P425/1
PURE MATHEMATICS
PAPER 1
July /August 2023
3 hours

KAYUNGA SECONDARY SCHOOLS EXAMINATIONS COMMITTEE (KASSEC)

JOINT MOCK EXAMINATION 2023

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer all the **Eight** questions in section A and five questions from section **B**.*
- *Any additional question (s) answered will **not** be marked*
- ***All** working **Must** be shown clearly*
- *Begin each question on a fresh page*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

TURN OVER

SECTION A (40 MARKS)

Answer all the questions in this section.

1. What values of x satisfy the inequality: $\frac{(x-2)^2-8}{5-4x} > 1.$ *(05 marks)*

2. Given that x and y are real numbers. Find the values of x and y which satisfy the equation: $\frac{2y+4i}{2x+y} - \frac{y}{x-i} = 0.$ *(05 marks)*

3. If $y = \frac{sinx}{x^2}$, prove that $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2+2)y = 0.$ *(05 marks).*

4. Given that $A(0,5,-3), B(2,3,-4), C(1,-2,2)$ are vertices of triangle. Find the area of the triangle. *(05 marks)*

5. Express $4x^2 - 24xy + 11y^2 = 0$ as a product of two straight lines and hence find the angle between them. *(05 marks)*

6. Form a differential equation given that $y = 2\cos(2x+\beta)$ and state its order. *(05 marks)*

7. Integrate $\int_2^3 \frac{3}{x^2-4x+5}dx$ to 4dps. *(05 marks)*

8. If $P(x,y)$is a point which moves such that $x = cos\theta$ and $y = cosec\theta - cot\theta$, Find the locus of point P. *(05 marks)*

SECTION B (60 MARKS)

*Attempt any **Five** in this section.*

9. (a) Prove that the roots of the equation: $(k+3)x^2 + (6-2k)x = 1-k$ are real if and only if, k is not greater than $\frac{3}{2}$. *(06 marks).*

(b) Solve the pair of simultaneous equations: $2^{x+y} = 6^y, 3^x = 6(2^y)$. *(06 marks)*

10. (a) The sum of the first n –terms of a certain series is$n^2 + 5n$, for all integral values of n. Find the first three terms and prove that the series is an arithmetic progression. (A.P). *(06 marks)*

(b) Use the knowledge of series to write $2.9\dot{6}\dot{0}$ as a fraction. *(06 marks)*

11. (a) Given the equation below;

$r = \begin{pmatrix}1\\1\\1\end{pmatrix} + m\,(4\underline{i} - \underline{j} - \underline{k}) + n\begin{pmatrix}2\\1\\0\end{pmatrix}$ Find the equation of the plane represented by equation above. ***(06 marks)***

(b) Find the perpendicular distance from $A\ (2, 3, 4)$ to the line.

$r = \begin{pmatrix}4\\0\\-1\end{pmatrix} + \lambda\begin{pmatrix}-2\\6\\-3\end{pmatrix}$ ***(06 marks)***

12. (a) Show that the equation $x^2 + 4x - 8y = 4$ represents a parabola of focus $(-2, 1)$. Find the tangent on the parabola that passes at its vertex. ***(06 marks)***

(b) The line $y = x - c$ touches the ellipse: $9x^2 + 16y^2 = 144$. Find the value of c and hence determine the point of contact. ***(06 marks)***

13. (a) Solve for x, $sinx + \sqrt{3}\,cosx = 1$ for $0 \leq x \leq 2\pi$. ***(04 marks)***

(b) Prove that: $\frac{sin3\theta}{1+2\,cos2\theta} = sin\theta$ and hence show that $\sin 15^0 = \frac{(\sqrt{3}-1)}{2\sqrt{2}}$. ***(08 marks)***

14. (a) Prove that, $\int_0^{\frac{\pi}{2}} \frac{sinx}{3sinx+4cosx}dx = \frac{3\pi}{50} + \frac{4}{25}In\left(\frac{4}{3}\right)$ ***(06 marks)***

(b) Integrate with respect of x,

(i) $\int xe^{2x^2}dx$ ***(03 marks)***

(ii) $\int x^2e^{2x}dx$ ***(03 marks)***

15. At 3:00pm, the temperature of a covid 19 patient was found to be 80⁰C and that of the surroundings was 20⁰C. At 3:03pm, the temperature of the patient had dropped to 42⁰C. the rate of cooling of the patient was directly proportional to the difference between its temperature Q and that of the surroundings.

(a) (i) Write a differential equation to represent the rate of cooling of the patient.

(ii) Solve the differential equation using the given conditions.

(b) Find the temperature of the patient at 3:05pm. ***(12 marks)***

16. (a) Find the gradient of the curve $y = x^2 - 25\log_{10} x$ at the point when $x = 10$. Give your answer to 3 s.f) ***(05 marks)***

(b) If $y = \tan\left[tan^{-1}\left(\frac{1}{2x}\right)\right]$. Show that $\frac{dy}{dx} = \frac{-2(1+y^2)}{1+4x^2}$. ***(07 marks)***

END