P425/1
PURE MATHEMATICS
Paper 1
Nov./Dec. 2024
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

This paper consists of **two** *Sections;* **A** *and* **B**.

Section **A** *is* **compulsory**.

Answer only **five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh page.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

 Turn Over

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SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

1. A committee of seven people is to be selected from 4 men and 6 women. If the committee must have **at least** two men, determine the total possible number of ways of selecting the committee. *(05 marks)*

2. A cylindrical can of capacity 1000 cm^3 is made from a thin sheet of metal. The can is open at the top and closed at the bottom. The radius of the bottom is x cm. Find the value of x that will minimise the area of the sheet to be used. (Leave π in your answer) *(05 marks)*

3. The equation of an ellipse is $4x^2 + 25y^2 + 8x - 100y + 4 = 0.$

   Determine the;

   (a) coordinates of the centre of the ellipse. *(03 marks)*

   (b) eccentricity of the ellipse. *(02 marks)*

4. Show that $\int_0^1 \left(\frac{1}{9 - x^2}\right) dx = \frac{1}{6} ln\, 2.$ *(05 marks)*

5. The population of a country increases in a geometric progression (G.P.) by 2.75 % per annum. Calculate the number of years it will take for the population to double. *(05 marks)*

6. Show that $\dfrac{1 - \cos 2x + 2 \sin x \cos^2 x}{1 + \cos 2x} = \sin x + \tan^2 x.$ *(05 marks)*

7. The point $C\,(a, 4, 5)$ divides the line joining points $A\,(1, 2, 3)$ and $B\,(6, 7, 8)$ in the ratio $\lambda : 3$. Using vectors, find the values of a and λ. *(05 marks)*

8. Find the area enclosed by the curve $y = x^2$ and the line $y = x$ from $x = 0$ to $x = 1$. *(05 marks)*

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SECTION B (60 MARKS)

Answer only **five** *questions from this section.*
All *questions carry equal marks.*

9. (a) Express $12\cos\theta + 16\sin\theta$ in the form $R\cos(\theta - \alpha)$ where R is a positive constant and α is an acute angle. *(06 marks)*

(b) Hence;
(i) find the maximum and minimum values of $12\cos\theta + 16\sin\theta$.
(ii) solve the equation $12\cos\theta + 16\sin\theta = 15$ for $0^\circ \leq \theta \leq 180^\circ$.
(06 marks)

10. (a) Given that the polynomial $x^3 - 13x + p$ is exactly divisible by $x - 4$, find the value of p.
Hence solve the equation $x^3 - 13x + p = 0$. *(06 marks)*

(b) Solve the inequality $\dfrac{x^2 - x - 18}{x+3} \geq \dfrac{x}{2}$. *(06 marks)*

11. (a) Use the substitution $x = \sin\theta$ to evaluate

$$\int_0^1 \frac{1+x}{\sqrt{1-x^2}}\,dx.$$ *(05 marks)*

(b) Given that $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$, find $\frac{dy}{dx}$ in terms of x. *(07 marks)*

12. A curve is defined by the equations $x = -t^3 + t^2 + 1$ and $y = t^2$. A tangent to the curve at a point $B\,(x, y)$ is parallel to the line $3y - 2x - 1 = 0$. Determine the;

(a) coordinates of B. *(09 marks)*

(b) equation of the tangent at B. *(03 marks)*

13. (a) Use Maclaurin's theorem to expand $ln(1 - 2x)$ in ascending powers of x as far as the term in x^3. *(06 marks)*

(b) Using small changes, find the approximate value of $\tan 46^\circ$ correct to **three** decimal places. *(06 marks)*

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14. (a) The point C in the complex plane corresponds to the complex number z such that $3|z - 2| = |z - 6i|$. Show that the locus of C is a circle.

(05 marks)

(b) Find the square root of $-5 + 12i$. (07 marks)

15. The coordinates of points P and Q are $(0, 2, 5)$ and $(-1, 3, 1)$ respectively.

Given that the equation of the line T is $\frac{x-3}{2} = \frac{2-y}{2} = 2 - z$;

(a) find the equation of a plane which contains the point P and is perpendicular to the line T. (03 marks)

(b) show that the point Q lies on the plane. (02 marks)

(c) determine the coordinates of the point R where the line T intersects with the plane. (04 marks)

(d) show that PR and QR are perpendicular. (03 marks)

16. The rate at which the quantity M of a commodity is sold is proportional to the difference between the amount initially present and the quantity sold at any time t. Initially 10 tonnes of the commodity were present. After one day, 2 tonnes were sold.

(a) Form a differential equation for the quantity of the commodity sold.

(02 marks)

(b) (i) Determine the expression for M in terms of t. (08 marks)

(ii) Calculate the quantity sold at the end of 5 days. (02 marks)

END

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SECTION A (40 MARKS)

Qn 1.

4 Men	6 Women	Number of ways
2	5	${}^4C_2 \times {}^6C_5 = 36$
3	4	${}^4C_3 \times {}^6C_4 = 60$
4	3	${}^4C_4 \times {}^6C_3 = 20$
Total number of ways =		116

$\therefore$ Number of ways $= \underline{116}$.

Qn 2

$$V = \pi x^2 h$$
$$1000 = \pi x^2 h$$
$$h = \frac{1000}{\pi x^2}$$
$$A = 2\pi x h + \pi x^2$$
$$A = 2\pi x\left(\frac{1000}{\pi x^2}\right) + \pi x^2$$
$$A = \frac{2000}{x} + \pi x^2$$

$\frac{dA}{dx} = 0$ for maximum volume

$$\frac{dA}{dx} = \frac{-2000}{x^2} + 2\pi x$$
$$\frac{-2000}{x^2} + 2\pi x = 0$$
$$2\pi x = \frac{2000}{x^2}$$
$$\left(x^3\right)^{1/3} = \left(\frac{2000}{2\pi}\right)^{1/3}$$
$$\underline{x = 10\pi^{-1/3}}$$

Qn 3(a)
$$4x^2 + 25y^2 + 8x - 100y + 4 = 0$$
$$4(x^2 + 2x) + 25(y^2 - 4y) + 4 = 0$$
$$4(x+1)^2 + 25(y-2)^2 - 4 - 100 + 4 = 0$$
$$\frac{4(x+1)^2}{100} + \frac{25(y-2)^2}{100} = \frac{100}{100}$$
$$\frac{(x+1)^2}{25} + \frac{(y-2)^2}{4} = 1$$

Centre $(-1, 2)$

(b) $b^2 = a^2(1-e^2)$

$b^2 = 4, \quad a^2 = 25$

$4 = 25(1-e^2)$

$4/25 = 1-e^2$

$e = \sqrt{1-\frac{4}{25}}$

$e = \frac{+\sqrt{21}}{5}$

Qn 4. $\int_0^1 \frac{1}{9-x^2}dx$

let $\frac{1}{(3+x)(3-x} = \frac{A}{(3+x)} + \frac{B}{(3-x)}$,

$1 = A(3-x) + B(3+x)$

putting $x=3$, $B = 1/6$

putting $x=-3$, $A = 1/6$.

$= \int_0^1 \frac{dx}{6(3+x)} + \int_0^1 \frac{dx}{6(3-x)}$

$= \left(\frac{1}{6}\ln(3+x) - \frac{1}{6}\ln(3-x)\right)\Big|_0^1$

$= \frac{1}{6}\ln\left(\frac{3+x}{3-x}\right)\Big|_0^1$

$= \frac{1}{6}\ln\left(\frac{3+1}{3-1}\right) - \frac{1}{6}\ln\left(\frac{3}{3}\right)$

$= \frac{1}{6}\ln(4/2)$

$= \frac{1}{6}\ln 2$ hence shown.

QN5 let P_0 be the initial population of the country

After 1 year, $P_1 = P_0\left(1 + \frac{2.75}{100}\right)^1$

$$P_1 = P_0(1.0275)^1$$

After 2 years, $P_2 = P_0(1.0275)^2$

After n years, $P_n = P_0(1.0275)^n$

Total population; $P = P_1 + P_2 + \cdots + P_n$.

$$P = P_0(1.0275^1 + 1.0275^2 + \cdots + 1.0275^n)$$

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad r = 1.0275, \quad a = 1.0275.$$

For the population to double, $P = 2P_0$.

$$2P_0 = \frac{P_0(1.0275)(1.0275^n - 1)}{(1.0275 - 1)}$$

$$\frac{0.055 P_0}{1.0275 P_0} = 1.0275^n - 1$$

$$\ln(1.0275^n) = \ln\left(\frac{0.055}{1.0275} + 1\right)$$

$$n = \frac{\ln\left(\frac{0.055}{1.0275} + 1\right)}{\ln(1.0275)}$$

$n = 1.9221$ years, $n \simeq 2$ years.

$\therefore$ The population will double in about 2 years.

Qn 6.

$$\frac{1-\cos 2x + 2\sin x\cos^2 x}{1+\cos 2x} = \sin x + \tan^2 x$$

$$\frac{1-(1-2\sin^2 x) + 2\sin x\cos^2 x}{1+2\cos^2 x - 1}$$

$$\frac{2\sin^2 x + 2\sin x\cos^2 x}{2\cos^2 x}$$

$$\frac{2\sin x\cos^2 x}{2\cos^2 x} + \frac{2\sin^2 x}{2\cos^2 x}$$

$\underline{\sin x + \tan^2 x}$ hence shown

Qn 7.

$$OC = \frac{3(OA) + \lambda(OB)}{3+\lambda}$$

$$\begin{pmatrix} a \\ 4 \\ 5 \end{pmatrix} = \frac{3\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}}{3+\lambda}$$

$$(3+\lambda)\begin{pmatrix} a \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3+6\lambda \\ 6+7\lambda \\ 9+8\lambda \end{pmatrix}$$

$$\begin{pmatrix} 3a+\lambda a \\ 12+4\lambda \\ 15+5\lambda \end{pmatrix} = \begin{pmatrix} 3+6\lambda \\ 6+7\lambda \\ 9+8\lambda \end{pmatrix}$$

$$12+4\lambda = 6+7\lambda$$

$$3\lambda = 6$$

$$\underline{\lambda = 2}$$

$$3a+\lambda a = 3+6\lambda$$

$$3a+2a = 3+6(2)$$

$$5a = 15$$

$$\underline{a = 3}$$

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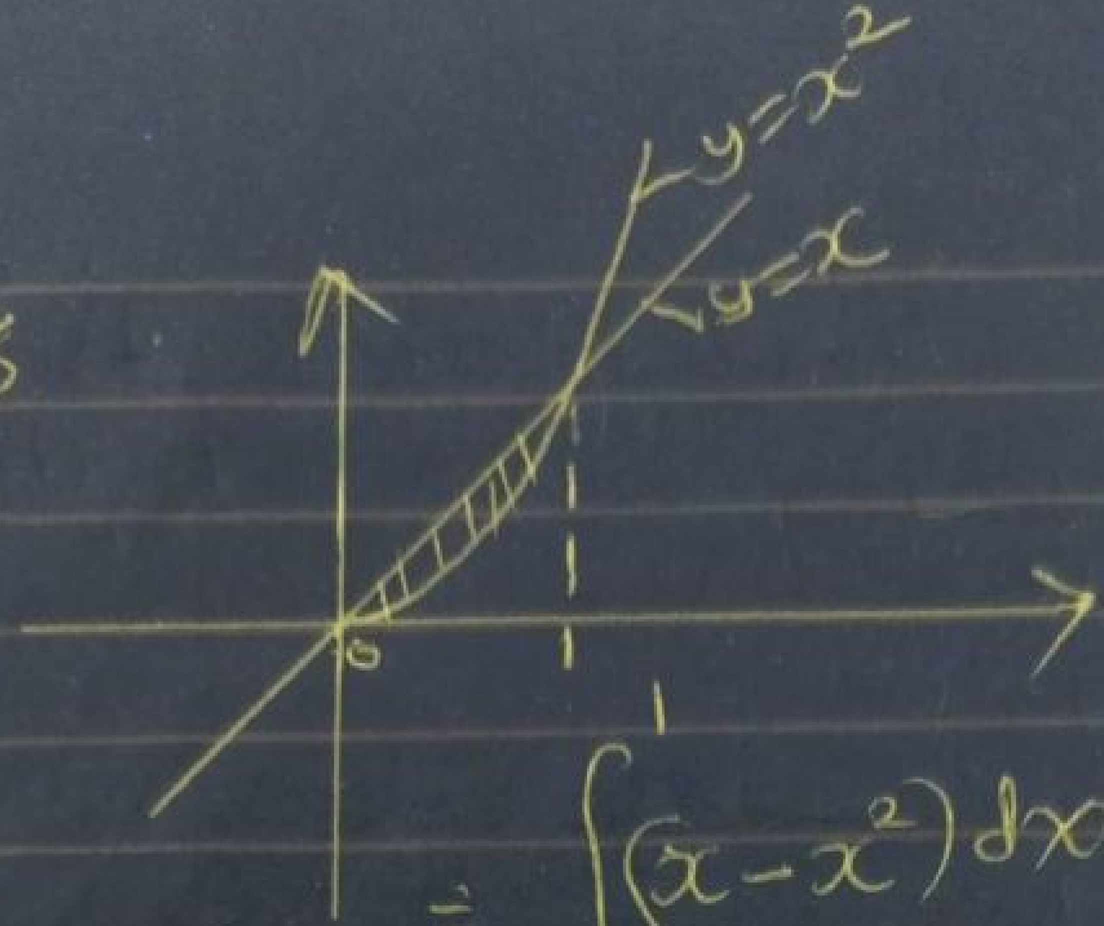
Qn 8



$$= \int_0^1 (x - x^2)\,dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{0^2}{2} - \frac{0^3}{3}\right)$$

$$= \frac{3-2}{6}$$

$$= \frac{1}{6} \text{ units}^2$$

SECTION B (60 MARKS)

Qn 9

(a) $12\cos\theta + 16\sin\theta = R\cos(\theta - \alpha)$

$12\cos\theta + 16\sin\theta = R\cos\alpha\cos\theta + R\sin\alpha\sin\theta$

$R\cos\alpha = 12$ — ①

$R\sin\alpha = 16$ — ②

$1^2 + 2^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 12^2 + 16^2$

$R = \sqrt{400}$

$R = 20.$

② ÷ ①

$\tan\alpha = \frac{16}{12}$

$\alpha = \tan^{-1}\left(\frac{16}{12}\right)$

$\alpha = 53.13°$

$\therefore 12\cos\theta + 16\sin\theta = 20\cos(\theta - 53.13°)$

(b) (i) For maximum value, $\cos(\theta - 53.13°) = 1$ and for minimum value, $\cos(\theta - 53.13°) = -1$

Maximum value $= 20(1)$

$= 20.$

Minimum value $= 20(-1)$

$= -20.$

(ii) $12\cos\theta + 16\sin\theta = 15$

$20\cos(\theta - 53.13°) = 15$

$\theta - 53.13° = \cos^{-1}\left(\frac{15}{20}\right)$

$\theta = 41.41° + 53.13°$

$\theta = 94.54°$ or $94.5°$

10(a) $x - 4 = 0$

$x = 4$ is a root

let $f(x) = x^3 - 13x + p$

$f(4) = 0$.

$4^3 - 13(4) + p = 0$.

$p = 4^3 - 13(4)$

$p = -12$

Hence using long division

$$
\begin{array}{r|l}
 & x^2 + 4x + 3 \\ \hline
x - 4 & x^3 - 13x - 12 \\
 & -(x^3 - 4x^2) \\ \hline
 & 4x^2 - 13x - 12 \\
 & -(4x^2 - 16x) \\ \hline
 & 3x - 12 \\
 & -(3x - 12) \\ \hline
 & \cdot
\end{array}
$$

$x^3 - 13x - 12 = 0$.

$x^2 + 4x + 3 = 0$

$x^2 + x + 3x + 3 = 0$

$x(x+1) + 3(x+1) = 0$.

$(x+1)(x+3) = 0$.

$x = -1$ or $x = -3$.

Hence the roots of the equation $x^3 - 13x - 12 = 0$ are $x = \{-1, -3, 4\}$

(b) $\dfrac{x^2 - x - 18}{x+3} - \dfrac{x}{2} \geq 0$

$\dfrac{2x^2 - 2x - 36 - x(x+3)}{2(x+3)} \geq 0$

$\dfrac{2x^2 - 2x - 36 - x^2 - 3x}{2(x+3)} \geq 0$.

$\dfrac{x^2 - 5x - 36}{2(x+3)} \geq 0$

$\dfrac{x^2 - 9x + 4x - 36}{2x+6} \geq 0$.

$\dfrac{x(x-9) + 4(x-9)}{(2x+6)} \geq 0$

$\dfrac{(x-9)(x+4)}{(2x+6)} \geq 0$.

Critical values. $(x-9)(x+4) = 0$

$x = 9, x = -4$

$2x + 6 = 0$, $x = -3$.

	$x < -4$	$-4 < x < -3$	$-3 < x < 9$	$x > 9$
$x - 9$	$-$	$-$	$-$	$+$
$x + 4$	$-$	$+$	$+$	$+$
$(x-9)(x+4)$	$+$	$-$	$-$	$+$
$2x + 6$.	$-$	$-$	$+$	$+$
$\dfrac{(x-9)(x+4)}{2x+6}$	$-$	$+$	$-$	$+$

since the critical values satisfy the inequality, the solutions are

$-4 \leq x \leq -3$ and $x \geq 9$

Qn 11

(a)

$$\int_0^1 \frac{1+x}{\sqrt{1-x^2}}\,dx.$$

x	0	1
θ	0	$\pi/2$

$x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

$$= \int_0^{\pi/2} \frac{(1+\sin\theta)\cdot\cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \int_0^{\pi/2} 1\, d\theta + \int_0^{\pi/2} \sin\theta\, d\theta$$

$$= (\theta - \cos\theta)\Big|_0^{\pi/2}$$

$$= \left(\frac{\pi}{2} - \cos\left(\frac{\pi}{2}\right)\right) - \left(0 - \cos(0)\right)$$

$$= \frac{\pi}{2}+1 \text{ or } \frac{2+\pi}{2} \text{ or } 2.5708$$

(b) $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$\sin y = \frac{x}{\sqrt{1+x^2}}$

$$\cos y\,\frac{dy}{dx} = \frac{(1+x^2)^{1/2}(1) - \frac{x}{2}(2x)(1+x^2)^{-1/2}}{\left((1+x^2)^{1/2}\right)^2}$$

$$\frac{\cos y}{\cos y}\frac{dy}{dx} = \frac{(1+x^2)^{1/2} - x^2(1+x^2)^{-1/2}}{(1+x^2)\cos y}$$

$\cos y = \sqrt{1-\sin^2 y}$

$= \sqrt{\left(1 - \frac{x^2}{1+x^2}\right)}$

$= \frac{1}{\sqrt{1+x^2}}$

$$\therefore \frac{dy}{dx} = \frac{(1+x^2-x^2)}{(1+x^2)^{1/2}} \times \frac{\sqrt{1+x^2}}{1+x^2} \quad \therefore \frac{dy}{dx} = \frac{1}{1+x^2}$$

Qn 12.

(a) $x = -t^3 + t^2 + 1$

$y = t^2$

$\frac{dx}{dt} = -3t^2 + 2t$

$\frac{dt}{dx} = \frac{1}{t(2-3t)}$

$\frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{dy}{dx} = 2t\left(\frac{1}{t(2-3t)}\right)$

$\frac{dy}{dx} = \frac{2}{2-3t}$.

$3y - 2x - 1 = 0$.

$y = \frac{2x}{3} + \frac{1}{3}$

$m = \frac{2}{3}$.

Since tangent is paralle to the line, gradient $\frac{dy}{dx} = m$.

$\frac{2}{3} = \frac{2}{2-3t}$

$2 - 3t = 3$

$t = -\frac{1}{3}$.

$x = -t^3 + t^2 + 1$

$x = \left(\frac{-1}{3}\right)^3 + \left(\frac{-1}{3}\right)^2 + 1$

$x = \frac{1}{27} + \frac{1}{9} + 1$

$x = \frac{1+3+27}{27}$

$x = \frac{31}{27}$

$y = t^2$

$y = \left(\frac{-1}{3}\right)^2$

$y = \frac{1}{9}$

$\therefore B\left(\frac{31}{27}, \frac{1}{9}\right)$

(b) $\frac{\left(y - \frac{1}{9}\right)}{\left(x - \frac{31}{27}\right)} = \frac{2}{3}$

$3\left(y - \frac{1}{9}\right) = 2\left(x - \frac{31}{27}\right)$

$3y - \frac{1}{3} = 2x - \frac{62}{27}$

$3y - 2x = \frac{1}{3} - \frac{62}{27}$

$81y - 54x = -53$

$81y - 54x + 53 = 0$

equation of the tangent at B.

Subject .... Paper code ..../...... Personal Number

13(a).

$f(x) = \ln(1-2x)$

$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$

$f(0) = \ln(1-2(0))$
$= 0$

$f'(x) = \frac{-2}{1-2x}$

$f'(0) = \frac{-2}{1-2(0)}$
$= -2$

$f''(x) = \frac{-4}{(1-2x)^2}$

$f''(0) = \frac{-4}{(1-2(0))^2}$
$= -4$

$f'''(x) = \frac{-16}{(1-2x)^3}$

$f'''(0) = \frac{-16}{(1-2(0))^3}$
$= -16$

$f(x) = 0 + \frac{(-2)x}{1} + \frac{(-4)x^2}{2\times1} + \frac{-16x^3}{3\times2\times1}$

$f(x) = -2x - 2x^2 - \frac{8x^3}{3}$

(b) $\tan 46°$.

$f(x+h) = f(x) + hf'(x)$

$f(x+h) = \tan 46°$.
$f(x) = \tan x$
$x = 45°$.
$h = 1° = \frac{\pi}{180}$

$f'(x) = \frac{1}{\cos^2 x}$

$= \left(\frac{1}{\cos 45}\right)^2$

$f'(x) = 2$

$f(x) = \tan 45$
$f(x) = 1$

$\therefore \tan 46° = 1 + \left(\frac{\pi}{180}\right) \times 2$

$= 1 + \frac{\pi}{90}$

$= 1.035$ (3dp cal)

Qn 14

(a) let $z = x+yi$

$3|x+yi-2| = |x+yi-6i|$

$3|(x-2)+yi| = |x+(y-6)i|$

$9((x-2)^2+y^2) = x^2+(y-6)^2$

$9(x^2-4x+4+y^2) = x^2+y^2-12y+36$

$9x^2-36x+36+9y^2 = x^2+y^2-12y+36$

$8x^2+8y^2-36x+12y = 0$

$x^2+y^2-\frac{36x}{8}+\frac{12y}{8} = 0$

$x^2+y^2-\frac{9}{2}x+\frac{3}{2}y = 0$

which is in the form

$x^2+y^2+2gx+2fy+c=0$

hence the locus is a circle

(b) let $\sqrt{-5+12i}$ be $a+bi$

squaring both sides

$(\sqrt{12i-5})^2 = (a+bi)^2$

$a^2+2abi-b^2 = -5+12i$

Equating real and imaginary parts.

$2ab = 12$ —①

$a = 6/b$ —$\cancel{①}_1$

$a^2-b^2 = -5$ —②

sub $\cancel{x}_1$ into eqn ②

$\frac{36}{b^2}-b^2 = -5$

let b^2 be p.

$\frac{36}{p}-\cancel{p} = -5$

$36-p^2 = -5p$

$p^2-5p-36=0$

$p^2+4p-9p-36=0$.

$p(p+4)-9(p+4)=0$

$(p+4)(p-9) = 0$.

$p=-4$ or $p=9$

but $b^2 = p$

$\pm\sqrt{b^2} = \pm\sqrt{-4}$

$b = \pm 2i$ X

$\sqrt{b^2} = \pm\sqrt{9}$

$b = \pm 3$ ✓

$\cancel{\sqrt{-5+12i}} =$

$a = 6/b$

$a = \frac{6}{\pm 3}$

$a = \pm 2$ ✓

also

$a = \frac{6}{\pm 2i}$

$a = \pm 3i$ X

$\sqrt{-5+12i} = \pm(2+3i)$

Qn 15.

(a) $P(0,2,5)$, $R(x,y,z)$, $d_1 = \begin{pmatrix}2\\-2\\-1\end{pmatrix}$

$PR \cdot d_1 = 0$.

$$\begin{pmatrix}x-0\\y-2\\z-5\end{pmatrix}\begin{pmatrix}2\\-2\\-1\end{pmatrix} = 0$$

$$2x - 2(y-2) - 1(z-5) = 0.$$

$$2x - 2y + 4 - z + 5 = 0.$$

$$\underline{2x - 2y - z + 9 = 0}.$$

<Equation of the plane

(b) $Q(-1,3,1)$, sub Q into equation of the plane

$$2x - 2y - z + 9 = 0.$$

$$2(-1) - 2(3) - (1) + 9 = 0.$$

$$-2 - 6 - 1 + 9 = 0$$

$$9 - 9 = 0.$$

$$0 = 0$$

Since LHS = RHS, Q lies on the plane

(c) let $\frac{x-3}{2} = \lambda$, $\frac{2-y}{2} = \lambda$, $2 - z = \lambda$

$x = 2\lambda + 3$, $y = 2 - 2\lambda$, $z = 2 - \lambda$

sub x, y, and z into $2x - 2y - z + 9 = 0$.

$$2(2\lambda+3) - 2(2-2\lambda) - (2-\lambda) + 9 = 0$$

$$4\lambda + 6 - 4 + 4\lambda - 2 + \lambda + 9 = 0.$$

$$9\lambda = -9$$

$$\lambda = -1$$

$x = 2(-1) + 3$ | $y = 2 - 2(-1)$ | $z = 2 - (-1)$

$x = 1$ | $y = 4$ | $z = 3$.

$\underline{R(1,4,3)}$

(d) $PR \cdot QR = 0$. for PR and QR to be perpendicular

$$\begin{pmatrix}1-0\\4-2\\3-5\end{pmatrix}\cdot\begin{pmatrix}1-1\\4-3\\3-1\end{pmatrix} = 0.$$

$$\begin{pmatrix}1\\2\\-2\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix} = 2 + 2 - 4$$

$$= 0.$$

$\therefore$ PR and QR are perpendicular.

Qn 16

(a) $\frac{dM}{dt} \propto (10-M)$

$\frac{dM}{dt} = k(10-M)$ where k is a constant.

(b)(i) $\frac{dM}{dt} = k(10-M)$

$\int \left(\frac{dM}{10-M}\right) = \int k\,dt$

$-\ln(10-M) = kt + c$

at $t=0$, $M=0$.

$-\ln(10-0) = k(0) + c$

$c = -\ln 10$.

$-\ln(10-M) = kt - \ln 10$.

$\ln\left(\frac{10}{10-M}\right) = kt$.

When $t=1$, $M=2$

$\ln\left(\frac{10}{10-2}\right) = k(1)$

$k = \ln\left(\frac{10}{8}\right)$

$k = \ln\left(\frac{5}{4}\right)$

$\ln\left(\frac{10}{10-M}\right) = t\ln\left(\frac{5}{4}\right)$

$\ln\left(\frac{10}{10-M}\right) = \ln\left(\frac{5}{4}\right)^t$

$\frac{10}{10-M} = \frac{5^t}{4^t}$

$10(4^t) = 5^t(10-M)$

$10(4^t) = 10(5^t) - M(5^t)$

$(5^t)(M) = 10(5^t) - 10(4^t)$

$M = \frac{10(5^t - 4^t)}{5^t}$

(ii) when $t = 5$ days

$$M = \frac{10(5^5 - 4^5)}{5^5}$$

$M = 6.7232$ tonnes.

-END-