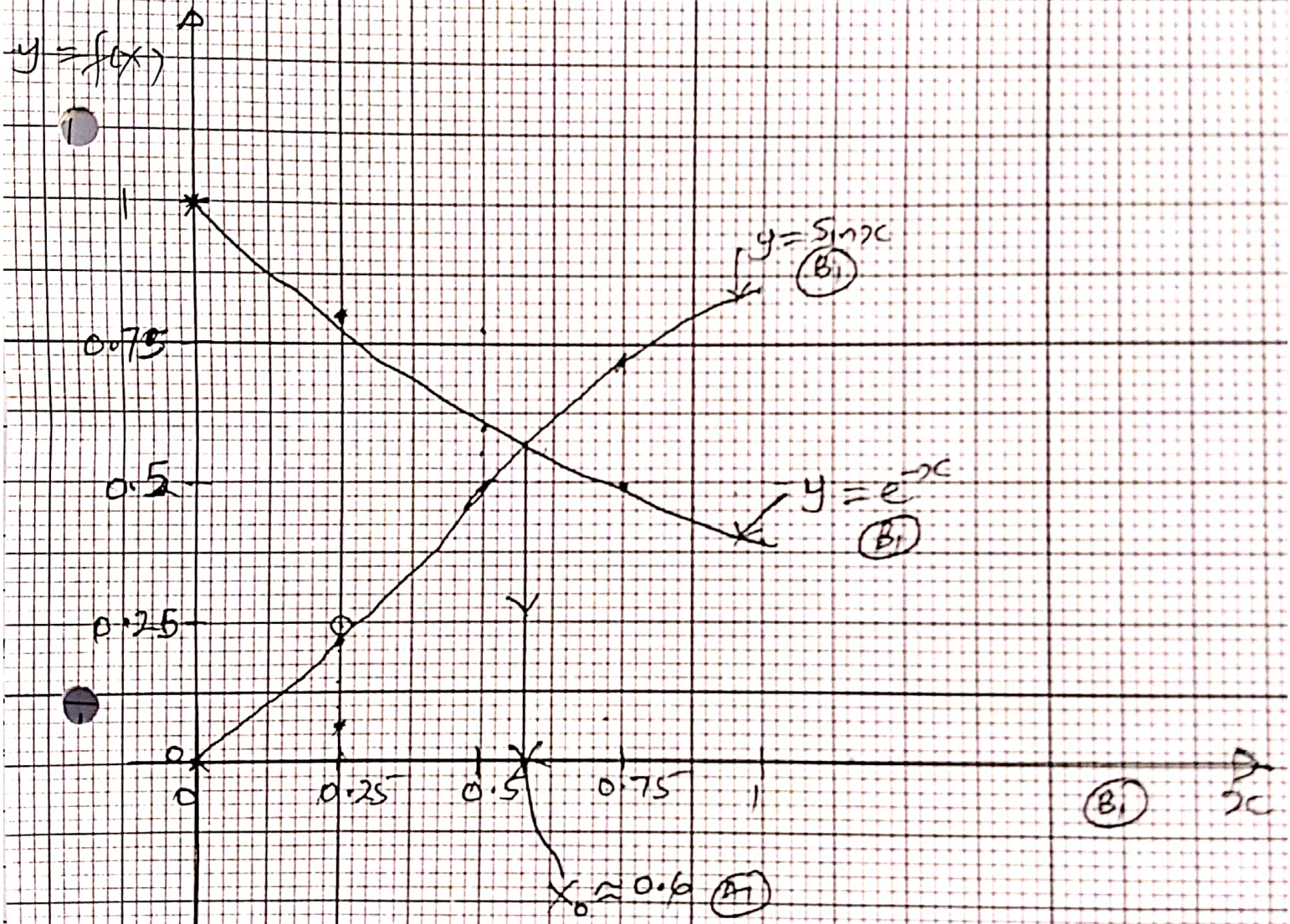


(14) (a)

x	0	0.25	0.5	0.75	1	B_1
$y = e^{-x}$	1	0.78	0.61	0.47	0.37	B_1
$y = \sin x$	0	0.25	0.48	0.68	0.84	B_1

$y = e^{-x}$ and $y = \sin x$ against values.



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at $t=0$, $\underset{\sim}{r}_{(t=0)} = \begin{pmatrix}1\\0\\1\end{pmatrix}$, $\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \underset{\sim}{c}$; $\underset{\sim}{c} = \begin{pmatrix}1\\0\\1\end{pmatrix}$ (B1)

$$\underset{\sim}{r}_{(t=t)} = \begin{pmatrix}\frac{3}{2}t^2+1\\-2t^2\\\frac{t^3}{3}+1\end{pmatrix} m \quad (B1)$$

$$\underset{\sim}{r}_{(t=25)} = \begin{pmatrix}\frac{3}{2}(2)^2+1\\-2(2)^2\\\frac{(2)^3}{3}+1\end{pmatrix} = \begin{pmatrix}7\\-8\\11/3\end{pmatrix} m \quad (B1)$$

$$\left|\underset{\sim}{r}_{(t=25)}\right| = \sqrt{(7)^2+(-8)^2+(11/3)^2} = 11.2448\,m \quad (M1)(A1)$$

(b)

$$\underset{\sim}{a}_{(t=t)} = \frac{d\underset{\sim}{v}}{dt} = \begin{pmatrix}3\\-4\\2t\end{pmatrix} ms^{-2} \quad (M1)$$

$$\underset{\sim}{a}_{(t=3s)} = \begin{pmatrix}3\\-4\\2\times3\end{pmatrix} = \begin{pmatrix}3\\-4\\6\end{pmatrix} ms^{-2} \quad (B1)$$

$$\left|\underset{\sim}{a}_{(t=3s)}\right| = \sqrt{(3)^2+(-4)^2+(6)^2} = 7.8102\,ms^{-2} \quad (M1)(A1)$$

(c)

$$\underset{\sim}{F} = m\underset{\sim}{a} = 3\begin{pmatrix}3\\-4\\6\end{pmatrix} = \begin{pmatrix}9\\-12\\18\end{pmatrix} N \quad (M1)(A1)$$

14 (b) $e^{-x} = \sin x$, $\sin x - e^{-x} = 0$, $f(x) = 0$

$f(x) = \sin x - e^{-x}$

$f'(x) = \cos x + e^{-x}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \left(\frac{\sin x_n - e^{-x_n}}{\cos x_n + e^{-x_n}}\right) \quad (M1)$$

$x_0 = 0.6$

$$x_1 = 0.6000 - \left(\frac{\sin 0.6000 - e^{-0.6000}}{\cos 0.6000 + e^{-0.6000}}\right) = 0.5885 \quad (M1)$$

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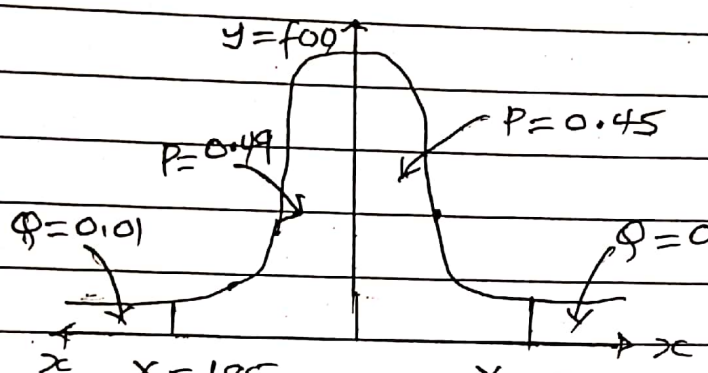
$x_2 = 0.5885 - \left(\frac{\sin 0.5885 - e^{-0.5885}}{\cos 0.5885 + e^{-0.5885}}\right) - 0.5885$ (M1)

$|x_1 - x_2| = 0$ (B1)

root ≈ 0.589 (3d.p.s) (A1)

(15) (a)



$-Z_{0.49} = -2.326$ (B1), $Z_{0.45} = 1.645$ (B1) $\quad Z = \frac{X-\mu}{\sigma}$

$1.645 = \frac{210 - \mu}{\sigma}$ (M1)

$1.6458 + \mu = 210$

$-2.326 = \frac{185 - \mu}{\sigma}$ (M1)

$\mu - 2.326\sigma = 185$ (−) $\quad \mu = 199.6437$

$\mu + 1.6458 = 210$ (+) $\quad \sigma = 6.2956$

(+) − (−) $\quad 3.9718 = 25$ (M1)

$\sigma = 6.2956$ (A1)

from (+) $\mu = 210 - 1.645 \times 6.2956 = 199.6437$ (M1) (A1)

(b) $P(182 < X < 195)$

$= P\left(\frac{182 - 199.6437}{6.2956} < Z < \frac{195 - 199.6437}{6.2956}\right)$ (M1)

$= P(-2.8025 < Z < -0.7376) = P(Z < -0.7376) -$

$P(Z < -2.8025) = 0.2304 - 0.0025 = 0.2279$ (B) (M) (A1)

OR $Q(-2.8025) - Q(-0.7376) = 0.4975 - 0.2696 = 0.2279$

10

49746
26962
22784

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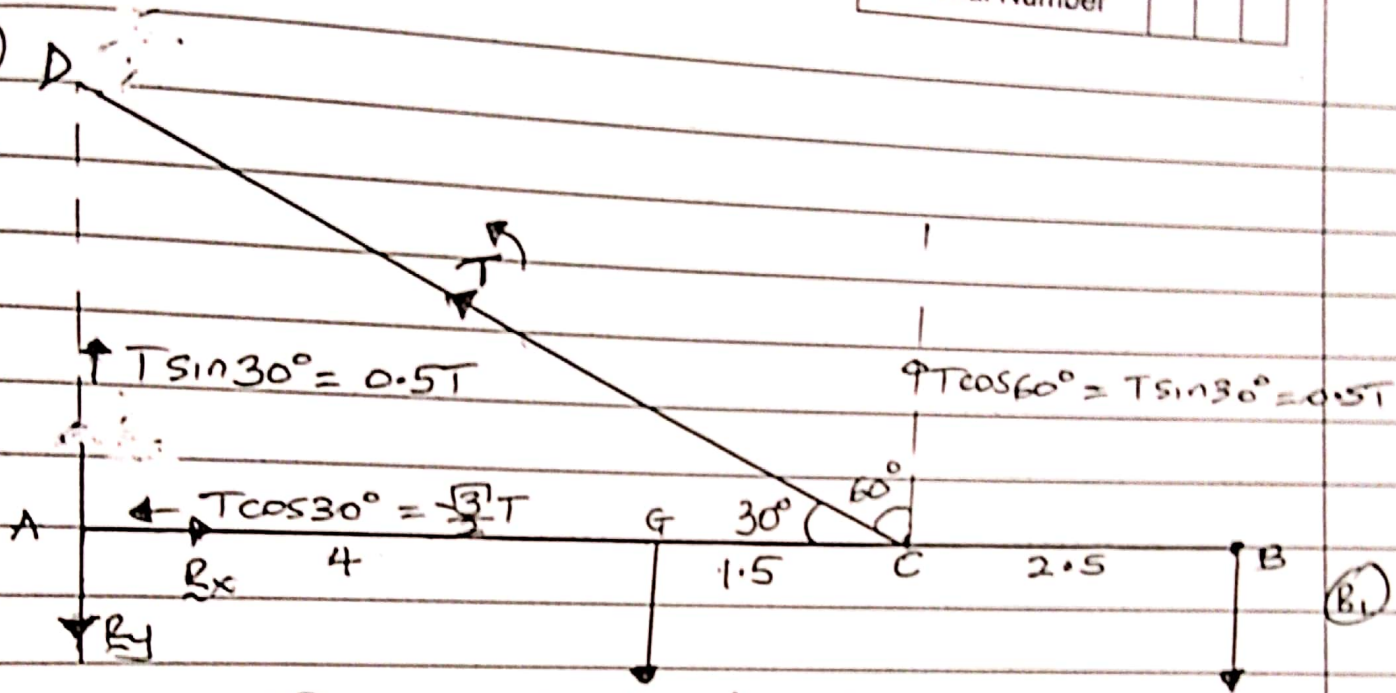
16 (a)



(M1) (M1) (M1) (B1)

↺ A: $5.5 \times 0.5T = 4 \times 40 + 8 \times 60$

$T = 232.7273N$ (A1)

(b) ↑ $0.5T = R_y + 40 + 60$ (B1)

$R_y = 0.5 \times 232.7273 - 100 = 16.3636N$

→ $R_x = \frac{\sqrt{3}}{2}T = \frac{\sqrt{3}}{2} \times 232.7273 = 201.5478N$ (B1)

$|R| = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(201.5478)^2 + (16.3636)^2} = 202.211N$ (M1) (A1)

$\theta = \tan^{-1}\left(\frac{16.3636}{201.5478}\right) = 4.64°$ below AB OR E 4.64° S (B1) (A1)

OR S 85.36° E