ST JOSEPHS SEMINARY

P425/2

S6 APPLIED MATHEMATICS REVISION QUESTIONS ON

NUMERIAL METHODS 2022

1(a)Use the trapezium rule with six ordinates to estimate $\int_0^2 xsinx^2dx$ correct to four decimal places.

(b)Calculate the exact value of $\int_0^2 xsinx^2dx$ correct to four decimal places.

(c)Calculate the percentage error made in your calculation in (a) and state how the error can be reduced.

2 The numbers x and y are approximated by X and Y with errors Δx and Δy respectively

(a) Show that the maximum relative error in $\frac{y^2}{x}$ is given by; $\left|\frac{\Delta x}{x}\right|+2\left|\frac{\Delta y}{y}\right|$

(b) If x=6.75 and y=4.285 are each rounded off to the given number of decimal places ,calculate the;

(i) Percentage error in $\frac{y^2}{x}$

(ii) Limits within which $\frac{y^2}{x+y}$

3(a) Find the exact value of $\int_1^{1.8} tan\frac{1}{2}$xdx. Correct to three decimal places.

(b)use the trapezium rile with seven ordinates to evaluate; $\int_1^{1.8} tan\frac{1}{2}xdx$.Correct to three decimal places.

(c) Calculate the percentage error made in your answer in (b) above. Suggest how the error may be reduced.

4(a) show by drawing graphs of the y=1-$e^x$ and y=$2^x$, that the equation $2^x$=1-$e^x$ has a root between -1.5 and 0.Hence find the root to 2 decimal places.

(c) Using NEWTON RAPHSON METHOD, show that the seventh root of a number K is $x_{n+1}=\frac{6x_n^7 + K}{7x_n^6}$, Hence, if K=66 and the initial approximation of the root is 1.9,find the root correct to three significant figures.

5(a) The numbers x and y are approximated with errors $\Delta x$ and Δy respectively. Deduce that the maximum error made in estimating $x^2$y is given by 2$|xy\Delta x|$+$|x^2y|$. Hence find the limits within which the exact value of $2.4^2$x 0.18 lies.

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(b) The relative error in measuring the volume of a cylinder is 0.125 and that in measuring the height is 0.05. Calculate the percentage error made in measuring the radius.

6(a) Show that Newtons Raphson iterative formula for solving the equation $2x^3$+5x-8=0 is

$x_{n+1} = \frac{4x_n^3+8}{6x_n^2+5}$ .

(b) Taking the first approximation to the largest positive root as 1.4, draw a flow chart diagram which reads and prints the number of iterations and the root with an error of less than 0.001. Carry out a dry run for the flow chart.

7(a) Show that the iterative formular based on Newton Raphson Method for solving the root of the equation $e^{2x}$+ 4x =5 is given by;

$x_{n+1}=\frac{e^{2x_n}(2x_n-1)+5}{2e^{2x_n}+4}$,n=0,1,2,3……………………..

(b)

(i) Construct a flow chart that:

-reads the initial approximation $x_0$,

-computes, using the iterative formula in (a) and prints the root of the equation $e^{2x}$+4x=5, and the number of iterations when the error is less than 1.0X$10^{-4}$

(ii) perform a dry run for the flow chart when $x_0$=0.5.

8(a) The table below shows the values of x and their corresponding natural logarithm.

| X | 5.0 | 5.2 | 5.4 | 5.7 | 6.0 |
|---|---|---|---|---|---|
| Inx | 1.609 | 1.647 | 1.686 | 1.740 | 1.792 |

Use linear interpolation or extrapolation to find

(i) In (5.56)
(ii) $e^{1.575}$

(b) A car consumed fuel amounting to shs 14,800,shs 15,600 and shs 17,200 in covering distances of 10km,20km, 30km and, 40km respectively. Estimate the;

(i) cost of fuel consumed for a distance of 45km,

(ii) distance travelled if fuel of shs 16,000 is used.

9(a) Using trapezium rule with five strips evaluate $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}}dx$, correct to three decimal places.

(b) Find the exact value of $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}}dx$

© Find the percentage error in the approximation in (a) above and suggest how the error can be reduced.

10(a) Given that y=xsinx  and x=2, find the absolute error in y giving your answer correct to three significant figures.

(b)The numbers x=1.5, y=-2.85 and z=10.345 were all rounded off to the given number of decimal places. Find the range within which the exact value of $\frac{1}{x}$ - $\frac{1}{y}$ + $\frac{y}{xz}$ lies.

11(a) Two positive decimal numbers X and Y were approximated with errors $E_1$ and $E_2$ respectively .Show that the maximum positive relative error in the approximation of the product $X^3Y^2$ is 3$\left|\frac{E_1}{X}\right|$+2$\left|\frac{E_2}{Y}\right|$.

(b) Given that X=5.64 and Y=10.0, rounded off to the given number of decimal places .Find the;

(i)maximum possible errors in X and Y.

(ii) percentage error made in the approximation of $X^3Y^2$.

12 (a) Use trapezium rule with five strips to estimate $\int_0^4 3^{2x}\,dx$ correct to 2 decimal places.

(b) Find the exact value of $\int_0^4 3^{2x}dx$ correct to 2 decimal places.

(c) Calculate the relative error made in (a) above.

13(a) Find the maximum possible error made in the expression 6.23 - 3.1 - $\frac{2.5\times4.1}{5}$ correct to three decimal significant figures.

14(a) The numbers x=3.7 and y=70 are each rounded off with percentage errors of 0.2 and 0.5 respectively. While z is calculated with relative error of 0.04.Find the interval within which the exact value of $\frac{x}{y-z}$ lies; correct your answer to 4 significant figures.

(b) The height and radius of a cylindrical water tank are given as H=3.5$\pm$0.2 and R=1.4$\pm$0.1 respectively.Determine in $m^3$, the least and greatest amount of water the tank can contain. Hence, calculate the maximum possible error in your calculation.