DISCRETE PROBABILITY DISTRIBUTION

This is a distribution which takes on only specific integer values.

Summary:

1. **Random variables:** These are numerical outcomes of an experiment associated with probabilities.
2. A table which associates each outcome with its probability is called a probability distribution.
3. A function that assigns probabilities is called a probability density function **(p·d·f).**
4. The probability that the random variable **X** takes the value **x** is written as **P(X = x).**
5. A discrete r·v **X** with p·d·f **P(X = x)** is such that**:**

   (i) The sum of all probabilities is **1.**

   $$\Rightarrow \quad \sum_{all\ x} P(X = x) = 1.$$

   (ii) Expectation, $E(X) = \sum_{all\ x} xP(X = x).$

   (iii) Variance, $Var(X) = E(X^2) - E^2(X).$

   where $E(X^2) = \sum_{all\ x} x^2 P(X = x), \qquad E^2(X) = [E(X)]^2.$

   (iv) Standard deviation $\sigma = \sqrt{variance}$

6. For a discrete r·v **X** and constants **a** and **b,**

(i) $E(a) = a$	$Var(a) = 0$
(ii) $E(aX) = aE(X)$	$Var(aX) = a^2Var(X)$
(iii) $E(aX + b) = aE(X) + b$	$Var(aX + b) = a^2Var(X)$

7. Mode is the value of **x** with the highest probability. There can be more than one mode.

8. The graph of the p·d·f **P(X = x)** is illustrated by vertical lines.

EXAMPLES:

1. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} 2^x k & , \quad x = 0, 1, 2, --- 6 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of **k.**

(ii) $P(X > 2)$

(iii) $P(1 < X < 3)$

(iv) $P(2 \leq X < 5)$

(v) $P(X = 2/X \geq 2)$

(vi) $P(X < 4/X > 1)$

(vii) $E(X)$

(viii) $Var(X)$

(ix) the standard deviation of **X.**

(x) E(4X – 3)

(xi) Var(4X – 3)

(xii) the mode of **X.**

[Ans: (i) $\frac{1}{127}$ **(ii)** $\frac{120}{127}$ **(iii)** $\frac{4}{127}$ **(iv)** $\frac{28}{127}$ **(v)** $\frac{1}{31}$ **(vi)** $\frac{3}{31}$ **(vii)** $\frac{642}{127}$

(viii) 1·61114 (ix) 1·26931 (x) $\frac{2187}{127}$ **(xi) 25·77824 (xii) 6]**

2.The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} \left(3/4\right)^{x} k & , \quad x = 0, 1, 2, 3, ---- \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of **k.**

(ii) $P(X > 4)$

[Ans: (i) $\frac{1}{4}$ **(ii)** $\frac{243}{1024}$ **]**

3. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} kx & , \quad x = 1, 2, ---- n \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Show that**:**

(i) $k = \dfrac{2}{n(n+1)}$.

(ii) the mean of **X** is $\dfrac{2n+1}{3}$.

(ii) the variance of **X** is $\dfrac{(n-1)(n+2)}{18}$.

(b) Given that the variance of **X** is **6,** find the value of **K.**

[Ans: (b) $\dfrac{1}{55}$**]**

4. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} k(n - x) & , \quad x = 1, 2, ---- n \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Show that**:**

(i) $k = \dfrac{2}{n(n-1)}$.

(ii) the mean of **X** is $\dfrac{n+1}{3}$.

(ii) the variance of **X** is $\dfrac{(n+1)(n-2)}{18}$.

(b) Given that **n = 5,** find $P(X \geq 2)$

[Ans: (b) $0 \cdot 6$**]**

5. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} \dfrac{1}{48}\log_{10} \beta^{-x} & , \quad x = 1, 2, 3 \\ \dfrac{x}{40}\log_{10}\lambda & , \quad x = 4, 5, 6 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Given that $P(2 \le X < 5) = \dfrac{49}{120}$, find the**:**

(i) values of β and λ

(ii) standard deviation of **X.**

(iii) $P(X < 5/X > 2)$

[Ans: (i) $\beta = 0{\cdot}001$, $\lambda = 100$ **(ii)** $1{\cdot}4476$ **(iii)** $\dfrac{13}{35}$ **]**

6. A discrete **r·v X** has the following probability distribution**.**

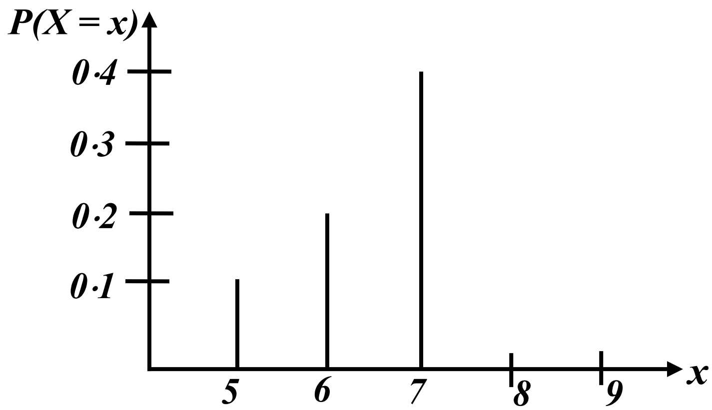
x	1	2	3	4
P(X = x)	0·2	0·1	0·4	0·3

(i) Sketch the probability distribution of **X.**

(ii) Obtain the probability distribution of **Y** such that $Y = \frac{1}{2}X(X + 2)$.

(iii) Find the mean and standard deviation of **Y.**

7. A r·v X takes values $5, 6, 7, 8, 9$ and its probability distribution is represented by the following incomplete vertical line graph.



(a) Given that $\mathbf{P(X = 8) = 2P(X = 9)}$, copy and complete the line graph.

(b) Find:

(i) $\mathbf{P(X \leq 8 / X \geq 6)}$

(ii) the mean of X

(iii) the variance of X.

8. A family plans to have **3** children**.** Given that **X** is the number of boys in the family**.**

(i) Write down the possible sample space and obtain the probability distribution.

(ii) Find the expected value and the standard deviation for the distribution in **(i)** above**.**

9. A team of **3** players is to be chosen from **4 boys** and **5 girls.** Find the**:**

(i) probability distribution for the number of girls in the team**.**

(ii) expected value and the standard deviation for the distribution in **(i)** above**.**

10. The chance of any one face of a thrown biased tetrahedral dice showing up is proportional to the number on it. If two such tetrahedral dice are thrown, determine the:

(i) probability that the faces show the same number or a sum greater than **5.**

(ii) probability distribution for the sum of the two numbers that show up. Hence state the most likely sum.

(iii) mean and standard deviation for the distribution in **(ii)** above .

11. Three balls are drawn at random without replacement from a bag containing **3** white and **5** red balls. Find the**:**

(i) most likely number of white balls drawn

(ii) expected number of white balls drawn.

[Ans: (i) 1 (ii) $\mathbf{\frac{9}{8}}$ **]**

12. A box contains **4red** and **3 blue** balls**.** A ball is drawn at random without replacement until a blue ball is drawn**.** Given that **X** represents the number of draws required to draw a blue ball**,** Find the**:**

(i) probability distribution of **X.**

(ii) mode, mean and variance of **X.**

13. Box **X** contains **4 red** and **3 green** sweets**.** Box **Y** contains **5 red** and **6 green** sweets. Box **X** is twice as likely to be picked as box **Y.** If a box is chosen at random and two sweets are removed from it**,** one at a time without replacement**.** Find the probability that the two sweets removed are of the same colour**.** Hence obtain the**:**

(i) probability table for the number of red sweets removed**.**

(ii) mean number of red sweets removed.

14. A shop sells electric bulbs coloured white, red and blue at **sh 200, 300** and **400** respectively**.** In stock the ratio of white, red and blue bulbs is **3 : 5 : 2** respectively**.** If **10** bulbs are randomly sold and the total sales being **T** shillings, calculate the expected value and standard deviation of **T.**

15. The chance of student **A, B** and **C** to watch a certain film is $\mathbf{\frac{1}{4}}$, $\mathbf{\frac{1}{3}}$ and $\mathbf{\frac{2}{5}}$ respectively. If they make independent decisions, construct a probability distribution for the number of students who watch the film. Hence obtain the mean for the distribution.

16. A question consists of two parts **A** and **B,** and the probability of a student getting part **A** correct is $\mathbf{\frac{2}{3}}$. If he gets **A** correct, the probability of getting **B** correct is $\mathbf{\frac{3}{4}}$, otherwise it is $\mathbf{\frac{1}{6}}$. There are three marks for a correct solution to part **A,** two marks for part **B** and a bonus mark if both parts are correct. Calculate the expected value and variance of the student's total mark for the question.

CUMULATIVE DISTRIBUTION FUNCTION *F(x)*

This function gives the accumulated probability up to **x.** It is obtained by summing up probabilities.

The cumulative distribution function is sometimes known as a distribution function and is denoted by $\mathbf{F(x) = P(X \le x)}$**.**

PROPERTIES OF *F(x)*

(i) The graph of **F(x)** is illustrated by horizontal lines.

(ii) The median, **m,** is the value of **x** corresponding to a cumulative probability of at least **0·5.**

$\Rightarrow$ If **m** is the median, then $\mathbf{F(m) \ge 0 \cdot 5}$**.**

(iii) The probability distribution **P(X = x)** can be obtained by subtracting the cumulative probabilities

EXAMPLES:

1. The p·d·f of a discrete **r·v X** is as follows**:**

$$\mathbf{P(X = x) = \begin{cases} \frac{1}{\beta}\binom{4}{x}\binom{6}{3-x} & , \quad x = 0, 1, 2, 3 \\ 0 & , \quad \text{otherwise} \end{cases}}$$

Find the**:**

(i) value of $\boldsymbol{\beta}$

(ii) cumulative distribution function of **X** and sketch it.

(iii) median of **X.**

2. The distribution function of a discrete r·v **X** is as follows**:**

X	**1**	**2**	**3**	**4**	**5**
F(x)	**0·2**	**0·32**	**0·67**	**0·9**	**1**

Find**:**

(i) $\mathbf{P(1 < X \le 3)}$

(ii) $\mathbf{P(1 < X < 4/X \ge 2)}$

(iii) the mean, median and mode of **X.**

(iv) Var **(3X)**

3. The cumulative distribution of a discrete r·v **X** is as follows:

$$\mathbf{F(x)} = \begin{cases} \mathbf{0} & , \quad \mathbf{x < 1} \\ \dfrac{\mathbf{(x + k)^2}}{\mathbf{36}} & , \quad \mathbf{x = 1, 2, 3, 4} \\ \mathbf{1} & , \quad \mathbf{x \ge 4} \end{cases}$$

Find**:**

(i) the value of **k** where **k > 0.**

(ii) the probability distribution of X.

(iii) $\mathbf{P(2 \le X < 4/X \ge 3)}$

(iii) Var **(3X)**

EER:

1. A discrete r·v **X** has the following probability distribution.

X	**1**	**2**	**3**	**4**
P(X = x)	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$

Find the mean and variance of **X.**

[Ans: $\frac{7}{3}$, $\frac{19}{18}$]

2. Three fair coins are tossed together. Find the**:**

(i) probability distribution for the number of heads obtained

(ii) mean and variance of the distribution in **(i)** above

[Ans: (ii) 1·5, 0·75]

3. A discrete r·v **X** has the following probability distribution.

X	**1**	**2**	**3**	**4**
P(X = x)	**0·45**	**0·1**	**0·2**	**k**

Find**:**

(i) the value of **k**

(ii) $P(2 \le X < 4)$

(iii) $P(X > 2)$

(iv) $P(1 < X < 3)$

(v) $P(X = 2/X \ge 2)$

(vi) $P(1 < X < 4/X \ge 2)$

(vii) the mode, mean, variance and median of **X.**

[Ans: (i) 0·25 (ii) 0·3 (iii) 0·45 (iv) 0·1
(v) $\frac{2}{11}$ (vi) $\frac{6}{11}$ (vii) 1, 2·25, 1·5875, 2]

4. The distribution function of a discrete r·v **X** is as follows**:**

x	1	2	3	4
F(x)	0·14	0·47	0·79	1

Find**:**

(i) $\mathbf{P(1 < X \le 3)}$

(ii) the mean, median and mode of **X.**

(iii) Var **(3X)**

[Ans: (i) 0·25 (ii) 2·6, 3, 2 (iii) 8·46]

5. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} \frac{1}{\beta}\binom{3}{x} & , \quad x = 0, 1, 2, 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of β

(ii) E(X)

(iii) E(2X + 6)

[Ans: (i) 8 (ii) 1·5 (iii) 9]

6. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X=x)=\begin{cases} 3^{-x}\lambda &, \quad x=1,2,3,\text{------} \\ 0 &, \quad \text{otherwise} \end{cases}$$

Find the**:**

(i) value of λ

(ii) $p(x<5/X\geq 3)$

[Ans: (i) 2 (ii) $\frac{8}{9}$]

7. A family plans to have **3** children**.** Given that **X** is the number of boys in the family**.**

(i) Construct the probability distribution of **X.**

(ii) Find the expected value and standard deviation of **X.**

[Ans: (ii) 1·5, 0·8660]

8. A random variable **X** has the following probability distribution**:**

P(X = 0) = P(X = 1) = P(X = 2) = a, P(X = 3) = P(X = 4) = P(X = 5) = b

and $P(X\geq 2) = 3P(X<2)$. Find**:**

(i) the values of **a** and **b.**

(ii) the mean, variance and median of **X.**

(iii) $P(X \geq 1/X < 4)$

[Ans: (i) $\frac{1}{8}$, $\frac{5}{24}$ (ii) 2·875, 2·7760, 3 (iii) $\frac{11}{14}$]

9. A discrete r·v **X** has the following p·d·f

$$\mathbf{P(X = x) = \begin{cases} kx & , \quad x = 1, 2, ---- n \\ 0 & , \quad \text{otherwise} \end{cases}}$$

Given that **E(X) = 7,** find**:**

(i) the values of **k** and **n.**

(ii) $\mathbf{P(2 < X < 7/X \geq 4)}$

[Ans: (i) 10, $\frac{1}{55}$ (ii) $\frac{15}{49}$]

10. A team of **3** players is to be chosen at random without replacement from **4 boys** and **5 girls.**

(i) Construct the probability distribution for the number of girls in the team**.**

(ii) Find the mode, mean and median for the distribution in **(i)** above**.**

[Ans: (ii) 2, $\frac{5}{3}$, 2]

11. Three balls are drawn at random without replacement from a bag containing **3** white and **5** red balls. The number of white balls drawn **X** were recorded as follows**:**

X	**0**	**1**	**2**	**3**
P(X = x)	$\frac{5}{28}$		$\frac{15}{56}$	

Copy and complete the above table**.** Hence find the

(i) expected number of white balls drawn.

(ii) probability of obtaining at least **2** white balls**.**

[Ans: (i) $\frac{9}{8}$ (ii) $\frac{2}{7}$]

12. Box **P** contains **3** white and **4** blue balls, while box **Q** contains **5** white and **3** blue balls. A ball is drawn at random from **P** and put into **Q,** and then a ball is taken from **Q** and put into **P.** Find the**:**

(i) probability that each box now contains the same number of balls of each colour as it did initially.

(ii) probability distribution for the number of white balls contained now in box **P.**

(iii) mean and variance for the distribution in **(i)** above

[Ans: (i) $\frac{34}{63}$ (iii) $\frac{200}{63}$, 0·4298]

13. A fair dice is thrown **3** times and the number of sixes **X** thrown were recorded as follows**:**

X	**0**	**1**	**2**	**3**
P(X = x)		**$^{25}/_{72}$**		

(i) Copy and complete the above table**.**

(ii) Find the mode and mean of **X.**

[Ans: (ii) 0, 0·5]

14. A random variable **X** has the following probability distribution**:**

P(X = 0) = P(X = 1) = 0·1, P(X = 2) = 0·2 and **P(X = 3) = P(X = 4) = 0·3.**

Find the**:**

(i) mean of **X**

(ii) variance of **X**

[Ans: (i) 2·6 (ii) 1·64]

15. A discrete r·v **X** has the following p·d·f.

$$P(X = x) = \begin{cases} 2P^{X} & , \quad x = 1, 2, 3, ------ \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of **P.**

(ii) $P(x < 5/X \geq 3)$

[Ans: (i) $\frac{1}{3}$ **(ii)** $\frac{8}{9}$**]**

16. The chances of a certain plant having **3, 4, 5** and **6** leaves are **0·4, 0·2, 0·3** and **0·1** respectively. Find the expected value and variance of the number of leaves on such a plant.

[Ans: 4·1, 1·09]

17. The cumulative distribution of a discrete r·v **X** is as follows:

$$F(x) = \begin{cases} 0 & , \quad x < 1 \\ \dfrac{(x + k)^2}{16} & , \quad x = 1, 2, 3 \\ 1 & , \quad x \geq 3 \end{cases}$$

Find the**:**

(i) value of **k** where **k > 0.**

(ii) probability distribution of **X.**

(iii) mean and variance of **X.**

[Ans: (i) 1 (ii) $\frac{35}{16}, \frac{167}{256}$ **]**

18. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} kx & , \quad x = 1, 2, ---- n \\ 0 & , \quad \text{otherwise} \end{cases}$$

Given that $P(X < 3) = \frac{1}{7}$, find the values of **k** and **n.**

[Ans: $\frac{1}{21}$, **6]**

19. A discrete **r·v X** has the following probability distribution**.**

x	1	2	3	4
P(X = x)	0·2	a	0·4	b

Given that the mean of **X** is **2·8,** find the**:**

(i) values of **a** and **b**

(ii) mode and median of **X**

(iii) probability distribution of **Y** where **Y = X(X −1)**

(iv) mean and standard deviation of **Y.**

[Ans: (i) 0·1, 0·3 (ii) 3, 3 (iv) 6·2, 4·4227]

20. The p·d·f of a discrete **r·v X** is as follows**:**

$$\mathbf{P(X = x) = \begin{cases} kx & , \quad x = 1, 2, ---- n \\ 0 & , \quad \text{otherwise} \end{cases}}$$

(a) Show that**:**

(i) $\mathbf{k = \dfrac{2}{n(n+1)}}$**.**

(ii) the mean of **X** is $\mathbf{\dfrac{2n+1}{3}}$**.**

(ii) the variance of **X** is $\mathbf{\dfrac{(n-1)(n+2)}{18}}$**.**

(b) Given that the variance of **X** is **6,** find the value of **K.**

[Ans: (b) $\mathbf{\dfrac{1}{55}}$**]**

21. A discrete r·v **X** takes the values **0, 1, 2,** and **3** only. Given that the mean of **X** is **1·4, P(X < 2) = 0·5** and $\mathbf{P(X \le 2) = 0 \cdot 9}$**,** find**:**

(i) P(X = 1)

(ii) P(X = 0)

[Ans: (i) 0·3 (ii) 0·2]

22. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X=x)=\begin{cases} \frac{1}{n} & , \quad x=1,2,----n \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Show that**:**

(i) the mean of **X** is $\frac{n+1}{2}$**.**

(ii) the variance of **X** is $\frac{n^2-1}{12}$**.**

(b) Given that the mean of **X** is **2·5,** find the value of **n.**

[Ans: (b) 4]

23. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X=x)=\begin{cases} \frac{x}{30}\log_{10}\beta & , \quad x=1,2,3,4,5 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of β.

(ii) $P(X<4/X\geq 2)$

[Ans: (i) 100 (ii) $\frac{5}{14}$]

24. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X=x)=\begin{cases} \frac{1+x}{kx} & , \quad x=1,2,----6 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of **k.**

(ii) the mean and standard deviation of **X.**

(iii) $P(X < 5/X \geq 3)$

[Ans: (i) $\frac{169}{20}$ **(ii)** $\frac{540}{169}$, **1·7449** **(iii)** $\frac{155}{297}$ **]**

25. A discrete r·v **X** has the following probability distribution**:**

P(1) = β, **P(2) = 2**β, **P(3) = 3**β, **P(4) = 4**β, and **P(5) = 5**β.

Find the**:**

(i) value of β

(ii) standard deviation of **X**

[Ans: (i) $\frac{1}{15}$ **(ii) 1·2472]**

26. A discrete **r·v X** has the following probability distribution**.**

x	1	2	3	4
P(X = x)	$3/16$	$1/2$	$1/4$	$1/16$

Find the mean and standard deviation of **X.** Hence obtain the mean and variance for the distribution **Y = 8X – 2.**

[Ans: $\frac{35}{16}$, **0·8077, 15·5, 41·75]**

27. The cumulative distribution of a discrete r·v **X** is as follows:

$$F(x) = \begin{cases} 0 & , \quad x < 1 \\ \dfrac{(x + k)^2}{64} & , \quad x = 1, 2, ---- 6 \\ 1 & , \quad x \geq 6 \end{cases}$$

Find**:**

(i) the value of **k** where **k > 0.**

(ii) the probability distribution of X.

(iii) $\mathbf{P(X < 5/X \geq 3)}$

(iv) Var **(3X)**

[Ans: (i) 2 (iii) $\frac{5}{12}$ (iv) 26·7517]

28. Find the expected number of boys on a committee of **3** members selected at random without replacement from **4 boys** and **3 girls.**

[Ans: $\frac{12}{7}$]

29. A box contains **3 red** and **5 white** balls. When a red ball is picked from the box, it is returned otherwise it is not returned. If two balls are picked in succession at random from the box, find the:

(i) probability of picking at least one red ball.

(ii) probability distribution for the number of white balls drawn.

(iii) mean and variance for the distribution in **(ii)** above.

[Ans: (i) $\frac{9}{14}$ (ii) $\frac{545}{448}$, 0·4509]

30. (a) A biased tetrahedral dice with faces numbered **1** to **4** is such that, the chance of any of its faces showing up is inversely proportional to the number on it . Find the probability that a prime number or an odd number occurs.

(b) If the r·v **X** is the number that shows up on the face of the dice in **(a)** above, find the:

(i) probability distribution of **X.** Hence state the most likely score

(ii) mean and variance of **X.**

(iii) $\mathbf{P(X = 2/X < 3)}$

[Ans: (a) $\mathbf{\frac{12}{25}}$ **(b) (i) 1 (ii)** $\mathbf{\frac{48}{25}}$, $\mathbf{\frac{696}{625}}$ **]**

31. A fair coin is tossed three times**.** Given that **X** is the number of heads obtained**,**

(i) write down the possible sample space and construct the probability distribution of **X.**

(ii) find the expected value and standard deviation of **X.**

[Ans: (ii) 1·5, 0·8660]

32. Find the probability distribution of the sum of the numbers when a pair of fair dice is tossed**.** Hence determine the mean and standard deviation for the distribution

[Ans: 7, 2·4152]

33. In a family**,** it is thrice as likely to have boys as girls**.** The number of boys **X** for a family planning to have **3** children are as follows**:**

X	**0**	**1**	**2**	**3**
P(X = x)		$\mathbf{^{9}/_{64}}$		$\mathbf{^{27}/_{64}}$

Copy and complete the above table**.** Hence find**:**

(i) the variance of **X.**

(ii) $\mathbf{P(\ X < 3/X > 0)}$

[Ans: (i) 0·5625 (ii) $\mathbf{^{4}/_{7}}$**]**

34. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} \left(\frac{1}{4}\right)^{x} k & , \quad x = 0, 1, 2, 3, ---- \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of **k.**

(ii) $P(X \geq 2 / X < 6)$

[Ans: (i) $\frac{3}{4}$ **(ii)** $\frac{17}{273}$ **]**

35. The p·d·f of a discrete **r·v X** is as follows**:**

$$P(X = x) = \begin{cases} k(2x + 1) & , \quad x = 0, 1, 2, 3 \\ k(11 - 2x) & , \quad x = 4, 5 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find**:**

(i) the value of **k**

(ii) $P(X > 1 / X \leq 4)$

(iii) **E(4X + 5)**

(iv) **Var(3X + 5)**

(v) the mode and median of **X**

[Ans: (i) $\frac{1}{20}$ **(ii)** $\frac{15}{19}$ **(iii) 15·2 (iv) 23·2 (v) 3, 3]**

36. The p·d·f of a discrete **r·v X** is as follows**:**

$$\mathbf{p(X = x) = \begin{cases} \dfrac{x}{k} & , \quad x = 1, 2, --- n \\ 0 & , \quad \text{otherwise} \end{cases}}$$

Given that the mean of **X** is **3,** find**:**

(i) the values of **n** and **k**

(ii) the standard deviation of **X**

(iii) P(X > 1)

(iv) P(X = 2/X ≥ 2)

[Ans: (i) 4, 10 (ii) 1 (iii) $\frac{9}{10}$ (iv) $\frac{2}{9}$]

37. The cumulative distribution of a discrete r·v **X** is as follows:

$$\mathbf{F(x) = \begin{cases} 0 & , \quad x < 1 \\ 1 - \left[1 - \dfrac{x}{4}\right]^{x} & , \quad x = 1, 2, 3, 4 \\ 1 & , \quad x \geq 4 \end{cases}}$$

Find**:**

(i) the probability distribution of **X** and sketch it.

(ii) P(X > m), where **m** is the median

(iii) mode, mean and variance of **X**

[Ans: (ii) 0·25 (iii) 2, $\frac{21}{64}$, 0·547]

38. The cumulative distribution of a discrete r·v **X** is as follows:

$$\mathbf{F(x) = \frac{x^2}{9}, \quad x = 1, 2, 3}$$

Find the expected value and variance of **X**

[Ans: $\mathbf{\frac{22}{9}, \frac{38}{81}}$**]**

39. A random variable **X** has the following probability distribution**:**

$\mathbf{P(X=0) = \frac{1}{8}, P(X=1) = P(X=2) = \frac{3}{8}}$ and $\mathbf{P(X=3) = \frac{1}{8}}$**.** Find the**:**

(i) mean of **X.**

(ii) variance of **X.**

[Ans: (i) 1·5 (ii) 0·75]

40. Two fair dice are thrown together. Find the**:**

(i) probability distribution for the positive difference in their scores.

(ii) mean and variance of the distribution in **(i)** above

[Ans: (ii) $\mathbf{\frac{35}{18}, \quad \frac{665}{324}}$ **]**

41. A set of five cards bearing the numbers **1** to **5** respectively is shuffled. Two cards are chosen at random without replacement. Given that **X** is the absolute difference between the numbers on two cards,

(i) construct the probability distribution of **X**

(ii) find the mean and variance of **X**

[Ans: (ii) 2, 1]

42. The chance of any one face of a thrown biased tetrahedral dice showing up is proportional to the number on it.

(a) Find the probability**:**

(i) with which each of the faces **1, 2, 3,** and **4** of the dice show up.

(ii) that an odd number or prime number shows up**.**

(b) If two such tetrahedral dice are thrown, find the**:**

(i) probability that the faces show the same number or a sum greater than **5.**

(ii) probability distribution for the sum of the two numbers that show up.

Hence state the most likely sum.

(iii) mean and variance of the distribution in **b(ii)** above .

[Ans: a(i) 0·1, 0·2, 0·3, 0·4 (ii) 0·6 b(i) 0·7 (ii) 6 (iii) 6, 2]

43. Two fair tetrahedral dice are thrown together. Find the**:**

(i) probability distribution for the sum of their scores.

(ii) mean and variance of the distribution in **(i)** above

[Ans: (ii) 5, 1·75]

44. Box **P** contains **3** red balls and **1** blue ball, while box **Q** contains **1** red ball and **1** blue ball. A ball is picked out of each bag and is then placed in the other bag. Given that **X** is the number of red balls in bag **P,**

(i) construct the probability distribution of **X.**

(ii) find the expected number of red balls in bag **P.**

[Ans: (ii) 2·75]

45. A discrete r·v **X** takes the values **2, 3** and **4** only. Given that the expected value and the standard deviation of **X** is **2·9** and **0·7** respectively, find the probability distribution of **X.** Hence obtain $\mathbf{P(X \le 3)}$.

[Ans: 0·8]

46. A discrete r·v **X** has the following distribution function:

$$\mathbf{F(x) = \frac{1}{4}(x - 2), \quad x = 3, 4, 5, 6}$$

Find the expected value and variance of **X**

[Ans: 4·5, 1·25]

10. In a game, a player wins **sh 500** and losses **sh 2000** with probabilities of **0·7** and **0·3** respectively**.** Calculate the player's expected gain or loss over **6** games**.**

1. A discrete r·v **X** has the following probability distribution.

X	0	1	2	3	4
P(X = x)	0·09	0·15	0·40	0·25	0·11

Find the mean and standard deviation of the distribution.

7. Find the probability distribution for the number of girls in a team of **3** players chosen at random without replacement from a group of **4** boys and **5** girls.

Hence obtain the expected value and standard deviation for the distribution

5. (a) A r·v **X** has the following p·d·f

$$\mathbf{f(x)} = \begin{cases} \boldsymbol{\beta x} & , \quad \mathbf{x = 1, 2, 3} \\ \boldsymbol{\beta(8 - x)} & , \quad \mathbf{x = 4, 5, 6, 7} \\ \mathbf{0} & , \quad \textbf{otherwise} \end{cases}$$

Find the**:**

(i) value of $\boldsymbol{\beta}$

(ii) E(3X – 5) and Var **(3X – 5)**

(iii) $\mathbf{P(X \leq 5/X > 2)}$

(f) X is a random variable such that**:**

$$p(X = x) = \begin{cases} \dfrac{1}{\beta}\dbinom{5}{x}\dbinom{7}{6-x} & , \quad x = 0, 1, 2, 3, 4, 5 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find the**:**

(i) value of β

(ii) $P(1 < X < 4 / X \geq 2)$

(iii) mode and median of **X.**

(iv) mean and standard deviation of **X.**