S475/1
SUBSIDIARY
MATHEMATICS
Paper 1
Nov./Dec. 2024
2 $^{2}/_{3}$ hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

SUBSIDIARY MATHEMATICS

Paper 1

2 hours 40 minutes

INSTRUCTIONS TO CANDIDATES:

This paper consists of two Sections; **A** *and* **B**.

Section **A** *is* **compulsory**.

Section **B** *consists of* **two** *Parts;* **one** *and* **two**. *Answer only* **four** *questions from this section, choosing* **at least one** *question from each part.*

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh page.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

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SECTION A

Answer **all** *the questions in this section.*

1. Given that $\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{y}{\sqrt{3}}$, express y in the form $a + b\sqrt{c}$ where a, b and c are integers. *(05 marks)*

2. Two independent events A and B are such that $P(A) = 0.25$ and $P(A \cup B) = 0.56$. Find:

   (a) $P(B)$ *(03 marks)*

   (b) $P(A \cap B)$ *(02 marks)*

3. The position vectors of **three** points P, Q and R are $-7\boldsymbol{i}$, $7\boldsymbol{j}$ and $\boldsymbol{i} + 6\boldsymbol{j}$ respectively.

   (a) Determine:

   (i) $\boldsymbol{PQ}$

   (ii) $\boldsymbol{QR}$

   (03 marks)

   (b) Use the results in (a) to show that $\boldsymbol{PQ}$ is perpendicular to $\boldsymbol{QR}$. *(02 marks)*

4. In a certain school, senior four students organised for a Geography study tour. 20 % of the students did not make it for the tour. Determine the probability that in a group of five students **at most** two did not make it for the tour. *(05 marks)*

5. The gradient of the curve $y = 3x^2 + 2x + 1$ at a point $P(x, y)$ is 14. Find the coordinates of P. *(05 marks)*

6. The table below shows the prices in Shillings (Shs) per unit of commodities A and B in the years 2021 and 2022.

COMMODITY	PRICE (Shs)	
	2021	2022
A	25000	28000
B	18000	24000

Using 2021 as the base year, calculate the simple aggregate price index for the commodities in 2022. *(05 marks)*

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7. Solve the equation $\text{Cosec}\theta = 3 + 4\text{Sin}\theta$ for $0^{o} \leq \theta \leq 90^{0}$. *(05 marks)*

8. A continuous random variable X has a probability density function (pdf) given by

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 4 \\ 0 & \text{elsewhere,} \end{cases}$$

where k is a constant.

Determine:

(a) the value of k. *(03 marks)*

(b) $P(1 < X < 3)$. *(02 marks)*

SECTION B (60 MARKS)

Answer only **four** *questions from this section, choosing at least* **one** *question from each part.* **All** *questions carry* **equal** *marks.*

PART ONE: PURE MATHEMATICS

9. (a) Solve the differential equation:

$$3x^2 + \frac{dy}{dx} - 4x = 0$$

given that $y = 6$ when $x = 1$. *(05 marks)*

(b) The rate of increase of the number of organisms x, in a controlled environment at any time t is proportional to the number of organisms present. Initially when time $t = 0$, the number of organisms present is x_0.

(i) Form a differential equation (DE) and solve the DE to show that $x = x_0 e^{kt}$, where k is a constant. *(07 marks)*

(ii) Find in terms of k, the time required for the number of organisms to double. *(03 marks)*

10. A farmer wishes to improve the harvest on a farm by adding fertilizers to both the maize and cabbage sections.The farmer plans to buy **at least** 5 kg of fertilizers for maize and **at least** 15 kg of fertilizers for cabbages. A kilogramme (kg) of fertilizers for maize costs Shs15,000 and a kilogramme of fertilizers for cabbages costs Shs10,000. The farmer has only Shs300,000 available for buying fertilizers. The quantity of fertilizers for cabbages should **not** exceed that for maize.

(a) If x represents the number of kilogrammes of fertilizers for maize and y the number of kilogrammes of fertilizers for cabbage, write **four** inequalities to represent the given information. *(04 marks)*

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(b) (i) Illustrate the inequalities formulated in (a) on a graph by shading the unwanted regions.

(ii) Use your graph to list all the possible combinations of fertilizers that the farmer could buy to minimise the cost.

(08 marks)

(c) Calculate the lowest amount of money the farmer will spend on buying fertilizers. *(03 marks)*

11. The curve $y = 2x^2 + 1$ and the line $y = 2x + 5$ intersect at two points.

(a) Find the coordinates of the points of intersection. *(05 marks)*

(b) Sketch, on the same axes, the graphs of the curve and the line. *(04 marks)*

(c) Use the sketch drawn in (b) to determine the area enclosed between the curve and the line. *(06 marks)*

12. (a) The roots of the equation $7x^2 - 2x + 1 = 0$ are a and b. Form a quadratic equation with integral coefficients whose roots are $\frac{1}{a}$ and $\frac{1}{b}$. *(07 marks)*

(b) Three consecutive numbers $p - 4$, $p + 2$ and $3p + 1$ are in a geometric progression (G.P.). Find the two possible values of the common ratio of the G.P. *(08 marks)*

PART TWO: STATISTICS

13. The table below shows the ages (x) years and intelligence quotient, IQ (y) of 10 scholars from a certain country.

	6.5	8	6.5	1	2	3	9	5	4	10
AGE (x) (years)	60	48	60	91	85	72	40	69	70	30
IQ (y)	185	181	142	196	174	157	150	193	170	160
	3	4	10	1	5	8	9	2	6	7

(a) (i) Calculate the rank correlation coefficient between the age and IQ of the scholars.

(ii) Comment on your result. *(07 marks)*

(b) (i) Plot a scatter diagram for the data.

(ii) On the same diagram, draw a line of best fit.

(iii) Use the diagram to find the value of x when $y = 165$. *(08 marks)*

14. The discrete random variable X has a probability distribution as:

$P(X = 0) = P(X = 4) = k$; $P(X = 1) = P(X = 3) = 2k$ and $P(X = 2) = 4k$.

Determine the;

(a) value of k. *(03 marks)*

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(b) mean, E(X). *(04 marks)*

(c) variance, $Var(X)$. *(05 marks)*

(d) $P(1 < X < 4)$. *(03 marks)*

15. The masses in kilograms (kg) of 52 girls measured to the nearest kg were recorded as shown in the table below.

Mass (kg)	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64	65 – 69	70 – 74
No. of Girls	3	2	7	18	18	3	1

(a) Draw a cumulative frequency curve (Ogive) for the given data. *(05 marks)*

Use the Ogive to:

(b) Determine;

(i) the number of girls whose mass is 57 kg or below. *(02 marks)*

(ii) the value of mass above which the upper 20 % of the girls weigh. *(03 marks)*

(c) Estimate the;

(i) median weight.

(ii) upper quartile. *(05 marks)*

16. The daily attendance of senior six students for two consecutive weeks was recorded as follows:

WEEK	**DAY**				
	MON	**TUE**	**WED**	**THUR**	**FRI**
ONE	21	25	31	31	26
TWO	30	39	40	38	40

(a) Calculate the five-day moving averages for the daily attendance. *(06 marks)*

(b) (i) Plot a graph of the daily attendance against the days of the weeks.

(ii) On the same axes, plot the five-day moving averages.

(iii) Comment on the trend of the daily attendance during the two weeks. *(06 marks)*

(c) Use your graph to estimate the expected attendance of the students for Monday of week three. *(03 marks)*

END

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UNEB 2024 SUBMATH

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1. $\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{y}{\sqrt{3}}$

$\frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{y}{\sqrt{3}}$

$\frac{\sqrt{6\times3}-\sqrt{12}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{y}{\sqrt{3}}$

$\frac{\sqrt{18}-\sqrt{12}}{3-2} = \frac{y}{\sqrt{3}}$

$\frac{9\sqrt{2}-2\sqrt{3}}{1} = \frac{y}{\sqrt{3}}$

$y = \sqrt{3}(9\sqrt{2}-2\sqrt{3})$

$y = 9\sqrt{6}-2\sqrt{9}$

$y = 9\sqrt{6}-2\times3$

$y = -6+9\sqrt{6}$

where $a=-6$, $b=9$ $c=6$.

2. $P(A)=0.25$ $P(A\cup B)=0.56$

from

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$0.56 = 0.25+P(B)-P(A\cap B)$

$P(B)-P(A\cap B) = 0.31$ —— ①

for Independent events $P(A\cap B)=P(A)\cdot P(B)$

$P(A\cap B) = 0.25P(B)$ —— ②

put 2 into 1

$P(B)-0.25P(B) = 0.31$

$0.75P(B) = 0.31$

$P(B) = \frac{31}{75}$ or 0.4133

ii) $P(A\cap B) = 0.25\times\frac{31}{25} = \frac{31}{300}$

3 $P = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$; $Q = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$ $R = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

(a) $PQ = OQ - OP$

$\begin{pmatrix} 0 \\ 7 \end{pmatrix} - \begin{pmatrix} -7 \\ 0 \end{pmatrix}$

$PQ = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$

(ii) $QR = OR - OQ$

$\begin{pmatrix} 1 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 7 \end{pmatrix}$

$QR = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(b) For perpendicular Vectors, their dot product $= 0$.

$PQ \cdot QR$

$\begin{pmatrix} 7 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= (7 \times 1) + (7 \times -1)$

$= 7 - 7$

$= 0$

Hence perpendicular.

4 Let X be number of students who did not make it for the tour.

$P = \frac{20}{100} = 0.2$

$q = 0.8$ $n = 5$

$P(X \leq 2) = P(X=1) + P(X=2) + P(X=0)$

${}^5C_1 (0.2)^1 (0.8)^4 + {}^5C_2 (0.2)^2 (0.8)^3$

$+ {}^5C_0 (0.2)^0 (0.8)^5$

$= 0.0081 + 0.0108$

$0.4096 + 0.2048 + 0.32768$

$= 0.0189$ or $\frac{189}{10000}$

$= 0.94208$

$= 0.9421$

5. $y = 3x^2 + 2x + 1$

$\frac{dy}{dx} = 6x + 2.$

But $\frac{dy}{dx} = 14.$

$14 = 6x + 2.$

$6x = 12$

$x = 2.$

from $y = 3x^2 + 2x + 1$

$y = 3(2)^2 + 2(2) + 1$

$y = 12 + 4 + 1$

$y = 17.$

$P = (2, 17).$

6. $SAPI = \frac{\Sigma P_1}{\Sigma P_0} \times 100.$

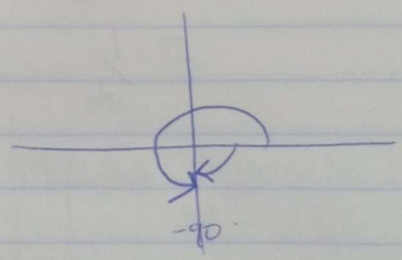
$\frac{28000 + 24000}{25000 + 18000} \times 100.$

$\frac{52000}{43000} \times 100.$

$= 120.9302.$ or $120.93.$

7. $\text{Cosec}\theta = 3 + 4\sin\theta$

$\frac{1}{\sin\theta} = 3 + 4\sin\theta.$



$1 = 3\sin\theta + 4\sin^2\theta.$

$4\sin^2\theta + 3\sin\theta - 1 = 0.$

$(\sin\theta + 1)(4\sin\theta - 1) = 0.$

either $\sin\theta = -1.$

$\theta = \sin^{-1}(-1). = -90°, 270°$ (Discard).

for $\sin\theta = \frac{1}{4}.$

$\theta = \sin^{-1}(\frac{1}{4}) = 14.48°.$

$\therefore \theta = 14.48°$ for $0° \leq \theta \leq 90°.$

8 $f(x) = \begin{cases} Kx, & 0 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$

$$K\int_0^4 x\,dx = 1$$

$$K\left[\frac{x^2}{2}\right]_0^4 = 1$$

$$K\left[\frac{(4)^2}{2} - \frac{0^2}{2}\right] = 1$$

$$K(8 - 0) = 1.$$

$$K = \frac{1}{8}.$$

b) $P(1 < x < 3) = K\int_1^3 x\,dx.$

$$K\left[\frac{x^2}{2}\right]_1^3.$$

$$K\left[\frac{3^2}{2} - \frac{1^2}{2}\right]$$

$$K\left[\frac{9}{2} - \frac{1}{2}\right]$$

$$K\left[\frac{8}{2}\right]$$

$$= 4K$$

$$4 \times \frac{1}{8}$$

$$= \frac{1}{2}. \quad \text{or } \frac{4}{8}$$

SECTION B

9(a) $3x^2 + \frac{dy}{dx} - 4x = 0$

$\frac{dy}{dx} = 4x - 3x^2$

$\int dy = \int (4x - 3x^2)\, dx$

$y = 2x^2 - x^3 + c$

$y = 6, \quad x = 1$

$6 = 2(1)^2 - (1)^3 + c$

$6 = 1 + c$

$c = 6 + 1$

$c = 5$

$y = 2x^2 - x^3 + 5$

(b) $\frac{dx}{dt} \propto x$

$\frac{dx}{dt} = kx$

$\int \frac{dx}{x} = \int k\, dt$

$\ln x = kt + c$

Initially $t = 0$, $x = x_0$

$\ln x_0 = k(0) + c$

$c = \ln x_0$

$\ln\left(\frac{x}{x_0}\right) = kt$

$\frac{x}{x_0} = e^{kt}$

$x = x_0 e^{kt}$

(ii) For organisms to double.

$x = 2x_0$

$\ln\left(\frac{2x_0}{x_0}\right) = kt$

$\frac{kt}{k} = \frac{\ln 2}{k}$

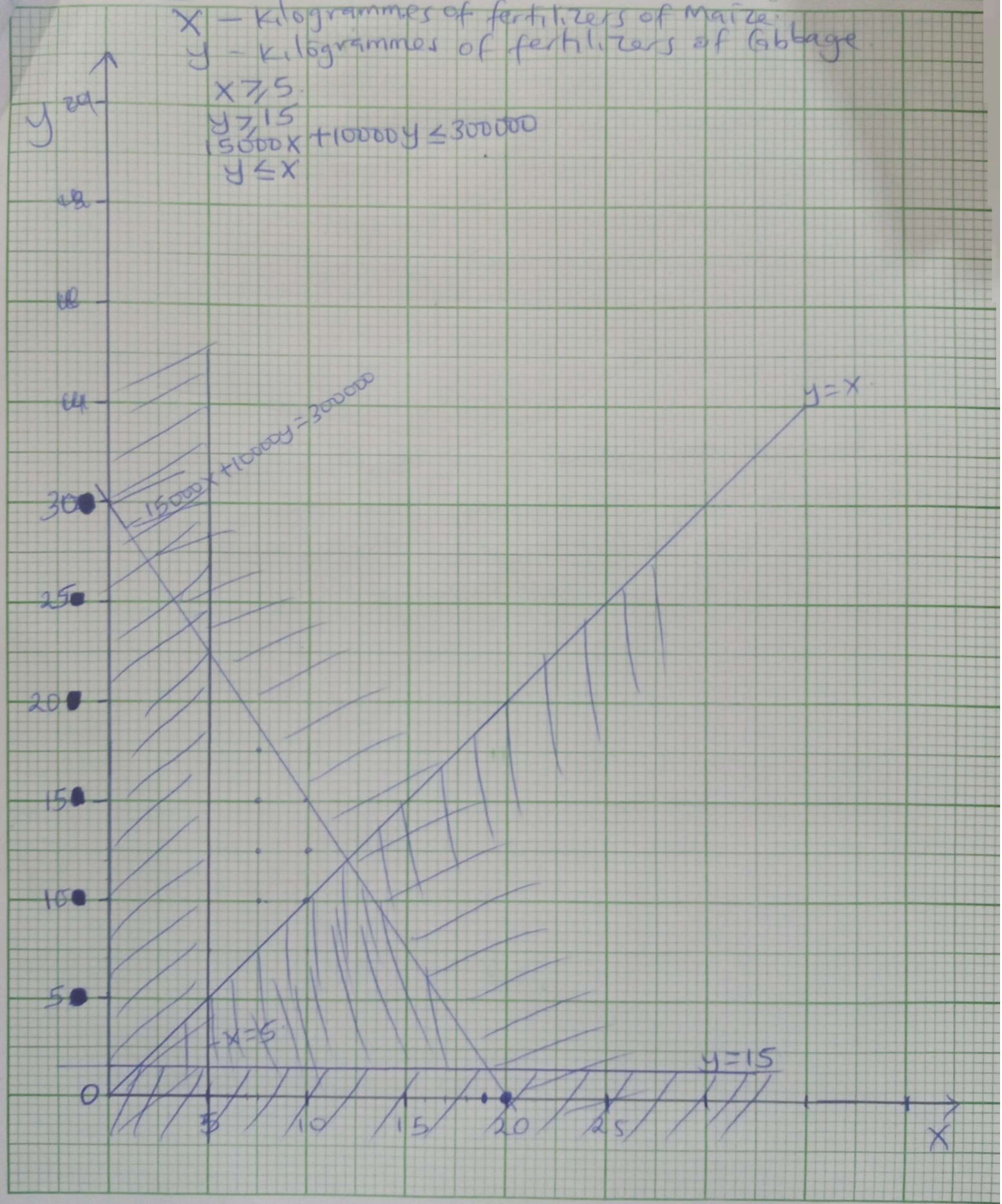
$t = \frac{1}{k} \ln 2$

Name: 10 Class: Stream:

Year: Term: Signature:

X – kilogrammes of fertilizers of Maize.
y – kilogrammes of fertilizers of Cabbage.

$x \geq 5$
$y \geq 15$
$15000x + 10000y \leq 300000$
$y \leq x$



EVER FORWARD

(-2, 1).

(x, y)	15000x + 10000y	
(5, 10)	(5x15000) + (10000x10)	175000
(5, 15)	5x15000 + 15000x1	225000
(5, 20)	(5x15000) + (20x10000	275000
(10, 10)	(10x15000) + 10x10000	250000
10, 15	(10x15000) + (15x10000)	300000
5 12.5	(5x15000) + (12.5x10000)	200000

The lowest amount of money ~~the~~ will be 175000 shillings to minimize the cost by buying 5kg of fertilizer for Maize and 10kg of fertilizer for Cabbage.

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11. $y = 2x^2 + 1$ and line $y = 2x + 5$.

At Point of Intersection, the curve = the line eqn

$2x^2 + 1 = 2x + 5$

$2x^2 - 2x + 1 - 5 = 0$

Sum -1 Pdt $= -2$.

$-2 \quad 1$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ and $x = -1$ are Points of Intersection.

(b) The curve $y = 2x^2 + 1$

y Intercept, $x = 0$

$y = 2(0)^2 + 1$

$y = 1$

$(0, 1)$

The x Intercept $y = 0$.

$2x^2 + 1 = 0$

$\frac{2x^2}{2} = \frac{-1}{2}$

$\sqrt{x^2} = \sqrt{-\frac{1}{2}}$

x is Immaginary, has no real roots.

Turning point:

$\frac{dy}{dx} = 4x$.

At T.P, $\frac{dy}{dx} = 0$

$4x = 0$

$x = 0$.

$y = 2(0)^2 + 1$

$y = 1$

$(0, 1)$ is the T.P.

Nature of the T.P.

$\frac{d^2y}{dx^2} = 4 > 0$

The Turning point is Minimum.

$y = 2x + 5$

w/n $x = 0$

$y = 5$.

$(0, 5)$

w/n $y = 1$,

$1 = 2x + 5$

$2x = -4$

$x = -2$.

$(-2, 1)$.

Name: _____ Class: _____ Stream: _____

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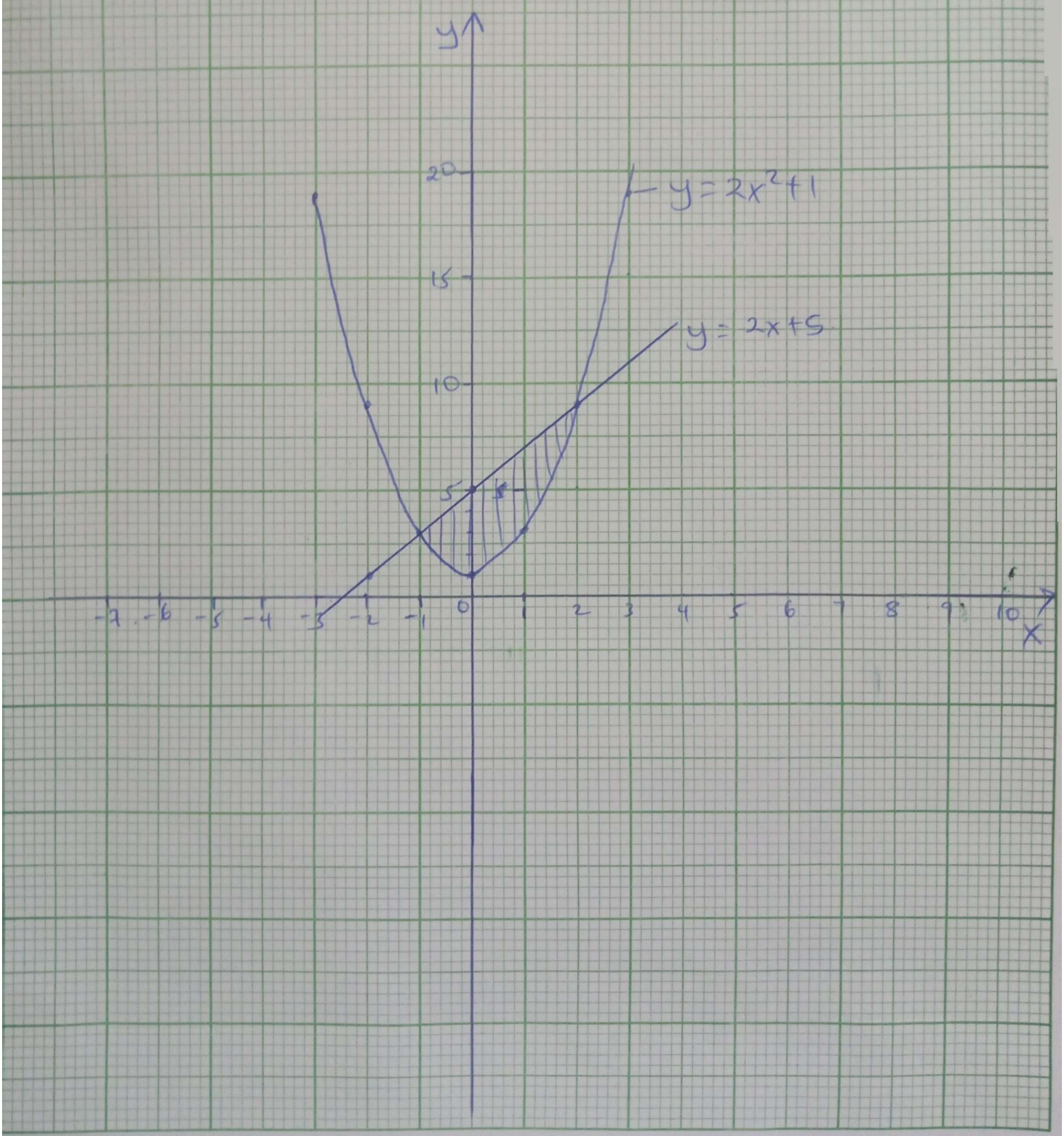
x	0	-1	1	2	-2	3
y	1	3	3	9	9	19



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Area between the line & the curve

$$\int_{-1}^{2} \left((2x+5) - (2x^2+1) \right) dx$$

$$\int_{-1}^{2} (2x - 2x^2 + 4)\, dx.$$

$(1-4) + 2/3$

$$\left[\frac{2x^2}{2} - \frac{2x^3}{3} + 4x \right]_{-1}^{2}$$

$$\left((2)^2 - \frac{2}{3}(2)^3 + 4(2) \right) - \left((-1)^2 - \frac{2}{3}(-1)^3 + 4(-1) \right)$$

$$\frac{20}{3} + \frac{7}{3}$$

$= 9$ square units.

(12)

2 $7x^2 - 2x + 1 = 0$.

$(a+b) = 2/7$.

$ab = 1/7$.

New roots

$\frac{1}{a}$ and $\frac{1}{b}$.

Sum of roots

$\frac{1}{a} + \frac{1}{b}$

$\frac{b+a}{ab}$

$2/7 \div 1/7$.

$2/7 \times 7/1$

$= 2$.

Product of roots.

$\left(\frac{1}{a}\right)\left(\frac{1}{b}\right)$.

$\frac{1}{ab}$

$\frac{1}{1/7}$.

$= 7$.

Equation.

$x^2 - 2x + 7 = 0$.

b. $p-4$, $p+2$, $3p+1$.

Common ratio, $\frac{3p+1}{p+2} = \frac{p+2}{p-4}$.

$(p+2)(p+2) = (3p+1)(p-4)$.

$p^2 + 4p + 4 = 3p^2 - 11p - 4$.

$2p^2 - 15p - 8 = 0$.

$p = \frac{15 \pm \sqrt{15^2 + 4 \times 2 \times 8}}{2 \times 2}$

$p = \frac{15 - 17}{4}$.

$p = \frac{15 + 17}{4}$

$p = -\frac{1}{2}$ or -0.5.

$p = 8$.

For $p = 8$,

Common ratio $= \frac{3(8)+1}{8+2} = \frac{25}{10}$ or $5/2$

for $p = -\frac{1}{2}$

Common ratio $= \frac{-0.5+2}{-0.5-4}$

$= -\frac{1}{2}$

13

X	Y	R_x	R_y	d $R_x - R_y$	d^2
60	185	6.5	3	3.5	12.25
48	181	8	4	4	16
60	142	6.5	10	-3.5	12.25
91	196	1	1	0	0
85	174	2	5	-3	9
72	157	3	8	-5	25
40	150	9	9	0	0
69	193	5	2	3	9
70	170	4	6	-2	4
30	160	10	7	3	9
					$\Sigma d^2 = 96.5$

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$1 - \frac{6 \times 96.5}{10(10^2-1)}$$

$$1 - \frac{576}{990}$$

$$1 - 0.581818$$

$$= 0.4182$$

There is a positive low correlation btn X & Y.

$$\Sigma X = \frac{625}{10} = 62.5 \simeq 63$$

$$\Sigma Y = \frac{1708}{10} = 170.8 \simeq 171$$

$$\Sigma X_L = \frac{40+48+30+60+60}{5} = \frac{238}{5} = \simeq 48$$

$$\Sigma Y_L \quad \frac{185+181+150+160+142}{5} = \frac{818}{5} \simeq 164$$

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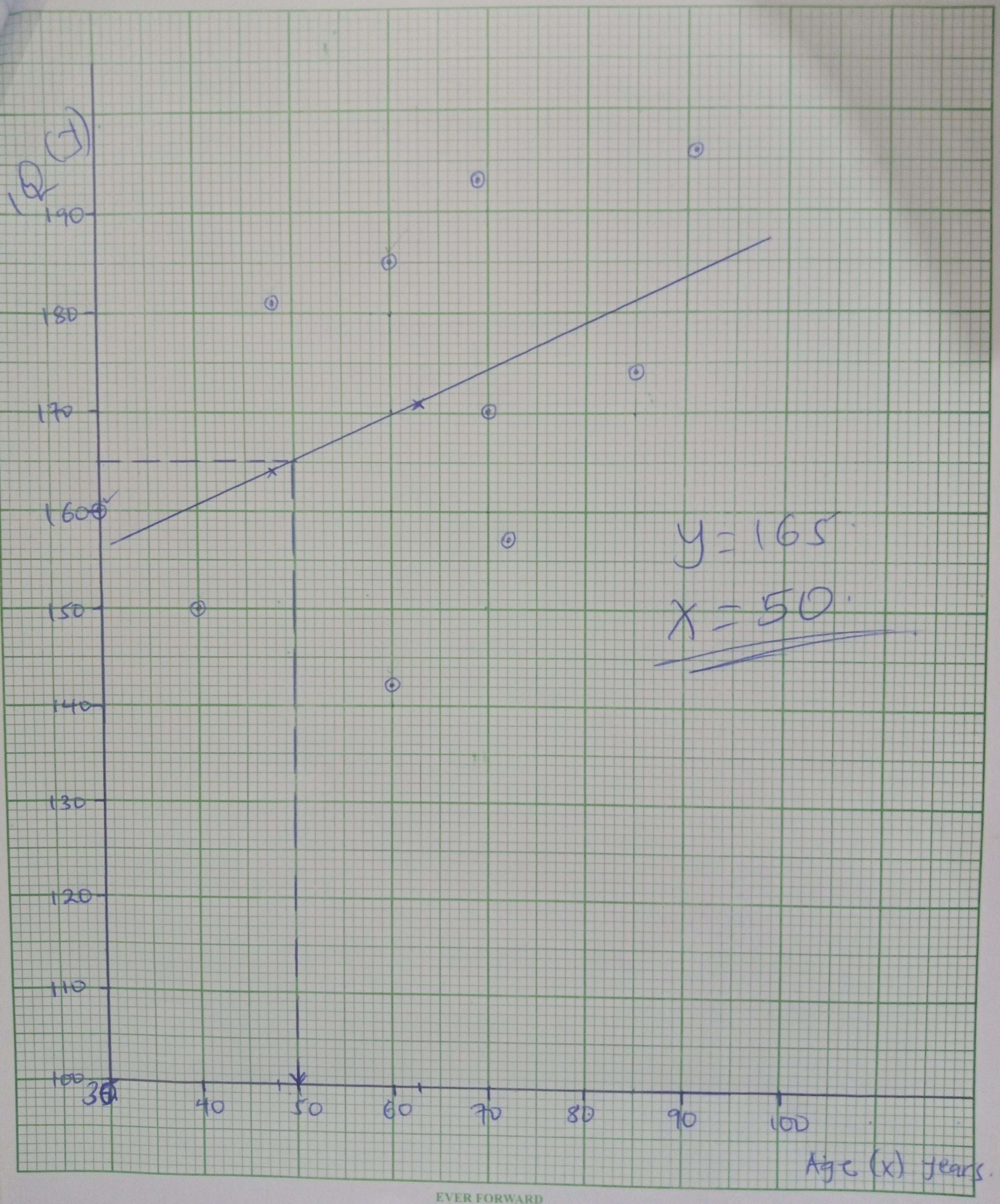

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(14)

4

x	$P(X=x)$	$xP(X=x)$	$x^2P(X=x)$
0	K	0	0
1	$2K$	$2K$	$2K$
2	$4K$	$8K$	$16K$
3	$2K$	$6K$	$18K$
4	K	$4K$	$16K$

a) Value of K.

$$\sum_{\text{All } x} P(X=x) = 1$$

$$K+2K+4K+2K+K = 1$$

$$K = \frac{1}{10}$$

b) Mean, $E(X) = \sum_{\text{All } x} xP(X=x)$

$$0+2K+8K+6K+4K$$

$$= 20K$$

$$= 20 \times \frac{1}{10}$$

$$E(X) = 2.$$

$$Var(X) = \sum_{\text{All } x} x^2P(X=x) - \sum_{\text{All } x} xP(X=x)$$

$$(0+2K+16K+18K+16K) - (2)^2$$

$$52K - 4.$$

$$\frac{52}{10} - 4.$$

$$5.2 - 4.$$

$$= 1.2 \quad \text{or} \quad \frac{6}{5}.$$

d) $P(1 < X < 4) = P(X=2) + P(X=3)$

$$4K + 2K.$$

$$= 6K$$

$$= \frac{6}{10} = 0.6$$

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1.5

Mass	f	cf	TCL
40-44	3	3	39.5 - 44.5
45-49	2	5	44.5 - 49.5
50-54	7	12	49.5 - 54.5
55-59	18	30	54.5 - 59.5
60-64	18	48	59.5 - 64.5
65-69	3	51	64.5 - 69.5
70-74	1	52	69.5 - 74.5

b i, 57kg or below = 23 girls

ii, Upper 20% = 100 - 20 = 80%

$\frac{80}{100} \times 52 = 42$ th girls position

= 62 kgs.

$\frac{20}{100} \times 52 = 10.4$

10 girls

52 - 10

= 42 girls

c Median $= \frac{1}{2} \times 52 = 26$th position

= 58

Upper quartile $= \frac{3}{4} \times 52 = 40.5$

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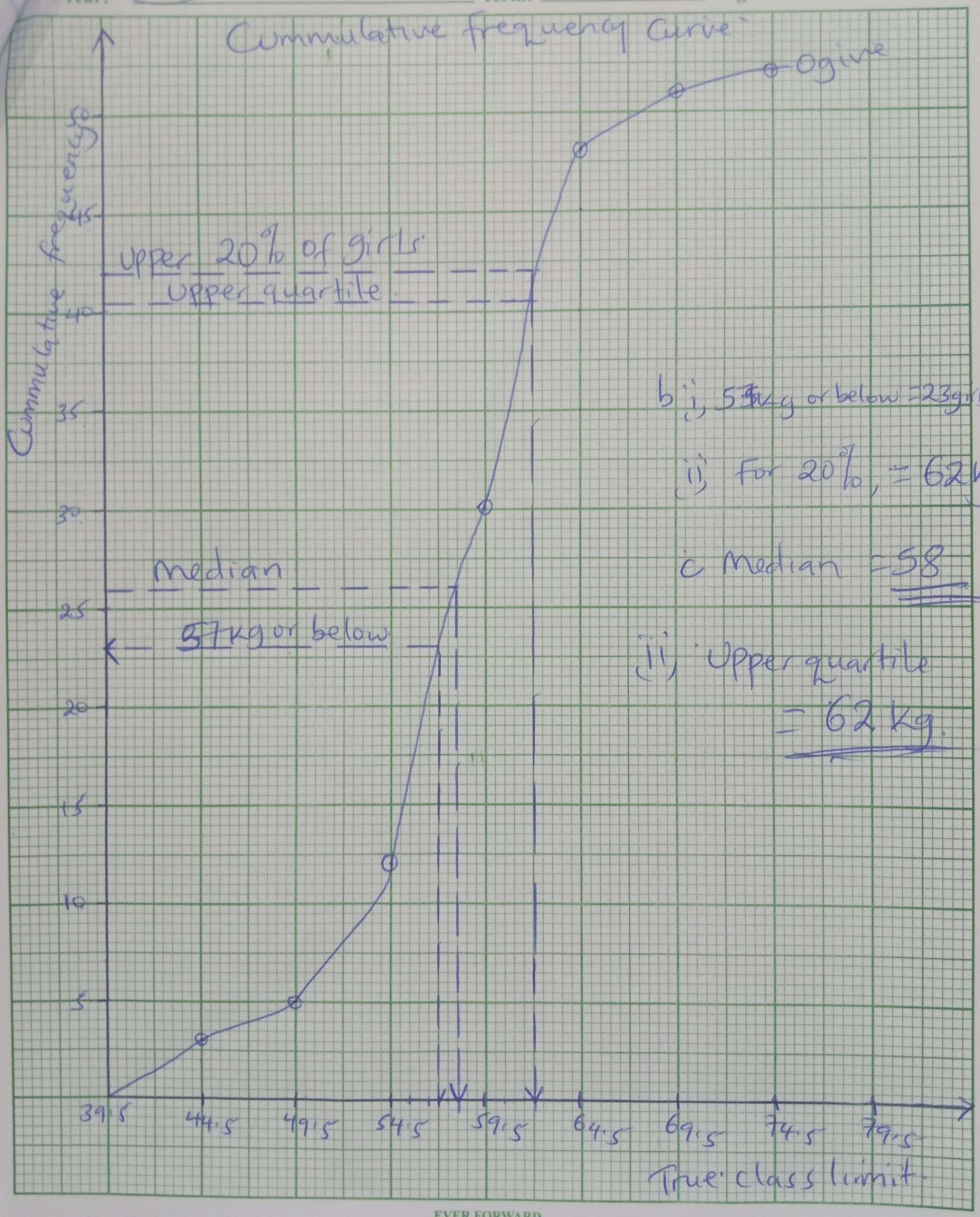


b i, 57kg or below = 23 girls

ii, For 20%, = 62kg

c Median = 58

ii, Upper quartile = 62 kg.

6.

Week			Moving Total	M.A.
ONE	M	21		
	T	25		
	W	31	134	26.8
	T	31	143	28.6
	F	26	157	31.4
TWO	M	30	166	33.2
	T	39	173	34.6
	W	40	187	37.4
	T	38		—
	F	40		
	M	x		

b. iii, The daily attendance is ~~slightly~~ Rapidly Increasing

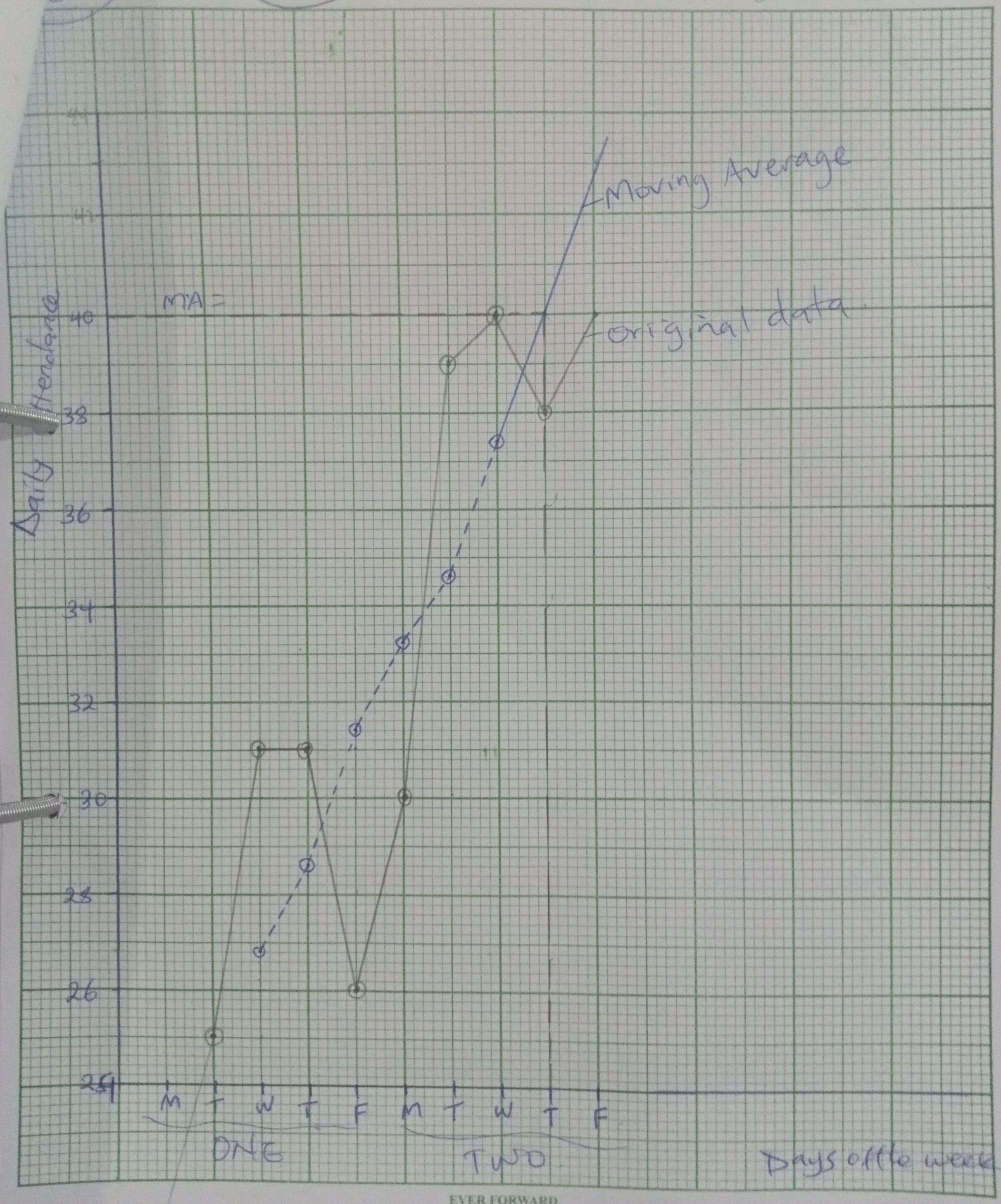
$$\frac{39+40+38+40+x}{5} = 40$$

$$157 + x = 200$$

$$x = 200 - 157$$

$$x = 43 \text{ students}$$

On Monday, the estimated number of students to attend is 43 students.

Name: 16 Class: Stream:

Year: Term: Signature: