Unit
05

CIRCULAR MOTION

Q.1 Define circular motion. Also give example. 111050

Ans. Circular Motion:

Definition:

"Motion of a body in a circular path is known as circular motion".

For example

1. Motion of electron around nucleus.
2. Motion of planets around the sun
3. A stone whirled around by a string
4. A car turning around a corner
5. Satellites orbits around the earth

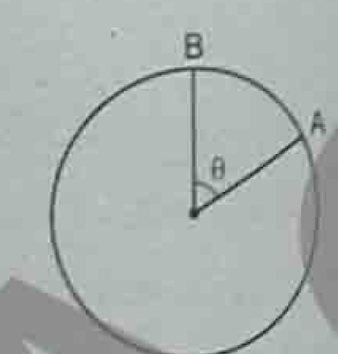
Q.2 Define angular displacement. Also write its unit. 111050

Ans. Angular Displacement:

"It is the angle subtended by an arc at the centre of the circle".

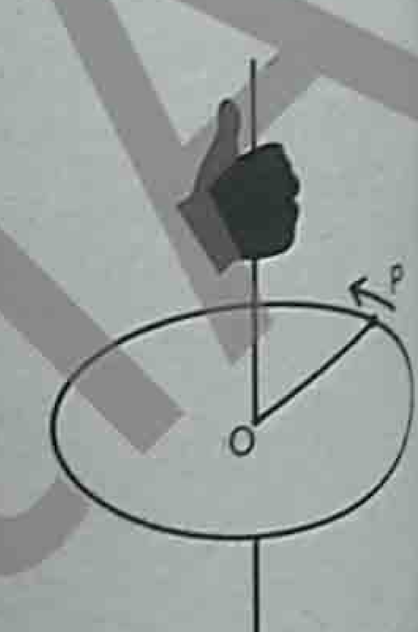
It is denoted by the symbol θ. For its small values, θ is a vector quantity and for large values θ is not a vector quantity.

Its direction is determined by right hand rule.



Right Hand Rule:

"Grasp the axis of rotation in the right hand such that the fingers are curled along the direction of rotation. The extended thumb along the axis indicates the direction of angular displacement".



Unit:

The units for angular displacement may be degree, revolution or radian, where radian is the supplementary S.I unit.

1 Degree:

"Angle subtended by the 360th part of a revolution at the centre of a circle is known as 1 degree."

Mathematically, $1^\circ = \frac{1}{360}\text{ rev}$

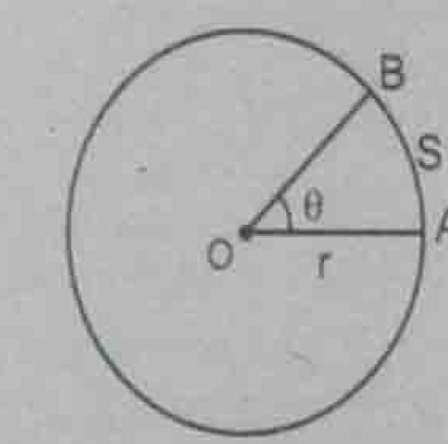
Q.3 Define Radian. Establish the relation $S = r\theta$ and show that 1 radian = 57.3° 11105

Ans. Definition of one Radian: (Board 2008)

"It is the angle at the centre of a circle for its circular arc whose length is equal to radius of that circle".

Relation between arc length 's' and angular displacement 'θ':

Consider a circle of radius 'r'. Let 's' be an arc length of this circle connecting the points A and B as shown in fig. By the definition of radian we can understand that:



$$1 \text{ radian} \propto r \Rightarrow 1 \text{ radian} = k\,r \text{ .... (1)}$$

$$\text{And } \theta \text{ radian} \propto s \Rightarrow \theta \text{ radian} = k\,s \text{ .... (2)}$$

Where k is the constant of proportionality.

By dividing equation (2) by equation (1) we get

$$\frac{\theta \text{radian}}{1\text{radian}} = \frac{ks}{kr}$$

$$\frac{\theta}{1} = \frac{s}{r}$$

$$\Rightarrow \quad s = r\theta$$

The above equation will hold only if θ is expressed in radian. Angular displacement is dimensionless because it is a ratio between same physical quantities.

Relation between Radian and Revolution:

The circumference of a circle is $2\pi r$ then θ is equal to 1 revolution. So last equations gives:

$$2\pi r = r\theta \Rightarrow 2\pi \text{ radian} = \theta$$

Relation between Revolution and Degree:

We know that 1 rev. = 360°

Therefore 2π radian = 360°. $\Rightarrow \pi$ radian = 180°

$$\Rightarrow 1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ$$

$$\Rightarrow 1 \text{ radian} = 57.3^\circ.$$

Relation between degree and Radian:

Take π radian = 180°

$$\Rightarrow 1^\circ = \left(\frac{\pi}{180}\right)\text{radian.}$$

$$\Rightarrow 1^\circ = 0.0174 \text{ radian.}$$

It shows that "size of one degree is smaller than size of one radian. 11105004

Q.4 Define angular velocity. Establish relation $v = \omega r$.

Ans. Angular Velocity:

Definition:

"Time rate of change of angular displacement is known as angular velocity".

Average Angular Velocity: Let Δθ is the angular displacement of body traversed in time "Δt" then the ratio $\frac{\Delta\theta}{\Delta t}$ is called average angular velocity written as below:

$$\boxed{\omega_{av} = \frac{\Delta\theta}{\Delta t}}$$

Instantaneous Angular Velocity:

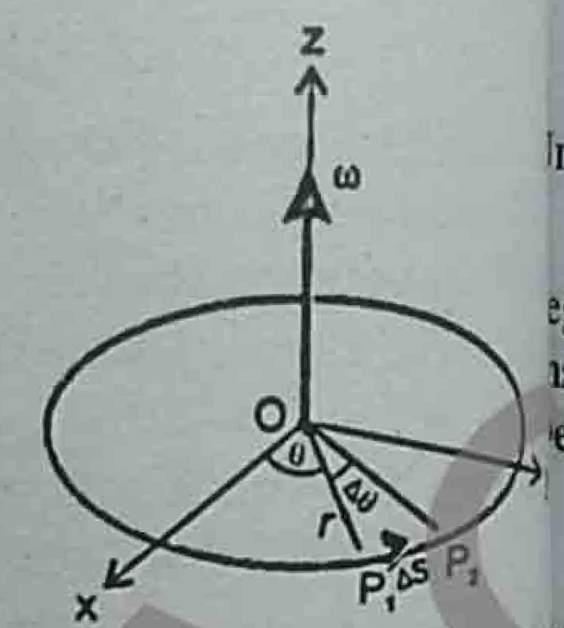
Angular velocity of the body at any instant of time is known as its instantaneous angular velocity a
its value is found by applying the limit $\Delta t \to 0$ on $\frac{\Delta\theta}{\Delta t}$ as below:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

It is a vector quantity. Its direction is determined by right hand rule illustrated below:
The S.I units for angular velocity are radian per second i.e. rad. s^{-1}.Some times it is also given i
terms of revolution per minute.

Relation between linear and angular velocities:

Consider a body moving along a circular path of radius 'r'. Let A is the initial position, and after time Δt seconds the body moves to the position B. The distance covered by the body is equal to length of the arc Δs, and its angular displacement is Δθ.



We know, $s = r\theta$

By applying Δ on both sides we get.

$$\Delta s = \Delta(r\theta)$$

Since r is constant, therefore

$$\Delta s = r\Delta\theta$$

Divide both sides by Δt

$$\frac{\Delta s}{\Delta t} = r \times \frac{\Delta\theta}{\Delta t}$$

Apply limit $\Delta t \to 0$ on above equation we get

$$\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t} = \lim_{\Delta t\to 0} r \times \frac{\Delta\theta}{\Delta t}$$

$$\Rightarrow \lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t} = r \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$$

By definition we know that $\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t} = v$ = tangential velocity and $\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} = \omega$ = angul
velocity.

Therefore $\boxed{v = r\omega}$

It is the relation between linear (or tangential) velocity and angular velocity for circul
motion.

In vector form this relation can be expressed as:

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Q.5 Define angular acceleration. How is it related with linear acceleration? 11105

Ans. Angular Acceleration:

Definition:

"The time rate of change of angular velocity is known as angular acceleration
It is denoted by the symbol 'α'.

In deriving the relation $v = r\omega$, ω is in manipulated as scalar because its axis of rotation fixed.
Also magnitude of linear velocity is taken as scalar.

Average Angular Acceleration:

If $\Delta\vec{\omega}$ is the change in angular velocity that occurs in time Δt seconds then the ratio $\frac{\Delta\vec{\omega}}{\Delta t}$ is
known as average angular acceleration written as below:

$$\vec{\alpha}_{av} = \frac{\Delta\vec{\omega}}{\Delta t}$$

Where $\Delta\vec{\omega} = \vec{\omega}_f - \vec{\omega}_i$ and time interval $\Delta t = t_f - t_i$, hence the above equation can also be
written as:

Formula:

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

Units:

Average angular acceleration is a vector pointing along the direction of $\Delta\vec{\omega}$. Its units are
degs^{-2}, rev s^{-2} or rad s^{-2}, where rad s^{-2} is the S.I unit of angular acceleration.

Instantaneous Angular Acceleration:

Definition:

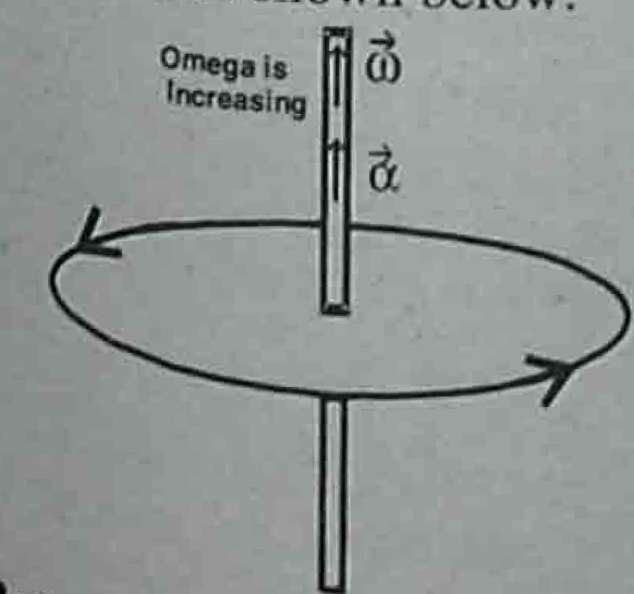
"The value of $\frac{\Delta\vec{\omega}}{\Delta t}$ when the time interval Δt is taken very, very small is called instantaneous angular acceleration".

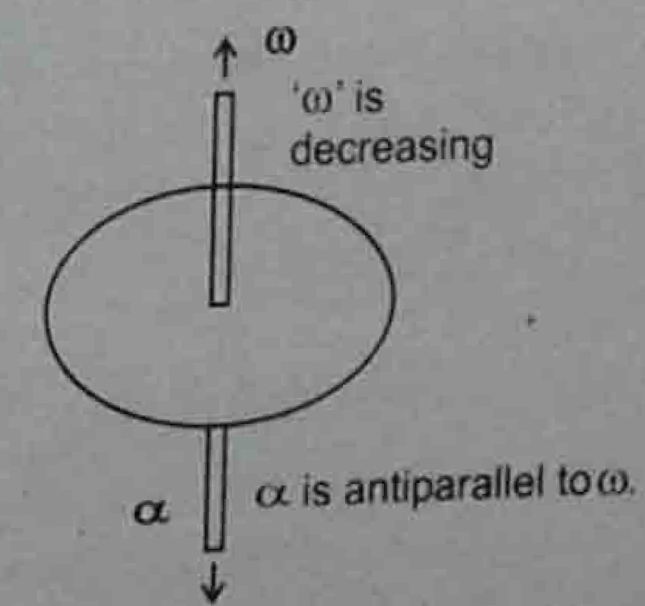
Formula:

Mathematically it is written as below:

$$\vec{\alpha}_{ins} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\omega}}{\Delta t}$$

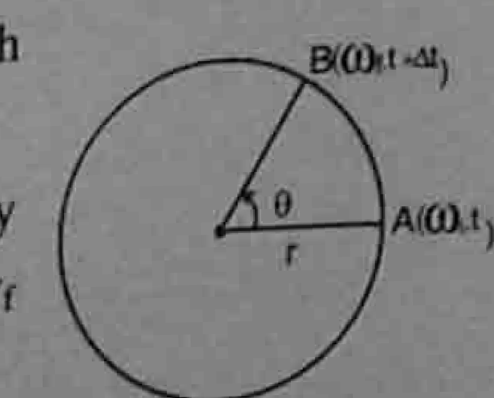
For circular motion in fixed plane the direction of instantaneous angular acceleration is along
the axis of rotation as shown below:





Relation Between Linear (or tangential) and Angular Accelerations:

Consider a rigid body moving in a circular path of radius 'r' with
variable angular velocity ω as shown in figure.



Let ω_i is the angular velocity at point A and ω_f is the angular velocity
after time Δt at the point B. The linear velocities at these points are v_i and v_f
respectively.

Using the relation $v = r\omega$ we have the following equations:

$$v_i = r\omega_i \text{ -------------------- } (1)$$

$$v_f = r\omega_f \text{ -------------------- } (2)$$

Subtracting equation (1) from equation (2)

$$\Rightarrow v_f - v_i = r\omega_f - r\omega_i$$

$$\Rightarrow \triangle v = r(\omega_f - \omega_i)$$

$$\Rightarrow \triangle v = r\triangle\omega$$

Divide both sides by $\triangle t$

$$\frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

Apply limit $\Delta t \to 0$ on both sides

$$\lim_{\triangle t \to o}\frac{\triangle v}{\triangle t} = \lim_{\triangle t \to o} r\frac{\triangle\omega}{\triangle t}$$

$$\Rightarrow \lim_{\triangle t \to o}\frac{\triangle v}{\triangle t} = r\lim_{\triangle t \to o}\frac{\triangle\omega}{\triangle t}$$

By definition we know that $\lim_{\triangle t \to o}\frac{\triangle v}{\triangle t} = a$ = tangential acceleration, and $\lim_{\triangle t \to o}\frac{\triangle\omega}{\triangle t} = \alpha$ = angu[lar] acceleration.

Therefore

$$\boxed{a = r\alpha}$$

It is the relation between linear (or tangential) and angular acceleration for circular motion. In [vector] form it can be written as below:

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

Q.6 Write the brief review of equation of uniformly accelerated body with ang[ular] acceleration.

Ans. Equations of Angular Motion:

When the body is moving with ***constant angular acceleration*** 'α' then we can write following equations for circular motion like we have for linear motion, by replacing the li[near] quantities by their angular counter parts that is v_i by ω_i, s by θ, v_f by ω_f and a by α.

The equations of angular motion are:

$$\omega_f = \omega_i + \alpha t \quad \text{.....................} (1)$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad \text{.....................} (2)$$

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \quad \text{.....................} (3)$$

The angular equations (1), (2), (3) hold true only in the case when the axis of rotati[on is] fixed.

Q.7 Define centripetal force. Derive the expression of centripetal force in terms of ang[ular] velocity.

Ans. Centripetal Force:

Definition:

"The force needed to bend the normally straight path of the body into a circular is know[n as] centripetal force".

Explanation:

In circular motion the direction of linear velocity changes continuously. Therefore some force is required to change the direction of this velocity i.e. the direction of motion. This required force is the centripetal force

Expression of Centripetal Force:

Consider a body moving in a circular path of radius 'r' with tangential velocity $\vec{v}$ and angular velocity ω, with O as centre of the circular path.

Let $\vec{v}_1$ is velocity of the body at point A and $\vec{v}_2$ is the velocity at point B. This body moves from point A to point B in a time Δt. Since velocity vector is changing so acceleration will be produced whose formula is written as below:

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right)$$

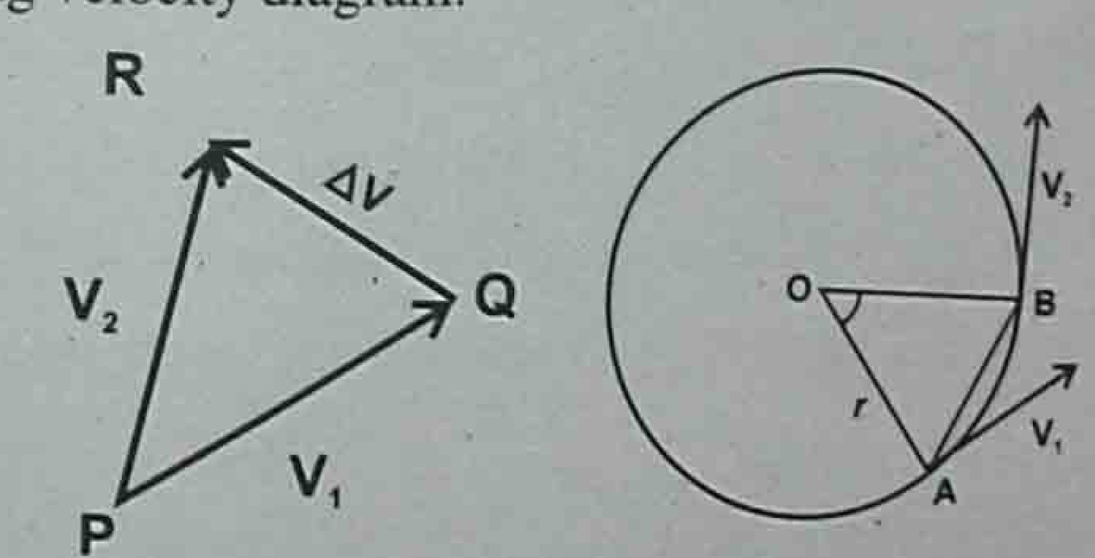
Magnitude of $\vec{a}$:

The time interval Δt can be found by using $s = v\,\Delta t$, because speed of the particle is constant in going from A to B:

Therefore $\Delta t = \frac{s}{v}$ .... (1)

For Δv we proceed as follows:

If velocity $\vec{v}_1$ is represented by vector $\overrightarrow{PQ}$ and $\vec{v}_2$ is represented by the vector $\overrightarrow{PR}$ then we can draw the following velocity diagram:



In triangle QPR the angle ∠QPR is Δθ. The radii of the circle are perpendicular to the tangents i.e. $\overline{OA}$ is perpendicular to $\vec{v}_1$ and $\overline{OB}$ is perpendicular to $\vec{v}_2$. Therefore the angles ∠AOB and ∠QPR are equal. Also $|\vec{v}_1| = |\vec{v}_2| = v$ and $\overline{OA} = \overline{OB}$. Hence both the triangles are isosceles similar triangles. Therefore ratios of these corresponding sides will be equal. i.e.

$$\frac{\Delta v}{v} = \frac{\overline{AB}}{r}$$

By applying the limit $\Delta t \to 0$ we see that $\overline{AB}$ = arc AB = s.

Therefore $\frac{\Delta v}{v} = \frac{s}{r}$

$$\Rightarrow \Delta v = \frac{s\,v}{r} \quad \text{...........} (2)$$

Now take formula of magnitude of the acceleration as below:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Put values from equation (1) and equation (2).

$$a = \frac{\left(\frac{sv}{r}\right)}{\left(\frac{s}{v}\right)}$$

$$a = \left(\frac{sv}{r}\right)\left(\frac{v}{s}\right)$$

Therefore $a = \dfrac{v^2}{r}$ -------------- (3)

This is the magnitude of acceleration vector $\vec{a}$.

Direction of a_c:-

From the isosceles similar triangles AOB and QPR we see that:

From Figure

$PQ \perp OA$

$PR \perp OB$

So $QR \perp AB$

Thus QR is || to perpendicular bisector of AB. The acceleration of object moving in circle is || to when AB → 0, so centripetal acceleration is directed along radius towards the centre of circle.

This means $\Delta\vec{v}$ is perpendicular to $\vec{v}_1$ (or $\vec{v}_2$). Keeping in view the diagram of above c and the direction of $\Delta\vec{v}$ we can see that $\Delta\vec{v}$ at point A is radially inward. Hence direction of acceleration vector $\vec{a}$ is along the radius and towards centre of the circle. Such a vector is ca centripetal acceleration denoted by the symbol $\vec{a}_c$. Its direction is written by the unit vector $(-\hat{r})$.

If this acceleration is multiplied by the mass of particle then by Newton's second law motion this force is called centripetal force whose magnitude is written as below:

$$F_c = ma_c$$

$$\boxed{F_c = m\frac{v^2}{r}}$$

For uniform circular motion $v = r\omega$, therefore the above relation can be written as below:

$$F_c = \frac{m(r^2\omega^2)}{r}$$

$$\boxed{F_c = mr\omega^2}$$

Thus centripetal force will be:

1. Directly proportional to mass of rotating body.
2. Directly proportional to square of velocity of rotating body.
3. Inversely proportional to the radius of the circular path.

Q.8 Define and explain moment of inertia of right body and its significance. OR What is moment of inertia? Find the expression for moment of inertia of a rigid body. 11105008

Ans. Moment of Inertia:

Definition:

"It is defined as resistance in a body against any change in its state of rest or state of uniform circular motion".

Explanation:

The term inertia is applied when the body is moving in a straight line whereas the term moment of inertia is related with the circular motion. In circular motion moment of inertia plays the same role as mass in linear motion.

Mathematically the moment of inertia is defined as the product of mass of the particle and square of distance from the axis of rotation.

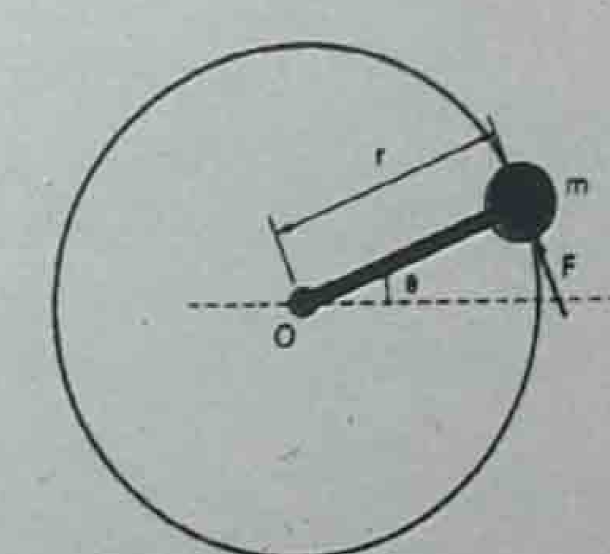
$$I = mr^2$$

S. I units are kg m^2 and its dimensions are $[ML^2]$

The moment of inertia depends upon

(i) The mass of body,

(ii) Distribution of mass of a body from axis of rotation.



Moment of inertia of a particle:

Consider a body of mass 'm' connected with the massless rod of length 'r'. The other end of the rod is pivoted at point O. Let the force of magnitude F is applied to the perpendicular of rod. By Newton's second law of motion.

$F = ma_T$ ------------------------- (1)

a_T is tangential acceleration.

Let α is the angular acceleration of the body.

$a_T = r\alpha$

$\therefore$ $F = mr\alpha$ ------------------------- (2)

Multiply 'r' on both sides

$rF = mr^2\alpha$

Where $rF = \tau =$ torque

$\tau = mr^2\alpha$

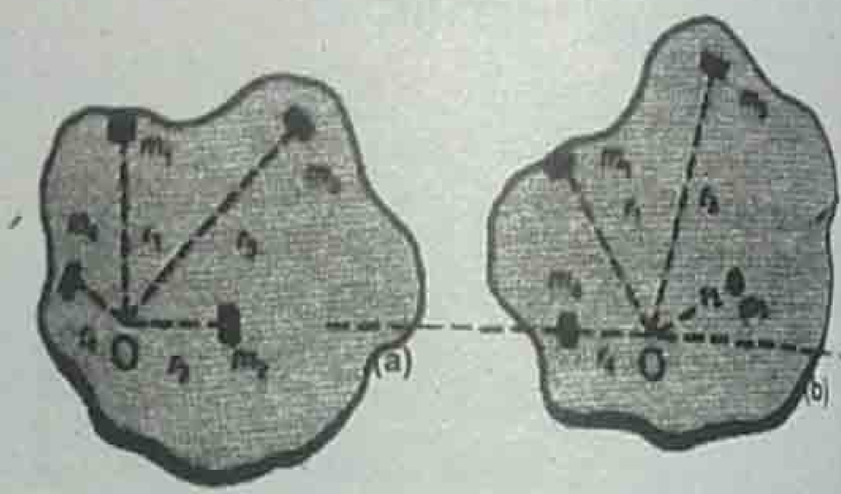
As $mr^2 = I$

$\therefore$ $\boxed{\tau = I\alpha}$

Moment of Inertia of Rigid Body

Definition of Rigid Body

"A body whose each particle is rotating with the same angular velocity 'ω' and same angular acceleration α is known as rigid body."



The moment of inertia of rigid body can be determined by dividing the rigid body which is rotating about the point O in number of small particles of masses m_1, m_2, m_3 ---------m_n.

Let the distances of these particles from the axis of rotation 'O' are r_1, r_2, ------r_n. The fo[rce] on the first particle is given by:

$$F_1 = m_1 a_1$$

Where $a_1 = r_1 \alpha_1$

Therefore $F_1 = m_1 r_1 \alpha_1$

Multiply by r_1 on both sides of above equation.

$$r_1 F_1 = m_1 r_1^2 \alpha_1$$

$$\Rightarrow \boxed{\tau_1 = m_1 r_1^2 \alpha_1}$$

Similarly the torques on other particles are written as:

$$\tau_2 = m_2 r_2^2 \alpha_1$$

$$\tau_3 = m_3 r_3^2 \alpha_3$$

$$\tau_n = m_n r_n^2 \alpha_n$$

The resultant torque on the rigid body

$$\tau = \tau_1 + \tau_2 + \tau_3 + \text{-----------------} \tau_n$$

$$\tau = m_1 r_1^2 \alpha_1 + m_2 r_2^2 \alpha_2 + m_3 r_3^2 \alpha_3 + \text{--------------} m_n r_n^2 \alpha_n$$

Because for rigid body

$$\alpha_1 = \alpha_2 = \alpha_3 = \ldots\ldots\ldots\ldots = \alpha_n = \alpha$$

$$\tau = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \text{--------------} m_n r_n^2 \alpha$$

$$\tau = (m_1 r_1^2 + m_2 r_2^2 + \text{--------} + m_n r_n^2)\, \alpha$$

$$\tau = \left(\sum_{i=1}^{i=n} m_i r_i^2 \right) \alpha$$

Where $\boxed{\sum_{i=1}^{i=n} m_i r_i^2 = I}$

I is moment of inertia of the rigid body.

$$\therefore \boxed{\tau = I\alpha}$$

Hence torque on the rigid body is always equal to the product of the moment of inertia I and angular acceleration α.

Q.9 Define and explain the term angular momentum. 11105009

Ans. Angular Momentum:

Definition:

"A particle is said to possess an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis".

The angular momentum $\vec{L}$ of a particle of mass 'm' moving with velocity $\vec{v}$ and momentum '$\vec{p}$' with respect to 'O' is defined as

$$\vec{L} = \vec{r} \times \vec{p}$$

Where $\vec{r}$ is position vector of particle with respect to point of rotation.

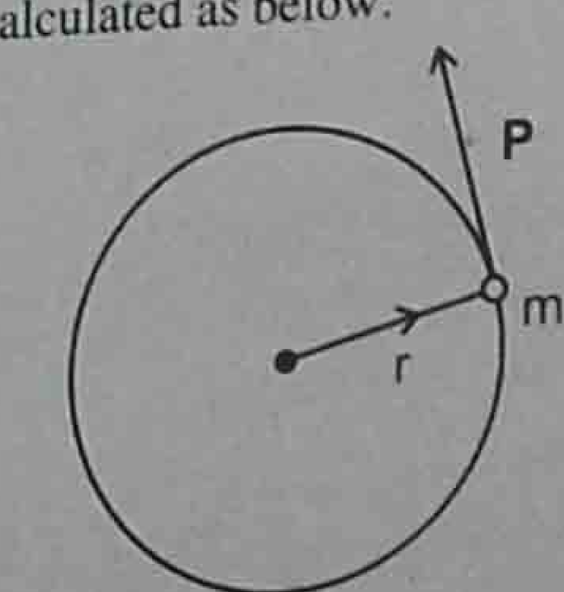
Types of Angular Momentum:

There are two types of angular momentum:

1. Spin angular momentum is due to spin motion.
2. Orbital angular momentum is due to orbital motion.

Angular Momentum of a Particle:

For a particle doing circular motion the angular momentum is calculated as below:



$$L = r\,p\,\mathrm{Sin}\theta$$

where $p = mv$

For circular motion $\theta = 90°$

$$L = rmv\,\mathrm{Sin}90°$$

but $\sin 90° = 1$

$$\therefore \quad L = mvr$$

as $v = r\omega$

$$\therefore \quad L = mr^2\omega$$

Where quantity mr^2 is a moment of inertia of the particle and is represented by I, therefore, the above equation is written as

$$\boxed{L = I\omega}$$

Hence angular momentum of the particle is equal to the product of moment of inertia and angular velocity. S.I unit of angular momentum are kg m^2s^{-1} or J-sec and its dimensions are $[ML^2T^{-1}]$.

Direction of Angular Momentum:

The direction of angular momentum is perpendicular to the plane containing the vector and $\overline{p}$ determined by right hand rule.

Angular Momentum of Rigid Body consisting of N-particles:

Consider a rigid body rotating about a fixed axis. The rigid body can be imagined to be made up of 'n' small pieces of masses m_1, m_2,……m_n at perpendicular distances r_1, r_2 ………, r_n from axis of rotation respectively. Each particle of the rigid body rotates with same angular velocity 'ω' that of the rigid body itself. The angular momentum of the particle of mass 'm_1' is.

$$L_1 = m_1 r_1^2 \omega_1$$

The angular momentum of particle of mass m_2

$$L_2 = m_2 r_2^2 \omega_2$$

The angular momentum of particle of mass m_n

$$L_n = m_n r_n^2 \omega_n$$

Total Angular momentum of rigid body 'L' is given by

$$L = L_1 + L_2 + \text{------------} + L_N$$

Put values in right hand side

$$L = m_1 r_1^2 \omega_1 + m_2 r_2^2 \omega_2 + \text{------------} + m_n r_n^2 \omega_n$$

As the body is rigid so all particles are moving with same angular velocity

$$\therefore \quad \omega_1 = \omega_2 = \text{--------} = \omega_n = \omega$$

So $\quad L = (m_1 r_1^2 + m_2 r_2^2 + \text{-----------} + m_n r_n^2)\,\omega$

Where $(m_1 r_1^2 + m_2 r_2^2 + \text{-----------} + m_n r_n^2) = \sum_{i=1}^{n} m_i r_i^2 = I$ is the moment of inertia of rigid body.

Hence $\boxed{L = I\omega}$

Q.10 State and explain the law of conservation of momentum and write its few applications in sports. 11105010

Ans. Law of Conservation of Angular Momentum:

Statement:

"In the absence of an external torque the total angular momentum of system remains constant".

$$\vec{L}_{total} = \vec{L}_1 + \vec{L}_2 + \ldots\ldots\ldots\ldots + \vec{L}_n = \text{Constant}$$

For the case of circular motion the law of conservation of angular momentum can also be written as

$$I\omega = \text{Constant} \qquad \text{or} \qquad I_1\omega_1 = I_2\omega_2$$

Where I and ω are respectively the moment of inertia and angular velocity of the system.

The absence of external torque the axis of rotation remain fixed.

Explanation: (Example of Diving)

The Law of conservation of angular moment is one of the fundamental principles. It has been verified from macroscopic to submicroscopic level. The effect of the Law is readily apparent if a single isolated system alters it moment of inertia. This is illustrated by somersaults executed by the diver during his motion:

It should be noted that angular momentum is a vector quantity with direction along the axis of rotation. The direction of angular momentum along the axis of rotation also remain the same if no external torque acts on the body.

When diver jumps off, he pushes the board with small angular velocity about an horizontal axis. Upon lifting off from the board he extends his arms and legs which means it has large moment of inertia "I" but the value of ω is reduced keeping the angular momentum constant.

When Arms and legs are drawn into closed tuck position, the moment of inertia decreases and hence the value of ω must increase to keep the angular momentum constant.

Application of Law of Conservation of Momentum:

Law of conservation of angular momentum has many applications such as acrobatics performed by the conservation of direction in the absence of external torque the axis of rotates remain fixed divers, ice skaters and ballet dancers. 11105011

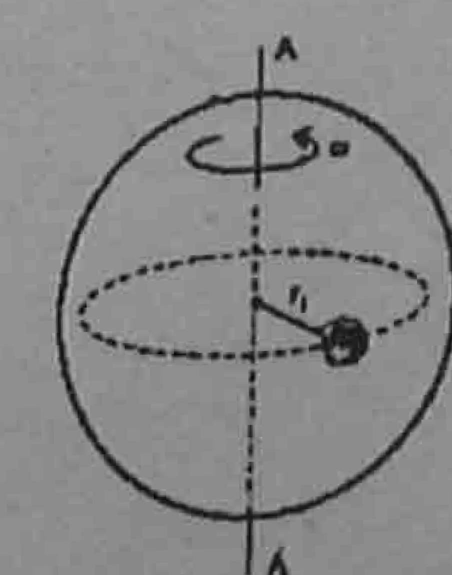
Q.11 Define Rotational K.E of rigid body. Derive its expression.

Ans. **Rotational Kinetic Energy:**

Definition:

"The energy associated with the rigid body due to its rotational motion is known as rotational kinetic energy".

Expression of Rotational Kinetic Energy:



Consider a rigid body consisting of 'n' number of particles of masses m_1, m_2, m_3,------------ m_n. The perpendicular distances of these particles are r_1, r_2, r_3 --------- r_n from the axis of rotation.

The particles are revolving with linear speeds. v_1, v_2, ……… v_n. The angular velocity of all these particles will be same"ω" but their linear speed will be different.

The K.E of first particle is:

$$(K.E)_1 = \frac{1}{2} m_1 v_1^2$$

Where

$$v_1 = r_1\omega$$

$$(K.E)_1 = \frac{1}{2} m_1 (r_1\omega)^2$$

$$(K.E)_1 = \frac{1}{2} m_1 r_1^2 \omega^2$$

Similarly

K.E of the remaining particles will be:-

$$(K.E)_2 = \frac{1}{2} m_2 r_2^2 \omega^2$$

$$(K.E)_n = \frac{1}{2} m_n r_n^2 \omega^2$$

Total K.E of rotation is:

$$(K.E)_{Rot} = (K.E)_1 + (K.E)_2 + \text{-----------------} + (K.E)_n$$

$$(K.E)_{Rot} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \text{-----------} + \frac{1}{2} m_n r_n^2 \omega^2$$

$$(K.E)_{Rot} = \frac{1}{2}\left(\sum_{i=1}^{i=n} m_i r_i^2\right)\omega^2$$

As $\left(\sum_{i=1}^{i=n} m_i r_i^2\right) = I$ = Moment of inertia of rigid body.

$$\therefore \quad \boxed{(K.E)_{Rot} = \frac{1}{2} I\omega^2}$$

This expression for kinetic energy of rotation is similar to the one that we have for line[ar] motion i.e. $\frac{1}{2} mv^2$, linear speed "v" is replaced by ω and "m" by I.

Q.12 Find the K.E of disc and hoop rolling on smooth surface of an incline plane. Using Law of conservation of energy, find their velocities at the bottom of inclined plane. 1110...

Ans. Rotational Kinetic Energy of a Disc and a Hoop:

A solid circular plate is known as disc and a hollow circular ring is known as hoop. Conside[r] a solid disc and hoop are moving on a smooth surface with constant angular velocity "ω", let "v" [be] their linear speed. When they are moving they have two types of motion:

1. Translational motion.
2. Rotational motion.

Related to these motions they have two types of kinetic energies: (i) translational K.E (ii) rotational K.E. The rotational kinetic energy of the disc is given by:

$$(K.E)_{rot} = \frac{1}{2} I\omega^2$$

For a disc the moment of inertia $I = \frac{1}{2} mr^2$.

$$\therefore \quad (K.E)_{rot} = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

$$(K.E)_{rot} = \frac{1}{4}mr^2\omega^2$$

$$(K.E)_{rot} = \frac{1}{4}m\left(r^2\omega^2\right)$$

Since $v = r\omega \quad \Rightarrow v^2 = r^2\omega^2$

$$\therefore \quad (K.E)_{rot} = \frac{1}{4}mv^2 \quad \text{-----------------------} \ (1)$$

This is the rotational kinetic energy of a rolling disc.

Rotational Kinetic Energy of Hoop (Ring):

Take $(K.E)_{rot} = \frac{1}{2} I\omega^2$

Where the moment of inertia of hoop is $I = mr^2$

$$\therefore \quad (K.E)_{rot} = \frac{1}{2}\left(mr^2\right)\omega^2$$

$$(K.E)_{rot} = \frac{1}{2}m(r^2\omega^2)$$

Since $r\omega = v \Rightarrow r^2\omega^2 = v^2$

$$\therefore \quad (K.E)_{rot} = \frac{1}{2}mv^2 \quad \text{-----------------} \ (2)$$

This is the rotational kinetic energy of a rolling hoop.

Using the conservation of energy we can find the velocity of disc and hoop moving down an inclined plane.

Consider disc and hoop at the top of an inclined plane of height "h". If no energy is lost in overcoming the friction the total potential energy at the top must be equal to the total kinetic energy at the bottom.

$$P.E = (K.E)_{tran} + (K.E)_{rot}$$

$$\text{Or} \quad mgh = \frac{1}{2}mv^2 + K.E_{rot} \quad \text{-------------------} \ (3)$$

Where "ω" and "v' are the angular and linear speeds at the bottom and "m" is the mass of the body.

Speed of Disc at the Bottom of Inclined plane:

As we know kinetic energy of rotation of disc from equation (1)

$$K.E\ rot = \frac{1}{4}mv^2$$

Putting this value in equation (3) we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{2mv^2 + mv^2}{4}$$

$$\text{or} \quad mgh = \frac{3mv^2}{4}$$

$$\text{or} \quad v^2 = \frac{4gh}{3}$$

$$\text{Hence} \quad v = \sqrt{\frac{4gh}{3}} \quad \text{------------------------- (4)}$$

This is the speed of disc at the bottom of frictionless incline of height "h".

Speed of Hoop at the Bottom of Inclined plane:

We know that rotational kinetic energy of rolling hoop is

$$K.E_{rot} = \frac{1}{2}mv^2$$

Putting in equation (3) we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$mgh = mv^2$$

$$\text{or} \quad gh = v^2$$

$$\therefore \quad \boxed{v = \sqrt{gh}} \quad \text{--------------------- (5)}$$

This is the speed of hoop at the bottom of frictionless incline of height "h".

By comparing equations (4) and (5) it is concluded that the velocity of disc at the bottom an inclined plane is greater than the velocity of hoop.

Conclusion:

When rigid bodies of different moment of inertia are allowed to roll down an inclined plane then the body of the smallest moment of inertia will have greater speed at the bottom and vice versa.

Q.13 What are Artificial Satellite? Find the expression of orbital speed of satellite orbiting very close to the earth and evaluate it. 11105013

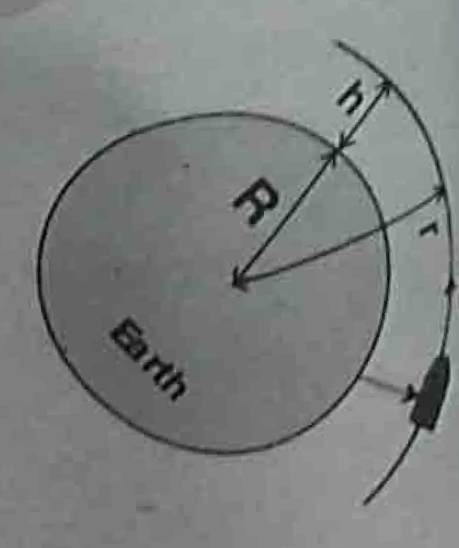
Ans. Artificial Satellite:

Definition:

"The man made objects orbiting around the earth are known as artificial satellites".

Explanation:

They are put in the orbit by rockets and gravity of the earth help the satellites to orbit around the earth. Once the satellite is placed in the orbit it will continue to move in that orbit under the gravitational force. The gravitational force between earth and satellite provide the necessary centripetal force, which is given by:



$$F_c = \frac{mv^2}{r}$$

Where m = mass of the satellite

v = orbital speed

r = radius of the orbit

Calculation of minimum velocity of satellite:

Let the satellite is orbiting around the earth and its height "h" above the earth is very small as compared to the radius of earth "R". The radius of the orbit of low flying satellite is almost equal to the radius of the earth because $r = R + h \Rightarrow r \cong R \because h << R$.

Since

$$F_c = F_g$$

$$\Rightarrow \quad \frac{mv^2}{R} = mg$$

$$v^2 = gR$$

$$v = \sqrt{gR}$$

substituting $g = 9.8\ ms^{-2}$ and $R = 6.4 \times 10^6$ m, we get

$$v = \sqrt{6.4 \times 10^6 \times 9.8}$$

$$v = 7.9 \times 10^3 \text{ m/s}$$

$$v = 7.9 \text{ km/s}$$

This is minimum velocity required to put the satellite into orbit near the earth and is called critical velocity.

Time Period:

"Time taken by the satellite to complete one revolution is known as its period of revolution".

Time 't' is given by

$$t = \frac{s}{v}$$

$$\Rightarrow \quad T = \frac{2\pi R}{v}$$

$$T = \frac{2 \times 3.14 \times 6.4 \times 10^6}{7.9 \times 10^3} = 5060 \text{ sec}$$

$$T = 84 \text{ min (approximately)}$$

The higher the satellite, the slower will be the required speed and longer it will take to complete one revolution around the earth. 11105014

Q.14 What do you mean by Global Positioning System.

Ans. Global Positioning System (GPs System):

Twenty four satellites orbiting around the earth at height of about 400 km form the Global Positioning system. A person can now use a pocket size instrument or mobile phone to find his position to an accuracy of 10 m on the earth's surface.

Q.15 Define real and apparent weight. Establish relation between real and apparent weight of body. Discuss all it cases in an elevator. 11105015

Ans. Real and Apparent Weight:

Real Weight:

The gravitational pull of the earth at a point on the body measures it real weight. It is designated by 'W'.

$$W = mg$$

Apparent Weight:

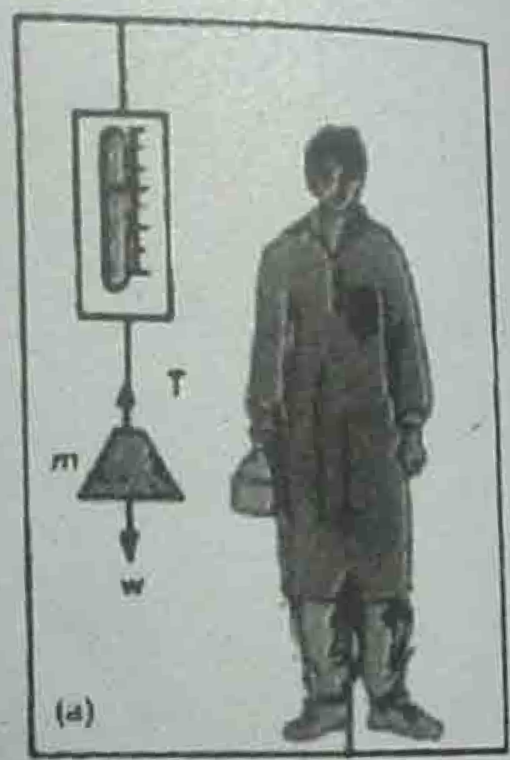
Apparent weight is equal and opposite to the force required to stop it from falling in the frame of reference. Apparent weight of the object is not equal to its true weight in accelerated frame of reference.

Special Cases:

Case – 1:

When elevator is stationary:

Let us consider frame of reference as an elevator. Consider a body of mass 'm' connected with a string whose other end is attached with the spring balance. Spring balance is supported by the ceiling of the elevator. The tension in the string is the apparent weight of the body. When elevator is at rest its acceleration will be zero and same will be the acceleration of the body.



The resultant force on the body is:

$F = T - mg$

$ma = T - mg$

as $a = 0$

$\therefore\ 0 = T - mg$

Or $T = mg$ ------------------- (1)

Hence real and apparent weights are equal in frame of reference having zero acceleration.

Case-2:

When elevator is moving upward with acceleration "a":

Now suppose that elevator moves upward with acceleration "a" as shown in fig. (b). The upward force which is the tension "T" in the string is greater than the downward force "W", the net force on the object is

$T - w = F$

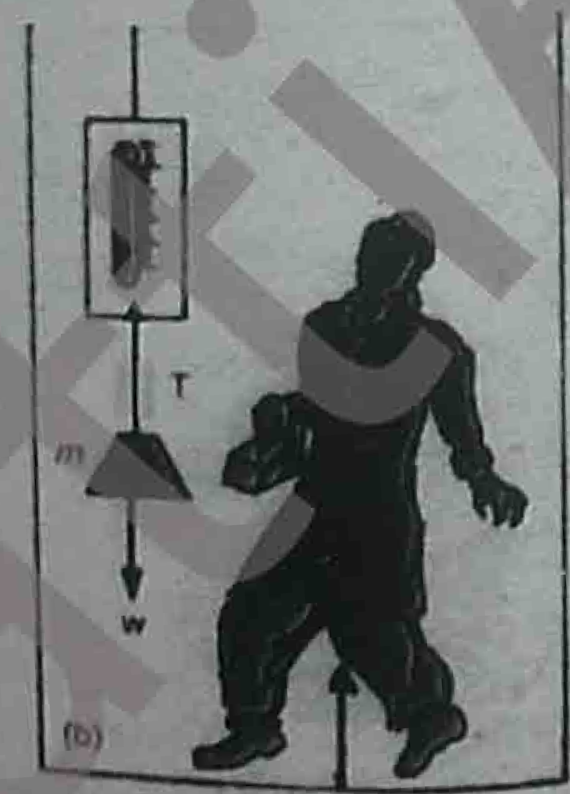
According to Newton's second law

$F = ma$

$\therefore\ T - w = ma$

Or $T = w + ma$ ------------------- (2)

This relation shows that the apparent weight "T" of the object has increased by an amount of "ma" as compare to real weight "W".



Acceleration upward

Case – 3:

When elevator is moving downward with acceleration 'a'.

Let the elevator is moving with acceleration "a" as shown in fig. (c). In this case "W" is greater than tension "T" in the string. Therefore the net force acting on the body is

$F = W - T$

as $F = ma$

$\therefore\ W - T = ma$

or $T = W - ma$ ------------------------ (3)

This relation shows that the apparent weight "T" of the object has decreased by and amount of "ma" as compare to real weight "W".

acceleration downward

Case – 4:

When elevator falls freely under gravity:

Now suppose that elevator is falling freely under the action of gravity. Then $a = g$

Substituting this value of "a" in equation (3) we get

$T = w - mg$

as $w = mg$

$\therefore\ T = mg - mg$

or $T = 0$

Hence apparent weight of the body becomes zero or we can say that the body has become weightless.

11105016

Q.16 Explain the weightlessness in satellite.

Ans. Weightlessness in Satellite and Gravity Free System:

A satellite orbiting around the earth is just like a freely falling frame of reference. The satellite do not strike the earth like the other freely falling body because curvature of its trajectory matches with earth's curvature. The centripetal force required is:

$$F_c = \frac{mv^2}{r} \text{ .... (1)}$$

This force is in fact the gravitational force on the satellite i.e. its weight

$\therefore\ F_C = w$

$\Rightarrow\ \frac{mv^2}{r} = mg$

$\frac{v^2}{r} = g$

Here $\frac{v^2}{r}$ is the centripetal acceleration "a_c" of the satellite.

Therefore the acceleration, of the satellite is equal to the acceleration due to gravity

i.e. $a_c = g$ ....(2)

Hence satellite is a freely falling frame of reference. Thus everything in satellite will appear to be weightless. So an orbiting satellite is a gravity free system. The earth satellite is a freely falling object. Its motion is similar to the motion of the projectile. When it is thrown at larger speed tangentially, then during its free fall to the earth, the curvature of the path decreases with horizontal tangentialy speed as shown in fig.

When projected at a critical velocity parallel to the earth, the curvature of the trajectory of satellite will match with the curvature of the earth. In this case the satellite will start revolving around the earth. The satellite has an acceleration equal to the acceleration due to gravity at all times when it is revolving around the earth, but the curvature of earth prevents it from hitting the earth.

Q.17 Derive an expression of orbital speed of satellite. How does it depends upon orbital radius? 11105017

Ans. Orbital Velocity:

Definition:

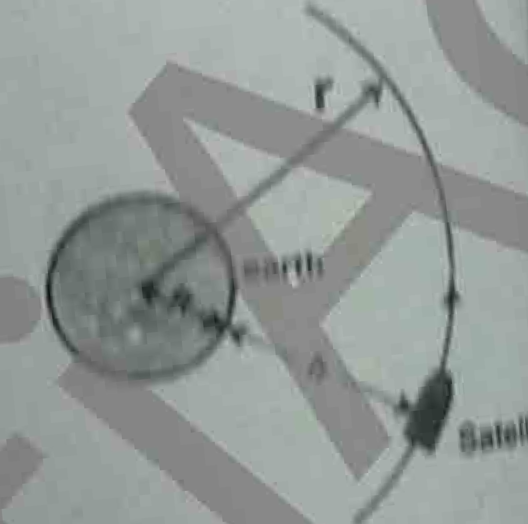
"The velocity of satellite orbiting around the earth is called its orbital velocity".

The artificial satellites when launched, will adopt nearly a circular path around earth. This type of motion is called orbital motion.

General Expression for orbital velocity:

Consider a satellite is moving around the earth in an orbit of radius 'r' with orbital speed v. The centripetal force required for keeping satellite in circular orbit is written as:

$$F_c = \frac{m_s v^2}{r}$$



The gravitational force between satellite and earth will furnish the required centripetal force.

i.e. $F_c = F_g$

or $\frac{m_s v^2}{r} = \frac{G m_s M}{r^2}$

Where "M" is the mass of the earth and "m_s" is the mass of satellite.

$$\therefore \quad v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}} \quad \text{..........} (1)$$

since $\sqrt{GM} = \text{constant}$

$$\therefore v = \text{constant} \times \frac{1}{\sqrt{r}}$$

$$\boxed{v \propto \frac{1}{\sqrt{r}}}$$

Hence orbital velocity of the satellite is inversely proportional to the square root of the radius of the orbit. When the velocity of satellite is less than as given by equation (1), it will not revolve around the earth and will fall back on earth i.e. gravitational field.

Artificial Gravity

Problem with Weightlessness

In space the weightlessness is a serious handicap for astronauts to perform their research work. In overcoming this difficulty a new situation is created in the spaceship so that the astronauts may perform experiments in normal manner as they do in earth's gravity.

How artificial gravity is produced?

The artificial gravity is produced by spinning the spaceship around its own axis. The objects and astronauts are pressed against the outer surface of the space station and exert a force on the floor of the spaceship in the same way as our feet on earth.

Expression for the Frequency of Spin

Space station is a hollow Circular Tube of large diameter revolving about a vertical axis. Let 'v' is its spinning speed, then the centripetal acceleration of astronaut in it is given by:

$$a_c = \frac{v^2}{R}$$

R is average radius of space station

as $v = \omega R$

$$\therefore \quad a_c = \frac{(\omega R)^2}{R}$$

$$a_c = \frac{\omega^2 R^2}{R}$$

$$a_c = \omega^2 R$$

$$\frac{a_c}{R} = \omega^2$$

$$\omega = \sqrt{\frac{a_c}{R}} \quad \text{..........} (1)$$

But $\omega = \frac{\theta}{t} \Rightarrow \omega = \frac{2\pi}{T}$

Where "T" is time for one revolution of the satellite or spaceship about its own axis.

Putting the value of ω in equation (1) we get

$$\frac{2\pi}{T} = \sqrt{\frac{a_c}{R}}$$

$$\Rightarrow \quad \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

If "f" is frequency of spin then $\frac{1}{T} = f$

So above equation becomes

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

This is the general formula for spin frequency to produce artificial gravity.

For earth like gravity we put $a_c = g$ in the above formula

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

It is the required frequency at which artificial gravity becomes equal to the true gravity of the earth. The astronauts and other objects inside the spaceship will have apparent weight equal to their real weight.

Q.18 What are Geostationary satellites? Derive an expression for the radius of Geo Stationary satellite. 11105018

Ans. Geo - Stationary Orbits:

Definition:

"The orbits of satellite in which its orbital motion is synchronized with the spin motion of the earth is called geo stationary orbits".

The satellites revolving in these orbits have the same angular velocity as that of the earth, and are known as synchronous satellite or geo-stationary satellite.

Explanation:

Why it appears stationary?

For geostationary satellites the orbital motion take place in same time in which earth complete its one rotation about its axis. Therefore the satellite remains always over the same point of the equator as the earth rotates around its axis. Therefore the satellite will appear stationary with respect to the point of the earth from where it is observed.

Expression for the orbital radius of geostationary satellites:

As we know that orbital speed of satellite is given by

$$v = \sqrt{\frac{GM}{R}} \text{ ---------- (1)}$$

But this speed must be equal to the average speed of the Satellite in one day i.e.

$$v = \frac{S}{t} \Rightarrow v = \frac{2\pi r}{t}$$

$$t = \frac{2\pi r}{v} \text{ ---------- (2)}$$

t = Period of revolution of Satellite = 24 Hour = 1 day

Putting (1) into (2)

$$t = \frac{2\pi r}{\sqrt{\frac{GM}{r}}}$$

$$t\sqrt{\frac{GM}{r}} = 2\pi r$$

Squaring on both sides we get

$$t^2 \frac{GM}{r} = 4\pi^2 r^2$$

$$\Rightarrow \frac{GMt^2}{4\pi^2} = r^3$$

Raising the power on both sides by $\frac{1}{3}$

$$\left[\frac{GMt^2}{4\pi^2}\right]^{\frac{1}{3}} = [r^3]^{\frac{1}{3}}$$

$$\Rightarrow r = \left[\frac{GMt^2}{4\pi^2}\right]^{\frac{1}{3}}$$

Putting

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$M = 6 \times 10^{24} \text{ kg}$$

$$t = 24 \times 3600 \text{ sec}$$

$$= 86400 \text{ sec}$$

$$\therefore \quad r = \left[\frac{6.67 \times 10^{-11} \times 10^{24} \times (86400)^2}{4 \times (3.14)^2}\right]^{\frac{1}{3}}$$

$$r = 4.23 \times 10^7 \text{ m}$$

$$\boxed{r = 4.23 \times 10^4 \text{ km}}$$

This is the value of orbital radius of geostationary satellite which may also be called geostationary radius.

Altitude of the Geostationary Orbit:

Let "h" is a height of satellite above the equator then

$$r = R + h$$

$$h = r - R$$

$$= 4.23 \times 10^4 - 6400$$

$$= 42300 - 6400$$

$$h = 35900 \text{ km}$$

or

$$\boxed{h \cong 36000 \text{ km}}$$

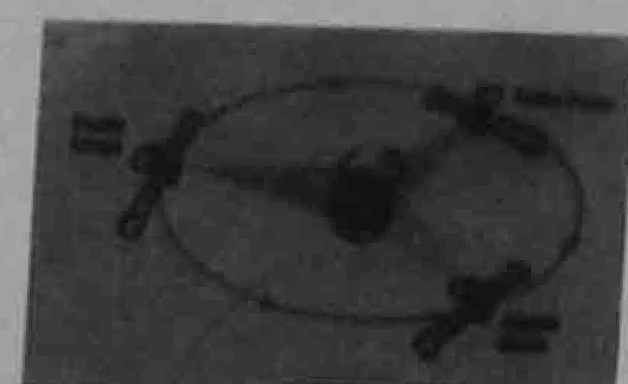
Applications of geostationary satellites:

Geostationary satellite are useful in many ways like

1. Worldwide Tele Communication.
2. Weather forecast.
3. Navigation.
4. Defense and other military purpose.

Q.19 What are communication satellites? 11105019

Ans. Communication Satellites **(Board 2009)**



A satellite communication system can be set up by placing several geostationary satellites in orbit over different points on the surface of Earth. One geostationary satellite covers 120° of longitude. So that the whole of the populated Earth surface can be covered by three correctly positioned satellites as shown in the fig.

Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the earth. The energy needed to amplify and retransmit the signals is provided by large solar panels fitted on the satellite. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellite from other countries.

The largest satellite system is managed by 126 countries' organization known as international telecommunication satellite organization (INTELSAT). *"An INTELSAT VI satellite operates at microwave frequencies of 4,6,11 and 14 GHz and has a capacity of 30,000 two way telephone circuits plus three T.V Channels".*

Q.20 Describe a brief view about Newton and Einstein theory about gravitation. W[hy] Einstein theory is considered to be most general one? 11105020

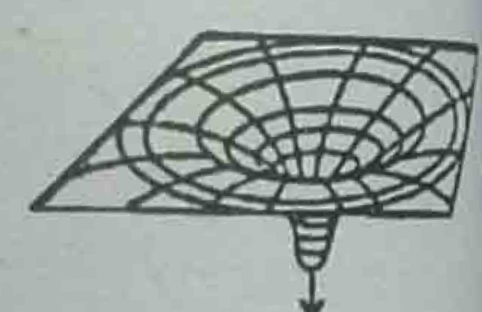
Ans. Newton's and Einstein's Views of Gravitation:

Newton's View:

According to Newton, it is the intrinsic property of matter that every particle of matter attra[cts] every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

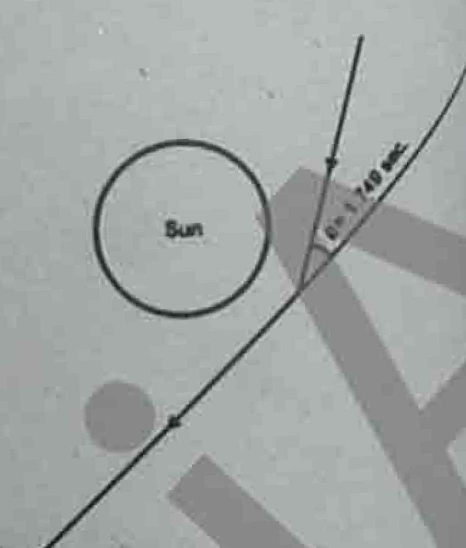
Einstein's View:

According to Einstein's theory, space time is curved, especially near the massive bodies. This [is] shown in diagram where the huge mass causes space itself to curve. Einstein says that bodies and lig[ht] rays move along geodesics in curved space time. Thus a body at rest or moving slowly near the grea[t] mass would follow a geodesic toward that body.



Weight

Einstein's theory gives us a physical picture of how gravity works. Newton discovered the inverse square law of gravity; but explicitly said that he offered no explanation of why gravity should follow an inverse square law. Einstein, theory not only says that gravity follows an inverse square law but it also tells us why this should be so.



Newton's theory, based on the idea of light as a stream of tiny particles, also suggested that a light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exactly twice as great as it is according to Newton's theory. When the bending of star light caused by the gravity of the Sun was measured during a solar eclipse in 1919, it was found to match with Einstein's prediction rather than Newton's. Due to these reasons Einstein theory is more comprehensive than Newton's theory.

Short Questions

5.1: Explain the difference between tangential velocity and angular velocity. If one of these is given for a wheel of known radius, how will you find the other? **(Board 2010)** 11105021

Ans. Tangential Velocity:

When the body is revolving in a circular path its linear velocity is always along the tangent at any point on the circle. This velocity is known as its tangential velocity. S.I units are $m\,s^{-1}$.

Angular velocity:

Angular displacement per unit time is known as angular velocity. It is along the axis of rotation. Its S.I units are $rad\,s^{-1}$.

Since $v = r\omega$

So by using this relation we can find the other if one of "v" or "ω" is given for a wheel of known radius "r".

5.2: Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path? **(Board 2009)** 11105022

Ans. When a body is moving in a circle with constant speed, the force, which keeps the body moving in a circular path, is always directed toward the centre of the circle. This force is known as centripetal force. The magnitude of this force is given by the following formula:

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

If this force is not furnished then particle will not follow a circular path, and may go along tangential path.

5.3: What is meant by moment of Inertia? Explain its significance: **(Board 2010)** 11105023

Ans. Moment of Inertia (I):

Product of mass "m" and square of its perpendicular distance "r" from the axis of rotation is called moment of inertia. It depends upon total mass of the body and distribution of this mass with respect to the axis of rotation.

That is $I = mr^2$

Its S.I units are $kg\,m^2$ and its dimensions are $[ML^2]$.

Significance of moment of Inertia:

The moment of inertia plays the same role in circular motion as the mass 'm' do in translational motion. It is in fact the resistance of rotating body against any change in its rotational motion.

5.4: What is meant by angular momentum? Explain the law of conservation of angular momentum. 11105024

Ans. Angular momentum:

A particle is said to posses angular momentum about a reference axis if it moves so that its angular position changes relative to that reference axis.

The value of angular momentum of a rotating body is defined as:

$$\vec{L} = \vec{r} \times \vec{p}$$

The law of conservation of angular momentum states that the total angular momentum of the system remains constant in the absence of an external torque.

Significance:

Law of conservation of angular momentum is a universal law. It is important in many sports like diving, gymnastics, ice-skating etc.

5.5: Show that angular momentum $L_o = mvr$. 11105025

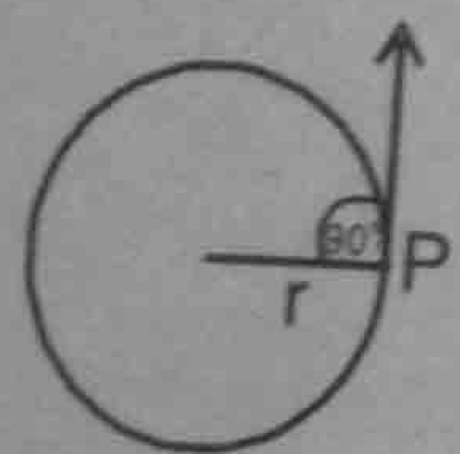
Ans. By definition of angular momentum we have

$$\vec{L}_0 = \vec{r} \times \vec{p}$$

If θ is angle between $\vec{r}$ and $\vec{p}$ then magnitude of this angular momentum is

$$L_o = rp\sin\theta$$

In circular motion $\theta = 90°$



therefore $L_o = rmv \sin 90°$

But $\sin 90° = 1$

Therefore $\boxed{L_o = mvr}$

5.6: Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it. (Board 2010) 11105026

Ans. Consider a Satellite moving in a circular orbit of radius "r". The centripetal acceleration of the satellite is given by

$$a_c = \frac{v^2}{r}$$

Close to the surface of Earth the value of this centripetal acceleration is in fact equal to "g".

Hence $g = \frac{v^2}{R}$

$$v = \sqrt{gR}$$

Put values

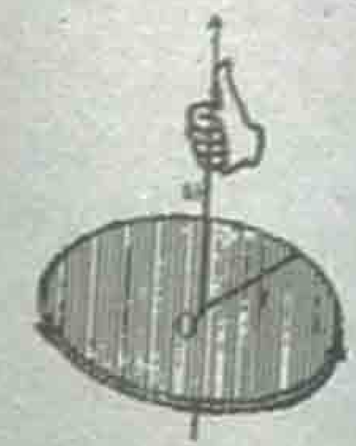
$$v = \sqrt{9.8 \times 64 \times 10^6}$$

$$v = 7.9 \text{ km s}^{-1}$$

This is the minimum velocity necessary to put a Satellite into an orbit close to the Earth and is also called critical velocity of the satellite.

5.7: State the direction of the following vectors in simple situations, angular momentum and angular velocity. 11105027

Ans. The directions of vectors of angular velocity and angular momentum by right hand rule is along the axis of rotation like those for (i) rotating car wheel (2) spinning top. (iii) revolving merry- go-round.



The direction of angular velocity "$\vec{\omega}$" and angular momentum $\vec{L}$ by right hand rule can be found as:

"Curl the fingers of the right hand around the rotation axis in the direction of rotation, the thumb points towards the direction of angular velocity $\vec{\omega}$ and the angular momentum $\vec{L}$ shown in figure.

5.8: Explain why an object orbiting the earth is said to be freely falling. Use your explanation to point out why objects appear with weightless under certain circumstances. 111050..

Ans. When an object is orbiting around the earth then the centripetal force required for keeping it in circular path is supplied by its weight.

i.e. $\frac{mv^2}{r} = mg$

$\therefore \frac{v^2}{r} = g$

as $a_c = \frac{v^2}{r}$

$\therefore a_c = g$

Therefore an object revolving around the earth behaves like a freely falling object.

The circumstances in which the revolving object is an enclosure like a space station then the bodies present in that revolving enclosure will be become weightless because of the acceleration equal to "g" towards the centre of orbit that is towards the centre of Earth.

5.9: When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain. 11105029

Ans. The mud flies off the tyre of a moving bicycle along the tangent to the tyre.

Explanation:

When tyre rotates then centripetal force is provided by adhesive force present between the tyre and mud. This adhesive force has certain maximum limiting value. When speed of the bicycle is increased, then centripetal force is increased and when its value becomes larger than the maximum limiting value of adhesive force then required centripetal force is not available to the mud particles. So the mud flies off the tyre of a moving bicycle along the tangent to the tyre.

5.10: A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom? 11105030

Ans. The velocities of disc and hoop reaching the bottom of the inclined plane are

$$V_{disc} = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4}{3}}\sqrt{gh} \quad \text{-------------- (1)}$$

$$V_{hoop} = \sqrt{gh} \quad \text{-------------- (2)}$$

Dividing (1) by (2)

$$\frac{V_{disc}}{V_{hoop}} = \frac{\sqrt{\frac{4}{3}}\sqrt{gh}}{\sqrt{gh}}$$

$$V_{disc} = \sqrt{\frac{4}{3}} V_{hoop}$$

$$V_{disc} = 1.15\, V_{hoop}$$

$$V_{disc} > V_{hoop}$$

Hence disc will be moving faster on reaching the bottom of inclined plane.

5.11: Why does a diver change his body positions before and after diving in the pool? (Board 2009) 11105031

Ans. The diver changes his body position to change his moment of inertia I, so that he can change his angular velocity. For this purpose when he takes off from the diving board he extends his arms and legs in order to have larger moment of inertia. This will decrease his angular velocity. When he pulls his legs and arms close to one another his moment of inertia is reduced. This will result in an increase in his angular velocity, so as to conserve angular momentum.

$$I\,\omega = \text{constant}$$

It shows large I implies a small ω and small I implies large ω. Hence large ω enables the diver to make more somersaults.

5.12: A student holds two dumb-bells with stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumb-bells towards his chest. What will be the effect on rate of rotation? 11105032

Ans. Due to this effect his moment of inertia "I_1" in position shown in fig (a) is greater than his moment of inertia "I_2" in position shown in fig (b). By law of conservation of angular momentum.

$$I_1\omega_i = I_2\omega_f$$

Where ω_i and ω_f are the angular velocities in initial and final position shown in diagram.

since $I_1 > I_2$

$\therefore \omega_i > \omega_f$

So the rate of rotation of the student will increase.

5.13: Explain how many minimum numbers of geo-stationary satellites are required for global coverage of T.V. transmission? 11105033

Ans. For the global coverage of T.V. Transmission, minimum three geo-stationary satellites are required. Since each of them cover 120° of longitude, so the whole populated Earth can be covered by the proper positioning of these three satellites to cover total of 360° of longitudes of the Earth. Presently these three geo-stationary satellites are located above Atlantic, Pacific, and Indian oceans.

Solved Examples

Example 1: An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest? 11105034

Solution: In this problem we have

$\omega_i = 3.0 \text{ rev s}^{-1}, \omega f = 0,$ $\quad t = 18.0$ s and $\alpha = ?$ $\quad \theta = ?$

As we know that

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{(0-3.0)\,\text{revs}^{-1}}{18.0\text{s}} = -0.167\text{revs}^{-2}$$

And from Eq 5.11, we have

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$= 3.0\text{revs}^{-1} \times 18.0\text{s} + \frac{1}{2}\left(-0.167\text{revs}^{-2}\right)\times(18.0\text{s})^2 = 27\text{rev}$$

Example 2: A 1000 kg car is turing round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of the circular path is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path? 11105035

Solution: The force required is the centripetal force.

So

$$F_c = \frac{mv^2}{r} = \frac{1000\text{kg}\times 100\text{m}2\text{s}^{-2}}{10\text{m}} = 1.0\times10^4\,\text{kgms}^{-2} = 1.0\times10^4\,\text{N}$$

This force must be supplied by the frictional force of the pavement on the wheels.

Example 3: A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. 5.7. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point? 11105036

Solution: For the ball to travel in a circle, the force acting on the ball must provide the required centripetal force. In this case, at point A, two forces act on the ball, the pull of the sting and the weight w of the ball. These forces act along the radius at A, and so their vector sum must furnish the required centripetal force. We, therefore, have

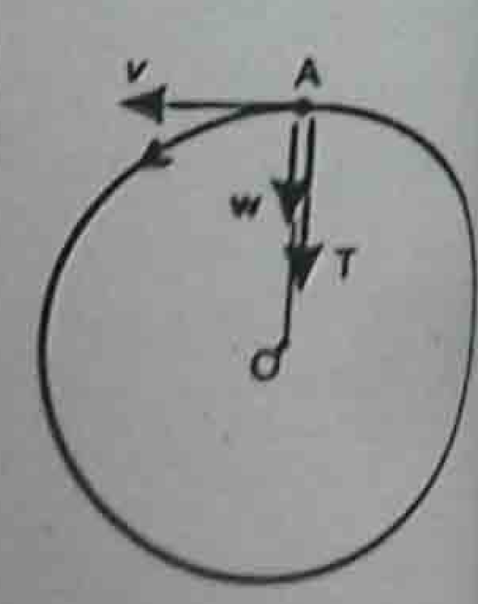


Fig. 5.7

$$T + w = \frac{mv^2}{r} \qquad \text{as} \quad w = mg$$

$$\therefore \quad T = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

If $\frac{v^2}{r} = g$, then T will be zero and the centripetal force is just equal to the weight.

Example 4: The mass of Earth is 6.00×10^{24} kg. The distance *r* from Earth to the Sun is 1.50×10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7s). 11105037

Solution: To find the Earth's orbital angular momentum we must first know its orbital speed from the given data. When the Earth moves around a circle of radius r, it travels a distance of $2\pi r$ in one year, its orbital speed v_o is thus

$$v_o = \frac{2\pi r}{T}$$

Orbital angular momentum of the Earth $= L_o = mv_o r$

$$= \frac{2\pi r^2 m}{T}$$

$$= \frac{2\pi(1.50\times10^{11}\text{m})^2\times(6.00\times10^{24}\text{kg})}{3.16\times10^7\text{s}}$$

$$= 2.67\times10^{40}\text{kg m}^2\text{s}^{-1}$$

The sign is positive because the revolution is counter clockwise.

Example 5: A disc without slipping rolls down a hill of height 10.0 m. If the disc starts from rest at the top of the hill, what is its speed at the bottom? 11105038

Solution: Using Eq.

$$v = \sqrt{\frac{4gh}{3}}$$

$$= \sqrt{\frac{4\times 9.80\text{ms}^{-2}\times 10.0\text{m}}{3}} = 11.4\text{ ms}^{-1}$$

Example 6: An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth M = 6.0×10^{24}kg and its radius R = 6400 km. 11105039

Solution:

As $\quad r = R + h = (6400 + 384000) = 390400$ km

Using $\quad v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\text{ Nm}^2\text{Kg}^{-2}\times 6\times10^{24}\text{ kg}}{390400\text{km}}}$

Also $\quad = 1.01\text{ kms}^{-1}$

$$T = \frac{2\pi r}{v} = 2\times3.14\times390400\text{ km}\times\frac{1}{1.01\text{ kms}^{-1}}\times\frac{1\text{ day}}{60\times60\times24\text{s}}$$

$= 27.5$ days

Example 7: Radio and TV signals bounce from a synchronous satellite. This satellite ... the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays ... the same spot on the Earth because the Earth is rotating at the same rate. (a) What ... orbital radius for a synchronous satellite? (b) What is its speed? 1110...

Solution:

From the Eq. $r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$

Where $G = 6.67\times10^{-11}\ N\,m^2\,kg^{-2}$

$M = 6.0\times10^{24} kg$

and $T = 24\times60\times60s$

Therefore, on substitution, we get

a) $r = \left(\frac{6.67\times10^{-11}\ Nm^2\,kg^{-2}\times6.0\times10^{24}\ kg\times(24\times60\times60s)^2}{4(3.14)^2}\right)^{1/3}$

$= 4.23\times10^7 m$

b) Substituting the value of r in equation $v = \frac{2\pi r}{T}$

We get,

$v = \frac{2\pi(4.23\times10^7\ m)}{86400s} = 3.1\ kms^{-1}$

Numerical Problems

5.1: A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of moon from the earth is 3.8×10^8m. 11105041

Solution:

Data:

Diameter = S = 2.50m

Divergence angle = θ = ?

Distance $r = 3.8\times10^8 m$

To calculate?

Divergence angle = θ = ?

Formula:

$S = r\theta$

$\theta = \frac{S}{r}$

$\theta = \frac{2.5}{3.8\times10^8} = 6.6\times10^{-9}$ rad.

5.2: A gramophone record turnt... accelerates from rest to an angular velo... of $45.0 rev\ min^{-1}$ in 1.60s. What is its aver... angular acceleration? 1110504...

Solution:

Data:

$\omega_i = 0$

$\omega_f = 45$ rev/sec

$= \frac{45}{60}\times2\pi$ rad/sec

$\omega_f = 4.7$ rad/sec

$t = 1.60$ sec

$\alpha = ?$

Formula:

Since $\alpha = \frac{\omega_f - \omega_i}{t}$

$\alpha = \frac{4.7 - 0}{1.60}$

$\boxed{\alpha = 2.95\ rad\ s^{-2}}$

5.3: A body of moment of inertia I=0.80kg m² about a fixed axis, rotates with a constant angular velocity of 100rad s⁻¹. Calculate its angular momentum L and torque to sustain this motion. 11105043

Solution:

$I = 0.80\ kg\ m^2$

$\omega = 100\ rad\ s^{-1}$

$\alpha = 0$

$L = ?$

$\tau = ?$

Formula:

Since $L = I\omega$

$\therefore L = 0.8\times100$

$\boxed{L = 80\ Js}$

As $\tau = I\times\alpha$

Since ω = constant

$\therefore \alpha = 0$

so $\tau = 0.80\times0$

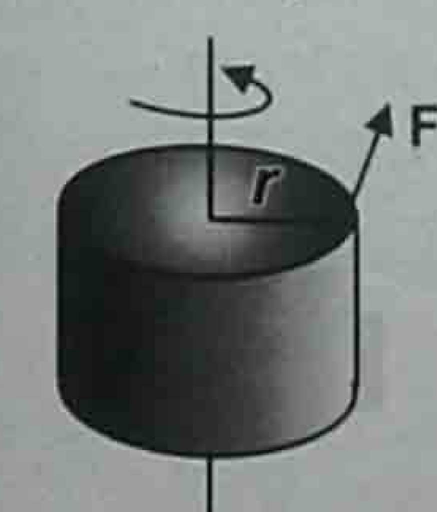
$\boxed{\tau = 0}$

5.4: Consider the rotating cylinder shown in Fig. Suppose that m = 5.0kg, F = 0.60N and r = 0.20m. Calculate:

(a) The torque acting on the cylinder.

(b) The angular acceleration of the cylinder. 11105044

(Moment of inertia of cylinder = $\frac{1}{2}mr^2$)



Data:

$m = 0.5kg$

$F = 0.60N$

$r = 0.2m$

$\tau = ?$

$\alpha = ?$

Formula:

$I = \frac{1}{2}mr^2$

$m = 5.0\ kg$

$r = 0.20\ m$

$F = 0.60\ N$

Formula:

$|\vec{\tau}| = |r\times F| = rF\sin\theta$

$\tau = 0.6\times0.2\times\sin90^\circ$

$\boxed{\tau = 0.12\ Nm}$

Now $I = \frac{1}{2}mr^2$

$= \frac{1}{2}\times5.0\times(0.02)^2$

$= 0.1 kg\ m^2$

Therefore

$\tau = I\alpha$

$\Rightarrow \alpha = \frac{\tau}{I} = \frac{0.12}{0.1}$

$\boxed{\alpha = 1.2\ rad\ s^{-2}}$

5.5: Calculate the angular momentum of a star of mass 2.0×10^{30}kg and radius 7.0×10^5km. If it makes one complete rotation about its axis once in 20 days, what is its K.E? (Board 2010) 11105045

Solution:

Data:

$r = 7.0\times10^5\ km$

$r = 7.0\times10^8\ m$

$m = 2.0\times10^{30}\ kg$

T = 20 days

$= 20\times24\times3600$ s

$= 1728\times10^3$ s

$T = 1.728\times10^6$ s

L = ?

$(K.E)_{rot} = ?$

Formula:

Since

$L = I\omega$ ------- (1)

Where

$I = \frac{2}{5}mr^2$

$= \frac{2}{5}\times2.0\times10^{30}\times(7.0\times10^8)^2$

$= \frac{2\times2\times49\times10^{46}}{5}$

$$I = 39.2 \times 10^{46}\ \text{kg.m}^2 \text{ -------(2)}$$

Now $\omega = \dfrac{\theta}{t}$

$\Rightarrow \quad \omega = \dfrac{2\pi}{T}$

$\therefore \quad \omega = \dfrac{2\pi}{1.728 \times 10^{6}}$

$= \dfrac{2 \times 3.14}{1.726 \times 10^{6}}$

$= \dfrac{6.28}{1.726} \times 10^{-6}$

$\omega = 3.63 \times 10^{-6}\ \text{rad s}^{-1}$ ------ (3)

Put values from equations (2) and (3) in equation (1) we get

$L = 39.2 \times 10^{46} \times 3.63 \times 10^{-6}$

$= 142.2 \times 10^{40}$

$\boxed{L = 1.42 \times 10^{42}\ \text{J s}}$

Now for K.E of rotation, we take

$K.E = \dfrac{1}{2} I\omega^2$

Put values from equations (2) and (3)

$K.E = \dfrac{1}{2} \times 39.2 \times 10^{46} (3.63 \times 10^{-6})^2$

$= \dfrac{1}{2} \times 39.2 \times 10^{46} \times 13.17 \times 10^{-12}$

$= \dfrac{1}{2} \times 39.2 \times 13.17 \times 10^{34}$

$= \dfrac{1}{2} \times 516.26 \times 10^{34}$

$= 258.13 \times 10^{34}$

$\boxed{K.E = 2.5 \times 10^{36}\ \text{J}}$

5.6: A 1000kg car travelling with a speed of 144 km h^{-1} rounds a curve of radius 100m. Find the necessary centripetal force. 11105046

Data:

$m = 1000\text{kg}$

$r = 100\text{m}$

$v = 144\text{km h}^{-1} = \dfrac{144 \times 1000}{3600}\ \text{m s}^{-1}$

$v = 40\text{m s}^{-1}$

$F_c = ?$

Formula:

Take $F_c = \dfrac{mv^2}{r}$

Put values

$F_c = \dfrac{1000 \times 1600}{100}$

$F_c = 16000\text{N}$

$\boxed{F_c = 1.6 \times 10^4}$

5.7: What is the least speed at which aeroplane can execute a vertical loop 1.0km radius so that there will be tendency for the pilot to fall down at highest point? 11105047

Solution:

Data:

$R = 1\text{km} = 10^3\text{m}$

$v = ?$

$F_c = W$

Solution:

There will be no tendency for the pilot to f down provided its weight furnishes required centripetal force.

$\therefore \quad \dfrac{mv^2}{r} = mg$

$v^2 = rg$

$v = \sqrt{rg}$

$= \sqrt{10^3 \times 9.8}$

$\boxed{v = 99\ \text{m s}^{-1}}$

5.8: The moon orbits the Earth so that th same side always faces the Ear Determine the ratio of its spin angul momentum (about its own axis) and orbital angular momentum. (In this cas treat the Moon as a particle orbiting th Earth). Distance between the Earth and th Moon is 3.85×10^8m. Radius of the Moon 1.74×10^6m. 11105048

Data:

Radius of the Moon = $R = 1.74 \times 10^6$m

Distance of the Moon from the Earth

$r = 3.85 \times 10^8$ m

$\dfrac{L_s}{L_o} = ?$

Solution:

Since $L_s = I\omega$

$\therefore \quad L_s = \dfrac{2}{5} MR^2\omega$ ------ (1)

Now $L_o = Mvr$

But $v = r\omega$

Therefore

$L_o = Mvr$

$= M\omega r.r$

$L_o = M\omega r^2$ --------------- (2)

Divide equation (1) by equation (2) we get

$\dfrac{L_s}{L_o} = \dfrac{\frac{2}{5} MR^2\omega}{M\omega r^2}$

$\dfrac{L_s}{L_o} = \dfrac{\frac{2}{5} R^2}{r^2}$

Put values in right hand side.

$\dfrac{L_s}{L_o} = \dfrac{2}{5}\left(\dfrac{1.74 \times 10^6}{3.85 \times 10^8}\right)^2$

$= \dfrac{2}{5}(0.45 \times 10^{-2})^2$

$= \dfrac{2}{5} \times 0.2025 \times 10^{-4}$

$= 0.081 \times 10^{-4}$

$\boxed{\dfrac{L_s}{L_o} = 8.1 \times 10^{-6}}$

5.9: The earth rotates on its axis once a day. Suppose by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then? (For Sphere I = 2/5 MR^2). 11105049

Data:

For sphere = $I = \dfrac{2}{5} MR^2$

Let:-

R = initial radius

$\therefore \quad R' = \text{final radius} = \dfrac{R}{2}$

ω = initial angular velocity

ω' = final angular velocity

Solution:

By Law of conservation of angular momentum.

$I\omega = I'\omega'$

$\Rightarrow \quad \dfrac{2}{5} MR^2\omega = \dfrac{2}{5} MR'^2\omega'$

$\dfrac{R^2}{R'^2} = \dfrac{\omega'}{\omega}$

$\dfrac{\omega'}{\omega} = \dfrac{R^2}{\left(\frac{R}{2}\right)^2}$

$\dfrac{\omega'}{\omega} = \dfrac{4R^2}{R^2}$

$\boxed{\omega' = 4\omega}$

Angular velocity of the Earth become 4 time of its present value.

As $\omega' = \dfrac{2\pi}{T'}$ and $\omega = \dfrac{2\pi}{T}$

Where T and T$'$ are the period of rotation of the earth before and after contraction, therefore above equation can be written as

$\dfrac{2\pi}{T'} = 4 \times \dfrac{2\pi}{T}$

$\therefore \quad T' = \dfrac{T}{4}$

As $\quad T = 24$ hrs

$\therefore \quad T' = \dfrac{24}{4}$

$\boxed{T' = 6\ \text{hrs}}$

That is the earth completes its one rotation about its axis in 6 hours instead of 24 hours.

5.10: What should be the orbiting speed to launch a satellite in a circular orbit 900km above the surface of the Earth? (Take mass of the earth as 6.0×10^{24} and its radius as 6400km). 11105050

Data:

$M = 6 \times 10^{24}$ kg

$R = 6400$ km

$h = 900$ km

$G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$

$r = R + h$

$= 6400 + 900$

$= 7300$ km

$= 7300 \times 10^3$ m

$r = 7.3 \times 10^6$ m

Formula:

The orbital speed of the satellite around the earth is given by

$$v=\sqrt{\frac{GM}{r}}$$

Where 'r' is the total distance from the centre of earth.

$$r=R+r_e$$

$$r=6400+900=7300km$$

$$r=7300\times10^3 m=7.3\times10^6$$

Substituting all the values we get

$$v=\sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{7.3\times10^6}}$$

$$v=\sqrt{\frac{4.002\times10^{13}}{7.3\times10^6}}$$

$$v=\sqrt{54.8\times10^6}$$

$$v=7.4\times10^3 m/s$$

$$\boxed{v=7.4\ km/s}$$

Unit 06 FLUID DYNAMICS

Q.1 Define the terms fluid dynamics, fluid and viscosity. 11106001

Ans. Fluid Dynamics:

"Fluid dynamics is the study of the behaviour of the fluids when they are moving".

Upto moderate velocities of the fluid, the fluid can be imagined in the form of thin layers. When fluid is flowing then it means its one layer is sliding over its other layer.

Fluids:

"The substances having the property to flow are known as fluids." The fluids are classified into: i. Gases ii. Liquids

Viscosity:

"Viscosity is a measure of the force needed to slide one layer of the fluid over its other layer".

Substances that do not flow easily like honey and thick tar have greater viscosity, and the substances that flow easily like water, petrol and air have smaller viscosity.

Q.2 Define Drag Force. What are the factors of its dependence? 11106002

Ans. Drag Force or Viscous Drag:

The opposing force offered by the fluid to the motion of the body moving in that fluid is known as drag force or viscous drag.

Factors of Dependence of Drag Force:

Drag force depends on the following factors:

1. **Viscosity of the fluid (η):**
   Greater is the viscosity; greater will be the drag force.
2. **Velocity of the body (v):**
   Greater is the velocity of the body in the fluid, greater is the drag force.
3. **Size of the body:**
   Greater size implies greater drag force and vice-versa.

Q.3 State the Stoke's law. 11106003

Ans. Stoke's Law:

"For a spherical body of radius 'r' moving slowly with velocity 'v' in a fluid of viscosity 'η' the drag force on the spherical body is given by the following formula

$$F=6\pi\eta rv$$

This equation is known as Stoke's Law because it was formulated by Stoke."

It may be remembered that at high speeds the drag force is not proportional to speed of the body.