APPLIED MATH PAPER 2

MOCKS P 425/2

MATHEMATICS. VACE.

Q.1

$n = 24, \quad p = 0.4, \quad q = 0.6$

$P(8 < x < 15) = ?$

$\mu = np$

$\mu = 24 \times 0.4 = 9.6$

$\sigma^2 = npq$

$= 9.6 \times 0.6$

$\sigma^2 = 5.76$

$\sigma = \sqrt{5.76}$

$\sigma = 2.4$ — B1

$P(8 < x < 15) = P(9 \leq x \leq 14)$

$= P(8.5 \leq x < 14.5)$.

Normalizing

$= P($

$= P\left(\frac{8.5 - 9.6}{2.4} \leq z < \frac{14.5 - 9.6}{2.4}\right)$ — M1 M1

$= P(-0.458 \leq z < 2.042)$. — B1

$= 0.1765 + 0.4794$

$= 0.6559$. — A1

05

Qn 2

(a)

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ M1

$a+6 = 0$

and $b-4 = 0$

$\therefore$ $a=-6$ and $b=4$. A1 A1

(b) $O\circlearrowright$ $G = \begin{vmatrix} -2 & -6 \\ -2 & 4 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ -1 & -4 \end{vmatrix}$ M1

$= -8+12 + -12 - -6$

$= -26\,Nm$

$= 26\,Nm$ clockwise. A1

(5)

Q.3 $h = \frac{2-1}{5} = 0.2$ $= \int_{1}^{2} x e^{-2x}$

x	y_0, y_5	y_1, y_2, y_3, y_4
1.0	0.13534	
1.2	~~0.10886~~	0.10886
1.4		0.08513
1.6		0.06522
1.8		0.04918
2.0	0.03663	
	0.17197	0.30839

B1B1B1

$$\int_1^2 x e^{-2x} dx \cong$$

$$\cong \frac{1}{2} \times 0.2 \left(0.17197 + 2 \times 0.30839\right)$$ M1

$$\cong 0.1 \left(0.78875\right.$$

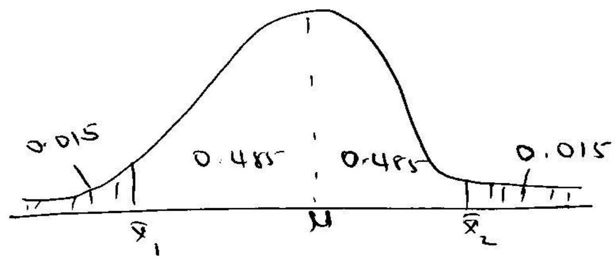
$$\cong 0.078875$$

$$\cong 0.0789 \quad 3\ s.f.$$ A1

(05)

Q4 $n = 100$, $\bar{x} = 76.0$, $s^2 = 144.00$

Find 97% Confidence limits.



P	Q	Z
0.485	0.015	2.17

B1

$\Rightarrow 2.17 = \dfrac{\bar{x}_2 - 76.0}{\frac{12}{\sqrt{100}}}$ M1

$\therefore \bar{x}_2 = 76 + 2.17 \times \frac{12}{10}$

$\bar{x}_2 = 76 + 2.17 \times 1.2$

$\bar{x}_2 = 78.604$

Upper limit $= 78.604$ A1

From $-2.17 = \dfrac{\bar{x}_1 - 76.0}{1.2}$ M1

$\bar{x}_1 = 76 - 2.17 \times 1.2$

$= 73.396$.

Lower limit $= 73.396$ A1

$\boxed{05}$

Q.5

$$s = ut + \frac{1}{2} a t^2$$

A → $x = \frac{1}{2} \times (2) t^2 \; \ldots \text{(i)}$ B1

B: $20 - x = \frac{1}{2}(5) t^2 \; \ldots \text{(ii)}$ B1

$$20 - x = \frac{5}{2} x$$ M1

$$20 - x = \frac{5}{2} x$$ B1

$$20 = \frac{7}{2} x$$

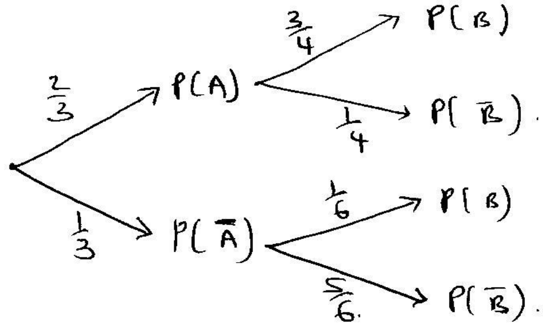
$$x = \frac{40}{7} \text{ m}$$ A1

or $x = 5.7143$ m.

(05)

Q. 6. A is for correct part of A

B " for correct part of B



both correct =

$$P(A \cap B) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

$$P(A \cap \bar{B}) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6} \quad \text{A correct}$$

$$P(\bar{A} \cap B) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18} \quad \text{B correct}$$

$$P(\bar{A} \cap \bar{B}) = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18} \quad \text{failed both}$$

B_2

Let x be mark scored.

x	$P(X=x)$	$xP(X=x)$
0	$\frac{5}{18}$	0
2	$\frac{1}{18}$	$\frac{2}{18}$
3	$\frac{1}{6}$	$\frac{3}{6}$
6	$\frac{1}{2}$	3

B1 M1

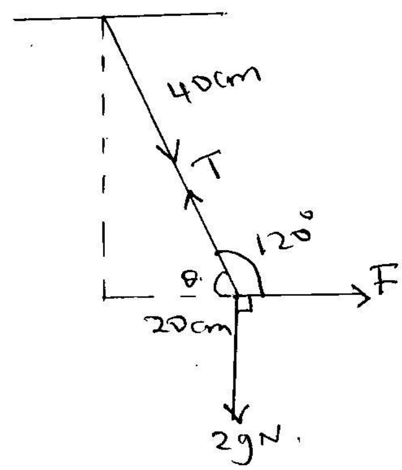
$$E(x) = 3.61$$

Expected total mark = 3.61

A1

05

Q. 7



B1

$\cos\theta = \frac{20}{40} = 0.5$

$\theta = 60^\circ$

$\frac{T}{\sin 90^\circ} = \frac{F}{\sin 150^\circ} = \frac{2g}{\sin 120^\circ}$ M1

$T = \frac{2\times 9.8}{\sin 120^\circ} = 22.6321N.$ A1

$F = \frac{2\times 9.8 \sin 150^\circ}{\sin 120^\circ}$ M1

$= 11.3161N.$ A1

(5)

Q.8

a.

x	1.1	1.16	1.2
f(x)	0.095	y	0.182

B1 (mobile)

$$\frac{y-0.095}{1.16-1.1} = \frac{0.182-0.095}{1.2-1.1}$$ M1

$y-0.095 = 0.87\times0.06 = 0.0522$

~~$y = 0.965$~~

$y = 0.1472$ A1

ii.

x	1.1	1.2	x_1
f(x)	0.095	0.182	0.24

$$\frac{x_1-1.2}{0.24-0.182} = \frac{1.2-1.1}{0.182-0.095}$$ M1

$x_1-1.2 = 0.066667$

$x_1 = 1.267$. A1

$\boxed{05}$

Q. 9

(a) $f(x) = e^{x} - 2x - 5 = 0$

$f(-3) = e^{-3} + 6 - 5 = 1.0498$ B1

$f(-2) = e^{-2} + 4 - 5 = -0.8647$ B1

Since $f(-3) > 0$ and $f(-2) < 0$

Implies there is a root

between -3 and -2.

OR $f(-3)\,f(-2) < 0$.

Implies there is a root between -3 and -2. A1

(b) $f(x) = e^{x} - 2x - 5$

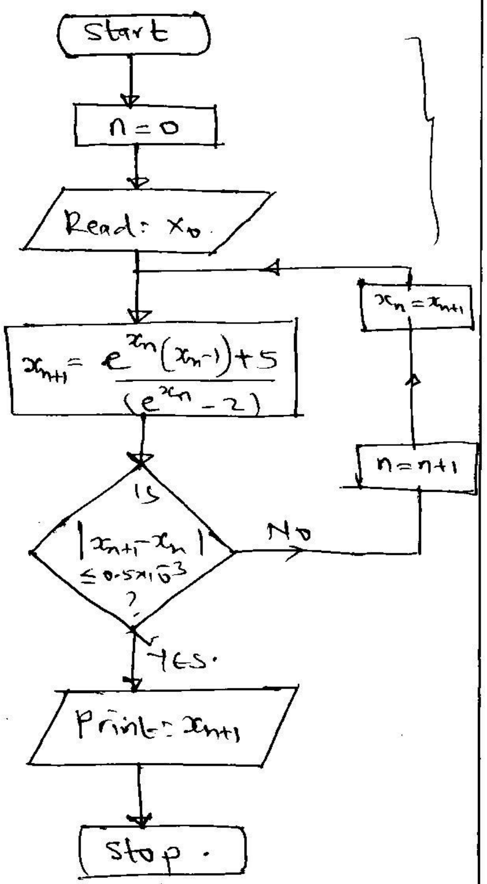
$f'(x) = e^{x} - 2$ B1

$$x_{n+1} = x_n - \frac{(e^{x_n} - 2x_n - 5)}{(e^{x_n} - 2)}$$ M1

$$= \frac{x_n e^{x_n} - 2x_n - e^{x_n} + 2x_n + 5}{e^{x_n} - 2}$$

$$= \frac{e^{x_n}(x_n - 1) + 5}{(e^{x_n} - 2)}$$ B1

Q. 9



B1

B1

B1

B1

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	-2.5	-2.4572	0.0428
1	-2.4572	-2.4572	0.0000

B1

$\therefore$ root $= -2.457$ A1

(112)

Q.10 $\mu = 130$ cm , $\sigma = 7$ cm.

$P(x > 144) =$

$P\left(z > \frac{144-130}{7}\right)$ M1

$P(z > 2) = 0.0228.$

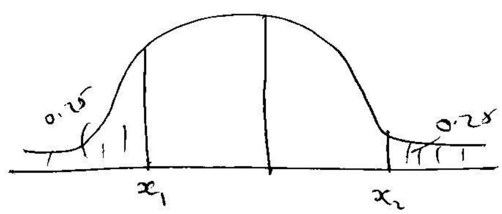
$\underline{P(x > 144) = 0.0228.}$ A1

(ii) $P(x < 123) =$

$P\left(z < \frac{123-130}{7}\right)$ M1

$P(z < -1)$

$\underline{= P(z < -1) = 0.1587}$ A1



p	ø	z
0.25	0.25	0.674

$\Rightarrow \quad 0.674 = \frac{x_2 - 130}{7}$ B1 either

$x_2 = 130 + 0.674 \times 7$ M1 either

$x_2 = 134.718$

Upper Quartile height = $\underline{134.718}$. ~~B1~~

then $-0.674 = \frac{x_1 - 130}{7}$ A1 upper

$x_1 = 130 - 0.674 \times 7$

$x_1 = 125.282$ A1 ~~the other~~ lower

Lower Quartile = $\underline{125.282}$

Q. 10

(c) $P(126.5 < X < 137)$

$= P\left(\frac{126.5 - 130}{7} < Z < \frac{137 - 130}{7}\right)$ M1

$= P(-0.5 < Z < 1)$.

$= 0.1915 + 0.3413$ B1

$= 0.5328$

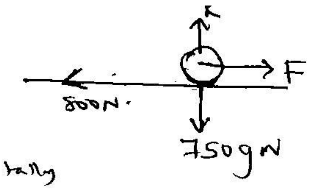
Number of pupils = M1

0.5328×4000.

$= 2131$ pupils. A1

12

Q.11



Horizontally

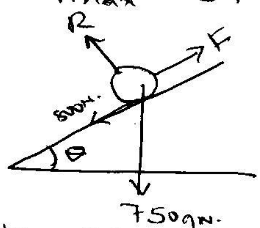
$F - 800 = 0$

$F = 800.$ B1

$F = \frac{P}{V} = \frac{30{,}000}{V_{max}} = 800$ M1

$V_{max} = 37.5\ ms^{-1}.$ A1

(a)



$\sin\theta = \frac{1}{10}.$ B1

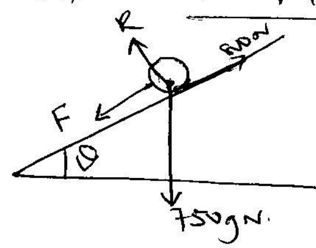
Resolving ∥ to plane

$F - 800 - 750g\sin\theta = 0.$ B1

$\frac{30{,}000}{V_{max}} = 800 + 75 \times 9.8$ M1

$V_{max} = 19.544\ ms^{-1}.$ A1

b.



B1

$F + 750g\sin\theta - 800 = 750a.$ B1

$\frac{30{,}000}{400} + 75g - 800 = 750a.$ M1

M1

$a = \frac{1}{75}\ ms^{-2} = 0.0133\ ms^{-2}.$

A1

12

Q.12

x	f	c	f.d	fx	fx^2	cf
42.5	4	5	0.8	170	7225	4
47.5	13	5	2.6	617.5	29331.25	17
52.5	17	5	3.4	892.5	46856.25	34
57.5	44	5	8.8	2530	145475	78 ←
65	59	10	5.9	3835	249275	137
75	7	10	0.7	525	39375	144

$\Sigma f = 144$

$\Sigma fx = 8{,}570$ $\quad \Sigma fx^2 = \cancel{386{,}1075} = 517537.5$

B1 B1

i, mean time $= \dfrac{8570}{144}$ M1

$= 59.514$ minutes (with or out units) A1

median time $= 55 + \left(\dfrac{72-34}{44}\right) \times 5.$

$= 59.318$ A1

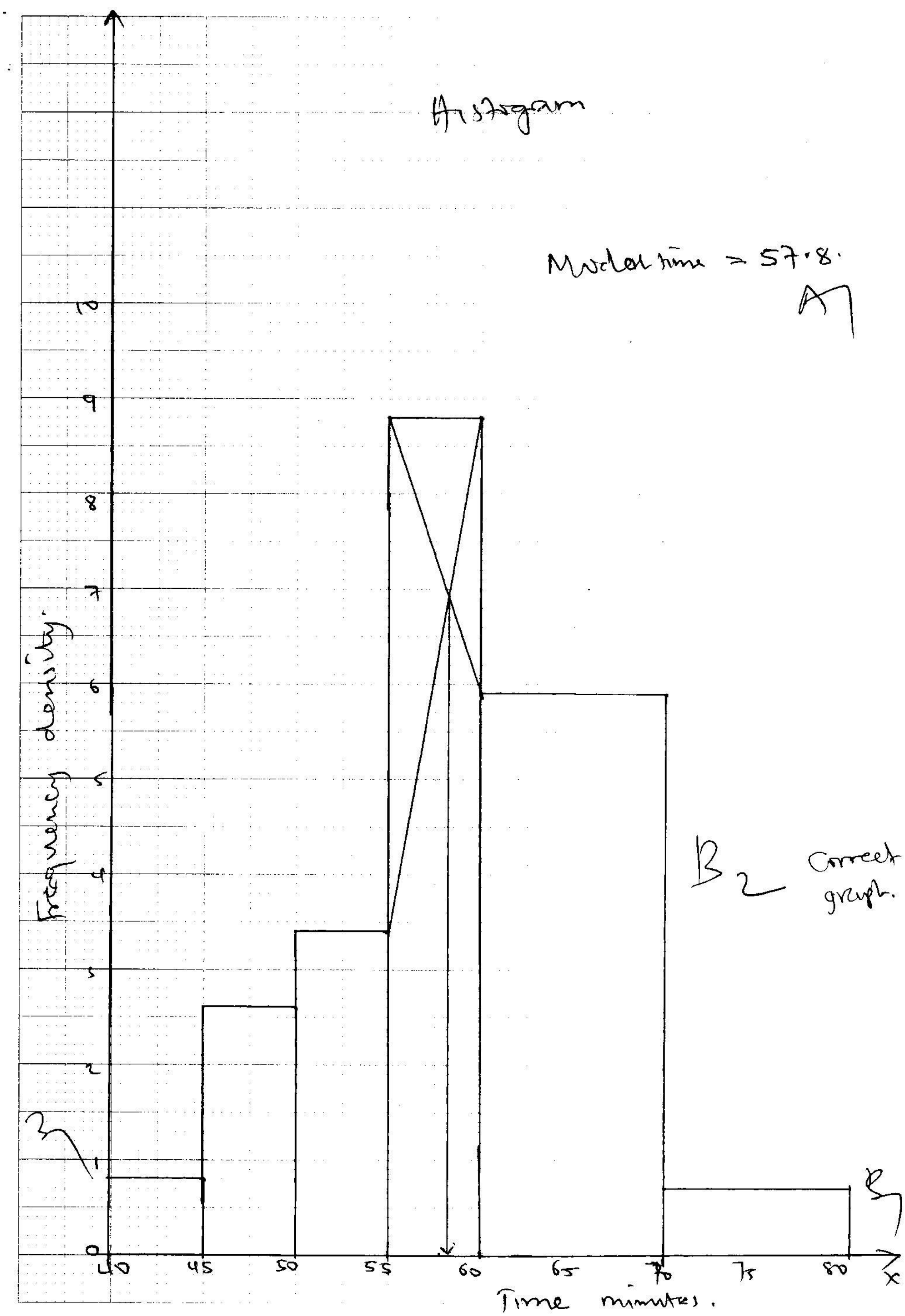
ii, $S.D = \sqrt{\cancel{\dfrac{3866102.5}{144}} - \left(\dfrac{8570}{144}\right)^2}$

$\therefore S.D = \sqrt{\dfrac{517537.5}{144} - \left(\dfrac{8570}{144}\right)^2}$ M1

$SD = \sqrt{3594.01042 - 3541.90297}$

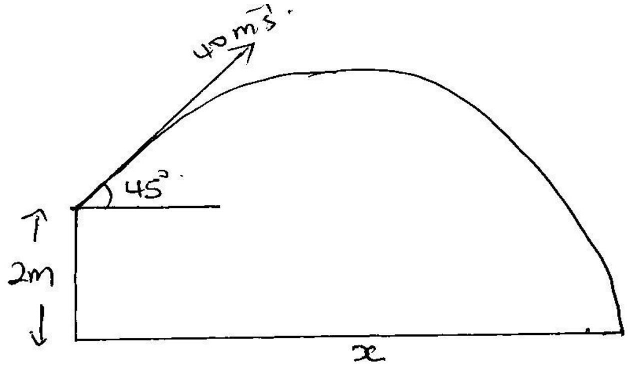
$S.D = \sqrt{52.1074461}$

$SD = 7.219$ minutes. A1



Histogram
Modal time = 57.8.
A1
B2 Correct graph.
B1
B1
Frequency density.
0
1
2
3
4
5
6
7
8
9
10
40
45
50
55
60
65
70
75
80
x
Time minutes.

Q.13



B1

$x = 40\cos 45° t$

$-2 = 40\sin 45° t - \frac{1}{2} g t^2$ M1

$4.9t^2 - 40\sin 45° t - 2 = 0.$ M1

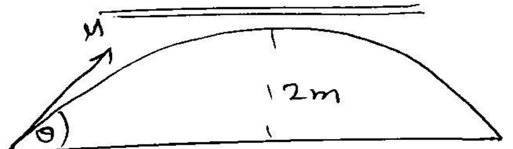
$t = \dfrac{40\sin 45° \pm \sqrt{(40\sin 45°)^2 - 4 \times 4.9 \times (-2)}}{2 \times 4.9}$ B1

$t = 5.822\,s$ or $-0.0699\,s.$ M1

$\therefore x = 40\cos 45° \times 5.822$

$= 165.2424m$ A1

b.



B1

$2 = \dfrac{u^2 \sin^2\theta}{2g}, \quad \sin^2\theta = \dfrac{4g}{u^2}$

$R = \dfrac{2u^2 \cos\theta \sin\theta}{g}$ M1

$R = \dfrac{2u^2 \cdot \sqrt{1 - \sin^2\theta} \cdot \sin\theta}{g}$ M1

$= \dfrac{2u^2}{g} \cdot \sqrt{\left(1 - \dfrac{4g}{u^2}\right)} \cdot \dfrac{2}{u}\sqrt{g}$ M1

$= 4u \cdot \dfrac{\sqrt{u^2 - 4g}}{u} \cdot \dfrac{\sqrt{g}}{g}.$ B1

$R = 4\sqrt{(u^2 - 4g)/g}..$ B1

12

Q.14

$$\text{Error} = \left(\frac{y+\Delta y}{z+\Delta z}\right) - \frac{y}{z}$$ 1M

$$= \frac{z(y+\Delta y) - y(z+\Delta z)}{z(z+\Delta z)}$$

$$= \frac{zy + z\Delta y - yz - y\Delta z}{z(z+\Delta z)}$$

$$= \frac{z\Delta y - y\Delta z}{z^2\left(1+\frac{\Delta z}{z}\right)}$$ 1M

If $\Delta z << z$ then $\frac{\Delta z}{z} \simeq 0$.

$$\text{Error} = \frac{z\Delta y - y\Delta z}{z^2}$$

$$R.E = \left|\left(\frac{z\Delta y - y\Delta z}{z^2}\right) \div \frac{y}{z}\right|$$ 1M

$$= \left|\frac{(z\Delta y - y\Delta z) \cdot z}{z^2 \cdot y}\right|$$

$$= \left|\frac{\Delta y}{y} - \frac{\Delta z}{z}\right|$$ 1M

$$\text{Max } R.E \le \left|\frac{\Delta y}{y}\right| + \left|-\frac{\Delta z}{z}\right|$$

$$= \left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta z}{z}\right|.$$ 1M

Q.14 (b)

$x = 20.131$, $y = 15.3$, $z = 9.5342$

Upper limit

$$= \frac{15.35 - 20.1355}{15.35 + 9.53425}$$

$$= -0.19231$$

lower limit $= \dfrac{15.25 - 20.1365}{15.25 + 9.53415}$

$$= -0.19716.$$

$\therefore$ lower limit $= -0.19716$

Upper limit $= -0.19231$

B1 M1

M1 3

3

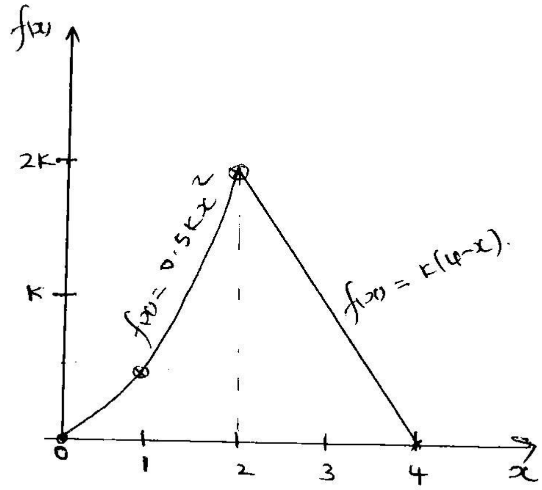
A1 A1

12

Q.15

x	0	1	2	4
$f(x)$	0	0.5K	2K	0

a)



B1 B1

B1

b) $$\int_0^2 \frac{1}{2}kx^2dx + \int_2^4 k(4-x)dx = 1$$

$$\frac{k}{2}\left[\frac{x^3}{3}\right]_0^2 + k\left[4x - \frac{x^2}{2}\right]_2^4 = 1$$ M1

$$\frac{k}{2}\left(\frac{8}{3}\right) + K(8-6) = 1$$ B1

$$\frac{4k}{3} + 2k = 1$$

$$10k = 3$$

$$K = 0.3 = \frac{3}{10}.$$ A1

Mean E(x) =

$$\int_0^2 x \cdot \frac{k}{2}x^2dx + \int_2^4 kx(4-x)dx$$

$$= \frac{k}{2}\int_0^2 x^3dx + k\int_2^4 (4x - x^2)dx$$

$$= \frac{k}{2}\left[\frac{x^4}{4}\right]_0^2 + K\left[2x^2 - \frac{x^3}{3}\right]_2^4$$ M1

$E(x) = 0.6 + 0.3\left(24 - \frac{56}{3}\right)$ B1

$E(x) = 2.2$ A1

Median.

$\int_2^4 k(4-x)\,dx$ = Area

$= \frac{1}{2} \times 2 \times 0.6 = 0.6.$

Hence median lies between 2 and 4.

$\int_m^4 k(4-x)\,dx = 0.5.$

$0.3\left[4x - \frac{x^2}{2}\right]_m^4 = 0.5.$ M1

$0.3\left(8 - 4m + \frac{m^2}{2}\right) = 0.5.$

$0.3\left(\frac{16 - 8m + m^2}{2}\right) = 0.5.$

$3(16 - 8m + m^2) = 10.$

$3m^2 - 24m + 38 = 0$

$m = \frac{24 \pm \sqrt{576 - 4 \times 3 \times 38}}{6}$ B1

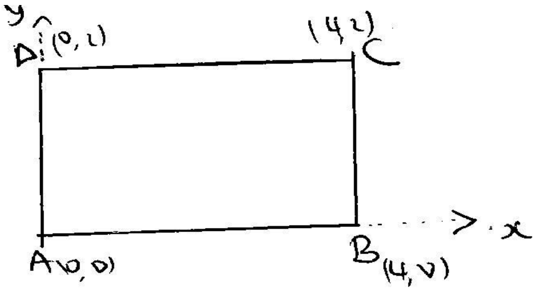
$m = 4 \pm 1.826$

median $= 2.174$

since 5.826 is outside the range. A1

12

Q. 16.



Total weight $= 3g + 5g + 1g + 7g = 16g\,N$. B1

Taking moment about AB and AD.

$$16g\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 3g\begin{pmatrix}0\\0\end{pmatrix} + 5g(4,0) + g\begin{pmatrix}4\\2\end{pmatrix} + 7g\begin{pmatrix}0\\2\end{pmatrix}$$ M1 M1

$16\bar{x} = 20 + 4$

$\bar{x} = 1.5$. B1

$16\bar{y} = 0 + 0 + 2 + 14$.

$\bar{y} = \frac{16}{16} = 1$. B1

Centre of gravity is 1.5cm from AD and

1cm from AB. A1

b)

Portion		C.O.G. from AB.
Cylinder	$8\pi a^2 h\rho$	h
Hole	$\pi a^2 h\rho$	$\frac{3}{2}h$.
Remainder	$7\pi a^2 h\rho$	$\bar{y}$.

B1 B1 B1

Taking moments about AB.

$$8\pi a^2 h\rho h = \pi a^2 h\rho \times \frac{3}{2}h + 7\pi a^2 h\rho \cdot \bar{y}.$$ M1 M1

$8h = \frac{3}{2}h + 7\bar{y}$.

$8h - \frac{3}{2}h = 7\bar{y}$.

$\bar{y} = \frac{13}{14}h$. A1

12