

A LEVEL

P425/1 , PAPER 1

PURE MATHEMATICS

Nov.Dec.2009. 3 hours.

SECTION A

1 Solve the simultaneous equations;

$$p + 2q - r = -1$$

$$3p - q + 2r = 16$$

$$2p + 3q + r = 3.$$

2. Give that $\sin(\theta - 45^\circ) = \cos(\theta + 45^\circ)$

Show that $\tan\theta = 1.$

Hence find θ if $0^\circ \leq \theta < 360^\circ$.

3. Differentiate e^{ax^2}

4. If $y = \frac{3-2x}{4+x^2}$, find the range of possible values of y for real x

5. The points $P(2, 3)$, $Q. (-11, 8)$ and $R. (-4, -5)$ are vertices of a parallelogram PQRS which has PR as a diagonal. Find the co-ordinates of vertex S.

6. Find $\int \frac{dx}{1-\cos x}$

7. Find the equation of a line through the point $(1, 3, -2)$ and perpendicular to the plane whose equation is $4x + 3y - 2z - 16 = 0$

8. Solve the differential equation

$$x(1-y) \frac{dy}{dx} + y = 0$$

Or SECTION B

9.(a) By using the binomial theorem expand $(8 - 24x)^{2/3}$ as far as the 4th term.

Hence evaluate $4^{2/3}$ to one decimal place.

(b) Find the coefficient of x in the expansion of

$$\left(x + \frac{2}{x^2}\right)^{10}$$

10.(a) Differentiate $\ln(1 - 2x^2)^{-1/2}$

With respect to x .

(b) Integrate

$$\frac{4x - x^3 + x^2 + 1}{x^3 + x} \text{ with respect to } x, \text{ we have; } x.$$

11. (a) use the factor formula to show that

$$\frac{\sin(A+2B) + \sin A}{\cos(A+2B)\cos A} = \tan(A+B)$$

(b) Express $y = 8\cos x + 6\sin x$ in the formula $R \cos(x - \alpha)$ where R is positive and α is acute.

Hence find the maximum and minimum values of

$$\frac{1}{8\cos x + 6\sin x + 15}$$

12(a) Given that

$$\frac{ix}{1+iy} = \frac{3x+iy}{x+3y}, \text{ find the}$$

Values of x and y ,

(b) if $z = x + iy$, find the equation of the locus

$$\left| \frac{z+3}{z-1} \right| = 4.$$

$z + 0$ and $x - y = 1$

(b) two lines are given by the parametric equations:

$$-i + 2j + k + t(i - 2j + 3k) \text{ and } -3i + pj + 7k + 5(i - j + 2k).$$

If the lines intersect, find the

(i) values of t , s and to co-ordinates of the points of intersection.

14. (a) Use Maclaurin's theorem to expand

$$\frac{1}{\sqrt{1+x}} \text{ up to the term in } x^3.$$

(b) Given that $e^x = \tan 2y$ show that

$$\frac{d^2y}{dx^2} = \frac{e^x - e^{3x}}{2(1 + e^{2x})}$$

15(a) Find the equation of the tangent and normal

to the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ at the point $P(2$

$\cos\theta, \sin\theta)$.

(b) If the tangent in (9) cuts the y -axis at point A and the x -axis at point C , find the co-ordinates of the points A , B and C .

16. In a certain process the rate of production of yeast is Rx grams per minute, where x grams is the amount produced and $R = 0.003$.

(a) show that the amount of yeast is doubled in about 230 minutes.

(b) If in addition yeast is removed at a constant rate of m grams per minute, find the

- (i) amount of yeast at time t minutes, given that when $t = 0$, $x = p$ grammas
- ii) Value of m if $p = 20,000$ grammas and the supply of yeast is exhausted in 100 minutes.

APPLIED MATHEMATICS

2009

Paper 2

SECTION A: (40 MARKS)

Answer all questions in this section.

1. If A and B are independent events:

(i) show that the events A and B' are also independent.

(ii) find $P(B)$ given that $P(A) = 0.4$ and $P(A \cup B) = 0.8$. (05 marks)

2. A car moves from Kampala to Jinja and back. Its average speed on the return journey is 4 kmh^{-1} greater than that on the outward journey and it takes 12 minutes less. Given that Kampala and Jinja are 80 km apart, find the average speed on the outward journey. (05 marks)

3. The table below shows the distance in kilometers (km) a truck can move with a given amount of fuel

in litres (l).

Distance (km)	20	28	33	42
Fuel (l)	10	13	21	24

Estimate

(a) how far the truck can move on 27.5 l of fuel,

(b) the amount of fuel required to cover 29.8 km.
(05 marks)

4. The random variable X has a probability function

$$F(x) = \begin{cases} k2^x & ; \quad x=0,1,2,3. \\ 0 & ; \quad \text{elsewhere.} \end{cases}$$

Find:

(a) the value of the constant k .

(b) $E(X)$. (05 marks).

5. A body of mass 8 kg rests on a rough plane inclined at θ to the horizontal.

If the coefficient of friction is μ , find the least

horizontal force in terms of μ , θ and g which will hold the body in equilibrium. (05 marks)

6. Use the trapezium rule with six ordinates to estimate $\int_1^2 \frac{\ln x}{x} dx$.

Give your answer correct to **three** decimal places. (05 marks)

7. The following information relates to three products sold by a company in the year 2001 and 2004

Product	2001		2004	
	Quantity in thousands	Selling price Per unit (£)	Quantity in thousands	Selling price Per unit (£)
<i>A</i>	76	0.60	72	0.18
<i>B</i>	52	0.75	60	1.00
<i>C</i>	28	1.10	40	1.32

Calculate the

- (a) percentage increase in sales over the period.
- (b) corresponding percentage increase in income over the period. *(05 marks)*

8. The velocity of a particle at any time t is given by an equation;

$$v(t) = a \omega \sin \omega t + b \omega \cos \omega t.$$

- (a) Find the expression for the displacement x at any time given that $x = 0$ when time $t = 0$.
- (b) show that the motion of the particle is Simple Harmonic.

(05 marks) **SECTION B: (60 MARKS)**

Answer any five questions from this section. All questions carry equal marks.

- (a) The dimensions of a rectangle are 6.2cm and 5.36cm.
 - (i) State the maximum possible error in each dimension.
 - (ii) Find the range within which the area of the rectangle lies.

(05 marks)

- (b) The numbers $a = 26.23$, $b = 13.18$ and $c = 5.1$

are calculated with percentage errors of 4, 3 and 2 respectively.

Find the limits to **two** decimal places within which the exact value of the expression $ab - \frac{b}{c}$ lies. (07 marks)

A pile driver of mass 1200 kg falls freely from a height of 3.6 m and strikes without rebounding, a pile of mass 800 kg. The blow drives the pile a distance of 36 cm into the ground.

Find the

(a) resistance of the ground. (08 marks)

(b) time for which the pile is in motion.
(04 marks)

[Assume the resistance of the ground to be uniform]

The table below shows the income of 40 factory workers in millions of shillings per annum.

1.	1.	1.	1.	5.	1.	2.	2.
0	1	0	2	4	6	0	5
2.	2.	1.	1.	1.	2.	3.	2.
1	2	3	7	8	4	0	2
2.	3.	4.	4.	3.	5.	5.	5.
7	5	0	4	9	0	4	3
4.	3.	3.	3.	5.	5.	5.	1.
4	7	6	9	2	1	7	5
1.	1.	3.	4.	2.	3.	5.	4.
6	9	4	3	6	8	3	0

(a) Form a frequency distribution table with class intervals of 0.5 million shillings starting with the lowest limit of 1 million shillings.

(02 marks)

(b) Calculate the

(i) mean income.

(ii) standard deviation. *(06 marks)*

(c) Draw a histogram to represent the above data. Use it to estimate the modal income. *(04*

marks)

12. Forces of magnitude 3N, 4N, 4N, 3N and 5N act along the lines AB , BC , CD , DA and AC respectively of the square $ABCD$ whose side has a length of a

units. The direction of the forces are indicated by the order of the letters.

(a) Find the magnitude and direction of the resultant force (09 marks)

(b) If the line of action of the resultant force cuts AB produced at E , find the length AE . (03 marks)

13. (a) A box contains two types of balls, red and black. When a ball is picked from the box, the probability that it is red is $\frac{7}{12}$. Two balls are selected at random from the box without replacement.

Find the probability that

(i) the second ball is black.

The first ball is red, given that the second one is black.

(b) An interview involves written, oral and practical tests. The probability that an interviewee passes the written test is 0.8, the oral test is 0.6 and the practical test is 0.7. What is the probability that the interviewee will pass

(i) the entire interview?

Exactly two of the interview tests?

14. (a) Show that the root of the equation $2x$

– $3\cos(x/2) = 0$ lies between 1 and 2.
(03 marks)

Use Newton Raphson's method to find the root of the equation in (a) above. Give your answer correct to **two** decimal places.

15. (a) The masses of soap powder in certain packets is normally distributed with mean 842 grams and variance 225 (grams).

Find the probability that a random sample of 120 packets has sample mean with mass

- (i) between 844 grams and 846 grams.
- (ii) Less than 843 grams.

(b) A random sample of size 76 electrical components produced by a certain manufacturer have resistances r_1, r_2, \dots, r_{76} ohms

Where $\sum r_i = 740$ and $\sum r_i^2 = 8,216$

Calculate the

- (i) unbiased estimate for the population variance.
- (iii) 91.86% confidence interval for the mean resistance of the electrical components produced.

[Give answers correct to **3** decimal places.] (06 marks)

16. Two particles P and Q move with constant velocities of $(4i + j - 2k) \text{ ms}^{-1}$ and $(6i + 3k) \text{ ms}^{-1}$ respectively. Initially P is at the point with position vector $(-i + 20j + 21k)\text{m}$ and Q is at the point with position vector $(i + 3k)\text{m}$.

Find the

(a) time for which the distance between P and Q is least. (08 marks)

(b) distance of P from the origin at the time when the distance between P and Q is least. (02 marks)

(c) least distance between P and Q . (02 marks)

END.

READ BELOW

NEXT PAGE!

**We can assist school Administrators
to train staff(teachers) on the use of
Artificial Intelligence (AI)**

To:

- (i) Write notes**
- (ii) Set tests**
- (iii) Set exams**
- (iv) Make lesson plans**
- (v) Prepare projects for the
new curriculum**

Reach us on: 0755 883 919

WE CAN ALSO:

Do the following:

- (i) Customized printing of all
academic work**

- (ii) Develop an e- library for the school with branded and customized books**
- (iii) Assist the school do massive e marketing using the same work**
- (iv) Train teachers tools like pdf editing and document encryption in pdfs etc**
- (v) Allow the school access to our library of more than 50,000 books and documents for the learners!**

Reach us on: 0755 883 919