

SECTION A (40 MARKS)

Answer all the questions in this section.

1. A committee of seven people is to be selected from 4 men and 6 women. If the committee must have **at least** two men, determine the total possible number of ways of selecting the committee. (05 marks)
2. A cylindrical can of capacity 1000 cm^3 is made from a thin sheet of metal. The can is open at the top and closed at the bottom. The radius of the bottom is $x \text{ cm}$. Find the value of x that will minimise the area of the sheet to be used. (Leave π in your answer) (05 marks)
3. The equation of an ellipse is $4x^2 + 25y^2 + 8x - 100y + 4 = 0$. Determine the;
(a) coordinates of the centre of the ellipse. $\text{focus} = (0, 0, 0)$ (03 marks)
(b) eccentricity of the ellipse. $e = \frac{b^2}{a^2(1-e^2)}$ (02 marks)
4. Show that $\int_0^1 \left(\frac{1}{9-x^2} \right) dx = \frac{1}{6} \ln 2$. (05 marks)
5. The population of a country increases in a geometric progression (G.P.) by 2.75 % per annum. Calculate the number of years it will take for the population to double. (05 marks)
6. Show that $\frac{1 - \cos 2x + 2 \sin x \cos^2 x}{1 + \cos 2x} = \sin x + \tan^2 x$. (05 marks)
7. The point $C(a, 4, 5)$ divides the line joining points $A(1, 2, 3)$ and $B(6, 7, 8)$ in the ratio $\lambda : 3$. Using vectors, find the values of a and λ . (05 marks)
8. Find the area enclosed by the curve $y = x^2$ and the line $y = x$ from $x = 0$ to $x = 1$. (05 marks)

SECTION B (60 MARKS)

Answer only **five** questions from this section.

All questions carry equal marks.

9. (a) Express $12\cos \theta + 16\sin \theta$ in the form $R \cos(\theta - \alpha)$ where R is a positive constant and α is an acute angle. (06 marks)

(b) Hence;

(i) find the maximum and minimum values of $12 \cos \theta + 16 \sin \theta$.

(ii) solve the equation $12 \cos \theta + 16 \sin \theta = 15$ for $0^\circ \leq \theta \leq 180^\circ$.

(06 marks)

10. (a) Given that the polynomial $x^3 - 13x + p$ is exactly divisible by $x - 4$, find the value of p .

Hence solve the equation $x^3 - 13x + p = 0$.

(06 marks)

- (b) Solve the inequality $\frac{x^2 - x - 18}{x + 3} \geq \frac{x}{2}$. (06 marks)

11. (a) Use the substitution $x = \sin \theta$ to evaluate

$$\int_0^1 \frac{1+x}{\sqrt{1-x^2}} dx. \quad (05 \text{ marks})$$

- (b) Given that $y = \sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right)$, find $\frac{dy}{dx}$ in terms of x . (07 marks)

12. A curve is defined by the equations $x = -t^3 + t^2 + 1$ and $y = t^2$. A tangent to the curve at a point $B(x, y)$ is parallel to the line $3y - 2x - 1 = 0$. Determine the;

(a) coordinates of B . (09 marks)

(b) equation of the tangent at B . (03 marks)

13. (a) Use Maclaurin's theorem to expand $\ln(1 - 2x)$ in ascending powers of x as far as the term in x^3 . (06 marks)

(b) Using small changes, find the approximate value of $\tan 46^\circ$ correct to three decimal places. (06 marks)

$$\begin{aligned} x(x^2 - 13) - 12 &= 0 \\ (x-12) &= 0 & (x^2 - 13) &= 0 \\ x &= 12 & \sqrt{x^2} &= 13 \end{aligned}$$

14. (a) The point C in the complex plane corresponds to the complex number z such that $3|z - 2| = |z - 6i|$. Show that the locus of C is a circle. (05 marks)
- (b) Find the square root of $-5 + 12i$. (07 marks)

15. The coordinates of points P and Q are $(0, 2, 5)$ and $(-1, 3, 1)$ respectively.

Given that the equation of the line T is $\frac{x-3}{2} = \frac{2-y}{2} = 2-z$;

- (a) find the equation of a plane which contains the point P and is perpendicular to the line T . (03 marks)
- (b) show that the point Q lies on the plane. (02 marks)
- (c) determine the coordinates of the point R where the line T intersects with the plane. (04 marks)
- (d) show that PR and QR are perpendicular. (03 marks)
16. The rate at which the quantity M of a commodity is sold is proportional to the difference between the amount initially present and the quantity sold at any time t . Initially 10 tonnes of the commodity were present. After one day, 2 tonnes were sold.
- (a) Form a differential equation for the quantity of the commodity sold. (02 marks)
- (b) (i) Determine the expression for M in terms of t . (08 marks)
- (ii) Calculate the quantity sold at the end of 5 days. (02 marks)

$$\frac{(2x-6) + 2i(y-3)}{(2+2i)(x-3)} = 3$$

4

$$\frac{\sqrt{x^2 + y^2 - 2^2}}{3\sqrt{(x+y-2)^2}} = \frac{\sqrt{x^2 + y^2 - 6i}}{x+y-6i}$$

END