

**P425/
PURE MATHEMATICS**

**Paper 1
July, 2024
3 Hours**

**RESOURCEFUL MOCKS-2024
Uganda Advanced Certificate of Education
PURE MATHEMATICS
Paper 1
3 hours**

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section **A** and any **five** from section **B**.*

Any additional question (s) answered will not be marked

*All necessary working **must** be shown clearly*

Begin each answer on a fresh sheet of paper

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

1. Solve the simultaneous equations:
 $x^2 + xy + 4y^2 = 6$ and $3x^2 + 8y^2 = 14$ (05 marks)
2. One side of a rectangle is three times the other. If the perimeter increases by 2%.
What is the percentage increase in the area? (05 marks)
3. The points A and B have position vectors \mathbf{a} and \mathbf{b} respectively relative to the origin, where $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = -4\mathbf{i} + s\mathbf{j} + t\mathbf{k}$. Find the possible values of s and t if $|\mathbf{AB}| = 7$ and $s = 2t$ (05 marks)
4. The first three terms of a G.P are $2x - 1$, $x + 1$ and $x - 1$ ($x \neq 0$). Find the value of x and the sum to infinity of the G.P. (05 marks)
5. If $t = \tan \theta$ and $\sec 2\theta + \tan 2\theta = k$, prove that $t = \frac{k-1}{k+1}$ (05 marks)
6. Show that: $\int_0^1 \frac{1}{(1+x^2)^2} dx = \frac{\pi}{8} + \frac{1}{4}$ (05 marks)
7. If $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$, show that $m = \frac{a}{c}$. (05 marks)
8. Find the value of k for which the lines $3x + 4y - k = 0$ and $12x - 5y + 29 = 0$ are equidistant from the point $(1, 3)$. (05 marks)

SECTION B(60 MARKS)

9. (a) If $\operatorname{cosec} A - \cot A = q$, then show that $\frac{q^2-1}{q^2+1} + \cos A = 0$ (05 marks)
(b) Solve the equation; $3\tan^3 \theta - 3\tan^2 \theta = \tan \theta - 1$ for $0 \leq \theta \leq 2\pi$ (07 marks)
10. (a) Find the cartesian equation of the plane passing through the points $P(1, 0, -2)$, $Q(3, -1, 1)$ and parallel to the line $\mathbf{r} = 3\mathbf{i} + (2\alpha - 1)\mathbf{j} + (5 - \alpha)\mathbf{k}$ (07 marks)
(b) Find the length of the perpendicular drawn from a point $(2, 3, -4)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ (05 marks)

11. (a) The expression $f(x) = 6x^2 + x + 7$ leaves the same remainder when divided by $x - a$ and $x + 2a$. Find the value of a for which $a > 0$.
(05 marks)
- (b) The polynomial $P(x) = \alpha x^3 - \mu x^2 + \beta x + 2$ gives a remainder -60 when divided by $x + 2$ and $f(3) = 35$. Given that $2x - 1$ is a factor of the polynomial. Find the values of α, μ and β . Hence evaluate $P(x) = 0$
(07 marks)
12. (a) The point $(2, 1)$ lies on the curve $Ax^2 + By^2 = 11$ where A and B are constants. If the gradient of the curve at that point is 6 . Find the values of A and B .
(05 marks)
- (b) A rectangular box without a lid is made from a thin cardboard. The sides of the base are $2x\text{cm}$ and $3x\text{cm}$ and the height of the box is $h\text{cm}$. If the total surface area is 200cm^2 , show that $h = \frac{20}{x} - \frac{3x}{5}\text{cm}$. And hence find the dimensions of the box to give maximum volume.
(07 marks)
13. (a) Find the values of x and y in: $\frac{x}{1+i} + \frac{y}{2-i} = 2 + 4i$ (06 marks)
- (b) Given that P is represented by $|Z - 2| = 2|Z + 1|$. Show that the locus of P is a circle and hence state its radius and centre.
(06 marks)
14. (a) Express $f(x) = \frac{32}{x^3 - 16x}$ into partial fractions. Hence find $\int f(x) dx$
(07 marks)
- (b) Show that: $\int_2^4 x \ln x dx = 14 \ln 2 - 3$ (05 marks)

15. (a) Find the length of the tangent to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ from the point $(5, 7)$. (05 marks)
- (b) Prove that the circles $x^2 + y^2 - 10x - 7y + 31 = 0$ and $x^2 + y^2 + 2x + 2y - 23 = 0$ touch each other externally. (07 marks)
16. (a) Solve the differential equation;
 $(1 + x^2) \frac{dy}{dx} = 1 + y^2$ for $y = 3$ and $x = 2$ (05 marks)
- (b) At time, t hours, the rate of decay of a radioactive element is directly proportional to its current mass.
- (i) Show that $N = N_0 e^{-kt}$ where N is the mass of the radioactive element and N_0 is the original mass.
- (ii) If the mass reduces to half the original mass in 4 hours, find the time it takes for the mass of the element to reach $\frac{1}{8}$ of the original mass. (07 marks)

END