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DIFFERENTIAL EQUATIONS.

Formation of DEs

1. $y = Ae^{3x} + Be^{-3x}$.
2. $y = acos2t + bsin2t$.
3. $y = e^{x}sinx$.
4. $y = e^{-2x}cosx$.
5. $y = Ae^{2x} + Be^{-x}$.
6. $x = e^{2t}(A + Bt)$.

Separable DEs

7. $\frac{dy}{dx} = y$
8. $\frac{dy}{dx} = \frac{2x-1}{y}$ given $y(0) = 2$.
9. $\frac{dy}{dx} = \frac{1-3y}{x+1}$ given that $y(2) = 0$.
10. $(x+2)\frac{dy}{dx} = y$
11. $\frac{dy}{dx} = (1+x)(1+y^2)$
12. $\frac{dv}{du} = v(v-1)$
13. $x\frac{dy}{dx} = x - 1$
14. $\frac{dy}{d\theta} = \tan y \tan \theta$
15. $e^t\frac{dx}{dt} = \sin t$
16. $(1+\cos 2\theta)\frac{dy}{d\theta} = 2,\ y\left(\frac{\pi}{4}\right) = 1$
17. $\frac{dy}{dx} = x(y-2)\ \ x = 0, y = 5$

Exact DEs

18. $x\frac{dy}{dx} + y = x^3$
19. $x^2\frac{dy}{dx} + 2xy = 1$
20. $2yx\frac{dy}{dx} + y^2 = e^{2x}$
21. $cosx\frac{dy}{dx} + ysinx = 4x$
22. $e^{x}y^2 + 2ye^{x}\frac{dy}{dx} = -cosec^2x$

Non exact DEs / linear DEs

23. $\frac{dy}{dx} + 3y = e^{2x}$
24. $\frac{dR}{dt} = e^{2t} + t$.
25. $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ given that $y(1) = 3$
26. $x\frac{dy}{dx} - 2y = 0$
27. $x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$
28. $tanx\frac{dy}{dx} - y = sin^2x$
29. $\frac{dy}{dx} + 2ytanx = cos^2x$: $y(0) = 2$
30. $\frac{dr}{d\theta} + 2r\cot\theta = cosec^2\theta$
31. $\frac{dy}{dx} + ycotx = cosx$.
32. $\frac{dy}{dx} + 2xy = x$.
33. $\frac{dy}{dx} - ytanx = x$.
34. $\frac{dy}{dx} + y + 3 = 2x$.

Homogeneous DEs.

35. $x^2\frac{dy}{dx} = y^2$.
36. $x\frac{dy}{dx} + y = x$.
37. $xy\frac{dy}{dx} = x^2 + y^2$.
38. $x\frac{dy}{dx} = x - y$.
39. $x^2\frac{dy}{dx} = y^2 + xy$.
40. $xy\frac{dy}{dx} = x^2 - y^2$.
41. $x^2\frac{dy}{dx} = y^2 + xy + x^2$.
42. $(x^2+y^2)\frac{dy}{dx} = xy$.
43. $x^2\frac{dy}{dx} = x^2 - y^2$.
44. $(x^2-y^2)\frac{dy}{dx} = xy$.
45. $x^2\frac{dy}{dx} = x^2 - xy + y^2$.
46. $(x+y)\frac{dy}{dx} = x - y, y(3) = -2$.

Special cases.

47. $\frac{dy}{dx} = \frac{2x+y-2}{2x+y+1}$.
48. $(x+y)\frac{dy}{dx} = x + y - 2$.
49. $\frac{dy}{dx} = \frac{x-y+1}{x-y+3}$.
50. $(2x-3y+3)\frac{dy}{dx} = 2x - 2y + 1$,

    use $z = 2x - 3y$
51. use $4x + y = z$ to solve

    $\frac{dy}{dx} = 4x + y$

Word problems.

52. The rate of cooling of a kettle of hot water is proportional to the difference between it's temperature and that of the room. When room temperature is 15^0c, the temperature of the kettle reduces from 95^oc to 55^0c in ten minutes. Find the time taken for the kettle to cool from 55^oc to 25^0c.
53. In a culture of bacteria, the rate of growth is proportional to the population present at a time t. the population doubles every day. Given that the initial P_0 is one million. Determine the number of days when the population will be 100millions.
54. A student walks to school at a speed proportional to the square root of the distance he still has to cover. If the student covered 900m in 100minutes and the school is 2500m from home, find how long he takes to get to school.
55. An athlete runs at a speed proportional to the square root of the distance he still has to cover. If the athlete starts running at 10 ms^{-1} and has a distance of 1600 m to cover, find how long he will take to cover the distance.
56. The rate of growth of a disease causing virus increases at a rate proportional to the number of virus present in the body if the number increases from 1000 to 2000 in 1 hour. How many days will be present after 1 and half hours' (ii)how long will it take the number of virus in the body to be 4000
57. North korea launched a missile. The acceleration of the missile, t seconds after it was fired is $(30t - 10)ms^{-2}$.Also at $t = 0, x = 0$ and $v = 2ms^{-1}$. Obtain the expression in terms of t, for velocity v, attained and determine, x travelled in t seconds. (ii) find the velocity of missile at the end of 5 seconds and the distance travelled up to the first 2 seconds.
58. A clinical thermometer whose reading is 25°c is placed in the mouth of the patient , the mouth of the temperature is T°c and its constant. The temperature Θ^0c indicated by the thermometer rises at a rate proportional to $T - \theta$ given that $\beta, \alpha,$ and λ are successive readings of Θ at equal intervals of time t, show that $T = \frac{\beta^2-\alpha\lambda}{2\beta-\alpha-\beta}$.

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Share a copy to every math tr to help his/her students.

c/o Tr. kabuzi

DIFFERENTIATION TOPICAL REVISION

Differentiating from first principles

1. x^2 and $\frac{2}{x^2}$
2. x^3
3. $\sqrt{x}$ and $\frac{1}{\sqrt{x}}$
4. $\cos x$ and $\sin x$
5. $\sin 2x$ and $\cos 3x$
6. $x^2 + \cos 3x$
7. $\cos^2 2x$,
8. $2x + \tan x$.
9. $\tan^{-1} x$.

Chain rule

10. $x = t^2, y = 4t - 1$
11. $y = 3t^2 + 2t, x = 1 - 2t$
12. $x = 2\sqrt{2}$, $y = 5t - 4$
13. $x = \frac{1}{t}$, $y = t^2 + 4t - 3$
14. $x = \frac{2}{3+\sqrt{t}}$, $y = \sqrt{t}$

Product rule

15. $(x^2+1)(x^3+2)$
16. $x^2(x+1)^3$
17. $(1+x)^{3/2}(x-1)^{5/4}$
18. $(x-1)\sqrt{x^2+1}$
19. $\sqrt{(x+1)(x-2)^3}$
20. $(x-1)^2\sqrt[3]{1-2x}$

Quotient rule

21. $\frac{x^2+1}{x^2-1}$
22. $\frac{x}{\sqrt{x^2+1}}$
23. $\sqrt{\frac{(x+2)^3}{x-1}}$
24. $\sqrt{\frac{(x+1)^3}{x+2}}$

Implicit functions

25. $x^2 + 2xy + y^2 = 8$
26. $x^2 - 3xy + y^2 - 2y + 4x = 0$
27. $3x^2 - 4xy = 7$
28. $x^2 + 3xy - y^2 = 0$
29. $x^3 - y^3 - 4x^2 + 3y = 11x + 4$

exponential functions

30. a) $4e^x$ b) e^{-2x}
31. c) e^{ax^2+b}
32. d) $e^{\sqrt{\cos x}}$ e) e^{xe^x}
33. f) $e^{\tan x^2}$
34. g) $e^{\sqrt{x^2}+1}$ h) $e^{-\cot x}$
35. a^x, $2^x, 3^x, 5^x$

logarithmic functions

36. $\ln(2x^3)$
37. $\ln(x^3+1)$
38. $\ln \sec x$
39. $\ln\left(\frac{1+\cos x}{1-\sin x}\right)$
40. $\frac{\ln x}{\sqrt{1+x^2}}$
41. $3x \ln x^2$
42. $\ln\cos x$
43. $\ln(\sec x + \tan x)$
44. $\ln\frac{(x+1)^2}{\sqrt{x-1}}$
45. $\frac{dy}{dx}(\ln x\sqrt{x^2-1})$
46. $\ln \sin^2 x$ **(b)** $\ln\tan(3x)$
47. $\ln 3\cos^2 x$ **(d)** $\ln\left(\frac{(x+1)^2}{x-1}\right)$
48. $\ln(x + \sqrt{(x^2-1)}$
49. $\sqrt[3]{\frac{x+1}{x-1}}$
50. $\frac{e^{x^2}\sqrt{\sin x}}{(2x+1)^3}$
51. x^x
52. $(\sin x)^x$
53. 2^x
54. $x10^{\sin x}$
55. $\ln(x)^x$
56. $x^{\sin x}$

inverse trigonometric fxns

57. $\cos^{-1} x$ b) $\sin^{-1} x$
58. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
59. $\tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
60. $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

Proofs

61. If $y = e^{2x}\cos 3x$ show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 13y = 0$
62. $y = xe^{-x}$ Show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$
63. Given that $y = \sin\sqrt{x}$, prove that $2\frac{dy}{dx} + y + 4x\frac{d^2y}{dx^2} = 0$
64. If $y = \tan xy$, prove that $\frac{dy}{dx} = \frac{y}{\cos^2 xy - x}$
65. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$. Show that $\frac{dy}{dx} = \frac{1}{1+x^2}$
66. if $e^x = \tan 2y$, then $\frac{d^2y}{dx^2} = \frac{e^x - e^{3x}}{2(1+e^{2x})^2}$.
67. If $x = \sin\theta$ and $y = 1 - \cos\theta$, show that $\left(\frac{d^2y}{dx^2}\right)^2 = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3$
68. If $y = \tan\left[2\tan^{-1}\left(\frac{x}{2}\right)\right]$ show that $\frac{dy}{dx} = \frac{4(1+y^2)}{(4+x^2)}$.
69. If $x = \theta - \sin\theta$, $y = 1 - \cos\theta$, show that $\frac{dy}{dx} = \cot\left(\frac{\theta}{2}\right)$.
70. If $y^x = x^y$, show that $\frac{dy}{dx} = \frac{y(x\ln y - y)}{x(y\ln x - x)}$.
71. If $e^x = \ln(x+y)$,Show that $\frac{d^2y}{dx^2} = (1+e^x)(1+\frac{dy}{dx})$.
72. If $y = \theta - \cos\theta$, $x = \sin\theta$. Show that $\frac{d^2y}{dx^2} = \frac{1+\sin\theta}{\cos^3\theta}$
73. Show that $\frac{d(\tan^{-1}x^x)}{dx} = \frac{(1+\ln x)x^x}{1+x^{2x}}$.

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There is still time to change your maths grade

INEQUALITIES

Simple inequalities.

1. $(0.6)^{2x} < 3.6.$
2. $(0.8)^{3x} > 4.$
3. $(0.7)^{-3t} < 4.2.$
4. $(0.4)^{-3x} < 3.6.$
5. $(0.8)^{2x} > 5$
6. $(0.8)^{-2x} > 4.$
7. $(0.3)^{2x} < 4.$

Quadratic inequalities.

8. $x^2 - 4x + 3 < 0$
9. $x^2 - x - 6 > 0$
10. $x^2 + x - 12 \leq 0$
11. $x^2 + 2x - 15 \geq 0$
12. $(x+2)(x-4) < x^2 - 6$
13. $2x^2 + 4x \geq x^2 + 5x + 6$
14. $4x^2 - 5x - 6 < 0$
15. $2x^2 + 5x - 3 < 0$

Modulus inequalities.

15. $|2x - 3| < 5.$
16. $|x + 3| > 4$
17. $|x + 3| \geq 2x$
18. $|2x + 1| > x + 5$
19. $|4x + 2| \leq x + 8$
20. $|x + 1| > |x - 3|$
21. $|2x - 3| > |x + 3|$
22. $|x - 3| < |2x + 5|.$
23. $|x + 3a| > |x - 2a|.$
24. $|x - 2| > 3|2x + 1|.$
25. $\left|\frac{x}{x-3}\right| < 2$
26. $\left|\frac{x}{x-3}\right| > x$
27. $\left|\frac{2x-4}{x+1}\right| \leq 4$

Rational inequalities.

28. $\frac{x+3}{x-1} > 2.$
29. $\frac{x-1}{x-3} \geq 2.$
30. $x < \frac{8}{x-2}$
31. $x \leq \frac{2}{x+1}.$
32. $x - 2 < \frac{6-x}{x}$
33. $\frac{3x^2-1}{x+2} \geq 2.$
34. $\frac{x^2-4}{x-1} \geq 0.$
35. $\frac{x+2}{x-2} + 1 \leq 0.$
36. $\frac{1}{x+1} \geq \frac{2}{x-3}.$
37. $\frac{12}{x-3} \leq x + 1.$
38. $\frac{2}{x-2} \leq \frac{1}{x+1}.$
39. $\frac{x+3}{x-2} > \frac{x+1}{x-3}.$
40. $\frac{x+1}{x-1} < \frac{x+3}{x+2}.$
41. $\frac{3x}{x-8} < \frac{2x-1}{x-5}.$
42. $\frac{(x-2)(x-1)}{(x-3)(x-4)} > 0.$
43. $\frac{x-2}{x+1} > \frac{x+1}{x+3}.$
44. $\frac{x-1}{2x^2-x-3} \leq 1.$
45. $\frac{x}{x-1} > \frac{1}{x+1}.$
46. $\frac{2}{x-3} < \frac{1}{1+x}.$
47. $\frac{x^2-4}{x-1} \geq 0.$
48. $\frac{x+2}{x-2} + 1 \leq 0.$
49. $\frac{(x+1)(x-2)}{(x-1)(x+2)} \geq 0.$
50. $\frac{x+1}{2x-1} \leq \frac{1}{x-3}.$

VECTOR PROBLEMS

Points on a triangle

1. show that the points (3,3,1), (8,7,4) and (11,4,5) are vertices of the triangle.
2. Show that if A (4,10,6) B (6,8, -2) C (1,10,3) are vertices of the triangle ABC, is a right angle.
3. Given that A (0, 5, -3), B (2, 3, -4) and C (1, -1, 2). Find the coordinates of D if ABCD is a rectangle or parallelogram.
4. Find the area of the triangle with vertices $A(1,30^0), B(2,60^0), C(3,90^0)$.

Ratio theorem

5. A point C(a,4,5) divide the line joining A (1,2,3) and B (6,7,8) in the ratio of α:3. Find the values of a *and* α.
6. Given points A (-3, -3), D (9,5) and B such that D divides AB in the ratio of 4:3. Find the coordinates of B if D divides AB (i) internally (ii) Externally.

Centroid, circumcenter and orthocenter

7. find the centroid of triangle whose vertices P (3,7,4), Q (1,2,3) R (2,0,5).
8. The triangle with vertices A (-2,1), B (3, -4) C (-6, -1) find its orthocenter and circumcenter.

Equation of a line

9. Find the vector and Cartesian equations of a line passing through the following points
(5, -4, 6) and (3, 7, 2)
(3, 4, -7) and (5, 1, 6)

Point on a line

10. Show that 4i – j – 12k lies on the line $\boldsymbol{r} = 2i + 3j + 4k + \lambda(i - 2j + 4k)$
11. The points A, B, C have position vectors $\begin{pmatrix}-4\\5\\-1\end{pmatrix}, \begin{pmatrix}5\\2\\3\end{pmatrix}, \begin{pmatrix}8\\1\\7\end{pmatrix}$. Find which of the three points lie in the line $\boldsymbol{r} = \begin{pmatrix}-1\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\2\end{pmatrix}$

Angle between two lines

12. $\boldsymbol{r} = 3i + 2j - 4k + \lambda(i + 2j + 2k)$
$\boldsymbol{r} = 5i - 2j + \mu(3i + 2j + 6k)$
13. $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-2}{-1}$ and $\frac{1-x}{2} = \frac{y-3}{1} = \frac{z-7}{2}$

Point of Intersection of two Lines

14. $\frac{x}{1} = \frac{y+2}{2} = \frac{z-5}{-1}$ & $\frac{x-1}{-1} = \frac{y+3}{-3} = \frac{z-4}{1}$
15. $\boldsymbol{r} = i - 2j + 3k + \lambda(2i + j - k)$
$\boldsymbol{r} = -i + 3j + 7k + \mu(-2i + j + 2k)$
16. show that the lines $r = i + 3j + k + \beta(-i - 2j + k)$ and $r = 2i - 3j + 4k + \alpha(3i - 2j + k)$ intersect

SKEW LINES

shortest distance between the following pairs of skew lines

17. $\frac{x-2}{0} = \frac{y+1}{1} = \frac{z}{2}$ and $\frac{x+1}{1} = \frac{y-1}{-3} = \frac{z-1}{-2}$
18. $r = \begin{pmatrix}-1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $r = \begin{pmatrix}0\\-1\\1\end{pmatrix} + \mu\begin{pmatrix}2\\1\\3\end{pmatrix}$

Length and the equation of the perpendicular drawn from the point

19. (2, 3, -4) to the line and $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$
20. (3, 0, 1) from line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z}{12}$.

Shortest distance between two parallel lines

$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-3}{2}$ and

21. $\frac{x+1}{1} = \frac{y-3}{-1} = \frac{z-1}{2}$
22. $r = \begin{pmatrix}2\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$
$r = \begin{pmatrix}1\\-1\\4\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\2\end{pmatrix}$

Equation of a Plane

23. Find the equation of a plane passing through (1, 2, 3), and is perpendicular to vector $4\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$
24. Find the equation of a plane passing through a point A with a position vector $-2\boldsymbol{i} + 4\boldsymbol{k}$ and is perpendicular to the vector $\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$.
25. Find the equation of a plane passing through points P (4, 2, 3), Q (5, 1, 4) and R (-2, 1, 1).
26. Find the Cartesian equation of the plane passing through the points A (1, 0, -2), B (3, -1, 1) parallel to the line
$r = 3i + (2\alpha - 1)j + (5 - \alpha)k$
27. determine in dot product form the equation of the plane containing the point (3,1,2) and the line $x - 1 = \frac{z-3}{3}, y = 2$.
28. Find the equation of the plane containing line $r = \begin{pmatrix}1\\-1\\0\end{pmatrix} + t\begin{pmatrix}-2\\1\\-1\end{pmatrix}$ and is parallel to the line $r = \begin{pmatrix}2\\1\\1\end{pmatrix} + S\begin{pmatrix}-1\\1\\2\end{pmatrix}$
29. Find the Cartesian equation of a plane passing through A (0, 3, -4) B (2, -1, 2) and C (7, 4, -1)
30. Find the Cartesian equation of the plane formed by the lines **r** = -2i + 5j – 11k + λ(3i + j + 3k) and **r** = 8i + 9j + λ(4i + 2j + 5k)

Angle between two planes

Find the angle between the planes

31. $2x + 3y + 5z = 7$ and $3x + 4y - z = 8$
32. $3x - 3y - z = 0$ and $x + 4y - 2z = 4$

Angle between a line and a plane

33. $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k} + \mu(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ and the plane $2x - y + z = 4$
34. $\frac{x-1}{-1} = \frac{y-8}{1} = \frac{z-3}{-2}$ and $7x - y + 5z = -5$

Point of intersection of a line and a plane

35. $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1}$ and $x + y + z = 19$
36. $\frac{x}{5} = \frac{y+2}{2} = \frac{z-1}{4}$ and $3x + 4y + 2z = 25$
37. $\frac{x+2}{-1} = \frac{y-2}{2} = z - 4$ and $2x - y + 3z = 10$

Line of intersection of two planes

38. $2x + 3y + 4z = 1$ and $x + y + 3z = 0$
39. $2x + 3y - z = 4$ and $x - y + 2z = 5$.

Perpendicular distance of a point from a plane

40. $(-2, 0, 6)$ from the plane $2x - y + 3z = 21$

Shortest Distance between Parallel Planes

41. $2x + 5y - 14z = 30$ and $2x + 5y - 14z = -15$
42. $x + 2y - z = -4$ *and* $x + 2y - z = 3$

CURVE SKETCHING II.

Curves with unknown

1. The curve $y = ax^3 + bx^2 + c$ has turning points at (0,4) and (-1,5). Determine the values of a,b and c. hence sketch the curve.
2. The curve with the equation $y = \frac{ax+b}{x(x+2)}$ where a and b are constants has a turning point at $(1, -2)$. Find values of a and b, (ii)find the equation of the asymptotes and hence sketch the curve

Curves with horizontal asymptotes

3. A curve is given by $y = \frac{x^2+2}{x^2-1}$. (i)determine the region where the curve is entirely contained. (ii)determine the turning points of the curve. (iii)state the asymptotes and intercept of the curve and sketch it.
4. Show that the curve $y = \frac{3x-9}{(x-2)(x+1)}$ doesn't lie between $\frac{1}{3} < y < 3$, determine the turning points and hence sketch the curve
5. Given that $y = \frac{4x-10}{x^2-4}$ find the range of values where the curve doesn't lie hence determine the stationary points of the curve state the equations of the three asymptotes of the curve , sketch the curve
6. Given that $y = \frac{4x-10}{x^2-4}$ is a curve. (i)find the range of y within which the curve doesn't lie. (ii)determine the stationary points of the curve. (iii)state the equation of asymptotes and sketch the curve.
7. Given that $y = \frac{x^2-6x+9}{x^2+x-2}$. Find the range of values of y where the curve doesn't exist for real x. hence determine the stationery points of the curve and determine the nature and sketch the curve.
8. Sketch the curve $y = \frac{x+1}{(x-1)(2x+1)}$. Showing clearly the nature of turning points.
9. A curve is given by $= \frac{(x+1)(x-3)}{x(x-2)}$. Show that for real x, y the curve can't lie between 1 and 4. (ii)hence determine the turning points and distinguish between them. (iii)state the asymptotes and hence sketch the curve.

Curves with slanting asymptotes

10. A curve is given by $y = \frac{x^2-7x+10}{x-6}$. Show that for real x, y cannot take on the values between 1 and 9. Hence find the turning points of the curve and sketch it
11. Sketch the curve $y = \frac{x^2-5x+6}{x-1}$ showing any asymptotes. Find the are enclosed by the curve, x-axis from $x = 2\ to\ x = 3$.
12. Given that $y = \frac{(x-2)^2}{x+1}$. Determine turning points of y, and hence determine the region where the curve is not defined. Sketch the curve.
13. Sketch the curve $y = \frac{x^2-6x+5}{2x-1}$

Curves without turning points

14. Given that the curve $y = \frac{x(x-3)}{(x-1)(x-4)}$. (i) show that the curve doesn't have turning points. (ii)find the asymptotes for the curve and sketch the curve.
15. The function $f(x), h(x) and\ g(x)$ are defined by $f(x) = x^2 - 1$, $g(x) = 2x + 1$ and $h(x) = \frac{f(x)}{g(x)}$. Show that $h(x)$ has no turning points. Obtain the asymptotes to the function $h(x)$ and sketch the curve.
16. Show that the curve $y = \frac{x+2}{x(x+3)}$ has no turning points, find the equation of the asymptotes and hence sketch the curve

Curves with unfactorizable denominators

17. A curve $y = \frac{2}{1+x^2}$. Determine the nature of the turning points on the curve, hence sketch the curve.
18. A curve $y = \frac{4}{2+x^2}$. Determine the nature of the turning points on the curve, hence sketch the curve.
19. A curve $y = \frac{2-3x}{x^2+3x+3}$. Determine the nature of the turning points on the curve, hence sketch the curve

Parametric equation of a curve

20. Curve parametrically given by $x = 2t$ and $y = \frac{2}{t} + 1$. Find the cartesian equation of the curve and sketch the curve
21. Find the Cartesian equation of the curve $x = \frac{1+t}{1-t}$ and $y = \frac{2t^2}{1-t}$ Hence sketch the curve ythe Cartesian equation of the curve
22. The curve is parametrically given by $x = 3t + 4$ and $y = \frac{t^2-t}{t+1}$. Find the cartesian equation for the curve and hence sketch it.

Curve and its inverse

23. Sketch the curve $f(x) = x^2(x + 2)$ and hence sketch the inverse of the curve.
24. Sketch the graph of $f(x) = 4 + 3x - x^2$ hence sketch the graph of $\frac{1}{f(x)}$
25. Show that $f(x) = \frac{x(x-5)}{(x-3)(x+2)}$ has no turning points. Sketch $y = f(x)$. If $g(x) = \frac{1}{f(x)}$, sketch $f(x)$ and $g(x)$ on the same axes

LINE AND CIRCLES

Centre and radius

1. Find the center and the radius of the circles below
   (i) $(x-1)^2+(y-2)^2=9$
   (ii) $(x+1)^2+(y-3)^2=25$
   (iii) $x^2+y^2-4x-2y=4$
   (iv) $2x^2+2y^2-2x+2y=1$
2. Find the equation of the circle with the following centers and radii
   (a) Center (2, 3) radius 1
   (b) Center (3, -4) radius 5
   (c) Center $\left(\frac{-3}{2},2\right)$ and radius $\frac{1}{2}$

equation of a circle

3. Find the equation of a circle whose center is (2, 1) and passes through (4, -3)
4. Find the equation of a circle passing through points (2, 3) and (4, 5) having its center on the line $y=4x+3$
5. The points (8, 4) and (2, 2) are end points of the diameter of the circle. Find the center, the radius and the equation of the circle
6. Find the equation of the circle whose center lies on the $x-2y+2=0$ which touches the positive axes.
7. Find the equation of the circle passing through the points
   a) A(-2, 1) B(6, 1) and C(-2, 7)
   b) A(-1, 4) B(2, 5) and C(0, 1)
   c) A(3, 1) B(8, 2) and C(2, 6)

Parametric eqautions of circle

8. Find the parametric equation of the circle
   a) $(x-4)^2+(y-3)^2=4$
   b) $(x+1)^2+(y-2)^2=9$
   c) $x^2+y^2-4x-2y+1=0$
   d) $x^2+y^2-6x+4y-12=0$
9. Find the Cartesian equation of the circle with parametric equations
   (i) $x=-2+3\cos\theta$ and $y=3+3\sin\theta$
   (ii) $x=2+2\cos\theta, y=1+2\sin\theta$
10. Given that $r=3cos\theta$ is an equation of a circle. Find its Cartesian form.

Length ofTangent to a circle

11. Find the length of the tangent from $x^2+y^2-4x-6y+9=0$ to (5,7) $x^2+y^2+6x+10y-2=0$, $(-2,3)$

Equation of tangent

12. Find the equation of the tangent to the circle $x^2+y^2+2x-2y-8=0$ at $(2,2)$
13. Find the equation of the tangent to the circle $2x^2+2y^2-8x-5y-1=0$ at C (1, -1)
14. Find the equation of the tangents to the circle $x^2+y^2-6x+4y-12=0$ which are parallel to the line $4x+3y+5=0$
15. The tangent to the circle $x^2+y^2-4x+6y-77=0$ at the point (5, 6) meets the axes at A and B. find A and B
16. Show that $y=mx+c$ is a tangent to the circle $x^2+y^2=a^2$
17. if $c^2=a^2(1+m^2)$
18. Show that the line $x+3y-1=0$ touches the circle $x^2+y^2-3x-3y+2=0$
19. For what values of c will the line $y=2x+c$ be tangent to the circle $x^2+y^2=5^2$
20. For what values of α, does the line $3x+4y=\alpha$ touch the circle $x^2+y^2-10x=0$?
21. Find the equation of the tangents to the circle $x^2+y^2-2x-4y-4=0$ which are parallel to line $3x-4y-1=0$
22. (ii) Which are perpendicular to the line $3x-4y-1=0$

Equation of common chord

23. Find the equation of the common chord of the circles
24. $x^2+y^2-4x-2y+1=0$
25. $x^2+y^2+4x-16y-10=0$
26. Show that the common chord of the circles $x^2+y^2=4$ and $x^2+y^2-4x-2y-4=0$ passes through the origin
27. Determine the equation of the chord of the two intersecting circles $x^2+y^2-4x-2y-31=0$ and $2x^2+2y^2-6x+8y-35=0$ and prove that the common chord is perpendicular to the line joining the two centres of the circles.
28. Find the point of intersection of the circles $x^2+y^2+2x+2y-23=0$ and $x^2+y^2-10x-7y+31=0$

Internal & external intersection

29. Prove that the circles $x^2+y^2-10x-7y+31=0$ and $x^2+y^2+2x+2y-23=0$ touch each other externally.
30. Prove that the following pairs of the circle touch each other externally or internally.
    (a) $x^2+y^2-2x=0$ and $x^2+y^2-8x+12=0$.
    (b) $x^2+y^2-2x-2y=18$ and $x^2+y^2-14x-8y+60=0$.
    (c) $x^2+y^2-12x-2y=12$ and $x^2+y^2-4x+4y+4=0$.
    (d) $x^2+y^2-4x+2y=8$ and $x^2+y^2+6x-13y+22=0$

Concentric circles

31. Prove that the circle $x^2+y^2-4x-6y=0$ and $x^2+y^2-4x-6y=3$ are concentric. Find the radius of the common center.

Orthogonal circles

32. Prove that the circles $x^2+y^2+4x-2y-11=0$ and $x^2+y^2-4x-8y+11=0$ are orthogonal
33. Show that the following pair of circle are orthogonal:
    $x^2+y^2-6x-8y+9=0, x^2+y^2=9$
34. Prove that if the circles $x^2+y^2-2a_1x-2b_1y+c_1=0$ and $x^2+y^2-2a_2x-2b_2y+c_2=0$, then $c_1+c_2=2a_1a_2+2b_1b_2$

Locus

35. What is the locus of point which is equidistant from the origin (0, 0) and the point (-2, 5)
36. Find the locus of a point which is equidistant from the line $x=-1$ and the origin.
37. A point p is twice as far from the line $x+y=5$ as from the point (3, 0). Find the locus of P.
38. Find the locus of a point which moves so that the sum of squares of its distances from $(-2,0)$ and
39. (2, 0) is 26
40. Find the locus of the point P which moves so that its distance from the point $(5,0)$ is a half its distance from the line $x-8=0$

SERIES AND PROOF INDUCTION 2023.

Arithmetic Progression.

1. The second term of an AP is -4 and the sixth term is 24. Find the fifteenth term and the sum of the first fifteen terms.
2. In an AP, the third term is -2, the sum of the first two terms is four times the second terms and the sum of the first n terms is -64. Find the value of n.
3. In an AP, the fourth term is 12. The difference between the 60^{th} and 50^{th} term is 25. Find the sum of the first 61 terms of an AP.
4. In an AP, the third term is -2, the sum of the first two terms is four times the second terms and the sum of the first n terms is -64. Find the value of n.
5. Show that $ln2^r$, $r \geq 1$ is an AP and hence find the sum of the first 10 terms of the progression.
6. Find the common difference, the nth term and the sum to $n - terms$ of the A.P given by $In\ 3 + \ln(3^2) + \ln(3^3) + \dots$

Geometric Progression.

7. In the GP, the difference between the fourth and the second term is 156. The difference between the fourth and the seventh term is 1404. Find the possible values of common ratio.
8. The first term of the GP is 3, the last term is 768. If the sum of the terms is 1533. Find the common ratio and the number of terms in the progression.
9. In the GP, the difference between the fourth and the second term is 156. The difference between the fourth and the seventh term is 1404. Find the possible values of common ratio.
10. The first term of the GP is $\sqrt{3} - 1$ and the sum of the first three terms is $3(\sqrt{3} - 1)$. Find the common ratio of the progression.
11. The sum to infinity $S\infty = 10$ and the first term is 7.5 of the GP. Find the common ratio and deduce the ratio of the 2^{nd} and the 5^{th} term.
12. Prove that. $1 - tan^2x + tan^4x + \dots = cos2x.$
13. How many term are in a geometric progression $2 + 4 + 8 + \dots + 128$

Sum of AP and GP

14. The first term of the GP is 3, the last term is 768. If the sum of the terms is 1533. Find the common ratio and the number of terms in the progression.
15. The angle of the triangle form a GP. If the smallest angle 20^0 determine the largest
16. The sum of the first n -terms of a certain progression is $n^2 + 5n$ for all integral value of n. find the first three terms and prove that the progression is an AP.
17. The sum of the first n-terms of a certain series is given by $S_n = \frac{1}{3}n(n^2 - 1)$. Find an expression for the nth term, hence find the fourth term.
18. Kabuzi deposited shs.900,000 at the beginning of each year from 2010 with a compound interest of 12% per annum. How much will he receive at the end of 2014

Combination AP and GP.

17. The sum of the first terms of an AP and GP is 12. The sum of the second terms of the terms of the same AP and GP is 27. The sum of the third terms of the same AP and GP is 66. If they have the same first term, find the sum of the fourth term.
18. The second, fourth and eighth term of An AP are in GP. If the sum of the third and the fifth term is 20. Find the sum of the first 4 terms of the progression.
19. The fourth term, seventh, sixteenth terms of an AP are in the GP. If the first six terms of the AP have the sum of 12, find the common difference of an AP and the common ratio of the GP.
20. The 2nd,$4th$ & $8th$ terms of an AP are in a GP. If the sum of the 3^{rd} & 5^{th} term is 20, find the sum of the first four terms.
21. The sum of the first terms of an AP and GP is 57. The sum of the second terms of the same AP and GP is 94. The sum of the third terms of the AP &GP is 171. If the common ratio of the GP is 2. Find the sum of the fourth terms of the AP and GP.
22. A sequence of numbers is formed by adding together the corresponding terms of an AP and GP with a common ratio of 2. The first term of the sequence is 43, the second term is 60 and the third term is 97. Find the fourth term.

Arithmetic and geometric mean.

23. The geometric mean of two numbers $a\ and\ b$ is equal four-fifth of the arithmetic mean of the two numbers. If $a = 6$, find b.
24. Given that the geometric mean of the numbers $4x - 3$ and $9x + 4$ is $6x - 1$, find the values of x.

Proof by induction

Category 1

23. $3^{2n} - 1$ is multiple of 8.
24. $6^n - 1$ is divisible by 5
25. $10^{2n-1} + 1$ is divisible by 11.
26. $10^{2n-1} + 1$ is divisible by 11.
27. $9^n + 7$ divisible by 8
28. $5^n + 4n - 1$ is divisible by 8.
29. $3^{2n+2} - 8n - 9$ is factor of 64
30. $2^n + 3^{2n-3}$ is divisible by 7

Category 2

Prove by induction

23. $1.2 + 2.3 + 3.4 + \dots + n(n + 1) = \frac{1}{3}n(n + 1)(n + 2)$.
24. $\sum_1^n r^3 = \frac{n^2}{4}(n + 1)^2$.
25. $\sum_{r=1}^n (r + 1)(r + 5) = \frac{n}{2}(n + 7)(2n + 7)$.
26. $\sum_{r=1}^n \frac{1}{(3r-2)(3r+1)} = \frac{n}{3n+1}$.
27. $\frac{1}{1\times2} + \frac{1}{2\times3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

Tr. Kabuzi

INTEGRATION TOPICAL REVISION

Exponential functions

1. $\int e^{3x}dx$
2. $\int e^{2x}dx$
3. $\int e^{-5x}dx$
4. $\int e^{2-3x}dx$
5. $\int e^{2x+5}dx$
6. $\int a^{x}dx$
7. $\int 2^{x}dx$. $\int 3^{x}dx$. $\int 5^{x}dx$

Linear functions

8. $\int \frac{dx}{x+1}$
9. $\int \frac{dx}{2x-3}$
10. $\int \frac{dx}{2-5x}$
11. $\int \frac{dx}{1-x}$
12. $\int \frac{dx}{3x+2}$
13. $\int \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)dx$.
14. $\int \frac{dx}{1-x^2}$

Function and its derivative

15. $\int (3x-1)^7 dx$
16. $\int x(x^2-3)^5 dx$
17. $\int (2x-3)(x^2-3x+7)^4\, dx$
18. $\int \frac{2x}{(4x^2-7)^2}dx$
19. $\int \frac{x^2-1}{\sqrt{x^3-3x}}dx$
20. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$
21. $\int_{e}^{e^{x}} \frac{dx}{x\ln x}$
22. $\int_{0}^{1} \frac{\tan^{-1}x dx}{1+x^2}$.
23. $\int_{1}^{2} \frac{8x+6\ dx}{(2x-1)^2(x+2)^2}$.
24. $\int \frac{e^x}{4-e^{2x}}dx$.
25. $\int \cos x\sqrt{\sin x}\, dx$
26. $\int \cos x \sin x\, dx$.
27. $\int \sec^5 x \tan x\, dx$
28. $\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}dx$.
29. $\int \int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$
30. $\int x\, cosec^2 x^2 dx$
31. $\int_{1}^{\sqrt{5}} \frac{x^2}{\sqrt{x^4-x^2}}dx$.

Factor formulae

32. $\int 2\cos 3x \cos x\ dx$
33. $\int \cos 3x \cos 5x\, dx$
34. $\int_{0}^{\pi/3} 2\sin 3x \cos x\, dx$
35. $\int \sin x \sin 3x\, dx$

Odd and Even powers

36. $\int \cos^3 x\, dx$
37. $\int \sin^2 x\, dx$
38. $\int \cos^2 x\, dx$
39. $\int \sin^4 x\, dx$
40. $\int \sin^2 2x\, dx$
41. $\int \cos^2 3x\, dx$
42. $\int \cos^4 2x\, dx$
43. $\int \sin^3 2x\, dx$
44. $\int \cos^5 3x\, dx$
45. $\int \cos^2\frac{x}{2}\sin^3\frac{x}{2}dx$
46. $\int \sec x \tan^3 x\, dx$

Tangent, Sine and cosine substitutions

47. $\int \frac{1}{a^2+b^2x^2}dx = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)+C$
48. $\int \frac{1}{\sqrt{a^2-b^2x^2}}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)+C$
49. $\int \frac{1}{4+x^2}dx$
50. $\int \frac{1}{1+16x^2}dx$
51. $\int \frac{1}{3+4x^2}dx$
52. $\int \frac{x}{1+x^4}dx$
53. $\int \frac{2x^3}{16+x^8}dx$
54. $\int \frac{1}{(x^2+9)^2}dx$
55. $\int \frac{1}{x^2-2x+5}dx$
56. $\int \frac{1}{2x^2+4x+11}dx$
57. $\int \frac{1}{4x^2-8x+7}dx$
58. $\int \frac{1}{\sqrt{9-4x^2}}dx$
59. $\int_{0}^{\sqrt{3}} \frac{1}{\sqrt{3-x^2}}dx$
60. $\int \frac{1}{\sqrt{4-(x-1)^2}}dx$
61. $\int \frac{x^2}{\sqrt{1-x^2}}dx$
62. $\int \frac{x}{\sqrt{4-x^4}}dx$
63. $\int \frac{4x^2}{\sqrt{1-x^6}}dx$.

Integration by parts

64. $\int x\cos x\, dx$
65. $\int x^2e^x dx$.
66. $\int x^2\sin^2 x\, dx$
67. $\int_{0}^{\frac{\pi}{2}} x^2 \sin x\, dx$
68. $\int x(\ln x)dx$
69. $\int \frac{(\ln x)}{x^2}dx$
70. $\int_{1}^{10} x\log_{10} x\, dx$
71. $\int x10^x\, dx$
72. $\int x\sin 2x\cos 2x\, dx$
73. $\int e^x \cos x\, dx$
74. $\int \tan^{-1}x\, dx$
75. $\int \sin^{-1}x\, dx$

t-formulae in integration

76. $\int \frac{1}{(1+\cos\theta)}d\theta$

    $\int \sec 2\theta\, d\theta$
77. $\int \frac{dx}{1+\sin x+\cos x}$.
78. $\int cosec\frac{x}{2}dx$ $\qquad \int \frac{1}{1+\sin 2x}dx$
79. $\int \frac{x}{1+\cos x^2}dx$.
80. $\int \frac{1}{5+3\cos\frac{1}{2}\theta}d\theta$ $\qquad \int \frac{1}{1+2\sin^2 x}dx$
81. $\int \frac{1}{(\cos 2x-3\sin^2 x)}\ dx$
82. $\int \frac{\sin^2 x}{1+\cos^2 x}dx$

Splitting numerator

83. $\int \frac{5x+7}{x^2+4x+8}dx$
84. $\int \frac{1+x}{1+x^2}dx$
85. $\int \frac{3x+4}{9x^2+6x+5}dx$
86. $\int \frac{2\cos x+9\sin x}{3\cos x+\sin x}dx$
87. $\int \frac{\sin x}{\cos x+\sin x}dx$
88. $\int \frac{x}{2x^2-x+1}dx$

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PARTIAL FRACTIONS

2023

Linear denominators

1. $\frac{3x+5}{(x-3)(2x+1)}$.
2. $\frac{3x}{(x-1)(x+2)}$.
3. $\frac{5x-12}{x(x-4)}$.
4. $\frac{x-1}{3x^2-11x+10}$.
5. $\frac{3x^2-21x+24}{(x+1)(x-2)(x-3)}$.
6. $\frac{32}{x^3-16x}$.
7. $\frac{68+11x}{(3+x)(16-x^2)}$.
8. $\frac{x-9}{x(x^2+2x-3)}$.
9. $\frac{1}{x^2-2x-3}$.
10. $\frac{3x-1}{x^2+x-6}$.
11. $\frac{x+8}{x^2+6x+8}$.
12. $\frac{2x+10}{(x-1)(x+2)(x+3)}$.
13. $\frac{6}{(x+3)(x-3)}$.
14. $\frac{x}{(2+x)(2-x)}$.
15. $\frac{x}{25-x^2}$.
16. $\frac{x-11}{(x+3)(x-4)}$.
17. $\frac{3x^2-21x+24}{(x+1)(x-2)(x-3)}$.
18. $\frac{3x+1}{x(x+1)}$.
19. $\frac{6x^2-26x+26}{(x-3)(x-2)(x-1)}$.
20. $\frac{x-9}{x(x-1)(x+3)}$.
21. $\frac{2y^2-y+14}{(4y^2-1)(y+3)}$.
22. $\frac{x+1}{3x^2-x-2}$.
23. $\frac{2x^2-5x+7}{(4x^2-9)(x+2)}$.
24. $\frac{2x^2+14x+10}{2x^2+9x+4}$.
25. $\frac{7x-8}{4x^2+3x-1}$.

Repeated denominators

26. $\frac{3x^3+x+1}{(x-2)(x+1)^3}$.
27. $\frac{3x^2+4x-1}{x^3+2x^2+x}$.
28. $\frac{36}{(x-1)^2(x+5)}$.
29. $\frac{6x}{(x-2)(x+4)^2}$.
30. $\frac{10+6x-3x^2}{(2x-1)(x+3)^2}$.
31. $\frac{3x+1}{(x-1)^2(x+2)}$.
32. $\frac{2(x+2)^2}{x^2(x^2+4)}$.
33. $\frac{1}{x^2(x+5)}$.
34. $\frac{3x+1}{(x+1)^2}$.
35. $\frac{2x+1}{(2x+3)^2}$.
36. $\frac{2x-1}{(x+2)^2}$.
37. $\frac{x^2-4}{(x+1)^2(x-5)}$.
38. $\frac{2x^2+3}{x^2(x-1)}$.
39. $\frac{5x^2+20x+6}{x^3+2x^2+x}$.
40. $\frac{5x+7}{x^2+4x+4}$.
41. $\frac{x^2-3x+1}{x^3-3x^2+2x}$.
42. $\frac{x+35}{x^2-25}$.
43. $\frac{2y^2-y+14}{(4y^2-1)(y+3)}$.
44. $\frac{x-3-2x^2}{x^2(x-1)}$.
45. $\frac{x+4}{(x+1)(x-2)^2}$.
46. $\frac{4x+3}{(x-1)^2}$.
47. $\frac{9x^2+4}{(2x+1)(x-2)^2}$.
48. $\frac{100}{x^2(10-x)}$.
49. $\frac{1+x}{(1-x)^2}$.
50. $\frac{3x+2}{(2x-1)^2(3-x)}$.
51.
52. $\frac{3x^4-x^2+1}{(x-2)^5}$.

Quadratic denominators.

53. $\frac{3x^2-2x+5}{(x-1)(x^2+5)}$.
54. $\frac{11x}{(2x-3)(2x^2+1)}$.
55. $\frac{6-3x}{(x+1)(x^2+3)}$.
56. $\frac{2x+1}{x^3-1}$.
57. $\frac{13x+7}{(x-4)(3x^2+2x+3)}$.
58. $\frac{5x}{(x-2)(x^2+x+1)}$.
59. $\frac{5x^2-6x-21}{(x-4)^2(2x-3)}$.
60. $\frac{4x^3+10x+4}{x(2x+1)}$.
61. $\frac{x^4+3x-1}{(x+2)(x-1)^2}$.
62. $\frac{2x^3+7x^2+2x-10}{(x+3)(2x-1)}$.
63. $\frac{3x+1}{(x-1)(x^2+1)}$.
64. $\frac{x^3}{x^2-4}$.
65. $\frac{1+x}{x^2(x^2+1)}$.
66. $\frac{24x^3(x-3)}{(x-1)(2x+1)}$.
67. $\frac{x^2}{x^4-1}$.
68. $\frac{x^2+6}{(x^2+4)(x^2+9)}$.
69. $\frac{6x^2-3x+1}{(4x+1)(x^2+1)}$.
70. $\frac{x^3-8x^2-1}{(x+3)(x-2)(x^2+1)}$.
71. $\frac{2(x+2)^2}{x^2(x^2+4)}$.
72. $\frac{6-9x}{27x^3+8}$.
73. $\frac{3+3x}{x^3-1}$.
74. $\frac{(x-2)^2}{x^3+1}$.
75. $\frac{x^2+x-3}{x^3+3x}$.
76. $\frac{x^3+5x^2-6x+6}{(x-1)^2(x^2+2)}$.
77. $\frac{6-9x}{27x^3+8}$.
78. $\frac{1}{x^4+5x^2+6}$.
79. $\frac{x^3-3}{(x-2)(x^2+1)}$.

Improper functions

80. $\frac{x^3-8}{x^5-3x^4-3x^3-x^2}$.
81. $\frac{x^2}{x^2-4}$.
82. $\frac{x^3-2x^2-1}{x^2-1}$.
83. $\frac{2x^3-2x^2-11x-8}{x^2-x-6}$.
84. $\frac{x^2+1}{x^2-x-2}$.
85. $\frac{x^4+2x}{(x-2)(x^2+1)}$.
86. $\frac{15-9x+x^2-2x^3}{(1-x)(x^2+4)}$.
87. $\frac{2x^3-x-1}{(x-3)(x^2+1)}$.
88. $\frac{x^3-1}{(x-2)(x^2+1)}$.
89. $\frac{x^3}{(x+4)(x-1)}$.
90. $\frac{x^4-x^3+x^2+1}{x^3+x}$.
91. $\frac{x^5}{(x-1)^3}$.
92. $\frac{x^2+10x+6}{x^2+2x-5}$.
93. $\frac{2x^4+x^3+20x^2+3x+31}{(x+1)(x^4+8x^2+16)}$
94. $\frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}$.

$\frac{x^5-x^4-2x^3+4x^2-15x}{(x^2+1)^2(x^2+4)}$

Tr. Kabuzi

RATES AND SMALL CHANGES

Applications of differentiation

1. If $p = 4s^2 - 10s + 7$, find the minimum value of p and the values of s which gives the minimum value of p.
2. Onyango wishes to fence a rectangular farm. He wants the sum of the length and the width of the farm to be 42 cm. Calculate the length and width of the farm for the area of the farm to be as maximum as possible.
3. A cylindrical can is made so that the sum of the height and the circumference of its base is 45π cm. Find the radius of the base of the cylinder if the volume of the can is maximum.
4. The length of a rectangular block is twice its width, and the total surface area is 108 cm². Show that if the width of the block is x cm, the volume is $\frac{4}{3}x(27 - x^2)$. Find the dimensions of the block if the volume is maximum.
5. A cylindrical volume V is to be cut from a solid sphere of radius R. Prove that the maximum volume of the cylindar, V is $V = \frac{4\pi R^3}{3\sqrt{3}}$
6. A rectangular block has a base x cm square. Its surface area is 150 cm². Prove that the volume of the block is $\frac{1}{2}(75x - x^3)$.

(a) Calculate the dimensions of the block when the volume is maximum.

(b) The maximum volume.

7. A variable rectangular flower garden has a constant perimeter of 40. Find the length of the side when the area is maximum.
8. ii)A variable rectangle has a constant area of 36 cm². Find the length of the sides when the perimeter is maximum.
9. Mukasa wishes to enclose a rectangular piece of land of area 1250 cm² whose one side is bound by a straight bank of a river. Find the least possible length of barbed wire required.
10. A closed right circular cylinder of base radius r cm and height h cm has volume of 54π cm³. Show that S, the total surface area of the cylinder, is given by $S = \frac{108\pi}{r} + 2\pi r^2$ hence find the radius and height which makes the surface area minimum.
11. A company that manufactures dog food wishes to pack the feed in closed cylindrical tins. What should be the dimensions of each tin if each is to have a volume of 250π cm³ and the minimum possible surface area?
12. A right circular cone of radius r cm has a maximum volume. The sum of its vertical height h and circumference of its base is 15 cm. If the radius varies, show that the maximum volume of the cone is $\frac{125}{3\pi}$ cm³

SMALL CHANGES

13. Given that $y = 3x^2 + 2x - 4$. Use small changes to find the small change in y when x increases from 2 to 2.02.
14. The radius of the circle increases from 5cm to 5.02cm. find the percentage increase in area of the circle
15. A cylinder of radius r and height $8r$. the radius increases from $4cm$ to $4.1cm$. find the approximate increase in volume

Approximating roots

16. use small changes to evaluate
    (i) $\sqrt[3]{28}$, $\sqrt{9.04}$, $\sqrt[3]{1003}$
    (ii) $sin30.5$, $cos47$, $sin56$, cot59.8⁰.

Percentages in small changes

17. An error of 3% is made in measuring the radius of the sphere. Find the percentage error in the volume
18. The height of a cylinder is 10 cm and the radius is 4 cm. Find the approximate percentage increase in the volume when the radius increases from 4 to 4.02 cm.
19. An error of 2.5% is made in measuring the area of a circle. What is the percentage error in the circumference?
20. The period T of a simple pendulum is calculated from the for $T = 2\pi\sqrt{\frac{l}{g}}$ where l is the length of the pendulum and g is the acceleration due to gravity constant. find the percentage change in the period caused by lengthening the pendulum by 2%.

Rates of change

21. A side of a cube is increasing at a rate of 6cm/s. Find the rate of increase in the volume of the cube when the length of the side is 8cm.
22. The volume of a cube is increasing at a rate of 2 cm³/s. Find the rate of change of the side of the base when the length is 3 cm.
23. The area of the circle is increasing at a rate of 3cm²/s. Find the rate of change of the circumference when its radius is 2cm.
24. A spherical balloon is inflated such that the rate at which its radius is increasing is 0.5cm/s. Find the rate at which:
25. the volume is increasing at the instant when $r = 5.0$cm
26. the surface area is increasing when $r = 8.5$ cm
27. The area of the circle is increasing at a rate of 3cm²/s. Find the rate of change of the circumference when its radius is 2cm

Rates in cones

28. A circular cone is held vertex downwards beneath a tap leaking at a rate of 2cm³/s. Find the rise of water level when the level is 6 cm. Given that the height of the cone is 18 cm and its radius is 12 cm.
29. An inverted cone with semi vertical angle of 30° is collecting water leaking from a tap at a rate of 2cm³/s. If the height of water collected is 10cm, find the rate at which the depth is decreasing at that instant.
30. An inverted right circular cone of vertical angle 120° is collecting water from a tap at a steady rate of 18π cm³/min. Find:
    (i)the depth of the water after 12 minutes
    (ii)the rate of increase of the depth at this instant.
31. A rectangular figure with sides $8cm$ by $5cm$, equal sides of xcm are removed from each corner and the edge are turned up to make an open box of volume Vcm³. Show that $V = 40x - 26x^2 + 4x^3$ and hence find the maximum possible volume and the value of x.

c/o Tr. Kabuzi

TRIGONOMETRY TOPICAL REVISION

1. Solve the equations for all values of x from 0° to 360°

(a) $\sin x = 2\cos x$
(b) $2\sin x - 3\cos x = 0$
(c) $\sin x\,(1 - 2\cos x) = 0$
(d) $\cos x(2\sin x + \cos x) = 0$
(e) $2\sin x\cos x + \sin x = 0$
(f) $4\sin x\cos x = 3\cos x$
(g) $\tan x = 4\sin x$
(h) $(2\sin x - 1)(\sin x + 1) = 0$
(i) $2\sin^2 x - \sin x - 1 = 0$
(j) $2\tan^2 x - \tan x - 6 = 0$

2. Solve the following equations from $0 \leq \theta \leq 360°$

(a) $\cos 2\theta + \cos\theta + 1 = 0$
(b) $\sin 2\theta\cos\theta + \sin^2\theta = 1$
(c) $2\sin\theta(5\cos 2\theta + 1) = 3\sin 2\theta$
(d) $3\cot 2\theta + \cot\theta = 1$
(e) $4\tan\theta\tan 2\theta = 1$

3. Solve the following equations for all values of x from -180° to 180°

(a) $\cos^2 x = \dfrac{3}{4}$
(b) $\sin 2x = 2\cos 2x$
(c) $\cos(x - 20) = 0.5$
(d) $\cos x(\sin x - 1) = 0$
(e) $3\sin^2 x = 2\sin x\cos x$
(f) $2\cos^2 x - 5\cos x + 2 = 0$
(g) $4 - \sin\theta = 4\cos^2\theta$
(h) $\sin^2\theta + \cos\theta + 1 = 0$
(i) $5 - 5\cos\theta = 3\sin^2\theta$
(j) $8\tan\theta = 3\cos\theta$
(k) $\sin^2\theta + 5\cos^2\theta = 0$
(l) $1 - \cos^2\theta = -2\sin\theta\cos\theta$

4. Solve the following equations for $0 \leq \theta \leq 360°$

(a) $2\cos\theta + 3\sin\theta - 2 = 0$
(b) $3\cos\theta - 4\sin\theta + 1 = 0$
(c) $3\cos\theta + 4\sin\theta = 2$
(d) $4\cos\theta\sin\theta + 15\cos 2\theta = 10$
(e) Express $4\sin x - 3\cos x$ in the form $R\sin(x - \propto)$

Find the maximum and minimum values of $6 + 4\sin x - 3\cos x$ and $\frac{1}{6+4\sin x-3\cos x}$.

(f) Express $y = 8\cos x + 6\sin x$ in the form $R\cos(x - \alpha)$ where R is positive and α is acute. Hence find the maximum and minimum values of

$$\frac{1}{8\cos x + 6\sin x + 15}$$

5. Solve the following equations from $x = 0°$ to 360° inclusive.

(a) $\cos x + \cos 5x = 0$
(b) $\sin 3x - \sin x = 0$
(c) $\sin 2x + \sin 3x + \sin 4x = 0$
(d) $\sin(x + 10) + \sin x = 0$
(e) $\cos(2x + 10) + \cos(2x - 10) = 0$
(f) $\cos(x + 20) - \cos(x - 70) = 0$

6. Show that:

(a) $\tan^{-1}\dfrac{1}{3} + \sin^{-1}\dfrac{1}{\sqrt{5}} = \dfrac{\pi}{4}$
(b) $2\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{7} = \dfrac{\pi}{4}$
(c) $2tan^{-1}\left(\frac{1}{3}\right) + tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$.
(d) $sec^{-1}\left(\frac{5}{3}\right) - sec^{-1}\left(\frac{5}{4}\right) = sec^{-1}\left(\frac{25}{24}\right)$.
(e) $sin(2sin^{-1}x + cos^{-1}) = \sqrt{1-x^2}$.

7. Solve the equation

(a) $\tan^{-1}3x + \tan^{-1}x = \dfrac{\pi}{4}$
(b) $\tan^{-1}(2x + 1) + \tan^{-1}(2x - 1) = \tan^{-1}2$
(c) $tan^{-1}x + tan^{-1}(1 - x) = tan^{-1}\left(\frac{9}{7}\right)$.
(d) $tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}tan^{-1}x$.
(e) $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = 32$
(f) Express $\sqrt{\frac{sin2x - cos2x - 1}{2 - 2sin2x}}$ in terms of $tanx$.
(g) $\sqrt{\frac{1-sin2x}{1+sin2x}}$ in terms of $tanx$.

8. Prove that

(a) $tan^{-1}a + tan^{-1}b + tan^{-1}c = \pi$ show that $a + b + c = abc$.
(b) $\frac{b-c}{b+c} = tan\left(\frac{A}{2}\right)tan\left(\frac{B-C}{2}\right)$.
(c) $\frac{a+b-c}{a+b+c} = tan\left(\frac{A}{2}\right)tan\left(\frac{B}{2}\right)$.
(d) $\frac{a^2-b^2+c^2}{a^2+b^2-c^2} = tanBtanC$
(e) $CosA + cosB + cosC = 1 + 4sin\left(\frac{A}{2}\right)sin\left(\frac{B}{2}\right)sin\left(\frac{C}{2}\right)$.
(f) For a triangle ABC, $tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}cot\left(\frac{C}{2}\right)$. hence solve the triangle if $C = 57^0, a = 9, b = 5.5$.
(g) $8cos3xcos2xcosx - 1 = \frac{sin7x}{sinx}$
(h) $\frac{1}{4-5sin^2x} = \frac{sec^2x}{4-tan^2x}$
(i) $sin2A + sin2B + sin2C = 4cosAcosBsinC$.
(j) $cos^2 2A + cos^2 2B + cos^2 2C - 1 = 2cos2Acos2Bcos2C$.
(k) $\frac{3sin\theta + sin2\theta}{1+3cos\theta+cos2\theta} = tan\theta$
(l) $\frac{sin3\theta - sin\theta}{sin5\theta + sin3\theta} = \frac{1}{4}sec^2\theta$
(m) $\frac{1}{a}cos^2\left(\frac{A}{2}\right) + \frac{1}{b}cos^2\left(\frac{B}{2}\right) + \frac{1}{c}cos^2\left(\frac{C}{2}\right) = \frac{(a+b+c)^2}{4abc}$
(n) $sec2x - tan2x = tan\left(\frac{\pi}{4} - x\right)$
(o) If $sin3x = p$, $sin^2x = \frac{3}{4} - q$ prove that $p^2 + 16q^3 = 12q^2$.
(p) $\frac{sinx + 2sin2x + sin3x}{sinx - 2sin2x + sin3x} = cot^2\left(\frac{x}{2}\right)$.

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ALGEBRA OF EQUATION.

Solve the following.

1. $x^2 + y^2 = 185$ *and* $x - y = 3$
2. $x^3 + y^3 = 4914$ *and* $x + y = 18$
3. $x^3 - y^3 = 218$ *and* $x - y = 2$
4. $2x + 3y = 5$ *and* $xy = 1$
5. $5x - y = 3$ *and* $y^2 - 6x^2 = 25$
6. $3x - 2y = 7$ *and* $xy = 20$

Homogeneous equations

7. $2x^2 - xy - y^2 = 8$ *and* $xy = 6$
8. $x^3 + y^3 = 35$ *and* $x^2y + xy^2 = 30$
9. $x^2 - 2xy + 8y^2 = 8$ & $3xy - 2y^2 = 4$

Simultaneous equations

$x + 2y - 3z = 3$
$2x - y - z = 11$
$3x + 2y + z = -5$

$3x + 2y - z = 19$
$4x - y + 2z = 4$
$2x + 4y - 5z = 32$

$4(x + 3y) - 2(4x + 3z) - 3(x - 2y - 4z) = 17$
$2(4x - 3y) + 5(x - 4z) + 4(x - 3y + 2z) = 23$
$3(y + 4z) + 4(2x - y - z) + 2(x + 3y - 2z) = 5$

$5(x + 2y) - 4(3x + 4z) - 2(x + 3y - 5z) = 16$

$2(3x - y) + 3(x - 4z) + 4(2x - 3y + z) = -16$
$4(4 - 2z) + 2(2x - 4y - 3) - 3(x + 4y - 2z) = -62$

Use of ratio theorem

10. $x - 3 = \frac{y-1}{2} = \frac{z-5}{-2}$ and $11x - 3y + 7z = 8$
11. $\frac{4x-3y}{4} = \frac{2y-x}{3} = \frac{z+4y}{2}$ and $6x + 6y + 2z = 6$
12. $\frac{x}{5} = \frac{y+2}{2} = z - 14$ and $3x + 4y + 2z - 15 = 0$
13. $\frac{x-y}{4} = \frac{z-y}{3} = 2z - x$ and $x + 3y + 2z = 4$

Logarithms

14. $\log_x 5 - \log_5 x = 32.$
15. $\log_4 x^2 - 6\log_x 4 - 1 = 0$
16. $\log_5 x + 2\log_x 5 = 3$
17. $3\log_2 x - \log_x 2 = 2$
18. $\log_x 5 + 4\log_5 x = 4$
19. $\log_x 27 - \log_{x^2} 81 = 1.$
20. $\log_4(6 - x) = \log_2 x.$
21. $(\log_3 x)(\log_{3x} 3) = \frac{3}{4}$
22. $\log_{25} 4x^2 = \log_5(3 - x^2).$

Indices.

21. $9^x - 3^{(x+1)} = 10$
22. $4^{2x} - 4^{x+1} + 4 = 0.$
23. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0.$
24. $2^{2x-1} + \frac{3}{2} = 2^{x+1}$
25. $x - 3x^{\frac{2}{3}} + 2x^{\frac{1}{3}} = 0.$
26. $9x^4 - 45x^2 = 324.$
27. $x^{\frac{4}{3}} + 16x^{\frac{-4}{3}} = 17.$
28. $2 - 5e^{-x} + 2e^{-2x} = 0.$

Rational equations

28. $\sqrt{2x + 1} - \sqrt{x} = \sqrt{x - 3}.$
29. $\sqrt{2x + 1} - \sqrt{x - 3} = 1.$
30. $\sqrt{x + 6} + \sqrt{4 - x} = \sqrt{1 - 3x}.$
31. $\sqrt{3 - x} - \sqrt{7 + x} = \sqrt{16 + 2x}.$
32. $2\sqrt{x - 1} - \sqrt{x + 4} = 1.$
33. $\sqrt{(2x + 3)} - \sqrt{(x + 1)} = \sqrt{x - 2}.$
34. $x^2 - 5x + 2\sqrt{x^2 - 5x + 3} = 12.$
35. $x^2 + 2\sqrt{x^2 + 6x} = 24 - 6x.$
36. $x^2 + x + 10\sqrt{x^2 + 3x + 10} = 2(20 - x).$

Special equations

37. $x^2 + 2x + \frac{12}{x^2+2x} = 7$
38. $x^2 - 2x + \frac{24}{x^2-2x} = 11$
39. $\frac{x^2+4x}{3} + \frac{84}{x^2+4x} = 11$
40. $(x^2 - 2x)^2 + 24 = 11(x^2 - 2x)$
41. $(3x^2 + 2x)^2 + 8 = 9(3x^2 + 2x)$
42. $x^2 + 2x = 34 + \frac{35}{x^2+2x}$
43. $\sqrt{3^x} + \frac{3}{\sqrt{3^x}} = 4$
44. use $x^2 - 4x = y$ to solve $2x^4 - 16x^3 + 77x^2 - 180x + 63 = 0.$
45. use $x^2 - x = y$ to solve $x^4 - 2x^3 - 7x^2 + 8x + 18 = 0.$

Solve the following

45. $8^{x-y} = 4^{x+y}$ & $5^{x^2-y^2} = 15625.$

46. $2^{x+y} = 6^y$ and $3^x = 6(2^y).$

48. $18\log_x y = 6y - x$ and $\log_y x = 2.$

47. $\log_2(x - 3y + 2) = 0$ and $\log_2(x + 1) - 1 = 2\log_2 y.$

48. $6\log_3 x + 6\log_{27} y = 7$
$4\log_9 x + 4\log_3 y = 9$

49. $\log_b a + 2\log_a b = 3$ and $\log_9 a + \log_9 b = 3.$

Symmetric equations.

49. $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0.$
50.. $4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0.$
50. $x^4 + 2x^3 - x^2 + 2x + 1 = 0.$
51. $x^4 + x^3 - 4x^2 + x + 1 = 0.$
53. $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0.$
52. 54. $x^4 + 1 - 3(x^3 + x) = 2x^2.$

Logarithms proofs.

55. Prove that $\log_8 x = \frac{2}{3}\log_4 x.$ Hence find $\log_8 6$ if $\log_4 3 = 0.7925.$
56. If $\log_4 m = a, \log_{12} m = b.$ Prove that $\log_3 48 = \frac{a+b}{a-b}.$
57. Given that $\log_3 x = p,$ $\log_{18} x = p.$ Prove that $\log_6 3 = \frac{q}{p-q}.$
58. Given that $\log_2 x + 2\log_4 y = 4$, show that $xy = 16.$ Hence solve for x and y in the equation. $\log_2 x + 2\log_4 y = 4$ and $\log_{10}(x + y) = 1.$
59. Given that $p = \log_2 3$ *and* $\log_4 5.$ Show that $\log_{45} 2 = \frac{1}{2(p+q)}.$
60. If $x^2 + y^2 = 7xy$, prove that $\log(x + y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y.$
61. Given that $a = \log_5 35$ *and* $b = \log_9 35.$ Show that $\log_5 21 = \frac{1}{2b}(2ab - 2b + a).$
62. If $log_4 m = a, log_{12} m = b.$ Prove that $log_3 48 = \frac{a+b}{a-b}$
63. If $a^2 + b^2 = 7ab$, show that $\log\frac{1}{3}(a + b) = \frac{1}{2}(\log a + \log b)$

Square root of surds

64. $6 + 14\sqrt{5}.$
65. $5 + 2\sqrt{6}.$
66. $18 - 12\sqrt{2}.$
67. $23 - 4\sqrt{15}.$

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QUADRATIC EQUATIONS AND POLYNOMIALS

Nature of roots

1. Determine the nature of the roots of the following equations:
(i) $x^2 - 6x + 9 = 0$
(ii) $x^2 - 2x + 1 = 0$
(iii) $x^2 - 6x + 10 = 0$
(iv) $4x^2 - 12x - 9 = 0$
2. Find the values of k for which the following equations have equal roots.
(i) $3x^2 + kx + 12 = 0$
(ii) $x^2 - 5x + k = 0$
3. Find the range of values k can take for $9x^2 + kx + 4 = 0$ to have two real distinct roots.
4. Find the values of k for which the equation $\frac{x^2-x+1}{x-1} = k$ has repeated roots. What are the repeated roots?

Maximum and Minimum values of a quadratic function

5. Find the greatest or least values of the following functions:
(i) $x^2 - 2x + 5$
(ii) $5 - 4x - x^2$
(iii) $x^2 - 3x + 5$
(iv) $2x^2 - 4x + 5$
(v) $7 + x - x^2$

Sum & Product of the roots of Quadratic Equations

6. If α and β are roots of the equation $x^2 - x - 3 = 0$, state the values of $\alpha + \beta$ and $\alpha\beta$ and find the values of:
(i) $\alpha^2 + \beta^2$
(ii) $(\alpha - \beta)^2$
(iii) $\alpha^3 + \beta^3$
7. If α and β are roots of the equation $2x^2 - 5x + 1 = 0$, find the values of:
(i) $\alpha + \beta$
(ii) $\alpha\beta$
(iii) $\alpha^2 + 3\alpha\beta + \beta^2$
(v) $\alpha^2 - 3\alpha\beta + \beta^2$.
(vi) $\alpha^3\beta + \alpha\beta^3$
(vii) $\frac{1}{\beta} + \frac{1}{\alpha}$
8. The roots of the equation $x^2 - 2x + 3 = 0$ are α and β. Find the equation whose roots are:
(i) $\alpha + 2, \beta + 2$
(ii) $\frac{1}{\beta}, \frac{1}{\alpha}$
9. If the roots of the equation $ax^2 + bx + c = 0$ differ by 4, show that $\frac{b^2}{4a} = 4a + c$.
10. The roots of the equation $x^2 - px + 8 = 0$ are α and
11. $\alpha + 2$. Find the two possible values of p.
12. The roots of the equation $x^2 + 2px + q = 0$ differ by 2. Show that $p^2 = 1 + q$
13. Prove that if the roots of the equation $ax^2 + bx + c = 0$ is three times the other, then $3b^2 = 16ac$.
14. The roots of the equation $x^2 - 12x + 4 = 0$ are m and n. Find the quadratic equation whose roots are $\sqrt{m}$ and $\sqrt{n}$.
15. Given that $\alpha^2 + \beta^2 = 25$ and $\alpha + \beta = 1$. Find the values of $\alpha - \beta$.
16. The roots of the equation $ax^2 + bx + c = 0$ are $\tan\alpha$ and $\tan\beta$. Express in terms of a, b and c. (i) $tan(\alpha + \beta)$ and $sec(\alpha + \beta)$.
17. The roots of the equation $x^2 + 2px + q = 0$ differ by 8. Show that $p^2 - 16 = q$.
18. The roots of the equation $ax^2 + bx + c = 0$ is a square of the other. Prove that $c(a - b)^3 = a(c - b)^3$.
19. Given that the polynomial $f(x) = Q(x)g(x) + R(x)$ where $Q(x)$ is a quotient, $g(x) = (x - \alpha)(x - \beta)$ and $R(x)$ is a remainder. Show that $R(x) = \frac{(x-\alpha)f(\alpha)+(\alpha-x)f(\beta)}{\alpha-\beta}$.
20. Given that α and β are roots of the equation $2x^2 - 8x + 2 = 0$, show that $\alpha^3 + \beta^3 = 52$. Hence that $\alpha^6 + \beta^3 = 27$.

Equations with common roots.

21. Given that equations $y^2 + py + q = 0$ and $y^2 + my + k = 0$ have a common root, show that $(q - k)^2 = (m - p)(pk - mq)$
22. Show that if the equations $x^2 + bx + c = 0$ and $x^2 + px + q = 0$ have a common root, then $(c - q)^2 = (b - p)(cp - bq)$
23. Show that if the equation $x^2 + ax + 1 = 0$ and $x^2 + x + b = 0$ have common roots, then $(b - 1)^2 = (a - 1)(1 - ab)$.

Remainder theorem

Find the remainders when
24. $3x^3 - 4x^2 + 5x - 8$ is divided by $x - 2$
25. $2x^3 - 3x^2 - 5x + 6$ is divided by $x + 2$
26. $2x^3 - 7x + 6$ is divided by $x - 3$
27. $x^5 + x - 9$ is divided by $x + 1$
28. $x + 2$ is a factor of $2x^3 + 6x^2 + bx - 5$. Find the remainder when the expression is divided by $2x - 1$
29. The remainder obtained when $2x^3 + ax^2 - 6x + 1$ is divided by $x + 2$ is twice the remainder obtainedwhenthe same expression is divided by $x - 3$. Find a
30. $x - 1$and $x + 1$ are factors of $x^3 + ax^2 + bx + c$ and it leaves a remainder of 12 when divided by $x - 2$.find the values of $a, b,$ and$c$.
31. When a polynomial $p(x)$is divided by $x - 1$, the remainder is 5 and when $p(x)$ is divided by $x - 2$, the remainder is 7. Find the remainder when the same expression is divided by $(x - 1)(x - 2)$.
32. When a polynomial$P(x)$ is divided by $x - 2$, the remainder is 4 and when $P(x)$ is divided by $x - 3$, the remainder is 7. Find the remainder when $P(x)$ is divided by $(x - 2)(x - 3)$
33. If $x^2 + 1$ a factor of $3x^4 + x^3 - 4x^2 + px + q$. Find the values of p and q

repeated factors

34. Given that the polynomial $f(x) = x^3 + 3x^2 - 9x + k$ has a repeated linear factor, find the possible values of k.
35. Given that $f(x) = (x - \alpha)^2 g(x)$, show that $f'(x)$ is divisible by $(x - \alpha)$ hence if$(x + 1)^2$ is a factor to the polynomial $p(x) = 2x^4 + 7x^3 + 6x^2 + ax + b$. Find the values of a and b, hence solve $p(x) = 0$.
36. A polynomial $P(x) = x^3 + 4ax^2 + bx + 3$ is divisible by $(x - 1)^2$. Use your results above to find the values of a and b. Hence solve the equation $p(x) = 0$

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COMPLEX ANALYSIS 2023

Modulus of a complex number

Find modulus of the following.

. $z = 1 + \sqrt{3}i$. $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$.

. Given that $z_1 = 5 - 12i$ and $z_2 = 3 - 4i$. Find $|z_1 z_2|$ and $\left|\frac{z_1}{z_2}\right|$

. $Z = \frac{(3i+1)(i-2)^2}{i-3}$.

Argument of a complex number

Find the principal argument of the following complex number

. $(a)\ 1 + i \quad (b) -1 - i\sqrt{3} \quad (c) -5$ and $z_5 = -5i \ (d) -\sqrt{3} + i \ (e)\sqrt{3} - i$

Argand diagram

Represent the following complex numbers on the argand diagram.

. $z_1 = 3 + 4i$, $z_2 = -2 + i$, $z_3 = -5 - 4i$, $z_4 = 2 - 3i$, $z_5 = -4 - 2i$,

Modulus- argument form (polar form)

Express the following complex numbers in modulus –argument

. $5 + 5i\sqrt{3}$

. $\sqrt{2} + i$

. $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$

. $-5i$

0. $-5 - 12i$

Square root of a complex

1. $35 - 12i$.
2. $5 - 12i$.
3. $16 - 30i$
4. $5 + 12i$.
5. Find the fourth roots of -16
6. Find the cube roots of 27i

Demoivres theorem

7. $(\cos\theta + i\sin\theta)^2(\cos\theta + i\sin\theta)^3$
8. $\frac{1}{(\cos\theta + i\sin\theta)^2}$
9. $\frac{\cos\theta + i\sin\theta}{(\cos\theta + i\sin\theta)^4}$

0. $\frac{\left(\cos\frac{\pi}{17} + i\sin\frac{\pi}{17}\right)^8}{\left(\cos\frac{\pi}{17} - i\sin\frac{\pi}{17}\right)^9}$

1. $\frac{(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)}{\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}$

2. $\left(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right)^{12}$

3. $\frac{\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)^8}{\left(\cos\frac{3\pi}{5} - i\sin\frac{3\pi}{5}\right)^3}$

Express in the form $a + bi$

24. $(1 - i\sqrt{3})^4$
25. $\frac{1}{(1 - i\sqrt{3})^3}$
26. $(\sqrt{3} + i)^{10}$ and $\frac{1}{(\sqrt{3} + i)^7}$

De-moivre's theorem proofs

27. $\tan 3\theta = \frac{3\tan\theta - 3\tan^3\theta}{1 - 3\tan^2\theta}$
28. $\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$
29. $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$
30. $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$
31. $\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$
32. $\frac{\cos 5x}{\cos x} = 1 - 12\sin^2 x + 16\sin^4 x$.

Roots of an equation

Show that the following are roots to the equations and hence find other roots

33. $2 + 3i$ and $z^3 - 6z^2 + 21z - 26 = 0$.
34. $1 + i$ and $z^4 + 3z^2 - 6z + 10 = 0$.
35. $z = 2 - i$ and $z^3 - 3z^2 + z + k = 0$, k is real. Find other roots.

Solving equation

36. $z_1 + (1 - i)z_2 = 0$ and $3z_2 - 3z_1 = 2 - 5i$
37. $Z_1 + 3Z_2 = 10 + i$ and $4Z_1 - i3Z_2 = 25 + 6i$.
38. $Z^3 - 1 = 0$.
39. $Z^3 + 27 = 0$.
40. $Z^4 - 6Z^2 + 25 = 0$.
41. $Z^3 = -\sqrt{2} + i\sqrt{6}$.

Find the values of x and y which satisfy the equation.

42. $\frac{2y + 4i}{2x + y} - \frac{y}{x - i} = 0$
43. $\frac{2y + 4i}{2x + y} - \frac{y}{x - i} = 0$
44. $\frac{x}{1 + i} + \frac{y}{2 - i} = 2 + 4i$
45. $\frac{xi}{1 + iy} = \frac{3x + 4i}{x + 3y}$
46. W is a complex number such that $W = \frac{p}{2 - i} + \frac{q}{1 + 3i}$ where p and q are real numbers. Given that $\arg W = \frac{\pi}{2}$ and $|W| = 7$, Find p and q.
47. Given that $Z_1 = 3 + i$ and $Z_2 = x + i$ and $arg(Z_1 Z_2) = \frac{\pi}{4}$, Find the value of x.
48. Given that $Z_1 = 2\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right]$ and $Z_2 = 3\left[\cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)\right]$, find the polar form of a complex number $Z = Z_1 Z_2$

Locus of a complex number

Describe locus of the following.

49. $|Z - 3| = 2$.
50. $|Z - 3 + i| = 4$.
51. $|Z - 3 + 3i| < 2$.
52. $2|Z + 1| = |Z - 2|$.
53. $|Z + 4i| = 3|Z - 4|$.
54. $\left|\frac{Z}{Z - 4}\right| = 5$.
55. $|z - 2| = 2|z + i|$.
56. $Arg(Z + 3) = \frac{\pi}{6}$.
57. $Arg(Z + 1 - 3i) = \frac{-\pi}{4}$.
58. $\arg(Z - 1 - 2i) = \frac{\pi}{4}$.

Shade the region of the following.

59. $|z - 1 - i| < 3$.
60. $|z - 2 + i| < 1$
61. $\left|\frac{Z - 1}{Z + 1}\right| > 2$.
62. $\frac{1}{3} \leq \arg(Z - 2) \leq \pi$
63. $0 \leq \arg(Z - 1 - 2i) < \frac{\pi}{4}$.
64. $arg\left(\frac{Z + 2}{Z - i}\right) = \frac{\pi}{3}$.
65. $arg\left(\frac{Z - 1}{Z + 1}\right) = \frac{\pi}{3}$. $arg\left(\frac{Z - 3}{Z - 2i}\right) = \frac{\pi}{4}$.

66. ***Purely real and pure imaginary numbers.***

67. Show that when $Re\left(\frac{Z + i}{Z + 2}\right) = 0$, the point $p(x, y)$ lies on the circle.
68. Show that $im\left(\frac{Z + i}{Z + 2}\right) = 0$ is equation of a straight line.
69. Re $\left(\frac{z + i}{(z + 2)}\right) = 0$,
70. Prove that if $\frac{Z - 6i}{Z + 8}$ is real, the locus of the point representing is a straight line.
71. Prove that if $\frac{Z - 2i}{2Z - 1}$ is purely imaginary, the locus of the point representing Z in the Argand diagram is a circle and find its radius

Minimum and maximum value of a complex number

72. Sketch on the argand diagram the locus of $|Z - 4| \leq 3$ and hence find the greatest and least value of $|Z|$.
73. Given that the complex number Z varies such that $|Z - 7| = 3$. Find the greatest and least value of $|Z - i|$.

c/o Kabuzi

CONIC SECTION 2023

Parabola

Find Vertices, directrices and foci of following and hence sketch

1. $y^2 = 12x$
2. $(y-2)^2 = 4(x-3)$
3. $y^2 + 8y = 4x - 12$
4. $y^2 + 6y + 4x + 2 = 0$

parametric equations

5. Show that any point (x, y) on the parabola $y^2 = 4ax$ can be represented by point $(ap^2, 2ap)$.

Equation of tangent and normal.

6. Find the equation of the normal and the tangent to the parabola $y^2 = 4ax$ at $(ap^2, 2ap)$
7. Prove that the line $y = 2x + 2$ touches the parabola $y^2 = 16x$ and find the coordinates of contact where this occurs.
8. Prove that the line $y = x + 6$ cuts the parabola $y^2 = 32x$ at two distinct points and find these two points.
9. Prove that the equation of the tangent to the parabola at $P(at^2, 2at)$ is $ty = x + at^2$.
   (b)the tangent to the above parabola $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ intersect at R. Find the coordinate of R. (C)if the tangents at P and Q are inclined to one another at 45^0.
   Show that the locus of R is the curve $y^2 = x^2 + 6ax + a^2$.Find the equation of the tangent drawn from the point (1, 3) to the parabola $y^2 = -16x$.
10. Prove that if the tangents to the parabola $y^2 = 4ax$ at the points P(ap^2, $2ap$) and Q(aq^2, $2aq$) meet at T(apq, $a(p+q)$)
11. Find the equation of the tangent to the parabola $y^2 = 4ax$ drawn from the point ($16a$, $17a$).
12. P(ap^2, $2ap$) and Q(aq^2, $2aq$) of the parabola $y^2 = 4ax$. If PQ is a focal chord, prove that the two tangents meet at the directrix.
13. PQ is a focal chord of the parabolas $y^2 = 4ax$ where $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$. Find the equation of the chord PQ. Given that PQ passes through the focus $F(a, 0)$, show that $pq = 1$.
14. Find the equation of the chord through the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ of the parabola $y^2 = 4ax$
15. Show that the chord cuts the directrix when $y = \frac{2a(t_2t_1 - 1)}{t_1 + t_2}$
16. Find the equation of the tangent to the parabola $y^2 = \frac{x}{16}$ at $\left(t^2, \frac{t}{4}\right)$.
17. If the tangents to the parabola in (a) above at P(p^2, $\frac{p}{4}$) and Q(q^2, $\frac{q}{4}$) meet at the line $y = 2$, show that $p + q = 16$.
18. Deduce that the midpoint of PQ lies on the line $y = 2$
19. Points P(ap^2, $2ap$) and Q(aq^2, $2aq$) lie on the parabola $y^2 = 4ax$. Find the locus of the midpoint of PQ given that P and Q are those for which $pq = 2a$.

ELLIPSE

20. Find the coordinates of the centre and foci of the given ellipses. Determine the length of the major and minor axes. Also determine the equation of the directrices.

(i) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(ii) $3x^2 + 6y^2 = 12$

(iii) $25x^2 + 9y^2 - 100x - 54y = 44$

(iv) $3x^2 + 4y^2 - 6x + 16y + 7 = 0$

(v) $4x^2 + 6y^2 + 36y + 55 = 0$

21. An ellipse has its foci at (-2, 3) and (6, 3) and its eccentricity e is $\frac{4}{5}$.
    Find the Cartesian equation of the ellipse.

Parametric equations

22. Show that any point on the ellipse is given by ($a\cos\theta$, $b\sin\theta$)
23. Show that $x = 1 + 4\cos\theta$ and $y = 3 + 5\sin\theta$ are parametric equations of an ellipse. Find the coordinates of the centre and foci

Tangent and normal to an ellipse

24. Find the equations of the tangent and normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point ($a\cos\theta$, $b\sin\theta$)
25. The normal at the point P($a\cos\theta$, $b\sin\theta$) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the coordinate axes at L and M respectively. Find the locus of the midpoint of LM.
26. Find the equation of the tangent and normal to the ellipse $\frac{x^2}{64} + \frac{y^2}{25} = 1$ at the point (2, 7)
27. Given that $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that $c^2 = a^2m^2 + b^2$. Hence find the equation of the tangent from (-3, 3) to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
28. Show that the line $5y = 4x + 25$ is a tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$
29. **b)** Find the equation of the normal to the ellipse at the point of contact.
30. **c)** Determine the eccentricity of the ellipse.
31. Find the equations of the tangents to the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ from (-2, 5)
32. Find the coordinates of the point of contact of the tangents to the ellipse.
33. Find the equations of the tangent and normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ at P($2\cos\theta$, $\sin\theta$)
34. If the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when $c \pm \sqrt{a^2m^2 + b^2}$. Hence find the equation of tangents to the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ from the point $(0, \sqrt{5})$.
35. If the tangent in (a) cut the x-axis at point A and y-axis a point B. the normal cuts the x-axis at C, find the coordinates of A, B and C.

Rectangular hyperbola

36. Show that any point on the rectangular hyperbola $xy = c^2$ is represented by $\left(ct, \frac{c}{t}\right)$
37. Find the equation of the tangent and normal to the rectangular hyperbola $xy = c^2$ at the point $\left(ct, \frac{c}{t}\right)$.
38. Find the equation of the chord joining $P(cp, c/p)$ and $Q(cq, c/q)$ on $xy = c^2$.
39. Prove that $y = -3x + 6$ is a tangent to the curve with parametric coordinates given by $(\sqrt{3}t, \sqrt{3}/t)$.

c/o. Kabuzi

INORMIAL AND MAUCLAURINS HEOREM

inomial theorem

. Expand $(1+2x)^{\frac{1}{2}}$ as far as the term inx^4.
. Find the first three terms of the expansion $(4+x)^{-\frac{1}{2}}$ in ascending powers of x. Deduce the approximate value of $\frac{1}{\sqrt{4.16}}$
. Expand $(1-x)^{\frac{1}{3}}$ as far as the term in x^3. Hence evaluate $\sqrt[3]{24}$
. Determine the expansion of $\left(1-\frac{x}{3}\right)^{\frac{1}{2}}$ as far as the term in x^3 Hence evaluate $\sqrt{8}$
. Determine the binomial expansion of $\left(\frac{1}{1-\frac{x}{2}}\right)^{\frac{1}{3}}$ in ascending powers of x up to the term with x^3, hence find $\sqrt[3]{\frac{1}{7}}$.
. Expand $(8+3x)^{\frac{1}{3}}$ in ascending powers of x as far as the term in x^3, stating the values of x for which the expansion is valid. Hence obtain the approximate value $\sqrt[3]{8.72}$ correct 4dp.
. Given that $\sqrt{1+px} = 1-2x+qx^2+\cdots$ find the values of p and q. (ii) estimate $\sqrt{3}$ to 2dps and find the percentage error made in approximating the expansion.
. Expand $(1+x)^{-2}$ in descending powers of x including the first four terms. If $x=9$, find the percentange error in using the first 3 terms of the expansion.
. Find the expansion of $(1+\frac{1}{x})^{1/2}$ as far as the term x^{-3}
0. Expand $(1-x)^3(2+x)^6$ up to the term in x^2.
1. Expand $\frac{(1+x)^2}{(1-\frac{x}{2})^3}$ up to and including the term in x^2.
12. Show that, if x is small enough for its cube and higher powers to be neglected, $\sqrt{\frac{1-x}{1+x}} = 1-x+\frac{1}{2}x^2+\ldots$
13. Use Binomial expansion to expand $\sqrt{\frac{1+2x}{1-2x}}$ up to and including the term in x^3. Hence find $\sqrt{\frac{1.02}{0.98}}$ to 4 decimal places. Hence deduce the square root of 51
14. Expand $\sqrt{\frac{1+5x}{1-5x}}$ *as* far as the term including x^3. Taking the first three terms , evaluate $\sqrt{14}$ to 3sf.
15. Expand $\sqrt{\frac{1+x}{1-x}}$ in ascending power of x up to the term in x^2. By substituting $x=\frac{1}{10}$, use your expansion to find the $\sqrt{11}$.
16. Expand $\left(\frac{1+3x}{2-x}\right)^{0.5}$ in ascending powers of x up to x^3 and taking $x=\frac{1}{5}$ evaluate $\sqrt{8}$ to 2sf.
17. Given that, the first three terms of the expansion in ascending powers of x of $(1+x+x^2)^n$ are the same as the first three terms in the expansion of $\left(\frac{1+ax}{1-3ax}\right)^3$, find the values of a and n.
18. Expand $\left(\frac{1+2x}{1-x}\right)^{\frac{1}{3}}$ up to the fourth term. By putting $x=\frac{1}{9}$, $\sqrt[3]{11}$ giving your answer to 3sf.
19. Find the independent term of x, $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$, $\left(x^2-\frac{2}{x^2}\right)^8$.
20. Find the coefficient of x^8 in expansion $\left(x^2+\frac{2y}{x}\right)^{10}$
21. Find the coefficient of x^{17} in $\left(x^3+\frac{1}{x^4}\right)^{15}$.

Maclaurins theorem

22. Use maclaurin's theorem to expand $ln(2+3x)$ as far as the term x^4. Hence evaluate $ln(2.03)$ correct to 4dp.
23. Applying maclaurin's theorem to establish a series for $ln(1+x)$. if $1+x=\frac{b}{a}$, show that $\frac{b^2-a^2}{2ab} = x-\frac{x^2}{2}+\frac{x^3}{2}$.
24. Use macluarin's expansion to expand $ln(1+ax)$ and use it to expand $ln(1+2x)$ up to the term x^3.
25. Use maclaurin's expansion to express ln $(sinx+cosx)$ as far as the term with x^2.
26. Use the expansion $ln\left(\frac{1+x}{1-x}\right)$ up to the term with x^3 to find ln2 correct to 3dp.
27. use maclurin's expansion of $e^{-x}sinx$ to the term x^3 to show that is given that $\frac{x}{3}(x^2-3x+3)$. Hence evaluate $e^{-\frac{\pi}{3}}sin\left(\frac{\pi}{3}\right)$ to 4dps.
28. Use maclaurin's theorem to expand $ln(1+sinx)$ as far as the term with x^3.
29. Use maclaurin's theorem to find the first three non-zero terms of the expansion $tan^{-1}x$ and $tan^{-1}2x$ hence use it to evaluate $tan^{-1}(0.1)$.