CODE HIGH SCHOOL

INTERNAL MOCK EXAMS

Uganda Advanced Certificate of Education
PURE MATHEMATICS
Paper 1
3 Hours

INSTRUCTIONS TO CANDIDATES

*Answer **all** the eight questions in section **A** and any **five** from section **B***

*Any addition question(s) answered will **not** be marked*

*All necessary working **must** be clearly shown*

Begin each answer on a fresh sheet of paper

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

*Answer **all** questions in this section*

1. Solve $\quad 4\sin 2x = 3\sin^2 x \quad$ for $-180^0 \leq x \leq 180^0$ **(5 marks)**

2. Solve for x in the equation $\log_{16}(21x - 5) = \log_4 2x$ **(5 marks)**

3. Differentiate $\sin(x^x)$ with respect to x **(5 marks)**

4. Find the term independent of x in the expansion of $\left| \dfrac{x^2}{2} - \dfrac{2}{x} \right|^9$ **(5 marks)**

5. Find the area bounded by the curve $\quad x = 2 - y - y^2 \quad$ and the $y -$ axis. **(5 marks)**

6. Points A and B are (-1, -2, 3) and (2, 1, -3) respectively. If point P divides the line AB externally in the ratio 3: 4. Find the Cartesian equation of the plane containing P and perpendicular to the line AB. **(5 marks)**

7. Find $\quad \displaystyle\int \dfrac{4x^2 + x + 1}{x^3 - 1} dx$ **(5 marks)**

8. Given that $\quad r - 6cos\theta + 4sin\theta = 0 \quad$ is a circle, find;
   - (i.) Cartesian equation of the circle.
   - (ii.) Coordinates of its Centre and the radius. **(5 marks)**

SECTION B (60 marks)

*Answer **any five** in this section*

9. (a) The polynomial $f(x)$ leaves a remainder of 3 when divided by $x + 3$ and a remainder of 7 when divided by $x - 2$. Find the remainder when $f(x)$ is divided by $x^2 + x - 6$. **(6 marks)**

   (b) The roots of the equation $16x^2 - x + 1 = 0$ are α^2 and β^2. Find the equation with integral coefficients whose roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$. **(6 marks)**

10. (a) Solve the equation $\quad cos6x + 1 = 2sin^2x \quad$ for $0^0 \leq x \leq 180^0$. **(5 marks)**

    (b) Given that $\quad f(\theta) = 4cos\theta + 3sin\theta$.

    (i) Express $f(\theta)$ in the form $R\sin(\theta + \beta)$, where R is a constant and β an acute angle.

    (ii) Determine the maximum value of $\quad 2 - f(\theta) \quad$ and the value of θ for which it occurs.

    (iii) Solve the equation $f(\theta) = \frac{5}{2}\sqrt{2} \quad$ for $0^0 \leq x \leq 360^0$. **(7 marks)**

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11. (a) Express the complex numbers $z_1 = 1 - 4i$ and $z_2 = 2 + i$ in the polar form. Hence find $z_1(z_2)^2$.

(6 marks)

(b) Find the values of x and y if $\frac{x}{3-i} - \frac{y}{5+2i} = \frac{4+17i}{17+i}$.

(6 marks)

12. (a) A line $r = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ a \\ 4 \end{pmatrix}$ is parallel to the plane $4x + 6y - 2z = 42$. Find;

(i) the value of a.

(ii) the shortest distance between the line and the plane. **(6 marks)**

(b) Find the acute angle between the line $r = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ and the plane $4x - 7y + 4z = 20$.

(6 marks)

13. (a) Find the locus of a point P which moves so that its distance from $(2,2)$ is half its distance from the line $x + y + 4 = 0$. **(6 marks)**

(b) Find the length of the tangents to the circle $x^2 + y^2 - 2x + 4y - 3 = 0$ from the centre

of the circle $x^2 + y^2 + 6x + 8y - 1 = 0$ **(6 marks)**

14. (a) Evaluate $\displaystyle\int_0^{\sqrt{3}} \frac{x^3}{\sqrt{1 + x^2}}\, dx$ **(6 marks)**

(b) Use the substitution $t = \tan \frac{1}{2} x$ to evaluate $\displaystyle\int_{\frac{1}{6}\pi}^{\pi} \frac{1}{1 - 2\sin x + \cos x}\, dx$ **(6 marks)**

15. A curve is given parametrically as $x = 2t$ and $y = \frac{1}{t} - t$.

(a.) Find the Cartesian equation of the curve. **(4 marks)**

(b.) Sketch the curve.

(c.) Find the area enclosed by the curve, x-axis and line x = 1. **(8 marks)**

16. (a) Solve the differential equation

$$\frac{y^5}{x}\frac{dy}{dx} = e^{y^3} \ln x$$ **(5 marks)**

(b) A spherical bubble is diminishing at a rate inversely proportional to its volume. After one hour, the volume halves.

(i) Form a differential equation in terms of V, t and the constant of proportionality k that mathematically describes the above situation.

(ii) When should we expect the bubble to have completely diminished? **(7 marks)**

END