P425/1
PURE MATHEMATICS
Paper 1
July/Aug. 2023
3 hours

UGANDA TEACHERS' EXAMINATIONS SCHEME

Uganda Advanced Certificate of Education

JOINT MOCK EXAMINATIONS

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B.**

Any additional question(s) answered will **not** *be marked.*

All *necessary working must be shown clearly.*

Begin each answer on a fresh sheet of paper.

squared paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn Over

SECTION A: (40 MARK)

Answer **all** *questions in this section.*

1. A perpendicular from point A(2, 1,-3) meets the plane $4x + 3y - 2z = 46$ at point N. Determine the coordinates of N. *(05 marks)*

2. The roots to a quadratic equation $2x^2 - 11x + 15 = 0$ are α and β where $\alpha > \beta$, form a quadratic equation whose roots are α and $-\beta$. *(05 marks)*

3. solve the equation $3\sin x + \cos 2x = 2$ for $0 \leq \theta \leq 2\pi$ *(05 marks)*

4. Expand $\sqrt{1 + 4x}$ upto the third nonzero term and hence find $\sqrt{17}$ correct to two decimal places. *(05 marks)*

5. Determine the equation of normal to the curve $3x^4 - 2xy^2 + 5x - y - 5 = 0$ at the point (1,1). *(05 marks)*

6. Find the equation of circle whose centre lies on the line y = x which passes in points (2,5) and (4,-1). *(05 marks)*

7. Find $\int \sin 7\theta \sin 11\theta d\theta$. *(05 marks)*

8. Given that $y = \tan\theta$ and $x = \sec^2\theta$, prove that $\frac{d^2y}{dx^2} = \frac{-1}{4}\cot^3\theta$. *(05 marks)*

SECTION B

Answer any **five** *questions from this section.*

9. (a) Evaluate $\int_0^1 x^2\,(2x^3+3)^4 dx$ *(06 marks)*

(b) Prove that if $y = \sqrt{\frac{1+\cos x}{1-\cos x}}$, $\frac{dy}{dx} = \frac{1}{\cos x - 1}$ *(06 marks)*

10. (a) The tangents to the parabola $y^2 = 4ax$ at the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ meet at point R. Find the coordinates of R *(07 marks)*

(b) Determine the focus of the parabola $y^2 - 2y - 8x - 17 = 0$ *(05 marks)*

11. Sketch a graph of $y = x^2 - 3x$ and hence find the volume of the solid generated when the area under this curve and x-axis is rotated by one revolution about the x-axis. *(12 marks)*

12. Prove that $\frac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x} = -\tan^2\left(\frac{x}{2}\right)$ and hence use it to

(a) Express $\tan^2 15^0$ in the form $p + q\sqrt{r}$ *(07 marks)*

(b) Solve the equation $\frac{2\sin x - 4\sin 2x + 2\sin 3x}{\sin x + 2\sin 2x + \sin 3x} + \sec^2\left(\frac{x}{2}\right) = 0$

for $0^0 \leq x \leq 360^0$ *(05 marks)*

13. Prove that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = y - 4 = \frac{z-5}{3}$ intersect. Hence find their point of intersection and the angle between them. *(12 marks)*

14. Express the function $f(x) = \frac{4x + 2x^2 + 5x^3 - 8}{x^2(4 - x^2)}$ into partial fractions and hence find $\int_1^{1.5} f(x)dx$. *(12 marks)*

Turn Over

15. (a) Find the complex number z such that $9z\bar{z} + 6i\bar{z} = 33 + 10i$ where $\bar{z}$ is the conjugate of z. *(06 marks)*

(b) Illustrate the region represented by $|z + 3i - 2| < 4$ where z is a complex number. *(06 marks)*

16. The temperature of a body is proportional to the difference in temperature T of the body at any time t and that of surroundings. If the temperature of the surroundings is 38^0C and the body cools from 90^0C to 80^0C in 10 minutes, find

(i) the temperature of this body in the next 8 minutes. *(09 marks)*

(ii) time taken for the body to cool to 65^0C. *(03 marks)*

END