

P425/2
APPLIED
MATHEMATICS
Paper 2
July/Aug. 2024
3 hours.



MATIGO EXAMINATIONS BOARD

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Applied Mathematics Paper 2

Time: 3 Hours

NAME: _____ **INDEX No:** _____

INSTRUCTIONS TO CANDIDATES:

- Answer **all** the **eight** questions in section **A** and only **five** questions from section **B**.
- Indicate the five questions attempted in section B in the table aside.
- Additional question(s) answered will **not** be marked.
- **All** working **must** be shown clearly.
- Graph paper is provided.
- Where necessary, take acceleration due to gravity, $g = 9.8 \text{ m s}^{-2}$.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Question		Mark
Section A		
Section B		
Total		

Section A (40 Marks)

Answer **all** the questions in this section.

Qn 1: The time, X , in seconds for phone calls made by eleven customers at a public telephone booth were recorded as follows:

110, 101, 91, 89, 122, 115, 106, 109, 112, 105, 106.

Find the:

- (i). mean value of X for the given data.
- (ii). interquartile range of the given data.

[5 Marks]

Qn 2: An electronic weighing scale is corrupted such that the actual weights of 24 g and 54 g are recorded as 35 g and 60 g respectively.

Using linear interpolation/extrapolation, estimate:

- (i). the weight which is recorded accurately.
- (ii). the weight which the scale records when it is unloaded.

[5 Marks]

Qn 3: A box of mass 4.5 kg is pulled across a rough horizontal plane by a string inclined at an angle 20° to the horizontal. When the tension in the string is 32 N, the particle's acceleration is 4.2 ms^{-2} . Find the coefficient of friction between the particle and the plane.

[5 Marks]

Qn 4: The table below shows the price relatives of a set of items A , B , C and D .

Item	A	B	C	D
Price relative	110	140	130	118
Weight	x	$2x$	y	$y + 6$

Given that the sum of the weights is 40 and that the weighted average price index is 126.7, calculate the numerical values of x and y .

[5 Marks]

Qn 5: Use trapezium rule with 6-ordinates to estimate $\int_{0.1}^{0.5} \frac{1}{2x+1} dx$, correct to **three** significant figures.

[5 Marks]

Qn 6: At 12 noon, ship *A* is 8 km due east of ship *B*.

Ship *A* is moving due north at a constant speed of 10 km h^{-1} .

Ship *B* is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to *A* as possible. Determine:

- (i). the bearing on which ship *B* moves.
- (ii). the shortest distance between the two ships. **[5 Marks]**

Qn 7: The cumulative mass function of a random variable, t , is given by:

$$F(t) = \begin{cases} 0 & ; \quad t < 0 \\ \frac{2at - t^2}{a^2} & ; \quad 0 \leq t \leq a \\ 1 & ; \quad t > a \end{cases}$$

By leaving the constant, a , in your answer,

- (i). Determine the probability mass function of t . **[2 Marks]**
- (ii). Find $P(20 \leq t \leq 40)$. **[3 Marks]**

Qn 8: A resultant of two forces $(2\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ N and $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N, acts on a rigid body at a point with position vector, $(\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

- (i). Determine the moment, about the origin $(0, 0)$, caused by this resultant force on the rigid body.
- (ii). Hence state the moment of the couple required to achieve rotational equilibrium of the rigid body. **[5 Marks]**

Section B (60 Marks)

Answer any **five** questions from this section. **All** questions carry equal marks.

Question 9:

The manager of a women clothing shop did a small survey on the amount a woman spends on clothes and her age. The findings were tabulated as follows:

Woman's age, X (in years)	16	21	36	44	25	55	23	38	30	32
Monthly expenditure on clothes, Y (in dollars)	280	300	180	116	280	128	250	150	246	190

- (a). Plot a scatter diagram for the data. **[3 Marks]**
- (b). Draw a line of best fit on the scatter diagram; and use it to estimate:

- (i). The age of a woman who spends 130 dollars monthly on clothes.
- (ii). The monthly expenditure on clothes for a 28-year-old woman. **[4 Marks]**
- (c). Calculate the rank correlation coefficient between the monthly expenditure and age.
Comment on your result at 5% level of significance. **[5 Marks]**

Question 10:

The numbers $x = 26.23$, $y = 13.18$ and $z = 5.1$ are calculated with percentage errors of 4, 3 and 2 respectively. Determine:

- (i). the errors in x , y and z . **[3 Marks]**
- (ii). the limits within which the exact value of the expression $xy - \frac{y}{z}$ lies; correct to 3 decimal places. **[4 Marks]**
- (iii). the percentage error in the expression in (ii) above. **[5 Marks]**

Question 11:

A projectile, P , travels in a vertical plane over level ground.

Its position vector, $\tilde{\mathbf{r}}$ at time, t seconds after projection is given by:

$$\tilde{\mathbf{r}} = 5\tilde{\mathbf{j}} + 30t\tilde{\mathbf{i}} + 40t\tilde{\mathbf{j}} - 5t^2\tilde{\mathbf{j}}$$

where distances are in metres and the origin is a point on the level ground.

- (a). Write down:
 - (i). the height from which P is projected. **[2 Marks]**
 - (ii). the value of acceleration due to gravity, g , in this case. **[2 Marks]**
- (b). Find the displacement of P from $t = 3$ to $t = 5$. **[4 Marks]**
- (c). Show that the equation of the trajectory is $y = 5 + \frac{4}{3}x + \frac{x^2}{180}$. **[4 Marks]**

Question 12:

A bag contains 30 white, 25 blue and 20 red balls. Three balls are drawn one after the other without replacement. If a random variable X is defined as “the number of blue balls picked”,

- (i). construct a probability distribution table for X .
- (ii). calculate the expectation of X , $E(X)$. **[12 Marks]**

Question 13:

- (a). Derive the Newton-Raphson formula for solving the equation $20 \cos x - x = 0$. Taking $x_0 = \frac{\pi}{2}$, show that $x_1 = \frac{10\pi}{21}$. [4 Marks]
- (b). Show that the iterative formula for finding the fifth root of a real number, N , can be given by:

$$x_{n+1} = \frac{1}{5} \left(4x_n + \frac{N}{x_n^4} \right)$$

Hence find $50^{\frac{1}{5}}$ to 3 decimal places using $x_0 = 2$. [8 Marks]

Question 14:

A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m. The other end of the string is attached to a fixed point O on a rough horizontal plane.

The coefficient of friction between P and the plane is $\frac{2}{5}$. The particle is held at rest at a point B on the plane, where $\overline{OB} = 1.5$ m.

When P is at B , the tension in the string is 20 N. The particle is released from rest.

- (a). Find the speed of P when $\overline{OP} = 1.2$ m. [7 Marks]
- (b). The particle comes to rest at the point C . Find the distance BC . [5 Marks]

Question 15:

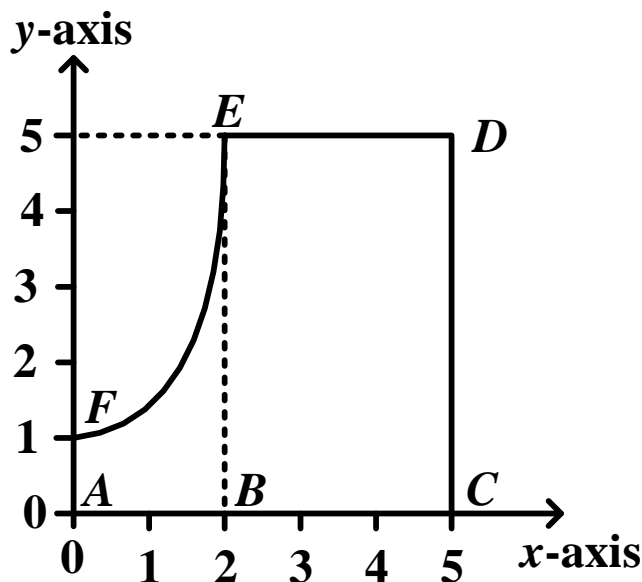
The Ugandan Traffic Service is doing a clamp down on speeding. During a recent speeding trap, they collected the following data:

	Speeding	Not speeding	Total
Male	398	a	615
Female	b	c	d
Total	603	397	1000

- (a). Calculate the values of a , b , c and d . [4 Marks]
- (b). Determine the probability that a driver selected at random is:
- A male driver and speeding.
 - A female driver given that she is not speeding. [4 Marks]
- (c). Are the events of being a male and speeding independent? Show ALL relevant working to support your answer. [4 Marks]

Question 16:

In the figure above, EF is a portion of the curve $y = x^2 + 1$ and $BCDE$ is a rectangle in which $\overline{BC} = 3$ cm and $\overline{CD} = 5$ cm.



- (a). Show, by integration, that the x-coordinate of the centroid of the portion $ABEF$ is 9. **[6 Marks]**
- (b). Find the distance of the centre of mass of the whole lamina $ABCDEF$ from the y –axis. **[6 Marks]**

END