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P425/1
PURE MATHEMATICS
Paper 1
3 hours

UGANDA ADVANCE CERTIFICATE OF EDUCATION

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

Answer **all** the **eight** questions in section **A** and any **five** questions from section **B**.

Any additional question(s) answered will **not** be marked.

All necessary working **must** be shown clearly.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

1. Solve the equation: $6\tan^2 x - 4\sin^2 x = 1$, for $0^0 \leq x \leq 360^0$. *(05 marks)*

2. If α and β are the roots of the quadratic equation $x^2 + px + q = 0$, express $(\alpha - \beta^2)(\beta - \alpha^2)$ in terms of p and q. *(05 marks)*

3. Find the equations of the lines through $(2, 3)$ which makes angle of 45^0 with the line $x - 2y = 1$. *(05 marks)*

4. If $x^2 + 2xy + 3y^2 = 1$, show that $(x + 3y)^3 \dfrac{d^2y}{dx^2} + 2 = 0$. *(05 marks)*

5. Use the substitution $y = mx$ to solve the equations $x^2 + 4xy + y^2 = 13$ and $2x^2 + 3xy = 8$. *(05 marks)*

6. Calculate the volume generated by rotating the area bounded by the curve $y = 2\cos\left(x - \dfrac{\pi}{3}\right)$, the y-axis, the x-axis and the line $x = \dfrac{\pi}{2}$ through 2π radians about x-axis. *(05 marks)*

7. The vector equation of the line, L is given by $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + \lambda(p\boldsymbol{i} + q\boldsymbol{j}) + \boldsymbol{k}$, where λ is a real parameter. Given that the point $(2, 4, 1)$ lies on L, find the;

(i) Values of p and q.

(ii) Angle between L and the positive x-axis. *(05 marks)*

8. Show that $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{4 - \tan^2 x} dx = \frac{1}{4}\ln 3.$ *(05 marks)*

SECTION B: (60 MARKS)

9. (a) Prove by induction that $8^n - 7n + 6$ is divisible by 7. *(06 marks)*

(b) Expand $\sqrt{\dfrac{1+5x}{1-5x}}$, as far as the term in x^3. Taking the first three terms and $x = \dfrac{1}{9}$, evaluate $\sqrt{14}$, correct to four significant figures. *(06 marks)*

10. (a) Solve the equation $16 \sin x \cos x = \tan x + \cot x$, for $0^0 \leq x \leq 180^0$. *(06 marks)*

(b) In a triangle ABC, prove that $\dfrac{bc}{ab + ac} = \dfrac{\text{cosec}(B + C)}{\text{cosec}B + \text{cosec}C}$. *(06 marks)*

11. (a) Differentiate with respect to x;

(i) $y = \sqrt{1 + 4x^2}$.

(ii) $\sin^2 5x$. *(06 marks)*

(b) If $y = \sqrt{\dfrac{x}{2x+1}}$, find the value of $\dfrac{dy}{dx}$ when $x = 4$. *(06 marks)*

12. (a) Express the complex number $z_1 = 4i$ and $z_2 = 2 - 2i$ in trigonometric form. Hence evaluate $\dfrac{z_1}{z_2}$. *(06 marks)*

(b) Find the values of x and y given that $\dfrac{x}{2+3i} - \dfrac{y}{3-2i} = \dfrac{6+2i}{1+8i}$. *(06 marks)*

13. A hemispherical bowl of radius r cm is initially full of water. The water runs out of the small hole at the bottom of the bowl at a constant rate which is such that it would empty the bowl in 24 seconds. Given that when the depth of the water is x, the volume is $\dfrac{1}{3}\pi x^2(3a - x)$ cm^3, prove that the depth is decreasing at a rate of $\dfrac{r^3}{36x(2a - x)}$ cm/s. Find after what time the depth is $\dfrac{1}{2}$ cm and the rate the water level is decreasing. *(12 marks)*

14. Show that if the chord joining the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ on the parabola $y^2 = 4ax$ passes through the focus, then $pq = -1$.

The tangent at point P meets the line through Q parallel to the axis of the parabola at R. Prove that the line $x + a = 0$ bisects PR. *(12 marks)*

15. The vector equations of two planes $\pi 1$ and $\pi 2$ are
$\boldsymbol{r} = 2\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k} + \alpha(-\boldsymbol{i} + 2\boldsymbol{k}) + \beta(\boldsymbol{i} + 2\boldsymbol{j} + 8\boldsymbol{k})$ and
$\boldsymbol{r} = -3\boldsymbol{i} - 3\boldsymbol{j} + \lambda(-\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}) + \mu(-3\boldsymbol{i} - 4\boldsymbol{j} - 2\boldsymbol{k})$ respectively.

(a) Find the Cartesian equation of each plane. (*05 marks*)

(b) If L is the line of intersection of the two planes above, find the;

(i) Equation of the line in vector form.

(ii) Coordinates of the foot of the perpendicular from the point $(-1, -5, -10)$ to line L. (*07 marks*)

16. (a) Solve the differential equation $\frac{dy}{dx} + ky = 2$ where k is a constant for which $y = 3$ when $x = 0$. (*05 marks*)

(b) A colony of bacteria which is initially of size 1500 increases at a rate proportional to its size such that after t hours, its population is N.

(i) Write down an equation connecting t and N.

(ii) If the size of the colony increases to 3000 in 20 hours, solve the equation to find N in terms of t.

(iii) What size is the colony when $t = 80$ hours?

(iv) How long did it take to the nearest minutes for the population to increase from 2000 to 3000? (*07 marks*)

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