P425/1
PURE MATHEMATICS
AUGUST - 2024
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS – AUGUST, 2024

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all the eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) will **not** be marked.*

*All working **must** be shown clearly.*

Begin each question on a fresh sheet of paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

 Turn Over

SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. Given that one root of the equation $4z^2 - 12z + p = 0$ is $k + 4i$, where k and p are real constants. Find the values of k and p. ***(5 marks)***

2. Solve the equation ; $2\sin(60^0 - x) = \sqrt{2}\cos(135^0 + x) + 1$

   for $-180^0 \leq x \leq 180^0$. ***(5 marks)***

3. If $y = \frac{\left(x-\frac{1}{3}\right)e^{2x}}{\cos x}$, find $\frac{dy}{dx}$. ***(5 marks)***

4. A circle of radius 5 units has its centre in the second quadrant on the line $x + y = 4$ and passes through point (3, 2). Find the equation of the circle. ***(5 marks)***

5. If $\log_8 x - \log_{16} x^2 + 3\log_{32} x = 2.6$, find the value of x. ***(5 marks)***

6. Show that $\int_0^3 x^2 \log_2(3x)\,dx = \frac{9}{2\ln 2}[4\ln 3 - 1]$. ***(5 marks)***

7. Find the acute angle between the lines $\frac{x-3}{5} = \frac{y+1}{-4} = \frac{z}{2}$ and

   $$y = \begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}1\\3\\2\end{pmatrix}$$ ***(5 marks)***

8. A shell is formed by rotating the area bounded by the portion of the parabola $y^2 = 4x$ for which $0 \leq x \leq 1$ and $y \geq 0$, through two right angles about the x-axis. Find the volume of the solid generated. ***(5 marks)***

SECTION B: (60 Marks)

Answer any five questions from this section. All questions carry equal marks.

9. (a.) When the quadratic expression $at^2 + bt + c$ is divided by $t + 1$, $t + 2$ and $t - 3$, the remainders are 6, 20 and 10 respectively. Find the values of a, b and c. *(7 marks)*

(b.) (i.) Simplify $\frac{2-\sqrt{3}+\sqrt{5}}{\sqrt{5}+\sqrt{3}}$.

(ii.) If $2x^2 + bx + 50 = 0$ has a repeated root, find the possible values of b.

(5 marks)

10. (a.) Prove that $\frac{\sin 2\theta - \cos 2\theta - 1}{2 - 2\sin 2\theta} = \frac{1}{\tan\theta - 1}$. *(3 marks)*

(b.) Solve $\sqrt{3}\cos 2\theta - \sin 2\theta + 1 = 0$, for $-180^0 \leq \theta \leq 180^0$. *(4 marks)*

(c.) Solve $\cos\theta - \cos 3\theta - \cos 7\theta + \cos 9\theta = 0$ for $0^0 \leq \theta \leq 90^0$. *(5 marks)*

1 **11.** (a.) Determine the equation of the normal to the curve $y = \frac{1}{x-2}$ at the point $\left(4, \frac{1}{2}\right)$. And find the coordinates of the other point where the normal meets the curve again.

(7 marks)

(b.) Find the maximum and minimum values of $x^2 e^{-x}$. *(5 marks)*

12. (a.) Find the integrals;

360+

(i.) $\int \ln\left(\frac{2}{x}\right)\, dx$.

360+120

480,

-240

(ii.) $\int (x\cos x)^2\, dx$. *(7 marks)*

(b.) Evaluate $\int_{-1}^{0} \frac{x}{\sqrt{1-3x}}\, dx$. *(5 marks)*

13. (a.) Find the ratio of the term in x^6 to the term in x^9 in the expansion of $(2x + 3)^{18}$ to its simplest form. *(3 marks)*

(b.) If the first four terms in the expansion of $(1 - x)^n$ are $1 - 6x + px^2 + qx^3$. Find the values of p and q. *(4 marks)*

(c.) The first term of an arithmetic progression is 5 and the common difference is 13. Find the sum to n terms of the progression. Hence find the least value of n for which the sum exceeds 1000. *(5 marks)*

$\overrightarrow{OR}$

Not popular

14. (a.) The line $y = x - c$ touches the ellipse $9x^2 + 16y^2 = 144$. Find the possible values of c and the coordinates of the points of contact. ***(7 marks)***

(b.) The point P lies on the ellipse $x^2 + 4y^2 = 1$ and N is the foot of the perpendicular from P to the line $x = 2$. Find the locus of the mid-point of PN as P moves on the ellipse. ***(5 marks)***

15. (a.) Given $\mathbf{OT} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$ and $\mathbf{OS} = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix}$. Find the coordinates of R such that TR : TS = 2 : 3. ***(5 marks)***

(b.) Show that the line $\frac{x-2}{4} = \frac{y-1}{9} = \frac{z+3}{5}$ is parallel to the plane $3x + 2y - 6z + 9 = 0$. Hence find the shortest distance between the line and the plane. ***(7 marks)***

16. (a.) Solve the differential equation $(x^2 + 1)\frac{dy}{dx} - xy = x$. given that y = 1 when x = 0. ***(4 marks)***

(b.) The rate of growth of the population of birds on an island is proportional to the number f birds present. If the birds doubled after 5 years. Form a differential equation for the rate of growth of the bird's population. Hence find after how long the birds will be 16 times the original number. ***(8 marks)***

$$3OR - \begin{pmatrix} 15 \\ 9 \\ -3 \end{pmatrix} = 2\begin{pmatrix} -3 \\ -3 \\ 8 \end{pmatrix}$$

$$OR = \begin{pmatrix} -6 \\ -6 \\ +16 \end{pmatrix} + \begin{pmatrix} 15 \\ 9 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 3 \\ -3 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

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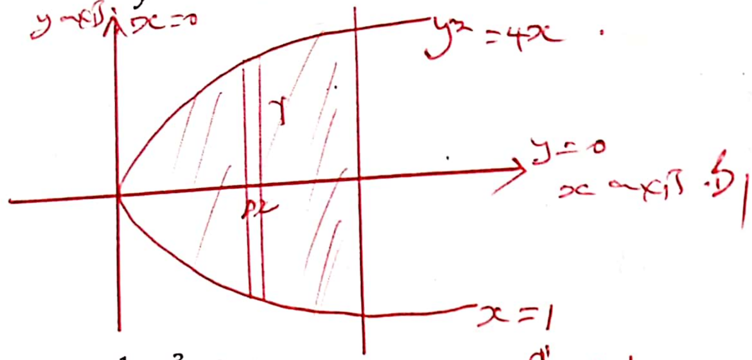
P425/1 - PURE MATHEMATICS

	SECTION A		
N0	Solutions	Marks	Comments
1	$4z^2 - 12z + p = 0$ $z = k + 4i \quad , z = k - 4i$ B1 $sum = 2k = \frac{12}{4} \quad , \quad k = \frac{3}{2}$ M1 $product = (k + 4i)(k - 4i) = \frac{p}{4}$ A1 $k^2 + 16 = \frac{p}{4}$ M1 $p = 4\left(\left(\frac{3}{2}\right)^2 + 16\right) = 9 + 64 = 73$ A1	B1 M1 A1 M1 A1	
2	$[2\sin(60^0 - x) = \sqrt{2}\cos(135^0 + x) + 1]$ $2[\sin60\cos x - \cos60\sin x] = \sqrt{2}[\cos135\cos x - \sin135\sin x] + 1$ M1 $2\left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right] = \sqrt{2}\left[\frac{-1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right] + 1$ M1 $(\sqrt{3} + 1)\cos x = 1$ $\cos x = \frac{1}{\sqrt{3}+1}$ A1 $x = 68.53^0$ M1 $, -68.53^0$ A1	 M1 M1 A1 M1 A1	
3	$y = \frac{\left(x - \frac{1}{3}\right)e^{2x}}{\cos x}$ $\ln y = \ln\left(x - \frac{1}{3}\right) + 2x - \ln\cos x$ M1 $= \ln\left(\frac{3x - 1}{3}\right) + 2x - \ln\cos x$ $\frac{1}{y}\frac{dy}{dx} = \frac{3}{3x - 1} + 2 + \frac{\sin x}{\cos x}$ M1 $= \frac{3\cos x + 2(3x - 1)\cos x + (3x - 1)\sin x}{(3x - 1)\cos x}$ M1 $\frac{dy}{dx} = \frac{3\cos x + 2(3x - 1)\cos x + (3x - 1)\sin x}{(3x - 1)\cos x} \cdot \frac{\left(x - \frac{1}{3}\right)e^{2x}}{\cos x}$ M1	 M1 M1 M1	

	$= \dfrac{3\cos x + 2(3x-1)\cos x + (3x-1)\sin x}{(3x-1)\cos x} \cdot \dfrac{(3x-1)e^{2x}}{3\cos x}$ M1 $= \dfrac{(1+6x)\cos x - (3x-1)\sin x\, e^{2x}}{3\cos^2 x}$ A1	M1 A1	
4	$x+y=4$, (3, 2) Let centre be (a, b) , radius = 5 a + b = 4 , b = 4 - a $(a-3)^2 + (4-a-2)^2 = 5^2$ $(a-3)^2 + (2-a)^2 = 25$ M1 $a^2 - 6a + 9 + 4 - 4a + a^2 = 25$ $2a^2 - 10a - 12 = 0$ $(a+1)(a-6) = 0$ $a = -1$, 6 A1 There fore $a = -1$ $b = 4 - -1 = 5 = \cancel{25}$ 5 Centre $(-1, 5)$ radius = 5 B1 Equation of circle $(x+1)^2 + (y-5)^2$ M1 = 25 $x^2 + y^2 + 2x - 10y + 1 = 0$ A1	M1 A1 B1 M1 A1	
5	$log_8 x - log_{16} x^2 + 3log_{32} x = 2.6$ $\dfrac{log_2 x}{log_2 8} - \dfrac{2log_2 x}{log_2 16} + \dfrac{3log_2 x}{log_2 32} = 2.6$ M1 $\left(\frac{1}{3} - \frac{2}{4} + \frac{3}{5}\right) log_2 x = 2.6$ M1 $\left(\frac{20-30+36}{60}\right) log_2 x = 2.6$	M1 M1	

	$log_2 x = 2.6\left(\frac{60}{26}\right)$ M1 $log_2 x = 6$ $x = 2^6 = 64$ A1	M1 M1 A1	
6	$\int_0^3 x^2 log_2(3x)\, dx$ $= \frac{1}{\ln 2}\int_0^3 x^2 \ln(3x)\ dx$ M1 $u = \ln(3x)\ ,\ \frac{du}{dx} = \frac{1}{x}\quad \frac{dv}{dx} = x^2\ ,\quad v = \frac{x^3}{3}$ $= \frac{1}{\ln 2}\left[\frac{x^3}{3}\ln(3x) - \int \frac{x^2}{3}\, dx\right]_0^3$ M1 $= \frac{1}{\ln 2}\left[\frac{x^3}{3}\ln(3x) - \frac{x^3}{9}\right]_0^3$ A1 $= \frac{1}{\ln 2}\left[9\ln 9 - \frac{9}{2} - 0\right]$ $= \frac{1}{ln2}\left[18ln3 - \frac{9}{2}\right]$ M1 $= \frac{3}{\ln 2}(6\ln 3 - 1)$ $= \frac{9}{2\ln 2}[2\ln 3 - 1]$ A1 $\frac{9}{2\ln 2}[4\ln 3 - 1]$	M1 M1 A1 M1 A1	
7	$\frac{x-3}{5} = \frac{y+1}{-4} = \frac{z}{2}\ ,\quad y = \begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}1\\3\\2\end{pmatrix}$ $b_1 = \begin{pmatrix}5\\-4\\2\end{pmatrix}\ ,\quad b_2 = \begin{pmatrix}1\\3\\2\end{pmatrix}$ B1 $\lvert b_1\rvert = \sqrt{25+16+4} = \sqrt{45}$ M1 $,\ \lvert b_2\rvert = \sqrt{1+9+4} = \sqrt{14}$ $b_1 . b_2 = \lvert b_1\rvert\lvert b_2\rvert\, cos\theta$ $\begin{pmatrix}5\\-4\\2\end{pmatrix}.\begin{pmatrix}1\\3\\2\end{pmatrix} = \sqrt{45}\sqrt{14} cos\theta$ M1 $cos\theta = \frac{\lvert 5-12+4\rvert}{\sqrt{45}\sqrt{14}} = \frac{3}{\sqrt{45}\sqrt{14}}$ M1 $\theta = 83.14^0$ A1	B1 M1 M1 M1 A1	only one correct answer.

	$0 \le x \le 1$		
8	$y^2 = 4x \quad , \quad 1 \le x \le 3 \quad , \quad y \ge 0$  vol. $= \frac{1}{2}\int_1^3 Y^2\ dx$ $X = \frac{1}{2}\int_0^1 \pi y^2 dx$ M1 $= \frac{1}{2}\int_1^3 4x\ dx$ $= \frac{\pi}{2}\int_0^1 4x\ dx$ A1 $= \frac{1}{2}\left[\frac{4x^2}{2}\right]_1^3$ $= \frac{\pi}{2}\left(2x^2\right)_0^1$ M1 $= \frac{1}{2}(18-2)$ $= \pi$ A1 $= 8$	B1 M1 A1 M1 A1	
	SECTION B		
9	(a.) $f(t) = at^2 + bt + c$ $f(-1) = a - b + c = 6$ $a - b + c = 6$ ---B1---------- (i) $f(-2) = 4a - 2b + c = 20$ $4a - 2b + c = 20$ --B1------- (ii) $f(3) = 9a + 3b + c = 10$ $9a + 3b + c = 10$ --B1--------- (iii) (ii) - (i) $3a - b = 14$ ------------ (iv) (iii) - (ii) $5a + 5b = -10$ --M1--------- (v) 5(iv) + (v) $20a = 60$ $a = 3$ A1 $b = 9 - 14 = -5$ A1 $c = 6 - 3 - 5 = -2$ A1	B1 B1 B1 M1 A1 A1 A1	
	(b.) (i.) $\frac{2-\sqrt{3}+\sqrt{5}}{\sqrt{5}+\sqrt{3}}$ $\frac{(2-\sqrt{3}+\sqrt{5})}{(\sqrt{5}+\sqrt{3})}\ \frac{(\sqrt{3}-\sqrt{5})}{(\sqrt{5}-\sqrt{3})}$ M1 $\frac{2\sqrt{5}-2\sqrt{3}-\sqrt{15}+3+5-\sqrt{15}}{5-3}$ M1 $\frac{8-2\sqrt{3}}{2}$ $= \frac{8+2\sqrt{2}-2\sqrt{15}-2\sqrt{3}}{2}$ $4 - \sqrt{3}$ M $= 4+\sqrt{5}-\sqrt{15}-\sqrt{3}$	M1 M1 A1	

	(ii.)	$2x^2 + bx + 50 = 0$ For repeated roots $b^2 = 4(2)(50)$ M1 $b^2 = 400$ $b = -20\ ,\ 20$ A1	M1 A1	
10	(a)	$\frac{sin2\theta - cos2\theta - 1}{2 - 2sin2\theta} = \frac{1}{tan\theta - 1}$ LHS $= \frac{2sin\theta cos\theta - 2cos^2\theta}{2 - 4cos\theta sin\theta}$ m1 $= \frac{2[sin\theta cos\theta - cos^2\theta]}{2[1 - 2cos\theta sin\theta]}$ $= \frac{cos\theta[sin\theta - cos\theta]}{cos^2\theta + sin^2\theta - 2cos\theta sin\theta}$ $\frac{\cos\theta[\sin\theta - \cos\theta]}{(\sin\theta - \cos\theta)^2} = \frac{cos\theta[sin\theta - cos\theta]}{(cos\theta - sin\theta)^2}$ m1 $= \frac{cos\theta}{sin\theta - cos\theta}$ $= \frac{1}{tan\theta - 1}$ B1 $\frac{\cos\theta}{\sin\theta - \cos\theta}$ $\frac{\cos\theta(\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}$ $\frac{\cos\theta \sin\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}$ $= \frac{\frac{1}{2}\sin 2\theta - \frac{1}{2}(1 + \cos 2\theta)}{1 - \sin 2\theta}$ $= \frac{\sin 2\theta - \cos 2\theta - 1}{2 - 2\sin 2\theta}$	 M1 M1 B1	
	(b.)	$\sqrt{3}cos2\theta - sin2\theta + 1 = 0$ $t = tan\theta$ $\sqrt{3}\ \frac{1 - t^2}{1 + t^2} - \frac{2t}{1 + t^2} + 1 = 0$ m1 $\sqrt{3} - \sqrt{3}t^2 - 2t + 1 + t^2 = 0$ $(1 - \sqrt{3})t^2 - 2t + \sqrt{3} + 1 = 0$ $t = \frac{2 \pm \sqrt{4 - 4(1 - \sqrt{3})(1 + \sqrt{3})}}{2(1 - \sqrt{3})}$ m1 $t = -0.7321,\ \ 1$ A1 (4dps) $tan\theta = -0.7321\ ,\ 1$ $\theta = \cancel{143.79^0}, \cancel{-36.21^0},\ \ 45^0,\ \ 135^0$ A1 $\sin 2\theta - \sqrt{3}\cos 2\theta = 1$ $R = 2$ $\alpha = 60$ $2\sin(2\theta - 60) = 1$ $\sin(2\theta - 60) = \frac{1}{2}$ $(2\theta - 60) = 30^0, 150^0, -210^0, -330^0$ $2\theta = 90^0, 210^0, -150^0, -270^0$ $\theta = \{45^0, 105^0, -75^0, -135^0\}$	M1 M1 A1 A1	For R formula. $\{+120^0, 240^0, -120^0, -240^0\}$. $\theta = \{45^0, 105^0, -75^0, -135^0\}$. ~~Atleast~~ ~~Atmost~~ Atleast 1 dp.
	(c.)	$cos\theta - cos3\theta - cos7\theta + cos9\theta = 0$ $cos9\theta + cos\theta - (cos7\theta + cos3\theta) = 0$ $2cos5\theta cos4\theta - 2cos5\theta cos2\theta = 0$ M1 $2cos5\theta[cos4\theta - cos2\theta] = 0$ $2cos5\theta\ .\ -2sin3\theta sin3\theta sin\theta = 0$ m1 $2\sin(60 - 2\theta) = -1$ $\sin(60 - 2\theta) = -\frac{1}{2}$	M1 M1	

$2\cos 5\theta = 0,\quad \cos 4\theta - \cos 2\theta = 0$

$2\cos^2 2\theta - \cos 2\theta - 1 = 0$

360

	$-4\cos 5\theta \sin 3\theta \sin\theta = 0$ $\cos 5\theta = 0, \quad \sin 3\theta = 0, \quad \sin\theta = 0$ m1 $\theta = 18^0, \quad 30^0, 54^0, 90^0$ A1 A1 18^0 54^0; $0^0, 60^0, 90^0$	M1 A1 A1	
11	(a.) $y = \frac{1}{x-2}$ $y = (x-2)^{-1}$ $\frac{dy}{dx} = -(x-2)^{-2} = \frac{-1}{(x-2)^{+2}}$ m1 At $\left(4, \frac{1}{2}\right)$, $\frac{dy}{dx} = \frac{-1}{(4-2)^2} = \frac{-1}{4}$ Gradient of the normal $= 4$ A1 $\quad m_1 m_2 = -1$, $-\frac{1}{4} m_2 = -1$, $m_2 = 4$ Equation of the normal $\frac{y-\frac{1}{2}}{x-4} = 4$ m1 $y - \frac{1}{2} = 4x - 16$ $8x - 2y - 31 = 0$ A1 Solving $y = \frac{1}{x-2}$ and $8x - 2y - 31 = 0$ simultaneously $8x - 2\left(\frac{1}{x-2}\right) - 31 = 0$ m1 $8x^2 - 47x + 60 = 0$ $x = \frac{47 \pm \sqrt{47^2 - 4.8.60}}{2.8}$ m1 $x = 4, \frac{15}{8}$ Therefore $x = \frac{15}{8}$, $y = \frac{1}{\frac{15}{8} - 2} = -8$ The other point is $\left(\frac{15}{8}, -8\right)$. A1	M1 A1 M1 A1 M1 M1 A1	$\frac{dy}{dx} = (x-2)^{-1}$ $\frac{-1(x-2)^{-2} \cdot 1}{}$ $\frac{-1}{(x-2)^2}$
	(b.) $y = x^2 e^{-x}$ $\frac{dy}{dx} = 2xe^{-x} - x^2 e^{-x} = xe^{-x}(2-x)$ m1	M1	

For maximum or minimum $\frac{dy}{dx} = 0$

$$xe^{-x}(2-x) = 0$$ M1 — M1

$$x = 0 \quad , \quad x = 2$$ A1 — A1

$$\frac{d^2y}{dx^2} = e^{-x}(2-3x)$$

When $x = 0$, $\frac{d^2y}{dx^2} = 2$ *positive*, min.

When $x = 2$, $\frac{d^2y}{dx^2} = -5e^{-2}$ *negative*, max.

Minimum value $= 0$ A1

Maximum value $+4e^{-2} = +0.54134$ A1 — A1 A1

2-6

$y = x^2e^{-x}$ $\quad y = 4e^{-2}$

$y = 0$

12 (a.) (i.) $\int ln\left(\frac{2}{x}\right) dx$

$$u = \ln\left(\frac{2}{x}\right) \quad , \quad \frac{dv}{dx} = 1$$

$$\frac{du}{dx} = \left(\frac{x}{2} \cdot \frac{-2}{x^2}\right) = \frac{-1}{x} \quad , \quad v = x$$ M1 — M1

$$\int ln\left(\frac{2}{x}\right) dx = xln\left(\frac{2}{x}\right) + \int 1\, dx$$ M1 — M1

$$= xln\left(\frac{2}{x}\right) + x + c$$ A1 — A1

(ii.) $\int (x \cos x)^2\, dx$

$$= \int x^2\cos^2 x\, dx$$ M1

$$u = x^2 \quad , \quad \frac{dv}{dx} = \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$ M1

$$\frac{du}{dx} = 2x \quad , \quad v = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right)$$ — M1

$\int (x \cos x)^2\, dx$

$$= \frac{1}{2}x^3 + \frac{1}{4}x^2 sin2x - \frac{x^3}{3} + \frac{1}{4}xcos\,2x - \int \frac{1}{4}cos\,2x\,dx$$ M1 — M1

$$= \frac{5}{6}x^3 + \frac{1}{4}x^2 sin\,2x + \frac{1}{4}xcos\,2x - \frac{1}{8}sin\,2x + c$$ A1 — A1

(b.) $\int_{-1}^{0} \frac{x}{\sqrt{1-3x}}\, dx$

	$u = 1 - 3x \qquad \frac{du}{dx} = -3$ M1 $x = 0, \quad u = 1$ $x = -1, \quad u = 4$	M1	
	$\int_4^1 \frac{(1-u)/3}{\sqrt{u}} \cdot \frac{-1}{3}\, du$ M1	M1	
	$\int_4^1 \frac{u^{\frac{1}{2}} - u^{\frac{-1}{2}}}{9}\, du$		
	$\frac{1}{9}\left[\frac{2u^{\frac{3}{2}}}{3} - 2u^{\frac{1}{2}}\right]_4^1$ A1	A1	
	$\frac{1}{9}\left[\left(\frac{2}{3} - 2\right) - \left(\frac{16}{3} - 4\right)\right]$ M1	M1	
	$= \frac{-8}{27}$ A1	A1	
13	*(a)* $(2x+3)^{18}$ $Ax^6 \quad : \quad Bx^9$		
	$18_{C_6}(2x)^6 . 3^{12} \quad : \quad 18_{C_9}(2x)^9 . 3^9$ M1 $18564 . 64 . 3^{12} . x^6 \quad : \quad 48620 . 2^9 . 3^9 . x^9$ $21 . 3^3 . x^6 \quad : \quad 55 . 2^3 . x^9$ M1	M1 M1	
	$567\, x^6 \quad : \quad 440\, x^9$ $567 \quad : \quad 440\, x^3$ A1	A1	
	(b.) $(1-x)^n = 1 - 6x + px^2 + qx^3$		
	$1 - nx + \frac{n(n-1)(-x)^2}{2!} + \frac{n(n-1)(n-2)(-x)^3}{3!}$ M1	M1	
	$1 - nx + \frac{n(n-1)x^2}{2} - \frac{n(n-1)(n-1)x^3}{6}$		

		$-nx = -6x$ $n = 6$ A1 $p = \dfrac{6(6-1)}{2} = 3\ .\ 5 = 15$ A1 $q = \dfrac{-6(6-1)(6-2)}{6} = -6\ .\ 5\ .\ 4 = -120$ A1	A1 A1 A1	
	(c.)	$a = 5\ ,\ d = 13$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2\ .\ 5 + (n-1)13]$ $= \frac{n}{2}(13n-3)$ B1 $S_n > 1000$ $\frac{n}{2}(13n-3) > 1000$ M1 $13n^2 - 3n > 2000$ $n > \dfrac{3 \pm\sqrt{3^2 - 4\ .\ 13\ .-12}}{2\ .\ 13}$ M1 $n > 12.52$ A1 $n = 13$ B1	B1 M1 M1 A1 B1	
14	(a.)	$9x^2 + 16y^2 = 144$ $y = x - c$ $9x^2 + 16(x-c)^2 = 144$ $9x^2 + 16x^2 - 32cx + 16c^2 = 144$ M1 $25x^2 - 32cx + 16c^2 - 144 = 0$ *For tangency* $(32c)^2 - 4\ .\ 25\,(16c^2 - 144) = 0$ M1 $1024c^2 - 1600c^2 + 14400 = 0$ $576c^2 = 14400$ $c^2 = 25$ $c = \pm 5$ A1	M1 M1	

	$25x^2 \pm 160x + 256 = 0$	~~A1~~	
	$(5x \pm 16)^2 = 0$ m1	M1	
	$x = \pm\frac{16}{5}$	A1	
	$y = \pm\frac{16}{5} \mp 5$		
	$y = \pm\frac{41}{5}$ A1	A1	
	$\left(\frac{16}{5}, \frac{41}{5}\right)$, $\left(\frac{-16}{5}, \frac{-41}{5}\right)$ -1.8 A1 B1	B1	
	(b.) $x^2 + 4y^2 = 1$	B1	
	let $P(x,y)$, $N(2,y)$ B1		
	mid $PN = \left(\frac{x+2}{2}, \frac{y+y}{2}\right)$	M1	
	$(X, Y) = \left(\frac{x+2}{2}, y\right)$ m1		
	$X = \frac{x+2}{2} \leftrightarrow x = 2X - 2$ m1	M1	
	$Y = y \leftrightarrow y = Y$		
	$x^2 + 4y^2 = 1$		
	$(2X-2)^2 + 4(Y)^2 = 1$ m1	M1	
	$4(X-1)^2 + 4(Y)^2 = 1$		
	Therefore the locus is $4(x-1)^2 + 4y^2 = 1$ A1	A1	
15	(a.) $\overrightarrow{OT} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$, $\overrightarrow{OS} = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix}$		
	$TR : TS = 2 : 3$	~~M1~~	
	$\frac{TR}{TS} = \frac{2}{3}$		
	$3TR = 2TS$		
	$3(OR - OT) = 2(OS - OT)$ m1	M1	
	$3OR = 2OS - 2OT + 3OT$		
	$= 2OS + OT$		

		$= 2\begin{pmatrix}5\\3\\-1\end{pmatrix} + \begin{pmatrix}2\\0\\7\end{pmatrix} = \begin{pmatrix}12\\6\\5\end{pmatrix}$ m1	M1	
		$OR = \frac{1}{3}\begin{pmatrix}12\\6\\5\end{pmatrix} = \begin{pmatrix}4\\2\\\frac{5}{3}\end{pmatrix}$ A1 $\begin{pmatrix}12\\6\\5\end{pmatrix}$	A1	
		$R\left(4,\ 2,\ \frac{5}{3}\right)$ B1 $\begin{pmatrix}3\\1\\13/3\end{pmatrix}$ 4	B1	
	(b.)	$\frac{x-2}{4} = \frac{y-1}{9} = \frac{z+3}{5}$, $3x + 2y - 6z + 9 = 0$		
		$\mathbf{b} = \begin{pmatrix}4\\9\\5\end{pmatrix}$, $\mathbf{n} = \begin{pmatrix}3\\2\\-6\end{pmatrix}$ B1	B1	$0 = 0$ B0
		$\mathbf{b}\cdot\mathbf{n} = \begin{pmatrix}4\\9\\5\end{pmatrix}\cdot\begin{pmatrix}3\\2\\-6\end{pmatrix}$ m1	M1	
		$= 12 + 18 - 30$ $= 0$ A1	A1	$12 - 18$
		Therefore the line is parallel to the plane. B1	B1	
		Shortest distance $= \frac{\lvert 3(2)+2(1)+6(3)\rvert}{\sqrt{3^2+2^2+6^2}}$ m1 $\frac{\lvert 3(2)+2(1)+6(3)\rvert}{\sqrt{3^2+2^2+6^2}}$	M1	
		$= \frac{26}{\sqrt{39}}$ m1 $= \frac{26+9}{\sqrt{49}} = \frac{35}{7}$	M1	
		$= \frac{2(13)}{\sqrt{3}\sqrt{13}}$ m1		
		$= \frac{2}{3}\sqrt{39} = 4.1633.$ A1 $= 5$	A1	
16	(a.)	$(x^2+1)\frac{dy}{dx} - xy = x$		
		$(x^2+1)\frac{dy}{dx} = xy + x$		
		$(x^2+1)\frac{dy}{dx} = x(y+1)$	M1	
		$\int \frac{dy}{y+1} = \int \frac{x}{x^2+1}\,dx$ m1		
		$ln(y+1) = \frac{1}{2}ln(x^2+1) + c$ A1	A1	
		$y = 1,\quad x = 0$		

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	$$ln2 = \frac{1}{2}ln1 + c$$ $$c = ln2$$ B1 $$ln(y+1) = \frac{1}{2}ln(x^2+1) + ln2$$ $$ln(y+1) = ln2(x^2+1)^{1/2}$$ $$y + 1 = 2(x^2+1)^{1/2}$$ A1 $$y = 2(x^2+1)^{1/2} - 1$$	B1 A1	
Correction	(b.) P — population of birds P_0 — initial population of birds t — time in years $$\frac{dp}{dt} \infty p$$ $$\frac{dp}{dt} = kp$$ B1 $$\int \frac{dp}{p} = \int k\,dt$$ M1 $$lnp = kt + c$$ A1 $$t = 0,\ p = p_0$$ $$lnp_0 = c$$ B1 $$ln\left(\frac{p}{p_0}\right) = kt$$ $$t = 5, \quad p = 2p_0$$ $$ln\left(\frac{2p_0}{p_0}\right) = 5k$$ $$k = \frac{1}{5}ln2$$ B1 $$ln\left(\frac{p}{p_0}\right) = \frac{t}{5}ln2$$ B1 $$p = 16p_0$$ $$ln\left(\frac{16p_0}{p_0}\right) = \frac{t}{5}ln2$$ M1 $$t = \frac{5ln16}{ln2} = \frac{20\,ln2}{ln\,2} = 20\ years$$ A1	B1 M1 A1 B1 B1 M1 A1	

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