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UNEB 2023

PROPOSED GUIDE APPLIED MATHEMATICS P425/2.

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SECTION A

QN.1

Let H represents the Head
T represents the Tail.

B_1 – Correct value of P(T)
B_1 – Correct value of P(H)

$P(T) + 2P(T) = 1$

$P(T) = \frac{1}{3}$, $P(H) = \frac{2}{3}$

Success, $p = \frac{1}{3}$, Failure, $q = \frac{2}{3}$, Number of trails, $n = 7$.

Let x be random variable for the number of tails that occured.

$P(x = 2) = {}^7C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^5$

$= \frac{224}{729}$ or 0.30727

B2 – Correct substitution into the formula

A_1 – Correct answer
Accept; decimals to atleast 4 d/ps.

05

QN: 2. (a)

$M_A u_A + M_B u_B = M_A v_A + M_B v_B$

$6 \times 4 - (2 \times 2) = 6 \times 2.6 + 2 v_B$

$v_B = 2.2 \, ms^{-1}$

m_1 – Correct substitution into the formula

A_1 – Correct velocity of B after collision

2(b)

Loss in kinetic energy

$= \frac{1}{2} \left[(6 \times 4^2 - 2 \times 2^2) - (6 \times 2.6^2 + 2 \times 2.2^2) \right]$

$= \frac{1}{2} (88 - 50.24)$

$= 18.88$ Joules

$m_1 B_1$ – Correct substitution into the formula

∴ Loss in kinetic energy = 18.88 Joules

A_1 – Correct answer

05

QN. 3.

$h = \frac{1}{2}$

x	fx	
0	0.1003	
$\frac{1}{2}$		0.0391
1		0.0801
$1\frac{1}{2}$		0.0602
2		0.0649
$2\frac{1}{2}$		0.0380
3	0.0327	
Sum	0.133	0.2823

05

$$\int_0^3 f(x)\,dx \simeq \frac{1}{4}(0.133 + 0.5646)$$

$$\simeq 0.1744$$

$$\simeq 0.174 \text{ to 3 dps}$$

Reject; equal sign (=)

B_1 – h value correct

B_1 – Correct sum in 2nd column.

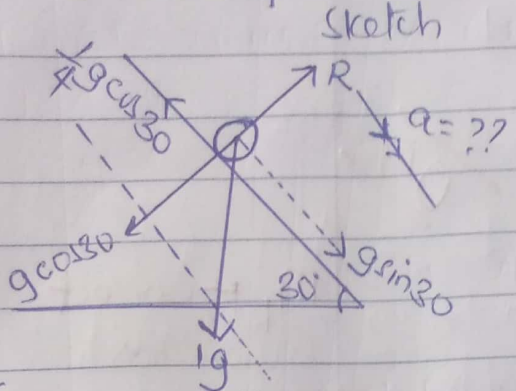
B_1 – correct sum in 3rd column.

M_1 – Correct substitution

A_1 – correct value

Reject; Answer not to 3dps

QN. 4



Force = ma.

$g\sin 30 - \frac{1}{4}g\cos 30 = a$

$a = 4.9 - \frac{1}{4}g\frac{\sqrt{3}}{2}$

$a = 2.77824\,ms^{-2}$

$v^2 = 0^2 + 2 \times 2.77824 \times 4$

$v = 4.71444\,ms^{-1}$

The ball reaches the ground with velocity of $4.71444\,ms^{-1}$

05

B_1 – Frictional force (Must be on the surface of the plane)

B_1 – Other relevant forces indicated using straight lines in proper directions.

B_1 – Correct value of acceleration to atleast 4dps

m_1 – Correct substitution.

A_1 – Correct velocity of the ball to atleast 4dps.

QN. 5

AGE (years)	f ('000)	w	f.d ('000)
0–10	15	10	1.5
10–20	19	10	1.9
20–30	16	10	1.6
30–40	18	10	1.8
40–60	30	20	1.5
60–80	6	20	0.3
80–90	1	10	0.1

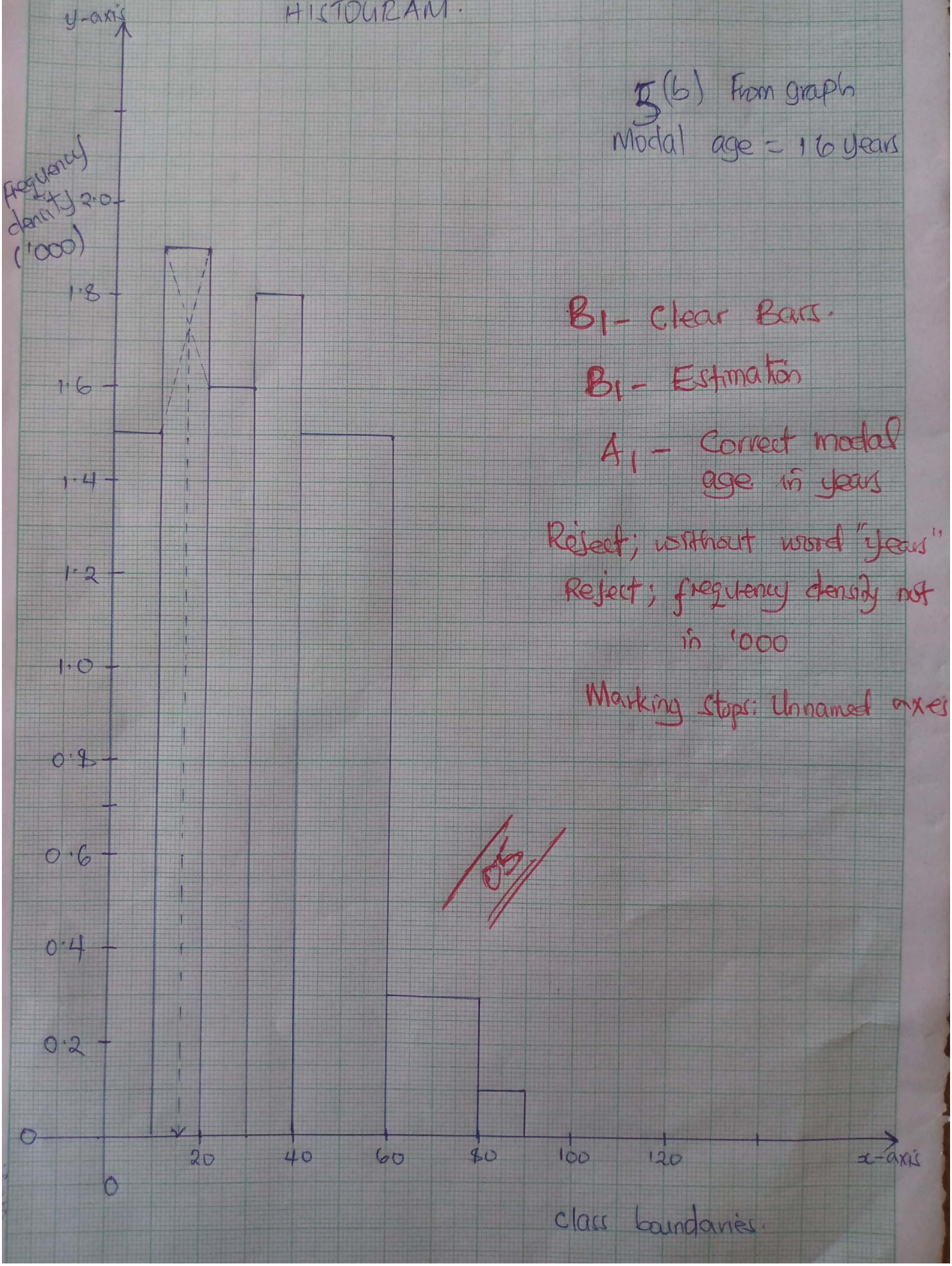
a) Shown on the graph.

B_1 – Class boundaries

B_1 – Column of f.d ('000)

5(a)

HISTOGRAM.



5(b) From graph

Modal age = 16 years

B_1 - Clear Bars.

B_1 - Estimation

A_1 - Correct modal age in years

Reject; without word "years"

Reject; frequency density not in '000

Marking Stops: Unnamed axes

05

QN. 6

$x = 6.45$, $y = 0.00215$ and $z = 2.7$

$$W = \frac{x + z^3}{\sqrt{y}}$$

05

$$W_{min} = \frac{6.445 + (2.65)^3}{\sqrt{0.002155}}$$

$W_{min} = 539.714728$

$$W_{max} = \frac{6.455 + (2.75)^3}{\sqrt{0.002145}}$$

$= 588.41365$

Interval in which $\frac{x + z^3}{\sqrt{y}}$ lies is $(539.714728, 588.41365)$

M_1 B Correct substitutions

B_1 – correct W_{min}

M_1 – correct substitution

B_1 – Correct W_{max}

A_1 – Correct interval.

QN. 7. (a)

$P(S') \times P(R') = P(S)$

$P(R')[1 - P(S)] = P(S)$

$P(R') = P(S)[1 + P(R')]$

$\frac{1}{4} = P(S)[1 + \frac{1}{4}]$

$P(S) = \frac{1}{5}$ or 0.2

B_1 – Independent expression

B_1 – Correct substitution

A_1 – Correct $P(S)$

7 (b)

05

$P(S' \cap R) = P(S') \times P(R)$

$= \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$ or 0.6

B_1 – using correct method

A_1 – Correct $P(S' \cap R)$

QN. 8

Let P be weight per unit area

Portion	Area	weight	Distance of c.o.g from AD
Square ABCD □	3600	3600P	30cm
Circle	400π	400πP	40 cm
Remainder	3600-400π	3600P-400πP	$\bar{x}$

Where $\bar{x}$ is the distance of c.o.g ~~from~~ of lamina from side AD.

Taking moment from side AD

D

$\bar{x}(3600-400\pi)P = 3600P \times 30 - 400\pi P \times 40$

A

$$\bar{x} = \frac{108000 - 16000\pi}{3600 - 400\pi}$$

$\bar{x} = 24.63746$cm

∴ The distance of centre of gravity of lamina from side AD is 24.63746cm

05

B_1 - Area of Square and circle

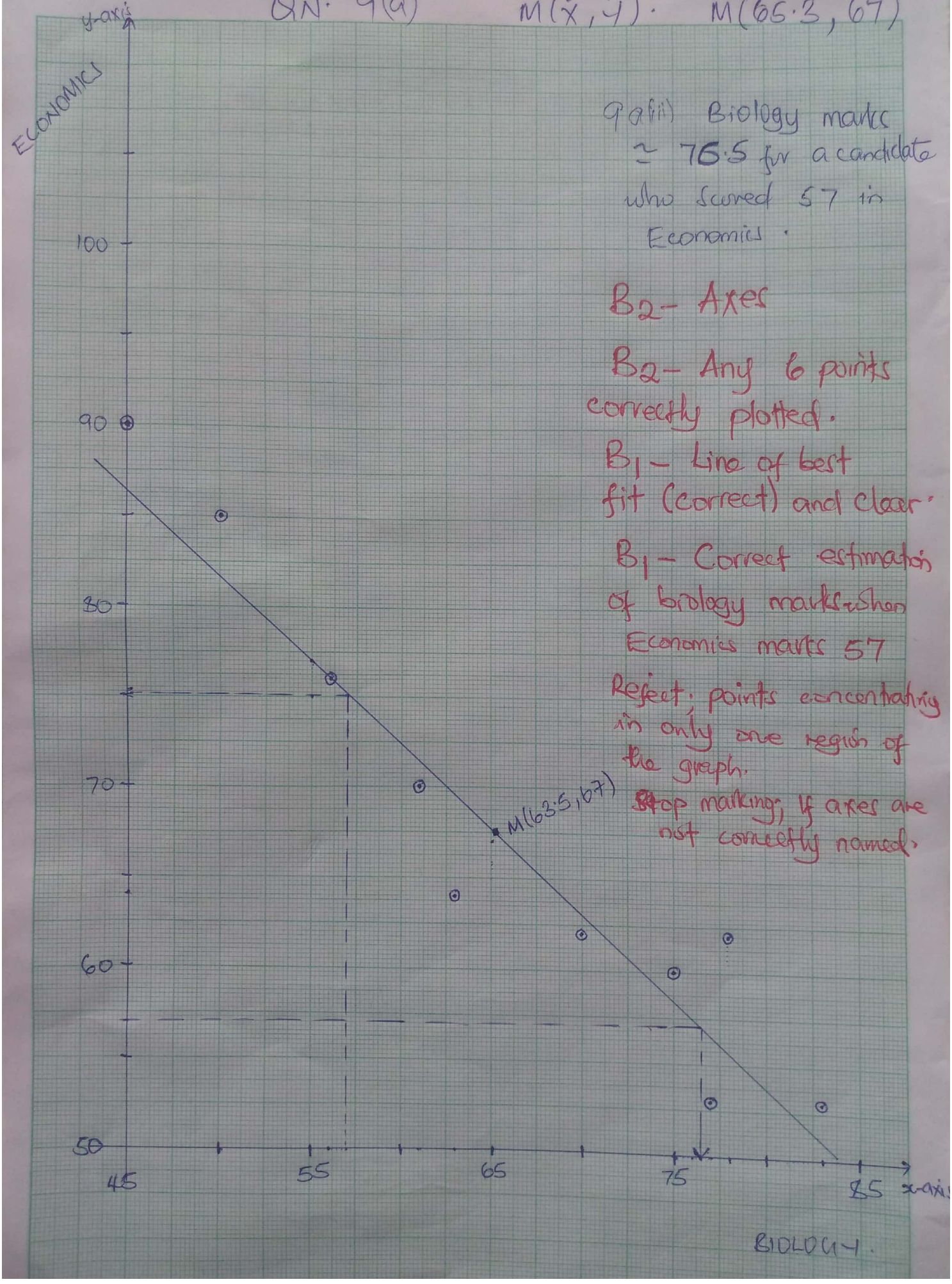
B_1 - Row of remainder (All values cor

B_1 - Correctly taking moment along AD

B_1 - Correct calculatus sum of moments

A_1 - Correct conclusion. Reject; without unit

QN. 9(a) $M(\bar{x}, \bar{y})$. M(65·3, 67)



9a(ii) Biology marks ≃ 76·5 for a candidate who scored 57 in Economics.

B_2 – Axes

B_2 – Any 6 points correctly plotted.

B_1 – Line of best fit (correct) and clear.

B_1 – Correct estimation of biology marks when Economics marks 57

Reject; points encountering in only one region of the graph.

Stop marking, if axes are not correctly named.

SECTION B

QN·9

9(a) shown on the graph.

9(b).

Let B represent Biology, E represents Economics.

	R_B	R_E	$d(R_B-R_E)$	d^2
A	10	1	9	81
B	6	5	1	1
C	8	3	5	25
D	7	4	3	9
E	3	8	-5	25
F	1	9.5	-8.5	72.25
G	4	6.5	-2.5	6.25
H	9	2	7	49
I	2	9.5	-7.5	56.25
J	5	6.5	-1.5	2.25
				$\Sigma d^2 = 327$

Using Spearman rank correlation,

$$\rho = 1 - \frac{6 \times 327}{10(99)}$$

$$\rho = -0.981818$$

Comment; There is very high negative correlation or since $|\rho| >$ Significant at 5% level, Significant at 1%

B_1 - R_B (correct ranking of Biology

B_1 - R_E (correct ranking of economics

B_1 - Σd^2 correct

Accept any other correct method and award accordingly.

$B1_1$ - Correct substitution into the method

A_1 - correct answer

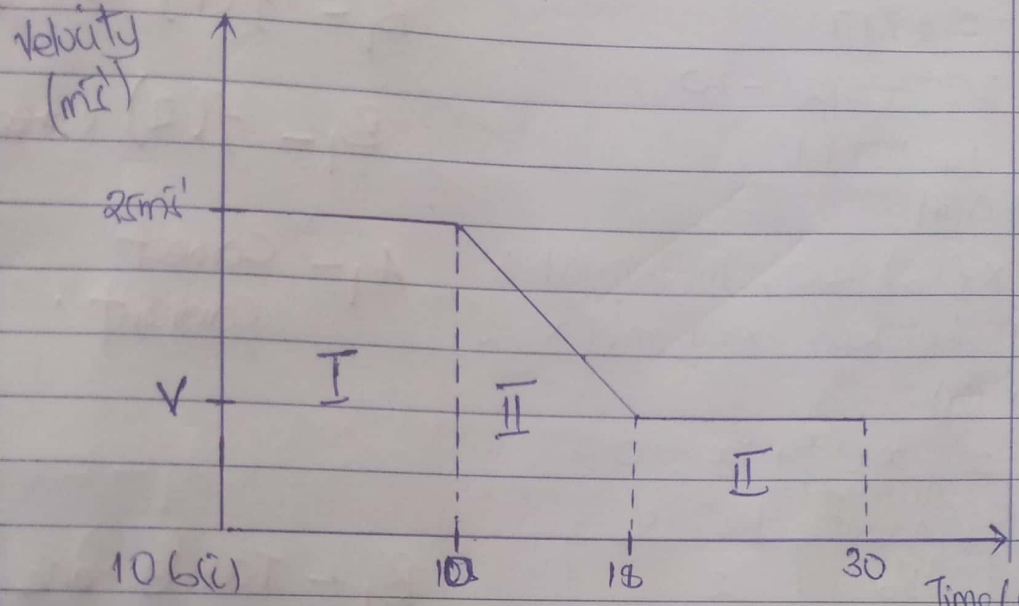
B_1 - For comment.

12 marks

QN. 10

10(a)

Velocity – Time graph



10 b(i)

Area I + Area II + Area III = 526

$250 + \frac{1}{2} \times 8(25+V) + 12V = 526$

$250 + 100 + 16V = 526$

$V = 11 ms^{-1}$

10 b(ii)

From $v = u + at$.

$11 = 25 + 8a$

$a = -1.75 ms^{-2}$ (Deceleration)

∴ The car decelerates at $1.75 ms^{-2}$

12 marks

B_2 – Velocity and Time axes correctly labelled.

B_1 – First part of motion (constant velocity)

B_1 – Decelerating part of motion.

M_1B_1 – Substituting in correct area method.

B_1 – Correct manipulation

A_1 – correct answer of V.

M_1B_1 – correct substitute into the equation

A_2 – correct deceleration with units.

QN. 11 a) (i)

$f(x) = xe^x + 5x - 10$

$f(1) = 1e^1 + 5(1) - 10$

$= -2.2817$

$f(2) = 2e^2 + 5(2) - 10$

$= 14.7781$

11 a(ii)

Since $f(1) \times f(2) < 0$, the equation has a root between $x=1$ and $x=2$.

11 (b)

Let $f(x) = xe^x + 5x - 10$.

Let the first approximation root be y_0.

1	y_0	2
-2.2817	0	14.7781

$$\frac{y_0 - 1}{2.2817} = \frac{1}{17.0598}$$

$y_0 = 1.1337$

$f(1.1337) = 1.1337e^{1.1337} + 5(1.1337) - 10$.

$= -0.8089$

Let the second approximation of root be y_1

1.1337	y_1	2
-0.8089	0	14.7781

$$\frac{y_1 - 1.1337}{0.8089} = \frac{2 - 1.1337}{14.7781 + 0.8089}$$

12

$y_1 = 1.1787$

∴ Root of the equation ≈ 1.179 3dps

B_1 – correct substitution of two values into the formula

B_1 – $f(1)$ correct

B_1 – $f(2)$ correct

A_1 – Correct comment.

B_1 – Table one

$m_1 B_1$ – correct value of y_0.

B_1 – $f(1.1337)$ correct

B_1 – Table two

B_1 – locating root

m_1 – correct method

A_1 – Root correct to 3dp.

QN. 12 (a)

For $-1 \leq x \leq 0$, $F(x) = \frac{1+x}{6}$.

$\frac{1}{6} < \frac{1}{2}$; Median doesn't lie in this range

B_1 - Testing range

For $0 \leq x \leq 2$, $F(x) = \frac{1+2x}{6}$

$\frac{4}{6} > \frac{1}{2}$; Median lies within this range.

let median be m.

M_1 - Correct method

$$\frac{1+2m}{6} = \frac{1}{2}.$$

$$1+2m = 3$$

$$m = 1.$$

$\therefore$ The median of the distribution = 1.

A_1 - correct median

12 (b).

For $-1 \leq x \leq 0$, $F(x) = \frac{1+x}{6}$

$$f(x) = \frac{d}{dx}\left(\frac{1+x}{6}\right)$$

B_1 - differentiating first function

$$f(x) = \frac{1}{6}.$$

For $0 \leq x \leq 2$, $F(x) = \frac{1+2x}{6}$

$$f(x) = \frac{d}{dx}\left(\frac{1+2x}{6}\right)$$

B_1 - Differentiating second function

$$f(x) = \frac{2}{6}$$

For $2 \leq x \leq 8/3$, $F(x) = \frac{4+3x}{12}$

$f(x) = \frac{d}{dx}\left(\frac{4+3x}{12}\right)$

$f(x) = 3/12$

For $x \geq 8/3$, $F(x) = 1$

$f(x) = \frac{d}{dx}(1)$

$f(x) = 0$

$$\therefore f(x) = \begin{cases} \frac{1}{6}; & -1 \leq x \leq 0 \\ 2/6; & 0 \leq x \leq 2 \\ 3/12; & 2 \leq x \leq 8/3 \\ 0; & \text{elsewhere} \end{cases}$$

A_1 – correct $f(x)$

12 (c)

$$P(1 \leq x \leq 2.5) = \frac{1}{3}\int_1^2 dx + \frac{1}{4}\int_2^{2.5} dx$$

$$= \frac{1}{3}[x]_1^2 + \frac{1}{4}[x]_2^{2.5}$$

$$= \frac{11}{24} \text{ or } 0.458333$$

B_1 – correct expression

B_1 – correct substitution

A_1 – correct probability.

Accept; answer to atleast 4dps.

12 (d)

Mean of $x = \frac{1}{6}\int_{-1}^{0} x\,dx + \frac{1}{3}\int_{0}^{2} x\,dx + \frac{1}{4}\int_{2}^{8/3} x\,dx$

$= \frac{1}{12}\left[x^2\right]_{-1}^{0} + \frac{1}{6}\left[x^2\right]_{0}^{2} + \frac{1}{8}\left[x^2\right]_{2}^{8/3}$

$= \frac{-1}{12} + \frac{4}{6} + \frac{7}{18}$

$= \frac{35}{36}$ or 0.97222

$\therefore$ Mean of $x = \frac{35}{36}$ or 0.972222

M_1 – correct method for finding mean.

B_1 – correct substitution

A_1 – correct Mean answer: Accept decimals to atleast 4dps.

12 marks

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Ethical.

QN. 13 (a)

Let the resultant force be R

$R = \begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}5\\2\end{pmatrix} + \begin{pmatrix}-2\\-11\end{pmatrix}$ Accept with or without units.

$R = (5\underset{\sim}{i} - 12\underset{\sim}{j})\ N.$

$|R| = \sqrt{5^2 + (-12)^2}$

$|R| = 13N$

13 (b).

Sum of moment, G

$G = \begin{vmatrix}2 & 2\\-3 & 3\end{vmatrix} + \begin{vmatrix}5 & -2\\2 & 3\end{vmatrix} + \begin{vmatrix}-2 & 3\\-11 & -2\end{vmatrix}$

$G = 12 + 19 + 37$

$= 68\ Nm$ clockwise

From $G - xFy + yFx = 0$

$-68 + 12x + 5y = 0$

$12x + 5y - 68 = 0$ is the equation of the line of action of resultant force.

(c)

$x = \frac{-G}{Fy}$, $x = \frac{68}{12} m$ or $5.6667\ m$.

∴ The resultant will cut cut the x-axis at 5.6667 m from the origin.

12

M_1 – correct method for finding resultant force

B_1 – correct method for finding magnitude of resultant force

A_1 – correct answer

B_1 – correct method for sum of moments.

B_1 – correct algebra

B_1 – correct value of G together with units.

B_1 – correct substitution.

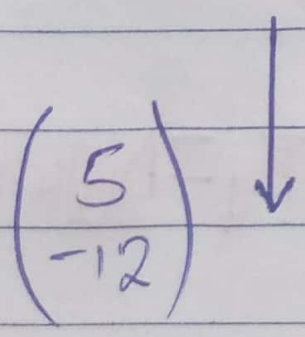
A_1 – correct equation

B_1 – Formula = $\frac{-G}{Fy}$ or letting $y = 0$

A_1 – value of distance to atleast 4dp. Accept; fractions.

13 (d).

Couple; equal but opposite force.



$$P + \begin{pmatrix} 5 \\ -12 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

B1 - correct

$$P = \begin{pmatrix} -5 \\ 12 \end{pmatrix} N.$$

∴ Force that should be added to form a couple is $(-5\underset{\sim}{i} + 12\underset{\sim}{j})N$. A1 - correct couple force.

QN. 14.

Let $x = N^{1/k}$ B_1 – Letting $x = N^{1/k}$.

$x^k - N = 0$

let $f(x) = x^k - N$, $f(x_n) = x_n^k - N$

$f'(x) = kx^{k-1}$, $f'(x_n) = kx_n^{k-1}$ B_1 – Differentiating

$$x_{n+1} = x_n - \frac{x_n^k - N}{kx_n^{k-1}}$$ B_1 – Correct Substitution into NRM

$$x_{n+1} = \frac{x_n \cdot x_n^{k-1} \cdot k - x_n^k + N}{kx_n^{k-1}}$$

$$x_{n+1} = \frac{(k-1)x_n^k + N}{kx_n^{k-1}}$$ B_1 – correct conclusion

$n = 0, 1, 2, 3, \ldots$

As required.

14 (b).

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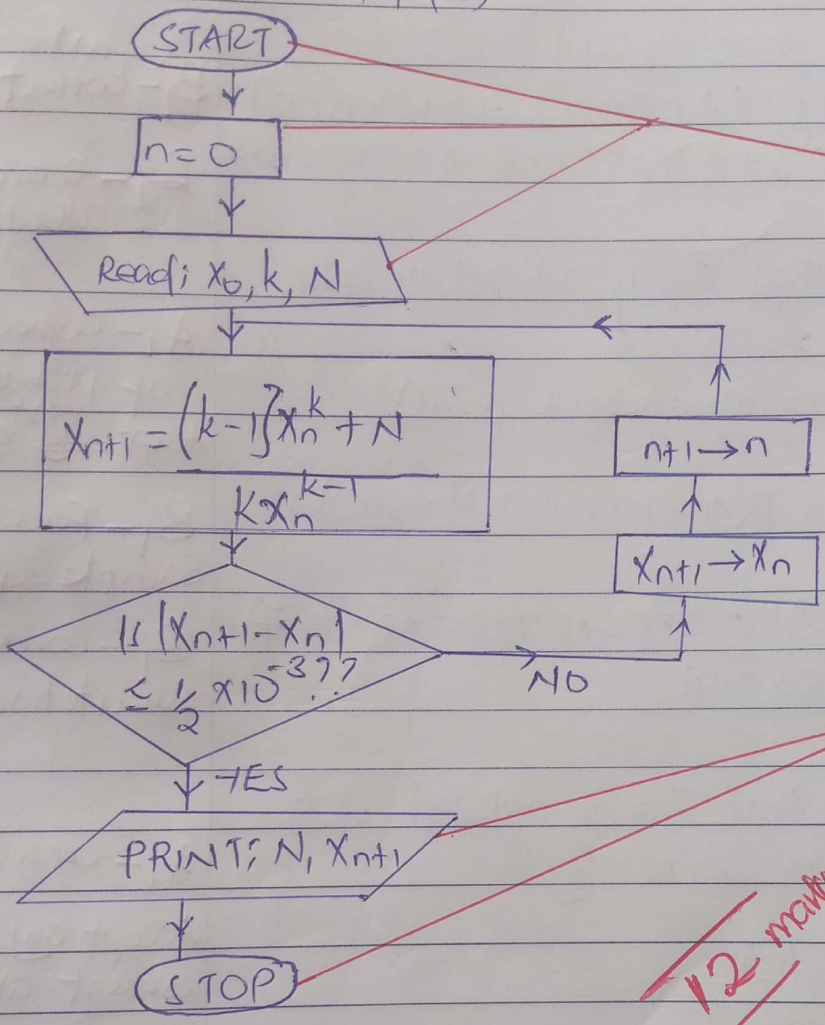
14 (b)



B_1

B_1 – correct.

B_1 – computation box and filling

B_1 – Loop with correct arrows

B_1 – Correct decision box.

B_1

Reject; Wrong shapes used

Reject; Without arrow direction and wrong arrows.

12 marks

14 (c)

Dry run. $N=13, X_0=1.6, k=4$.

n	X_n	X_{n+1}	Is $\lvert X_{n+1} - X_n \rvert$ less
0	1.6	1.99346	NO
1	1.99346	1.90536	NO
2	1.90536	1.89886	NO
3	1.89886	1.89882	YES

When $N=13$, Root $\simeq$ 1.899 (to 3 dps)

Correct

B_1 – X_n values

B_1 – $|X_{n+1} - X_n|$

A_1 – Correct not to atleast 3 dps.

Subject .....

QN. 15

15(a)(i) P(only one check point)

$= P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= 0.3 \times 0.5 \times 0.3 + 0.7 \times 0.5 \times 0.3 + 0.7 \times 0.5 \times 0.7$

$= 0.395$

∴ The probability that she is delayed at only one check point = 0.395

15 a(ii) P(two or more check point)

$= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C) + P(A \cap B \cap C)$

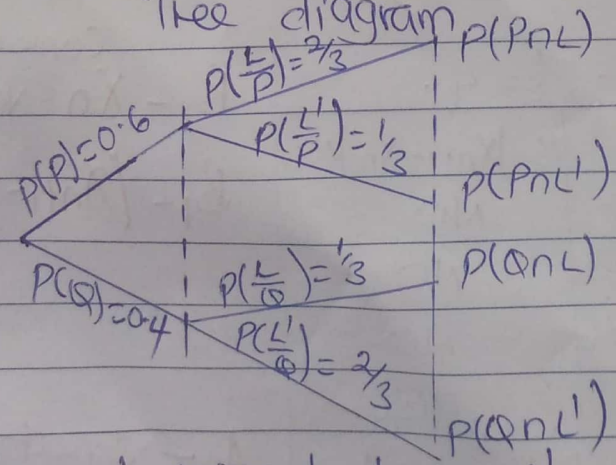
$= (0.3 \times 0.5 \times 0.3) + (0.3 \times 0.5 \times 0.7) + (0.7 \times 0.5 \times 0.7) + (0.3 \times 0.5 \times 0.7)$

$= 0.5$

∴ The probability that she is delayed in two or more check points is $\frac{1}{2}$.

15(b)

Tree diagram



where L represents Late, L' = Not late

B1 - all correct probabilities

B1 - Correct substitution

A1 - Correct answer of probability Accept ~~only~~ at least 3 dps.

B1 - For correct Sample Spaces.

B1 - Correct substitution

A1 - correct probability

Accept any other correct alternative

M1 - Correct method (clear Probability tree diagram)

P 15 6(i)

$P(\text{he is late}) = P(P \cap \frac{L}{P}) + P(Q \cap \frac{L}{Q})$

$= (0.6 \times \frac{2}{3}) + (0.4 \times \frac{1}{3})$

$= \frac{8}{15}$ or 0.53333

15 6(ii).

$P(\frac{P}{L'}) = \frac{P(P \cap L')}{P(L')} = \frac{P(P \cap \frac{L'}{P})}{P(L')}$

~~$= P$~~

$= \frac{0.6 \times \frac{1}{3}}{1 - \frac{8}{15}}$

$= \frac{3}{7}$ or 0.42857.

12 marks

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B_1 – Correct $P(P \cap \frac{L}{P})$ and $P(Q \cap \frac{L}{Q})$

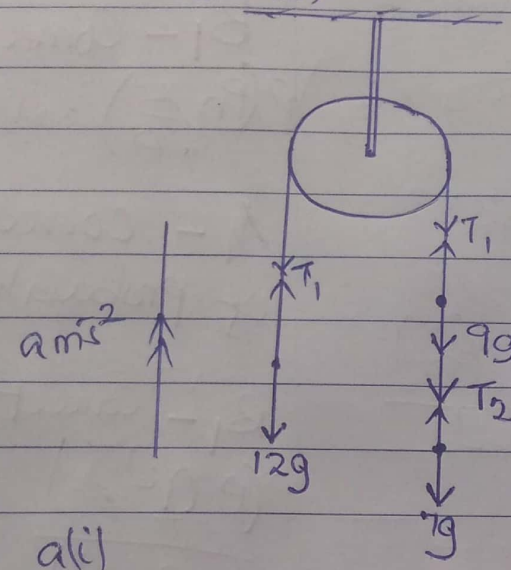
A_1 – Correct answer of probability.

B_1 – Correct expression $\frac{P(P \cap \frac{L'}{P})}{P(L')}$

A_1 – correct answer of probability Accept to atleast 4 d.p.

QN. 16

Sketch.



Let T_1 and T_2 be Tensions in the Strings.

Reject; strings not taut

16 a(i)

For 12kg mass

$T_1 - 12g = 12a$ —(i)

For 9kg mass

$T_2 + 9g - T_1 = 9a$ —(ii)

For 7kg mass

$7g - T_2 = 7a$ —(iii)

Eqn(i) + eqn(ii)

$T_2 + 9g - 12a - 12g = 9a$

$7g - 7a - 3g - 12a = 9a$

$a = \frac{4(9.8)}{28}$

$a = 1.4 ms^{-2}$

16a(ii) Tensions

$T_1 = 12(1.4) + 12 \times 9.8$

$T_1 = 134.4N$

$T_2 = 7(9.8) - 7 \times 1.4$

$T_2 = 58.8N$

$\therefore T_1 = 134.4N$ and $T_2 = 58.8N$.

16 (b)

From $v = u + at$

$v = 0 + 1.4 \times 1.5$

$v = 2.1 ms^{-1}$

$\therefore$ The velocity of 12kg mass after 1.5 seconds is $2.1 ms^{-1}$

12 marks

B_1 – For correct diagramatical set up.

B_1 – For Tensions/forces indicated; strings not taut

B_1 – Correct Expression of 12kg mass

B_1 – correct expression of 9kg mass

B_1 – Correct expression of 7kg mass

B_1 – Correct Substitution

A_1 – correct tension T_1

A_1 – correct tension T_2

A_1 – Correct acceleration with units.

M_1 – correct substitution

A_1 – Correct velocity with units