P425/1
PURE MATHEMATICS
Paper 1
April /May. 2021
3 hours

REVISION EXAMINATIONS, 2021

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in Section* **A** *and* **five** *questions from Section* **B**

All necessary calculations must be done on the **same page** *as the rest of the answer. Therefore no paper should be given for rough work.*

Begin each answer on a fresh page.

Mathematical tables with a list of formulae and squared paper are provided.

Silent, non-programmable scientific calculators may be used.

State the degree of accuracy at the end of the answer to each question attempted using a calculator or tables; and indicate **Cal** *for calculator, or* **Tab** *for mathematical tables.*

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SECTION A (40 MARKS)

1. Show that the equation of the normal to the curve $x^2 + 2y - 2 = e^{y}$ at the point where $y = 0$ and $x > 0$ is $2\sqrt{3}\,y - x + \sqrt{3} = 0$. (*05 marks*)

2. The position vectors of points *A*, *B* and *C* are respectively $i - j + 2k$, $2i + j + 4k$ and $3i + 4k$. Determine the:

(i) angle *BAC* to the nearest degree.

(ii) area of triangle *ABC* (*05 marks*)

3. Find the constant term in the expansion of $\left(2x - \frac{1}{x^2}\right)^6$ (*05 marks*)

4. Solve the equation $\sin 3x \sin x = 2\cos 2x + 1$ for $0° \leq x \leq 360°$ (*05 marks*)

5. *A* and *B* are points of contact of the tangents from the point *P* (1, 1) to the circle $x^2 + y^2 - 4x - 6y + 12 = 0$. If the chord *AB* subtends an angle 2θ at the centre of the circle, find the values of $\tan \theta$. (*05 marks*)

6. In an arithmetic progression, the sum of the squares of the five consecutive terms equals 20 times the square of the middle term and the product of the five terms equals 80. Find the middle term. (*05 marks*)

7. A box is to be constructed in such a way that it must have a fixed volume of 800 cm^3 and a square base. If the box is to be open-ended at one end, find the dimensions of the box that will require the least amount of material. (*05 marks*)

8. Find the area enclosed by the curves $y = \sin 2x$ and $y = \cos^2 x$ over the region $0 \leq x \leq \pi$ (*05 marks*)

SECTION B (60 MARKS)

9. (a) *ABCD* is a square of side length 1m. Points *E* and *F* are taken on sides *AB* and *AD* such that $AE = AF = x$. Show that the area, y of the quadrilateral *CDFE* is given by $y = \frac{1+x-x^2}{2}$. Hence find the maximum area of the quadrilateral. (*05 marks*)

(b) If $y = x\sin^{-1}x$, show that $\frac{d^2y}{dx^2} = \frac{2-x^2}{(1-x^2)\sqrt{1-x^2}}$ (*07 marks*)

10. (a) Show that the general solution of the differential equation $x(x+1)\frac{dy}{dx} = y(y+1)$ is $y = \frac{kx}{1+(1-k)x}$ (*05 marks*)

(b) It is found out that the rate at which the rumour spreads in a population is jointly proportional to the number, x, of people who have heard the rumour and $N - x$, who have not yet heard it. If initially one person hears the rumour, show that

$$x = \frac{N}{1+(N-1)e^{-Nkt}}.$$ (*07 marks*)

11. (a) If $2\sin 2x + \cos 2x = a$, show that $(1+a)\tan^2 x - 4\tan x + a = 1$.

Hence or otherwise show that it $\tan x_1$ and $\tan x_2$ are the roots of the quadratic equation, then $\tan(x_1 + x_2) = 2$. (*08 marks*)

(b) From a point *X*, 200m due south of a cliff, the angle of elevation of the top of the cliff is 30°. From a point *Y* due east of the cliff, the angle of elevation of the top of the cliff is 20°. How far apart to the nearest metre are the points *X* and *Y*? (*04 marks*)

12. (a) Sketch the curve $y = \tan^{-1}x$. (*05 marks*)

(b) Determine the area enclosed by the curve above, the line $x = \sqrt{3}$ and the $x-$axis. (*04 marks*)

(c) Differentiate $x^2\tan^{-1}x$ with respect to x. (*03 marks*)

13. (a) Show that the equation of the tangent to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $P(a \sec\theta, b \tan\theta)$ is $bx \sec\theta - ay \tan\theta = ab$ (*04 marks*)

(b) Show that if the line $y = mx + c$ is a tangent to the rectangular hyperbola $x^2 - y^2 = a^2$, then $c^2 = a^2(m^2 - 1)$ and the co-ordinates of the point of contact T are $(-ma^2/c, -a^2/c)$. If the line meets the asymptotes of the hyperbola at P and Q, show that T is the midpoint of PQ. (*08 marks*)

14. (a) Determine the Cartesian equation of the plane defined by the vector equation $\boldsymbol{r} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ (*04 marks*)

(b) Find the equation of the plane which passes through the point $A\,(4, 2, 1)$ and

(i) contains the vector joining the points $B(3, -2, 4)$ and $C(5, 0, 1)$

(ii) is perpendicular to the planes $5x - 2y + 6z + 1 = 0$ and $2x - y - z = 4$. (*08 marks*)

15. (a) Prove that $\int_0^{\pi/2} x^2 \sin 2x \, dx = \frac{\pi^2 - 4}{8}$ (*05 marks*)

(b) Express $\frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)}$ into partial fractions. Hence evaluate to three decimal places $\int_3^5 \frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)} \, dx$. (*07 marks*)

16. (a) Find a complex number Z which satisfies the equation $Z(1 + \sqrt{2}i) = 1 - \sqrt{2}\,i$ (*05 marks*)

(b) A complex number z satisfies the equation $zz^* - 3z - 2z^* = 2i$ where z^* denotes a complex conjugate of z. Find the two possible values of z giving your answer in the form $a + bi$. (*07 marks*)

END