

A-level math paper 2:

Index numbers

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This is the percentage ratio of one quantity to the other, e.g. price index

Index number is a technique of measuring changes in a variable or group of variables with respect to time, geographical location or other characteristics. It is a statistical measure of change in a representative group of individual data points.

For example, if the price of a certain commodity rises from shs. 100 in the year 2007 to shs. 150 in the year 2017, the price index number will be 150 showing that there is a 50% increase in the prices over this period

Simple index numbers

The simple index numbers include;

- (i) Price index or price relative
Price relative = $\frac{P_1}{P_0} \times 100$ where P_1 = price the current year and P_0 – price in the base year
- (ii) Wage index
Wage index = $\frac{W_1}{W_0} \times 100$
- (iii) Quantity index (quantum) index
Quantum index = $\frac{Q_1}{Q_0} \times 100$

Example 1

A loaf of bread cost shs. 1200/= in 2008 and shs. 1800/= in 2014. Taking 2008 as the base year, find the price relative in 2014

Solution

$$\text{Price relative} = \frac{P_1}{P_0} \times 100 = \frac{1800}{1200} \times 100 = 150$$

Example 2

In 2020, the price index of a commodity using 2019 as the base was 180. In 2021, the price index using 2020 as the base year was 150. What is the price index in 2021 using 2019 as the base year?

Solution

$$\frac{P_{2020}}{P_{2019}} \times 100 = 180$$

$$\frac{P_{2020}}{P_{2019}} = 1.80 \dots\dots\dots (i)$$

$$\frac{P_{2021}}{P_{2020}} \times 100 = 150$$

$$\frac{P_{2021}}{P_{2020}} = 1.50 \dots\dots\dots (ii)$$

Eqn. (i) x Eqn. (ii)

$$\frac{P_{2020}}{P_{2019}} \cdot \frac{P_{2021}}{P_{2020}} = 1.8 \times 1.5$$

$$\frac{P_{2021}}{P_{2019}} = 2.7$$

$$\frac{P_{2021}}{P_{2019}} \times 100 = 2.7 \times 100 = 270$$

∴ the price index in 2021 using 2019 as the base year = 270

Example 3

The wage of a nurse in Uganda in 2010 was 350,000/=. the wage of the nurse in 2015 was increase by shs. 150,000/=. Using 2010 as the a base year calculate the nurses wage index in 2015.

$$\text{Wage index} = \frac{W_1}{W_0} \times 100 = \frac{500,000}{350,000} \times 100 = 142.9$$

Price indices

Price indices are divided into

- (a) Simple price index
- (b) Simple Aggregate price index
- (c) Weighed price index

- (a) Simple price index

This is the average of the price relative

$$\text{It is given by simple price index} = \frac{\sum \frac{P_1}{P_0}}{n} \times 100$$

Where n = number of items.

- (b) Simple aggregate price index

$$\text{It is given by simple aggregate price index} = \left(\frac{\sum P_1}{\sum P_0} \times 100 \right)$$

Example 4

The table shows the prices Of bread and meat per kg in 2000 and 2008

	Year	
item	2000	2008
Beans	700	1200
Meat	2500	4500

Using 2000 as the base year, find

- (a) Price relatives of each commodity
- (b) Simple price index
- (c) Simple aggregate price index

Solution

$$(a) \text{ Price index} = \frac{P_1}{P_0} \times 100$$

$$\text{For beans: P.R} = \frac{1200}{700} \times 100 = 171.43$$

$$\text{For meat: P.R} = \frac{4500}{2500} \times 100 = 180$$

$$(b) \text{ Simple price index (S.P.I)} = \frac{171.43+180}{2} = 175.72$$

$$(c) \text{ Simple aggregate price index (S.A.P.I)} = \frac{1200+450}{700+2500} \times 100 = 178.13$$

Example 5

In 2014 the price of a shirt, a dress and a pair of shoes were shs. 20,000; shs. 35,000 and shs. 45,000 respectively. Given that in 2017 the prices were shs. 25,000, shs. 50,000 and shs. y respectively. Find the value of y if the aggregate price index was 130 while taking 2014 as the base year.

Solution

$$\text{Simple aggregate price index} = \left(\frac{\sum P_1}{\sum P_0} \times 100 \right)$$

$$\left(\frac{25,000 + 50,000 + y}{20,000 + 35,000 + 45,000} \right) \times 100 = 130$$

$$\left(\frac{75,000 + y}{1000} \right) = 130$$

$$y = \text{shs. } 55,000$$

Weighted price index (composite index)

If the weight or quantity in the base year and current year are the same, we use

(i) Weighted aggregate price index

$$\text{Weighted aggregate price index} = \left(\frac{\sum P_1 w}{\sum P_0 w} \right) \times 100$$

Example 6

The table below shows the prices (shs.) and amounts of items bought for assembling a phone in 2012 and 2015

Items	Prices (shs)		Quantity
	2012	2015	
Transistor	12,000	18,000	8
Resistor	16,500	21,000	22
Capacitor	15,000	17,000	9
Diode	16,000	18,000	2
Circuit	20,000	25,000	1

Calculate the composite price index for a phone taking 2012 as the base year

$$\text{W.A.P.I} = \left(\frac{18,000 \times 8 + 21,000 \times 22 + 17,000 \times 9 + 18,000 \times 2 + 25,000 \times 1}{12,000 \times 8 + 16,500 \times 22 + 15,000 \times 9 + 16,000 \times 2 + 20,000 \times 1} \right) \times 100 = 126.94$$

Example 7

The table below shows the prices (shs) and amount of items bought for making a cake in 2008 and 2009.

Items	Prices (shs)		Quantity
	2008	2009	
Flour per kg	6,000	7,800	3
Sugar per kg	5,000	4,000	1
Milk per litre	1,000	1,500	2
Eggs per egg	200	300	8

(a) Calculate the weighted aggregate price index taking 2008 as the base year

(b) In 2009, the cost of making a cake was shs. 80,000/=. Using the weighted aggregate price index above, find the cost of the cake in 2008.

Solution

$$(a) \text{ W.A.P.I} = \left(\frac{7800 \times 3 + 4000 \times 1 + 1500 \times 2 + 3 \times 8}{6000 \times 3 + 5000 \times 1 + 1000 \times 2 + 200 \times 8} \right) \times 100 = 123.3083$$

$$(b) \frac{P_1}{P_0} \times 100 = 123.3083$$

$$\frac{80,000}{P_0} \times 100 = 123.3083$$

$$P_0 = \text{shs. } 64,878.0333$$

(ii) Average weighted price index

$$\text{Average weighted price index} = \frac{\sum \frac{P_1}{P_0} w}{\sum w} \times 100$$

When the price relative (P.R) is given then

$$\text{Average weighted price index} = \frac{\sum (P.R \times w)}{\sum w}$$

Example 8

The table shows the expenditure (Ug. Shs.) of a student during the first and second terms

Items	Expenditure (shs.)		Amount
	1 st term	2 nd term	
clothing	46,500	49,350	5
Pocket money	55,200	37,500	3
Books	80,000	97,500	8

Using the first term expenditure as the base, find the average weighted price index

$$\begin{aligned} \text{Average weighted price index} &= \frac{\sum \frac{P_1}{P_0} w}{\sum w} \times 100 \\ &= \left(\frac{\frac{49,350}{46,500} \times 5 + \frac{37,500}{55,200} \times 3 + \frac{97,500}{80,000} \times 8}{5+3+8} \right) \times 100 \\ &= 106.841 \end{aligned}$$

Example 9

The table below shows the price relatives together with their weights for a certain family

Item	Weight	Price relative
Food	172	120
Water	160	124
Housing	170	125
Electricity	210	135
Clothing	140	104

Find the:

- (i) simple price index
- (ii) cost of living

Solution

$$(i) \text{ S.P.I} = \frac{\sum \left(\frac{P_1}{P_0} \right)}{n} = \frac{120+124+125+135+104}{5} = 121.6$$

$$(ii) \text{ Cost of living} = \frac{\sum (P.R \times w)}{\sum w} = \frac{120 \times 172 + 124 \times 160 + 125 \times 170 + 135 \times 210 + 104 \times 140}{172+160+170+210+140} = 122.82$$

Weighted aggregate price indices/ Paache's theory/value index

If the weight or quantity in the base year and current year are different, we use

$$\text{Weighted aggregate price index} = \frac{\sum P_1 W_1}{\sum P_0 W_0} \times 100$$

Example 10

The table below shows the prices of items per kg in the year 2001 and 2002

Item	2001 = 100		2002	
	Price (shs.)	Quantity (kg)	Price (shs.)	Quantity (kg)
Rice	2800	20	3200	30
Millet	1500	10	1900	10
Beans	2000	5	2500	70

Calculate for 2002

- (i) Price index
- (ii) Simple aggregate price index
- (iii) Simple aggregate quantity index
- (iv) Weighted aggregate price index

Solution

- (i) Price index = $\frac{P_{2002}}{P_{2001}} \times 100$
For rice, price index = $\frac{3200}{2800} \times 100 = 114.29$
For millet, price index = $\frac{1900}{1500} \times 100 = 126.67$
For beans, price index = $\frac{2500}{2000} \times 100 = 125$
- (ii) S.A.P.I = $\left[\frac{3200+1900+2500}{2800+1500+2000} \right] \times 100 = 120.63$
- (iii) S.A.Q.I = $\frac{\sum Q_{2002}}{\sum Q_{2001}} \times 100$
 $= \left[\frac{30+10+70}{20+20+5} \right] \times 100 = 314.29$
- (iv) W.A.P.I = $\frac{\sum P_1 W_1}{\sum P_0 W_0} \times 100$
 $= \left[\frac{3200 \times 30 + 1900 \times 10 + 2500 \times 70}{2800 \times 20 + 1500 \times 10 + 2000 \times 5} \right] \times 100$
 $= 358.02$

Revision exercise

1. UNEB 2020/2/5

The table below shows the price indices of beans, maize, rice and meat with corresponding weights

Item	Price index 2008 (2007 = 100%)	Weight
Beans	105	4
Maize	x	7
Rice	104	2
Meat	113	5

Calculate the;

- (a) Value of x given that the price indices of maize in 2007 and 2008 using 2006 as the base year are 112 and 130 respectively. **(116.0714)**

- (b) Weighted price index for 2008 using 2007 as the base year (**111.4167**)

2. UNEB 2018/2/5

The price index of an article in 2000 based on 1998 was 130. The price index for the article in 2005 based on 2000 was 80. Calculate

- (a) Price index of the article in 2005 based on 1998. (104)
 (b) Price of the article in 1998 if the price of the article was 45,000 in 2005.

UNEB 2017/2/7

The table below shows the price (shs.) and amount of items bought weekly by a restaurant in 2002 and 2003.

Items	Price (shs.)		Amount
	2002	2003	
Mil k per litre	400	500	200
Eggs per tray	2500	3000	18
Cooking oil per litre	2400	2100	2
Flour per packet	2000	2200	15

Calculate

- (a) the weighted aggregate price index taking 2002 as the base year. (**119.63**)
 (b) In 2003, the restaurant spent shs. 450,000/=. Using the weighted aggregate price index, find how the restaurant could have spent in 2002. (**376,096.95**)

3. The table below shows the prices (shs.) an amount of items bought in 2006 and 2007

Items	Price (shs.)		Amount
	2006	2007	
Mil k per litre	300	400	1
Eggs per tray	2500	3000	3
Cooking oil per litre	3000	8000	4
Flour per packet	1500	1800	15

Taking 2006 as the base year

- (a) Calculate the simple aggregate price index (**180.92**)
 (b) Calculate the weighted aggregate price index (**161.702**)
4. The table below shows the prices in US dollars and weights of five components of an engine, in 1998 and 2005

components	weight	Prices (US D)	
		1998	2005
A	6	35	60
B	5	70	135
C	3	43	105
D	2	180	290
E	1	480	800

- (a) Taking 1998 as the base year, calculate the
- (i) Simple aggregate price index (**172.03**)
 (ii) Price relative of each component (**171.4, 192.9, 244.2, 161.1, 166.7**)
 (iii) Weighted price index (**178.55**)

- (b) Using the price index in (a)(i) estimate the cost of the engine in 1998 if the cost of the engine in 2005 was 1600 USD (**896.11**)

5. The table below shows the prices and amounts of items bought in 2004 and 2005

Item	Prices (shs)		Amount
	2004	2005	
A	635	887.5	6
B	720	815	4
C	730	1045	3
D	362	503	7

- (a) Calculate the simple aggregate price index (**132.836**)
 (b) Calculate the weighted aggregate price index (**133.52**)
 (c) Calculate the price of an item costing 500 in 2004 using weighted aggregate price index above. (**667.64**)
6. The table below shows the prices of items per kg in the year 2005 and 2007

Item	Posho	Beans	Rice	Beef	Chicken
Price in 2005	1200	2000	1200	4000	8000
Price in 2007	1600	2500	1600	6000	9500

Calculate for 2007 using 2005a as the base year

- (a) Simple price index (**132.0833**)
 (b) Simple aggregate price index (**129.2683**)
7. The table below shows the prices in the year 2010 and 2018

Item	Price in 2010	Price in 2018
Flour per kg	3000	5400
Eggs per dozen	5,000	7800

Calculate for 2018 using 2010 as the base year

- (i) Simple price index (**168**)
 (ii) Simple aggregate price index (**165**)
8. The table below shows the prices and quantities of four items in the year 2020 and 2021

Item	Price per unit		Quantities	
	2020	2021	2020	2021
A	100	120	36	42
B	110	100	96	88
C	50	65	10	12
D	80	85	11	10

- (a) Calculate the price index (120, 90.91, 130, 106.25)
 (b) Simple aggregate price index (108.82)
 (c) Weighted aggregate price index (99.55)
 (d) Cost in 2021, A, B, C and Dare ingredients to make chapatti and in 2020a price of chapatti costed shilling 600 using index in (iii) above. (**shs. 597.3**)