PROPOSED MARKING GUIDE UACE 2024
APPLIED MATHEMATICS UMTA
P425/2

<table>
<tr><th>NO</th><th>SOLUTION</th><th>MKS</th><th>COMMENT</th></tr>
<tr><td>1</td><td>Let E =Economy class
G= Good night
$\mathrm{P}(E) = 0.82$, $\mathrm{P}(G/E) = x$
$\mathrm{P}(E') = 0.18$, $\mathrm{P}(G/E') = 0.9$
a) $\mathrm{P}(G) = \mathrm{P}(E)\cdot\mathrm{P}(G/E) + \mathrm{P}(E')\cdot\mathrm{P}(G/E')$
$0.285 = 0.82x + 0.18 \times 0.9$
$0.285 = 0.82x + 0.162$
$0.82x = 0.123$
$x = \frac{3}{20}$ or 0.15
b) $\mathrm{P}(E/G') = \frac{P(E\cap G')}{P(G')}$
$= \frac{0.82\times0.85}{1-0.285}$
$= \frac{697}{715}$ or 0.9748</td><td></td><td></td></tr>
<tr><td></td><td></td><td>05</td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>05</td><td></td></tr>
<tr><td>3</td><td><table>
<tr><td>P_0</td><td>P_1</td><td>$\frac{P_1}{P_0} \times 100$</td></tr>
<tr><td>6,000</td><td>7,800</td><td>$\frac{7,800}{6,000} \times 100 = 130$</td></tr>
<tr><td>5,000</td><td>4,000</td><td>$\frac{4,000}{5,000} \times 100 = 80$</td></tr>
<tr><td>500</td><td>700</td><td>$\frac{700}{500} \times 100 = 140$</td></tr>
<tr><td>2,000</td><td>2,500</td><td>$\frac{2,500}{2,000} \times 100 = 125$</td></tr>
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$\text{S.P.I} = \frac{130+80+140+125}{4}$

$= 118.75$

Comment: the cost increased by 18.75%

05

4

Let $y = \frac{x}{2+\cos x}$

$h = \frac{1.0-0.2}{4} = 0.2$

x	y	
1.0	0.394	
1.2		0.508
1.4		0.645
1.6		0.812
1.8		1.015
2.0	1.263	
	1.657	**2.980**

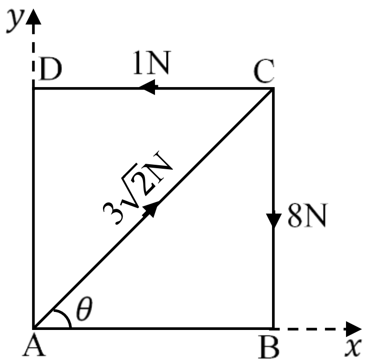
$$\int_{0.2}^{1.0} \frac{x}{2+\cos x} dx \approx \frac{1}{2} \times 0.2[1.657 + 2(2.980)]$$

$$\approx 0.7617$$

$$\approx 0.76(2\text{sfs})$$

05

5



$\tan\theta = \frac{a}{a}$

$\theta = \tan^{-1}(1)$

	$\theta = 45^0$ $(\rightarrow);\ F_x = 3\sqrt{2}\cos 45^0 - 1$ $= 2\text{N}$ $(\uparrow);\ F_y = 3\sqrt{2}\sin 45^0 - 8$ $= -5\text{ N}$ $\overset{\curvearrowleft}{\text{A}}$; $\text{G} = 1 \times a - 8 \times a = -7a$ From $G - xF_y + yF_x = 0$ $-7a + 5x + 2y = 0$ $\therefore 2y + 5x - 7a = 0$		
		05	
6	$f(x) = \frac{4}{9}(x - x^3)$ $f'(x) = \frac{4}{9}(1 - 3x^2)$ For mode, $f'(x) = 0$ $\frac{4}{9}(1 - 3x^2) = 0$ $x = \pm\frac{1}{\sqrt{3}}$ $f''(x) = \frac{4}{9}(-6x)$ $f''(x) = -\frac{8}{9}x$ When $x = \frac{1}{\sqrt{3}}; f''\left(\frac{1}{\sqrt{3}}\right) = -\frac{4}{9} \times \frac{1}{\sqrt{3}} = -\frac{4}{9\sqrt{3}}$ When $x = -\frac{1}{\sqrt{3}}; f''\left(-\frac{1}{\sqrt{3}}\right) = -\frac{4}{9} \times -\frac{1}{\sqrt{3}} = \frac{4}{9\sqrt{3}}$ Mode $= \frac{1}{\sqrt{3}}$, since $f''\left(\frac{1}{\sqrt{3}}\right) < 0$		
		05	

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Diagram

Parallel to the plane;

$F = \mu R + 49 \sin 35^0$ ………………………..(i)

Perpendicular to the plane;

$R = 49 \cos 35^0$ ………………(ii)

Putting (ii) into (i)

$F = 49\mu \cos 35^0 + 49 \sin 35^0$

But $\mu = \tan \theta$

$F = 49 \tan 20^0 \cos 35^0 + 49 \sin 35^0$

F= 42.7144 N

05

8

B	***A***
10	8
2	6
-	4
0	2

$\therefore A = 2$

05

9 a)

$\boldsymbol{c.b}$	$\boldsymbol{f}$	$\boldsymbol{x}$	$\boldsymbol{fx}$	$\boldsymbol{c.f}$	$\boldsymbol{c}$	$\boldsymbol{f.d}$
-0.5 – 9.5	5	4.5	22.5	5	10	0.5
9.5 – 14.5	8	12	96	13	5	1.6
14.5 – 19.5	32	17	544	45	5	6.4
19.5 – 29.5	41	24.5	1004.5	86	10	4.1
29.5 – 39.5	16	34.5	552	102	10	1.6
39.5 – 49.5	2	44.5	89	104	10	0.2
$\sum$	104		2308			

(i) Mean , $\bar{x} = \frac{\sum fx}{\sum f}$

$$= \frac{2308}{104}$$

$$= 22.1923$$

≈ 22 cigarettes

(ii)

19.5	25	29.5
45	A	86

$$\frac{A-45}{25-19.5} = \frac{86-45}{29.5-19.5}$$

$$A = 67.55$$

$$\approx 68$$

$\therefore$ 68 patients smoked 25 cigarettes and below

b) ***See graph***

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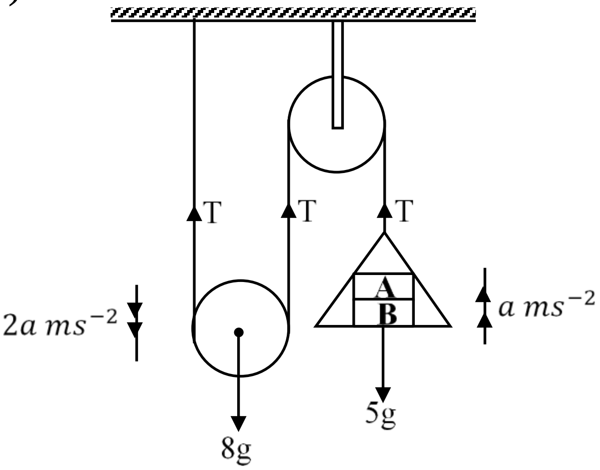
10 a) (i)

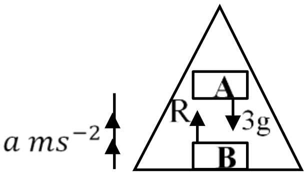


For 8 kg mass:

$$8g - 2T = 8 \times 2a$$

$4g - T = 8a$ ………………(i)

For 5 kg mass:

	$T - 5g = 5a$ ………………(ii) (i)+(ii); $3g = 13a$ $a = \frac{3\times9.8}{13}$ $= \frac{147}{65}\ ms^{-2}$ or 2.2615 ms^{-2} ∴ Acceleration of particle, C $= 2 \times \frac{3\times9.8}{13}$ $= \frac{294}{65}\ ms^{-2}$ or 4.5231 (ii) From (i); $4g - T = 8a$ $T = 4 \times 9.8 - 8 \times \frac{3\times9.8}{13}$ T $= \frac{1372}{65}$ N or 21.1077 N b)  $R - 3g = 3a$ R= $3(a+g)$ R $= 3\left(\frac{147}{65} + 9.8\right)$ R= $\frac{2352}{65}$ N or 36.1846 N		
		12	
11	Let $f(x) = x^2 - 3x + 1$ $f(2) = 2^2 - 3(2) + 1$ $= -1$		

$f(3) = 3^2 - 3(3) + 1$

$= 1$

$\therefore$ Since $f(2) \cdot f(3) < 0$, then 2<root<3

2	x_0	3
-1	0	1

$\frac{x_0-2}{0+1} = \frac{3-2}{1+1}$

$x_0 = 2.5$

$f(2.5) = (2.5)^2 - 3(2.5) + 1 = -0.25$

2.5	x_1	3
-0.25	0	1

$\frac{x_1-2.5}{0+0.25} = \frac{3-2.5}{1+0.25}$

$x_1 = 2.6$

$f(2.6) = (2.6)^2 - 3(2.6) + 1 = -0.04$

2.6	x_2	3
-0.04	0	1

$\frac{x_2-2.6}{0+0.04} = \frac{3-2.6}{1+0.04}$

$x_2 = 2.615$

$f(2.615) = (2.615)^2 - 3(2.615) + 1 = -0.006775$

2.615	x_3	3
-0.006775	0	1

$\frac{x_3-2.615}{0+0.006775} = \frac{3-2.615}{1+0.006775}$

$x_3 = 2.618$

$|x_3 - x_2| = 0.003 < 0.005$

$\therefore$ root = 2.62(2dps)

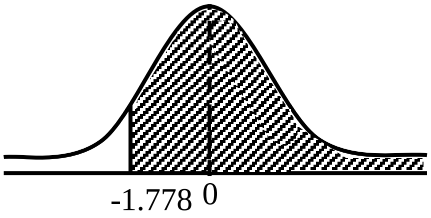
12

12 a) Let X be marks scored.

$\mu = 54, \delta = 9$

$P(X \geq 38) = P\left(z > \frac{38-54}{9}\right)$

$= P(z > -1.778)$



$= 0.5 + P(0 < z < 1.778)$

$= 0.5 + 0.4623$

$= 0.9623$

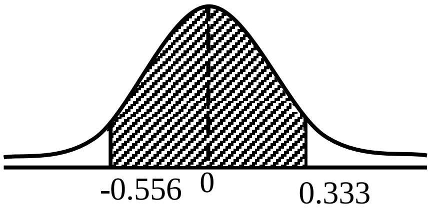
$\therefore$ Number of students $= 0.9623 \times 400$

$= 384.92$

≈ 385

b) $P(49 < X < 57) = P\left(\frac{49-54}{9} < z < \frac{57-54}{9}\right)$

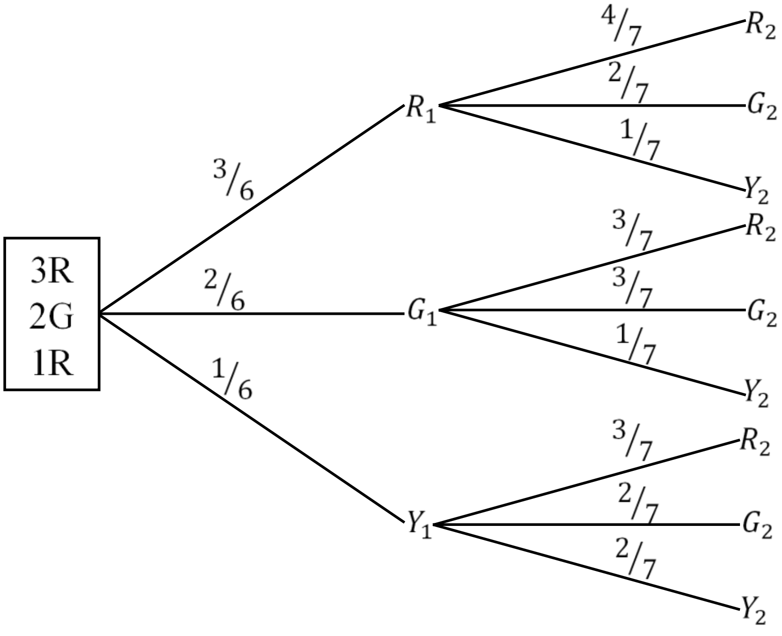
$= P(-0.556 < z < 0.333)$



$= P(0 < z < 0.556) + P(0 < z < 0.333)$

$= 0.2109 + 0.1304$

	$= 0.3413$ c) $p = 0.3413, q = 0.6587, n = 10$ Let Y be the number of students who scored between 49 and 57. $\mathrm{P}(Y \geq 1) = 1 - \mathrm{P}(Y = 0)$ $= 1 - {}^{10}C_0 \cdot (0.3413)^0 \cdot (0.6587)^{10}$ $= 1 - 0.0154$ $= 0.9846$		
		12	
13			
		12	
14	a) Let $T = X\sqrt{Y}$ Squaring both sides. $T^2 = X^2Y$ $(T + \Delta T)^2 = (X + \Delta X)^2(Y + \Delta Y)$ $T^2 + 2T\Delta T + (\Delta T)^2 = (X^2 + 2X\Delta X + (\Delta X)^2)(Y + \Delta Y)$ ΔX and ΔT are too small, then $(\Delta T)^2 \approx 0, (\Delta X)^2 \approx 0$ $T^2 + 2T\Delta T = X^2Y + X^2\Delta Y + 2XY\Delta X + 2X\Delta X\Delta Y$ ΔY is also too small, then $2X\Delta X\Delta Y \approx 0$ $2T\Delta T = X^2\Delta Y + 2XY\Delta X$ $\Delta T = \frac{X^2\Delta Y + 2XY\Delta X}{2T}$ R.E $= \frac{\Delta T}{T}$ R.E $= \frac{X^2\Delta Y + 2XY\Delta X}{2T^2}$		

	$= \frac{X^2\Delta Y+2XY\Delta X}{2X^2Y}$ $= \frac{\Delta \mathrm{Y}}{2Y}+\frac{\Delta X}{X}$ $\left\lvert\frac{\Delta T}{T}\right\rvert = \left\lvert\frac{\Delta \mathrm{Y}}{2Y}+\frac{\Delta X}{X}\right\rvert \leq \left\lvert\frac{\Delta X}{X}\right\rvert + \frac{1}{2}\left\lvert\frac{\Delta Y}{Y}\right\rvert$ $\therefore \left\lvert\frac{\Delta T}{T}\right\rvert_{max} = \left\lvert\frac{\Delta X}{X}\right\rvert + \frac{1}{2}\left\lvert\frac{\Delta Y}{Y}\right\rvert$ b) $\Delta X = 0.0005, \Delta Y = 0.05$ %age error $= \left\lvert\left\lvert\frac{\Delta X}{X}\right\rvert + \frac{1}{2}\left\lvert\frac{\Delta Y}{Y}\right\rvert\right\rvert \times 100$ $= \left(\frac{0.0005}{1.824}+\frac{1}{2}\left(\frac{0.05}{3.9}\right)\right)\times 100$ $= 0.6684\%$		
		12	
15	 a) A b) B		
		12	
16	a) Let k be weight per unit area. Area of square $= 8 \times 8 = 64\ cm^2$		

Area of right-angled triangle $= \frac{1}{2} \times 3 \times 4 = 6\ cm^2$

Area of semi-circle $= \frac{1}{2} \times \pi \times 2^2 = 2\pi\ cm^2$

Area of the remainder $= 64 - (6 + 2\pi)$

$= (58 - 2\pi)\ cm^2$

Figure	**Weight**	**Distance of C.O.G**	
		AB	**AD**
Square	$64k$	4	4
Right-angled	$6k$	$^{10}/_{3}$	3
Semi-circle	$2\pi k$	2	$\frac{8+12\pi}{3\pi}$
Remainder	$(58 - 2\pi)k$	$\bar{y}$	$\bar{x}$

Taking moments about AB;

$(58 - 2\pi)k \times \bar{x} = 64k \times 4 - 6k \times 3 - 2\pi k \times \frac{8+12\pi}{3\pi}$

$(58 - 2\pi)\bar{x} = 256 - 18 - \frac{16}{3} - 8\pi$

$(58 - 2\pi)\bar{x} = \frac{698}{3} - 8\pi$

$(58 - 2\pi)\bar{x} = \frac{698-24\pi}{3}$

$\bar{x} = \frac{698-24\pi}{174-6\pi}$

$\bar{x} = 4.0129\ cm$

Taking moments about AD;

$(58 - 2\pi)k \times \bar{y} = 64k \times 4 - 6k \times \frac{10}{3} - 2\pi k \times 2$

$(58 - 2\pi)\bar{y} = 256 - 20 - 4\pi$

$(58 - 2\pi)\bar{y} = 236 - 4\pi$

$\bar{y} = \frac{236-4\pi}{58-2\pi}$

$\bar{y} = 4.3203\ cm$

	$\therefore G(\bar{x}, \bar{y}) = G(4.0129, 4.3203)$ b) $\tan\theta = \frac{\bar{y}}{8-\bar{x}}$ $\tan\theta = \frac{4.3203}{8-4.0129}$ $\theta = \tan^{-1}\left(\frac{4.3203}{3.9871}\right)$ $\theta = 47.30^0$ $\therefore$ Required angle $= 90^0 - 47.30^0$ $= 42.70^0$		
		12	