SMACON

PURE MATHEMATICS 2024 FINAL DISCUSSION

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1. (a) (i) Express each of the following complex numbers $Z_1 = (1-i)(1+2i), Z_2 = \frac{2+6i}{3-i}$ and $Z_3 = \frac{-4i}{1-i}$ in the form $a+bi$.
   (ii) find the modulus and argument of $Z_1Z_2Z_3$ given in (a) (i) above.
   b) find the square root of $12i - 5$

2. (a) The first term of an arithmetic progression (A.P) is 73 and the 9th is 25. Determine
   (i) The common difference of the A.P
   (ii) The number of terms that must be added to give a sum of 96.

   (b) A geometric progression (G.P) and an arithmetic progression (A.P) have the same first term. The sum of their first, second and third terms are 6,10.5 and 18 respectively. Calculate the sum of their fifth terms.

3. (a) Show that $f(x) = \frac{x(x-5)}{(x-3)(x+2)}$ has no turning points. Sketch the curve $y = f(x)$.
   (b) If $g(x) = \frac{1}{f(x)}$,sketch the curve $y = g(x)$ on the same axes. Show the asymptotes and where $f(x)$ and $g(x)$ intersect.

4. (a) i) If $x^2 secx - xy + 2y^2 = 15, find\ \frac{dy}{dx}$.
   ii) Given that $y = \theta - cos\theta; x = sin\theta$; show that $\frac{d^2y}{dx^2} = sec^3\theta + tan\theta sec^2\theta$.
   (b) Determine the maximum and minimum values of x^2e^{-x}.

5. (a) find the equation of the circle and its radius circumscribing the triangle whose vertices are A (1,3), B (4, -5) and C (9, -1).
   (b) If the tangent to the circle at A (1,3) meets the x-axis at P(h,0) and y-axis at Q (0, k), find the values of h and k

6. (a) Given that $7tan\theta + cot\theta = 5sec\theta$, derive the quadratic equation for $sin\theta$. Hence or otherwise find all the values of θ in the interval $0° \leq \theta \leq 180°$ which satisfy the given equation, giving your answers to the nearest $0.1°$, where necessary.
   (b) The acute angles A and B are such that $cosA = {}^1/_2$, $sinB = {}^1/_3$. Show without the use of tables or calculators that $tan(A+B) = \frac{8\sqrt{2}+9\sqrt{3}}{5}$.

7. (a) In the triangle ABC, P is the point on BC such that BP: PC=$\gamma : \mu$, show that $(\gamma + \mu)AP = \gamma AC + \mu AB$.

(b) Three non collinear points A, B and C have position vectors a, b and c respectively with respect to an origin O. The point M on AC is such that AM:MC=2:1 and the point N on AB is such that AN: NB=2:1.

(i) Show that BM=$\frac{1}{3}(a-b)+\frac{2}{3}c$, and find a similar expression for CN

(ii) The lines BM and CN intersect at L. Given that BL= rBM and CL= sCN, where r and s are scalers, express BL and CL in terms of r, s, a, b and c. Hence by using triangle BLC, or otherwise, find r and s.

8. (a) Find the general solution of the equation

$$x\frac{dy}{dx} - 2y = (x-2)e^x$$

(b) The rate of cooling of a body is given by the equation $\frac{dT}{dt} = -k(T-10)$ where T is the temperature in degree Centigrade, k is a constant and t is the time in minutes. When $t = 0, T = 90$ and when $t = 5, T = 60$. Find T when $t = 10$.

9. (a) Show that $\int tan^n xdx = \frac{tan^{n-2}x}{n-1} - \int tan^{n-2}xdx$. Hence or otherwise evaluate $\int tan^4 xdx$.

(b) Using calculus of small increments or otherwise, find $\sqrt{98}$ correct to one decimal place.

(c) Find the area of the region bounded by the curve $y = \frac{1}{x(2x+1)}$, the x-axis and the lines $x = 1, x = 2$

10. a) (i) Find the equation of the chord through the points $(at_1{}^2, 2at_1)$ and $(at_2{}^2, 2at_2)$ of the parabola $y^2 = 4ax$

(ii) Show that the chord cuts the directrix when $y = \frac{2a(t_2t_1-1)}{t_1+t_2}$

(b) Find the equation of the normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ and determine its point of intersection with the directrix.

11. a) Show that $tan\left(\frac{x+y}{2}\right) - tan\left(\frac{x-y}{2}\right) = \frac{2siny}{cosx+cosy}$

(b) Find in radians the solution of the equation $cosx + sin2x = cos3x$,

for $0 \leq x \leq 2\pi$

12. Given that $tan^{-1}\alpha = x$ and $tan^{-1}\beta = y$ by expressing α and β as tangent ratios, of x and y, Show that $x + y = tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)$. Hence or otherwise:

(i) Solve for x in $tan^{-1}\left(\frac{1}{x-1}\right) + tan^{-1}(x+1) = tan^{-1}(-2)$

(ii) Without using tables or calculators, determine the value of;

$$tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{5}\right) + tan^{-1}\left(\frac{1}{8}\right)$$

13.(a) A and B are points whose position vectors are $a = 2i + k$ and $b = i - j + 3k$ respectively. Determine the position vector of the point P that divides AB in the ratio 4:1.

(b) Given that $a = i - 3j + 3k$ and $b = -l - 3j + 2k$, determine

(i) the equation of the plane containing a and b

(ii) the angle the line $\frac{x-4}{4} = \frac{y}{3} = \frac{z-1}{2}$ with the plane in (i) above.

14. The equation of a circle, centre O is given by $x^2 + y^2 + Ax + By + C = 0$, where A, B and C are constants. Given that $4A = 3B, 3A = 2C \; and \; C = 9$,

a) Determine (i) the coordinates of the centre of the circle
   (ii) the radius of the circle
b) A tangent is drawn from the point $Q(3, 2)$ to the circle. Find
   (i) the coordinates of P, the point where the tangent meets the circle.
   (ii) the area of the triangle QPO.

15. a) If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{16}$, find x

b) In an arithmetic progression $u_1 + u_2 + u_3 + \cdots, u_4 = 15$ and $u_{16} = -3$. Find the greatest integer N such that $u_N \geq 0$. Determine the sum of the first N terms of the progression.

c) Determine the expansion of $\frac{x+4}{x^2-1}$ in ascending powers of x up to the term containing x^r for $|x| < 1$.

16. a) Given that $z = 3 + 4i$, find the value of the expression $z + \frac{25}{z}$.

b) Given that $\left|\frac{z-1}{z+1}\right| = 2$, show that the locus of the complex number is

$x^2 + y^2 + \frac{10x}{3} + 1 = 0$. Sketch the locus.

17. a) Differentiate:

i. $e^{ax} sinbx$.

ii. $\frac{(x+2)(x+1)^2}{(x+3)^3}$, giving your answer in the simplest form

b) Given that $y = e^{tan^{-1}x}$, show that $(1 + x^2)\frac{dx^2}{d^2y} + (2x - 1)\frac{dy}{dx} = 0$. Hence or otherwise, determine the first four non-zero terms of the Maclaurin's expansion of y.

c) Determine the equation of the normal to the curve $y = \frac{1}{x}$ at the point $x = 2$. Find the coordinates of the other point where the normal meets the curve again.

18. (a) A chord AB subtends an angle θ in radians at the centre O of a circle of radius r. The area of the circle is three times the area of the minor segment AB. Show that

$3\theta = 3sin\theta + 2\pi$.

(b) Given that $tanx = secx - \frac{1}{3}$, find the value of:

(i) $cosx$

(ii) $tanx$

19. (a) Integrate $\frac{2x}{\sqrt{x^2+4}}$ with respect to x.

(b) Evaluate $\int_0^{\pi/6}(sinxsin3x)dx$

(c) Using the substitution $x = 3sin\theta$, evaluate $\int_0^3 \sqrt{\left(\frac{3+x}{3-x}\right)}\, dx$

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