S475/1

SUBSIDIARY MATHEMATICS

March, 2024

PAPER 1

2 HOURS 40 MINUTES

UGANDA ADVANCED CERTIFICATE OF EDUCATION

SUBSIDIARY MATHEMATICS

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INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** questions in Section A and any ***FOUR*** from Section B with at least one question from each part.
- All working must be shown clearly.
- No paper should be given for rough work.
- Squared papers and mathematical tables are provided.
- Silent, non programmable scientific calculators may be used.
- Where necessary, use $g = 9.8ms^{-2}$

SECTION A: (40 MARKS)

Attempt ***all*** questions from this section

1. Given that α and β are roots of the equation $3x^2 - 2x - 1 = 0$, without solving the equation, find the value of:

(i) $\alpha^2 + \beta^2$

(ii) $\alpha - \beta$ *(05 marks)*

2. The position vectors of points A, B and C are $-\boldsymbol{i} + 2\boldsymbol{j}$, $\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{i} + 4\boldsymbol{j}$ respectively. Use vectors to prove that $\boldsymbol{BA}$ and $\boldsymbol{BC}$ are perpendicular. *(05 marks)*

3. Show that the derivative of $\frac{2x+1}{x+1}$ is equal to $\frac{1}{(x+1)^2}$ hence or otherwise evaluate $\int_0^2 \frac{4}{(x+1)^2}\boldsymbol{dx}$ *(05 marks)*

4. A committee of 5 members is to be chosen from 4 girls and 5 boys. Determine the number of possible committees if there must be atleast a boy and a girl on each committee. *(05 marks)*

5. The probability that Moses will pass a Math test is $\frac{4}{5}$. The probability that Ali will pass the same test is ⅔. Find the probability that:

(i) only one will pass

(ii) atleast one will fail. *(05 marks)*

6. The heights of 500 students in S.1 class of a certain school are normally distributed, with a mean of $120cm$ and a standard deviation of $5cm$. Find the number of students whose height lies between $122cm$ and $126\ cm$. *(05 marks)*

7. Given $\boldsymbol{x \sim B(15, 0.2)}$ find the:

(i) $P(x = 2)$

(ii) $P(x \leq 3)$ *(05 marks)*

8. The matrix $\boldsymbol{P} = \begin{pmatrix} \boldsymbol{\sin 2\theta} & \boldsymbol{\cos\theta} \\ \boldsymbol{\cos\theta} & \boldsymbol{2} \end{pmatrix}$ is a singular matrix. Find the values of $\boldsymbol{\theta}\ for\ \boldsymbol{0^0 \leq \theta \leq 180^0}$. *(05 marks)*

SECTION B (60 MARKS)

Attempt **FOUR** questions from this section with at least one from each part.

PART ONE: PURE MATHEMATICS

9. (a) Given that $P = \frac{1-sin\theta}{1+sin\theta}$, show that $P = (sec\theta - tan\theta)^2$ hence deduce that if $\theta = 60^o$ then $P = 7 - 4\sqrt{3}$. *(09 marks)*

(b) Solve the equation $3\,cos^2x - 2sin^2x - sinx + 1 = 0$
For $0^o \leq x \leq 360^o$ *(06 marks)*

10. The rate at which the value x (shs) of an article depreciates with time t $(years)$ is proportional to x. *(04 marks)*

(i) Form a differential equation in terms of x and t and obtain it's general solution.

(ii) Initially the value of the article was 20 million and after two years it was 18 million, find its price after 5 years. *(11 marks)*

11. The gradient function of a curve is $-\boldsymbol{2x + k}$ and the value of the gradient at $(\mathbf{4}, \mathbf{0})$ is $-\mathbf{7}$.

(a) Find the value of $\boldsymbol{k}$. *(02 marks)*

(b) Obtain the equation of the curve and hence sketch the curve. *(08 marks)*

(c) Find the area between the curve and the x-axis *(05 marks)*

12. An export company is to transport 300tonnes of pineapples. Two Cargo planes are Available. A Boeing which can carry 30 tonnes of pineapples per flight and an Airbus which can carry 20 tonnes of pineapples per flight . The Airbus has to make moreflights than theBoeing. The Boeing has to make at least 3 flights. The company has 150,000 US dollars for transport costs. The cost per flight is 12,000 dollars for Boeing and 9,000 dollars for Airbus.

If $\boldsymbol{x}$ is the number of flights made by the Boeing and $\boldsymbol{y}$ the number of flights made by the Airbus;

(a) White down four inequalities satisfying the given conditions.

(b) Plot graphs of the inequalities you have formed on the same axes and shade the Unwanted regions.

(c) Find the number of flights each plane should make if the cost of transport is to be minimum. *(15 marks)*

PART TWO: STATISTICS

13. The weights (kg) of 100 recruits in the police force were recorded as shown.

Weight(kg)	50 –	55 –	60 –	65 –	70 –	75 –	80 –
Number of recruits	6	11	17	28	20	15	3

(a) Calculate the:

(i) Mean weight

(ii) Standard deviation *(10 marks)*

(b) Draw a cumulative frequency curve and use it to determine the number of recruits whose height exceeds $67.5kg$. *(05 marks)*

14. The prices and price relatives of some items in the months of June and July respectively of 2017 required by a certain family are given in the table below.

Item	**June price (Ushs)**	**July Price relatives (June=100)**	**Weight**
Posho (1kg)	1200	140	3
Beans (1kg)	2,000	120	3
Matooke (a bunch)	20,000	95	2
Rice (1 kg)	3,500	110	1
Cooking oil (1 litre)	1,500	90	1

(a) Taking beans as the base, calculate the prices relatives for June.

(b) Determine the price of each item in July.

(c) Calculate the average weighted cost of the items in July. *(15marks)*

15. The table below shows the monthly sales of bags of sugar by a certain wholesale shop in the year 2017.

Month	***Sales***	***Month***	***Sales***	***Month***	***Sales***
January	84	**May**	92	**September**	100
February	64	**June**	70	**October**	81
March	61	**July**	63	**November**	72
April	82	**August**	85	**December**	96

(a) Calculate the four – monthly moving total and hence the four moving averages for the data. *(06 marks)*

(b) (i) On the same axes draw graphs of moving averages and actual sales.Comment on the sales over the period.

(ii) Determine the number of bags of sugar expected to be sold in January 2018. *(09 marks)*

16. The probability that a candidate completes an examination in x *hours* after starting is given by the p.d.f

$$\begin{cases} \boldsymbol{f(x) = k(9 - x);\ 0 \le x \le 3} \\ 0 \qquad \text{otherwise.} \end{cases}$$

where k is constant,

(a) determine the value of k.

(b) sketch f(x).

(c) find the probability that a candidate selected at random took less than 2 ½ hours.

(d) Find the mean value. *(15 marks)*

END