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UACE MATHEMATICS PAPER 2 2019 guide

SECTION A

1. The table shows the masses of bolts bought by a carpenter.

Mass (grams)	98	99	100	101	102	103	104
Number of bolts	8	11	14	20	17	6	4

Calculate the:

- (a) median mass
 (b) mean mass of the bolt (05mark)
2. A uniform rod AB of length 3m and mass 8kg is freely hinged to a vertical wall at A. A string BC of length 4m attached at b and to point C on the wall, keeps the rod in equilibrium. If C is 5m vertically above A, find the
- (a) Tension in the string (03marks)
 (b) Magnitude of the normal reaction at A. (02marks)
3. Use the trapezium rule with seven coordinates to estimate $\int_0^3 [(1.2)^x - 1]^{\frac{1}{2}} dx$ correct to 2 decimal places (05marks)
4. A discrete random variable X has the following probability distribution

X	0	1	2	3	4	5
P(X = x)	0.11	0.17	0.2	0.13	p	0.09

Find the

- (a) Value of p (02marks)
 (b) Expected value of X (03marks)
5. A stone is thrown vertically upwards with velocity 16ms^{-1} from a point H meters above the ground level. The stone hits the ground 4 seconds later. Calculate the
- (a) Value of H (03marks)
 (b) Velocity of the stone as it hits the ground (02marks)
6. The table below shows the commuter bus fare from stages A to B, C, D and E

Stage	A	B	C	D	E
Distance (km)	0	12	16	19	23
Fare (shs)	0	1300	1700	2200	2500

- (a) Jane boarded from A and stopped at a place 2km after E. How much did she pay? (03marks)
 (b) Okello paid shs 2000. How far from A did the bus leave him? (02marks)

7. The amount of meat sold by a butcher is normally distributed with a mean of 43kg and standard deviation 4kg. Determine the probability that the amount of meat sold is between 40kg and 50kg. (05marks)
8. A particle is moving with simple harmonic motion (SHM). When the particle is 15m from the equilibrium, its speed is 6ms^{-1} . When the particle is 13 m from equilibrium, its speed is 9ms^{-1} . Find the amplitude of the motion (05marks)

SECTION B

9. Car A is 80m North West of point O. Car B is 50m N 30° E of O. Car A s moving at 20ms^{-1} while car B is moving at 10ms^{-1} each on a straight road towards O. Determine the
 - (a) Initial distance between the two cars (03mark)
 - (b) Velocity of A relative to B (05marks)
 - (c) The shortest distance between the two cars as they approach O (04marks)

10. The table below shows the marks obtained in a mathematic test by a group of student

marks	5 -<15	15-<25	25-<35	35-<45	45-<55	55-<65	65-<75	75-<100
Number of students	5	7	19	17	7	4	2	3

- (a) Construct a cumulative frequency (O give) for the data (05 marks)
 - (b) Use your Ogive to find the
 - (i) Range between the 10th and 70th percentiles
 - (ii) Probability that a student selected at random scored below 50 marks. (07 marks)
11. (a) Show that the equation $x - 3\sin x = 0$ has a root between 2 and 3. (03marks)
- (b) Show that Newton- Raphson iterative formula for estimating the root of the equation in (a) is given by

$$X_{n+1} = \frac{3(\sin x_n - x_n \cos x_n)}{1 - 3 \cos x_n}, n = 0, 1, 2, \dots$$

Hence find the root of the equation corrected to 2 decimal places (09 marks)

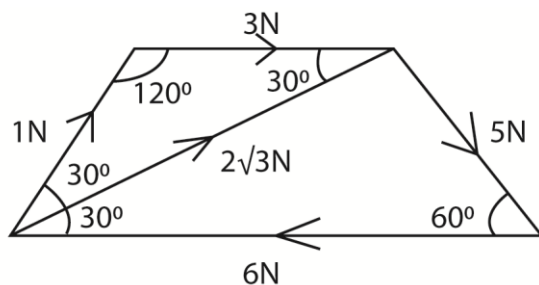
12. A force $F = (2t \mathbf{i} + \mathbf{j} - 3t \mathbf{k})\text{N}$ acts on a particle of mass 2kg. The particle is initially at a point (0,0,0) and moving with a velocity $(\mathbf{i} + 2\mathbf{j} - \mathbf{k})\text{ms}^{-1}$. Determine the:
 - (a) Magnitude of the acceleration of the particle after 2 seconds (04marks)
 - (b) Velocity of the particle after 2seconds (04marks)
 - (c) Displacement of the particle after 2 seconds (04marks)
13. Two events A and B are such that $P(B) = \frac{1}{8}$, $P(A \cap B) = \frac{1}{10}$ and $P(B/A) = \frac{1}{3}$

Determine the

- (a) $P(A)$ (03marks)
 - (b) $P(A \cup B)$ (03marks)
 - (c) $P(A/\bar{B})$ (06marks)
14. (a) Given that $y = e^x$ and $x = 0.62$ correct to two decimal places, find the interval within which the exact value of y lies. (05marks)
- (b) Show that the maximum possible relative error in $y \sin^2 x$ is

$$\left| \frac{\Delta y}{y} \right| + 2 \cot x |\Delta x|$$
, where Δx and Δy are errors in x and y respectively

Hence find the percentage error in calculating $y \sin^2 x$ if $y = 5.2 \pm 0.05$ and $x = \frac{\pi}{6} \pm \frac{\pi}{360}$ (07 marks)
15. The diagram below shows a trapezium rule ABCD, AD = DC = CB = 1 and AB = 2 meters. Forces of magnitude 1N, 3N, 5N, 6N and $2\sqrt{3}$ N respectively.



- (a) Calculate the magnitude of the resultant force and the angle it makes with side AB (09marks)
- (b) Given that the line of action of the resultant force meets AB at X, find AX. (03marks)
16. A biased die with faces labelled 1, 2, 3, 4, 5 and 6 is tossed 45 times
Calculate the probability that 2 will appear;
- (a) More than 18 times (07marks)
- (b) Exactly 11 times (05marks)

Solutions

SECTION A

17. The table shows the masses of bolts bought by a carpenter.

Mass (grams)	98	99	100	101	102	103	104
Number of bolts	8	11	14	20	17	6	4
fx	784	1084	1400	2020	1734	618	416
c.f	8	19	33	53	70	76	80

$$\sum f = 80, \sum fx = 8061$$

Calculate the:

(a) median mass

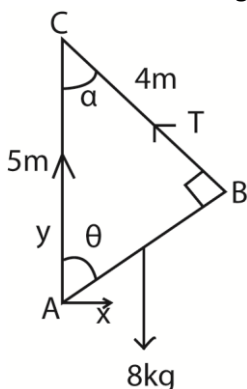
$$\text{Median position} = \left(\frac{N}{2}\right)^{th} \text{ value} = \left(\frac{80}{2}\right)^{th} \text{ value} = 40^{th} \text{ value}$$

$$\therefore \text{median} = 101$$

(b) mean mass of the bolt (05mark)

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{8061}{80} = 100.76g$$

18. A uniform rod AB of length 3m and mass 8kg is freely hinged to a vertical wall at A. A string BC of length 4m attached at B and to point C on the wall, keeps the rod in equilibrium. If C is 5m vertically above A, find the
- (c) Tension in the string (03marks)



$$AB^2 + 4^2 = 5^2$$

$$AB = \sqrt{(25 - 16)} = 3$$

Let T be tension in the string, from the diagram

$$\cos \theta = \frac{3}{5}, \cos \alpha = \frac{4}{5}$$

Equation of moment about A

$$T \times 3 = 8g \times 1.5 \cos \alpha$$

$$3T = 8 \times 9.8 \times \frac{4}{5}; T = 31.36\text{N}$$

\therefore tension in the string is 31.36N

(d) Magnitude of the normal reaction at A. (02marks)

$$x = T \cos \theta = 31.36 \times \frac{3}{5} = 18.816\text{N}$$

\therefore the magnitude of normal reaction at A is 18.816N

19. Use the trapezium rule with seven coordinates to estimate

$$\int_0^3 [(1.2)^x - 1]^{\frac{1}{2}} dx \text{ correct to 2 decimal places (05marks)}$$

Solution

For 7 ordinates, there are 6 subintervals

$$\text{Width, } h = \frac{b-a}{\text{subinterval}} = \frac{3-0}{6} = 0.5$$

$$\text{Let } y = \sqrt{(1.2)^x - 1}$$

x	y	
0	0	
0.5		0.309
1		0.447
1.5		0.561
2		0.663
2.5		0.760
3	0.853	
Sum	0.853	2.74

Using the trapezium rule

$$\int_0^3 [(1.2)^x - 1]^{\frac{1}{2}} dx = \frac{0.5}{2} [0.853 + 2(2.74)] = 1.58$$

20. A discrete random variable X has the following probability distribution

X	0	1	2	3	4	5
P(X = x)	0.11	0.17	0.2	0.13	p	0.09

Find the

(c) Value of p (02marks)

$$\text{Using } \sum P(X = x) = 1$$

$$0.11 + 0.17 + 0.2 + 0.13 + p + 0.09 = 1$$

$$p = 0.3$$

(d) Expected value of X (03marks)

$$E(X) = \sum x \cdot P(X = x)$$

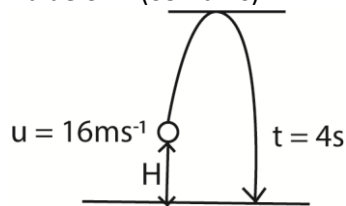
$$= 0 \times 0.11 + 1 \times 0.17 + 2 \times 0.2 + 3 \times 0.13 + 4 \times 0.3 + 5 \times 0.09$$

$$= 2.61$$

21. A stone is thrown vertically upwards with velocity 16ms^{-1} from a point H meters above the ground level. The stone hits the ground 4 seconds later.

Calculate the

(c) Value of H (03marks)



Using $s = ut + \frac{1}{2}at^2$; $s = -H$ (below point of projection), $u = 16\text{ms}^{-1}$, $a = -g$, $t = 4\text{s}$

$$-H = 16 \times 4 - \frac{1}{2} \times 9.8 \times 4^2$$

$$H = 14.4\text{m}$$

(d) Velocity of the stone as it hits the ground (02marks)

Using $v = u + at$; $v = -v$ (below point of projection), $a = -g$, $t = 4\text{s}$

$$-v = 16 - 9.8 \times 4$$

$$v = 23.2\text{ms}^{-1}$$

\therefore the velocity of the stone as it hits the ground is 23.2ms^{-1}

22. The table below shows the commuter bus fare from stages A to B, C, D and E

Stage	A	B	C	D	E
Distance (km)	0	12	16	19	23
Fare (shs)	0	1300	1700	2200	2500

(c) Jane boarded from A and stopped at a place 2km after E. How much did she pay?

(03marks)

2km after E = 25km from A, let x be the fare

Extract

D	E	
19	23	25
2200	2500	x

Using linear extrapolation

$$\frac{x - 2500}{25 - 23} = \frac{2500 - 2200}{23 - 19}$$

$$x = \text{sh } 2650$$

(d) Okello paid shs 2000. How far from A did the bus leave him? (02marks)

Let y be the distance

Extract

C		D
16	y	19
1700	200	2200

Using linear extrapolation

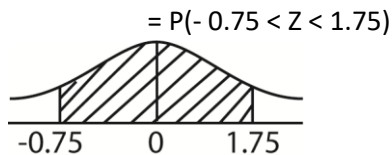
$$\frac{y - 16}{2000 - 1700} = \frac{19 - 16}{2200 - 1700}$$

$$y = 17.8\text{km}$$

23. The amount of meat sold by a butcher is normally distributed with a mean of 43kg and standard deviation 4kg. Determine the probability that the amount of meat sold is between 40kg and 50kg. (05marks)

$X \sim N(43, 4)$

$$P(40 < x < 50) = P\left(\frac{40 - 43}{4} < Z < \frac{50 - 43}{4}\right)$$



$$P(40 < x < 50) = P(-0.75 < Z < 0) + P(0 < Z < 1.75)$$

By property of symmetry

$$\begin{aligned} P(40 < x < 50) &= P(-0.75 < Z < 0) + P(0 < Z < 1.75) \\ &= 0.2735 + 0.4599 \\ &= 0.733 \end{aligned}$$

24. A particle is moving with simple harmonic motion (SHM). When the particle is 15m from the equilibrium, its speed is 6ms^{-1} . When the particle is 13 m from equilibrium, its speed is 9ms^{-1} . Find the amplitude of the motion (05marks)

$$v^2 = \omega^2(A^2 - x^2)$$

$$6^2 = \omega^2(A^2 - 15^2) \dots\dots (i)$$

$$9^2 = \omega^2(A^2 - 13^2) \dots\dots (ii)$$

Dividing (i) by (ii)

$$\frac{36}{81} = \frac{A^2 - 225}{A^2 - 169}$$

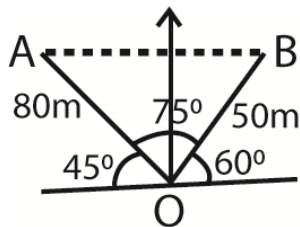
$$\text{Amplitude } A = 16.4256\text{m}$$

SECTION B

25. Car A is 80m North West of point O. Car B is 50m N 30° E of O. Car A is moving at 20ms^{-1} while car B is moving at 10ms^{-1} each on a straight road towards O. Determine the

(d) Initial distance between the two cars (03mark)

Method I: using geometrical approach



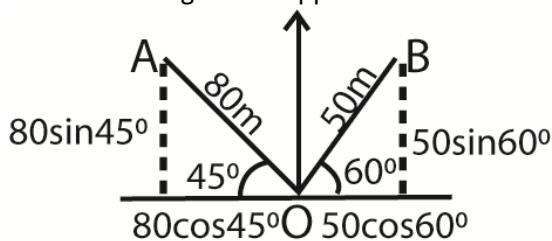
$$\text{Initial distance} = \overline{AB}$$

Using cosine rule

$$\overline{AB}^2 = 80^2 + 50^2 - 2 \times 80 \times 50 \cos 75^\circ$$

$$\overline{AB} = 82.64\text{m}$$

Method II using vector approach



$$\overline{AB} = \overline{OB} - \overline{OA}$$

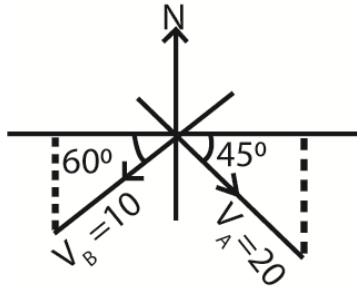
$$= \begin{pmatrix} 50 \cos 60^\circ \\ 50 \sin 60^\circ \end{pmatrix} - \begin{pmatrix} -80 \cos 45^\circ \\ 80 \sin 45^\circ \end{pmatrix} = \begin{pmatrix} 81.569 \\ -13.267 \end{pmatrix}$$

$$\overline{AB} = \sqrt{81.569^2 + (-13.267)^2} = 82.64m$$

(e) Velocity of A relative to B (05marks)

Method I: using vector approach

Vector	Direction	Magnitude
V_A	South east	$20ms^{-1}$
V_B	$S30^{\circ}W$	$10ms^{-1}$
${}_A V_B$?	?



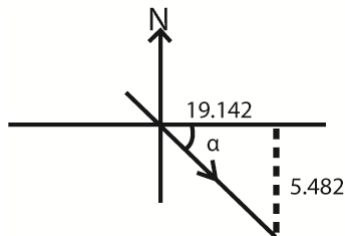
$${}_A V_B = V_A - V_B$$

$$= \begin{pmatrix} 20\cos 45^{\circ} \\ -20\sin 45^{\circ} \end{pmatrix} - \begin{pmatrix} 10\cos 60^{\circ} \\ -10\sin 60^{\circ} \end{pmatrix} = \begin{pmatrix} 19.142 \\ -5.482 \end{pmatrix}$$

$$\therefore {}_A V_B = 19.142i - 5.482j$$

Or : expressing it in terms of magnitude and direction

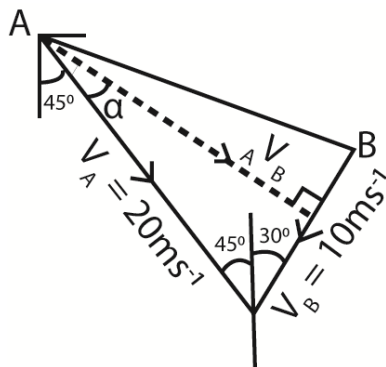
$$|{}_A V_B| = \sqrt{(19.142)^2 + (-5.482)^2} = 19.912m$$



$$\alpha = \tan^{-1}\left(\frac{5.482}{19.142}\right) = 15.98^{\circ}$$

Hence the relative velocity of A relative B is $19.912ms^{-1}$ in the direction $E15.98^{\circ}S$

Method II: Using geometric approach



Using cosine rule

$$|{}_A V_B|^2 = 20^2 + 10^2 - 2 \times 20 \times 10 \cos 75^{\circ}$$

$$|{}_AV_B| = 19.912 \text{ms}^{-1}$$

$$\frac{|{}_AV_B|}{\sin 75^\circ} = \frac{10}{\sin \alpha}$$

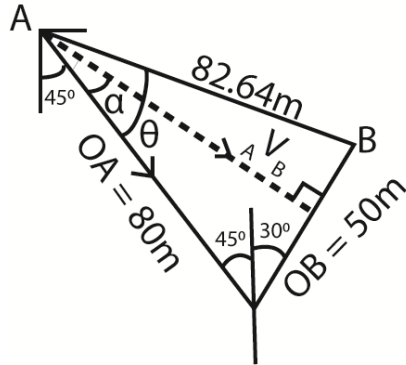
$$\alpha = 29.02^\circ$$

$$45^\circ + 29.02^\circ = 74.02^\circ$$

\therefore The velocity of A relative to B is 19.912ms^{-1} due $S74.02^\circ E$

- (f) The shortest distance between the two cars as they approach O (04marks)

Using geometrical approach



$$\frac{50}{\sin \theta} = \frac{82.64}{\sin 75^\circ}; \theta = 35.76^\circ$$

$$< BAD = 35.76 - 29.02 = 6.74^\circ$$

$$\sin 6.74^\circ = \frac{|{}_Ar_B|}{AB}$$

$${}_Ar_B = 82.64 \sin 6.74^\circ = 9.699 \text{m}$$

\therefore The shortest distance between the two cars they approach O is 9.699m

26. The table below shows the marks obtained in a mathematic test by a group of student

marks	5 -<15	15-<25	25-<35	35-<45	45-<55	55-<65	65-<75	75-<100
Number of students	5	7	19	17	7	4	2	3

- (c) Construct a cumulative frequency (O give) for the data (05 marks)

Class boundaries	F	Cf
5 – 15	5	5
15 - 25	7	12
25 – 35	19	31
35 – 45	17	48
45 – 55	7	55
55 – 65	4	59
65 – 75	2	61
75 – 85	3	64

- (d) Use your Ogive to find the

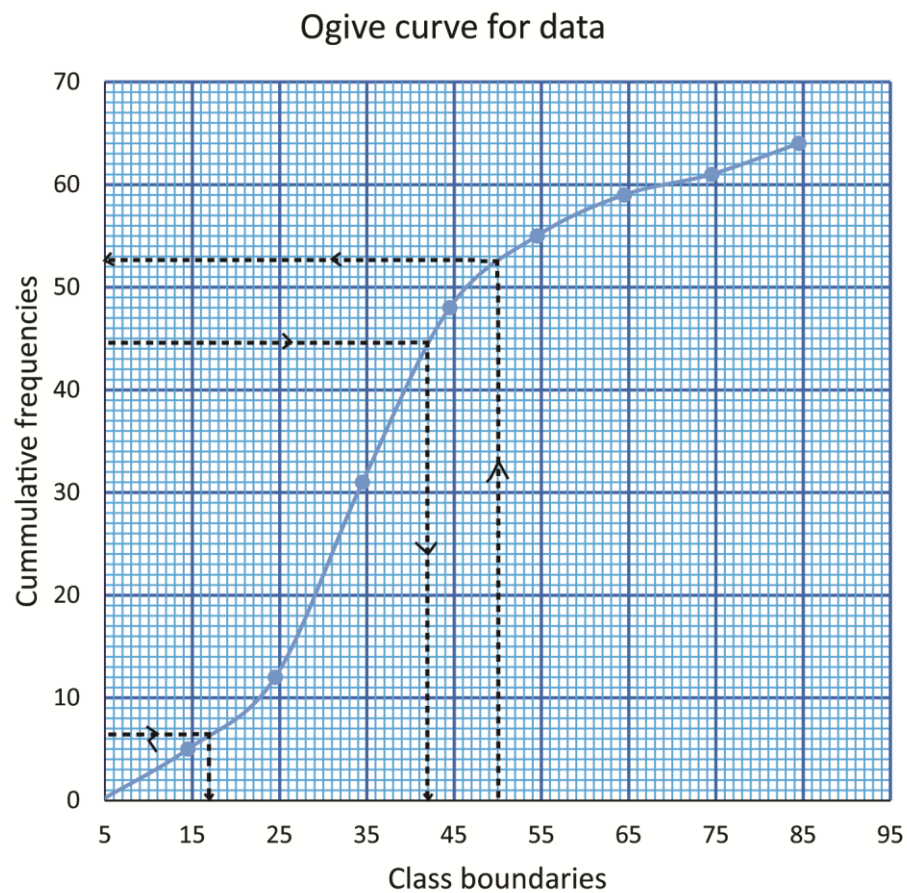
- (iii) Range between the 10th and 70th percentiles

$$10^{\text{th}} \text{ percentile} = \left(\frac{10}{100} \times 64 \right)^{\text{th}} \text{ value} = 6.4^{\text{th}} \text{ value}$$

From the graph below $P_{10} = 17$

$$10^{\text{th}} \text{ percentile} = \left(\frac{70}{100} \times 64 \right)^{\text{th}} \text{ value} = 44.8^{\text{th}} \text{ value}$$

From the graph below $P_{70} = 43$



Percentile range = $43 - 17 = 26$

- (iv) Probability that a student selected at random scored below 50 marks. (07 marks)

From the graph number of students who scored below 50 marks = 52

$$\text{Probability} = \frac{52}{64} = 0.8125$$

27. (a) Show that the equation $x - 3\sin x = 0$ has a root between 2 and 3. (03marks)

$$f(x) = x - 3\sin x$$

$$f(2) = 2 - 3\sin 2 = -0.7279$$

$$f(3) = 3 - 3\sin 3 = 2.5766$$

$$\text{since } f(2) \cdot f(3) = -1.8755 < 0$$

there exist a root of $x - 3\sin x = 0$ between 2 and 3

- (b) Show that Newton- Raphson iterative formula for estimating the root of the equation in (a) is given by

$$X_{n+1} = \frac{3(\sin x_n - x_n \cos x_n)}{1 - 3 \cos x_n}, n = 0, 1, 2 \dots$$

Hence find the root of the equation corrected to 2 decimal places (09 marks)

$$f'(x) = 1 - 3\cos x$$

$$\begin{aligned} x_{n+1} &= x_n - \frac{f(x)}{f'(x)} \\ &= x_n - \frac{x_n - 3 \sin x_n}{1 - 3 \cos x_n} \end{aligned}$$

$$= \frac{x_n - 3x_n \cos x_n - x_n + 3 \sin x_n}{1 - 3 \cos x_n}$$

$$x_{n+1} = \frac{3(\sin x_n - x_n \cos x_n)}{1 - 3 \cos x_n}$$

Taking $x_0 = \frac{2+3}{2} = 2.5$

$$x_1 = \frac{3(\sin 2.5 - 2.5 \cos 2.5)}{1 - 3 \cos 2.5} = 2.293$$

Error = $|2.293 - 2.5| = 0.207 > 0.005$

$$x_2 = \frac{3(\sin 2.293 - 2.5 \cos 2.293)}{1 - 3 \cos 2.293} = 2.279$$

Error = $|2.279 - 2.293| = 0.014 > 0.005$

$$x_3 = \frac{3(\sin 2.279 - 2.5 \cos 2.279)}{1 - 3 \cos 2.279} = 2.279$$

Error = $|2.279 - 2.279| = 0.000 < 0.005$

$\therefore \text{root} = 2.279 = 2.28(2D)$

28. A force $F = (2t \mathbf{i} + \mathbf{j} - 3t \mathbf{k})\text{N}$ acts on a particle of mass 2kg. The particle is initially at a point (0,0,0) and moving with a velocity $(\mathbf{i} + 2\mathbf{j} - \mathbf{k})\text{ms}^{-1}$. Determine the:

(d) Magnitude of the acceleration of the particle after 2 seconds (04marks)

$$F = (2t \mathbf{i} + \mathbf{j} - 3t \mathbf{k}) = \begin{pmatrix} 2t \\ 1 \\ -3t \end{pmatrix} \text{N}$$

$$\mathbf{a} = \frac{F}{m} = \frac{1}{2} \begin{pmatrix} 2t \\ 1 \\ -3t \end{pmatrix} = \begin{pmatrix} t \\ 0.5 \\ -1.5t \end{pmatrix} \text{ms}^{-1}$$

At $t = 2\text{s}$

$$\underline{a} = 2\mathbf{i} + 0.5\mathbf{j} - 3\mathbf{k}$$

$$|\underline{a}| = \sqrt{2^2 + 0.5^2 + (-3)^2} = 3.64\text{ms}^{-2}$$

(e) Velocity of the particle after 2seconds (04marks)

$$\underline{v} = \int \underline{a} dt = \int \begin{pmatrix} t \\ 0.5 \\ -1.5t \end{pmatrix} dt = \begin{pmatrix} 0.5t^2 \\ 0.5t \\ -7.5t^2 \end{pmatrix} + C$$

At $t = 0$ initial velocity = $(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + C \Rightarrow C = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$\therefore \underline{v} = \begin{pmatrix} 0.5t^2 + 1 \\ 0.5t + 2 \\ -7.5t^2 - 1 \end{pmatrix}$$

At $t = 2\text{s}$

$$\underline{v} = \begin{pmatrix} 0.5(2)^2 + 1 \\ 0.5(2) + 2 \\ -7.5(2)^2 - 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \text{ms}^{-1}$$

(f) Displacement of the particle after 2 seconds (04marks)

$$\underline{r} = \int \underline{v} dt = \int \begin{pmatrix} 0.5t^2 + 1 \\ 0.5t + 2 \\ -7.5t^2 - 1 \end{pmatrix} dt = \begin{pmatrix} \frac{t^3}{6} + t \\ \frac{t^2}{4} + 2t \\ -t - \frac{t^3}{4} \end{pmatrix} + C$$

$$\text{At } t = 0; \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + C$$

$$C = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} t^3 + t \\ \frac{t^2}{4} + 2t \\ -t - \frac{t^3}{4} \end{pmatrix}$$

At $t = 2s$

$$\underline{r} = \begin{pmatrix} \frac{2^3}{6} + 2 \\ \frac{2^2}{4} + 2 \times 2 \\ -2 - \frac{2^3}{4} \end{pmatrix} = \begin{pmatrix} \frac{10}{3} \\ 5 \\ -4 \end{pmatrix} m$$

29. Two events A and B are such that $P(B) = \frac{1}{8}$, $P(A \cap B) = \frac{1}{10}$ and $P(B/A) = \frac{1}{3}$

Determine the

(d) $P(A)$ (03marks)

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$\frac{1}{3} = \frac{1}{10} \div P(A)$$

$$P(A) = 3 \times \frac{1}{10} = \frac{3}{10}$$

(e) $P(A \cup B)$ (03marks)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{10} + \frac{1}{8} - \frac{1}{10} = \frac{13}{40}$$

(f) $P(A/\bar{B})$ (06marks)

$$\begin{aligned} P(A/\bar{B}) &= \frac{P(A \cap \bar{B})}{P(\bar{B})} \\ &= \frac{P(A) - P(A \cap B)}{1 - P(B)} \\ &= \frac{\frac{3}{10} - \frac{1}{10}}{1 - \frac{1}{8}} \\ &= \frac{\frac{2}{10}}{1 - \frac{1}{8}} \\ &= \frac{8}{35} \end{aligned}$$

30. (a) Given that $y = e^x$ and $x = 0.62$ correct to two decimal places, find the interval within which the exact value of y lies. (05marks)

$$e_x = 0.005$$

$$y_{max} = e^{0.625} = 1.8682$$

$$y_{min} = e^{0.615} = 1.8497$$

$$\text{The interval} = (1.8497, 1.8682)$$

(c) Show that the maximum possible relative error in $y \sin^2 x$ is

$$\left| \frac{\Delta y}{y} \right| + 2 \cot x |\Delta x|, \text{ where } \Delta x \text{ and } \Delta y \text{ are errors in } x \text{ and } y \text{ respectively}$$

$$\text{Hence find the percentage error in calculating } y \sin^2 x \text{ if } y = 5.2 \pm 0.05 \text{ and } x = \frac{\pi}{6} \pm \frac{\pi}{360}$$

(07 marks)

$$z = y \sin^2 x$$

$$e_z = \Delta y \sin^2 x + 2y \Delta x \cos x \sin x$$

$$\frac{e_z}{z} = \frac{\Delta y \sin^2 x}{y \sin^2 x} + \frac{2y \Delta x \cos x \sin x}{y \sin^2 x}$$

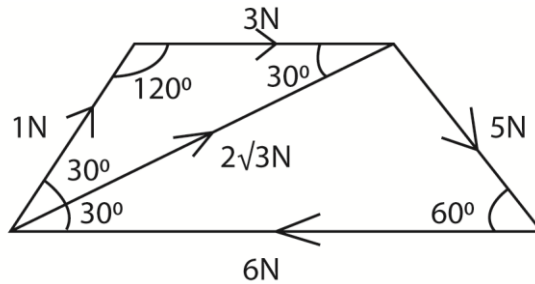
$$\left| \frac{e_z}{z} \right| = \left| \frac{\Delta y}{y} + 2 \cot x \cdot \Delta x \right|$$

$$\leq \left| \frac{\Delta y}{y} \right| + 2 \cot x \cdot |\Delta x|$$

$$\therefore \text{Maximum possible error is } \left| \frac{\Delta y}{y} \right| + 2 \cot x \cdot |\Delta x|$$

$$\text{percentage error} = \left[\frac{0.05}{5.2} + 2 \cot \frac{\pi}{6} \cdot \left| \frac{\pi}{360} \right| \right] \times 100\% = 3.9845\%$$

31. The diagram below shows a trapezium rule ABCD, AD = DC = CB = 1 and AB = 2 meters. Forces of magnitude 1N, 3N, 5N, 6N and $2\sqrt{3}$ N respectively.



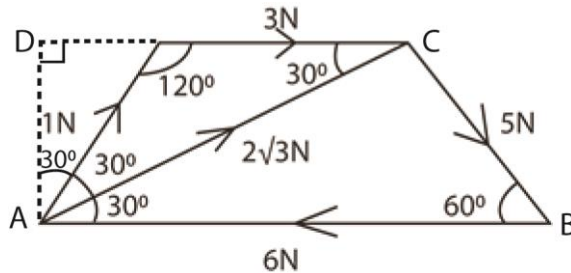
- (c) Calculate the magnitude of the resultant force and the angle it makes with side AB (09marks)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 2\sqrt{3} \cos 30^\circ \\ 2\sqrt{3} \sin 30^\circ \end{pmatrix} + \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} + \begin{pmatrix} 5 \cos 60^\circ \\ -5 \sin 60^\circ \end{pmatrix} = \begin{pmatrix} 3 \\ -\sqrt{3} \end{pmatrix}$$

$$\text{Resultant force, } R = \sqrt{(3)^2 + (-\sqrt{3})^2} = 3.464\text{N}$$

$$\text{Direction, } \alpha = \tan^{-1} \left(\frac{\sqrt{3}}{3} \right) = 30^\circ$$

- (d) Given that the line of action of the resultant force meets AB at X, find AX. (03marks)



Equation of the line action of the resultant is given by $G - xY + yX = 0$

Taking moments about A

$$G = -3 \times 1 \cos 30^\circ - 5 \times 2 \cos 30^\circ$$

$$= -3 \times \frac{\sqrt{3}}{2} - 10 \times \frac{\sqrt{3}}{2} = \frac{-13\sqrt{3}}{2}$$

By substitution

$$\frac{-13\sqrt{3}}{2} + \sqrt{3}x + 3y = 0$$

The line of action of the resultant cuts AB when $y = 0$

$$\frac{-13\sqrt{3}}{2} + \sqrt{3}x + 3 \times 0 = 0$$

$$x = 6.5\text{m}$$

$$\text{Hence } \overline{AX} = 6.5\text{m}$$

32. A biased die with faces labelled 1, 2, 3, 4, 5 and 6 is tossed 45 times
Calculate the probability that 2 will appear;

(c) More than 18 times (07marks)

$$n=45, p=\frac{2}{6}=\frac{1}{3}, q=\frac{2}{3}$$

$$\mu=np=45 \times \frac{1}{3}=15$$

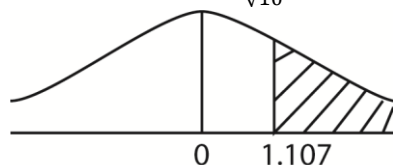
$$\sigma=\sqrt{npq}=\sqrt{45 \times \frac{1}{3} \times \frac{2}{3}}=\sqrt{10}$$

Changing binomial to normal distribution.

$$P(X > x) = P(X > 18 + 0.5) = P(X > 18.5)$$

Standardizing using $z = \frac{\bar{x} - \mu}{\sigma}$

$$P(X > 18.5) = P\left(z > \frac{18.5-15}{\sqrt{10}}\right) = P(z > 1.107)$$



$$\begin{aligned} P(z > 1.107) &= 0.5 - P(0 < z < 1.107) \\ &= 0.5 - 0.3658 \\ &= 0.1342 \end{aligned}$$

$$\therefore P(X > 18) = 0.1342$$

(d) Exactly 11 times (05marks)

$$P(X = 11) = P(11 - 0.5 < X < 11 + 0.5)$$

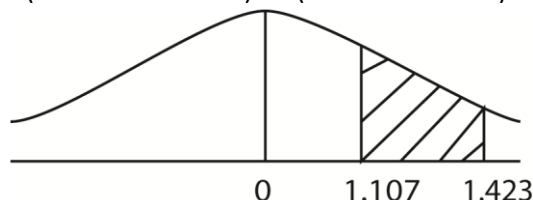
$$= P(10.5 < X < 11.5)$$

$$= P\left(\frac{10.5-15}{\sqrt{10}} < z < \frac{11.5-15}{\sqrt{10}}\right)$$

$$= P(-1.423 < z < 1.107)$$

By symmetry

$$P(-1.423 < z < 1.107) = P(1.107 < z < 1.423)$$



$$\begin{aligned} P(1.107 < z < 1.423) &= P(0 < z < 1.423) - P(0 < z < 1.107) \\ &= 0.4226 - 0.3658 \\ &= 0.0568 \end{aligned}$$

Thank you