***P425/1***
***PURE***
***MATHEMATICS***
***May, 2022***
***Paper 1***
***3 hours***

**INTERNAL MOCK EXAMINATIONS 2022**

**Uganda Advanced Certificate of Education**

PURE MATHEMATICS

**Paper 1**

3 hours

**INSTRUCTIONS TO CANDIDATES:**

*Attempt **ALL** the eight questions in section **A** and five from section **B.***

*All working must be shown clearly*

*Begin each answer on a fresh sheet of paper.*

*Mathematical tables with a list of formulae and squared papers are provided.*

*Silent non – programmable scientific calculators may be used.*

*State the degree of accuracy at the end of the answer to each question attempted using a calculator or tables and indicate **“cal”** for calculator or **“Tab”** for Mathematical tables.*

## SECTION A (40 MARKS)

**(attempt all questions)**

1. If $\alpha$ *and* $\beta$ are roots of $px^2 + qx + r = 0$. Express $(\alpha - 2\beta)(\beta - 2\alpha)$ in terms of p, q and r. Hence deduce that for one root to be twice the other $9pr = 2q$. ***(05 mks)***

2. Solve the equation $\sqrt{2x+3} - \sqrt{x+1} = \sqrt{x-2}$ ***(05 mks)***

3. Solve $5\cos\theta - 3\sin\theta = 4$ $for$ $0^o \le \theta \le 360^o$ ***(05 mks)***

4. Solve simultaneously to find the solution P(x. y. z) if;
$x + y + z = 0$
$2y + 2z + x = 2$
$2z + 2x + y = 4$ ***(05 mks)***

5. Find the point of intersection of the line; $\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-2}{3}$ with the plane $2x + 7y + 5z - 3 = 0$. ***(05 mks)***

6. Given that $y = x^{-x}$, find the value of $\frac{dy}{dx}$ when $x = 2$. ***(05 mks)***

7. The remainder obtained when $f(x) = 2x^3 + mx^2 - 6x + 1$ is divided by $x + 2$ is twice the remainder when $f(x)$ is divided by $(x + 1)$. Find the value of m. ***(05 mks)***

8. Solve the differential equation $\frac{dy}{dx} = 3y + 2xe^{3x}$ given that y = 1 when $x = 0$.
***(05 mks)***

## SECTION B (60 MARKS)

9. (a) Prove that $cos3A = cosA(2cos2A - 1)$ ***(03 mks)***
(b) Solve $sin4\theta + \sin 2\theta = sin\theta + sin3\theta$ $for$ $0^o \le \theta \le 90^o$.
***(05 mks)***
(c) If A, B and C are angles of a triangle, show that;
$$\sin(B + C - A) + \sin(A + B - C) + \sin(C + A - B) = 4\sin A\, sinB\, sinC$$
***(04 mks)***

10. Express $\frac{x^4 + 2x}{(x-1)(x^2+1)}$ hence evaluate $\int_2^3 \frac{x^4 + 2x}{(x-1)(x^2+1)}\,dx$ to 4 significant figures. ***(12 mks)***

11. (a) Given that $x = \sin\theta$, $y = 1 - \cos\theta$ prove that;

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3$$ ***(06 mks)***

(b) A curve is represented parametrically by $x = (t^2 - 1)^2$ and $y = t^3$.

Find; $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. ***(06 mks)***

12. (a) Z is a complex number such that $Z = \frac{p}{2-i} + \frac{q}{1+3i}$ where p and q are real numbers.

If $\text{Arg}(Z) = \frac{\pi}{4}$ and $|Z| = 7$. Find the values of p and q. ***(06 mks)***

(b) Show that the locus of a complex number Z, such that $\left|\frac{Z-1-i}{Z-1+i}\right| = 2$ as a circle and hence determine the area of the circle $\left(\pi = \frac{22}{7}\right)$. ***(06 mks)***

13. (a) Show that if $y = e^{4x}\cos 3x$, then, $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$ ***(05 mks)***

(b) A body is placed in a room which is kept at a constant temperature. The temperature of the body falls at a rate $K\theta^{o\,o}$ per minute, where K is a constant and $\theta$ is the difference between the temperature of the body and that of the room at time, t.

(i) Form a differential equation in terms of K, t and $\theta$ hence show that $\theta = \theta_o e^{-Kt}$, where $\theta_o$ is the temperature difference at $t = 0$.

***(03 mks)***

(ii) If the temperature of the body falls 5°C in the first minute, and 4°C in the second minute, show that the fall of temperature in the third minute is 3.2°C. ***(04 mks)***

14. (a) Differentiate the following functions with respect to $x$.

(i) $\frac{(x-1)e^{4x}}{(x+1)^3}$

(ii) $10^{\sqrt{1-x^2}}$ ***(07 mks)***

(b) If $x^2 + y^2 = 2y$, prove that; ***(05 mks)***

$$\frac{d^2y}{dx^2} = \frac{1}{(1-y)^3}$$

15. (a) Find the acute angle between the lines whose equations are;

$$\frac{x-2}{-4} = \frac{y-3}{3} = \frac{z+1}{-1} \quad \textit{and} \quad \frac{x-3}{2} = \frac{y-1}{6} = \frac{z+1}{-5}$$ ***(05 mks)***

(b) Lines $L_1$ and $L_2$ have vector equations;

$$\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \text{ respectively.}$$

Find the position vector of the point of intersection. ***(07 mks)***

16. (a)Form the equation of a circle that passes through the points A(5, 7), B (1, 3) and ( 2,2) ***(06 mks)***

(b) Expand $(1-2x)^{\frac{2}{3}}$ in ascending powers of $x$ up to and including the term in $x^3$.

Hence evaluate $(28)^{\frac{2}{3}}$ correct to 4 decimal places. ***(06 mks)***

**END**