

P425/1

Pure Mathematics

Paper one

JULY/AUG 2024

3 HOURS

**ASSHU ANKOLE JOINT MOCK EXAMINATIONS 2024**

**Uganda Advanced Certificate of Education**

**PURE MATHEMATICS**

**PAPER ONE**

**3 HOURS**

**INSTRUCTIONS TO CANDIDATES**

- Answer **all** the **eight** questions in section **A** and any **five** questions in section **B**.
- Any additional question(s) answered will **not** be marked.
- All necessary working **must** be shown clearly.
- Begin each question on a fresh sheet of paper.
- Indicate the questions attempted
- Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.



### SECTION A (40 marks)

1. Solve the simultaneous equations  

$$\begin{aligned} 2x + y - 3z &= 7 \\ 4x - 2y + z &= 15 \\ 3x + 3y + 2z &= 1 \end{aligned}$$
(05 marks)
2. Evaluate  $\int_0^{\pi/2} \sin 3x \cos 5x \, dx$ . (05 marks)
3. Solve the equation  $4 \cos x + 3 \cos \frac{x}{2} = 1$  for  $0^\circ \leq x \leq 360^\circ$  (05 marks)
4. Find the acute angle between the lines  $\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z+3}{1}$  and  $\underline{r} = (2 + 2\lambda)\underline{i} + (1 + 3\lambda)\underline{j} + (6\lambda - 1)\underline{k}$ . (05 marks)
5. Differentiate  $\log_2 \left( \frac{e^{x^2}}{\sin 2x} \right)$  with respect to  $x$  (05 marks)
6. Solve for  $x$ :  $\log_4 x = \log_2 (3 - 2x)$  (05 marks)
7. Points A(0, 2) and B(4, -2) lie on the circumference of a circle. Points C(-3, -3) and D(7, 2) lie outside the circle but the centre of the circle lies on line CD. Find the equation of the circle. (05 marks)
8. A curve is represented by parametric equations  $x = \sqrt{t^2 + 3}$  and  $y = 3t + 4$ . Find the equation of the tangent to the circle at point (2, 7). (05 marks)

### SECTION B (60 marks)

Answer any five questions.

9. Show that
  - i.  $\int_0^1 \frac{3x+9}{x^2+5x+4} \, dx = \ln 5$  (06 marks)
  - ii.  $\int_0^{2\pi/3} \frac{3dx}{5+4\cos x} = \pi/3$ . (06 marks)
10. (a) Express  $(-1 + i\sqrt{3})^8$  in the form  $x + iy$  (05 marks)  
 (b) Find the Cartesian equation of the curve given as  $|z - 2| = 2|z + 1 - 3i|$ , show by leaving unshaded, the region  $|z - 2| > 2|z + 1 - 3i|$  on the Argand diagram. (07 marks)
11. (a) Find, in vector form, the equation of a line passing through the point (1, 1, 3) and perpendicular to the plane  $2x + 3y + 3z = 7$ . (03 marks)  
 (b) Find the position vector of the point of intersection of the lines  

$$\underline{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} \text{ and } \underline{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}.$$
 Write down the vector equation of the plane containing lines  $\underline{r}_1$  and  $\underline{r}_2$  hence or



otherwise find the Cartesian equation of the plane containing lines  $r_1$  and  $r_2$ .

12. (a) If  $\alpha$  and  $\beta$  are roots of the equation  $2x^2 - 7x + 1 = 0$ . Show that; (09 marks)

$$\left( \sqrt{\frac{\alpha}{\beta}} - \sqrt{\frac{\beta}{\alpha}} \right)^2 = \frac{41}{2} \quad (05 \text{ marks})$$

- (b) Given that  $(x - 2)^2$  is a factor of the polynomial  $f(x) = x^4 + ax^3 + bx^2 + cx + 4$  and  $f(x)$  leaves a remainder of 2 when divided by  $(x - 1)$ . Find the values of  $a$ ,  $b$  and  $c$ . (07 marks)

13. (a)  $A$  is an acute angle and  $B$  is obtuse such that  $\tan A = \frac{4}{3}$  and  $\tan B = -2$ , without using tables or a calculator. Find the values of;

i)  $\sin(A - B)$

ii)  $\cos(A + B)$  (06 marks)

- (b) Prove that, in any triangle  $ABC$ ,  $\frac{a+b-c}{a+b+c} = \tan \frac{A}{2} \tan \frac{B}{2}$ . (06 marks)

14. (a) If  $y = \frac{5x+3}{\sqrt{1-2x^2}}$ . Find  $\frac{dy}{dx}$  (05 marks)

- (b) A cylindrical tin without a lid is made of a sheet metal. If 5 is the area of the sheet used, without waste,  $V$  is the volume of the tin and  $r$  is the radius of the cross-section, prove that  $2V = Sr - \pi r^3$ . If  $S$  is given, prove that the volume is maximum when the ratio of the height to diameter is 1 : 2. (07 marks)

15. (a) A curve is represented by parametric equations  $x = 4\cos\theta$ ,  $y = 3\sin\theta$ ; show that the Cartesian equations of the curve represents the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ . (03 marks)

- (b) The Normal to the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  at  $P(4\cos\theta, 3\sin\theta)$  meets the  $x$ - and  $y$ -axes at  $A$  and  $B$  respectively. Find the equation of the normal. If  $M$  is the mid-point of  $AB$ , show that the locus of point  $M$  is also an ellipse. (09 marks)

16. (a) Solve the differential equation  $\frac{dy}{dx} = e^{2x+y}$ . (04 marks)

- (b) Mbarara city's population is growing in such a way that at time  $t$  years, the rate at which the population is increasing is proportional to size,  $N$ , of the population at that time,  $t$ . If the population increases from 10,000 to 20,000 in five years. What will be the population in the next five years? (08 marks)

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