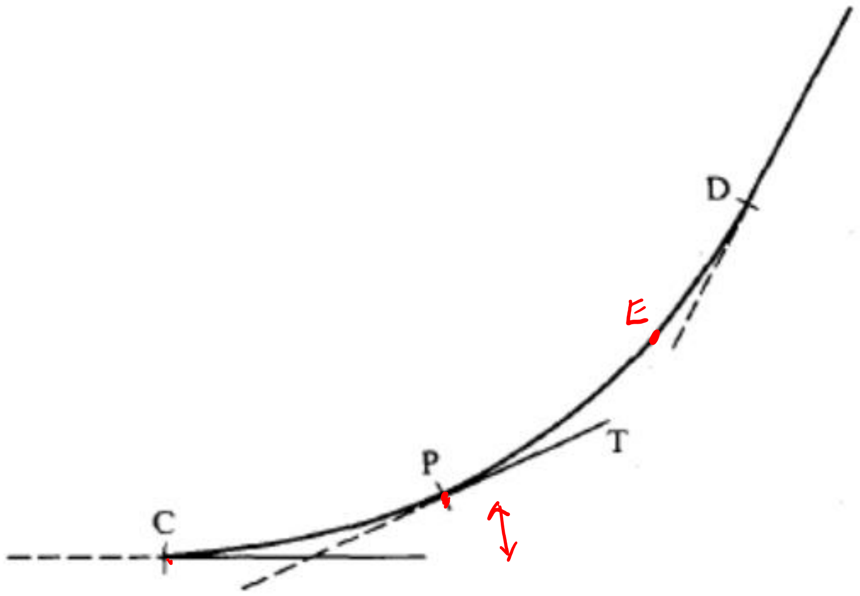
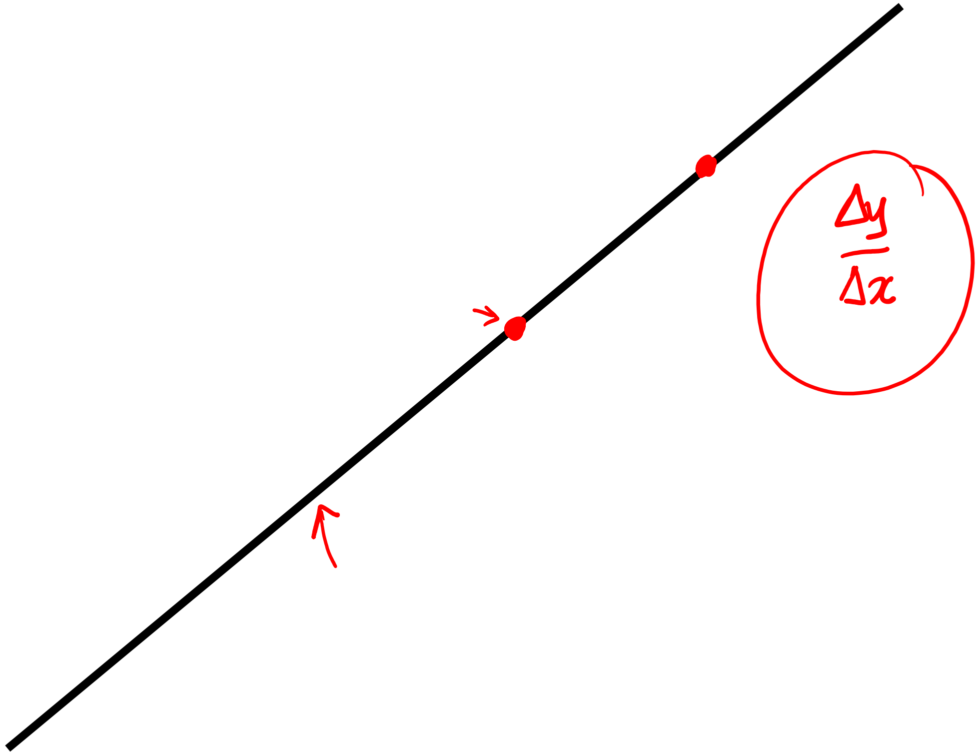
DIFFERENTIATION

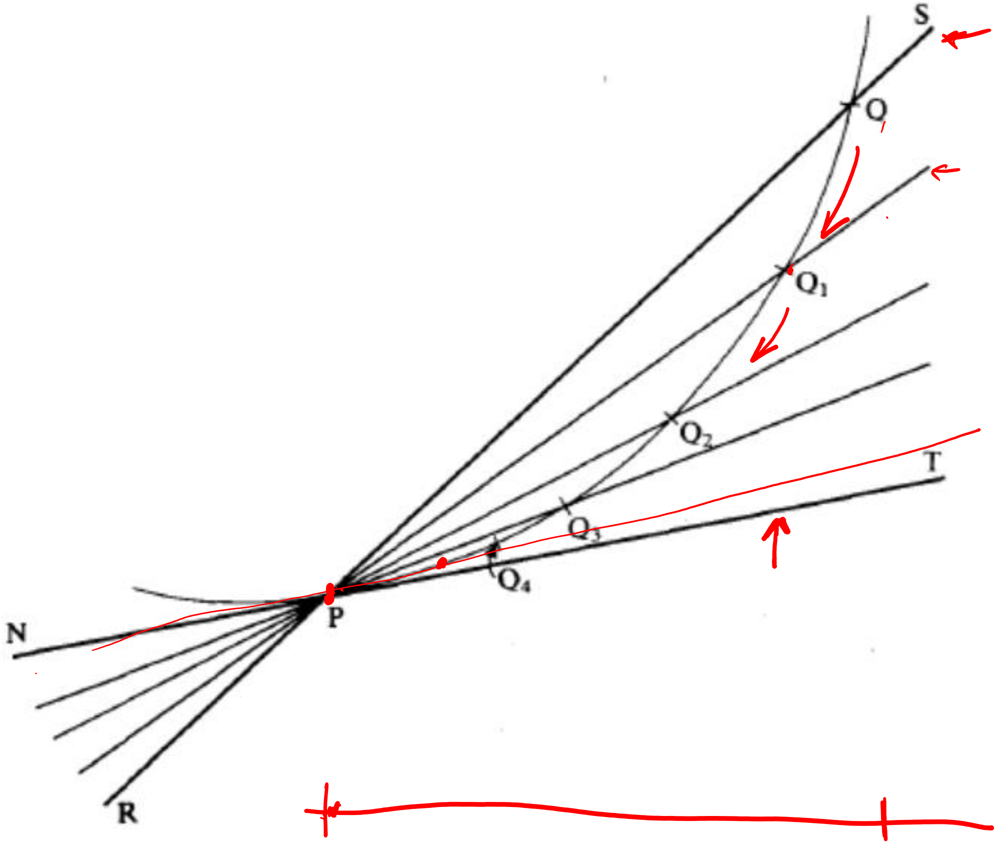
INTRODUCTION

GRADIENT OF A CURVE





GRADIENT OF A CURVE



Curve NPQ

Chord RS

at point P.

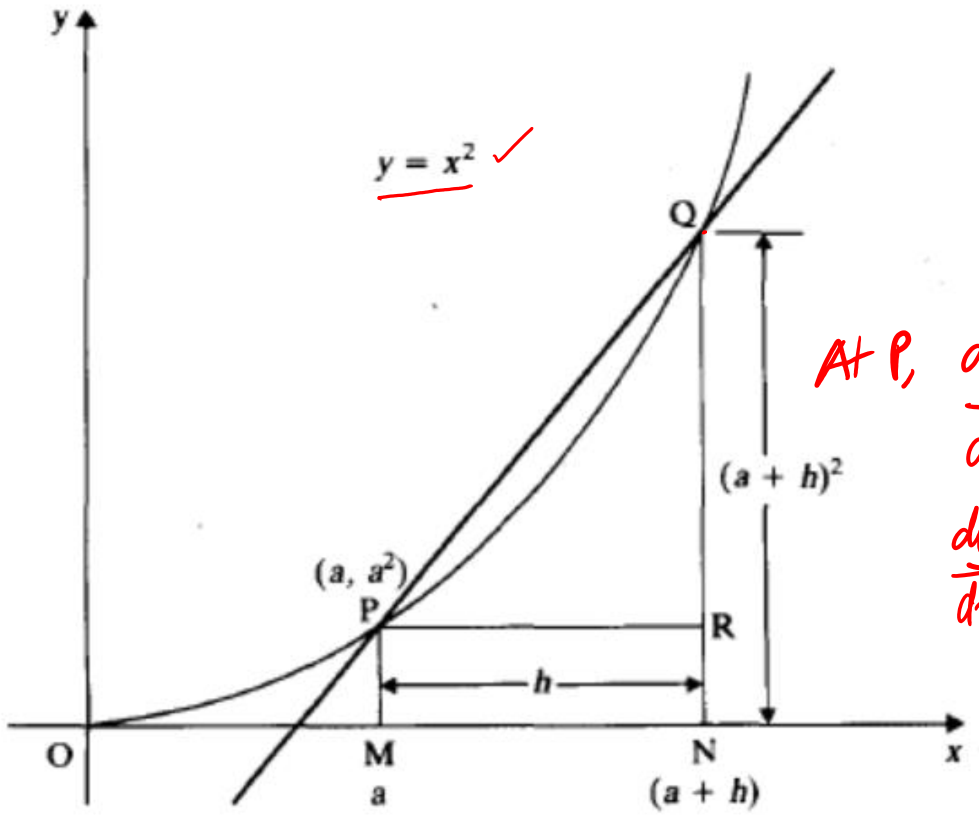
NT

Gradient of PQ $= \dfrac{\Delta y}{\Delta x} \rightarrow$ Gradient of tangent

as Q moves closer to P

PQ_1, PQ_2, PQ_3, NT

DIFFERENTIATION FROM FIRST PRINCIPLES



At P, $\frac{dy}{dx} = 2a$

$\frac{dy}{dx} = 2x$

Gradient of the curve at P:

$$\frac{\Delta y}{\Delta x} = \frac{(a+h)^2 - a^2}{(a+h) - a}$$

$$= \frac{a^2 + 2ah + h^2 - a^2}{h}$$

$$= \frac{2ah + h^2}{h}$$

$$= 2a + h$$

As $h \to 0$, $\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$ at P

As h tends to zero, Gradient of PQ $\to$ Gradient of curve at P.

x^3

$$\frac{\Delta y}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{(x+\Delta x) - x}$$

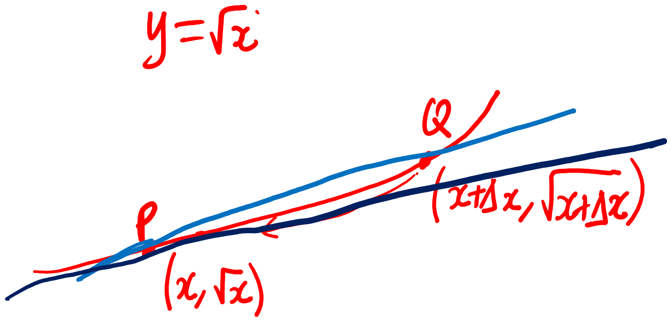
$$= \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$

$$= \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

$$= 3x^2 + 3x\Delta x + (\Delta x)^2$$

As $\Delta x \to 0$, $\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$ $\quad \therefore \frac{dy}{dx} = 3x^2$

$\sqrt{x}$



$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{x+\Delta x} - \sqrt{x}}{(x+\Delta x) - x}$$

$$= \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$= \frac{(\sqrt{x+\Delta x} - \sqrt{x})(\sqrt{x+\Delta x} + \sqrt{x})}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})}$$

$$= \frac{x+\Delta x - x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})}$$

$$= \frac{\Delta x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} = \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

As $\Delta x \to 0$, $\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}+\sqrt{x}}$$

$$= \underline{\underline{\frac{1}{2\sqrt{x}}}}$$

WHAT DO WE OBSERVE?

x^2 $\quad 2x$

x^3 $\quad 3x^2$

$\sqrt{x}$ $\quad \frac{1}{2\sqrt{x}}$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$y = k = kx^0$

$\frac{dy}{dx} = 0$

$y = kx^1$

$\frac{dy}{dx} = k$

$y = ax^2 + bx + c$

$\frac{dy}{dx} = \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c)$

$= 2ax + b$

PRODUCT RULE

$P(x, uv)$ $\quad Q(x+\Delta x, (u+\Delta u)(v+\Delta v))$

$$y = uv$$

$$\frac{\Delta y}{\Delta x} = \frac{(u+\Delta u)(v+\Delta v) - uv}{\Delta x}$$

$$= \frac{uv + u\Delta v + v\Delta u + \Delta u\Delta v - uv}{\Delta x}$$

$$= \frac{u\Delta v}{\Delta x} + \frac{v\Delta u}{\Delta x} + \frac{\Delta u\Delta v}{\Delta x}$$

As $\Delta x \to 0$, $\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$ but also $\frac{\Delta u}{\Delta x} \to \frac{du}{dx}$, $\frac{\Delta v}{\Delta x} \to \frac{dv}{dx}$, $\Delta u\Delta v \to 0$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

QUOTIENT RULE

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Differentiate $\frac{x+4}{(x+3)^3}$

$$y = \frac{x+4}{(x+3)^3} \qquad u = x+4 \quad , \quad v = (x+3)^3 = x^3 + 9x^2 + 27x + 27$$

$$\frac{du}{dx} = 1 \quad , \quad \frac{dv}{dx} = 3x^2 + 18x + 27$$

$$y = \frac{x+4}{x^3 + 9x^2 + 27x + 27}$$

$$\frac{dy}{dx} = \frac{(x+3)^3 (1) - (x+4)(3x^2+18x+27)}{(x+3)^6}$$

$$=$$