ACEITEKA 2024 MARKING GUIDE
PURE MATHEMATICS

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SECTION A (40 MARKS)

1. $64x^{\frac{2}{3}} + x^{-\frac{2}{3}} = 20$

$64x^{\frac{2}{3}} + \frac{1}{x^{\frac{2}{3}}} = 20$ M_1

; let $x^{\frac{2}{3}} = m$

$64m + \frac{1}{m} = 20$

$64m^2 + 1 = 20m$

$64m^2 - 20m + 1 = 0$; solve M_1

$m = \frac{1}{4}$ and $m = \frac{1}{16}$

Either $x^{\frac{2}{3}} = 4^{-1 \times \frac{3}{2}}$

$x^{\frac{2}{3} \times \frac{3}{2}} = 4^{-\frac{3}{2}}$ M_1

$x = \frac{1}{(4^3)^{\frac{1}{2}}} = \frac{1}{8}$

or $x^{\frac{2}{3}} = \frac{1}{16}$

$x = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{(16^3)^{\frac{1}{2}}}$ M_1

$x = \frac{1}{64}$

$\therefore x = \frac{1}{8}$ and $x = \frac{1}{64}$ A_1

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(2)

$$\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{3/2}}\,dx$$

$\theta = \sin^{-1}(x/2)$

$$= \int_0^{\sqrt{3}} \frac{1}{\left(\sqrt{4-x^2}\right)^3}\,dx$$

x	0	$\sqrt{3}$
θ	0	$\pi/3$

Let $x = 2\sin\theta$, $x^2 = 4\sin^2\theta$

$dx = 2\cos\theta\, d\theta$

$$= \int_0^{\pi/3} \frac{1}{\left(\sqrt{4-4\sin^2\theta}\right)^3} \cdot 2\cos\theta\, d\theta \quad M_1$$

$$= \int_0^{\pi/3} \frac{1}{8\cos^3\theta} \cdot 2\cos\theta\, d\theta \quad M_1$$

$$= \frac{1}{4}\int_0^{\pi/3} \frac{1}{\cos^2\theta}\, d\theta$$

$$= \frac{1}{4}\int_0^{\pi/3} \sec^2\theta\, d\theta \quad M_1$$

$$= \frac{1}{4}\tan\theta \Big|_0^{\pi/3} \quad M_1$$

$$= \frac{1}{4}\tan\frac{\pi}{3} - \frac{1}{4}\tan 0$$

$$= \frac{1}{4}(\sqrt{3}) - 0$$

$$= \sqrt{3}/4 \quad A_1$$

05

(3.) $-\sqrt{5} \leq \cos x + 2\sin x \leq 5$

Consider

$\cos x + 2\sin x = R\cos(x-\alpha)$

let $\cos(x-\alpha) = \cos x\cos\alpha + \sin x \sin\alpha$

$\Rightarrow R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$ --- (i)

$R\cos(x-\alpha) = \cos x + 2\sin x$ --- (ii)

Compare (i) and (ii) M_1

$R\cos\alpha = 1$ --- (*)

$R\sin\alpha = 2$ --- (**)

(**) ÷ (*)

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{2}{1}$

$\tan\alpha = 2$

$\alpha = \tan^{-1}(2)$

$\alpha = 63.4^\circ$

$(*)^2 + (**)^2$

$(R\sin\alpha)^2 + (R\cos\alpha)^2 = 1^2 + 2^2$

$R^2 = 5$

$R = \sqrt{5}$

$\Rightarrow \cos x + 2\sin x = \sqrt{5}\cos(x - 63.4^\circ)$ M_1

Minimum

$\cos(x - 63.4^\circ) = -1$ M_1

$\text{Min} = -\sqrt{5}$

Max ; $\cos(x - 63.4^\circ) = 1$ M_1

$\text{Max} = \sqrt{5}$

$\therefore -\sqrt{5} \leq \cos x + 2\sin x \leq \sqrt{5}$ A_1 (05)

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4. $3Z + w = 9 + 11i$ --- (i)

$iw - Z = -8 - 2i$ --- (ii)

(i) + 3(ii)

$$\begin{array}{rl} & 3Z + w = 9 + 11i \\ + & -3Z + i3w = -24 - 6i \\ \hline & w + 3wi = -15 + 5i \end{array}$$ M_1

$w + 3wi = 5i - 15$

$w(3i + 1) = 5i - 15$

$w = \dfrac{5i - 15}{3i + 1}$ M_1

Rationalise the denominator.

$w = \dfrac{(5i - 15)(3i - 1)}{(3i + 1)(3i - 1)}$

$= \dfrac{-15 - 5i - 45i + 15}{-9 - 1}$

$w = \dfrac{-50i}{-10} = 5i$ M_1

from (ii)

$Z = iw + 8 + 2i$

$Z = i(5i) + 8 + 2i$

$= -5 + 8 + 2i$

$= 3 + 2i$ M_1

$\therefore$ $Z = 3 + 2i$ and $w = 5i$ A_1

05

PICFARE

5. let

$$f(x) = (x+1)\sin^{-1}(x); \quad f(0) = 0 \quad M_1$$

$$f'(x) = (x+1)\cdot\frac{1}{\sqrt{1-x^2}} + \sin^{-1}(x); \quad f'(0) = 1 \quad M_1$$

$$f''(x) = \frac{\sqrt{1-x^2} - (1+x) - x(1-x^2)^{-1/2}}{1-x^2}$$

$$f''(x) = \frac{\sqrt{1-x^2} + x(1+x)(1-x^2)^{-1/2}}{1-x^2}$$

$$f''(0) = 1 \quad M_1$$

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \cdots$$

$$(x+1)\sin^{-1}(x) = 0 + x(1) + \frac{x^2}{2}(1) + \cdots \quad M_1$$

$$= x + \frac{x^2}{2} + \cdots \quad A_1$$

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(6).

$3y^2 - 18y + 2x + 37 = 0$

$y^2 - 6y + \frac{2}{3}x + \frac{37}{3} = 0$

Complete squares

$y^2 - 6y + \frac{2}{3}x + \frac{37}{3} = 0$

$(y-3)^2 = -\frac{2}{3}x - \frac{37}{3} + (-3)^2$ M_1

$(y-3)^2 = -\frac{2}{3}x - \frac{19}{3}$

$(y-3)^2 = -\frac{2}{3}(x-5)$

$(y-k)^2 = 4a(x-h)$

(h,k) - vertex

$(h+a, k)$ - focus

$x = h-a$; Equation of the directrix

$(h,k) = (3,5)$ - vertex A_1

but $4a = -\frac{2}{3}$

$a = -\frac{1}{6}$

focus;

$(h+a, k) = (3 - \frac{1}{6}, 5) = (\frac{17}{6}, 5)$ A_1

03

ignore;

Length of the latus rectum

$2(2a+k)$

$= 2[2 \cdot -\frac{1}{6} + 5]$

$= 2[-\frac{1}{3} + 5]$

$= \frac{28}{3}$

(i) Equation of the directrix.

$x = h - a$

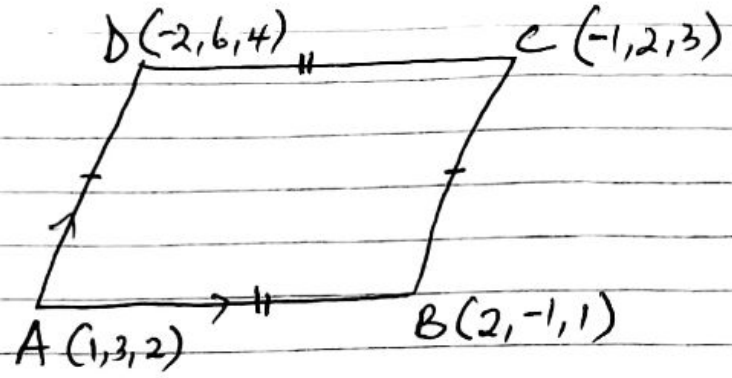
$x = 3 - -\frac{1}{6}$ M_1

$x = \frac{19}{6}$ A_1

02

05

7.



$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{AB} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix} \quad M_1$$

$$\vec{AD} = \vec{OD} - \vec{OA}$$

$$= \begin{pmatrix} -2 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix} \quad M_1$$

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} i & j & k \\ 1 & -4 & -1 \\ -3 & 3 & 2 \end{vmatrix}$$

$$= \underset{\sim}{i} \begin{vmatrix} -4 & -1 \\ 3 & 2 \end{vmatrix} - \underset{\sim}{j} \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} + \underset{\sim}{k} \begin{vmatrix} 1 & -4 \\ -3 & 3 \end{vmatrix}$$

$$= (-8 - -3)\underset{\sim}{i} - \underset{\sim}{j}(2-3) + \underset{\sim}{k}(3-12)$$

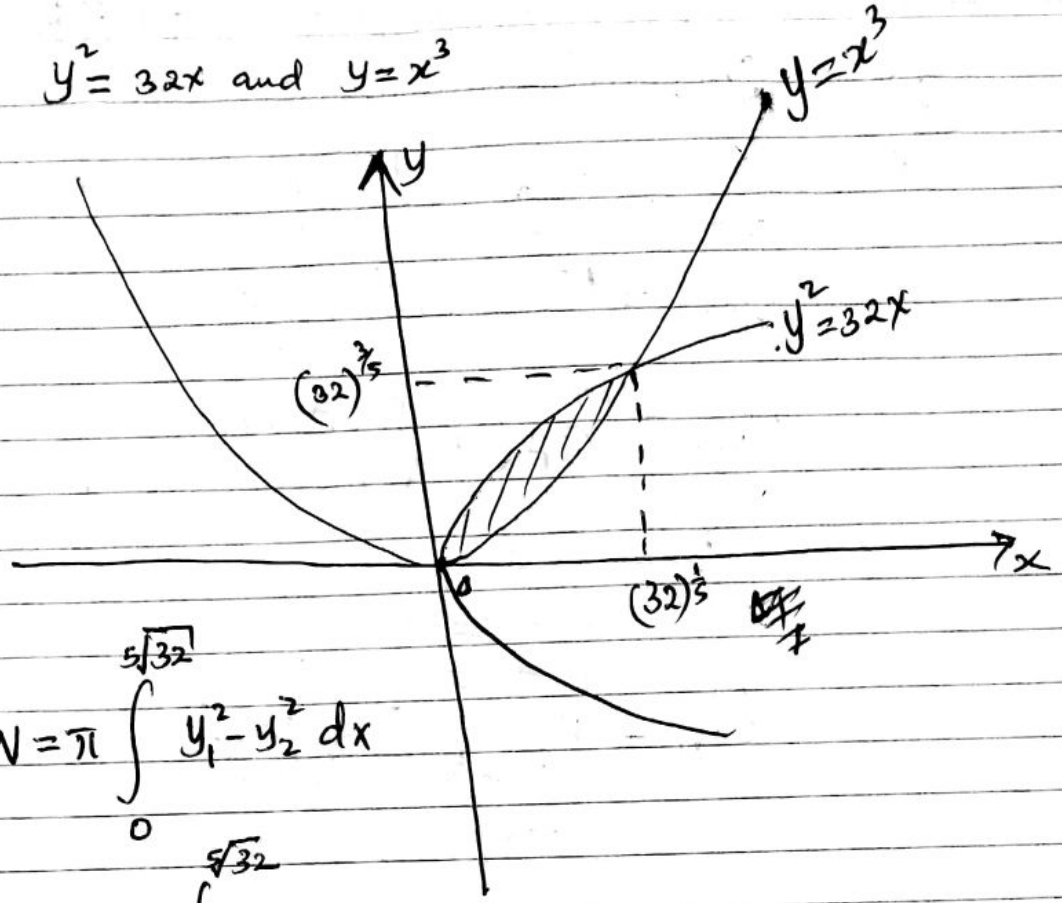
$$\vec{AB} \times \vec{AD} = -5\underset{\sim}{i} + \underset{\sim}{j} - 9\underset{\sim}{k} \quad M_1 \; M_1$$

$$|\vec{AB} \times \vec{AD}| = \sqrt{(-5)^2 + (1)^2 + (-9)^2}$$

Area $= \sqrt{107}$ square units. A_1

(05)

(8.) $y^2 = 32x$ and $y = x^3$



$$V = \pi \int_0^{\sqrt[5]{32}} y_1^2 - y_2^2 \, dx$$

$$V = \pi \int_0^{\sqrt[5]{32}} 32x - (x^3)^2 \, dx \quad M_1$$

$$= \pi \int_0^{\sqrt[5]{32}} 32x - x^6 \, dx$$

$$= \pi \left[16x^2 - \frac{x^7}{7} \right]_0^{\sqrt[5]{32}} \quad M_1$$

$$= \pi \left[\left[16(\sqrt[5]{32})^2 - \frac{1}{7}(\sqrt[5]{32})^7 \right] - (0) \right] \quad M_1$$

$$= \pi \left[64 - \frac{128}{7} \right] \quad M_1$$

$$= \left(\frac{448 - 128}{7} \right) \pi$$

$$V = \frac{320}{7} \pi \text{ Cubic units} \quad A_1$$

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SECTION B:

(Any five questions from this section)

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(a)

$$U_{r+1} = {}^{n}C_{r}\, a^{n-r} b^{r}$$

$$U_{r+1} = {}^{17}C_{r}\, (3x)^{17-r} \left(\tfrac{2}{3}\right)^{r}$$

$$U_{r+1} = {}^{17}C_{r}\, (3)^{17-r} \left(\tfrac{2}{3}\right)^{r} (x)^{17-r} \quad M_1$$

Coefficient of x^7

$$17 - r = 7$$

$$r = 10$$

$$U_{11} = {}^{17}C_{10}\, (3)^{7} \left(\tfrac{2}{3}\right)^{10} x^{7}$$

$$= 737583.4074\, x^{7} \quad M_1$$

Coefficient of x^8

$$17 - r = 8$$

$$r = 9$$

$$U_{10} = {}^{17}C_{9}\, 3^{8} \cdot \left(\tfrac{2}{3}\right)^{9} x^{8} \quad M_1$$

$$= 4148906.667\, x^{8} \quad M_1$$

$$\frac{x^7}{x^8} = \frac{737583.4074}{4148906.667} = \frac{8}{45}$$

$$x^7 : x^8 = 8 : 45 \quad A_1$$

05

9 (b) (i)

$(1+x)^{-2}$

$(x+1)^{-2} = \left(x\left[1+\frac{1}{x}\right]\right)^{-2}$

$(x+1)^{-2} = x^{-2}\left[1+\frac{1}{x}\right]^{-2}$ M_1

Recall:

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \cdots$

$\left(1+\frac{1}{x}\right)^{-2} = 1 + -2\left(\frac{1}{x}\right) + \frac{-2(-3)}{2}\left(\frac{1}{x}\right)^2 + \cdots$ M_1

$= 1 - \frac{2}{x} + \frac{3}{x^2} + \cdots$

$\left(1+\frac{1}{x}\right)^{-2} = 1 - 2x^{-1} + 3x^{-2} + \cdots$

$(1+x)^{-2} = x^{-2}\left[1 - 2x^{-1} + 3x^{-2} + \cdots\right]$ M_1

$= x^{-2} - 2x^{-3} + 3x^{-4}$ A_1

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(ii) $x = 9$

Exact value $= (1+x)^{-2}$

$= (1+9)^{-2} = 10^{-2}$

$= \frac{1}{100}$

Inexact value $= \frac{1}{x^2} - \frac{2}{x^3}$

$= \frac{1}{9^2} - \frac{2}{9^3} = \frac{1}{81} - \frac{2}{729} = \frac{7}{729}$ M_1

% error $= \frac{\frac{1}{100} - \frac{7}{729}}{\left(\frac{1}{100}\right)} \times 100\ \%$ M_1

$= 3.9781\ \%$ A_1

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07

10.

(a) $\dfrac{x^3+9x^2+28x+28}{(x+3)^2}$

$\Rightarrow \dfrac{x^3+9x^2+28x+28}{x^2+6x+9}$

$$
\begin{array}{r|l}
 & x+3 \\
\hline
x^2+6x+9 & x^3+9x^2+28x+28 \\
 & -\ x^3+6x^2+9x \\
\hline
 & 3x^2+19x+28 \\
 & -\ 3x^2+18x+27 \\
\hline
 & x+1
\end{array}
$$

$\dfrac{x^3+9x^2+28x+28}{(x+3)^2} \equiv x+3 + \dfrac{x+1}{x^2+6x+9}$ $m_1 m_2$

$\equiv x+3 + \dfrac{x+1}{(x+3)^2}$

Consider;

$\dfrac{x+1}{(x+3)^2} = \dfrac{A}{x+3} + \dfrac{B}{(x+3)^2}$ m_1

$x+1 = A(x+3) + B$

When $x=-3$

$-2 = B$

When $x=0$

$1 = 3A+B$

$1 = 3A-2$

$A=1$

$\dfrac{x+1}{(x+3)^2} = \dfrac{1}{x+3} + \dfrac{-2}{(x+3)^2}$ m_1

$\Longrightarrow \dfrac{x^3+9x^2+28x+28}{(x+3)^2} \equiv (x+3) + \dfrac{1}{x+3} - \dfrac{2}{(x+3)^2}$ A_1 06

10(b) $\displaystyle\int_0^1 \frac{x^3+9x^2+28x+28}{(x+3)^2}\,dx = \frac{1}{3}\left(10+\ln\frac{4}{3}\right)$

$$\int_0^1 \frac{x^3+9x^2+28x+28}{(x+3)^2}\,dx = \int_0^1 (x+3)+\frac{1}{x+3}-\frac{2}{(x+3)^2}\,dx$$

$$= \int_0^1 (x+3)\,dx + \int_0^1 \frac{1}{x+3}\,dx - 2\int_0^1 \frac{1}{(x+3)^2}\,dx$$

$$= \frac{x^2}{2}+3x+\ln(x+3) - 2\int_0^1 \frac{1}{(x+3)^2}\,dx \quad M_1$$

$$= \frac{x^2}{2}+3x+\ln(x+3) - 2\cdot\frac{-1}{x+3}\Big|_0^1$$

$$= \left[\frac{x^2}{2}+3x+\ln(x+3)+\frac{2}{x+3}\right]\Big|_0^1 \quad M_1\ M_1$$

$$\cancel{= \left(\frac{1}{2}+3+\ln 4\right) - (}$$

$$= \left(\frac{1}{2}+3+\ln 4+\frac{2}{4}\right) - \left(0+0+\ln 3+\frac{2}{3}\right) \quad M_1$$

$$= \frac{1}{2}+3+\frac{2}{4}-\frac{2}{3}+\ln 4-\ln 3 \quad M_1$$

$$= \frac{10}{3}+\ln\frac{4}{3}$$

06

$$= \frac{1}{3}(10)+\ln\frac{4}{3} \quad A_1$$

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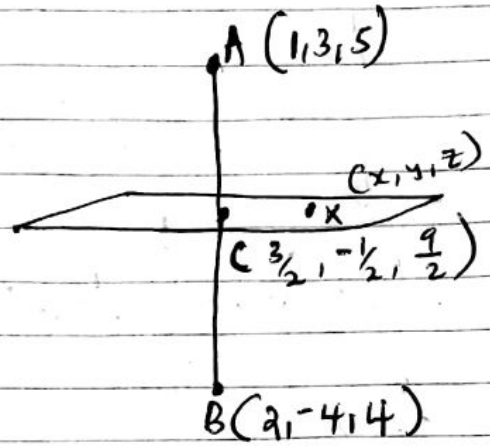
11.

(a) $A(1,3,5)$ and $B(2,-4,4)$



Plane is perpendicular to AB

Midpoint of AB $= \left(\frac{1+2}{2}, \frac{3-4}{2}, \frac{5+4}{2}\right)$

$= \left(\frac{3}{2}, -\frac{1}{2}, \frac{9}{2}\right)$ M_1

So the position vector of any point $X(x,y,z)$ that lies on the plane must satisfy the equation

$$(\vec{OX} - \vec{OC}) \cdot \vec{AB} = 0$$

$$\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3/2 \\ -1/2 \\ 9/2 \end{pmatrix}\right] \cdot \left[\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}\right] = 0 \quad M_1$$

$$\begin{pmatrix} x - 3/2 \\ y + \frac{1}{2} \\ z - 9/2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -7 \\ -1 \end{pmatrix} = 0 \quad M_1$$

$$x - 3/2 - 7y - 7/2 - z + 9/2 = 0$$

$$x - 7y - z - 1/2 = 0$$

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$$2x - 14y - 2z = 1 \quad A_1$$

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11(a) Alternatively;

Let the parametric point be $X(x, y, z)$ on the plane which is equidistant from points A and B.

Distance between $A(1,3,5)$ and $X(x,y,z)$

$$\overline{AX} = \sqrt{(x-1)^2 + (y-3)^2 + (z-5)^2} \quad M_1$$

Distance btn $B(2,-4,4)$ and $X(x,y,z)$

$$\overline{BX} = \sqrt{(x-2)^2 + (y+4)^2 + (y-4)^2} \quad M_1$$

Since $\overline{AX} = \overline{BX}$

$$\sqrt{(x-1)^2 + (y-3)^2 + (z-5)^2} = \sqrt{(x-2)^2 + (y+4)^2 + (z-4)^2} \quad M_1$$

$$x^2 - 2x + 1 + y^2 - 6y + 9 + z^2 - 10z + 25 = x^2 - 4x + 4 + y^2 + 8y + 16 + z^2 - 8z + 16$$

$$-2x - 6y - 10z + 35 = -4x + 8y - 8z + 36$$

$$2x - 4y - 2z = 1 \quad A_1$$

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11 (b) Plane

(i) $4x+5y+6z=87$

line; $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z+4}{2}$

Let $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z+4}{2}=m$

$x=2+3m,\ y=3+4m,\ z=-4+2m$ M_1

$4x+5y+6z=87$

$4(2+3m)+5(3+4m)+6(-4+2m)=87$ M_1

$8+12m+15+20m-24+12m=87$

$-1+44m=87$

$44m=88$

$m=2$ M_1

$x=2+3(2)=8$

$y=3+4(2)=11$ M_1

$z=-4+2(2)=0$

$P(x,y,z)=(8,11,0)$ A_1 05

(ii) $\underset{\sim}{n}\cdot\underset{\sim}{d}=|\underset{\sim}{n}||\underset{\sim}{d}|\sin\theta$

$\begin{pmatrix}4\\5\\6\end{pmatrix}\cdot\begin{pmatrix}3\\4\\2\end{pmatrix}=\sqrt{4^2+5^2+6^2}\cdot\sqrt{3^2+4^2+2^2}\ \sin\theta$ M_1

$12+20+12=\sqrt{77}\cdot\sqrt{29}\ \sin\theta$

$\frac{44}{\sqrt{2233}}=\frac{\sqrt{2233}}{\sqrt{2233}}\sin\theta$ M_1 03

$\theta=\sin^{-1}\left(\frac{44}{\sqrt{2233}}\right)=68.6^\circ$ A_1

12

12

(a)

$$\frac{x}{2x-1} + \left(\frac{x}{2x-1}\right)^2 + \left(\frac{x}{2x-1}\right)^3 + \cdots = \frac{3}{4}$$

The series is an G.P

Common ratio; r

$$r = \frac{x}{2x-1}$$

Sum of G.P = 3/4

$$S_n = \frac{a(r^n-1)}{r-1}, \quad U_n = a r^{n-1}$$

$$n \to \infty$$

$$S_\infty = \frac{a}{1-r}$$

Since; $S_\infty = 3/4$

$$a = \frac{x}{2x-1} \quad \text{and} \quad r = \frac{x}{2x-1}$$

$$\frac{a}{1-r} = S_\infty$$

$$\left(\frac{x}{2x-1}\right) \div \left[1 - \left(\frac{x}{2x-1}\right)\right] = 3/4 \quad M_1$$

$$\left(\frac{x}{\cancel{2x-1}}\right) \times \left(\frac{\cancel{2x-1}}{2x-1-x}\right) = 3/4$$

M_1

$$\frac{x}{x-1} = 3/4$$

$$4x = 3x - 3$$

$$x = -3 \quad A_1$$

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12 (b) $f(x) = ax^4 + 7x^3 + x^2 + bx - 3$

factors $(x-1)$ and $(x+1)$.

for $(x-1)$; $x = 1$, $f(1) = 0$

$a(1)^4 + 7(1)^3 + (1)^2 + b(1) - 3 = 0$

$a + 7 + 1 + b - 3 = 0$

$a + b = -5$ --- (i) M1

For $(x+1)$

$x + 1 = 0$; $x = -1$

$f(-1) = 0$

$f(-1) = a - 7 + 1 - b + -3 = 0$

$a - b = 9$ --- (ii) M1

eqn (i) + (ii)

$$+\left|\begin{array}{l} a + b = -5 \\ a - b = 9 \end{array}\right| \quad \text{M1}$$

$2a = 4$

$a = 2$,

from eqn (i)

$b = -5 - a$

$b = -5 - 2 = -7$

$a = 2$ and $b = -7$ A1

$f(x) \geq 0$

$f(x) = 2x^4 + 7x^3 + x^2 - 7x - 3$

$2x^4 + 7x^3 + x^2 - 7x - 3 > 0$

factor $(x-1)$ and $(x+1)$

$(x-1)(x+1) = x^2 - 1$

$$
\begin{array}{r|l}
 & 2x^2 + 7x + 3 \\
\hline
x^2 - 1 & 2x^4 + 7x^3 + x^2 - 7x - 3 \\
 & 2x^4 - 2x^2 \\
\hline
 & 7x^3 + 3x^2 - 7x - 3 \\
 & 7x^3 - 7x \\
\hline
 & 3x^2 - 3 \\
 & 3x^2 - 3 \\
\hline
\end{array}
$$

M1

$f(x) = (x-1)(x+1)(2x^2 + 7x + 3)$

$f(x) = (x-1)(x+1)(x+3)(2x+1)$ M1

$\Rightarrow (x-1)(x+1)(x+3)(2x+1) > 0$

Critical values.

$x = 1,\ x = -1,\ x = -3,\ x = -\frac{1}{2}$

$x = \{-3, -1, -\frac{1}{2}, 1\}$ M1 M1

	$x < -3$	$-3 < x < -1$	$-1 < x < -\frac{1}{2}$	$-\frac{1}{2} < x < 1$	$x > 1$
$x-1$	−	−	−	−	+
$x+1$	−	−	+	+	+
$x+3$	−	+	+	+	+
$2x+1$	−	−	−	+	+
$f(x)$	+	−	+	−	+

09

Solution set: $x < -3$, $-1 < x < -\frac{1}{2}$ and $x > 1$ A1

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(a)

General equation of a Circle

$x^2+y^2+2gx+2fy+c=0$

At; $(-2,-4)$

$4+16-4g+-8f+c=0$

$20-4g-8f+c=0$

$4g+8f-c=20$ --- (i) M_1

At $(3,1)$

$9+1+6g+2f+c=0$

$6g+2f+c=-10$ ---- (ii) M_1

At $(-2,0)$

$4+0+-4g+0+c=0$

$4g-c=4$ ---- (iii) M_1

from (iii)

$c=4g-4$ ——— (x)

(x) into (i) and (ii)

$4g+8f-(4g-4)=20$ M_1

$8f+4=20$

$8f=16$

$f=2$

Eqn (ii)

$6g+2(2)+4g-4=-10$

$6g+4+4g-4=-10$

$10g=-10$

$g=-1$

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$c = 4g - 4$

$= 4(-1) - 4$

$c = -8$

Centre

$(-g, -f) = (+1, -2)$

$(-g, -f) = (1, -2)$ M1

Radius $r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-1)^2 + (-2)^2 - -8}$

$= \sqrt{1 + 4 + 8}$

$r = \sqrt{13}$ M1

07

Circle $(x-1)^2 + (y+2)^2 = 13$ A1

Alternatively;

Since $g = -1$, $f = 2$ and $c = -8$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 - 2x + 4y - 8 = 0$

$x^2 - 2x + y^2 + 4y = 8$

$(x-1)^2 + (y+2)^2 = 8 + (-1)^2 + (2)^2$

$(x-1)^2 + (y+2)^2 = 13$

Centre $(1, -2)$

radius $= \sqrt{13}$

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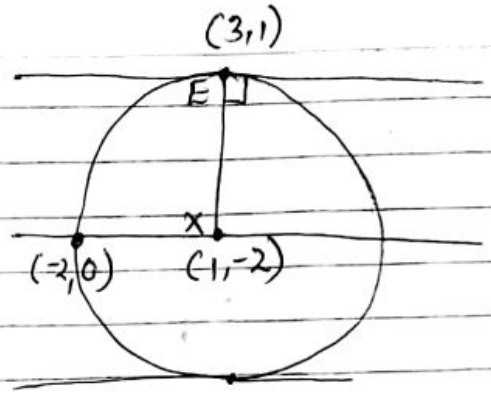
13 (b)



Equation of the normal to x to the tangent

Gradient, $m_1 = \frac{1 - -2}{3-1}$

Consider: (3,1) $m_1 = 3/2$

$3/2 = \frac{y-1}{x-3}$

$2y = 3x - 7$

$y = 3/2\,x - 7/2$

Since the normal is perpendicular to the tangent

Grad (Normal) x Grad (tangent) = -1

$m_1 \cdot m_2 = -1$

$3/2 \cdot m_2 = -1$ M_1

Gradient of tangent, $m_2 = -2/3$

Consider (3, 1)

$-2/3 = \frac{y-1}{x-3}$

$y = -2/3\,x + 3$: Equation of tangent M_1

Since (-2,0) is point on the Circle

Equation of the line (-2,0) and the Centre (1,-2)

Grad, $m = \frac{0 - -2}{-2, -1} = -2/3$

Consider point (1,-2)

$-2/3 = \frac{y - -2}{x-1}$ M_1

$3(y+2) = -2(x-1)$

$3y + 6 = -2x - 2$

$3y = -2x - 8$

$y = -\frac{2}{3}x - \frac{8}{3}$ M_1

Since their gradients are the same

$m = -\frac{2}{3}$ A_1

they are parallel. /05

Gradient of tangent = Gradient of the line $(-2,0)$ ~~and~~ to the Centre $(1,-2)$

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14 (a)

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Let $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

let $x = 45^\circ$ and $\delta x = -0.4^\circ$

$y = \cos 45^\circ$ M_1

$y = \frac{1}{\sqrt{2}}$

$y = \cos x$; $\frac{dy}{dx} = -\sin x$ M_1

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y = \frac{dy}{dx} \cdot \delta x$

$= -\sin x \cdot -0.4^\circ$

$= -\sin 45^\circ \cdot \frac{-2\pi}{900}$ M_1

$\delta y = \frac{1}{\sqrt{2}} \cdot \frac{2\pi}{900}$

$\cos(x + \delta y) = y + \delta y$

$\cos(45^\circ - 0.4^\circ) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{2\pi}{900}$ M_1

$= \frac{1}{\sqrt{2}}\left(1 + \frac{2\pi}{900}\right)$ 05

$\cos 44.6^\circ = \frac{\sqrt{2}}{2}\left(\frac{900 + 2\pi}{900}\right)$ A_1

14 (b)

$$P = 8000\left[1 - \sin(2\pi t - 3)\right]$$

$$P = 8000 - 8000\sin(2\pi t - 3)$$

$$\frac{dP}{dt} = -8000 \cdot 2\pi \cos(2\pi t - 3) \quad M_1$$

$$= -1600\pi \cos(2\pi t - 3)$$

At maximum

$$\frac{dP}{dt} = 0$$

$$-1600\pi \cos(2\pi t - 3) = 0$$

$$\cos(2\pi t - 3) = 0$$

$$2\pi t - 3 = \cos^{-1}(0) \quad M_1$$

$$\cancel{2\pi t - 3 = \pi/2, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}}$$

$$\cancel{2\pi t = \pi/2 + 3, 3/2\,\pi + 3, 5\pi/2 + 3, 7\pi/2 + 3}$$

$$2\pi t - 3 = \pi/2 \quad M_1$$

$$2\pi t = \pi/2 + 3$$

$$2\pi t = \frac{\pi + 6}{2}$$

$$t = \frac{\pi + 6}{2} \div 2\pi$$

$$t = \frac{\pi + 6}{4\pi} = \frac{\pi}{4\pi} + \frac{6}{4\pi}$$

$$t = \left(\frac{1}{4} + \frac{3}{2\pi}\right) s \quad A_1$$

$$P = 8000\left[1 - \sin\left(\cancel{2\pi} \cdot \frac{\pi + 6}{2\cancel{4\pi}} - 3\right)\right]$$

$$= 8000\left[1 - \sin\left(\frac{\pi + 6}{2} - \frac{3}{1}\right)\right] \quad M_1$$

$$= 8000\left[1 - \sin(\pi/2)\right]$$

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$= 8000\left[1 - \sin \frac{\pi}{2}\right]$ M_1

$= 8000\left[1 - 1\right]$

$P = 0\ Nm^{-2}$ A_1 07

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15 (a) $\frac{4\sin^2\theta}{\operatorname{cosec}\theta} + \frac{3}{\operatorname{cosec}^2\theta \sec\theta} = \sin^2\theta$

$4\sin^2\theta \frac{1}{\operatorname{cosec}\theta} + 3 \cdot \frac{1}{\operatorname{cosec}^2\theta} \cdot \frac{1}{\sec\theta} = \sin^2\theta$

$\frac{4\sin^2\theta \cdot \sin\theta}{\sin^2\theta} + \frac{3\sin^2\theta\cos\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta}$ M_1

$4\sin\theta + 3\cos\theta = 1$

Using R- formula

let

~~$4\sin\theta + 3\cos\theta = R\sin(\theta + x)$~~

~~$R\sin(\theta + x) = R\sin\theta\sin x + R\cos\theta$~~

$4\sin\theta + 3\cos\theta = R\sin(\theta + x)$

~~Let~~ $\Rightarrow R\sin(\theta + x) = R\sin\theta\cos x + R\cos\theta\sin x$

$R\sin\theta\cos x + R\cos\theta\sin x = 4\sin\theta + 3\cos\theta$

Compare

$R\cos x = 4$ --(i) and $R\sin x = 3$ --(ii)

(ii)/(i)

$\frac{R\sin x}{R\cos x} = \frac{3}{4}$ M_1 M_1

$\tan x = 3/4$

$x = \tan^{-1}(3/4)$

$x = 36.9°$

$(i)^2 + (ii)^2$

$(R\sin x)^2 + (R\cos x)^2 = 3^2 + 4^2$

$R^2 = 25$

$R = 5$

$4\sin\theta + 3\cos\theta = 5\sin(\theta + 36.9°)$ M_1

But $R\sin(\theta + 36.9) = 1$

$5\sin(\theta + 36.9) = 1$

$\theta + 36.9° = \sin^{-1}\left(\frac{1}{5}\right)$ M_1

$\theta + 36.9° = 11.5°, 168.5°, 371.5°, 528.5°$

$\theta = -25.4°, 131.6°, 334.6°, 491.6°$

06

$\theta = \{131.6°, 334.6°\}$ A_1

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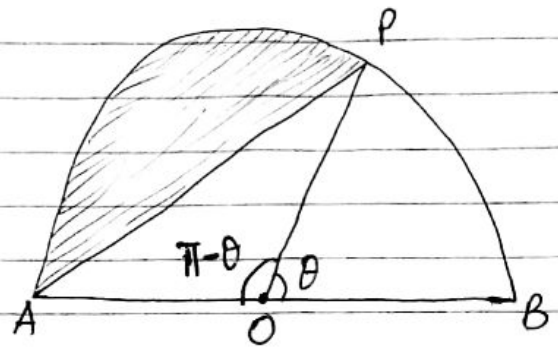
15 (b) $3\theta = 2(\pi - \sin\theta)$



Area of sector POB = 2 (Area of the shaded segment)

Area of semi circle APB $= \frac{\pi r^2}{2}$

Area of sector POB (radians) $= \frac{\theta}{2} r^2$ M_1

Area of shaded region (segment)

$$A = \frac{1}{2} r^2 \left[(\pi - \theta) - \sin(\pi - \theta) \right] \quad M_1 M_1$$

But;

$$\frac{\theta}{2} r^2 = \frac{1}{2} r^2 \left[(\pi - \theta) - \sin(\pi - \theta) \right] \quad M_1$$

$$\theta = 2\left[\pi - \theta - \sin(\pi - \theta)\right]$$

Now $\sin(\pi - \theta) = \sin\pi \cos\theta - \cos\pi \sin\theta$

$\sin(\pi - \theta) = \sin\theta$

Implying; $\theta = 2\left[\pi - \theta - \sin\theta\right]$

$$\theta = 2\pi - 2\theta - 2\sin\theta \quad M_1$$

$$3\theta = 2\pi - 2\sin\theta$$

$$3\theta = 2(\pi - \sin\theta) \quad A_1$$

Hence showed

06

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16

(a) $(x+2)^2 \frac{dy}{dx} + 3y^2 = 0$

$(x+2)^2 \frac{dy}{dx} = -3y^2$

$(x+2)^2 dy = -3y^2 dx$

$\frac{dy}{y^2} = \frac{-3dx}{(x+2)^2}$ M_1

$\int y^{-2} dy = -3 \int \frac{dx}{(x+2)^2}$

$-\frac{1}{y} = -3 \frac{-1}{(x+2)} + c$ M_1

$-\frac{1}{y} = \frac{3}{(x+2)} + c$

Taking; $y(0) = -2$.

$-\frac{1}{-2} = \frac{3}{2} + c$

$\frac{1}{2} - \frac{3}{2} = c$ M_1

$c = \frac{-2}{2} = -1$; $c = -1$

$-y^{-1} = \frac{3}{x+2} - 1$

04

$\Rightarrow -\frac{1}{y} = \frac{3}{x+2} - 1$ A_1

16 (b)

(i) $\frac{dM}{dt} \propto M$

$\frac{dM}{dt} = -kM$ M1

Separate variables.

$\int \frac{dM}{M} = \int -k\,dt$

$\ln M = -kt + c$ A1

(ii) Initially

$t=0,\ M = M_0$

$\ln M_0 = -k(0) + c$

$c = \ln M_0$

$\ln M = -kt + \ln M_0$

$\ln M - \ln M_0 = -kt$ M1

$\ln M/M_0 = -kt$

$M/M_0 = e^{-kt}$

$M = M_0 e^{-kt}$ A1

(iii) $t = 2$ hours

$t = 1$ hour 10% of radioactive material is lost.

$M_0 = 100\%$

$M = 90\%$, $t = 1$ hour

$$\frac{90}{100} = \frac{100}{100} e^{-k}$$

M1

$$0.9 = e^{-k}$$

$$\ln 0.9 = \ln e^{-k}$$

$$-k = \ln 0.9$$

$$k = -\ln 0.9$$

$$M = M_0 e^{t \ln 0.9}$$ M1

for $t = 2$ hours.

$$M = 100\, e^{2 \ln 0.9}$$ M1

$$= 100\, e^{\ln (0.9)^2}$$

$$= (0.9)^2 \times 100$$

$$M = 81\%$$ A1

08

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Alternatively; After 1 hour, $M_0 = 90\%$

$$M = M_0 e^{t \ln 0.9}$$

$$M = 90 e^{\ln 0.9}$$

$$= 90 \times 0.9$$

$$= 81\%$$

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Thank you !!!

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God bless you all

For God and My Country

@ Uganda

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