

SECTION A (40 Marks)

Attempt **all** questions in this section.

1. Solve $2 \cos 2\theta - 5 \sin 2\theta = 4$ for $0 \leq \theta \leq \pi$ (5 marks)
2. Given that $y = \log_5 \left\{ e^x \left(\frac{x+2}{x-2} \right)^{\frac{4}{3}} \right\}$. Find $\frac{dy}{dx}$ (5 marks)
3. Solve the equations $x^2 - 10x + y^2 = 25$ and $x - y + 1 = 0$ (5 marks)
4. If the lines $3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} + \lambda(10\mathbf{i} + 5\mathbf{j} - 5\mathbf{k})$ and $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu[9\mathbf{i} + (t - 2)\mathbf{j} + 3\mathbf{k}]$ intersect, find the value of t . (5 marks)
5. A right circular cone of radius r cm has a maximum volume. The sum of its vertical height h and the circumference is 15cm. find the maximum volume of the cone. (5 marks)
6. A point Z moves such that its distance from two points $P(2, 0)$ and $Q(6, 8)$ are in the ratio $PZ : PQ = 3 : 5$. Show that the locus of Z is a circle. (5 marks)
7. Displacement of a particle from a fixed point after time t is given by $y = e^{-t} \sin t$. Show that the particle is instantaneously at rest at time $t = \frac{\pi}{4}$ seconds. (5 marks)
8. Expand $(1-x)^{\frac{1}{3}}$ as far as the term including x^3 . Use your expansion to estimate the $\sqrt[3]{24}$ correct to 4 significant figures. (5 marks)

SECTION B (60 Marks)

Attempt five questions in this section.

9. a) If $(1 - i)$ is a root of $z^4 - 6z^3 + 23z^2 - 34z + 26 = 0$ find the other roots. (6 marks)

b) Given that $z = \frac{(3+4i)(2-3i)}{(-i+3)}$,

i. express z in terms of $a + bi$ (4 marks)

ii. find the argument of z . (2 marks)

10. a) Line L_1 is the intersection of the two planes whose equations are $x + 5y + 2z = 6$ and $y - 3x - z + 2 = 0$. Find the Cartesian equation of the line L_1 . (6 marks)

b) Show that lines $\mathbf{r} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k} - \mathbf{i})$ and $\mathbf{r} = 5\mathbf{i} - 2\mathbf{k} + \beta(2\mathbf{k} - \mathbf{i} - \mathbf{j})$ intersect. (6 marks)

11. a) Given that $\sin A = \frac{4}{5}$ and $\sin B = \frac{5}{13}$ where both A and B are acute angles. Find the value of;

i. $\cot(A + B)$

ii. $\sin(A - B)$ (6 marks)

b) Use $t = \tan \frac{x}{2}$, to solve $3 \sin x + 4 \cos x = 2$ for

$-360^\circ \leq x \leq 360^\circ$ (6 marks)

12. a) Given that $y = \frac{1+\sin^2 x}{1+\cos^2 x}$, prove that $\frac{dy}{dx} = \frac{3\sin 2x}{(\cos^2 x + 1)^2}$ (4 marks)

b) Differentiate $x^2 e^{-bx} 10^{\sin x}$ with respect to x . (4 marks)

c) A curve is represented by the parametric equations $x = 3t$ and $y = \frac{4}{1+t^2}$. Determine the equation of the tangent to the curve (3, 2) (4 marks)

13. a) The line $x + y = c$ is a tangent to the curve $x^2 + y^2 - 4y + 2 = 0$. Find the coordinates of the points of contact of the tangent for each value of c . (6 marks)

b) the equation of the curve is given by $y^2 - 6y + 20x + 49 = 0$. Sketch the curve showing clearly the vertex, focus, directive, and the axis. (6 marks)

END.