P425/2
Applied
Math.
Paper 2
July 2024
3 Hours.

ACEITEKA JOINT MOCK EXAMINATIONS 2024

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS PAPER 2

Time: 3 Hours

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in section A and only **five** questions in section B.*
- *Indicate the five questions attempted in section B in the table aside.*
- *Additional question(s) answered will **not** be marked.*
- ***All** working **must** be shown clearly.*
- *Graph paper is provided.*
- *Where necessary, take acceleration due to gravity, $g = 9.8\ m\ s^{-2}$.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

Question		Mark
Section A		
Section B		
Total		

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Five numbers have a median of 30. The biggest and second biggest numbers are twice the second smallest and smallest numbers respectively. The range of the numbers is equal to the second biggest number. Determine the numbers if their mean is 7.4. **[5 Marks]**

Qn 2: A golf player hits a ball from a point O on horizontal ground with velocity of $v\sqrt{13}\ \text{m s}^{-1}$ at an angle θ above the horizontal; where $\tan\theta = \frac{3}{2}$. The ball first hits the ground at a point A, where $\overline{OA} = 240$ m. Find:

(a). the value of v. **[3 Marks]**

(b). the Cartesian equation of the trajectory of the ball. **[2 Marks]**

Qn 3: The amount of a liquid remaining in a leaking drum for distances 20, 28, 33 and 42 km is 24, 21, 13 and 10 litres respectively.
Using linear interpolation/extrapolation, estimate the:

(a). Distance covered if 27 litres remained in the drum. **[3 Marks]**

(b). The amount of the liquid that remained in the drum if a distance of 29 km was covered. **[2 Marks]**

Qn 4: A discrete random variable X has a probability distribution function given by:

$$P(X = x) = \begin{cases} \frac{x}{k} & ; \quad x = 1, 2, 3, \ldots\ldots, n \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

If $P(X = 2) = \frac{2}{15}$, find the value of k and n. **[5 Marks]**

Qn 5: A particle is moving on the inside of smooth surface of a fixed spherical bowl of radius 2 m. It describes a horizontal circle at a distance 1.2 m below the centre of the bowl. Find:

(a). the speed of the particle. **[3 Marks]**

(b). the time taken by the particle to perform a complete revolution. **[2 Marks]**

Qn 6: Given that $a = 84.1$ and $b = 4.3$ are rounded off with corresponding percentage errors of 0.05 and 0.5, find the percentage error in $(a - b)$; correct to 3 significant figures. **[5 Marks]**

Qn 7: A biased die is tossed such that the probability of obtaining a six is $\frac{1}{10}$. If it is tossed 120 times, find the probability that there are less than 15 sixes. **[5 Marks]**

Qn 8: A uniform ladder of weight, W, and length, $2a$, rests in limiting equilibrium with one end on a rough horizontal ground and the other end on a rough

vertical wall. The coefficients of friction between the ladder and the ground and between the ladder and the wall are μ and λ respectively.
If the ladder makes an angle, θ, with the ground; where $\tan\theta = \frac{5}{12}$, show that $5\mu + 6\lambda\mu - 6 = 0$. [5 Marks]

Section B (60 Marks)

Answer any **five** *questions from this section.*
All *questions carry equal marks.*

Question 9:
Nine voters in Jinja and Kampala were asked to give the government a score out of 100, on each of the nine issues. The results are shown below.

ISSUES	A	B	C	D	E	F	G	H	I
JINJA	45	38	65	80	70	45	25	95	77
KAMPALA	73	82	61	43	48	65	90	30	48

(a). Plot a scatter diagram for the data. [3 Marks]
(b). Draw a line of best fit on the scatter diagram; and use it to estimate:
(i). The voter's score in Kampala on an issue which was given a score of 89 in Jinja.
(ii). The voter's score in Jinja on an issue which was given a score of 55 in Kampala. [4 Marks]
(c). Calculate the rank correlation coefficient between the voters in the two districts. Comment on your result at 5% level of significance. [5 Marks]

Question 10:
The engine of a car of mass 1000 kg works at a constant rate of 15 kW when travelling along a straight level road with maximum speed of 120 km h^{-1}.
(a). Calculate the total resistance to the motion of the car. [2 Marks]
(b). The resistance to motion is directly proportional to the square of the speed. The car now moves up a road of inclination θ, where $\sin\theta = \frac{1}{25}$. Find:
(i). the rate at which the engine is working when the car is moving at a constant speed of 40 km h^{-1}.
(ii). the time the car takes to come to momentary rest if its engine is shut off at the instant when the speed is 40 km h^{-1}. [10 Marks]

Question 11:

(a). Use trapezium rule with strip width of 1.6 to estimate the value of $\int_2^{10} \frac{x}{\sqrt{x-1}} dx$, correct to **three** decimal places.

[4 Marks]

(b). (i). Evaluate $\int_2^{10} \frac{x}{\sqrt{x-1}} dx$, correct to **three** decimal places.

(ii). Calculate the error in your estimation in (a) above.

(iii). Suggest how the error may be reduced. **[8 Marks]**

Question 12:

(a). Two players A and B take turns to toss a tetrahedral die until a 4 appears. A person who first throws a 4 wins the game. Assuming that A throws first, find the probability that A wins the game.

[5 Marks]

(b). A spare parts dealer receives spare parts from three different suppliers, X, Y and Z. Of the spare parts received, $\frac{2}{5}$ are from X, $\frac{7}{20}$ from Y and the rest from Z. It is known that $\frac{2}{25}$ of the spare parts supplied by X, $\frac{1}{10}$ supplied by Y and $\frac{1}{20}$ supplied by Z; are defectives. If a spare part is bought for the dealer, find the probability that the spare part is:

(i). defective.

(ii). either from Y or it is defective.

(iii). from X given that it is not defective.

-3	-2	-1	0	1	2	3
-53	-19	-3	1	-1	-3	1

[7 Marks]

Question 13:

At time, t seconds, the velocities, $\underset{\sim}{v_1}$ and $\underset{\sim}{v_2}$ of two particles P_1 and P_2 respectively, are given by:

$$\underset{\sim}{v_1} = \left(2t\underset{\sim}{i} - 3t^2\underset{\sim}{j}\right) \text{ m s}^{-1}, \qquad \underset{\sim}{v_2} = \left[t^3\underset{\sim}{i} + (2t-3)\underset{\sim}{j}\right] \text{ m s}^{-1}$$

(a). Find the non-zero value of t for which the acceleration of P_1 and P_2 are perpendicular.

[3 Marks]

(b). Obtain the velocity of P_1 relative to P_2 when their accelerations are perpendicular.

[2 Marks]

(c). Given that P_1 and P_2 are at the origin when $t = 0$, find the distance between P_1 and P_2 when their acceleration are perpendicular.

[7 Marks]

Question 14:

8 − 12 + 1

(a). By plotting graphs of $y = 2\sin x$ and $y = x$ on the same axes, obtain to 2 decimal places the roots of the equation $x - 2\sin x = 0$ in the interval $-2 \leq x \leq 2$.

−4 + 1

27 − 27 + 1

[8 Marks]

(b). The equation $x^3 - 3x^2 + 1 = 0$ has one negative root and two positive roots. Use a suitable table of values to locate the interval in which each of the root lies.

[4 Marks]

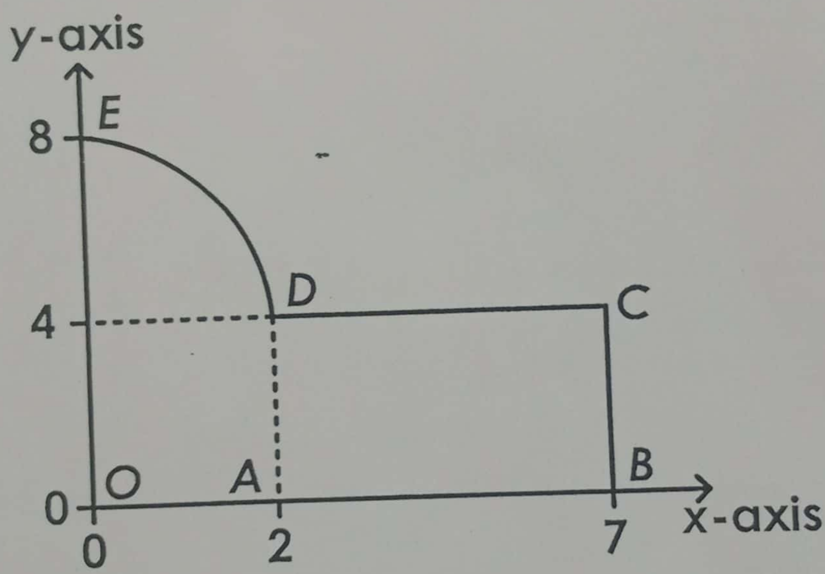
Question 15:
Biscuits are produced with weight, W grams, where $W \sim N(10, 4)$ and are packed at random into boxes consisting of 25 biscuits.
Find the probability that:

(a). a biscuit chosen at random weighs less than 9.5 g. **[4 Marks]**

(b). the contents of a box weigh between 247 g and 253 g. **[4 Marks]**

(c). the mean weight of the biscuits in the box is greater than 10.2 g. **[4 Marks]**

Question 16:
In the figure above, ED is a portion of the curve $y = 8 - x^2$ and $ABCD$ is a rectangle in which $\overline{BC} = 4$ cm and $\overline{CD} = 5$ cm.



(a). Show, by integration, that the x-coordinate of the centroid of the portion $OADE$ is $\frac{9}{10}$. **[6 Marks]**

(b). Find the distance of the centre of mass of the whole lamina $OBCDE$ from OE. **[6 Marks]**

END

SNo.	Working	Marks
1	Let the numbers in ascending order be: $X = \{a, b, c, d, e\}$ Median, $c = 30$ but, $e = 2b$, and, $d = 2a$ $\therefore X = \{a, b, 30, 2a, 2b\}$ also, Range = Second biggest number $2b - a = 2a$ $2b = 3a$ $b = 1.5a$ $\therefore X = \{a, 1.5a, 30, 2a, 3a\}$ $\sum X = a + 1.5a + 30 + 2a + 3a = 7.5a + 30$ but, Mean $= \frac{\sum X}{n} = 36$ $\frac{7.5a+30}{5} = 36$ 7·4 $7.5a + 30 = 180$ $7.5a = 150$ $a = 20$ The numbers in ascending order are: $X = \{a, 1.5a, c, d, 2a, 3a\}$ $= \{20, 1.5 \times 20, 30, 2 \times 20, 3 \times 20\}$ $= \{20, 30, 30, 40, 60\}$	B1-relation btn 1st and 2nd number M1-equating mean B1-1st number M1-substitution A1-output
		05
2	(a). $\tan\theta = \frac{3}{2}$, $\Rightarrow \sin\theta = \frac{3}{\sqrt{3^2+2^2}} = \frac{3}{\sqrt{13}}$, $\cos\theta = \frac{2}{\sqrt{13}}$ Range $= \overline{OA}$ $\frac{2u^2 \sin\theta\cos\theta}{g} = 240$ $2u^2\sin\theta\cos\theta = 240g$ $2 \times \left(v\sqrt{13}\right)^2 \times \frac{3}{\sqrt{13}} \times \frac{2}{\sqrt{13}} = 240 \times 9.8$ $12v^2 = 2352$ $v^2 = 196$ $v = 14$ [Accept: $v = 14\ \mathrm{m\,s^{-1}}$.] (b).	B1-sin θ and cos θ (or $\theta = 56.31°$) M1-substitution A1-value of v

		$y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}$ $y = x \times \dfrac{3}{2} - \dfrac{9.8 \times x^2\left[1+\left(\frac{3}{2}\right)^2\right]}{2\times\left(14\sqrt{13}\right)^2}$ $y = \dfrac{3}{2}x - \dfrac{1}{160}x^2$ $160y = 240x - x^2$	**M1**-substitution **A1**-simplifired output
			05
3	(a).		

Distance	x	20	28
Amount left	27	24	21

my

B1-extracting out necessary values

$$\frac{x-20}{28-20} = \frac{27-24}{21-24}$$ my

$$\frac{x-20}{8} = -1$$

M1-equating quotients

$$x = 12 \text{ km}$$ A1

A1-output

(b).

Distance	28	29	33
Amount left	21	y	13

$$\frac{y-13}{21-13} = \frac{29-33}{28-33}$$ my

$$\frac{y-13}{8} = 0.8$$

M1-equating quotients

$x = 19.4$ litres remaining A1

A1-output

05

4

$$P(X=2) = \frac{2}{15}$$
$$\frac{2}{k} = \frac{2}{15}$$
$$k = 15$$

B1-value of k

$$\sum_{all\ x} P(X=x) = 1$$
$$\frac{1}{15} + \frac{2}{15} + \frac{3}{15} + \cdots + \frac{n}{15} = 1$$
$$1 + 2 + 3 + \cdots + n = 15$$

M1-addition and equating to 1

$$\frac{1}{2}n(n+1) = 15$$
$$n(n+1) = 30$$
$$n^2 + n - 30 = 0$$

M1-fsummation in terms of n

<table>
<tr><td></td><td>

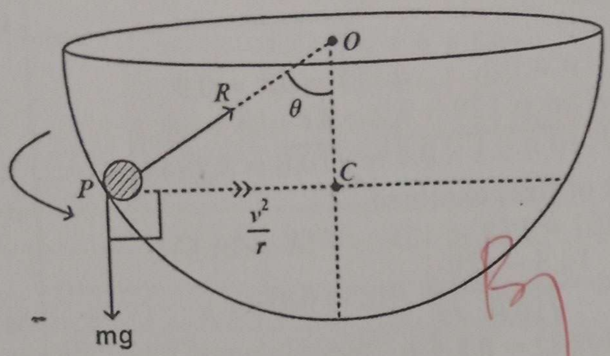
$$n = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (30)}}{2 \times 1}$$

$n = 5,$ or, $n = -6$

but, $n \neq -6,$ $\therefore n = 5$

</td><td>M1-solving

A1-conclusion</td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>5</td><td>



B1

(a).

For the horizontal circle,

$\overline{OP} = 2\text{ m}, \quad \overline{OC} = 1.2\text{ m}$

Radius, $r = \overline{PC} = \sqrt{2^2 - 1.2^2} = 1.6\text{ m}$ B1

Speed, $v = \sqrt{rg\tan\theta}$

$$= \sqrt{1.6 \times 9.8 \times \frac{1.6}{1.2}} = \sqrt{\frac{1568}{75}} = \frac{28\sqrt{6}}{15} \approx 4.5724\text{ m s}^{-1}$$ M1 A1

(b).

Period, $T = \frac{2\pi r}{v} = \frac{2\pi \times 1.6}{\sqrt{\frac{1568}{75}}} = 2.1987\text{ s}$ A1

</td><td>B1-radius

M1 A1-substitution and output

M1 A1-substitution and output</td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>6</td><td>

$a = 84.1, \quad \Rightarrow \Delta a = \frac{0.05}{100} \times 84.1 = 0.04205$

$b = 4.3, \quad \Rightarrow \Delta b = \frac{0.5}{100} \times 4.3 = 0.0215$

Absolute eror in $(a - b) = |\Delta a| + |\Delta b|$

$= |0.04205| + |0.0215| = 0.06355$

Exact value $= a - b = 84.1 - 4.3 = 79.8$

Percentage error $= \frac{\text{Absolute eror}}{\text{Exact value}} \times 100$

$= \frac{0.06355}{79.8} \times 100 = 0.079637 \approx 0.0796\text{ (3 s.f)}$

ALT:

Working value, $(a - b) = 84.1 - 4.3 = 79.8$

$(a - b)_{\min} = a_{\min} - b_{\max}$

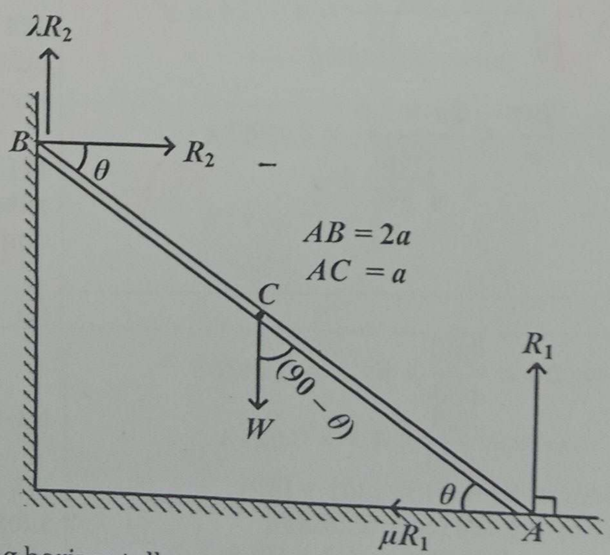
$= (84.1 - 0.04205) - (4.3 + 0.0215)$

$= 79.73645$

</td><td>B1-both errors

M1 A1-substitution and output

M1 A1-substitution and output to 3 s.f</td></tr>
</table>

	$(a-b)_{\max} = a_{\max} - b_{\min}$ $= (84.1 + 0.04205) - (4.3 - 0.0215)$ $= 79.86355$ $\text{Maximum error} = \dfrac{(a-b)_{\max} - (a-b)_{\min}}{2}$ $= \dfrac{79.86355 - 79.73645}{2} = 0.06355$ $\text{Percentage error} = \dfrac{\text{Maximum error}}{\text{Working value}} \times 100$ $= \dfrac{0.06355}{79.8} \times 100 = 0.079637 \approx 0.0796\ (3\text{ s.f})$	**M1 A1**- substitution and output **M1 A1**- substitution and output to 3 s.f
		05
7	$n = 120, \quad p = 0.1, \quad q = 1 - 0.1 = 0.9$ $\mu = np = 120 \times 0.1 = 12$ $\sigma = \sqrt{npq} = \sqrt{120 \times 0.1 \times 0.9} = \sqrt{10.8} = 3.2863$ B1 Let $X\sim$ be the number of sixes obtained. $P(\text{less than } 15) = P(X < 15) \longrightarrow P(X < 14.5)$ $= P\left(Z < \dfrac{14.5 - 12}{\sqrt{10.8}}\right) = P(Z < 0.761)$ M1 $= 0.5 + \phi(0.761) = 0.5 + 0.2767 = 0.7767$ B1 M1 A1	**B1**-both μ and σ **M1**- standardisation **B1 M1 A1**- table value, addition and output
		05
8	 Resolving horizontally, $R_2 = \mu R_1$ Resolving vertically, $R_1 + \lambda R_2 = W$ $R_1 + \lambda \times \mu R_1 = W$	**M1**-resolving vertically and horizontally

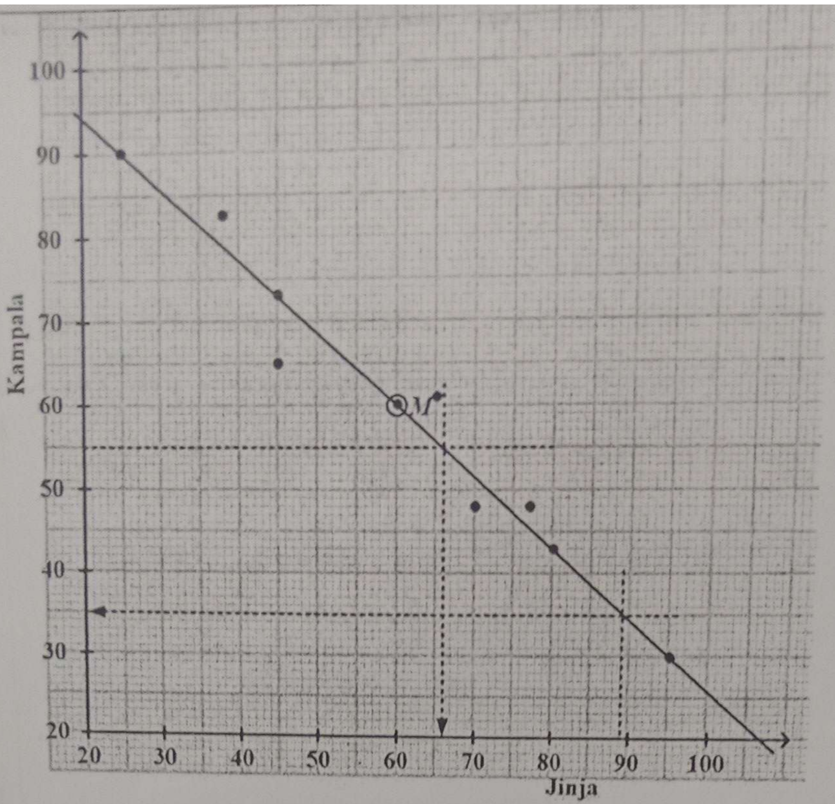
	$$(1+\lambda\mu)R_1 = W$$ $$R_1 = \frac{W}{1+\lambda\mu}$$ Taking moments about B, $$R_1 \times 2a\cos\theta = \mu R_1 \times 2a\sin\theta + W \times a\cos\theta$$ $$2R_1 = 2\mu R_1 \tan\theta + W$$ $$2\times\left(\frac{W}{1+\lambda\mu}\right) = 2\mu\times\left(\frac{W}{1+\lambda\mu}\right)\times\frac{5}{12} + W$$ $$\left(\frac{2}{1+\lambda\mu}\right) = \frac{5}{6}\mu\left(\frac{1}{1+\lambda\mu}\right) + 1$$ $$2 = \frac{5}{6}\mu + 1 + \lambda\mu$$ $$12 = 5\mu + 6 + 6\lambda\mu$$ $$12 = 5\mu + 6 + 6\lambda\mu$$ $$5\mu + 6\lambda\mu - 6 = 0, \quad \text{as required}$$	**B1**-one reaction eliminated **M1**-taking moments about a convenient point **M1**-substitution for R_1 **B1**-required expression
		05
9	(table below)	**B1**-both ranking correct **B1**-$\sum d^2$ correct

x	y	R_x	R_y	d	d^2
45	73	6.5	3	3.5	12.25
38	82	8	2	6	36
65	61	5	5	0	0
80	43	2	8	- 6	36
70	48	4	6.5	- 2.5	6.25
45	65	6.5	4	2.5	6.25
25	90	9	1	8	64
95	30	1	9	- 8	64
77	48	3	6.5	- 3.5	12.25
$\sum x =$ 540	$\sum y =$ 540				$\sum d^2 = 237$

(a).

$$\bar{x} = \frac{\sum x}{n} = \frac{540}{9} = 60, \qquad \bar{y} = \frac{\sum y}{n} = \frac{540}{9} = 60$$

$\therefore$ Mean point is $M(60, 60)$ — **B1**-meant point



		B1-both axes labelled and with uniform scale B1 B1-plotting B1-line of best fit
	(b). (i). The voter's score in Kampala on an issue which was given a score of 89 in Jinja is 35.	B1-estimation
	(ii). The voter's score in Jinja on an issue which was given a score of 55 in Kampala is 66.	B1-estimation
	(c). $\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 237}{9(9^2-1)} = -0.975$ **Comment:** Significant at 5%. (or Significant at 1%)	**M1 A1**-substitution and output **B1**-comment
		12
10	(a). $u = 120 \text{ km h}^{-1} = \frac{120 \times 1000}{3600} = \frac{100}{3} \text{ m s}^{-1}$ At maximum speed, acceleration is zero. $\because$ Resistance = Tractive force $\therefore \text{Resistance} = \frac{P}{u} = 15000 \div \frac{100}{3} = 450 \text{ N}$ M1 A1	**M1 A1**-division and output
	(b). (i). $\text{Resistance} = kv^2$ $450 = k \times \left(\frac{100}{3}\right)^2$ $k = \frac{81}{200}$ B1 When moving up the incline,	**B1**-value of proportionality constant

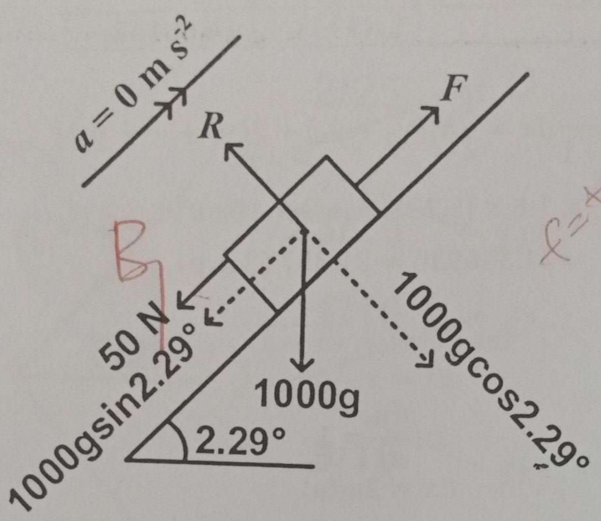
$$v = 40\ \text{km h}^{-1} = \frac{40 \times 1000}{3600} = \frac{100}{9}\ \text{m s}^{-1}$$

$$\text{New resistance} = kv^2 = \frac{81}{200} \times \left(\frac{100}{9}\right)^2 = 50\ \text{N}$$ B1

$$\sin\theta = \frac{1}{25}, \qquad \Rightarrow \theta = 2.29°$$

B1-new resistance

B1



f = tractive force

Resolving along the plane,

$$F = 50 + 1000g \sin 2.29°$$

$$= 50 + 1000 \times 9.8 \times \frac{1}{25} = 442\ \text{N}$$ B1

$$\text{Power of the engine} = Fv = 442 \times \frac{100}{9} = \frac{44200}{9}\ \text{W}$$ M1 A1

M1 B1-resolving and output
M1 A1-multiplication and output

(ii).
When the engine is switched off,

$$0 - (50 + 1000g \sin 2.29°) = 1000a$$ M1

$$0 - \left(50 + 1000 \times 9.8 \times \frac{1}{25}\right) = 1000a$$

$$-442 = 1000a$$

$$a = -0.442\ \text{m s}^{-2}$$ B1

$$v = u + at$$

$$0 = \frac{100}{9} - 0.442t$$ M1

$$t = 25.1383\ \text{s}$$

A1

M1-resolving and equating to ma
A1-acceleration

M1-substitution
A1-required time

11	(a).	12
	$y_n = \dfrac{x_n}{\sqrt{x_n - 1}}, \qquad h = 1.6$	

n	x_n	y_0, y_5	$y_1, \dots y_4$
0	2	2.00000	
1	3.6		2.23263
2	5.2		2.53734
3	6.8		2.82355
4	8.4		3.08790
5	10	3.33333	
Total		5.33333	10.68142

B1-x_n values

B1-y_n values

$$\int_2^{10} \frac{x}{\sqrt{x-1}} dx \approx \frac{1}{2} h[(y_0 + y_4) + 2(y_1 + \dots + y_3)]$$
$$\approx \frac{1}{2} \times 1.6 \times [5.33333 + 2 \times 10.68142]$$
$$\approx 21.356936 \approx 21.357 \text{ (3 d.p)}$$

M1-substitution
A1-output to 3 d.p

(b). (i).

let, $u = \sqrt{x-1}$
$$u^2 = x - 1$$
$$2u\frac{du}{dx} = 1$$
$$dx = 2udu$$

x	u
2	1
10	3

B1-changing variable and limits

$$\int_2^{10} \frac{x}{\sqrt{x-1}} dx = \int_1^3 \frac{u^2+1}{u} \times 2udu = 2\int_1^3 (u^2+1)\, du$$
$$= 2\left[\frac{1}{3}u^3 + u\right]_1^3$$
$$= 2\left[\left(\frac{1}{3} \times 3^3 + 3\right) - \left(\frac{1}{3} \times 1^3 + 1\right)\right] = \frac{64}{3} \approx 21.333 \text{ (3 d.p)}$$

M1-substitution

M1-integration

M1 A1-substitution and output

(ii).

Error $= |21.333 - 21.357| = 0.024$

M1 A1-substitution and output

(iii).

The error may be reducd by increasing the number of ordinates. (reducing ~~increasing~~) width of strips

B1-suggestion

12	(a).	12

$$P(\text{win}) = P(\text{throws a } 4) = \frac{1}{4}$$
$$P(\text{lose}) = 1 - \frac{1}{4} = \frac{3}{4}$$

B1-prob of win and lose

$P(A \text{ wins})$
$= P(A_1) + P(A_1' \cap B_1' \cap A_2) + P(A_1' \cap B_1' \cap A_2' \cap B_2' \cap A_3) + \cdots$

$$= \frac{1}{4} + \left(\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}\right) + \left(\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}\right) + \cdots$$

M1-substitution

$$= \frac{1}{4}\left[1 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^4 + \cdots\right]$$

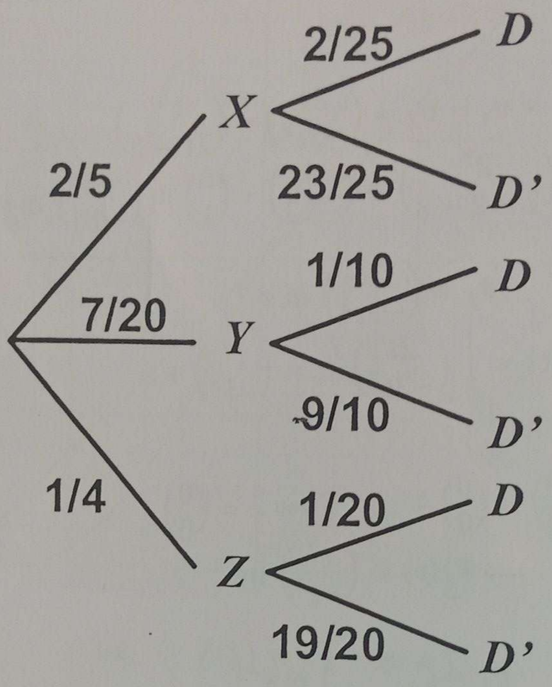
M1-factorising to obtain a G.P

This is a G.P with $a = 1$ and $r = \left(\frac{3}{4}\right)^2$

$$P(A \text{ wins}) = \frac{1}{4}\times\frac{a}{1-r} = \frac{1}{4}\times\frac{1}{1-0.75^2} = \frac{4}{7}$$

M1 A1-substitution and output

(b).



B1-tree diagram

(i).

$$P(D) = P(X\cap D) + P(Y\cap D) + P(Z\cap D)$$

$$= \left(\frac{2}{5}\times\frac{2}{25}\right) + \left(\frac{7}{20}\times\frac{1}{10}\right) + \left(\frac{1}{4}\times\frac{\cancel{19}}{\cancel{20}}\right)\ \left(\frac{1}{20}\right)$$

$$= \frac{4}{125} + \frac{7}{200} + \frac{19}{80} = \frac{609}{2000} = 0.3045$$

M1-substitution
A1-output

(ii).

$$P(Y\cup D) = P(Y) + P(D) - P(Y\cap D)$$

$$= \frac{7}{20} + \frac{609}{2000} - \frac{7}{200} = \frac{1239}{2000} = 0.6195$$

M1 A1-substitution and output

(iii).

$$P(X/D') = \frac{P(X\cap D')}{P(D')}$$

$$= \left(\frac{2}{5}\times\frac{23}{25}\right) \div \left(1 - \frac{609}{2000}\right) = \frac{736}{1391} \approx 0.5291$$

M1 A1-substitution and output

12

13 (a).

$$\underset{\sim}{v_1} = \begin{pmatrix} 2t \\ -3t^2 \end{pmatrix} \text{ m s}^{-1}, \quad \Rightarrow \underset{\sim}{a_1} = \frac{d}{dt}\begin{pmatrix} 2t \\ -3t^2 \end{pmatrix} = \begin{pmatrix} 2 \\ -6t \end{pmatrix} \text{ m s}^{-2}$$

B1-both

accelerations

$$\underset{\sim}{v_2} = \begin{pmatrix} t^3 \\ 2t-3 \end{pmatrix} \text{ m s}^{-1}, \quad \Rightarrow \underset{\sim}{a_2} = \frac{d}{dt}\begin{pmatrix} t^3 \\ 2t-3 \end{pmatrix} = \begin{pmatrix} 3t^2 \\ 2 \end{pmatrix} \text{ m s}^{-2}$$

For perpendicular accelerations,

$$\underset{\sim}{a_1}.\underset{\sim}{a_2} = 0$$

$$\begin{pmatrix} 2 \\ -6t \end{pmatrix}.\begin{pmatrix} 3t^2 \\ 2 \end{pmatrix} = 0$$

$$6t^2 - 12t = 0$$

$$6t(t-2) = 0$$

$$t = 0, \quad \text{or,} \quad t = 2$$

$$\text{but,} \quad t \neq 0, \quad \therefore t = 2\text{ s}$$

M1-dotting and equating to 0

A1-conclusion

(b).

When $t = 2$,

$$_1\underset{\sim}{v_2} = \underset{\sim}{v_1} - \underset{\sim}{v_2} = \begin{pmatrix} 2t \\ -3t^2 \end{pmatrix} - \begin{pmatrix} t^3 \\ 2t-3 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times 2 \\ -3\times 2^2 \end{pmatrix} - \begin{pmatrix} 2^3 \\ 2\times 2-3 \end{pmatrix} = \begin{pmatrix} 4 \\ -12 \end{pmatrix} - \begin{pmatrix} 8 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -13 \end{pmatrix} \text{ m s}^{-1}$$

M1 A1-substitution and output

(c).

$$\underset{\sim}{v_1} = \begin{pmatrix} 2t \\ -3t^2 \end{pmatrix} \text{ m s}^{-1}$$

$$\underset{\sim}{r_1}(t) = \int \begin{pmatrix} 2t \\ -3t^2 \end{pmatrix} dt = \begin{pmatrix} t^2 \\ -t^3 \end{pmatrix} + \underset{\sim}{c}$$

When $t = 0, \underset{\sim}{r_1}(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \underset{\sim}{c}, \quad \Rightarrow \underset{\sim}{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore \underset{\sim}{r_1}(t) = \begin{pmatrix} t^2 \\ -t^3 \end{pmatrix} \text{ m}$$

M1-integration with constant seen

B1-position vector, $\underset{\sim}{r_1}(t)$

$$\underset{\sim}{v_2} = \begin{pmatrix} t^3 \\ 2t-3 \end{pmatrix} \text{ m s}^{-1}$$

$$\underset{\sim}{r_2}(t) = \int \begin{pmatrix} t^3 \\ 2t-3 \end{pmatrix} dt = \begin{pmatrix} 0.25t^4 \\ t^2 - 3t \end{pmatrix} + \underset{\sim}{c}$$

When $t = 0, \underset{\sim}{r_2}(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \underset{\sim}{c}, \quad \Rightarrow \underset{\sim}{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore \underset{\sim}{r_2}(t) = \begin{pmatrix} 0.25t^4 \\ t^2 - 3 \end{pmatrix} \text{ m}$$

M1-integration with constant seen

B1-position vector, $\underset{\sim}{r_2}(t)$

When $t = 2$,

$$_1\underset{\sim}{r_2}(t) = \underset{\sim}{r_1}(t) - \underset{\sim}{r_2}(t) = \begin{pmatrix} t^2 \\ -t^3 \end{pmatrix} - \begin{pmatrix} 0.25t^4 \\ t^2 \end{pmatrix}$$

$$_1\underset{\sim}{r_2}(2) = \begin{pmatrix} 2^2 \\ -2^3 \end{pmatrix} - \begin{pmatrix} 0.25\times 2^4 \\ 2^2 - 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -12 \end{pmatrix} \text{ m}$$

$$\text{Distance} = \left|{}_1\underset{\sim}{r_2}(2)\right| = \sqrt{0^2 + (-12)^2} = 12 \text{ m}$$

B1-relative displacement in vector form

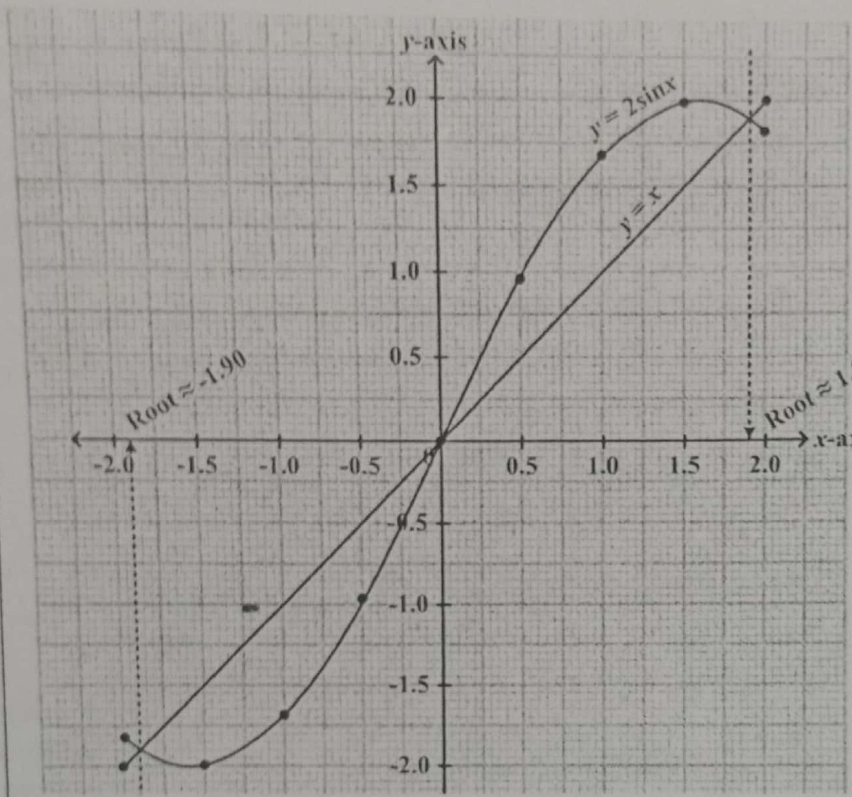
M1 A1-substitution and output

12

14	(a).	

x	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0
$2\sin x$	-1.82	-1.99	-1.68	-0.96	0.00	0.96	1.68	1.99	1.82

B1-values of x
B1-values of $2\sin x$ (max. 2 d.p)



B1-axes
B1-plotting $y = 2\sin x$
B1-curve $y = 2\sin x$
B1-line $y = x$
B1-negative root (2 d.p)
B1-positive root (2 d.p)

(b).

let, $\quad f(x) = x^3 - 3x + 1$

x	-3	-2	-1	0	1	2	3
$f(x)$	-17	-1	3	1	-1	3	19

B1-values of $f(x)$

Since
$f(-2).f(-1) < 0, \quad \Rightarrow -2 < \text{root} < -1,$ negative root
$f(0).f(1) < 0, \quad \Rightarrow 0 < \text{root} < 1,$ positive root
$f(1).f(2) < 0, \quad \Rightarrow 1 < \text{root} < 2,$ positive root

B1-negative root
B1-positive root
B1-positive root

12

15 (a).

$$P(W < 9.5) = P\left(Z < \frac{9.5 - 10}{\sqrt{4}}\right) = P(Z < -0.25)$$

$$= 0.5 - \phi(-0.25) = 0.5 - \phi(0.25) = 0.5 - 0.0987 = 0.4013$$

M1-standardising
B1 M1 A1-table value, subtraction, output

(b).

$$P(247 < 25W < 253) = P(9.88 < W < 10.12)$$

$$= P\left(\frac{9.88 - 10}{\sqrt{4}} < Z < \frac{10.12 - 10}{\sqrt{4}}\right) = P(-0.06 < Z < 0.06)$$

$$= 2 \times \phi(0.06) = 2 \times 0.0239 = 0.0478$$

M1- standardising
B1 M1 A1- table value, doubling, output

(c).

$$P(\overline{W} > 10.2) = P\left(Z > \frac{10.2 - 10}{\sqrt{4/25}}\right) = P(Z > 0.5)$$

$$= 0.5 - \phi(0.5) = 0.5 - 0.1915 = 0.3085$$

M1- standardising
B1 M1 A1- table value, subtraction, output

12

16 (a).

Taking moments about the y-axis,

$$\overline{x} = \frac{\int_0^2 xy\,dx}{\int_0^2 y\,dx}$$

$$\int_0^2 xy\,dx = \int_0^2 x(8 - x^2)\,dx = \int_0^2 (8x - x^3)\,dx$$

$$= \left[4x^2 - \frac{1}{4}x^4\right]_0^2 = \left(4 \times 2^2 - \frac{1}{4} \times 2^4\right) - 0 = 12$$

M1 B1- integration and output

$$\int_0^2 y\,dx = \int_0^2 (8 - x^2)\,dx$$

$$= \left[8x - \frac{1}{3}x^3\right]_0^2 = \left(8 \times 2 - \frac{1}{3} \times 2^3\right) - 0 = \frac{40}{3}$$

M1 B1- integration and output

$$\overline{x} = \frac{\int_0^2 xy\,dx}{\int_0^2 y\,dx} = 12 \div \frac{40}{3} = \frac{9}{10}, \quad \text{as required}$$

M1 B1-division and output

(b).

Let ρ be the weight per unit area.

Figure	Weight	Distance from side OA
OADE	$= \frac{40}{3}\rho$	$= \frac{9}{10}$
ABCD	$= (5 \times 4)\rho = 20\rho$	$= 2 + \frac{5}{2} = 4.5$
Whole lamina	$= \frac{100}{3}\rho$	$\overline{x}$

B1-weight column

B1-distance column

By taking moments about side OE,

$$\left(\frac{40}{3}\rho \times \frac{9}{10}\right) + (20\rho \times 4.5) = \frac{100}{3}\rho \times \overline{x}$$

$$12 + 90 = \frac{100}{3} \times \overline{x}$$

$$102 = \frac{100}{3} \times \overline{x}$$

M1 M1 M1- correct product of each moment

	$$\bar{x} = \frac{153}{50} = 3.06 \text{ units}$$ The centre of gravity of the lamina is 3.06 units from the side OE.	**B1**-output
		12

END