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UACE MATHEMATICS PAPER 1 2019 guide

SECTION A

1. Show that the modulus of $\frac{(1-i)^6}{(1+i)} = 4\sqrt{2}$ (05marks)
2. Solve $2\cos 2\theta - 5\cos \theta = 4$ for $0^\circ \leq \theta \leq 360^\circ$. (05marks)
3. Using the substitution $u = \tan^{-1}x$; show $\int_0^1 \frac{\tan^{-1}x}{1+x^2} dx = \frac{\pi^2}{32}$ (05marks)
4. Given the plane $4x + 3y - 3z - 4 = 0$
 - (a) Show that the point A(1,1,1) lies on the plane (02marks)
 - (b) Find the perpendicular distance from the plane to the point B(1, 5,1) (03marks)
5. Find the equation of the tangent to the curve $y = \frac{a^3}{x^2}$ at the point $(\frac{a}{t}, at^2)$. (05marks)
6. Given that $\alpha + \beta = \frac{-1}{3}$ and $\alpha\beta = \frac{2}{3}$, form a quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ (05marks)
7. Find the area enclosed between the curve $y = 2x^2 - 4x$ and x-axis. (05marks)
8. Given that $Q = \sqrt{80 - 0.1P}$ and $E = \frac{-dQ}{dP} \cdot \frac{P}{Q}$; find E when $P = 600$

SECTION B

9. (a) Determine the perpendicular distance of the point (4, 6) from the line $2x + 4y - 3 = 0$ (03marks)
- (b) Show that the angle θ , between two lines with gradient λ_1 and λ_2 is given by $\theta = \tan^{-1}\left(\frac{\lambda_1 - \lambda_2}{1 + \lambda_1\lambda_2}\right)$. Hence find the acute angle between the lines $x + y + 7 = 0$ and $\sqrt{3}x - y + 5 = 0$ (09 marks)
10. (a) Given that $26\left(1 - \frac{1}{26^2}\right)^{1/2} = a\sqrt{3}$, find the value of a (05marks)
- (b) Solve the simultaneous equations:

$$2x = 3y = 4z$$

$$x^2 - 9y^2 - 4z + 8 = 0$$
 (07marks)
11. Express $7\cos 2\theta + 6\sin 2\theta$ in form $R\cos(2\theta - \alpha)$, where R is a constant and α is an acute angle. (05marks)
- Hence solve $7\cos 2\theta + 6\sin 2\theta = 5$ for $0^\circ \leq \theta \leq 180^\circ$. (07marks)
12. (a) given that $y = \ln \left\{ e^x \left(\frac{x-2}{x+2} \right)^{\frac{3}{4}} \right\}$, show that $\frac{dy}{dx} = \frac{x^2-1}{x^2-4}$ (05marks)
- (b) Evaluate $\int \frac{dx}{x^2\sqrt{(25-x^2)}}$ (07marks)
13. Four points have coordinates A(3, 4, 7), B(13, 9, 2), C(1, 2, 3) and D(10, k, 6). The lines AB and CD intersect at P.

Determine the;

- (i) Vector equation of the lines AB and CD. (06marks)
- (ii) Value of k (04marks)
- (iii) Coordinates of P (02marks)

14. Expand $\sqrt{\left(\frac{1+2x}{1-x}\right)}$ up to the term x^2 . Hence find the value of $\sqrt{\left(\frac{1.04}{0.98}\right)}$ to four significant figures. (12marks)
15. (a) Differentiate $y = 2x^2 + 3$ from first principles. (04marks)
 (b) A rectangular sheet is 50cm long and 40cm wide. A square of x cm is cut off from each corner. The remaining sheet is folded to form an open box. Find the maximum volume of the box (08marks)
16. (a) Find $\int \frac{\ln x}{x^2} dx$ (04marks)
 (c) Solve the differential equation $\frac{dy}{dx} + y \cot x = x$, given that $y = 1$ when $x = \frac{\pi}{2}$. (08marks)

Solutions

SECTION A

1. Show that the modulus of $\frac{(1-i)^6}{(1+i)} = 4\sqrt{2}$ (05marks)

Method I

$$\left| \frac{(1-i)^6}{1+i} \right| = \frac{|1-i|^6}{|1+i|} = \frac{(\sqrt{1^2 + (-1)^2})^6}{\sqrt{1^2 + 1^2}} = \frac{(\sqrt{2})^6}{\sqrt{2}} = (\sqrt{2})^5 = (\sqrt{2})^4 \sqrt{2} = 4\sqrt{2}$$

Method II

$$\begin{aligned} \frac{(1-i)^6}{(1+i)} &= \frac{1 + {}^6C_1(-i)^2 + {}^6C_2(-i)^4 + {}^6C_3(-i)^6 + {}^6C_4(-i)^8 + {}^6C_5(-i)^{10} + {}^6C_6(-i)^{12}}{1+i} \\ &= \frac{1 - 6i - 15 + 20i + 15 - 6i - 1}{1+i} \\ &= \frac{8i}{1+i} \times \frac{1-i}{1-i} \\ &= \frac{8i+8}{1+1} = 4 + 4i \\ |4 + 4i| &= \sqrt{4^2 + 4^2} = \sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2} \end{aligned}$$

2. Solve $2\cos 2\theta - 5\cos \theta = 4$ for $0^\circ \leq \theta \leq 360^\circ$. (05marks)

$$2\cos 2\theta - 5\cos \theta = 4$$

$$2(2\cos^2 \theta - 1) - 5\cos \theta = 4$$

$$4\cos^2 \theta - 5\cos \theta - 6 = 0$$

$$\cos \theta = \frac{5 \pm \sqrt{(-5)^2 - 4(4)(-6)}}{2(4)} = \frac{-3}{4}, 2$$

$$\text{Either } \cos \theta = \frac{-3}{4}$$

$$\therefore \theta = 138.59^\circ \text{ and } \theta = 221.41^\circ$$

3. Using the substitution $u = \tan^{-1}x$; show $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx = \frac{\pi^2}{32}$ (05marks)

$$u = \tan^{-1}x$$

$$\frac{du}{dx} = \frac{1}{1+x^2}$$

$$du = \frac{dx}{1+x^2}$$

Changing limits

| x | U |
|---|-----------------|
| 0 | 0 |
| 1 | $\frac{\pi}{4}$ |

By change of variable;

$$\begin{aligned}\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx &= \int_0^{\frac{\pi}{4}} u du = \left[\frac{u^2}{2} \right]_0^{\frac{\pi}{4}} \\ &= \frac{1}{2} x \left(\frac{\pi}{4} \right)^2 \\ &= \frac{\pi^2}{32}\end{aligned}$$

4. Given the plane $4x + 3y - 3z - 4 = 0$

(a) Show that the point A(1,1,1) lies on the plane (02marks)

Substitute A(1, 1, 1) into the equation of plane

$$4 \times 1 + 3 \times 1 - 3 \times 1 - 4 = 0$$

Hence the point lies on the plane

(b) Find the perpendicular distance from the plane to the point B(1, 5,1) (03marks)

$$d = \frac{|4 \times 1 + 3 \times 5 - 3 \times 1 - 4|}{\sqrt{4^2 + 3^2 + (-3)^2}} = 2.058$$

5. Find the equation of the tangent to the curve $y = \frac{a^3}{x^2}$ at the point $(\frac{a}{t}, at^2)$. (05marks)

$$\text{Given that } y = \frac{a^3}{x^2}, \frac{dy}{dx} = \frac{-2a^3}{x^3}$$

$$\text{Gradient } m = \frac{-2a^3}{\frac{a^3}{t^3}} = -2t^3$$

$$\text{From } y - y_1 = m(x - x_1)$$

$$y - at^2 = -2t^3 \left(x - \frac{a}{t} \right)$$

$$\therefore y = 3at^2 - 2t^3x$$

6. Given that $\alpha + \beta = \frac{-1}{3}$ and $\alpha\beta = \frac{2}{3}$, form a quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ (05marks)

$$\text{Sum of the roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-1}{3}\right)^2 - 2 \times \frac{2}{3}}{\frac{2}{3}} = \frac{-11}{6}$$

$$\text{Product of the roots} = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$$

Equation

$$x^2 + (\text{sum})x + \text{product} = 0$$

$$x^2 - \frac{11}{6}x + 1 = 0$$

or

$$6x^2 - 11x + 6 = 0$$

7. Find the area enclosed between the curve $y = 2x^2 - 4x$ and x-axis. (05marks)

On x-axis, $y = 0$

$$2x^2 - 4x = 0$$

Either $x = 0$ or 2

$$\text{Area enclosed} = \int_0^2 (2x^2 + 4x) dx$$

$$= \left[\frac{2x^3}{3} + 2x^2 \right]_0^2$$

$$= \frac{2(2)^3}{3} + 2(2)^2 = \frac{28}{3}$$

$$\therefore \text{area} = \frac{28}{3} \text{ square units}$$

8. Given that $Q = \sqrt{80 - 0.1P}$ and $E = \frac{-dQ}{dP} \cdot \frac{P}{Q}$; find E when $P = 600$

$$\frac{dQ}{dP} = \frac{-0.1}{2\sqrt{80-0.1P}}$$

$$E = \frac{0.1}{2\sqrt{80-0.1(600)}} \times \frac{600}{\sqrt{80-0.1(600)}} = 1.5$$

SECTION B

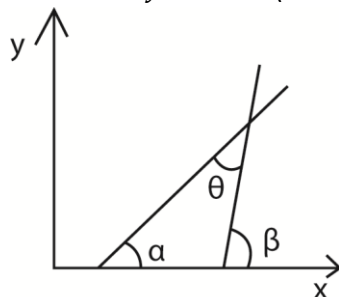
9. (a) Determine the perpendicular distance of the point (4, 6) from the line $2x + 4y - 3 = 0$ (03marks)

$$\text{Perpendicular distance, } d = \frac{|2(4) + 4(6) - 3|}{\sqrt{2^2 + 4^2}} = \frac{29}{\sqrt{20}} = 6.4846$$

- (b) Show that the angle θ , between two lines with gradient λ_1 and λ_2 is given by

$$\theta = \tan^{-1} \left(\frac{\lambda_1 - \lambda_2}{1 + \lambda_1 \lambda_2} \right). \text{ Hence find the acute angle between the lines } x + y + 7 = 0 \text{ and}$$

$$\sqrt{3}x - y + 5 = 0 \text{ (09 marks)}$$



$$\tan \alpha = \lambda_2, \tan \beta = \lambda_1$$

$$\alpha + \theta = \beta; \theta = \beta - \alpha$$

$$\tan \theta = \tan (\beta - \alpha)$$

$$= \frac{\tan \beta - \tan \alpha}{1 + \tan \alpha \tan \beta}$$

$$= \left(\frac{\lambda_1 - \lambda_2}{1 + \lambda_1 \lambda_2} \right)$$

$$\therefore \theta = \tan^{-1} \left(\frac{\lambda_1 - \lambda_2}{1 + \lambda_1 \lambda_2} \right)$$

$$\text{But } \lambda_1 = -1 \text{ and } \lambda_2 = \sqrt{3}$$

$$\theta = \tan^{-1} \left(\frac{-1 - \sqrt{3}}{1 + (-1 \times \sqrt{3})} \right) = 75^\circ$$

10. (a) Given that $26 \left(1 - \frac{1}{26^2} \right)^{1/2} = a\sqrt{3}$, find the value of a (05marks)

$$26 \left(1 - \frac{1}{26^2} \right)^{1/2} = a\sqrt{3}$$

$$26 \left(\frac{26^2 - 1}{26^2} \right)^{1/2} = a\sqrt{3}$$

$$(675)^{\frac{1}{2}} = a\sqrt{3}$$

$$a = \left(\frac{675}{3}\right)^{\frac{1}{2}} = \pm 15$$

(b) Solve the simultaneous equations:

$$2x = 3y = 4z$$

$$x^2 - 9y^2 - 4z + 8 = 0 \text{ (07marks)}$$

$2x = 3y = 4z$, substituting $4z = 2x$ and $y = \frac{2x}{3}$ into the equation $x^2 - 9y^2 - 4z + 8 = 0$

$$x^2 - (2x)^2 - 2x + 8 = 0$$

$$-3x^2 - 2x + 8 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(-3)(8)}}{2(-3)}; x = -2 \text{ or } x = \frac{4}{3}$$

$$\text{When } x = -2; y = \frac{2x(-2)}{3} = \frac{-4}{3}; z = \frac{2x(-2)}{4} = -1$$

$$(x, y, z) = (-2, \frac{-4}{3}, -1)$$

$$\text{When } x = \frac{4}{3}; y = \frac{2x(\frac{4}{3})}{3} = \frac{8}{9}; z = \frac{2x(\frac{4}{3})}{4} = \frac{2}{3}$$

$$(x, y, z) = (\frac{4}{3}, \frac{8}{9}, \frac{2}{3})$$

Alternatively

$2x = 3y = 4z$, substituting $4z = 3y$ and $x = \frac{3y}{2}$ into the equation $x^2 - 9y^2 - 4z + 8 = 0$

$$(\frac{3}{2}y)^2 - 9y^2 - 3y + 8 = 0$$

$$9y^2 - 36y^2 - 12y + 32 = 0$$

$$-27y^2 - 12y + 32 = 0$$

$$y = \frac{12 \pm \sqrt{(-12)^2 - 4(-27)(32)}}{2(-27)}; y = \frac{-4}{3} \text{ or } y = \frac{8}{9}$$

$$\text{When } y = \frac{-4}{3}; x = \frac{3}{2} \times \frac{-4}{3} = -2; z = \frac{3}{4} \times \frac{-4}{3} = -1$$

$$(x, y, z) = (-2, \frac{-4}{3}, -1)$$

$$\text{When } y = \frac{8}{9}; x = \frac{3}{2} \times \frac{8}{9} = \frac{4}{3}; z = \frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$$

$$(x, y, z) = (\frac{4}{3}, \frac{8}{9}, \frac{2}{3})$$

Alternatively

$2x = 3y = 4z$, substituting $2x = 4z$ or $x = 2z$ and $3y = 4z$ or $y = \frac{4z}{3}$ into the equation

$$(2z)^2 - (4z)^2 - 4z + 8 = 0$$

$$4z^2 - 16z^2 - 4z + 8 = 0$$

$$-12z^2 - 4z + 8 = 0$$

$$z = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)}; z = -1 \text{ or } z = \frac{2}{3}$$

$$\text{When } z = -1; y = \frac{4(-1)}{3} = \frac{-4}{3}; x = 2(-1) = -2$$

$$(x, y, z) = (-2, \frac{-4}{3}, -1)$$

$$\text{When } z = \frac{2}{3}; y = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}; x = 2 \times \frac{2}{3} = \frac{4}{3}$$

$$(x, y, z) = (\frac{4}{3}, \frac{8}{9}, \frac{2}{3})$$

11. Express $7\cos 2\theta + 6\sin 2\theta$ in form $R\cos(2\theta - \alpha)$, where R is a constant and α is an acute angle. (05marks)

$$7\cos 2\theta + 6\sin 2\theta \equiv R\cos(2\theta - \alpha)$$

$$7\cos 2\theta + 6\sin 2\theta \equiv R\cos 2\theta \cos \alpha + R\sin 2\theta \sin \alpha$$

Comparing both sides

$$R\cos \alpha = 7 \dots\dots\dots (i)$$

$$R\sin \alpha = 6 \dots\dots\dots (ii)$$

(i)2 + (ii)2 gives

$$R = \sqrt{7^2 + 6^2} = \sqrt{85}$$

From equation (i)

$$\sqrt{85}\cos \alpha = 7$$

$$\alpha = \cos^{-1} \left(\frac{7}{\sqrt{85}} \right) = 40.6^\circ$$

Hence solve $7\cos 2\theta + 6\sin 2\theta = 5$ for $0^\circ \leq \theta \leq 180^\circ$. (07marks)

$$\therefore 7\cos 2\theta + 6\sin 2\theta = \sqrt{85} \cos(2\theta - 40.6^\circ) = 5$$

$$2\theta - 40.6 = \cos^{-1} \left(\frac{5}{\sqrt{85}} \right) = 57.16^\circ, 302.84^\circ$$

$$\theta = 48.88^\circ, 171.72^\circ$$

12. (a) given that $y = \ln \left\{ e^x \left(\frac{x-2}{x+2} \right)^{\frac{3}{4}} \right\}$, show that $\frac{dy}{dx} = \frac{x^2-1}{x^2-4}$ (05marks)

$$y = \ln \left\{ e^x \left(\frac{x-2}{x+2} \right)^{\frac{3}{4}} \right\}$$

$$= \ln e^x + \frac{3}{4} \{ \ln(x-2) + \ln(x+2) \}$$

$$= x \ln e + \frac{3}{4} \{ \ln(x-2) + \ln(x+2) \}$$

$$\frac{dy}{dx} = 1 + \frac{3}{4} \left\{ \frac{1}{x-2} - \frac{1}{x+2} \right\} = 1 + \frac{3}{4} \left\{ \frac{x+2-x+2}{x^2-4} \right\} = 1 + \frac{3}{4} \left\{ \frac{4}{x^2-4} \right\} = 1 + \frac{3}{x^2-4} = \frac{x^2-4+3}{x^2-4} = \frac{x^2-1}{x^2-4}$$

(b) Evaluate $\int \frac{dx}{x^2\sqrt{(25-x^2)}}$ (07marks)

$$\int \frac{dx}{x^2\sqrt{(25-x^2)}}$$

$$\text{Let } x = 5\sin\theta$$

$$\Rightarrow dx = 5\cos\theta d\theta$$

$$\int \frac{dx}{x^2\sqrt{(25-x^2)}} = \int \frac{5\cos\theta d\theta}{25\sin^2\theta\sqrt{25-25\sin^2\theta}}$$

$$= \frac{1}{25} \int \operatorname{cosec}^2\theta d\theta$$

$$= -\frac{1}{25} \cot\theta + C$$

$$\text{But } \sin\theta = \frac{x}{5}, \cos\theta = \left(\frac{\sqrt{25-x^2}}{5} \right)$$

$$\therefore \int \frac{dx}{x^2\sqrt{(25-x^2)}} = -\frac{1}{25} \left(\frac{5\sqrt{25-x^2}}{x^2} \right) + C$$

13. Four points have coordinates

A(3, 4, 7), B(13, 9, 2), C(1, 2, 3) and D(10, k, 6). The lines AB and CD intersect at P.

Determine the;

- (i) Vector equation of the lines AB and CD. (06marks)

$$\underline{r} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} + \lambda \left[\begin{pmatrix} 13 \\ 9 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} \right]$$

$$\underline{r} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix}$$

- (ii) Value of k (04marks)

Equating corresponding entries

$$3 + 10\lambda = 1 + 9\mu \dots\dots\dots (i)$$

$$4 + 5\lambda = 2 + \mu k - 2\mu \dots\dots\dots (ii)$$

$$7 - 5\lambda = 3 + 3\mu \dots\dots\dots (iii)$$

Solving equation (i) and (iii) simultaneously

$$9\mu - 10\lambda = 2$$

$$3\mu + 5\lambda = 4$$

$$\mu = \frac{2}{3}; \lambda = \frac{2}{5}$$

substituting into equation (ii)

$$4 + 5\left(\frac{2}{5}\right) = 2 - 2\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)k$$

$$\therefore k = 8$$

- (iii) Coordinates of P (02marks)

$$\text{Let } \underline{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} + \frac{2}{5} \begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix}$$

$$x = 3 + 4 = 7$$

$$y = 4 + 2 = 6$$

$$z = 7 - 2 = 5$$

$$\therefore P(7, 6, 5)$$

14. Expand $\sqrt{\left(\frac{1+2x}{1-x}\right)}$ up to the term x^2 . Hence find the value of $\sqrt{\left(\frac{1.04}{0.98}\right)}$ to four significant figures.

(12marks)

$$\sqrt{\left(\frac{1+2x}{1-x}\right)} = (1 + 2x)^{\frac{1}{2}}(1 - x)^{-\frac{1}{2}}$$

$$\text{using } (1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots$$

$$\sqrt{\left(\frac{1+2x}{1-x}\right)} = \left(1 + x - \frac{1}{2}x^2\right)\left(1 + \frac{1}{2}x + \frac{3}{8}x^2\right)$$

$$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + x + \frac{1}{2}x^2 - \frac{1}{2}x^2$$

$$= 1 + \frac{3}{2}x + \frac{3}{8}x^2$$

$$\therefore \sqrt{\left(\frac{1+2x}{1-x}\right)} \approx 1 + \frac{3}{2}x + \frac{3}{8}x^2$$

substituting for $x = 0.02$

$$\sqrt{\left(\frac{1.04}{0.98}\right)} = \sqrt{\frac{1+2(0.02)}{1-0.02}}$$

$$= 1 + \frac{3}{2}(0.02) + \frac{3}{8}(0.02)^2 = 1.030$$

15. (a) Differentiate $y = 2x^2 + 3$ from first principles. (04marks)

$$y = 2x^2 + 3$$

$$y + \delta y = 2(x + \delta x)^2 + 3$$

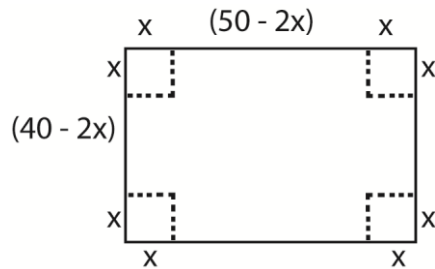
$$\delta y = 2x^2 + 4x\delta x + 2(\delta x)^2 + 3 - 2x^2 - 3$$

$$\delta y = 4x\delta x + 2(\delta x)^2$$

$$\lim_{\delta x \rightarrow 0} \left(\frac{\delta y}{\delta x} \right) = \lim_{\delta x \rightarrow 0} (4x + 2\delta x)$$

$$\frac{\delta y}{\delta x} = 4x$$

- (b) A rectangular sheet is 50cm long and 40cm wide. A square of x cm is cut off from each corner. The remaining sheet is folded to form an open box. Find the maximum volume of the box (08marks)



$$\text{Volume } V = x(50 - 2x)(40 - 2x)$$

$$V = 2000x - 180x^2 + 4x^3$$

$$\frac{\delta V}{\delta x} = 2000 - 360x + 12x^2$$

$$\text{For maximum volume, } \frac{\delta V}{\delta x} = 0$$

$$\therefore 3x^2 - 90x + 500 = 0$$

$$x = \frac{90 \pm \sqrt{(-90)^2 - 4(3)(500)}}{2(3)}$$

$$x = 7.3624 \text{ or } x = 22.6376$$

$$\therefore x = 7.3624 \text{ cm}$$

$$V_{\max} = 2000x(7.3624) - 180(7.3624)^2 + 4(7.3624)^3$$

$$= 6564.22554 \text{ cm}^3$$

16. (a) Find $\int \frac{\ln x}{x^2} dx$ (04marks)

$$\text{Let } u = \ln x, \frac{du}{dx} = \frac{1}{x}$$

$$\frac{dv}{dx} = \frac{1}{x^2}, v = \int x^{-2} dx = \frac{-1}{x}$$

Using integration by parts

$$\int u \frac{dv}{dx} dx = u \cdot v - \int v \frac{du}{dx} dx$$

$$\int \frac{\ln x}{x^2} dx = \frac{-1}{x} \ln x - \int \frac{-1}{x} \cdot \frac{1}{x} dx$$

$$= \frac{-1}{x} \ln x + \int \frac{1}{x^2} dx$$

$$= \frac{-\ln x}{x} + \int x^{-2} dx$$

$$= \frac{-\ln x}{x} - \frac{1}{x} + C$$

$$\therefore \int \frac{\ln x}{x^2} dx = \frac{-\ln x}{x} - \frac{1}{x} + C$$

(c) Solve the differential equation $\frac{dy}{dx} + y \cot x = x$, given that $y = 1$ when $x = \frac{\pi}{2}$. (08marks)

$$\frac{dy}{dx} + y \cot x = x$$

Integrating factor,

$$I.F = e^{\int \cot x \, dx} = e^{\int \frac{\cos x}{\sin x} dx} = e^{\ln \sin x} = \sin x$$

Multiplying all terms by integrating factor

$$\sin x \frac{dy}{dx} + y \cot x \sin x = x \sin x$$

$$\sin x \frac{dy}{dx} + y \frac{\cos x}{\sin x} \sin x = x \sin x$$

$$\sin x \frac{dy}{dx} + y \cos x = x \sin x$$

$$\frac{d(y \sin x)}{dx} dx = x \sin x$$

Integrating with respect to x

$$\int \frac{d(y \sin x)}{dx} dx = \int x \sin x dx$$

$$y \sin x = \int x \sin x dx$$

$$\text{Let } u = x, \frac{du}{dx} = 1$$

$$\frac{dv}{dx} = \sin x, v = \int \sin x \, dx = -\cos x$$

Using integration by parts on RHS

$$y \sin x = -x \cos x + \int \cos x \, dx$$

$$y \sin x = -x \cos x + \sin x + C$$

$$\text{But } y = 1, x = \frac{\pi}{2}$$

$$1 \sin\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) - \left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right) + C$$

$$C = 0$$

By substitution,

$$\therefore y \sin x = \sin x - x \cos x$$