

P425/1  
PURE  
MATHEMATICS  
Paper 1  
July /Aug. 2023  
3 hours



UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

**INSTRUCTIONS TO CANDIDATES:**

*Answer **all** questions in section A and any **five** from section B.*

*All necessary working must be shown clearly.*

*Silent non – programmable scientific calculators and mathematical tables may be used.*

*Any extra question(s) attempted in section **B** will **not** be marked.*

### SECTION A (40 MARKS)

Answer ALL questions in this section

1. Use the Echelon method to solve the simultaneous equations:

$$2x - y + 3z = 14.$$

$$x + 4y - z = -5$$

$$3x + y + 4z = 17$$

(05 marks)

2. Prove the identity:

$$\sin 5A \cos 3A - \cos 7A \sin A = \sin 4A \cos 2A$$

(05 marks)

3. Calculate the total area bounded by the curve  $y = 3x^2 - 6x$ , the  $x$ -axis and the lines  $x = -1$  and  $x = 2$ . (05 marks)

4. Find a unit vector perpendicular to the vectors;

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

(05 marks)

5. A circle whose centre lies in the first quadrant touches the positive  $x$ -axis at +4, and touches the line  $3y = 4x$ . Find the radius of the circle, and state the coordinates of its centre. (05 marks)

6. Given that  $x$  and  $y$  are real numbers such that:

$$xz + y\bar{z} = 7i - 2, \text{ where } z = 2 + i, \text{ find the modulus of } x + iy.$$

(05 marks)

7. Differentiate the function  $x \sin x$  from first principles. (05 marks)

8. A curve is represented by the parametric equations;

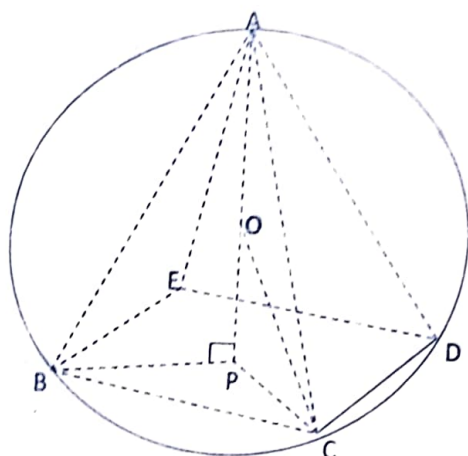
$$x = t^2: y = 5t - 7, \text{ find the equation of the tangent to the curve at the point } (4, 3).$$

(05 marks)

### SECTION B (60 MARKS)

9. Given the lines  $\mathbf{r}_1 = \begin{pmatrix} 6 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$  and  $\mathbf{r}_2 = \begin{pmatrix} 7 \\ 3 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ .
- Find the coordinates of their point of intersection. (04 marks)
  - Calculate the acute angle between the lines. (04 marks)
  - Find the Cartesian equation of the plane containing the lines. (04 marks)
10. (a) The roots of the equation  $x^2 + px + (p + 9) = 0$  differ by 3, find the possible values of  $p$ . (05 marks)
- (b) Use the remainder theorem to find the remainder when the polynomial  $P(x) = x^3 - 3x^2 + 2x - 5$  is divided by  $(x - 2)^2$ . (07 marks)
11. (a) Given that  $\cos(\theta + 60^\circ) = \sin\theta$ , show that  $\tan\theta = 2 - \sqrt{3}$ ; hence or otherwise solve for  $\theta$  in the interval  $[0^\circ, 360^\circ]$ . (06 marks)
- (b) Given that  $A, B$  and  $C$  are angles of a triangle. Prove that;  
 $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A \sin B \cos C$ . (06 marks)
12. (a) Use small changes to evaluate  $\tan 46^\circ$  to 4 dps. (05 marks)
- (b) Evaluate:  $\int_4^5 \frac{x^3}{x^2 - 9} dx$  to dps. (07 marks)
13. (a) The  $n^{\text{th}}$  term of a series is  $3^n + 4n$ . Calculate the sum of the first 20 terms of the series. (05 marks)
- (b) Expand  $\sqrt{1 - 4x}$  up to the term in  $x^4$ . State the range of values of  $x$  within which the expansion is convergent. Hence evaluate;  
 $\sqrt{15}$  to 4dps. (07 marks)

14.



ABCDE is right pyramid with a square base. The pyramid is completely inscribed in a sphere of radius  $\overline{OC} = 6\text{cm}$ , where O is the centre of the sphere. P is the centre of the square base BCDE as shown.

Given that  $\overline{OP} = x$ .

- (a) Show that the volume of the pyramid; (07 marks)

$$V = \frac{2}{3}(6 + x)^2(6 - x)\text{ cm}^3$$

- (b) Calculate the maximum volume of the pyramid. (05 marks)

15. (a) Show that the equation of the chord joining the point  $P(p^2, 2p)$  and  $Q(q^2, 2q)$  on the parabola  $y^2 = 4x$  is

$$2x - (p + q)y + 2pq = 0 \quad (04 \text{ marks})$$

- (b) If the chord in (a) above passes through the point  $R(4, 0)$  show that  $pq = -4$ , hence:

- (i) show that the chord  $\overline{PQ}$  makes a right angle at the origin  $O(0,0)$ .

- (ii) find the locus of the mid-point of  $\overline{PQ}$ . (08 marks)

16. In a certain game reserve, there are 80 elephants. Poachers start killing the elephants at a rate which is directly proportional to the number of elephants remaining in the forest. After one month 40 elephants have been killed. Let  $x$  be the number of elephants killed after  $t$  months.

- (a) Show that;  $\ln\left(\frac{80}{80-x}\right) = t\ln 2$  (07 marks)

- (b) Calculate the:

- (i) number of elephants killed after 2 months.

- (ii) time taken to kill 75 elephants, and in this case state the average number of elephants killed per day. (05 marks)

END