P425/1
PURE MATHEMATICS
Paper 1
July/ August, 2024
3 Hours

MATIGO EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer **all the eight** questions in section **A** and any **five** from section **B**.*
- *Any additional question(s) answered will **not** be marked.*
- *All working **must** be shown clearly.*
- *Begin each question on a fresh sheet of paper.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

Turn Over

SECTION A (40 MARKS)

Answer all the questions in this section.

1. Solve the equation:

$4^x - 2^{x+3} + 15 = 0$ (05 marks)

2. Show that; (05 marks)

$$\frac{tan\theta}{sin\theta} - \frac{sin\theta}{tan\theta} = tan\theta sin\theta$$

3. The point K has position vector $3i + 2j - 5k$ and a point L has position vector $i + 3j + 2k$. Find the position vector of a point M which divides $\overrightarrow{KL}$ in the ratio of 4:3. (05 marks)

4. Show that the line $3x - 4y + 14 = 0$ is a tangent to a circle $x^2 + y^2 + 4x + 6y - 3 = 0$. (05 marks)

5. Solve the inequality; $|x - 2| < 3x - 4$ (05 marks)

6. The volume of a spherical balloon is increasing at a rate of $0.05m^3s^{-1}$. How fast is the surface area increasing when the radius is 0.4m? (05 marks)

7. Find the equations of the lines through the point (2,3) which makes an angle $45^0$with the line $2x - y + 3 = 0$ (05 marks)

8. Solve the differential equation. (05 marks)

$$\frac{dy}{dx} = \frac{1 + 4y^2}{e^x}$$

SECTION B (60 MARKS)

*Answer any **five** questions from this section*

***All** questions carry equal marks*

9. With respect to the origin O, the points A, B, C and D have position vectors given by.

$$\overrightarrow{\mathbf{OA}} = 4i + k, \overrightarrow{\mathbf{OB}} = 5i - 2j - 2k, \overrightarrow{\mathbf{OC}} = i + j, \quad \overrightarrow{\mathbf{OD}} = -i - 4k$$

(i) Calculate the acute angle between the lines AB and CD

(ii) Prove that the lines AB and CD intersect

(iii) The point P has position vector $i + 5j + 6k$, show that the perpendicular distance from P to the line AB is equal to $\sqrt{3}$

(12 marks)

10. (a) Show that $\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} = \frac{4}{\sin\theta\tan\theta}$, Hence solve for $0° < \theta < 360°$, the equation; $\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3$ (07 marks)

(b) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{1}{4}\pi$ (05 marks)

11. (a) Find a Geometric progression (GP) for which the sum of first two terms is -4 and the fifth term is 4 times the third term. (04 marks)

(b) The quadratic equation $x^2 - 7kx + k^2$, where k is a positive constant, has roots α and β where $\alpha > \beta$

(i) Show that $\alpha - \beta = 3k\sqrt{5}$

(ii) Hence form a quadratic equation with roots $\alpha + 1$ and $\beta - 1$ in the form $x^2 + px + q = 0$ where p and q should be given in terms of k

12. (a) The gradient at any point (x, y) on the curve is $\sqrt{(1+2x)}$. The curve passes through the point (4,11). Find the equation of the curve. (05 marks)

(b) A rectangular box is to be made from a piece of cardboard 24cm long and 9cm wide, by cutting out identical squares from the four corners and turning up the sides. What are the dimensions of the box of maximum volume? (07 marks)

13. (a) Integrate with respect to x

$$\int x\tan^2 x\,dx$$

(06 marks)

(b) Show that $\int x\sin^{-1}x\,dx = \frac{1}{4}(2x^2 - 1)\sin^{-1}x + \frac{1}{4}x\sqrt{(1-x^2)} + c$

(06 marks)

14. (a) Find the two square roots of the complex number $-3 + 4i$, giving your answers in the form $x + iy$, where x and y are real. (06 marks)

(b) The complex number z is given by

$$z = \frac{-1+3i}{2+i}$$

(i) Express z in the form $x + iy$, where x and y are real. (04 marks)

(ii) Show z on a sketch of an Argand diagram. (02 marks)

15. (a) The tangent to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 8ax$ at P, Q. Find the locus of the midpoint of PQ. (05 marks)

(b) The chord of contact of the point (x_1, y_1) with respect to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Cut the axes at L and M. if the locus of the midpoint of LM is the circle $x^2 + y^2 = 1$, find the locus of (x_1, y_1). (07 marks)

16. (a) Solve $Iny + \frac{x}{y}\frac{dy}{dx} = secx\ tanx$ for $y(0) = \frac{\pi}{2}$ (05 marks)

(b) A tank contains a solution of salt in water. Initially the tank contains 1000litres of water with 10kg of salt dissolved in it. The mixture is poured off at a rate of 20 litres per minute, and simultaneously pure water is added at a rate of 20 litres per minute. All the time the tank is stirred to keep the mixture uniform.

(i) Find the mass of salt in the tank after 5 minutes. (03 marks)

(ii) The tank must be toppled up by adding more salt when the mass of salt in the tank falls to 5kg, after how many minutes will it need topping up? (04 marks)

END

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