PROPOSED
MARKING GUIDE
UTEC P425/2
APPLIED MATHEMATICS 2023

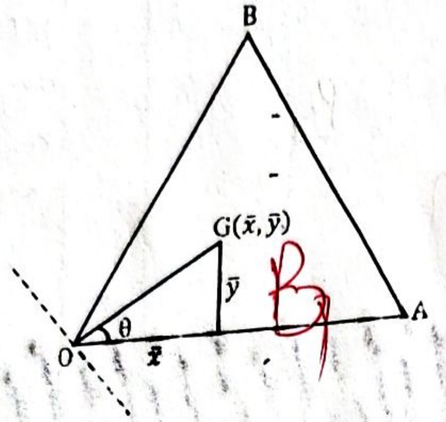
NO	SOLUTION	Mks	Comments
1	(a) From $P(A' \cup B) = 1 - P(A \cap B')$ $\frac{1}{2} = 1 - \frac{5}{8}P(B')$ $\frac{5}{8}P(B') = \frac{1}{2} \qquad \therefore P(B') = \frac{4}{5}$ B1 $\Rightarrow P(A \cup B') = 1 - P(A' \cap B)$ M1 $= 1 - \left(\frac{3}{8} \times \frac{1}{5}\right)$ $= \frac{37}{40}$ A1 (b) $P(A' \cup B') = P(A \cap B)^{1}$ $= 1 - P(A \cap B)$ $= 1 - \left(\frac{5}{8} \times \frac{1}{5}\right)$ M1 $= \frac{7}{8}$ A1		Accept any other correct method.
		05	
2	i) $\begin{array}{c\|c\|c} 0.5 & 0.8 & 1.2 \\ \hline A & -0.24 & 0.18 \end{array}$ M1 B1 $\frac{A+0.24}{0.5-0.8} = \frac{-0.24-0.18}{0.8-1.2}$ $A = -0.555$ A1 ii) $\begin{array}{c\|c\|c} 0.8 & B & 1.2 \\ \hline -0.24 & -0.12 & 0.18 \end{array}$ B1 $\frac{B-0.8}{-0.12+0.24} = \frac{1.2-0.8}{0.18+0.24}$ $B = 0.9143$ A1		Mark any other correct method
		05	

3 (a) $G(\bar{x},\bar{y}) = G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ M1

$=G\left(\frac{0+9+6}{3}, \frac{0+0+6}{3}\right)$

$=G(5, 2)$ A1

(b)



$\tan\theta = \frac{2}{5}$ M1

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta = 21.80^0$ A1

05

4

R_H	R_M	d	d^2
1	2	−1	1
2	1	1	1
3	4	−1	1
4	3	1	1
5	7	−2	4
6	5	1	1
7	6	1	1
			$\sum d^2 = 10$

B1 (d) B1 (d^2) for any 3 correct values of d

$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$ M1

$\rho = 1 - \frac{6\times 10}{7(7^2-1)}$

$\rho = 0.8214$ A1

Very high positive correlation
Or
Significant at 5% B1
Or
Not significant at 1%

05

5 Velocity at $t = 1s$;

$v_{(t=1)} = 12i + (8+23)j = 12i + 31j\ ms^{-1}$ M1

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	Speed at $t = 1$s; $v = \sqrt{12^2 + 31^2} = \sqrt{1105}\, ms^{-1}$ B1 Velocity at $t = 3$ s $v_{(t=3)} = 12(3)^2 i + (8 \times 3 + 23)j = 108\, i + 47j\, ms^{-1}$ M1 Speed at $t = 3$s; $v = \sqrt{108^2 + 47^2} = \sqrt{13873}\, ms^{-1}$ B1 $\Rightarrow$ Average speed $= \frac{\sqrt{1105}+\sqrt{13873}}{2} = 75.5126\, ms^{-1}$ A1		
		05	
6	$e_x = 0.005, e_y = 0.0005$ Max value $= (xy)_{max}$ $= (1.25 + 0.005) \times (1.600 + 0.0005)$ $= 2.0086$ M1 Min value $= (xy)_{min}$ $= (1.25 - 0.005) \times (1.600 - 0.0005)$ $= 1.9914$ M1 Interval $= 1.9914 \leq xy \leq 2.0086$ A1 Or $= [1.9914, 2.0086]$ Maximum error $= \frac{1}{2}(2.0086 - 1.9914)$ M1 $= 0.0086$ A1		Square bracket
		05	
7	$P(H) = 3\,P(T)$ B1 $P(H) + P(T) = 1$ $3P(T) + P(T) = 1$ $4P(T) = 1$ $\therefore P(T) = \frac{1}{4} = 0.25, P(H) = 0.75$ M1 Let X = Number of heads that occurs $X \sim B(15, 0.75)$ $P(X \geq 7) = P(X' \leq 8)$ M1 $= 1 - P(X' \geq 9)$ M1		Accept use of the formulae

	$= 1 - 0.0042$ $= 0.9958$ A1	05	
8	From $v = u + at$ $0 = 12 + 5a$ $a = -2.4\ ms^{-2}$ M1 $s = s_{(t=5)} - s_{(t=4)}$ M1 $s = \left(12 \times 5 - \frac{1}{2} \times 2.4 \times 5^2\right) - \left(12 \times 4 - \frac{1}{2} \times 2.4 \times 4^2\right)$ M1 $s = 30 - 28.8$ M1 $s = 1.2\ m$ A1	05	Mark any other correct method
9	(table and working below)		× atleast 4 values correct

$c.b$	f	x	fx	c	$f.d$	$c.f$
$0-10$	8	5	40	10	0.8	8
$10-15$	10	12.5	125	5	2	18
$15-25$	25	20	500	10	2.5	43
$25-40$	15	32.5	487.5	15	1	58
$40-50$	4	45	180	10	0.4	62
$50-60$	2	55	110	10	0.2	64
Σ	64		1442.5			

B1 B1 B1

(a) (i) mean $= \frac{\Sigma fx}{\Sigma f}$ M1

$= \frac{1442.5}{64}$

$= 22.5391$ A1

(ii) mode $= l_1 + \left(\frac{d_1}{d_1 + d_2}\right) \times c$

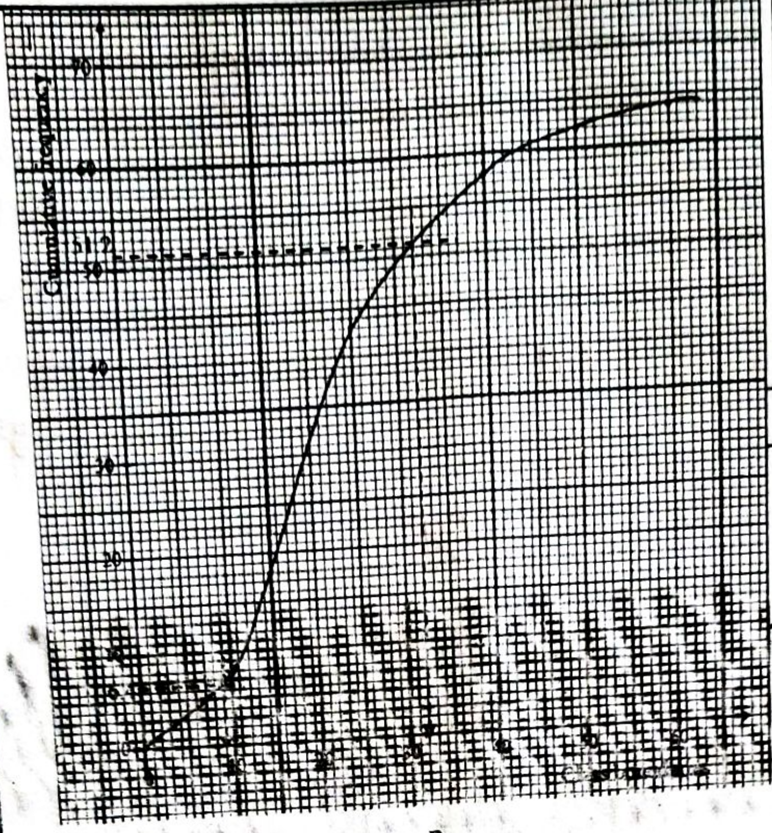
$= 15 + \left(\frac{0.5}{0.5+1.5}\right) \times 10$ M1

$15 + 2.5 = 17.5$ A1

$= \cancel{21.25}$

17.5

(b)



B1 B1 for correct scales both

B1 B1 for plotting an ogive

Percentile deviation $= P_{80} - P_{10}$

$$= \left(\frac{80}{100} \times 64\right)^{th} - \left(\frac{10}{100} \times 64\right)^{th}$$

B1 $= 51.2^{th} - 6.4^{th}$ B1

B1 $= 32 - 9$

23 $= 23$ B1

12

10 (a) Let $y = 2x + \cos x$, $h = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$ B1

Atleast a minimum of 2dps is rewarded mark

x	y B1	B1
0	1.00000	
$\pi/12$		1.48952
$\pi/6$		1.91322
$\pi/4$		2.27790
$\pi/3$		2.59440
$5\pi/12$		2.87681
$\pi/2$	3.14159	
Total	**4.14159**	**11.15185**

Only correct values is awarded a mark in each column

$\int_0^{\pi/2}(2x+\cos x)dx \approx \frac{1}{2}\times\frac{\pi}{12}[4.14159+2(11.15185)]$ M1

≈ 3.461680366 A1

≈ 3.4617 (4dps) B1

For only correct value of error will be awarded 1 mark for suggestion

(b) Exact $= [x^2+\sin x]_0^{\frac{\pi}{2}}$ M1

$= \left(\frac{\pi^2}{4}+\sin\left(\frac{\pi}{2}\right)\right) - 0$ M1

$= 3.4674011$

≈ 3.4674 (4dps) A1

%age error $= \frac{|3.4674-3.4617|}{3.4674}\times 100$ M1

$= 16.4388\,\%$ or $16.44\,\%$ A1

$= 0.1644$

It can be minimized by increasing the number of ordinates B1

12

11 (a) From $v^2 = \omega^2(a^2-x^2)$

When $x = 3m, v = 8\ ms^{-1}$

$64 = \omega^2(a^2-9)$ ....(i) M1

When $x = 4\ m, v = 6\ ms^{-1}$

$36 = \omega^2(a^2-16)$ ....(ii) M1

$(i) \div (ii)$;

$\frac{64}{36} = \frac{a^2-9}{a^2-16}$

$16(a^2-16) = 9(a^2-9)$ M1

$16a^2 - 256 = 9a^2 - 81$

$7a^2 = 175$

$a^2 = 25$ $\therefore a = 5\ m$ A1

From (i); $64 = \omega^2(25-9)$

$\omega^2 = 4$ $\therefore \omega = 2$ rads^{-1} A1

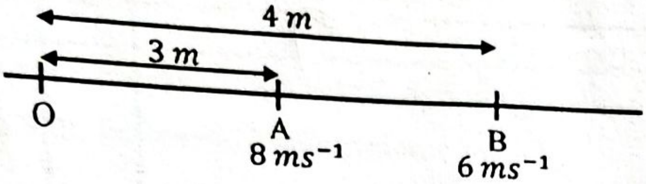
From $T = \frac{2\pi}{\omega}$ M1

$T = \frac{2\pi}{2}$ M1

$T = \pi = 3.1416$ s A1

	$\int_0^{\pi/2}(2x+\cos x)dx \approx \frac{1}{2}\times\frac{\pi}{12}[4.14159 + 2(11.15185)]$ M1 ≈ 3.461680366 A1 ≈ 3.4617 (4dps) B1 (b) Exact $= [x^2 + \sin x]_0^{\frac{\pi}{2}}$ M1 $= \left(\frac{\pi^2}{4} + \sin\left(\frac{\pi}{2}\right)\right) - 0$ M1 $= 3.4674011$ ≈ 3.4674 (4dps) A1 %age error $= \frac{\lvert 3.4674-3.4617\rvert}{3.4674} \times 100$ M1 $= 16.4388\,\%$ or $16.44\,\%$ A1 $= 0.1644$ It can be minimized by increasing the number of ordinates B1		For only correct value of error will be awarded a mark for suggestion
		12	
11	(a) From $v^2 = \omega^2(a^2 - x^2)$ When $x = 3m, v = 8\,ms^{-1}$ $64 = \omega^2(a^2-9)$ ....(i) M1 When $x = 4\,m, v = 6\,ms^{-1}$ $36 = \omega^2(a^2-16)$ ....(ii) M1 (i) ÷ (ii); $\frac{64}{36} = \frac{a^2-9}{a^2-16}$ $16(a^2-16) = 9(a^2-9)$ M1 $16a^2 - 256 = 9a^2 - 81$ $7a^2 = 175$ $a^2 = 25 \qquad \therefore a = 5\,m$ A1 From (i); $64 = \omega^2(25-9)$ $\omega^2 = 4 \qquad \therefore \omega = 2$ rads^{-1} A1 From $T = \frac{2\pi}{\omega}$ M1 $T = \frac{2\pi}{2}$ M1 $T = \pi = 3.1416$ s A1		

(b)



From $x = a\sin\omega t$

Time at point A from centre, O

$3 = 5\sin(2t_1)$ M1

$t_1 = \frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

Time at point B from centre, O

$4 = 5\sin(2t_2)$ M1

$t_2 = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

$\frac{1}{2}(0.9273) - 0.6435$

$0.4636 - $ ~~[illegible]~~

Time from A to B,

$t = t_2 - t_1$

$t = \frac{1}{2}\left[\sin^{-1}\left(\frac{4}{5}\right) - \sin^{-1}\left(\frac{3}{5}\right)\right]$ M1

$\frac{1}{2}(0.9273) - (0.6435)$

$= 0.1419\,s$

$t = 8.1301$ s A1

12

12 (a) $F(2);\ 3a = a + 2b$ B1

$2a = 2b$

$a = b$

$F(3) = 1;$

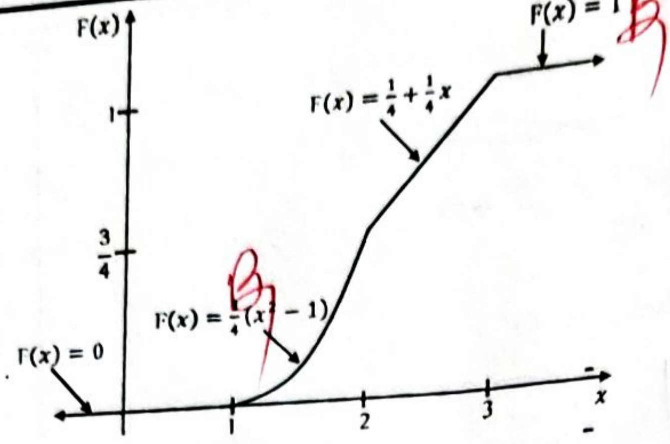
$a + 3b = 1$ M1

$b + 3b = 1$

$4b = 1 \qquad \therefore b = \frac{1}{4}, a = \frac{1}{4}$ A1

M1 for solving

A1 for correct values and B



(b) $P(X < 2.5/X < 1.5) = \frac{P(X<2.5 \cap X>1.5)}{P(X>1.5)}$

$= \frac{P(1.5<X<2.5)}{P(X>1.5)}$ M1

$= \frac{F(2.5)-F(1.5)}{1-F(1.5)}$

$= \frac{\left(\frac{1}{4}+\frac{1}{4}(2.5)\right)-\left(\frac{1}{4}(1.5^2-1)\right)}{1-\frac{1}{4}(1.5^2-1)}$ M1

$= \left(\frac{7}{8}-\frac{5}{16}\right) \div \left(1-\frac{5}{16}\right)$

$= \frac{9}{16} \times \frac{16}{11}$

$= \frac{9}{11}$ A1

(c) For $1 \le x \le 2, f(x) = \frac{d}{dx}\left[\frac{1}{4}(x^2-2)\right] = \frac{x}{2}$

For $2 \le x \le 3, f(x) = \frac{d}{dx}\left[\frac{1}{4}+\frac{1}{4}x\right] = \frac{1}{4}$

For $x \ge 1, f(x) = \frac{d}{dx}(1) = 0$

$$f(x) = \begin{cases} \frac{x}{2}, 1 \le x \le 2 \\ \frac{1}{4}, 2 \le x \le 3 \\ 0, elsewhere \end{cases}$$ B1

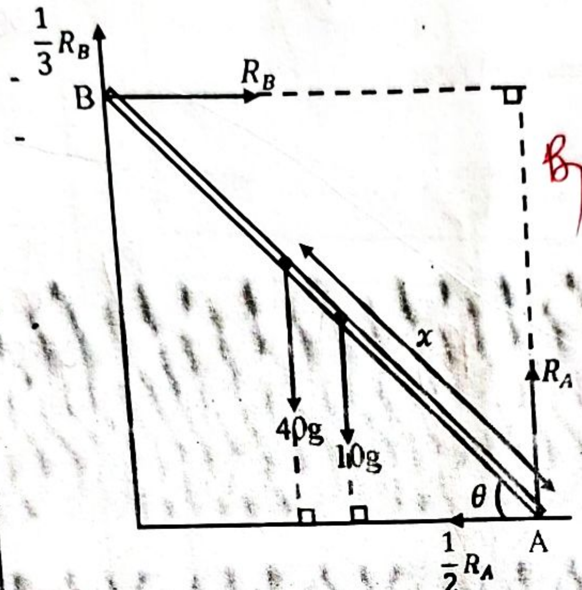
$E(x) = \int_1^2 \frac{1}{2}x^2\,dx + \int_2^3 \frac{1}{4}x\,dx$ M1

$E(x) = \left[\frac{x^3}{6}\right]_1^2 + \left[\frac{x^2}{8}\right]_2^3$

$E(x) = \frac{1}{6}(8-1) + \frac{1}{8}(9-4)$

$\frac{5}{8} + \frac{7}{6}$

$\frac{15+28}{24} = \frac{43}{24}$

	$E(x) = \cancel{\frac{29}{24}}$ $\frac{43}{24} = 1.7917$ A1		
		12	
13	(a) Let $2l$ =length of the ladder, x = distance the man climbs before the ladder slides. Let $\theta = \tan^{-1}\frac{3}{4}$; $\tan\theta = \frac{3}{4}$  B1	B1	for accurate sketch
	$(\rightarrow)$; $R_B = \frac{1}{2}R_A$…………(i) M1 $(\uparrow)$; $R_A + \frac{1}{3}R_B = 40g + 10g$ ~~A1~~ $R_A + \frac{1}{3}R_B = 50g$ ---- ii)	M1	for resolving any direction
	$2R_B + \frac{1}{3}R_B = 50g$ $\frac{7}{3}R_B = 50g$ M1 $R_B = \frac{3\times50\times9.8}{7} = 210\text{N}$	M1	for resolving the 2 eqns
	From (i); $R_A = 2 \times 210 = 420\text{N}$ M1 Taking moments about point A; $40g \times x\cos\theta + 10g \times l\cos\theta = R_B \times 2l\sin\theta + \frac{1}{3}R_B \times 2l\cos\theta$ M1 Dividing through by $\cos\theta$;	M1	reaction at any moment

$40gx + 10gl = 2lR_B \tan\theta + \frac{2}{3}lR_B$

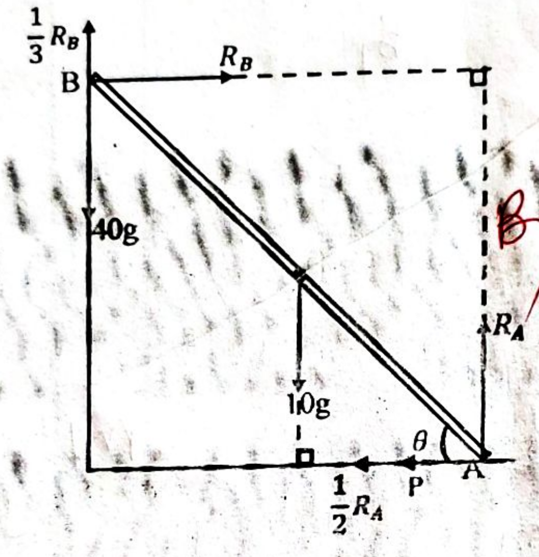
$40gx = \frac{2}{3} \times l \times 210 + 2 \times l \times 210 \times \frac{3}{4} - 10 \times 9.8 \times l$

$40gx = 140l + 315l - 98l$

$40gx = 357l$ A1 for correct value of x.

$x = \frac{357}{392}l = \frac{51}{56}l$ m or $= 0.9107l$ m from A A1

(b) Let P be the minimum horizontal force



B1 for the accurate sketch

M1 Resolving → ↑

$(\uparrow)$; $\frac{1}{3}R_B + R_A = 50g$

$R_B = 150g - 3R_A$

$R_B = 150 \times 9.8 - 3R_A$

$R_B = 1470 - 3R_A$ ....M1......(i)

$(\rightarrow)$; $R_B = P + \frac{1}{2}R_A$ ....(ii)

$(i) = (ii)$;

$1470 - 3R_A = P + \frac{1}{2}R_A$ M1 B1 solving

$\frac{7}{2}R_A = 1470 - P$

$R_A = 420 - \frac{2}{7}P$ M1

Taking moments about point B;

$10g \times l\cos\theta + \frac{1}{2}R_A \times 2l\sin\theta + P \times 2l\sin\theta = R_A \times 2l\cos\theta$ M1

$10 \times 9.8 + R_A \tan\theta + 2P\tan\theta = 2R_A$

$98 + \frac{3}{4}R_A + 2P \times \frac{3}{4} = 2R_A$ A1

$98 + \frac{3}{2}P = \frac{5}{4}R_A$

$392 + 6P = 5R_A$

But $R_A = 420 - \frac{2}{7}P$

$\Rightarrow 392 + 6P = 5\left(420 - \frac{2}{7}P\right)$

$392 + 6P = 2100 - \frac{10}{7}P$

$\frac{52}{7}P = 1708 \qquad \therefore P = \frac{2989}{13}N$ or $229.9231N$ A1

Any where for simplify

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14 (a) Let X = weights of the goats sold

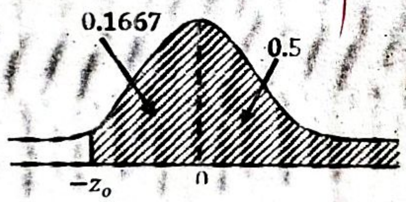
$\mu = 16$ kg, $\delta = ?$ $\mu = 26$ kg

$P(X > 20) = \frac{8}{12} = 0.6667$ B1

$P\left(z > \frac{20-16}{\delta}\right) = 0.6667$

Let $\frac{20-16}{\delta} = z_0$

$P(z > z_0) = 0.6667$ M1



$P(0 < z < z_0) = 0.1667$

$z_0 = -0.431$ M1

$\frac{20-16}{\delta} = -0.431$ M1

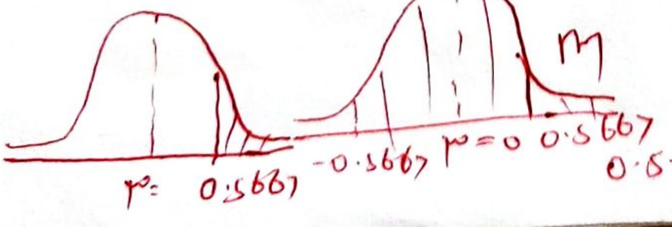
$-0.431\delta = 4$ M1 $\quad \therefore \delta = -9.28074 \approx -9$ A1

Helo members check on that number in my opinion I think there is a mistake somewhere

B1

b) $P(\bar{X} > 15) = P\left(Z > \frac{25-16}{-9}\right)$ M1

12



$0.5 + P(0 < Z < 0.5667)$ M1

$0.5 - 0.2109$

$= 0.2891$ A1

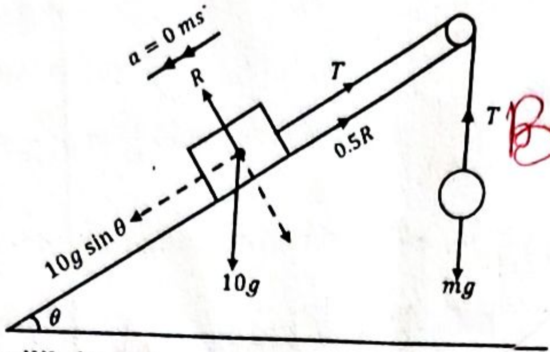
$= \cancel{0.7109}$

by use of table

15 (a) From $\theta = \tan^{-1}\frac{4}{3}$, $\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

For minimum:

This is when the particle is just at the point of moving down the plane



B1

B1 for correct dr

Must use a straight sketch represent force

At equilibrium;

$T = mg$ ……….(i)

Along the plane;

$T + 0.5R = 10g\sin\theta$ M1

But $R = 10g\cos\theta$

$mg + 0.5 \times 10g\cos\theta = 10g\sin\theta$ M1

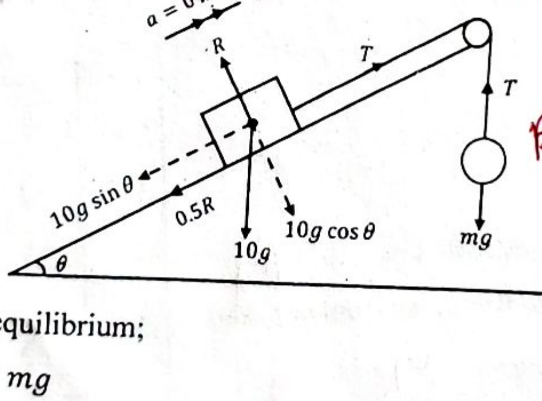
M1 for simplifying and substitute

$m = 10 \times \frac{4}{5} - 5 \times \frac{3}{5}$

$m = 8 - 3 = 5$ kg B1

For maximum;

This is when the particle is just at the point of moving up the plane



B1

B1 for the sketch

At equilibrium;

$T = mg$

Along the plane;

$T = 0.5R + 10g \sin\theta$ M1

But $R = 10g \cos\theta$

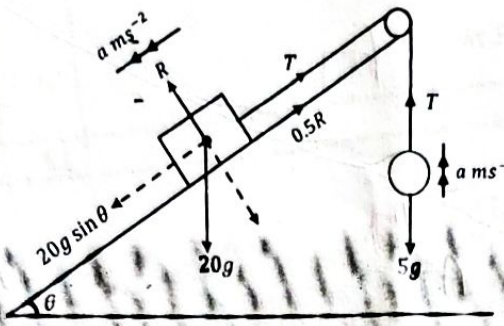
$mg = 0.5 \times 10g \times \cos\theta + 10g \times \sin\theta$ M1

$m = 5 \times \frac{3}{5} + 10 \times \frac{4}{5}$

$m = 3 + 8 = 11$ kg A1

A1 for accuracy mark

(b) When the mass of B is 5kg



For 5 **kg mass;**

$T - 5g = 5a$ M1

$T = 5a + 5g$ ....(i)

M1 Resultant force of 5kg and 20kg

For 20 kg mass;

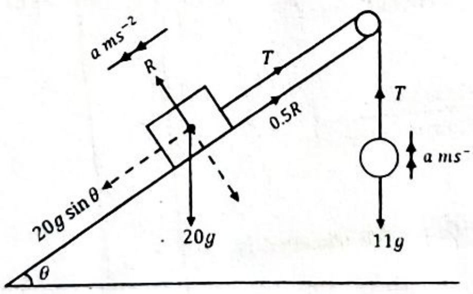
$20g \sin\theta - 0.5R - T = 20a$

But $R = 20g \cos\theta$

$20 \times 9.8 \times \frac{4}{5} - 0.5 \times 20 \times 9.8 \times \frac{3}{5} - 5a - 5 \times 9.8 = 20a$

$25a = 49$ $\therefore a = 1.96\ ms^{-2}$ A1

When he mass of B is 11 kg;



For 11 kg mass;

$T - 11g = 11a$

$T = 11a + 11g$ M1

For 20 kg mass;

$20g \sin\theta - 0.5R - T = 20a$

But R= $20g \cos\theta$

$20 \times 9.8 \times \frac{4}{5} - 0.5 \times 20 \times 9.8 \times \frac{3}{5} - 11a - 11 \times 9.8 = 20a$

$31a = -9.8;$ $\qquad a = -\frac{49}{155}\ ms^{-2}$

$\therefore a = \frac{49}{155}\ ms^{-2}$ or $0.3161\ ms^{-2}$ A1

16 (a) Let $f(x) = x \sin x - 1$ | 12

$f(1) = 1\sin(1) - 1 = -0.15853$ M1

$f(1.5) = 1.5\sin(1.5) - 1 = 0.49624$ Accept ans in sentence form

$\therefore$ Since $f(1) \cdot f(1.5) < 0$, $1 <$ root < 1.5 A1

1	x_0	1.5
−0.15853	0	0.49624

$\frac{x_0 - 1}{0 + 0.15853} = \frac{1.5 - 1}{0.49624 + 0.15853}$ M1

$x_0 = 1.121057776$

$x_0 \approx 1.12106$ A1

(b) $f'(x) = x\cos x + \sin x$ M1

$x_{n+1} = x_n - \left(\frac{x_n \sin x_n - 1}{x_n \cos x_n + \sin x_n}\right)$ B1 M1 only mark the root given to 4dps

Taking $x_0 = 1.12106$ A1

$x_1 = 1.12106 - \left(\frac{1.12106\sin(1.12106) - 1}{1.12106\cos(1.12106) + \sin(1.12106)}\right)$

$= 1.11415$ M1

$x_2 = 1.11415 - \left[\frac{1.11415\sin(1.11415) - 1}{1.11415\cos(1.11415) + \sin(1.11415)}\right]$

$= 1.11416$ M1

Since $|x_2 - x_1| = 0.00001 < 0.00005$, then the root is 1.1142 A1

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NB:

1. *The solutions in this Guide were according to my opinion.*
2. *I accept to own any mistakes detected*
3. *Try out the numbers and we compare the solutions thanx.*

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