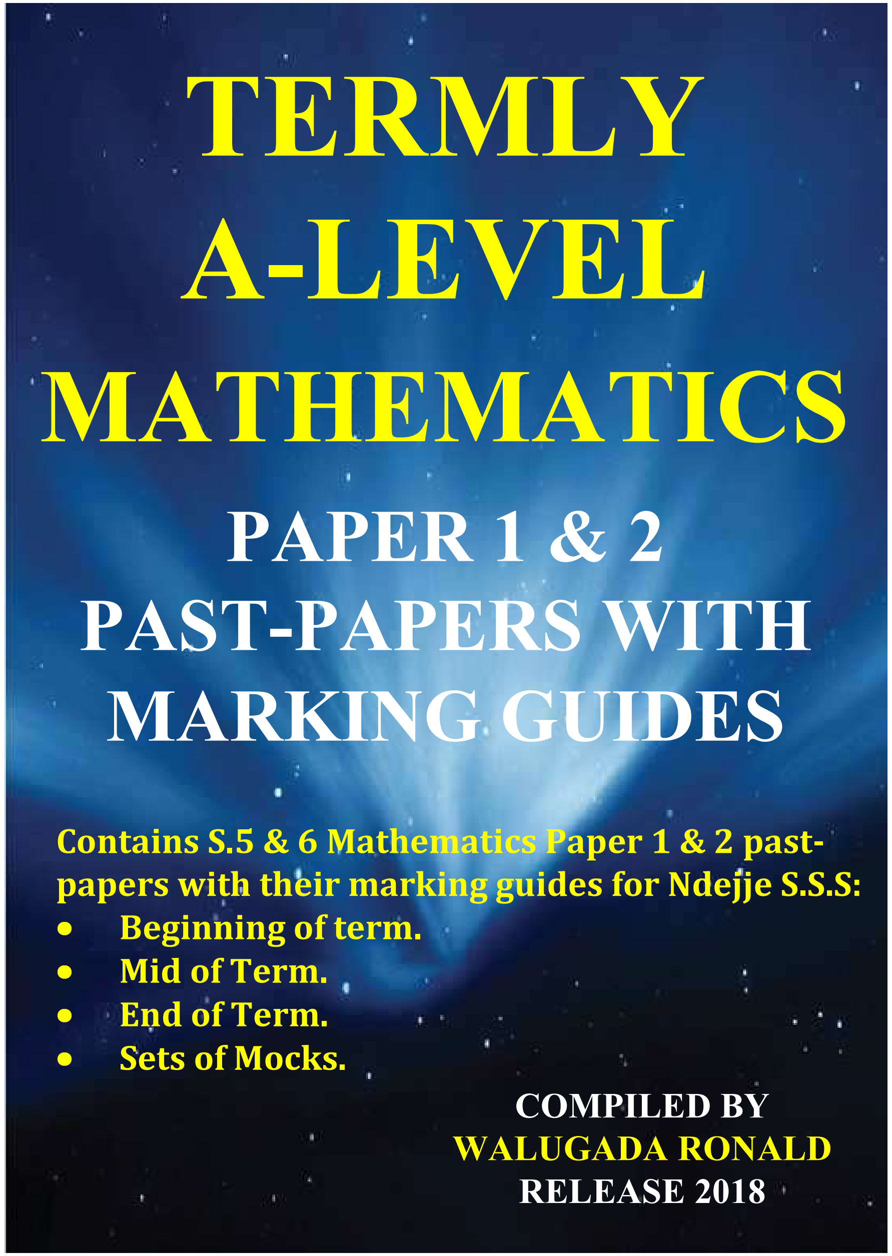

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

TERMLY A-LEVEL MATHEMATICS PAPER 1 & 2

PAST PAPERS WITH MARKING GUIDES

Contains S.5 & 6 Mathematics paper 1 & 2 past-papers with their marking guides for Ndejje S.S.S:

- Beginning of term,
- Mid of Term,
- End of Term,
- Sets of Mocks,
- UACE UNEB 2018-2016.

BY
WALUGADA RONALD
Release 2018

PREFACE

I take an opportunity to present this compilation of Senior 5 and 6 Principal Mathematics Past Papers for Ndejje S.S.S, for the year 2018, to all Advanced Level Mathematics Students and Teachers in various parts of the country.

The objective of this book is to equip the students with the necessary techniques while preparing for exams of Principal Mathematics Paper 1 and 2.

Students very often find a challenge on how to correctly and precisely present their answers in an exam without leaving out the important steps needed by the examiners. This problem has been well addressed by the author by providing compiled sets of past papers with their marking guides in an organized, precise and exhaustive manner as the reader will appreciate.

Each solution has been presented as simply, well organized and precisely as possible to suit the students' understanding as well as the requirements of examiners. Alternative methods and approach of solutions have been provided in the where necessary.

This book, no doubt, will be of benefit to both the student and the teacher.

Any suggestions for improvement of this text or for orders of copies of this book, notify the author on the following contacts:

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I shall be grateful.
Thank you.

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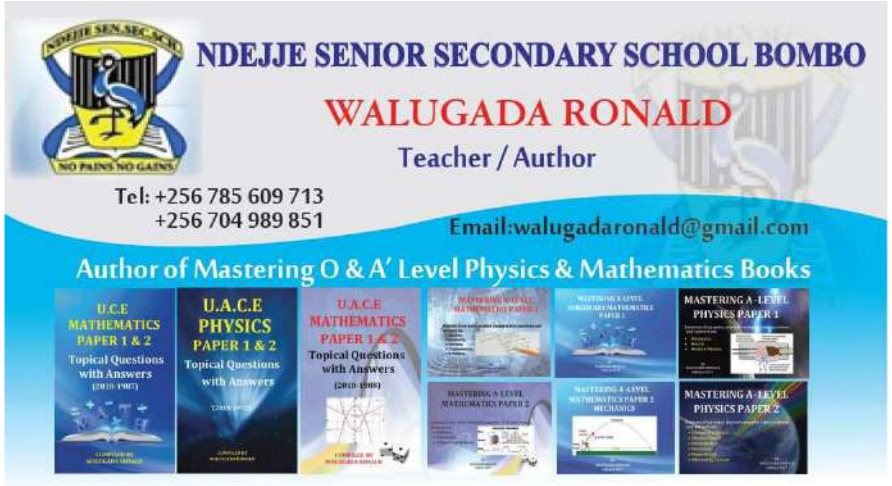

OTHER BOOKS BY THE SAME AUTHOR SO FAR

- MASTERING A-LEVEL MATHEMATICS PAPER 1.
- MASTERING A-LEVEL MATHEMATICS PAPER 2.
- MASTERING A-LEVEL SUBSIDIARY MATHEMATICS PAPER 1.
- MASTERING A-LEVEL PHYSICS PAPER 1.
- MASTERING A-LEVEL PHYSICS PAPER 2.
- TERMLY O-LEVEL MATH PAPER 1 & 2 PAST PAPERS WITH MARKING GUIDES.
- TERMLY A-LEVEL SUBSIDIARY MATH PAPER 1 PAST-PAPERS WITH MARKING GUIDES.
- U.A.C.E MATH 1 & 2 TOPICAL QUESTIONS WITH ANSWERS.
- U.C.E MATH 1 & 2 TOPICAL QUESTIONS WITH ANSWERS.
- U.A.C.E PHYSICS 1 & 2 TOPICAL QUESTIONS WITH ANSWERS.

AKNOWLEDGEMENT

The author acknowledges the support he received from various persons at one stage or another in the compilation, typing, proofreading, typesetting, reproduction, collating and binding of this book. Their efforts were not in vain. Upon their request, their names and forms of contribution shall not be made public.

DEDICATION

I dedicate this book to my beloved wife, Mrs. Walugada Esther, for standing beside me while coming up with this piece of work; all teachers of Mathematics at Ndejje Senior Secondary School; and the past, present and future A-level Mathematics students in Uganda; who have been the inspiration behind this book.

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P425/1
PURE
MATHEMATICS
PAPER 1
April 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 1 MOT 1 2018
Time: 3 Hours

NAME: ______________________________ **COMB:** ________

INSTRUCTIONS:
- Attempt ALL questions in this paper.
- Show your working clearly.

Qn 1: Simplify:

(i). $\frac{27^{n+2}-6\bullet 3^{3n+3}}{3^{n}\bullet 9^{n+2}}$

(ii). $\frac{\left(x^{3/2}+x^{1/2}\right)\left(x^{1/2}-x^{-1/2}\right)}{\left(x^{3/2}-x^{1/2}\right)^{2}}$. [10]

Qn 2: Solve the equations:

(i). $\log_3 x + 3\log_x 3 = 4$.

(ii). $2^{2x+1} - 5(2^x) + 2 = 0$. [12]

Qn 3: Given one root is the square of the other in the equation $ax^2 + bx + c = 0$, prove that $c(a-b)^3 = a(c-b)^3$. [5]

Qn 4: Prove that:

(i). $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{2}{\sin\theta}$. [3]

(ii). $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$. [3]

Qn 5: When the polynomial $f(x)$ is divided by $(x-1)$, the remainder is 7 and when divided by $(x-3)$, the remainder is 13. Find the remainder when $f(x)$ is divided by $x^2 - 4x + 3$. [5]

Qn 6: Given that $x^2 + px + q$ and $3x^2 + q$ have a common factor $(x-b)$; where p, q and b are non-zero, show that $3p^2 + 4q = 0$. [5]

Qn 7: Given that $(x+2)$ is a factor of $2x^3 + 6x^2 + qx - 5$, find the remainder when the expression is divided by $(2x-1)$. [5]

Qn 8: Given that α and β are the roots of the equation $x^2 - bx + c = 0$.

(i). Show that $(\alpha^2+1)(\beta^2+1) = (c-1)^2 + b^2$.

(ii). Find, in terms of b and c, a quadratic equation whose roots are $\frac{\alpha}{\alpha^2+1}$ and $\frac{\beta}{\beta^2+1}$. [10]

Qn 9: If $t = \tan\frac{1}{2}\theta$, show that $\sin\theta = \frac{2t}{1+t^2}$ and derive and expression of $\cos\theta$ in terms of t. Hence or otherwise, solve the equation $3\sin\theta + \cos\theta = 2$ for values of θ in the range $0^\circ \le \theta \le 180^\circ$. [10]

Qn 10: (a). Given that $\tan B = \frac{4}{3}$ and that B is acute. Without using tables or calculator, find the value of:

(i). $\cos 2B$,

(ii). $\tan\frac{B}{2}$.

(b). Express $8\cos^4\theta$ in the form $a\cos 4\theta + b\cos 2\theta + c$, giving the numerical values of the constants a, b and c. [10]

Qn 11: Show that, for all values of θ,

$$\frac{\cot^2\theta}{1+\cot^2\theta} = \cos^2\theta$$

Hence or otherwise, find the solution in the range $^{-}180^\circ \le \theta \le {}^{+}180^\circ$ of the equation

$$\frac{\cot^2\theta}{1+\cot^2\theta} = 2\cos 2\theta$$

[10]

Qn 12: (i). Solve for the value of x in the equation

$$2\sqrt{(2x-12)} - \sqrt{(2x-3)} = 3$$

(ii). Rationalise the denominator of;

$$\frac{1+\sin 45^\circ}{1-\sin 45^\circ}$$

(iii). Prove that

$$\log_6 x = \frac{\log_3 x}{1+\log_3 2}$$

Hence, given that $\log_3 2 = 0.631$, find the value of $\log_6 4$ correct to 3 s.f. [12]

*****END*****

MARKING GUIDE

SNo.	Working	Marks
1	(i).	
	$\frac{27^{n+2} - 6 \bullet 3^{3n+3}}{3^n \bullet 9^{n+2}} = \frac{(3^3)^{n+2} - 6 \times 3^{3n+3}}{3^n \times (3^2)^{n+2}}$	B1
	$= \frac{3^{3n} \times 3^6 - 2 \times 3^1 \times 3^{3n} \times 3^3}{3^n \times 3^{2n} \times 3^4}$	B1
	$= \frac{3^{3n} \times 3^6 - 2 \times 3^{3n} \times 3^4}{3^{3n} \times 3^4} = \frac{3^{3n} \times 3^4(3^2 - 2)}{3^{3n} \times 3^4}$	B1
	$= 3^2 - 2 = 7$	B1 B1
	(ii).	
	$\frac{\left(x^{3/2} + x^{1/2}\right)\left(x^{1/2} - x^{-1/2}\right)}{\left(x^{3/2} + x^{1/2}\right)^2}$	
	$= \frac{x^{3/2}\left(x^{1/2} - x^{-1/2}\right) + x^{1/2}\left(x^{1/2} - x^{-1/2}\right)}{x^3 - 2 \times x^{3/2} \times x^{1/2} + x^1}$	B1
	$= \frac{x^2 - x^1 + x^1 - x^0}{x^3 - 2x^2 + x^1} = \frac{x^2 - 1}{x(x^2 - 2x + 1)}$	B1 B1
	$= \frac{(x-1)(x+1)}{x(x-1)^2} = \frac{(x+1)}{x(x-1)}$	B1 B1
		10
2	(i).	
	$\log_3 x + 3\log_x 3 = 4$	
	$\log_3 x + \frac{3}{\log_3 x} = 4$	B1
	Let $y = \log_3 x$	
	$y + \frac{3}{y} = 4$	
	$y^2 + 3 = 4y$	
	$y^2 - 4y + 3 = 0$	M1
	sum $= -4$, product $= 3$, factors $= (-1, -3)$	
	$(y-1)(y-3) = 0$	
	$(y-1) = 0$, or, $(y-3) = 0$	
	$y = 1$, or, $y = 3$	A1
	$\log_3 x = 1$, or, $\log_3 x = 3$	
	$3^1 = x$, or, $3^3 = x$	M1
	$3 = x$, or, $27 = x$	
	$x = 3$, or, $x = 27$	A1 A1
	(ii).	
	$2^{2x+1} - 5(2^x) + 2 = 0$	
	$(2^x)^2 \times 2 - 5(2^x) + 2 = 0$	B1
	Let $y = 2^x$	
	$y^2 \times 2 - 5y + 2 = 0$	
	$2y^2 - 5y + 2 = 0$	
	sum $= -5$, product $= 2 \times 2 = 4$, factors $= (-4, -1)$	

	$2y^2 - 4y - y + 2 = 0$	
	$2y(y-2) - (y-2) = 0$	M1
	$(2y-1)(y-2) = 0$	
	$(2y-1) = 0$, or, $(y-2) = 0$	
	$2y = 1$, or, $y = 2$	
	$y = \frac{1}{2}$, or, $y = 2$	A1
	$2^x = 2^{-1}$, or, $2^x = 2^1$	M1
	$x = -1$, or, $x = 1$	A1 A1
		12
3	Let one of the roots be β; then the other root is β^2	
	$\beta + \beta^2 = -\frac{b}{a} \longrightarrow (1)$	
	$\beta \times \beta^2 = \frac{c}{a}$, $\Rightarrow \beta^3 = \frac{c}{a} \longrightarrow (2)$	B1
	Equation (1) + (2) gives,	
	$\beta + \beta^2 + \beta^3 = \frac{c}{a} - \frac{b}{a}$	B1
	$\beta(1 + \beta + \beta^2) = \frac{c-b}{a}$	
	$\beta\left(1 - \frac{b}{a}\right) = \frac{c-b}{a}$	
	$\beta\left(\frac{a-b}{a}\right) = \frac{c-b}{a}$	B1
	$\beta = \frac{(c-b)}{(a-b)}$	
	$\beta^3 = \frac{(c-b)^3}{(a-b)^3}$	B1
	$\frac{c}{a} = \frac{(c-b)^3}{(a-b)^3}$	B1
	$c(a-b)^3 = a(c-b)^3$, as required	
		05
4	(i).	
	$L.H.S = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$	B1
	$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta} = \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$	B1
	$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta} = \frac{2}{\sin\theta}$	B1
	(ii).	
	$L.H.S = \tan^2 A - \sin^2 A = \frac{\sin^2 A}{\cos^2 A} - \sin^2 A$	B1
	$= \frac{\sin^2 A - \sin^2 A\cos^2 A}{\cos^2 A}$	B1 B1
	$= \frac{\sin^2 A\,(1 - \cos^2 A)}{\cos^2 A} = \frac{\sin^2 A \sin^2 A}{\cos^2 A} = \frac{\sin^4 A}{\cos^2 A} = \sin^4 A \sec^2 A$	

Compiled By Walugada Ronald

		06
5	for $(x-1)=0, \quad x=1, \quad \Rightarrow R(1)=7$	
	for $(x-3)=0, \quad x=3, \quad \Rightarrow R(3)=13$	
	$g(x)=x^2-4x+3=(x-1)(x-3)$	
	Let $R(x)=mx+n$ be the remainder when $f(x)$ is divided by x^2-4x+3.	B1
	$R(1)=m(1)+n, \quad \Rightarrow 7=m+n \quad \longrightarrow (1)$	B1
	$R(3)=m(3)+n, \quad \Rightarrow 13=3m+n \quad \longrightarrow (2)$	
	Equation $(2)-(1)$ gives,	M1
	$6=2m, \quad \Rightarrow m=3$	
	From equation (1)	A1
	$n=7-m=7-3=4$	B1
	$\therefore R(x)=3x+4$	
		05
6	$b^2+pb+q=0 \quad \longrightarrow (1)$	
	$3b^2+q=0 \quad \longrightarrow (2)$	
	Equation $3\times(1)-(2)$ gives,	
	$\begin{array}{r\|l} 3b^2+3pb+3q=0 & \rightarrow 3\times(1) \\ -\quad 3b^2+q=0 & \rightarrow (2) \\ \hline 3pb+2q=0 & \end{array}$	M1
	$b=-\dfrac{2q}{3p}$	A1
	$3\left(-\dfrac{2q}{3p}\right)^2+q=0$	M1
	$\dfrac{4q^2}{3p^2}+q=0$	M1
	$4q^2+3p^2q=0$	
	$4q+3p^2=0$	
	$3p^2+4q=0, \quad$ as required	B1
		05
7	$(x+2)=0,$	
	for $(x+2)=0, \quad x=-2$	
	$R(-2)=2(-2)^3+6(-2)^2+q(-2)-5$	M1
	$0=-16+24-2q-5$	
	$0=3-2q$	
	$q=1.5$	A1
	for $(2x-1)=0, \quad x=0.5$	
	$R(0.5)=2(0.5)^3+6(0.5)^2+1.5(0.5)-5$	M1 B1
	$=0.25+1.5+0.75-5=-2.5$	A1
		05
8	(i).	
	$x^2-bx+c=0$	
	$\alpha+\beta=b$	
	$\alpha\beta=c$	B1

	$L.H.S=(\alpha^2+1)(\beta^2+1)=\alpha^2\beta^2+\alpha^2+\beta^2+1$	
	$=(\alpha\beta)^2+(\alpha+\beta)^2-2\alpha\beta+1$	B1
	$=c^2+b^2-2c+1$	
	$=c^2-2c+1+b^2$	M1
	$=(c-1)^2+b^2$	
	(ii).	A1
	product of new roots $=\dfrac{\alpha}{\alpha^2+1}\times\dfrac{\beta}{\beta^2+1}$	M1
	$=\dfrac{\alpha\beta}{(\alpha^2+1)(\beta^2+1)}=\dfrac{c}{(c-1)^2+b^2}$	A1
	sum of new roots $=\dfrac{\alpha}{\alpha^2+1}+\dfrac{\beta}{\beta^2+1}=\dfrac{\alpha(\beta^2+1)+\beta(\alpha^2+1)}{(\alpha^2+1)(\beta^2+1)}$	
	$=\dfrac{\alpha\beta^2+\alpha+\alpha^2\beta+\beta}{(c-1)^2+b^2}=\dfrac{\alpha\beta(\beta+\alpha)+(\alpha+\beta)}{(c-1)^2+b^2}=\dfrac{cb+b}{(c-1)^2+b^2}$	M1 M1 A1
	The required equation is:	
	$x^2+(\text{sum})x+(\text{product})=0$	
	$x^2+\left[\dfrac{cb+b}{(c-1)^2+b^2}\right]x+\left[\dfrac{c}{(c-1)^2+b^2}\right]=0$	B1
	$[(c-1)^2+b^2]x^2+b(c+1)x+c=0$	
		10
9	$\sin\theta=\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1}=\dfrac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\sin^2\left(\frac{\theta}{2}\right)+\cos^2\left(\frac{\theta}{2}\right)}$	M1
	$=\dfrac{\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right]\times\frac{1}{\cos^2\left(\frac{\theta}{2}\right)}}{\left[\sin^2\left(\frac{\theta}{2}\right)+\cos^2\left(\frac{\theta}{2}\right)\right]\times\frac{1}{\cos^2\left(\frac{\theta}{2}\right)}}$	M1
	$=\dfrac{2\tan\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}=\dfrac{2t}{1+t^2}$	A1
	$\cos\theta=\dfrac{\cos^2\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)}{1}=\dfrac{\cos^2\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)}{\sin^2\left(\frac{\theta}{2}\right)+\cos^2\left(\frac{\theta}{2}\right)}$	M1
	$=\dfrac{\left[\cos^2\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)\right]\times\frac{1}{\cos^2\left(\frac{\theta}{2}\right)}}{\left[\sin^2\left(\frac{\theta}{2}\right)+\cos^2\left(\frac{\theta}{2}\right)\right]\times\frac{1}{\cos^2\left(\frac{\theta}{2}\right)}}$	M1
	$=\dfrac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}=\dfrac{1-t^2}{1+t^2}$	A1
	For the hence part:	
	$3\sin\theta+\cos\theta=2$	
	$\dfrac{3\times 2t}{1+t^2}+\dfrac{1-t^2}{1+t^2}=2$	M1
	$6t+1-t^2=2+2t^2$	
	$3t^2-6t+1=0$	

	$t = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{6 \pm \sqrt{24}}{6}$	
	$t = \frac{6 + \sqrt{24}}{6}$, or, $t = \frac{6 - \sqrt{24}}{6}$	
	$\tan\left(\frac{\theta}{2}\right) = 0.1835$, or, $\tan\left(\frac{\theta}{2}\right) = 1.8165$	B1
	$\left(\frac{\theta}{2}\right) = 10.40°$, or, $\left(\frac{\theta}{2}\right) = 61.17°$	M1
	$\theta = 20.80°$, or, $\theta = 122.34°$	A1
		10
10	(a). (i).	
	$\cos 2B = \frac{1 - \tan^2 B}{1 + \tan^2 B} = \frac{1 - \left(\frac{4}{3}\right)^2}{1 + \left(\frac{4}{3}\right)^2} = -\frac{7}{9} \times \frac{9}{25} = -\frac{7}{25}$	M1 B1 A1
	(ii).	
	Let $t = \tan\frac{B}{2}$	
	$\tan B = \frac{4}{3}$	
	$\frac{2t}{1 - t^2} = \frac{4}{3}$	M1
	$6t = 4 - 4t^2$	
	$4t^2 + 6t - 4 = 0$	
	$2t^2 + 3t - 2 = 0$	
	sum $= 3$, product $= -2 \times 2 = -4$, factors $= (-1, 4)$	
	$2t^2 - t + 4t - 2 = 0$	M1
	$t(2t - 1) + 2(t - 1) = 0$	
	$(t + 2)(2t - 1) = 0$	
	$(t + 2) = 0$, or, $(2t - 1) = 0$	
	$t = -2$, or, $t = 0.5$	A1
	$\tan\frac{B}{2} = -2$, or, $\tan\frac{B}{2} = 0.5$	
	for acute B, $\tan\frac{B}{2} \neq -2$, $\Rightarrow \tan\frac{B}{2} = 0.5$	B1
	(b).	
	$8\cos^4\theta = 8[\cos^2\theta]^2 = 8\left[\frac{1}{2}(1 + \cos 2\theta)\right]^2$	M1
	$= 2(1 + 2\cos 2\theta + \cos^2 2\theta)$	
	$= 2 + 4\cos 2\theta + 2\cos^2 2\theta$	
	$= 2 + 4\cos 2\theta + (1 + \cos 4\theta)$	M1
	$= 3 + 4\cos 2\theta + \cos 4\theta$	
	$\therefore a = 1, \quad b = 4, \quad c = 3$	A1
		10
11	$L.H.S = \frac{\cot^2\theta}{1 + \cot^2\theta} = \frac{\csc^2\theta - 1}{\csc^2\theta}$	B1 B1

	$= 1 - \frac{1}{\csc^2\theta} = 1 - \sin^2\theta = \cos^2\theta$	B1 B1
	For the hence part:	
	$\frac{\cot^2\theta}{1 + \cot^2\theta} = 2\cos 2\theta$	
	$\cos^2\theta = 2\cos 2\theta$	M1
	$\frac{1}{2}(1 + \cos 2\theta) = 2\cos 2\theta$	M1
	$1 + \cos 2\theta = 4\cos 2\theta$	
	$1 = 3\cos 2\theta$	
	$\cos 2\theta = \frac{1}{3}$	B1
	$2\theta = \pm 70.53°, \pm 289.47°$	M1
	$\theta = \pm 35.265°, \pm 144.74°$	A1 A1
		10
12	(i).	
	$2\sqrt{(2x - 12)} - \sqrt{(2x - 3)} = 3$	
	$\left(2\sqrt{(2x - 12)}\right)^2 = \left(3 + \sqrt{(2x - 3)}\right)^2$	M1
	$4(2x - 12) = 9 + 6\sqrt{(2x - 3)} + (2x - 3)$	
	$8x - 48 = 6 + 6\sqrt{(2x - 3)} + 2x$	
	$6x - 54 = 6\sqrt{(2x - 3)}$	
	$x - 9 = \sqrt{(2x - 3)}$	
	$x^2 - 18x + 81 = 2x - 3$	
	$x^2 - 20x + 84 = 0$	M1
	$x = \frac{20 \pm \sqrt{(-20)^2 - 4 \times 1 \times 84}}{2 \times 1} = \frac{20 \pm 8}{2}$	
	$x = \frac{20 + 8}{2} = 14$, or, $x = \frac{20 - 8}{2} = 6$	A1
	but $x \neq 6$, $\Rightarrow x = 14$	B1
	(ii).	
	$\frac{1 + \sin 45°}{1 - \sin 45°} = \frac{(1 + 0.5\sqrt{2}) \times (1 + 0.5\sqrt{2})}{(1 - 0.5\sqrt{2}) \times (1 + 0.5\sqrt{2})}$	B1 M1
	$= \frac{1 + \sqrt{2} + 0.5}{1 - 0.5} = \frac{1.5 + \sqrt{2}}{0.5} = 3 + 2\sqrt{2}$	A1
	(iii).	
	$L.H.S = \log_6 x = \frac{\log_3 x}{\log_3 6} = \frac{\log_3 x}{\log_3 3 + \log_3 2} = \frac{\log_3 x}{1 + \log_3 2}$	B1 B1
	For the hence part:	
	$\log_6 4 = \frac{\log_3 4}{1 + \log_3 2} = \frac{2\log_3 2}{1 + \log_3 2} = \frac{2 \times 0.631}{1 + 0.631} = \frac{1.262}{1.631} \approx 0.7738$	M1 M1 B1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
April 2018
2 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 2 MOT 1 2018
Time: 2 Hours

NAME: ______________________________ **COMB:** __________

INSTRUCTIONS:
- Attempt ALL questions in both sections.
- Show your working clearly.

Section A

Qn 1: For a particular set of observations, $\sum f = 20$, $\sum fx^2 = 16143$ and $\sum fx = 563$. Find the value of the:
(i). mean,
(ii). standard deviation.

Qn 2: The resultant of forces $\underset{\sim}{\boldsymbol{F_1}} = 3\underset{\sim}{\boldsymbol{i}} + (a-c)\underset{\sim}{\boldsymbol{j}}$, $\underset{\sim}{\boldsymbol{F_2}} = (2a+3c)\underset{\sim}{\boldsymbol{i}} + 5\underset{\sim}{\boldsymbol{j}}$ and $\underset{\sim}{\boldsymbol{F_3}} = 4\underset{\sim}{\boldsymbol{i}} + 6\underset{\sim}{\boldsymbol{j}}$ acting on a particle is $10\underset{\sim}{\boldsymbol{i}} + 12\underset{\sim}{\boldsymbol{j}}$. Find:
(i). the value of a and c.
(ii). the magnitude of force $\underset{\sim}{\boldsymbol{F_2}}$.

Qn 3: A set of digits consists of $\boldsymbol{m}$ zeros and $\boldsymbol{n}$ ones.
(a). Find the mean of this set of data.
(b). Hence show that the standard deviation of the set of digits is:

$$\frac{\sqrt{mn}}{m+n}$$

Qn 4: Find in the form $a\underset{\sim}{\boldsymbol{i}} + b\underset{\sim}{\boldsymbol{j}}$ the velocity of a plane flying from $A(10, 50)$ to $B(130, -110)$ at a speed of 100 m s^{-1}.

Qn 5: A bag contained five balls each bearing one of the numbers 1, 2, 3, 4, 5. A ball was drawn from the bag, its number noted and then replaced. This was done 50 times in all and the table below shows the resulting frequency distribution. If the mean is 2.7, find the value of x and y.

Number	1	2	3	4	5
Frequency	x	11	y	8	9

Qn 6: A force vector $\underset{\sim}{\boldsymbol{F}} = p\underset{\sim}{\boldsymbol{i}} + 12\underset{\sim}{\boldsymbol{j}}$ has a magnitude of 13 units.
(a). Find the two possible values of p.
(b). For each value of p, find the unit force vector.

Qn 7: The data below shows the weights of some students in a senior five class.

Mass	$-< 45$	$-< 50$	$-< 55$	$-< 60$	$-< 65$	$-< 70$	$-< 75$
Number of students	3	30	39	33	13	1	1

Calculate the mean mass of the students.

Qn 8: A line has a vector equation $\underset{\sim}{\boldsymbol{r}} = (1+\lambda)\underset{\sim}{\boldsymbol{i}} + (3-5\lambda)\underset{\sim}{\boldsymbol{j}}$.
(a). State its direction vector.
(b). State the coordinates of the point for which $x = 3$.

Section B

Question 9:
The table below is the distribution of weights of a group of animals;

Mass (kg)	**Frequency**
21 – 25	10
26 – 30	20
31 – 35	15
36 – 40	10
41 – 50	30
51 – 65	45
66 – 75	5

(a). Construct a histogram for the above data and use it to estimate the mode.
(b). Calculate the median for the above data.

Question 10:

(a). Find the magnitude of the resultant of forces $\underset{\sim}{F_1}$, $\underset{\sim}{F_2}$ and $\underset{\sim}{F_3}$ if;

$\underset{\sim}{F_1} = 3\underset{\sim}{i} - 6\underset{\sim}{j} - 4\underset{\sim}{k}$.

$\underset{\sim}{F_2}$ is parallel to vector $\overrightarrow{AB}$ and is of magnitude 12 N.

$\underset{\sim}{F_3}$ is parallel to vector $\overrightarrow{OM}$ and is of magnitude $3\sqrt{13}$ N and given that the position vectors of A and B are $\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and $5\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ respectively; and M is the midpoint of $\overrightarrow{AB}$.

(b). Two forces $\underset{\sim}{F_1}$ and $\underset{\sim}{F_2}$ act on a particle at a point A with position vector $3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$.

$\underset{\sim}{F_1}$ is of magnitude 14 N and is in direction of the vector $6\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$.

$\underset{\sim}{F_2} = 4\underset{\sim}{i} + 7\underset{\sim}{j} + 6\underset{\sim}{k}$.

Find a vector equation for the line of action of the resultant of $\underset{\sim}{F_1}$ and $\underset{\sim}{F_2}$.

END

MARKING GUIDE

SNo.	Working	Marks
1	(i). Mean, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{563}{20} = 28.15$	M1 A1
	(ii). Standard deviation, $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ $= \sqrt{\frac{16143}{20} - (28.15)^2} = \sqrt{14.7275} \approx 3.8376$	M1 M1 A1
		05
2	(i). $\underset{\sim}{F} = \underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3}$ $\binom{10}{12} = \binom{3}{a-c} + \binom{2a+3c}{5} + \binom{4}{6}$ $\binom{10}{12} = \binom{7+2a+3c}{11+a-c}$	M1
	By comparison, $10 = 7 + 2a + 3c, \quad \Rightarrow 2a + 3c = 3 \quad \longrightarrow (1a)$ $12 = 11 + a - c, \quad \Rightarrow a = c + 1 \quad \longrightarrow (1b)$	B1-both equations
	Substituting $(1b)$ into $(1a)$ gives; $2(c+1) + 3c = 3$ $5c = 1, \quad \Rightarrow c = 0.2$ From $(1b)$, $a = c + 1 = 0.2 + 1 = 1.2$	B1-both values of a and c
	(ii). $\underset{\sim}{F_2} = \binom{2a+3c}{5} = \binom{2\times 0.2 + 3\times 1.2}{5} = \binom{4}{5}$ $\lvert\underset{\sim}{F_2}\rvert = \sqrt{4^2+5^2} = \sqrt{41} \approx 6.403$ N	M1 A1
		05
3	(a). $\bar{x} = \frac{\sum fx}{\sum f} = \frac{0\times m + 1\times n}{m+n} = \frac{n}{m+n}$	M1 A1
	(b). Variance, $\sigma^2 = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$ $= \frac{0^2\times m + 1^2\times n}{m+n} - \left(\frac{n}{m+n}\right)^2$	B1-for $\sum fx^2$
	$= \frac{n(m+n) - n^2}{(m+n)^2} = \frac{mn + n^2 - n^2}{(m+n)^2} = \frac{mn}{(m+n)^2}$	M1
		B1

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{mn}{(m+n)^2}} = \frac{\sqrt{mn}}{m+n}$$

05

4

$$|\underset{\sim}{v}| = 100\text{ m s}^{-1}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}130\\-110\end{pmatrix} - \begin{pmatrix}10\\50\end{pmatrix} = \begin{pmatrix}120\\-160\end{pmatrix}$$ **M1 B1**

$$|\overrightarrow{AB}| = \sqrt{120^2 + (-160)^2} = \sqrt{40000} = 200$$ **B1**

$$\underset{\sim}{v} = \frac{|\underset{\sim}{v}|}{|\overrightarrow{AB}|}\overrightarrow{AB} = \frac{100}{200}\begin{pmatrix}120\\-160\end{pmatrix} = \begin{pmatrix}60\\-80\end{pmatrix}\text{ m s}^{-1}$$ **M1 A1**

05

5

$$\sum f = x + 11 + y + 8 + 9$$

$$50 = x + y + 28$$

$$x + y = 22 \quad \longrightarrow (1)$$ **B1**

$$\text{Mean} = \frac{(1\times x) + (2\times 11) + (3\times y) + (4\times 8) + (5\times 9)}{50}$$ **M1**

$$2.7 = \frac{x + 22 + 3y + 32 + 45}{50}$$

$$135 = x + 3y + 99$$

$$x + 3y = 36 \quad \longrightarrow (2)$$ **B1**

Equation (2) − (1) gives;

$$\begin{array}{r|l} & x + 3y = 36 \;\rightarrow (1) \\ + & x + y = 22 \;\rightarrow (2) \\ \hline & 2y = 14 \end{array}$$ **M1**

$$y = 7$$

$$x = 22 - y = 22 - 7 = 15$$ **A1**

05

6 (a).

$$|\underset{\sim}{F}| = \sqrt{p^2 + 12^2} = 13$$ **M1**

$$p^2 + 144 = 169$$

$$p^2 = 25$$

$$p = \pm 5$$ **A1 A1**

(b).

for $p = -5$, $\hat{\underset{\sim}{F}} = \frac{1}{|\underset{\sim}{F}|}\underset{\sim}{F} = \frac{1}{13}\begin{pmatrix}-5\\12\end{pmatrix} = \begin{pmatrix}-5/13\\12/13\end{pmatrix}$ **B1**

for $p = 5$, $\hat{\underset{\sim}{F}} = \frac{1}{13}\begin{pmatrix}5\\12\end{pmatrix} = \begin{pmatrix}5/13\\12/13\end{pmatrix}$ **B1**

05

7

Class	f	x	fx
−< 45	3	42.5	127.5

−< 50	30	47.5	1425
−< 55	39	52.5	2047.5
−< 60	33	57.5	1897.5
−< 65	13	62.5	812.5
−< 70	1	67.5	67.5
−< 75	1	72.5	72.5
Total	120		6450

B1-for x column

B1-for fx column

B1-for $\sum fx$

Mean mass, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{6450}{120} = 53.75$ **M1 A1**

05

8 (a).

$$\underset{\sim}{r} = \begin{pmatrix}1+\lambda\\3-5\lambda\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\-5\end{pmatrix}$$ **M1**

$$\text{Direction vector} = \begin{pmatrix}1\\-5\end{pmatrix}$$ **B1**

(b).

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1+\lambda\\3-5\lambda\end{pmatrix}$$

when $x = 3$, $\quad 1 + \lambda = 3, \quad \Rightarrow \lambda = 3 - 1 = 2$ **B1**

$$y = 3 - 5\lambda = 3 - 5\times 2 = -7$$ **M1**

Coordinates are $(3, -7)$ **A1**

05

9

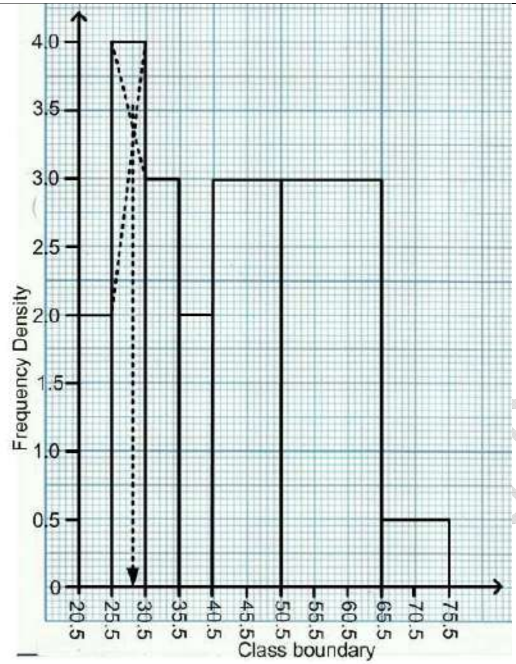
Class	f	Class boundary	c	f/c	C.F
21 – 25	10	20.5 – 25.5	5	2	10
26 – 30	20	25.5 – 30.5	5	4	30
31 – 35	15	30.5 – 35.5	5	3	45
36 – 40	10	35.5 – 40.5	5	2	55
41 – 50	30	40.5 – 50.5	10	3	85
51 – 65	45	50.5 – 65.5	15	3	130
66 – 75	5	65.5 – 75.5	10	0.5	135
Total					

B1-for f/c

B1-for C.F

(a).

B2-for axes



B2-for bars

M1-finding mode

A1

M1

From the histogram, the estimated mode is 27.5. **A1**

(b).

$N = 135, N/2 = 67.5, L_m = 40.5, C.F_b = 55, f_m = 30, c = 10$

$$\text{Median} = L_m + \left(\frac{N/2 - C.F_b}{f_m}\right)c = 40.5 + \left(\frac{67.5 - 55}{30}\right) \times 10$$

$$= 40.5 + 4.167 = 44.667$$

10

10 (a).

$$\underset{\sim}{F_1} = \begin{pmatrix} 3 \\ -6 \\ -4 \end{pmatrix} \text{N}, \quad \overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix}$$ **B1**

$$|\overrightarrow{AB}| = \sqrt{4^2 + (-4)^2 + (-2)^2} = \sqrt{36} = 6$$ **B1**

$$\Rightarrow \underset{\sim}{F_2} = \frac{|\underset{\sim}{F_2}|}{|\overrightarrow{AB}|}\overrightarrow{AB} = \frac{12}{6}\begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \\ -4 \end{pmatrix} \text{N}$$ **B1**

For midpoint, $\overrightarrow{OM} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 6 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$$ **M1 B1**

$$|\overrightarrow{OM}| = \sqrt{3^2 + 0 + 2^2} = \sqrt{13} \approx 3.606$$ **B1**

$$\Rightarrow \underset{\sim}{F_3} = \frac{|\underset{\sim}{F_3}|}{|\overrightarrow{OM}|}\overrightarrow{OM} = \frac{3\sqrt{13}}{\sqrt{13}}\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \\ 6 \end{pmatrix} \text{N}$$ **B1**

Resultant force, $\underset{\sim}{F} = \underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3}$

$$= \begin{pmatrix} 3 \\ -6 \\ -4 \end{pmatrix} + \begin{pmatrix} 8 \\ -8 \\ -4 \end{pmatrix} + \begin{pmatrix} 9 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 20 \\ -14 \\ -2 \end{pmatrix} \text{N}$$ **B1**

$$|\underset{\sim}{F}| = \sqrt{20^2 + (-14)^2 + (-2)^2} = 10\sqrt{6} \approx 24.4949 \text{ N}$$ **M1 A1**

(b).

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \quad \underset{\sim}{F_2} = \begin{pmatrix} 4 \\ 7 \\ 6 \end{pmatrix} \text{N}, \quad |\underset{\sim}{F_1}| = 14 \text{ N}, \quad \underset{\sim}{d_1} = \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix}$$

$$|\underset{\sim}{d_1}| = \sqrt{6^2 + 3^2 + (-2)^2} = \sqrt{49} = 7$$

$$\Rightarrow \underset{\sim}{F_1} = \frac{|\underset{\sim}{F_1}|}{|\underset{\sim}{d_1}|}\underset{\sim}{d_1} = \frac{14}{7}\begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \\ -4 \end{pmatrix} \text{N}$$ **B1**

Resultant force, $\underset{\sim}{F} = \underset{\sim}{F_1} + \underset{\sim}{F_2} = \begin{pmatrix} 12 \\ 6 \\ -4 \end{pmatrix} + \begin{pmatrix} 4 \\ 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 16 \\ 13 \\ 2 \end{pmatrix} \text{N}$ **B1**

Line of action, $\underset{\sim}{r} = \overrightarrow{OA} + \lambda\underset{\sim}{F} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 16 \\ 13 \\ 2 \end{pmatrix}$ **M1 A1**

14

END

P425/1
PURE
MATHEMATICS
PAPER 1
May 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 1 BOT 2 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Attempt ALL questions in this paper.
- Show your working clearly.

Qn 1: (a). If the roots of $ax^2 + bx + c = 0$ differ by 3, show that
$$b^2 = 9a^2 + 4ac.$$
(b). If α and β are roots of the quadratic equation $x^2 - 4x + 2 = 0$, find the quadratic equation that has roots; $(\alpha + 2), (\beta + 2)$. [12]

Qn 2: The expression $6x^3 - 23x^2 + ax + b$ gives a remainder of 11 when divided by $(x - 3)$ and a remainder of $^{-}21$ when divided by $(x + 1)$. Find the values of a and b; hence factorize the expression. [10]

Qn 3: (a). Given that $\log_2 x + 2\log_4 y = 4$, show that $xy = 16$. Hence solve for x and y in the simultaneous equation:
$$\log_{10}(x + y) = 1$$
$$\log_2 x + 2\log_4 y = 4$$
[8]

(b). If $5^x \bullet 25^{2y} = 1$ and $3^{5x} \bullet 9^y = \frac{1}{9}$, determine the value of x and y. [7]

Qn 4: (a). Prove by induction that
$$\cos\theta + \cos 3\theta + \cos 5\theta + \cdots + \cos(2n - 1)\theta = \frac{\sin 2n\theta}{2\sin\theta}$$

(b). Show that for all positive integral values of n, $7^n + 2^{2n+1}$ is divisible by 3. [12]

Qn 5: (a). Differentiate from first principles $f(x) = 2x^2 + 5x - 3$. Hence find $f'(2)$.
(b). Find $\frac{dy}{dx}$ for each of the functions:
(i). $y = 3x - \frac{5}{x} + \frac{6}{x^2}$,
(ii). $y = (2x + 3)(x + 2)$. [12]

Qn 6: (a). A cylinder of volume, V, is to be cut from a solid sphere of radius, R. Prove that the maximum value of V is $\frac{4\pi R^3}{3\sqrt{3}}$.
(b). A rectangular block has a square base of area x^2 cm^2. Its total surface area is 150 cm^2. Prove that the volume of the block is $\frac{1}{2}(75x - x^3)$ cm^3. Hence find the dimensions of the block when its volume is a maximum. [12]

Qn 7: (a). Prove the identity:
$$\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \operatorname{cosec}\theta - \cot\theta.$$
(b). Solve the equation $4\cos\theta - 3\sec\theta = 2\tan\theta$ for $^{-}180° \le \theta \le {}^{+}180°$. [9]

Qn 8: Given that α and β are acute angles with $\sin\alpha = \frac{7}{25}$ and $\cos\beta = \frac{5}{13}$; find, without using tables or calculator, $\sin(\alpha + \beta)$ and $\tan(\alpha + \beta)$. [8]

Qn 9: The sum of the first two terms of a G.P is 9 and the sum to infinity of the G.P is 25. If the G.P has a positive common ratio, r, find r and the first term. [5]

Qn 10: Determine the equation of the tangent and the normal to the curve $y = (x + 1)(2x + 3)$ at a point $(2, 21)$. [5]

END

MARKING GUIDE

SNo.	Working	Marks
1	(a).	
	Let one of the roots be α. Then the other root is $(\alpha-3)$.	
	sum of roots, $\alpha+(\alpha-3)=-\frac{b}{a}$	B1
	$2\alpha-3=-\frac{b}{a}$	
	$2\alpha=3-\frac{b}{a}$	
	$2\alpha=\frac{3a-b}{a}$	
	$\alpha=\frac{3a-b}{2a}$	B1
	product of roots, $\alpha(\alpha-3)=\frac{c}{a}$	B1
	$\alpha^2-3\alpha=\frac{c}{a}$	
	$\left(\frac{3a-b}{2a}\right)^2-3\left(\frac{3a-b}{2a}\right)=\frac{c}{a}$	M1
	$\frac{9a^2-6ab+b^2}{4a^2}-\frac{9a-3b}{2a}=\frac{c}{a}$	B1
	$9a^2-6ab+b^2-2a(9a-3b)=4ac$	
	$9a^2-6ab+b^2-18a^2+6ab=4ac$	
	$b^2-9a^2=4ac$	
	$b^2=9a^2+4ac$, as required	B1
	(b).	
	$x^2-4x+2=0$	
	$\alpha+\beta=-(-4)=4$	
	$\alpha\beta=2$	B1
	For the required equation,	
	sum of roots $=(\alpha+2)+(\beta+2)=\alpha+\beta+4=4+4$	M1 B1
	$=8$	
	product of roots $=(\alpha+2)(\beta+2)=\alpha\beta+2\alpha+2\beta+4$	M1 B1
	$=\alpha\beta+2(\alpha+\beta)+4=2+2\times4+4=14$	
	The required equation is given by:	
	$x^2-(\text{sum of roots})x+(\text{product of roots})=0$	
	$x^2-8x+14=0$	A1
		12
2		
	let $f(x)=6x^3-23x^2+ax+b$	
	for $(x-3)=0$, $x=3$	B1
	$f(3)=6\times3^3-23\times3^2+3a+b=11$	M1
	$162-207+3a+b=11$	
	$3a+b=56 \longrightarrow (1)$	B1
	for $(x+1)=0$, $x=-1$	B1
	$f(3)=6\times(-1)^3-23\times(-1)^2-a+b=-21$	M1
	$-6-23-a+b=-21$	
	$-a+b=8 \longrightarrow (2)$	B1
	Equation (1) − (2) gives,	
	$3a+b=56 \longrightarrow (1)$	
	$-\quad -a+b=8 \longrightarrow (2)$	M1
	$4a=48$	
	$a=12$	A1
	From equation (2),	
	$b=8+a=8+12=20$	M1 A1
		10
3	(a).	
	$\log_2 x+2\log_4 y=4$	
	$\log_2 x+2\left(\frac{\log_2 y}{\log_2 4}\right)=4$	
	$\log_2 x+2\left(\frac{\log_2 y}{2\log_2 2}\right)=4$	M1
	$\log_2 x+\log_2 y=4$	
	$\log_2 xy=4$	B1
	$xy=2^4$	
	$xy=16$	B1
	For the hence part,	
	$\log_{10}(x+y)=1$	
	$\log_2 x+2\log_4 y=4$	
	$x+y=10 \longrightarrow (1)$	
	$xy=16 \longrightarrow (2)$	
	$\log_{10}(x+y)=1, \quad \Rightarrow x+y=10 \longrightarrow (1)$	
	$\log_2 x+2\log_4 y=4, \quad \Rightarrow xy=16 \longrightarrow (2)$	B1
	Substituting for y in equation (2) gives,	
	$x(10-x)=16$	
	$10x-x^2=16$	M1
	$x^2-10x+16=0$	
	$x=\frac{10\pm\sqrt{(-10)^2-4\times1\times16}}{2\times1}=\frac{10\pm6}{2}$	M1
	$x=\frac{10+6}{2}=8$, or, $x=\frac{10-6}{2}=2$	
	for $x=8$, $y=10-8=2$	A1
	for $x=2$, $y=10-2=8$	A1
	(b).	
	$5^x\bullet 25^{2y}=1$	
	$5^x\times(5^2)^{2y}=5^0$	
	$5^{x+4y}=5^0$	M1
	$x+4y=0$	
	$x=-4y \longrightarrow (1)$	B1
	$3^{5x}\bullet 9^y=\frac{1}{9}$	

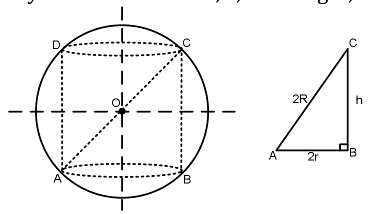
	$3^{5x}\times(3^2)^y=3^{-2}$	M1
	$3^{5x+2y}=3^{-2}$	
	$5x+2y=-2 \quad \longrightarrow (2)$	B1
	Substituting equation (1) into (2) gives,	
	$5\times(-4y)+2y=-2$	M1
	$-18y=-2$	
	$y=\frac{2}{18}=\frac{1}{9}$	A1
	From equation (2),	
	$x=-4\times\frac{1}{9}=-\frac{4}{9}$	A1
		15
4	(a).	
	$\cos\theta+\cos 3\theta+\cos 5\theta+\cdots+\cos(2n-1)\theta=\frac{\sin 2n\theta}{2\sin\theta}$	
	For $n=1$,	
	$L.H.S=\cos\theta$	
	$R.H.S=\frac{\sin 2\times 1\times\theta}{2\sin\theta}=\frac{\sin 2\theta}{2\sin\theta}=\frac{2\sin\theta\cos\theta}{2\sin\theta}=\cos\theta$	B1
	True for $n=1$ since $L.H.S=R.H.S$	
	For $n=2$,	
	$L.H.S=\cos\theta+\cos 3\theta=2\cos\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{\theta-3\theta}{2}\right)$	B1
	$=2\cos 2\theta\cos(-\theta)=2\cos 2\theta\cos\theta$	
	$R.H.S=\frac{\sin 2\times 2\theta}{2\sin\theta}=\frac{\sin 4\theta}{2\sin\theta}=\frac{2\sin 2\theta\cos 2\theta}{2\sin\theta}$	
	$=\frac{2(2\sin\theta\cos\theta)\cos 2\theta}{2\sin\theta}=2\cos\theta\cos 2\theta$	B1
	True for $n=2$ since $L.H.S=R.H.S$	
	Suppose it's true for $n=k$, the series becomes:	
	$\cos\theta+\cos 3\theta+\cos 5\theta+\cdots+\cos(2k-1)\theta=\frac{\sin 2k\theta}{2\sin\theta} \quad \longrightarrow (1)$	
	For $n=k+1$,	
	$R.H.S=\frac{\sin 2(k+1)\theta}{2\sin\theta}$	
	$L.H.S=\cos\theta+\cos 3\theta+\cos 5\theta+\cdots+\cos(2k-1)\theta+\cos(2k+1)\theta$	
	$=\frac{\sin 2k\theta}{2\sin\theta}+\cos(2k+1)\theta$	
	$=\frac{\sin 2k\theta+2\sin\theta\cos(2k+1)\theta}{2\sin\theta}$	M1
	$=\frac{\sin 2k\theta+\sin[(2k+1)\theta+\theta]-\sin[(2k+1)\theta+\theta]}{2\sin\theta}$	M1
	$=\frac{\sin 2k\theta+\sin[(2k+2)\theta]-\sin(2k\theta)}{2\sin\theta}$	
	$=\frac{\sin 2(k+1)\theta}{2\sin\theta}$	B1
	True for $n=(k+1)$ since $L.H.S=R.H.S$.	

	Since its true for $n=1, n=2, n=k$ and $n=k+1$, then it's true for all positive integers of n.	
	(b).	
	let $f(n)=7^n+2^{2n+1}$	
	$f(1)=7^1+2^{2\times1+1}=7+8=15=3\times5$, true for $n=1$	B1
	$f(2)=7^2+2^{2\times2+1}=49+32=81=3\times27$, true for $n=1$	B1
	Suppose it's true for $n=k$, then,	
	$f(k)=7^k+2^{2k+1}=3A$	
	$7^k=3A-2^{2k+1} \quad \longrightarrow (1)$	B1
	where A is a positive integer.	
	For $n=k+1$,	
	$f(k+1)=7^{k+1}+2^{2k+3}=7^1(7^k)+2^{2k+3}$	
	but $7^k=3A-2^{2k+1}$	
	$f(k+1)=7(3A-2^{2k+1})+2^{2k+3}$	M1
	$=21A-7\times2^{2k}\times2+2^{2k}\times2^3$	M1
	$=21A-14\times2^{2k}+2^{2k}\times8$	
	$=21A-2^{2k}(14-8)$	
	$=21A-6\times2^{2k}$	
	$=3(7A-2\times2^{2k})$	
	$=3(7A-2^{2k+1})$	
	True of $n=k+1$ since $f(k+1)$ is a multiple of 3.	B1
	Since its true for $n=1, n=2, n=k$ and $n=k+1$, then 7^n+2^{2n+1} is divisible by 3 for all positive integral values of n.	
		12
5	(a).	
	$f(x)=2x^2+5x-3$	
	$f(x+\delta x)=2(x+\delta x)^2+5(x+\delta x)-3$	B1
	$=2x^2+4x(\delta x)+2(\delta x)^2+5x+5(\delta x)-3$	
	$=2x^2+5x-3+4x(\delta x)+2(\delta x)^2+5(\delta x)$	B1
	$f'(x)=\lim_{\delta x\to0}\left[\frac{f(x+\delta x)-f(x)}{\delta x}\right]$	
	$=\lim_{\delta x\to0}\left[\frac{4x(\delta x)+2(\delta x)^2+5(\delta x)}{\delta x}\right]$	M1
	$=\lim_{\delta x\to0}[4x+2(\delta x)+5]=[4x+2\times0+5]=4x+5$	M1 B1
	For the hence part:	
	$f'(2)=4\times2+5=13$	M1 A1
	(b). (i).	
	$y=3x-\frac{5}{x}+\frac{6}{x^2}$	
	$y=3x-5x^{-1}+6x^{-2}$	
	$\frac{dy}{dx}=3\times x^{1-1}-5\times(-1\times x^{-1-1})+6\times(-2\times x^{-2-1})$	
	$\frac{dy}{dx}=3+5x^{-2}-12x^{-3}$	B1 B1 B1
	(ii).	
	$y=(2x+3)(x+2)$	

$$y = 2x^2 + 3x + 4x + 6$$
$$y = 2x^2 + 7x + 6 \qquad \textbf{M1}$$
$$\frac{dy}{dx} = 4x + 7 \qquad \textbf{B1}$$

12

6 (a). Let the cylinder be of radius, r, and height, h.



B1

By Pythagoras theorem,

$$(2r)^2 = (2R)^2 - h^2, \qquad \Rightarrow r^2 = R^2 - \frac{h^2}{4} \qquad \textbf{B1}$$

Volume of cylinder,

$$V = \pi r^2 h = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi R^2 h - \frac{\pi h^3}{4} \qquad \textbf{M1}$$

$$\frac{dV}{dh} = \pi R^2 - \frac{3\pi h^2}{4} \qquad \textbf{M1}$$

For maximum volume, $\frac{dV}{dh} = 0$

$$\pi R^2 - \frac{3\pi h^2}{4} = 0 \qquad \textbf{M1}$$

$$R^2 = \frac{3\pi h^2}{4}, \qquad \Rightarrow h = \frac{2R}{\sqrt{3}} \qquad \textbf{A1}$$

$$V_{max} = \pi R^2\left(\frac{2R}{\sqrt{3}}\right) - \pi\left(\frac{2R}{\sqrt{3}}\right)^3 = \pi R^3\left(\frac{2}{\sqrt{3}} - \frac{2}{3\sqrt{3}}\right) \qquad \textbf{M1}$$

$$= \pi R^3\left(\frac{6-2}{3\sqrt{3}}\right) = \pi R^3\left(\frac{4}{3\sqrt{3}}\right) = \frac{4\pi R^3}{3\sqrt{3}} \qquad \textbf{B1}$$

(b).

Let h be the height of the block.

$$\text{total surface area} = 2(l \times w + l \times h + w \times h) = 150$$
$$2(x \times x + x \times h + x \times h) = 150$$
$$x^2 + 2hx = 75 \qquad \textbf{M1}$$
$$2hx = 75 - x^2$$
$$h = \frac{75 - x^2}{2x} \qquad \textbf{B1}$$
$$\text{volume} = l \times w \times h = x^2 \times \left(\frac{75 - x^2}{2x}\right) \qquad \textbf{M1}$$
$$= \frac{75x - x^3}{2} = \frac{1}{2}(75x - x^3)\ \text{cm}^3, \qquad \text{as required} \qquad \textbf{B1}$$

12

7 (a).

$$L.H.S = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} \qquad \textbf{M1}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} = \frac{1-\cos\theta}{\sin\theta} \qquad \textbf{M1 B1}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta - \cot\theta \qquad \textbf{B1}$$

(b).

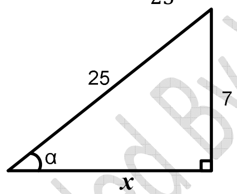
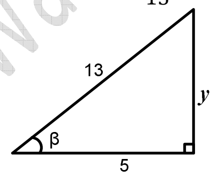
$$4\cos\theta - 3\sec\theta = 2\tan\theta$$
$$4\cos\theta - \frac{3}{\cos\theta} = \frac{2\sin\theta}{\cos\theta} \qquad \textbf{M1}$$
$$4\cos^2\theta - 3 = 2\sin\theta$$
$$4(1-\sin^2\theta) - 3 = 2\sin\theta$$
$$4 - 4\sin^2\theta - 3 = 2\sin\theta$$
$$4\sin^2\theta + 2\sin\theta - 1 = 0 \qquad \textbf{B1}$$
$$\sin\theta = \frac{-2 \pm \sqrt{2^2 - 4\times 4\times(-1)}}{2\times 4} = \frac{-2 \pm \sqrt{20}}{8} \qquad \textbf{M1}$$

either $\sin\theta = -0.80902, \quad \Rightarrow \theta = -54°, -126°$ **A1**

or $\sin\theta = 0.30902, \quad \Rightarrow \theta = 18°, 162°$ **A1**

09

8

$\sin\alpha = \frac{7}{25}$  $x = \sqrt{25^2 - 7^2} = 24$ $\cos\alpha = \frac{24}{25}$	$\cos\beta = \frac{5}{13}$  $x = \sqrt{13^2 - 5^2} = 12$ $\sin\beta = \frac{12}{13}$

B1 B1

$$\therefore \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \frac{7}{25}\times\frac{5}{13} + \frac{24}{25}\times\frac{12}{13} = \frac{7}{65} + \frac{288}{325} = \frac{323}{325} \qquad \textbf{M1 A1}$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \frac{24}{25}\times\frac{5}{13} - \frac{7}{25}\times\frac{12}{13} = \frac{24}{65} - \frac{84}{325} = \frac{36}{325} \qquad \textbf{M1 B1}$$
$$\therefore \tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{323}{325} \div \frac{36}{325} = \frac{323}{36} \qquad \textbf{M1 A1}$$

08

9

	$a + ar = 9$ $a(1 + r) = 9$ $a = \frac{9}{1 + r}$	**B1**
	$S_\infty = \frac{a}{1 - r} = 25$ $a = 25(1 - r)$	**B1**
	Equating the two equations gives: $\frac{9}{1 + r} = 25(1 - r)$	**M1**
	$9 = 25(1 - r^2)$ $r^2 = 1 - \frac{9}{25}$ $r = \frac{4}{5} = 0.8$	**A1**
	First term, $a = 25(1 - 0.8) = 5$	**A1**
		05
10	$y = (x + 1)(2x + 3)$ $y = 2x^2 + 3x + 2x + 3$ $y = 2x^2 + 5x + 3$ Gradient function, $\frac{dy}{dx} = 4x + 5$	**M1**
	At point $(2, 21)$, Gradient of tangent $= 4 \times 2 + 5 = 13$ The required equation of the tangent is given by: $\frac{y - 21}{x - 2} = 13$ $y - 21 = 13x - 26$	**M1**
	$y = 13x + 5$	**A1**
	Gradient of normal $= -\frac{1}{13}$ The required equation of the normal is given by: $\frac{y - 21}{x - 2} = -\frac{1}{13}$	**M1**
	$y - 21 = -\frac{1}{13}x + \frac{2}{13}$ $y = -\frac{1}{13}x + \frac{275}{13}$	**A1**
		05

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
May 2018
$2\frac{1}{2}$ hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 2 BOT 2 2018
Time: 2 Hours 30 Minutes

NAME: **COMB:**

INSTRUCTIONS:
- Attempt ALL questions in both sections.
- Show your working clearly.

Section A (40 Marks)

Qn 1: A particle has an initial position vector $\left(4\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}} + 9\underset{\sim}{\boldsymbol{k}}\right)$ m. The particle moves with a constant velocity of $\left(3\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} - 5\underset{\sim}{\boldsymbol{k}}\right)$ m s^{-1}. Find:

(a). the position vector of the particle after t seconds. [2]
(b). how far the particle is from the origin after 5 seconds. [3]

Qn 2: In 1991, the index number of the value of commodity was 135 when 1989 was taken as base year. The value of the commodity in 1991 was shs 5,400 and in 1990 was shs 4,600. Find:
(a). the value of the commodity in 1989. [2]
(b). the index number of the value of the commodity in 1990 when 1989 was taken as base year. [3]

Qn 3: Three force $\underset{\sim}{\boldsymbol{F_1}} = 2\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}} - \underset{\sim}{\boldsymbol{k}}$, $\underset{\sim}{\boldsymbol{F_2}} = 3\underset{\sim}{\boldsymbol{i}} - 7\underset{\sim}{\boldsymbol{j}} + 6\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{F_3}} = \underset{\sim}{\boldsymbol{i}} + 5\underset{\sim}{\boldsymbol{j}} - 4\underset{\sim}{\boldsymbol{k}}$ are acting on a particle which is given a displacement, $12\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}} + 2\underset{\sim}{\boldsymbol{k}}$. Find:

(a). the resultant of forces, $\underset{\sim}{\boldsymbol{F_1}}$, $\underset{\sim}{\boldsymbol{F_2}}$ and $\underset{\sim}{\boldsymbol{F_3}}$. [2]

(b). work done by the resultant force. [3]

Qn 4: The numbers $a, b, 8, 5, 7$ have a mean of 6 and a variance 2. Find the values of a and b if $a > b$. [5]

Qn 5: Calculate the magnitude of the horizontal and vertical components of a tension of 8 N in a string of length 10 m which has one end fastened to the top of a flag pole of height 6 m and the other end fixed to the ground. [5]

Qn 6: The table shows the speeds of 200 vehicles passing a particular point.

Speeds (km h^{-1})	30 –	40 –	50 –	60 –	70 –
Frequency	14	30	52	71	33

Find the mean speed. [5]

Qn 7: A car A, travelling at a constant velocity of 25 m s^{-1}, overtakes a stationary car B. Two seconds later, car $\boldsymbol{B}$ sets off in pursuit; accelerating at 6 m s^{-2}. How far does $\boldsymbol{B}$ travel before catching up with $\boldsymbol{A}$? [5]

Qn 8: Calculate a weighted price index for the following figures for 1994 based on 1990. (Give your answer to the nearest integer.)
Hence comment on your results.

Item	1990 price (£)	1994 price (£)	Weight, w
Food	55	60	4
Housing	48	52	2
Transport	16	20	1

[5]

Section B (48 Marks)

Question 9:

(a). An object with constant speed of 13 m s^{-1} moves in a direction $10\underset{\sim}{\boldsymbol{i}} - 24\underset{\sim}{\boldsymbol{j}}$ from a point with position vector $\underset{\sim}{\boldsymbol{i}} - \underset{\sim}{\boldsymbol{j}}$. Find the position vector of the object after 3 seconds. [5]

(b). $\boldsymbol{P}$ and $\boldsymbol{Q}$ are particles moving with constant velocities $\underset{\sim}{\boldsymbol{v}_P}$ and $\underset{\sim}{\boldsymbol{v}_Q}$ respectively.

$$\underset{\sim}{\boldsymbol{v}_P} = 3\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} - \underset{\sim}{\boldsymbol{k}}, \quad \text{and,} \quad \underset{\sim}{\boldsymbol{v}_Q} = 3\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}} + 5\underset{\sim}{\boldsymbol{k}}$$

$\boldsymbol{P}$ is a point with position vector ${}^{-}16\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}} + 4\underset{\sim}{\boldsymbol{k}}$ when $t = 0$ and one second later, $\boldsymbol{Q}$ passes through the point with position vector ${}^{-}5\underset{\sim}{\boldsymbol{i}} - 10\underset{\sim}{\boldsymbol{j}} - 9\underset{\sim}{\boldsymbol{k}}$. Show that the particles collide at a point with position vector $\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}}$. [7]

Question 10:
The table gives the frequency distribution of heights (in cm) of 400 children in a certain school.

Height (cm)	Frequency
< 110	27
< 120	58
< 130	130
< 140	105
< 150	50
< 160	25
< 170	5

(a). Draw a cumulative frequency curve for the above data. Hence, estimate:
(i). the median,
(ii). The interquartile range,
(iii). The 10th to 90th percentile range.
(b). Calculate the modal height. [12]

Question 11:

(a). $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are three points lying in that order on a straight road with $\overline{\boldsymbol{AB}} = 5$ km and $\overline{\boldsymbol{BC}} = 4$ km. A man runs from $\boldsymbol{A}$ to $\boldsymbol{B}$ at 20 km h^{-1} and then walks from $\boldsymbol{B}$ to $\boldsymbol{C}$ at 8 km h^{-1}. Find:
(i). the total time taken to travel from $\boldsymbol{A}$ to $\boldsymbol{C}$.
(ii). The average speed of the man for the journey from $\boldsymbol{A}$ to $\boldsymbol{C}$. [5]

(b). At $t = 0$, a body is projected from an origin $\boldsymbol{O}$ with an initial velocity of 10 m s^{-1}. The body moves along straight line with a constant acceleration of ${}^{-}2$ m s^{-2}.
(i). Find the displacement of the body from the point $\boldsymbol{O}$ when $t = 7$ s. [2]
(ii). How far from $\boldsymbol{O}$ does the body come to instantaneous rest and what is the value of t then. [3]
(iii). Find the distance travelled by the body during the time interval, $t = 0$ to $t = 7$ seconds. [2]

Question 12:

The following table gives the test results for 10 children.

Child	A	B	C	D	E	F	G	H	I	J
Math (x)	1	8	15	18	23	28	33	39	45	45
English (y)	3	14	8	20	19	17	36	26	14	29

(a). Draw a scatter diagram for the above data. Hence comment on the relationship between the Maths and English marks.

(b). Draw a line of best fit on the scatter diagram and use it to estimate the Math mark of a student who scored 30 marks in English.

(c). Calculate the rank correlation coefficient for the above data. Hence comment on your result at 5% level of significance. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	(a). $\overrightarrow{OP} = \begin{pmatrix}4\\3\\9\end{pmatrix}$ m, $\quad \underset{\sim}{v} = \begin{pmatrix}3\\-2\\-5\end{pmatrix}$ m s^{-1}	
	$\overrightarrow{OP'} = \overrightarrow{OP} + t\underset{\sim}{v} = \begin{pmatrix}4\\3\\9\end{pmatrix} + t\begin{pmatrix}3\\-2\\-5\end{pmatrix} = \begin{pmatrix}4+3t\\3-2t\\9-5t\end{pmatrix}$ m	M1 A1
	(ii). When $t = 5$, $\overrightarrow{OP'} = \begin{pmatrix}4\\3\\9\end{pmatrix} + 5\begin{pmatrix}3\\-2\\-5\end{pmatrix} = \begin{pmatrix}4\\3\\9\end{pmatrix} + \begin{pmatrix}15\\-10\\-25\end{pmatrix} = \begin{pmatrix}19\\-7\\-16\end{pmatrix}$ m	M1 A1
	Distance from origin $= \sqrt{19^2 + (-7)^2 + (-16)^2} = \sqrt{666}$ $= 25.807$ m	A1
		05
2	(a). $\frac{P_{1991}}{P_{1989}} = 1.35$	
	$\frac{5400}{P_{1989}} = 1.35$	M1
	$P_{1989} = \frac{5400}{1.35} = 4{,}000$	A1
	(b). $I_{1990} = \frac{P_{1990}}{P_{1989}} \times 100 = \frac{4600}{4000} \times 100 = 115$	M1 M1 A1
		05
3	(a). $\underset{\sim}{F} = \underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3} = \begin{pmatrix}2\\4\\-1\end{pmatrix} + \begin{pmatrix}3\\-7\\6\end{pmatrix} + \begin{pmatrix}12\\4\\2\end{pmatrix} = \begin{pmatrix}6\\2\\1\end{pmatrix}$	M1 A1
	(b). Work done $= \underset{\sim}{F}.\underset{\sim}{s} = \begin{pmatrix}6\\2\\1\end{pmatrix}.\begin{pmatrix}12\\4\\2\end{pmatrix} = 72 + 8 + 2 = 82$ J	M1 M1 A1
		05
4	Mean $= \frac{\sum x}{n}$	
	$\frac{a+b+8+5+7}{5} = 6$	
	$a + b + 20 = 6 \times 5$	
	$a + b = 10 \quad \longrightarrow (1)$	B1
	Variance $= \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$	

$$\frac{a^2+b^2+8^2+5^2+7^2}{5}-6^2=2$$ **B1**

$$\frac{a^2+b^2+138}{5}=38$$

$$a^2+b^2+138=38\times 5$$

$$a^2+b^2=52 \quad \longrightarrow (2)$$

Substituting equation (1) into (2) gives;

$$a^2+(10-a)^2=52$$

$$a^2+100-20a+a^2=52$$

$$2a^2-20a+48=0$$

$$a=\frac{20\pm\sqrt{(-20)^2-4\times 2\times 48}}{2\times 2}$$ **M1**

$a=4$, or, $a=6$ **B1**

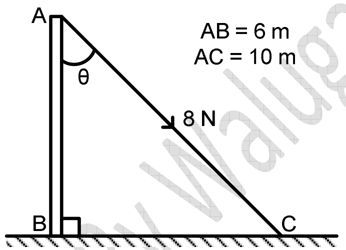
For $a=4$, $b=10-4=6$

For $a=6$, $b=10-6=4$

But $a>b$, $\Longrightarrow a\neq 4$, $\therefore a=6$ and $b=4$ **A1**

05

5



B1

$$\overline{BC}=\sqrt{10^2-6^2}=8\text{ m}$$

Horizontal component $=8\sin\theta=8\times\frac{8}{10}=6.4$ N **M1 A1**

Vertical component $=8\cos\theta=8\times\frac{6}{10}=4.8$ N **M1 A1**

05

6

Class	Class boundary	f	x	fx
30 –	30 – 40	14	35	490
40 –	40 – 50	30	45	1350
50 –	50 – 60	52	55	2850
60 –	60 – 70	71	65	4615
70 –	70 – 80	33	75	2475
Total		200		11790

B1-class boundary

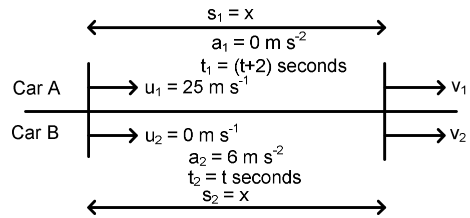
B1-for x

B1-for $\sum fx$

$$\text{Mean}=\frac{\sum fx}{\sum f}=\frac{11790}{200}=58.95$$ **M1 A1**

05

7



For the car A,

$$s_1=u_1t_1+\frac{1}{2}a_1t_1^2$$

$$x=25(t+2)+0$$

$$x=25t+50 \quad \longrightarrow (1)$$ **B1**

For the car B,

$$s_2=u_2t_2+\frac{1}{2}a_2t_2^2$$

$$x=0+\frac{1}{2}\times 6t^2$$

$$x=3t^2 \quad \longrightarrow (2)$$ **B1**

Equating equations (1) and (2) gives, **M1**

$$3t^2=25t+50$$

$$3t^2-25t-50=0$$

$$t=\frac{25\pm\sqrt{(-25)^2-4\times 3\times(-50)}}{2\times 3}$$

$t=10$, or, $t=-\frac{5}{3}$

But $t=-\frac{5}{3}$, $\Longrightarrow t=10$ s

From equation (2),

$$x=3t^2=3\times 10^2=300\text{ m}$$ **M1 A1**

05

8

$$\text{Weighted price index}=\frac{\sum\left(\frac{P_{1994}}{P_{1990}}\times W\right)}{\sum W}\times 100$$

$$=\frac{\left(\frac{60}{55}\times 4\right)+\left(\frac{52}{48}\times 2\right)+\left(\frac{20}{16}\times 1\right)}{4+2+1}\times 100$$ **M1 M1 M1**

$=111.1472\approx 111$ (nearest integer) **A1**

The cost of living index increased by 11%. **B1**

05

9 (a).

Velocity vector, $$\underset{\sim}{v}=\frac{13}{\sqrt{10^2+(-24)^2}}\begin{pmatrix}10\\-24\end{pmatrix}$$ **M1**

$$= \begin{pmatrix} 5 \\ -12 \end{pmatrix} \text{ m s}^{-1}$$ **B1**

Position vector, $\underset{\sim}{r}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 5 \\ -12 \end{pmatrix}$ **M1**

$$\underset{\sim}{r}(t=3) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 3\begin{pmatrix} 5 \\ -12 \end{pmatrix} = \begin{pmatrix} 16 \\ -37 \end{pmatrix} \text{ m}$$ **M1 A1**

(b).

$$\underset{\sim}{r_Q}(t=0) = \begin{pmatrix} -5 \\ -10 \\ -9 \end{pmatrix} - 1\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -8 \\ -14 \\ -14 \end{pmatrix} \text{ m}$$ **B1**

$$\underset{\sim}{r_Q}(t) = \begin{pmatrix} -8 \\ -14 \\ -14 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -8+3t \\ -14+4t \\ -14+5t \end{pmatrix}$$ **B1**

$$\underset{\sim}{r_P}(t) = \begin{pmatrix} 16 \\ 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} -5 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 16-5t \\ 4-2t \\ 4-t \end{pmatrix}$$ **B1**

For collision,

$$\underset{\sim}{r_P}(t) = \underset{\sim}{r_Q}(t)$$

$$\begin{pmatrix} 16-5t \\ 4-2t \\ 4-t \end{pmatrix} = \begin{pmatrix} -8+3t \\ -14+4t \\ -14+5t \end{pmatrix}$$ **M1**

$16-5t = -8+3t, \quad \Rightarrow t = 3$

$4-2t = -14+4t, \quad \Rightarrow t = 3$ **B1**

$4-t = -14+5t, \quad \Rightarrow t = 3$

$$\underset{\sim}{r_P}(t=3) = \begin{pmatrix} 16-5\times 3 \\ 4-2\times 3 \\ 4-3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \text{ m}$$ **M1 B1**

Hence they collide at $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$.

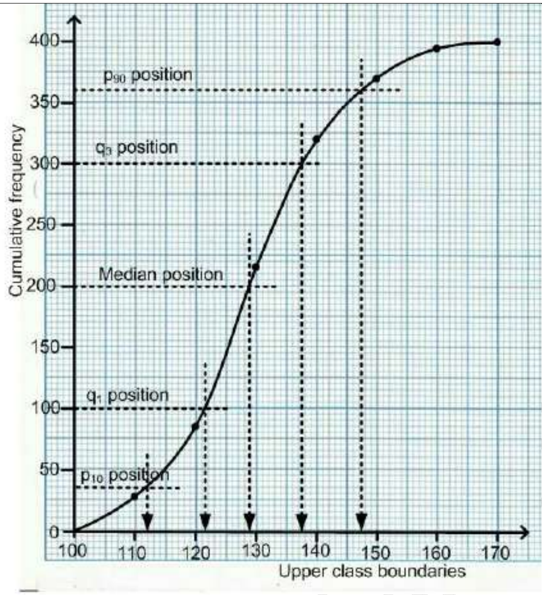
12

10 (a).

Class	f	C.F	Class boundary
< 110	27	27	100 – 110
< 120	58	85	110 – 120
< 130	130	215	120 – 130
< 140	105	320	130 – 140
< 150	50	370	140 – 150
< 160	25	395	150 – 160
< 170	5	400	160 – 170

B1-class boundary

B1-for C.F



B1-axes well labeled and suitable scale.

B1-correctly plotted points.

B1-smooth curve

(i).

Median position: $\frac{1}{2}N = \frac{1}{2}\times 400 = 200$

From the ogive, Median: $q_2 = 129$ cm **A1**

(ii).

Lower quartile position: $\frac{1}{4}N = \frac{1}{4}\times 400 = 100$

Upper quartile position: $\frac{3}{4}N = \frac{3}{4}\times 400 = 300$

From the ogive,

Lower quartile: $q_1 = 121.5$

Upper quartile: $q_3 = 137.5$

∴Interquartile range: $q_3 - q_1 = 137.5 - 121.5 = 16$ cm **M1 A1**

(iii).

10^{th} percentile position: $\frac{10}{100}N = \frac{10}{100}\times 400 = 40$

90^{th} percentile position: $\frac{90}{100}N = \frac{90}{100}\times 400 = 360$

From the ogive,

10^{th} percentile: $P_{10} = 112$

90^{th} percentile: $P_{90} = 147.5$

∴ 10^{th} to 90^{th} percentile range: $P_{90} - P_{10}$

$= 147.5 - 112 = 35.5$ cm **M1 A1**

(b).

$\Delta_1 = 130 - 58 = 72, \Delta_2 = 130 - 105 = 25, L_m = 120, c = 10$

$$\text{Mode} = L_m + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)c = 120 + \left(\frac{72}{72+25}\right)\times 10$$ **M1**

$= 120 + 7.4226 = 127.4226$ **A1**

12

11 (a). (i).

$$\text{Time taken from A to B} = \frac{\text{Distance}}{\text{Speed}} = \frac{5}{20} = 0.25 \text{ hours}$$

$$\text{Time taken from B to C} = \frac{\text{Distance}}{\text{Speed}} = \frac{4}{8} = 0.5 \text{ hours}$$

$$\text{Total time taken from A to C} = 0.25 + 0.5 = 0.75 \text{ hours}$$

M1 M1 A1

(ii).

$$\text{Average speed from A to C} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{5+4}{0.75} = 12 \text{ km h}^{-1}$$

M1 A1

(b). (i).

$$u = 10 \text{ m s}^{-1}, a = -2 \text{ m s}^{-1}, t = 7 \text{ s}$$

$$s = ut + \frac{1}{2}at^2 = 10 \times 7 - \frac{1}{2} \times 2 \times 7^2 = 21 \text{ m}$$

M1 A1

(ii).

$$u = 10 \text{ m s}^{-1}, a = -2 \text{ m s}^{-1}, v = 0 \text{ m s}^{-1}$$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 10^2}{2 \times {}^{-}2} = 25 \text{ m}$$

M1 A1

(iii).

Time taken to reach instantaneous rest is given by:

$$t = \frac{v-u}{a} = \frac{0-10}{{}^{-}2} = 5 \text{ s}$$

B1

Therefore time take after instantaneous rest $= 7 - 5 = 2$ s.

Extra distance covered is given by:

$$s = ut + \frac{1}{2}at^2 = 10 \times 2 + \frac{1}{2} \times (-2) \times 2^2 = 4 \text{ m}$$

$$\text{Total Distance} = 25 + 4 = 29 \text{ m}$$

M1 A1

12

12 (a).

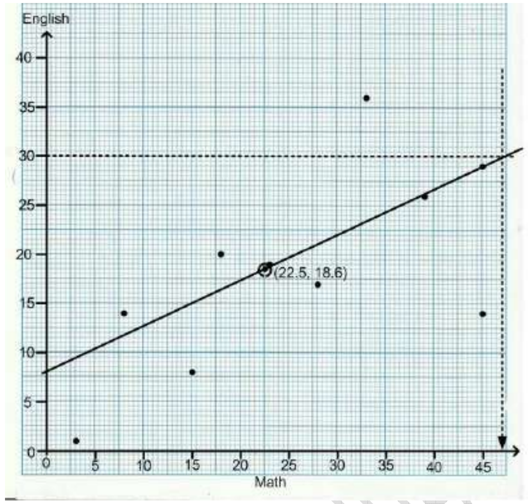
	x	y	R_x	R_y	d	d^2
A	1	3	10	10	0	0
B	8	14	9	7.5	1.5	2.25
C	15	8	8	9	-1	1
D	18	20	7	4	3	9
E	23	19	6	5	1	1
F	28	17	5	6	-1	1
G	33	36	4	1	3	9
H	39	26	3	3	0	0
I	45	14	1.5	7.5	-6	36
J	45	29	1.5	2	-0.5	0.25
Total	225	186				$\sum d^2 = 59.5$

B1-$\sum d^2$

$$\bar{x} = \frac{225}{10} = 22.5, \quad \bar{y} = \frac{186}{10} = 18.6$$

$$\Rightarrow \text{Mean point,} \quad (\bar{x}, \bar{y}) = (22.5, 18.6)$$

B1



B1-axes well labeled and suitable scale

B2-plotting scatter points including mean point

B1-drawing line of best fit

Comment: There is a positive linear relationship. **B1**

(ii). The estimate Math mark of a student who scored 30 marks in English is 47 marks. **B1**

(b).

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 59.5}{10(10^2-1)} = 0.6394$$

M1 A1

Comment: Insignificant at 5%. **B1**

12

END

P425/1
PURE
MATHEMATICS
PAPER 1
July 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 1 MOT 2 2018
Time: 3 Hours

NAME: ______________________ **COMB:** ________

INSTRUCTIONS:
- Attempt ALL questions in this paper.
- Show your working clearly.

Qn 1: Three consecutive terms of an A.P have a sum of 36 and a product of 1428. Find the three terms. [4]

Qn 2: Determine the term independent of x in the binomial expansion of $\left(\frac{3}{x^2} - 2x\right)^6$. [4]

Qn 3: Given that α and β are acute angles with $\sin\alpha = \frac{7}{25}$ and $\cos\beta = \frac{5}{13}$; find, without using tables or calculator, $\sin(\alpha+\beta)$ and $\tan(\alpha+\beta)$. [8]

Qn 4: The sum of the first two terms of a G.P is 9 and the sum to infinity of the G.P is 25. If the G.P has a positive common ratio, r, find r and the first term. [5]

Qn 5: Determine the equation of the tangent and the normal to the curve $y = (x+1)(2x+3)$ at a point $(2, 21)$. [5]

Qn 6: Expand $\sqrt{\left(\frac{1+x}{1-x}\right)}$ in ascending powers of x, upto and including the term in x^3. State the value of x for which the expansion is valid and by substituting $x = 0.2$, find an approximation for $\sqrt{1.5}$. [7]

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

Qn 7: (a). If the roots of $ax^2 + bx + c = 0$ differ by 3, show that
$$b^2 = 9a^2 + 4ac.$$
(b). If α and β are roots of the quadratic equation $x^2 - 4x + 2 = 0$, find the quadratic equation that has roots; $(\alpha+2), (\beta+2)$. [12]

Qn 8: The expression $6x^3 - 23x^2 + ax + b$ gives a remainder of 11 when divided by $(x-3)$ and a remainder of $^-21$ when divided by $(x+1)$. Find the values of a and b; hence factorize the expression. [10]

Qn 9: (a). Given that $\log_2 x + 2\log_4 y = 4$, show that $xy = 16$. Hence solve for x and y in the simultaneous equation:
$$\log_{10}(x+y) = 1$$
$$\log_2 x + 2\log_4 y = 4$$
(b). If $5^x \bullet 25^{2y} = 1$ and $3^{5x} \bullet 9^y = \frac{1}{9}$, determine the value of x and y. [12]

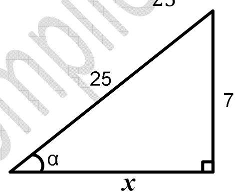
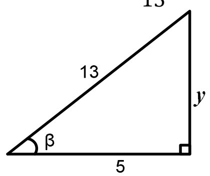
Qn 10: (a). Differentiate from first principles $f(x) = 2x^2 + 5x - 3$. Hence find $f'(2)$.
(b). Find $\frac{dy}{dx}$ for each of the functions:
(i). $y = 3x - \frac{5}{x} + \frac{6}{x^2}$,
(ii). $y = (2x+3)(x+2)$. [12]

Qn 11: (a). A cylinder of volume, V, is to be cut from a solid sphere of radius, R. Prove that the maximum value of V is $\frac{4\pi R^3}{3\sqrt{3}}$.
(b). A rectangular block has a square base of area x^2 cm². Its total surface area is 150 cm². Prove that the volume of the block is $\frac{1}{2}(75x - x^3)$ cm³. Hence find the dimensions of the block when its volume is a maximum. [12]

Qn 12: (a). Prove the identity:
$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta.$$
(b). Solve the equation $4\cos\theta - 3\sec\theta = 2\tan\theta$ for $^-180° \leq \theta \leq {}^+180°$. [9]

*****END*****

MARKING GUIDE

SNo.	Working	Marks
1	$\text{sum} = a + (a+d) + (a+2d) = 36$ $3a + 3d = 36$ $a + d = 12$ $a = 12 - d$	 B1
	$\text{product} = a(a+d)(a+2d) = 1428$ $(12-d)(12-d+d)(12-d+2d) = 1428$ $12(12-d)(12+d) = 1428$ $144 - d^2 = 119$ $d^2 = 25$ $d = 5$	 M1 B1
	First term, $a = 12 - d = 12 - 5 = 7$ Second term, $a + d = 7 + 5 = 12$ Third term, $a + 2d = 7 + 2 \times 5 = 17$	 A1-all three terms
		04
2	General term $= {}^6C_r \times \left(\frac{3}{x^2}\right)^r (2x)^{6-r}$ $= {}^6C_r \times 3^r \times 2^{6-r} \times x^{-2r} \times x^{6-r}$	 M1
	For the term independent of x, $-2r + 6 - r = 0$ $6 - 3r = 0$ $r = 2$	 B1
	Required term $= {}^6C_2 \times 3^2 \times 2^{6-2}$ $= 15 \times 9 \times 16 = 2160$	M1 A1
		04
3	$\sin\alpha = \frac{7}{25}$  $x = \sqrt{25^2 - 7^2} = 24$ $\cos\alpha = \frac{24}{25}$	
	$\cos\beta = \frac{5}{13}$  $x = \sqrt{13^2 - 5^2} = 12$ $\sin\beta = \frac{12}{13}$	B1 B1
	$\therefore \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $= \frac{7}{25} \times \frac{5}{13} + \frac{24}{25} \times \frac{12}{13} = \frac{7}{65} + \frac{288}{325} = \frac{323}{325}$	M1 A1

SNo.	Working	Marks
	$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $= \frac{24}{25} \times \frac{5}{13} - \frac{7}{25} \times \frac{12}{13} = \frac{24}{65} - \frac{84}{325} = \frac{36}{325}$	M1 B1
	$\therefore \tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{323}{325} \div \frac{36}{325} = \frac{323}{36}$	M1 A1
		08
4	$a + ar = 9$ $a(1+r) = 9$ $a = \frac{9}{1+r}$	 B1
	$S_\infty = \frac{a}{1-r} = 25$ $a = 25(1-r)$	 B1
	Equating the two equations gives: $\frac{9}{1+r} = 25(1-r)$ $9 = 25(1-r^2)$ $r^2 = 1 - \frac{9}{25}$ $r = \frac{4}{5} = 0.8$ First term, $a = 25(1-0.8) = 5$	 M1 A1 A1
		05
5	$y = (x+1)(2x+3)$ $y = 2x^2 + 3x + 2x + 3$ $y = 2x^2 + 5x + 3$ Gradient function, $\frac{dy}{dx} = 4x + 5$	 M1
	At point $(2, 21)$, Gradient of tangent $= 4 \times 2 + 5 = 13$ The required equation of the tangent is given by: $\frac{y-21}{x-2} = 13$ $y - 21 = 13x - 26$ $y = 13x + 5$	 M1 A1
	Gradient of normal $= -\frac{1}{13}$ The required equation of the normal is given by: $\frac{y-21}{x-2} = -\frac{1}{13}$ $y - 21 = -\frac{1}{13}x + \frac{2}{13}$ $y = -\frac{1}{13}x + \frac{275}{13}$	 M1 A1
		05

6	$\sqrt{\left(\frac{1+x}{1-x}\right)} = (1+x)^{1/2}(1-x)^{-1/2}$	
	$(1+x)^{1/2} \approx 1 + \frac{1}{2}x + \frac{1}{2}\times\frac{-1}{2}\times\frac{x^2}{2!} + \frac{-1}{4}\times\frac{-3}{2}\times\frac{x^3}{3!} + \cdots$	B1
	$\approx 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots$	
	$(1-x)^{-1/2} \approx 1 - \frac{1}{2}(-x) - \frac{1}{2}\times\frac{-3}{2}\times\frac{(-x)^2}{2!} + \frac{3}{4}\times\frac{-5}{2}\times\frac{(-x)^3}{3!} + \cdots$	B1
	$\approx 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \cdots$	
	$(1+x)^{1/2}(1-x)^{-1/2}$ $\approx \left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots\right)\left(1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \cdots\right)$	M1
	$\approx \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3\right) + \left(\frac{1}{2}x + \frac{1}{4}x^2 + \frac{3}{16}x^3\right) + \left(\frac{1}{8}x^2 + \frac{1}{16}x^3\right) + \cdots$	
	$\approx 1 + x + \frac{3}{4}x^2 + \frac{9}{16}x^3 + \cdots$	A1
	The expansion is valid for $\lvert x\rvert < 1$.	B1
	putting $x = 0.2$ gives	
	$\sqrt{\left(\frac{1+0.2}{1-0.2}\right)} \approx 1 + 0.2 + \frac{3}{4}\times(0.2)^2 + \frac{9}{16}\times(0.2)^3$	M1
	$\sqrt{\left(\frac{1.2}{0.8}\right)} \approx 1 + 0.2 + 0.03 + 0.0045$	
	$\sqrt{1.5} \approx 1.2345$	A1
		07
7	(a).	
	Let one of the roots be α. Then the other root is $(\alpha - 3)$.	
	sum of roots, $\alpha + (\alpha - 3) = -\frac{b}{a}$	B1
	$2\alpha - 3 = -\frac{b}{a}$	
	$2\alpha = 3 - \frac{b}{a}$	
	$2\alpha = \frac{3a-b}{a}$	
	$\alpha = \frac{3a-b}{2a}$	B1
	product of roots, $\alpha(\alpha - 3) = \frac{c}{a}$	B1

	$\alpha^2 - 3\alpha = \frac{c}{a}$	M1
	$\left(\frac{3a-b}{2a}\right)^2 - 3\left(\frac{3a-b}{2a}\right) = \frac{c}{a}$	
	$\frac{9a^2 - 6ab + b^2}{4a^2} - \frac{9a-3b}{2a} = \frac{c}{a}$	B1
	$9a^2 - 6ab + b^2 - 2a(9a - 3b) = 4ac$	
	$9a^2 - 6ab + b^2 - 18a^2 + 6ab = 4ac$	
	$b^2 - 9a^2 = 4ac$	B1
	$b^2 = 9a^2 + 4ac$, as required	
	(b).	
	$x^2 - 4x + 2 = 0$	
	$\alpha + \beta = -(-4) = 4$	
	$\alpha\beta = 2$	B1
	For the required equation,	
	sum of roots $= (\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 4 + 4$ $= 8$	M1 B1
	product of roots $= (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4$ $= \alpha\beta + 2(\alpha + \beta) + 4 = 2 + 2\times 4 + 4 = 14$	M1 B1
	The required equation is given by:	
	$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$	
	$x^2 - 8x + 14 = 0$	A1
		12
8	let $f(x) = 6x^3 - 23x^2 + ax + b$	
	for $(x - 3) = 0$, $x = 3$	B1
	$f(3) = 6\times 3^3 - 23\times 3^2 + 3a + b = 11$	M1
	$162 - 207 + 3a + b = 11$	
	$3a + b = 56 \quad \longrightarrow (1)$	B1
	for $(x + 1) = 0$, $x = -1$	B1
	$f(3) = 6\times(-1)^3 - 23\times(-1)^2 - a + b = -21$	M1
	$-6 - 23 - a + b = -21$	
	$-a + b = 8 \quad \longrightarrow (2)$	B1
	Equation (1) − (2) gives,	
	$\begin{array}{r\|c\|l} & 3a + b = 56 & \longrightarrow (1) \\ - & -a + b = 8 & \longrightarrow (2) \\ \hline & 4a = 48 & \end{array}$	M1
	$a = 12$	A1
	From equation (2),	
	$b = 8 + a = 8 + 12 = 20$	M1 A1
		10
9	(a).	
	$\log_2 x + 2\log_4 y = 4$	
	$\log_2 x + 2\left(\frac{\log_2 y}{\log_2 4}\right) = 4$	

$$\log_2 x + 2\left(\frac{\log_2 y}{2\log_2 2}\right) = 4$$ **M1**
$$\log_2 x + \log_2 y = 4$$
$$\log_2 xy = 4$$
$$xy = 2^4$$
$$xy = 16$$ **B1**

For the hence part,
$$\log_{10}(x+y) = 1$$
$$\log_2 x + 2\log_4 y = 4$$

$$x + y = 10 \quad \longrightarrow (1)$$
$$xy = 16 \quad \longrightarrow (2)$$
$$\log_{10}(x+y) = 1, \quad \Rightarrow x + y = 10 \quad \longrightarrow (1)$$
$$\log_2 x + 2\log_4 y = 4, \quad \Rightarrow xy = 16 \quad \longrightarrow (2)$$

Substituting for y in equation (2) gives,
$$x(10 - x) = 16$$ **M1**
$$10x - x^2 = 16$$
$$x^2 - 10x + 16 = 0$$
$$x = \frac{10 \pm \sqrt{(-10)^2 - 4\times1\times16}}{2\times1} = \frac{10 \pm 6}{2}$$ **M1**
$$x = \frac{10+6}{2} = 8, \quad \text{or,} \quad x = \frac{10-6}{2} = 2$$
$$\text{for } x = 8, \quad y = 10 - 8 = 2$$ **A1**
$$\text{for } x = 2, \quad y = 10 - 2 = 8$$

(b).
$$5^x \bullet 25^{2y} = 1$$
$$5^x \times (5^2)^{2y} = 5^0$$
$$5^{x+4y} = 5^0$$ **M1**
$$x + 4y = 0$$
$$x = -4y \quad \longrightarrow (1)$$ **B1**

$$3^{5x} \bullet 9^y = \frac{1}{9}$$
$$3^{5x} \times (3^2)^y = 3^{-2}$$
$$3^{5x+2y} = 3^{-2}$$ **M1**
$$5x + 2y = -2 \quad \longrightarrow (2)$$

Substituting equation (1) into (2) gives, **B1**
$$5\times(-4y) + 2y = -2$$
$$-18y = -2$$ **M1**
$$y = \frac{2}{18} = \frac{1}{9}$$ **A1**

From equation (2),
$$x = -4\times\frac{1}{9} = -\frac{4}{9}$$ **A1**

12

10 (a).
$$f(x) = 2x^2 + 5x - 3$$
$$f(x+\delta x) = 2(x+\delta x)^2 + 5(x+\delta x) - 3$$ **B1**

$$= 2x^2 + 4x(\delta x) + 2(\delta x)^2 + 5x + 5(\delta x) - 3$$
$$= 2x^2 + 5x - 3 + 4x(\delta x) + 2(\delta x)^2 + 5(\delta x)$$ **B1**
$$f'(x) = \lim_{\delta x\to0}\left[\frac{f(x+\delta x) - f(x)}{\delta x}\right]$$
$$= \lim_{\delta x\to0}\left[\frac{4x(\delta x) + 2(\delta x)^2 + 5(\delta x)}{\delta x}\right]$$ **M1**
$$= \lim_{\delta x\to0}[4x + 2(\delta x) + 5] = [4x + 2\times0 + 5] = 4x + 5$$ **M1 B1**

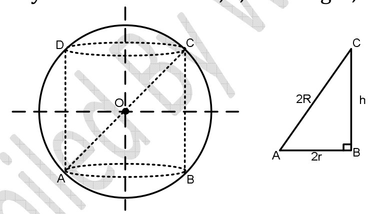
For the hence part:
$$f'(2) = 4\times2 + 5 = 13$$ **M1 A1**

(b). (i).
$$y = 3x - \frac{5}{x} + \frac{6}{x^2}$$
$$y = 3x - 5x^{-1} + 6x^{-2}$$
$$\frac{dy}{dx} = 3\times x^{1-1} - 5\times(-1\times x^{-1-1}) + 6\times(-2\times x^{-2-1})$$ **B1 B1 B1**
$$\frac{dy}{dx} = 3 + 5x^{-2} - 12x^{-3}$$

(ii).
$$y = (2x+3)(x+2)$$
$$y = 2x^2 + 3x + 4x + 6$$
$$y = 2x^2 + 7x + 6$$ **M1**
$$\frac{dy}{dx} = 4x + 7$$ **B1**

12

11 (a). Let the cylinder be of radius, r, and height, h.



B1

By Pythagoras theorem,
$$(2r)^2 = (2R)^2 - h^2, \quad \Rightarrow r^2 = R^2 - \frac{h^2}{4}$$ **B1**

Volume of cylinder,
$$V = \pi r^2 h = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi R^2 h - \frac{\pi h^3}{4}$$ **M1**
$$\frac{dV}{dh} = \pi R^2 - \frac{3\pi h^2}{4}$$ **M1**

For maximum volume, $\frac{dV}{dh} = 0$
$$\pi R^2 - \frac{3\pi h^2}{4} = 0$$ **M1**
$$R^2 = \frac{3\pi h^2}{4}, \quad \Rightarrow h = \frac{2R}{\sqrt{3}}$$ **A1**

	$V_{max} = \pi R^2\left(\frac{2R}{\sqrt{3}}\right) - \pi\left(\frac{2R}{\sqrt{3}}\right)^3 = \pi R^3\left(\frac{2}{\sqrt{3}} - \frac{2}{3\sqrt{3}}\right)$	M1
	$= \pi R^3\left(\frac{6-2}{3\sqrt{3}}\right) = \pi R^3\left(\frac{4}{3\sqrt{3}}\right) = \frac{4\pi R^3}{3\sqrt{3}}$	B1
	(b).	
	Let h be the height of the block.	
	total surface area $= 2(l \times w + l \times h + w \times h) = 150$	
	$2(x \times x + x \times h + x \times h) = 150$	M1
	$x^2 + 2hx = 75$	
	$2hx = 75 - x^2$	
	$h = \frac{75 - x^2}{2x}$	B1
	volume $= l \times w \times h = x^2 \times \left(\frac{75 - x^2}{2x}\right)$	M1
	$= \frac{75x - x^3}{2} = \frac{1}{2}(75x - x^3)$ cm^3, as required	B1
		12
12	(a).	
	$L.H.S = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$	M1
	$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} = \frac{1-\cos\theta}{\sin\theta}$	M1 B1
	$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta - \cot\theta$	B1
	(b).	
	$4\cos\theta - 3\sec\theta = 2\tan\theta$	
	$4\cos\theta - \frac{3}{\cos\theta} = \frac{2\sin\theta}{\cos\theta}$	M1
	$4\cos^2\theta - 3 = 2\sin\theta$	
	$4(1-\sin^2\theta) - 3 = 2\sin\theta$	
	$4 - 4\sin^2\theta - 3 = 2\sin\theta$	
	$4\sin^2\theta + 2\sin\theta - 1 = 0$	B1
	$\sin\theta = \frac{-2 \pm \sqrt{2^2 - 4\times 4\times(-1)}}{2\times 4} = \frac{-2\pm\sqrt{20}}{8}$	M1
	either $\sin\theta = -0.80902$, $\Rightarrow \theta = -54°, -126°$	A1
	or $\sin\theta = 0.30902$, $\Rightarrow \theta = 18°, 162°$	A1
		09

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
July 2018
$2\frac{1}{2}$ hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 2 MOT 2 2018
Time: 2 Hours 30 Minutes

NAME: ____________________ **COMB:** ________

INSTRUCTIONS:
- Attempt ALL questions in both sections.
- Show your working clearly.
- Where necessary, use $g = 9.8$ m s^{-2}.

Section A (40 Marks)

Qn 1: Two events M and N are such that $P(M) = 0.7$, $P(M \cap N) = 0.45$ and $P(M' \cap N') = 0.18$. Find:
(a). $P(N')$, [3]
(b). $P(M$ or N but not M and $N)$. [2]

Qn 2: A ball is thrown vertically upwards to a height of 10 m. Find:
(a). the time taken for the ball to reach this height. [3]
(b). the initial speed of the ball. [2]

Qn 3: Five students obtained the following A-level grades in mid-term and end of term II examinations in a certain subject.

Mid-term (M)	A	B	C	D	E
End of term (E)	B	A	C	D	E

(i). Determine the rank correlation coefficient between mid-term and End of term examinations. [4]
(ii). Comment on your result in (a) above. [1]

Qn 4: Find in the form $a\underset{\sim}{i} + b\underset{\sim}{j}$, a force of magnitude 65 N acting along the line

$\underset{\sim}{r} = \underset{\sim}{i} + \lambda\left(5\underset{\sim}{i} - 12\underset{\sim}{j}\right)$. [5]

Qn 5: The table below is the distribution of weights of a group of animals.

Mass (kg)	Frequency
21 – 25	10
26 – 30	20
31 – 35	15
36 – 40	10
41 – 50	30
51 – 65	45
66 – 75	5

Calculate the standard deviation for the given data. [6]

Qn 6: A particle is moving so that at any instant, its velocity, $\underset{\sim}{v}$, is given by

$\underset{\sim}{v} = \left(3t\underset{\sim}{i} - 4\underset{\sim}{j} + t^2\underset{\sim}{k}\right)$ m s^{-1}. When $t = 0$, it is at the point $(1, 0, 1)$. Find:

(a). the displacement of the particle when $t = 2$ s. [3]
(b). the speed of the particle when $t = 2$ s. [2]

Qn 7: The table below shows the prices in shillings of flour and eggs in 1990 and 2000.

Item	Price (shs)	
	1990	2000
Flour (1 kg)	3,000	5,400
Eggs (1 dozen)	5,000	7,800

Calculate the simple aggregate price index for the above data; taking 1990 as the base year. [5]

Qn 8: A particle travels with speed 50 m s^{-1} from the point $(3, -7)$ in a direction $7\underset{\sim}{i} - 24\underset{\sim}{j}$. Find its position vector after

(a). t seconds. [3]
(b). 3 seconds. [2]

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

Section A (40 Marks)

Question 9:

X and Y are two independent events such that $P(X') = 0.6$ and $P(X \cap Y) = 0.2$.

(a). Show that X' and Y' are also independent. [4]
(b). Find:
(i). $P(Y)$, [2]
(ii). $P(X' \cap Y')$, [2]
(c). A and B are two events such that $P(A) = \frac{2}{5}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Find:
(i). $P(A \cap B)$, [2]
(ii). $P(A \cup B)$. [2]

Question 10:

(a). Find the equation of the line of action of the resultant of forces F_1 and F_2 if the equations of the lines of action of F_1 and F_2 are respectively

$\underset{\sim}{r_1} = 2\underset{\sim}{i} - 7\underset{\sim}{j} + \underset{\sim}{k} + \lambda\left(\underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k}\right)$ and $\underset{\sim}{r_2} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + \mu\left(\underset{\sim}{i} + 5\underset{\sim}{j} + 5\underset{\sim}{k}\right)$

and the magnitudes of F_1 and F_2 are $\sqrt{21}$ and $2\sqrt{51}$ respectively. [8]

(b). Find the value of λ if $\lambda\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $5\underset{\sim}{i} - \lambda\underset{\sim}{j} + \underset{\sim}{k}$ are perpendicular vectors. [4]

Question 11:

(a). Two ordinary dice are thrown. Find the possibility space for the sum of the scores obtained. [2]
Hence find the probability that the sum of the scores obtained:
(i). is a multiple of 5, [1]
(ii). is greater than 9, [1]
(iii). is a multiple of 5 or greater than 9, [2]
(iv). is a multiple of 5 and is greater than 9. [2]
(b). Given that A and B are mutually exclusive events and that $P(A) = \frac{2}{5}$ and $P(B) = \frac{1}{2}$, find:
(i). $P(A \cup B)$, [2]
(ii). $P(A' \cap B')$. [2]

Question 12:

(a). A, B and C are three points lying in that order on a straight line. A body is projected from B towards A with a speed 3 m s^{-1}. The body experiences an acceleration of 1 m s^{-2} towards C. If $\overline{BC} = 20$ m, find:

(i). the time taken to reach C.

(ii). the distance travelled by the body from the moment of projection until it reaches C. [6]

(b). Two stations A and B are a distance of $6\boldsymbol{X}$ m apart along a straight track. A train starts from rest at A and accelerates uniformly to a speed of $\boldsymbol{V}\text{ m s}^{-1}$; covering a distance of $\boldsymbol{X}$ m.
The train then maintains this speed until is has travelled a further $3\boldsymbol{X}$ m. It then retards uniformly to rest at B.

(i). Make a sketch of the velocity-time graph for the motion.

(ii). Hence show that if $\boldsymbol{T}$ is the time taken for the train to travel from A to B, then $\boldsymbol{T} = \frac{9X}{V}$ seconds. [6]

END

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

MARKING GUIDE

SNo.	Working	Marks
1	(a). $P(M' \cap N') = P(M \cup N)' = 0.18$ $P(M \cup N) = 1 - P(M \cup N)' = 1 - 0.18 = 0.82$ but, $P(M \cup N) = P(M) + P(N) - P(M \cap N)$ $1 - 0.18 = 0.7 + P(N) - 0.45$ $P(N) = 0.57$ $P(N') = 1 - 0.57 = 0.43$ (b). $P(M \text{ or } N \text{ but not } M \text{ and } N) = P(M \cup N) - P(M \cap N)$ $= 0.82 - 0.45 = 0.37$	M1 B1 B1 M1 A1
		05
2	(a). At maximum height, $v = 0\text{ m s}^{-1}, s = 10\text{ m}$ $v = u - gt$ $0 = u - gt$ $u = gt \longrightarrow (1)$ $s = ut - \frac{1}{2}gt^2$ $10 = ut - \frac{1}{2}gt^2 \longrightarrow (2)$ Substituting equation (1) into (2) gives: $10 = gt^2 - \frac{1}{2}gt^2$ $10 = \frac{1}{2}gt^2$ $20 = 9.8t^2$ $t^2 = \frac{20}{9.8}$ $t = \frac{10}{7}\text{ s}$ (b). $u = gt = 9.8 \times \frac{10}{7} = 14\text{ m s}^{-1}$	B1 M1 A1 M1 A1
		05
3	(table below)	B1-correct ranking B1-for $\sum d^2$

M	E	R_M	R_E	d	d^2
A	B	1	2	-1	1
B	A	2	1	1	1
C	C	3	3	0	0
D	D	4	4	0	0
E	E	5	5	0	0
					$\sum d^2 = 2$

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\times 2}{5(5^2-1)} = 0.9$$ **M1 A1**

Comment: Insignificant at 1%. **B1**

05

4

Direction vector $= \begin{pmatrix}5\\-12\end{pmatrix}$ **B1**

Force vector, $\underset{\sim}{F} = \dfrac{65}{\sqrt{5^2+(-12)^2}}\begin{pmatrix}5\\-12\end{pmatrix}$ **M1 M1 M1**

$= \pm 5\begin{pmatrix}5\\-12\end{pmatrix} = \pm\begin{pmatrix}25\\-60\end{pmatrix}$ N **A1**

05

5

Class	f	x	fx	fx^2
21 – 25	10	23	230	5290
26 – 30	20	28	560	15680
31 – 35	15	33	495	16335
36 – 40	10	38	380	14440
41 – 50	30	45.5	1365	62107.5
51 – 65	45	58	2610	151380
66 – 75	5	70.5	352.5	24851.25
Total			5992.5	290083.75

B1-for $\sum fx$

B1-for $\sum fx^2$

Standard deviation $= \sqrt{\dfrac{\sum fx^2}{\sum f} - \left(\dfrac{\sum fx}{\sum f}\right)^2}$

$= \sqrt{\dfrac{290083.75}{135} - \left(\dfrac{5992.5}{135}\right)^2} = 13.3565$ **M1 M1 A1**

05

6

(a).

$\underset{\sim}{s} = \int \underset{\sim}{v}\,dt = \int\left(3t\underset{\sim}{i} - 4\underset{\sim}{j} + t^2\underset{\sim}{k}\right)dt$ **M1**

$\underset{\sim}{s} = \left(\frac{3}{2}t^2\underset{\sim}{i} - 4t\underset{\sim}{j} + \frac{1}{3}t^3\underset{\sim}{k}\right) + \underset{\sim}{c}$ **M1**

When $t = 0$, $\underset{\sim}{s} = \begin{pmatrix}1\\0\\1\end{pmatrix}$ m

$\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \underset{\sim}{c}$, $\quad \Rightarrow \underset{\sim}{c} = \begin{pmatrix}1\\0\\1\end{pmatrix}$

$\therefore \underset{\sim}{s} = \left(\frac{3}{2}t^2 + 1\right)\underset{\sim}{i} - 4t\underset{\sim}{j} + \left(\frac{1}{3}t^3 + 1\right)\underset{\sim}{k}$

When $t = 2$,

$\therefore \underset{\sim}{s} = \left(\frac{3}{2}\times 2^2 + 1\right)\underset{\sim}{i} - (4\times 2)\underset{\sim}{j} + \left(\frac{1}{3}\times 2^3 + 1\right)\underset{\sim}{k}$

$\therefore \underset{\sim}{s} = 7\underset{\sim}{i} - 8\underset{\sim}{j} + \frac{11}{3}\underset{\sim}{k}$ **A1**

(b). When $t = 2$,

$\underset{\sim}{v} = 3\times 2\underset{\sim}{i} - 4\underset{\sim}{j} + 2^2\underset{\sim}{k} = \left(6\underset{\sim}{i} - 4\underset{\sim}{j} + 4\underset{\sim}{k}\right)$ m s^{-1} **B1**

Speed, $\left|\underset{\sim}{v}\right| = \sqrt{6^2 + (-4)^2 + 4^2} = \sqrt{68}$ **B1**

$= 2\sqrt{17}$ m s^{-1}

05

7

Simple aggregate price index $= \dfrac{\sum P_{2000}}{\sum P_{1990}} \times 100$

$= \dfrac{5400 + 7800}{3000 + 5000} \times 100$ **M1 M1 M1**

$= \dfrac{13200}{8000} \times 100 = 165$ **B1 A1**

05

8

(a).

Velocity vector, $\underset{\sim}{v} = \dfrac{50}{\sqrt{7^2 + (-24)^2}}\begin{pmatrix}7\\-24\end{pmatrix}$ **M1**

$= 2\begin{pmatrix}7\\-24\end{pmatrix} = \begin{pmatrix}14\\-48\end{pmatrix}$ m s^{-1} **B1**

Position vector, $\underset{\sim}{r}(t) = \begin{pmatrix}3\\-7\end{pmatrix} + t\begin{pmatrix}14\\-48\end{pmatrix} = \begin{pmatrix}3+14t\\-7-48t\end{pmatrix}$ **B1**

(b). At $t = 3$,

$\underset{\sim}{r}(3) = \begin{pmatrix}3 + 14\times 3\\-7 - 48\times 3\end{pmatrix} = \begin{pmatrix}45\\-15\end{pmatrix}$ m **M1 A1**

05

9

(a). For events X' and Y' to be independent,

$P(X'\cap Y') = P(X').P(Y')$

$L.H.S = P(X'\cap Y') = P(X\cup Y)' = 1 - P(X\cup Y)$ **M1**

$= 1 - \{P(X) + P(Y) - P(X\cap Y)\}$

$= 1 - \{P(X) + P(Y) - P(X).P(Y)\}$ **M1**

$= 1 - P(X) - P(Y) + P(X).P(Y)$

$= [1 - P(X)] - P(Y)[1 - P(X)] = P(X') - P(Y)P(X')$ **M1**

$= P(X')[1 - P(Y)] = P(X').P(Y')$, hence shown **A1**

(b). (i).

$P(X) = 1 - P(X') = 1 - 0.6 = 0.4$

but, $P(X\cap Y) = P(X).P(Y)$

$0.2 = 0.4P(Y)$, $\quad \Rightarrow P(Y) = \dfrac{0.2}{0.4} = 0.5$ **M1 A1**

(ii).

$P(X'\cap Y') = P(X').P(Y') = 0.6\times(1 - 0.5) = 0.3$ **M1 A1**

(c). (i).

$P(A\cap B) = P(A).P(B/A) = \dfrac{2}{5}\times\dfrac{2}{3} = \dfrac{4}{15}$ **M1 A1**

(ii).

	$P(A\cap B)=P(B).P(A/B)$ $\frac{4}{15}=\frac{1}{2}P(B), \quad \Rightarrow P(B)=\frac{8}{15}$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $=\frac{2}{5}+\frac{8}{15}-\frac{4}{15}=\frac{2}{3}$	M1 A1
		12
10	(a). Direction vector for $\underset{\sim}{\boldsymbol{F_1}}=\begin{pmatrix}1\\4\\2\end{pmatrix}$ $\underset{\sim}{\boldsymbol{F_1}}=\frac{\sqrt{21}}{\sqrt{1^2+4^2+2^2}}\begin{pmatrix}1\\4\\2\end{pmatrix}=\begin{pmatrix}1\\4\\2\end{pmatrix}$ N	M1 B1
	Direction vector for $\underset{\sim}{\boldsymbol{F_2}}=\begin{pmatrix}1\\5\\5\end{pmatrix}$ $\underset{\sim}{\boldsymbol{F_2}}=\frac{2\sqrt{51}}{\sqrt{1^2+5^2+5^2}}\begin{pmatrix}1\\5\\5\end{pmatrix}=2\begin{pmatrix}1\\5\\5\end{pmatrix}=\begin{pmatrix}2\\10\\10\end{pmatrix}$ N	M1 B1
	Resultant force $=\underset{\sim}{\boldsymbol{F_1}}+\underset{\sim}{\boldsymbol{F_2}}=\begin{pmatrix}1\\4\\2\end{pmatrix}+\begin{pmatrix}2\\10\\10\end{pmatrix}=\begin{pmatrix}3\\14\\12\end{pmatrix}$ N	B1
	The line of action of the resultant force passes through the point of intersection of the lines of action of the two forces." At the point of intersection, $\underset{\sim}{\boldsymbol{r_1}}=\underset{\sim}{\boldsymbol{r_2}}$ $\begin{pmatrix}2\\-7\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\2\end{pmatrix}=\begin{pmatrix}3\\0\\-4\end{pmatrix}+\mu\begin{pmatrix}1\\5\\5\end{pmatrix}$	B1
	$2+\lambda=3+\mu, \quad \Rightarrow \mu=\lambda-1$ $-7+4\lambda=5\mu, \quad \Rightarrow -7+4\lambda=5(\lambda-1), \quad \Rightarrow \lambda=2$	M1 B1
	Point of intersection $=\begin{pmatrix}2\\-7\\1\end{pmatrix}+2\begin{pmatrix}1\\4\\2\end{pmatrix}=\begin{pmatrix}4\\1\\5\end{pmatrix}$	B1
	The line of action of the resultant force is: $\underset{\sim}{\boldsymbol{r}}=\begin{pmatrix}4\\1\\5\end{pmatrix}+\beta\begin{pmatrix}3\\14\\12\end{pmatrix}$	B1
	(b). For perpendicular vectors, $\begin{pmatrix}\lambda\\2\\-1\end{pmatrix}.\begin{pmatrix}5\\-\lambda\\1\end{pmatrix}=0$	M1
	$5\lambda-2\lambda-1=0$ $3\lambda=1$ $\lambda=\frac{1}{3}$	A1
		12
11	(a).	

Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

B2

$n(S)=36$

Let $A=\{\text{multiple of 5}\}$, $B=\{\text{greater than 9}\}$,

(i).

$$P(A)=\frac{n(A)}{n(S)}=\frac{7}{36}$$ B1

(ii).

$$P(B)=\frac{n(B)}{n(S)}=\frac{6}{36}=\frac{1}{6}$$ B1

(iii).

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{3}{36}=\frac{1}{12}$$ M1 A1

(iv).

$$P(A\cup B)=\frac{n(A\cup B)}{n(S)}=\frac{10}{36}=\frac{5}{18}$$ M1 A1

Alternatively:

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{7}{36}+\frac{1}{6}-\frac{1}{12}=\frac{5}{18}$$

(b). (i).

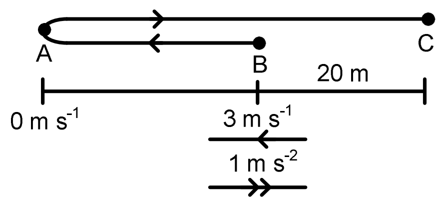
$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{2}{5}+\frac{1}{2}-0=\frac{9}{10}$$ M1 A1

(ii).

$$P(A'\cap B')=P(A\cup B)'=1-P(A\cup B)=1-\frac{9}{10}=\frac{1}{10}$$ M1 A1

12

12 (a).



(i). For motion BAC,

$$s=-20\text{ m}, a=-1\text{ m s}^{-2}, u=3\text{ m s}^{-1}$$

$$s=ut+\frac{1}{2}at^2$$

$$-20 = 3t + \frac{1}{2}\times(-1)\times t^2$$ **M1**

$$-40 = 6t - t^2$$

$$t^2 - 6t - 40 = 0$$

$$t = \frac{6 \pm \sqrt{(-6)^2 - 4\times 1\times(-40)}}{2\times 1}$$ **B1**

$t = 10$, or, $t = -4$ **M1**

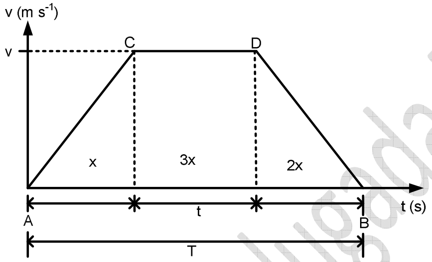
but $t \neq -4$, $\Rightarrow t = 10$ s

For motion BA, **A1**

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 3^2}{2\times(-1)} = 4.5 \text{ m}$$ **B1**

Total distance $= s_{BA} + s_{AC} = 4.5 + (4.5 + 20) = 29$ m **B1**

(b).



B2-all correct

Distance $CD = vt = 3x$, $\Rightarrow t = \frac{3x}{v}$

Total distance covered $= \frac{1}{2}v(T + t) = 6x$ **B1**

$$vT + vt = 12x$$ **M1**

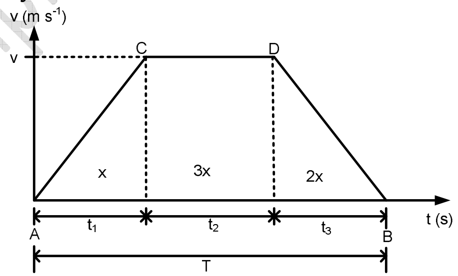
$$vT + v\times\frac{3x}{v} = 12x$$

$$vT + 3x = 12x$$

$$vT = 9x$$

$T = \frac{9x}{v}$ seconds, as required

Alternatively: **A1**



Distance AC $= \frac{1}{2}\times t_1\times v = x$, $\Rightarrow t_1 = \frac{2x}{v}$

Distance CD $= t_2\times v = 3x$, $\Rightarrow t_2 = \frac{3x}{v}$

Distance DA $= \frac{1}{2}\times t_1\times v = 2x$, $\Rightarrow t_3 = \frac{4x}{v}$,

$T = t_1 + t_2 + t_3 = \frac{2x}{v} + \frac{3x}{v} + \frac{4x}{v} = \frac{9x}{v}$ seconds, as required

12

END

P425/1
PURE MATHEMATICS
PAPER 1
August 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 1 EOT 2 2018
Time: 3 Hours

NAME: ________________ **COMB:** ________

INSTRUCTIONS:
- Attempt ALL questions in this paper.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Without using mathematical tables or a calculator, find the value of:

$$\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{8\sqrt{5}}$$

[5]

Qn 2: Find the equation of the normal to the curve $x^2y+3y^2-4x-12=0$ at the point $(1,2)$. [5]

Qn 3: Solve the simultaneous equations:

$$x-2y-2z=0$$
$$2x+3y+z=1$$
$$3x-y-3z=3$$

[5]

Qn 4: Given the parametric equations $x=4t^2$ and $y=8t$, find $\frac{d^2y}{dx^2}$. [5]

Qn 5: Given that x^2+qx+p and x^2+mx+n have a common factor of $(x-k)$, show that $k=\frac{n-p}{q-m}$. [5]

Qn 6: The sum of the first two terms of the geometric progression (G.P) is 9 and the sum to infinity of the G.P is 25. If the G.P has a positive common ratio, r, find r and the first term. [5]

Qn 7: A 2% error is made in measuring the radius of a sphere. Find the percentage error in surface area. [5]

Qn 8: Find the area lying in the first quadrant and bounded by the curve $y=2x^2+1$, the y-axis and the lines $y=2$ and $y=5$. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). Find the volume generated when the area enclosed by $y^2=x$, $x=0$ and $y=2$ is rotated about $y=2$. [6]

(b). On the same axes, sketch the curves $y=x^2+3x$ and $y=4x-x^2$ and hence find the area enclosed by the two curves. [6]

Question 10:

Find the first derivatives, with respect to x, of the following functions in their simplest form:

(a). $3x^2-6x+\frac{2}{x^2}$. [3]

(b). $\frac{\cos 2x}{1+\sin 2x}$. [6]

(c). $\tan^2 4x$. [3]

Question 11:

(a). Prove that: $\sin(2\sin^{-1}x+\cos^{-1}x)=\sqrt{(1-x^2)}$. [8]

(b). Prove that $\frac{\cos A+\cos B}{\sin A+\sin B}=\cot\left(\frac{A+B}{2}\right)$. [4]

Question 12:

A hemispherical bowl is being filled with water at a uniform rate when the height of the water is h cm, the volume is $\pi\left(rh^2-\frac{1}{3}h^3\right)$ cm^3, r cm being the radius of the hemisphere. Find the rate at which the water level is rising when it is half way to the top, given that $r=6$ and that the bowl fills in 1 minute.

[12]

Question 13:

(a). Given that α and β are roots of the equation $x^2 - 4x + 2 = 0$, find the quadratic equation that has the roots $(\alpha + 2)$ and $(\beta + 2)$. [5]

(b). Find the range of values k can take for $kx^2 + 3x + k - 4 = 0$ to have two real distinct roots. [7]

Question 14:

Solve:

(a). $\log_x 5 + 4\log_5 x = 4$. [5]

(b). $5^{x+2} + 7^{y+1} = 3468$
$7^y = 5^x - 76$ [7]

Question 15:

Given that $\cos A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$ where A and B are acute, find the value of:

(a). $\tan(A+B)$.

(b). $\operatorname{cosec}(A+B)$.

(c). $\sec(A-B)$. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	$\frac{(\sqrt{5}-2)^2 - (\sqrt{5}+2)^2}{8\sqrt{5}} = \frac{(\sqrt{5}-2+\sqrt{5}+2)(\sqrt{5}-2-\sqrt{5}-2)}{8\sqrt{5}}$ $= \frac{2\sqrt{5}\times(-4)}{8\sqrt{5}} = \frac{-8\sqrt{5}}{8\sqrt{5}} = -1$	B1 B1 M1 M1 A1
		05
2	$\frac{d}{dx}(x^2y + 3y^2 - 4x - 12) = \frac{d}{dx}(0)$ $x^2\frac{dy}{dx} + 2xy + 6y\frac{dy}{dx} - 4 = 0$ $(x^2+6y)\frac{dy}{dx} = 4 - 2xy$ $\frac{dy}{dx} = \frac{4-2xy}{x^2+6y}$ At the point $(1, 2)$, Gradient of tangent $= \frac{4 - 2\times1\times2}{1^2 + 6\times2} = 0$ Gradient of the normal $= \frac{-1}{0} = -\infty$ The required equation of the normal is given by: $\frac{y-2}{x-1} = \frac{-1}{0}$ $0 = -x + 1$ $x = 1$	B1 B1 B1 M1 A1
		05
3	$x - 2y - 2z = 0 \quad \longrightarrow (1)$ $2x + 3y + z = 1 \quad \longrightarrow (2)$ $3x - y - 3z = 3 \quad \longrightarrow (3)$ Equation (1) + 2 × (2) gives, $x - 2y - 2z = 0 \quad \longrightarrow (1)$ $+\ 4x + 6y + 2z = 2 \quad \longrightarrow 2\times(2)$ $5x + 4y = 2 \quad \longrightarrow (4)$ Equation 3 × (2) + (3) gives, $6x + 9y + 3z = 3 \quad \longrightarrow 3\times(2)$ $+\ 3x - y - 3z = 3 \quad \longrightarrow (3)$ $9x + 8y = 6 \quad \longrightarrow (5)$ Equation 2 × (4) − (5) gives, $10x + 8y = 4 \quad \longrightarrow 2\times(4)$ $-\ 9x + 8y = 6 \quad \longrightarrow (5)$ $x = -2$ From equation (4), $5\times(-2) + 4y = 2$ $-10 + 4y = 2$ $4y = 12$	M1 A1 M1

Compiled By Walugada Ronald

	$y=3$ From equation (1), $-2-2\times 3-2z=0$ $-8-2z=0$ $2z=-8$ $z=-4$	A1 A1
		05
4	$x=4t^2, \qquad \Rightarrow \frac{dx}{dt}=8t$ $y=8t, \qquad \Rightarrow \frac{dy}{dt}=8$ $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=8\times\frac{1}{8t}=\frac{1}{t}$ $\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx}=-\frac{1}{t^2}\times\frac{1}{8t}=-\frac{1}{8t^3}$	B1 B1 B1 M1 A1
		05
5	for $(x-k)=0, \qquad x=k$ $\therefore k^2+qk+p=k^2+mk+n$ $qk+p=mk+n$ $qk-mk=n-p$ $k(q-m)=n-p$ $k=\frac{n-p}{q-m}$	B1 B1 B1 B1 B1
		05
6	$a+ar=9$ $a(1+r)=9$ $a=\frac{9}{1+r}$ $S_\infty=\frac{a}{1-r}=25$ $a=25(1-r)$ Equating the two equations gives: $\frac{9}{1+r}=25(1-r)$ $9=25(1-r^2)$ $r^2=1-\frac{9}{25}$ $r=\frac{4}{5}=0.8$ First term, $\qquad a=25(1-0.8)=5$	 B1 B1 M1 M1 A1 A1
		05
7	Let r be the radius of the sphere. surface area, $\qquad A=4\pi r^2, \qquad \Rightarrow \frac{dA}{dr}=8\pi r$ $\delta A\approx\frac{dA}{dr}\times\delta r=8\pi r\times\frac{2}{100}r=0.16\pi r^2$	 B1 M1 B1

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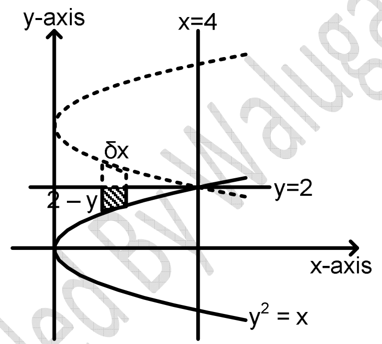
	Percentage error in surface area is given by: $=\frac{0.16\pi r^2}{4\pi r^2}\times 100\%=4\%$	M1 A1
		05
8	$y=2x^2+1$ $x=\frac{1}{\sqrt{2}}(y-1)^{1/2}$ area $=\int_2^5 x\,dy=\frac{1}{\sqrt{2}}\int_2^5 (y-1)^{1/2}\,dy$ $=\frac{1}{\sqrt{2}}\left[\frac{2}{3\times 1}(y-1)^{3/2}\right]_2^5$ $=\frac{2}{3\sqrt{2}}\left[(5-1)^{3/2}-(2-1)^{3/2}\right]=\frac{2}{3\sqrt{2}}[8-1]$ $=\frac{14}{3\sqrt{2}}=3.2998$ sq. units	B1 M1 M1 B1 A1
		05
9	(a).  element of volume, $\qquad \delta v=\pi(2-y)^2\delta x$ total volume generated, $v=\int_0^4 \pi(2-y)^2\,dx=\pi\int_0^4\left(2-\sqrt{x}\right)^2 dx$ $=\pi\int_0^4\left(4-4x^{1/2}+x\right)dx=\pi\left[4x-\frac{8}{3}x^{3/2}+\frac{1}{2}x^2\right]_0^4$ $=\pi\left(16-\frac{64}{3}+8\right)-0=\frac{8}{3}\pi$ cubic units (b). For $y=x^2+3x$ when $x=0, \qquad y=0$ when $y=0, \qquad x^2+3x=0$ $x(x+3)=0$ $x=0, \qquad$ or, $\qquad x=-3$ Intercepts are: $(0,0), (-3,0)$.	B1 B1 M1 M1 B1 A1 B1

For $y = 4x - x^2$

when $x = 0, \quad y = 0$

when $y = 0, \quad 4x - x^2 = 0$

$x(4 - x) = 0$

$x = 0, \quad \text{or}, \quad x = 4$

Intercepts are: $(0, 0), (4, 0)$. **B1**

At the point of intersection,

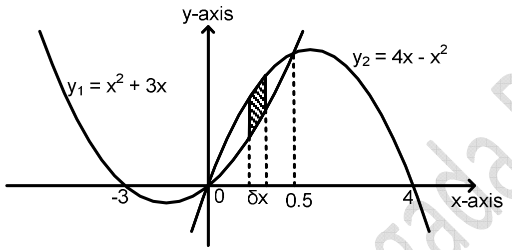
$x^2 + 3x = 4x - x^2$

$2x^2 - x = 0$

$x(2x - 1) = 0$ **B1**

$x = 0, \quad \text{or}, \quad x = 0.5$

B1



For the hence part:

$$\text{area} = \int_0^{0.5} (y_2 - y_1)\, dx$$

$$= \int_0^{0.5} [4x - x^2 - (x^2 + 3x)]\, dx = \int_0^{0.5} (x - 2x^2)\, dx$$ **M1**

$$= \left[\frac{1}{2}x^2 - \frac{2}{3}x^3\right]_0^{0.5} = \left(\frac{1}{2} \times (0.5)^2 - \frac{2}{3} \times (0.5)^3\right) - 0$$ **M1**

$$= \frac{1}{24} \text{ square units}$$ **A1**

12

10 (a).

$$\text{let } y = 3x^2 - 6x + \frac{2}{x^2}$$

$$\frac{dy}{dx} = 6x - 6 - \frac{4}{x}$$ **B1 B1 B1**

(b).

$$\text{let } y = \frac{\cos 2x}{1 + \sin 2x}$$

$$u = \cos 2x, \quad \Rightarrow \frac{du}{dx} = -2\sin 2x$$ **B1**

$$v = 1 + \sin 2x, \quad \Rightarrow \frac{dv}{dx} = 2\cos 2x$$ **B1**

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + \sin 2x) \times (-2\sin 2x) - \cos 2x \times 2\cos 2x}{(1 + \sin 2x)^2}$$ **M1 B1**

$$= \frac{-2\sin 2x - 2\sin^2 2x - 2\cos^2 2x}{(1 + \sin 2x)^2}$$

$$= \frac{-2\sin 2x - 2(\sin^2 2x + \cos^2 2x)}{(1 + \sin 2x)^2}$$

$$= \frac{-2\sin 2x - 2}{(1 + \sin 2x)^2} = \frac{-2(\sin 2x + 1)}{(1 + \sin 2x)^2} = \frac{-2}{1 + \sin 2x}$$ **B1 A1**

(c).

$$\text{let } y = \tan^2 4x = (\tan 4x)^2$$

$$\frac{dy}{dx} = 2(\tan 4x)^1 \times 4\sec^2 4x = 8\tan 4x \sec^2 4x$$ **B1 B1 A1**

12

11 (a).

$$\text{let } A = \sin^{-1} x, \quad \Rightarrow \sin A = x$$

$$\therefore \cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - x^2}$$ **B1**

$$\text{let } B = \cos^{-1} x, \quad \Rightarrow \cos B = x$$

$$\therefore \sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - x^2}$$ **B1**

$$L.H.S = \sin(2\sin^{-1} x + \cos^{-1} x) = \sin(2A + B)$$ **B1**

$$= \sin 2A \cos B + \cos 2A \sin B$$ **B1**

$$= 2\sin A \cos A \cos B + (1 - 2\sin^2 A)\sin B$$ **B1**

$$= 2x\sqrt{1 - x^2} \times x + (1 - 2x^2)\sqrt{1 - x^2}$$ **B1**

$$= 2x^2\sqrt{1 - x^2} + \sqrt{1 - x^2} - 2x^2\sqrt{1 - x^2}$$ **B1**

$$= \sqrt{1 - x^2}$$ **B1**

(b).

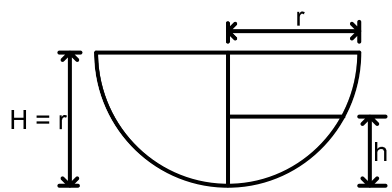
$$L.H.S = \frac{\cos A + \cos B}{\sin A + \sin B} = \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$ **B1 B1**

$$= \frac{\cos\left(\frac{A+B}{2}\right)}{\sin\left(\frac{A+B}{2}\right)} = \cot\left(\frac{A + B}{2}\right)$$ **B1 B1**

12

12

$$t = 1 \text{ min} = 60\, s, H = r = 6 \text{ cm}$$



When the bowl is full, $h = H = 6$

$$\text{volume,} \quad v = \pi\left(rh^2 - \frac{1}{3}h^3\right) = \pi\left(6 \times 6^2 - \frac{1}{3} \times 6^3\right)$$ **B1 B1**

$$= 144\pi \text{ cm}^3$$

$$\frac{dv}{dt} = \frac{v}{t} = \frac{144\pi}{1 \times 60} = 2.4\pi\ \text{cm}^3\text{s}^{-1}$$ **M1 B1 B1**

When the bowl is half way full, $h = \frac{1}{2}H = \frac{1}{2} \times 6 = 3$ cm **B1**

volume, $v = \pi\left(rh^2 - \frac{1}{3}h^3\right)$

$$\frac{dv}{dh} = \pi(2rh - h^2) = \pi(2 \times 6 \times 3 - 3^2) = 27\pi$$ **M1 B1 B1**

$$\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt} = \frac{1}{27\pi} \times 2.4\pi = \frac{4}{45}\text{cm s}^{-1}$$ **M1 M1 A1**

12

13 (a).

$$x^2 - 4x + 2 = 0$$
$$\alpha + \beta = -(-4) = 4$$
$$\alpha\beta = 2$$

For the required equation,

sum of roots $= (\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 4 + 4 = 8$ **M1 B1**

product of roots $= (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4 = \alpha\beta + 2(\alpha + \beta) + 4 = 2 + 2 \times 4 + 4 = 14$ **M1 B1**

The required equation is given by:

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$
$$x^2 - 8x + 14 = 0$$ **A1**

(b).

$$kx^2 + 3x + k - 4 = 0$$

For two real distinct roots,

$$b^2 - 4ac > 0$$
$$3^2 - 4k(k - 4) > 0$$ **M1 B1**
$$9 - 4k^2 + 16k > 0$$
$$-4k^2 + 16k + 9 > 0$$ **M1**
$$-4k^2 + 18k - 2k + 9 > 0$$
$$2k(9 - 2k) + (9 - 2k) > 0$$
$$(2k + 1)(9 - 2k) > 0$$ **M1**

	$k < -0.5$	$-0.5 < k < 4.5$	$k > 4.5$
$(2k + 1)$	−	+	+
$(9 - 2k)$	+	+	+
$(2k + 1)(9 - 2k)$	−	+	−

M1 M1

The range of values k is:

$$-0.5 < k < 4.5$$ **A1**

12

14 (a).

$$\log_x 5 + 4\log_5 x = 4$$
$$\frac{1}{\log_5 x} + 4\log_5 x = 4$$ **B1**

let $y = \log_5 x$

$$\frac{1}{y} + 4y = 4$$ **B1**
$$1 + 4y^2 = 4y$$
$$4y^2 - 4y + 1 = 0$$
$$(2y - 1)^2 = 0$$ **M1**
$$y = 0.5$$ **B1**

but $y = \log_5 x$

$$\therefore \log_5 x = 0.5, \quad \Rightarrow x = 5^{0.5} = \sqrt{5}$$ **A1**

(b).

$$5^{x+2} + 7^{y+1} = 3468$$
$$7^y = 5^x - 76$$

let $A = 5^x$, $B = 7^y$

$$5^{x+2} + 7^{y+1} = 3468$$
$$5^2 \times 5^x + 7^1 \times 7^y = 3468$$
$$25A + 7B = 3468 \quad \longrightarrow (1)$$ **B1**
$$7^y = 5^x - 76, \quad \Rightarrow B = A - 76 \quad \longrightarrow (2)$$ **B1**

Substituting (2) into (1) gives:

$$25A + 7(A - 76) = 3468$$ **M1**
$$25A + 7A - 532 = 3468$$
$$32A = 4000$$
$$A = 125$$ **B1**

but $A = 5^x$, $\Rightarrow 5^x = 5^3$, $\therefore x = 3$ **A1**

From equation (2),

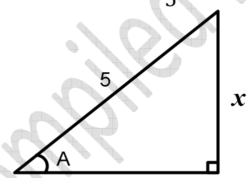
$$B = 125 - 76 = 49$$ **B1**

but $B = 7^y$, $\Rightarrow 7^y = 7^2$, $\therefore y = 2$ **A1**

12

15

$\cos A = \frac{3}{5}$

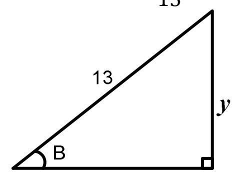


$x = \sqrt{5^2 - 3^2} = 4$

$\sin A = \frac{4}{5}$

$\tan A = \frac{4}{3}$

$\cos B = \frac{12}{13}$



$x = \sqrt{13^2 - 12^2} = 5$

$\sin B = \frac{5}{13}$

$\tan B = \frac{5}{12}$

B1 B1

(a).

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$ **B1**

	$= \left(\frac{4}{3}+\frac{5}{12}\right) \div \left(1-\frac{4}{3}\times\frac{5}{12}\right) = \frac{7}{4} \div \frac{4}{9} = \frac{63}{16}$	M1 M1 A1
	(b).	
	$\operatorname{cosec}(A+B) = \frac{1}{\sin(A+B)}$	
	$\sin(A+B) = \sin A\cos B + \cos A \sin B$	B1
	$= \frac{4}{5}\times\frac{12}{13}+\frac{3}{5}\times\frac{5}{13} = \frac{48}{65}+\frac{3}{13} = \frac{63}{65}$	M1
	$\therefore \operatorname{cosec}(A+B) = \frac{65}{63}$	A1
	(c).	
	$\sec(A-B) = \frac{1}{\cos(A-B)}$	
	$\cos(A-B) = \cos A\cos B + \sin A\sin B$	B1
	$= \frac{3}{5}\times\frac{12}{13}+\frac{4}{5}\times\frac{5}{13} = \frac{36}{65}+\frac{4}{13} = \frac{56}{65}$	M1
	$\therefore \sec(A-B) = \frac{65}{56}$	A1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
August 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 2 EOT 2 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B.**
- Show your working clearly.
- Where necessary, use $g = 9.8\text{ m s}^{-2}$.

Section A (40 Marks)

Qn 1: Given that $P(A/B) = \frac{2}{5}, P(B) = \frac{1}{4}, P(A) = \frac{1}{3}$, find

(a). $P(B/A)$,
(b). $P(A \cap B)$. [5]

Qn 2: A plane flies from $A(10, 50)$ to $B(130, -110)$ at a speed of 100 m s^{-1}, find the velocity of the plane in the form $a\underset{\sim}{i} + b\underset{\sim}{j}$. [5]

Qn 3: The table below shows the height distribution of seedlings:

Height (cm)	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29
Frequency	18	24	46	32	30

Calculate:
(i). the modal height.
(ii). the number of seedlings of height less than 17 cm. [5]

Qn 4: A particle has an initial position vector $\left(4\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}} + 9\underset{\sim}{\boldsymbol{k}}\right)$ m. The particle moves with constant velocity $\left(3\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} - 5\underset{\sim}{\boldsymbol{k}}\right)$ m s^{-1}. Find:

(a). the position vector of the particle at any time, t.
(b). the position vector of the particle after 5 seconds. [5]

Qn 5: A class performed an experiment to estimate the diameter of a circular object. A sample of five students had the following results in centimeters:

3.12, 3.16, 2.94, 3.33 and 3.00.

Determine the sample:
(a). mean.
(b). standard deviation. [6]

Qn 6: A car of mass 900 kg tows a caravan of mass 700 kg along a level road. The engine of the car exerts a forward force of 2.4 kilo Newtons and there is no resistance to motion. Find the acceleration and the tension in the tow bar. [5]

Qn 7: In how many ways can the letters of the "**FACETIOUS**" be arranged in a line. What is the probability that an arrangement begins with "**F**" and ends with "**S**". [5]

Qn 8: A particle with an initial velocity 2 m s^{-1} moves in a straight line with a constant acceleration of 3 m s^{-2} for 5 seconds. Find the final velocity and distance. [4]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The table below shows marks obtained by 120 students in a test:

Marks	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
No. of students	5	15	35	10	25	8	7	5

(a). Calculate:
  (i). the mean.
  (ii). standard deviation of the marks.
(b). Plot an ogive for this data and use it to estimate:
  (i). the median,
  (ii). the number of students with marks that are within one standard deviation of the mean. [12]

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Question 10:
A lift travels vertically upwards from rest at floor A to rest at floor B, which is 20 m above A, in three stages as follows:
At first, the lift accelerates from A at 2 m s^{-2} for 2 s. It then travels at a constant speed and finally it decelerates uniformly, coming to rest at B after a total time of $6\frac{1}{2}$ s.

(a). Sketch the velocity-time graph for this motion.
(b). Find the magnitude of the constant deceleration.
(c). The mass of the lift and its contents is 500 kg. Find the tension in the lift cable during the stage of motion when the lift is:
  (i). accelerating upwards.
  (ii). moving with constant speed. [12]

Question 11:
The height (cm) and ages (years) of a random sample of ten farmers are given in the table below.

Heights, X (cm)	156	151	152	160	146	157	149	142	158	140
Ages, Y (years)	47	38	44	55	46	49	45	30	45	30

(a). (i). Calculate the rank correlation coefficient.
  (ii). Comment on your result.
(b). Plot a scatter diagram for the data and comment on the relationship of the data. Draw a line of best fit.
(c). Use your graph to find Y when $X = 147$. [12]

Question 12:
A particle of mass 2 kg moves such that its displacement, $\underset{\sim}{\boldsymbol{s}}$, is given by:

$$\underset{\sim}{\boldsymbol{s}} = \begin{pmatrix} t^2 - 4t - 5 \\ t^2 - 4t + 3 \end{pmatrix}$$

(a). Find the expression for its velocity, $\underset{\sim}{\boldsymbol{v}}$, in terms of t.
(b). Hence calculate the speed of the particle at $t = 2$ seconds.
(c). Find the acceleration of the body and hence calculate the magnitude of the force acting on the particle. [12]

Question 13:

(a). In a game a player tosses three fair coins. He wins £ 10 if 3 heads occur, £ x if 2 heads occur, £ 3 if 1 head occurs and £ 2 if no heads occur. Express, in terms of x, his expected gain from each game. Given that he pays £ 4.50 to play each game, calculate the value of x for which the game is fair.

(b). A committee of 3 is to be chosen from 4 girls and 7 boys.

(i). Form a probability distribution for the number of girls on the committee and show that the experiment in random.

(ii). Find the expected number of girls on the committee. [12]

Question 14:

(a). Particles of mass m_1 and m_2 ($m_2 > m_1$) are connected by a light inextensible string passing over a smooth fixed pulley. The particles hang vertically and are released from rest. Show that the acceleration of the system is $\frac{(m_2-m_1)g}{m_1+m_2}$ and that the tension in the string is $\frac{2m_1m_2g}{m_1+m_2}$.

(b). Two particles A and B are connected by a light inextensible string passing over a smooth fixed pulley. The masses of A and B are $\frac{11}{2}m$ and $\frac{9}{2}m$ respectively. With A and B hanging freely, the system is released from rest with particle A a distance, d, above the floor. If a time, t, elapses before A hits the floor, show that $20d = t^2g$. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	(a). $$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{P(B).P(A/B)}{P(A)} = \frac{1}{4}\times\frac{2}{5}\div\frac{1}{3} = \frac{3}{10}$$	M1 M1 A1
	(b). $$P(A\cap B) = P(B).P(A/B) = \frac{1}{4}\times\frac{2}{5} = \frac{1}{10}$$	M1 A1
		05
2	$$\lvert \underset{\sim}{v} \rvert = 100 \text{ m s}^{-1}$$ $$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}130\\-110\end{pmatrix} - \begin{pmatrix}10\\50\end{pmatrix} = \begin{pmatrix}120\\-160\end{pmatrix}$$	B1
	$$\lvert\overrightarrow{AB}\rvert = \sqrt{120^2 + (-160)^2} = 200$$	M1
	$$\underset{\sim}{v} = \frac{\lvert \underset{\sim}{v}\rvert}{\lvert\overrightarrow{AB}\rvert}\overrightarrow{AB} = \frac{100}{200}\begin{pmatrix}120\\-160\end{pmatrix} = \begin{pmatrix}60\\-80\end{pmatrix} \text{ m s}^{-1}$$	M1 B1
	$$\Rightarrow \underset{\sim}{v} = 60\underset{\sim}{i} - 80\underset{\sim}{j} \text{ m s}^{-1}$$	A1
		05
3	(table below) (i). $\Delta_1 = 46 - 24 = 22$, $\Delta_2 = 46 - 32 = 14$ $$\text{Modal height} = L_m + \left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right)c$$ $$= 14.5 + \left(\frac{22}{22+14}\right)\times 5 = 14.5 + \frac{55}{18} = 17.5556$$	M1 A1
	(ii). Let N_1 be the number of seedlings of height less than 17 cm. (table below)	

Class	f	$C.F$	Class boundaries
5 – 9	18	18	4.5 – 9.5
10 – 14	24	42	9.5 – 14.5
15 – 19	46	88	14.5 – 19.5
20 – 24	32	120	19.5 – 24.5
25 – 29	30	150	24.5 – 29.5

Upper class boundary	14.5	17	19.5
Cumulative frequency	42	N_1	88

$$\frac{N_1 - 42}{88 - 42} = \frac{17 - 14.5}{19.5 - 14.5}$$ **M1 M1**

$$N_1 = \frac{2.5}{5} \times 46 + 42 = 65$$ **A1**

Alternatively:

$$17 = L_m + \left(\frac{N_1 - C.F_b}{f_m}\right)c$$
$$17 = 14.5 + \left(\frac{N_1 - 42}{46}\right) \times 5$$
$$2.5 = 5\left(\frac{N_1 - 42}{46}\right)$$
$$23 = N_1 - 42$$
$$N_1 = 65$$

05

4 (a).

$$\underset{\sim}{r}(t) = \overrightarrow{OP} + t\underset{\sim}{v} = \begin{pmatrix}4\\3\\9\end{pmatrix} + t\begin{pmatrix}3\\-2\\-5\end{pmatrix} = \begin{pmatrix}4+3t\\3-2t\\9-5t\end{pmatrix} \text{ m}$$ **M1 A1**

(b).

$$\underset{\sim}{r}(t=5) = \begin{pmatrix}4\\3\\9\end{pmatrix} + 5\begin{pmatrix}3\\-2\\-5\end{pmatrix} = \begin{pmatrix}4\\3\\9\end{pmatrix} + \begin{pmatrix}15\\-10\\-25\end{pmatrix}$$ **M1 M1**

$$= \begin{pmatrix}19\\-7\\-16\end{pmatrix} \text{ m}$$ **A1**

05

5 (a).

Sample mean, $\bar{x} = \dfrac{3.12 + 3.16 + 2.94 + 3.33 + 3.00}{5}$ **M1 M1**

$$= \frac{15.55}{5} = 3.11$$ **B1**

(b).

$$\sum x^2 = 3.12^2 + 3.16^2 + 2.94^2 + 3.33^2 + 3^2 = 48.4525$$

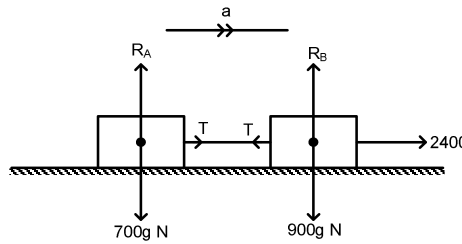
Sample standard deviation, $\sigma_s = \sqrt{\left(\dfrac{n}{n-1}\right)\left(\dfrac{\sum x^2}{n} - \bar{x}^2\right)}$

$$= \sqrt{\left(\frac{5}{5-1}\right)\left[\frac{48.4525}{5} - (3.11)^2\right]}$$ **M1 M1**

$$= \sqrt{1.25 \times 0.0184} = \sqrt{0.023} \approx 0.1517$$ **A1**

06

6



Considering the whole system,

$$2400 = (900 + 700)a$$ **M1**

$$a = \frac{2400}{1600} = 1.5 \text{ m s}^{-2}$$ **A1**

Considering the caravan only,

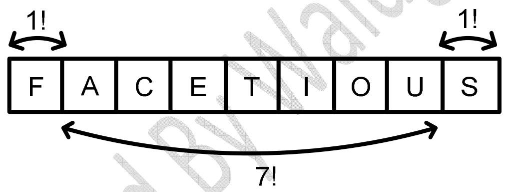
$$T - 0 = 700a$$
$$T = 700 \times 1.5$$ **M1 M1**
$$T = 1050 \text{ N}$$ **A1**

05

7 Without restriction,

number of ways $= 9! = 362{,}880$ ways **M1 A1**

When the arrangement begins with "**F**" and ends with "**S**",



number of ways $= 1! \times 7! \times 1! = 1 \times 5040 \times 1 = 5{,}040$ ways **B1**

required probability $= \dfrac{5{,}040}{362{,}880} = \dfrac{1}{72} \approx 0.0139$ **M1 A1**

05

8

$$v = u + at = 2 + 3 \times 5 = 17 \text{ m s}^{-1}$$ **M1 A1**

$$s = ut + \frac{1}{2}at^2 = 2 \times 5 + \frac{1}{2} \times 3 \times 5^2 = 47.5 \text{ m}$$ **M1 A1**

04

9

Class	f	x	fx	fx^2	Class boundaries	C.F
10 – 14	5	12	60	720	9.5 – 14.5	
15 – 19	15	17	255	4335	14.5 – 19.5	
20 – 24	35	22	770	16940	19.5 – 24.5	
25 – 29	10	27	270	7290	24.5 – 29.5	

B1-for C.F column

30 – 34	25	32	800	25600	29.5 – 34.5	
35 – 39	8	37	296	10952	34.5 – 39.5	
40 – 44	7	42	294	12348	39.5 – 44.5	
45 – 49	5	47	235	11045	44.5 – 49.5	
Total	110		2980	89230		

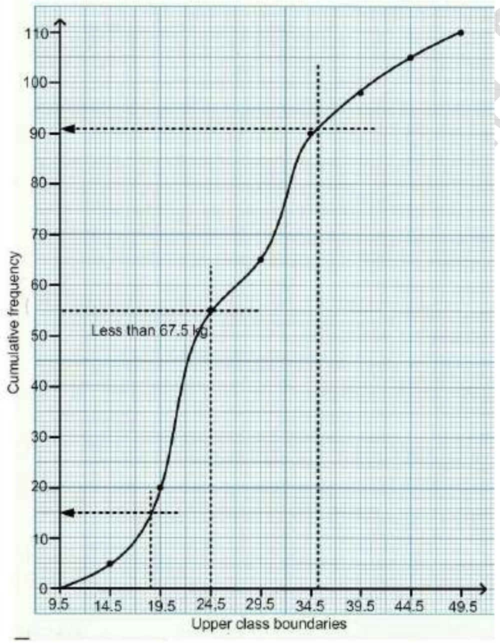
(a). (i).

$$\overline{x} = \frac{\sum fx}{\sum f} = \frac{2980}{110} = 27.0909$$ **M1 A1**

(ii).

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{89230}{110} - \left(\frac{2980}{110}\right)^2} = 8.79002$$ **M1 M1 A1**

(b).



B1-axes

B1- plotting points

B1-smooth curve

(i).

$$\text{Median position} = \frac{1}{2} \times 110 = 55$$

$$\therefore \text{Median} = 24.5$$ **B1**

(ii).

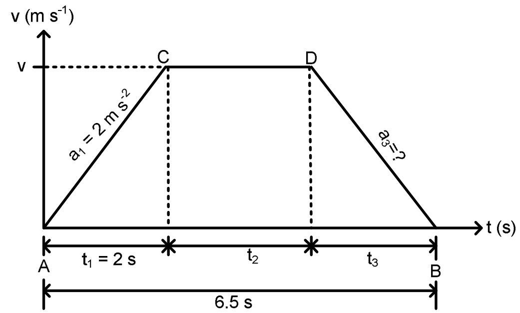
Position of one standart deviation of the mean

$$= 27.0909 \pm 8.79002 \approx [18.3, 35.9]$$ **B1**

$$\therefore \text{One standart deviation of the mean} = 91 - 15 = 76$$ **A1**

12

10 (a).



B1 B1 B1

(ii).

For the first stage,

$$v = u + at = 0 + 2 \times 2 = 4 \text{ m s}^{-1}$$

$$\text{total distance} = \frac{1}{2} \times 4 \times (t_2 + 6.5) = 20$$ **B1**

$$2t_2 + 13 = 20$$

$$2t_2 = 7$$

$$t_2 = 3.5 \text{ s}$$ **B1**

$$t_3 = 6.5 - 2 - 3.5 = 1 \text{ s}$$ **B1**

For the last stage,

$$v = u + at$$

$$0 = 4 + a_3 \times 1$$ **M1**

$$a_3 = -4 \text{ m s}^{-2}$$

$$\therefore \text{deceleration} = 4 \text{ m s}^{-2}$$ **A1**

(b). (i).

$$T - 500g = 500a$$

$$T - 500 \times 9.8 = 500 \times 2$$ **M1**

$$T = 5900 \text{ N}$$ **A1**

(ii).

$$T - 500g = 500 \times 0$$

$$T - 500 \times 9.8 = 0$$ **M1**

$$T = 4900 \text{ N}$$ **A1**

12

11 (a).

	x	y	R_x	R_y	d	d^2
J	140	30	10	9.5	0.5	0.25
H	142	30	9	9.5	-0.5	0.25
E	146	46	8	4	4	16

B1-for d

G	149	45	7	5.5	1.5	2.25	**B1**-for d^2
B	151	38	6	8	-2	4	
C	152	44	5	7	-2	4	**B1**-for $\sum d^2$
A	156	47	4	3	1	1	
F	157	49	3	2	1	1	
I	158	45	2	5.5	-3.5	12.25	
D	160	55	1	1	0	0	
Total	1511	429				$\sum d^2 = 41$	

(i).

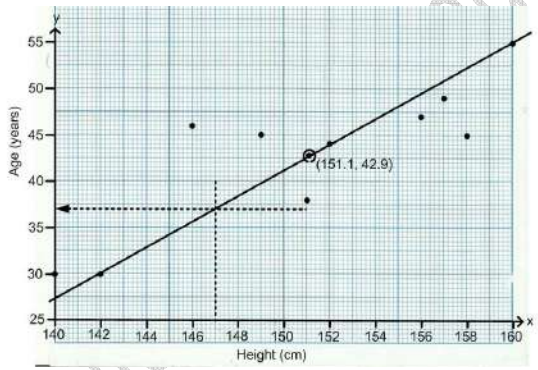
$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\times 41}{10(10^2-1)} = 1 - 0.2485 = 0.7515$$ **M1 A1**

(ii). Significant at 5%. (or: Not significant at 1%.) **B1**

(b).

$$\bar{x} = \frac{1511}{10} = 151.1, \qquad \bar{y} = \frac{429}{10} = 42.9$$

$$\Rightarrow \text{Mean point,} \qquad (\bar{x}, \bar{y}) = (151.1, 42.9)$$



B1-axes

B2-points

B1-line of best fit

Comment: There is a positive linear relationship between X and Y. **B1**

(c). $Y = 37$ when $X = 147$. **A1**

12

12 (a).

$$\underset{\sim}{v} = \frac{d}{dt}\begin{pmatrix} t^2 - 4t - 5 \\ t^2 - 4t + 3 \end{pmatrix} = \begin{pmatrix} 2t - 4 \\ 2t - 4 \end{pmatrix}$$ **M1 M1 B1**

(b).

$$\underset{\sim}{v}(t = 2) = \begin{pmatrix} 2\times 2 - 4 \\ 2\times 2 - 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ **M1 B1**

$$\text{speed} = \sqrt{0^2 + 0^2} = 0 \text{ m s}^{-1}$$ **M1 A1**

(c).

$$\underset{\sim}{a} = \frac{d}{dt}\begin{pmatrix} 2t - 4 \\ 2t - 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \text{ m s}^{-2}$$ **M1 B1**

$$\underset{\sim}{F} = m\underset{\sim}{a} = 2\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \text{ N}$$ **B1**

$$|\underset{\sim}{F}| = \sqrt{2^2 + 2^2} = 4\sqrt{2} \approx 5.6568 \text{ N}$$ **M1 A1**

12

13 (a).

Coin 3 / Coin 1,2	H	T
HH	HHH	HHT
TH	THH	THT
HT	HTH	HTT
TT	TTH	TTT

B1

.

x	$P(X = x)$	$xP(X = x)$
1	$\frac{1}{8}$	$\frac{2}{8}$
2	$\frac{3}{8}$	$\frac{9}{8}$
x	$\frac{3}{8}$	$\frac{3x}{8}$
10	$\frac{1}{8}$	$\frac{10}{8}$
Sums	1	$\frac{21 + 3x}{8}$

B1-$P(X = x)$

B1-$xP(X = x)$

$$\text{Expected gain} = \frac{21 + 3x}{8}$$ **M1**

The game is fair if:

$$\text{Profit} = 0$$

$$\frac{21 + 3x}{8} - 4.5 = 0$$ **M1 M1**

$$\frac{21 + 3x}{8} = 4.5$$

$$21 + 3x = 36$$

$$3x = 15$$

$$x = 5$$ **A1**

(b).

Let X be the number of girls on the committee.

$$P(X = 0) = \frac{{}^4C_0 \times {}^7C_3}{{}^{11}C_3} = \frac{7}{33}$$

$$P(X = 1) = \frac{{}^4C_1 \times {}^7C_2}{{}^{11}C_3} = \frac{84}{165}$$ **B1**

$$P(X=2)=\frac{{}^4C_2 \times {}^7C_1}{{}^{11}C_3}=\frac{42}{165},$$

$$P(X=3)=\frac{{}^4C_3 \times {}^7C_0}{{}^{11}C_3}=\frac{4}{165}$$ **B1**

$$\sum_{all\,x} P(X=x)=\frac{7}{33}+\frac{84}{165}+\frac{42}{165}+\frac{4}{165}=1$$ **B1**

Hence the experiment is random.

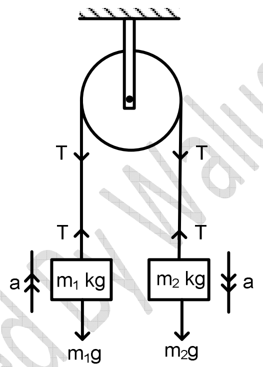
(ii).

$$E(X)=\sum_{all\,x} xP(X=x)$$

$$=0\times\frac{7}{33}+1\times\frac{84}{165}+2\times\frac{42}{165}+3\times\frac{4}{165}=\frac{12}{11}$$ **M1 A1**

12

14 (a).



$$T-m_1g=m_1a \quad \longrightarrow (1)$$ **B1**

$$m_2g-T=m_2a \quad \longrightarrow (2)$$ **B1**

Equation (1) + (2) gives:

$$m_2g-m_1g=m_1a+m_2a$$ **M1**

$$(m_2-m_1)g=(m_1+m_2)a$$

$$a=\frac{(m_2-m_1)g}{m_1+m_2}, \quad \text{as required}$$ **B1**

Substituting for a into equation (1) gives:

$$T=m_1(g+a)$$

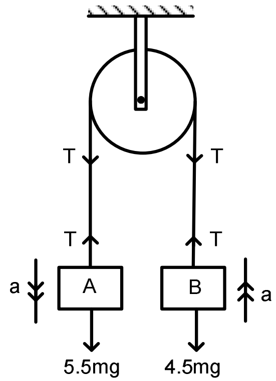
$$=m_1\left(g+\frac{m_2g-m_1g}{m_1+m_2}\right)$$ **M1**

$$=m_1\left(\frac{m_1g+m_2g+m_2g-m_1g}{m_1+m_2}\right)$$ **M1**

$$=m_1\left(\frac{2m_2g}{m_1+m_2}\right)$$

$$=\frac{2m_1m_2g}{m_1+m_2}, \quad , \quad \text{as required}$$ **B1**

(b).



$$5.5mg-T=5.5ma \quad \longrightarrow (1)$$ **B1**

$$T-4.5mg=4.5ma \quad \longrightarrow (2)$$ **B1**

Equation (1) + (2) gives:

$$mg=10ma$$

$$a=\frac{g}{10}\ \text{m s}^{-2}$$ **B1**

$$s=ut+\frac{1}{2}at^2$$

$$d=0+\frac{1}{2}\times\frac{g}{10}\times t^2$$

$$d=\frac{gt^2}{20}$$ **M1**

$$20d=gt^2, \quad \text{as required}$$ **B1**

12

END

P425/1
PURE
MATHEMATICS
PAPER 1
Sept 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 1 BOT 3 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Attempt **ALL** questions in section A and any **five** from section B.
- Show your working clearly.

Section A (40 Marks)

Qn 1: Solve the simultaneous equations:
$$4x - 5y + 7z = -14$$
$$9x + 2y + 3z = 47$$
$$x - y - 5z = 11$$
[5]

Qn 2: Solve the equation: $\tan 2x = \cot 3x$ for $0° \leq x \leq 180°$. [5]

Qn 3: The coefficients of the first three terms of the expansion of $\left(1 + \frac{x}{2}\right)^n$ are in an A.P. Find the value of n. [5]

Qn 4: Differentiate $\left(\frac{1+2x}{1+x}\right)^2$ with respect to x. [5]

Qn 5: Solve for x: $\log_3 x + \log_x 3 = \frac{10}{3}$. [5]

Qn 6: Find the equation of the circle whose end diameter is the line joining the points $A(1, 3)$ and $B(-2, 5)$. [5]

Qn 7: Solve for x in the equation: $4^{2x} - 4^{x+1} + 4 = 0$. [5]

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

Qn 8: A container is in the form of an inverted right circular cone. Its height is 100 cm and base radius is 40 cm. The container is full of water and has a small hole at its vertex. Water is flowing through the hole at a rate of $100\ \mathrm{m^3\ s^{-1}}$. Find the rate at which the water level in the container is falling when the height of water in the container is halved. [5]

Section B (60 Marks)

Question 9:

(a). Find the three numbers in a Geometric progression with a sum of 28 and a product of 512. [7]

(b). The roots of the quadratic equation $x^2 + px + q = 0$ are α and β, show that the quadratic equation whose roots are $(\alpha^2 - q\alpha)$ and $(\beta^2 - q\beta)$ is given by:
$$x^2 - (p^2 + pq - 2q)x + q^2(p + q + 1) = 0$$
[5]

Question 10:

A circle whose centre is in the first quadrant touches the $x-$ and $y-$axes and the line $8x - 15y = 120$. Find the:

(a). equation of the circle. [10]

(b). point at which the circle touches the $x-$axis. [2]

Question 11:

(a). Find the locus of a point which moves such that the ratio of its distance from the point $A(2, 4)$ to its distance from the point $B(-5, 3)$ is $2 : 3$. [6]

(b). Find the equation of the normal to the curve $x^2y + 3y^2 - 4x - 12 = 0$ at the point $(1, 2)$. [6]

Question 12:

(a). Find the first three terms of the expansion $(2 - x)^6$ and hence use it to find 1.998^6 to 2 d.p. [7]

(b). Expand $(1 - 3x + 2x^2)^5$ in ascending powers of x as far as the x^2 term. [5]

Question 13:

The equations of a curve is parametrically given by $x = \frac{1+t}{1-t}$ and $y = \frac{2t^2}{1-t}$.
Find $\frac{d^2y}{dx^2}$. [12]

Question 14:

(a). Prove that in any triangle ABC,

$$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}=1-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

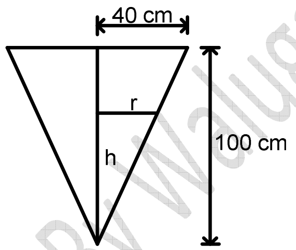
(b). Show that $4\cos 3\theta\cos\theta+1=\frac{\sin 5\theta}{\sin\theta}$. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	$4x-5y+7z=-14 \quad \longrightarrow (1)$	
	$9x+2y+3z=47 \quad \longrightarrow (2)$	
	$x-y-5z=11 \quad \longrightarrow (3)$	
	Equation $5\times(3)-(1)$ gives,	
	$5x-5y-25z=55 \quad \longrightarrow 5\times(3)$	
	$-\ 4x-5y+7z=-14 \quad \longrightarrow (1)$	
	$x-32z=69 \quad \longrightarrow (4)$	
	Equation $2\times(3)+(2)$ gives,	
	$2x-2y-10z=22 \quad \longrightarrow 2\times(3)$	
	$+\ 9x+2y+3z=47 \quad \longrightarrow (2)$	
	$11x-7z=69 \quad \longrightarrow (5)$	
	Equation $(5)-11\times(4)$ gives,	
	$11x-7z=69 \quad \longrightarrow (5)$	
	$-\ 11x-352z=759 \quad \longrightarrow 11\times(4)$	
	$345z=-690$	M1
	$z=-2$	A1
	From equation (4),	
	$x-32\times(-2)=69$	M1-mobile
	$x=5$	A1
	From equation (3),	
	$5-y-5\times(-2)=11$	
	$5-y+10=11$	
	$y=4$	A1
		05
2	$\tan 2x=\cot 3x$	
	$\frac{\sin 2x}{\cos 2x}=\frac{\cos 3x}{\sin 3x}$	M1
	$\sin 2x\sin 3x=\cos 2x\cos 3x$	
	$\cos 2x\cos 3x-\sin 2x\sin 3x=0$	B1
	$\cos(2x+3x)=0$	M1
	$\cos 5x=0$	
	$5x=90°,270°,450°,630°,810°$	B1
	$x=18°,54°,90°,126°,162°$	A1
		05
3	$\left(1+\frac{x}{2}\right)^n\approx 1+n\left(\frac{x}{2}\right)+\frac{n(n-1)}{2!}\left(\frac{x}{2}\right)^2+\cdots$	
	$\approx 1+\frac{n}{2}x+\frac{n(n-1)}{8}x^2+\cdots$	B1
	For the coefficients to be in A.P,	
	$\frac{n(n-1)}{8}-\frac{n}{2}=\frac{n}{2}-1$	M1 M1
	$\frac{n(n-1)}{8}=n-1$	
	$n(n-1)=8(n-1)$	

	$n(n-1)-8(n-1)=0$ $(n-8)(n-1)=0$ $n=8,\quad$ or, $\quad n=1$ but $n\neq 1,\quad \Rightarrow n=8$	**B1** **A1**
		05
4	let $y=\left(\dfrac{1+2x}{1+x}\right)^2$ $u=(1+2x)^2,\quad \Rightarrow \dfrac{du}{dx}=2(1+2x)\times 2=4(1+2x)$ $v=(1+x)^2,\quad \Rightarrow \dfrac{dv}{dx}=2(1+x)\times 1=2(1+x)$ $\dfrac{dy}{dx}=\dfrac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ $=\dfrac{(1+x)^2\times 4(1+2x)-(1+2x)^2\times 2(1+x)}{(1+x)^4}$ $=\dfrac{(1+x)(1+2x)[4(1+x)-2(1+2x)]}{(1+x)^4}$ $=\dfrac{(1+2x)[4+4x-2-4x]}{(1+x)^3}=\dfrac{2(1+2x)}{(1+x)^3}$	**B1** **B1** **M1 M1** **A1**
		05
5	$\log_3 x+\log_x 3=\dfrac{10}{3}$ $\log_3 x+\dfrac{1}{\log_3 x}=\dfrac{10}{3}$ let $y=\log_3 x$ $y+\dfrac{1}{y}=\dfrac{10}{3}$ $3y^2+3=10y$ $3y^2-10y+3=0$ $3y^2-9y-y+3=0$ $3y(y-3)-(y-3)=0$ $(3y-1)(y-3)=0$ $y=\dfrac{1}{3}\ ,\quad$ or, $\quad y=3$ but $y=\log_3 x$ $\log_3 x=\dfrac{1}{3}\ ,\quad$ or, $\quad \log_3 x=3$ $\therefore x=\sqrt[3]{3},\quad$ or, $\quad x=3^3=27$	**B1** **M1** **B1** **M1** **A1**
		05
6	Centre $=$ Midpoint of $AB=\left(\dfrac{1+(-2)}{2},\dfrac{3+5}{2}\right)$ $=(-0.5,4)$ Radius $=$ Length of $AB=\sqrt{(-2-1)^2+(5-3)^2}$ $=\sqrt{13}$ units	**B1** **B1**

	The required equation of the circle is given by: $(x+0.5)^2+(y-4)^2=\left(\sqrt{13}\right)^2$ $x^2+x+0.25+y^2-8y+16=13$ $x^2+y^2+x-8y+3.25=0$ $4x^2+4y^2+4x-32y+13=0$	**M1** **B1** **A1**
		05
7	$4^{2x}-4^{x+1}+4=0$ $(4^x)^2-4^1\times 4^x+4=0$ let $y=4^x$ $y^2-4y+4=0$ $(y-2)^2=0$ $y=2$ but $y=4^x$ $\therefore 4^x=2,\quad \Rightarrow 2^{2x}=2^1$ $2x=1,\quad \Rightarrow x=0.5$	**M1** **B1** **M1** **B1** **A1**
		05
8	 $\dfrac{dv}{dt}=100\text{ cm}^3\text{s}^{-1},\quad h=\dfrac{1}{2}\times 100=50$ cm $\dfrac{r}{h}=\dfrac{40}{100}\ ,\quad \Rightarrow r=0.4h$ volume, $\quad v=\dfrac{1}{3}\pi r^2h=\dfrac{1}{3}\pi(0.4h)^2h=\dfrac{2}{15}\pi h^3$ $\dfrac{dv}{dh}=\dfrac{2}{5}\pi h^2=\dfrac{2}{5}\pi\times 50^2=1000\pi$ $\dfrac{dh}{dt}=\dfrac{dh}{dv}\times\dfrac{dv}{dt}=\dfrac{1}{1000\pi}\times 100=\dfrac{1}{10\pi}\text{ cm s}^{-1}$	**B1** **B1** **B1** **M1 A1**
		05
9	(a). sum $=a+ar+ar^2=28$ $a(1+r+r^2)=28\quad\longrightarrow (1)$ product $=a\times ar\times ar^2=28$ $a^3r^3=512$ $ar=8\quad\longrightarrow (2)$ Equation (1) $\div$ (2) gives:	**B1** **B1**

$$\frac{a(1+r+r^2)}{ar} = \frac{28}{8}$$ **M1**

$$1 + r + r^2 = 3.5r$$
$$r^2 - 2.5r + 1 = 0$$
$$2r^2 - 5r + 2 = 0$$
$$2r^2 - 4r - r + 2 = 0$$
$$2r(r-2) - (r-2) = 0$$
$$(2r-1)(r-2) = 0$$
$$r = 0.5, \quad \text{or}, \quad r = 2$$ **B1**

$$\text{when } r = 0.5, \quad a = \frac{8}{0.5} = 16$$ **B1**

First term, $a = 16$
Second term $= 16 \times 0.5 = 8$
Third term $= 8 \times 0.5 = 4$ **A1-all the three terms**

$\therefore$ The three numbers are: 16, 8 and 4.

Alternatively:

$$\text{when } r = 2, \quad a = \frac{8}{2} = 4$$

First term, $a = 4$
Second term $= 4 \times 2 = 8$
Third term $= 8 \times 2 = 16$

$\therefore$ The three numbers are: 4, 8 and 16.

(b).

$$x^2 + px + q = 0$$
$$\alpha + \beta = -p, \quad \alpha\beta = q$$

For $(\alpha^2 - q\alpha)$ and $(\beta^2 - q\beta)$ as roots, **B1**

$$\text{product} = (\alpha^2 - q\alpha)(\beta^2 - q\beta)$$
$$= \alpha^2\beta^2 - q\alpha^2\beta - q\alpha\beta^2 + q^2\alpha\beta$$
$$= (\alpha\beta)^2 - q\alpha\beta(\alpha+\beta) + q^2\alpha\beta$$
$$= q^2 - q \times q \times (-p) + q^2 \times q$$
$$= q^2 + q^2p + q^3$$ **M1**
$$= q^2(1 + p + q)$$

$$\text{sum} = (\alpha^2 - q\alpha) + (\beta^2 - q\beta)$$ **B1**
$$= \alpha^2 - q\alpha + \beta^2 - q\beta$$
$$= (\alpha^2 + \beta^2) - q(\alpha + \beta)$$
$$= (\alpha+\beta)^2 - 2\alpha\beta - q(\alpha+\beta)$$
$$= (-p)^2 - 2q - q \times (-p)$$
$$= p^2 - 2q + qp$$ **M1**

The required equation is: **B1**

$$x^2 - (\text{sum})x + (\text{product}) = 0$$
$$x^2 - (p^2 + pq - 2q)x + q^2(p + q + 1) = 0, \quad \text{as required}$$ **B1**

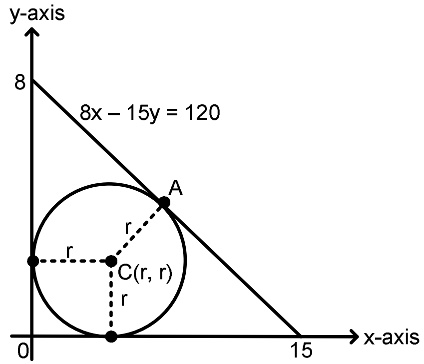
12

10 (a).

For $8x - 15y = 120$,

when $x = 0$, $0 - 15y = 120$, $\Rightarrow y = 8$ **B1**

when $y = 0$, $8x - 0 = 120$, $\Rightarrow x = 15$ **B1**



B1 B1

$$\text{Length } AC, \quad r = \left|\frac{8r - 15r - 120}{\sqrt{8^2 + (-15)^2}}\right|$$ **M1**

$$r = \left|\frac{-7r - 120}{17}\right|$$
$$r = \left|\frac{-(7r + 120)}{17}\right|$$ **M1**
$$r = \frac{7r + 120}{17}$$
$$17r = 7r + 120$$
$$10r = 120$$ **B1**
$$r = 10$$ **A1**

Centre $(10, 10)$, Radius $= 10$ units

The required equation of the circle is given by:

$$(x-10)^2 + (y-10)^2 = 10^2$$ **M1**
$$x^2 - 20x + 100 + y^2 - 20y + 100 = 100$$ **B1**
$$x^2 + y^2 - 20x - 20y + 100 = 0$$ **A1**

(b).

The circle touches the x –axis at point $(10, 0)$. **A1**

12

11 (a). Let the variable point be $P(x, y)$

$$\overline{AP} : \overline{PB} = 2 : 3$$
$$3\overline{AP} = 2\overline{PB}$$
$$3\sqrt{(x-2)^2 + (y-4)^2} = 2\sqrt{(x+5)^2 + (y-3)^2}$$ **M1**
$$9(x^2 - 4x + 4 + y^2 - 8y + 16) = 4(x^2 + 10x + 25 + y^2 - 6y + 9)$$ **M1 M1**
$$9x^2 - 36x + 9y^2 - 72y + 180 = 4x^2 + 40x + 4y^2 - 24y + 136$$
$$5x^2 + 5y^2 - 76x - 48y + 44 = 0$$ **B1**

$$\text{Radius} = \sqrt{\left(\frac{-76}{5}\right)^2 + \left(\frac{-48}{5}\right)^2 - \frac{44}{5}} = \sqrt{314.4}$$ **B1**

The locus is a circle with centre $\left(\frac{-76}{5}, \frac{-48}{5}\right)$ and radius $\sqrt{314.4}$ **B1** units.

(b).

$$\frac{d}{dx}(x^2y+3y^2-4x-12)=\frac{d}{dx}(0)$$ **M1 M1**

$$x^2\frac{dy}{dx}+2xy+6y\frac{dy}{dx}-4=0$$

$$(x^2+6y)\frac{dy}{dx}=4-2xy$$

$$\frac{dy}{dx}=\frac{4-2xy}{x^2+6y}$$ **B1**

At the point $(1,2)$,

$$\text{Gradient of tangent}=\frac{4-2\times1\times2}{1^2+6\times2}=0$$ **B1**

$$\text{Gradient of the normal}=\frac{-1}{0}=-\infty$$

The required equation of the normal is given by:

$$\frac{y-2}{x-1}=\frac{-1}{0}$$ **M1**

$$0=-x+1$$

$$x=1$$ **A1**

12

12 (a).

$$(2-x)^6=2^6\left(1-\frac{x}{2}\right)^6$$ **B1**

$$\approx 2^6\left[1+6\left(-\frac{x}{2}\right)+\frac{6\times5}{2!}\times\left(-\frac{x}{2}\right)^2+\cdots\right]$$ **M1 M1 M1**

$$\approx 64\left[1-3x+\frac{15}{4}x^2+\cdots\right]$$

$$\approx 64-192x+240x^2+\cdots$$ **B1**

For the hence part:

$$1.998^6=(2-0.002)^6$$

$$\approx 64-192\times0.002+240\times(0.002)^2=63.62\ (2\ \text{d.p})$$ **M1 A1**

(b).

$$(1-3x+2x^2)^5=[1+(2x^2-3x)]^5$$ **B1**

$$\approx 1+5(2x^2-3x)+\frac{5\times4}{2!}\times(2x^2-3x)^2+\cdots$$ **M1 M1**

$$\approx 1+5(2x^2-3x)+10(4x^4-12x^3+9x^2)+\cdots$$

$$\approx 1+10x^2-15x+40x^4-120x^3+90x^2+\cdots$$ **B1**

$$\approx 1+10x^2-15x+90x^2+\cdots$$

$$\approx 1-15x+100x^2+\cdots$$ **B1**

12

13

$$x=\frac{1+t}{1-t}$$

$$\frac{dx}{dt}=\frac{(1-t)\times1-(1+t)\times(-1)}{(1-t)^2}$$ **M1 M1**

$$=\frac{1-t+1+t}{(1-t)^2}=\frac{2}{(1-t)^2}$$ **B1 B1**

$$y=\frac{2t^2}{1-t}$$

$$\frac{dy}{dt}=\frac{(1-t)\times2t-2t^2\times(-1)}{(1-t)^2}$$ **M1 M1**

$$=\frac{2t-2t^2+2t^2}{(1-t)^2}=\frac{2t}{(1-t)^2}$$ **B1 B1**

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\frac{2t}{(1-t)^2}\times\frac{(1-t)^2}{2}=t$$ **M1 B1**

$$\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx}=1\times\frac{(1-t)^2}{2}=\frac{1}{2}(1-t)^2$$ **M1 B1**

12

14 (a).

$$L.H.S=\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}$$

$$=\frac{1}{2}(1-\cos A)+\frac{1}{2}(1-\cos B)+\frac{1}{2}(1-\cos C)$$ **B1**

$$=\frac{1}{2}\{3-\cos A-\cos B-\cos C\}$$

$$=\frac{1}{2}\left[3-\left(1-2\sin^2\frac{A}{2}\right)-(\cos B+\cos C)\right]$$ **B1**

$$=\frac{1}{2}\left[2+2\sin^2\frac{A}{2}-2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right]$$ **B1**

For angles of a triangle, A, B, C,

$$\sin\left(\frac{B+C}{2}\right)=\sin\left(90-\frac{A}{2}\right)=\cos\frac{A}{2}$$

$$\cos\left(\frac{B+C}{2}\right)=\cos\left(90-\frac{A}{2}\right)=\sin\frac{A}{2}$$

$$\Rightarrow L.H.S=\frac{1}{2}\left[2+2\sin^2\frac{A}{2}-2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)\right]$$ **B1**

$$=1+\sin^2\frac{A}{2}-\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$$

$$=1+\sin\frac{A}{2}\left[\sin\frac{A}{2}-\cos\left(\frac{B-C}{2}\right)\right]$$

$$=1+\sin\frac{A}{2}\left[\cos\left(\frac{B+C}{2}\right)-\cos\left(\frac{B-C}{2}\right)\right]$$ **B1**

$$=1+\sin\frac{A}{2}\left[-2\sin\frac{B}{2}\sin\frac{C}{2}\right]$$ **B1**

$$=1-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\ ,\qquad\text{as required}$$ **B1**

(b).

$$4\cos3\theta\cos\theta+1=\frac{\sin5\theta}{\sin\theta}$$

$$L.H.S=4\cos3\theta\cos\theta+1$$

$$=4\cos3\theta\cos\theta+\frac{\sin\theta}{\sin\theta}$$ **B1**

$$=\frac{4\cos3\theta\cos\theta\sin\theta+\sin\theta}{\sin\theta}$$ **B1**

	$= \dfrac{2\cos 3\theta \sin 2\theta + \sin\theta}{\sin\theta}$	B1
	$= \dfrac{\sin 5\theta - \sin\theta + \sin\theta}{\sin\theta}$	B1
	$= \dfrac{\sin 5\theta}{\sin\theta}$, as required	B1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Sept 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 2 BOT 3 2018
Time: 3 Hours

NAME:________________________________ COMB:__________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)

Qn 1: The events A and B are such that $P(A/B) = 0.4$, $P(B/A) = 0.25$, $P(A \cap B) = 0.12$. find $P(A \cup B')$. [5]

Qn 2: At the same instant, two children who are standing 24 m apart begin to cycle directly towards each other. James starts from rest at a point A riding with a constant acceleration of 2 m s^{-2} and William rides with a constant speed of 2 m s^{-1}. Find how long it is before they meet. [5]

Qn 3: Eight candidates seeking to join a certain school were given Maths (x) and English (y) tests and their scores were recorded as follows:

x	54	35	62	87	53	71	50	55
y	60	47	65	83	56	74	63	57

(a). Draw a scatter diagram and comment on your results.
(b). Find x when $y = 55$. [5]

Qn 4: A ball is projected vertically upwards and when it is at a height of 10 m, it takes 8 seconds to return to its point of projection. Find the speed with which the ball was projected. [5]

Qn 5: The probability that a fisherman catches fish on a clear day is $\frac{2}{5}$ and on cloudy day is $\frac{7}{10}$. If the probability that the day is cloudy is $\frac{3}{5}$, find the probability that the day is cloudy given that the fisherman does not catch fish. [5]

Qn 6: A plane flying from $A(10, 50)$ to $B(130, -110)$ at a speed of 100 m s^{-1}, find the velocity of the plane in the form $a\underset{\sim}{\boldsymbol{i}} + b\underset{\sim}{\boldsymbol{j}}$. [5]

Qn 7: A discrete random variable X has a probability distribution given below:

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	a	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{2}a$

Find the:
(i). value of a.
(ii). $E(X^2)$. [5]

Qn 8: A particle has an initial position vector $\left(4\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}} + 9\underset{\sim}{\boldsymbol{k}}\right)$ m. The particle moves with constant velocity $\left(3\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} - 5\underset{\sim}{\boldsymbol{k}}\right)$ m s^{-1}. Find:
(a). the position vector of the particle at any time, t.
(b). the position vector of the particle after 5 seconds. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The table below shows the cumulative marks scored by 100 students in a Maths test.

Marks (%)	<10	<20	<30	<40	<50	<60	<70	<80	<90	<100
Cumulative frequency	5	9	16	26	41	60	77	90	98	100

(a). Calculate the:
(i). Mean mark,
(ii). Standard deviation. [9]
(b). Draw a cumulative frequency curve and use it to estimate the:
(i). Median mark.
(ii). pass mark if 70 students passed. [6]

Question 10:
Two tetrahedral dice, with faces labeled 1, 2, 3 and 4 are thrown and the number on which each lands is noted. The score is the sum of the two numbers. Find the probability that:
(a). the score is even, given that at least one die lands on three.
(b). at least one die lands on three given that the score is even. [12]

Question 11:
The table below shows the expenditure of restaurant for the years 2014 and 2016.

Item	Price (shs)		Weight
	2014	2016	
Milk (per litre)	1,000	1,300	0.5
Eggs (per tray)	6,500	8,300	1
Sugar (per kg)	3,000	3,800	2
Blue band	7,000	9,000	1

Taking 2014 as the base year, calculate for 2016 the:
(a). Price relative for each item.
(b). Simple aggregate price index.
(c). Weighted aggregate price index and comment on your result.
(d). In 2016, the restaurant spent shs 45,000 on buying these items. Using the index obtained in (c), find how much money the restaurant could have spent in 2014. [12]

Question 12:
A car is started from rest accelerated uniformly for 120 seconds and then maintained a speed of 50 km h^{-1}. Another car started 120 seconds later from the same spot and this car too accelerated uniformly for 120 seconds and then maintained a speed of 75 km h^{-1}.
(a). Sketch a velocity-time graph and find when and where the second car overtook the first.
(b). The car maintained the speed of 50 km h^{-1} for 10 minutes. It then decelerated uniformly for further 2.5 minutes before coming to rest. How far has the car travelled from the start? [12]

Question 13:

A discrete random variable X has its probability distribution given below:

x	1	2	5	7	10
$P(X = x)$	$2c + k$	$2(c + k)$	$5c + k$	$5c + 0.1$	$6c + 0.1$

Given that $P(X = 1) = P(X = 5)$, find:

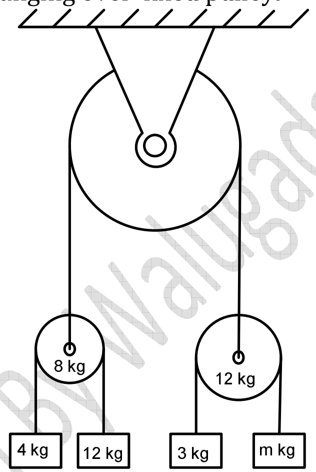
(a). the values of c and k.

(b). the mean and mode of X.

(c). the standard deviation of X. [12]

Question 14:

The diagram below shows two pulleys of masses 8 kg and 12 kg connected by a light inextensible string hanging over fixed pulley.



The acceleration of 4 kg and 12 kg masses are $^g/_2$ upwards and $^g/_2$ downwards respectively. The acceleration of the 3 kg and $\boldsymbol{m}$ kg masses are $^g/_3$ upwards and $^g/_3$ downwards respectively. The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the:

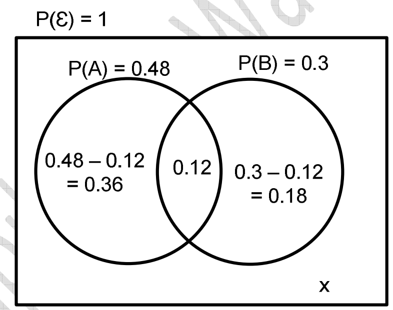
(a). tensions in the strings, [9]

(b). value of $\boldsymbol{m}$. [3]

END

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

MARKING GUIDE

SNo.	Working	Marks
1	$P(A/B) = \dfrac{P(A\cap B)}{P(B)}$	
	$\Rightarrow P(B) = \dfrac{P(A\cap B)}{P(A/B)} = \dfrac{0.12}{0.4} = 0.3$	B1
	$P(B/A) = \dfrac{P(B\cap A)}{P(A)}$	
	$\Rightarrow P(A) = \dfrac{P(B\cap A)}{P(B/A)} = \dfrac{0.12}{0.25} = 0.48$	B1
	$P(A\cap B') = P(A) - P(A\cap B) = 0.48 - 0.12 = 0.36$	B1
	$P(A\cup B') = P(A) + P(B') - P(A\cap B')$ $= 0.48 + (1 - 0.3) - 0.36 = 0.82$	M1 A1
	Alternatively:	
	$P(A/B) = \dfrac{P(A\cap B)}{P(B)}$	
	$\Rightarrow P(B) = \dfrac{P(A\cap B)}{P(A/B)} = \dfrac{0.12}{0.4} = 0.3$	B1
	$P(B/A) = \dfrac{P(B\cap A)}{P(A)}$	
	$\Rightarrow P(A) = \dfrac{P(B\cap A)}{P(B/A)} = \dfrac{0.12}{0.25} = 0.48$	B1
		
	$P(\varepsilon) = 0.48 + 0.18 + x = 1$	M1
	$x = 0.34$	B1
	$P(A\cup B') = 0.48 + 0.34 = 0.82$	A1
		05
2	Let x be the distance, in metres, travelled by James before they meet.	
	For motion of James, $s = x, u = 0, a = 2\text{ m s}^{-2}$	
	$s = ut + \frac{1}{2}at^2$	

$$x = 0 + \frac{1}{2} \times 2 \times t^2$$

$$x = t^2 \quad \longrightarrow (1)$$ **B1**

For motion of William, $s = 24 - x, a = 0, u = 2\text{ m s}^{-1}$

$$s = ut + \frac{1}{2}at^2$$

$$24 - x = 2t + 0$$

$$x = 24 - 2t \quad \longrightarrow (2)$$ **B1**

Substituting equation (2) into (1) gives,

$$24 - 2t = t^2$$ **M1**

$$t^2 + 2t - 24 = 0$$

$$t = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-24)}}{2 \times 1}$$ **M1**

$$t = 4, \quad \text{or,} \quad t = -6$$

$$\text{but } t \neq -6, \quad \Longrightarrow t = 4\text{ s}$$ **A1**

05

3 (a).

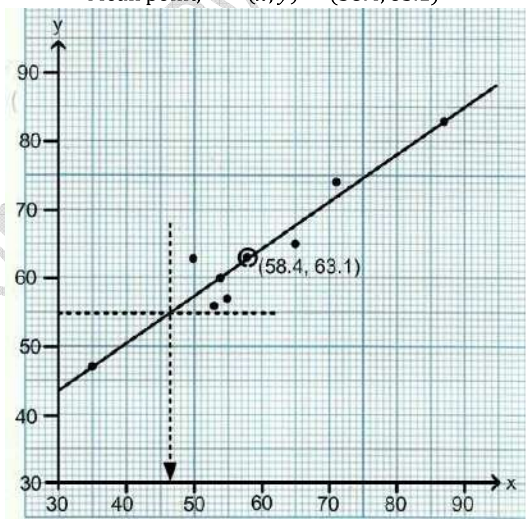
$$\overline{x} = \frac{54 + 35 + 62 + 87 + 53 + 71 + 50 + 55}{8}$$

$$= \frac{467}{8} = 58.375 \approx 58.4$$

$$\overline{y} = \frac{60 + 47 + 65 + 83 + 56 + 74 + 63 + 57}{8}$$

$$= \frac{505}{8} = 63.125 \approx 63.1$$

Mean point, $(\overline{x}, \overline{y}) = (58.4, 63.1)$ **B1**



B1-all points correctly plotted with correct scale and labeling axes

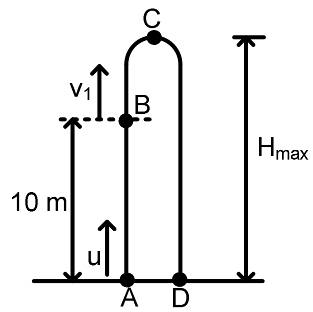
B1-line of best fit

Comment: There is a positive linear relationship between the Maths and English tests. **B1**

(b). When $y = 55, x = 46.5$. **A1**

05

4



For motion BCD, $s = -10\text{ m}, t = 8\text{ s}, u = v_1$

$$s = ut - \frac{1}{2}gt^2$$

$$-10 = 8v_1 - \frac{1}{2} \times 9.8 \times 8^2$$ **M1 M1**

$$-10 = 8v_1 - 313.6$$

$$v_1 = \frac{303.6}{8} = 37.95\text{ m s}^{-1}$$ **B1**

For motion AB, $s = 10\text{ m}, v = 37.95\text{ m s}^{-1}$

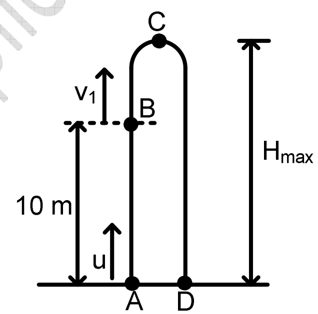
$$v^2 = u^2 - 2gs$$

$$37.95^2 = u^2 - 2 \times 9.8 \times 10$$ **M1**

$$u^2 = 1636.2025$$

$$u = 40.45\text{ m s}^{-1}$$ **A1**

Alternatively:



For motion BC, $u = v_1, v = 0, t = t_1$

$$v = u - gt$$

$$0 = v_1 - 9.8t_1$$

$$t_1 = \frac{v_1}{9.8}$$

For motion CD, $u = 0, v = u_1, t = t_2$

$$v = u + gt$$
$$u_1 = 0 - 9.8t_2$$
$$t_2 = \frac{u_1}{9.8}$$

B1-for both t_1 and t_2

but, $t_1 + t_2 = 8$

$$\frac{v_1}{9.8} + \frac{u_1}{9.8} = 8$$

M1-substitution

$$v_1 + u_1 = 78.4$$
$$v_1 = 78.4 - u_1$$

For motion AB, $u = u_1, v = v_1, s = 10\text{ m}$

$$v^2 = u^2 - 2gs$$
$${v_1}^2 = {u_1}^2 - 2 \times 9.8 \times 10$$
$$(78.4 - u_1)^2 = {u_1}^2 - 196$$

M1

$$6146.56 - 156.8u_1 + {u_1}^2 = {u_1}^2 - 196$$
$$6342.56 = 156.8u_1$$

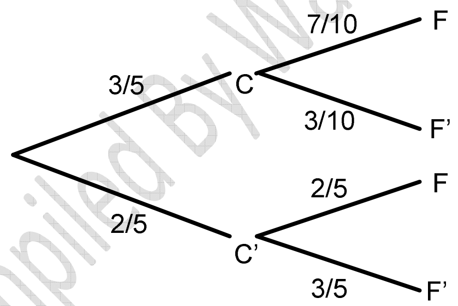
M1

$$u_1 = 40.45\text{ m s}^{-1}$$

A1

05

5 Let C denote event of a cloudy day and F denote event of catching fish.



B2-all correct tree diagram

$$P(C/F') = \frac{P(C \cap F')}{P(F')} = \frac{\left(\frac{3}{5} \times \frac{7}{10}\right)}{\left(\frac{3}{5} \times \frac{3}{10} + \frac{2}{5} \times \frac{3}{5}\right)}$$

M1 M1

$$= \frac{9}{80} \div \frac{21}{80} = \frac{3}{7} \approx 0.4286$$

A1

Alternatively:

$$P(F/C') = \frac{2}{5}, P(F/C) = \frac{7}{10}, P(C) = \frac{3}{5}$$
$$P(F) = P(F \cap C) + P(F \cap C')$$
$$= P(C).P(F/C) + P(C').P(F/C')$$

$$= \frac{3}{5} \times \frac{7}{10} + \left(1 - \frac{3}{5}\right) \times \frac{2}{5}$$

B1 B1

$$= 0.42 + 0.16 = \frac{29}{50} = 0.58$$

M1

$$P(C/F') = \frac{P(C \cap F')}{P(F')} = \frac{P(C) - P(C \cap F)}{1 - P(F)}$$
$$= \frac{0.6 - 0.42}{1 - 0.58} = \frac{3}{7} \approx 0.4286$$

M1 A1

05

6

$$\overrightarrow{\boldsymbol{AB}} = \begin{pmatrix} 130 \\ -110 \end{pmatrix} - \begin{pmatrix} 10 \\ 50 \end{pmatrix} = \begin{pmatrix} 120 \\ -160 \end{pmatrix}\text{ m}$$

M1 B1

$$|\underset{\sim}{\boldsymbol{v}}| = 100\text{ m s}^{-1}$$

$$\underset{\sim}{\boldsymbol{v}} = \frac{|\underset{\sim}{\boldsymbol{v}}|}{|\overrightarrow{\boldsymbol{AB}}|}.\overrightarrow{\boldsymbol{AB}} = \frac{100}{\sqrt{120^2 + (-160)^2}}.\begin{pmatrix} 120 \\ -160 \end{pmatrix}$$

M1 M1

$$= \begin{pmatrix} 60 \\ -80 \end{pmatrix}\text{ m s}^{-1}$$
$$\underset{\sim}{\boldsymbol{v}} = 60\underset{\sim}{\boldsymbol{i}} - 80\underset{\sim}{\boldsymbol{j}}\text{ m s}^{-1}$$

A1

05

7

x	$P(X = x)$	$xP(X = x)$	$x^2P(X = x)$
-1	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$
0	a	0	0
1	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{2}$
3	$\frac{3a}{2}$	$\frac{9a}{2}$	$\frac{27a}{2}$
Sums	$\frac{5}{8} + \frac{5a}{2}$	$\frac{3}{4} + \frac{9a}{2}$	$\frac{7}{4} + \frac{27a}{2}$

B1-$\sum x^2P(X = x)$

(a).

$$\sum_{all\ x} P(X = x) = 1$$
$$\frac{5}{8} + \frac{5a}{2} = 1$$
$$\frac{5a}{2} = 1 - \frac{5}{8}$$

M1-suming and equating to 1

$$\frac{5a}{2} = \frac{3}{8}$$
$$a = \frac{3}{8} \times \frac{2}{5} = \frac{3}{20} = 0.15$$

A1

(b).

$$E(X^2) = \sum_{all\ x} x^2 P(X = x) = \frac{7}{4} + \frac{27a}{2} = \frac{7}{4} + \frac{27}{2} \times \frac{3}{20}$$ **M1**

$$= \frac{151}{40} = 3.775$$ **A1**

05

8 (a).

$$\underset{\sim}{r}(t) = \underset{\sim}{r_0} + t\underset{\sim}{v} = \begin{pmatrix} 4 \\ 3 \\ 9 \end{pmatrix} + t\begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 4+3t \\ 3-2t \\ 9-5t \end{pmatrix} \text{ m}$$ **M1 A1**

(b).

$$\underset{\sim}{r}(t = 5) = \begin{pmatrix} 4+3\times 5 \\ 3-2\times 5 \\ 9-5\times 5 \end{pmatrix} = \begin{pmatrix} 19 \\ -7 \\ -16 \end{pmatrix} \text{ m}$$ **M1 M1 A1**

05

9 (a).

Marks	$C.F$	f	x	fx	fx^2	Class boundaries
<10	5	5	5	25	125	0 – 10
<20	9	4	15	60	900	10 – 20
<30	16	7	25	175	4375	20 – 30
<40	26	10	35	350	12250	30 – 40
<50	41	15	45	675	30375	40 – 50
<60	60	19	55	1045	57475	50 – 60
<70	77	17	65	1105	71825	60 – 70
<80	90	13	75	975	73125	70 – 80
<90	98	8	85	680	57800	80 – 90
<100	100	2	95	190	18050	90 – 100
Total		100		5280	326,300	

B1-for f

B1-for x

B1-for class boundary

(i).

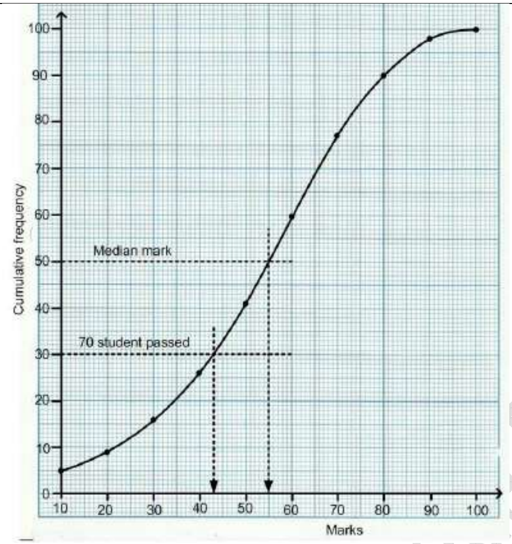
$$\text{Mean mark} = \frac{\sum fx}{\sum f} = \frac{5280}{100} = 52.8$$ **M1 A1**

(ii).

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{326300}{100} - \left(\frac{5280}{100}\right)^2} = \sqrt{475.16} \approx 21.7982$$ **M1 A1**

(b).



B1-plotting all points correctly and smooth ccurve

B1-labelling axes with uniform scale

(i).

$$\text{Median position} = \frac{1}{2}N = \frac{1}{2} \times 100 = 50$$

$$\Rightarrow \text{Median mark} = 55$$ **B1**

(ii).

$$\text{Pass mark} = 43$$ **B1**

12

10

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

B2-all correct

$$n(S) = 16$$

Let A = {even score}, B = {at least one die lands on three},

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{16} = 0.5$$ **M1 B1**

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{16} = 0.4375$$ **M1 B1**

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{16} = 0.1875$$ **M1 B1**

(a).

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.1875}{0.4375} = \frac{3}{7} \approx 0.4286$$ **M1 A1**

(b).

$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{0.1875}{0.5} = \frac{3}{8} = 0.375$$ **M1 A1**

12

11 (a).

$$\text{Price relative} = \frac{P_{2016}}{P_{2014}}$$

Item	Price relative	
Milk (per litre)	$=\frac{1300}{1000} = 1.3$	**B1**
Eggs (per tray)	$=\frac{8300}{6500} = 1.277$	**B1**
Sugar (per kg)	$=\frac{3800}{3000} = 1.267$	**B1**
Blue band	$=\frac{9000}{7000} = 1.286$	**B1**

Accept: Price relative $= \frac{P_{2016}}{P_{2014}} \times 100$

(b).

$$\text{Simple aggregate price index} = \frac{\sum P_{2016}}{\sum P_{2014}} \times 100$$

$$= \frac{1300 + 8300 + 3800 + 9000}{1000 + 6500 + 3000 + 7000} \times 100$$ **M1**

$$= \frac{22400}{17500} \times 100 = 128$$ **A1**

Accept: S. A. P. I $= \frac{\sum P_{2016}}{\sum P_{2014}}$

(c).

$$\text{Weighted aggregate price index} = \frac{\sum(P_{2016} \times W)}{\sum(P_{2014} \times W)} \times 100$$

$$= \frac{1300 \times 0.5 + 8300 \times 1 + 3800 \times 2 + 9000 \times 1}{1000 \times 0.5 + 6500 \times 1 + 3000 \times 2 + 7000 \times 1} \times 100$$ **M1**

$$= \frac{650 + 8300 + 7600 + 9000}{500 + 6500 + 6000 + 7000} \times 100$$

$$= \frac{25550}{20000} \times 100 = 127.75$$ **A1**

The prices increased by 27.75% between 2014 and 2016. **B1**

(d).

$$I = \frac{P_{2016}}{P_{2014}} \times 100$$

$$127.75 = \frac{45000}{P_{2014}} \times 100$$ **M1 B1**

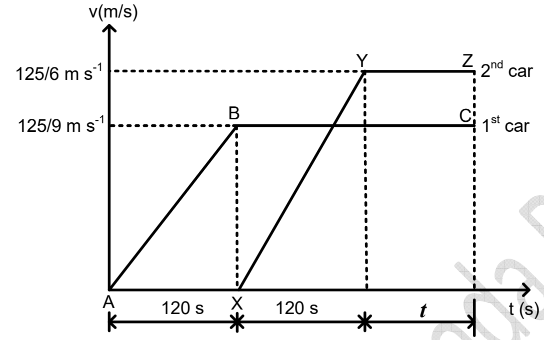
$$P_{2014} = \text{shs } 35225.049$$ **A1**

12

12 W

$$50 \text{ km h}^{-1} = \frac{50 \times 1000}{3600} = \frac{125}{9} \text{m s}^{-1}$$

$$75\text{km h}^{-1} = \frac{75 \times 1000}{3600} = \frac{125}{6} \text{m s}^{-1}$$



B1-1st car

B1-2nd car

B1-labelling axes

$$s_{ABC} = \frac{1}{2} \times \frac{125}{9}[(240 + t) + (120 + t)] = \frac{125}{18}[360 + 2t]$$ **B1**

$$s_{XYZ} = \frac{1}{2} \times \frac{125}{6}[(120 + t) + t] = \frac{125}{12}[120 + 2t]$$ **B1**

Both cars will cover the same distance. i.e.

$$s_{ABC} = s_{XYZ}$$

$$\frac{125}{18}[360 + 2t] = \frac{125}{12}[120 + 2t]$$ **M1**

$$2(360 + 2t) = 3(120 + 2t)$$

$$720 + 4t = 360 + 6t$$

$$360 = 2t$$

$$t = 180 \text{ s}$$ **A1**

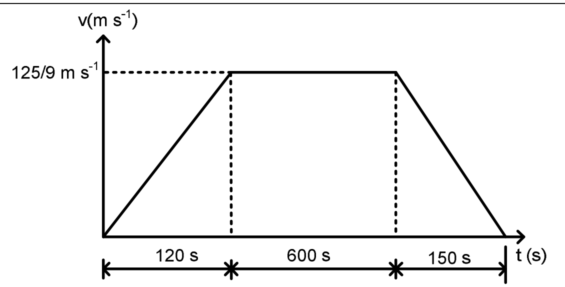
$$s_{XYZ} = \frac{125}{12}[120 + 2 \times 180] = 5000 \text{ m}$$ **M1 A1**

The second car overtook the first car at a spot 5000 metres (or 5 km) from the starting point and it took him 300 seconds (or 5 minutes) of travel.

(b).

$$10 \text{ minutes} = 10 \times 60 = 600 \text{ s}$$

$$2.5 \text{ minutes} = 2.5 \times 60 = 150 \text{ s}$$



B1

Total distance from start

$$= \frac{1}{2} \times \frac{125}{9}[(120 + 600 + 150) + 600]$$ **M1**

$$= 10208.333 \text{ m}$$ **A1**

12

13

x	$P(X=x)$	$P(X=x)$	$xP(X=x)$	$x^2P(X=x)$
1	$2c+k$	0.2	0.2	0.2
2	$2c+2k$	0.4	0.8	1.6
5	$5c+k$	0.2	1	5
7	$5c+0.1$	0.1	0.7	4.9
10	$6c+0.1$	0.1	1	10
Sum	$20c+4k+0.2$		3.7	21.7

B4 -(B1 for each of the last four columns)

(a).

$$P(X=1) = P(X=5)$$
$$2c + k = 5c + k$$ **M1**
$$3c = 0$$
$$c = 0$$ **B1**

but, $\displaystyle\sum_{all\,x} P(X=x) = 1$

$$20c + 4k + 0.2 = 1$$ **M1**
$$0 + 4k = 0.8$$
$$k = 0.2$$ **A1**

(b).

Mean, $\displaystyle E(X) = \sum_{all\,x} xP(X=x) = 3.7$ **B1**

Mode = 2 **B1**

(c).

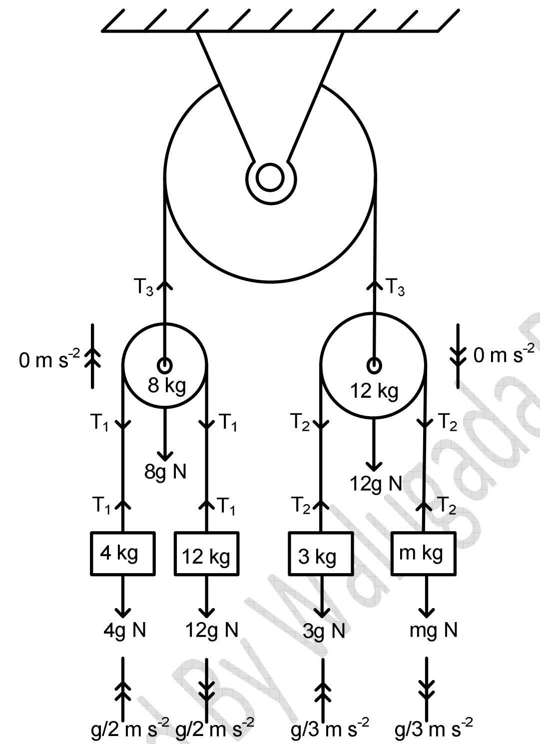
Variance, $\displaystyle Var(X) = \sum_{all\,x} x^2P(X=x) - [E(X)]^2$

$$= 21.7 - (3.7)^2 = 8.01$$

Standard deviation $= \sqrt{8.01} = 2.8302$ **M1 A1**

12

14



(a).

Obtaining T_1	**Alternatively**
For the 4 kg mass, $T_1 - 4g = 4 \times \frac{g}{2}$; $T_1 = 4g + 2g$; $T_1 = 6g = 6 \times 9.8$; $T_1 = 58.8$ N	For the 12 kg mass, $12g - T_1 = 12 \times \frac{g}{2}$; $T_1 = 12g - 6g$; $T_1 = 6g = 6 \times 9.8$; $T_1 = 58.8$ N

M1 M1

B1

S

Obtaining T_2	**Alternatively**
For the 3 kg mass, $T_2 - 3g = 3 \times \frac{g}{3}$; $T_2 = 3g + g$; $T_2 = 4g = 4 \times 9.8 = 39.2$ N	

M1 M1

B1

<table>
<tr><td></td><td>S
<table><tr><th>Obtaining T_3</th><th>Alternatively</th></tr><tr><td>For the 8 kg movable pulley,
$T_3 = 2T_1 + 8g$
$T_3 = 2 \times 58.8 + 8 \times 9.8$
$T_3 = 196$ N</td><td>For the 12 kg movable pulley,
$T_3 = 2T_2 + 12g$
$T_3 = 2 \times 39.2 + 12 \times 9.8$
$T_3 = 196$ N</td></tr></table></td><td>M1 M1

B1</td></tr>
<tr><td></td><td>(b). For the m kg mass,
$$mg - T_2 = m \times \frac{g}{3}$$
$$9.8m - 39.2 = \frac{9.8}{3}m$$
$$\frac{98}{15}m = 39.2$$
$$m = 39.2 \times \frac{15}{98} = 6 \text{ kg}$$</td><td>M1

M1

A1</td></tr>
<tr><td></td><td></td><td>12</td></tr>
</table>

END

P425/1
PURE
MATHEMATICS
PAPER 1
Nov 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 1 EOT 3 2018
Time: 3 Hours

NAME: ______________________________ **COMB:** ____________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B.**
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: In a triangle ABC, $a = 7$ cm, $b = 4$ cm and $c = 5$ cm. Find the value of:
(a). $\cos A$,
(b). $\sin A$. [5]

Qn 2: Determine the equation of the tangent to the curve $y^3 + y^2 - x^4 = 1$ at the point $(1, 1)$. [5]

Qn 3: Show that $2\log 4 + \frac{1}{2}\log 25 - \log 20 = 2\log 2$. [5]

Qn 4: The region bounded by the curve $y = x^2 - 2x$ and the x −axis from $x = 0$ to $x = 2$ is rotated about the x −axis. Calculate the volume of the solid formed. [5]

Qn 5: A point P moves such that its distances from two points $A(-2, 0)$ and $B(8, 6)$ are in the ratio $AP: PB = 3: 2$. Show that the locus of P is a circle. [5]

Qn 6: Express the function $f(x) = x^2 + 12x + 32$, in the form $a(x + b)^2 + c$. Hence find the minimum value of the function $f(x)$. [5]

Qn 7: Determine the term independent of x in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{20}$. [5]

Qn 8: Given that $(x+2)$ is a factor of $2x^3 + 6x^2 + bx - 5$, find the remainder when the expression is divided by $(2x-1)$. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). The first three terms of a Geometric progression (G.P) are 4, 8 and 16. Determine the sum of the first ten terms of the G.P. [4]

(b). An Arithmetic progression (A.P) has a common difference of 3. A Geometric progression (G.P) has a common ratio of 2.
A sequence is formed by subtracting the terms of the A.P from the corresponding terms of the G.P. The third term of the sequence is 4. The sixth term of the sequence is 79. Find the first term of the:
(i). A.P,
(ii). G.P. [8]

Question 10:

(a). Show that

$$\tan 4\theta = \frac{4t(1-t^2)}{t^4 - 6t^2 + 1}$$

where $t = \tan\theta$. [6]

(b). Solve the equation: $\sin x + \sin 5x = \sin 2x + \sin 4x$, for $0° < x < 180°$. [6]

Question 11:

(a). Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x. [6]

(b). The volume, V, of a sphere of radius r, is $\frac{4\pi r^3}{3}$ and surface area, A, is $4\pi r^2$. The volume is increased at a steady rate of 3 cm^3 s^{-1}. find:
(i). $\frac{dr}{dt}$,
(ii). Calculate the value of $\frac{dA}{dt}$ in cm^2 s^{-1} at the instant when radius is 12 cm. [6]

Question 12:

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Express $5 + 12i$ in polar form. Hence, evaluate $\sqrt[3]{(5+12i)}$, giving your answer in the form $a + ib$ where a and b are corrected to **two** decimal places.
[12]

Question 13:

(a). Given that $5^x \bullet 25^{2y} = 1$ and $3^{5x} \bullet 9^y = \frac{1}{9}$. Calculate the value of x and y.

(b). Solve the equation:

$$2^{2x+1} - 2^{x+1} + 1 = 2^x$$

[12]

Question 14:

α^2 and β^2 are the roots of $x^2 - 21x + 4 = 0$, and α and β are both positive, find:
(i). $\alpha\beta$,
(ii). $\alpha + \beta$,
(iii). the equation with roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [12]

Question 15:

(a). In the expansion of $(1 + ax)^n$, the first three terms are

$$1 - \frac{5}{2}x + \frac{75}{8}x^2.$$

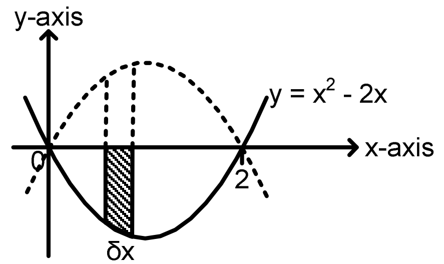
Find n and a, and state the range of values of x for which the expansion is valid.

(b). Expand $(1+x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^2; and hence find an approximation for $\sqrt{1.08}$. Deduce that $\sqrt{12} \approx 3.464$.
[12]

END

MARKING GUIDE

SNo.	Working	Marks
1	(a). $a^2 = b^2 + c^2 - 2bc\cos A$ $7^2 = 4^2 + 5^2 - 2 \times 4 \times 5\cos A$ $49 = 41 - 40\cos A$ $8 = -40\cos A$ $\cos A = -\frac{1}{5}$ (b). $\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(-\frac{1}{5}\right)^2} = \sqrt{\frac{24}{25}} = \frac{1}{5}\sqrt{24}$	M1 B1 A1 M1 A1
		05
2	$\frac{d}{dx}(y^3 + y^2 - x^4) = \frac{d}{dx}(1)$ $3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 4x^3 = 0$ $(3y^2 + 2y)\frac{dy}{dx} = 4x^3$ $\frac{dy}{dx} = \frac{4x^3}{3y^2 + 2y}$ At the point $(1, 1)$, Gradient of tangent $= \frac{4 \times 1^3}{3 \times 1^2 + 2 \times 1} = \frac{4}{5}$ The required equation of the tangent is given by: $\frac{y-1}{x-1} = \frac{4}{5}$ $y - 1 = \frac{4}{5}x - \frac{4}{5}$ $y = \frac{4}{5}x + \frac{1}{5}$	M1 B1 B1 M1 A1
		05
3	$2\log 4 + \frac{1}{2}\log 25 - \log 20$ $= \log 4^2 + \log\sqrt{25} - \log 20$ $= \log 16 + \log 5 - \log 20$ $= \log\left(\frac{16 \times 5}{20}\right)$ $= \log 4$ $= \log 2^2$ $= 2\log 2$	B1 B1 M1 B1 A1
		05
4		



SNo.	Working	Marks
	element of volume, $\delta v = \pi y^2 \delta x$ total volume, $v = \int_0^2 \pi y^2\, dx = \pi\int_0^2 (x^2 - 2x)^2\, dx$ $= \pi\int_0^2 (x^4 - 4x^3 + 4x^2)\, dx$ $= \pi\left[\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3\right]_0^2$ $= \pi\left(\frac{1}{5} \times 2^5 - 2^4 + \frac{4}{3} \times 2^3\right) - 0 = \frac{16}{15}\pi$ cubic units	M1 M1 M1 M1 A1
		05
5	Let the variable point be $P(x, y)$ $\overline{AP}:\overline{PB} = 3:2$ $2\overline{AP} = 3\overline{PB}$ $2\sqrt{(x+2)^2 + (y-0)^2} = 3\sqrt{(x-8)^2 + (y-6)^2}$ $4(x^2 + 4x + 4 + y^2)$ $= 9(x^2 - 16x + 64 + y^2 - 12y + 36)$ $4x^2 + 16x + 16 + 4y^2$ $= 9x^2 - 144x + 9y^2 - 108y + 900$ $5x^2 + 5y^2 - 160x - 108y + 884 = 0$ Since x^2 and y^2 have the same coefficients and the rest of the terms are linear, then the locus is a circle.	M1 M1 M1 B1 B1
		05
6	$f(x) = \left(x + \frac{12}{2}\right)^2 - \left(\frac{12}{2}\right)^2 + 32 = (x+6)^2 - 4$ Hence, $[f(x)]_{\min} = 0^2 - 4 = -4$	M1 M1 B1 M1 A1
		05
7	General term $= {}^{20}C_r \times (2x^3)^r\left(\frac{1}{x}\right)^{20-r}$ $= {}^{20}C_r \times 2^r \times x^{3r} \times x^{-(20-r)}$ For the term independent of x, $3r - (20 - r) = 0$ $4r - 20 = 0$ $4r = 20$ $r = 5$	M1 M1 B1

$$\text{Required term} = {}^{20}C_5 \times 2^5 = 15504 \times 32 = 496{,}128$$ **M1 A1**

05

8

$$\text{let } f(x) = 2x^3 + 6x^2 + bx - 5$$
$$\text{for } (x+2) = 0, \quad x = -2$$ **B1**
$$f(-2) = 2 \times (-2)^3 + 6 \times (-2)^2 + b \times (-2) - 5 = 0$$ **M1**
$$-16 + 24 - 2b - 5 = 0$$
$$2b = 3$$
$$b = 1.5$$ **B1**
$$\text{for } (2x-1) = 0, \quad x = 0.5$$
$$\text{Remainder} = f(0.5)$$
$$= 2 \times (0.5)^3 + 6 \times (0.5)^2 + 1.5 \times (0.5) - 5$$ **M1**
$$= 0.25 + 1.5 + 0.75 - 5$$
$$= -2.5$$ **A1**

05

9 (a).

$$a = 4, \quad r = \frac{8}{4} = 2$$ **B1 B1**
$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$
$$S_{10} = 4\left(\frac{2^{10} - 1}{2 - 1}\right) = 4092$$ **M1 A1**

(b).

Term	G.P	A.P	sequence
First term	a_1	a_2	$a_1 - a_2$
Third term	$a_1 \times 2^2 = 4a_1$	$a_2 + 2 \times 3 = a_2 + 6$	$4a_1 - a_2 - 6$
Sixth term	$a_1 \times 2^5 = 32a_1$	$a_2 + 5 \times 3 = a_2 + 15$	$32a_1 - a_2 - 15$

B1

B1

$$4a_1 - a_2 - 6 = 4, \quad \Rightarrow 4a_1 - a_2 = 10 \quad \longrightarrow (1)$$ **B1**
$$32a_1 - a_2 - 15 = 79, \quad \Rightarrow 32a_1 - a_2 = 94 \quad \longrightarrow (2)$$ **B1**

Equation (2) − (1) gives:

$$\begin{array}{r|l} & 32a_1 - a_2 = 94 \longrightarrow (2) \\ - & 4a_1 - a_2 = 10 \longrightarrow (1) \\ \hline & 28a_1 = 84 \end{array}$$ **M1**
$$a_1 = 3$$ **A1**

From equation (1),

$$4 \times 3 - a_2 = 10, \quad \Rightarrow a_2 = 2$$ **M1 A1**

(i). The first term of the A.P is 2.

(ii). The first term of the A.P is 3.

12

10 (a).

$$L.H.S = \tan 4\theta = \tan(2\theta + 2\theta) = \frac{\tan 2\theta + \tan 2\theta}{1 - \tan 2\theta \tan 2\theta}$$ **M1**

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$$= \frac{2\tan 2\theta}{1 - \tan^2 2\theta} = \frac{2\left(\frac{2t}{1-t^2}\right)}{1 - \left(\frac{2t}{1-t^2}\right)^2} = \frac{\left(\frac{4t}{1-t^2}\right)}{\left[\frac{(1-t^2)^2 - 4t^2}{(1-t^2)^2}\right]}$$ **B1 M1 B1**
$$= \frac{4t(1-t^2)}{1 - 2t^2 + t^4 - 4t^2} = \frac{4t(1-t^2)}{t^4 - 6t^2 + 1}, \quad \text{as required}$$ **B1 B1**

(b).

$$\sin x + \sin 5x = \sin 2x + \sin 4x$$
$$\sin 5x + \sin x = \sin 4x + \sin 2x$$
$$2\sin\left(\frac{5x + x}{2}\right)\cos\left(\frac{5x - x}{2}\right) = 2\sin\left(\frac{4x + 2x}{2}\right)\cos\left(\frac{4x - 2x}{2}\right)$$ **M1 M1**
$$\sin 3x \cos 2x = \sin 3x \cos x$$
$$\sin 3x \cos 2x - \sin 3x \cos x = 0$$
$$\sin 3x\,(\cos 2x - \cos x) = 0$$ **B1**
$$\sin 3x\left[-2\sin\left(\frac{2x + x}{2}\right)\sin\left(\frac{2x - x}{2}\right)\right] = 0$$ **M1**
$$-2\sin 3x \sin\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right) = 0$$
$$\sin 3x = 0, \quad \text{or}, \quad \sin\left(\frac{3x}{2}\right) = 0, \quad \text{or}, \quad \sin\left(\frac{x}{2}\right) = 0$$ **B1**
$$\text{for } \sin 3x = 0, \quad 3x = 0°, 180°, 360°, 540°,$$
$$\Rightarrow x = 0°, 60°, 120°, 180°$$
$$\text{for } \sin\left(\frac{3x}{2}\right) = 0, \quad \frac{3x}{2} = 0°, 180°, \quad \Rightarrow x = 0°, 120°$$
$$\text{for } \sin\left(\frac{x}{2}\right) = 0, \quad \frac{x}{2} = 0°, \quad \Rightarrow x = 0°$$

For the range $0° < x < 180°$,

$$x = 0°, 60°, 120°, 180°$$ **B1**

12

11 (a).

$$\text{let } y = \frac{x^3}{\sqrt{(1 - 2x^2)}}$$
$$u = x^3, \quad \Rightarrow \frac{du}{dx} = 3x^2$$ **B1**
$$v = (1 - 2x^2)^{\frac{1}{2}}, \quad \Rightarrow \frac{dv}{dx} = \frac{1}{2}(1 - 2x^2)^{-\frac{1}{2}} \times (-4x)$$
$$= -2x(1 - 2x^2)^{-\frac{1}{2}}$$ **B1**
$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$
$$= \frac{(1 - 2x^2)^{\frac{1}{2}} \times 3x^2 + x^3 \times 2x(1 - 2x^2)^{-\frac{1}{2}}}{(1 - 2x^2)}$$ **M1 M1**
$$= \frac{3x^2(1 - 2x^2)^{\frac{1}{2}} + 2x^4(1 - 2x^2)^{-\frac{1}{2}}}{(1 - 2x^2)}$$

	$= \dfrac{x^2(1-2x^2)^{-\frac{1}{2}}\,[3(1-2x^2)+2x^2]}{(1-2x^2)}$	B1
	$= \dfrac{x^2(3-4x^2)}{(1-2x^2)^{3/2}}$	A1
	(b). (i).	
	$V = \dfrac{4\pi r^3}{3}, \quad \Rightarrow \dfrac{dV}{dr} = 4\pi r^2$	M1
	$\dfrac{dr}{dt} = \dfrac{dr}{dV} \times \dfrac{dV}{dt} = \dfrac{1}{4\pi r^2} \times 3 = \dfrac{3}{4\pi r^2}$	M1 B1
	(ii).	
	$A = 4\pi r^2, \quad \Rightarrow \dfrac{dA}{dr} = 8\pi r$	M1
	$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt} = 8\pi r \times \dfrac{3}{4\pi r^2} = \dfrac{6}{r}$	B1
	At the instant when radius is 12 cm,	
	$\dfrac{dA}{dt} = \dfrac{6}{12} = 0.5 \text{ cm}^2\text{ s}^{-1}$	A1
		12
12	let $z = 5 + 12i$	
	$\lvert z\rvert = \sqrt{5^2 + 12^2} = 13$	B1
	$\arg(z) = \tan^{-1}\left(\dfrac{12}{5}\right) = 67.38°$	B1
	$\therefore z = 13(\cos 67.38° + i\sin 67.38°)$	B1
	For the hence part:	
	$\sqrt[3]{(5+12i)} = \sqrt[3]{z}$	
	$= 13^{\frac{1}{3}}\left[\cos\left(\dfrac{67.38+360n}{3}\right) + i\sin\left(\dfrac{67.38+360n}{3}\right)\right]$	
	$= 13^{\frac{1}{3}}[\cos(22.46+120n) + i\sin(22.46+120n)]$	B1
	When $n = 0$	
	$z_1 = 13^{\frac{1}{3}}[\cos 22.46° + i\sin 22.46°]$	
	$= 13^{\frac{1}{3}}[0.9241 + 0.3820i] = 2.17 + 0.90i$	M1 A1
	When $n = 1$	
	$z_2 = 13^{\frac{1}{3}}[\cos(22.46+120) + i\sin(22.46+120)]$	
	$= 13^{\frac{1}{3}}[\cos 142.46° + i\sin 142.46°]$	M1
	$= 13^{\frac{1}{3}}[-0.7929 + 0.6093i] = -1.86 + 1.43i$	B1 A1
	When $n = 2$	
	$z_2 = 13^{\frac{1}{3}}[\cos(22.46+240) + i\sin(22.46+240)]$	M1
	$= 13^{\frac{1}{3}}[\cos 262.46° + i\sin 262.46°]$	
	$= 13^{\frac{1}{3}}[-0.1312 - 0.9913i] = -0.03 - 2.33i$	B1 A1
		12

13	(a).	
	$5^x \bullet 25^{2y} = 1$	
	$5^x \times (5^2)^{2y} = 5^0$	M1
	$5^{x+4y} = 5^0$	
	$x + 4y = 0$	
	$x = -4y \quad \longrightarrow (1)$	B1
	$3^{5x} \bullet 9^y = \dfrac{1}{9}$	
	$3^{5x} \times (3^2)^y = 3^{-2}$	M1
	$3^{5x+2y} = 3^{-2}$	
	$5x + 2y = -2 \quad \longrightarrow (2)$	B1
	Substituting equation (1) into (2) gives,	
	$5 \times (-4y) + 2y = -2$	M1
	$-18y = -2$	
	$y = \dfrac{2}{18} = \dfrac{1}{9}$	A1
	From equation (2),	
	$x = -4 \times \dfrac{1}{9} = -\dfrac{4}{9}$	A1
	(b).	
	$2^{2x+1} - 2^{x+1} + 1 = 2^x$	
	$2^1 \times (2^x)^2 - 2^1 \times 2^x + 1 = 2^x$	M1
	let $y = 2^x$	
	$2y^2 - 2y + 1 = y$	B1
	$2y^2 - 3y + 1 = 0$	
	$2y^2 - 2y - y + 1 = 0$	
	$2y(y-1) - (y-1) = 0$	M1
	$(2y-1)(y-1) = 0$	
	$y = 0.5, \quad$ or, $\quad y = 1$	
	but $y = 2^x$	
	$\therefore$ for $y = 0.5, \quad 2^x = 2^{-1}, \quad \Rightarrow x = -1$	A1
	$\therefore$ for $y = 1, \quad 2^x = 2^0, \quad \Rightarrow x = 0$	A1
		12
14	(i).	
	$x^2 - 21x + 4 = 0$	
	sum of roots, $\quad \alpha^2 + \beta^2 = 21$	B1
	product of roots, $\quad \alpha^2\beta^2 = 4$	B1
	$\Rightarrow \alpha\beta = \sqrt{\alpha^2\beta^2} = \sqrt{4} = 2$	B1
	(ii).	
	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	
	$21 = (\alpha+\beta)^2 - 2 \times 2$	M1
	$(\alpha+\beta)^2 = 25$	B1
	$\Rightarrow \alpha + \beta = \sqrt{25} = 5$	M1 A1
	(iii).	
	sum of roots $= \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{\beta^2+\alpha^2}{\alpha^2\beta^2} = \dfrac{21}{4}$	M1 B1

	$$\text{product of roots} = \frac{1}{\alpha^2}\times\frac{1}{\beta^2} = \frac{1}{\alpha^2\beta^2} = \frac{1}{4}$$	M1 B1
	The required equation is	
	$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$	
	$$x^2 - \frac{21}{4}x + \frac{1}{4} = 0$$	A1
	$$4x^2 - 21x + 1 = 0$$	
		12
15	(a).	
	$$(1+ax)^n \approx 1 + n(ax) + \frac{n(n-1)}{2!}\times(ax)^2 + \cdots$$	
	$$\approx 1 + nax + \frac{n(n-1)a^2x^2}{2} + \cdots$$	B1
	By comparison,	
	$$an = -\frac{5}{2}, \quad \Rightarrow a = \frac{-5}{2n} \quad \longrightarrow (1)$$	B1
	$$\frac{1}{2}n(n-1)a^2 = \frac{75}{8} \quad \longrightarrow (2)$$	B1
	Substituting equation (1) into (2) gives:	
	$$\frac{1}{2}n(n-1)\left(\frac{-5}{2n}\right)^2 = \frac{75}{8}$$	M1
	$$\frac{1}{2}n(n-1)\times\frac{25}{4n^2} = \frac{75}{8}$$	
	$$\frac{25}{8n}(n-1) = \frac{75}{8}$$	
	$$(n-1) = 3n$$	
	$$2n = -1$$	
	$$n = -0.5$$	A1
	From equation (1),	
	$$a = \frac{-5}{2\times(-0.5)} = 5$$	A1
	The expansion is valid for $\lvert x\rvert < \frac{1}{5}$.	
	(b).	
	$$(1+x)^{\frac{1}{2}} \approx 1 + \frac{1}{2}x + \frac{1}{2}\times\frac{-1}{2}\times\frac{x^2}{2!} + \cdots$$	
	$$\approx 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$$	B1
	For the hence part:	
	$$\sqrt{1.08} = (1+0.08)^{\frac{1}{2}} \approx 1 + \frac{1}{2}\times 0.08 - \frac{1}{8}\times(0.08)^2 = 1.0392$$	M1 A1
	For the deducing part:	
	$$\sqrt{1.08} = \sqrt{\frac{27}{25}} = \sqrt{\frac{9\times 3}{25}} = \frac{3}{5}\sqrt{3}$$	M1
	$$\text{but } \sqrt{1.08} \approx 1.0392$$	
	$$\frac{3}{5}\sqrt{3} \approx 1.0392$$	

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	$$\sqrt{3} \approx 1.0392\times\frac{5}{3}$$	
	$$\sqrt{3} \approx 1.732$$	B1
	$$\therefore \sqrt{12} = \sqrt{4\times 3} = 2\sqrt{3} \approx 2\times 1.732$$	B1
	$$\approx 3.464, \qquad \text{as required}$$	
		12

END

Compiled By Walugada Ronald

P425/2
APPLIED
MATHEMATICS
PAPER 2
Nov 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.5 MATH 2 EOT 3 2018
Time: 3 Hours

NAME:____________________________________ **COMB:**__________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: The sizes of shoes sold in a certain shop in a given week are shown in the table below:

Size	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0
Number of pairs of shoes sold	4	9	11	8	10	7	2	3

Find the:

(i). Mean, [2]

(ii). Standard deviation of the sizes of shoes sold. [3]

Qn 2: Events X and Y are such that $3P(X \cap Y) = 2P(X' \cap Y) = P(X' \cap Y') = a$ and $P(X) = \frac{3}{5}$. Use a venn diagram to find:

(i). the value of a.

(ii). $P(X' \cup Y)$. [5]

Qn 3: The table below gives values of x and the corresponding values of $f(x)$.

x	0.1	0.2	0.3	0.4	0.5	0.7
$f(x)$	4.21	3.83	3.25	2.85	2.25	1.43

Use linear interpolation/extrapolation to find:

(a). $f(x)$ when $x = 0.6$. [3]

(b). the value of x when $f(x) = 0.75$. [2]

Qn 4: A ball is projected vertically upwards and it returns to its point of projection 3 seconds later. Find the:

(a). speed with which the ball was projected.

(b). greatest height reached. [5]

Qn 5: The table below shows the grades scored by 8 candidates in Applied Maths and overall score in mathematics on Mocks of a certain school.

Applied Maths	D1	D2	C4	C6	C6	F9	C5	C6
Overall grade	A	C	B	D	E	F	C	O

(a). Calculate the rank correlation coefficient for the data.

(b). Test whether it is significant at 5% level. [5]

Qn 6: A continuous random variable has a p.d.f given as:

$$f(x) = \begin{cases} k(x+3) & ; \quad -1 < x \leq 1, \\ k(5-x) & ; \quad 1 < x \leq 3, \\ 0 & ; \quad \text{elsewhere.} \end{cases}$$

Sketch $f(y)$ and hence find the value of k. [5]

Qn 7: The table below shows the wages of workers in thousands of shillings.

Category	Monthly wage		Number of workers
	2016	2017	
1	1200	1920	180
2	1500	2850	165
3	1650	3300	100
4	1700	4250	55

(a). Calculate the weighted aggregate index number for the monthly wage in 2017.

(b). Comment on your results in (a) above. [5]

Qn 8: A particle moves in the $x - y$ plane such that its position vector at any time t is given by $\underset{\sim}{r} = (3t^2 - 1)\underset{\sim}{i} + (4t^3 + t - 1)\underset{\sim}{j}$. Find

(a). Its speed,
(b). The magnitude of acceleration,
after $t = 2$. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

The table below shows the percentage of sand, y, in the soil at different depths, x (in cm).

Soil depth (x)	35	65	55	25	45	75	20	90	51	60
Percentage of sand (y)	86	70	84	92	79	68	96	58	86	77

(a). (i). Calculate the rank correlation coefficient between the two variables.
(ii). Comment on the significance at 5% level. [5]
(b). (i). Draw a scatter diagram for the data and comment on our result.
(ii). Draw the line of best fit; hence estimate the:
- percentage of sand in the soil at a depth of 31 cm.
- depth of the soil with 54% sand. [7]

Question 10:

(a). The resultant of the forces $F_1 = \begin{pmatrix} 3 \\ a - c \end{pmatrix}$, $F_2 = \begin{pmatrix} 2a + 3c \\ 5 \end{pmatrix}$, $F_3 = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ acting on a particle is $\begin{pmatrix} 10 \\ 12 \end{pmatrix}$. Find the:
(i). Values of a and c.
(ii). Magnitude of force F_2. [5]
(b). Five forces of magnitudes 3 N, 4 N, 4 N, 3 N and 5 N act along the lines AB, BC, CD, DA and AC respectively, of a square ABCD of side 1 m. the direction of the forces is given by the order of the letters. Taking AB and AD as reference axes, find the magnitude and direction of the resultant force. [7]

Question 11:

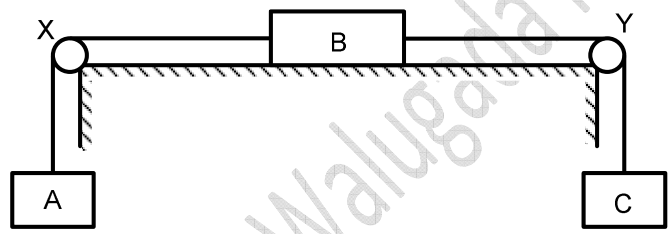
The probability density function of a continuous random variable X is given as

$$f(x) = \begin{cases} \frac{2}{13}(x + 1) & ; \quad 0 \le x \le 2, \\ \frac{2}{13}(5 - x) & ; \quad 2 \le x \le 3, \\ 0 & ; \quad \text{elsewhere.} \end{cases}$$

(a). Calculate the:
(i). $P(X < 2.5)$, [3]
(ii). Mean of X. [3]
(b). Determine the cumulative distribution function, $F(x)$. [6]

Question 12:

The diagram below shows masses A, B and C of masses 3, 4 and 6 kg, respectively connected by light strings which pass over smooth pulleys X and Y. Mass B rests on a horizontal rough table; the coefficient of friction between the table and the mass B being 0.5.



The system is released from rest.

(a). Determine the:
(i). acceleration of the masses,
(ii). tensions in the strings,
(b). After mass C has dropped through a distance of 2 m, the string connecting it to mass B snaps. Determine the time and velocity at which this occurs. [12]

Question 13:

(a). Use the trapezium rule to estimate $\int_0^1 (3x + 5)\, dx$, using 5 sub-intervals. Give your answer correct to 2 decimal places.
(b). Find the exact value of $\int_0^1 (3x + 5)\, dx$.
(c). (i). Determine the percentage error in the two calculations in (a) and (b) above.
(ii). State how the percentage error in (c)(i) can be reduced. [12]

Question 14:
The table below is an extract from the table of $\sin x$.

x	0.1	0. 2	0.3	0.4	0.5
$\sin x$	0.0998	0.1987	0.2955	0.3894	0.4794

Use linear interpolation to find:
(i). $\sin 0.29$,
(ii). $\sin 0.52$,
(iii). $\sin^{-1}(0.2598)$,
(iv). $\sin^{-1}(0.4900)$. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1		

Size (x)	f	fx	fx^2
7.5	4	30	225
8.0	9	72	576
8.5	11	93.5	794.75
9.0	8	72	648
9.5	10	95	902.5
10.0	7	70	700
10.5	2	21	220.5
11.0	3	33	363
Total	54	486.5	4429.75

B1-for $\sum fx$

B1-for $\sum fx^2$

(i).

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{486.5}{54} = 9.0093$$

B1

(ii).

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{4429.75}{54} - \left(\frac{486.5}{54}\right)^2} = 0.9304$$

M1 A1

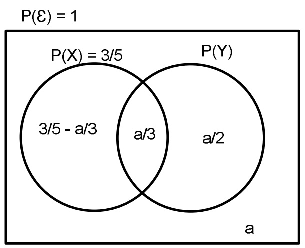
05

2

$$P(X \cap Y) = \frac{a}{3}, \quad P(X' \cap Y) = \frac{a}{2}, \quad P(X' \cap Y') = a,$$

$$P(X) = \frac{3}{5}$$

(i).



B1

(ii).

$$P(\varepsilon) = \frac{3}{5} + \frac{a}{2} + a = 1$$ **M1**

$$1.5a = 0.4, \quad \Rightarrow a = \frac{4}{15}$$ **A1**

$$P(X' \cup Y) = \frac{a}{3} + \frac{a}{2} + a = \frac{11}{6}a = \frac{11}{6} \times \frac{4}{15} = \frac{22}{45}$$ **M1 A1**

05

3 (a).

x	0.5	0.6	0.7
$f(x)$	2.25	y_1	1.43

$$\frac{y_1 - 2.25}{1.43 - 2.25} = \frac{0.6 - 0.5}{0.7 - 0.5}$$ **M1**

$$y_1 = 0.5 \times (-0.82) + 2.25 = 1.84$$ **A1**

When $x = 0.6$, $f(x) = 1.84$.

(b).

x	0.5	0.7	x_2
$f(x)$	2.25	1.43	0.75

B1

$$\frac{x_2 - 0.5}{0.7 - 0.5} = \frac{0.75 - 2.25}{1.43 - 2.25}$$ **M1**

$$x_2 = \frac{75}{41} \times 0.2 + 0.5 = \frac{71}{82} \approx 0.866$$ **A1**

When $f(x) = 0.75$, $x = 0.866$.

05

4 (a). $t = 3$ s

$$s = ut - \frac{1}{2}gt^2$$

$$0 = 3u - \frac{1}{2} \times 9.8 \times 3^2$$ **M1 B1** (B1 is for equating to zero)

$$0 = 3u - 44.1$$

$$u = \frac{44.1}{3} = 14.7 \text{ m s}^{-1}$$ **A1**

(b).

$$v^2 = u^2 - 2gs$$

$$0 = 14.7^2 - 2 \times 9.8 \times s$$ **M1**

$$0 = 216.09 - 19.6s$$

$$s = \frac{216.09}{19.6} = 11.025 \text{ m}$$ **A1**

05

5 (a).

x	y	R_x	R_y	d	d^2
D1	A	1	1	0	0
D2	C	2	3.5	-1.5	2.25
C4	B	3	2	1	1
C6	D	6	5	1	1
C6	E	6	6	0	0
F9	F	8	8	0	0
C5	C	4	3.5	0.5	0.25
C6	O	6	7	-1	1
					$\sum d^2 = 5.5$

B1-for all values of d^2

B1-for $\sum d^2$

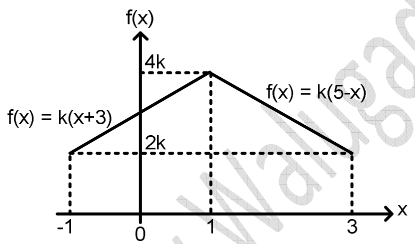
$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 5.5}{8(8^2-1)} = 1 - 0.0655 = 0.9345$$ **M1 A1**

Comment: Significant at 5%. **B1**

05

6

$$f(-1) = k(-1+3) = 2k$$
$$f(1) = k(1+3) = 4k$$
$$f(3) = k(5-3) = 2k$$ **B1**



B1-for line f(x)=k(x+3)

B1-for line f(x)=k(5-x)

$$\text{Total area} = 4 \times 2k + \frac{1}{2} \times 4 \times 2k = 1$$ **M1**

$$8k + 4k = 1$$

$$12k = 1, \quad \Rightarrow k = \frac{1}{12}$$ **A1**

05

7 (a).

$$\text{Weighted aggregate index number} = \frac{\sum P_{2017}}{\sum P_{2016}} \times 100$$

$$= \frac{1920 \times 180 + 2850 \times 165 + 3300 \times 100 + 4250 \times 55}{1200 \times 180 + 1500 \times 165 + 1650 \times 100 + 1700 \times 55} \times 100$$ **B1 B1 M1**

$$= \frac{1{,}379{,}600}{722{,}000} \times 100 = 191.0803$$ **A1**

(b).

Percentage increase $= 191.0803 - 100 = 91.0803$

The wages increased by 91.0803% between 2016 and 2017. **B1**

05

8 (i).

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$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = \begin{pmatrix} 6t \\ 12t^2 + 1 \end{pmatrix}$$ **M1**

when $t = 2$, $\underset{\sim}{v} = \begin{pmatrix} 6 \times 2 \\ 12 \times 2^2 + 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 49 \end{pmatrix}$ **B1**-mobile

speed, $\left|\underset{\sim}{v}\right| = \sqrt{12^2 + 49^2} = \sqrt{2545} = 50.4480 \text{ m s}^{-1}$ **A1**

(ii).

$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = \begin{pmatrix} 6 \\ 24t \end{pmatrix}$$

when $t = 2$, $\underset{\sim}{a} = \begin{pmatrix} 6 \\ 24 \times 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 48 \end{pmatrix}$ **M1**

magnitude of acceleration,

$$\left|\underset{\sim}{a}\right| = \sqrt{6^2 + 48^2} = \sqrt{2340} = 48.3735 \text{ m s}^{-2}$$ **A1**

05

9 (a).

x	y	R_x	R_y	d	d^2
35	86	3	7.5	-4.5	20.25
65	70	8	3	5	25
55	84	6	6	0	0
25	92	2	9	-7	49
45	79	4	5	-1	1
75	68	9	2	7	49
20	96	1	10	-9	81
90	58	10	1	9	81
51	86	5	7.5	-2.5	6.25
60	77	7	4	3	9
$\sum x$ =521	$\sum y$ =796				$\sum d^2$ =321.5

B1-for all values of d^2

B1-for $\sum d^2$

(a). (i).

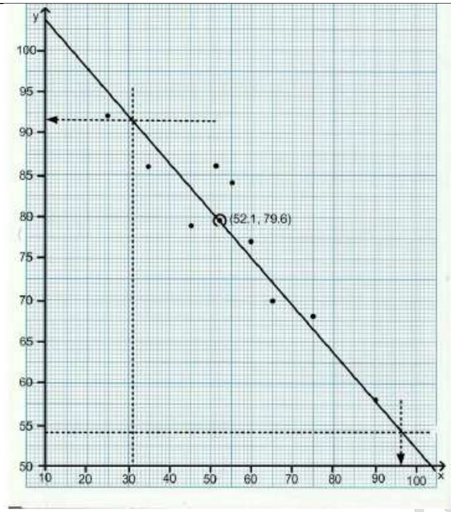
$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 321.5}{10(10^2-1)} = -0.9485$$ **M1 A1**

(ii). Significant at 5%. **B1**

(b). (i).

$$\bar{x} = \frac{\sum x}{n} = \frac{521}{10} = 52.1, \qquad \bar{y} = \frac{\sum y}{n} = \frac{796}{10} = 79.6,$$

$$\Rightarrow (\bar{x}, \bar{y}) = (52.1, 79.6)$$ **B1**



B1-all correctly plotted points

B1-line of best fit

B1-uniform scale and labeling on both axes

Negative linear relationship. **B1**

(ii).

- When $x = 31$ cm, $y = 91.5\%$. **B1**
- When $y = 54\%$, $x = 96.5$ cm. **B1**

12

10 (a).

$$F = \begin{pmatrix} 3 \\ a-c \end{pmatrix} + \begin{pmatrix} 2a+3c \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} 7 + 2a + 3c \\ 11 + a - c \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \end{pmatrix}$$ **M1**

$$7 + 2a + 3c = 10, \quad \Rightarrow 2a + 3c = 3 \quad \longrightarrow (1)$$

$$11 + a - c = 12, \quad \Rightarrow a = c + 1 \quad \longrightarrow (2)$$

Substituting (2) into (1) gives:

$$2(c+1) + 3c = 3$$

$$2c + 2 + 3c = 3$$ **M1**

$$5c = 1$$

$$c = \frac{1}{5}$$

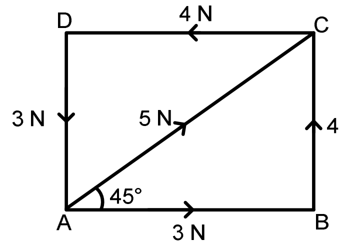
From equation (2), **A1**

$$a = \frac{1}{5} + 1 = \frac{6}{5} = 1.2$$ **A1**

$$F_2 = \begin{pmatrix} 2 \times 1.2 + 3 \times 0.2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$ **B1**

$$|F_2| = \sqrt{3^2 + 5^2} = \sqrt{34} \approx 5.8310 \text{ N}$$ **B1**

(b).

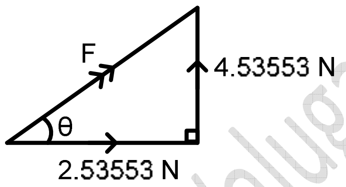


B1-sketch

$$F = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}0\\4\end{pmatrix} + \begin{pmatrix}-4\\0\end{pmatrix} + \begin{pmatrix}0\\-3\end{pmatrix} + \begin{pmatrix}5\cos 45°\\5\sin 45°\end{pmatrix}$$ **M1**

$$= \begin{pmatrix}-1+2.5\sqrt{2}\\1+2.5\sqrt{2}\end{pmatrix} = \begin{pmatrix}2.53553\\4.53553\end{pmatrix}\text{ N}$$ **B1**

Magnitude, $|F| = \sqrt{(2.53553)^2 + (4.53553)^2} \approx 5.19615\text{ N}$ **M1 A1**



Direction, $\theta = \tan^{-1}\left(\frac{4.53553}{2.53553}\right)$

$= 60.7932°$ above the positve horizontal **B1**

12

11 (a). (i).

$$P(X < 2.5) = \int_0^2 \frac{2}{13}(x+1)\,dx + \int_2^{2.5} \frac{2}{13}(5-x)\,dx$$

$$= \frac{2}{13}\left[\frac{1}{2}x^2 + x\right]_0^2 + \frac{2}{13}\left[5x - \frac{1}{2}x^2\right]_2^{2.5}$$ **M1**

$$= \frac{2}{13}\left[\left(\frac{1}{2}\times 2^2 + 2\right) + \left(5\times 2.5 - \frac{1}{2}\times(2.5)^2\right) - \left(5\times 2 - \frac{1}{2}\times 2^2\right)\right]$$ **M1**

$$\frac{2}{13}\left(4 + \frac{75}{8} - 8\right)$$

$$= \frac{2}{13}\times\frac{43}{8} = \frac{43}{52} \approx 0.8269$$ **A1**

(ii).

$$E(X) = \int_{all\ x} xf(x)\,dx$$

$$= \int_0^2 \frac{2}{13}(x^2 + x)\,dx + \int_2^3 \frac{2}{13}(5x - x^2)\,dx$$

$$= \frac{2}{13}\left[\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_0^2 + \frac{2}{13}\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_2^3$$ **M1**

$$= \frac{2}{13}\left[\left(\frac{1}{3}\times 2^3 + \frac{1}{2}\times 2^2\right) + \left(\frac{5}{2}\times 3^2 - \frac{1}{3}\times 3^3\right) - \left(\frac{5}{2}\times 2^2 - \frac{1}{3}\times 2^3\right)\right]$$ **M1**

$$= \frac{2}{13}\left(\frac{14}{3} + \frac{27}{2} - \frac{22}{3}\right)$$

$$= \frac{2}{13}\times\frac{65}{6} = \frac{5}{3} \approx 1.6667$$ **A1**

(b).

For $x < 0$,

$$F(x) = 0$$
$$F(0) = 0$$

For $0 \le x \le 2$,

$$F(x) = 0 + \int_0^x \frac{2}{13}(t+1)\,dt = \frac{2}{13}\left[\frac{1}{2}t^2 + t\right]_0^x = \frac{2}{13}\left(\frac{1}{2}x^2 + x\right)$$ **B1**

$$F(2) = \frac{2}{13}\left(\frac{1}{2}\times 2^2 + 2\right) = \frac{8}{13}$$ **B1**

For $2 \le x \le 3$

$$F(x) = \frac{8}{13} + \int_2^x \frac{2}{13}(5-t)\,dt = \frac{8}{13} + \frac{2}{13}\left[5t - \frac{1}{2}t^2\right]_2^x$$ **M1**

$$= \frac{8}{13} + \frac{2}{13}\left[\left(5x - \frac{1}{2}x^2\right) - \left(5\times 2 - \frac{1}{2}\times 2^2\right)\right]$$ **M1**

$$= \frac{8}{13} + \frac{2}{13}\left(5x - \frac{1}{2}x^2 - 8\right)$$

$$= \frac{2}{13}(8 + 10x - x^2 - 16)$$

$$= \frac{1}{13}(10x - x^2 - 8)$$ **B1**

$$F(x) = \frac{1}{13}(10\times 3 - 3^2 - 8) = 1$$

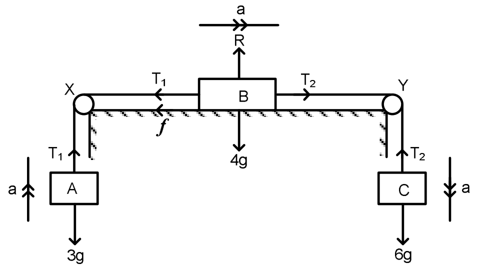
For $x > 3$

$$F(x) = 1$$

$$\therefore F(x) = \begin{cases} 0 & ;\quad x < 0, \\ \frac{2}{13}\left(\frac{1}{2}x^2 + x\right) & ;\quad 0 \le x \le 2, \\ \frac{1}{13}(10x - x^2 - 8) & ;\quad 2 \le x \le 3, \\ 1 & ;\quad x > 3. \end{cases}$$ **B1**

12

12 (a). (i).



Friction, $f = \mu R = 0.5 \times 4g = 0.5 \times 4 \times 9.8 = 19.6\text{ N}$ **B1**

For the 4 kg mass gives,

$$T_2 - (T_1 + f) = 4a$$

$$T_2 - T_1 - 19.6 = 4a \quad \longrightarrow (1)$$ **B1**

For the 3 kg mass gives,

$$T_1 - 3g = 3a$$

$$T_1 - 3 \times 9.8 = 3a$$

$$T_1 - 29.4 = 3a \quad \longrightarrow (2)$$ **B1**

For the 6 kg mass gives,

$$6g - T_2 = 6a$$

$$6 \times 9.8 - T_2 = 3a$$

$$58.8 - T_2 = 6a \quad \longrightarrow (3)$$ **B1**

Equation (2) + (3) gives;

$$\begin{array}{r|c|l} & T_1 - 29.4 = 3a & \longrightarrow (2) \\ + & 58.8 - T_2 = 6a & \longrightarrow (3) \\ \hline & T_1 - T_2 + 29.4 = 9a & \longrightarrow (4) \end{array}$$

Equation (1) + (4) gives;

$$\begin{array}{r|c|l} & T_2 - T_1 - 19.6 = 4a & \longrightarrow (1) \\ + & T_1 - T_2 + 29.4 = 9a & \longrightarrow (2) \\ \hline & 9.8 = 13a & \longrightarrow (3) \end{array}$$ **M1**

$$\Rightarrow a = \frac{9.8}{13} = 0.7538\text{ m s}^{-2}$$ **A1**

(ii). From equation (2),

$$T_1 = 29.4 + 3a = 29.4 + 3 \times \frac{9.8}{13} = 31.662\text{ N}$$ **B1**

From equation (3),

$$T_2 = 58.8 - 6a = 58.8 - 6 \times \frac{9.8}{13} = 54.277\text{ N}$$ **B1**

(b).

$$u = 0\text{ m s}^{-2}, \quad a = 0.7538\text{ m s}^{-2}, \quad s = 2\text{ m}$$

$$v = \sqrt{u^2 + 2as} = \sqrt{0 + 2 \times \frac{9.8}{13} \times 2} = \sqrt{\frac{196}{65}} \approx 1.736\text{ m s}^{-1}$$ **M1 A1**

$$t = \frac{v - u}{a} = \frac{1.736 - 0}{0.7538} = 2.303\text{ s}$$ **M1 A1**

12

13 (a).

$$y_n = (3x_n + 5), \qquad h = \frac{1 - 0}{5} = \frac{1}{5} = 0.2$$ **B1**

n	x_n	y_0, y_5	$y_1, \dots y_4$
0	0	5	
1	0.2		5.6
2	0.4		6.2
3	0.6		6.8
4	0.8		7.4
5	1	8	
sums		13	26

B2-all y-values correct

$$\int_0^1 (3x + 5)\,dx \approx \frac{1}{2}h[(y_0 + y_4) + 2(y_1 + \cdots + y_3)]$$

$$\approx \frac{1}{2} \times \frac{1}{5}[13 + 2 \times 26] = 6.50\ (2\text{ d.p})$$ **M1 A1**

(b).

$$\int_0^1 (3x + 5)\,dx = \left[\frac{3}{2}x^2 + 5x\right]_0^1 = \left(\frac{3}{2} + 5\right) - 0$$ **M1 M1**

$$= \frac{13}{2} \approx 6.50\ (2\text{ d.p})$$ **A1**

(c). (i).

Absolute error $= |6.50 - 6.50| = 0$ **B1**

Percentage error $= \dfrac{0}{6.50} \times 100 = 0.00\ (2\text{ d.p})$ **M1 A1**

(ii). The percentage error in (c)(i) is zero and can't be reduced farther. **B1**

12

14 (i).

x	0.2	0.29	0.3
$\sin x$	0.1987	y_1	0.2955

$$\frac{y_1 - 0.1987}{0.2955 - 0.1987} = \frac{0.29 - 0.2}{0.3 - 0.2}$$ **M1 M1**

$$y_1 = \frac{0.09}{0.1} \times 0.0968 + 0.1987 = 0.28582$$ **A1**

$$\therefore \sin 0.29 = 0.28582$$

(ii).

x	0.4	0.5	0.52
$\sin x$	0.3894	0.4794	y_2

$$\frac{y_2 - 0.3894}{0.4794 - 0.3894} = \frac{0.52 - 0.4}{0.5 - 0.4}$$ **M1 M1**

$$y_2 = \frac{0.12}{0.1} \times 0.09 + 0.3894 = 0.4974$$ **A1**

$$\therefore \sin 0.52 = 0.4974$$

(iii).

x	0. 2	x_1	0.3
$\sin x$	0.1987	0.2598	0.2955

$$\frac{x_1 - 0.2}{0.3 - 0.2} = \frac{0.2598 - 0.1987}{0.2955 - 0.1987}$$ **M1 M1**

$$x_1 = \frac{0.0611}{0.0968} \times 0.1 + 0.2 = 0.2631$$ **A1**

$$\therefore \sin^{-1}(0.2598) = 0.2631$$

(iv).

x	0.4	0.5	x_2
$\sin x$	0.3894	0.4794	0.4900

$$\frac{x_2 - 0.4}{0.5 - 0.4} = \frac{0.4900 - 0.3894}{0.4794 - 0.3894}$$ **M1 M1**

$$x_2 = \frac{0.1006}{0.09} \times 0.1 + 0.4 = 0.5118$$ **A1**

$$\therefore \sin^{-1}(0.4900) = 0.5118$$

12

END

P425/1
PURE MATHEMATICS
PAPER 1
Feb 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 1 MOCK SET 1 2018
Time: 3 Hours

NAME: ______________________ **COMB:** ________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B.**
- Show your working clearly.

Section A (40 Marks)

*Answer **all** the questions in this section.*

Qn1: Solve for the value of x: $\sqrt{3x+1} + \sqrt{4x+5} = \sqrt{16x+9}$. [5]

Qn2: Find the Cartesian equation of the locus $Z$of $|Z - 2 + i| = 1$. [5]

Qn3: A cable 10 m long is divided into ten pieces whose lengths are in a G.P. the length of the longest piece is 8 times the length of the shortest piece. Calculate to the nearest centimeter the length of the third piece. [5]

Qn4: Solve the equation: $2\cos\alpha + 3\sin\alpha = 5$for $-\pi \le \alpha \le \pi$. [5]

Qn5: Using small changes, show that $(244)^{\frac{1}{4}} = 3\frac{1}{405}$. [5]

Qn6: Solve the simultaneous equations:

$$xy = 2$$
$$2\log(x-1) = \log y$$ [5]

Qn7: If the x-axis and y-axis are tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, show that $c = g^2 = f^2$. [5]

Qn8: Solve the equation: $\log_3 x + \log_x 3 = 4$. [5]

<u>Section B (60 Marks)</u>

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). If $z_1 = \frac{2i}{1+3i}$ and $z_2 = \frac{3+2i}{5}$, find $|z_1 - z_2|$. [6]

(b). Given the complex number $z = x + yi$;

(i). Find $\frac{z+i}{z+2}$.

(ii). Show that the locus of $\frac{z+i}{z+2}$ is a straight line when its imaginary part is zero. State the gradient of the line. [6]

Question 10:

(a). Solve the equation $\cos 2x = 4\cos^2 x - 2\sin^2 x$ for $0° \le x \le 180°$. [6]

(b). Show that if $\sin(x+\alpha) = p\sin(x-\alpha)$, then $\tan x = \left(\frac{p+1}{p-1}\right)\tan\alpha$. Hence solve the equation $\sin(x+20°) = 2\sin(x-20°)$for $0° \le x \le 180°$. [6]

Question 11:

Given that $x = \frac{t^2}{1+t^3}$and $= \frac{t^3}{1+t^3}$, find $\frac{d^2y}{dx^2}$. [12]

Question 12:

(a). The sum of n terms of a series is $2^n - 1$.

(i). Show that the terms of this series are in a G.P.

(ii). Hence find the first term, the common ratio and the sum of the second set of n terms. [6]

(b). From a group of 6 boys and 4 girls of the Science club, 5 members are to be selected to represent the club in a science workshop. In how many ways can the selection be done if:

(i). there must be exactly two girls in the science workshop.

(ii). one boy and one girl must be in the science workshop. [6]

Question 13:

(a). If $x^2 + 3y^2 = k$, where k is a constant, find $\frac{dy}{dx}$ at a point $(1, 2)$. [5]

(b). A cylinder of volume, V, is to be cut from a solid sphere of radius, R. Prove that the maximum volume of $V$is $\frac{4\pi R^3}{3\sqrt{3}}$. [7]

END

MARKING GUIDE

SNo.	Working	Marks
1	$\left(\sqrt{3x+1}+\sqrt{4x+5}\right)^2 = \left(\sqrt{16x+9}\right)^2$ $3x+1+2\sqrt{(3x+1)(4x+5)}+4x+5 = 16x+9$ $2\sqrt{12x^2+19x+5} = 9x+3$ $4(12x^2+19x+5) = 81x^2+54x+9$ $33x^2-22x-11=0$ $x = \frac{22\pm\sqrt{(-22)^2-4\times33\times(-11)}}{2\times33} = \frac{22\pm44}{66}$ $x = \frac{22-44}{66} = -\frac{1}{3}$, or, $x = \frac{22+44}{66} = 1$	B1 M1 M1 M1 A1
		05
2	Let $z = x+yi$ $\lvert Z-2+i\rvert = 1$ $\lvert(x-2)+(y+1)i\rvert = 1$ $\sqrt{(x-2)^2+(y+1)^2} = 1$ $(x-2)^2+(y+1)^2 = 1$ $x^2-4x+4+y^2+2y+1 = 1$ $x^2+y^2-4x+2y+4 = 0$ The locus is a circle. centre $= (2,-1)$, radius $= \sqrt{1} = 1$ unit	B1 M1 M1 A1 B1
		05
3	Let the first term, a, be the shortest piece and the last term, μ_{10}, be the longest piece. $\mu_n = ar^{n-1}$, $S_n = a\left(\frac{r^n-1}{r-1}\right)$ $\mu_{10} = 8a$, $\Rightarrow ar^9 = 8a$, $\Rightarrow r^9 = 2^3$ $r^3 = 2$, $\Rightarrow r = \sqrt[3]{2} = 1.2599$ $S_{10} = 10$, $\Rightarrow a\left(\frac{\left(\sqrt[3]{2}\right)^{10}-1}{\sqrt[3]{2}-1}\right) = 10$ $34.931a = 10$, $\Rightarrow a = 0.286$ The length of the third piece, μ_3, is given by: $\mu_3 = ar^2 = 0.286\times\left(\sqrt[3]{2}\right)^2 = 0.454$ m $= 45.4$ cm ≈ 45 cm (0 d.p)	B1 M1 B1 M1 A1
		05
4	$2\cos\alpha + 3\sin\alpha \equiv R\cos(\alpha-\beta) = R\cos\alpha\cos\beta + R\sin\alpha\sin\beta$ By comparison, $R\cos\beta = 2$, $R\sin\beta = 3$ $\frac{R\sin\beta}{R\cos\beta} = \frac{3}{2}$, $\Rightarrow \tan\beta = 1.5$, $\Rightarrow \beta = 56.31°$ $R = \sqrt{3^2+2^2} = \sqrt{13}$ $2\cos\alpha + 3\sin\alpha \equiv \sqrt{13}\cos(\alpha - 56.31°)$	B1 B1 B1

$$2\cos x + 3\sin x = 5$$

$$\sqrt{13}\cos(\alpha - 56.31°) = 5$$

$$\cos(\alpha - 56.31°) = \frac{5}{\sqrt{13}}$$ **M1**

$(\alpha - 56.31°)$ is undefined **A1**

05

5

let, $y = x^{\frac{1}{5}}$, and, $y + \delta y \approx (x + \delta x)^{\frac{1}{5}} = (244)^{\frac{1}{5}}$

if, $x = 243$, $\Rightarrow \delta x = 244 - 243 = 1$, $y = (243)^{\frac{1}{5}} = 3$

$$\frac{dy}{dx} = \frac{1}{5}x^{-\frac{4}{5}} = \frac{1}{5}(243)^{-\frac{4}{5}} = \frac{1}{405}$$ **B1**

$$\delta y \approx \frac{dy}{dx} \times \delta x = \frac{1}{405} \times 1 = \frac{1}{405}$$ **M1 B1**

$$(244)^{\frac{1}{5}} \approx y + \delta y = 3 + \frac{1}{405} = 3\frac{1}{405}$$ **M1 A1**

05

6

$$xy = 2$$

$$2\log(x-1) = \log y$$

$xy = 2$, $\Rightarrow y = \frac{2}{x}$

$2\log(x-1) = \log y$, $\Rightarrow \log(x-1)^2 = \log\left(\frac{2}{x}\right)$

$$(x-1)^2 = \left(\frac{2}{x}\right)$$ **M1**

$$x^2 - 2x + 1 = \frac{2}{x}$$

$$x^3 - 2x^2 + x = 2$$

$$x^3 - 2x^2 + x - 2 = 0$$

let, $f(x) = x^3 - 2x^2 + x - 2$

$f(2) = 8 - 8 + 2 - 2 = 0$, $\Rightarrow (x-2)$ is a factor **B1**

By synthetic method:

$$\begin{array}{c|cccc|} & 1 & -2 & 1 & -2 \\ x = 2 & & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$ **M1**

$$\Rightarrow f(x) = (x-2)(x^2+1)$$

$(x^2 + 1) = 0$ has no real roots

$\therefore x = 2$, and, $y = \frac{2}{x} = \frac{2}{2} = 1$ **A1 A1**

05

7

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Considering tangent $y = 0$,

$$x^2 + 2gx + c = 0$$ **B1**

For tangency,

$$b^2 - 4ac = 0$$

$$(2g)^2 - 4 \times 1 \times c = 0$$

$$4g^2 - 4c = 0$$

$$c = g^2 \longrightarrow (1)$$ **B1**

Considering tangent $x = 0$,

$$y^2 + 2fy + c = 0$$ **B1**

For tangency,

$$b^2 - 4ac = 0$$

$$(2f)^2 - 4 \times 1 \times c = 0$$

$$4f^2 - 4c = 0$$

$$c = f^2 \longrightarrow (2)$$

Combining equations (1) and (2) gives: **B1**

$$c = g^2 = f^2$$ **B1**

05

8

$$\log_3 x + \log_x 3 = 4$$

$$\log_3 x + \frac{1}{\log_3 x} = 4$$ **B1**

Let $y = \log_3 x$

$$y + \frac{1}{y} = 4$$

$$y^2 + 1 = 4y$$

$$y^2 - 4y + 1 = 0$$

$$y = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 1}}{2 \times 1} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$ **M1 M1**

$\Rightarrow \log_3 x = 2 \pm \sqrt{3}$, $\Rightarrow x = 3^{(2 \pm \sqrt{3})}$ **M1**

$x = 3^{(2-\sqrt{3})} \approx 1.342$, or, $x = 3^{(2+\sqrt{3})} \approx 60.345$ **A1**

05

9

(a).

$$z_1 - z_2 = \frac{2i}{1+3i} - \frac{3+2i}{5} = \frac{2i \times 5 - (3+2i)(1+3i)}{5(1+3i)}$$ **M1**

$$= \frac{10i - 3 - 9i - 2i + 6}{5(1+3i)} = \frac{3-i}{5(1+3i)}$$ **B1**

$$= \frac{(3-i) \times (1-3i)}{5(1+3i) \times (1-3i)} = \frac{3 - 9i - i - 3}{5(1^2 + 3^2)} = \frac{-10i}{50} = -\frac{1}{5}i$$ **B1**

$$|z_1 - z_2| = \left|-\frac{1}{5}i\right| = \frac{1}{5}\text{units}$$ **M1 A1**

(b). (i). Let $Z = x + yi$.

$$\frac{z+i}{z+2} = \frac{x + (y+1)i}{(x+2) + yi} = \frac{\{x + (y+1)i\} \times \{(x+2) - yi\}}{\{(x+2) + yi\} \times \{(x+2) - yi\}}$$ **M1**

$$= \frac{x(x+2) - xyi + (x+2)(y+1)i + y(y+1)}{(x+2)^2 + y^2}$$ **M1**

$$= \frac{x^2 + 2x - xyi + i(xy + x + 2y + 2) + y^2 + y}{(x+2)^2 + y^2}$$ **M1**

$$= \frac{x^2 + y^2 + 2x + y + i(x + 2y + 2)}{(x+2)^2 + y^2}$$ **A1**

(ii).

$$\text{imaginary part} = \frac{x + 2y + 2}{(x+2)^2 + y^2} = 0$$ **M1 M1**

$$x + 2y + 2 = 0$$
$$y = -\frac{1}{2}x - 1$$

The locus is a straight line since its in the form $y = mx + c$. **A1**

The gradient of the line is $-\frac{1}{2}$.

12

10 (a).

$$\cos 2x = 4\cos^2 x - 2\sin^2 x$$
$$1 - 2\sin^2 x = 4\cos^2 x - 2\sin^2 x \quad \textbf{B1}$$
$$1 = 4\cos^2 x \quad \textbf{M1}$$
$$\cos x = \pm 0.5 \quad \textbf{M1}$$
$$x = 60°, 120°, 240°, 300° \quad \textbf{A1}$$

(b).

$$\sin(x+\alpha) = p\sin(x-\alpha)$$
$$\sin x\cos\alpha + \cos x\sin\alpha = p\sin x\cos\alpha - p\cos x\sin\alpha \quad \textbf{B1}$$
$$\tan x\cos\alpha + \sin\alpha = p\tan x\cos\alpha - p\sin\alpha$$
$$\sin\alpha + p\sin\alpha = p\tan x\cos\alpha - \tan x\cos\alpha \quad \textbf{B1}$$
$$(p+1)\sin\alpha = (p-1)\tan x\cos\alpha \quad \textbf{B1}$$
$$(p+1)\tan\alpha = (p-1)\tan x$$
$$\Rightarrow \tan x = \left(\frac{p+1}{p-1}\right)\tan\alpha, \quad \text{hence shown} \quad \textbf{B1}$$

For the hence part

$$\sin(x+20°) = 2\sin(x-20°)$$
$$\tan x = \left(\frac{2+1}{2-1}\right)\tan 20° \quad \textbf{M1}$$
$$\tan x = 1.0919 \quad \textbf{M1}$$
$$x = 47.52°, -132.48° \quad \textbf{A1 A1}$$

12

11

$$\frac{dx}{dt} = \frac{(1+t^3)(2t) - t^2(3t^2)}{(1+t^3)^2} = \frac{2t + 2t^4 - 3t^4}{(1+t^3)^2} \quad \textbf{B1}$$
$$= \frac{2t - t^4}{(1+t^3)^2} = \frac{t(2-t^3)}{(1+t^3)^2} \quad \textbf{B1}$$
$$\frac{dy}{dt} = \frac{(1+t^3)(3t^2) - t^3(3t^2)}{(1+t^3)^2} = \frac{3t^2 + 3t^5 - 3t^5}{(1+t^3)^2} = \frac{3t^2}{(1+t^3)^2} \quad \textbf{B1 B1}$$
$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx} = \frac{3t^2}{(1+t^3)^2}\times\frac{(1+t^3)^2}{t(2-t^3)} = \frac{3t}{(2-t^3)} \quad \textbf{M1 A1}$$
$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{3t}{(2-t^3)}\right]\times\frac{dt}{dx} = \frac{(2-t^3)(3) - (3t)(-3t^2)}{(2-t^3)^2}\times\frac{(1+t^3)^2}{t(2-t^3)} \quad \textbf{M1 M1}$$
$$= \frac{6 - 3t^3 + 9t^3}{(2-t^3)^2}\times\frac{(1+t^3)^2}{t(2-t^3)} = \frac{(6+6t^3)(1+t^3)^2}{t(2-t^3)^3} \quad \textbf{M1 M1}$$
$$= \frac{6(1+t^3)(1+t^3)^2}{t(2-t^3)^3} = \frac{6(1+t^3)^3}{t(2-t^3)^3} \quad \textbf{A1}$$

12

12 (a). (i).

$$S_n = 2^n - 1$$

$$S_1 = 2^1 - 1 = 1, \quad S_2 = 2^2 - 1 = 3 \quad \textbf{B1}$$
$$S_3 = 2^3 - 1 = 7, \quad S_4 = 2^4 - 1 = 15$$
$$\mu_1 = S_1 = 1, \quad \mu_2 = S_2 - S_1 = 3 - 1 = 2 \quad \textbf{B1}$$
$$\mu_3 = S_3 - S_2 = 7 - 3 = 4, \quad \mu_4 = S_4 - S_3 = 15 - 7 = 8$$
$$\frac{\mu_2}{\mu_1} = \frac{2}{1} = 2, \quad \frac{\mu_3}{\mu_2} = \frac{4}{2} = 2, \quad \frac{\mu_4}{\mu_3} = \frac{8}{4} = 2 \quad \textbf{M1}$$

Since a common ratio exists, then the series is a geometric progression. **B1**

(ii). For the second set of n terms,

$$\text{First term,} \quad \mu_{n+1} = S_{n+1} - S_n = (2^{n+1} - 1) - (2^n - 1)$$
$$= 2^{n+1} - 2^n = 2(2^n) - 2^n = (2^n)(2-1) = 2^n$$
$$\text{Common ratio,} \quad r = 2 \quad \textbf{A1}$$
$$S_n = 2^n - 1, \quad \Rightarrow S_{2n} = 2^{2n} - 1$$
$$\text{Sum of the second set of } n \text{ terms} = S_{2n} - S_n$$
$$= (2^{2n} - 1) - (2^n - 1) = 2^{2n} - 2^n$$
$$= (2^n)^2 - 2^n = 2^n(2^n - 1) \quad \textbf{M1 A1}$$

(b). (i). The number of ways of selecting so that there must be exactly two girls in the science workshop is:

$$= {}^4C_2 \times {}^6C_3 = 6 \times 20 = 120\text{ways} \quad \textbf{M1 A1}$$

(ii). The number of ways of selecting so that one boy and one girl must be in the science workshop is:

$$= {}^3C_0 \times {}^5C_3 + {}^3C_1 \times {}^5C_2 + {}^3C_2 \times {}^5C_1 + {}^3C_3 \times {}^5C_0 \quad \textbf{M1}$$
$$= 10 + 30 + 15 + 1 = 56\text{ways} \quad \textbf{M1 A1}$$

12

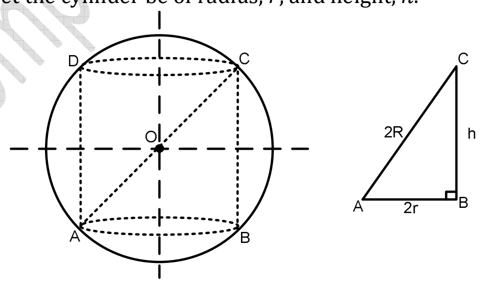
13 (a).

$$x^2 + 3y^2 = k$$
$$2x + 6y\frac{dy}{dx} = 0 \quad \textbf{B1}$$
$$\frac{dy}{dx} = \frac{-2x}{6y} = -\frac{x}{3y}$$

At a point $(1, 2)$,

$$\frac{dy}{dx} = -\frac{1}{3\times 2} = -\frac{1}{6} \quad \textbf{M1 A1}$$

(b). Let the cylinder be of radius, r, and height, h.



B1

By Pythagoras theorem,

$$(2r)^2 = (2R)^2 - h^2, \quad \Rightarrow r^2 = R^2 - \frac{h^2}{4} \quad \textbf{B1}$$

	Volume of cylinder, $V = \pi r^2 h = \pi\left(R^2 - \frac{h^2}{4}\right)h$	
	$= \pi R^2 h - \frac{\pi h^3}{4}$	M1
	$\frac{dV}{dh} = \pi R^2 - \frac{3\pi h^2}{4}$	M1
	For maximum volume, $\frac{dV}{dh} = 0$	
	$\pi R^2 - \frac{3\pi h^2}{4} = 0$	M1
	$R^2 = \frac{3\pi h^2}{4}$, $\Rightarrow h = \frac{2R}{\sqrt{3}}$	A1
	$V_{max} = \pi R^2\left(\frac{2R}{\sqrt{3}}\right) - \pi\left(\frac{2R}{\sqrt{3}}\right)^3 = \pi R^3\left(\frac{2}{\sqrt{3}} - \frac{2}{3\sqrt{3}}\right)$	M1
	$= \pi R^3\left(\frac{6-2}{3\sqrt{3}}\right) = \pi R^3\left(\frac{4}{3\sqrt{3}}\right) = \frac{4\pi R^3}{3\sqrt{3}}$	A1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Feb 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 2 MOCK SET 1 2018
Time: 3 Hours

NAME: ________________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)
Answer **all** *the questions in this section.*

Qn 1: The distribution below shows the weights of babies in Gombe hospital:
3, 5, 3, 9, 6, 8, 20, 19, 24, 14, 12
Find the:
(i). upper quartile.
(ii). Standard deviation. [5]

Qn 2: A particle is projected from a point $P(3, 4)$ with velocity $\left(\underset{\sim}{i} + 2\underset{\sim}{j}\right) \text{ m s}^{-1}$ and it moves freely under gravity. Find its velocity and position vector 1.5 s later. [5]

Qn 3: The table below gives values of x and the corresponding values of $f(x)$.

x	0.1	0.2	0.3	0.4	0.5	0.7
$f(x)$	4.21	3.83	3.25	2.85	2.25	1.43

Use linear interpolation/extrapolation to find:
(a). $f(x)$ when $x = 0.6$. [3]
(b). the value of x when $f(x) = 0.75$. [2]

Qn 4: Opio sits for four examinations in his class. If the probability of passing an examination in his class is 0.25, find the probability that he passes:
(i). Only two examinations.
(ii). Less than half of the examination. [5]

Qn 5: A car decelerated from a speed of 20 m s^{-1} to rest in 8 seconds, falling short of its parking slot by 20 m. By how much longer should the car have decelerated from the same speed so as to just reach the parking slot? [5]

Qn 6: A bag contains 15 white, 5 red and 5 blue balls. Three balls are drawn at random one at a time without replacement. Determine the probability that the first ball is blue and the third one is red. [5]

Qn 7: Given that $y = \frac{1}{x} + x$ and $x = 2.4$ correct to **one** decimal place, find the limits within which y lies. [5]

Qn 8: A particle initially moving with a constant velocity is acted upon by a retardation force. If after t seconds its position vector is $\underset{\sim}{\boldsymbol{r}} = \left(0.5t^2\left(\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right) + t\left(2\underset{\sim}{\boldsymbol{i}} + 5\underset{\sim}{\boldsymbol{j}}\right) + 6\underset{\sim}{\boldsymbol{i}} - 22\underset{\sim}{\boldsymbol{j}}\right)$ metres, find the time after which its speed will reduce to 15 m s^{-1}. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The table below shows the frequency distribution of marks obtained in paper one of the mathematics contest by Ndejje SSS students.

Marks (%)	10 –	20 –	30 –	40 –	50 –	60 –	70 –	80 – 90
Frequency	18	34	58	42	24	10	6	8

(a). Calculate the:
(i). Mean mark. [2]
(ii). Standard deviation. [2]
(iii). Number of students who scored above 54%. [2]
(b). Draw a cumulative frequency curve and use it to estimate the:
(i). 5th decile.
(ii). Number of students that would not qualify for paper two if the pass mark is fixed at 40%.

(iii). Least mark if the top 10% of the students are to be awarded. [6]

Question 10:
At 8:00 am, a bus initially parked at stage A, starts moving along a straight road with acceleration, $a = (4t)$ km h^{-2}, which acceleration continues until $t = 5$ hours, where upon it cease and the bus uniformly retards at 20 km h^{-2} to rest at stage B.
(a). Determine the:
(i). time when the bus reaches stage B. [5]
(ii). Distance between A and B. [5]
(b). Sketch a velocity-time graph to represent the above journey of the bus. [2]

Question 11:

(a). A discrete random variable X has a function given by $P(X \leq x) = \frac{x^2}{9}$, for $x = 1, 2, 3$.
 (i). Write out the probability distribution for X.
 (ii). Find the mode.
 (iii). Find $E(3X + 2)$. [6]

(b). A and B are two independent events with $P(A) > P(B)$ such that $P(A \cap B) = \frac{1}{3}$ and $P(A \cup B) = 0.9$. Find:
 (i). $P(A)$.
 (ii). $P(B)$. [6]

Question 12:

A stone projected from a point O on the horizontal ground moves freely under gravity and hits the ground again at point A. Referred to O as the origin, OA as the x-axis and the upward vertical at O as the y-axis, the equation of the path of the stone is given by $40y = 40x - x^2$, where x and y are measured in metres. Calculate the:

(i). Distance $\overline{OA}$.
(ii). Greatest height above OA attained by the stone.
(iii). Magnitude and direction of the velocity at O.
(iv). Time taken by the stone to reach A from O. [12]

Question 13:

The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows. 5% of the residents have over 90 cows.

(a). Determine the values of the mean and standard deviation of the cows. [8]
(b). If there are 200 residents, find how many have more than 80 cows. [4]

Question 14:

(a). Given that $x = 3.7$ and $y = 70$ are each rounded off with percentage error of 0.2 and 0.05 respectively while number $z = 26.23$ is calculated with relative error of 0.04. Find the interval in which the exact value of $\frac{x}{y-z}$ lies, correct to 4 significant figures. [6]

(b). The height and radius of a cylindrical water tank are given as $H = 3.5 \pm 0.2$ m and $R = 1.4 \pm 0.1$ m respectively. Determine, in m^3, the least and greatest ammount of water the tank can contain; hence calculate the maximum possible error in your calculation. [6]

Question 15:

A non-uniform rod AB of mass 10 kg has its centre gravity at a distance $\frac{1}{4}\overline{AB}$ from B. The rod is smoothly hinged at A. It is maintained in equilibrium at 60° to above the horizontal by a light inextensible string tied at B and at a right angle to AB. Calculate the magnitude and direction of the reaction at A. [12]

*****END*****

MARKING GUIDE

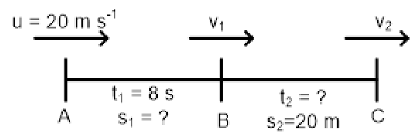
SNo.	Working	Marks
1	(i).	
	3, 3, 5, 6, 8, 9, 12, 14, 19, 20, 24	M1
	upper quartile position $= \frac{3}{4}(N+1) = \frac{3}{4}\times 12 = 9^{\text{th}}$	
	upper quartile, $q_3 = 19$	A1
	(ii).	
	$\sum X = 3+3+5+6+8+9+12+14+19+20+24 = 123$	
	$\sum X^2 = 3^2+3^2+5^2+6^2+8^2+9^2+12^2+14^2+19^2+20^2+24^2 = 1901$	B1
	$\sigma = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2} = \sqrt{\frac{1901}{11} - \left(\frac{123}{11}\right)^2} = \sqrt{47.785} = 6.913$	M1 A1
		05
2	$\overrightarrow{OP} = \begin{pmatrix}3\\4\end{pmatrix}, \underset{\sim}{u} = \begin{pmatrix}1\\2\end{pmatrix}\text{ m s}^{-1}, \underset{\sim}{a} = \begin{pmatrix}0\\-9.8\end{pmatrix}\text{ m s}^{-2}$	
	$\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t = \begin{pmatrix}1\\2\end{pmatrix} + 1.5\begin{pmatrix}0\\-9.8\end{pmatrix} = \begin{pmatrix}1\\-12.7\end{pmatrix}\text{ m s}^{-1}$	M1 B1
	$\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2 = 1.5\begin{pmatrix}1\\2\end{pmatrix} + \frac{1}{2}\times 1.5^2\begin{pmatrix}0\\-9.8\end{pmatrix} = \begin{pmatrix}1.5\\-8.025\end{pmatrix}\text{ m}$	M1 B1
	Position vector $= \overrightarrow{OP} + \underset{\sim}{s} = \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}1.5\\-8.025\end{pmatrix} = \begin{pmatrix}4.5\\-4.025\end{pmatrix}\text{ m}$	A1
		05
3	(a).	

x	0.5	0.6	0.7
$f(x)$	2.25	y_1	1.43

SNo.	Working	Marks
		B1
	$\frac{y_1 - 2.25}{1.43 - 2.25} = \frac{0.6 - 0.5}{0.7 - 0.5}$	M1
	$y_1 = 0.5 \times (-0.82) + 2.25 = 1.84$	A1
	When $x = 0.6, f(x) = 1.84$.	
	(b).	

x	0.5	0.7	x_2
$f(x)$	2.25	1.43	0.75

SNo.	Working	Marks
	$\frac{x_2 - 0.5}{0.7 - 0.5} = \frac{0.75 - 2.25}{1.43 - 2.25}$	M1
	$x_2 = \frac{75}{41} \times 0.2 + 0.5 = \frac{71}{82} \approx 0.866$	A1
	When $f(x) = 0.75, x = 0.866$.	
		05
4	(i). $n = 4, p = 0.25, q = 1 - 0.25 - 0.75$	

SNo.	Working	Marks
	$P(X = 2) = 0.2109$	B1 A1
	(ii).	
	$P(X < 2) = 1 - P(X \geq 2) = 1 - 0.2617 = 0.7383$	M1 B1 A1
	Alternatively:	
	$P(X < 2) = P(X = 0) + P(X = 1) = 0.3164 + 0.4219 = 0.7383$	
		05
5	 	
	For motion AB, $v_1 = 0$	
	$v_1 = u + a_1t_1, \quad \Rightarrow a_1 = \frac{v_1 - u}{t_1} = \frac{0 - 20}{8} = -2.5\text{ m s}^{-2}$	B1
	$s_{AB} = ut_1 + \frac{1}{2}a_1{t_1}^2 = 20 \times 8 + \frac{1}{2}\times(-2.5)\times 8^2 = 80\text{ m}$	B1
	For motion AC, $v_2 = 0, s_{AC} = 80 + 20 = 100\text{ m}$	
	${v_1}^2 = u^2 + 2a_2s_{AC}$	
	$a_2 = \frac{{v_1}^2 - u^2}{2s_{AC}} = \frac{0 - 20^2}{2 \times 100} = -2\text{ m s}^{-2}$	B1
	$v_1 = u + a_1t_1$	
	$t_{AC} = \frac{v_2 - u}{a_2} = \frac{0 - 20}{-2} = 10\text{ s}$	B1
	extra time, $t_{BC} = t_{AC} - t_{AB} = 10 - 8 = 2\text{ s}$	A1
		05
6		

$P(\text{first blue and third red})$

$= P(B_1 \cap W_2 \cap R_3) + P(B_1 \cap R_2 \cap R_3) + P(B_1 \cap B_2 \cap R_3)$

$= \frac{5}{25} \times \frac{15}{24} \times \frac{5}{23} + \frac{5}{25} \times \frac{5}{24} \times \frac{4}{23} + \frac{5}{25} \times \frac{4}{24} \times \frac{5}{23}$ **M1 B1 B1 B1**

$= \frac{1}{24} \approx 0.04167$ **A1**

05

7

$x = 2.4, \quad e_x = 0.05$

$y_{max} = \frac{1}{x_{min}} + x_{max} = \frac{1}{(2.4 - 0.05)} + (2.4 + 0.05) = 2.8755$ **M1 B1**

$y_{min} = \frac{1}{x_{max}} + x_{min} = \frac{1}{(2.4 + 0.05)} + (2.4 - 0.05) = 2.7582$ **M1 B1**

Upper limit $= 2.8755$, Lower limit $= 2.7582$ **A1**

05

8

$\underset{\sim}{r} = 0.5t^2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ -22 \end{pmatrix}$ m, $\quad |\underset{\sim}{v}| = 15 \text{ m s}^{-1}$

velocity, $\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} t+2 \\ t+5 \end{pmatrix} \text{m s}^{-1}$ **M1**

$|\underset{\sim}{v}| = \sqrt{(t+2)^2 + (t+5)^2} = 15 \text{ m s}^{-1}$ **M1**

$t^2 + 4t + 4 + t^2 + 10t + 25 = 225$

$2t^2 + 14t - 196 = 0$

$t^2 + 7t - 98 = 0$ **B1**

$t = 7, \quad \text{or}, \quad t = -14$

but, $\quad t \neq -14, \quad \Longrightarrow t = 7 \text{ s}$ **A1**

05

9 (a).

Class	f	x	fx	fx^2	C.F	Class boundaries
10 –	18	15	270	4050	18	10 – 20
20 –	34	25	850	21250	52	20 – 30
30 –	58	35	2030	71050	110	30 – 40
40 –	42	45	1890	85050	152	40 – 50
50 –	24	55	1320	72600	176	50 – 60
60 –	10	65	650	42250	186	60 – 70
70 –	6	75	450	33750	192	70 – 80
80 – 90	8	85	680	57800	200	80 – 90
Total			8140	387800		

B1-for fx^2

(i).

Mean mark, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{8140}{200} = 40.7$ **A1**

(ii).

Standard deviation $= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

$= \sqrt{\frac{387800}{200} - 40.7^2} = \sqrt{282.51} \approx 16.81$ **M1 A1**

(iii). Let N_1 be the number of students who scored below 54%.

$54 = L_1 + \left(\frac{N_1 - C.F_b}{f_1}\right)c$

$54 = 50 + \left(\frac{N_1 - 152}{24}\right) \times 10$ **M1**

$4 = 10\left(\frac{N_1 - 152}{24}\right),$

$\Longrightarrow N_1 = 0.4 \times 24 + 152 = 161.6$

The number of stiudents who scored above 54% **A1**

$= 200 - 161.4 = 38.6 \approx 39$students (rounding off)

Or: The number of stiudents who scored above 54%

$= 200 - 161.4 = 38.6 \approx 38$students(truncating)

Alternatively:

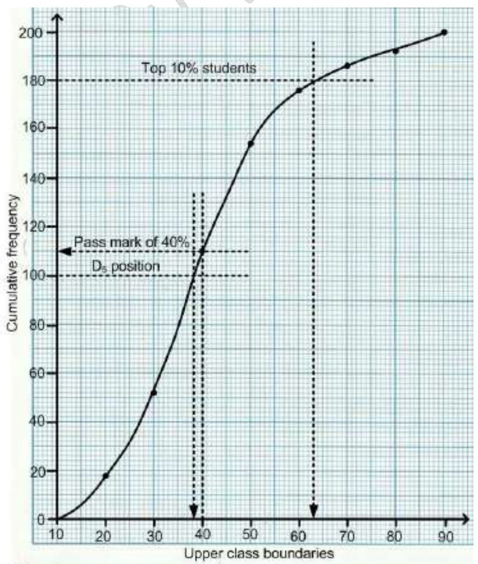
$C.F$	152	N_1	176
Upper class boundary	50	54	60

$\frac{N_1 - 152}{176 - 152} = \frac{54 - 50}{60 - 50}$

$\frac{N_1 - 152}{24} = 0.4$,

$\Longrightarrow N_1 = 0.4 \times 24 + 152 = 161.6$

(b).



B1-axes

B1-smooth curve

B1-estimations

(c). (i). 5th decile position: $\frac{5}{10}N = \frac{5}{10} \times 200 = 100$

From the ogive, 5th decile: $D_5 = 38\%$. **A1**

(ii). If the pass mark is fixed at 40% the number of students that would not qualify for paper two would be 110 students. **A1**

(iii). Number of students awarded $= \frac{10}{100} \times 200 = 20$ students.

From the ogive, the least mark if the top 10% of the students are to be awarded is 63%. **A1**

12

10 (a). (i).

$$v = \int 4t\, dt = 2t^2 + c$$ **M1**

When $t = 0, v = 0, \Rightarrow c = 0$

$$v = 2t^2$$

$$\text{when } t = 5, \quad v = 2 \times 5^2 = 50 \text{ km h}^{-1}$$ **B1**

During retardation, , $u = 50 \text{ km h}^{-1}, v = 0\text{km h}^{-1}, a = -20 \text{ km h}^{-2}$

$$v = u + at, \quad \Rightarrow t = \frac{v-u}{a} = \frac{0-50}{-20} = 2.5 \text{ hours}$$ **B1**

$$\text{Total time taken,} \quad T = 5 + 2.5 = 7.5 \text{ hours}$$ **B1**

In 24 hour clock,

$$\begin{array}{r|l|l} & 0800 & \text{hours} \\ + & 0730 & \text{hours} \\ \hline & 1530 & \text{hours} \end{array}$$ **A1**

The bus reaches stage B at 3:30 pm.

(ii).

From $t = 0$ to $t = 5$ hours,

$$s_1 = \int_0^5 v\, dt = \int_0^5 2t^2\, dt = \left[\frac{2}{3}t^3\right]_0^5 = \frac{2}{3} \times 5^3 - 0 = \frac{250}{3} \approx 83.333 \text{ km}$$ **M1 B1**

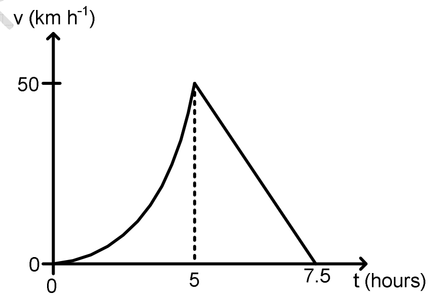
From $t = 5$ to $t = 7.5$ hours,

$u = 50 \text{ km h}^{-1}, v = 0\text{km h}^{-1}, a = -20 \text{ km h}^{-2}$

$$v^2 = u^2 + 2as_2, \quad \Rightarrow s_2 = \frac{v^2 - u^2}{2a} = \frac{0^2 - 50^2}{-2 \times 20} = 62.5 \text{ km}$$ **M1 B1**

$$\text{Total distance,} \quad s = s_1 + s_2 = \frac{250}{3} + 62.5 = \frac{875}{6} \approx 145.83 \text{ km}$$ **A1**

(b).



B1 B1

12

11 (a). (i).

x	$P(X \le x)$	$P(X = x)$	$xP(X = x)$
1	1/9	1/9	1/9
2	4/9	3/9	2/3
3	1	5/9	5/3
Total		1	22/9

B1-for $P(X = x)$
B1-for $xP(X = x)$

(ii).

$$\text{Mode} = 3$$ **A1**

(iii).

$$E(3X + 2) = 3E(X) + 2 = 3 \times \frac{22}{9} + 2 = \frac{28}{3} \approx 9.333$$ **M1 A1**

(b). Let $P(A) = x$ and $P(B) = y$

$$P(A \cap B) = \frac{1}{3}, \quad \Rightarrow xy = \frac{1}{3}, \quad \Rightarrow y = \frac{1}{3x}$$ **M1**

$$P(A \cup B) = 0.9$$

$$P(A) + P(B) - P(A \cap B) = 0.9$$

$$x + y - \frac{1}{3} = 0.9$$ **M1**

$$x + \frac{1}{3x} - \frac{37}{30} = 0$$

$$30x^2 + 10 - 37x = 0$$ **M1**

$$30x^2 - 37x + 10 = 0$$

$$x = \frac{5}{6}, \quad \text{or,} \quad x = 0.4$$ **B1**

$$\text{when } x = \frac{5}{6}, \quad y = \frac{1}{3} \times \frac{6}{5} = \frac{3}{5} = 0.6$$ **B1**

$$\text{when } x = 0.4, \quad y = \frac{1}{3 \times 0.4} = \frac{5}{6}$$ **B1**

$$\text{for } x > y, \quad x \ne \frac{5}{6}, \quad \Rightarrow x = \frac{5}{6}, \quad y = 0.4$$

$$\therefore P(A) = \frac{5}{6}, \quad P(B) = 0.4$$ **A1**

12

12 (i). At point A, $y = 0$

$$\text{from,} \quad 40y = 40x - x^2, \quad 0 = x(40 - x)$$

$$x = 0, \quad \text{or,} \quad x = 40$$ **M1**

$$\Rightarrow \text{Distance } \overline{OA} = 40 \text{ m}$$ **A1**

(ii). At the greatest height, $\frac{dy}{dx} = 0$

$$40y = 40x - x^2, \quad \Rightarrow \frac{dy}{dx} = 40 - 2x = 0, \quad \Rightarrow x = 20$$ **M1 B1**

$$\text{when } x = 20, \quad y = x - \frac{1}{40}x^2 = 20 - \frac{1}{40} \times 20^2 = 10$$

$$\Rightarrow \text{Greatest height} = 10 \text{ m}$$ **A1**

(iii).

$$y = x - \frac{1}{40}x^2$$

The general equation of trajectory is given by,

$$y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$$

Comparing coefficients of x,

$$\tan\theta = 1, \qquad \Rightarrow \theta = 45°$$ **M1 A1**

Comparing coefficients of x^2,

$$\frac{g(1+\tan^2\theta)}{2u^2} = \frac{1}{40}$$ **M1**

$$\frac{9.8(1+1)}{2u^2} = \frac{1}{40}$$

$$\frac{9.8}{u^2} = \frac{1}{40}$$

$$u^2 = 40 \times 9.8\,, \qquad \Rightarrow u = \sqrt{392} = 19.80 \text{ m s}^{-1}$$ **B1**

The initial velocity of the particle point O is 19.80 m s^{-1} in the direction $45°$ above the horizontal. **A1**

(iv).

$$\text{Distance } \overline{OA} = ut\cos\theta = 40$$

$$19.8t\cos 45° = 40, \qquad \Rightarrow t = \frac{40}{19.8\cos 45°} = 2.857 \text{ s}$$ **M1 A1**

12

13 (a).

$$P(X < 60) = 0.15, \qquad \Rightarrow P(Z < -z_1) = 0.15$$

$$-z_1 = 1.036, \qquad z_1 = -1.036$$ **M1**

$$\text{but, } z_1 = \frac{60-\mu}{\sigma} = -1.036, \qquad \Rightarrow 60 - \mu = -1.036\sigma \;\rightarrow (1)$$ **M1**

Also, $P(X > 90) = 0.05, \qquad \Rightarrow P(Z > z_2) = 0.05,$

$$\Rightarrow z_2 = 1.645$$ **M1**

$$\text{but, } z_2 = \frac{90-\mu}{\sigma} = 1.645, \qquad \Rightarrow 90 - \mu = 1.645\sigma \;\rightarrow (2)$$ **M1**

Equation (2) − (1) gives;

$$30 = 2.681\sigma, \qquad \Rightarrow \sigma = \frac{30}{2.681} = 11.19$$ **M1 A1**

From equation (1);

$$\mu = 60 + 1.036\sigma = 60 + 1.036 \times 11.19 = 71.59$$ **M1 A1**

∴ Mean, $\mu = 71.59$, Standard deviation, $\sigma = 11.19$

(b).

$$P(X > 80) = P\left(Z > \frac{80 - 71.59}{11.19}\right) = P(Z > 0.752)$$ **M1**

$$= 0.5 - \phi(0.752) = 0.5 - 0.2740 = 0.226$$ **M1 B1**

$$\text{Number of residents} = 0.226 \times 200 = 45.2$$ **A1**

12

14 (a).

$$x = 3.7, \qquad y = 70, \qquad z = 26.23$$

$$\%e_x = 0.2, \qquad \Rightarrow e_x = \frac{0.2 \times 3.7}{100} = 0.0074$$ **B1**

$$\%e_y = 0.05, \qquad \Rightarrow e_y = \frac{0.05 \times 70}{100} = 0.035$$ **B1**

$$\frac{e_z}{z} = 0.04, \qquad \Rightarrow e_z = 0.04 \times 26.23 = 1.0492$$ **B1**

$$\left(\frac{x}{y-z}\right)_{max} = \frac{(3.7 + 0.0074)}{(70 - 0.035) - (26.23 + 1.0492)} = 0.08685$$ **M1 B1**

$$\left(\frac{x}{y-z}\right)_{min} = \frac{(3.7 - 0.0074)}{(70 + 0.035) - (26.23 - 1.0492)} = 0.08232$$ **M1 B1**

$$\text{interval} = [0.08232, 0.08685]$$ **A1**

(b).

$$H = 3.5 \pm 0.2, \qquad R = 1.4 \pm 0.1, \qquad V = \pi R^2 H$$

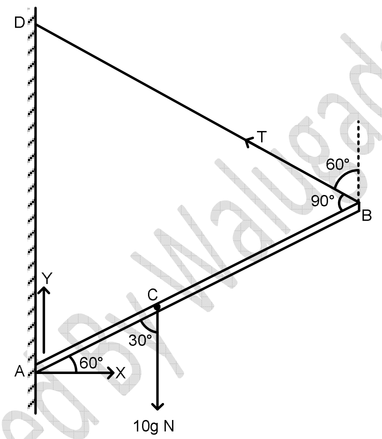
$$V_{max} = \pi(1.4 + 0.1)^2(3.5 + 0.2) = 26.1538 \text{ m}^3$$ **B1**

$$V_{min} = \pi(1.4 - 0.1)^2(3.5 - 0.2) = 17.5207 \text{ m}^3$$ **B1**

$$\text{maximum possible error} = \frac{26.1538 - 17.5207}{2} = 4.31655 \text{ m}^3$$ **M1 A1**

12

15



B1

$$\text{let,} \qquad \overline{AB} = l, \qquad \Rightarrow \overline{AC} = \frac{1}{4}\overline{AB} = 0.75l$$

Taking moments about A, **M1**

$$T \times l = 10g \times 0.75l\sin 30°$$

$$T = 10 \times 9.8 \times 0.75\sin 30° = 36.75 \text{ N}$$ **M1 B1**

Resolving vertically,

$$Y + T\cos 60° = 10g$$

$$Y + 36.75\cos 60° = 10 \times 9.8$$ **M1**

$$Y = 79.625 \text{ N}$$ **B1**

Resolving horizontally,

$$X = T\sin 60° = 36.75\sin 60° = 31.826\text{N}$$ **M1 B1**

$$\text{Reaction at A} = \sqrt{X^2 + Y^2} = \sqrt{31.826^2 + 79.625^2} \approx 85.75 \text{ N}$$ **M1 A1**

$$\text{Direction} = \tan^{-1}\left(\frac{Y}{X}\right) = \tan^{-1}\left(\frac{79.625}{31.826}\right)$$ **M1**

$$= 68.21° \text{ above the horizontal}$$ **A1**

12

END

P425/1
PURE
MATHEMATICS
PAPER 1
April 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 1 MOCK SET 2 2018
Time: 3 Hours

NAME: ______________________ **COMB:** ________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn1: Given that $\sin 2x = \sqrt{3}\cos^2 x$, solve for$x$ for $-180° \le x \le 180°$. [5]

Qn2: Given that $y = Ae^{3x} + Be^{-2x}$, show that: $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$. [5]

Qn3: Find the area of the triangle with vertices $(2, 1)$, $(3, -2)$ and $(-4, -1)$. [5]

Qn4: If α and β are roots of $px^2 + qx + r = 0$, express $(\alpha - 2\beta)(\beta - 2\alpha)$ in terms of p, q and r. Hence deduce that for one root to be twice the other, $9pr = 2q^2$. [5]

Qn5: The surface area of a sphere is decreasing at a rate of $0.9\ \mathrm{m}^2/\mathrm{s}$ when the radius is 0.6 m. Find the rate of change of the volume of the sphere at this instant. [5]

Qn 6: Obtain Arg (Z) and $|Z|$ for the complex number: $Z = \frac{(1-i)(i-2)}{1+i}$. [5]

Qn7: Differentiate: $\cos(x^2e^x)$ with respect to x. [5]

Qn8: Find the locus of the point $P(x, y)$ which moves such that its distance from the point $A(5, 3)$ is equal to twice its distance from $x = 2$. [5]

Section B (60 Marks)
Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
(a). Function $f(x) = b + ax - 4x^2 + 8x^3$ gives a remainder of -19 when divided by $(x + 1)$ and a remainder of 2 when divided by $(2x - 1)$. Find the value of $a$and b.
(b). The roots of the equation $x^2 - 4x + 2 = 0$ are $\alpha$and β. Form the equation whose roots are $(\alpha + 2\beta)$and $(\beta + 2\alpha)$. [12]

Question 10:
Sketch the curve: $y = \frac{x^2 - x - 6}{x - 1}$. [12]

Question 11:
(a). Solve $3\sin x + 4\cos x = 2$ for $-180° \le x \le 180°$.
(b). Show that in any triangle ABC, $\frac{a^2 - b^2}{c^2} = \frac{\sin(A-B)}{\sin(A+B)}$. [12]

Question 12:
(a). On the same axes, sketch the curve $y = x(x + 2)$ and $y = x(4 - x)$.
(b). Find the area enclosed by the two curves in (a) above.
(c). Determine the volume generated when the area enclosed by the two curves in (a) above is rotated about the x-axis. [12]

Question 13:
(a). Express $\frac{3x^2 + x + 1}{(x-2)(x+1)^2}$ into partial fractions.
(b). Hence evaluate $\int_3^4 \frac{3x^2 + x + 1}{(x-2)(x+1)^2}\,dx$. Give your answer to 3 decimal places. [12]

Question 14:
(a). A line and a plane are given by the equations $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1}$ and $2x - y + 3z = 20$ respectively. Find:
(i). the point of intersection of the line and the plane.

(ii). the acute angle between them.

(b). Find the Cartesian equation of the plane through the points $A(0,3,-4)$, $B(2,-1,2)$ and $C(7,4,-1)$. [12]

Question 15:

(a). The points $A(-3,0)$, $B(3,0)$ and $P(x,y)$ are such that $\overline{PB} = 2\overline{PA}$. Show that the locus of P is a circle. Hence find its centre and radius.

(b). A circle with centre P and radius r touches externally both circles $x^2+y^2=4$ and $x^2+y^2-6x+8=0$. Prove that the x-coordinates of P is given by $\frac{r}{3}+2$. [12]

Question 16:

(a). Find the locus defined by $\text{Arg}\left(\frac{Z-1}{Z+1}\right)=\frac{\pi}{4}$ where Z is a complex number $x+iy$.

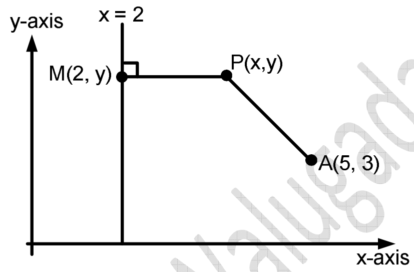
(b). The complex number $Z=x+iy$ satisfies the expression $\frac{Z}{Z+2}=2-i$. Find the modulus of Z. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	$\sin 2x = \sqrt{3}\cos^2 x$	
	$2\sin x\cos x - \sqrt{3}\cos^2 x = 0$	
	$\cos x\left(2\sin x - \sqrt{3}\cos x\right) = 0$	B1
	$\cos x = 0,$ or, $\left(2\sin x - \sqrt{3}\cos x\right) = 0$	M1
	for, $\cos x = 0,$ $x = \pm 90°$	A1
	for, $2\sin x - \sqrt{3}\cos x = 0$	
	$2\sin x = \sqrt{3}\cos x$	
	$\tan x = \frac{\sqrt{3}}{2}$	M1
	$x = \pm 90°, 40.89°, -139.11°$	A1
	For $-180° \leq x \leq 180°, x = 40.89°, -139.11°$	
		05
2	$y = Ae^{3x} + Be^{-2x}$	
	$\frac{dy}{dx} = 3Ae^{3x} - 2Be^{-2x}$	B1
	$\frac{d^2y}{dx^2} = 9Ae^{3x} + 4Be^{-2x}$	B1
	$\frac{d^2y}{dx^2} = 3Ae^{3x} - 2Be^{-2x} + 6Ae^{3x} + 6Be^{-2x}$	B1
	$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 6y$	B1
	$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$	B1
	Alternatively:	
	$y = Ae^{3x} + Be^{-2x}$	
	$ye^{2x} = Ae^{5x} + B$	
	$2ye^{2x} + e^{2x}\frac{dy}{dx} = 5Ae^{5x}$	
	$2ye^{-3x} + e^{-3x}\frac{dy}{dx} = 5A$	
	$\left(2y + \frac{dy}{dx}\right)e^{-3x} = 5A$	
	$\left(2\frac{dy}{dx} + \frac{d^2y}{dx^2}\right)e^{-3x} - 3e^{-3x}\left(2y + \frac{dy}{dx}\right) = 0$	
	$\left(2\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) - 3\left(2y + \frac{dy}{dx}\right) = 0$	
	$2\frac{dy}{dx} + \frac{d^2y}{dx^2} - 6y - 3\frac{dy}{dx} = 0$	
	$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$	
		05
3		

	(2, 1) (3, -2) (-4, -1) (2, 1) $\text{area} = \left\lvert\frac{(2\times{}^-2+3\times{}^-1+{}^-4\times 1)-(1\times 3+{}^-2\times{}^-4+{}^-1\times 2)}{2}\right\rvert$ $\text{area} = \left\lvert\frac{-11-9}{2}\right\rvert = \lvert -10\rvert = 10 \text{ sq. units}$	B1 B1 M1 M1 A1
		05
4	$px^2+qx+r=0$ $x^2+\frac{q}{p}x+\frac{r}{p}=0$ sum of roots, $(\alpha+\beta)=-\frac{q}{p}$ product of roots, $\alpha\beta=\frac{q}{r}$ $(\alpha-2\beta)(\beta-2\alpha)=\alpha\beta-2\alpha^2-2\beta^2+4\alpha\beta=5\alpha\beta-2(\alpha^2+\beta^2)$ $=5\alpha\beta-2[(\alpha+\beta)^2-2\alpha\beta]=9\alpha\beta-2(\alpha+\beta)^2$ $=\frac{9r}{p}-2\left(-\frac{q}{p}\right)^2=\frac{9r}{p}-\frac{2q^2}{p^2}$ For the hence part $(\alpha-2\beta)(\beta-2\alpha)=\frac{9r}{p}-\frac{2q^2}{p^2}$ $[\alpha-2(2\alpha)][2\alpha-2\alpha]=\frac{9pr-2q^2}{p^2}$ $0=\frac{9pr-2q^2}{p^2}$ $0=9pr-2q^2$ $9pr=2q^2$	B1 M1 B1 M1 A1
		05
5	$A=4\pi r^2, \quad \Rightarrow \frac{dA}{dr}=8\pi r=8\pi\times 0.6=4.8\pi$ $V=\frac{4}{3}\pi r^3, \quad \Rightarrow \frac{dV}{dr}=4\pi r^2=4\pi\times(0.6)^2=1.44\pi$ $\frac{dA}{dt}=0.9 \text{ m}^2/\text{s}$ $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dA}\times\frac{dA}{dt}=1.44\pi\times\frac{1}{4.8\pi}\times 0.9=0.27 \text{ m}^3/\text{s}$	B1 B1 M1 B1 A1
		05
6	$\lvert Z\rvert=\frac{\lvert 1-i\rvert\lvert i-2\rvert}{\lvert 1+i\rvert}=\frac{\sqrt{1^2+1^2}\times\sqrt{1^2+(-2)^2}}{\sqrt{1^2+1^2}}=\frac{\sqrt{2}\times\sqrt{5}}{\sqrt{2}}=\sqrt{5}$	M1 A1

	$\text{Arg}(Z)=\text{Arg}(1-i)+\text{Arg}(i-2)-\text{Arg}(1+i)$ $=\left[-\tan^{-1}\left(\frac{1}{1}\right)\right]+\left[180°-\tan^{-1}\left(\frac{1}{2}\right)\right]-\tan^{-1}\left(\frac{1}{1}\right)$ $=-45°+153.43°-45°=63.43°$	B1 M1 A1
		05
7	let $y=\cos(x^2e^x)$, and, $u=x^2e^x$, $\Rightarrow y=\cos u$ $\frac{du}{dx}=x^2e^x+2xe^x=x(x+2)e^x, \quad \frac{dy}{du}=-\sin u$ $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=-\sin(x^2e^x)\times x(x+2)e^x$ $=-x(x+2)e^x\sin(x^2e^x)$	B1 B1 B1 M1 A1
		05
8	 $\overline{AP}=2\overline{MP}$ $\sqrt{(x-5)^2+(y-3)^2}=2\sqrt{(x-2)^2}$ $(x^2-10x+25)+(y^2-6y+9)=4(x^2-4x+4)$ $(x^2-10x+25)+(y^2-6y+9)=4x^2-16x+16$ $y^2-6y-3x^2+6x+18=0$ $3x^2-y^2-6x+6y-18=0$	B1 M1 M1 M1 A1
		05
9	(a). $f(x)=b+ax-4x^2+8x^3$ For $(x+1)=0, x=-1$ $f(-1)=b+a(-1)-4(-1)^2+8(-1)^3=-19$ $b-a-4-8=-19$ $b-a=-7 \quad \rightarrow (1)$ For $(2x-1)=0, x=\frac{1}{2}$ $f(0.5)=b+a(0.5)-4(0.5)^2+8(0.5)^3=2$ $b+0.5a-1+1=2$ $b+0.5a=2 \quad \rightarrow (2)$ Equation (2) − (1) gives: $1.5a=9, \quad \Rightarrow a=6$ From equation (1), $b=a-7=6-7=-1$ (b).	B1 B1 M1 A1 A1

$$px^2 + qx + r = 0$$
$$x^2 + \frac{q}{p}x + \frac{r}{p} = 0$$

sum of roots, $(\alpha + \beta) = 4$ **B1**

product of roots, $\alpha\beta = 2$

for the new equation,

sum of new roots $= (\alpha + 2\beta) + (2\alpha + \beta)$

$= 3\alpha + 3\beta = 3(\alpha + \beta) = 3 \times 4 = 12$ **M1 B1**

product of new roots $= (\alpha + 2\beta)(2\alpha + \beta) = 2\alpha^2 + \alpha\beta + 4\alpha\beta + 2\beta^2$

$= 2(\alpha^2 + \beta^2) + 5\alpha\beta = 2[(\alpha + \beta)^2 - 2\alpha\beta] + 5\alpha\beta$ **M1**

$= 2(\alpha + \beta)^2 + \alpha\beta = 2 \times 4^2 + 2 = 34$ **M1 B1**

$x^2 - (\text{sum of new roots})x + (\text{product of new roots}) = 0$

$x^2 - 12x + 34 = 0$ **B1**

12

10

Asymptotes

For vertical asymptote,

$(x - 1) = 0, \quad \Rightarrow x = 1$ **B1**

For slanting asymptote,

$$y = \frac{x^2 - x - 6}{x - 1} = \frac{x(x - 1) - 6}{x - 1} = x - \frac{6}{x - 1}$$ **B1**

Slanting asymptote is $y = x$. **B1**

Intercepts

when, $x = 0, \quad y = \frac{-6}{-1} = 6$ **B1**

when, $y = 0, \quad x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0, \quad \Rightarrow x = -2, \quad$ or, $\quad x = 3$ **B1 B1**

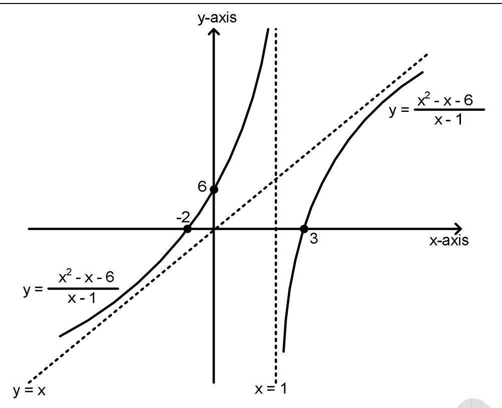
Intercepts are $(0,6)$, $(-2,0)$, $(3,0)$. **B1**

Region where the curve lies

	$x < -2$	$-2 < x < 1$	$1 < x < 3$	$x > 3$
$(x - 3)$	-	-	-	+
$(x + 2)$	-	+	+	+
$(x - 1)$	-	-	+	+
y	-	+	-	+

B1

Sketch



B1-curve 1

B1-curve 2

B1-axes

12

11

(a).

$$3\sin x + 4\cos x = 2$$

$3\sin x + 4\cos x \equiv R\sin(x + \alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$

By comparison,

$R\cos\alpha = 3, \quad R\sin\alpha = 4$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{4}{3}, \quad \Rightarrow \tan\alpha = \frac{4}{3}, \quad \Rightarrow \alpha = 53.13°$ **B1** **B1**

$R = \sqrt{3^2 + 4^2} = 5$ **B1**

$3\sin x + 4\cos x \equiv 5\sin(x + 53.13°)$ **M1**

$5\sin(x + 53.13°) = 2$

$\sin(x + 53.13°) = 0.4$

$(x + 53.13°) = 23.58°, 156.42°$

$x = -29.55°, 103.29°$ **A1 A1**

(b).

From sine rule, $a = k\sin A$, $b = k\sin B$, $c = k\sin C$.

$$L.H.S = \frac{b^2 - c^2}{a^2} = \frac{(k\sin B)^2 - (k\sin C)^2}{(k\sin A)^2} = \frac{\sin^2 B - \sin^2 C}{\sin^2 A}$$ **B1**

$$= \frac{(\sin B - \sin C)(\sin B + \sin C)}{\sin^2 A}$$ **B1**

$$= \frac{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right) \times 2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\sin^2(B + C)}$$ **B1**

$$= \frac{2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B-C}{2}\right) \times 2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B+C}{2}\right)}{\sin^2(B + C)}$$ **B1**

$$= \frac{\sin\left\{2\left(\frac{B-C}{2}\right)\right\}\sin\left\{2\left(\frac{B+C}{2}\right)\right\}}{\sin^2(B + C)}$$ **B1**

$$= \frac{\sin(B - C)\sin(B + C)}{\sin^2(B + C)} = \frac{\sin(B - C)}{\sin(B + C)}$$ **B1**

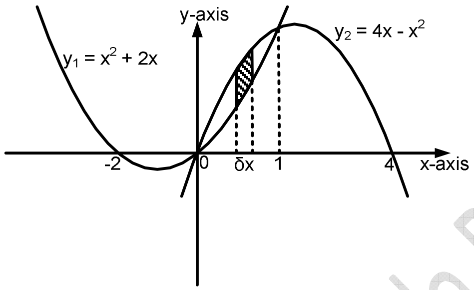
12

12

(a).

At the points of intersection,

$$x(x+2) = x(4-x)$$
$$x^2 + 2x = 4x - x^2$$
$$2x^2 - 2x = 0$$
$$2x(x-1) = 0$$
$$x = 0, \quad \text{or,} \quad x = 1$$ **B1**



B1-curve 1

B1-curve 2

(b).

$$\text{area} = \int_0^1 (y_2 - y_1)\,dx = \int_0^1 [4x - x^2 - (2x^2 + 2x)]\,dx$$ **M1**

$$= \int_0^1 (2x - 2x^2)\,dx = \left[x^2 - \frac{2}{3}x^3\right]_0^1 = \left(1 - \frac{2}{3}\right) - 0 = \frac{1}{3} \text{ square units}$$ **M1 M1 A1**

(c).

$$\text{volume} = \pi \int_0^1 (y_2 - y_1)^2\,dx = \pi \int_0^1 (2x - 2x^2)^2\,dx$$ **B1 B1**

$$= \pi \int_0^1 (4x^2 - 8x^3 + 4x^4)\,dx = \pi\left[\frac{4}{3}x^3 - 2x^4 + \frac{4}{5}x^5\right]_0^1$$ **M1 M1**

$$= \pi\left(\frac{4}{3} - 2 + \frac{4}{5}\right) - 0 = \frac{2\pi}{15} \text{ cubic units}$$ **M1 A1**

12

13

(a).

$$\frac{3x^2 + x + 1}{(x-2)(x+1)^2} \equiv \frac{A}{(x-2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$ **B1**

$$3x^2 + x + 1 \equiv A(x+1)^2 + B(x-2)(x+1) + C(x-2)$$ **B1**

Putting $x = -1$; $3 - 1 + 1 = -3C$, $\Rightarrow C = -1$ **B1**

Putting $x = 2$; $12 - 2 + 1 = 9A$, $\Rightarrow A = \frac{5}{3}$ **B1**

Coefficient of x^2; $3 = A + B$, $\Rightarrow B = 3 - \frac{5}{3} = \frac{4}{3}$ **B1**

$$\frac{3x^2 + x + 1}{(x-2)(x+1)^2} \equiv \frac{5}{3(x-2)} + \frac{4}{3(x+1)} - \frac{1}{(x+1)^2}$$ **B1**

(b).

$$\int_3^4 \frac{3x^2 + x + 1}{(x-2)(x+1)^2}\,dx$$

$$= \frac{5}{3}\int_3^4 \frac{1}{(x-2)}\,dx + \frac{4}{3}\int_3^4 \frac{1}{(x+1)}\,dx - \frac{4}{3}\int_3^4 \frac{1}{(x+1)^2}\,dx$$ **M1**

$$= \left[\frac{5}{3}\ln(x-2) + \frac{4}{3}\ln(x+1) + \frac{1}{(x+1)}\right]_3^4$$ **M1**

$$= \left(\frac{5}{3}\ln 2 + \frac{4}{3}\ln 5 + \frac{1}{5}\right) - \left(\frac{5}{3}\ln 1 + \frac{4}{3}\ln 4 + \frac{1}{4}\right)$$ **M1**

$$= \frac{5}{3}\ln 2 + \frac{4}{3}\ln\left(\frac{5}{4}\right) - \frac{1}{20} \approx 1.40277 = 1.403 \ (3\text{ d.p})$$ **A1 B1**

12

14

(a). (i).

$$\text{let,} \quad \lambda = \frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1}$$

$$\lambda = \frac{x+1}{2}, \quad \Rightarrow \quad x = 2\lambda - 1$$
$$\lambda = \frac{y-3}{5}, \quad \Rightarrow \quad y = 5\lambda + 3$$ **B1**
$$\lambda = \frac{z+1}{-1}, \quad \Rightarrow \quad z = -\lambda - 1$$

$$2(2\lambda - 1) - (5\lambda + 3) + 3(-\lambda - 1) = 20$$ **M1**
$$4\lambda - 2 - 5\lambda - 3 - 3\lambda - 3 = 20$$
$$-4\lambda = 28$$
$$\lambda = -7$$ **A1**
$$x = 2(-7) - 1 = -15$$
$$y = 5(-7) + 3 = -32$$
$$z = -(-7) - 1 = 6$$

Point of intersection is: $(-15, -32, 6)$ **B1**

(ii).

Direction vector, $\underset{\sim}{d} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$, Normal vector, $\underset{\sim}{n} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

Let θ be the angle between the given line and plane.

$$\sin\theta = \frac{\underset{\sim}{n}.\underset{\sim}{d}}{|\underset{\sim}{n}||\underset{\sim}{d}|} = \frac{\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}.\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}}{\sqrt{(4+25+1)(4+1+9)}} = \frac{4 - 5 - 3}{\sqrt{30 \times 14}} = -0.1952$$ **M1 M1**

$$\theta = 78.74°$$ **A1**

(b).

$$\overrightarrow{AP} = \begin{pmatrix} x \\ y-3 \\ z+4 \end{pmatrix}, \overrightarrow{AB} = \begin{pmatrix} 2 \\ -1-3 \\ 2+4 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 7 \\ 4-3 \\ -1+4 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix}$$ **B1 B1**

$$(\overrightarrow{AP}).(\overrightarrow{AB} \times \overrightarrow{AC}) = 0$$

$$\begin{vmatrix} x & y-3 & z+4 \\ 2 & -4 & 6 \\ 7 & 1 & 3 \end{vmatrix} = 0$$ **M1**

$$x\begin{vmatrix} -4 & 6 \\ 1 & 3 \end{vmatrix} - (y-3)\begin{vmatrix} 2 & 6 \\ 7 & 3 \end{vmatrix} + (z+4)\begin{vmatrix} 2 & -4 \\ 7 & 1 \end{vmatrix} = 0$$

$$x(-12-6)-(y-3)(6-42)+(z+4)(2+28)=0$$ **M1**
$$-18x+36(y-3)+30(z+4)=0$$
$$-18x+36(y-3)+30(z+4)=0$$
$$-18x+36y-108+30z+120=0$$
$$-18x+36y+30z+12=0$$
$$3x-6y-5z-2=0$$ **A1**

Alternatively:

$$\overrightarrow{AB}=\begin{pmatrix}2\\-1-3\\2+4\end{pmatrix}=\begin{pmatrix}2\\-4\\6\end{pmatrix},\qquad \overrightarrow{AC}=\begin{pmatrix}7\\4-3\\-1+4\end{pmatrix}=\begin{pmatrix}7\\1\\3\end{pmatrix}$$
$$\underset{\sim}{r}=\overrightarrow{OA}+\lambda\overrightarrow{AB}+\mu\overrightarrow{AC}$$
$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\3\\-4\end{pmatrix}+\lambda\begin{pmatrix}2\\-4\\6\end{pmatrix}+\mu\begin{pmatrix}7\\1\\3\end{pmatrix}$$
$$x=2\lambda+7\mu \quad\longrightarrow(1)$$
$$y=3-4\lambda+\mu \quad\longrightarrow(2)$$
$$z=-4+6\lambda+3\mu \quad\longrightarrow(3)$$

Equation $2\times(1)+(2)$ gives,
$$\begin{array}{r|l|l} & 2x=4\lambda+14\mu & \to 2\times(1)\\ + & y=3-4\lambda+\mu & \to(2)\\ \hline & 2x+y=3+15\mu & \to(4)\end{array}$$

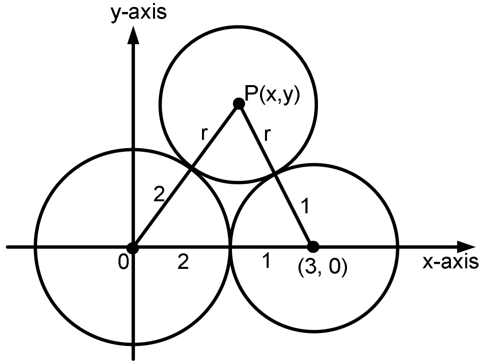
Equation $3\times(1)-(3)$ gives,
$$\begin{array}{r|l|l} & 3x=6\lambda+21\mu & \to 3\times(1)\\ - & z=-4+6\lambda+3\mu & \to(3)\\ \hline & 3x-z=4+18\mu & \to(5)\end{array}$$

Equation $3\times(1)-(3)$ gives,
$$\begin{array}{r|l|l} & 36x+18y=54+270\mu & \to 18\times(4)\\ - & 45x-15z=60+270\mu & \to 15\times(5)\\ \hline & -9x+18y+15z=-6 & \end{array}$$
$$3x-6y-5z=2$$

12

15 (a).
$$\overline{PB}=2\overline{PA}$$
$$\sqrt{(3-x)^2+y^2}=2\sqrt{(-3-x)^2+y^2}$$ **B1**
$$9-6x+x^2+y^2=4(9+6x+x^2+y^2)$$ **M1**
$$9-6x+x^2+y^2=36+24x+4x^2+4y^2$$
$$0=27+30x+3x^2+3y^2$$ **M1**
$$x^2+y^2+10x+9=0$$
Centre $(-5,0)$, Radius $=\sqrt{(-5)^2-9}=4$ units **A1** **B1 B1**

(b).
For $x^2+y^2=4$,
Centre $=(0,0)$, Radius $=\sqrt{4}=2$ units
For $x^2+y^2-6x+8=0$,
Centre $=(3,0)$, Radius $=\sqrt{(-3)^2-8}=1$ unit



B1

$$x^2+y^2=(2+r)^2 \quad\longrightarrow(1)$$ **B1**
$$(x-3)^2+y^2=(1+r)^2 \quad\longrightarrow(2)$$ **B1**

Equation $(1)-(2)$ gives:
$$x^2-(x-3)^2=(2+r)^2-(1+r)^2$$ **M1**
$$x^2-(x^2-6x+9)=(2+r+1+r)(2+r-1-r)$$ **M1**
$$6x-9=3+2r$$
$$6x=12+2r$$
$$x=2+\frac{r}{3}$$ **A1**

12

16 (a).
$$\frac{Z-1}{Z+1}=\frac{(x-1)+iy}{(x+1)+yi}=\frac{\{(x-1)+iy\}\times\{(x+1)-iy\}}{\{(x+1)+yi\}\times\{(x+1)-iy\}}$$ **B1**
$$=\frac{(x-1)(x+1)-iy(x-1)+iy(x+1)+y^2}{(x+1)^2+y^2}$$
$$=\frac{x^2-1-ixy+iy+ixy+iy+y^2}{(x+1)^2+y^2}$$
$$=\frac{x^2+y^2-1+2yi}{(x+1)^2+y^2}$$ **B1**

real part $=\dfrac{x^2+y^2-1}{(x+1)^2+y^2}$, imaginary part $=\dfrac{2y}{(x+1)^2+y^2}$

$$\text{Arg}\left(\frac{Z-1}{Z+1}\right)=\tan^{-1}\left(\frac{\text{imaginarypart}}{\text{realpart}}\right)=\frac{\pi}{4}$$
$$\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right)=\frac{\pi}{4}$$ **M1**
$$\frac{2y}{x^2+y^2-1}=\tan\frac{\pi}{4}=1$$
$$2y=x^2+y^2-1$$ **M1**
$$x^2+y^2-2y-1=0$$ **A1**

(b).
$$\frac{Z}{Z+2}=2-i$$

	$$\frac{x+iy}{(x+2)+iy} = 2-i$$	
	$$x+iy = (2-i)[(x+2)+iy]$$	B1
	$$x+iy = 2(x+2)+2iy-i(x+2)+y$$	
	$$x+iy = 2x+4+2iy-ix-2i+y$$	
	$$x+iy = (2x+y+4)+i(2y-x-2)$$	
	Comparing real parts,	
	$$x = 2x+y+4$$	
	$$x+y=-4 \quad \longrightarrow (1)$$	
	Comparing real parts,	B1
	$$y = 2y-x-2$$	
	$$y-x = 2 \quad \longrightarrow (2)$$	
	Equation (1) + (2) gives:	B1
	$$2y=-2, \qquad \Longrightarrow y=-1$$	
	$$x = y-2 = -1-2 = -3$$	B1
	$$\therefore Z = -3-i, \qquad \lvert Z\rvert = \sqrt{(-3)^2+(-1)^2} = \sqrt{10}$$	M1 A1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
April 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 2 MOCK SET 2 2018
Time: 3 Hours

NAME:______________________________________ **COMB:**__________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B.**
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)

Answer **all** *the questions in this section*

Qn 1: ***AB*** is a vertical pole of height 77.5 m, with ***A*** standing on the horizontal ground. A particle is projected from the top ***B*** with initial velocity $12\underset{\sim}{\boldsymbol{i}} + 9\underset{\sim}{\boldsymbol{j}}$ m s^{-1}.

(i). How long does the particle take in air before striking the ground?
(ii). At what distance from ***A*** does the particle strike the ground? [5]

Qn 2: Two events A and B are such that $P(A'/B') = \frac{2}{7}$ and $P(B) = \frac{2}{3}$. Find the:

(i). $P(A \cup B)$,
(ii). $P(A \cap B')$. [5]

Qn 3: Given that $y = \sin\theta$ and θ is measured with a maximum possible error of 2%. If $\theta = 30°$, determine the:

(i). absolute error in y,
(ii). interval within which the value of y lies. Give your answer correct to 4 significant figures. [5]

Qn 4: Forces of $\left(\underset{\sim}{i} + \underset{\sim}{j}\right)$ N, $\left(-4\underset{\sim}{i} + \underset{\sim}{j}\right)$ N and $\left(3\underset{\sim}{i} - 2\underset{\sim}{j}\right)$ N act at points with position vectors $\left(2\underset{\sim}{i} + 2\underset{\sim}{j}\right)$, $\left(-\underset{\sim}{i} + \underset{\sim}{j}\right)$ and $\left(4\underset{\sim}{i} - 2\underset{\sim}{j}\right)$ respectively. Show that the forces reduce to a couple and find the moment of the couple. [5]

Qn 5: A random variable X is uniformly distibuted over the interval $[-3, 9]$. Find the:
(i). Var(X),
(ii). $P(|X| > 1.5)$. [5]

Qn 6: The temperatures (°C) of a cooling body measured every 10 minutes were recorded as 82, 70, 56, and 42. If the body's initial temperature is 93°C , find, using linear interpolation/extrapolation, the:
(i). time taken for the body to cool to 63°C.
(ii). temperature of the body after 45 minutes. [5]

Qn 7: The force, F, acting on a particle of mass 2 kg is given by $F = 5 + 4t$ N, where t is the time in seconds. Given that initially the particle is moving at a speed of 5 m s^{-1}, find the speed of the particle when $t = 2$. [5]

Qn 8: The table below shows the average monthly wage in thousands of shillings of workers in category in 2014 and 2016 for a certain soft drinks factory.

Category	Monthly wage		Index number	Number of workers
	2014	2016		
1	120	192	160	180
2	150	285	X	165
3	Y	330	200	100
4	170	Z	250	55

Taking 2014 = 100%, find the:
(i). values of X, Y and Z.
(ii). Weighted index number for the monthly wage of the whole factory in 2016. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The heights (cm) of senior six candidates in a certain school were recorded as in the table below.

Height (cm)	Frequency
148 - < 152	5
152 - < 156	8
156 - < 160	12
160- < 164	15
164 - < 168	6
168 - < 172	4

(a). Calculate the:
(i). Mean,
(ii). Standard deviation. [7]
(b). Calculate the unbiased estimate of the variance and hence construct a 95% confidence interval for the mean height of all the senior six candidates. [5]

Question 10:
(a). The numbers 2.6754, 4.802, 15.18 and 0.925 are rounded off to the given number of decimal places. Find the range within which the exact value of: $2.6754\left(4.802 - \frac{15.18}{0.925}\right)$ can be expected to lie. [6]
(b). The numbers a, b, c and d are all rounded with errors e_1 , e_2 , e_3 and e_4 respectively. Show that the expression for the maximum absolute error, e_z in $z = \frac{ab}{c+d}$ is: $\frac{ab}{c+d}\left(\left|\frac{e_1}{a}\right| + \left|\frac{e_2}{b}\right| + \left|\frac{e_3+e_4}{c+d}\right|\right)$. [6]

Question 11:
Two vehicles $\boldsymbol{P}$ and $\boldsymbol{Q}$ travel equal distance of 36 km in the same time of 36 minutes. Vehicle $\boldsymbol{P}$ moves at a constant speed for the first 14.4 km and is then brought to rest with a uniform retardation. Vehicle $\boldsymbol{Q}$ starts from rest and accelerates uniformly to a speed of 90 km h^{-1}; travels steadily at this speed; and is then brought to rest under a uniform retardation.
(a). Sketch the velocity-time graphs for each vehicle's motion. [2]
(b). Calculate the:
(i). Initial speed of $\boldsymbol{P}$, [3]
(ii). Retardation of $\boldsymbol{P}$ in m s^{-2}, [3]
(iii). Distance $\boldsymbol{Q}$ travels at a steady speed. [4]

Question 12:

(a). A random variable R has the probability distribution function given below:

$$P(R=r)=\begin{cases}\frac{k}{17}\left[2^{(9-r)}\right] & ; \quad r=0,1,2,\dots,n \\ 0 & ; \quad \text{elsewhere.}\end{cases}$$

Given that $P(R=n)=\frac{1}{255}$;

(i). Find the value of n and show that $k=\frac{1}{60}$. [6]

(ii). Find $P(R>2/R\neq 2)$. [2]

(b). A biased tetra-hedral die, whose faces are labelled 1, 2, 3, 4 is such that getting an even face is twice as likely as getting an odd one. Find the probability of getting a 4 not less than twice in the 10 throws. [4]

Question 13:

(a). Use the trapezium rule with six ordinates to find the approximate value of $\int_2^5 xe^{-x}\,dx$ correct to 3 significant figures.

(b). Find the area bounded by the curve $y=xe^{-x}$ between $x=2$ and $x=5$.

(c). Find the percentage error in (a) and (b) above. [12]

Question 14:

(a). Two particles of masses $2\boldsymbol{m}$ and $3\boldsymbol{m}$ are moving towards each other with speeds $4\boldsymbol{u}$ and $2\boldsymbol{u}$ respectively. After the impact, the direction of motion of the heavier particle is reversed and moves with speed $\boldsymbol{u}$. Find the loss in kinetic energy during impact. [6]

(b). An engine raises water from an underground reservoir through a height of 12 m and delivers it at a speed of 5 m s^{-1} through a square pipe of internal length of 35 cm. By taking a mass of 1 litre of water to be 1 kg, find the:

(i). mass of water raised per second,

(ii). Effective power of the engine. [6]

Question 15:

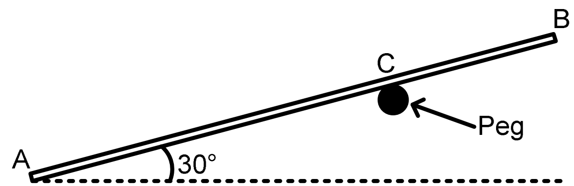
The mean life of a certain make of dry cells is 150 days and standard deviation 32 days. Their duration is normally distributed.

(a). Find the probability that the cells will last between 125 and 210 days.

(b). If there are 300 dry cells, calculate how many will need replacement after 225 days.

(c). After how many days will a quarter of the cells have expired? [12]

Question 16:



A uniform rod $\boldsymbol{AB}$ of weight 12 N is smoothly hinged at $\boldsymbol{A}$, and rests in equilibrium against a horizontal peg at $\boldsymbol{C}$, where $\overline{\boldsymbol{AC}}=\frac{3}{4}\overline{\boldsymbol{AB}}$**.** The rod makes 30° with the horizontal.

(a). Find the reactions at $\boldsymbol{C}$, and at $\boldsymbol{A}$. [6]

(b). If the peg is removed and a force is applied at $\boldsymbol{B}$, to keep the rod at rest in the same position, find the minimum value of the force required; and the new magnitude of reaction at $\boldsymbol{A}$. [6]

END

MARKING GUIDE

SNo.	Working	Marks
1	(i). $\underset{\sim}{\boldsymbol{u}} = \begin{pmatrix}12\\9\end{pmatrix}\ \mathrm{m\,s^{-1}}, \underset{\sim}{\boldsymbol{a}} = \begin{pmatrix}0\\-9.8\end{pmatrix}\ \mathrm{m\,s^{-2}}, \underset{\sim}{\boldsymbol{s}} = \begin{pmatrix}x\\-77.5\end{pmatrix}\ \mathrm{m}$ $\underset{\sim}{\boldsymbol{s}} = \underset{\sim}{\boldsymbol{u}}t + \frac{1}{2}\underset{\sim}{\boldsymbol{a}}t^2$ $\begin{pmatrix}x\\-77.5\end{pmatrix} = \begin{pmatrix}12\\9\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}0\\-9.8\end{pmatrix}t^2$	M1
	$-77.5 = 9t - 4.9t^2$ $4.9t^2 - 9t - 77.5 = 0$ $t = \dfrac{9 \pm \sqrt{(-9)^2 - 4 \times 4.9 \times (-77.5)}}{2 \times 4.9} = \dfrac{9 \pm 40}{9.8}$	M1
	$t = \dfrac{9+40}{9.8} = 5$, or, $t = \dfrac{9-40}{9.8} = -3.1633$ but $t \neq -3.1633$, $\Rightarrow t = 5$ s	A1
	(ii). $x = 12t = 12 \times 5 = 60$ m	M1 A1
		05
2	(i). $P(B') = 1 - \frac{2}{3} = \frac{1}{3}$ $P(A' \cap B') = P(B').P(A'/B') = \frac{1}{3} \times \frac{2}{7} = \frac{2}{21} \approx 0.0952$	B1
	$P(A \cup B) = 1 - P(A \cup B)' = 1 - P(A' \cap B') = 1 - \frac{2}{21} = \frac{19}{21}$ ≈ 0.9048	M1 A1
	(ii). $P(A \cap B') = P(B') - P(A' \cap B') = \frac{1}{3} - \frac{2}{21} = \frac{5}{21} \approx 0.2381$	M1 A1
		05
3	(i). $\theta = 30° = \frac{\pi}{6}$, $\delta\theta = \frac{2}{100} \times \frac{\pi}{6} = \frac{\pi}{300}$	B1
	$y = \sin\theta$, $\Rightarrow \dfrac{dy}{dx} = \cos\theta = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ but, $\dfrac{\delta y}{\delta x} \approx \dfrac{dy}{dx}$ $\Rightarrow \delta y = \dfrac{dy}{dx} \times \delta x = \frac{\sqrt{3}}{2} \times \frac{\pi}{300} = 0.009069$ (4 s. f)	B1
	(ii). $y = \sin\theta = \sin\left(\frac{\pi}{6}\right) = 0.5$ $y_{min} = 0.5 - 0.009069 = 0.4909$ (4 s. f)	B1
	$y_{max} = 0.5 + 0.009069 = 0.5091$ (4 s. f)	B1
	Interval $= [0.4909, 0.5091]$	B1
	Alternatively:	

	(i). $\delta\theta = \frac{2}{100} \times 30 = 0.6°$	B1
	$y_{max} = \sin(30° + 0.6°) = \sin 30.6° = 0.5090$ (4 s. f)	B1
	$y_{min} = \sin(30° - 0.6°) = \sin 29.4° = 0.4909$ (4 s. f)	B1
	absolute error in $y = \dfrac{0.5090 - 0.4909}{2} = 0.00905$	B1
	(ii). Interval $= [0.4909, 0.5090]$	A1
		05
4	Resultant force, $\underset{\sim}{\boldsymbol{F_R}} = \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}-4\\1\end{pmatrix} + \begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ N	B1
	$G = \sum\left(\underset{\sim}{\boldsymbol{r}} \times \underset{\sim}{\boldsymbol{F}}\right) = \begin{vmatrix}2 & 2\\1 & 1\end{vmatrix} + \begin{vmatrix}-1 & 1\\-4 & 1\end{vmatrix} + \begin{vmatrix}4 & -2\\3 & -2\end{vmatrix}$	M1
	$= (2-2) + (-1+4) + (-8+6) = 1$ N m	B1
	Since the resultant force is zero but the resultant moment is not zero, the system is equivalent to a couple.	B1
	Resultant moment about O = 1 N m anticlockwise $\Rightarrow$ Required moment of a couple = 1 N m clockwise	A1
		05
5	(i). $Var(X) = \dfrac{(b-a)^2}{12} = \dfrac{(9 - {}^{-}3)^2}{12} = 12$	M1 A1
	(ii). $f(x) = \dfrac{1}{b-a} = \dfrac{1}{9 - {}^{-}3} = \dfrac{1}{12}$	B1
	$P(\lvert X\rvert > 1.5) = 1 - P(\lvert X\rvert < 1.5) = 1 - P(-1.5 < X < 15)$ $= 1 - \frac{1}{12} \times (1.5 - {}^{-}1.5) = 1 - \frac{1}{4} = \frac{3}{4}$	M1 A1
		05
6	(i). $\begin{array}{\|l\|c\|c\|c\|}\hline \text{Temperature} & 70 & 63 & 56 \\ \hline \text{Time} & 20 & y_0 & 30 \\ \hline \end{array}$	B1
	$\dfrac{y_0 - 20}{30 - 20} = \dfrac{63 - 70}{56 - 70}$	M1
	$\dfrac{y_0 - 20}{10} = 0.5$ $y_0 = 20 + 0.5 \times 10 = 25$ minutes	A1
	(ii). $\begin{array}{\|l\|c\|c\|c\|}\hline \text{Temperature} & 56 & 42 & x_0 \\ \hline \text{Time} & 30 & 40 & 45 \\ \hline \end{array}$ $\dfrac{x_0 - 42}{56 - 42} = \dfrac{45 - 40}{30 - 40}$	M1

$$\frac{x_0 - 42}{14} = -0.5$$
$$t_0 = 42 - 0.5 \times 14 = 35°C$$ **A1**

05

7

$$m = 2\text{ kg}, \quad u = 5\text{ m s}^{-1}, \quad F = 5 + 4t$$
$$a = \frac{F}{m} = \frac{5 + 4t}{2} = 2.5 + 2t$$ **B1**
$$v = \int (2.5 + 2t)\,dt$$
$$v = 2.5t + t^2 + c$$ **B1**

When $t = 0, v = 5$,
$$5 = 0 + 0 + c, \qquad \Rightarrow c = 5$$ **B1**
$$\therefore v = 2.5t + t^2 + 5$$

When $t = 2$,
$$v = 2.5 \times 2 + 2^2 + 5 = 14\text{ m s}^{-1}$$ **M1 A1**

05

8

(i).
$$I = \frac{P_{2016}}{P_{2014}} \times 100$$

For category 2, $X = \frac{285}{150} \times 100 = 190$ **B1**

For category 3, $200 = \frac{330}{Y} \times 100,$
$$\Rightarrow Y = \frac{330 \times 100}{200} = 165$$ **B1**

For category 4, $250 = \frac{Z}{170} \times 100,$
$$\Rightarrow Z = \frac{250 \times 170}{100} = 425$$ **B1**

(ii).
$$\text{Weighted index number} = \frac{\sum I \times W}{\sum W}$$
$$= \frac{160 \times 180 + 190 \times 165 + 200 \times 100 + 250 \times 55}{180 + 165 + 100 + 55}$$ **M1**
$$= \frac{93900}{500} = 187.8$$ **A1**

05

9

Class	f	x	fx	fx^2
148 - < 152	5	150	750	112500
152 - < 156	8	154	1232	189728
156 - < 160	12	158	1896	299568
160- < 164	15	162	2430	393660
164 - < 168	6	166	996	165336
168 - < 172	4	170	680	115600
Total	50		7984	1276392

B1-for x

B1-for fx

B1-for fx^2

(a).

Mean, $\bar{x} = \frac{\sum fx}{\sum f} = \frac{7984}{50} = 159.68$ **M1 A1**

Standard deviation, $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$
$$= \sqrt{\frac{1276392}{500} - (159.68)^2} = \sqrt{30.1376} \approx 5.4898$$ **M1 M1 A1**

(b).
$$\sigma^2{}_{\text{unbiased}} = \left(\frac{n}{n-1}\right)\sigma^2 = \left(\frac{50}{50-1}\right) \times 30.1376 = 30.7527$$ **B1**

For the 95% confidence interval,
$$\phi(z_{\alpha/2}) = \frac{0.95}{2} = 0.475, \qquad \Rightarrow z_{\alpha/2} = \phi^{-1}(0.475) = 1.96$$ **B1**
$$\mu = \bar{x} \pm z_{\alpha/2}\sqrt{\frac{\sigma^2{}_{\text{unbiased}}}{n}} = 159.68 \pm 1.96 \times \sqrt{\frac{30.7527}{50}}$$ **M1**
$$= 159.68 \pm 1.5371$$
Confidence interval $= [158.1429, 161.2171]$ **A1**

12

10

(a).
$$\left[2.6754\left(4.802 - \frac{15.18}{0.925}\right)\right]_{min}$$
$$= (2.6754 - 0.00005)\left[(4.802 - 0.0005) - \frac{15.18 + 0.005}{0.925 - 0.0005}\right]$$ **M1**
$$= -31.097184$$ **B1**
$$\left[2.6754\left(4.802 - \frac{15.18}{0.925}\right)\right]_{max}$$
$$= (2.6754 + 0.00005)\left[(4.802 + 0.0005) - \frac{15.18 - 0.005}{0.925 + 0.0005}\right]$$ **M1**
$$= -31.019281$$ **B1**
$$\text{Range} = [{}^{-}31.097184, {}^{-}31.019281]$$ **A1**

(b). Let A, B, C and D be the exact values of a, b, c and d respectively.
$$\Rightarrow Z = \frac{AB}{C + D}$$
$$z + e_z = \frac{(a + e_1)(b + e_2)}{(c + e_3) + (d + e_4)}$$ **M1**
$$z + e_z = \frac{ab + ae_2 + be_1 + e_1e_2}{(c + d) + (e_3 + e_4)}$$
suppose $e_1 \ll a$ and $e_2 \ll b$, $\quad \Rightarrow e_1e_2 \approx 0$
$$z + e_z = \frac{[ab + ae_2 + be_1] \times [(c + d) - (e_3 + e_4)]}{[(c + d) + (e_3 + e_4)] \times [(c + d) - (e_3 + e_4)]}$$ **B1**
$$z + e_z = \frac{(c + d)[ab + ae_2 + be_1] - (e_3 + e_4)[ab + ae_2 + be_1]}{(c + d)^2 - (e_3 + e_4)^2}$$

$$\text{suppose } e_1 \ll a, e_2 \ll b, e_3 \ll c \text{ and } e_4 \ll d,$$

$$\Rightarrow (e_3+e_4)^2 \approx 0, \quad (e_3+e_4)e_1 \approx 0, \quad (e_3+e_4)e_2 \approx 0$$ **B1**

$$z + e_z = \frac{(c+d)[ab + ae_2 + be_1] - ab(e_3+e_4)}{(c+d)^2}$$

$$z + e_z = \frac{ab}{c+d} + \frac{be_1}{c+d} + \frac{ae_2}{c+d} - \frac{ab(e_3+e_4)}{(c+d)^2}$$

$$e_z = \frac{be_1}{c+d} + \frac{ae_2}{c+d} - \frac{ab(e_3+e_4)}{(c+d)^2}$$ **B1**

$$e_z = \frac{ab}{c+d}\left(\frac{e_1}{a} + \frac{e_2}{b} - \frac{e_3+e_4}{c+d}\right)$$

$$|e_z| = \frac{ab}{c+d}\left(\left|\frac{e_1}{a} + \frac{e_2}{b} - \frac{e_3+e_4}{c+d}\right|\right)$$ **M1**-absolute error

$$|e_z| \le \frac{ab}{c+d}\left(\left|\frac{e_1}{a}\right| + \left|\frac{e_2}{b}\right| + \left|\frac{e_3+e_4}{c+d}\right|\right)$$ **M1**-triangular inequality

$$\text{maximum absolute error} = \frac{ab}{c+d}\left(\left|\frac{e_1}{a}\right| + \left|\frac{e_2}{b}\right| + \left|\frac{e_3+e_4}{c+d}\right|\right)$$ **B1**-concusion as required

12

11 (a).

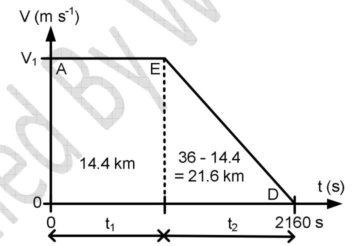
$$90 \text{ km h}^{-1} = \frac{90 \times 1000}{3600} = 25 \text{ m s}^{-1}$$

$$36 \text{ minutes} = 36 \times 60 = 2160 \text{ s}$$

$$36 \text{ km} = 36 \times 1000 = 36000 \text{ m}$$

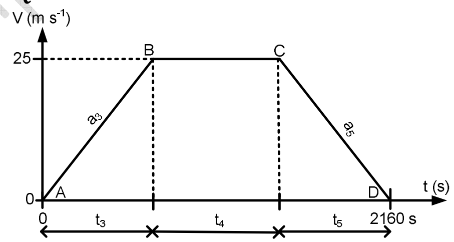
$$14.4 \text{ km} = 14.4 \times 1000 = 14400 \text{ m}$$

For vehicles ***P***



B1

For vehicles ***Q***



B1 B1

(b). (i).
For vehicles ***P***,

$$\text{Distance } AE = v_1 t_1 = 14400$$

$$\text{Distance } AD = \frac{1}{2}v_1(t_1 + 2160) = 36000$$ **M1**

$$v_1 t_1 + 2160 v_1 = 72000$$

$$14400 + 2160 v_1 = 72000$$

$$2160 v_1 = 57600$$ **M1**

$$v_1 = \frac{80}{3} \approx 26.667 \text{ m s}^{-1}$$ **A1**

(ii).

$$v^2 = u^2 + 2as$$

$$0 = \left(\frac{80}{3}\right)^2 + 2a \times 21600$$ **M1**

$$a = -\left(\frac{80}{3}\right)^2 \times \frac{1}{2 \times 21600} = -\frac{4}{243} \approx -0.0165 \text{ m s}^{-2}$$ **M1 A1**

(iii).
For vehicles ***Q***,

$$\text{Distance } AD = \frac{1}{2} \times 25(t_4 + 2160) = 36000$$ **M1**

$$t_4 + 2160 = 2880$$

$$t_4 = 720 \text{ s}$$

$$\text{Distance } BC = 720 \times 25 = 18000 \text{ m}$$ **M1 A1**

12

12 (a). (i).

$$P(R = n) = \frac{k}{17}(2^{9-n}) = \frac{1}{255}$$

$$\frac{512k}{17}(2^{-n}) = \frac{1}{255}, \quad \Rightarrow k(0.5^n) = \frac{1}{7680} \quad \longrightarrow (1)$$ **B1**

$$\sum_{all\ x} P(R = r) = 1$$

$$\frac{k}{17}[2^9 + 2^8 + 2^7 + \cdots + 2^{9-n}] = 1$$

$$\frac{k}{17}\left[a\left(\frac{1-r^n}{1-r}\right)\right] = 1, \quad \text{where}, \quad a = 2^9, \quad r = \frac{2^8}{2^9} = 0.5$$ **B1**

$$\frac{k}{17}\left[2^9 \times \left(\frac{1-0.5^n}{1-0.5}\right)\right] = 1$$

$$\frac{1024k}{17}(1 - 0.5^n) = 1$$ **M1**

$$k(1 - 0.5^n) = \frac{17}{1024} \quad \longrightarrow (2)$$ **B1**

(2) ÷ (1) gives

$$\frac{k(1-0.5^n)}{k(0.5^n)} = \frac{17}{1024} \times 7680$$ **M1**

$$\frac{1}{(0.5^n)} - 1 = 127.5$$

$$0.5^{-n} = 128.5$$

$$\log(0.5^{-n}) = \log(128.5)$$

$$n = -\frac{\log 128.5}{\log 0.5} = 7.0056 \approx 7$$ **B1**

From equation (1),

$$k(0.5^7) = \frac{1}{7680}$$

$$k = \frac{1}{7680 \times 0.5^7} = \frac{1}{60}, \quad \text{hence shown}$$ **M1 A1**

(ii).

$$P(R > 2/R \neq 2) = \frac{P(R > 2)}{P(R \neq 2)} = \frac{1 - P(R \leq 2)}{1 - P(R = 2)}$$ **B1**

$$= \frac{1 - \frac{1}{60\times 17}[2^9 + 2^8 + 2^7]}{1 - \frac{2^7}{60\times 17}} = \frac{31}{255} \div \frac{223}{255} = \frac{31}{223} \approx 0.139$$ **M1**

(b).

$$P(\text{even}) = \frac{2}{3}, \qquad P(\text{odd}) = \frac{1}{3}$$

$$p = P(\text{getting a } 4) = \frac{2}{3}, \qquad q = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(X \nless 2) = 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - {}^{10}C_0 \times \left(\frac{2}{3}\right)^0 \times \left(\frac{1}{3}\right)^{10} - {}^{10}C_1 \times \left(\frac{2}{3}\right)^1 \times \left(\frac{1}{3}\right)^9$$

$$= 1 - \frac{1}{59049} - \frac{20}{59049} = 0.9996$$ **M1 A1**

12

13 (a).

$$y_n = x_n e^{-x_n}, \qquad h = \frac{5-2}{6-1} = 0.6$$ **B1-for h**

n	x_n	y_0, y_5	$y_1, \dots y_4$
0	2.0	0.2707	
1	2.6		0.1931
2	3.2		0.1304
3	3.8		0.0850
4	4.4		0.0540
5	5.0	0.0337	
sums		0.3044	0.4625

B1-for x_n
B1-for y_0, y_5
B1-for $y_1, \dots y_4$

$$\int_2^5 xe^{-x}\,dx \approx \frac{1}{2}h[(y_0 + y_5) + 2(y_1 + \dots + y_4)]$$

$$\approx \frac{1}{2} \times 0.6[0.3044 + 2 \times 0.4625] = 0.36882 \approx 0.369 \text{ (3 s.f)}$$ **M1 A1**

(b).

Sign	Differentiating	Integrating
+	x	e^{-x}
-	1	$-e^{-x}$
+	0	e^{-x}

B1

$$\int_2^5 xe^{-x}\,dx = \Big[\ x \times (-e^{-x}) - 1 \times e^{-x}\Big]_2^5 = \Big[\ -(x+1)e^{-x}\Big]_2^5$$ **M1**

$$= -[6e^{-5} - 3e^{-2}] = 0.366 \ \text{(3 s.f)}$$ **A1**

(c).

$$\text{Absolute error} = |0.366 - 0.369| = 0.003$$ **M1**

$$\%\text{age error} = \frac{0.03}{0.366} \times 100\% = 0.820 \ \text{(3 s.f)}$$ **M1 A1**

12

14 (a).

$$\text{momentum before collision} = \text{momentum after collision}$$
$$(2m)(4u) + (3m)(-2u) = (2m)(v) + (3m)(u)$$
$$8mu - 6mu = 2mv + 3mu$$
$$2u = 2v + 3u$$
$$-u = 2v$$
$$v = -0.5u$$ **M1** **B1**

Before collision, kinetic energy is:

$$= \frac{1}{2}(2m)(4u)^2 + \frac{1}{2}(3m)(-2u)^2 = 16mu^2 + 6mu^2 = 22mu^2$$ **B1**

After collision, kinetic energy is:

$$= \frac{1}{2}(2m)(-0.5u)^2 + \frac{1}{2}(3m)(u)^2 = 0.25mu^2 + 1.5mu^2 = 1.75mu^2$$ **B1**

The loss in kinetic energy during impact is:

$$= 22mu^2 - 1.75mu^2 = 20.25mu^2$$ **M1 A1**

(b). (i).

$$h = 12 \text{ m}, v = 5 \text{ m s}^{-1}, l = 35 \text{ cm} = 0.35 \text{ m}$$

Volume of water delivered per second is:

$$= v \times l^2 = 5 \times (0.35)^2 = 0.6125 \text{ m}^3\text{s}^{-1}$$ **B1**

$$\text{but } 1 \text{ m}^3 = 1000 \text{ litres}, \quad \Rightarrow 1 \text{ m}^3 = 1000 \text{ kg}$$

Mass of water delivered per second is:

$$m = 0.6125 \times 1000 = 612.5 \text{ kg s}^{-1}$$ **B1**

(ii).

$$\text{Effective engine power} = \text{K.E per second} + \text{P.E per second}$$
$$= \frac{1}{2}mv^2 + mgh$$
$$= \frac{1}{2} \times 612.5 \times 5^2 + 612.5 \times 9.8 \times 12$$
$$= 7656.25 + 72030 = 79686.25 \text{ W}$$

M1 M1 M1 A1
(M1 for K.E, M1 for P.E, M1 for addition)

12

15

$$\mu = 150, \qquad \sigma = 32$$

(a).

$$P(125 < X < 210) = P\left(\frac{125 - 150}{32} < Z < \frac{210 - 150}{32}\right)$$ **B1**

$$= P(-0.781 < Z < 1.875) = \phi(0.781) + \phi(1.875)$$

$= 0.2826 + 0.4697 = 0.7523$ **M1 A1**

(ii).

$P(\text{replacing a dry cell}) = P(X \leq 225) = P\left(Z < \dfrac{225 - 150}{32}\right)$ **B1**

$= P(Z < 2.344) = 0.5 + \phi(2.344) = 0.5 + 0.4905 = 0.9905$ **M1 M1**

Number of cells that need replacement $= 0.9905 \times 300 = 297.15$ **M1 A1**

≈ 297

(iii).

$P(Z < z_1) = 0.25$

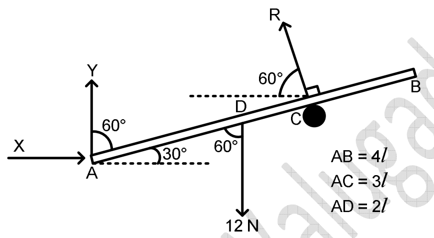
$z_1 = -\phi^{-1}(0.25) = -0.674$ **B1**

$\Rightarrow \dfrac{x_1 - 150}{32} = -0.674$ **M1**

$x_1 = 150 - 32 \times 0.674 = 128.432 \approx 128$ days **M1 A1**

12

16 (a).



B1

Taking moments about A,

$R \times 3l = 12 \times 2l \sin 60°$ **M1**

$R = \dfrac{24 \sin 60°}{3} = 4\sqrt{3} \approx 6.928$ N

∴ The reaction at C is 6.928 N in the direction 60° above the negative horizontal. **A1**

Resolving vertically,

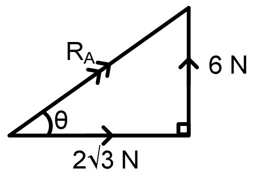
$Y + R \sin 60° = 12$

$Y + 4\sqrt{3} \sin 60° = 12$ **M1**

$Y = 12 - 6 = 6$ N

Resolving horizontally,

$X = R \cos 60° = 4\sqrt{3} \cos 60° = 2\sqrt{3} \approx 3.464$ N **B1**

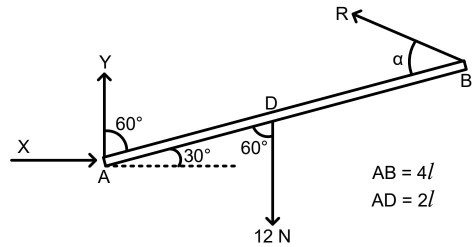


$R_A = \sqrt{(2\sqrt{3})^2 + 6^2} = \sqrt{18} \approx 4.243$ N **M1**

$\tan\theta = \dfrac{6}{2\sqrt{3}}, \quad \Rightarrow \theta = 60°$ **B1**

∴ The reaction at A is 4.243 N in the direction 60° above the positive horizontal. **A1**

(b).



Taking moments about A,

$R \times 4l \sin\alpha = 12 \times 2l \sin 60°$

$R = \dfrac{24 \sin 60°}{4 \sin\alpha} = \dfrac{3\sqrt{3}}{\sin\alpha}$ **M1**

when R is minimum, $\sin\alpha = 1$, $\Rightarrow \alpha = 90°$

$R_{\min} = 3\sqrt{3} \approx 5.196$ N

Resolving vertically,

$Y + R \sin 60° = 12$

$Y + 3\sqrt{3} \sin 60° = 12$ **M1**

$Y = 12 - 4.5 = 7.5$ N

Resolving horizontally,

$X = R \cos 60° = 3\sqrt{3} \cos 60° = 1.5\sqrt{3} \approx 2.598$ N **M1**

New magnitude, $R_A = \sqrt{(1.5\sqrt{3})^2 + (3\sqrt{3})^2} = \sqrt{63} \approx 7.937$ N **A1**

12

END

P425/1
PURE
MATHEMATICS
PAPER 1
June 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 1 MOCK SET 3 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn 1: Solve the inequality: $\frac{x-1}{x-2} \geq \frac{x-2}{x+3}$. [5]

Qn 2: Given that the roots of the equation $x^2 - 2x + 10 = 0$ are α and β, determine the equation whose roots are $\frac{1}{(2+\alpha)^2}$ and $\frac{1}{(2+\beta)^2}$. [5]

Qn 3: ABCD is a square inscribed in a circle $x^2 + y^2 - 4x - 3y = 36$. Find the length of diagonals and the area of the square. [5]

Qn 4: Solve $\cos(\theta + 35°) = \sin(\theta + 25°)$ for $0° \leq \theta \leq 360°$. [5]

Qn 5: Find $\int \frac{dx}{3 - 2\cos x}$. [5]

Qn 6: Find the values of k for which $\frac{x^2 - x + 1}{x - 1} = k$ has repeated roots. What are the repeated roots? [5]

Qn 7: PQRS is a quadrilateral with vertices $P(1, -2)$, $Q(4, -1)$, $R(5, 2)$ and $S(2, 1)$. Show that the quadrilateral is a rhombus. [5]

Qn 8: The gradient of a certain curve is given by kx. If the curve passes through the point $(2, 3)$ and the tangent that this point makes is an angle of $\tan^{-1} 6$ with the positive direction of the x-axis, find the equation of the curve. [5]

Section B (60 Marks)
Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
(a). Integrate $\frac{4x^2}{\sqrt{1-x^6}}$ with respect to x.
(b). Evaluate $\int_1^3 \frac{x^2+1}{x^3+4x^2+3x} dx$. [12]

Question 10:
(a). Solve $4^x - 2^{x+1} - 15 = 0$.
(b). Five millions shillings are invested each year, at a rate of 15% interest. In how many years will it accumulate to more than shs 50 million? [12]

Question 11:
(a). Given that $\boldsymbol{OP} = 4\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 5\underset{\sim}{\boldsymbol{k}}$ and $\boldsymbol{OQ} = \underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{k}}$, find the coordinates of the point R such that $\overrightarrow{\boldsymbol{PR}} : \overrightarrow{\boldsymbol{PQ}} = 1:2$ and the points P, Q and R are collinear.
(b). Show that the vector $5\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}}$ is perpendicular to the line $\underset{\sim}{\boldsymbol{r}} = \underset{\sim}{\boldsymbol{i}} - 4\underset{\sim}{\boldsymbol{j}} + t\left(2\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}} - 4\underset{\sim}{\boldsymbol{k}}\right)$.
(c). Find the equation of the plane through a point with position vector $5\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} + 3\underset{\sim}{\boldsymbol{k}}$ perpendicular to the vector $3\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}} - \underset{\sim}{\boldsymbol{k}}$. [12]

Question 12:
(a). Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.
(b). Find all the solutions to $2\sin 3\theta = 1$ for θ between $0°$ and $360°$. Hence find the solutions for $8x^3 - 6x + 1 = 0$. [12]

Question 13:
(a). (i). If $x^2 \sec x - xy + 2y^2 = 15$, find $\frac{dy}{dx}$.
(ii). Given that $y = \theta - \cos\theta$; $x = \sin\theta$; show that $\frac{d^2y}{dx^2} = \frac{1+\sin\theta}{\cos^3\theta}$.
(b). Determine the maximum and minimum values of $x^2 e^{-x}$. [12]

Question 14:

(a). Use Maclaurin's theorem to expand $\ln(1+\sin x)$ as far as the term in x^3.

(b). Expand $(1-x)^{\frac{1}{3}}$ as far as the term in x^3. Use your expansion to deduce $\sqrt[3]{24}$ correct to three significant figures. [12]

Question 15:

(a). A tangent from the point $T(t^2, 2t)$ touches the curve $y^2 = 4x$. Find:

(i). the equation of the tangent,

(ii). the equation of line L parallel to the normal at $(t^2, 2t)$ and passes through $(1, 0)$,

(iii). The point of intersection, X, of the line L and the tangent.

(b). A point $P(x, y)$ is equidistant from X and T in (a) above. Show that the locus of P is $t^4 - 3t - 2(x+y) = 0$. [12]

Question 16:

A research to investigate the effect of a certain chemical on a virus infection crops revealed that the rate at which the virus population is destroyed is directly proportional to the population at that time. Initially, the population was P_0. At t months later, it was found to be P.

(a). Form a differential equation connecting P and t.

(b). Given that the virus populations reduced to one third of the initial population in 4 months, solve the equation in (a) above. [12]

END

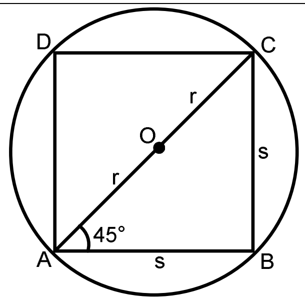
MARKING GUIDE

SNo.	Working	Marks
1	$\dfrac{x-1}{x-2} \geq \dfrac{x-2}{x+3}$	
	$\dfrac{x-1}{x-2} - \dfrac{x-2}{x+3} \geq 0$	M1
	$\dfrac{(x-1)(x+3)-(x-2)^2}{(x-2)(x+3)} \geq 0$	M1
	$\dfrac{(x^2+2x-3)-(x^2-4x+4)}{(x-2)(x+3)} \geq 0$	
	$\dfrac{6x-7}{(x-2)(x+3)} \geq 0$	B1
	The critical values are: $x=-3, x=\frac{7}{6}, x=2$.	

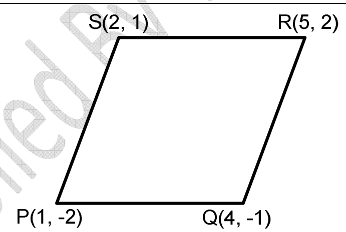
	$x<-3$	$-3<x<\frac{7}{6}$	$\frac{7}{6}<x<2$	$x>2$
$(6x-7)$	-	-	+	+
$(x-2)$	-	-	-	+
$(x+3)$	-	+	+	+
$\dfrac{6x-7}{(x-2)(x+3)}$	-	+	-	+

B1

SNo.	Working	Marks
	The solution set is $-3 \leq x \leq \frac{7}{6}$ and $x \geq 2$.	B1
		05
2	$x^2-2x+10=0$	
	For the given equation, sum of roots, $\alpha+\beta = -(-2) = 2$ product of roots, $\alpha\beta = 10$	
	For the new equation, product of roots $= \dfrac{1}{(2+\alpha)^2} \times \dfrac{1}{(2+\beta)^2} = \dfrac{1}{(2+\alpha)^2(2+\beta)^2}$	
	$= \dfrac{1}{[(2+\alpha)(2+\beta)]^2} = \dfrac{1}{[4+2\alpha+2\beta+\alpha\beta]^2}$	M1
	$= \dfrac{1}{[4+2(\alpha+\beta)+10]^2} = \dfrac{1}{[14+2\times 2]^2} = \dfrac{1}{324}$	B1
	sum of roots $= \dfrac{1}{(2+\alpha)^2} + \dfrac{1}{(2+\beta)^2} = \dfrac{(2+\beta)^2+(2+\alpha)^2}{[(2+\alpha)(2+\beta)]^2}$	
	$= \dfrac{4+4\beta+\beta^2+4+4\alpha+\alpha^2}{324} = \dfrac{8+4(\alpha+\beta)+\beta^2+\alpha^2}{324}$	M1
	$= \dfrac{8+4\times 2+(\alpha+\beta)^2-2\alpha\beta}{324} = \dfrac{16+2^2-2\times 10}{324} = 0$	B1
	The new equation is	

	$x^2-(\text{sum})x+(\text{product})=0$	
	$x^2-0x+\frac{1}{324}=0$	
	$324x^2+1=0$	A1
		05
3		
	$x^2+y^2-4x-3y=36$	
	Comparing with the general equation of the circle: $x^2+y^2+2gx+2fy+c=0$.	
	$2g=-4$ $\quad 2f=-3$ $\quad c=-36$	
	$g=-2$ $\quad g=-1.5$	M1
	Radius, $\quad r=\sqrt{g^2+f^2-c}=\sqrt{4+2.25+36}=\sqrt{42.25}$	M1
	$=6.5$ units	
	Length of each diagonal, $\quad l=2r=2\times 6.5=13$ units	A1
	By Pythagoras theorem,	
	$l^2=s^2+s^2, \quad \Rightarrow l^2=2s^2, \quad \Rightarrow s^2=\frac{1}{2}l^2$	
	Area of square $=s^2=\frac{1}{2}l^2=\frac{1}{2}\times 13^2=84.5\text{ cm}^2$	M1 A1
		05
4	$\cos(\theta+35°)=\sin(\theta+25°)$	M1
	$\cos\theta\cos 35°-\sin\theta\sin 35°=\sin\theta\cos 25°+\cos\theta\sin 25°$	B1
	$\cos 35°-\tan\theta\sin 35°=\tan\theta\cos 25°+\sin 25°$	
	$\cos 35°-\sin 25°=\tan\theta\cos 25°+\tan\theta\sin 35°$	
	$\cos 35°-\sin 25°=\tan\theta(\cos 25°+\sin 35°)$	B1
	$\tan\theta=\frac{\cos 35°-\sin 25°}{\cos 25°+\sin 35°}\approx 0.2679$	M1
	$\theta=15°, 195°$	A1
		05
5	let, $\quad t=\tan\left(\frac{x}{2}\right)$,	
	$\Rightarrow \frac{dt}{dx}=\frac{1}{2}\sec^2\left(\frac{x}{2}\right)=\frac{1}{2}\left[1+\tan^2\left(\frac{x}{2}\right)\right]=\frac{1}{2}(1+t^2)$	B1
	$dx=\frac{2\,dt}{1+t^2}$	

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

	$\int\frac{dx}{3-2\cos x}=\int\frac{\left(\frac{2\,dt}{1+t^2}\right)}{3-2\left(\frac{1-t^2}{1+t^2}\right)}=\int\frac{\left(\frac{2\,dt}{1+t^2}\right)}{\left[\frac{3(1+t^2)-2(1-t^2)}{1+t^2}\right]}$	M1
	$=\int\frac{2\,dt}{3+3t^2-2+2t^2}=2\int\frac{1}{1+5t^2}dt$	B1
	$\frac{2}{\sqrt{5}}\tan^{-1}(t\sqrt{5})+c=\frac{2}{\sqrt{5}}\tan^{-1}\left[\sqrt{5}\tan\left(\frac{x}{2}\right)\right]+c$	B1 A1
		05
6	$\frac{x^2-x+1}{x-1}=k$	
	$x^2-x+1=k(x-1)$	
	$x^2-(k+1)x+(k+1)=0$	M1
	Comparing with the general quadratic equation $ax^2+bx+c=0$,	
	$a=1, b=-(k+1), c=(k+1)$	
	For repeated roots,	
	$b^2-4ac=0$	
	$(k+1)^2-4(k+1)=0$	
	$(k+1)(k-3)=0$	M1
	$k=3$, $\quad$ or, $\quad k=-1$	
	$x=-\frac{b}{2a}=\frac{(k+1)}{2a}$	B1
	for $k=3$, $\quad x=\frac{(k+1)}{2a}=\frac{3+1}{2\times 1}=2$	A1
	for $k=-1$, $\quad x=\frac{(k+1)}{2a}=\frac{-1+1}{2\times 1}=0$	A1
		05
7		
	$PQ=\begin{pmatrix}4\\-1\end{pmatrix}-\begin{pmatrix}1\\-2\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$, $\quad SR=\begin{pmatrix}5\\2\end{pmatrix}-\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$	B1
	$PS=\begin{pmatrix}2\\1\end{pmatrix}-\begin{pmatrix}1\\-2\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$, $\quad QR=\begin{pmatrix}5\\2\end{pmatrix}-\begin{pmatrix}4\\-1\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$	B1
	$\lvert PQ\rvert=\sqrt{3^2+1^2}=\sqrt{10}$, $\quad \lvert PS\rvert=\sqrt{1^2+3^2}=\sqrt{10}$	B1
	$PQ.PS=\begin{pmatrix}3\\1\end{pmatrix}.\begin{pmatrix}1\\3\end{pmatrix}=3+3=6$	
	Since $PQ \nearrow\!\!\nearrow SR$, $PS \nearrow\!\!\nearrow QR$, $\lvert PQ\rvert=\lvert PS\rvert$ and $PQ.PS\neq 0$ it implies	B1
	that the quadrilateral is a rhombus.	B1
		05
8	$\frac{dy}{dx}=kx, \quad \Rightarrow y=\frac{1}{2}kx^2+c$	M1

At $(2,3)$, $x = 2$ and $y = 3$,

$$3 = \frac{1}{2}k \times 2^2 + c, \quad \Rightarrow 3 = 2k + c \quad \longrightarrow (1)$$ **M1**

Also, at $(2,3)$, gradient is 6,

$$\frac{dy}{dx} = kx, \quad \Rightarrow 6 = k \times 2, \quad \Rightarrow k = 3$$ **B1**

From equation (1),

$$3 = 2k + c, \quad \Rightarrow 3 = 2 \times 3 + c, \quad \Rightarrow c = 3 - 6 = -3$$ **B1**

The equation of the curve is given by:

$$y = \frac{1}{2}kx^2 + c, \quad \Rightarrow y = \frac{1}{2} \times 3x^2 - 3, \quad \Rightarrow y = \frac{3}{2}x^2 - 3$$ **A1**

05

9 (a).

$$\text{let } x^6 = \sin^2 u, \quad x^3 = \sin u, \quad u = \sin^{-1} x^3$$

$$3x^2 = \cos u \frac{du}{dx}, \quad dx = \frac{\cos u \, du}{3x^2}$$ **B1**

$$\int \frac{4x^2}{\sqrt{1 - x^6}} dx = \int \frac{4x^2}{\sqrt{1 - \sin^2 u}} \times \frac{\cos u \, du}{3x^2} = \frac{4}{3}\int \frac{\cos u}{\cos u} du$$ **M1**

$$= \frac{4}{3}\int 1 \, du = \frac{4}{3}u + c = \frac{4}{3}\sin^{-1} x^3 + c$$ **B1 M1 A1**

(b).

$$\frac{x^2 + 1}{x^3 + 4x^2 + 3x} = \frac{x^2 + 1}{x(x^2 + 4x + 3)} = \frac{x^2 + 1}{x(x + 1)(x + 3)}$$

$$\frac{x^2 + 1}{x(x + 1)(x + 3)} \equiv \frac{A}{x} + \frac{B}{(x + 1)} + \frac{C}{(x + 3)}$$

$$x^2 + 1 \equiv A(x + 1)(x + 3) + Bx(x + 3) + Cx(x + 1)$$

putting $x = 0$; $1 = A(1 \times 3)$, $\Rightarrow A = \frac{1}{3}$ **B1**

putting $x = -1$; $2 = B(-1)(2)$, $\Rightarrow B = -1$ **B1**

putting $x = -3$; $10 = C(-3)(-2)$, $\Rightarrow C = \frac{5}{3}$ **B1**

$$\int_1^3 \frac{x^2 + 1}{x^3 + 4x^2 + 3x} dx$$

$$= \frac{1}{3}\int_1^3 \frac{1}{x} dx - \int_1^3 \frac{1}{(x + 1)} dx + \frac{5}{3}\int_1^3 \frac{1}{(x + 3)} dx$$ **M1**

$$= \left[\frac{1}{3}\ln x - \ln(x + 1) + \frac{5}{3}\ln(x + 3)\right]_1^3$$ **M1**

$$= \left[\frac{1}{3}\ln 3 - \ln 4 + \frac{5}{3}\ln 6\right] - \left[\frac{1}{3}\ln 1 - \ln 2 + \frac{5}{3}\ln 4\right]$$ **M1**

$$= 1.9662 - 1.6173 = 0.3489$$ **A1**

12

10 (a).

$$4^x - 2^{x+1} - 15 = 0$$

$$2^{2x} - 2(2^x) - 15 = 0$$

$$\text{let,} \quad y = 2^x$$

$$y^2 - 2y - 15 = 0$$

$$y^2 - 2y - 15 = 0$$ **M1**

$$\text{sum} = -2, \quad \text{product} = -15, \quad \text{factors} = -5, 3$$

$$(y - 5)(y + 3) = 0$$ **M1**

$$y = 5, \quad \text{or,} \quad y = -3$$ **B1**

$$\text{for,} \quad y = 5, \quad 2^x = 5, \quad \Rightarrow x = \frac{\log 5}{\log 2} = 2.322$$ **A1**

$$\text{for,} \quad y = -3, \quad 2^x = -3, \quad \Rightarrow x \text{ is undefined}$$ **A1**

(b).

$$R = 15, P = 5000000, A_{total} > 50000000$$

$$\text{Total amount,} \quad A_{total} = \sum_1^n A_n, \quad \text{where } A_n = P\left(1 + \frac{R}{100}\right)^n$$ **M1**

$$\Rightarrow A_{total} = A_1 + A_2 + \cdots + A_n$$

$$= P[(1 + 0.15)^1 + (1 + 0.15)^2 + \cdots + (1 + 0.15)^n]$$ **M1**

$$= P[1.15 + 1.15^2 + \cdots + 1.15^n]$$

$$= P\left[\frac{a(r^n - 1)}{r - 1}\right], \quad \text{where } a = r = 1.15$$

$$\text{but,} \quad A_{total} > 50000000$$

$$5{,}000{,}000\left[\frac{1.15(1.15^n - 1)}{1.15 - 1}\right] > 50{,}000{,}000$$ **M1**

$$\left[\frac{1.15(1.15^n - 1)}{0.15}\right] > 10$$

$$1.15(1.15^n - 1) > 1.5$$

$$1.15^n > \frac{1.5}{1.15} + 1$$

$$1.15^n > 2.3043$$ **B1**

$$n > \frac{\log 2.3043}{\log 1.15}$$ **M1**

$$n > 5.973$$ **B1**

$$n = 6 \text{ terms}$$ **A1**

12

11 (a).

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix}1\\0\\2\end{pmatrix} - \begin{pmatrix}4\\-3\\5\end{pmatrix} = \begin{pmatrix}-3\\3\\-3\end{pmatrix}$$ **B1**

$$\overrightarrow{PR}:\overrightarrow{PQ} = 1:2, \quad \Rightarrow \frac{\overrightarrow{PR}}{\overrightarrow{PQ}} = \frac{1}{2}, \quad \Rightarrow \overrightarrow{PR} = \frac{1}{2}\overrightarrow{PQ}$$ **M1**

$$\overrightarrow{OR} - \overrightarrow{OP} = \frac{1}{2}\begin{pmatrix}-3\\3\\-3\end{pmatrix}$$ **B1**

$$\overrightarrow{OR} - \begin{pmatrix}4\\-3\\5\end{pmatrix} = \begin{pmatrix}-1.5\\1.5\\-1.5\end{pmatrix}$$ **A1**

	$\overrightarrow{\boldsymbol{OR}} = \begin{pmatrix}-1.5\\1.5\\-1.5\end{pmatrix} + \begin{pmatrix}4\\-3\\5\end{pmatrix} = \begin{pmatrix}2.5\\-1.5\\3.5\end{pmatrix}, \quad \Rightarrow R(2.5, -1.5, 3.5)$	M1 M1
	(b).	
	For perpendicular vectors,	
	$\begin{pmatrix}5\\-2\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix} = 0$	
	$10 - 6 - 4 = 0$	M1
	$0 = 0,$ hecne shown	B1
	(c).	
	Position vector, $\underset{\sim}{a} = \begin{pmatrix}5\\-2\\3\end{pmatrix}$	B1
	Normal vector, $\underset{\sim}{n} = \begin{pmatrix}3\\4\\-1\end{pmatrix}$	
	The equation of the plane is given by:	
	$\underset{\sim}{r}.\underset{\sim}{n} = \underset{\sim}{n}.\underset{\sim}{a}$	
	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}3\\4\\-1\end{pmatrix} = \begin{pmatrix}3\\4\\-1\end{pmatrix}.\begin{pmatrix}5\\-2\\3\end{pmatrix}$	M1
	$3x + 4y - z = 15 - 8 - 3$	M1
	$3x + 4y - z = 4$	A1
		12
12	(a).	
	$\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	
	$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$	M1
	$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$	M1
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$	B1
	(b).	B1
	$2\sin 3\theta = 1$	
	$\sin 3\theta = 0.5$	B1
	$3\theta = 30°, 150°, 390°, 510°, 750°, 870°$	B1
	$\theta = 10°, 50°, 130°, 170°, 250°, 290°$	A1
	For the hence part,	
	$8x^3 - 6x + 1 = 0$	
	let, $x = \sin\theta$	
	$8\sin^3\theta - 6\sin\theta + 1 = 0$	B1
	$6\sin\theta - 8\sin^3\theta = 1$	
	$2(3\sin\theta - 4\sin^3\theta) = 1$	
	$2\sin 3\theta = 1$	
	$\theta = 10°, 50°, 130°, 170°, 250°, 290°$	
	$x = \sin\theta$	
	$x_1 = \sin 10° = \sin 170° = 0.1736$	A1
	$x_2 = \sin 50° = \sin 130° = 0.7660$	A1

	$x_3 = \sin 250° = \sin 290° = -0.9397$	A1
		12
13	(a). (i).	
	$x^2 \sec x - xy + 2y^2 = 15$	
	$\frac{d}{dx}(x^2 \sec x - xy + 2y^2) = \frac{d}{dx}(15)$	
	$x^2 \sec x \tan x + 2x \sec x - \left(x\frac{dy}{dx} + y\right) + 4y\frac{dy}{dx} = 0$	M1M1
	$x \sec x\,(x \tan x + 2) - y = x\frac{dy}{dx} - 4y\frac{dy}{dx}$	B1
	$x \sec x\,(x \tan x + 2) - y = (x - 4y)\frac{dy}{dx}$	
	$\frac{dy}{dx} = \frac{x \sec x\,(x \tan x + 2) - y}{x - 4y}$	A1
	(ii).	
	$y = \theta - \cos\theta, \quad \Rightarrow \frac{dy}{d\theta} = 1 + \sin\theta$	
	$x = \sin\theta, \quad \Rightarrow \frac{dx}{d\theta} = \cos\theta$	M1
	$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{1 + \sin\theta}{\cos\theta}$	
	$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \times \frac{d\theta}{dx} = \frac{d}{d\theta}\left(\frac{1 + \sin\theta}{\cos\theta}\right) \times \frac{1}{\cos\theta}$	
	$= \left[\frac{\cos\theta\cos\theta - (1 + \sin\theta)(-\sin\theta)}{\cos^2\theta}\right] \times \frac{1}{\cos\theta}$	M1
	$= \left[\frac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos^3\theta}\right] = \frac{1 + \sin\theta}{\cos^3\theta}$	B1
	(b).	
	let, $y = x^2e^{-x}, \quad \Rightarrow \frac{dy}{dx} = 2xe^{-x} - x^2e^{-x} = (2x - x^2)e^{-x}$	M1
	for stationary points, $\frac{dy}{dx} = 0$	
	$(2x - x^2)e^{-x} = 0$	
	$(2x - x^2) = 0,$ or, $e^{-x} = 0$	
	for, $(2x - x^2) = 0, \quad x = 0,$ or, $x = 2$	
	for, $e^{-x} = 0, \quad x$ is undefined	B1
	$\frac{d^2y}{dx^2} = (2 - 2x)e^{-x} - (2x - x^2)e^{-x} = (2 - 4x + x^2)e^{-x}$	M1
	when, $x = 0, \quad \frac{d^2y}{dx^2} = (2 - 0 + 0)e^0 = 2,$	A1
	hence minimum	
	minimum value, $y_{min} = 0$	
	when, $x = 2, \quad \frac{d^2y}{dx^2} = (2 - 8 + 4)e^{-2} = -0.271,$	A1
	hence maximum	
	maximum value, $y_{max} = 2^2e^{-2} = 5413$	

		12
14	(a).	
	Let, $f(x)=\ln(1+\sin x)$, $\Rightarrow f(0)=\ln(1+0)=0$	
	$f'(x)=\dfrac{\cos x}{1+\sin x}$, $\Rightarrow f'(0)=\dfrac{1}{1+0}=1$	B1
	$f''(x)=\dfrac{(1+\sin x)(-\sin x)-\cos^2 x}{(1+\sin x)^2}=\dfrac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$	
	$=\dfrac{-\sin x-(\sin^2 x+\cos^2 x)}{(1+\sin x)^2}=\dfrac{-\sin x-1}{(1+\sin x)^2}$	B1
	$=\dfrac{-(\sin x+1)}{(1+\sin x)^2}=\dfrac{-1}{1+\sin x}$	
	$\Rightarrow f''(0)=\dfrac{-1}{(1+0}=-1$	B1
	$f'''(x)=-(-1)(1+\sin x)^{-2}\cos x=\dfrac{\cos x}{(1+\sin x)^2}$	
	$\Rightarrow f'''(0)=\dfrac{1}{(1+0)^2}=1$	B1
	By Maclaurin's theorem,	
	$f(x)=f(0)+xf'(0)+\dfrac{x^2}{2!}f''(0)+\dfrac{x^3}{3!}f'''(0)+\cdots$	
	$=0+x\times 1+\dfrac{x^2}{2!}\times(-1)+\dfrac{x^3}{3!}\times 1+\cdots$	B1
	$\therefore \ln(1+\sin x)=x-\dfrac{1}{2}x^2+\dfrac{1}{6}x^3+\cdots$	A1
	(b).	
	$(1-x)^{\frac{1}{3}}=1+\dfrac{1}{3}(-x)+\dfrac{1}{3}\times\dfrac{-2}{3}\times\dfrac{(-x)^2}{2!}+\dfrac{1}{3}\times\dfrac{-2}{3}\times\dfrac{-5}{3}\times\dfrac{(-x)^3}{3!}+\cdots$	B1 B1
	$=1-\dfrac{1}{3}x-\dfrac{1}{9}x^2-\dfrac{5}{81}x^3+\cdots$	A1
	For the hence part,	
	$\sqrt[3]{24}=(27-3)^{\frac{1}{3}}=\left[27\left(1-\dfrac{3}{27}\right)\right]^{\frac{1}{3}}=3\left(1-\dfrac{1}{9}\right)^{\frac{1}{3}}$	B1
	By comparison, $x=\frac{1}{9}$	
	$\sqrt[3]{24}=3\left[1-\dfrac{1}{3}\times\left(\dfrac{1}{9}\right)-\dfrac{1}{9}\times\left(\dfrac{1}{9}\right)^2-\dfrac{5}{81}\times\left(\dfrac{1}{9}\right)^3\right]$	M1
	$\sqrt[3]{24}=3\times 0.9615=2.88\ (3\text{ s.f})$	A1
		12
15	(a). (i).	
	$y^2=4x$, $\Rightarrow \dfrac{d}{dx}(y^2)=\dfrac{d}{dx}(4x)$	
	$2y\dfrac{dy}{dx}=4$, $\Rightarrow \dfrac{dy}{dx}=\dfrac{2}{y}$	M1

	At point $T(t^2,2t)$, $x=t^2, y=2t$	
	gradient of the tangent $=\dfrac{dy}{dx}=\dfrac{2}{2t}=\dfrac{1}{t}$	B1
	The equation of the tangent is given by,	
	$\dfrac{y-2t}{x-t^2}=\dfrac{1}{t}$	
	$y-2t=\dfrac{1}{t}(x-t^2)$	
	$y=\dfrac{1}{t}x+t$	A1
	(ii).	
	gradient of line L $=-1\div\dfrac{1}{t}=-t$	B1
	The equation of the line L is given by,	
	$\dfrac{y-0}{x-1}=-t$	
	$y=-t(x-1)$	
	$y=-xt+t$	A1
	(iii). At the point of intersection,	
	$\dfrac{1}{t}x+t=-xt+t$	M1
	$\dfrac{1}{t}x=-xt$	
	$x(1+t^2)=0$, $\Rightarrow x=0$	B1
	when $x=0$, $y=-xt+t=0+t=t$	
	point of intersection is $X(0,t)$	A1
	(b).	
	$X(0,t)$, $P(x,y)$, $T(t^2,2t)$	
	P is the midpoint of X and T,	
	$x=\dfrac{0+t^2}{2}=\dfrac{t^2}{2}$, $y=\dfrac{t+2t}{2}=\dfrac{3t}{2}$	M1
	$(x+y)=\dfrac{t^2}{2}+\dfrac{3t}{2}$	M1
	$2(x+y)=t^2+3t$	B1
	$t^2+3t-2(x+y)$	A1
	Alternatively:	
	$X(0,t)$, $P(x,y)$, $T(t^2,2t)$	
	$\lvert XP\rvert=\lvert PT\rvert$	
	$\sqrt{(y-t)^2+(x-0)^2}=\sqrt{(y-2t)^2+(x-t^2)^2}$	
	$\sqrt{y^2-2ty+t^2+x^2}=\sqrt{y^2-4ty+4t^2+x^2-2xt^2+t^4}$	
	$y^2-2ty+t^2+x^2=y^2-4ty+4t^2+x^2-2xt^2+t^4$	
	$0=-2ty+3t^2-2xt^2+t^4$	
	$t^4+3t^2-2ty-2xt^2=0$	
	$t^4+3t^2-2t(xt+y)=0$	
	$t^3+3t-2(xt+y)=0$	
		12

16	(a).	
	$\frac{dP}{dt} \propto P, \quad \Rightarrow \frac{dP}{dt} = -kP$	**B1 B1**
	(b).	
	$\int \frac{dP}{P} = \int -k\,dt$	**M1**
	$\ln P = -kt + c \quad \rightarrow (i)$	
	When $t = 0, P = P_0$	
	$\ln P_0 = -k \times 0 + c, \quad \Rightarrow c = \ln P_0$	**B1**
	Equation (i) becomes	
	$\ln P = -kt + \ln P_0 \quad \rightarrow (ii)$	**B1**
	When $t = 4, P = \frac{1}{3}P_0$	
	$\ln\left(\frac{1}{3}P_0\right) = -4k + \ln P_0$	
	$\ln\left(\frac{1}{3}P_0\right) - \ln P_0 = -4k$	**B1**
	$\ln\left(\frac{1}{3}\right) = -4k$	**B1**
	$k = 0.25 \ln 3$	**B1**
	Equation (ii) becomes	
	$\ln P = -0.25t \ln 3 + \ln P_0$	**B1**
	$\ln\left(\frac{P}{P_0}\right) = -0.25t \ln 3$	**B1**
	$\frac{P}{P_0} = e^{-0.25t \ln 3}$	**B1**
	$P = P_0 e^{-0.25t \ln 3}$	**B1**
	$P = P_0 e^{-0.275t}$	
	Alternatively:	
	(a).	
	$\frac{dP}{dt} \propto P, \quad \Rightarrow \frac{dP}{dt} = -kP$	
	(b).	
	$\int \frac{dP}{P} = \int -k\,dt$	
	$\ln P = -kt + c$	
	$P = Ae^{-kt} \quad \rightarrow (i), \quad$ where $A = e^c$ and is a constant	
	When $t = 0, P = P_0$	
	$P_0 = Ae^0, \quad \Rightarrow A = P_0$	
	Equation (i) becomes	
	$P = P_0 e^{-kt} \quad \rightarrow (ii)$	
	When $t = 4, P = \frac{1}{3}P_0$	
	$\frac{1}{3}P_0 = P_0 e^{-4k}$	
	$\frac{1}{3} = e^{-4k}$	

	$-4k = \ln\left(\frac{1}{3}\right)$	
	$k = 0.25 \ln 3$	
	Equation (ii) becomes	
	$P = P_0 e^{-0.25t \ln 3}$	
	$P = P_0 e^{-0.275t}$	
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
June 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 2 MOCK SET 3 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)
*Answer **all** the questions in this section*

Qn 1: Forces $\begin{pmatrix}-3\\1\end{pmatrix}$ N, $\begin{pmatrix}2\\9\end{pmatrix}$ N, and $\begin{pmatrix}4\\-6\end{pmatrix}$ N act on a body of mass 2 kg. Find the magnitude of the acceleration of the body. [5]

Qn 2: The table below shows the times to the nearest second taken by 100 students to solve a problem.

Times (s)	30 – 49	50 – 64	65 – 69	70 – 74	75 – 79
No. of students	10	30	25	20	15

Calculate the mean time of the distribution, correct to **one** decimal place. [5]

Qn 3: (i). Use the trapezium rule with equal strips of width $\frac{\pi}{6}$ to find an approximation for $\int_0^{\pi} x \sin x \, dx$. Give your answer to **4** significant figures. [4]

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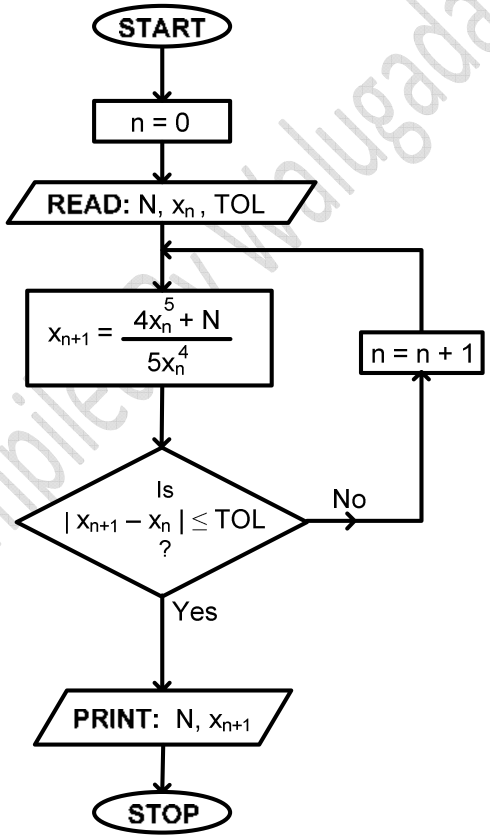
(ii). Comment on how you could obtain a better approximation to the value of the integral using the trapezium rule. [1]

Qn 4: $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are points on a straight road such that $\overline{\boldsymbol{AB}} = \overline{\boldsymbol{BC}} = 0.2$ m. a cyclist moving with uniform acceleration passes $\boldsymbol{A}$ and then notices that it takes him 10 s and 15 s to travel between $\boldsymbol{AB}$ and $\boldsymbol{BC}$ respectively. Find:

(i). his acceleration, [3]
(ii). The velocity with which he passes $\boldsymbol{A}$. [2]

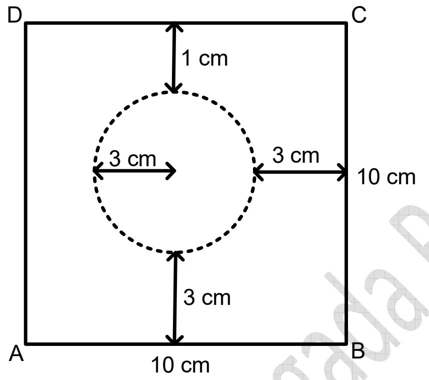
Qn 5: In Ndejje S.S.S, 15% of the students like posho. Find the probability that in a sample of 300 students in the school, over 50 students like posho. [5]

Qn 6: Study the flow chart below:



(i). Using the flow chart, perform a dry run for $x_0 = 2.1$ and $N = 50, TOL = 0.0005$. [4]
(ii). What is the purpose of the flowchart? [1]

Qn 7: Find the centre of gravity of the remainder of the square $ABCD$ if the circle of radius 3 cm is removed as shown below. [5]



Qn 8: A discrete random variable X has the following probability distribution.

x	1	2	3
$P(X = x)$	0.1	0.6	0.3

Find:
(i). $E(5X + 3)$,
(ii). $Var(5X + 3)$. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
X is a random variable such that:

$$f(x) = \begin{cases} \beta(1-2x) & ; \quad -1 \le x \le 0, \\ \beta(1+2x) & ; \quad 0 \le x \le 2, \\ 0 & ; \quad \text{elsewhere.} \end{cases}$$

(a). (i). Sketch the p.d.f, $f(x)$. [3]
(ii). Determine the value of the constant, β. [3]
(b). Find the:
(i). Mean of X, [3]
(ii). 60th percentile. [3]

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Question 10:
(a). Forces of magnitude 3 N, 4 N, 2 N, 1 N, $\boldsymbol{P}$ N, and $\boldsymbol{Q}$ N act along the sides AB, BC, CD, DE, EF and FA. The direction of the forces is given by the order of the letters. Find the value of $\boldsymbol{P}$ and $\boldsymbol{Q}$ if the resultant of the six forces acts along CE. [6]
(b). A uniform ladder of weight, $\boldsymbol{W}$, rests with end **B** against a smooth vertical wall and end **A** on a smooth floor. The ladder is prevented from slipping by a light string attached to the ladder at a point **C** to the point **O** where angle $\boldsymbol{ACO} = 90°$. Show that the tension in the string is given by:

$$T = \frac{\boldsymbol{W}\cos\alpha}{2(\sin^2\alpha - \cos^2\alpha)}$$

Where α is the angle of inclination to the horizontal. [6]

Question 11:
Given the equation $3x^3 + x - 5 = 0$;
(a). (i). Show that the equation has a root between $x = 1$ and $x = 1.5$. [3]
(ii). Hence use linear interpolation to obtain an approximation of the root. [3]
(b). Use Newton Raphson's formula to find the root of the equation by performing two iterations, correct to two decimal places. [6]

Question 12:
Eight candidates seeking admission to a university course sat for a written and oral test. The scores were as shown in the table below:

Written (X)	55	54	35	62	87	53	71	50
Oral (Y)	57	60	47	65	83	56	74	63

(a). (i). Draw a scatter diagram for this data. [3]
(ii). Draw a line of best fit on your scatter diagram. [1]
(b). Use the line of best fit to estimate the value of Y when $X = 70$. [2]
(c). Calculate the rank correlation coefficient. Comment on your result.[6]

Question 13:
(a). At time, t, the position vector of a particle of mass 2 kg is $\left(\cos t\,\underset{\sim}{\boldsymbol{i}} + t^2\underset{\sim}{\boldsymbol{j}}\right)$ m. Show that the force acting on the particle when $t = \pi$ seconds is of magnitude $2\sqrt{5}$ N. [7]

(b). A particle is moving so that at any instant, its velocity vector, $\underset{\sim}{v}$, is given by $\underset{\sim}{v} = \left(3t\underset{\sim}{i} + 4\underset{\sim}{j} + t^2\underset{\sim}{k}\right)$ m s^{-1}. When $t = 0$, it is at the point $(1, 0, 1)$.
Show that the magnitude of its acceleration at $t = 2$ seconds is 5 m s^{-2}. [5]

Question 14:
The numbers $\boldsymbol{A}$ and $\boldsymbol{B}$ are rounded off to $\boldsymbol{a}$ and $\boldsymbol{b}$ with errors $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ respectively.
(a). Show that the absolute relative error in the product $\boldsymbol{AB}$ is given by:

$$\frac{|\boldsymbol{a}||\boldsymbol{e_2}| + |\boldsymbol{b}||\boldsymbol{e_1}|}{\boldsymbol{ab}}.$$

[5]

(b). Given that $\boldsymbol{A} = 6.43$ and $\boldsymbol{B} = 37.2$ are rounded off to the given number of decimal places indicated;
(i). State the maximum possible errors in $\boldsymbol{A}$ and $\boldsymbol{B}$. [2]
(ii). Determine the absolute error in $\boldsymbol{AB}$. [2]
(iii). Find the limits within which the product $\boldsymbol{AB}$ lies. Give your answer to **4** decimal places. [3]

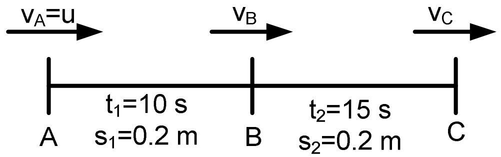
Question 15:
(a). The marks of a certain electric light bulb is known to be normally distributed with a mean life of 2000 hours and a standard deviation of 120 hours. Estimate the probability that the life of such bulb will be:
(i). greater than 2015 hours,
(ii). Between 1850 hours and 2090 hours. [8]
(b). Wavah industry manufactures light bulbs that have a length of life time that are approximately normally distributed with a standard deviation of 40 hours. If a random sample of 36 bulbs have an average life of 780 hours, find the 99.9% confidence interval for the mean of the entire bulbs. [4]

END

MARKING GUIDE

<table>
<tr><th>SNo.</th><th>Working</th><th>Marks</th></tr>
<tr><td>1</td><td>Resultant force, $\underset{\sim}{F} = \begin{pmatrix}-3\\1\end{pmatrix} + \begin{pmatrix}2\\9\end{pmatrix} + \begin{pmatrix}4\\-6\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}$ N
Acceleration, $\underset{\sim}{a} = \frac{\underset{\sim}{F}}{m} = \frac{1}{2}\begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}1.5\\2\end{pmatrix}$ m s^{-2}
Magnitude of acceleration, $\left|\underset{\sim}{a}\right| = \sqrt{(1.5)^2 + 2^2} = 2.5$ m s^{-2}</td><td>M1 B1
B1
M1 A1</td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>2</td><td>
<table>
<tr><th>Class</th><th>f</th><th>x</th><th>fx</th></tr>
<tr><td>30 – 49</td><td>10</td><td>39.5</td><td>395</td></tr>
<tr><td>50 – 64</td><td>30</td><td>57</td><td>1710</td></tr>
<tr><td>65 – 69</td><td>25</td><td>67</td><td>1675</td></tr>
<tr><td>70 – 74</td><td>20</td><td>72</td><td>1440</td></tr>
<tr><td>75 – 79</td><td>15</td><td>77</td><td>1155</td></tr>
<tr><td>Total</td><td></td><td></td><td>6375</td></tr>
</table>
Mean $= \frac{\sum fx}{\sum f} = \frac{6375}{100} = 63.75 \approx 63.8$ (1 d.p)</td><td>B1-for x values
B1-for fx values
B1-for $\sum fx$
M1 A1</td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>3</td><td>(i).
$y_n = x_n \sin x_n$, $h = \frac{\pi}{6}$
<table>
<tr><th>n</th><th>x_n</th><th>y_0, y_6</th><th>$y_1, \dots y_5$</th></tr>
<tr><td>0</td><td>0</td><td>0</td><td></td></tr>
<tr><td>1</td><td>$\frac{\pi}{6}$</td><td></td><td>0.261799</td></tr>
<tr><td>2</td><td>$\frac{2\pi}{6}$</td><td></td><td>0.906900</td></tr>
<tr><td>3</td><td>$\frac{3\pi}{6}$</td><td></td><td>1.570796</td></tr>
<tr><td>4</td><td>$\frac{4\pi}{6}$</td><td></td><td>1.813799</td></tr>
<tr><td>5</td><td>$\frac{5\pi}{6}$</td><td></td><td>1.308997</td></tr>
<tr><td>6</td><td>π</td><td>0</td><td></td></tr>
<tr><td>sums</td><td></td><td>0</td><td>5.862291</td></tr>
</table>
Approximation for $\int_0^{\pi} x \sin x\, dx = \frac{1}{2}h[(y_0 + y_5) + 2(y_1 + \cdots + y_4)]$
$= \frac{1}{2} \times \frac{\pi}{6}[0 + 2 \times 5.862291] = 3.069488 \approx 3.069$ (4 s.f)
(ii). By increasing on the number of strips.</td><td>B1
B1-for x_n values
B1-for y_n values
M1 B1
B1</td></tr>
<tr><td></td><td></td><td>05</td></tr>
</table>

4



(i).
For motion AB, $s = 0.2$ m, $t = 10$ s,

$$s = ut + \frac{1}{2}at^2$$
$$0.2 = 10u + \frac{1}{2}a \times 10^2$$
$$0.2 = 10u + 50a$$
$$0.02 = u + 5a \quad \longrightarrow (1)$$

For motion AC, $s = 0.2 + 0.2 = 0.4$ m, $t = 10 + 15 = 25$ s, **B1**

$$s = ut + \frac{1}{2}at^2$$
$$0.4 = 25u + \frac{1}{2}a \times 25^2$$
$$0.4 = 25u + 312.5a$$
$$0.016 = u + 12.5a \quad \longrightarrow (2)$$

Equations $(1) - (2)$ gives; **B1**

$$0.004 = -7.5a$$
$$a = -\frac{0.004}{7.5} = -0.000533 \text{ m s}^{-2}$$ **A1**

(ii). From equation (1)

$$u = 0.02 - 5a = 0.02 - 5 \times \frac{-0.004}{7.5} = \frac{17}{750}$$ **M1 A1**
$$\approx 0.02267 \text{ m s}^{-1}$$

05

5

Let $X\sim$ be the number of students who like posho

$$n = 300, p = 0.15, q = 1 - 0.15 = 0.85$$
$$\mu = np = 300 \times 0.15 = 45$$ **B1**
$$\sigma = \sqrt{npq} = \sqrt{300 \times 0.15 \times 0.85} = \sqrt{38.25} = 6.185$$ **B1**
$$P(\text{over 50 like posho}) = P(X > 50)$$

By continuity correction,

$$P(X > 50) \rightarrow P(X > 50.5)$$
$$= P\left(Z > \frac{50.5 - 45}{\sqrt{38.25}}\right) = P(Z > 0.889)$$ **B1**
$$= 0.5 - \phi(0.889) = 0.5 - 0.3130 = 0.1870$$ **M1 B1**

05

6

(i).

$$\Rightarrow x_{n+1} = \frac{4x_n^{\,5} + 50}{5x_n^{\,4}}$$

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$	
0	2.1	2.19419	0.09419	**B1**
1	2.19419	2.18677	0.00742	**B1**
2	2.18677	2.18672	0.00005	**B1**

$$\therefore N = 50, \qquad x_{n+1} = 2.187 \text{ (3 d.p)}$$ **B1**

(ii).

$$x_{n+1} = \frac{4x_n^{\,5} + N}{5x_n^{\,4}}$$

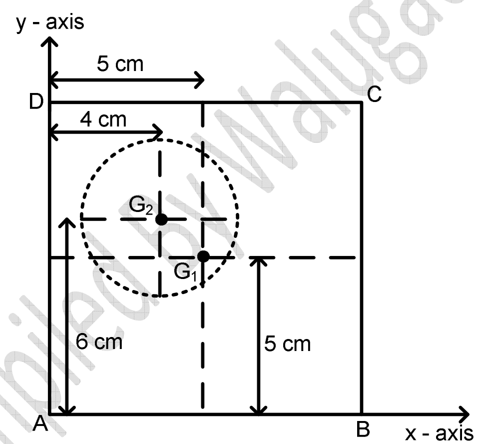
As $n \to \infty$, $x_n \to x$ where x it the root.

$$x = \frac{4x^5 + N}{5x^4}$$
$$5x^5 = 4x^5 + N$$
$$x^5 = N$$
$$x = \sqrt[5]{N}$$

The purpose of the flow chart is to find the fifth root of a number, N. **B1**

05

7



Let ρ be the weight per unit area.

Figure	Area	Weight w_i	Coordinates of the centre of gravity x_i	y_i
Square	$10 \times 10 = 100$	100ρ	$\frac{1}{2} \times 10 = 5$	$\frac{1}{2} \times 10 = 5$
Circle	$\pi \times 3^2 = 9\pi$	$9\pi\rho$	$1 + 3 = 4$	$3 + 3 = 6$
Remaining lamina	$100 - 9\pi$	$(100 - 9\pi)\rho$	$\bar{x}$	$\bar{y}$

B1-for x_i and y_i

By taking moments about y-axis,

$$100\rho \times 5 = 9\pi\rho \times 4 + (100 - 9\pi)\rho\bar{x}$$ **M1**

$$500 = 36\pi + (100 - 9\pi)\bar{x}$$

$$\bar{x} = \frac{500 - 36\pi}{100 - 9\pi} \approx 5.394$$ **A1**

By taking moments about x-axis,

$$100\rho \times 5 = 9\pi\rho \times 6 + (100 - 9\pi)\rho\bar{y}$$

$$500 = 54\pi + (100 - 9\pi)\bar{y}$$ **M1**

$$\bar{y} = \frac{500 - 54\pi}{100 - 9\pi} \approx 4.606$$ **A1**

The distance of the centre of gravity, G, of the remaining lamina is 5.394 cm from AD and 4.606 cm from AB.

05

8

x	$P(X = x)$	$xP(X = x)$	$x^2P(X = x)$
1	0.1	0.1	0.1
2	0.6	1.2	2.4
3	0.3	0.9	2.7
Total	1	2.2	5.2

B1-for $xP(X = x)$

B1-for $x^2P(X = x)$

(i).

$$E(5X + 3) = 5E(X) + 3 = 5 \times 2.2 + 3 = 14$$ **A1**

(ii).

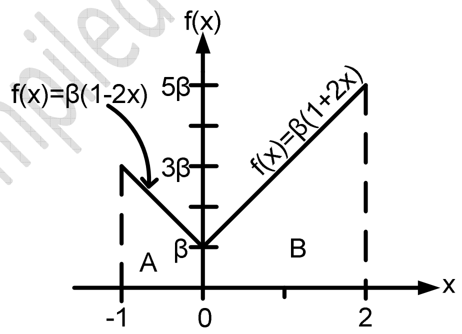
$$Var(5X + 3) = 5^2 \times Var(X) = 25[5.2 - (2.2)^2] = 9$$ **M1 A1**

05

9 (a). (i).

$-1 \leq x \leq 0,\quad f(-1) = \beta(1 + 2) = 3\beta,\quad f(0) = \beta(1 - 0) = \beta$

$0 \leq x \leq 2,\quad f(0) = \beta(1 + 0) = \beta,\quad f(2) = \beta(1 + 4) = 5\beta$ **B1**



B1-for $f(x) = \beta(1 - 2x)$

B1-for $f(x) = \beta(1 + 2x)$

(ii).

total area under the graph = area A + area B

$$1 = \frac{1}{2} \times (0 + 1)(3\beta + \beta) + \frac{1}{2} \times (2 - 0)(\beta + 5\beta)$$ **M1**

$$1 = 2\beta + 6\beta, \qquad \Rightarrow \beta = \frac{1}{8}$$ **A1**

(b).(i).

$$f(x) = \begin{cases} \frac{1}{8}(1 - 2x) & ; \quad -1 \leq x \leq 0, \\ \frac{1}{8}(1 + 2x) & ; \quad 0 \leq x \leq 2, \\ 0 & ; \quad elsewhere. \end{cases}$$

$$E(X) = \int_{all\ x} xf(x)\,dx$$

$$= \frac{1}{8}\int_{-1}^{0}(x - 2x^2)\,dx + \frac{1}{8}\int_{0}^{2}(x + 2x^2)\,dx$$

$$= \frac{1}{8}\left[\frac{1}{2}x^2 - \frac{2}{3}x^3\right]_{-1}^{0} + \frac{1}{8}\left[\frac{1}{2}x^2 + \frac{2}{3}x^3\right]_{0}^{2}$$ **M1**

$$= \frac{1}{8}\left\{0 - \left(\frac{1}{2} + \frac{2}{3}\right)\right\} + \frac{1}{8}\left\{\left(2 + \frac{16}{3}\right) - 0\right\}$$ **M1**

$$= -\frac{7}{48} + \frac{11}{12} = \frac{17}{16} = 1.0625$$ **A1**

(ii).

Let p be the 60th percentile

$$\int_{-1}^{p} f(x)\,dx = 0.6$$

$$\text{but } \int_{-1}^{0} \frac{1}{8}(1 - 2x)\,dx = 2\beta = 2 \times \frac{1}{8} = 0.25$$

$$\Rightarrow \int_{-1}^{p} f(x)\,dx = \int_{-1}^{0} \frac{1}{8}(1 - 2x)\,dx + \int_{0}^{p} \frac{1}{8}(1 + 2x)\,dx$$

$$0.6 = 0.25 + \frac{1}{8}\left[\ x + x^2\right]_{0}^{p}$$ **M1**

$$0.35 = \frac{1}{8}(p + p^2) - 0$$

$$2.8 = p + p^2$$

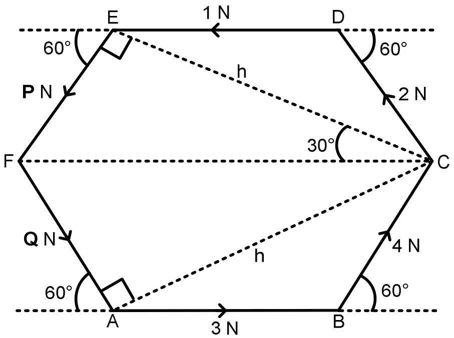
$$p = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-2.8)}}{2 \times 1}$$ **M1**

$$p = 1.246, \quad \text{or,} \quad p = -2.246$$

For the interval; $0 \leq x \leq 2, p \neq -2.246$. Thus 60th percentile = 1.246. **B1** **A1**

12

10 (a).



$$F_R = \binom{3}{0} + \binom{4\cos 60°}{4\sin 60°} + \binom{-2\cos 60°}{2\sin 60°} + \binom{-1}{0} + \binom{-P\cos 60°}{-P\sin 60°} + \binom{Q\cos 60°}{-Q\sin 60°}$$ **M1 M1**

$$F_R = \binom{3 - 0.5P + 0.5Q}{3\sqrt{3} - 0.5\sqrt{3}P - 0.5\sqrt{3}Q}$$

From ΔABC, $h = \sqrt{(2a)^2 + (2a)^2 - 2 \times 2a \times 2a\cos 120°}$

$= 2a\sqrt{3}$ m

Total moment of forces about C = Moment of resultant about C

$(1 \times 2a\sin 120°) + (P \times 2a\sqrt{3}) + (Q \times 2a\sqrt{3}) + (3 \times 2a\sin 120°) = F_R \times 0$ **M1**

$$a\sqrt{3} + 2aP\sqrt{3} + 2aQ\sqrt{3} + 3a\sqrt{3} = 0$$
$$4a\sqrt{3} + 2a(P+Q)\sqrt{3} = 0$$
$$2 + P + Q = 0$$
$$Q = -2 - P$$

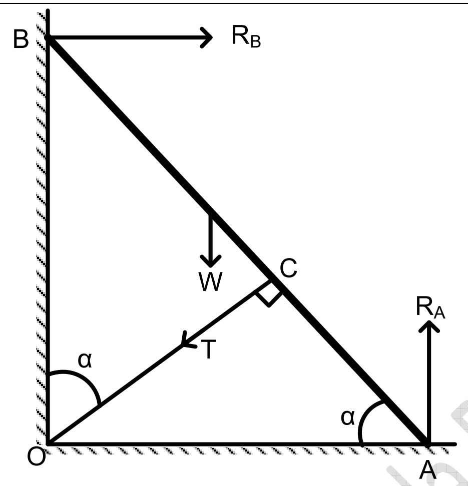
Direction, $\theta = \tan^{-1}\left(\frac{3\sqrt{3} - 0.5\sqrt{3}P - 0.5\sqrt{3}Q}{3 - 0.5P + 0.5Q}\right) = 30°$ **M1**

$$\frac{3\sqrt{3} - 0.5\sqrt{3}P - 0.5\sqrt{3}Q}{3 - 0.5P + 0.5Q} = \tan 30°$$

$$3\sqrt{3} - 0.5\sqrt{3}P - 0.5\sqrt{3}Q = \frac{1}{\sqrt{3}}(3 - 0.5P + 0.5Q)$$ **B1**

$$9 - 1.5P - 1.5Q = 3 - 0.5P + 0.5Q$$
$$6 - P - 2Q = 0$$
$$6 - P - 2(-2 - P) = 0$$
$$6 - P + 4 + 2P = 0$$
$$10 + P = 0$$
$$P = -10$$ **A1**
$$Q = -2 - P = -2 + 10 = 8$$ **A1**

(b).



B1

$AB = l$, $OA = l\sin\theta$, $OB = l\cos\theta$

Resolving vertically,

$$R_A = T\cos\alpha + W$$

Resolving horizontally,

$$R_B = T\sin\alpha$$

Taking moments about O,

$$R_A \times l\cos\alpha = R_B \times l\sin\alpha + W \times \frac{l\cos\alpha}{2}$$

$$(T\cos\alpha + W)\cos\alpha = T\sin\alpha \times \sin\alpha + W \times \frac{\cos\alpha}{2}$$ **M1 M1**

$$T\cos^2\alpha + W\cos\alpha = T\sin^2\alpha + \frac{W\cos\alpha}{2}$$

$$\frac{W\cos\alpha}{2} = T(\sin^2\alpha - \cos^2\alpha)$$ **B1**

$$\frac{W\cos\alpha}{2(\sin^2\alpha - \cos^2\alpha)} = T$$ **A1**

12

11 (a). (i).

let, $f(x) = 3x^3 + x - 5$

$f(1) = 3(1)^3 + 1 - 5 = -1$ **M1**

$f(1.5) = 3(1.5)^3 + 1.5 - 5 = 6.625$ **M1**

Since $f(1).f(1.5) < 0$, $\Rightarrow 1 < \text{root} < 1.5$ **B1**

(ii).

x	1	x_0	1.5
$f(x)$	-1	0	6.625

$$\frac{x_0 - 1}{0 - (-1)} = \frac{1.5 - 1}{6.625 - (-1)}$$ **M1 M1**

$$x_0 = \frac{0.5}{7.625} \times 1 + 1 = 1.0656$$ **A1**

(b).

$$f(x) = 3x^3 + x - 5$$
$$f'(x) = 9x^2 + 1$$ **B1**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3x_n{}^3 + x_n - 5}{9x_n{}^2 + 1}$$ **M1**

$$= \frac{9x_n{}^3 + x_n - 3x_n{}^3 - x_n + 5}{9x_n{}^2 + 1} = \frac{6x_n{}^3 + 5}{9x_n{}^2 + 1}$$

Using $x_0 = 1.0656$

$$x_1 = \frac{6(1.0656)^3 + 5}{9(1.0656)^2 + 1} = 1.0927$$ **M1 B1**

$$x_2 = \frac{6(1.0927)^3 + 5}{9(1.0927)^2 + 1} = 1.0921$$ **M1 B1**

$\therefore$ Root $= 1.09$ (2 d.p) **A1**

12

12 (a).

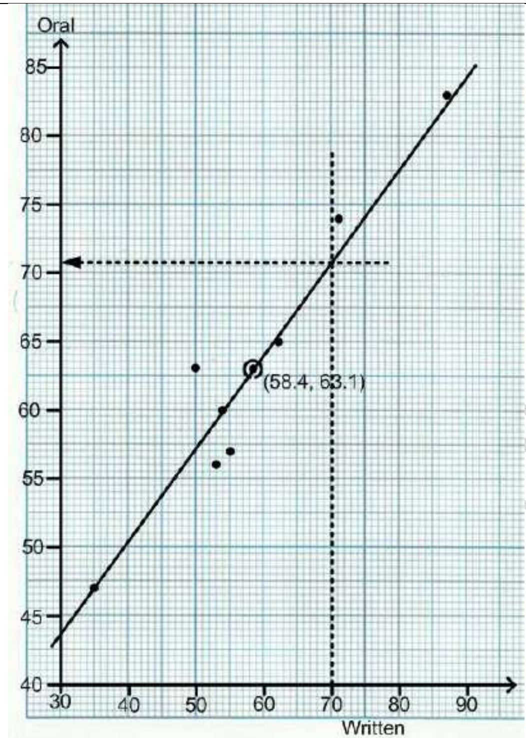
x	y	R_x	R_y	d	d^2
55	57	4	6	-2	4
54	60	5	5	0	0
35	47	8	8	0	0
62	65	3	3	0	0
87	83	1	1	0	0
53	56	6	7	-1	1
71	74	2	2	0	0
50	63	7	4	3	9
467	505				$\sum d^2$ =14

B1-correct ranking

B1-$\sum d^2$

$$\overline{x} = \frac{467}{8} = 58.375 \approx 58.4, \quad \overline{y} = \frac{505}{8} = 63.125 \approx 63.1$$

$\Longrightarrow$ Mean point, $(\overline{x}, \overline{y}) = (58.4, 63.1)$ **B1 B1**



B1-axes

B1-plotting

B1-line of best fit thru the mean point

M1-dotted line for estimation

(b). $Y = 70.75$ when $X = 70$. **B1**

(c).

$$\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 14}{8(8^2 - 1)} = 0.8333$$ **M1 A1**

Comment: Significant at 5% (or: Insignificant at 1%.) **B1**

12

13 (a).

$$m = 2\text{ kg}, \quad \underset{\sim}{r} = \begin{pmatrix} \cos t \\ t^2 \end{pmatrix} \text{ m}$$

$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = \begin{pmatrix} -\sin t \\ 2t \end{pmatrix}$$ **M1**

$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = \begin{pmatrix} -\cos t \\ 2 \end{pmatrix}$$ **M1**

When $t = \pi$,

when $t = \pi$, $\underset{\sim}{a} = \begin{pmatrix} -\cos\pi \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ m s^{-2} **M1 B1**

$$\underset{\sim}{F} = m\underset{\sim}{a} = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \text{ N}$$ **B1**

$$\left|\underset{\sim}{F}\right| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}\text{ N}, \quad \text{as required}$$ **M1 B1**

(b).

	$\underset{\sim}{v} = \begin{pmatrix} 3t \\ 4 \\ t^2 \end{pmatrix} \text{ m s}^{-1}$ $\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = \begin{pmatrix} 3 \\ 0 \\ 2t \end{pmatrix} \text{ m s}^{-2}$ when $t = 2$, $\quad \underset{\sim}{a} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \text{ m s}^{-2}$ $\lvert \underset{\sim}{a} \rvert = \sqrt{3^2 + 0 + 4^2} = \sqrt{25} = 5 \text{ m s}^{-2}$, as required	 M1 M1 B1 M1 A1
		12
14	(a). $A = a + e_1, \quad B = b + e_2$ exact, $Z = (AB)$, approximate, $z = (ab)$ error, $e = (AB) - (ab)$ $= \{(a + e_1)(b + e_2)\} - (XY)$ $= \{ab + ae_2 + be_1 + e_1e_2\} - (ab)$ $= ae_2 + be_1 + e_1e_2$ suppose $e_1 \ll a$ and $e_2 \ll b$, $\implies e_1e_2 \approx 0$ $\therefore e = be_1 + ae_2$ $\left\lvert \frac{e}{z} \right\rvert = \left\lvert \frac{be_1 + ae_2}{ab} \right\rvert$ $\left\lvert \frac{e}{Z} \right\rvert \leq \frac{\lvert b \rvert \lvert e_1 \rvert + \lvert a \rvert \lvert e_2 \rvert}{ab}$ $\therefore$ maximum relative error in the product $AB = \frac{\lvert a \rvert \lvert e_2 \rvert + \lvert b \rvert \lvert e_1 \rvert}{ab}$ (b). (i). Maximum error is A, $\quad e_1 = 0.005$ Maximum error is B, $\quad e_2 = 0.05$ (ii). $(AB)_{max} = (6.43 + 0.005)(37.2 + 0.05) = 239.70375$ $(AB)_{min} = (6.43 - 0.005)(37.2 - 0.05) = 238.68875$ Absolute error in $AB = \frac{239.70375 - 238.68875}{2} = 0.5075$ (iii). Upper limit $= (AB)_{max} = 239.70375 \approx 239.7038$ (4 dp) Lower limit $= (AB)_{min} = 238.68875 \approx 238.6888$ (4 dp)	 B1 B1 B1 B1 B1 B1 B1 B1 B1 M1 A1 B1-for rounding off to 4 d.p
		12
15	(a). (i). $P(X > 2015) = P\left(z > \frac{2015 - 2000}{120}\right) = P(z > 0.125)$ $= 0.5 - \phi(0.125)$ $= 0.5 - 0.0498 = 0.4502$ (ii). $P(1850 < X < 2090) = P\left(\frac{1850 - 2000}{120} < z < \frac{2090 - 2000}{120}\right)$ $= P(-1.250 < z < 0.750)$	 M1 M1 B1 A1 M1

	$= \phi(1.250) + \phi(0.750)$ $= 0.3944 + 0.2734 = 0.6678$ (b). For the 99% confidence interval, $\phi(z_{\alpha/2}) = \frac{0.99}{2} = 0.475, \quad \implies z_{\alpha/2} = \phi^{-1}(0.475) = 1.96$ Confidence limits $= \overline{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $= 780 \pm 1.96 \times \frac{40}{\sqrt{36}} = 780 \pm 13.0667$ Confidence interval $= [766.9333, 793.0667]$	M1 B1 B1 A1 B1 M1 A1
		12

END

P425/1
PURE
MATHEMATICS
PAPER 1
July 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 1 MOCK SET 4 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn 1: Given that $p^2 = qr$, show that $2\log_q p \log_r p = \log_q p + \log_r p$. [5]

Qn 2: Evaluate: $\int_1^4 \frac{(1+\sqrt{x})^5}{\sqrt{x}}\,dx$. [5]

Qn 3: Find the equations of the tangents to the parabola $y^2 = 6x$ which pass through the point $(10, -8)$. [5]

Qn 4: Solve the equation: $\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$. [5]

Qn 5: If $y = 3^x$, find $\frac{d^2y}{dx^2}$ when $x = -1$. [5]

Qn 6: Solve the simultaneous equations:
$$\cos x + \cos y = 1$$
$$\sec x + \sec y = 4$$
for $0° < x, y < 180°$. [5]

Qn 7: Show that the lines $\boldsymbol{L_1}$, vector equation $\underset{\sim}{\boldsymbol{r}} = \begin{pmatrix}2\\5\end{pmatrix} + \lambda\begin{pmatrix}2\\-3\end{pmatrix}$ and $\boldsymbol{L_2}$, vector equation $\underset{\sim}{\boldsymbol{r}} = \begin{pmatrix}3\\-3\end{pmatrix} + \mu\begin{pmatrix}3\\2\end{pmatrix}$ are perpendicular and find the position vector of their point of intersection. [5]

Qn 8: Find the percentage increase in the volume of a cube when all the edges of the cube are increased by 2%. [5]

Section B (60 Marks)
Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
(a). Given that Z_1 and Z_2 are complex numbers, solve the simultaneous equations:
$$4Z_1 + 3Z_2 = 23$$
$$Z_1 + iZ_2 = 6 + 8i$$
[6]

(b). If $Z = x + yi$, find the locus given by
$$\left|\frac{Z-1}{Z+1-i}\right| = \frac{2}{5}$$
[6]

Question 10:
Given that $x = \frac{1-t}{1+t}$ and $y = (1-t)(1+t)^2$, find $\frac{d^2y}{dx^2}$. [12]

Question 11:
(a). Find the coordinates of the foot of the perpendicular from the point $(2, -6)$ to the line $3y - x + 2 = 0$. [4]
(b). A circle touches both x −axis and the line $4x - 3y + 4 = 0$. Its centre is in the first quadrant and lies on the line $x - y - 1 = 0$. Prove that its equation is $x^2 + y^2 - 6x - 4y + 9 = 0$. [8]

Question 12:
Express $y = \frac{64x^2 - 148x + 78}{(4x-5)^3}$ into partial fractions. Hence find $\int_4^6 y\,dx$. [12]

Question 13:
Given that $y = \frac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x}$, prove that $y = -\tan^2\frac{x}{2}$. hence find the value of $\tan^2 15°$ in the form $p + q\sqrt{r}$ where p, q and r are integers and and solve $2y + \sec^2\frac{x}{2} = 0$ for $0° \le x \le 360°$. [12]

Question 14:

(a). Solve the differential equation

$$\frac{dy}{dx} = \sqrt{\frac{y}{x+1}}$$

Given that $y = 9$ when $x = 3$. [5]

(b). At time t minutes, the rate of change of temperature of a cooling liquid is proportional to the temperature, T °C of that liquid at that time. Initially $T = 80$.

(i). Show that $T = 80e^{-kt}$,

(ii). If $T = 20$ when $t = 6$, find the time at which the temperature will reach 10°C . [7]

Question 15:

(a). One root of the equation $x^2 - 6x + k = 0$ is three times the other. Find the roots and the value of k. [5]

(b). If x is so small that x^5 and higher powers of x can neglected such that

$$(1+x)^6(1-3x^3)^{10} \approx 1 + ax + bx^2 + cx^4$$

Find the values of a, b and c. [7]

Question 16:

(a). Find the Cartesian equation of the plane containing the point $(1, 3, 1)$ and parallel to the vectors $\underset{\sim}{\boldsymbol{i}} - \underset{\sim}{\boldsymbol{j}} + 3\underset{\sim}{\boldsymbol{k}}$ and $2\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}} - 3\underset{\sim}{\boldsymbol{k}}$. [7]

(b). Find the angle between the plane in (a) above and the line

$$\frac{x-6}{5} = \frac{y-1}{-1} = \frac{z+1}{4}$$

[5]

END

MARKING GUIDE

SNo.	Working	Marks
1	$p^2 = qr$	
	$\log_p p^2 = \log_p(qr)$	M1
	$2\log_p p = \log_p q + \log_p r$	M1
	$2 = \frac{1}{\log_q p} + \frac{1}{\log_r p}$	M1
	$2 = \frac{\log_r p + \log_q p}{\log_q p \log_r p}$	M1
	$2\log_q p \log_r p = \log_r p + \log_q p$, as required	B1
		05
2	let $u = 1+\sqrt{x}$, $\Rightarrow du = \frac{1}{2\sqrt{x}}dx$, $\Rightarrow dx = 2\sqrt{x}du$	B1
	x: 1, 4; u: 2, 3	B1
	$\int_1^4 \frac{(1+\sqrt{x})^5}{\sqrt{x}}dx = \int_2^3 \frac{u^5}{\sqrt{x}} \times 2\sqrt{x}du = \int_2^3 2u^5\,du$	
	$= \left[\frac{1}{3}u^6\right]_2^3 = \frac{1}{3}(3^6 - 2^6) = \frac{665}{3}$	M1 M1 A1
		05
3	$4a = 6$, $\Rightarrow a = \frac{3}{2}$	
	Equation of the tangent is: $y = mx + \frac{a}{m}$	
	At $(10, -8)$, $-8 = 10m + \frac{3}{2m}$	
	$20m^2 + 16m + 3 = 0$	M1
	$m = \frac{-16 \pm \sqrt{16^2 - 4\times 20\times 3}}{2\times 20} = \frac{-16 \pm 4}{40}$	M1
	$m = \frac{-16-4}{40} = -\frac{1}{2}$, or, $m = \frac{-16+4}{40} = -\frac{3}{10}$	B1
	The tangents are: $y = -\frac{1}{2}x - 3$, and, $y = -\frac{3}{10}x - 5$	A1 A1
		05
4	$\left(\sqrt{x+5} + \sqrt{x+21}\right)^2 = \left(\sqrt{6x+40}\right)^2$	
	$x + 5 + x + 21 + 2\sqrt{x^2 + 26x + 105} = 6x + 40$	M1
	$2\sqrt{x^2 + 26x + 105} = 4x + 14$	

	$\sqrt{x^2+26x+105} = 2x+7$	
	$x^2+26x+105 = 4x^2+28x+49$	
	$3x^2+2x-56 = 0$	M1
	$x = \dfrac{-2 \pm \sqrt{2^2 - 4\times 3\times(-56)}}{2\times 3} = \dfrac{-2\pm 26}{6}$	M1
	$x = \dfrac{-2-26}{6} = -\dfrac{14}{3}$, or, $x = \dfrac{-2+26}{6} = 4$	B1
	$x \neq -\dfrac{14}{3}$, $\Rightarrow x = 4$	A1
		05
5	$y = 3^x$	
	$\ln y = \ln 3^x$	
	$\ln y = x\ln 3$	
	$\dfrac{1}{y}\dfrac{dy}{dx} = \ln 3$	M1
	$\dfrac{dy}{dx} = y\ln 3$	A1
	$\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx}\ln 3$	
	$\dfrac{d^2y}{dx^2} = (y\ln 3)\ln 3 = 3^x(\ln 3)^2$	M1
	$\left.\dfrac{d^2y}{dx^2}\right\|_{x=-1} = 3^{-1}(\ln 3)^2 = \dfrac{1}{3}(\ln 3)^2 \approx 0.4023$	M1 A1
		05
6	$\cos x + \cos y = 1$	
	$\sec x + \sec y = 4$	
	for, $\sec x + \sec y = 4$	
	$\dfrac{1}{\cos x} + \dfrac{1}{\cos y} = 4$	
	$\dfrac{\cos x + \cos y}{\cos x \cos y} = 4$	M1
	$\dfrac{1}{\cos x\,(1-\cos x)} = 4$	
	$4\cos x - 4\cos^2 x = 1$	
	$4\cos^2 x - 4\cos x + 1 = 0$	B1
	$(2\cos x - 1)^2 = 0$	M1
	$\cos x = 0.5$, $\Rightarrow x = 60°$	A1
	$\cos y = 1 - 0.5 = 0.5$, $\Rightarrow y = 60°$	A1
		05
7	$\underset{\sim}{\boldsymbol{d}_1} = \begin{pmatrix}2\\-3\end{pmatrix}$, $\underset{\sim}{\boldsymbol{d}_2} = \begin{pmatrix}3\\2\end{pmatrix}$	
	For lines $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ to be perpendicular,	
	$\underset{\sim}{\boldsymbol{d}_1}.\underset{\sim}{\boldsymbol{d}_2} = 0$	

	$\begin{pmatrix}2\\-3\end{pmatrix}.\begin{pmatrix}3\\2\end{pmatrix} = 0$	M1
	$6 - 6 = 0$	
	$0 = 0$, hence shown	B1
	$\begin{pmatrix}2+2\lambda\\5-3\lambda\end{pmatrix} = \begin{pmatrix}3+3\mu\\-3+2\mu\end{pmatrix}$	M1
	$2\lambda - 3\mu = 1 \quad \longrightarrow (1)$	
	$3\lambda + 2\mu = 8 \quad \longrightarrow (2)$	
	Equation $3\times(1) - 2\times(2)$ gives	
	$6\lambda - 9\mu = 3$	
	$-\ 6\lambda + 4\mu = 16$	M1
	$-13\mu = -13$	
	$\mu = 1$	A1
	Position vector $= \begin{pmatrix}3+3\times 1\\-3+2\times 1\end{pmatrix} = \begin{pmatrix}6\\-1\end{pmatrix}$	
		05
8	Let the length of the sides be x.	
	$v = x^3$, $\Rightarrow \dfrac{dv}{dx} = 3x^2$	M1
	$\dfrac{\delta v}{\delta x} \approx \dfrac{dv}{dx}$, $\Rightarrow \delta v \approx 3x^2\delta x$	B1
	percentage increase in volume $= \dfrac{\delta v}{v}\times 100 \approx \dfrac{3x^2\delta x}{x^3}\times 100$	M1
	$\approx 3\left(\dfrac{\delta x}{x}\times 100\right)$	M1
	$\approx 3(2)$	
	$\approx 6\%$	A1
		05
9	(a).	
	$4Z_1 + 3Z_2 = 23 \quad \longrightarrow (1)$	
	$Z_1 + iZ_2 = 6 + 8i \quad \longrightarrow (2)$	
	Equation $(1) - 4\times(2)$ gives	
	$4Z_1 + 3Z_2 = 23$	
	$-\ 4Z_1 + 4iZ_2 = 24 + 32i$	
	$(3-4i)Z_2 = -1 - 32i$	M1
	$Z_2 = \dfrac{(-1-32i)\times(3+4i)}{(3-4i)\times(3+4i)}$	M1
	$Z_2 = \dfrac{-2-96i-4i+12}{9+16} = \dfrac{125-100i}{25} = 5-4i$	M1 A1
	From equation (2),	
	$Z_1 = 6+8i - i(5-4i) = 6+8i-5i-4 = 2+3i$	M1 A1
	(b).	
	$\left\|\dfrac{x+yi-1}{x+yi+1-i}\right\| = \dfrac{2}{5}$	
	$5\|(x-1)+yi\| = 2\|(x+1)+(y-1)i\|$	

Compiled By Walugada Ronald

$$5\sqrt{(x-1)^2+y^2}=2\sqrt{(x+1)^2+(y-1)^2}$$ **M1**

$$25[(x-1)^2+y^2]=4[(x+1)^2+(y-1)^2]$$ **M1**

$$25[x^2-2x+1+y^2]=4[x^2+2x+1+y^2-2y+1]$$ **M1 M1**

$$25x^2-50x+25+25y^2=4x^2+8x+4+4y^2-8y+4$$

$$21x^2+21y^2-58x+8y+17=0$$ **M1 A1**

12

10

$$x=\frac{1-t}{1+t}$$

$$\frac{dx}{dt}=\frac{-(1+t)-(1-t)}{(1+t)^2}=\frac{-1-t-1+t}{(1+t)^2}=\frac{-2}{(1+t)^2}$$

M1 M1 B1 (M2-correct use of quotient rule, B1-simplification and output)

$$y=(1-t)(1+t)^2$$

$$\frac{dy}{dt}=-(1+t)^2+2(1+t)(1-t)$$

$$=(1+t)(-1-t+2-2t)=(1+t)(1-3t)$$

M1 B1 (M1-correct use of product rule, B1-correct output)

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=(1+t)(1-3t)\times\frac{(1+t)^2}{-2}$$

$$=-\frac{1}{2}(1-3t)(1+t)^3$$

M1-chain rule

A1-correct output

$$\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx}$$

$$=-\frac{1}{2}[3(1+t)^2(1-3t)-3(1+t)^3]\times\frac{(1+t)^2}{-2}$$

$$=\frac{1}{4}(1+t)^4[3(1-3t)-3(1+t)]$$

$$=\frac{1}{4}(1+t)^4[3-9t-3-3t]$$

$$=-3t(1+t)^4$$

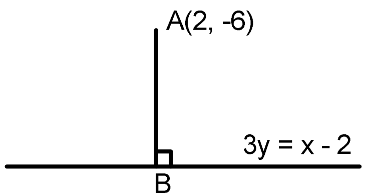
M1 M1 M1 (chain rule+product rule)

M1-simplification

A1-

12

11 (a).



$$3y-x+2=0,\qquad\Rightarrow y=\frac{1}{3}x-\frac{2}{3}$$

$$\therefore \text{Gradient of } AB=-3$$

$$\frac{y+6}{x-2}=-3$$

$$y+6=-3x+6$$

$$y=-3x$$

B1-eqn of perpendicular

At the foot B,

$$\frac{1}{3}x-\frac{2}{3}=-3$$

M1

$$x-2=-9x$$

$$10x=2,\qquad\Rightarrow x=\frac{1}{5}$$

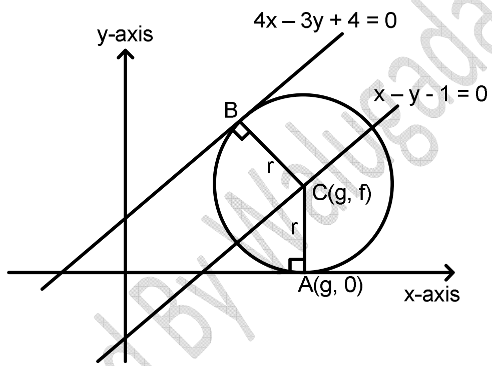
$$y=-3\times\frac{1}{5}=-\frac{3}{5}$$

B1

$$\therefore B\left(\frac{1}{5},\frac{-3}{5}\right)$$

A1

(b).



$$\overline{BC}=\frac{|4g-3f+4|}{\sqrt{4^2+(-3)^2}}=\frac{4g-3f+4}{5}$$

$$\overline{AC}=\sqrt{(g-g)^2+(f-0)^2}=f$$

but, $\overline{BC}=\overline{AC}$

$$\frac{4g-3f+4}{5}=f$$

M1

$$4g-3f+4=5f$$

$$4g-8f+4=0\quad\longrightarrow(1)$$

B1

Also, centre (g,f) lie on the line $x-y-1=0$

$$g-f-1=0\quad\longrightarrow(2)$$

B1

Equation $(1)-4\times(2)$ gives

$$\begin{array}{r|l|} & 4g-8f+4=0 \\ - & 4g-4f-4=0 \\ \hline & -4f+8=0 \end{array}$$

M1

$$f=2$$

B1

From equation (2),

	$g = 2 + 1 = 3$	B1
	The equation of the circle is given by:	
	$(x-3)^2 + (y-2)^2 = 2^2$	M1
	$x^2 - 6x + 9 + y^2 - 4y + 4 = 4$	
	$x^2 + y^2 - 6x - 4y + 9 = 0$	B1
		12
12	$\frac{64x^2 - 148x + 78}{(4x-5)^3} \equiv \frac{A}{4x-5} + \frac{B}{(4x-5)^2} + \frac{C}{(4x-5)^3}$	
	$64x^2 - 148x + 78 \equiv A(4x-5)^2 + B(4x-5) + C$	M1
	$64x^2 - 148x + 78 \equiv A(16x^2 - 40x + 25) + B(4x-5) + C$	
	For $x = \frac{5}{4}$;	
	$100 - 185 + 78 = C, \quad \Rightarrow C = -7$	B1
	Coefficient of x^2 ;	
	$64 = 16A, \quad \Rightarrow A = 4$	B1
	Coefficient of x^0 ;	
	$78 = 25A - 5B + C, \quad \Rightarrow 78 = 100 - 5B - 7$	M1
	$15 = 5B, \quad \Rightarrow B = 3$	B1
	$\therefore \frac{64x^2 - 148x + 78}{(4x-5)^3} \equiv \frac{4}{4x-5} + \frac{3}{(4x-5)^2} - \frac{7}{(4x-5)^3}$	A1
	For the hence part:	
	$\int_4^6 y\,dx = \int_4^6 \left(\frac{4}{4x-5} + \frac{3}{(4x-5)^2} - \frac{7}{(4x-5)^3}\right) dx$	
	$= \left[\ln(4x-5) - \frac{3}{4(4x-5)} + \frac{7}{8(4x-5)^2}\right]_4^6$	M1 M1 M1
	$= (\ln 19 - \ln 11) - \frac{3}{4}\left(\frac{1}{19} - \frac{1}{11}\right) + \frac{7}{8}\left(\frac{1}{19^2} - \frac{1}{11^2}\right)$	M1 M1
	$= 0.57044$	A1
		12
13	$y = \frac{\sin 3x + \sin x - 2\sin 2x}{\sin 3x + \sin x + 2\sin 2x}$	
	$= \frac{2\sin 2x \cos x - 2\sin 2x}{2\sin 2x \cos x + 2\sin 2x}$	M1-factor formula
	$= \frac{2\sin 2x\,(\cos x - 1)}{2\sin 2x\,(\cos x + 1)}$	M1-factorisation
	$= \frac{\cos x - 1}{\cos x + 1}$	
	$= \frac{\left\{1 - 2\sin^2\left(\frac{x}{2}\right)\right\} - 1}{\left\{2\cos^2\left(\frac{x}{2}\right) - 1\right\} + 1}$	M1-numerator M1-denominator
	$= \frac{-2\sin^2\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}$	
	$y = -\tan^2\left(\frac{x}{2}\right)$, as required	B1
	For the hence part	

	for $\tan^2 15°$, $\quad \Rightarrow \frac{x}{2} = 15°$, $\quad \Rightarrow x = 30°$	
	$y = \frac{\sin 90° + \sin 30° - 2\sin 60°}{\sin 90° + \sin 30° + 2\sin 60°} = \frac{1 + 0.5 - 2 \times 0.5\sqrt{3}}{1 + 0.5 + 2 \times 0.5\sqrt{3}} = \frac{1.5 - \sqrt{3}}{1.5 + \sqrt{3}}$	M1
	$= \frac{(1.5 - \sqrt{3}) \times (1.5 - \sqrt{3})}{(1.5 + \sqrt{3}) \times (1.5 - \sqrt{3})}$	M1
	$= \frac{2.25 - 3\sqrt{3} + 3}{2.25 - 3} = \frac{5.25 - 3\sqrt{3}}{-0.75} = -7 + 4\sqrt{3}$	M1
	but, $\quad y + \tan^2\left(\frac{x}{2}\right) = 0$	
	$(-7 + 4\sqrt{3}) + \tan^2 15° = 0$	
	$\tan^2 15° = 7 - 4\sqrt{3}$	A1
	$2y + \sec^2\left(\frac{x}{2}\right) = 0$	
	$-2\tan^2\left(\frac{x}{2}\right) + \left[1 + \tan^2\left(\frac{x}{2}\right)\right] = 0$	M1
	$-\tan^2\left(\frac{x}{2}\right) + 1 = 0$	
	$\tan^2\left(\frac{x}{2}\right) = 1$	
	$\tan\left(\frac{x}{2}\right) = \pm 1$	
	$\frac{x}{2} = 45°, 135°$	B1
	$x = 90°, 270°$	A1
		12
14	(a).	
	$\frac{dy}{dx} = \sqrt{\frac{y}{x+1}}$	
	$\int \frac{dy}{\sqrt{y}} = \int \frac{dx}{\sqrt{x+1}}$	
	$2\sqrt{y} = 2\sqrt{x+1} + c$	M1
	But $y = 9$ when $x = 3$,	
	$2\sqrt{9} = 2\sqrt{3+1} + c, \quad \Rightarrow c = 2$	M1 B1
	$2\sqrt{y} = 2\sqrt{x+1} + 2$	
	$\sqrt{y} = \sqrt{x+1} + 1$	A1
	(b).	
	$\frac{dT}{dt} \propto -T, \quad \Rightarrow \frac{dT}{dt} = -kT$	B1
	$\int \frac{dT}{T} = \int -k\,dt$	
	$\ln T = -kt + c$	M1
	(i). But initially $T = 80$	
	$\ln 80 = -k(0) + c, \quad \Rightarrow c = \ln 80$	B1
	$\ln T = -kt + \ln 80$	
	$\ln T - \ln 80 = -kt$	

	$\ln\left(\frac{T}{80}\right) = -kt$ $\frac{T}{80} = e^{-kt}$ $T = 80e^{-kt}$, as required (ii). $T = 20$ when $t = 6$, $\ln\left(\frac{20}{80}\right) = -6k, \quad \Rightarrow k = -\frac{1}{6}\ln(0.25)$ $\ln\left(\frac{T}{80}\right) = \frac{t}{6}\ln(0.25)$ When $T = 10°C$, $\ln\left(\frac{10}{80}\right) = \frac{t}{6}\ln(0.25)$ $t = \frac{6\ln 0.125}{\ln 0.25} = 9$ minutes	B1 M1 A1 M1 A1
		12
15	(a). $x^2 - 6x + k = 0$ sum of roots, $\alpha + 3\alpha = 6$ $4\alpha = 6$ $\Rightarrow \alpha = \frac{3}{2}$, and, $3\alpha = \frac{9}{2}$ The roots are: $\frac{3}{2}$ and $\frac{9}{2}$. product of roots, $\alpha \times 3\alpha = k$ $k = 3\alpha^2 = 3 \times \left(\frac{3}{2}\right)^2 = \frac{27}{4}$ (b). $(1+x)^6$ $= 1 + {}^6C_1\,(x) + {}^6C_2\,(x^2) + {}^6C_3\,(x^3) + {}^6C_4\,(x^4) + \cdots$ $= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + \cdots$ $(1-2x^3)^{10} = 1 + {}^{10}C_1\,(-2x^3) + \cdots$ $= 1 - 20x^3 + \cdots$ $(1+x)^6(1-2x^3)^{10}$ $= (1-20x^3)(1 + 6x + 15x^2 + 20x^3 + 15x^4)$ $= 1 + 6x + 15x^2 + 20x^3 + 15x^4 - 20x^3 - 120x^4$ $= 1 + 6x + 15x^2 - 105x^4$ $\therefore a = 6, \quad b = 15, \quad c = -105$	M1 A1 A1 M1 A1 M1 B1 M1 M1 B1 A1 A1
		12
16	(a). Direction vectors, $\underset{\sim}{\mathbf{d}}_1 = \begin{pmatrix}1\\-1\\3\end{pmatrix}$, $\underset{\sim}{\mathbf{d}}_2 = \begin{pmatrix}2\\1\\-3\end{pmatrix}$ Normal vector, $\underset{\sim}{\mathbf{n}} = \underset{\sim}{\mathbf{d}}_1 \times \underset{\sim}{\mathbf{d}}_2 = \begin{vmatrix}\underset{\sim}{\mathbf{i}} & -\underset{\sim}{\mathbf{j}} & \underset{\sim}{\mathbf{k}}\\1 & -1 & 3\\2 & 1 & -3\end{vmatrix}$	M1

	$= \underset{\sim}{\mathbf{i}}\begin{vmatrix}-1 & 3\\1 & -3\end{vmatrix} - \underset{\sim}{\mathbf{j}}\begin{vmatrix}1 & 3\\2 & -3\end{vmatrix} + \underset{\sim}{\mathbf{k}}\begin{vmatrix}1 & -1\\2 & 1\end{vmatrix}$ $= \underset{\sim}{\mathbf{i}}(3-3) - \underset{\sim}{\mathbf{j}}(-3-6) + \underset{\sim}{\mathbf{k}}(1+2) = 9\underset{\sim}{\mathbf{j}} + 3\underset{\sim}{\mathbf{k}}$ The Cartesian equation of the plane through the origin is given by: $\underset{\sim}{\mathbf{r}}.\underset{\sim}{\mathbf{n}} = \underset{\sim}{\mathbf{n}}.\underset{\sim}{\mathbf{p}}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}0\\9\\3\end{pmatrix} = \begin{pmatrix}0\\9\\3\end{pmatrix}.\begin{pmatrix}1\\3\\1\end{pmatrix}$ $9y + 3z = 0 + 27 + 3$ $9y + 3z = 30$ $3y + z = 10$ (b). Direction vector, $\underset{\sim}{\mathbf{d}} = \begin{pmatrix}5\\-1\\4\end{pmatrix}$, Normal vector, $\underset{\sim}{\mathbf{n}} = \begin{pmatrix}0\\9\\3\end{pmatrix}$ Let θ be the angle between the given line and plane. $\lvert\underset{\sim}{\mathbf{n}}\rvert = \sqrt{9^2 + 3^2} = \sqrt{90} = 3\sqrt{10}$ $\lvert\underset{\sim}{\mathbf{d}}\rvert = \sqrt{5^2 + (-1)^2 + 4^2} = \sqrt{42}$ $\begin{pmatrix}0\\9\\3\end{pmatrix}.\begin{pmatrix}5\\-1\\4\end{pmatrix} = 0 - 9 + 12 = 3$ $\sin\theta = \frac{\underset{\sim}{\mathbf{n}}.\underset{\sim}{\mathbf{d}}}{\lvert\underset{\sim}{\mathbf{n}}\rvert\lvert\underset{\sim}{\mathbf{d}}\rvert} = \frac{3}{3\sqrt{10} \times \sqrt{42}} = \frac{1}{\sqrt{420}}$ $\theta = 2.797°$	M1 M1 A1 M1 M1 A1 B1 B1 B1 M1 A1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
July 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 2 MOCK SET 4 2018
Time: 3 Hours

NAME: ______________________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn 1: A ball is projected vertically upwards and when it is at a height of 10 m, it takes 8 seconds to return to its point of projection. Find the speed with which the ball was projected. [5]

Qn 2: Use trapezium rule with 4 sub-intervals to estimate

$$\int_0^{\frac{\pi}{2}} \cos x \, dx$$

correct to **three** decimal places. [5]

Qn 3: A continuous random variable X has a cumulative distribution function;

$$P(X \le x) = \begin{cases} \frac{1}{64}x^3 & ; \quad 0 \le x \le \beta, \\ 1 & ; \quad x \ge \beta. \end{cases}$$

Find the:
(a). value of the constant β. [2]
(b). probability density function. [3]

Qn 4: A body of mass 4 kg is moving with an initial velocity of 5 m s^{-1} on a plane experiences a resistance of 0.4 N in a distance of 40 m. Find the loss in kinetic energy. [5]

Qn 5: The events A and B are such that $P(A/B) = 0.4$, $P(B/A) = 0.25$, $P(A \cap B) = 0.12$. Find $P(A \cup B')$. [5]

Qn 6: Given below are values of $f(x)$ for given values of x.
$f(0.4) = -0.9613$, $f(0.6) = -0.5108$ and $f(0.8) = -0.2231$.
Use linear interpolation to determine $f^{-1}(-0.4308)$ correct to **2** decimal places. [4]

Qn 7: The data below shows the duration of telephone calls received by teacher of Masaka Secondary school. The time was measured in minutes.

1.1	4.3	2.6	1.4	1.6	1.8	2.4	3.3
3.9	4.7	2.7	4.8	2.9	1.4	3.6	2.6
2.8	1.8	2.5	4.4	3.2	4.4	4.1	3.2
2.1	1.6	1.4	1.2	2.8	3.4	4.6	4.8
2.7	3.9	2.3	4.0	1.4	3.7	2.2	1.9

(a). Form a frequency distribution table for the data with a class interval of 0.5 starting with a class of 1.0.
(b). Calculate the median time. [6]

Qn 8: Forces of magnitude 10 N, 15 N and 20 N act away from a common point in the directions $S\ 30°\ E$, $E\ 60°\ N$ and North-West respectively. Find the resultant force. [5]

Section B (60 Marks)
Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The table below shows the expenditure of restaurant for the years 2014 and 2016.

Item	Price (shs)		Weight
	2014	2016	
Milk (per litre)	1,000	1,300	0.5
Eggs (per tray)	6,500	8,300	1
Sugar (per kg)	3,000	3,800	2
Blue band	7,000	9,000	1

Taking 2014 as the base year, calculate for 2016 the:

(a). Price relative for each item.

(b). Simple aggregate price index.

(c). Weighted aggregate price index and comment on your result.

(d). In 2016, the restaurant spent shs 45,000 on buying these items. Using the index obtained in (c), find how much money the restaurant could have spent in 2014. [12]

Question 10:

(a). Three forces $\left(-2\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}}\right)$ N, $\left(3\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}}\right)$ N and $\left(-\underset{\sim}{\boldsymbol{i}} - \underset{\sim}{\boldsymbol{j}}\right)$ N act at the points $(2, 0)$, $(0, 3)$ and $(1, 1)$ respectively. Show that these forces reduce to a couple. [5]

(b). $ABCD$ is a rectangle with $\overline{AB} = 3$ m and $C\hat{A}B = 30°$. Forces of 10 N, 20 N and 20 N act along AC, AD and DB respectively. Calculate the magnitude and direction of the resultant force hence find where its line of action cuts AB. [7]

Question 11:

Given the equation $x^3 - 6x^2 + 9x + 2 = 0$;

(a). Find graphically the root of the equation which lies between -1 and 0. [5]

(b). (i). Show that Newton Raphson formula for approximating the root of the equation is given by

$$x_{n+1} = \frac{2}{3}\left[\frac{{x_n}^3 - 3{x_n}^2 - 1}{{x_n}^2 - 4x_n + 3}\right], \qquad \text{where } n = 0, 1, 2, \ldots\ldots$$

[3]

(ii). Use the formula in (b)(i) above, with an initial approximation in (a) above to find the root of the given equation correct to **two** decimal places. [4]

Question 12:

The displacement of a particle of mass 2 kg is given by:

$$\underset{\sim}{\boldsymbol{s}} = \begin{pmatrix} t^3 - 4 \\ t^2 - t \\ 2\sin t + \cos 2t \end{pmatrix} \text{ metres.}$$

Find the:

(a). average velocity between $t = 1$ s and $t = 3$ s. [3]

(b). magnitude of the forced when $t = 4$ s. [6]

(c). kinetic energy when $t = 2$ s. [3]

Question 13:

The drying time of a newly manufactured paint is normally distributed with mean 110.5 minutes and standard deviation 12 minutes.

(a). Find the probability that the paint dries for less than 104 minutes. [3]

(b). If random sample of 20 tins of the paint was taken, find the probability that the mean drying time of the sample is more than 112 minutes. [4]

(c). A random sample of 16 tins taken from a different type pf paint of standard deviation 15 minutes is found to have a mean time of 105.5 minutes, determine the 90% confidence limits the mean of time of this type of paint. [5]

Question 14:

The numbers x and y are approximated by X and Y with errors Δx and Δy respectively.

(a). Show that the maximum relative error in $\frac{y^2}{x}$ is given by

$$\left|\frac{\Delta x}{X}\right| + 2\left|\frac{\Delta y}{Y}\right|$$

[5]

(b). If $x = 4.95$ and $y = 2.013$ are each rounded off to the given number of decimal places, calculate the:

(i). percentage error in $\frac{y^2}{x}$,

(ii). limits within which $\frac{y^2}{y-x}$ is expected to lie. Give your answer to **three** decimal places. [7]

Question 15:

(a). A pump draws water from a tank and issues it at a speed of 10 m s^{-1} from the end of a hose of cross-sectional area 5 cm^2, situated 4 m above the level from which the water is drawn. Find the rate at which the pump is working. [5]

(b). A car of mass 800 kg moves against a constant resistance R N. the maximum speeds of the car up and down an incline of 1 in 16 are 14 m s^{-1} and 42 m s^{-1} respectively. If the rate at which the engine is working is H kW, find the:

(i). values of R and H,

(ii). acceleration at the instant when the speed is 17.5 m s^{-1} on level ground. [7]

Question 16:

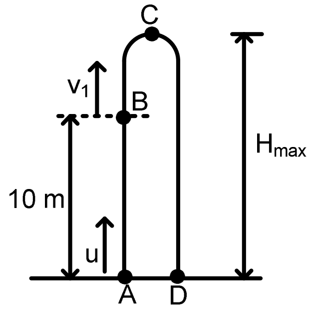
A box contains 4 pink counters, 3 green counters and 3 yellow counters. Three counters are drawn at random one after the other without replacement.

(a). Find the probability that the third counter drawn is green and the first two are of the same colour. [4]

(b). Find the expected number of pink counters drawn. [8]

END

MARKING GUIDE

SNo.	Working	Marks
1		
	For motion BCD, $s = -10$ m, $t = 8$ s, $u = v_1$	
	$s = ut - \frac{1}{2}gt^2$	
	$-10 = 8v_1 - \frac{1}{2} \times 9.8 \times 8^2$	M1
	$-10 = 8v_1 - 313.6$	
	$v_1 = \frac{303.6}{8} = 37.95 \text{ m s}^{-1}$	B1
	For motion AB, $s = 10$ m, $v = 37.95 \text{ m s}^{-1}$	
	$v^2 = u^2 - 2gs$	
	$37.95^2 = u^2 - 2 \times 9.8 \times 10$	M1 B1
	$u^2 = 1636.2025$	
	$u = 40.45 \text{ m s}^{-1}$	A1
		05
2	$y_n = \cos x_n$, $h = \frac{\frac{\pi}{2} - 0}{4} = \frac{\pi}{8}$	B1-fraction
	(table below)	B1-for x_n B1-for y_n to 4d.p or more

n	x_n	y_0, y_4	$y_1, \ldots y_3$
0	0	1	
1	$\pi/8$		0.92388
2	$\pi/4$		0.70711
3	$(3\pi)/8$		0.38268
4	$\pi/2$	0	
sums		1	2.01367

$$\int_0^{\frac{\pi}{2}} \cos x \, dx \approx \frac{1}{2}h[(y_0 + y_4) + 2(y_1 + \cdots + y_3)]$$

$$\approx \frac{1}{2}\times\frac{\pi}{8}[1+2\times 2.01367] = 0.98712 = 0.987\ (3\text{ d.p})$$ **M1 A1**

05

3 (a).

$$P(X\le\beta) = \frac{1}{64}\beta^3 = 1$$ **M1**

$$\beta^3 = 64, \quad \Rightarrow \beta = 4$$ **A1**

(b).

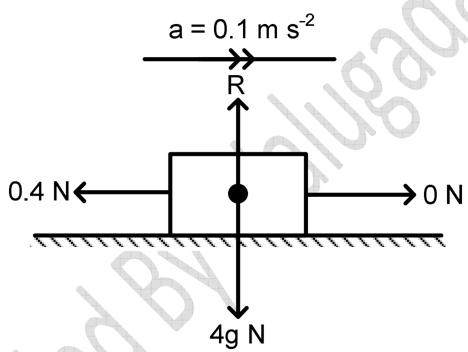
$$P(X=x) = \frac{d}{dx}\left(\frac{1}{64}x^3\right) = \frac{3}{64}x^2, \quad \text{for } 0\le x\le 4$$ **M1**

$$P(X=x) = \frac{d}{dx}(1) = 0, \quad \text{elsewhere}$$ **M1**

$$\therefore P(X=x) = \begin{cases}\frac{3}{64}x^2 & ;\ 0\le x\le 4,\\ 0 & ;\ \text{elsewhere.}\end{cases}$$ **A1**

05

4



$$F = ma$$
$$0 - 0.4 = 4a, \quad \Rightarrow a = 0.1\text{ m s}^{-2}$$ **M1**
$$v^2 = u^2 + 2as$$
$$v = \sqrt{5^2 - 2\times 0.1\times 40} = 4.123\text{ m s}^{-1}$$ **M1 B1**
$$\text{Loss in kinetic energy} = \frac{1}{2}m(u^2 - v^2)$$
$$= \frac{1}{2}\times 4\times(5^2 - 4.123^2) = 16.0017\text{ J}$$ **M1 A1**

Alternatively:

Loss in kinetic energy = Work done against resistances
$= 0.4\times 40$
$= 16\text{ J}$

05

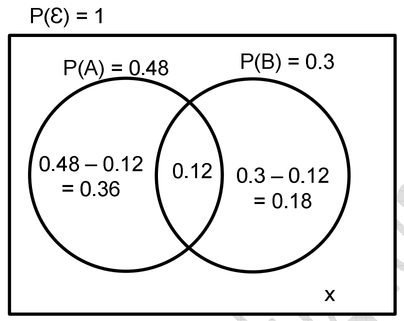
5

$$P(B) = \frac{P(A\cap B)}{P(A/B)} = \frac{0.12}{0.4} = 0.3$$ **B1**

$$P(A) = \frac{P(B\cap A)}{P(B/A)} = \frac{0.12}{0.25} = 0.48$$ **B1**
$$P(A\cap B') = P(A) - P(A\cap B) = 0.48 - 0.12 = 0.36$$ **B1**
$$P(A\cup B') = P(A) + P(B') - P(A\cap B')$$
$$= 0.48 + (1-0.3) - 0.36 = 0.82$$ **M1 A1**

Alternatively:

$$P(B) = \frac{P(A\cap B)}{P(A/B)} = \frac{0.12}{0.4} = 0.3$$
$$P(A) = \frac{P(B\cap A)}{P(B/A)} = \frac{0.12}{0.25} = 0.48$$



$$P(\varepsilon) = 0.48 + 0.18 + x = 1, \quad \Rightarrow x = 0.34$$
$$P(A\cup B') = 0.48 + 0.34 = 0.82$$

05

6

x	0.6	x	0.8
$f(x)$	-0.5108	-0.4308	-0.2231

B1

$$\frac{x-0.6}{0.8-0.6} = \frac{-0.4308-(-0.5108)}{-0.2231-(-0.5108)}$$ **M1 M1**
$$y = \frac{0.0872}{0.2949}\times 0.2 + 0.6 = 0.6591$$
$$\Rightarrow f^{-1}(-0.4308) = 0.66(2\text{ d.p})$$ **A1**

04

7 (a).

Class	Tally	f	$C.F$	Class boundaries
1.0 – 1.4	~~////~~ /	6	6	0.95 – 1.45
1.5 – 1.9	~~////~~	5	11	1.45 – 1.95
2.0 – 2.4	////	4	15	1.95 – 2.45
2.5 – 2.9	~~////~~ ///	8	23	2.45 – 2.95

B1-for class

B1-for f

B1-for C.F

3.0 – 3.4	////	4	27	2.95 – 3.45
3.5 – 3.9	////	4	31	3.45 – 3.95
4.0 – 4.4	////\	5	36	3.95 – 4.45
4.5 – 4.9	////	4	40	4.45 – 4.95

(b).

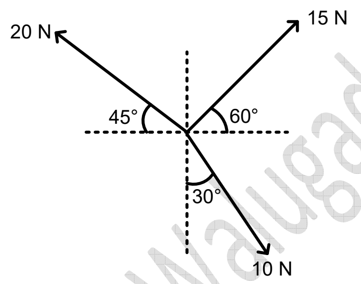
$$\text{Median position} = \frac{1}{2}N = 0.5 \times 40 = 20$$ **B1**

$$\text{Median} = L_m + \left(\frac{N/2 - C.F_b}{f_m}\right)c$$

$$= 2.45 + \left(\frac{20 - 15}{8}\right) \times 0.5 = 2.7625$$ **M1 A1**

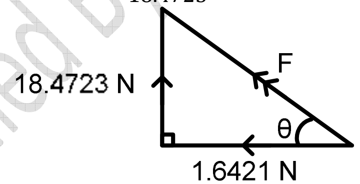
06

8



$$\underset{\sim}{F} = \begin{pmatrix} 10\sin 30° \\ -10\cos 30° \end{pmatrix} + \begin{pmatrix} 15\cos 60° \\ 15\sin 60° \end{pmatrix} + \begin{pmatrix} -20\cos 45° \\ 20\sin 45° \end{pmatrix}$$ **B1** **B1**

$$= \begin{pmatrix} -1.6421 \\ 18.4723 \end{pmatrix} \text{ N}$$



Magnitude, $\left|\underset{\sim}{F}\right| = \sqrt{(-1.6421)^2 + (18.4723)^2} = 18.5451 \text{ N}$ **M1 A1**

Direction, $\theta = \tan^{-1}\left(\frac{18.4723}{1.6421}\right) = 84.920°$

The resultant for is 21.7029 N in the direction 84.920° above the negative horizontal. **B1**

05

9 (a).

$$\text{Price relative} = \frac{P_{2016}}{P_{2014}}$$

Item	Price relative	
Milk (per litre)	$=\frac{1300}{1000} = 1.3$	**B1**
Eggs (per tray)	$=\frac{8300}{6500} = 1.277$	**B1**
Sugar (per kg)	$=\frac{3800}{3000} = 1.267$	**B1**
Blue band	$=\frac{9000}{7000} = 1.286$	**B1**

Accept: Price relative $= \frac{P_{2016}}{P_{2014}} \times 100$

(b).

$$\text{Simple aggregate price index} = \frac{\sum P_{2016}}{\sum P_{2014}} \times 100$$

$$= \frac{1300 + 8300 + 3800 + 9000}{1000 + 6500 + 3000 + 7000} \times 100$$ **M1**

$$= \frac{22400}{17500} \times 100 = 128$$ **A1**

Accept: $\text{S. A. P. I} = \frac{\sum P_{2016}}{\sum P_{2014}}$

(c).

$$\text{Weighted aggregate price index} = \frac{\sum(P_{2016} \times W)}{\sum(P_{2014} \times W)} \times 100$$

$$= \frac{1300 \times 0.5 + 8300 \times 1 + 3800 \times 2 + 9000 \times 1}{1000 \times 0.5 + 6500 \times 1 + 3000 \times 2 + 7000 \times 1} \times 100$$ **M1 M1**

$$= \frac{650 + 8300 + 7600 + 9000}{500 + 6500 + 6000 + 7000} \times 100$$

$$= \frac{25550}{20000} \times 100 = 127.75$$ **A1**

The prices increased by 27.75% between 2014 and 2016. **B1**

Accept: $\text{W. A. P. I} = \frac{\sum(P_{2016} \times W)}{\sum(P_{2014} \times W)}$

(d).

$$I = \frac{P_{2016}}{P_{2014}} \times 100$$

$$127.75 = \frac{45000}{P_{2014}} \times 100, \quad \Rightarrow P_{2014} = \text{shs } 35225.048$$ **M1 A1**

12

10 (a).

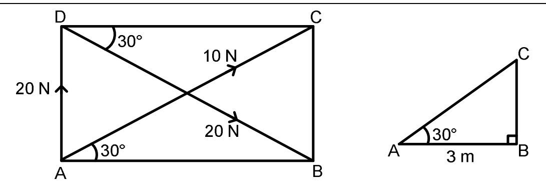
Resultant force, $\underset{\sim}{F_R} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{N}$ **M1**

$$G = \sum\left(\underset{\sim}{r} \times \underset{\sim}{F}\right) = \begin{vmatrix} 2 & 0 \\ -2 & -3 \end{vmatrix} + \begin{vmatrix} 0 & 3 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}$$ **M1**

$$= (-6 - 0) + (0 - 9) + (-1 + 1) = -15 \text{ N m}$$

Since the resultant force is zero but the resultant moment is not zero, the forces reduce to a couple. **B1** **B1**

(b).

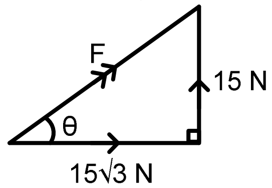


B1

$$\tan 30° = \frac{\overline{BC}}{3}, \qquad \Rightarrow \overline{BC} = 3\tan 30° = \sqrt{3}\text{ m}$$

$$\underset{\sim}{F} = \begin{pmatrix} 0 \\ 20 \end{pmatrix} + \begin{pmatrix} 20\cos 30° \\ -20\sin 30° \end{pmatrix} + \begin{pmatrix} 10\cos 30° \\ 10\sin 30° \end{pmatrix} = \begin{pmatrix} 15\sqrt{3} \\ 15 \end{pmatrix}\text{ N}$$ **M1 M1**

Magnitude, $\left|\underset{\sim}{F}\right| = \sqrt{\left(15\sqrt{3}\right)^2 + 15^2} = 30$ N **M1 A1**



$$\theta = \tan^{-1}\left(\frac{15}{15\sqrt{3}}\right) = 30°$$

The direction of the resultant is due $E\ 30°\ N$. (or 30° above the positive horizontal).

$$\underset{\sim}{F} = \begin{pmatrix} 15\sqrt{3} \\ 15 \end{pmatrix}\text{ N}, \qquad \Rightarrow \sum F_x = 15\sqrt{3}\text{ N}, \qquad \sum F_y = 15\text{ N}$$

$$G = A \circlearrowleft = -20 \times \sqrt{3}\cos 30° = -30\text{ N m}$$ **B1**

The equation of the line of action is given by:

$$G - x\sum F_y + y\sum F_x = 0$$

$$-30 - 15x + 15\sqrt{3}y = 0$$

At the point where the line of action cuts AB, $y = 0$,

$$\Rightarrow -30 - 15x = 0, \qquad \Rightarrow x = -2\text{ m}$$ **M1**

The line of action of the resultant cuts BA produced at a distance 2 m from A. **A1**

12

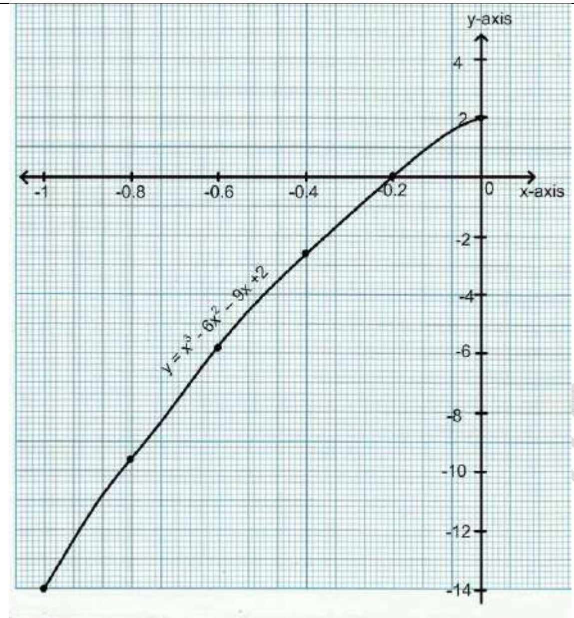
11 (a).

let, $y = x^3 - 6x^2 + 9x + 2$

x	-1	-0.8	-0.6	-0.4	-0.2	0
y	-14	-9.6	-5.8	-2.6	0.0	2

B1

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018



B1-ploting points

B1-drawing curve

B1-labelling curve and axes.

From the graph, the root $x_0 = -0.2$. **B1**

(b). (i).

$$f(x) = x^3 - 6x^2 + 9x + 2$$

$$f(x) = x^3 - 6x^2 + 9x + 2, \qquad \Rightarrow f'(x) = 3x^2 - 12x + 9$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad \text{where } n = 0, 1, 2, \dots\dots$$ **M1**

$$x_{n+1} = x_n - \frac{{x_n}^3 - 6{x_n}^2 + 9x_n + 2}{3{x_n}^2 - 12x_n + 9}$$

$$= \frac{x_n(3{x_n}^2 - 12x_n + 9) - ({x_n}^3 - 6{x_n}^2 + 9x_n + 2)}{3{x_n}^2 - 12x_n + 9}$$ **M1**

$$= \frac{3{x_n}^3 - 12{x_n}^2 + 9x_n - {x_n}^3 + 6{x_n}^2 - 9x_n - 2}{3{x_n}^2 - 12x_n + 9}$$ **M1**

$$= \frac{2{x_n}^3 - 6{x_n}^2 - 2}{3{x_n}^2 - 12x_n + 9} = \frac{2({x_n}^3 - 3{x_n}^2 - 1)}{3({x_n}^2 - 4x_n + 3)}$$

$$\therefore x_{n+1} = \frac{2}{3}\left[\frac{{x_n}^3 - 3{x_n}^2 - 1}{{x_n}^2 - 4x_n + 3}\right], \qquad \text{as required}$$ **B1**

(ii). From the graph, $x_0 = -0.2$

$$x_1 = \frac{2}{3}\left[\frac{(-0.2)^3 - 3(-0.2)^2 - 1}{(-0.2)^2 - 4(-0.2) + 3}\right] = -0.1958$$ **M1**

$$x_2 = \frac{2}{3}\left[\frac{(-0.1958)^3 - 3(-0.1958)^2 - 1}{(-0.1958)^2 - 4(-0.1958) + 3}\right] = -0.1958$$ **M1**

$$|x_2 - x_1| = |-0.1958 - (-0.1958)| = 0 < 0.005$$

The root is -0.20 (2 d.p). **A1**

12

12 (a).

when $t = 1$ s, $\underset{\sim}{s_1} = \begin{pmatrix} 1^3 - 4 \\ 1^2 - 3 \\ 2\sin 1 + \cos 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1.2668 \end{pmatrix}$ m — **B1**

when $t = 3$ s, $\underset{\sim}{s_2} = \begin{pmatrix} 3^3 - 4 \\ 3^2 - 3 \\ 2\sin 3 + \cos 6 \end{pmatrix} = \begin{pmatrix} 23 \\ 6 \\ 1.2424 \end{pmatrix}$ m — **B1**

$$\text{Average velocity} = \frac{\underset{\sim}{s_2} - \underset{\sim}{s_1}}{3 - 1}$$

$$= \frac{1}{2}\left[\begin{pmatrix} 23 \\ 6 \\ 1.2424 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \\ 1.2668 \end{pmatrix}\right] = \begin{pmatrix} 13 \\ 3 \\ -0.0122 \end{pmatrix} \text{ m s}^{-1}$$ — **M1 A1**

(b).

$$\underset{\sim}{v} = \frac{d\underset{\sim}{s}}{dt} = \begin{pmatrix} 3t^2 \\ 2t - 1 \\ 2\cos t - 2\sin 2t \end{pmatrix} \text{ m s}^{-1}$$ — **M1**

$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = \begin{pmatrix} 6t \\ 2 \\ -2\sin t - 4\cos 2t \end{pmatrix} \text{ m s}^{-2}$$ — **M1**

$$\underset{\sim}{F} = m\underset{\sim}{a} = 2\begin{pmatrix} 6t \\ 2 \\ -2\sin t - 4\cos 2t \end{pmatrix} = \begin{pmatrix} 12t \\ 4 \\ -4\sin t - 8\cos 2t \end{pmatrix} \text{ N}$$ — **M1**

When $t = 4$,

$$\underset{\sim}{F} = \begin{pmatrix} 12 \times 4 \\ 4 \\ -4\sin 4 - 8\cos 8 \end{pmatrix} = \begin{pmatrix} 48 \\ 4 \\ -4.19121 \end{pmatrix} \text{ N}$$ — **B1**

$$\left|\underset{\sim}{F}\right| = \sqrt{48^2 + 4^2 + (-4.19121)^2} = 48.34838 \text{ N}$$ — **M1 A1**

(c).

When $t = 2$,

$$\underset{\sim}{v} = \begin{pmatrix} 3 \times 2^2 \\ 2 \times 2 - 1 \\ 2\cos 2 - 2\sin 4 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 0.68131 \end{pmatrix} \text{ m s}^{-1}$$ — **B1**

$$\text{Kinetic energy} = \frac{1}{2}m\underset{\sim}{v}.\underset{\sim}{v} = \frac{1}{2} \times 2\begin{pmatrix} 12 \\ 3 \\ 0.68131 \end{pmatrix}.\begin{pmatrix} 12 \\ 3 \\ 0.68131 \end{pmatrix}$$ — **M1**

$$= 144 + 9 + 0.46418 = 153.46418 \text{ J}$$ — **A1**

12

13 (a).

$$P(X < 104) = P\left(Z < \frac{104 - 110.5}{12}\right)$$

$$= P(Z < -0.542) = 0.5 - \phi(0.542)$$ — **M1**

$$= 0.5 - 0.2061 = 0.2939$$ — **B1 A1**

(b).

$$P(\overline{X} > 112) = P\left(Z > \frac{112 - 110.5}{(12/\sqrt{20})}\right)$$ — **M1**

$$= P(Z < 0.559) = 0.5 + \phi(0.559)$$ — **B1**

$$= 0.5 + 0.2119 = 0.7119$$ — **B1 A1**

(c).

$$P\left(0 < Z < z_{\alpha/2}\right) = \frac{0.90}{2} = 0.45$$

$$z_{\alpha/2} = \phi^{-1}(0.45) = 1.645$$ — **B1**

$$\text{Confidence Limits} = \overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 105.5 \pm 1.645 \times \frac{15}{\sqrt{16}}$$

$$= 105.5 \pm 6.16875$$

$$\text{Lower limit} = 105.5 - 6.16875 = 99.33125$$ — **M1 A1**

$$\text{Upper limit} = 105.5 + 6.16875 = 111.66875$$ — **M1 A1**

12

14 (a).

$$\Delta x = x - X, \quad \Rightarrow x = X + \Delta x$$

$$\Delta y = y - Y, \quad \Rightarrow y = Y + \Delta y$$

$$\text{let,} \quad z = \frac{y^2}{x}, \quad \Rightarrow Z = \frac{Y^2}{X}$$

$$\therefore \Delta z = \frac{y^2}{x} - \frac{Y^2}{X}$$

$$= \frac{(Y + \Delta y)^2}{X + \Delta x} - \frac{Y^2}{X}$$ — **M1**

$$= \frac{X(Y + \Delta y)^2 - Y^2(X + \Delta x)}{X(X + \Delta x)}$$

$$= \frac{X[Y^2 + 2Y\Delta y + (\Delta y)^2] - Y^2X - Y^2\Delta x}{X^2\left(1 + \frac{\Delta x}{X}\right)}$$

$$= \frac{X[Y^2 + 2Y\Delta y + (\Delta y)^2] - Y^2X - Y^2\Delta x}{X^2\left(1 + \frac{\Delta x}{X}\right)}$$

suppose $\Delta x \ll X$ and $\Delta y \ll Y$, $\Rightarrow (\Delta y)^2 \approx 0$ and $\left(\frac{\Delta x}{X}\right) \approx 0$ — **B1**

$$\Delta z = \frac{2XY\Delta y - Y^2\Delta x}{X^2}$$

$$\frac{\Delta z}{Z} = \left(\frac{2XY\Delta y - Y^2\Delta x}{X^2}\right) \div \frac{Y^2}{X} = \frac{2XY\Delta y - Y^2\Delta x}{Y^2X} = \frac{2\Delta y}{Y} - \frac{\Delta x}{X}$$ — **M1**

$$\left|\frac{\Delta z}{Z}\right| = \left|\frac{2\Delta y}{Y} - \frac{\Delta x}{X}\right| \leq \left|\frac{2\Delta y}{Y}\right| + \left|\frac{\Delta x}{X}\right|$$ — **B1**

$$\Rightarrow \text{maximum relative error} = 2\left|\frac{\Delta y}{Y}\right| + \left|\frac{\Delta x}{X}\right|$$ — **B1**

Alternatively:

$$z = \frac{y^2}{x}$$

$$Z + \Delta z = \frac{(Y + \Delta y)^2}{X + \Delta x}$$

$$Z + \Delta z = \frac{(Y + \Delta y)^2 \times (X - \Delta x)}{(X + \Delta x) \times (X - \Delta x)}$$

$$= \frac{[Y^2 + 2Y\Delta y + (\Delta y)^2](X - \Delta x)}{X^2 - (\Delta x)^2}$$

$$= \frac{X[Y^2 + 2Y\Delta y + (\Delta y)^2] - [Y^2 + 2Y\Delta y + (\Delta y)^2]\Delta x}{X^2 - (\Delta x)^2}$$

suppose $\Delta x \ll X$ and $\Delta y \ll Y$,

$$\Rightarrow (\Delta y)^2 \approx 0, \Delta y\Delta x \approx 0, (\Delta x)^2 \approx 0$$

$$Z + \Delta z = \frac{X[Y^2 + 2Y\Delta y] - Y^2\Delta x}{X^2}$$

$$Z + \Delta z = \frac{XY^2 + 2XY\Delta y - Y^2\Delta x}{X^2}$$

$$Z + \Delta z = \frac{Y^2}{X} + \frac{2Y\Delta y}{X} - \frac{Y^2\Delta x}{X^2}$$

$$\Delta z = \frac{2Y\Delta y}{X} - \frac{Y^2\Delta x}{X^2}$$

$$\frac{\Delta z}{Z} = \left(\frac{2Y\Delta y}{X} - \frac{Y^2\Delta x}{X^2}\right) \times \frac{X}{Y^2}$$

$$\frac{\Delta z}{Z} = \frac{2\Delta y}{Y} - \frac{\Delta x}{X}$$

$$\left|\frac{\Delta z}{Z}\right| = \left|\frac{2\Delta y}{Y} - \frac{\Delta x}{X}\right| \le \left|\frac{2\Delta y}{Y}\right| + \left|-\frac{\Delta x}{X}\right|$$

$$\left|\frac{\Delta z}{Z}\right| \le 2\left|\frac{\Delta y}{Y}\right| + \left|\frac{\Delta x}{X}\right|$$

$$\text{maximum relative error} = 2\left|\frac{\Delta y}{Y}\right| + \left|\frac{\Delta x}{X}\right|$$ **B1**

(b). (i).

$$\Delta y = 0.0005, \qquad \Delta X = 0.005$$

$$\text{maximum relative error in } \frac{y^2}{x} = 2\left|\frac{\Delta y}{Y}\right| + \left|\frac{\Delta x}{x}\right|$$ **M1**

$$\text{maximum percentage error} = \left(2\left|\frac{0.0005}{2.013}\right| + \left|\frac{0.005}{4.95}\right|\right) \times 100$$ **A1**

$$= 0.1507$$

(ii).

$$\text{Upper limit} = \left(\frac{y^2}{y-x}\right)_{\max} = \frac{{y_{\min}}^2}{(y-x)_{\min}}$$

$$= \frac{(2.013 - 0.0005)^2}{(2.013 - 0.0005) - (4.95 + 0.005)}$$ **M1**

$$= -1.376434 \approx -1.376 \text{ (3 d.p)}$$ **A1**

$$\text{Lower limit} = \left(\frac{y^2}{y-x}\right)_{\min} = \frac{{y_{\max}}^2}{(y-x)_{\max}}$$

$$= \frac{(2.013 + 0.0005)^2}{(2.013 + 0.0005) - (4.95 - 0.005)}$$ **M1**

$$= -1.382972 \approx -1.383 \text{ (3 d.p)}$$ **A1**

Alternatively:

(b). (i).

$$\text{Working value} = \frac{y^2}{x} = \frac{2.013^2}{4.95} = 0.81862$$

$$\left(\frac{y^2}{x}\right)_{\max} = \frac{(2.013 + 0.0005)^2}{(4.95 - 0.005)} = 0.819855$$

$$\left(\frac{y^2}{x}\right)_{\min} = \frac{(2.013 - 0.0005)^2}{(4.95 + 0.005)} = 0.817388$$

$$\text{absolute error} = \frac{|0.819855 - 0.817388|}{2} = 0.0012335$$

$$\text{absolute percentage error} = \left|\frac{0.0012335}{0.81862}\right| \times 100 = 0.1507$$

(ii).

$$\text{maximum relative error in } \frac{y^2}{y-x} = 2\left|\frac{\Delta y}{Y}\right| + \left|\frac{\Delta y + \Delta x}{Y - X}\right|$$

$$= 2\left|\frac{0.0005}{2.013}\right| + \left|\frac{0.0005 + 0.005}{2.013 - 4.95}\right|$$

$$= 0.000497 + 0.001873 = 0.00237$$

$$\text{maximum percentage error} = 0.00237 \times 100 = 0.237$$

$$\text{Working value} = \frac{y^2}{y-x} = \frac{2.013^2}{2.013 - 4.95} = -1.379697$$

$$\text{maximum absolute error} = 0.00236943 \times |-1.379697|$$

$$= 0.00326910$$

$$\text{Upper limit} = -1.379697 + 0.00326910 = -1.3764279$$

$$\approx -1.376 \text{ (3 d.p)}$$

$$\text{Lower limit} = -1.379697 - 0.00326910 = -1.3829661$$

$$\approx -1.383 \text{ (3 d.p)}$$

12

15 (a).

$$v = 10 \text{ m s}^{-1}, A = 5 \text{ cm}^2 = 0.0005 \text{ m}^2, h = 4 \text{ m}, \rho = 1000 \text{ kg m}^{-3}$$

Mass of water raised and issued per second

$$m = Av\rho = 0.0005 \times 10 \times 1000 = 5 \text{ kg s}^{-1}$$ **B1**

Potential energy given to raise the water

$$P.E = mgh = 5 \times 9.8 \times 4 = 196 \text{ J s}^{-1}$$ **M1**

Kinetic energy given to raise the water

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 10^2 = 250 \text{ J s}^{-1}$$ **M1**

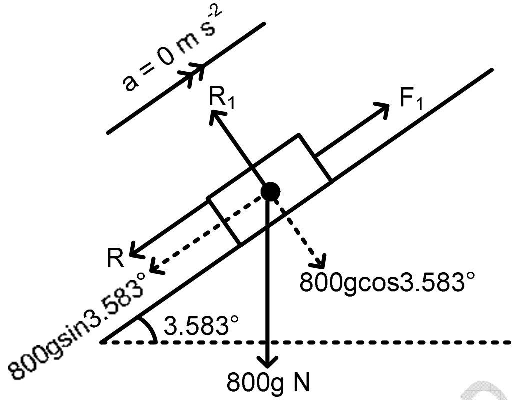
Rate at which the pump is working,

$$P_{\text{total}} = P.E + K.E = 196 + 250 = 446 \text{ J s}^{-1}$$ **A1**

(b). (i).

$$\theta = \sin^{-1}\left(\frac{1}{16}\right) = 3.583°, \qquad P = H \text{ kW} = 10000H \text{ W}$$

For upward motion along the plane,



B1

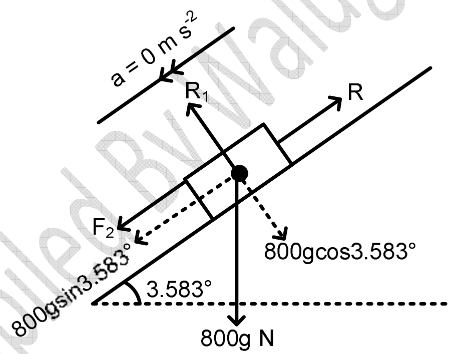
Resolving parallel to the plane gives,

$$F_1 = 800g\sin 3.583° + R$$

$$\frac{1000H}{14} = 800 \times 9.8 \times \frac{1}{16} + R$$

$$1000H = 6860 + 14R \quad \longrightarrow (1)$$

For downward motion along the plane, **B1**



Resolving parallel to the plane gives,

$$F_1 + 800g\sin 3.583° = R$$

$$\frac{1000H}{42} + 800 \times 9.8 \times \frac{1}{16} = R$$

$$1000H + 20580 = 42R$$

$$1000H = 42R - 20580 \quad \longrightarrow (2)$$ **B1**

Equating equations (1) and (2) gives,

$$6860 + 14R = 42R - 20580$$ **M1**

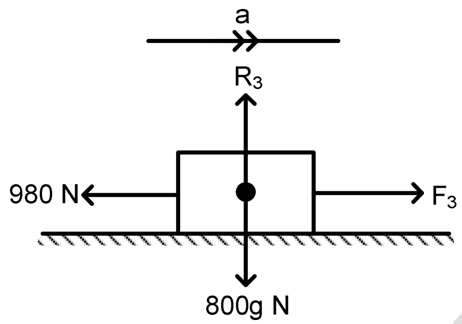
$$28R = 27440$$

$$R = 980\text{ N}$$ **A1**

From equation (1),

$$H = \frac{6860 + 14 \times 980}{1000} = 20.58\text{ kW}$$ **A1**

(ii).



when $v = 17.5\text{ m s}^{-1}$, $\quad F_3 = \frac{1000H}{v} = \frac{1000 \times 20.58}{17.5} = 1176\text{ N}$

Resolving parallel to the plane,

$1176 - 980 = 800a, \quad \Rightarrow a = 0.245\text{ m s}^{-2}$ **M1 A1**

12

16 (a).

$$P(\text{all green}) = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$$ **M1**

$$P(\text{first two pink and third is green}) = \frac{4}{10} \times \frac{3}{9} \times \frac{3}{8} = \frac{1}{20}$$ **M1**

$$P(\text{first two yellow and third is green}) = \frac{3}{10} \times \frac{2}{9} \times \frac{3}{8} = \frac{1}{40}$$ **M1**

$$\Rightarrow P(\text{first two same colour and third is green}) = \frac{1}{120} + \frac{1}{20} + \frac{1}{40}$$

$$= \frac{1}{12} \approx 0.0833$$ **A1**

(b). Let x denote number of pink counters.

x	$P(X = x)$	$xP(X = x)$
0	$\frac{{}^4C_0 \times {}^6C_3}{{}^{10}C_3} = \frac{1 \times 20}{120} = \frac{20}{120} = \frac{1}{6}$	0
1	$\frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{4 \times 15}{120} = \frac{60}{120} = \frac{1}{2}$	$\frac{1}{2}$
2	$\frac{{}^4C_2 \times {}^6C_1}{{}^{10}C_3} = \frac{6 \times 6}{120}$	$\frac{3}{5}$

B1-for $P(X = 0)$

B1-for $P(X = 1)$

B1-for $P(X = 2)$

B1-for

			$=\frac{36}{120}=\frac{3}{10}$		$P(X=3)$
		3	$\frac{{}^4C_3 \times {}^6C_0}{{}^{10}C_3}=\frac{4\times 1}{120}$ $=\frac{4}{120}=\frac{1}{30}$	$\frac{1}{10}$	**B1**-for all $xP(X=x)$
		Total	1	$\frac{6}{5}$	**B1**-for $\sum xP(X=x)$
	(ii).		$E(X)=\frac{6}{5}$ ≈ 1 pink counter		**B1** **A1**
					12

END

P425/1
PURE
MATHEMATICS
PAPER 1
Sept 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 1 MOCK SET 5 2018
Time: 3 Hours

NAME: ______________________ **COMB:** ________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B.**
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section*

Qn 1: Differentiate $\cos^2 x$ with respect to x from first principles. [5]

Qn 2: The roots of the equation $3x^2+2x-5=0$ are $\alpha$and β. Find the value of $\alpha^4+\beta^4$. [5]

Qn 3: Find the shortest distance of a point $A(1,6,3)$ from the line $\underset{\sim}{\boldsymbol{r}}=\underset{\sim}{\boldsymbol{i}}+\underset{\sim}{\boldsymbol{j}}+\underset{\sim}{\boldsymbol{k}}+\beta\left(-\underset{\sim}{\boldsymbol{i}}+\underset{\sim}{\boldsymbol{j}}+2\underset{\sim}{\boldsymbol{k}}\right)$. [5]

Qn 4: Find the values of x lying between $-180°$ and $180°$ that satisfy the equation $10\sin^2 x+10\sin x\cos x=\cos^2 x+2$. [5]

Qn 5: Find: $\int 2^{\sqrt{x-1}}\,dx$. [5]

Qn 6: Find the coefficient of x^{17} in the expansion of $\left(x^3+\frac{1}{x^4}\right)^{15}$. [5]

Qn 7: Show that the equation $y^2 - 4y = 4x$ represents a parabola; hence determine its focus and directrix. [5]

Qn 8: Solve the differential equation: $\frac{dy}{dx} = xy \ln x$given $y = x = 1$. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). Given that $x = \frac{3t-1}{t}$ and $= \frac{t^2+4}{t}$, show that $\frac{d^2y}{dx^2} = 2t^3$. [6]

(b). Determine the area of the largest rectangular piece of land that can be enclosed by 100 m of fencing if part of an existing wall is used. [6]

Question 10:

(a). Given that $z_1 = 3 + i$, $z_2 = x + i$ and $Arg(z_1 z_2) = \frac{\pi}{4}$, find the value of x. [5]

(b). Solve: $z^4 - 6z^2 + 25 = 0$. [7]

Question 11:

(a). The position vectors of points A, B and C are $\frac{1}{4}\left(\underset{\sim}{\boldsymbol{a}} + 3\underset{\sim}{\boldsymbol{b}}\right)$, $\frac{1}{2}\left(3\underset{\sim}{\boldsymbol{a}} - \underset{\sim}{\boldsymbol{b}}\right)$ and $\frac{1}{8}\left(3\underset{\sim}{\boldsymbol{a}} + 5\underset{\sim}{\boldsymbol{b}}\right)$ respectively. Prove that the points lie on a straight line and determine the ratio AB:BC. [6]

(b). Find the equation of a plane which contains the line $\frac{x-1}{2} = \frac{y+4}{-3} = \frac{z+1}{-1}$and passes through the point $(2, 3, -1)$. [6]

Question 12:

(a). Show that: $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{7}{9}\right)$. [5]

(b). Express $3\cos x + 4\sin x$, in the form $R\cos(x - \alpha)$, where R is positive and α is an acute angle. Hence, find the maximum and minimum values and state clearly where they occur. [7]

Question 13:

Show that for real x, the function $f(x) = \frac{x^2-x-6}{x-1}$ can take all real values. Hence sketch the curve of $f(x)$. [12]

Question 14:

(a). If the equations $x^2 + ax + p = 0$ and $cx^2 + 2ax - 3p = 0$ have a common root, show that $p(c+3)^2 = 5a^2(c-2)$. [5]

(b). Prove by induction that

$$\sum_{r=1}^{n} r^2(r+1) = \frac{n}{12}(n+1)(n+2)(3n+1);$$

where$n$ is a whole number. [7]

Question 15:

(a). The equation of an ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b > 0$. A tangent drawn to the upper part of the ellipse at (m, n) cuts the x-axis at a point of distance, c, from the origin. Show that $\frac{a^2}{b^2} = \frac{mc-m^2}{n^2}$. [6]

(b). The normal to the curve $xy = 4$ at the point $P\left(2p, \frac{2}{p}\right)$ meets the curve again at point Q. Find the coordinates of point Q. [6]

Question 16:

(a). Solve the equation $ye^y\frac{dy}{dx} = e^{-x} = 0$. [4]

(b). John walks towards a trading centre which is 1000 m away at a rate which is proportional to the distance he still has to cover. He starts by walking at a speed of 1 m s^{-1} from his home towards the trading centre. How many minutes does he take to cover 600 m from his home? [8]

END

MARKING GUIDE

SNo.	Working	Marks
1	Let, $y=\cos^2 x \quad \to (i)$ $y+\delta y=\cos^2(x+\delta x) \quad \to (ii)$ Subtracting (i) from (ii) gives; $\delta y=\cos^2(x+\delta x)-\cos^2 x$ $=[\cos(x+\delta x)-\cos x][\cos(x+\delta x)+\cos x]$	M1
	$\delta y=\left[-2\sin\left(\frac{2x+\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)\right]\left[2\cos\left(\frac{2x+\delta x}{2}\right)\cos\left(\frac{\delta x}{2}\right)\right]$	M1
	Suppose δx is a small angle expressed in radians, then $\sin\left(\frac{\delta x}{2}\right)\approx\frac{\delta x}{2}$ and $\cos\left(\frac{\delta x}{2}\right)\approx 1$. $\delta y=\left[-2\left(\frac{\delta x}{2}\right)\sin\left(\frac{2x+\delta x}{2}\right)\right]\left[2\cos\left(\frac{2x+\delta x}{2}\right)\right]$	M1
	$\delta y=-2(\delta x)\sin\left(\frac{2x+\delta x}{2}\right)\cos\left(\frac{2x+\delta x}{2}\right)$ $\delta y=-(\delta x)\sin(2x+\delta x)$ $\frac{\delta y}{\delta x}=-\sin(2x+\delta x)$	B1
	$\therefore \frac{dy}{dx}=\lim_{\delta x\to 0}\left(\frac{\delta y}{\delta x}\right)=-\sin(2x+0)=-\sin 2x$	B1
		05
2	$3x^2+2x-5=0$ $x^2+\frac{2}{3}x-\frac{5}{3}=0$ sum of roots, $\alpha+\beta=-\frac{2}{3}$ product of roots, $\alpha\beta=-\frac{5}{3}$	B1 –both sum and product
	$\alpha^4+\beta^4=(\alpha^2)^2+(\beta^2)^2=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$	M1
	$=[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2=\left[\left(-\frac{2}{3}\right)^2-2\left(-\frac{5}{3}\right)\right]^2-2\left(-\frac{5}{3}\right)^2$	M1 B1
	$=\left[\frac{4}{9}-\frac{50}{9}\right]^2-\frac{50}{9}=\frac{2116}{81}-\frac{50}{9}=\frac{1666}{81}\approx 20.568$	A1
		05
3	$\overrightarrow{OA}=\begin{pmatrix}1\\6\\3\end{pmatrix}$, $\overrightarrow{OP}=\begin{pmatrix}1\\1\\1\end{pmatrix}$, $\underset{\sim}{\boldsymbol{d}}=\begin{pmatrix}-1\\1\\2\end{pmatrix}$ $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}1\\1\\1\end{pmatrix}-\begin{pmatrix}1\\6\\3\end{pmatrix}=\begin{pmatrix}0\\-5\\-2\end{pmatrix}$	B1
	$AP\times\underset{\sim}{\boldsymbol{d}}=\begin{vmatrix}\underset{\sim}{\boldsymbol{i}} & -\underset{\sim}{\boldsymbol{j}} & \underset{\sim}{\boldsymbol{k}}\\0 & -5 & -2\\-1 & 1 & 2\end{vmatrix}$	M1

	$=\underset{\sim}{\boldsymbol{i}}\begin{vmatrix}-5 & -2\\1 & 2\end{vmatrix}-\underset{\sim}{\boldsymbol{j}}\begin{vmatrix}0 & -2\\-1 & 2\end{vmatrix}+\underset{\sim}{\boldsymbol{k}}\begin{vmatrix}0 & -5\\-1 & 1\end{vmatrix}$ $=\underset{\sim}{\boldsymbol{i}}(-10+2)-\underset{\sim}{\boldsymbol{j}}(0-2)+\underset{\sim}{\boldsymbol{k}}(0-5)=-8\underset{\sim}{\boldsymbol{i}}+2\underset{\sim}{\boldsymbol{j}}-5\underset{\sim}{\boldsymbol{k}}$	B1
	$\lvert AP\times\underset{\sim}{\boldsymbol{d}}\rvert=\sqrt{64+4+25}=\sqrt{93}$ $\lvert\underset{\sim}{\boldsymbol{d}}\rvert=\sqrt{1+1+4}=\sqrt{6}$	M1 –both magnitudes correct
	Shortesrt distance $=\frac{\lvert\overrightarrow{AP}\times\underset{\sim}{\boldsymbol{d}}\rvert}{\lvert\underset{\sim}{\boldsymbol{d}}\rvert}=\frac{\sqrt{93}}{\sqrt{6}}\approx 3.937$ units	M1 A1
	Alternatively: Let N be the point where the foot of the perpendicular from A meets line r. $\overrightarrow{OA}=\begin{pmatrix}1\\6\\3\end{pmatrix}$, $\underset{\sim}{\boldsymbol{r}}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\beta\begin{pmatrix}-1\\1\\2\end{pmatrix}$, $\Rightarrow\overrightarrow{ON}=\begin{pmatrix}1-\beta\\1+\beta\\1+2\beta\end{pmatrix}$, $\underset{\sim}{\boldsymbol{d}}=\begin{pmatrix}-1\\1\\2\end{pmatrix}$	B1
	$\overrightarrow{AN}=\overrightarrow{ON}-\overrightarrow{OA}=\begin{pmatrix}1-\beta\\1+\beta\\1+2\beta\end{pmatrix}-\begin{pmatrix}1\\6\\3\end{pmatrix}=\begin{pmatrix}-\beta\\-5+\beta\\-2+2\beta\end{pmatrix}$ $\overrightarrow{AN}.\underset{\sim}{\boldsymbol{d}}=0$ $\begin{pmatrix}-\beta\\-5+\beta\\-2+2\beta\end{pmatrix}.\begin{pmatrix}-1\\1\\2\end{pmatrix}=0$ $\beta-5+\beta-4+4\beta=0$	M1
	$-9+6\beta=0, \quad \Rightarrow\beta=1.5$	B1
	$\overrightarrow{AN}=\begin{pmatrix}-\beta\\-5+\beta\\-2+2\beta\end{pmatrix}=\begin{pmatrix}-1.5\\-5+1.5\\-2+2\times 1.5\end{pmatrix}=\begin{pmatrix}1.5\\-3.5\\1\end{pmatrix}$ Shortesrt distance, $\lvert\overrightarrow{AN}\rvert=\sqrt{1.5^2+(-3.5)^2+1}$ $=\sqrt{15.5}\approx 3.937$ units	M1 A1
		05
4	$10\sin^2 x+10\sin x\cos x=\cos^2 x+2$ Dividing throughout by $\cos^2 x$ gives: $10\tan^2 x+10\tan x=1+2\sec^2 x$	M1
	$10\tan^2 x+10\tan x=1+2(1+\tan^2 x)$ $10\tan^2 x+10\tan x=1+2+2\tan^2 x$ $8\tan^2 x+10\tan x-3=0$	B1
	$\tan x=\frac{-10\pm\sqrt{10^2-4\times 8\times(-3)}}{2\times 8}=\frac{-10\pm 14}{16}$	M1
	$\tan x=\frac{-10-14}{16}=-1.5$, $\quad\Rightarrow x=123.69°, -56.31°$	A1 –both angles seen

or, $\tan x = \frac{-10 \pm 14}{16} = 0.25,$

$\Rightarrow x = 14.04°, 165.96°$ — **A1**–both angles seen

05

5

let, $u = \sqrt{x-1}, \quad \frac{du}{dx} = \frac{1}{2}(x-1)^{-\frac{1}{2}},$ — **M1**

$\Rightarrow dx = 2\sqrt{x-1}\,du$

$$\int 2^{\sqrt{x-1}}\,dx = \int 2^u \times 2\sqrt{x-1}\,du = \int 2^u \times 2u\,du$$ — **M1**

$$= \int (2u)(2^u)\,du$$

Sign	Differentiation	Integration
+	$2u$	2^u
−	2	$\frac{2^u}{\ln 2}$
+	0	$\frac{2^u}{(\ln 2)^2}$

M1

$$\int (2u)(2^u)\,du = \frac{(2u)(2^u)}{\ln 2} - \frac{(2)(2^u)}{(\ln 2)^2} + c$$ — **M1**

$$= \frac{2(\sqrt{x-1})(2^{\sqrt{x-1}})}{\ln 2} - \frac{2(2^{\sqrt{x-1}})}{(\ln 2)^2} + c$$ — **A1**

05

6

$$\left(x^3 + \frac{1}{x^4}\right)^{15} = \sum_{r=0}^{15} ({}^{15}C_r)(x^3)^r \left(\frac{1}{x^4}\right)^{15-r}$$

general term $= ({}^{15}C_r)(x^3)^r\left(\frac{1}{x^4}\right)^{15-r} = ({}^{15}C_r)(x^3)^r(x^{-4})^{15-r}$ — **M1 B1**

$= ({}^{15}C_r)x^{3r-4(15-r)} = ({}^{15}C_r)x^{7r-60}$ — **B1**

For the term in x^{17},

$7r - 60 = 17, \quad \Rightarrow r = \frac{77}{7} = 11$ — **M1**

term in $x^{17} = {}^{15}C_r = {}^{15}C_{11} = 1365$ — **A1**

05

7

$y^2 - 4y = 4x$

$(y-2)^2 - 4 = 4x$

$(y-2)^2 = 4(x+1)$ — **M1**

This is in the form $Y^2 = 4aX$ hence it's a parabola.

$Y = y - 2, \quad X = x + 1, \quad 4a = 4$ hence $a = 1$ — **M1**

Focus

$(X, Y) = (a, 0) = (1, 0)$

$X = (x+1) = 1, \quad \Rightarrow x = 0$ — **M1**-both x and y values

$Y = (y-2) = 0, \quad \Rightarrow y = 2$

The focus $(x, y) = (0, 2)$. — **A1**-coordinate

Directrix

$X = -a = -1$

$(x+1) = -1, \quad \Rightarrow x = -2$

The directrix on the is line $x = -2$. — **A1**

05

8

$$\frac{dy}{dx} = xy\ln x$$

$$\frac{dy}{y} = x\ln x\,dx$$

$$\int \frac{1}{y}dy = \int x\ln x\,dx$$

for $\int x\ln x\,dx$, let $u = \ln x$ and $\frac{dv}{dx} = x, \quad \Rightarrow \frac{du}{dx} = \frac{1}{x},$ — **M1**

$v = \frac{1}{2}x^2$

$$\int x\ln x\,dx = uv - \int v\frac{du}{dx}dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2 \times \frac{1}{x}dx$$

$$= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$$ — **M1**

from, $\int \frac{1}{y}dy = \int x\ln x\,dx$

$$\ln y = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$$ — **M1**

When $x = 1, y = 1$,

$\ln 1 = \frac{1}{2}\ln 1 - \frac{1}{4} + c, \quad \Rightarrow c = \frac{1}{4}$ — **M1**

$$\ln y = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + \frac{1}{4}$$

$$4\ln y = 2x^2\ln x - x^2 + 1$$ — **B1**

05

9 (a).

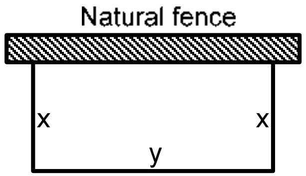
$y = \frac{t^2+4}{t}, \quad \Rightarrow \frac{dy}{dt} = \frac{t \times 2t - (t^2+4) \times 1}{t^2}$

$= \frac{2t^2 - t^2 - 4}{t^2} = \frac{t^2-4}{t^2}$ — **M1**- for $\frac{dy}{dt}$

$x = \frac{3t-1}{t} = 3 - \frac{1}{t}, \quad \Rightarrow \frac{dx}{dt} = \frac{1}{t^2}$ — **M1**- for $\frac{dx}{dt}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t^2-4}{t^2} \times t^2 = t^2 - 4$ — **M1**- for $\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx} = \frac{d}{dt}(t^2-4) \times t^2 = 2t \times t^2 = 2t^3$ — **M1 B1** (b). Let x and y be the dimensions that will give him the maximum possible area of the land.



$$\text{perimeter} = x + y + x = 100$$
$$y + 2x = 100, \qquad \Rightarrow y = 100 - 2x$$
$$\text{area}, \qquad A = xy = x(100 - 2x) = 100x - 2x^2$$ **M1**

$$\frac{dA}{dx} = 100 - 4x$$ **M1** **M1**

Area is maximum when $\frac{dA}{dx} = 0$

$$100 - 4x = 0, \qquad \Rightarrow x = \frac{100}{4} = 25 \text{ m}$$ **M1 B1**

$$y = 100 - 2x = 100 - 2 \times 25 = 50 \text{ m}$$ **B1**

$$\text{maximum area} = xy = 25 \times 50 = 1250 \text{ m}^2$$ **A1**

12

10 (a).

$$z_1 z_2 = (3 + i)(x + i) = 3x + 3i + xi - 1 = (3x - 1) + (3 + x)i$$ **M1**

$$\text{Arg}(z_1 z_2) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{3 + x}{3x - 1}\right) = \frac{\pi}{4}$$ **M1**

$$\frac{3 + x}{3x - 1} = \tan\frac{\pi}{4}$$ **M1**

$$\frac{3 + x}{3x - 1} = 1$$ **M1**

$$3 + x = 3x - 1$$
$$4 = 2x, \qquad \Rightarrow x = 2$$ **A1**

Alternatively:

$$z_1 = 3 + i, \qquad \Rightarrow \text{Arg}(z_1) = \tan^{-1}\left(\frac{1}{3}\right)$$

$$z_1 = x + i, \qquad \Rightarrow \text{Arg}(z_2) = \tan^{-1}\left(\frac{1}{x}\right)$$ **M1**

$$\text{Arg}(z_1 z_2) = \frac{\pi}{4}$$

$$\text{Arg}(z_1) + \text{Arg}(z_2) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{4}, \qquad \Rightarrow \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{x}}{1 - \frac{1}{3} \times \frac{1}{x}}\right) = \frac{\pi}{4}$$ **M1 M1**

$$\frac{\left(\frac{x+3}{3x}\right)}{\left(\frac{3x-1}{3x}\right)} = \tan\frac{\pi}{4}, \qquad \Rightarrow \frac{x + 3}{3x - 1} = 1$$ **M1**

$$x + 3 = 3x - 1$$
$$4 = 2x, \qquad \Rightarrow x = 2$$ **A1**

(b).

$$z^4 - 6z^2 + 25 = 0$$

$$z^2 = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 25}}{2 \times 1} = \frac{6 \pm \sqrt{-64}}{2} = \frac{6 \pm 8i}{2} = 3 \pm 4i$$ **M1**

$$z^2 = 3 - 4i, \qquad \text{or}, \qquad z^2 = 3 + 4i$$

let, $z = a + bi$, $\Rightarrow z^2 = (a + bi)^2 = (a^2 - b^2) + 2abi$ **M1**

for, $z^2 = 3 - 4i$, $(a^2 - b^2) + 2abi = 3 - 4i$

By comparison,

$$2ab = -4, \qquad \Rightarrow b = \frac{-4}{a}$$
$$a^2 - b^2 = 3$$
$$a^2 - \frac{16}{a^2} = 3$$
$$a^4 - 3a^2 - 16 = 0$$

$$a^2 = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-16)}}{2 \times 1} = \frac{3 \pm \sqrt{73}}{2}$$ **M1**

for, $a^2 = \frac{3 + \sqrt{73}}{2} \approx 5.772$, $\Rightarrow a = \sqrt{5.772} = \pm 2.402$ **M1**

for, $a^2 = \frac{3 - \sqrt{73}}{2} \approx -2.772$, $\Rightarrow a$ is undefined **B1**

$$b = \frac{-4}{a} = \frac{-4}{\pm 2.402} = \mp 1.665$$

$$\Rightarrow z = 2.402 - 1.665i, \qquad \text{or}, \qquad z = -2.402 + 1.665i$$

Also,

for, $z^2 = 3 + 4i$, $(a^2 - b^2) + 2abi = 3 + 4i$

By comparison, **A1**

$$2ab = 4, \qquad \Rightarrow b = \frac{4}{a}$$

$$b = \frac{4}{a} = \frac{4}{\pm 2.402} = \pm 1.665$$

$$\Rightarrow z = 2.402 + 1.665i, \qquad \text{or}, \qquad z = -2.402 - 1.665i$$ **A1**

12

11 (a).

$$OA = \frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right), \qquad OB = \frac{1}{2}\left(3\underset{\sim}{a} - \underset{\sim}{b}\right), \qquad OC = \frac{1}{8}\left(3\underset{\sim}{a} + 5\underset{\sim}{b}\right)$$

$$AB = OB - OA = \frac{1}{2}\left(3\underset{\sim}{a} - \underset{\sim}{b}\right) - \frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right)$$

$$= \frac{2\left(3\underset{\sim}{a} - \underset{\sim}{b}\right) - \left(\underset{\sim}{a} + 3\underset{\sim}{b}\right)}{4} = \frac{6\underset{\sim}{a} - 2\underset{\sim}{b} - \underset{\sim}{a} - 3\underset{\sim}{b}}{4}$$ **M1**

$$=\frac{5\underset{\sim}{a}-5\underset{\sim}{b}}{4}=\frac{5}{4}\left(\underset{\sim}{a}-\underset{\sim}{b}\right)=1.25\left(\underset{\sim}{a}-\underset{\sim}{b}\right)$$ **B1**

$$BC=OC-OB=\frac{1}{8}\left(3\underset{\sim}{a}+5\underset{\sim}{b}\right)-\frac{1}{2}\left(3\underset{\sim}{a}-\underset{\sim}{b}\right)$$

$$=\frac{\left(3\underset{\sim}{a}+5\underset{\sim}{b}\right)-4\left(3\underset{\sim}{a}-\underset{\sim}{b}\right)}{8}=\frac{3\underset{\sim}{a}+5\underset{\sim}{b}-12\underset{\sim}{a}+4\underset{\sim}{b}}{8}$$ **M1**

$$=\frac{9\underset{\sim}{b}-9\underset{\sim}{a}}{8}=\frac{9}{8}\left(\underset{\sim}{b}-\underset{\sim}{a}\right)=-1.125\left(\underset{\sim}{a}-\underset{\sim}{b}\right)$$ **B1**

$$\frac{AB}{BC}=\frac{1.25\left(\underset{\sim}{a}-\underset{\sim}{b}\right)}{-1.125\left(\underset{\sim}{a}-\underset{\sim}{b}\right)}=-\frac{10}{9}\,,\qquad \Rightarrow AB=-\frac{10}{9}BC$$ **M1**

Since line **AB** is a multiple of line **BC** and both lines have a common point **B**, then points **A, B** and **C** lie on a straight. **B1**

(b).

From the Cartesian equation of the line,

$$\text{position vector, }\quad \boldsymbol{OA}=\begin{pmatrix}1\\-4\\-1\end{pmatrix},\qquad \text{direction vector,}$$

$$\underset{\sim}{d}=\begin{pmatrix}2\\-3\\-1\end{pmatrix}$$ **B1**

The point on the plane is $B(2,3,-1)$

$$\boldsymbol{AB}=\boldsymbol{OB}-\boldsymbol{OA}=\begin{pmatrix}2\\3\\-1\end{pmatrix}-\begin{pmatrix}1\\-4\\-1\end{pmatrix}=\begin{pmatrix}1\\7\\0\end{pmatrix}$$ **M1**

$$\underset{\sim}{n}=\overrightarrow{AB}\times\underset{\sim}{d}=\begin{vmatrix}\underset{\sim}{i} & -\underset{\sim}{j} & \underset{\sim}{k}\\ 1 & 7 & 0\\ 2 & -3 & -1\end{vmatrix}$$ **M1**

$$=\underset{\sim}{i}\begin{vmatrix}7 & 0\\-3 & -1\end{vmatrix}-\underset{\sim}{j}\begin{vmatrix}1 & 0\\2 & -1\end{vmatrix}+\underset{\sim}{k}\begin{vmatrix}1 & 7\\2 & -3\end{vmatrix}$$

$$=\underset{\sim}{i}(-7-0)-\underset{\sim}{j}(-1-0)+\underset{\sim}{k}(-3-14)=-7\underset{\sim}{i}+\underset{\sim}{j}-17\underset{\sim}{k}$$ **M1**

The equation of the plane is given by:

$$\underset{\sim}{r}.\underset{\sim}{n}=\underset{\sim}{n}.\boldsymbol{OB}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}-7\\1\\-17\end{pmatrix}=\begin{pmatrix}-7\\1\\-17\end{pmatrix}.\begin{pmatrix}2\\3\\-1\end{pmatrix}$$

$$-7x+y-17z=-14+3+17$$ **M1**

$$-7x+y-17z=6$$

$$7x-y+17z=-6$$ **A1**

12

12 (a).

$$\text{let,}\qquad \alpha=\tan^{-1}\left(\frac{1}{2}\right),\qquad \beta=\tan^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \tan\alpha=\frac{1}{2}\,,\qquad \tan\beta=\frac{1}{5}$$ **M1**

$$\text{but, }\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$=\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}=\left(\frac{7}{10}\right)\Big/\left(\frac{9}{10}\right)=\frac{7}{9}$$ **M1 B1**

$$\Rightarrow \alpha+\beta=\tan^{-1}\left(\frac{7}{9}\right)$$ **M1**

$$\therefore \tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)=\tan^{-1}\left(\frac{7}{9}\right),\qquad \text{hence proved}$$ **B1**

(b)

$$3\cos x+4\sin x\equiv R\cos(x-\alpha)=R\cos x\cos\alpha+R\sin x\sin\alpha$$

By comparison,

$$R\cos\alpha=3\ \rightarrow(1a),\qquad R\sin\alpha=4\ \rightarrow(1b)$$ **M1-comparison**

Dividing (1b) by (1a) gives;

$$\frac{R\sin\alpha}{R\cos\alpha}=\frac{4}{3}\,,\qquad \Rightarrow\tan\alpha=\frac{4}{3},\qquad \Rightarrow\alpha=53.13°$$ **M1-both α and R.**

$$R=\sqrt{4^2+3^2}=5$$

$$3\cos x+4\sin x\equiv 5\cos(x-53.13)$$ **M1**

Maximum value:

$$\{3\cos x+4\sin x\}_{max}=5\times 1=5$$ **A1**

It occurs when:

$$\cos(x-53.13)=1,\qquad \Rightarrow(x-53.13)=0,\qquad \Rightarrow x=53.13°$$ **A1**

Minimum value:

$$\{3\cos x+4\sin x\}_{min}=5\times -1=-5$$ **A1**

It occurs when:

$$\cos(x-53.13)=-1,\qquad \Rightarrow(x-53.13)=180,$$ **A1**

$$\Rightarrow x=233.13°$$

The maximum value of $3\cos x+4\sin x$ is 5 and it occurs when $x=53.13°$. The minimum value of $3\cos x+4\sin x$ is -5 and it occurs when $x=233.13°$.

12

13 **Slanting asymptote**

$$y=\frac{x^2-x-6}{x-1}=\frac{x(x-1)-6}{x-1}=\frac{x(x-1)}{x-1}-\frac{6}{x-1}=x-\frac{6}{x-1}$$ **M1**

$$\Rightarrow y=x\text{ is the slanting asymptote}$$ **B1**

Vertical asymptote

$$y=\frac{x^2-x-6}{x-1}$$

$$\text{as } y\to\infty,\ (x-1)\to 0$$

$$\Rightarrow x=1\text{ is the vetical asymptote}$$ **M1**

Turning points

$$\frac{dy}{dx}=\frac{(x-1)(2x-1)-(x^2-x-6)(1)}{(x-1)^2}$$ **M1**

$$= \frac{2x^2 - 3x + 1 - x^2 + x + 6}{(x-1)^2} = \frac{x^2 - 2x + 7}{(x-1)^2}$$

At turning points, $\frac{dy}{dx} = 0$

$$\frac{x^2 - 2x + 7}{(x-1)^2} = 0, \qquad \Rightarrow x^2 - 2x + 7 = 0$$

M1-equating to zero

x is undefined hence no turning point

B1

Intercepts

$$y = \frac{x^2 - x - 6}{x - 1} = \frac{(x-3)(x+2)}{x-1}$$

when, $\quad x = 0, \quad y = \frac{-6}{-1} = 6$

$\Rightarrow (0, 6)$, is the y − intercept

M1-for y-intercept

when, $\quad y = 0, \quad (x-3)(x+2) = 0,$

$\Rightarrow x = 3$, or $x = -2$

$\Rightarrow (-2, 0)$and $(3, 0)$, are the x − intercept

M1 for x-intercept

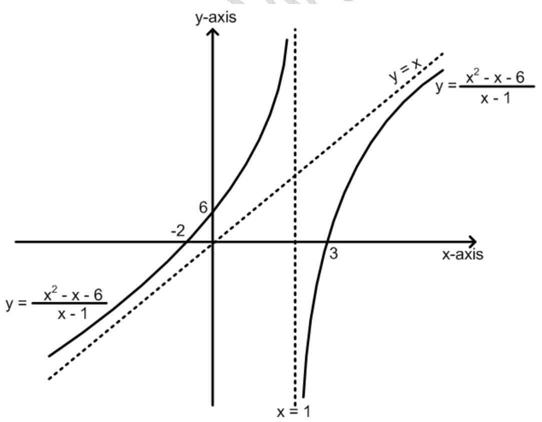
The Critical values include: $x = -2, x = 1, x = 3$

Region where the curve lies:

	$x < -2$	$-2 < x < 1$	$1 < x < 3$	$x > 3$
$(x-3)$	-	-	-	+
$(x+2)$	-	+	+	+
$(x-1)$	-	-	+	+
y	-	+	-	+

M1

Sketch of the curve



B1 –curve 1 correct with intercepts

B1 –curve 2 correct with intercepts

B1 –both asymptotes (should be dotted)

12

14 Let the common root be β. It implies that $x = \beta$

from, $\quad x^2 + ax + p = 0, \quad \beta^2 + a\beta + p = 0 \quad \rightarrow (1a)$

from, $\quad cx^2 + 2ax - 3p = 0, \quad c\beta^2 + 2a\beta - 3p = 0 \quad \rightarrow (1b)$

$c \times (1a) - (1b)$ gives,

$$\begin{array}{r|l} c\beta^2 + ac\beta + pc = 0 & \rightarrow c \times (1a) \\ - \quad c\beta^2 + 2a\beta - 3p = 0 & \rightarrow (1b) \\ \hline a\beta(c-2) + p(c+3) = 0 & \end{array}$$

M1

$$\beta = -\frac{p(c+3)}{a(c-2)} \rightarrow (2a)$$

B1

Substituting $(2a) in (1a)$ gives,

$$\beta^2 + a\beta + b = 0$$

$$\left\{-\frac{p(c+3)}{a(c-2)}\right\}^2 + a\left\{-\frac{p(c+3)}{a(c-2)}\right\} + b = 0$$

M1

Multiplying throughout by the LCM $\{a^2(c-2)^2\}$ gives

$$p^2(c+3)^2 - a^2p(c+3)(c-2) + pa^2(c-2)^2 = 0$$

$$p(c+3)^2 - a^2(c+3)(c-2) + a^2(c-2)^2 = 0$$

M1

$$p(c+3)^2 = a^2(c+3)(c-2) - a^2(c-2)^2$$

$$p(c+3)^2 = a^2(c-2)\{(c+3) - (c-2)\}$$

$$p(c+3)^2 = 5a^2(c-2)$$

B1

(b).

$$\sum_{r=1}^{n} r^2(r+1) = \frac{n}{12}(n+1)(n+2)(3n+1)$$

$$1^2 \times 2 + 2^2 \times 3 + \cdots + n^2(n+1) = \frac{n}{12}(n+1)(n+2)(3n+1)$$

For $n = 1$,

$L.H.S = 1^2 \times 2 = 2$	$R.H.S = \frac{1}{12} \times 2 \times 3 \times 4 = 2$
True for $n = 1$ since $L.H.S = R.H.S = 2$	

For $n = 2$,

$L.H.S = 1^2 \times 2 + 2^2 \times 3 = 14$	$R.H.S = \frac{2}{12} \times 3 \times 4 \times 7 = 14$
True for $n = 2$ since $L.H.S = R.H.S = 14$	

B1 –both n=1 and n=2

Suppose it's true for $n = k$, the series becomes:

$$1^2 \times 2 + 2^2 \times 3 + \cdots + k^2(k+1) = \frac{k}{12}(k+1)(k+2)(3k+1) \rightarrow (1)$$

B1-for n=k (assumption must be seen)

For $n = (k+1)$,

$$R.H.S = \frac{(k+1)}{12}(k+2)(k+3)(3k+4)$$

$$L.H.S = \{1^2 \times 2 + 2^2 \times 3 + \cdots + k^2(k+1)\} + (k+1)^2(k+2) \rightarrow (2)$$

M1

Substituting equation (1) into (2) gives:

$$L.H.S = \frac{k}{12}(k+1)(k+2)(3k+1) + (k+1)^2(k+2)$$

M1

$$=\frac{(k+1)(k+2)[k(3k+1)+12(k+1)]}{12}$$

$$=\frac{(k+1)(k+2)[3k^2+k+12k+12]}{12}$$

$$=\frac{(k+1)(k+2)[3k^2+13k+12]}{12}$$ **M1**

$$=\frac{(k+1)(k+2)[3k^2+9k+4k+12]}{12}$$ **M1**

$$=\frac{(k+1)(k+2)[3k(k+3)+4(k+3)]}{12}$$

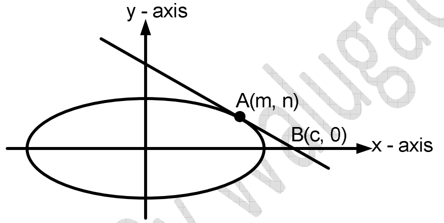
$$=\frac{(k+1)(k+2)(k+3)(3k+4)}{12}$$ **B1**

True for $n=(k+1)$ since $.H.S=R.H.S=\frac{(k+1)(k+2)(k+3)(3k+4)}{12}$.

Since its true for $n=1, n=2, n=k$ and $n=(k+1)$, then it's true for all positive whole numbers of n.

12

15 (a).



The equation of the tangent is given by:

$$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\frac{2x}{a^2}+\left(\frac{2y}{b^2}\right)\frac{dy}{dx}=0, \qquad \Rightarrow \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$ **M1 B1**

but, Gradient of tangent at A = Gradient of line AB

$$-\frac{b^2m}{a^2n}=\frac{n-0}{m-c}$$ **M1**

$$-b^2m(m-c)=a^2n^2$$

$$\frac{-m(m-c)}{n^2}=\frac{a^2}{b^2}$$ **M1**

$$\frac{mc-m^2}{n^2}=\frac{a^2}{b^2}, \qquad \text{hence shown}$$ **B1**

(b).

$$\frac{d}{dx}(xy)=\frac{d}{dx}(4)$$

$$y+x\frac{dy}{dx}=0, \qquad \Rightarrow \frac{dy}{dx}=\frac{-y}{x}$$

$$\text{gradient of tangent}=-\frac{2}{p}\div 2p=\frac{-1}{p^2}$$

$$\text{gradient of normal}=p^2$$ **B1**

Equation of the normal at point P is,

$$\frac{y-\frac{2}{p}}{x-2p}=p^2, \qquad \Rightarrow y=p^2x-2p^3+\frac{2}{p}$$ **M1**

from, $xy=4$

$$x\left(p^2x-2p^3+\frac{2}{p}\right)=4$$ **M1**

$$p^2x^2-2p^3x+\frac{2x}{p}=4$$

$$p^3x^2-2p^4x+2x=4p$$

$$p^3x^2+(2-2p^4)x-4p=0$$

$$x=\frac{-(2-2p^4)\pm\sqrt{(2-2p^4)^2-4p^3\times(-4p)}}{2\times p^3}$$ **M1**

$$=\frac{2p^4-2\pm\sqrt{4p^8+8p^4+4}}{2p^3}=\frac{2p^4-2\pm\sqrt{4(p^8+2p^4+1)}}{2p^3}$$

$$=\frac{2p^4-2\pm\sqrt{4(p^4+1)^2}}{2p^3}=\frac{2p^4-2\pm 2(p^4+1)}{2p^3}$$

$$x=\frac{2p^4-2-2(p^4+1)}{2p^3}=\frac{2p^4-2-2p^4-2}{2p^3}=\frac{-2}{p^3}$$ **B1**

or, $$x=\frac{2p^4-2+2(p^4+1)}{2p^3}=\frac{2p^4-2+2p^4+2}{2p^3}=\frac{4p^4}{2p^3}$$

$$=2p$$

when $x=\frac{-2}{p^3}$,

$$y=p^2\left(\frac{-2}{p^3}\right)-2p^3+\frac{2}{p}=\frac{-2}{p}-2p^3+\frac{2}{p}=-2p^3$$ **M1**

The coordinates of Q is $\left(\frac{-2}{p^3}, -2p^3\right)$. **A1**

12

16 (a).

$$e^y\frac{dy}{dx}-e^{-x}=0$$

$$e^y dy=e^{-x}dx$$ **M1**

$$\int e^y\,dy=\int e^{-x}\,dx$$ **M1**

$$e^y=-e^{-x}+c$$

$$e^y=-Ae^{-kx}, \qquad \text{where } c=Ae^k$$ **B1**

(b).

Sol:

	$v \propto (1000-x), \quad \Rightarrow v = -k(1000-x),$ $\Rightarrow \frac{dx}{dt} = -k(1000-x)$	M1
	Where k is the proportionality constant. $\frac{dx}{1000-x} = -k\,dt, \quad \Rightarrow \int \frac{dx}{1000-x} = -\int k\,dt,$	M1
	$\Rightarrow -\ln(1000-x) = -kt + c \quad \to (1)$ When $t = 0$ s, $v = 1$ m s^{-1}	M1
	$1 = -k(1000-0), \quad \Rightarrow k = \frac{-1}{1000} = -0.001$	B1
	Also, when $t = 0$ s, $x = 0$ m $-\ln(1000-0) = 0 + c, \quad \Rightarrow c = -\ln 1000$	B1
	Equation (1) becomes, $-\ln(1000-x) = 0.001t - \ln 1000$	M1
	$-0.001t = \ln\left(\frac{1000-x}{1000}\right)$ $t = 1000\ln\left(\frac{1000}{1000-x}\right)$	B1
	When $x = 600$ m, $t = 1000\ln\left(\frac{1000}{1000-600}\right) = 1000\ln\left(\frac{5}{3}\right) = 510.83$ s	M1
	$t = \frac{510.83}{60} = 8.514$ minutes	A1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Sept 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 2 MOCK SET 5 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B.**
- Show your working clearly.
- Where necessary, use $g = 9.8$ m s^{-2}.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn 1: At the same instant, two children who are standing 24 m apart begin to cycle directly towards each other. James starts from rest at a point A riding with a constant acceleration of 2 m s^{-2} and William rides with a constant speed of 2 m s^{-1}. Find how long it is before they meet. [5]

Qn 2: Given that $X \sim B(15, 0.2)$, find:
(a). $P(X > 8)$, [2]
(b). the mode of X. [3]

Qn 3: Use the trapezium rule with 5 strips to estimate
$$\int_{-1}^{1} (3+2x)^5\,dx$$
correct to **three** decimal places. [5]

Qn 4: ABC is an equilateral triangle. Forces of 7 N, $8\sqrt{3}$ N and $8\sqrt{3}$ N act along BA, CA and CB respectively in the direction indicated by the order of the letters. Find the resultant force. [5]

Qn 5: The marks of **40** students in a test were as follows:

Marks	30 −	40 −	50 −	70 −	80 −	90 −
Number of students	8	5	12	9	6	0

Calculate the standard deviation of the marks. [5]

Qn 6: Given that $g(0.9) = 0.2661$, $g(1.0) = 0.2420$ and $g(1.1) = 0.2179$. Use linear interpolation or extrapolation to estimate:
(a). $g(0.96)$, [3]
(b). $g^{-1}(0.2372)$. [2]

Qn 7: The probability that a fisherman catches fish on a clear day is $\frac{2}{5}$ and on cloudy day is $\frac{7}{10}$. If the probability that the day is cloudy is $\frac{3}{5}$, find the probability that the day is cloudy given that the fisherman does not catch fish. [5]

Qn 8: A bullet of mass 8 g is fired towards a fixed wooden block an enters the block when traveling horizontally at 300 m s^{-1}. Find how far the bullet penetrates if the wood provides a constant resistance of 1800 N. [5]

<u>Section B (60 Marks)</u>

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
(a). If $x = 4.95$ and $y = 2.013$ are each rounded off to the given number of decimal places, calculate the maximum possible error in $\frac{x}{x-y}$. [6]
(b). The height and radius of a cylinder are measured as h and r with maximum possible errors Δ_1 and Δ_2 respectively. Show that the maximum percentage error made in calculating the volume is

$$\left(\left|\frac{\Delta_1}{h}\right| + 2\left|\frac{\Delta_2}{r}\right|\right) \times 100$$

[6]

Question 10:
The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows, 5% of the residents have over 90 cows.

(a). Determine the values of the mean and standard deviation of the cows. [8]
(b). If there are 200 residents, find how many have more than 80 cows. [4]

Question 11:
A uniform rectangular plate $OABC$ has mass $4m$, $\overline{OA} = \overline{BC} = 2d$ and $\overline{OC} = \overline{AB} = d$. Particles of mass $2m$, m and $3m$ are attached at A, B and C respectively on the plate. Find, in terms of d, the distance of the centre of mass of the loaded plate:
(a). from OA,
(b). from OC.
The corner O of the loaded plate is freely hinged to a fixed point and the plate rests in equilibrium.
(c). Calculate the angle between OC and the downward vertical. [12]

Question 12:
The table below shows the percentage of sand, y, in the soil at different depths, x (in cm).

Soil depth (x)	35	65	55	25	45	75	20	90	51	60
Percentage of sand (y)	86	70	84	92	79	68	96	58	86	77

(a). (i). Calculate the rank correlation coefficient between the two variables.
(ii). Comment on the significance at 5% level. [5]
(b). (i). Draw a scatter diagram for the data and comment on our result.
(ii). Draw the line of best fit; hence estimate the:
- percentage of sand in the soil at a depth of 31 cm.
- depth of the soil with 54% sand. [7]

Question 13:
(a). A pump draws water from a tank and issues it at a speed of 10 m s^{-1} from the end of a hose of cross-sectional area 5 cm^2, situated 4 m above the level from which the water is drawn. Find the rate at which the pump is working. [5]
(b). A car of mass 800 kg moves against a constant resistance R N. The maximum speeds of the car up and down an incline of 1 in 16 are 14 m s^{-1} and 42 m s^{-1} respectively. If the rate at which the engine is working is H kW, find the:

(i). values of R and H,
(ii). acceleration at the instant when the speed is $17.5\ \text{m s}^{-1}$ on level ground. [7]

Question 14:
Show graphically that the equation $x^3 + 5x^2 - 3x - 4 = 0$ has roots between 0 and -1. Hence use Newton Raphson's method to find the root of the equation, correct to **2** decimal places. [12]

Question 15:
(a). A, B and C are three aircrafts. $\boldsymbol{A}$ has velocity $\left(200\underset{\sim}{\boldsymbol{i}} + 170\underset{\sim}{\boldsymbol{j}}\right)$ m s^{-1}. To the pilot of $\boldsymbol{A}$, it appears that $\boldsymbol{B}$ has velocity $\left(50\underset{\sim}{\boldsymbol{i}} - 270\underset{\sim}{\boldsymbol{j}}\right)$ m s^{-1}. To the pilot of $\boldsymbol{B}$, it appears that $\boldsymbol{C}$ has velocity $\left(50\underset{\sim}{\boldsymbol{i}} + 170\underset{\sim}{\boldsymbol{j}}\right)$ m s^{-1}. Find, the vector form of the velocities of $\boldsymbol{B}$ and $\boldsymbol{C}$. [4]
(b). Two ships $\boldsymbol{A}$ and $\boldsymbol{B}$ have the following position vector, $\underset{\sim}{\boldsymbol{r}}$, and velocity vector, $\underset{\sim}{\boldsymbol{v}}$ at the times stated:

$\underset{\sim}{\boldsymbol{r}}_A = \left(-2\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}}\right)$ km, $\quad \underset{\sim}{\boldsymbol{v}}_A = \left(12\underset{\sim}{\boldsymbol{i}} - 4\underset{\sim}{\boldsymbol{j}}\right)$ km h^{-1}, $\quad$ at 11: 45 am

$\underset{\sim}{\boldsymbol{r}}_B = \left(8\underset{\sim}{\boldsymbol{i}} + 7\underset{\sim}{\boldsymbol{j}}\right)$ km, $\quad \underset{\sim}{\boldsymbol{v}}_B = \left(2\underset{\sim}{\boldsymbol{i}} - 14\underset{\sim}{\boldsymbol{j}}\right)$ km h^{-1}, $\quad$ at 12: 00 noon

If the two ships do not alter their velocities, find their least distance of separation. [8]

Question 16:
Two tetrahedral dice, with faces labeled 1, 2, 3 and 4 are thrown and the number on which each lands is noted. The score is the sum of the two numbers. Find the probability that:
(a). the score is even, given that at least one die lands on three.
(b). at least one die lands on three given that the score is even. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	Let x be the distance, in metres, travelled by James before they meet. For motion of James, $u = 0, a = 2\ \text{m s}^{-2}$ $s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times t^2 = t^2$ For motion of William, $a = 0, u = 2\ \text{m s}^{-1}$ $s_2 = ut + \frac{1}{2}at^2 = 2t + 0 = 2t$ but, $s_1 + s_2 = 24$ $t^2 + 2t = 24$ $t^2 + 2t - 24 = 0$ $t = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-24)}}{2 \times 1}$ $t = 4$, or, $t = -6$ but $t \neq -6$, $\Longrightarrow t = 4$ s	 B1 B1 M1 M1 A1
		05
2	(a). $P(X > 8) = P(X \geq 9) = 0.0008$ (b). $E(X) = np = 15 \times 0.2 = 3$ $P(X = 2) = 0.2309$ $P(X = 3) = 0.2501$ $P(X = 4) = 0.1876$ $\Longrightarrow$ Mode = 3	 M1 A1 M1 B1 A1
		05
3	$y_n = (3 + 2x_n)^5$, $\quad h = \frac{1-(-1)}{5} = \frac{2}{5} = 0.4$	B1

n	x_n	y_0, y_5	$y_1, \dots y_4$
0	-1	1	
1	-0.6		18.89568
2	-0.2		118.81376
3	0.2		454.35424
4	0.6		1306.91232
5	1	3125	
sums		3126	1898.976

B1-for all x_n

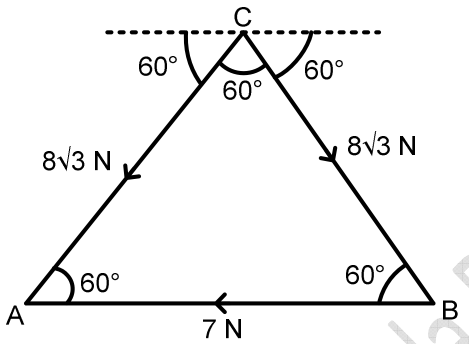
B1-for all y_n (4 d.p or more)

$$\int_{-1}^{1} (3 + 2x)^5\, dx \approx \frac{1}{2}h[(y_0 + y_4) + 2(y_1 + \cdots + y_3)]$$

$$\approx \frac{1}{2} \times \frac{2}{5}[3126 + 2 \times 189.976] = 1384.7904 \approx 1384.790 \text{ (3 d.p)}$$ **M1 A1**

05

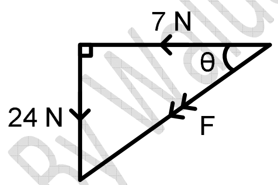
4



B1-force diagram

$$\underset{\sim}{F} = \begin{pmatrix} -7 \\ 0 \end{pmatrix} + \begin{pmatrix} 8\sqrt{3}\cos 60° \\ -8\sqrt{3}\sin 60° \end{pmatrix} + \begin{pmatrix} -8\sqrt{3}\cos 60° \\ -8\sqrt{3}\sin 60° \end{pmatrix} = \begin{pmatrix} -7 \\ -24 \end{pmatrix}$$ **B1**

$$\left|\underset{\sim}{F}\right| = \sqrt{(-7)^2 + (-24)^2} = 25 \text{ N}$$ **M1 B1**



$$\tan\theta = \frac{24}{7}, \quad \Rightarrow \theta = 73.740°$$

The resultant force is 25 N in the direction 73.740° below the negative horizontal. **A1**

05

5

Marks	f	x	fx	fx^2	Class boundary
30 –	8	35	280	9800	30 – 40
40 –	5	45	225	10125	40 – 50
50 –	12	60	720	43200	50 – 70
70 –	9	75	675	50625	70 – 80
80 –	6	85	510	43350	80 – 90
90 –	0	–	0	0	90 –
Total	40		2410	157100	

M1-for fx values

M1-for fx^2 values

M1-for both $\sum fx$ and $\sum fx^2$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{157100}{40} - \left(\frac{2410}{40}\right)^2} = \sqrt{297.4375} \approx 17.2464$$ **M1 A1**

05

6 (a).

x	0.9	0.96	1.0
$g(x)$	0.2661	y	0.2420

B1

$$\frac{y - 0.2661}{0.2420 - 0.2661} = \frac{0.96 - 0.9}{1 - 0.9}$$ **M1**

$$y = \frac{0.06}{0.1} \times (-0.0241) + 0.2661 = 0.25164$$ **A1**

$$\therefore g(0.96) = 0.25164 \text{ (4 d.p)}$$

(b).

x	1.0	x	1.1
$g(x)$	0.2420	0.2372	0.2179

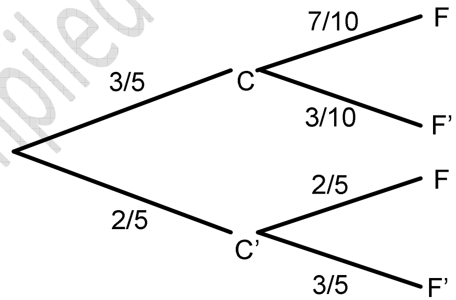
$$\frac{x - 1}{1.1 - 1} = \frac{0.2372 - 0.2420}{0.2179 - 0.2420}$$ **M1**

$$y = \frac{-0.0048}{-0.0241} \times 0.1 + 1 = 1.0199$$ **A1**

$$\therefore g^{-1}(0.2372) = 1.0199$$

05

7 Let C denote event of a cloudy day and F denote event of catching fish.



$$P(C/F') = \frac{P(C \cap F')}{P(F')}$$

$$P(C \cap F') = \frac{3}{5} \times \frac{7}{10} = \frac{9}{80}$$ **M1**

$$P(F') = \frac{3}{5} \times \frac{3}{10} + \frac{2}{5} \times \frac{3}{5} = \frac{21}{80}$$ **M1 M1**

$$P(C/F') = \frac{P(C\cap F')}{P(F')} = \frac{9}{80} \div \frac{21}{80} = \frac{3}{7} \approx 0.4286$$ **M1 A1**

Alternatively:

$$P(F/C') = \frac{2}{5}, P(F/C) = \frac{7}{10}, P(C) = \frac{3}{5}$$

$$P(F) = P(F\cap C) + P(F\cap C') = P(C).P(F/C) + P(C').P(F/C')$$

$$= \frac{3}{5}\times\frac{7}{10} + \left(1-\frac{3}{5}\right)\times\frac{2}{5} = 0.42 + 0.16 = \frac{29}{50} = 0.58$$

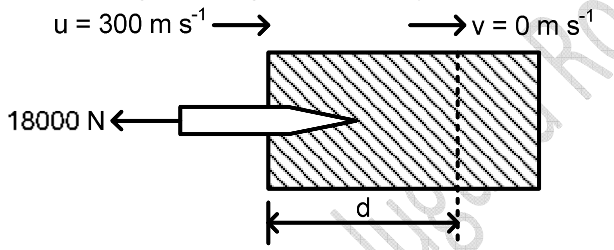
$$P(C/F') = \frac{P(C\cap F')}{P(F')} = \frac{P(C) - P(C\cap F)}{1-P(F)} = \frac{0.6-0.42}{1-0.58} = \frac{3}{7}$$

$$\approx 0.4286$$

05

8

$$m = 8\text{ g} = 0.008\text{ kg}, u = 300\text{ m s}^{-1}, f = 1800\text{ N}$$



$$\text{Work done} = -f\times d = -1800d$$ **M1**

$$\text{Change in kinetic energy} = \frac{1}{2}m(v^2-u^2)$$

$$= \frac{1}{2}\times 0.008\times(0-300^2) = -360\text{ J}$$ **M1 M1**

Since the work done is equal to the change in kinetic energy,

$$-1800d = -360$$ **M1**

$$d = \frac{360}{1800} = 0.2\text{ m}$$ **A1**

Alternatively:

$$m = 8\text{ g} = 0.008\text{ kg}, u = 300\text{ m s}^{-1}, F_r = 1800\text{ N}, v = 0\text{ m s}^{-1}$$

Using Newton's second law of motion,

$$0 - 18000 = 0.008a, \quad \Rightarrow a = \frac{-1800}{0.008} = -225000\text{ m s}^{-2}$$

Using the third equation of motion,

$$v^2 = u^2 + 2as$$

$$0^2 = 300^2 + 2\times(-225000)\times d$$

$$0 = 90{,}000 - 450{,}000d$$

$$d = \frac{90{,}000}{450{,}000} = 0.2\text{ m}$$

05

9 (a).

$$\Delta x = 0.005, \quad \Delta y = 0.0005$$

$$\left(\frac{x}{x-y}\right)_{\max} = \frac{x_{\max}}{(x-y)_{\min}}$$

$$= \frac{(4.95+0.005)}{(4.95-0.005)-(2.013+0.0005)} = 1.690261$$ **M1 B1**

$$\left(\frac{x}{x-y}\right)_{\min} = \frac{x_{\min}}{(x-y)_{\max}}$$

$$= \frac{(4.95-0.005)}{(4.95+0.005)-(2.013-0.0005)} = 1.680544$$ **M1 B1**

$$\text{Maximum possible error} = \frac{|1.690261-1.680544|}{2}$$ **M1**

$$= \frac{0.009717}{2} = 0.0048585$$ **A1-4 d.p or more**

(b).

$$\text{volume,} \quad v = \frac{\pi}{3}r^2h$$

$$H = h+\Delta_1, \quad R = r+\Delta_2$$

$$\text{Exact value} = \frac{\pi}{3}R^2H, \quad \text{Approximate value} = \frac{\pi}{3}r^2h$$

$$\text{Error} = \frac{\pi}{3}R^2H - \frac{\pi}{3}r^2h$$

$$= \frac{\pi}{3}[(r+\Delta_2)^2(h+\Delta_1) - r^2h]$$ **M1**

$$= \frac{\pi}{3}\left[(r^2+2r\Delta_2+\Delta_2{}^2)(h+\Delta_1) - r^2h\right]$$

$$= \frac{\pi}{3}\left[r^2h + 2rh\Delta_2 + h\Delta_2{}^2 + r^2\Delta_1 + 2r\Delta_2\Delta_1 + \Delta_2{}^2\Delta_1 - r^2h\right]$$

Assuming $\Delta_1 \ll h$ and $\Delta_2 \ll r$, **B1**

$$\Rightarrow \Delta_1\Delta_2 \approx 0, \Delta_2{}^2 \approx 0 \text{ and } \Delta_2{}^2\Delta_1 \approx 0$$

$$\text{Error} = \frac{\pi}{3}(2rh\Delta_2 + r^2\Delta_1)$$

$$\text{Absolute error} = \frac{\pi}{3}|2rh\Delta_2 + r^2\Delta_1|$$

$$\text{Relative error} = \frac{\pi}{3}|2rh\Delta_2 + r^2\Delta_1| \div \frac{\pi}{3}r^2h$$ **M1**

$$= \left|\frac{2rh\Delta_2 + r^2\Delta_1}{r^2h}\right| = \left|\frac{2\Delta_2}{r} + \frac{\Delta_1}{h}\right| \le 2\left|\frac{\Delta_2}{r}\right| + \left|\frac{\Delta_1}{h}\right|$$ **B1 B1**

$$\therefore \text{Maximum percentage error} = \left(\left|\frac{\Delta_1}{h}\right| + 2\left|\frac{\Delta_2}{r}\right|\right)\times 100$$ **B1**

12

10 (a).

$$P(X<60) = 0.15, \quad P(Z<-z_1) = 0.15$$

$$0.5 - \phi(-z_1) = 0.15$$

$$\phi(-z_1) = 0.5 - 0.15 = 0.35$$

$$-z_1 = 1.036, \quad \Rightarrow z_1 = -1.036$$

$$\text{but}, z_1 = \frac{60-\mu}{\sigma} = -1.036, \quad \Rightarrow 60-\mu = -1.036\sigma \quad \rightarrow (1)$$ **M1 B1**

$$P(X>90) = 0.05, \quad P(Z>z_2) = 0.05$$

$$z_2 = 1.645$$

but, $z_2 = \dfrac{90-\mu}{\sigma} = 1.645, \quad \Longrightarrow 90-\mu = 1.645\sigma \quad \rightarrow (2)$ **M1 B1**

Equation (2) − (1) gives;

$$30 = 2.681\sigma, \quad \Longrightarrow \sigma = \frac{30}{2.681} = 11.190$$ **M1 A1**

Equation (1)

$$\mu = 60 + 1.036\sigma = 60 + 1.036 \times 11.190 = 71.593$$ **A1**

(b). $\mu = 71.593, \sigma = 11.190$

$$P(X > 80) = P\left(Z > \frac{80 - 71.593}{11.190}\right) = P(Z > 0.751)$$ **M1**

$$= 0.5 - \phi(0.751)$$

$$= 0.5 - 0.2737$$ **B1-table value**

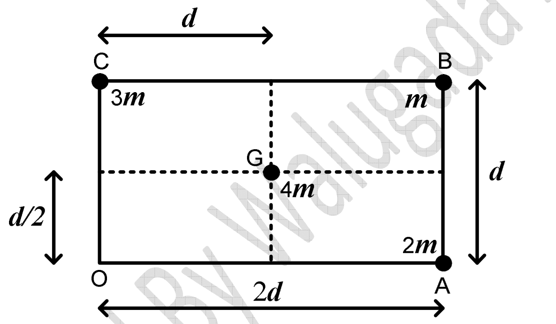
$$= 0.2263$$ **B1**

Number of residents with more than 80 cows

$$= 0.2263 \times 200 = 45.26 \approx 45 \text{ cows}$$ **M1 A1**

12

11



$O(0,0), \quad A(2l,0), \quad B(2l,d), \quad C(0,d), \quad G(l,0.5d)$

$$\sum m_i \begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} \sum m_i$$

$$2m\begin{pmatrix} 2d \\ 0 \end{pmatrix} + m\begin{pmatrix} 2d \\ d \end{pmatrix} + 3m\begin{pmatrix} 0 \\ d \end{pmatrix} + 4m\begin{pmatrix} d \\ 0.5d \end{pmatrix}$$ **M1 M1-for LHS**

$$= (2m + m + 3m + 4m)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$ **M1 M1-for RHS**

$$\begin{pmatrix} 4d \\ 0 \end{pmatrix} + \begin{pmatrix} 2d \\ d \end{pmatrix} + \begin{pmatrix} 0 \\ 3d \end{pmatrix} + \begin{pmatrix} 4d \\ 2d \end{pmatrix} = 10\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$ **B1**

$$\begin{pmatrix} 10d \\ 6d \end{pmatrix} = \begin{pmatrix} 10\bar{x} \\ 10\bar{y} \end{pmatrix}$$ **B1**

(a).

$$10\bar{x} = 10d, \quad \Longrightarrow \bar{x} = d$$

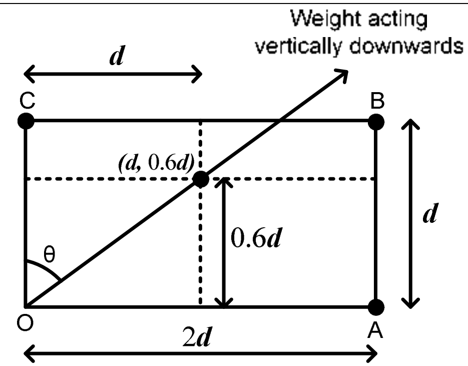
$\therefore$ Centre of mass from OA $= d$ **A1**

(b).

$$10\bar{y} = 6d, \quad \Longrightarrow \bar{x} = 0.6d$$

$\therefore$ Centre of mass from OC $= 0.6d$ **A1**

(c).



$$\tan\theta = \frac{d}{0.6d} = \frac{5}{3}, \quad \Longrightarrow \theta = 59.036°$$ **M1 M1 A1**

12

12 (a).

x	y	R_x	R_y	d	d^2
35	86	3	7.5	-4.5	20.25
65	70	8	3	5	25
55	84	6	6	0	0
25	92	2	9	-7	49
45	79	4	5	-1	1
75	68	9	2	7	49
20	96	1	10	-9	81
90	58	10	1	9	81
51	86	5	7.5	-2.5	6.25
60	77	7	4	3	9
$\sum x = 521$	$\sum y = 796$				$\sum d^2 = 321.5$

B1-for R_x

B1-for R_y

(a). (i).

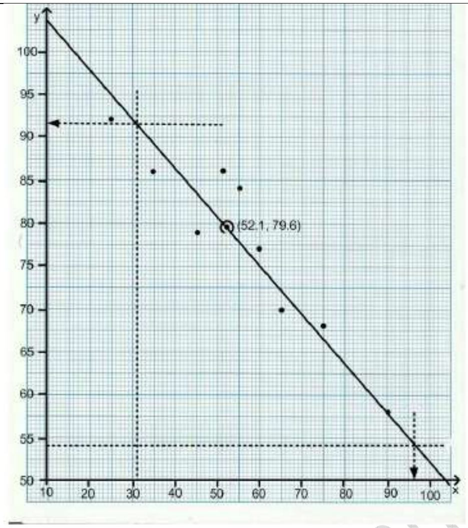
$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 321.5}{10(10^2-1)} = -0.9485$$ **M1 A1**

(ii). Significant at 5%. **B1**

(b). (i).

$$\bar{x} = \frac{\sum x}{n} = \frac{521}{10} = 52.1, \quad \bar{y} = \frac{\sum y}{n} = \frac{796}{10} = 79.6,$$

$$\Longrightarrow (\bar{x}, \bar{y}) = (52.1, 79.6)$$



B1-all axes labelled and on scale

B2-all points

B1-line of best fit

Comment: Negative linear relationship. **B1**

(ii).

- When $x = 31\ cm, y = 91.5\%$. **A1**
- When $y = 54\%, x = 96.5$ cm. **A1**

12

13 (a).

$v = 10\text{ m s}^{-1}, A = 5\text{ cm}^2 = 0.0005\text{ m}^2, h = 4\text{ m}, \rho = 1000\text{ kg m}^{-3}$

Mass of water raised and issued per second

$$m = Av\rho = 0.0005\ \times 10 \times 1000 = 5\text{ kg s}^{-1}$$ **B1**

Potential energy given to raise the water

$$P.E = mgh = 5 \times 9.8 \times 4 = 196\text{ J s}^{-1}$$ **M1**

Kinetic energy given to raise the water

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 10^2 = 250\text{ J s}^{-1}$$ **M1**

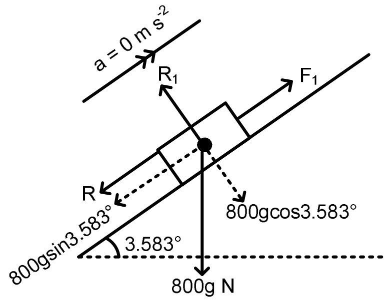
Rate at which the pump is working,

$$\text{P}_{\text{total}} = P.E + K.E = 196 + 250 = 446\text{ J s}^{-1}$$ **A1**

(b). (i).

$$\theta = \sin^{-1}\left(\frac{1}{16}\right) = 3.583°, \qquad P = H\text{ kW} = 10000H\text{ W}$$

For upward motion along the plane,



B1

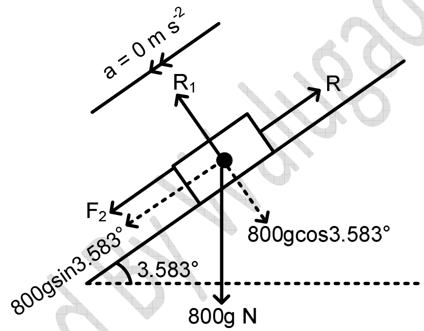
Resolving parallel to the plane gives,

$$F_1 = 800g\sin 3.583° + R$$

$$\frac{1000H}{14} = 800 \times 9.8 \times \frac{1}{16} + R$$

$$1000H = 6860 + 14R \quad \longrightarrow (1)$$ **B1**

For downward motion along the plane,



Resolving parallel to the plane gives,

$$F_1 + 800g\sin 3.583° = R$$

$$\frac{1000H}{42} + 800 \times 9.8 \times \frac{1}{16} = R$$

$$1000H + 20580 = 42R$$

$$1000H = 42R - 20580 \quad \longrightarrow (2)$$ **B1**

Equating equations (1) and (2) gives,

$$6860 + 14R = 42R - 20580$$ **M1**

$$28R = 27440$$

$$R = 980\text{ N}$$ **A1**

From equation (1),

$$H = \frac{6860 + 14 \times 980}{1000} = 20.58\text{ kW}$$ **A1**

(ii).

when $v = 17.5 \text{ m s}^{-1}$, $\quad F_3 = \dfrac{1000H}{v} = \dfrac{1000 \times 20.58}{17.5} = 1176 \text{ N}$

Resolving parallel to the plane,

$1176 - 980 = 800a, \qquad \Rightarrow a = 0.245 \text{ m s}^{-2}$ **M1 A1**

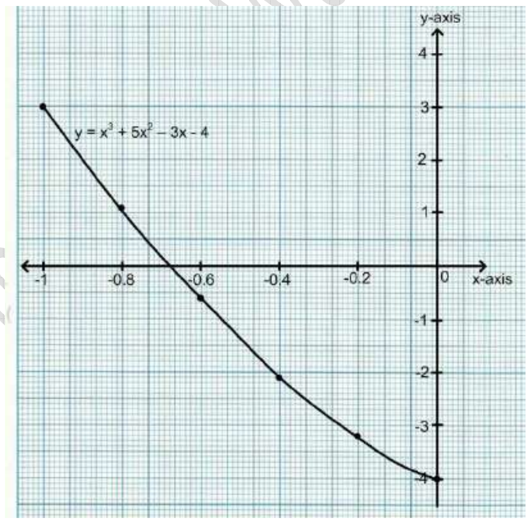
12

14 (a).

let, $\quad y = x^3 + 5x^2 - 3x - 4$

x	-1	-0.8	-0.6	-0.4	-0.2	0
y	3	1.1	-0.6	-2.1	-3.2	-4

B1
B1



B1-ploting points

B1-drawing curve

B1-labelling curve and axes.

From the graph, the root $x_0 = -0.68$.

(b). (i).

$f(x) = x^3 + 5x^2 - 3x - 4, \qquad \Rightarrow f'(x) = 3x^2 + 10x - 3$ **B1**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad \text{where } n = 0, 1, 2, \dots\dots$$

$$x_{n+1} = x_n - \frac{x_n{}^3 + 5x_n{}^2 - 3x_n - 4}{3x_n{}^2 + 10x_n - 3}$$ **M1**

From the graph, $x_0 = -0.68$

$$x_1 = (-0.68) - \frac{(-0.68)^3 + 5 \times (-0.68)^2 - 3 \times (-0.68) - 4}{3 \times (-0.68)^2 + 10 \times (-0.68) - 3}$$ **M1 B1**

$= -0.6755$

x_1 **M1**

$$= (-0.6755) - \frac{(-0.6755)^3 + 5 \times (-0.6755)^2 - 3 \times (-0.6755) - 4}{3 \times (-0.6755)^2 + 10 \times (-0.6755) - 3} = -0.6755$$ **M1**

The root is -0.68 (2 d.p). **A1**

12

15 (a).

$$\underset{\sim}{v}_A = \begin{pmatrix} 200 \\ 170 \end{pmatrix} \text{ m s}^{-1}, \qquad {}_B\underset{\sim}{v}_A = \begin{pmatrix} 50 \\ -270 \end{pmatrix} \text{ m s}^{-1},$$

$${}_C\underset{\sim}{v}_B = \begin{pmatrix} 50 \\ 170 \end{pmatrix} \text{ m s}^{-1}$$

$$\underset{\sim}{v}_B = {}_B\underset{\sim}{v}_A + \underset{\sim}{v}_A = \begin{pmatrix} 50 \\ -270 \end{pmatrix} + \begin{pmatrix} 200 \\ 170 \end{pmatrix} = \begin{pmatrix} 250 \\ -100 \end{pmatrix} \text{ m s}^{-1}$$ **M1 A1**

$$\underset{\sim}{v}_C = {}_C\underset{\sim}{v}_B + \underset{\sim}{v}_B = \begin{pmatrix} 50 \\ 170 \end{pmatrix} + \begin{pmatrix} 250 \\ -100 \end{pmatrix} = \begin{pmatrix} 300 \\ 70 \end{pmatrix} \text{ m s}^{-1}$$ **M1 A1**

(b).

Taking 12: 00 noon as the reference/starting time, then

$$OA = \underset{\sim}{r}_A + \frac{15}{60}\underset{\sim}{v}_A = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 12 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ km}$$ **B1-initialising**

$$OB = \underset{\sim}{r}_B = \begin{pmatrix} 8 \\ 7 \end{pmatrix} \text{ km},$$

$$\Rightarrow {}_A\underset{\sim}{r}_B = OA - OB = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ 7 \end{pmatrix} = \begin{pmatrix} -7 \\ -5 \end{pmatrix} \text{ km}$$ **B1**

$${}_A\underset{\sim}{v}_B = \underset{\sim}{v}_A - \underset{\sim}{v}_B = \begin{pmatrix} 12 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -14 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix} \text{ km h}^{-1}$$ **B1**

$${}_A\underset{\sim}{r}_B(t) = {}_A\underset{\sim}{r}_B + t\,{}_A\underset{\sim}{v}_B = \begin{pmatrix} -7 \\ -5 \end{pmatrix} + t\begin{pmatrix} 10 \\ 10 \end{pmatrix} = \begin{pmatrix} 10t - 7 \\ 10t - 5 \end{pmatrix} \text{ km}$$ **M1**

Method 1:

$$t_{min} = \frac{\left|{}_A\underset{\sim}{r}_B \cdot {}_A\underset{\sim}{v}_B\right|}{\left|{}_A\underset{\sim}{v}_B\right|^2} = \frac{\left|\begin{pmatrix} -7 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 10 \end{pmatrix}\right|}{10^2 + 10^2}$$ **M1**

$$= \frac{|-70 - 50|}{200} = \frac{120}{200} = 0.6 \text{ hours}$$ **B1**

$${}_A\underset{\sim}{r}_B(0.6) = \begin{pmatrix} -7 \\ -5 \end{pmatrix} + 0.6\begin{pmatrix} 10 \\ 10 \end{pmatrix} = \begin{pmatrix} -7 \\ -5 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \text{ km}$$

$$D_{min} = \left|{}_A\underset{\sim}{r}_B(0.6)\right| = \sqrt{(-1)^2 + 1^2} = \sqrt{2} = 1.414 \text{ km}$$ **M1 A1**

The least distance between A and B is 1.414 km.

Method 2:

Forclosestapproach, $\quad {}_A\underset{\sim}{r}_B(t) \cdot {}_A\underset{\sim}{v}_B = 0$

$$\begin{pmatrix}10t-7\\10t-5\end{pmatrix}\cdot\begin{pmatrix}10\\10\end{pmatrix}=0$$
$$100t-70+100t-50=0$$
$$t=\frac{120}{200}=0.6\text{ hours}$$
$$D_{min}=\left|{}_A\underset{\sim}{r}_B\,(0.6)\right|=\sqrt{(-1)^2+1^2}=\sqrt{2}=1.414\text{ km}$$

Method 3:

$$\left|{}_A\underset{\sim}{r}_B\,(t)\right|^2=(10t-7)^2+(10t-5)^2$$
$$\text{Forclosestapproach,}\quad \frac{d}{dt}\left|{}_A\underset{\sim}{r}_B\,(t)\right|^2=0$$
$$2(10t-7)(10)+2(10t-5)(10)=0$$
$$200t-140+200t-100=0$$
$$t=\frac{240}{400}=0.6\text{ hours}$$
$${}_A\underset{\sim}{r}_B\,(0.6)=\begin{pmatrix}10\times 0.6-7\\10\times 0.6-5\end{pmatrix}=\begin{pmatrix}-1\\1\end{pmatrix}\text{ km}$$
$$D_{min}=\left|{}_A\underset{\sim}{r}_B\,(0.6)\right|=\sqrt{(-1)^2+1^2}=\sqrt{2}=1.414\text{ km}$$

Method 4:

Let D be the distance between A and B at any time, t.

Forclosestapproach, $D=\left|{}_A\underset{\sim}{r}_B\,(t)\right|$ isminimum

$$D=\left|{}_A\underset{\sim}{r}_B\,(t)\right|=\sqrt{(10t-7)^2+(10t-5)^2}$$
$$=\sqrt{100t^2-140t+49+100t^2-100t+25}$$
$$=\sqrt{200t^2-240t+74}=\sqrt{200[t^2-1.2t+0.37]}$$
$$=\sqrt{200[(t-0.6)^2-0.36+0.37]}$$
$$D=\sqrt{200(t-0.6)^2+2}$$

$D$is minimum when $(t-0.6)^2=0,\ \Longrightarrow t=0.6$ hours.

$$\Longrightarrow D_{min}=\sqrt{0+2}\approx 1.414\text{ km}$$

12

16

Die 2 / Die 1	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

B1 B1

$$n(S)=16$$

Let $A=\{\text{even score}\}$, $B=\{\text{at least one die lands on three}\}$,

$$P(A)=\frac{n(A)}{n(S)}=\frac{8}{16}=0.5$$ **M1**

$$P(B)=\frac{n(B)}{n(S)}=\frac{7}{16}=0.4375$$ **M1**

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{3}{16}=0.1875$$ **M1 A1**

(a).

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{0.1875}{0.4375}=\frac{3}{7}\approx 0.4286$$ **M1 M1 A1**

(b).

$$P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{0.1875}{0.5}=\frac{3}{8}=0.375$$ **M1 M1 A1**

12

END

P425/1
PURE
MATHEMATICS
PAPER 1
Sept 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 1 MOCK SET 6 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section*

Qn 1: The sum of the second and third terms of a Geometric Progrssion (G.P) is 48. The sum of the fifth and sixth terms is 1296. Find the common ratio, the first term and the sum of the first 12 terms of the G.P. [5]

Qn 2: Use De Moivre's theorem to prove that
$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. [5]

Qn 3: When a polynomial $P(x)$ is divided by $x^2 - 5x - 14$, the remainder is $2x + 5$. Find the remainder when $P(x)$ is divided by:
(i). $x - 7$,
(ii). $x + 2$. [5]

Qn 4: OAB is a triangle in which $= \underset{\sim}{\boldsymbol{a}}$, $OB = \underset{\sim}{\boldsymbol{b}}$. $C$is a point on AB such that $AC: CB = 3: 1$. D is the midpoint of OA. $DC$and OB, both produced meet in point T. Find vector OT in terms of $\underset{\sim}{\boldsymbol{a}}$and $\underset{\sim}{\boldsymbol{b}}$. [5]

Qn 5: Find the integral $\int x\cos^2 x\,dx$. [5]

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

Qn 6: Given that $y = x + a$ is a tangent to the curve $y = ax^2 + bx + c$ at the point $(2, 4)$. Find the values of the constants a, b and c. [5]

Qn 7: Find the volume of the solid of revolution generated when the area under $y = \frac{1}{x-2}$ from $x = 3$ to $x = 4$ is rotated through four right angles about the x-axis. [5]

Qn 8: In triangle ABC, $AB = x - y$, $BC = x + y$ and $CA = x$, show that $\cos A = \frac{x-4y}{2(x-y)}$. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). Find the centroid of the triangle whose sides are given by the equations $x + y = 11$, $y = x - 1$ and $3y = x - 3$. [5]

(b). ABCD is a rhombus such that the coordinates $A(-3, -4)$and $C(5, 4)$. Find the equation of the diagonal BD of the rhombus. If the gradient of side BC is 2, obtain the coordinates of B and D, prove that the area of the rhombus is $21\frac{1}{3}$ square units. [7]

Question 10:

Show that $\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)}dx = \frac{\pi}{20}$. [12]

Question 11:

(a). Using Maclaurin's theorem, expand $e^{-x}\sin x$ upto the term in x^3. Hence evaluate $e^{-\frac{\pi}{3}}\sin\frac{\pi}{3}$ to four significant figures. [5]

(b). The curve $y = x^3 + 8$ cuts the x and y axesat the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B and C hence find the area enclosed between the curve and the line. [7]

Question 12:

(a). The position vectors of the points P and Q are $4\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 5\underset{\sim}{\boldsymbol{k}}$and $\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$respectively. Find the coordinates of the point R such that $PQ: PR = 2: 1$. [4]

(b). If the vector $5\underset{\sim}{\boldsymbol{i}} - \lambda\underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}}$is perpendicular to the line

$\underset{\sim}{r} = \underset{\sim}{i} - 4\underset{\sim}{j} + t(2\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k})$. Find the value of λ. [3]

(c). Obtain the equation of the plane that passes through $(1, -2, 2)$ and it's perpendicular to the line $\frac{x-9}{4} = \frac{y-6}{-1} = \frac{z-8}{1}$. [5]

Question 13:

The parametric equations $x = \frac{1+t}{1-t}$ and $y = \frac{2t^2}{1-t}$ represent a curve.

(i). Find the cartesian equation of the curve. [4]
(ii). Determine the turning points of the curve and their nature. [3]
(iii). State the asymptotes and intercepts of the curve. [3]
(iv). Hence sketch the curve. [2]

Question 14:

(a). Determine the maximum value of the expression $6\sin x - 3\cos x$. [3]
(b). Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$. [3]
(c). In a triangle ABC, prove that $\sin B + \sin C - \sin A = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [6]

Question 15:

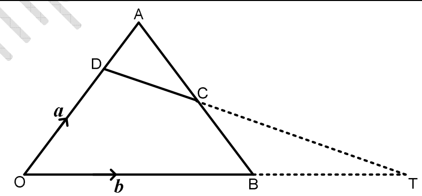
(a). Simplify $(2+5i)^2 + 5\frac{(7+2i)}{3-4i} - i(4-6i)$ expressing your answer in the form $a + bi$. [5]
(b). If $= x + yi$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Show that the locus of $Arg\left(\frac{z-1}{z-i}\right) = \frac{\pi}{3}$ is a circle. Find its centre and radius. [7]

Question 16:

(a). Using the substitution $y = ux$, solve the differential equation
$x^2\frac{dy}{dx} = x^2 + xy + y^2$. [4]
(b). The rate at which a liquid runs from a container is proportional to the square root of the depth of the opening below the surface of the liquid. A cylindrical petrol storage tank is sunk in the ground with its axis vertical. There is a leak in the tank at an unknown depth. The level of the petrol in the tank originally full is found to drop by 20 cm in 1 hour and by 19 cm in the next hour. Find the depth at which the leak is located. [8]

END

MARKING GUIDE

SNo.	Working	Marks
1	$ar + ar^2 = 48, \quad \Rightarrow ar(1+r) = 48 \quad \to (1)$ $ar^4 + ar^5 = 1296, \quad \Rightarrow ar^4(1+r) = 1296 \quad \to (2)$ (2) ÷ (1) gives; $\frac{ar^4(1+r)}{ar(1+r)} = \frac{1296}{48}, \quad \Rightarrow r^3 = 27, \quad \Rightarrow r = 3$ From (1); $a = \frac{48}{r(1+r)} = \frac{48}{3(1+3)} = 4$ $\Rightarrow S_{12} = 4\left(\frac{3^{12}-1}{3-1}\right) = 1062880$	B1–for eqns 1 & 2 M1 -solving A1–for r. M1 A1
		05
2	$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ $\cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta$ $+ 5\cos\theta\sin^4\theta + i\sin^5\theta = \cos 5\theta + i\sin 5\theta$ By comparison, $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta$ $+ 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$ $= -10\cos^3\theta + 11\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	B1-equating M1-expanding M1–equating real parts M1-simplification A1
		05
3	(i). $g(x) = x^2 - 5x - 14 = (x-7)(x+2)$ let, $R(x) = 2x + 5$ for, $(x-7) = 0, x = 7, \quad \Rightarrow R(7) = 2(7) + 5 = 19$ (ii). for, $(x+2) = 0, x = -2, \quad \Rightarrow R(-2) = 2(-2) + 5 = 1$	B1 M1 A1 M1 A1
		05
4	 $OD = DA = \frac{1}{2}\underset{\sim}{a}, \quad AB = OB - OA = \underset{\sim}{b} - \underset{\sim}{a}$ $AC : CB = 3 : 1, \quad \Rightarrow AC = \frac{3}{4}AB = \frac{3}{4}\underset{\sim}{b} - \frac{3}{4}\underset{\sim}{a}$ $DT = \mu DC = \mu(DA + AC) = \mu\left[\frac{1}{2}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b} - \frac{3}{4}\underset{\sim}{a}\right] = \frac{3}{4}\mu\underset{\sim}{b} - \frac{1}{4}\mu\underset{\sim}{a}$	B1–vector diagram B1 –for AC B1 –for DT

$$OT = \lambda OB = \lambda \underset{\sim}{\boldsymbol{b}}$$

$$OT = OD + DT$$

$$\lambda \underset{\sim}{\boldsymbol{b}} = \frac{1}{2}\underset{\sim}{\boldsymbol{a}} + \frac{3}{4}\mu \underset{\sim}{\boldsymbol{b}} - \frac{1}{4}\mu \underset{\sim}{\boldsymbol{a}}$$

Comparing coefficients of $\underset{\sim}{\boldsymbol{a}}$ gives:

$$0 = \frac{1}{2} - \frac{1}{4}\mu, \qquad \Rightarrow \mu = 2$$

B1 –for μ

Comparing coefficients of $\underset{\sim}{\boldsymbol{b}}$ gives:

$$\lambda = \frac{3}{4}\mu = \frac{3}{4} \times 2 = \frac{3}{2}, \qquad \Rightarrow OT = \lambda \underset{\sim}{\boldsymbol{b}} = \frac{3}{2}\underset{\sim}{\boldsymbol{b}}$$

B1 –for OT

05

5

$$\int x\cos^2 x\,dx = \int x\left(\frac{1+\cos 2x}{2}\right)dx$$

$$= \frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos 2x\,dx$$

B1

Sign	Differentiation	Integration
$+$	x	$\cos 2x$
$-$	1	$\frac{1}{2}\sin 2x$
$+$	0	$-\frac{1}{4}\cos 2x$

B1 **B1** **B1**

$$\int x\cos^2 x\,dx = \frac{1}{2}x^2 + \frac{1}{2}\left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right] + c$$

M1 M1– substitution & simplification

$$= \frac{1}{2}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + c$$

A1

05

6

$$y = ax^2 + bx + c, \qquad \Rightarrow \frac{dy}{dx} = 2ax + b$$

B1 – for dy/dx

At point (2, 4),

$$y = x + a, \qquad \Rightarrow 4 = 2 + a, \qquad \Rightarrow a = 2$$

$$\text{gradient}, \quad \frac{dy}{dx} = 2 \times 2 \times 2 + b = 1, \qquad \Rightarrow b = -7$$

M1–solving
A1–for a
A1 –for b

$$y = ax^2 + bx + c, \qquad \Rightarrow 4 = 2(2)^2 + (-7)(2) + c$$

$$\Rightarrow 4 = 8 - 14 + c, \qquad \Rightarrow c = 10$$

$$\therefore a = 2, \qquad b = -7, \qquad c = 10$$

A1 –for c

05

7

$$\text{Volume} = \pi\int_3^4 y^2\,dx = \pi\int_3^4 (x-2)^{-2}\,dx = \pi\left[\frac{(x-2)^{-1}}{-1}\right]_3^4$$

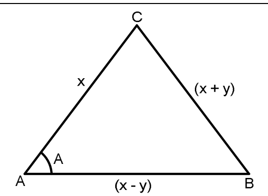
M1 M1

$$= \pi\left[\frac{1}{2-x}\right]_3^4 = \pi\left(-\frac{1}{2} + 1\right) = \frac{1}{2}\pi \text{ cubic units}$$

M1 M1 A1

05

8



B1

By cosine rule,

$$(x+y)^2 = x^2 + (x-y)^2 - 2x(x-y)\cos A$$

$$x^2 + 2xy + y^2 = x^2 + x^2 - 2xy + y^2 - 2x(x-y)\cos A$$

M1 – substitution

$$4xy - x^2 = -2x(x-y)\cos A$$

$$x - 4y = 2(x-y)\cos A$$

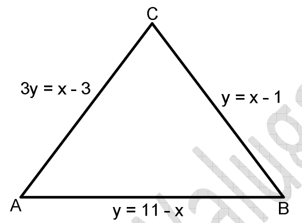
M1 M1– simplification

$$\cos A = \frac{x-4y}{2(x-y)}$$

A1

05

9



B1

At point A,

$$3(x-11) = x - 3, \qquad \Rightarrow x = 15$$

$$\text{when,} \quad x = 15, \qquad y = 15 - 11 = 4, \qquad \Rightarrow A(15, 4)$$

At point B,

$$11 - x = x - 1, \qquad \Rightarrow x = 6$$

$$\text{when,} \quad x = 6, \qquad y = 6 - 1 = 5, \qquad \Rightarrow B(6, 5)$$

At point C,

$$3(x-1) = x - 3, \qquad \Rightarrow x = 0$$

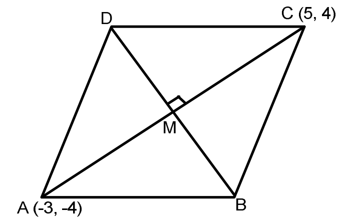
$$\text{when,} \quad x = 0, \qquad y = 0 - 1 = -1, \qquad \Rightarrow C(0, -1)$$

M1 B1

$$\text{centroid} = \left(\frac{15+6+0}{3}, \frac{4+5-1}{3}\right) = \left(7, \frac{8}{3}\right)$$

M1 A1

(b).



$$\text{Gradient of AC} = \frac{-4-4}{-3-5} = 1, \qquad \Rightarrow \text{Gradient of BD} = -1$$

B1 –gradient AC

Midpoint of AC, $M\left(\frac{-3+5}{2},\frac{-4+4}{2}\right)=(1,0)$

The equation of line BD is given by:

$$\frac{y-0}{x-1}=-1, \quad \Rightarrow y=-x+1$$ **B1** –equation BD

The equation of line BC is given by:

$$\frac{y-4}{x-5}=2, \quad \Rightarrow y=2x-6$$ **B1** –equation BC

At point B,

$$-x+1=2x-6, \quad \Rightarrow x=\frac{7}{3}$$

$$x=\frac{7}{3}, \quad y=-\frac{7}{3}+1=-\frac{4}{3}, \quad \Rightarrow B\left(\frac{7}{3},-\frac{4}{3}\right)$$ **B1**

$$\text{Midpoint of AC}=\left(\frac{\frac{7}{3}+x}{2},\frac{-\frac{4}{3}+y}{2}\right)=(1,0)$$

$$\frac{7}{3}+x=2, \quad \Rightarrow x=\frac{1}{3}$$

$$-\frac{4}{3}+y=0, \quad \Rightarrow y=\frac{4}{3}$$

$$\Rightarrow D\left(\frac{1}{3},\frac{4}{3}\right)$$ **B1** –for D

The coordinates of B and D are $B\left(\frac{7}{3},-\frac{4}{3}\right)$and $D\left(\frac{1}{3},\frac{4}{3}\right)$.

$$AC=OC-OA=\begin{pmatrix}5\\4\end{pmatrix}-\begin{pmatrix}-3\\-4\end{pmatrix}=\begin{pmatrix}8\\8\end{pmatrix}$$

$$MB=OB-OM=\frac{1}{3}\begin{pmatrix}7\\-4\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}=\frac{1}{3}\begin{pmatrix}4\\-4\end{pmatrix}$$

$$\text{Area}=|AC||MB|=\sqrt{8^2+8^2}\times\frac{1}{3}\sqrt{4^2+4^2}=\frac{64}{3}$$ **M1 A1**

$$\approx 21.33 \text{ sq. units}$$

Alternatively:

A(-3, -4)

B(7/3, -4/3)

C(5, 4)

A(-3, -4)

$$\text{Area}=\left|\left(3\times\frac{4}{3}+\frac{7}{3}\times 4-5\times 4\right)-\left(-4\times\frac{7}{3}-5\times\frac{4}{3}-4\times 3\right)\right|$$

$$\text{Area}=\left|-\frac{20}{3}+28\right|=\frac{64}{3}\approx 21.33 \text{ sq. units}$$

12

10

$$\frac{x^2+6}{(x^2+4)(x^2+9)}\equiv\frac{Ax+B}{(x^2+4)}+\frac{Cx+D}{(x^2+9)}$$ **M1**

$$x^2+6\equiv(Ax+B)(x^2+9)+(Cx+D)(x^2+4)$$ **M1**

Comparing coefficients of;

x^0, $9B+4D=6 \longrightarrow (1a)$

x^1, $9A+4C=0 \longrightarrow (1b)$ **M1**

x^2, $B+D=1 \longrightarrow (1c)$

x^3, $A+C=0 \longrightarrow (1d)$

Equation $(1a)-(1c)$ gives:

$$5B=2, \quad \Rightarrow B=\frac{2}{5}$$ **A1**

From equation $(1c)$;

$$D=1-B=1-\frac{2}{5}=\frac{3}{5}$$ **A1**

Equation $(1b)-(1d)$ gives:

$$5A=0, \quad \Rightarrow A=0$$ **A1**

From equation $(1c)$;

$$C=-A=0$$

$$\frac{x^2+6}{(x^2+4)(x^2+9)}\equiv\frac{2}{5(x^2+4)}+\frac{3}{5(x^2+9)}$$ **B1**

$$\int_0^1\frac{x^2+6}{(x^2+4)(x^2+9)}dx=\frac{2}{5}\int_0^1\frac{1}{(x^2+4)}dx+\frac{3}{5}\int_0^1\frac{1}{(x^2+9)}dx$$ **M1**

$$=\frac{2}{5}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^1+\frac{3}{5}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_0^1$$ **M1**

$$=\frac{2}{5}\left[\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)-0\right]+\frac{3}{5}\left[\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\right)-0\right]$$ **M1**

$$=\frac{1}{5}\tan^{-1}\left(\frac{1}{2}\right)+\frac{1}{5}\tan^{-1}\left(\frac{1}{3}\right)=\frac{1}{5}\left[\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)\right]$$

let, $\alpha=\tan^{-1}\left(\frac{1}{2}\right)$, $\Rightarrow \tan\alpha=\frac{1}{2}$

let, $\beta=\tan^{-1}\left(\frac{1}{2}\right)$, $\Rightarrow \tan\beta=\frac{1}{3}$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\left(\frac{1}{2}+\frac{1}{3}\right)/\left(1-\frac{1}{2}\times\frac{1}{3}\right)=\frac{5}{6}\div\frac{5}{6}$$

$$=1$$

$$\Rightarrow(\alpha+\beta)=\tan^{-1}1=\frac{\pi}{4}$$

$$\int_0^1\frac{x^2+6}{(x^2+4)(x^2+9)}dx=\frac{1}{5}\left[\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)\right]$$ **M1**

$$=\frac{1}{5}(\alpha+\beta)=\frac{1}{5}\times\frac{\pi}{4}=\frac{\pi}{20}$$ **B1**

12

11 (a).

Let, $f(x)=e^{-x}\sin x$, $\Rightarrow f(0)=e^0\sin 0=0$

$$f'(x)=e^{-x}\cos x-e^{-x}\sin x=e^{-x}(\cos x-\sin x),$$

$$\Rightarrow f'(0)=1$$ **B1**

$$f''(x)=e^{-x}(-\sin x-\cos x)-e^{-x}(\cos x-\sin x)$$

$$=-2e^{-x}\cos x, \quad \Rightarrow f''(0)=-2$$ **B1**

$$f'''(x)=2e^{-x}\sin x+2e^{-x}\cos x=2e^{-x}(\sin x+\cos x),$$

$$\Rightarrow f'''(0)=2$$

By Maclaurin's theorem,

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$$
$$= 0 + x \times 1 + \frac{x^2}{2!} \times (-2) + \frac{x^3}{3!} \times 2 + \cdots$$

M1

$$\therefore e^x \sin x = x - x^2 + \frac{1}{3}x^3 + \cdots$$

A1

For the hence part,

$$e^{-\frac{\pi}{3}} \sin\frac{\pi}{3} = \frac{\pi}{3} - \left(\frac{\pi}{3}\right)^2 + \frac{2}{3}\left(\frac{\pi}{3}\right)^3 \approx 0.3334 \text{ (4 s.f)}$$

M1 A1

(b).

$$y = x^3 + 8$$

when, $y = 0$, $0 = x^3 + 8$, $x = -2$, $\Rightarrow A(-2, 0)$

when, $x = 0$, $y = 0 + 8 = 8$, $\Rightarrow B(0, 8)$

B1 –for A & B

The equation of line AB is given by:

$$\frac{y-8}{x-0} = \frac{0-8}{-2-0}, \quad \Rightarrow y = 4x + 8$$

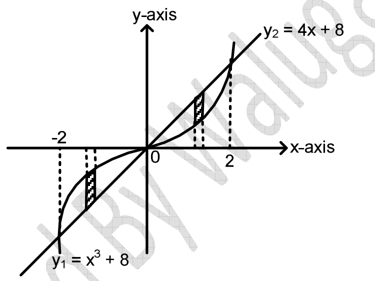
When the line AB meets the curve,

$$4x + 8 = x^3 + 8, \quad \Rightarrow x(4 - x^2) = 0$$
$$x = 0, \quad \text{or,} \quad x = \pm 2$$

when, $x = 2$, $y = 8 + 8 = 16$, $\Rightarrow C(2, 16)$

B1

(ii).



B1

$$\text{Area} = \int_{-2}^{0}(y_1 - y_2)\,dx + \left|\int_0^2 (y_2 - y_1)\,dx\right|$$
$$\int_{-2}^{0}(y_1 - y_2)\,dx = \int_{-2}^{0}(x^3 - 4x)\,dx = \left[\frac{1}{4}x^4 - 2x^2\right]_{-2}^{0}$$
$$= 0 - (4 - 8) = 4$$
$$\int_0^2 (y_2 - y_1)\,dx = \int_0^2 (4x - x^3)\,dx = \left[2x^2 - \frac{1}{4}x^4\right]_0^2$$

M1

$$= (4 - 8) - 0 = -4$$

A1

$$\text{Area} = \int_{-2}^{0}(y_1 - y_2)\,dx + \left|\int_0^2 (y_2 - y_1)\,dx\right| = 4 + |-4|$$
$$= 8 \text{ sq. units}$$

12

12 (a).

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

$$PQ = OQ - OP = \begin{pmatrix}1\\2\\0\end{pmatrix} - \begin{pmatrix}4\\-3\\5\end{pmatrix} = \begin{pmatrix}-3\\5\\5\end{pmatrix}$$

B1 –for PQ

$$PQ : PR = 2:1, \quad \Rightarrow \frac{PQ}{PR} = \frac{2}{1}, \quad \Rightarrow PR = \frac{1}{2}PQ$$

B1 –for PR

$$OR = OP + \frac{1}{2}PQ = \begin{pmatrix}4\\-3\\5\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-3\\5\\5\end{pmatrix} = \begin{pmatrix}2.5\\-0.5\\7.5\end{pmatrix}$$

B1 –for OR

The coordinates are $R(2.5, -0.5, 7.5)$.

B1 –for coordinate R

(b). For perpendicular vectors,

$$\begin{pmatrix}5\\-\lambda\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix} = 0$$

M1 M1– dotting and equating to zero.

$$10 - 3\lambda - 4 = 0, \quad \Rightarrow \lambda = 2$$

B1 –for λ

(c).

Normal vector, $\underset{\sim}{n} = \begin{pmatrix}4\\-1\\1\end{pmatrix}$

B1

Position vector, $\underset{\sim}{a} = \begin{pmatrix}1\\-2\\2\end{pmatrix}$

B1

$$\underset{\sim}{r}.\underset{\sim}{n} = \underset{\sim}{n}.\underset{\sim}{a}$$
$$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}4\\-1\\1\end{pmatrix} = \begin{pmatrix}4\\-1\\1\end{pmatrix}.\begin{pmatrix}1\\-2\\2\end{pmatrix}$$

B1

$$4x - y + z = 4 + 2 + 2$$

M1

$$4x - y + z = 8$$

A1

12

13 (i).

From, $x = \frac{1+t}{1-t}$, $x - tx = 1 + t$, $\Rightarrow t = \frac{x-1}{x+1}$

B1 –for t

$$t^2 = \frac{x^2 - 2x + 1}{x^2 + 2x + 1}$$
$$y = \frac{2t^2}{1-t} = \frac{2\left(\frac{x-1}{x+1}\right)^2}{\left(1 - \frac{x-1}{x+1}\right)} = \frac{2\left(\frac{x-1}{x+1}\right)^2}{\left(\frac{2}{x+1}\right)} = \frac{(x-1)^2}{(x+1)}$$

M1 – substitution

$$\Rightarrow y = \frac{(x-1)^2}{(x+1)}$$

A1

(ii).

$$\frac{dy}{dx} = \frac{(x+1) \times 2(x-1) - (x-1)^2}{(x+1)^2}$$

For turning points, $\frac{dy}{dx} = 0$

$$\frac{2(x+1)(x-1) - (x-1)^2}{(x+1)^2} = 0$$

M1

$$(x-1)[2(x+1) - (x-1)] = 0$$
$$(x-1)(x+3) = 0$$

$$x = 1, \quad \text{or,} \quad x = -3$$

$$\text{when,} \quad x = 1, \quad y = \frac{(1-1)^2}{(1+1)} = 0$$

$$\text{when,} \quad x = -3, \quad y = \frac{(-3-1)^2}{(-3+1)} = -8$$

A1

The turning points are: $(1, 0)$and $(-3, -8)$.

B1 –turning points

x	L	-3	R		L	1	R
Sign of $\frac{dy}{dx}$	+	0	−		−	0	+
Nature		Max				Min	

(iii).

$$y = \frac{(x-1)^2}{(x+1)} = \frac{x^2 - 2x + 1}{x+1}$$

By synthetic method

	1	−2	1
$x = -1$		−1	3
	1	−3	4

$$y = x - 3 + \frac{4}{x+1}, \qquad \Rightarrow y = x - 3 \text{ is the slanting asymptote}$$

B1 –slanting asymptote

Vertical asymptote

$$as\ y \to \infty, (x+1) \to 0$$

$$\Rightarrow x = -1 \text{ is the vetical asymptote}$$

B1 –vertical asymptote

Intercepts

$$y = \frac{(x-1)^2}{(x+1)}$$

$$\text{when,} \quad x = 0, \quad y = 1$$

$$\text{when,} \quad y = 0, \quad (x-1)^2 = 0, \quad \Rightarrow x = 1$$

The intercepts are $(0, 1)$and $(1, 0)$.

B1 – intercepts

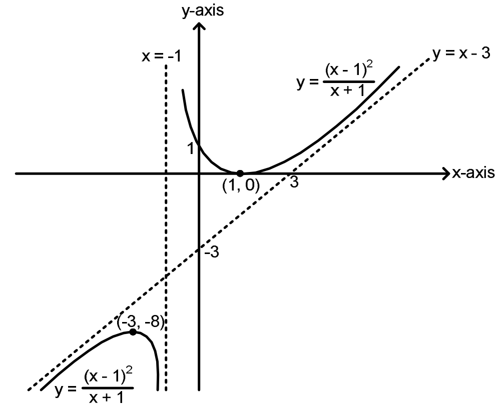
(iv). The Critical values include: $x = -1, x = 1$.

Region where the curve lies:

	$x < -1$	$-1 < x < 1$	$x > 1$
$(x-1)^2$	+	+	+
$(x+1)$	−	+	+
y	−	+	+

B1

Sketch of the curve



B1 B1

12

14 (a).

$$6\sin x - 3\cos x \equiv R\sin(x - \alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$$

B1

By comparison,

$$R\cos\alpha = 6 \ \to (1a), \qquad R\sin\alpha = 3 \ \to (1b)$$

$(1b) \div (1a)$ gives:

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{6}, \qquad \Rightarrow \tan\alpha = 0.5, \qquad \Rightarrow \alpha = 26.57°$$

B1 –for α

$$R = \sqrt{3^2 + 6^2} = \sqrt{45}$$

$$\Rightarrow 6\sin x - 3\cos x \equiv \sqrt{45}\sin(x - 26.57°)$$

B1 –for R

Maximum value:

$$\{6\sin x - 3\cos x\}_{max} = \sqrt{45} \times 1 = \sqrt{45} \approx 6.708$$

B1

(b).

$$L.H.S = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \frac{1 + \tan 11°}{1 - \tan 11°} = \frac{\tan 45° + \tan 11°}{\tan 45° - \tan 11°}$$

B1

$$= \tan(45 + 11)° = \tan 56°$$

B1

(c).

$$L.H.S = \sin B + \sin C - \sin A$$

$$= \left[2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right] - 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$

B1

For angles of a triangle, A, B, C,

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90 - \frac{A}{2}\right) = \cos\left(\frac{A}{2}\right)$$

B1

$$\cos\left(\frac{B+C}{2}\right) = \cos\left(90 - \frac{A}{2}\right) = \sin\left(\frac{A}{2}\right)$$

B1

$$\Rightarrow L.H.S = \left[2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)\right] - 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$

$$= 2\cos\left(\frac{A}{2}\right)\left[\cos\left(\frac{B-C}{2}\right) - \sin\left(\frac{A}{2}\right)\right]$$

B1

$$= 2\cos\left(\frac{A}{2}\right)\left[\cos\left(\frac{B-C}{2}\right) - \cos\left(\frac{B+C}{2}\right)\right]$$

B1

	$= 2\cos\left(\frac{A}{2}\right)\left[-2\sin\left(\frac{B}{2}\right)\sin\left(-\frac{C}{2}\right)\right]$	B1
	$= 4\cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$	
		12
15	(a).	
	$(2+5i)^2 + 5\frac{(7+2i)}{3-4i} - i(4-6i)$	B1
	$= 4 + 20i - 25 + \frac{(35+10i)(3+4i)}{9+16} - 4i - 6$	B1
	$= 16i - 27 + \frac{105 + 140i + 30i - 40}{25}$	B1
	$= \frac{(400i - 675) + (65 + 170i)}{25}$	B1
	$= \frac{570i - 610}{25} = \frac{114i}{5} - \frac{122}{5} = 22.8i - 24.4$	B1
	(b).	
	$\frac{z-1}{z-i} = \frac{(x-1)+yi}{x+(y-1)i} = \frac{\{(x-1)+yi\}\times\{x-(y-1)i\}}{\{x+(y-1)i\}\times\{x-(y-1)i\}}$	B1
	$= \frac{(x-1)x - (x-1)(y-1)i + xyi + y(y-1)}{x^2+(y-1)^2}$	
	$= \frac{x^2 - x - (xy - x - y + 1)i + xyi + y^2 - y}{x^2+(y-1)^2}$	
	$= \frac{(x^2+y^2-x-y) - (-x-y+1)i}{x^2+(y-1)^2}$	B1
	real part $= \frac{(x^2+y^2-x-y)}{x^2+(y-1)^2}$	
	imaginary part $= \frac{-(-x-y+1)}{x^2+(y-1)^2}$	
	$Arg\left(\frac{z-1}{z-i}\right) = \tan^{-1}\left(\frac{\text{imaginary part}}{\text{real part}}\right) = \frac{\pi}{3}$	
	$\tan^{-1}\left(\frac{x+y-1}{x^2+y^2-x-y}\right) = \frac{\pi}{3}$	M1
	$\frac{x+y-1}{x^2+y^2-x-y} = \tan\frac{\pi}{3} = \sqrt{3}$	
	$x + y - 1 = \sqrt{3}(x^2+y^2-x-y)$	
	$x^2\sqrt{3} + y^2\sqrt{3} - x(1+\sqrt{3}) - y(1+\sqrt{3}) + 1 = 0$	A1
	The locus is a circle.	
	By comparison with the general equation: $x^2+y^2+2gx+2fy+c=0$	
	$2g = -\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) = -\left(\frac{3+\sqrt{3}}{3}\right), \quad \Rightarrow g = -\left(\frac{3+\sqrt{3}}{6}\right)$	B1
	$f = g = -\left(\frac{3+\sqrt{3}}{6}\right) \approx -0.7887, \quad c = \frac{1}{\sqrt{3}}$	

	centre $= (-g, -f) = \left(\frac{3+\sqrt{3}}{6}, \frac{3+\sqrt{3}}{6}\right)$	M1 A1
	radius $= \sqrt{g^2+f^2-c} = \sqrt{\left(\frac{3+\sqrt{3}}{6}\right)^2 + \left(\frac{3+\sqrt{3}}{6}\right)^2 - \frac{1}{\sqrt{3}}}$	
	$= 0.8165$ units	
		12
16	(a).	
	$x^2\frac{dy}{dx} = x^2 + xy + y^2$	
	but, $\quad y = ux, \quad \Rightarrow \frac{dy}{dx} = \left(u + x\frac{du}{dx}\right)$	B1
	Substituting for y and $\frac{dy}{dx}$ gives:	
	$x^2\left(u + x\frac{du}{dx}\right) = x^2 + ux^2 + u^2x^2$	M1
	$ux^2 + x^3\frac{du}{dx} = (1+u+u^2)x^2$	
	$u + x\frac{du}{dx} = 1 + u + u^2$	
	$x\frac{du}{dx} = 1 + u^2$	M1
	$\int \frac{du}{1+u^2} = \int \frac{1}{x}dx$	
	$\tan^{-1}u = \ln x + c$	
	$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$	A1
	(b). Let h be the depth of the opening below the surface of the liquid at any time, t. Let h_0 be the initialdepth of the opening below the surface of the liquid when the tank is full.	
	$\frac{dh}{dt} \propto \sqrt{h}$	B1
	$\frac{dh}{dt} = -kh^{\frac{1}{2}}$	
	$\int h^{-\frac{1}{2}}dh = -\int k\,dt$	B1
	$2\sqrt{h} = -kt + c$	
	When $t = 0, h = h_0$	
	$2\sqrt{h_0} = c$	B1
	$2\sqrt{h} = -kt + 2\sqrt{h_0}$	
	When $t = 1, h = h_0 - 20$	
	$2\sqrt{h_0 - 20} = -k + 2\sqrt{h_0}$	B1
	$-k = 2\sqrt{h_0-20} - 2\sqrt{h_0}$	
	$2\sqrt{h} = 2t\left(\sqrt{h_0-20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$	
	When $t = 2, h = h_0 - 20 - 19 = h_0 - 39$	
	$2\sqrt{h_0-39} = 4\left(\sqrt{h_0-20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$	M1
	$\sqrt{h_0-39} = 2\sqrt{h_0-20} - \sqrt{h_0}$	

	$h_0 - 39 = 4(h_0 - 20) - 4\sqrt{{h_0}^2 - 20h_0} + h_0$	M1
	$4\sqrt{{h_0}^2 - 20h_0} = 4h_0 - 41$	M1
	$16{h_0}^2 - 320h_0 = 16{h_0}^2 - 328h_0 + 1681$	
	$8h_0 = 1681$	
	$h_0 = 210.125\text{cm}$	A1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Sept 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 2 MOCK SET 6 2018
Time: 3 Hours

NAME: ____________________ **COMB:** ________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn 1: Two events A and B are such that $P(A) = \frac{8}{15}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{5}$.
Calculate the probabilities that:
(a). Both events occur.
(b). Only one of the two events occurs.
(c). Neither events occurs. [5]

Qn 2: A small block of weight w rests on a smooth plane of inclination θ to the horizontal. Find the value of θ if:
(a). A force of $\frac{1}{2}w$ parallel to the plane is required to keep the block in equilibrium.
(b). A horizontal force of $\frac{1}{3}w$ keeps the block in equilibrium. [5]

Qn 3: Show that the centre of gravity of a uniform solid right circular cone of height h is at a distance $\frac{1}{4}h$ from the base. [5]

Qn 4: Using linear interpolation twice, find the $\sqrt[3]{15}$, correct to 2 decimal places. [5]

Qn 5: Show that, with an initial speed u m s^{-1}, the maximum horizontal distance that a particle can travel from the point of projection is twice the maximum height it can reach above the point of projection. [5]

Qn 6: The discrete random variable X can take values 0, 1, 2 and 3 only. Given $P(X \leq 2) = 0.9$, $P(X \leq 1) = 0.5$ and $E(X) = 1.4$, find:
(a). $P(X = 1)$,
(b). $P(X = 0)$. [5]

Qn 7: The table below shows the length of lectures (to the nearest minute) recorded by a student.

Length of lecture (minutes)	50 – 53	54 – 55	56 – 59	60 – 67
Frequency density	5	13	7.5	1.5

Calculate the mean length of time for the lectures attended during the month. [5]

Qn 8: Locate each of the three roots of the equation $x^3 - 3x + 1 = 0$. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The random variable X has probability density function

$$f(x) = \begin{cases} 3x^k & ; \quad 0 \leq x \leq 1, \\ 0 & ; \quad \text{otherwise} \end{cases}$$

where k is a positive integer. Find:
(a). The value of k,
(b). The mean of X,
(c). The value of m such that $P(X \leq m) = 0.5$. [12]

Question 10:
The speeds of cars passing a certain point on a motorway can be taken to be normally distributed. Observations show that of cars passing the point, 95%are travelling at less than 85 km h^{-1} and 10% are travelling at less than 55 km h^{-1}.
(a). Find the average and standard deviation of the speeds of the cars passing a certain point. [6]
(b). If a random sample of 25 cars is selected, find the:
(i). probability that their mean speed is not more than 70 km h^{-1}. [3]
(ii). 95% confidence interval for the mean speed. [3]

Question 11:
A car has a maximum power of 200 kW. Its maximum speed on a level road is twice its maximum speed up a hill inclined at $\sin^{-1}\left(\frac{1}{15}\right)$ to the horizontal against a resistance to motion of 1600 N in each case. Find the:
(a). mass of the car.
(b). acceleration of the car at the instant when its speed is 30 km h^{-1} on the level with the engine working at full power, assuming the resistance to motion is unchanged. [12]

Question 12:
(a). Prove that, if a particle moving with linear SHM of amplitude A has velocity v when distant x from the centre of its path, then $v = \omega\sqrt{A^2 - x^2}$ where ω is a constant. [6]
(b). A point travelling with linear SHM has sppeds 3 m s^{-1} and 2 m s^{-1} when distant 1 m and 2 m respectively from the centre of oscillation. Calculate the amplitude and the maximum velocity of the point. [6]

Question 13:
Show that the iterative formula for finding the 4th root of a number N is given by:

$$x_{n+1} = \frac{3}{4}\left(x_n + \frac{N}{3x_n^{\ 3}}\right), \qquad \text{for } n = 0, 1, 2, 3, \ldots$$

Draw a flow chart that:
(i). reads the number N and the initial approximation, x_0,
(ii). computes and prints N the its fourth root after 3 iterations and give the root correct to 3d.p.
Perform a dry run for $N = 54$ and $x_0 = 2.5$. [12]

Question 14:
(a). Use the trapezium rule to estimate the area of 5^{2x} between the x-axis, $x = 0$ and $x = 1$, using 5 sub-intervals. Give your answer correct to 3 decimal places.
(b). Find the exact value of $\int_0^1 5^{2x}\, dx$.
(c). Determine the percentage error in the two calculations in (a) and (b) above. [12]

Question 15:

A uniform ladder of length $2l$ and weight w rests in a vertical plane with one end against a rough vertical wall and the other against a rough horizontal surface, the angles of friction at each end being $\tan^{-1}\left(\frac{1}{3}\right)$ and $\tan^{-1}\left(\frac{1}{2}\right)$ respectively.

(a). If the ladder is in limiting equilibrium at either end, find θ, the angle of inclination of the ladder to the horizontal. [6]

(b). A man of weight 10 times that of the ladder begins to ascend it, how far will he climb before the ladder slips? [6]

Question 16:

The cumulative distribution of the ages (in years) of the employees of a company is given in the table below:

Age	<15	<20	<30	<40	<50	<60	<65	<100
Cumulative frequency	0	17	39	69	87	92	98	98

(a). Find the:

(i). Mean and median age.

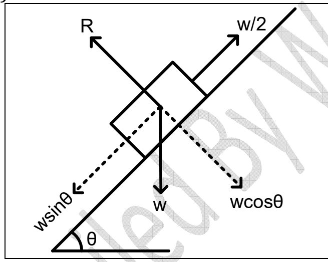
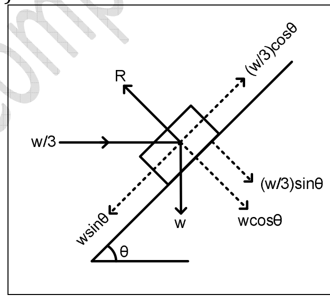
(ii). Middle 70% age range.

(b). Represent the above information on a histogram and use it to estimate the modal age. [12]

END

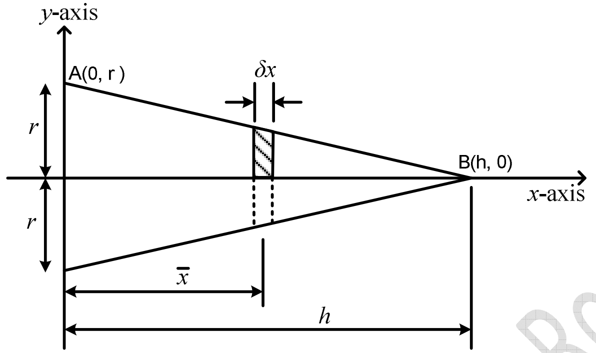
A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

MARKING GUIDE

SNo.	Working	Marks
1	(a). $P(\text{both occur}) = P(A\cap B) = P(B).P(A/B) = \frac{1}{3}\times\frac{1}{5} = \frac{1}{15}$ ≈ 0.0667	A1
	(b). $P(\text{only one occurs}) = P(A\cup B) - P(A\cap B)$ but, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $= \frac{8}{15} + \frac{1}{3} - \frac{1}{15} = \frac{4}{5} = 0.8$ $P(\text{only one occurs}) = P(A\cup B) - P(A\cap B)$ $= \frac{4}{5} - \frac{1}{15} = \frac{11}{15} \approx 0.7333$ **Alternatively:** $P(\text{only one occurs}) = P(A\cap B') + P(A'\cap B)$	M1 A1
	(c). $P(\text{neither event occurs}) = P(A'\cap B') = 1 - P(A\cup B)$ $= 1 - \frac{4}{5} = \frac{1}{5} = 0.2$	M1 A1
		05
2	(a).  For equilibrium along the plane, $w\sin\theta = \frac{1}{2}w$ $\sin\theta = 0.5$ $\theta = 30°$	M1 A1
	(b).  For equilibrium along the plane, $w\sin\theta = \frac{1}{3}w\cos\theta$ $\tan\theta = \frac{1}{3}$ $\theta = 18.435°$	B1-force diagram M1 A1

05

3 Consider a cone subdividing into small discs of thickness, δx as shown below.



B1

The equation of line AB is given by:

$$\text{Gradient} = \frac{y-0}{x-h} = \frac{r-0}{0-h}, \qquad \Rightarrow y = \frac{r}{h}(h-x)$$

Let σ be the weight per unit volume.

$$\bar{x}\sigma\int_0^h \pi y^2\,dx = w\bar{x} = \sigma\int_0^h \pi y^2 x\,dx$$

$$\bar{x}\sigma\int_0^h \pi\times\frac{r^2}{h^2}(h-x)^2\,dx = \sigma\int_0^h \pi\times\frac{r^2}{h^2}(h-x)^2 x\,dx$$

$$\bar{x}\int_0^h (h-x)^2\,dx = \int_0^h (h-x)^2 x\,dx$$

$$\bar{x}\int_0^h (h^2-2hx+x^2)\,dx = \int_0^h (h^2x-2hx^2+x^3)\,dx$$ **M1**

$$\bar{x}\left[h^2x-hx^2+\frac{1}{3}x^3\right]_0^h = \left[\frac{1}{2}h^2x^2-\frac{2}{3}hx^3+\frac{1}{4}x^4\right]_0^h$$ **M1**

$$\bar{x}\left(h^3-h^3+\frac{1}{3}h^3\right)-0 = \left(\frac{1}{2}h^4-\frac{2}{3}h^4+\frac{1}{4}h^4\right)-0$$ **M1**

$$\frac{1}{3}h^3\,\bar{x} = \frac{1}{12}h^4$$

$$\bar{x} = \frac{1}{4}h, \qquad \text{hence proved}$$ **B1**

Alternatively:

Consider a cone subdividing into small discs of radius, a, and thickness, δx as shown below.

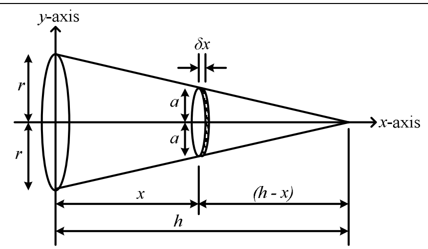


Fig. 1

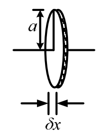


Fig. 2

By using similarity of figures,

$$\frac{a}{r} = \frac{h-x}{h}, \qquad \Rightarrow a = \left(\frac{h-x}{h}\right)r$$

Let σ be the weight per unit volume.

weight of the elementary disc, $\delta w = \sigma\delta V = \sigma\pi a^2\delta x$

$$= \sigma\pi\left(\frac{h-x}{h}\right)^2 r^2\delta x$$

weight of the cone, $w = \sigma V = \frac{1}{3}\sigma\pi r^2 h$

Moment of the whole cone is equal to the sum of moments of the discs about the y-axis:

$$w\bar{x} = \int_0^h \sigma\pi\left(\frac{h-x}{h}\right)^2 r^2 x\,dx$$

$$\frac{1}{3}\sigma\pi r^2 h\bar{x} = \frac{\sigma\pi r^2}{h^2}\int_0^h (h^2x-2hx^2+x^3)\,dx$$

$$\frac{1}{3}h^3\,\bar{x} = \left[\frac{1}{2}h^2x^2-\frac{2}{3}hx^3+\frac{1}{4}x^4\right]_0^h$$

$$\frac{1}{3}h^3\,\bar{x} = \left(\frac{1}{2}h^4-\frac{2}{3}h^4+\frac{1}{4}h^4\right)-0$$

$$\frac{1}{3}h^3\,\bar{x} = \frac{1}{12}h^4$$

$$\bar{x} = \frac{1}{4}h, \qquad \text{hence proved}$$

05

4

$$\text{let } x = \sqrt[3]{15}$$

$$x^3 = 15$$

$$x^3-15 = 0$$

$$f(x) = x^3-15$$

$$f(2.4) = (2.4)^3-15 = -1.176$$

$$f(2.5) = (2.5)^3-15 = 0.625$$

There is a root between 2.4 and 2.5. **B1**-both f(2.4) and f(2.5)

x	2.4	x_1	2.5
$f(x)$	-1.176	0	0.625

$$\frac{x_1 - 2.4}{2.5 - 2.4} = \frac{0-(-1.176)}{0.625-(-1.176)}$$ **M1**

$$x_1 = 2.4 + \frac{1.176}{1.801} \times 0.1 = 2.4653$$

$$f(2.4653) = (2.4653)^3 - 15 = -0.0166$$ **B1**

x	2.4653	x_2	2.5
$f(x)$	-0.0166	0	0.625

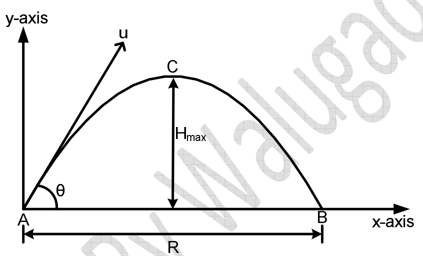
$$\frac{x_2 - 2.4653}{2.5 - 2.4653} = \frac{0-(-0.0166)}{0.625-(-0.0166)}$$ **M1**

$$x_2 = 2.4653 + \frac{0.0166}{0.6416} \times 0.0347 = 2.4662$$

$\therefore$ root $= -2.47$ (2 d.p) **A1**

05

5



Using $s = ut - \frac{1}{2}gt^2$ for vertical motion of ACB,

$$0 = ut\sin\theta - \frac{1}{2}gt^2$$

$$t = \frac{2u\sin\theta}{g}$$

For horizontal motion AB,

$$R = ut\cos\theta = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$$ **M1**

for R_{max}, $\sin 2\theta = 1$, $\Rightarrow \theta = 45°$

$$\therefore R_{max} = \frac{u^2}{g} \longrightarrow (1)$$ **M1**

Using $v^2 = u^2 - 2gs$ for vertical motion upto maximum height,

$$0 = (u\sin\theta)^2 - 2gH_{max}$$

$$H_{max} = \frac{u^2\sin^2\theta}{2g}$$

But $\theta = 45°$,

$$H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin^2 45°}{2g} = \frac{0.5u^2}{2g} = \frac{u^2}{4g} \longrightarrow (2)$$ **M1**

$(1) \div (2)$ gives,

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$$\frac{R_{max}}{H_{max}} = \frac{u^2}{g} \div \frac{u^2}{4g}$$ **M1**

$$\frac{R_{max}}{H_{max}} = 4$$

$$R_{max} = 4H_{max}$$ **B1**

05

6

x	$P(X \le x)$	$P(X = x)$	$xP(X = x)$
0	a	a	0
1	0.5	$0.5 - a$	$0.5 - a$
2	0.9	$0.9 - 0.5 = 0.4$	0.8
3	1	$1 - 0.9 = 0.1$	0.3
Sums		1	$1.6 - a$

B1-P(X=x)
B1-xP(X=x)

$E(X) = \sum xP(X = x) = 1.4,\quad \Rightarrow 1.6 - a = 1.4,\quad \Rightarrow a = 0.2$ **M1**

(a).

$$P(X = 0) = a = 0.2$$ **A1**

(b).

$$P(X = 1) = 0.5 - a = 0.5 - 0.2 = 0.3$$ **A1**

05

7

Length	f/c	c	f	x	fx
50 – 53	5	4	20	51.5	1030
54 – 55	13	2	26	54.5	1417
56 – 59	7.5	4	30	57.5	1725
60 – 67	1.5	8	12	63.5	762
Total			88		4934

B1-for c
B1-for f
B1-for fx

$$\text{Mean length} = \frac{\sum fx}{\sum f} = \frac{4934}{88} = 56.068$$ **M1A1**

05

8

let, $f(x) = x^3 - 3x + 1$

x	-3	-2	-1	0	1	2	3
$f(x)$	-17	-1	3	1	-1	3	19

B1 B1

Since

$f(-2).f(-1) < 0,\quad \Rightarrow -2 < \text{root} < -1$ **B1**
$f(0).f(1) < 0,\quad \Rightarrow 0 < \text{root} < 1$ **B1**
$f(1).f(2) < 0,\quad \Rightarrow 1 < \text{root} < 2$ **B1**

05

9 (a).

$$\int_{all\ x} f(x)\,dx = 1$$

$$\int_0^1 3x^k\,dx = 1$$ **M1**

$$\left[\frac{3x^{k+1}}{k+1}\right]_0^1 = 1$$ **M1**

$$\frac{3\times 1^{(k+1)}}{k+1} - 0 = 1$$ **M1**

$$\frac{3}{k+1} = 1$$ **B1**

$$k+1 = 3$$

$$k = 2$$ **A1**

(b).

$$f(x) = \begin{cases} 3x^2 & ; \quad 0 \le x \le 1, \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$E(X) = \int_{all\ x} x f(x)\,dx = \int_0^1 3x^3\,dx$$ **M1**

$$= \left[\frac{3x^4}{4}\right]_0^1 = \frac{3}{4} - 0 = \frac{3}{4}$$ **M1 A1**

(c).

$$P(X \le m) = 0.5$$

$$\int_0^m 3x^2\,dx = 0.5$$ **M1**

$$\left[\frac{3x^3}{3}\right]_0^m = 0.5$$ **M1**

$$m^3 - 0 = 0.5$$ **M1**

$$m = 0.7937$$ **A1**

12

10 (a).

$$P(X < 85) = 0.95 \quad , \qquad P(Z < z_1) = 0.95$$

$$0.5 + \phi(z_1) = 0.95$$

$$\phi(z_1) = 0.95 - 0.5 = 0.45$$

$$z_1 = 1.645$$

$$\text{but, } z_1 = \frac{85-\mu}{\sigma} = 1.645, \qquad \Rightarrow 85 - \mu = 1.645\sigma \quad \longrightarrow (1)$$ **B1**

$$P(X < 55) = 0.1 \quad , \qquad P(Z < z_1) = 0.1$$

$$0.5 - \phi(z_1) = 0.1$$

$$\phi(z_1) = 0.5 - 0.1 = 0.4$$

$$z_1 = -\phi^{-1}(0.4) = -1.282$$

$$\text{but, } z_1 = \frac{55-\mu}{\sigma} = -1.282, \qquad \Rightarrow 55 - \mu = -1.282\sigma \quad \longrightarrow (2)$$ **B1**

Equation (1) − (2) gives;

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$$30 = 2.927\sigma, \qquad \Rightarrow \sigma = \frac{30}{2.927} = 10.249$$ **M1 A1**

From equation (1)

$$\mu = 85 - 1.645\sigma = 85 - 1.645 \times 10.249 = 68.140$$

(b). (i). $\mu = 68.140, \sigma = 10.249, n = 25$ **M1 A1**

$$P(\overline{X} \le 70) = P\left(Z < \frac{70 - 68.140}{10.249/\sqrt{25}}\right) = P(Z < 0.907)$$ **M1**-z-value

$$= 0.5 + \phi(0.907) = 0.5 + 0.3177$$ **B1**-table value

$$= 0.8177$$ **A1**

(ii). For the 95% confidence interval,

$$\phi(z_{\alpha/2}) = \frac{0.95}{2} = 0.475, \qquad \Rightarrow z_{\alpha/2} = \phi^{-1}(0.475) = 1.96$$

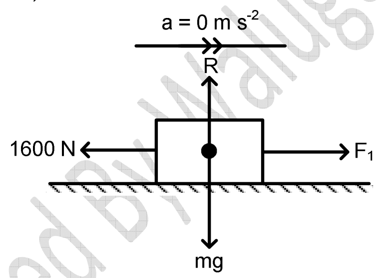
$$\text{Confidence limits} = \overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$= 68.140 \pm 1.96 \times \frac{10.249}{\sqrt{25}} = 68.140 \pm 4.0176$$ **M1 B1**

$$\text{Confidence interval} = [64.1224, 72.1576]$$ **A1**

12

11 (a). On level road,



Let v_1 be the maximum speed on level road.
Resolving parallel to the plane gives:

$$F_1 - 1600 = m \times 0$$ **M1**

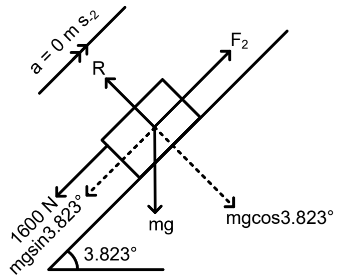
$$F_1 = 1600 \text{ N}$$ **B1**

$$\text{Maximum power,} \qquad P_{\max} = F_1 v_1$$

$$v_1 = \frac{P_{\max}}{F_1} = \frac{200000}{1600} = 125 \text{ m s}^{-1}$$ **M1**

On inclined road,

$$\sin^{-1}\left(\frac{1}{15}\right) = 3.823°$$



B1

$$v_2 = \frac{1}{2}v_1 = \frac{1}{2}\times 125 = 62.5 \text{ m s}^{-1}$$ **M1**

Maximum power, $P_{\max} = F_2 v_2$

$$F_2 = \frac{P_{\max}}{v_2} = \frac{200000}{62.5} = 3200 \text{ N}$$ **M1**

Resolving parallel to the plane gives:

$$F_2 - (1600 + mg\sin 3.823^\circ) = m \times 0$$ **M1 M1**

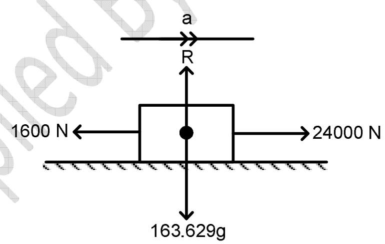
$$3200 - 1600 - 9.8m\sin 3.823^\circ = 0$$
$$1600 = 9.8m\sin 3.823^\circ$$
$$m = \frac{1600}{9.8\sin 3.823^\circ} = 2448.6941 \text{ kg}$$ **A1**

(b).

$$v_3 = 30 \text{ km h}^{-1} = \frac{30\times 1000}{3600} = \frac{25}{3} \text{ m s}^{-1}$$

Maximum power, $P_{\max} = F_3 v_3$

$$F_3 = \frac{P_{\max}}{v_3} = \frac{200000\times 3}{25} = 24000 \text{ N}$$ **M1**



Resolving parallel to the plane gives:

$$24000 - 1600 = 163.629a$$ **M1 M1**

$$22400 = 2448.6941a$$
$$a = 9.1477 \text{ m s}^{-2}$$ **A1**

12

12 (a).

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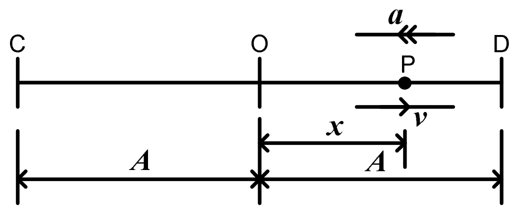


for s. h. m, $a \propto x$, $\Rightarrow a = -\omega^2 x \longrightarrow (1)$

also from calculus, $a = v\dfrac{dv}{dx}$ **B1-both equations for acceleration**

Substituting for $\boldsymbol{a}$ into equation (1) gives; **M1-equating**

$$v\frac{dv}{dx} = -\omega^2 x$$
$$vdv = -\omega^2 xdx$$
$$\int v\,dv = -\omega^2 \int x\,dx$$ **M1**

$$\frac{1}{2}v^2 = -\frac{1}{2}\omega^2 x^2 + c$$ **B1**

But $v = 0$ when $x = A$

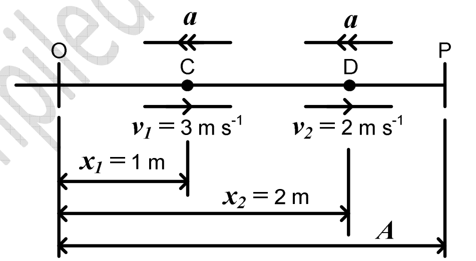
$$0 = -\frac{1}{2}\omega^2 A^2 + c, \quad \Rightarrow c = \frac{1}{2}\omega^2 A^2$$ **M1**

Substituting for $\boldsymbol{c}$ gives;

$$\frac{1}{2}v^2 = -\frac{1}{2}\omega^2 x^2 + \frac{1}{2}\omega^2 A^2$$
$$v^2 = -\omega^2 x^2 + \omega^2 A^2$$
$$v^2 = \omega^2(A^2 - x^2)$$ **B1**

$$v = \sqrt{\omega^2(A^2 - x^2)}$$
$$v = \omega\sqrt{A^2 - x^2}$$

(b).



(i).

For motion OA,

$$v_1{}^2 = \omega^2(A^2 - x_1{}^2)$$ **B1**

$$3^2 = \omega^2(A^2 - 1^2)$$
$$9 = \omega^2(A^2 - 1) \longrightarrow (1)$$

For motion OB, **B1**

$$v_2{}^2 = \omega^2(A^2 - x_2{}^2)$$
$$2^2 = \omega^2(A^2 - 2^2)$$

$$4 = \omega^2(A^2 - 4) \quad \longrightarrow (2)$$

Equation (1) ÷ (2) gives:

$$\frac{9}{4} = \frac{\omega^2(A^2-1)}{\omega^2(A^2-4)}$$ **M1**

$$9(A^2-4) = 4(A^2-1)$$
$$9A^2 - 36 = 4A^2 - 4$$
$$5A^2 = 32$$
$$A^2 = 6.4$$
$$A = 2.5298 \text{ m}$$ **A1**

The amplitude of motion is 2.5298 m.

(ii).

From equation (1),

$$9 = \omega^2(A^2-1)$$
$$9 = \omega^2(6.4-1)$$
$$\omega^2 = \frac{9}{6.4}$$
$$\omega = 1.291 \text{ rad s}^{-1}$$
$$\therefore v_{max} = \omega A = 1.291 \times 2.5298 = 3.266 \text{ m s}^{-1}$$ **M1 A1**

12

13 (i). **Iterative formula**

let, $\quad x = \sqrt[4]{N}, \quad \Longrightarrow x^4 = N, \quad \Longrightarrow x^4 - N = 0$

$$f(x) = x^4 - N, \qquad f'(x) = 4x^3$$ **B1**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0, 1, 2, \ldots$$

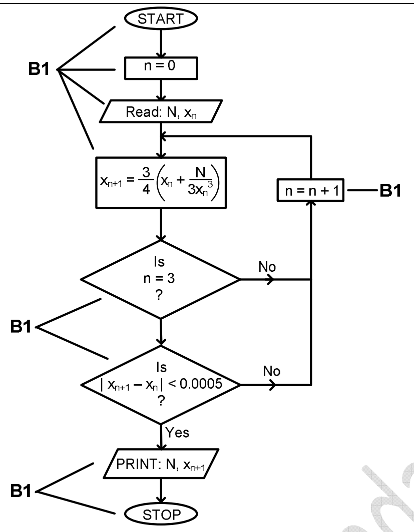
$$x_{n+1} = x_n - \frac{{x_n}^4 - N}{4{x_n}^3}$$ **M1**

$$= \frac{4{x_n}^4 - ({x_n}^4 - N)}{4{x_n}^3} = \frac{3{x_n}^4 + N}{4{x_n}^3}$$ **B1**

$$= \frac{3{x_n}^4}{4{x_n}^3} + \frac{N}{4{x_n}^3} = \frac{3}{4}\left(x_n + \frac{N}{3{x_n}^3}\right)$$

$$x_{n+1} = \frac{3}{4}\left(x_n + \frac{N}{3{x_n}^3}\right), \qquad \text{for } n = 0, 1, 2, 3, \ldots$$ **B1**

(ii). **Flow chart**



* **B1** for loop back (n=n+1) is given only if the arrows from the decision boxes are correct

(iii). **Dry run**

$$N = 54, \qquad \Longrightarrow x_{n+1} = \frac{3}{4}\left(x_n + \frac{54}{3{x_n}^3}\right)$$

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$	
0	2.5	2.739	0.239	**B1**
1	2.739	2.71124	0.02776	**B1**
2	2.71124	2.71081	0.00043	**B1**

$$\Longrightarrow \text{The root, } x_{n+1} = 2.711 \text{ (3 d.p)}$$ **A1**

12

14 (i).

$$y_n = 5^{2x_n}, \qquad h = \frac{1-0}{5} = \frac{1}{5} = 0.2$$ **B1**

n	x_n	y_0, y_5	$y_1, \ldots y_4$
0	0	1	
1	0.2		1.90365
2	0.4		3.62390
3	0.6		6.89865
4	0.8		13.13264
5	1	25	
sums		26	25.55884

B1-for x_n

B1-for all y_n correct

$$\int_0^1 5^{2x}\,dx \approx \frac{1}{2}h[(y_0 + y_4) + 2(y_1 + \cdots + y_3)]$$

$$\approx \frac{1}{2}\times\frac{1}{5}[26 + 2\times 25.55884] = 7.711768$$ **M1**

$$\approx 7.712 \text{ (3 d.p)}$$ **A1 -3 d.p**

(ii).

$$\text{let } y = 5^{2x}$$
$$\ln y = 2x\ln 5$$
$$\frac{1}{y}\frac{dy}{dx} = 2\ln 5$$
$$\frac{dy}{dx} = (2\ln 5)(5^{2x})$$ **M1**

$$\frac{d}{dx}(5^{2x}) = (2\ln 5)(5^{2x}), \quad \Rightarrow \int 5^{2x}\,dx = \frac{5^{2x}}{2\ln 5} + c$$ **B1**

$$\int_0^1 5^{2x}\,dx = \left[\frac{5^{2x}}{2\ln 5}\right]_0^1$$
$$= \frac{25}{2\ln 5} - \frac{1}{2\ln 5} = \frac{12}{\ln 5} \approx 7.456 \text{ (3 d.p)}$$ **M1 A1**

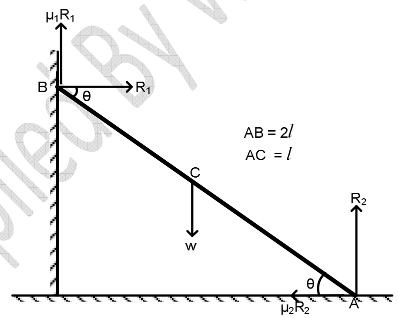
(iii).

$$\text{absolute error} = |7.456 - 7.712| = 0.256$$ **M1- Magnitudes must be seen**

$$\text{percentage error} = \frac{0.256}{7.456}\times 100 = 3.433$$ **M1 A1 No % sign on the final answer**

12

15 (a).



$$\mu_1 = \frac{1}{3}, \quad \mu_2 = \frac{1}{2}$$

Resolving horizontally,

$$R_1 = \mu_2 R_2, \quad \Rightarrow R_1 = \frac{1}{2}R_2, \quad \Rightarrow R_2 = 2R_1$$ **M1**

Resolving vertically,

$$\mu_1 R_1 + R_2 = w, \quad \Rightarrow \frac{1}{3}R_1 + 2R_1 = w, \quad \Rightarrow \frac{7}{3}R_1 = w$$ **M1**

Taking moments about A,

$$\mu_1 R_1 \times 2l\cos\theta + R_1 \times 2l\sin\theta = w \times l\cos\theta$$

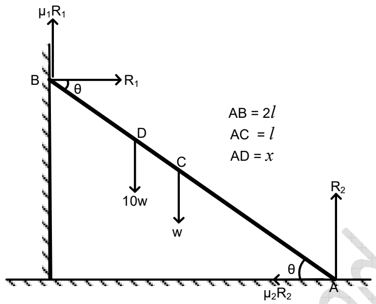
$$\frac{1}{3}R_1 \times 2\cos\theta + R_1 \times 2\sin\theta = \frac{7}{3}R_1 \times \cos\theta$$ **M1 M1**

$$\frac{2}{3}\cos\theta + 2\sin\theta = \frac{7}{3}\cos\theta$$ **B1**

$$2\sin\theta = \frac{5}{3}\cos\theta$$

$$\tan\theta = \frac{5}{6}, \Rightarrow \theta = 39.81°$$ **A1**

(b).



Resolving horizontally,

$$R_1 = \mu_2 R_2, \quad \Rightarrow R_1 = \frac{1}{2}R_2, \quad \Rightarrow R_2 = 2R_1$$ **M1**

Resolving vertically,

$$\mu_1 R_1 + R_2 = w + 10w, \quad \Rightarrow \frac{1}{3}R_1 + 2R_1 = 11w$$ **M1**

$$\frac{7}{3}R_1 = 11w$$

Taking moments about A,

$$\mu_1 R_1 \times 2l\cos\theta + R_1 \times 2l\sin\theta = w \times l\cos\theta + 10w \times x\cos\theta$$ **M1 M1**

$$\frac{2}{3}R_1 l + 2R_1 l\tan\theta = wl + 10wx$$

$$\frac{2}{3}R_1 l + 2R_1 l \times \frac{5}{6} = \frac{7}{33}R_1 l + \frac{70}{33}R_1 x$$ **B1**

$$\frac{2}{3}l + \frac{5}{3}l = \frac{7}{33}l + \frac{70}{33}x$$

$$x = \frac{33}{70}\left(\frac{2}{3} + \frac{5}{3} - \frac{7}{33}\right)l = \frac{33}{70}\times\frac{70}{33}l = l$$

The man will climb up to a distance l along the ladder from A before the ladder slips. **A1**

12

16 (a).

Class	$C.F$	f	x	fx	c	f/c	Class boundaries
<15	0	0	7.5	0	15	0	0 – 15
<20	17	17	17.5	297.5	5	3.4	15 – 20
<30	39	22	25	550	10	2.2	20 – 30
<40	69	30	35	1050	10	3	30 – 40
<50	87	18	45	810	10	1.8	40 – 50
<60	92	5	55	275	10	0.5	50 – 60
<65	98	6	62.5	375	5	1.2	60 – 65
<100	98	0	82.5	0	35	0	65 – 100
Total				3357.5			

B1-for frequency density (f/c)

(i).

$$\text{Mean age} = \frac{\sum fx}{\sum f} = \frac{3357.5}{98} = 34.2602$$ **M1 A1**

$$\text{Median position} = \frac{1}{2}N = \frac{1}{2}\times 98 = 49$$

$$\text{Median age} = L_m + \left(\frac{N/2 - C.F_b}{f_m}\right)c$$

$$= 30 + \left(\frac{49-39}{30}\right)\times 10 = 33.333$$ **M1 A1**

(ii).

$$\text{Middle 70\% age range} = P_{85} - P_{15}$$

$$15^{\text{th}} \text{ percentile position} = \frac{15}{100}N = \frac{15}{100}\times 98 = 14.7$$

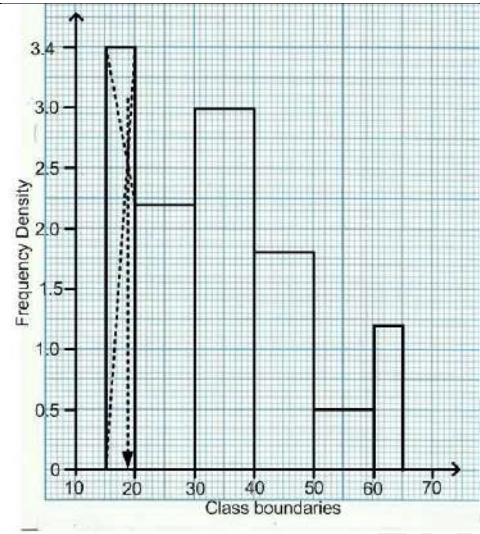
$$P_{15} = 15 + \left(\frac{14.7-0}{17}\right)\times 5 = 19.3235$$ **B1**

$$85^{\text{th}} \text{ percentile position} = \frac{85}{100}N = \frac{85}{100}\times 98 = 83.3$$

$$P_{85} = 40 + \left(\frac{83.3-69}{18}\right)\times 10 = 47.9444$$ **B1**

$$\text{Middle 70\% age range} = 47.9444 - 19.3235 = 28.6209$$ **M1 A1**

(b).



B1-axes correctly labelled with uniform scale

B1-for bars

$\therefore$ Modal age = 19

A1-lines for estimation of mode must be seen on histogram

12

END

P425/1
PURE
MATHEMATICS
PAPER 1
Oct. 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 1 MOCK SET 7 2018

Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section*

Qn 1: Solve the equation $3\cos 4\theta + 7\cos 2\theta = 0$ for $0° \leq \theta \leq 180°$. [5]

Qn 2: Find the solution set for which $\log_2 x - \log_x 4 \leq 1$. [5]

Qn 3: Integrate $\frac{x^3}{\sqrt{1+x^2}}$ with respect to x. [5]

Qn 4: The points P, Q and R have position vectors $2\underset{\sim}{\boldsymbol{a}} - 5\underset{\sim}{\boldsymbol{b}}$, $5\underset{\sim}{\boldsymbol{a}} - \underset{\sim}{\boldsymbol{b}}$ and $11\underset{\sim}{\boldsymbol{a}} + 7\underset{\sim}{\boldsymbol{b}}$ respectively. Show that P, Q and R are collinear and state the ratio $\boldsymbol{PQ}:\boldsymbol{QR}$. [5]

Qn 5: Given that $y = 5^{-2x}\sin 2x$, find $\frac{dy}{dx}$. [5]

Qn 6: Using Maclaurin's theorem, expand $y = x + \ln(1+x)$ as far as the term in x^3. [5]

Qn 7: Determine the equation of the circle with centre at $(1, 5)$ and has a tangent passing through the points $A(-1, 2)$ and $B(0, -2)$. [5]

Qn 8: By eliminating the constants A and B, form a differential equation for which $y = Ae^{3t} + Be^{-2t}$ is a solution. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). Express $\frac{x^2+5x+11}{(x+1)(x^2+4)}$ in partial fractions. [7]

(b). Evaluate: $\int_1^4 \frac{x^2+5x+11}{(x+1)(x^2+4)}\,dx$. [5]

Question 10:

(a). Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x+\alpha)$. Hence find the maximum value of $10\sin x\cos x + 12\cos 2x$. [6]

(b). Prove that in any triangle ABC,

(i). $a = b\cos C + c\cos B$,

(ii). $a\sin\left(\frac{B-C}{2}\right) = (b-c)\cos\frac{A}{2}$. [6]

Question 11:

(a). Solve the simultaneous equations.

$$2a - 3b + c = 10$$
$$a + 4b + 2c + 3 = 0$$
$$5a - 2b - c = 7$$

[6]

(b). Given that the equations $y^3 - 2y + 4 = 0$ and $y^2 + y + c = 0$ have a common root, show that $c^3 + 4c^2 + 14c + 20 = 0$. [6]

Question 12:

(a). Calculate the perpendicular distance from the point $(1, -2, 3)$ from the line with equation $\underset{\sim}{\boldsymbol{r}} = 2\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}} + t\left(2\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}} - 2\underset{\sim}{\boldsymbol{k}}\right)$. [5]

(b). Determine the equation of a plane through the point $(1, -3, 2)$ and contains the vectors $\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 3\underset{\sim}{\boldsymbol{k}}$ and $-\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 2\underset{\sim}{\boldsymbol{k}}$. [7]

Question 13:

(a). Find the turning points to the curve $y = \frac{2x}{(2-x)^2}$. Distinguish between the turning points.

(b). Sketch the graph of the curve $y = \frac{2x}{(2-x)^2}$. [12]

Question 14:

(a). A geometric progression has the first term 10 and sum to infinity of 12.5. How many terms of the progression are needed to make a sum which exceeds 10? [6]

(b). Prove by induction that $\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$. [6]

Question 15:

(a). Show that the curve $y^2-8y=-4x$ represents a parabola. Sketch the parabola and state its focus. [6]

(b). The points $P(at^2,2at)$ and $Q(aT^2,2aT)$ lie on the parabola with equation $y^2=4ax$. Determine the locus of the mid-point of the line segment PQ, for when $tT=2a$. [6]

Question 16:

(a). Solve the equation $x\frac{dy}{dx}+y=xe^{-2x}$. [5]

(b). The rate at which the temperature of a body placed in an oven increases at any instant is proportional to the amount by which the temperature of the oven exceeds the temperature of the body at that instant. The temperature of the oven is 120°C. Given that the temperature of the body rises from 50°C to 80°C in 6 minutes, how long does the temperature of the body take to rise from 90°C to 99°C ? [7]

END

MARKING GUIDE

SNo.	Working	Marks
1	$3\cos4\theta+7\cos2\theta=0$	
	$3(2\cos^2 2\theta-1)+7\cos2\theta=0$	B1-identity
	$6\cos^2 2\theta-3+7\cos2\theta=0$	
	$6\cos^2 2\theta+7\cos2\theta-3=0$	B1-quad eqn
	$\cos2\theta=\frac{-7\pm\sqrt{7^2-4\times6\times(-3)}}{2\times6}$	M1
	$\cos2\theta=\frac{1}{3}$, or, $\cos2\theta=-1.5$	B1
	for $\cos2\theta=\frac{1}{3}$, $2\theta=70.53°, 289.47°$,	
	$\Rightarrow\theta=35.27°, 144.74°$	
	$\cos2\theta=-1.5$, θ is undefined	A1
		05
2	$\log_2 x-\log_x 4\le1$	
	$\log_2 x-2\log_x 2\le1$	
	$\log_2 x-\frac{2}{\log_2 x}\le1$	M1
	Let $y=\log_2 x$	
	$y-\frac{2}{y}\le1$	
	$y-\frac{2}{y}-1\le0$	
	$\frac{y^2-y-2}{y}\le0$	M1-getting LCM
	$\frac{y^2+y-2y-2}{y}\le0$	
	$\frac{y(y+1)-2(y+1)}{y}\le0$	
	$\frac{(y-2)(y+1)}{y}\le0$	
	The Critical values include: $y=-1, y=0, y=2$	B1-all critical values
	Region where the curve lies	

	$x<-1$	$x=-1$	$-1<x<0$	$x=0$	$0<x<2$	$x=2$	$x>2$
$(y+1)$	−	0	+	+	+	+	+
$(y-2)$	−	−	−	−	−	0	+
y	−	−	−	0	+	+	+

M1

	$\begin{array}{|c|c|c|c|c|c|c|c|}\hline \frac{(y-2)(y+1)}{y} & - & 0 & + & \infty & - & 0 & + \\ \hline \end{array}$ The solution set is: $y < -1$ and $0 < y \le 2$ $y < -1,$ and, $0 < y \le 2$ $\log_2 x < -1,$ and, $0 < \log_2 x \le 2$ $x < 2^{-1},$ and, $2^0 < x \le 2^2$ $x < \frac{1}{2},$ and, $1 < x \le 4$	**A1**-both ranges
		05
3	$\int \frac{x^3}{\sqrt{1+x^2}}dx$ let $u = 1 + x^2$, $\frac{du}{dx} = 2x$ $x^2 = u - 1$, $dx = \frac{du}{2x}$ $\int \frac{x^3}{\sqrt{1+x^2}}dx = \int \frac{x^3}{\sqrt{u}}.\frac{du}{2x} = \frac{1}{2}\int \frac{x^2}{\sqrt{u}}du = \frac{1}{2}\int \frac{u-1}{\sqrt{u}}du$ $= \frac{1}{2}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du = \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) + c$ $= \frac{1}{3}u^{\frac{3}{2}} - u^{\frac{1}{2}} + c = \frac{1}{3}(1+x^2)^{\frac{3}{2}} - (1+x^2)^{\frac{1}{2}} + c$	 **M1** **M1** **M1 B1** **A1**
		05
4	$OP = 2\underset{\sim}{\mathbf{a}} - 5\underset{\sim}{\mathbf{b}}$, $OQ = 5\underset{\sim}{\mathbf{a}} - \underset{\sim}{\mathbf{b}}$, $OR = 11\underset{\sim}{\mathbf{a}} + 7\underset{\sim}{\mathbf{b}}$ $PQ = OQ - OP = \left(5\underset{\sim}{\mathbf{a}} - \underset{\sim}{\mathbf{b}}\right) - \left(2\underset{\sim}{\mathbf{a}} - 5\underset{\sim}{\mathbf{b}}\right) = 3\underset{\sim}{\mathbf{a}} + 4\underset{\sim}{\mathbf{b}}$ $QR = OR - OQ = \left(11\underset{\sim}{\mathbf{a}} + 7\underset{\sim}{\mathbf{b}}\right) - \left(5\underset{\sim}{\mathbf{a}} - \underset{\sim}{\mathbf{b}}\right) = 6\underset{\sim}{\mathbf{a}} + 8\underset{\sim}{\mathbf{b}}$ $= 2\left(3\underset{\sim}{\mathbf{a}} + 4\underset{\sim}{\mathbf{b}}\right)$ Since $\boldsymbol{PQ} = 2\boldsymbol{QR}$ and they share a common point P, then P, Q and R are collinear. $\boldsymbol{PQ}:\boldsymbol{QR} = 1:2$	 **B1** **B1** **B1** **M1 A1**
		05
5	let $u = 5^{-2x}$, $v = \sin 2x$ for $u = 5^{-2x}$ $\ln u = -2x \ln 5$ $\frac{1}{u}\frac{du}{dx} = -2\ln 5$ $\frac{du}{dx} = (-2\ln 5)(5^{-2x})$ for $v = \sin 2x$ $\frac{dv}{dx} = 2\cos 2x$	 **M1** **M1**

A-LEVEL MATHEMATICS PAST PAPER WITH MARKING GUIDES 2018

	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = (5^{-2x})(2\cos 2x) + (\sin 2x)(-2\ln 5)(5^{-2x})$ $= 2(5^{-2x})[\cos 2x - (\ln 5)\sin 2x]$	**M1 M1** **A1**-collecting like terms
		05
6	$f(x) = x + \ln(1+x)$, $\Rightarrow f(0) = 0 + \ln 1 = 0$ $f'(x) = 1 + (1+x)^{-1}$, $\Rightarrow f'(0) = 1 + 1 = 2$ $f''(x) = -(1+x)^{-2}$, $\Rightarrow f''(0) = -1 \times 1 = -1$ $f'''(x) = 2(1+x)^{-3}$, $\Rightarrow f'''(0) = 2 \times 1 = 2$ $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$ $x + \ln(1+x) = 0 + x \times 2 + \frac{x^2}{2!} \times (-1) + \frac{x^3}{3!} \times 2 + \cdots$ $= 2x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots$	 **B1** **B1** **B1** **M1** **A1**
		05
7	The equation of the tangent is given by: $\frac{y-2}{x-(-1)} = \frac{-2-2}{0-(-1)}$ $y - 2 = -4(x+1)$ $4x + y + 2 = 0$ Radius $= \left\lvert\frac{4 \times 1 + 5 + 2}{\sqrt{4^2 + 1^2}}\right\rvert = \frac{11}{\sqrt{17}}$ The required equation of the circle is given by: $(x-1)^2 + (y-5)^2 = \left(\frac{11}{\sqrt{17}}\right)^2$ $x^2 - 2x + 1 + y^2 - 10y + 25 = \frac{121}{17}$ $17x^2 + 17y^2 - 34x - 170y + 321 = 0$	 **M1** **B1** **B1**-radius **M1** **A1**
		05
8	$y = Ae^{3t} + Be^{-2t}$ $\frac{dy}{dt} = 3Ae^{3t} - 2Be^{-2t}$ $\frac{d^2y}{dt^2} = 9Ae^{3t} + 4Be^{-2t}$ $\frac{d^2y}{dt^2} = 3Ae^{3t} - 2Be^{-2t} + 6Ae^{3t} + 6Be^{-2t}$ $\frac{d^2y}{dx^2} = \frac{dy}{dx} + 6y$ $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 0$ **Alternatively:** $y = Ae^{3t} + Be^{-2t}$ $ye^{2t} = Ae^{5t} + B$	 **M1** **M1** **M1** **M1** **B1**

$$2ye^{2t} + e^{2t}\frac{dy}{dt} = 5Ae^{5t}$$
$$2ye^{-3t} + e^{-3t}\frac{dy}{dt} = 5A$$
$$\left(2y + \frac{dy}{dt}\right)e^{-3t} = 5A$$
$$\left(2\frac{dy}{dt} + \frac{d^2y}{dt^2}\right)e^{-3t} - 3e^{-3t}\left(2y + \frac{dy}{dt}\right) = 0$$
$$\left(2\frac{dy}{dt} + \frac{d^2y}{dt^2}\right) - 3\left(2y + \frac{dy}{dt}\right) = 0$$
$$2\frac{dy}{dt} + \frac{d^2y}{dt^2} - 6y - 3\frac{dy}{dt} = 0$$
$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 0$$

05

9 (a).

$$\frac{x^2+5x+11}{(x+1)(x^2+4)} \equiv \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+4)}$$ **M1**

$$x^2+5x+11 \equiv A(x^2+4) + (Bx+C)(x+1)$$

Putting $x = -1;\ 1-5+11 = 5A,\ \Rightarrow A = 2$ **A1**

Putting $x = 0;\ 11 = 8 + C,\ \Rightarrow C = 3$ **A1**

Putting $x = 1;\ 4+5+11 = 10+2B+6,\ \Rightarrow B = 2$ **A1**

$$\frac{x^2+5x+11}{(x+1)(x^2+4)} \equiv \frac{2}{(x+1)} + \frac{2x+3}{(x^2+4)}$$ **B1**

(b).

$$\int_1^4 \frac{x^2+5x+11}{(x+1)(x^2+4)}dx = \int_1^4 \frac{2}{x+1}dx + \int_1^4 \frac{2x+3}{x^2+4}dx$$
$$= \int_1^4 \frac{2}{x+1}dx + \int_1^4 \left(\frac{2x}{x^2+4} + \frac{3}{x^2+4}\right)dx$$
$$= \left[2\ln(x+1) + \ln(x^2+4) + \frac{3}{2}\tan^{-1}\frac{x}{2}\right]_1^4$$ **M1 M1 M1** (M1 for each integration)

$$= \left(2\ln 5 + \ln 20 + \frac{3}{2}\tan^{-1}2\right) - \left(2\ln 2 + \ln 5 + \frac{3}{2}\tan^{-1}\frac{1}{2}\right)$$ **B1 B1 M1**

$$\approx 4.1841$$ **A1**

(B1- substituting lower limit, B1- substituting upper limit, M1- subtracting)

12

10 (a).

$$10\sin x\cos x + 12\cos 2x \equiv R\sin(2x+\alpha)$$
$$5\sin 2x + 12\cos 2x \equiv R\sin 2x\cos\alpha + R\cos 2x\sin\alpha$$ **M1**

By comparison,

$$R\cos\alpha = 5 \quad \longrightarrow (1), \qquad R\sin\alpha = 12 \quad \longrightarrow (2)$$ **M1**

(2) ÷ (1) gives,

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{12}{5}\ , \quad \Rightarrow \tan\alpha = \frac{12}{5}, \quad \Rightarrow \alpha = 67.38°$$ **M1**

$$R = \sqrt{5^2 + 12^2} = 13$$ **M1**

$$10\sin x\cos x + 12\cos 2x \equiv 13\sin(2x + 67.38°)$$ **B1**

$$\therefore \text{maximum value} = 13 \times 1 = 13$$ **B1**

(b).

From sine rule, $a = k\sin A\,,\ b = k\sin B\,,\ c = k\sin C$

(i).

$$R.H.S = b\cos C + c\cos B$$
$$= k\sin B\cos C + k\sin C\cos B$$ **M1**
$$= k(\sin B\cos C + \sin C\cos B)$$
$$= k\sin(B+C)$$ **M1**
$$= k\sin(180° - A)$$
$$= k\sin A$$
$$= a$$ **B1**
$$\therefore a = b\cos C + c\cos B$$

(ii).

$$R.H.S = (b-c)\cos\frac{A}{2}$$
$$= (k\sin B - k\sin C)\cos\frac{A}{2}$$ **M1**
$$= k(\sin B - \sin C)\cos\frac{A}{2}$$
$$= k\left(2\cos\frac{B+C}{2}\sin\frac{B-C}{2}\right)\cos\frac{A}{2}$$ **M1-factor formula**
$$= 2k\cos\left(90° - \frac{A}{2}\right)\sin\left(\frac{B-C}{2}\right)\cos\frac{A}{2}$$
$$= 2k\sin\frac{A}{2}\sin\left(\frac{B-C}{2}\right)\cos\frac{A}{2}$$
$$= k\left(2\sin\frac{A}{2}\cos\frac{A}{2}\right)\sin\left(\frac{B-C}{2}\right)$$
$$= k\sin A\sin\left(\frac{B-C}{2}\right)$$
$$= a\sin\left(\frac{B-C}{2}\right)$$
$$\therefore a\sin\left(\frac{B-C}{2}\right) = (b-c)\cos\frac{A}{2}$$ **B1**

12

11 (a).

$$2a - 3b + c = 10 \quad \longrightarrow (1)$$
$$a + 4b + 2c + 3 = 0 \quad \longrightarrow (2)$$
$$5a - 2b - c = 7 \quad \longrightarrow (3)$$

Equation $(1) - 2 \times (2)$ gives,

$$\begin{array}{r|l|} & 2a - 3b + c = 10 \\ - & \underline{2a + 8b + 4c = -6} \\ & -11b - 3c = 16 \end{array} \quad \longrightarrow (4)$$ **M1**

Equation $5 \times (1) - 2 \times (3)$ gives,

$$\begin{array}{r|l|} & 10a - 15b + 5c = 50 \\ - & \underline{10a - 4b - 2c = 14} \\ & -11b + 7c = 36 \end{array} \quad \longrightarrow (5)$$ **M1**

Equation $(4) - (5)$ gives,

$$\begin{array}{r|l|} & -11b - 3c = 16 \\ - & \underline{-11b + 7c = 36} \\ & -10c = -20 \end{array}$$ **M1**

$$c = 2$$ **A1**

From equation (2),

$$-11b + 7 \times 2 = 36$$
$$b = -2$$ **A1**

From equation (2),

$$a + 4(-2) + 2(2) + 3 = 0$$
$$a = 1$$ **A1**

(b).

Let the common root be α

$$\alpha^3 - 2\alpha + 4 = 0 \quad \longrightarrow (1)$$
$$\alpha^2 + \alpha + c = 0 \quad \longrightarrow (2)$$

Equation $(1) - \alpha \times (2)$ gives,

$$\begin{array}{r|l|} & \alpha^3 + \alpha^2 + \alpha c = 0 \\ - & \underline{\alpha^3 - 2\alpha + 4 = 0} \\ & \alpha^2 + \alpha(c + 2) - 4 = 0 \end{array} \quad \longrightarrow (3)$$ **M1**

Equation $(3) - (2)$ gives,

$$\begin{array}{r|l|} & \alpha^2 + \alpha(c + 2) - 4 = 0 \\ - & \underline{\alpha^2 + \alpha + c = 0} \\ & \alpha(c + 1) - 4 - c = 0 \end{array}$$ **M1**

$$\alpha = \frac{c + 4}{c + 1}$$ **B1**

From equation (2)

$$\left(\frac{c + 4}{c + 1}\right)^2 + \frac{c + 4}{c + 1} + c = 0$$ **M1**

$$(c + 4)^2 + (c + 4)(c + 1) + c(c + 1)^2 = 0$$
$$(c^2 + 8c + 16) + (c^2 + 4c + c + 4) + (c^3 + 2c^2 + 1c) = 0$$
$$(c^2 + 8c + 16) + (c^2 + 4c + c + 4) + (c^3 + 2c^2 + 1c) = 0$$ **M1-expansion**
$$c^3 + 4c^2 + 14c + 20 = 0$$ **B1**

12

12 (a).

$$\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \qquad \Rightarrow \overrightarrow{\mathbf{OA}} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \mathbf{d} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$P(1, -2, 3), \qquad \Rightarrow \overrightarrow{\mathbf{OP}} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$$\overrightarrow{\mathbf{AP}} = \overrightarrow{\mathbf{OP}} - \overrightarrow{\mathbf{OA}} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$ **B1**

$$\overrightarrow{\mathbf{AP}} \times \mathbf{d} = \begin{vmatrix} \mathbf{i} & -\mathbf{j} & \mathbf{k} \\ -1 & 1 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$ **M1**

$$= \mathbf{i}\begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} -1 & 2 \\ 2 & -2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix}$$
$$= \mathbf{i}(-2 - 2) - \mathbf{j}(2 - 4) + \mathbf{k}(-1 - 2) = -4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$$ **B1**

$$\left|AP \times \mathbf{d}\right| = \sqrt{16 + 4 + 9} = \sqrt{29}$$
$$\left|\mathbf{d}\right| = \sqrt{4 + 1 + 4} = 3$$ **B1-both magnitudes**

$$\text{Shortesrt distance} = \frac{\left|\overrightarrow{\mathbf{AP}} \times \mathbf{d}\right|}{\left|\mathbf{d}\right|} = \frac{\sqrt{29}}{3} \approx 1.7951 \text{ units}$$ **M1 A1**

(b).

$$\text{normal vector,} \quad \mathbf{n} = \begin{vmatrix} \mathbf{i} & -\mathbf{j} & \mathbf{k} \\ 1 & -3 & 3 \\ -1 & -3 & 2 \end{vmatrix}$$ **M1**

$$= \mathbf{i}\begin{vmatrix} -3 & 3 \\ -3 & 2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 1 & -3 \\ -1 & -3 \end{vmatrix}$$
$$= \mathbf{i}(-6 + 9) - \mathbf{j}(2 + 3) + \mathbf{k}(-3 - 3) = 3\mathbf{i} - 5\mathbf{j} - 6\mathbf{k}$$ **B1**

$$\mathbf{r}.\mathbf{n} = \mathbf{p}.\mathbf{n}$$
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}.\begin{pmatrix} 3 \\ -5 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}.\begin{pmatrix} 3 \\ -5 \\ -6 \end{pmatrix}$$ **M1 M1**
$$3x - 5y - 6z = 3 + 15 - 12$$ **B1**
$$3x - 5y - 6z = 6$$ **A1**

Alternatively:

$$\mathbf{r} = \overline{OA} + \lambda\mathbf{d_1} + \mu\mathbf{d_2}$$
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}$$
$$x = 1 + \lambda - \mu \quad \longrightarrow (1)$$
$$y = -3 - 3\lambda - 3\mu \quad \longrightarrow (2)$$
$$z = 2 + 3\lambda + 2\mu \quad \longrightarrow (3)$$

Equation $3 \times (1) + (2)$ gives,

$$\begin{array}{r|c|l} & 3x = 3+3\lambda-3\mu & \\ + & y = -3-3\lambda-3\mu & \\ \hline & 3x+y=-6\mu & \to (4)\end{array}$$

Equation $3\times(1)-(3)$ gives,

$$\begin{array}{r|c|l} & 3x = 3+3\lambda-3\mu & \\ - & z = 2+3\lambda+2\mu & \\ \hline & 3x-z=1-5\mu & \to (5)\end{array}$$

Equation $5\times(4)-6\times(5)$ gives,

$$\begin{array}{r|c|} & 15x+5y=-30\mu \\ - & 18x-6z=6-30\mu \\ \hline & -3x+5y+6z=-6 \end{array}$$

$$3x-5y-6z=6$$

12

13 (a).

$$y=\frac{2x}{(2-x)^2}$$

$$\frac{dy}{dx}=\frac{(2-x)^2(2)-2x\times 2(2-x)(-1)}{(2-x)^4}$$ **M1**

$$=\frac{(2-x)(4-2x+4x)}{(2-x)^4}=\frac{4+2x}{(2-x)^3}$$ **B1**

For turning points, $\frac{dy}{dx}=0$

$$\frac{4+2x}{(2-x)^3}=0$$ **M1**

$$4+2x=0$$

$$x=-2$$

$$y=\frac{2\times(-2)}{(2+2)^2}=-\frac{1}{4}=-0.25$$ **A1**

$\Longrightarrow (-2,-0.25)$ is the turning points

(b).

Horizontal asymptote

$as\ x\to\infty,\quad y\to 0$

$\Longrightarrow y=0,$ is the horizontal asymptote **B1**

Vertical asymptotes

$as\ y\to\infty,\quad (2-x)^2\to 0$

$\Longrightarrow x=2,$ is the vetical asymptote **B1**

Intercepts

when, $x=0,\quad y=0$

$\Longrightarrow (0,0)$ is the intercept **B1**

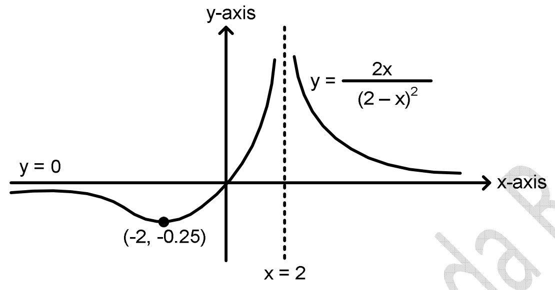
The Critical values include: $x=0, x=2$

Region where the curve lies

	$x<0$	$0<x<2$	$x>2$
$2x$	$-$	$+$	$+$
$(2-x)^2$	$+$	$+$	$+$
y	$-$	$+$	$+$

B1

Sketch of the curve



B1-curve 1

B1-curve 2

B1-axes labelled

12

14 (a).

$$S_\infty=\frac{a}{1-r}$$

$$12.5=\frac{10}{1-r}$$ **M1**

$$12.5-12.5r=10$$

$$12.5r=2.5$$

$$r=\frac{1}{5}=0.2$$ **A1**

for $S_n>10$

$$a\left(\frac{1-r^n}{1-r}\right)>10$$

$$10\left[\frac{1-(0.2)^n}{1-0.2}\right]>10$$ **M1**

$$\frac{1-(0.2)^n}{0.8}>1$$

$$1-(0.2)^n>0.8$$

$$0.2>(0.2)^n$$

$$\log 0.2>n\log 0.2$$ **M1**

$$-0.69897>-0.69897n$$

$$\frac{-0.69897}{-0.69897}<n$$

$$n>1$$ **A1**

$$n=2$$ **B1**

The least number of terms is 2.

(b).

$$\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

For $n=1$,

$L.H.S=\frac{1}{3\times 5}=\frac{1}{15}$	$R.H.S=\frac{1}{3(2+3)}=\frac{1}{15}$
True for $n=1$ since $L.H.S=R.H.S=\frac{1}{15}$	

B1

For $n=2$,

$L.H.S=\frac{1}{3\times 5}+\frac{1}{5\times 7}=\frac{2}{21}$	$R.H.S=\frac{2}{3(4+3)}=\frac{2}{21}$
True for $n=2$ since $L.H.S=R.H.S=\frac{2}{21}$	

Suppose it's true for $n=k$, the series becomes:

$$\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2k+1)(2k+3)}=\frac{k}{3(2k+3)}\quad\longrightarrow(1)$$ **B1**

For $n=(k+1)$,

$$R.H.S=\frac{k+1}{3(2k+2+3)}=\frac{k+1}{3(2k+5)}$$

$$L.H.S=\left[\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{(2k+1)(2k+3)}\right]+\frac{1}{(2k+3)(2k+5)}\quad\longrightarrow(2)$$

Substituting (1) in (2) gives:

$$L.H.S=\frac{k}{3(2k+3)}+\frac{1}{(2k+3)(2k+5)}$$

$$=\frac{1}{(2k+3)}\left[\frac{k}{3}+\frac{1}{(2k+5)}\right]$$ **M1**

$$=\frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{3(2k+5)}\right]$$

$$=\frac{2k^2+2k+3k+3}{3(2k+3)(2k+5)}$$

$$=\frac{2k(k+1)+3(k+1)}{3(2k+3)(2k+5)}$$

$$=\frac{(2k+3)(k+1)}{3(2k+3)(2k+5)}$$ **M1-factorisation**

$$=\frac{(k+1)}{3(2k+5)}$$ **A1**

True for $n=(k+1)$ since $L.H.S=R.H.S=\frac{(k+1)}{3(2k+5)}$

Since its true for $n=1, n=2, n=k$ and $n=(k+1)$, then it's true for all positive integers of n. **B1**

12

15 (a).

$$y^2-8y=-4x$$

$$(y-4)^2-16=-4x$$
$$(y-4)^2=-4x+16$$ **M1**
$$(y-4)^2=-4(x-4)$$

This is in the form $Y^2=4aX$ hence it's a parabola. **A1**

$$Y=y-4,\quad X=x-4$$
$$4a=-4,\quad\Rightarrow a=-1$$ **B1-value of a**

Vertex

$$(X,Y)=(0,0)$$
$$X=(x-4)=0,\quad\Rightarrow x=4$$
$$Y=(y-4)=0,\quad\Rightarrow y=4$$

The vertex is $(x,y)=(4,4)$. **B1-vertex**

Intercepts

When $x=0$,

$$(y-4)^2=16$$
$$y-4=\pm 4$$
$$y=4-4=0,\quad\text{or,}\quad y=4+4=8$$

When $y=0$,

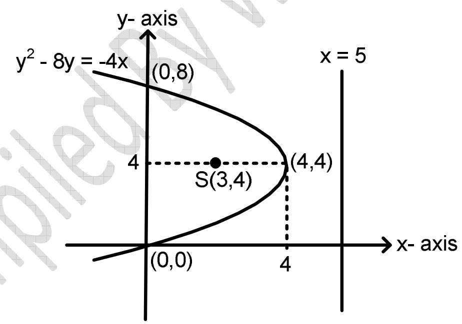
$$16=-4x+16,\quad\Rightarrow x=0$$

The intercepts are: $(0,0),(0,8)$.

Focus

$$(X,Y)=(a,0)=(-1,0)$$
$$X=(x-4)=-1,\quad\Rightarrow x=3$$
$$Y=(y-4)=0,\quad\Rightarrow y=4$$ **B1-focus**

The focus is $(x,y)=(3,4)$.



B1

(b).

$$P(at^2,2at)$$
$$Q(aT^2,2aT)$$
$$y^2=4ax$$

The coordinates of the midpoint of $\overline{PQ}$, M are,

$$=\left\{\frac{at^2+aT^2}{2},\frac{2at+2aT}{2}\right\}$$

$$= \left\{\frac{1}{2}a(t^2+T^2)\,,a(t+T)\right\}$$ **M1**

$$y = a(t+T), \qquad \Rightarrow (t+T) = \frac{y}{a} \quad \longrightarrow (1)$$ **B1**

$$x = \frac{1}{2}a(t^2+T^2)$$

$$x = \frac{1}{2}a[(t+T)^2 - 2tT]$$ **M1**

$$x = \frac{1}{2}a(t+T)^2 - atT$$

$$x = \frac{1}{2}a(t+T)^2 - atT \quad \longrightarrow (2)$$ **B1**

Substituting (1) into (2) gives,

$$x = \frac{1}{2}a\left(\frac{y}{a}\right)^2 - atT$$ **M1**

$$x = \frac{1}{2a}y^2 - atT$$

but $tT = 2a$

$$x = \frac{1}{2a}y^2 - 2a^2$$

$$2ax = y^2 - 4a^3$$

$$y^2 = 2ax + 4a^3$$ **A1**

12

16 (a).

$$x\frac{dy}{dx} + y = xe^{-2x}$$

$$\frac{dy}{dx} + \frac{y}{x} = e^{-2x}$$

Integrating factor, $R = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$ **B1**

$$x\frac{dy}{dx} + y = xe^{-2x}$$

$$\frac{d}{dx}(xy) = xe^{-2x}$$

$$\int \frac{d}{dx}(xy) = \int xe^{-2x}\,dx$$ **M1**

Sign	Differentiation	Integration
+	x	e^{-2x}
−	1	$-\frac{1}{2}e^{-2x}$
+	0	$\frac{1}{4}e^{-2x}$

B1

$$xy = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$$ **A1 A1**

(b).

Let θ be the temperature of the body at a time t.

$$\frac{d\theta}{dt} \propto (120-\theta)$$

$$\frac{d\theta}{dt} = k(120-\theta)$$ **B1**

Where k is the proportionality constant

$$\int \frac{d\theta}{(120-\theta)} = \int k\,dt$$ **M1**

$$-\ln(120-\theta) = kt + c$$ **A1**

When $t = 0, \theta = 50$°C,

$$-\ln(120-50) = k \times 0 + c\,, \qquad \Rightarrow c = -\ln 70$$ **B1**

$$-\ln(120-\theta) = kt - \ln 70$$

When $t = 6, \theta = 80$°C,

$$-\ln(120-80) = 6k - \ln 70$$

$$-\ln 40 + \ln 70 = 6k$$

$$k = \frac{1}{6}\ln\left(\frac{7}{4}\right)$$ **B1**

$$-\ln(120-\theta) = \frac{t}{6}\ln\left(\frac{7}{4}\right) - \ln 70$$

$$\ln\left(\frac{70}{120-\theta}\right) = \frac{t}{6}\ln\left(\frac{7}{4}\right)$$

When $\theta = 90$°C,

$$\ln\left(\frac{70}{120-90}\right) = \frac{t}{6}\ln\left(\frac{7}{4}\right)$$

$$t = \frac{6\ln\left(\frac{7}{3}\right)}{\ln\left(\frac{7}{4}\right)} = 9.0844 \text{ minutes}$$

When $\theta = 99$°C,

$$\ln\left(\frac{70}{120-99}\right) = \frac{t}{6}\ln\left(\frac{7}{4}\right)$$

$$t = \frac{6\ln\left(\frac{10}{3}\right)}{\ln\left(\frac{7}{4}\right)} = 12.9086 \text{ minutes}$$

time taken $= 12.9086 - 9.0844 = 3.8242$ minutes **M1 A1**

12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Oct. 2018
3 hours

NDEJJE SENIOR SECONDARY SCHOOL
S.6 MATH 2 MOCK SET 7 2018
Time: 3 Hours

NAME: ______________________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.
- Where necessary, use $g = 9.8 \text{ m s}^{-2}$.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn 1: A Continuous random variable has accumulative probability function given by
$$F(x) = \begin{cases} \log_2(x^k) & ; \quad 0 \le x \le e, \\ 1 & ; \quad x \ge e. \end{cases}$$
(i). Show that $k = \ln 2$.
(ii). Obtain the p.d.f of X. [5]

Qn 2: A stone is projected from the top a cliff of height 25 m with an initial speed of 12 m s^{-1} at an angle of $60°$ to the vertical. Find the time it takes the stone to hit the sea-level. [5]

Qn 3: Given that $y = \sec 45° \pm 10\%$, find the limit within which the exact value of y lies. [5]

Qn 4: The table below shows the cost of ingredients used for making Chapats for two different birthday parties for 2015 and 2017.

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Ingredients	**Cost**	
	2015	**2017**
Salt	200	350
Baking flour	3800	4600
Cooking oil	1500	1800

By taking 2015 as a base year, calculate the price relative for each ingredient and hence, obtain the average index number. [5]

Qn 5: A random variable X is such that $X \sim N(102, 16)$.
Find $P(|x - 100| < 7.2)$. [5]

Qn 6: Given the information in the table below:

x	2.5	3.0	3.2	3.4	3.8
$f(x)$	1.56	1.82	1.95	2.05	2.13

Use linear interpolation/extrapolation to find:
(i). $f(4.1)$
(ii). $f^{-1}(1.72)$. [5]

Qn 7: A hose pipe of cross section area 12 cm^2 is located at a height of 6m above the ground, draws and issues water at a speed of 4.5 m s^{-1}. Find the rate at which the water is oozing out of the pipe (Take density of water to be 1000 kg m^{-3}). [5]

Qn 8: Find the centre of gravity of a uniform lamina whose shape is the area bounded by $y = x^2$, the $x -$ axis and $x = 4$. [5]

Section B (60 Marks)
Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
(a). Use the trapezoidal rule with five ordinates to evaluate $\int_0^{\frac{\pi}{4}} \frac{2}{\sqrt{1-x^2}}\,dx$ to 3 decimal places.
(b). Find the exact value of $\int_0^{\frac{\pi}{4}} \frac{2}{\sqrt{1-x^2}}\,dx$ to 3 decimal places.
(c). Find the absolute error in the function and state one way how this error can be reduced. [12]

Question 10:

Two equal forces each of magnitude $\boldsymbol{P}$ N have an angle, 2α, between them. If their resultant is twice that when the same forces have an angle, 2β, between them,

(a). Prove that $8\cos^2\beta - \cos 2\alpha - 1 = 0$.

(b). If $\alpha = \frac{\pi}{4}$, find β. [12]

Question 11:

A random variable X has its p.d.f given by

$$P(x = x) = \begin{cases} \frac{k}{x} & ; \quad x = 1, 2, 3, \\ 0 & ; \quad \text{elsewhere.} \end{cases}$$

Find:

(i). the value of Constant k,

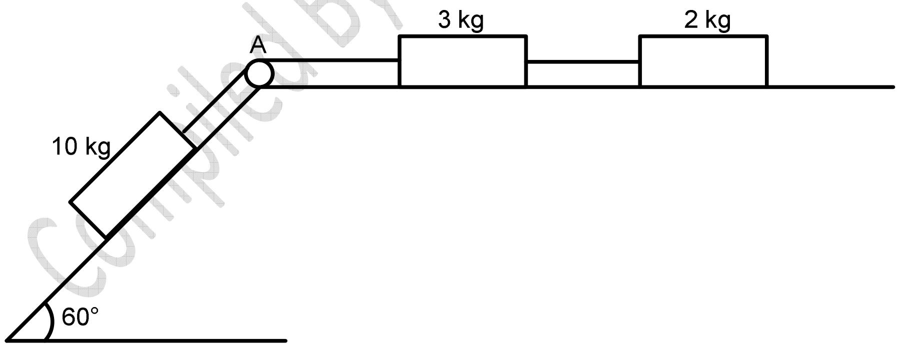
(ii). $E(X + 1)^2$,

(iii). Median,

(iv). 3rd decile. [12]

Question 12:

The figure below shows a mass of 10 kg placed on a smooth incline of inclination 60° attached to a mass of 3 kg placed on a rough horizontal table by means of an inelastic string passing over a smooth pulley and connected to a second mass of 2 kg on the table by means of another string. The coefficient of friction of the table is $\frac{1}{3}$.



(a). Find the acceleration of each mass.

(b). Find the tension in each string.

(c). Reaction on the pulley. [12]

Question 13:

Annet stays in Kenya and Bob stays in Uganda. The probability that Annet will go to china in December this year is $\frac{3}{5}$ and that of Bob is $\frac{3}{8}$.

(a). Find the probability that they are likely to be in different countries next year.

(b). The probability that patience passes Biology, Chemistry and Mathematics is 0.7, 0.8 and 0.65 respectively.

(i). Find the probability that she passes at most one subject.

(ii). If we know that she passed atmost one subject. What is the probability that she passed Mathematics. [12]

Question 14:

(a). Construct a flow chart that computes and prints the average of the squares of the first six counting numbers. Perform a dry run for your flow chart.

(b). Locate graphically the positive root of the equation $e^{-x} = 4 - 3x$ and hence, use linear interpolation to find the root of the equation to 2 decimal places. [12]

Question 15:

(a). A cyclist A appears to be moving at a velocity of 10 m s^{-1} on a bearing of 330° to a cyclist B moving with a velocity of $\sqrt{8}$ m s^{-1} on a bearing of 045°. Find the true velocity of the cyclist.

(b). A particle of mass 2 kg, moves on a space curve accelerating at a rate of

$$\underset{\sim}{\boldsymbol{a}} = \left(\cos 2t\, \underset{\sim}{\boldsymbol{i}} + \sin 2t\, \underset{\sim}{\boldsymbol{j}} + t\underset{\sim}{\boldsymbol{k}}\right).$$

Find the power developed after 3 seconds. [12]

Question 16:

The table below shows height in centimetres of 25 students in a certain school.

Height (cm)	< 10	< 20	< 25	< 30	< 50	<55	< 65
Number of students	0	3	7	15	17	23	25

Calculate

(i). Mean height

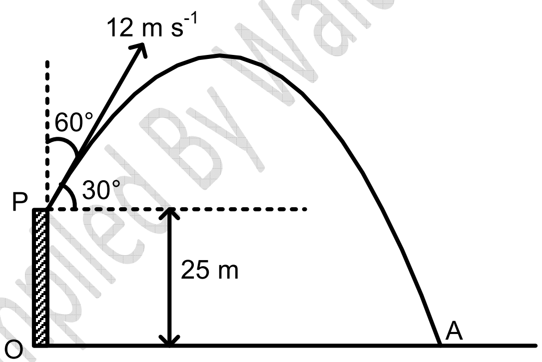
(ii). Variance

(iii). Mode

(iv). Middle 70% of the height. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	(i). For $0 \le x \le e$,	
	$F(x) = \log_2(x^k) = \dfrac{k \log_e x}{\log_e 2} = \dfrac{k \ln x}{\ln 2}$	M1
	$F(e) = \dfrac{k \ln e}{\ln 2} = 1$	
	$\dfrac{k}{\ln 2} = 1$	
	$k = \ln 2$	B1
	(ii).	
	$\dfrac{d}{dx}\left(\dfrac{k \ln x}{\ln 2}\right) = \dfrac{k}{x \ln 2} = \dfrac{\ln 2}{x \ln 2} = \dfrac{1}{x}$	M1
	$\dfrac{d}{dx}(1) = 0$	M1
	$\therefore f(x) = \begin{cases} \dfrac{1}{x} & ; \quad 0 \le x \le e, \\ 0 & ; \quad \text{elsewhere.} \end{cases}$	A1
		05
2		
	For vertical motion,	
	$y = ut \sin\theta - \dfrac{1}{2} g t^2$	
	$-25 = 12t \sin 30° - \dfrac{1}{2} \times 9.8 t^2$	M1
	$-25 = 6t - 4.9t^2$	
	$4.9t^2 - 6t - 25 = 0$	B1
	$t = x = \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 4.9 \times (-25)}}{2 \times 4.9}$	M1
	$t = 2.953, \quad$ or, $\quad t = -1.728$	B1

SNo.	Working	Marks
	butt $\ne -1.728, \quad \Rightarrow t = 2.953$ s	A1
		05
3	$y = \sec 45° \pm 10\%$	
	$y = \sec(45° \pm 0.1°)$	B1
	Lower limit $= y_{\min} = \sec 44.9° = 1.4118$	M1 A1
	Upper limit $= y_{\max} = \sec 45.1° = 1.4167$	M1 A1
		05
4		

Ingredients	Price relatives
Salt	$\dfrac{350}{200} = 1.75$
Baking flour	$\dfrac{4600}{3800} = 1.211$
Cooking oil	$\dfrac{1800}{1500} = 1.2$

Marks: B1, B1, B1

For the hence part:

Average index number $= \dfrac{1.75 + 1.211 + 1.2}{3} \times 100 = 138.7$ — M1 A1

05

SNo.	Working	Marks
5	$\mu = 102, \quad \sigma^2 = 16, \quad \sigma = 4$	
	$P(\lvert x - 100\rvert < 7.2) = P(-7.2 < X - 100 < 7.2)$	M1
	$= P(100 - 7.2 < X < 100 + 7.2)$	
	$= P(92.8 < X < 107.2)$	
	$= P\left(\dfrac{92.8 - 102}{4} < Z < \dfrac{107.2 - 102}{4}\right)$	M1
	$= P(-2.3 < Z < 1.3)$	B1
	$= \phi(2.3) + \phi(1.3)$	
	$= 0.4893 + 0.4032 = 0.8925$	M1 A1
		05
6		

x	3.4	3.8	4.1
$f(x)$	2.05	2.13	y

B1

$$\frac{y - 2.05}{2.13 - 2.05} = \frac{4.1 - 3.4}{3.8 - 3.4}$$ M1

$$x_1 = 2.05 + \frac{0.7}{0.4} \times 0.08 = 2.19$$

$$\therefore f(4.1) = 2.19$$ A1

(ii).

x	3.0	x	3.2
$f(x)$	1.82	1.72	1.95

$$\frac{x-3}{3.2-3}=\frac{1.72-1.82}{1.95-1.82}$$ **M1**

$$x=3+\frac{-0.1}{0.13}\times 0.2=2.8462$$ **A1**

$$\therefore f^{-1}(1.72)=2.8462$$

05

7. $v=4.5\text{ m s}^{-1}, A=12\text{ cm}^2=0.0012\text{ m}^2, h=6\text{ m}, \rho=1000\text{ kg m}^{-3}$

Mass of water raised and issued per second

$$m=Av\rho=0.0012\times 4.5\times 1000=5.4\text{ kg s}^{-1}$$ **B1**

Potential energy given to raise the water

$$P.E=mgh=5.4\times 9.8\times 6=317.52\text{ J s}^{-1}$$ **M1**

Kinetic energy given to raise the water

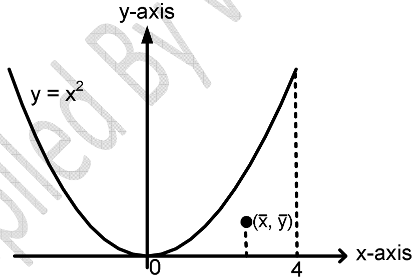
$$K.E=\frac{1}{2}mv^2=\frac{1}{2}\times 5.4\times(4.5)^2=54.675\text{ J s}^{-1}$$ **M1**

Rate at which the pump is working,

$$P_{total}=P.E+K.E=317.52+54.675=372.195\text{ J s}^{-1}$$ **M1 A1**

05

8. when $y=0$, $x=0$



Let ρ be the weight per unit area.

Taking moments about the y-axis,

$$\bar{x}\rho\int_0^4 y\,dx=\rho\int_0^4 xy\,dx$$

$$\bar{x}\int_0^4 x^2\,dx=\int_0^4 x^3\,dx$$

$$\bar{x}\left[\frac{1}{3}x^3\right]_0^4=\left[\frac{1}{4}x^4\right]_0^4$$ **M1**

$$\frac{64}{3}\bar{x}=64$$

$$\bar{x}=3$$

Taking moments about the x-axis,

$$\bar{y}\rho\int_0^4 y\,dx=\rho\int_0^4\frac{1}{2}y^2\,dx$$ **B1**

$$\bar{y}\int_0^4 x^2\,dx=\int_0^4\frac{1}{2}x^4\,dx$$

$$\bar{y}\left[\frac{1}{3}x^3\right]_0^4=\left[\frac{1}{10}x^5\right]_0^4$$ **M1**

$$\frac{64}{3}\bar{y}=\frac{1024}{10}$$

$$\bar{y}=\frac{24}{5}$$ **B1**

The centre of gravity is $(\bar{x},\bar{y})=\left(3,\frac{24}{5}\right)$. **A1**

05

9. (a).

$$y_n=\frac{2}{\sqrt{1-{x_n}^2}},\quad h=\frac{\frac{\pi}{4}-0}{5-1}=\frac{\pi}{16}$$ **B1**

n	x_n	y_0, y_4	$y_1, \dots y_3$
0	0	2	
1	$\frac{\pi}{16}$		2.0397
2	$\frac{2\pi}{16}$		2.1747
3	$\frac{3\pi}{16}$		2.4749
4	$\frac{4\pi}{16}$	3.2311	
sums		5.2311	6.6893

B1-for all x_n

B1-for all y_n

$$\int_0^{\frac{\pi}{4}}\frac{2}{\sqrt{1-x^2}}dx\approx\frac{1}{2}h[(y_0+y_4)+2(y_1+\dots+y_3)]$$

$$\approx\frac{1}{2}\times\frac{\pi}{16}[5.2311+2\times 6.6893]=1.8270\approx 1.827\ (3\text{ d.p})$$ **M1 A1**

(b).

$$\int_0^{\frac{\pi}{4}}\frac{2}{\sqrt{1-x^2}}dx=\left[\ 2\sin^{-1}x\right]_0^{\frac{\pi}{4}}$$ **M1**

$$=2\sin^{-1}\frac{\pi}{4}-2\sin^{-1}0=1.807\ (3\text{ d.p})$$ **M1 B1 A1**

(c).

$$\text{Absolute error}=|1.807-1.827|=0.02$$ **M1 A1**

The absolute error can be reduced by increasing the number of ordinates. **B1**

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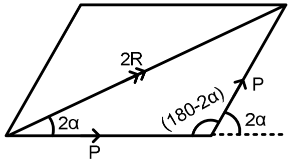
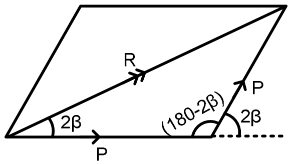
12

10





B1 B1

By cosine rule,
Case 1:

$$(2R)^2 = P^2 + P^2 - 2P^2\cos(180° - 2\alpha)$$

$$4R^2 = 2P^2(1 + \cos 2\alpha) \quad \longrightarrow (1)$$

M1 B1

Case 2:

$$R^2 = P^2 + P^2 - 2P^2\cos(180° - 2\beta)$$

$$R^2 = 2P^2(1 + \cos 2\beta) \quad \longrightarrow (2)$$

M1 B1

Equation (1) ÷ (2) gives;

$$\frac{4R^2}{R^2} = \frac{2P^2(1+\cos 2\alpha)}{2P^2(1+\cos 2\beta)}$$

$$4 = \frac{1+\cos 2\alpha}{1+\cos 2\beta}$$

M1

$$4 + 4\cos 2\beta = 1 + \cos 2\alpha$$

$$4 + 4(2\cos^2\beta - 1) = 1 + \cos 2\alpha$$

B1

$$4 + 8\cos^2\beta - 4 = 1 + \cos 2\alpha$$

$$8\cos^2\beta = 1 + \cos 2\alpha$$

$$8\cos^2\beta - \cos 2\alpha - 1 = 0$$

B1

(b).
When $\alpha = \frac{\pi}{4}$

$$8\cos^2\beta - \cos\left(2\times\frac{\pi}{4}\right) - 1 = 0$$

M1

$$8\cos^2\beta - 0 - 1 = 0$$

$$8\cos^2\beta = 1$$

$$\cos\beta = \pm\frac{1}{\sqrt{8}}$$

B1

$$\beta = 69.3°, \quad \text{or,} \quad \beta = 110.7°$$

A1

12

11

x	$P(X=x)$	$P(X=x)$	$xP(X=x)$	$x^2P(X=x)$	$P(X\le x)$
1	k	$\frac{6}{11}$	$\frac{6}{11}$	$\frac{6}{11}$	$\frac{6}{11}$
2	$\frac{k}{2}$	$\frac{3}{11}$	$\frac{6}{11}$	$\frac{12}{11}$	$\frac{9}{11}$
3	$\frac{k}{3}$	$\frac{2}{11}$	$\frac{6}{11}$	$\frac{18}{11}$	1
Sums	$\frac{11}{6}k$	1	$\frac{18}{11}$	$\frac{36}{11}$	

B1-$P(X=x)$ with k
B1-$P(X=x)$ with k substituted
B1-$xP(X=x)$
B1-$x^2P(X=x)$
B1-$P(X\le x)$

(i).

$$\sum_{all\ x} P(X=x) = 1$$

$$\frac{11}{6}k = 1$$

M1

$$k = \frac{6}{11}$$

A1

(ii).

$$E(X+1)^2 = E(X^2 + 2X + 1) = E(X^2) + 2E(X) + 1$$

M1

$$= \frac{36}{11} + 2\times\frac{18}{11} + 1 = \frac{83}{11}$$

M1 A1

(iii).

Median = 1

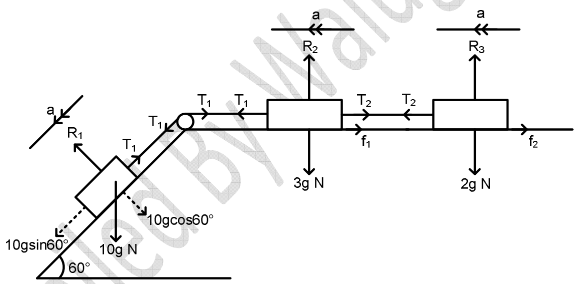
B1

3rd decile = 1

B1

12

12



B1

(a).

$$f_2 = \mu R_2 = \frac{1}{3}\times 3g = \frac{1}{3}\times 3\times 9.8 = 9.8\text{ N}$$

$$f_3 = \mu R_3 = \frac{1}{3}\times 2g = \frac{1}{3}\times 2\times 9.8 = \frac{98}{15}\text{ N}$$

For 10 kg mass,

$$10g\sin 60° - T_1 = 10a$$

$$98\sin 60° - T_1 = 10a \quad \longrightarrow (1)$$

B1

For 3 kg mass,

$$T_1 - (T_2 + f_2) = 3a$$

$$T_1 - T_2 - 9.8 = 3a \quad \longrightarrow (2)$$

B1

For 2 kg mass,

$$T_2 - f_3 = 2a$$

$$T_2 - \frac{98}{15} = 2a \quad \longrightarrow (3)$$ **B1**

Equation (1) + (2) + (3) gives,

$$98\sin 60° - 9.8 - \frac{98}{15} = 10a + 3a + 2a$$ **M1**

$$68.5372 = 15a$$

$$a = 4.5691 \text{ m s}^{-2}$$ **A1**

(b).

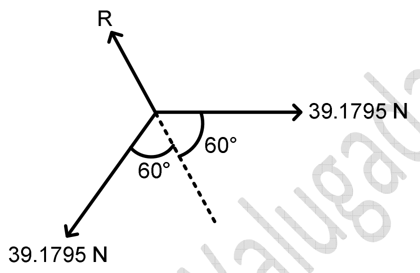
From equation (3)

$$T_2 = \frac{98}{15} + 2a = \frac{98}{15} + 2 \times 4.5691 = 15.6715 \text{ N}$$ **M1 A1**

From equation (1)

$$T_1 = 98\sin 60° - 10a = 98\sin 60° - 10 \times 4.5691 = 39.1795 \text{ N}$$ **M1 A1**

(c).



$$R = 2 \times 39.1795\cos 60° = 39.1795 \text{ N}$$ **M1 A1**

12

13 (a).

$$P(A) = \frac{3}{5}, \quad P(A') = \frac{2}{5}, \quad P(B) = \frac{3}{8}, \quad P(B') = \frac{5}{8}$$

$$P(\text{different countries}) = P(A' \cap B') + P(A \cap B') + P(A' \cap B)$$

$$= \frac{2}{5} \times \frac{5}{8} + \frac{3}{5} \times \frac{5}{8} + \frac{2}{5} \times \frac{3}{5}$$ **M1 M1 M1**

$$= \frac{31}{40}$$ **A1**

(b).

$$P(B) = 0.7, \quad P(B') = 0.3, \quad P(C) = 0.8, \quad P(C') = 0.2$$

$$P(M) = 0.65, \quad P(M') = 0.35, \quad P(B) = \frac{3}{8}, \quad P(B') = \frac{5}{8}$$

(i). Let T denote event that she passes at most one subject.

$$P(T) = P(B \cap C' \cap M') + P(B' \cap C \cap M') + P(B' \cap C' \cap M) + P(B' \cap C' \cap M')$$

$$= 0.7 \times 0.2 \times 0.35 + 0.3 \times 0.8 \times 0.35 + 0.3 \times 0.2 \times 0.65 + 0.3 \times 0.2 \times 0.35$$ **M1 M1 M1 M1**

$$= 0.049 + 0.084 + 0.039 + 0.021 = 0.193$$ **A1**

(ii).

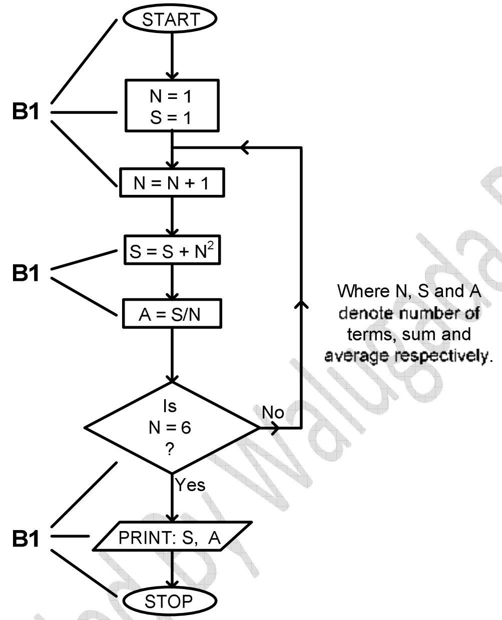
$$P(M/T) = \frac{P(M \cap T)}{P(T)} = \frac{P(B' \cap C' \cap M)}{P(T)}$$

$$= \frac{0.3 \times 0.2 \times 0.65}{0.193} = \frac{0.039}{0.193} = \frac{39}{193} \approx 0.2021$$ **M1 M1 A1**

12

14 (a).

Flow chart:



B3

Dry run:

N	S	A
1	1	1
2	5	2.5
3	14	14/3
4	30	7.5
5	55	11
6	91	91/6

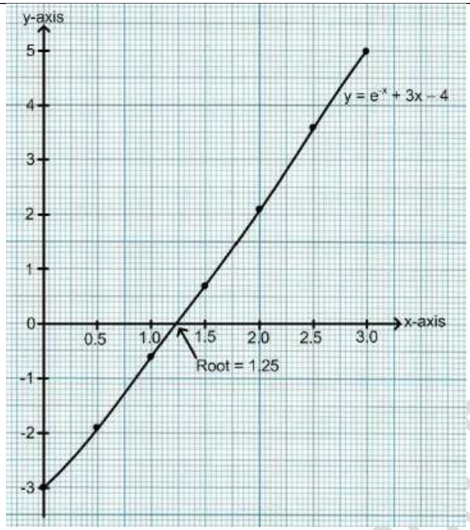
B1 B1

(b).

let, $y = e^{-x} + 3x - 4$

x	0	0.5	1.0	1.5	2.0	2.5	3.0
y	-3	-1.9	-0.6	0.7	2.1	3.6	5.0

B1



B1 B1

From the graph, the root $x_0 = 1.25$.

(b). (i).

$$f(x) = e^{-x} + 3x - 4$$
$$f(1.25) = e^{-1.45} + 3 \times 1.45 - 4 = 0.0365$$

x	1	x_1	1.25
$f(x)$	-0.6	0	0.0365

$$\frac{x_1 - 1}{1.25 - 1} = \frac{0 - (-0.6)}{0.0365 - (-0.6)}$$
$$x_1 = 1 + \frac{0.6}{0.6365.} \times 0.25 = 1.2357$$ **M1**
$$|x_1 - x_0| = |1.2357 - 1.25| = 0.0143$$

$$f(1.228) = e^{-1.2357} + 3 \times 1.2357 - 4 = -0.00227$$

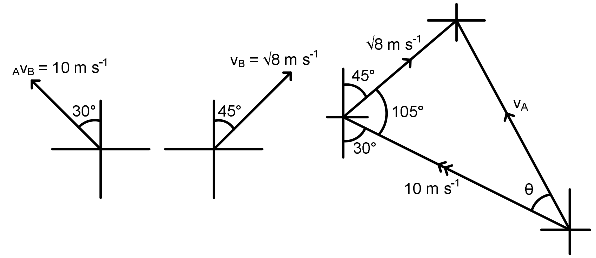
x	1.2357	x_2	1.25
$f(x)$	-0.00227	0	0.0365

$$\frac{x_2 - 1.2357}{1.25 - 1.2357} = \frac{0 - (-0.00227)}{0.0365 - (-0.00227)}$$
$$x_2 = 1.2357 + \frac{0.00227}{0.03877} \times 0.0143 = 1.2365$$ **M1**
$$|x_2 - x_1| = |1.2365 - 1.2357| = 0.0008$$
$$\therefore \text{root} = 1.24 \text{ (2 d.p)}$$ **B1**

A1

12

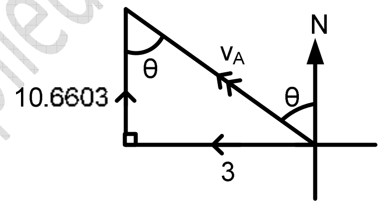
15 (a). **Method 1: Geometrical method**



$$v_A{}^2 = 10^2 + (\sqrt{8})^2 - 2 \times 10 \times \sqrt{8}\cos 105°$$ **M1**
$$v_A = \sqrt{122.6410} = 11.0743 \text{ m s}^{-1}$$ **M1 A1**
$$\frac{\sin\theta}{\sqrt{8}} = \frac{\sin 105°}{11.0743}$$ **M1**
$$\sin\theta = \frac{\sqrt{8}\sin 105°}{11.0743} = 0.2467, \quad \Rightarrow \theta = 14.282°$$ **B1**
$$\text{Bearing} = 270 + 60 + 14.282 = 344.282°$$ **A1**

Method 2: Vector Method

$$_A\underset{\sim}{v}_B = \begin{pmatrix} -10\cos 60° \\ 10\sin 60° \end{pmatrix} \text{ m s}^{-1}, \quad \underset{\sim}{v}_B = \begin{pmatrix} \sqrt{8}\cos 45° \\ \sqrt{8}\sin 45° \end{pmatrix} \text{ m s}^{-1}$$
$$_A\underset{\sim}{v}_B = \underset{\sim}{v}_A - \underset{\sim}{v}_B$$
$$\underset{\sim}{v}_A = {_A\underset{\sim}{v}_B} + \underset{\sim}{v}_B = \begin{pmatrix} -10\cos 60° \\ 10\sin 60° \end{pmatrix} + \begin{pmatrix} \sqrt{8}\cos 45° \\ \sqrt{8}\sin 45° \end{pmatrix}$$
$$= \begin{pmatrix} -3 \\ 10.6603 \end{pmatrix} \text{ m s}^{-1}$$ **M1 A1**
$$|\underset{\sim}{v}_B| = \sqrt{(-3)^2 + (10.6603)^2} = 11.0744 \text{ m s}^{-1}$$ **M1 A1**



$$\tan\theta = \frac{3}{10.6603}, \quad \Rightarrow \theta = 15.718°$$ **B1**
$$\text{Bearing} = 360 - 15.718 = 344.282°$$ **A1**

(b).

$$\boldsymbol{P} = \int_0^3 m\underset{\sim}{\boldsymbol{a}}.\underset{\sim}{\boldsymbol{a}}\, dt$$

but $\underset{\sim}{a}.\underset{\sim}{a} = \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} . \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} = \cos^2 2t + \sin^2 2t + t^2 = 1 + t^2$ **B1 B1**

$$\boldsymbol{P} = \int_0^3 m\underset{\sim}{a}.\underset{\sim}{a}\,dt = 2\int_0^3 (1+t^2)\,dt = 2\left[t + \frac{1}{3}t^3\right]_0^3$$ **M1 M1**

$$= 2\left(3 + \frac{1}{3} \times 3^3\right) - 0 = 24\text{ W}$$ **M1 A1**

12

16 (a).

Marks	$C.F$	f	c	f/c	x	fx	fx^2	Class boundaries
<10	0	0						0 – 10
<20	3	3	10	0.3	15	45	675	10 – 20
<25	7	4	5	0.8	22.5	90	2025	20 – 25
<30	15	8	5	1.6	27.5	220	6050	25 – 30
<50	17	2	20	0.1	40	80	3200	30 – 50
<55	23	6	5	1.2	52.5	315	16537.5	50 – 55
<65	25	2	10	0.2	60	120	7200	55 – 65
Total		25				870	35687.5	

B1-for f/c

B1-for fx

B1-for fx^2

(i).

$$\text{Mean height} = \frac{\sum fx}{\sum f} = \frac{870}{25} = 34.8$$ **M1 A1**

(ii).

$$\text{Variance} = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2 = \frac{35687.5}{25} - \left(\frac{870}{25}\right)^2 = 216.46$$ **M1 A1**

(iii).

$$\Delta_1 = 1.6 - 0.8 = 0.8, \qquad \Delta_2 = 1.6 - 0.1 = 1.5$$

$$\text{Mode} = L_m + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)c = 25 + \left(\frac{0.8}{0.8 + 1.5}\right) \times 5 = 26.7391$$ **M1 A1**

(iv).

$$100 - \alpha = 70$$
$$\alpha = 30$$
$$\frac{\alpha}{2} = 15$$
$$\text{Lower limit} = P_{\frac{\alpha}{2}} = P_{15}$$
$$15^{\text{th}}\text{ percentile position} = \frac{15}{100}N = \frac{15}{100} \times 25 = 3.75$$

$$P_{15} = L_1 + \left(\frac{3.75 - C.F_b}{f_1}\right)c$$
$$= 20 + \left(\frac{3.75 - 3}{4}\right) \times 5 = 20.9375$$ **B1**
$$\text{Upper limit} = P_{\left(100 - \frac{\alpha}{2}\right)} = P_{85}$$
$$85^{\text{th}}\text{ percentile position} = \frac{85}{100}N = \frac{85}{100} \times = 21.25$$
$$P_{85} = L_2 + \left(\frac{21.25 - C.F_b}{f_2}\right)c$$
$$= 40 + \left(\frac{21.25 - 17}{6}\right) \times 5 = 53.5417$$ **B1**
$$\text{Middle 70\% age range} = P_{85} - P_{15}$$
$$= 53.5417 - 20.9375 = 32.6042$$ **A1**

12

END

P425/1
PURE
MATHEMATICS
PAPER 1
Nov./Dec. 2018
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD
S.6 MATH 1 UNEB 2018
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)
Answer **all** *the questions in this section*

Qn 1: In a triangle ABC, $a = 7$ cm, $b = 4$ cm and $c = 5$ cm. Find the value of:
(a). $\cos A$,
(b). $\sin A$. [5]

Qn 2: Determine the angle between the line $\frac{x+4}{8} = \frac{y-2}{2} = \frac{z+1}{-4}$ and the plane $4x + 3y - 3z + 1 = 0$. [5]

Qn 3: Find $\int x^2 e^x \, dx$. [5]

Qn 4: Express the function $f(x) = x^2 + 12x + 32$, in the form $a(x+b)^2 + c$. Hence find the minimum value of the function $f(x)$. [5]

Qn 5: A point P moves such that its distances from two points $A(-2, 0)$ and $B(8, 6)$ are in the ratio $AP:PB = 3:2$. Show that the locus of P is a circle. [5]

Qn 6: Determine the equation of the tangent to the curve $y^3 + y^2 - x^4 = 1$ at the point $(1, 1)$. [5]

Qn 7: Show that $2\log 4 + \frac{1}{2}\log 25 - \log 20 = 2\log 2$. [5]

Qn 8: The region bounded by the curve $y = x^2 - 2x$ and the x −axis from $x = 0$ to $x = 2$ is rotated about the x −axis. Calculate the volume of the solid formed. [5]

Section B (60 Marks)
Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The position vectors of the vertices of a triangle are $\underset{\sim}{\boldsymbol{o}}$, $\underset{\sim}{\boldsymbol{r}}$ and $\underset{\sim}{\boldsymbol{s}}$, where $\boldsymbol{O}$ is the origin. Show that its area (A) is given by

$$4A^2 = \left|\underset{\sim}{\boldsymbol{r}}\right|^2 \left|\underset{\sim}{\boldsymbol{s}}\right|^2 - \left(\underset{\sim}{\boldsymbol{r}} \cdot \underset{\sim}{\boldsymbol{s}}\right)^2$$

[6]

Hence, find the area of a triangle when $\underset{\sim}{\boldsymbol{r}} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\underset{\sim}{\boldsymbol{s}} = \begin{pmatrix}1\\4\end{pmatrix}$. [6]

Question 10:
Express $5 + 12i$ in polar form. Hence, evaluate $\sqrt[3]{(5+12i)}$, giving your answer in the form $a + ib$ where a and b are corrected to **two** decimal places. [12]

Question 11:
(a). Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x. [6]
(b). The period, T of a swing of a simple pendulum of length, l is given by the equation

$$T^2 = \frac{4\pi^2 l}{g}$$

where g is the acceleration due to gravity.
An error of 2% is made in measuring the length, l. Determine the resulting percentage error in the period, T. [6]

Question 12:
(a). Show that $\tan 4\theta = \frac{4t(1-t^2)}{t^4 - 6t^2 + 1}$, where $t = \tan\theta$. [6]
(b). Solve the equation:
$\sin x + \sin 5x = \sin 2x + \sin 4x$ for $0° < x < 90°$. [6]

Question 13:

(a). The first three terms of a Geometric progression (G.P) are 4, 8 and 16. Determine the sum of the first ten terms of the G.P. [4]

(b). An Arithmetic progression (A.P) has a common difference of 3. A Geometric progression (G.P) has a common ratio of 2.
A sequence is formed by subtracting the terms of the A.P from the corresponding terms of the G.P. the third term of the sequence is 4. The sixth term of the sequence is 79. Find the first term of the:
(i). A.P,
(ii). G.P. [8]

Question 14:

Evaluate:

(a). $\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x \, dx$. [6]

(b). $\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2}$. [6]

Question 15:

The line $= mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a). Obtain an expression for c in terms of a, b, and m. [6]

(b). Calculate the gradients of the tangents to the ellipse through the point $\left(\sqrt{(a^2+b^2)}, 0\right)$. [6]

Question 16:

The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding. The temperature of the body is initially 60°C. After 15 minutes, the temperature of the body is 50°C. The temperature of the surrounding is 10°C.

(a). Form a differential equation for the temperature of the body. [9]

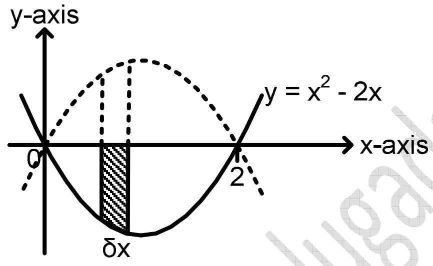
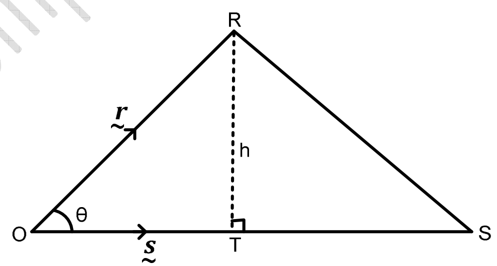
(b). Determine the time it takes for the temperature of the body to reach 30°C. [3]

END

MARKING GUIDE

<table>
<tr><th>SNo.</th><th>Working</th><th>Marks</th></tr>
<tr><td>1</td><td>(a).
$a^2 = b^2 + c^2 - 2bc\cos A$
$7^2 = 4^2 + 5^2 - 2\times 4\times 5\cos A$
$49 = 41 - 40\cos A$
$8 = -40\cos A$
$\cos A = -\frac{1}{5}$
(b).
$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(-\frac{1}{5}\right)^2} = \sqrt{\frac{24}{25}} = \frac{1}{5}\sqrt{24}$</td><td>M1
B1
A1
M1 A1</td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>2</td><td>direction vector, $\underset{\sim}{\boldsymbol{d}} = \begin{pmatrix}8\\2\\-4\end{pmatrix}$, normal vector, $\underset{\sim}{\boldsymbol{n}} = \begin{pmatrix}4\\3\\-3\end{pmatrix}$
Let θ be the angle between the given line and plane.
$\sin\theta = \frac{\underset{\sim}{\boldsymbol{n}}.\underset{\sim}{\boldsymbol{d}}}{\lvert\underset{\sim}{\boldsymbol{n}}\rvert\lvert\underset{\sim}{\boldsymbol{d}}\rvert}$
$\sin\theta = \frac{\begin{pmatrix}4\\3\\-3\end{pmatrix}.\begin{pmatrix}8\\2\\-4\end{pmatrix}}{\sqrt{4^2+3^2+(-3)^2}\times\sqrt{8^2+2^2+(-4)^2}}$
$\sin\theta = \frac{32+6+12}{\sqrt{34}\times\sqrt{84}}$
$\theta = 69.3255°$</td><td>B1
M1 M1 M1
M1
A1</td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>3</td><td><table><tr><th>Sign</th><th>Differentiation</th><th>Integration</th></tr><tr><td>+</td><td>x^2</td><td>e^x</td></tr><tr><td>−</td><td>x</td><td>e^x</td></tr><tr><td>+</td><td>1</td><td>e^x</td></tr><tr><td>−</td><td>0</td><td>e^x</td></tr></table>
$\therefore \int x^2e^x\,dx = x^2e^x - xe^x + e^x + c$
Alternatively:
let $u = x^2$ and $\frac{dv}{dx} = e^x$, $\Rightarrow \frac{du}{dx} = x$, $v = e^x$</td><td>M1 M1
B1 B1 B1</td></tr>
</table>

	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ $\int x^2e^x\,dx = x^2e^x - \int xe^x\,dx$ for $u = x$ and $\frac{dv}{dx} = e^x, \quad \Rightarrow \frac{du}{dx} = 1, \quad v = e^x$ $\therefore \int x^2e^x\,dx = x^2e^x - \left[xe^x - \int e^x\,dx\right]$ $= x^2e^x - xe^x + e^x + c$	
		05
4	$f(x) = \left(x + \frac{12}{2}\right)^2 - \left(\frac{12}{2}\right)^2 + 32 = (x+6)^2 - 4$ Hence, $[f(x)]_{\min} = 0^2 - 4 = -4$	**M1 M1 B1** **M1 A1**
		05
5	Let the variable point be $P(x, y)$ $\overline{AP}:\overline{PB} = 3:2$ $2\overline{AP} = 3\overline{PB}$ $2\sqrt{(x+2)^2 + (y-0)^2} = 3\sqrt{(x-8)^2 + (y-6)^2}$ $4(x^2 + 4x + 4 + y^2)$ $= 9(x^2 - 16x + 64 + y^2 - 12y + 36)$ $4x^2 + 16x + 16 + 4y^2$ $= 9x^2 - 144x + 9y^2 - 108y + 900$ $5x^2 + 5y^2 - 160x - 108y + 884 = 0$ Since x^2 and y^2 have the same coefficients and the rest of the terms are linear, then the locus is a circle.	**M1** **M1 M1** **B1** **B1**
		05
6	$\frac{d}{dx}(y^3 + y^2 - x^4) = \frac{d}{dx}(1)$ $3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 4x^3 = 0$ $(3y^2 + 2y)\frac{dy}{dx} = 4x^3$ $\frac{dy}{dx} = \frac{4x^3}{3y^2 + 2y}$ At the point $(1, 1)$, Gradient of tangent $= \frac{4 \times 1^3}{3 \times 1^2 + 2 \times 1} = \frac{4}{5}$ The required equation of the tangent is given by: $\frac{y-1}{x-1} = \frac{4}{5}$ $y - 1 = \frac{4}{5}x - \frac{4}{5}$ $y = \frac{4}{5}x + \frac{1}{5}$	**M1** **B1** **B1** **M1** **A1**

		05
7	$2\log 4 + \frac{1}{2}\log 25 - \log 20$ $= \log 4^2 + \log\sqrt{25} - \log 20$ $= \log 16 + \log 5 - \log 20$ $= \log\left(\frac{16 \times 5}{20}\right)$ $= \log 4$ $= \log 2^2$ $= 2\log 2$	**B1 B1** **M1** **B1** **A1**
		05
8	 element of volume, $\delta v = \pi y^2 \delta x$ total volume, $v = \int_0^2 \pi y^2\,dx = \pi\int_0^2 (x^2 - 2x)^2\,dx$ $= \pi\int_0^2 (x^4 - 4x^3 + 4x^2)\,dx$ $= \pi\left[\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3\right]_0^2$ $= \pi\left(\frac{1}{5} \times 2^5 - 2^4 + \frac{4}{3} \times 2^3\right) - 0 = \frac{16}{15}\pi$ cubic units	**M1** **M1** **M1** **B1 A1**
		05
9	 Area $= \frac{1}{2} \times$ base $\times$ height	

$$A = \frac{1}{2}|\overrightarrow{OS}||\overrightarrow{RT}|$$

$$2A = |\overrightarrow{OS}||\overrightarrow{OR}\sin\theta|$$ **M1**

$$2A = \frac{1}{2}\left|\underset{\sim}{s}\right|\left|\underset{\sim}{r}\sin\theta\right|$$

$$2A = \left|\underset{\sim}{s}\right|\left|\underset{\sim}{r}\right||\sin\theta|$$ **B1**

Squaring both sides gives,

$$4A^2 = \left|\underset{\sim}{r}\right|^2\left|\underset{\sim}{s}\right|^2|\sin^2\theta|$$ **M1**

$$4A^2 = \left|\underset{\sim}{r}\right|^2\left|\underset{\sim}{s}\right|^2|1-\cos^2\theta|$$ **B1**

$$4A^2 = \left|\underset{\sim}{r}\right|^2\left|\underset{\sim}{s}\right|^2 - \left|\underset{\sim}{r}\right|^2\left|\underset{\sim}{s}\right|^2|\cos^2\theta|$$

$$4A^2 = \left|\underset{\sim}{r}\right|^2\left|\underset{\sim}{s}\right|^2 - \left(\left|\underset{\sim}{s}\right|\left|\underset{\sim}{r}\right||\cos\theta|\right)^2$$ **B1**

$$\text{but } \underset{\sim}{r}.\underset{\sim}{s} = \left|\underset{\sim}{s}\right|\left|\underset{\sim}{r}\right|\cos\theta$$

$$\therefore 4A^2 = \left|\underset{\sim}{r}\right|^2\left|\underset{\sim}{s}\right|^2 - \left(\underset{\sim}{r}.\underset{\sim}{s}\right)^2, \quad \text{as required}$$ **B1**

(b).

$$4A^2 = \left|\underset{\sim}{r}\right|^2\left|\underset{\sim}{s}\right|^2 - \left(\underset{\sim}{r}.\underset{\sim}{s}\right)^2$$

$$4A^2 = \left(\sqrt{2^2+3^2}\right)^2\left(\sqrt{1^2+4^2}\right)^2 - \left[\begin{pmatrix}2\\3\end{pmatrix}.\begin{pmatrix}1\\4\end{pmatrix}\right]^2$$ **M1 M1 M1**

$$4A^2 = 13\times 17 - (2+12)^2$$ **M1**

$$4A^2 = 25$$

$$2A = 5$$ **B1**

$$A = 2.5 \text{ sq. units}$$ **A1**

12

10

$$\text{let } z = 5 + 12i$$

$$|z| = \sqrt{5^2+12^2} = 13$$ **B1**

$$\arg(z) = \tan^{-1}\left(\frac{12}{5}\right) = 67.38°$$ **B1**

$$\therefore z = 13(\cos 67.38° + i\sin 67.38°)$$ **B1**

For the hence part:

$$\sqrt[3]{(5+12i)} = \sqrt[3]{z}$$

$$= 13^{\frac{1}{3}}\left[\cos\left(\frac{67.38+360n}{3}\right) + i\sin\left(\frac{67.38+360n}{3}\right)\right]$$ **B1**

$$= 13^{\frac{1}{3}}[\cos(22.46+120n) + i\sin(22.46+120n)]$$

When $n = 0$

$$z_1 = 13^{\frac{1}{3}}[\cos 22.46° + i\sin 22.46°]$$

$$= 13^{\frac{1}{3}}[0.9241 + 0.3820i] = 2.17 + 0.90i$$ **M1** **A1**

When $n = 1$

$$z_2 = 13^{\frac{1}{3}}[\cos(22.46+120) + i\sin(22.46+120)]$$ **M1**

$$= 13^{\frac{1}{3}}[\cos 142.46° + i\sin 142.46°]$$

$$= 13^{\frac{1}{3}}[-0.7929 + 0.6093i] = -1.86 + 1.43i$$ **B1** **A1**

When $n = 2$

$$z_2 = 13^{\frac{1}{3}}[\cos(22.46+240) + i\sin(22.46+240)]$$ **M1**

$$= 13^{\frac{1}{3}}[\cos 262.46° + i\sin 262.46°]$$ **B1**

$$= 13^{\frac{1}{3}}[-0.1312 - 0.9913i] = -0.03 - 2.33i$$ **A1**

12

11 (a).

$$\text{let } y = \frac{x^3}{\sqrt{(1-2x^2)}}$$

$$u = x^3, \quad \Rightarrow \frac{du}{dx} = 3x^2$$ **B1**

$$v = (1-2x^2)^{\frac{1}{2}}, \quad \Rightarrow \frac{dv}{dx} = \frac{1}{2}(1-2x^2)^{-\frac{1}{2}} \times (-4x)$$

$$= -2x(1-2x^2)^{-\frac{1}{2}}$$ **B1**

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(1-2x^2)^{\frac{1}{2}} \times 3x^2 + x^3 \times 2x(1-2x^2)^{-\frac{1}{2}}}{(1-2x^2)}$$ **M1 M1**

$$= \frac{3x^2(1-2x^2)^{\frac{1}{2}} + 2x^4(1-2x^2)^{-\frac{1}{2}}}{(1-2x^2)}$$

$$= \frac{x^2(1-2x^2)^{-\frac{1}{2}}[3(1-2x^2)+2x^2]}{(1-2x^2)}$$ **B1**

$$= \frac{x^2(3-4x^2)}{(1-2x^2)^{3/2}}$$ **A1**

(b).

$$\frac{d}{dl}(T^2) = \frac{d}{dl}\left(\frac{4\pi^2 l}{g}\right)$$

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}$$ **M1 M1**

$$\frac{dT}{dl} = \frac{4\pi^2}{2gT}$$ **B1**

$$\text{but,} \quad \delta T \approx \frac{dT}{dl} \times \delta l$$

$$\delta T = \frac{4\pi^2}{2gT} \times \frac{2}{100}l$$ **M1**

$$\delta T = \frac{4\pi^2 l}{100gT}$$

$$\text{percentage error in } T = \frac{\delta T}{T} \times 100 = \left(\frac{4\pi^2 l}{100gT} \div T\right) \times 100$$ **M1**

$$= \frac{4\pi^2 l}{gT^2} = \frac{4\pi^2 l}{g} \times \frac{g}{4\pi^2 l} = 1$$ **A1**

12

12 (a).

$$L.H.S = \tan 4\theta = \tan(2\theta + 2\theta) = \frac{\tan 2\theta + \tan 2\theta}{1 - \tan 2\theta \tan 2\theta}$$ **M1**

$$= \frac{2\tan 2\theta}{1 - \tan^2 2\theta} = \frac{2\left(\frac{2t}{1-t^2}\right)}{1 - \left(\frac{2t}{1-t^2}\right)^2} = \frac{\left(\frac{4t}{1-t^2}\right)}{\left[\frac{(1-t^2)^2 - 4t^2}{(1-t^2)^2}\right]}$$ **B1 M1 B1**

$$= \frac{4t(1-t^2)}{1 - 2t^2 + t^4 - 4t^2} = \frac{4t(1-t^2)}{t^4 - 6t^2 + 1}, \quad \text{as required}$$ **B1 B1**

(b).

$$\sin x + \sin 5x = \sin 2x + \sin 4x$$
$$\sin 5x + \sin x = \sin 4x + \sin 2x$$
$$2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) = 2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)$$ **M1 M1**
$$\sin 3x \cos 2x = \sin 3x \cos x$$
$$\sin 3x \cos 2x - \sin 3x \cos x = 0$$
$$\sin 3x(\cos 2x - \cos x) = 0$$ **B1**
$$\sin 3x\left[-2\sin\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right] = 0$$ **M1**
$$-2\sin 3x \sin\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right) = 0$$
$$\sin 3x = 0, \quad \text{or,} \quad \sin\left(\frac{3x}{2}\right) = 0, \quad \text{or,} \quad \sin\left(\frac{x}{2}\right) = 0$$ **B1**

for $\sin 3x = 0$, $\quad 3x = 0°, 180°, 360°, 540°,$
$$\Rightarrow x = 0°, 60°, 120°, 180°$$
for $\sin\left(\frac{3x}{2}\right) = 0$, $\quad \frac{3x}{2} = 0°, 180°$, $\quad \Rightarrow x = 0°, 120°$

for $\sin\left(\frac{x}{2}\right) = 0$, $\quad \frac{x}{2} = 0°$, $\quad \Rightarrow x = 0°$

For the range $0° < x < 180°$, **B1**
$$x = 0°, 60°, 120°, 180°$$

12

13 (a).

$$a = 4, \quad r = \frac{8}{4} = 2$$ **B1 B1**
$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$
$$S_{10} = 4\left(\frac{2^{10} - 1}{2 - 1}\right) = 4092$$ **M1 A1**

(b).

Term	G.P	A.P	sequence
First term	a_1	a_2	$a_1 - a_2$
Third term	$a_1 \times 2^2 = 4a_1$	$a_2 + 2 \times 3 = a_2 + 6$	$4a_1 - a_2 - 6$
Sixth term	$a_1 \times 2^5 = 32a_1$	$a_2 + 5 \times 3 = a_2 + 15$	$32a_1 - a_2 - 15$

B1
B1

$$4a_1 - a_2 - 6 = 4, \quad \Rightarrow 4a_1 - a_2 = 10 \quad \rightarrow (1)$$ **B1**
$$32a_1 - a_2 - 15 = 79, \quad \Rightarrow 32a_1 - a_2 = 94 \quad \rightarrow (2)$$ **B1**

Equation (2) − (1) gives:
$$\begin{array}{r|l} 32a_1 - a_2 = 94 & \rightarrow (2) \\ -\ \ 4a_1 - a_2 = 10 & \rightarrow (1) \\ \hline 28a_1 = 84 & \end{array}$$ **M1**
$$a_1 = 3$$ **A1**

From equation (1),
$$4 \times 3 - a_2 = 10, \quad \Rightarrow a_2 = 2$$ **M1 A1**

(i). The first term of the A.P is 2.
(ii). The first term of the A.P is 3.

12

14 (a).

$$\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x\, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 8x + \sin 2x)\, dx$$ **M1**
$$= \frac{1}{2}\left[-\frac{1}{8}\cos 8x - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$ **M1**
$$= -\frac{1}{2}\left[\left(\frac{1}{8}\cos 4\pi + \frac{1}{2}\cos\pi\right) - \left(\frac{1}{8}\cos 0 + \frac{1}{2}\cos 0\right)\right]$$ **M1**
$$= -\frac{1}{2}\left[\left(\frac{1}{8} - \frac{1}{2}\right) - \left(\frac{1}{8} + \frac{1}{2}\right)\right]$$ **B1**
$$= \frac{1}{2}$$ **A1**

(b).

let $4x^2 = 9\tan^2 u$, $\quad \Rightarrow x = \frac{3}{2}\tan u \;$ and $u = \tan^{-1}\left(\frac{2x}{3}\right)$

$$\frac{dx}{du} = \frac{3}{2}\sec^2 u, \quad \Rightarrow dx = \frac{3}{2}\sec^2 u\, du$$ **B1**
$$\int \frac{dx}{9 + 4x^2}dx = \int \frac{1}{9 + 9\tan^2 u}\cdot\left(\frac{3}{2}\sec^2 u\, du\right) = \frac{3}{2}\int \frac{\sec^2 u}{9\sec^2 u}du$$ **M1**
$$= \int \frac{1}{6}du = \frac{1}{6}u + c = \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + c$$ **B1**
$$\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9 + 4x^2} = \frac{1}{6}\left[\tan^{-1}\left(\frac{2x}{3}\right)\right]_0^{\frac{\sqrt{3}}{2}}$$
$$= \frac{1}{6}\left[\tan^{-1}\left(\frac{2}{3} \times \frac{\sqrt{3}}{2}\right) - \tan^{-1} 0\right]$$ **M1**
$$= \frac{1}{6}\left(\frac{\pi}{6} - 0\right) = \frac{\pi}{36}$$ **M1 A1**

12

15 (a).

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{a^2} + \frac{(mx + c)^2}{b^2} = 1$$

	$b^2x^2 + a^2(mx+c)^2 = a^2b^2$	M1
	$b^2x^2 + a^2m^2x^2 + 2a^2mcx + a^2c^2 = a^2b^2$	
	$(b^2 + a^2m^2)x^2 + 2a^2mcx + a^2(c^2 - b^2) = 0 \quad \longrightarrow (1)$	B1
	Comparing equation (1) with the general quadratic equation $Ax^2 + Bx + C = 0$, it implies that	
	$A = (b^2 + a^2m^2), \quad B = 2a^2mc, \quad$ and, $C = a^2(c^2 - b^2)$	
	Therefore, for tangency,	
	$B^2 - 4AC = 0$	
	$(2a^2mc)^2 - 4a^2(c^2 - b^2)(b^2 + a^2m^2) = 0$	
	$4a^4m^2c^2 = 4a^2(c^2 - b^2)(b^2 + a^2m^2)$	M1
	$a^2m^2c^2 = b^2c^2 + a^2m^2c^2 - b^4 + a^2m^2b^2$	
	$0 = b^2c^2 - b^4 + a^2m^2b^2$	M1
	$0 = c^2 - b^2 + a^2m^2$	B1
	$c^2 = b^2 + a^2m^2$	
	(b).	B1
	$y = mx + c$	
	$0 = m \times \sqrt{(a^2+b^2)} \pm \sqrt{(b^2 + a^2m^2)}$	M1 M1
	$-m\sqrt{(a^2+b^2)} = \pm\sqrt{(b^2 + a^2m^2)}$	
	$m^2(a^2+b^2) = (b^2 + a^2m^2)$	
	$m^2a^2 + m^2b^2 = b^2 + a^2m^2$	M1
	$m^2b^2 = b^2$	
	$m^2 = 1$	B1
	$m = \pm 1$	A1 A1
	The gradients are 1 and -1.	
		12
16	(a). Let θ be the temperature of the body at a time t.	
	$\dfrac{d\theta}{dt} \propto (\theta - 10)$	B1
	$\dfrac{d\theta}{dt} = -k(\theta - 10)$	B1
	Where k is the proportionality constant.	
	$\displaystyle\int \frac{d\theta}{(\theta - 10)} = \int -k\, dt$	
	$\ln(\theta - 10) = -kt + c$	M1 M1
	But $\theta = 60°C$ when $t = 0$,	
	$\ln(60 - 10) = -k \times 0 + c, \quad \Rightarrow c = \ln 50$	M1 B1
	$\ln(\theta - 10) = -kt + \ln 50$	
	$\ln\left(\dfrac{\theta - 10}{50}\right) = -kt$	
	Also $\theta = 50°C$ when $t = 15$,	
	$\ln\left(\dfrac{50 - 10}{50}\right) = -15k$	M1
	$\ln 0.8 = -15k$	
	$k = -\dfrac{\ln 0.8}{15}$	B1
	$\ln\left(\dfrac{\theta - 10}{50}\right) = t\dfrac{\ln 0.8}{15}$	B1

	Accept: $\theta = 10 + 50e^{-kt}$, where $k = -\frac{\ln 0.8}{15}$	
	(b). When $\theta = 30°C$,	
	$\ln\left(\dfrac{30 - 10}{50}\right) = t\dfrac{\ln 0.8}{15}$	M1
	$\ln 0.4 = t\dfrac{\ln 0.8}{15}$	B1
	$t = \dfrac{15 \ln 0.4}{\ln 0.8} = 61.5943$ minutes	A1
		12

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Nov./Dec. 2018
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

S.6 MATH 2 UNEB 2018

Time: 3 Hours

NAME: ____________________ **COMB:** ________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section*

Qn 1: A stone is thrown vertically upwards with a velocity of 21 m s^{-1}.
Calculate the:

(a). maximum height attained by the stone. [3]

(b). time the stone takes to reach the maximum height. [2]

Qn 2: Two events A and B are such that $P(A/B) = \frac{2}{5}$, $P(B) = \frac{1}{4}$ and $P(A) = \frac{1}{5}$.
Find:
(a). $P(A \cap B)$. [2]
(b). $P(A \cup B)$. [3]

Qn 3: The table below shows how T varies with S.

T	-2.9	-0.1	2.9	3.1
S	30	20	12	9

Use linear interpolation/extrapolation to estimate the value of:
(a). T when $S = 26$. [3]
(b). S when $T = 3.4$. [2]

Qn 4: A particle of mass 15 kg is pulled up a smooth slope by a light inextensible string parallel to a slope . The slope is 10.5 m long inclined at $\sin^{-1}\left(\frac{4}{7}\right)$ to the horizontal. The acceleration of the particle is 0.98 m s^{-2}. Determine the:
(a). tension in the string. [3]
(b). work done against gravity when the particle reaches the end of the slope. [2]

Qn 5: The price index of an article in 2000 based on 1998 was 130. The price index of the article in 2005 based on 2000 was 80. Calculate the:
(a). price index of the article in 2005 based on 1998. [3]
(b). price of the article in 1998 if the article was 45,000 in 2005. [2]

Qn 6: Two numbers A and B have maximum possible errors e_a and e_b respectively.
(a). Write an expression for the maximum possible error in their sum.
(b). If $A = 2.03$ and $B = 1.547$, find the maximum possible error in $A + B$. [5]

Qn 7: In an equilateral triangle PQR, three forces of magnitude 5 N, 10 N and 8 N act along the sides PQ, QR and PR respectively. Their forces are in the order of the letters. Find the magnitude of the resultant force. [5]

Qn 8: A biased coin is such that a head is three times as likely to occur as a tail. The coin is tossed 5 times. Find the probability that at most two tails occur. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
The frequency distribution below shows the ages of 240 students admitted to a certain University.

Age (years)	Number of students
$18-<19$	24
$19-<20$	70
$20-<24$	76
$24-<26$	48
$26-<30$	16
$30-<32$	6

(a). Calculate the mean age of the students. [4]
(b). (i). Draw the histogram for the given data.
(ii). Use the histogram to estimate the modal age. [8]

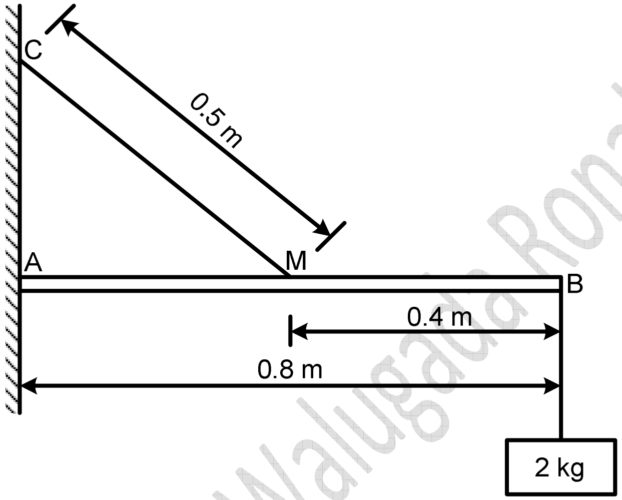
Question 10:
A particle of mass 4 kg starts from rest at a point $\left(2\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}}\right)$ m. It moves with acceleration $\underset{\sim}{\boldsymbol{a}} = \left(4\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}} - 3\underset{\sim}{\boldsymbol{k}}\right) \text{m s}^{-2}$ when a constant force, $\underset{\sim}{\boldsymbol{F}}$, acts on it. Find the:
(a). force $\underset{\sim}{\boldsymbol{F}}$. [2]
(b). velocity at any time t. [4]
(c). work done by the force $\underset{\sim}{\boldsymbol{F}}$after 6 seconds. [6]

Question 11:
(a). Use trapezium rule with 6-ordinates to estimate the value of $\int_0^{\frac{\pi}{2}}(x + \sin x)\,dx$, correct to **three** decimal places. [6]
(b). (i). Evaluate $\int_0^{\frac{\pi}{2}}(x + \sin x)\,dx$, correct to **three** decimal places. [3]
(ii). Calculate the error in your estimation in (a) above. [2]
(iii). Suggest how the error may be reduced. [1]

Question 12:
A random variable X has a normal distribution where $P(X > 9) = 0.9192$ and $P(X < 11) = 0.7580$. Find:

(a). the values of the mean and standard deviation. [8]
(b). $P(X > 10)$. [4]

Question 13:
The figure below shows a uniform beam of length 0.8 metres and mass 1 kg. the beam is hinged at A and has a load of mass 2 kg attached at B.



The beam is held in a horizontal position by a light inextensible string of length 0.5 metres. The string joins the mid-point M of the beam of a point C vertically above A. Find the:
(a). tension in the string. [8]
(b). magnitude and direction of the force exerted by the hinge. [4]

Question 14:
(a). Draw on the same axes the graphs of the curves $y = 2 - e^{-x}$ and $y = \sqrt{x}$ for $2 \leq x \leq 5$. [5]
(b). Determine from your graphs the interval within which the root of the equation $e^{-x} + \sqrt{x} - 2 = 0$ lies. Hence, use Newton Raphson's method to find the root of the equation correct to **3** decimal places. [7]

Question 15:
The table below shows the number of red and green balls put in three identical boxes A, B and C.

Boxes	A	B	C
Red balls	4	6	3
Green balls	2	7	5

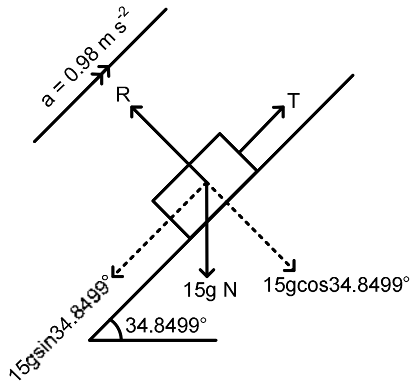
A box is chosen at random and two balls are then drawn from it successively without replacement. If the random variable X is "the number of green balls drawn",
(a). draw a probability distribution table for X. [6]
(b). calculate the mean and variance of X. [6]

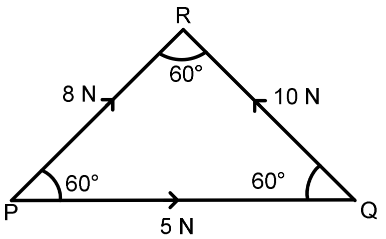
Question 16:
At 10:00 am, ship A and ship B are 16 km apart. Ship A is on a bearing $N\ 35°\ E$ from ship B. Ship A is travelling at 14 km h^{-1} on a bearing $S\ 29°\ E$. Ship B is travelling at 17 km h^{-1} on a bearing $N\ 50°\ E$. Determine the:
(a). velocity of ship B relative to ship A. [5]
(b). closest distance between the two ships and the time when it occurs. [7]

END

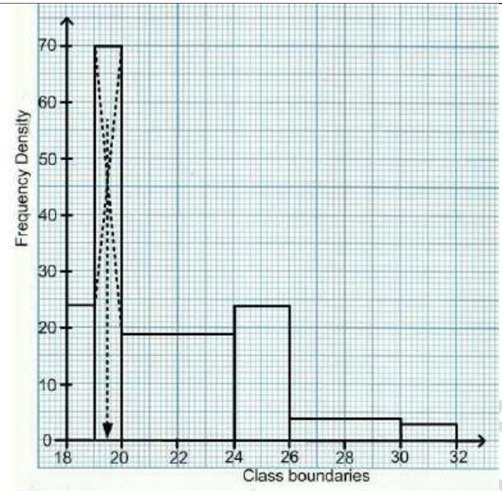
MARKING GUIDE

SNo.	Working	Marks
1	(a). $v^2 = u^2 - 2gs$ $0 = 21^2 - 2 \times 9.8 \times h_{\max}$ $441 = 19.6h_{\max}$ $h_{\max} = \frac{441}{19.6} = 22.5\text{ m}$ (b). $v = u - gt$ $0 = 21 - 9.8 \times t$ $9.8t = 21$ $t = 2.1429\text{ s}$	M1 M1 A1 M1 A1
		05
2	(a). $P(A \cap B) = P(B).P(A/B) = \frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$ (b). $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= \frac{1}{5} + \frac{1}{4} - \frac{1}{10} = \frac{7}{20}$	M1 A1 M1 M1 A1
		05
3	(a). $\begin{array}{\|c\|c\|c\|}\hline -2.9 & T & -0.1 \\ \hline 30 & 26 & 20 \\ \hline \end{array}$ $\frac{T-(-2.9)}{-0.1-(-2.9)} = \frac{26-30}{20-30}$ $\frac{T+2.9}{2.8} = \frac{-4}{-10}$ $T = 0.4 \times 2.8 - 2.9 = -1.78$ (b). $\begin{array}{\|c\|c\|c\|}\hline 2.9 & 3.1 & 3.4 \\ \hline 12 & 9 & S \\ \hline \end{array}$ $\frac{S-9}{12-9} = \frac{3.4-3.1}{2.9-3.1}$ $\frac{S-9}{3} = \frac{0.3}{-0.2}$ $S = -1.5 \times 3 + 9 = 4.5$	B1 M1 A1 M1 A1
		05
4	$\sin^{-1}\left(\frac{4}{7}\right) = 34.8499°$	

	 (a). $T - 15g\sin\theta = 15 \times 0.98$ $T - 15 \times 9.8 \times \frac{4}{7} = 14.7$ $T - 84 = 14.7$ $T = 98.7\text{ N}$ (b). Vertical height, $h = 10.5\sin\theta = 10.5 \times \frac{4}{7} = 6\text{ m}$ Work done against gravity $= mgh$ $= 15 \times 9.8 \times 6 = 882\text{ J}$	M1 M1 A1 M1 A1
		05
5	(a). $\frac{P_{2000}}{P_{1998}} \times 100 = 130, \quad \Rightarrow P_{1998} = \frac{P_{2000}}{1.3}$ $\frac{P_{2005}}{P_{2000}} \times 100 = 80, \quad \Rightarrow P_{2005} = 0.8P_{2000}$ $I_{2005} = \frac{P_{2005}}{P_{1998}} \times 100 = 0.8P_{2000} \times \frac{1.3}{P_{2000}} \times 100 = 104$ (b). $I_{2005} = \frac{P_{2005}}{P_{1998}} \times 100$ $104 = \frac{45{,}000}{P_{1998}} \times 100$ $P_{1998} = \frac{4{,}500{,}000}{104} = \text{shs } 43{,}269.23077$	M1 M1 A1 M1 A1
		05
6	(a). Maximum possible error in sum $= \|e_a\| + \|e_b\|$ (b). $A = 2.03, \quad \Rightarrow e_a = 0.005$	B1 B1

	$B = 1.547, \quad \Rightarrow e_a = 0.0005$ Maximum possible error in $(A+B)$ $= \|0.005\| + \|0.0005\| = 0.0055$	B1 M1 A1
		05
7	 Let F be the resultant force, $\underset{\sim}{F} = \begin{pmatrix}5\\0\end{pmatrix} + \begin{pmatrix}-10\cos 60^\circ\\10\sin 60^\circ\end{pmatrix} + \begin{pmatrix}8\cos 60^\circ\\8\sin 60^\circ\end{pmatrix} = \begin{pmatrix}4\\15.5885\end{pmatrix}$ N Magnitude, $\left\|\underset{\sim}{F}\right\| = \sqrt{4^2 + (15.5885)^2} \approx 16.0935$ N	B1 B1 M1 M1 A1
		05
8	let $P(T) = t, \quad \Rightarrow P(H) = 3t$ but $t + 3t = 1, \quad \Rightarrow t = \frac{1}{4}$ $\therefore p = 0.25$, $\quad q = 1 - 0.25 = 0.75$, $\quad n = 5$ Let $X \sim B(5,\ 0.25)$ $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$ $= 0.2373 + 0.3955 + 0.2637$ $= 0.8965$	 B1 M1 M1M1 A1
		05
9	(table below) (a). Mean age $= \frac{\sum fx}{\sum f} = \frac{5315}{240} = 22.1458$ years (b). (i).	B1-for x B1-for fx B1-for $\sum fx$ B1-class bloundaries B1-for f/c M1 A1

Age (years)	f	x	fx	Class boundaries	c	f/c
18–< 19	24	18.5	444	18 – 19	1	24
19–< 20	70	19.5	1365	19 – 20	1	70
20–< 24	76	22.0	1672	20 – 24	4	19
24–< 26	48	25.0	1200	24 – 26	2	24
26–< 30	16	28.0	448	26 – 30	4	4
30–< 32	6	31.0	186	30 – 32	2	3
Total	240		5315			

	 (ii). From the histogram, the modal age is 19.5 years.	B1-axes B2-bars M1-attempting to find mode A1
		12
10	(a). $\underset{\sim}{F} = m\underset{\sim}{a} = 4\begin{pmatrix}4\\2\\-3\end{pmatrix} = \begin{pmatrix}16\\8\\-12\end{pmatrix}$N (b). $\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t = \begin{pmatrix}0\\0\\0\end{pmatrix} + t\begin{pmatrix}4\\2\\-3\end{pmatrix}$ $= \left(4t\underset{\sim}{i} + 2t\underset{\sim}{j} - 3t\underset{\sim}{k}\right)$ m s^{-1} (c). When $t = 6$s, displacement, $\underset{\sim}{s} = \underset{\sim}{u}t + \frac{1}{2}\underset{\sim}{a}t^2 = 6\begin{pmatrix}0\\0\\0\end{pmatrix} + 18\begin{pmatrix}4\\2\\-3\end{pmatrix} = \begin{pmatrix}72\\36\\-54\end{pmatrix}$m Work done $= \underset{\sim}{F}.\underset{\sim}{s} = \begin{pmatrix}16\\8\\-12\end{pmatrix}.\begin{pmatrix}72\\36\\-54\end{pmatrix}$ $= 1152 + 288 + 648 = 2088$ J	 M1 A1 M1 M1 A1 M1 M1M1 B1 M1 M1 A1
		12
11	(a). $y_n = x_n + \sin x_n$, $\quad h = \frac{\frac{\pi}{2} - 0}{6 - 1} = \frac{\pi}{10}$ (table below)	 B1 B1-for x_n

n	x_n	y_0, y_5	$y_1, \ldots y_4$
0	0	0	
1	$\frac{\pi}{10}$		0.62318

2	$\frac{\pi}{5}$		1.21610
3	$\frac{3\pi}{10}$		1.75149
4	$\frac{2\pi}{5}$		2.20769
5	$\frac{\pi}{2}$	2.57080	
sums		2.57080	5.79846

B1-for y_0, y_5

B1-for $y_1, \ldots y_4$

$$\int_0^{\frac{\pi}{2}} (x+\sin x)\,dx \approx \frac{1}{2}h[(y_0+y_4)+2(y_1+\cdots+y_3)]$$

$$\approx \frac{1}{2}\times\frac{\pi}{10}\times[2.57080+2\times 5.79846] = 2.22546$$ **M1**

$$\approx 2.225 \text{ (3 d.p)}$$

(b). (i). **A1**

$$\int_0^{\frac{\pi}{2}} (x+\sin x)\,dx = \left[\frac{1}{2}x^2-\cos x\right]_0^{\frac{\pi}{2}}$$ **M1**

$$= \left[\left(\frac{1}{2}\times\frac{\pi^2}{4}-\cos\frac{\pi}{2}\right)-\left(\frac{1}{2}\times 0-\cos 0\right)\right]$$ **M1**

$$= \frac{\pi^2}{8}-0+1 = 2.234 \text{ (3 d.p)}$$ **B1**

(ii).

$$\text{Error} = |2.234-2.225| = 0.009$$ **M1 A1**

(iii). The error may be reduced by increasing the number of ordinates. **B1**

12

12 (a).

$$P(X>9) = 0.9192$$
$$P(Z>z_1) = 0.9192$$
$$0.5+\phi(-z_1) = 0.9192$$
$$\phi(-z_1) = 0.4192$$
$$-z_1 = \phi^{-1}(0.4192) = -1.400$$
$$z_1 = -1.400$$ **B1**
$$\frac{9-\mu}{\sigma} = -1.4$$
$$9-\mu = -1.4\sigma \quad \longrightarrow (1)$$ **B1**

$$P(X<11) = 0.7580$$
$$P(Z<z_2) = 0.7580$$
$$0.5+\phi(z_2) = 0.7580$$
$$\phi(z_2) = 0.2580$$ **B1**
$$z_2 = \phi^{-1}(0.2580)$$
$$z_2 = 0.700$$ **B1**
$$\frac{11-\mu}{\sigma} = 0.7$$
$$11-\mu = 0.7\sigma \quad \longrightarrow (2)$$ **B1**

Equation (2) − (1) gives,

$$2 = 2.1\sigma$$ **M1**
$$\sigma = 0.9524$$ **A1**

From equation (1),

$$\mu = 9+1.4\sigma = 9+1.4\times 0.9524 = 10.3334$$ **M1 A1**

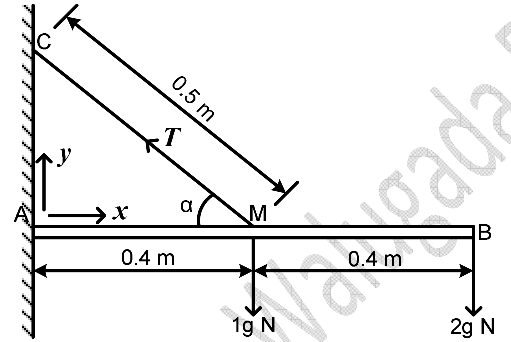
The value of the mean is 10.3334 and the standard deviation is 0.9524.

(b).

$$P(X>10) = P\left(Z>\frac{10-10.3334}{0.9524}\right) = P(Z>0.350)$$ **M1**
$$= 0.5-\phi(0.350) = 0.5-0.1368 = 0.3632$$ **M1 A1**

12

13 (a).



From triangle AMC,

$$\cos\alpha = \frac{0.4}{0.5} = 0.8, \qquad \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-(0.8)^2} = 0.6$$ **B1**

Taking moments about A,

$$T\times 0.4\sin\alpha = 1g\times 0.4+2g\times 0.8$$
$$T\times 0.4\times 0.6 = 9.8\times 0.4+2\times 9.8\times 0.8$$ **M1 M1**
$$0.24T = 19.6$$
$$T = 81.6667 \text{ N}$$ **A1**

(b).

Resolving vertically,

$$y+81.6667\sin\alpha = 1g+2g$$
$$y+81.6667\times 0.6 = 3\times 9.8$$ **M1 M1**
$$y = 29.4-49.00$$
$$y = -19.6 \text{ N}$$ **B1**

Resolving horizontally,

$$x = 81.6667\cos\alpha$$
$$x = 81.6667\times 0.8$$ **M1**
$$x = 65.3334 \text{ N}$$

Let F be the force exerted by the hinge,

$$F = \sqrt{(65.3334)^2 + (19.6)^2} = 68.2101\text{ N}$$ **M1 A1**

$$\tan\theta = \frac{19.6}{65.3334}, \qquad \Rightarrow \theta = 16.6992°$$ **M1 A1**

The force exerted by the hinge is 68.2101 N in the direction 16.6992° below the positive horizontal.

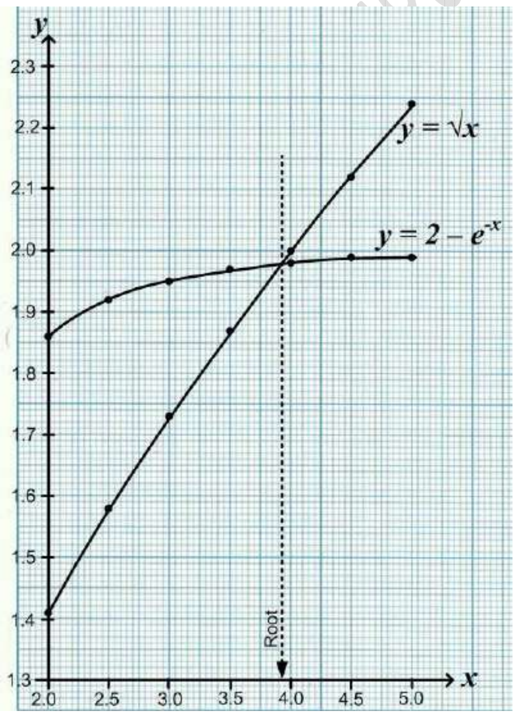
12

14

let, $y_1 = 2 - e^{-x}$, $\quad y_2 = \sqrt{x}$

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
y_1	1.86	1.92	1.95	1.97	1.98	1.99	1.99
y_2	1.41	1.58	1.73	1.87	2.00	2.12	2.24

B1
B1



B1-$y = 2 - e^{-x}$

B1-$y = \sqrt{x}$

B1-axes

M1-indicating where the root is.

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From the graph, the roots lies between $x = 3.5$ and $x = 4.0$.

For the hence part:

$$f(x) = e^{-x} + \sqrt{x} - 2, \qquad \Rightarrow f'(x) = e^{-x} + \frac{1}{2\sqrt{x}} = \frac{2e^{-x}\sqrt{x} + 1}{2\sqrt{x}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad \text{where } n = 0, 1, 2, \dots\dots$$

$$x_{n+1} = x_n - \frac{\left(e^{-x_n} + \sqrt{x_n} - 2\right) \times 2\sqrt{x_n}}{2e^{-x_n}\sqrt{x_n} + 1}$$ **M1 B1**

The initial approximation of the root $x_0 = 3.5 + 8.5 \times 0.05 = 3.925$. **B1**

$$x_1 = 3.925 - \frac{\left(e^{-3.925} + \sqrt{3.925} - 2\right) \times 2\sqrt{3.925}}{2e^{-3.925} \times \sqrt{3.925} + 1} = 3.92168$$ **B1**

$$|x_1 - x_0| = |3.92168 - 3.925| = 0.00332 > 0.0005$$

$$x_2 = 3.92168 - \frac{\left(e^{-3.92168} + \sqrt{3.92168} - 2\right) \times 2\sqrt{3.92168}}{2e^{-3.92168} \times \sqrt{3.92168} + 1}$$ **B1**

$$= 3.92120$$

$$|x_2 - x_1| = |3.92120 - 3.92168| = 0.00048 < 0.0005$$

$$\therefore \text{Root} = 3.921 \text{ (3 d.p)}$$ **A1**

12

15 (a).

$$P(X = 0) = P(\text{no green from A}) + P(\text{no green from B}) + P(\text{no green from C})$$

$$= \frac{1}{3} \times \frac{{}^2C_0 \times {}^4C_2}{{}^6C_2} + \frac{1}{3} \times \frac{{}^7C_0 \times {}^6C_2}{{}^{13}C_2} + \frac{1}{3} \times \frac{{}^5C_0 \times {}^3C_2}{{}^8C_2}$$ **M1**

$$= \frac{1}{3}\left(\frac{1 \times 6}{15} + \frac{1 \times 15}{78} + \frac{1 \times 3}{28}\right) = \frac{1273}{5460}$$ **B1**

$$P(X = 1) = P(\text{1 green from A}) + P(\text{1 green from B}) + P(\text{1 green from C})$$

$$= \frac{1}{3} \times \frac{{}^2C_1 \times {}^4C_1}{{}^6C_2} + \frac{1}{3} \times \frac{{}^7C_1 \times {}^6C_1}{{}^{13}C_2} + \frac{1}{3} \times \frac{{}^5C_1 \times {}^3C_1}{{}^8C_2}$$ **M1**

$$= \frac{1}{3}\left(\frac{2 \times 4}{15} + \frac{7 \times 6}{78} + \frac{5 \times 3}{28}\right) = \frac{8777}{16380}$$ **B1**

$$P(X = 2) = P(\text{2 green from A}) + P(\text{2 green from B}) + P(\text{2 green from C})$$

$$= \frac{1}{3} \times \frac{{}^2C_2 \times {}^4C_0}{{}^6C_2} + \frac{1}{3} \times \frac{{}^7C_2 \times {}^6C_0}{{}^{13}C_2} + \frac{1}{3} \times \frac{{}^5C_2 \times {}^3C_0}{{}^8C_2}$$ **M1**

$$= \frac{1}{3}\left(\frac{1 \times 1}{15} + \frac{21 \times 1}{78} + \frac{10 \times 1}{28}\right) = \frac{946}{4095}$$ **B1**

Alternatively:

$$P(X = 0) = P(\text{no green from A}) + P(\text{no green from B}) + P(\text{no green from C})$$

$$= \frac{1}{3} \times \frac{4}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{6}{13} \times \frac{5}{12} + \frac{1}{3} \times \frac{3}{8} \times \frac{2}{7} = \frac{2}{15} + \frac{5}{78} + \frac{1}{28} = \frac{1273}{5460}$$

$$P(X=2) = P(\text{2 green from A}) + P(\text{2 green from B}) + P(\text{2 green from C})$$

$$= \frac{1}{3}\times\frac{2}{6}\times\frac{1}{5}+\frac{1}{3}\times\frac{7}{13}\times\frac{6}{12}+\frac{1}{3}\times\frac{5}{8}\times\frac{4}{7} = \frac{1}{45}+\frac{7}{78}+\frac{5}{42} = \frac{946}{4095}$$

$$P(X=1) = 1-\left(\frac{1273}{5460}+\frac{946}{4095}\right) = \frac{8777}{16380}$$

x	$P(X=x)$	$xP(X=x)$	$x^2P(X=x)$
0	$\frac{1273}{5460}$	0	0
1	$\frac{8777}{16380}$	$\frac{8777}{16380}$	$\frac{8777}{16380}$
2	$\frac{946}{4095}$	$\frac{1892}{4095}$	$\frac{3784}{4095}$
Total		$\frac{467}{468}$	$\frac{2657}{1820}$

B1-for $P(X=x)$

(b).

$$E(X) = \sum_{all\,x} xP(X=x) = \frac{467}{468} \approx 0.9979$$ **M1 A1**

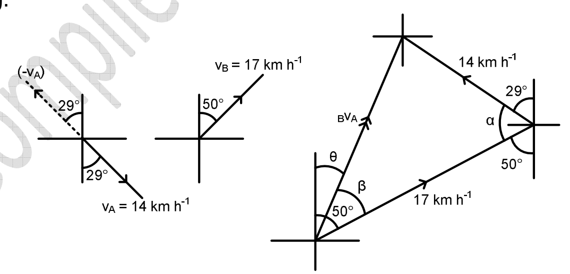
$$Var(X) = \sum_{all\,x} x^2P(X=x) - \left[\sum_{all\,x} xP(X=x)\right]^2$$

$$= \frac{2657}{1820} - \left(\frac{467}{468}\right)^2 = 0.4642$$ **M1 M1 A1**

12

16 **Method 1: Geometrical method**

(a).



B1 B1

$$\alpha = 180-(50+29) = 101°$$

$${}_Bv_A = \sqrt{17^2+14^2-2\times17\times14\cos101°} = 23.9964 \text{ km h}^{-1}$$ **M1 B1**

$$\frac{\sin\beta}{14} = \frac{\sin 101°}{{}_Bv_A}, \quad \Rightarrow \sin\beta = \frac{14\sin101°}{23.9964} = 0.5727,$$

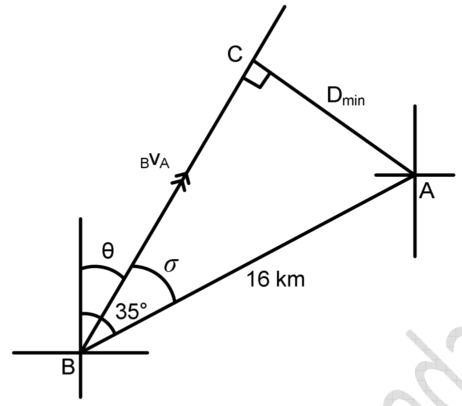
$$\Rightarrow \beta = 34.9388°$$

$$\beta = 50 - 34.9388 = 15.0612°$$

Direction = N 15.0612° E **A1**

The velocity of B relative to A is 23.9964 km h^{-1} in the direction N 15.0612° E.

(b).



B1-for Dmin

B1-for relative path

$$\sigma = 35 - 15.0612 = 19.9388°$$

$$D_{min} = AC = 16\sin 19.9388° = 5.4563 \text{ km}$$ **M1 A1**

$$t_{min} = \frac{BC}{{}_Bv_A} = \frac{16\cos19.9388°}{23.9964} = 0.6268 \text{ hours} = 38 \text{ minutes}$$ **M1 B1**

1000 hours
\+ 0038 hours
1038 hours

The ships are nearest each other at 10:38 am and the closest distance between them is 5.4563 km. **A1**

Method 2: Vector method

(a).

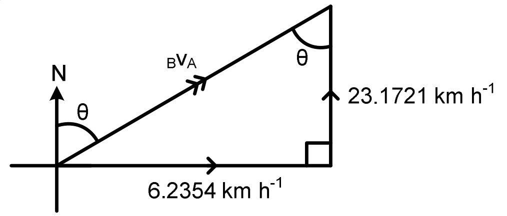
$$\underset{\sim}{v}_A = \begin{pmatrix}14\sin29°\\-14\cos29°\end{pmatrix} \text{ km h}^{-1}, \quad \underset{\sim}{v}_B = \begin{pmatrix}17\sin50°\\17\cos50°\end{pmatrix} \text{ km h}^{-1}$$

$${}_B\underset{\sim}{v}_A = \underset{\sim}{v}_B - \underset{\sim}{v}_A = \begin{pmatrix}17\sin50°\\17\cos50°\end{pmatrix} - \begin{pmatrix}14\sin29°\\-14\cos29°\end{pmatrix}$$

$$= \begin{pmatrix}6.2354\\23.1721\end{pmatrix} \text{ km h}^{-1}$$

Magnitude, $\left|{}_B\underset{\sim}{v}_A\right| = \sqrt{(6.2354)^2+(23.1721)^2}$

$$= 23.9964 \text{ km h}^{-1}$$



$$\tan\theta = \frac{6.2354}{23.1721}, \qquad \Rightarrow \theta = 15.0610°$$

Direction of ${}_Bv_A$ = N 15.0610° E

(b).

$${}_B\underset{\sim}{\boldsymbol{r}}_A = \begin{pmatrix}-16\sin 35°\\-16\cos 35°\end{pmatrix} = \begin{pmatrix}-9.1772\\-13.1064\end{pmatrix}\text{ km}$$

$${}_B\underset{\sim}{\boldsymbol{r}}_A(t) = {}_B\underset{\sim}{\boldsymbol{r}}_A + t\,{}_B\underset{\sim}{\boldsymbol{v}}_A = \begin{pmatrix}-9.1772\\-13.1064\end{pmatrix} + t\begin{pmatrix}6.2354\\23.1721\end{pmatrix}$$

$$t_{min} = \frac{\left|{}_B\underset{\sim}{\boldsymbol{r}}_A \cdot {}_B\underset{\sim}{\boldsymbol{v}}_A\right|}{\left|{}_B\underset{\sim}{\boldsymbol{v}}_A\right|^2} = \frac{\left|\begin{pmatrix}-9.1772\\-13.1064\end{pmatrix}\cdot\begin{pmatrix}6.2354\\23.1721\end{pmatrix}\right|}{(23.9964)^2}$$

$$= \frac{|-57.2235 - 303.7028|}{(23.9964)^2} = 0.6268\text{ hours} = 38\text{ minutes}$$

$${}_B\underset{\sim}{\boldsymbol{r}}_A(0.6268) = \begin{pmatrix}-9.1772\\-13.1064\end{pmatrix} + 0.6268\begin{pmatrix}6.2354\\23.1721\end{pmatrix}$$

$$= \begin{pmatrix}-5.2689\\1.4179\end{pmatrix}\text{ km}$$

$$D_{min} = \left|{}_B\underset{\sim}{\boldsymbol{r}}_A(0.6268)\right| = \sqrt{(-5.2689)^2 + (-1.4179)^2}$$

$$= 5.4563\text{ km}$$

$$\begin{array}{rl} & 1000\ \text{hours}\\ + & \underline{0038\ \text{hours}}\\ & 1038\ \text{hours}\end{array}$$

∴ The ships are nearest each other at 10:38 am and the closest distance between them is 5.4563 km.

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END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Nov./Dec. 2017
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD
S.6 MATH 2 UNEB 2017
Time: 3 Hours

NAME: ______________________ **COMB:** __________

INSTRUCTIONS:
- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section*

Qn 1: A particle is projected from a point O with speed 20 m s^{-1} at an angle of 60° to the horizontal. Express in vector form its velocity $\boldsymbol{v}$ and its displacement $\boldsymbol{r}$, from O at any time $\boldsymbol{t}$ seconds. [5]

Qn 2: The probability that a patient suffering from a certain disease recovers is 0.4. If 15 people contracted the disease, find the probability that:
(a). more than 9 will recover. [2]
(b). between five and eight will recover. [3]

Qn 3: The table below gives values of x and the corresponding values of $f(x)$.

x	0.1	0.2	0.3	0.4	0.5	0.7
$f(x)$	4.21	3.83	3.25	2.85	2.25	1.43

Use linear interpolation/extrapolation to find:

(a). $f(x)$ when $x = 0.6$. [3]
(b). the value of $x$when $f(x) = 0.75$. [2]

Qn 4: In a square ABCD, three forces of magnitudes 4 N, 10 N and 7 N act along AB, AD and CA respectively. Their directions are in the order of the letters. Find the magnitude of the resultant force. [5]

Qn 5: A box A contains 1 white ball and 1 blue ball. Box B contains only 2 white balls. If a ball is picked at random, find the probability that it is:
(a). white. [3]
(b). from box A given that it is white. [2]

Qn 6: Given that $y = \frac{1}{x} + x$ and $x = 2.4$ correct to **one** decimal place, find the limits within which y lies. [5]

Qn 7: The table below shows the retail prices (shs) and ammount of each item bought weekly by a restaurant in 2002 and 2003.

Item	**Price (Shs)**		**Ammount bought**
	2002	**2003**	
Milk (per litre)	400	500	200
Eggs (per tray)	2,500	3,000	18
Cooking oil (per litre)	2,400	2,100	2
Baking flour (per packet)	2,000	2,200	15

(a). Taking 2002 as the base year, calculate the weighted aggregate price index. [3]
(b). In 2003, the restaurant spent shs. 450,000 on buying these items. Using the weighted aggregate price index obtained in (a), calculate what the restaurant could have spent in 2002. [2]

Qn 8: The engine of a lorry of mass 5000 kg is working at a steady rate of 350 kW against a constant resistance force of 1000 N. The lorry ascends a slope of inclination $\theta°$ to the horizontal. If the maximum speed of the lorry is 20 m s^{-1}, find the value of θ. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:
A discrete random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} kx & ; \quad x = 1, 2, 3, 4, 5, \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

where$k$ is a constant.
Determine:
(a). the value of k. [3]
(b). $P(2 < X < 5)$. [2]
(c). Expectation, $E(X)$. [3]
(d). Variance, $Var(X)$. [4]

Question 10:
A particle of mass 3 kg is acted upon by a force $\underset{\sim}{\boldsymbol{F}} = 6\underset{\sim}{\boldsymbol{i}} - 36t^2\underset{\sim}{\boldsymbol{j}} + 54t\underset{\sim}{\boldsymbol{k}}$ Newtons at time t. At time $t = 0$, the particle is at the position vector $\underset{\sim}{\boldsymbol{i}} - 5\underset{\sim}{\boldsymbol{j}} - \underset{\sim}{\boldsymbol{k}}$and its velocity is $3\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}}$m s^{-1}. Determine the:
(a). position vector of the particle at time $t = 1$ second. [9]
(b). distance of the particle from the origin at time $t = 1$ second. [3]

Question 11:
A student used the trapezium rule with five sub-intervals to estimate $\int_2^3 \frac{x}{(x^2-3)}\,dx$ correct to **three** decimal places.
Determine:
(a). the value the student obtained. [6]
(b). the actual value of the integral. [3]
(c). (i). the error the student made in the estimate.
(ii). How the student can reduce the error. [3]

Question 12:
The times taken for S5 students to have their lunch to the nearest minute are given in the table below.

Time (minutes)	3 – 4	5 – 9	10 – 19	20 – 29	30 – 44
Number of students	2	7	16	21	9

(a). Calculate the mean time for the students to have lunch. [4]
(b). (i). Draw a histogram for the given data.

(ii). Use your histogram to estimate the modal time for the students to have their lunch. [8]

Question 13:

A non-uniform rod AB of mass 10 kg has its centre gravity at a distance $\frac{1}{4}\overline{AB}$ from B. The rod is smoothly hinged at A. It is maintained in equilibrium at 60° to above the horizontal by a light inextensible string tied at B and at a right angle to AB. Calculate the magnitude and direction of the reaction at A. [12]

Question 14:

By plotting graphs of $y = x$ and $y = 4\sin x$ on the same axes, show that the root of the equation $x - 4\sin x = 0$ lies between 2 and 3.

Hence use Newton Raphson's method to find the root of the equation correct to **3** decimal places. [12]

Question 15:

The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows. 5% of the residents have over 90 cows.

(a). Determine the values of the mean and standard deviation of the cows. [8]

(b). If there are 200 residents, find how many have more than 80 cows. [4]

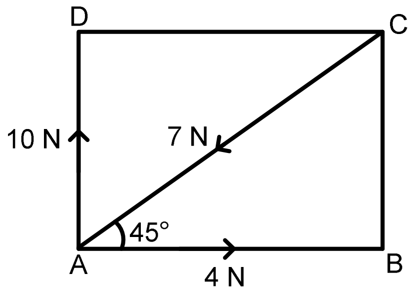
Question 16:

At 12 noon, a ship A is moving with constant velocity of 20.4 km h^{-1} in the direction $N\ \theta°\ E$, where $\tan\theta = \frac{1}{5}$. A second ship B is 15 km due north of A. Ship B is moving with constant velocity of 5 km h^{-1} in the direction $S\ \alpha°\ W$, where $\tan\alpha = \frac{3}{4}$. If the shortest distance between the ships is 4.2 km, find the time to the nearest minute when the distance between the ships is shrtest. [12]

END

MARKING GUIDE

SNo.	Working	Marks
1	$\underset{\sim}{v} = \begin{pmatrix} u\cos\theta \\ u\sin\theta - gt \end{pmatrix}$ $= \begin{pmatrix} 20\cos 60° \\ 20\sin 60° - 9.8t \end{pmatrix} = \begin{pmatrix} 10 \\ 10\sqrt{3} - 9.8t \end{pmatrix}$	M1 B1
	$\underset{\sim}{r} = \begin{pmatrix} ut\cos\theta \\ ut\sin\theta - 0.5gt^2 \end{pmatrix}$ $= \begin{pmatrix} 20t\cos 60° \\ 20t\sin 60° - 0.5\times 9.8t^2 \end{pmatrix}$	M1 M1
	$= \begin{pmatrix} 10t \\ 10t\sqrt{3} - 4.9t^2 \end{pmatrix}$	B1
		05
2	(a). $n = 15,\quad p = 0.4,\quad q = 1 - 0.4 = 0.6$ $P(X > 9) = P(X \geq 10) = 0.0338$	M1 A1
	(b). $P(5 < X < 8) = P(X = 6) + P(X = 7)$ $= 0.2066 + 0.1771 = 0.3837$	M1 M1 A1
		05
3	(a). x: 0.5, 0.6, 0.7; $f(x)$: 2.25, y_1, 1.43 $\frac{y_1 - 2.25}{1.43 - 2.25} = \frac{0.6 - 0.5}{0.7 - 0.5}$ $y_1 = 0.5\times(-0.82) + 2.25 = 1.84$ When $x = 0.6$, $f(x) = 1.84$.	B1-mobile M1 A1
	(b). x: 0.5, 0.7, x_2; $f(x)$: 2.25, 1.43, 0.75 $\frac{x_2 - 0.5}{0.7 - 0.5} = \frac{0.75 - 2.25}{1.43 - 2.25}$ $x_2 = \frac{75}{41}\times 0.2 + 0.5 = \frac{71}{82} \approx 0.866$ When $f(x) = 0.75$, $x = 0.866$.	M1 A1
		05
4	Let F_R be the resultant force	



B1

$$F_R = \binom{4}{0} + \binom{0}{10} + \binom{-7\cos 45°}{-7\sin 45°}$$
$$= \binom{4-3.5\sqrt{2}}{10-3.5\sqrt{2}}$$
$$= \binom{-0.9497}{5.0503}\text{ N}$$

M1-horizontal component
M1- vertical component

Magnitude, $|F_R| = \sqrt{(-0.9497)^2 + (5.0503)^2} \approx 5.139$ N **M1 A1**

05

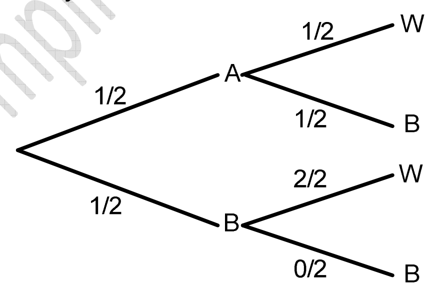
5 (a).

$$P(W/A) = \frac{1}{2}\ , \quad P(W/B) = 1\ , \quad P(A) = P(B) = \frac{1}{2}$$
$$P(\text{white}) = P(A\cap W) + P(B\cap W)$$
$$= P(A).P(W/A) + P(B).P(W/B)$$
$$= \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times 1$$ **M1 M1**
$$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$ **A1**

(b).

$$P(A/W) = \frac{P(A\cap W)}{P(W)} = \frac{1}{4}\div\frac{3}{4} = \frac{1}{3}$$ **M1 A1**

Alternatively:



$$P(\text{white}) = P(A\cap W) + P(B\cap W) = \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{2}{2} = \frac{3}{4}$$

(b).

$$P(A/W) = \frac{P(A\cap W)}{P(W)} = \left(\frac{1}{2}\times\frac{1}{2}\right)\div\frac{3}{4} = \frac{1}{3}$$

05

6

$$x = 2.4, \quad e_x = 0.05$$ **B1**

$$\text{Upper limit} = y_{max} = \frac{1}{(2.4-0.05)} + (2.4+0.05) = 2.8755$$
$$= 2.9 \ \ (1\text{ d.p})$$ **M1 A1**

$$\text{Lower limit} = y_{min} = \frac{1}{(2.4+0.05)} + (2.4-0.05) = 2.7582$$
$$= 2.8 \ \ (1\text{ d.p})$$ **M1 A1**

05

7 (a).

$$\text{Weighted aggregate price index} = \frac{\sum(P_{2003}\times W)}{\sum(P_{2002}\times W)}\times 100$$
$$= \frac{500\times 200 + 3000\times 18 + 2100\times 2 + 2200\times 15}{400\times 200 + 2500\times 18 + 2400\times 2 + 2000\times 15}\times 100$$ **M1 M1**
$$= \frac{100000 + 54000 + 4200 + 33000}{80000 + 45000 + 4800 + 30000}\times 100$$
$$= \frac{191200}{159800}\times 100 = 119.65$$ **A1**

(b).

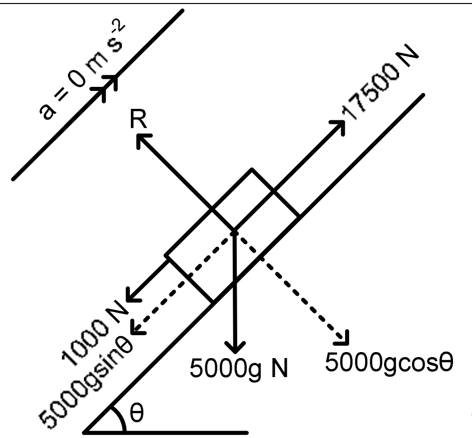
$$I = \frac{P_{2003}}{P_{2003}}\times 100$$
$$119.65 = \frac{450000}{P_{2003}}\times 100$$ **M1**
$$P_{2003} = \frac{450000}{119.65}\times 100 = 376{,}096.95$$ **A1**

05

8

$$v = 20\text{ m s}^{-1}, \quad P = 350\text{ kW},$$
$$\Rightarrow F = \frac{P}{v} = \frac{350000}{20} = 17500\text{ N}$$ **B1**

At maximum speed, acceleration is zero.



B1

Resolving parallel to the plane gives:

$$17500-(1000+5000g\sin\theta)=5000a$$
$$16500-5000\times 9.8\sin\theta=0$$
$$16500-49000\sin\theta=0$$
$$\sin\theta=\frac{16500}{49000}$$
$$\theta=19.68°$$

M1 M1

A1

05

9 (a).

x	$P(X=x)$	$xP(X=x)$	$x^2P(X=x)$
1	k	k	k
2	$2k$	$4k$	$8k$
3	$3k$	$9k$	$27k$
4	$4k$	$16k$	$64k$
5	$5k$	$25k$	$125k$
Total	$15k$	$55k$	$225k$

B1-$P(X=x)$

B1-$xP(X=x)$

B1-$x^2P(X=x)$

$$\sum_{all\,x}P(X=x)=1,\quad \Rightarrow 15k=1,\quad \Rightarrow k=\frac{1}{15}$$ **M1 B1**

(b).

$$P(2<X<5)=P(X=3)+P(X=4)$$
$$=3k+4k=7k=\frac{7}{15}\approx 0.4667$$ **M1 A1**

(c).

$$E(X)=\sum_{all\,x}xP(X=x)=55k=\frac{55}{15}=\frac{11}{3}\approx 3.6667$$ **M1 A1**

(d).

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$$Var(X)=\sum_{all\,x}x^2P(X=x)-[E(X)]^2=225k-\left(\frac{11}{3}\right)^2$$ **M1**
$$=\frac{225}{15}-\left(\frac{11}{3}\right)^2=\frac{14}{9}\approx 1.5556$$ **B1 A1**

12

10 (a).

$$m=3\text{ kg},\quad \underset{\sim}{F}=\begin{pmatrix}6\\-36t^2\\54t\end{pmatrix}\text{N}$$
$$\overrightarrow{OP}=\begin{pmatrix}1\\-5\\-1\end{pmatrix}\text{m},\quad \underset{\sim}{u}=\begin{pmatrix}3\\3\\0\end{pmatrix}\text{m s}^{-1}$$
$$\underset{\sim}{a}=\frac{\underset{\sim}{F}}{m}=\frac{1}{3}\begin{pmatrix}6\\-36t^2\\54t\end{pmatrix}=\begin{pmatrix}2\\-12t^2\\18t\end{pmatrix}$$ **M1**
$$\underset{\sim}{v}=\int\begin{pmatrix}2\\-12t^2\\18t\end{pmatrix}dt=\begin{pmatrix}2t\\-4t^3\\9t^2\end{pmatrix}+c$$ **M1**

When $t=0,\ \underset{\sim}{v}=\underset{\sim}{u}=\begin{pmatrix}3\\3\\0\end{pmatrix}$

$$\begin{pmatrix}3\\3\\0\end{pmatrix}=0+c,\quad \Rightarrow c=\begin{pmatrix}3\\3\\0\end{pmatrix}$$ **B1**
$$\underset{\sim}{v}=\begin{pmatrix}2t\\-4t^3\\9t^2\end{pmatrix}+\begin{pmatrix}3\\3\\0\end{pmatrix}=\begin{pmatrix}2t+3\\-4t^3+3\\9t^2\end{pmatrix}$$ **B1**
$$\underset{\sim}{s}=\int\begin{pmatrix}2t+3\\-4t^3+3\\9t^2\end{pmatrix}dt=\begin{pmatrix}t^2+3t\\-t^4+3t\\3t^3\end{pmatrix}+c$$ **M1**

When $t=0,\ \underset{\sim}{s}=\overrightarrow{OP}=\begin{pmatrix}1\\-5\\-1\end{pmatrix}$

$$\begin{pmatrix}1\\-5\\-1\end{pmatrix}=0+c,\quad \Rightarrow c=\begin{pmatrix}1\\-5\\-1\end{pmatrix}$$
$$\underset{\sim}{s}=\begin{pmatrix}t^2+3t\\-t^4+3t\\3t^3\end{pmatrix}+\begin{pmatrix}1\\-5\\-1\end{pmatrix}=\begin{pmatrix}t^2+3t+1\\-t^4+3t-5\\3t^3-1\end{pmatrix}$$ **B1**

When $t=1$, **B1**

$$\underset{\sim}{s}=\begin{pmatrix}1+3+1\\-1+3-5\\3-1\end{pmatrix}=\begin{pmatrix}5\\-3\\2\end{pmatrix}$$

Position vector $=\overrightarrow{OP}+\underset{\sim}{s}=\begin{pmatrix}1\\-5\\-1\end{pmatrix}+\begin{pmatrix}5\\-3\\2\end{pmatrix}=\begin{pmatrix}6\\-8\\1\end{pmatrix}$ **B1**

(b).

Distance from origin $=\sqrt{6^2+(-8)^2+1^2}=\sqrt{101}\approx 10.05\text{m}$ **M1 A1**

M1 A1

12

11 (a).

$$y_n = \frac{x_n}{x_n^2 - 3}, \qquad h = \frac{3-2}{5} = 0.2$$ **B1**

n	x_n	y_0, y_5	$y_1, \dots y_4$
0	2	2	
1	2.2		1.19565
2	2.4		0.86957
3	2.6		0.69149
4	2.8		0.57851
5	3	0.5	
sums		2.5	3.33522

B1- x_n values

B1- x_n values

$$\int_2^3 \frac{x}{(x^2-3)}dx \approx \frac{1}{2}h[(y_0 + y_5) + 2(y_1 + \cdots + y_4)]$$
$$\approx \frac{1}{2} \times 0.2[2.5 + 2 \times 3.33522] = 0.91704$$
$$\approx 0.917 \text{ (3 d.p)}$$ **M1 B1 A1**

(b).

$$\text{Actual value} = \int_2^3 \frac{x}{(x^2-3)}dx = \left[\frac{1}{2}\ln(x^2-3)\right]_2^3$$ **M1**
$$= \frac{1}{2}\ln 6 - \frac{1}{2}\ln 1 = 0.896 \text{ (3 d.p)}$$ **M1 A1**

(c). (i).

Error made $= |0.896 - 0.917| = 0.021$ **M1 A1**

(ii). The student can reduce the error by increasing the number of sub-intervals. **B1**

12

12 (a).

Class	f	x	fx	c	f/c	Class boundaries
3 – 4	2	3.5	7	2	1	2.5 – 4.5
5 – 9	7	7.0	49	5	1.4	4.5 – 9.5
10 – 19	16	14.5	232	10	1.6	9.5 – 19.5
20 – 29	21	24.5	514.5	10	2.1	19.5 – 29.5
30 – 44	9	37.0	333	15	0.6	29.5 – 44.5
Total	55		1135.5			

B1-x values

B1- fx values

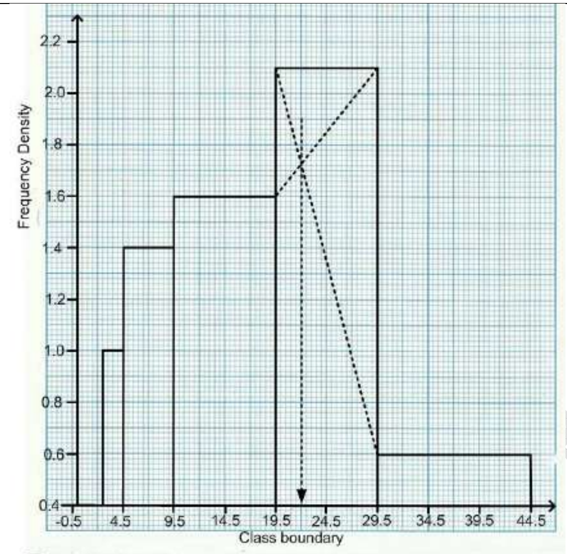
B1- f/c values

B1- $\sum fx$

(a).

$$\text{Mean time,} \quad \bar{x} = \frac{\sum fx}{\sum f} = \frac{1135.5}{55} = 20.65$$ **M1 A1**

(b). (i).



B1-horizontal axis

B1-vertical axis

B2-bars

B1-attempting to find mode

(ii). The estimated the modal time is 22. **A1**

12

13

B1-all forces shown clearly

let, $\quad \overline{AB} = l, \quad \Rightarrow \overline{AC} = \frac{3}{4}\overline{AB} = 0.75l$

Taking moments about A,

$$T \times l = 10g \times 0.75l \sin 30°$$
$$T = 10 \times 9.8 \times 0.75 \sin 30° = 36.75 \text{ N}$$ **M1 M1 B1**

Resolving vertically,

$$Y + T\cos 60° = 10g$$

$$Y + 36.75\cos 60° = 10 \times 9.8$$ **M1 M1**

$$Y = 79.625\text{ N}$$ **B1**

Resolving horizontally,

$$X = T\sin 60° = 36.75\sin 60° = 31.826\text{N}$$ **M1 B1**

$$\text{Reaction at A} = \sqrt{X^2 + Y^2} = \sqrt{31.826^2 + 79.625^2} \approx 85.75\text{ N}$$ **M1 A1**

$$\text{Direction} = \tan^{-1}\left(\frac{Y}{X}\right) = \tan^{-1}\left(\frac{79.625}{31.826}\right)$$

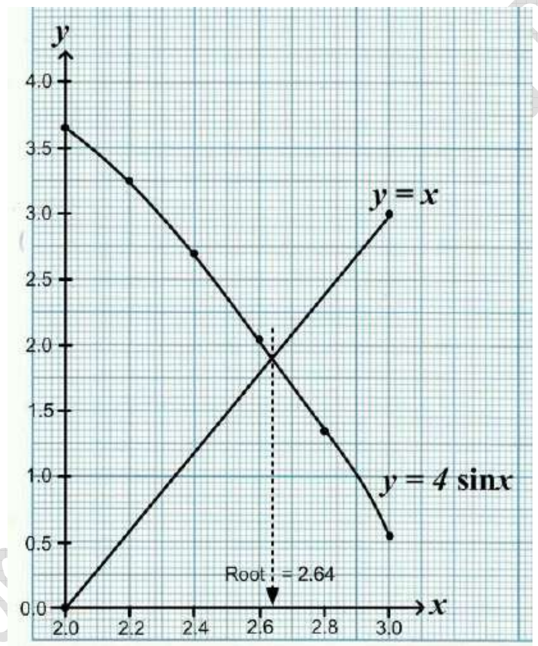
$$= 68.21° \text{ above the horizontal}$$ **B1**

12

14 (a).

x	2.0	2.2	2.4	2.6	2.8	3.0
$4\sin x$	3.64	3.23	2.70	2.06	1.34	0.56

B1
B1



B1-$y = 4\sin x$

B1-$y = x$

B1-axes

M1-indicating where the root is.

For the hence part:

$$f(x) = x - 4\sin x$$

$$f(x) = x - 4\sin x, \quad f'(x) = 1 - 4\cos x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

$$x_{n+1} = x_n - \frac{x_n - 4\sin x_n}{1 - 4\cos x_n}$$ **M1 B1**

$$x_0 = 2.64$$

$$x_1 = 2.64 - \frac{2.64 - 4\sin 2.64}{1 - 4\cos 2.64} = 2.48099$$ **B1**

$$|x_1 - x_0| = |2.48099 - 2.64| = 0.15901 > 0.0005$$

$$x_2 = 2.48099 - \frac{2.48099 - 4\sin 2.48099}{1 - 4\cos 2.48099} = 2.47459$$ **B1**

$$|x_2 - x_1| = |2.47459 - 2.48099| = 0.0064 > 0.0005$$ **B1**

$$x_3 = 2.64 - \frac{2.47459 - 4\sin 2.47459}{1 - 4\cos 2.47459} = 2.47458$$

$$|x_3 - x_2| = |2.47458 - 2.47459| = 0.00001 < 0.0005$$

$$\therefore \text{Root} = 2.475 \ \ (3\text{ d.p})$$ **A1**

12

15 (a).

$$P(X < 60) = 0.15, \quad \Rightarrow P(Z < -z_1) = 0.15$$

$$-z_1 = 1.036, \quad z_1 = -1.036$$ **B1**

$$\text{but, } z_1 = \frac{60 - \mu}{\sigma} = -1.036, \quad \Rightarrow 60 - \mu = -1.036\sigma \quad \rightarrow (1)$$ **B1**

$$\text{Also,} \quad P(X > 90) = 0.05, \quad \Rightarrow P(Z > z_2) = 0.05,$$

$$\Rightarrow z_2 = 1.645$$ **B1**

$$\text{but, } z_2 = \frac{90 - \mu}{\sigma} = 1.645, \quad \Rightarrow 90 - \mu = 1.645\sigma \quad \rightarrow (2)$$ **B1**

Equation (2) − (1) gives;

$$30 = 2.681\sigma, \quad \Rightarrow \sigma = \frac{30}{2.681} = 11.19$$ **M1 A1**

From equation (1);

$$\mu = 60 + 1.036\sigma = 60 + 1.036 \times 11.19 = 71.59$$ **M1 A1**

$\therefore$ Mean, $\mu = 71.59$, Standard deviation, $\sigma = 11.19$

(b).

$$P(X > 80) = P\left(Z > \frac{80 - 71.59}{11.19}\right) = P(Z > 0.752)$$ **M1**

$$= 0.5 - \phi(0.752) = 0.5 - 0.2740 = 0.226$$ **B1**

$$\text{Number of residents} = 0.226 \times 200 = 45.2$$ **M1**

≈ 45 residents (truncated) **A1**

or, ≈ 45 residents (rounded off)

12

16

$$\theta = \tan^{-1}\left(\frac{1}{5}\right) = 11.3099°$$

$$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.8699°$$

$${}_A\underset{\sim}{\boldsymbol{r}}_B = \underset{\sim}{\boldsymbol{r}}_A - \underset{\sim}{\boldsymbol{r}}_B = \begin{pmatrix}0\\0\end{pmatrix} - \begin{pmatrix}0\\15\end{pmatrix} = \begin{pmatrix}0\\-15\end{pmatrix}\text{ km}$$

$${}_A\underset{\sim}{\boldsymbol{v}}_B = \underset{\sim}{\boldsymbol{v}}_A - \underset{\sim}{\boldsymbol{v}}_B = \begin{pmatrix}20.4\sin 11.3099°\\20.4\cos 11.3099°\end{pmatrix} - \begin{pmatrix}-5\sin 36.8699°\\-5\cos 36.8699°\end{pmatrix}$$

$$= \begin{pmatrix}7.00075\\24.00385\end{pmatrix}\text{ km h}^{-1}$$

	${}_A\underset{\sim}{\boldsymbol{r}}_B(t) = {}_A\underset{\sim}{\boldsymbol{r}}_B + t\,{}_A\underset{\sim}{\boldsymbol{v}}_B = \begin{pmatrix}0\\-15\end{pmatrix} + t\begin{pmatrix}7.00075\\24.00385\end{pmatrix}$ $= \begin{pmatrix}7.00075t\\24.00385t-15\end{pmatrix}$ $\left\|{}_A\underset{\sim}{\boldsymbol{r}}_B(t)\right\| = \sqrt{(7.00075t)^2 + (24.00385t-15)^2} = 4.2$ $49.0105t^2 + 576.1848t^2 + 720.1155t + 225 = 17.64$ $625.1953t^2 + 720.1155t + 207.36 = 0$ $t = \dfrac{-720.1155 \pm \sqrt{(720.1155)^2 - 4\times 625.1953 \times 207.36}}{2\times 625.1953}$ $t = -0.5742$, or, $t = -0.5776$	
		12

END

P425/1
PURE
MATHEMATICS
PAPER 1
Nov./Dec. 2016
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD
S.6 MATH 1 UNEB 2016
Time: 3 Hours

NAME: ______________________ **COMB:** ________

INSTRUCTIONS:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Show your working clearly.

Section A (40 Marks)

Answer **all** *the questions in this section*

Qn 1: Without using mathematical tables or a calculator, find the value of:

$$\frac{(\sqrt{5}-2)^2 - (\sqrt{5}+2)^2}{8\sqrt{5}}$$

[5]

Qn 2: Find the angle between the lines $2x - y = 3$ and $11x + 2y = 13$. [5]

Qn 3: Evaluate: $\int_{\frac{1}{2}}^{1} 10x\sqrt{1-x^2}\,dx$. [5]

Qn 4: Solve the equation $\frac{dy}{dx} = 1 + y^2$ given that $y = 1$when $x = 0$. [5]

Qn 5: Given that $2x^2 + 7x - 4$, $x^2 + 3x - 4$ and $7x^2 + ax - 8$ have a common factor, find the:

(a). factors of $2x^2 + 7x - 4$ and $x^2 + 3x - 4$,

(b). value of a in $7x^2 + ax - 8$. [5]

Qn 6: Solve the equation $\sin 2\theta + \cos 2\theta \cos 4\theta = \cos 4\theta \cos 6\theta$ for $0 \leq \theta \leq \frac{\pi}{4}$. [5]

Qn 7: Using small changes, show that $(244)^{\frac{1}{5}} = 3\frac{1}{405}$. [5]

Qn 8: The points $A(2, -1, 0)$, $B(-2, 5, -4)$ and C are on a straight line such that $3\boldsymbol{AB} = 2\boldsymbol{AC}$. Find the coordinates of C. [5]

Section B (60 Marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). If $Z_1 = \frac{2i}{1+3i}$ and $Z_2 = \frac{3+2i}{5}$, find $|Z_1 - Z_2|$. [6]

(b). Given the complex number $Z = x + iy$,

(i). find $\frac{Z+i}{Z+2}$.

(ii). show that the locus of $\frac{Z+i}{Z+2}$ is a straight line when its imaginary part is zero. State the gradient of the line. [6]

Question 10:

(a). Solve the equation $\cos 2x = 4\cos^2 x - 2\sin^2 x$ for $0° \leq x \leq 180°$. [6]

(b). Show that if $\sin(x + \alpha) = P\sin(x - \alpha)$ then: $\tan x = \left(\frac{P+1}{P-1}\right)\tan\alpha$. Hence solve the euation $\sin(x + 20°) = 2\sin(x - 20°)$ for $0° \leq x \leq 360°$. [6]

Question 11:

Given that $x = \frac{t^2}{1+t^3}$ and $= \frac{t^3}{1+t^3}$, find $\frac{d^2y}{dx^2}$. [12]

Question 12:

(a). Line A is the intersection of two planes whose equations are

$$3x - y + z = 2, \quad \text{and,} \quad x + 5y + 2z = 6.$$

Find the Cartesian equation of the line. [5]

(b). Given that line B is perpendicular to the plane $3x - y + z = 2$ and passes through the point $C(1, 1, 0)$, find the:

(i). Cartesian equation of line B.

(ii). angle between line B and line A in (a) above. [7]

Question 13:

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(a). Find: $\int \frac{1+\sqrt{x}}{2\sqrt{x}}\,dx$. [3]

(b). The gradient of the tangent at any point on a curve is $-\frac{2y}{x}$. The curve passes through the point $(2, 4)$. Find the equation of the curve. [9]

Question 14:

(a). The points $P(at_1{}^2, 2at_1)$ and $Q(at_2{}^2, 2at_2)$ are on the parabola $y^2 = 4ax$. OP is perpendicular to OQ, where O is the origin. Show that $t_1t_2 + 4 = 0$. [4]

(b). The normal to the rectangular hyperbola $xy = 8$ at a point $(4, 2)$ meets the asymptotes at M and N. Find the length of MN. [8]

Question 15:

(a). Prove by induction:

$$1 \times 3 + 2 \times 4 + \cdots + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$$

for all integral values of n. [6]

(b). A man deposits shs. 150,000 at the beginning of every year in a micro-finance bank with the understanding that at the end of seven years, he is paid back his money with 5% per annum compound interest. How much does he receive? [6]

Question 16:

(a). If $x^2 + 3y^2 = k$, where k is a constant, find $\frac{dy}{dx}$ at the point $(1, 2)$. [4]

(b). A rectangular field of area 7200 m^2 is said to be fenced using a wire mesh. On one side of the field, is a straight river. This side of the field is not to be fenced. Find the dimensions of the field that will minimize the amount of wire mesh to be used. [8]

*****END*****

MARKING GUIDE

SNo.	Working	Marks
1	$\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{8\sqrt{5}}=\frac{(\sqrt{5}-2-\sqrt{5}-2)(\sqrt{5}-2+\sqrt{5}+2)}{8\sqrt{5}}$ $=\frac{-4\times 2\sqrt{5}}{8\sqrt{5}}=-1$	M1 M1-numerator B1 B1 A1
		05
2	for, $2x-y=3, \quad \Rightarrow m_1=2$ for, $11x+2y=13, \quad \Rightarrow m_2=-\frac{11}{2}=-5.5$ $\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$ $\tan\theta=\frac{2-(-5.5)}{1+2\times(-5.5)}$ $\tan\theta=-0.75$ $\theta=143.1301°$	B1 B1 M1 B1 A1
		05
3	but, $\frac{d}{dx}(1-x^2)^{\frac{3}{2}}=-2x\times\frac{3}{2}(1-x^2)^{\frac{1}{2}}$ $\frac{d}{dx}(1-x^2)^{\frac{3}{2}}\times\frac{10}{-3}=-3x(1-x^2)^{\frac{1}{2}}\times\frac{10}{-3}$ $-\frac{10}{3}\int\frac{d}{dx}(1-x^2)^{\frac{3}{2}}\,dx=\int 10x(1-x^2)^{\frac{1}{2}}\,dx$ $\int_{\frac{1}{2}}^{1}10x\sqrt{1-x^2}\,dx=\left[-\frac{10}{3}(1-x^2)^{\frac{3}{2}}\right]_{\frac{1}{2}}^{1}$ $=-\frac{10}{3}\left(0-(0.75)^{\frac{3}{2}}\right)$ $=-\frac{10}{3}\left(0-\frac{3\sqrt{3}}{8}\right)$ $=\frac{5\sqrt{3}}{4}\approx 2.1651$ **ALT:** let, $u=\sqrt{1-x^2}$ $u^2=1-x^2$ $2u\frac{du}{dx}=-2x$ $dx=\frac{-u}{x}du$ $\begin{array}{c\|c} x & u \\ \hline \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \hline 1 & 0 \end{array}$ $\int_{\frac{1}{2}}^{1}10x\sqrt{1-x^2}\,dx=\int_{\frac{\sqrt{3}}{2}}^{0}10xu\times\frac{-u}{x}du$ $=-\int_{\frac{\sqrt{3}}{2}}^{0}10u^2\,du=\left[\frac{-10}{3}u^3\right]_{\frac{\sqrt{3}}{2}}^{0}$ $=0+\frac{10}{3}\times\left(\frac{\sqrt{3}}{2}\right)^3$ $=\frac{10}{3}\times\frac{3\sqrt{3}}{8}$ $=\frac{5\sqrt{3}}{4}\approx 2.1651$	B1 B1 M1 M1 A1
		05
4	$\frac{dy}{dx}=1+y^2$ $\int\frac{dy}{1+y^2}=\int 1\,dx$ $\tan^{-1}y=x+c$ But $y=1$ when $x=0$, $\tan^{-1}1=0+c$ $c=\frac{\pi}{4}$ $\tan^{-1}y=x+\frac{\pi}{4}$ $y=\tan\left(x+\frac{\pi}{4}\right)$	M1 M1 B1-must be in radians B1 A1
		05
5	(a). $2x^2+7x-4=2x^2+8x-x-4$ $=2x(x+4)-(x+4)$ $=(2x-1)(x+4)$ $x^2+3x-4=x^2+4x-x-4$ $=x(x+4)-(x+4)$ $=(x-1)(x+4)$ (b). The common factor is $(x+4)$, let, $f(x)=7x^2+ax-8$ $f(-4)=7(-4)^2+a(-4)-8=0$ $112-4a-8=0$ $104=4a$ $a=26$	B1 B1 B1 M1 A1
		05
6	$\sin 2\theta+\cos 2\theta\cos 4\theta=\cos 4\theta\cos 6\theta$ $\sin 2\theta=\cos 4\theta\cos 6\theta-\cos 2\theta\cos 4\theta$ $\sin 2\theta=\cos 4\theta(\cos 6\theta-\cos 2\theta)$	B1

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	$\sin 2\theta = \cos 4\theta\,(-2\sin 4\theta \sin 2\theta)$ $\sin 2\theta + 2\sin 4\theta \sin 2\theta \cos 4\theta = 0$ $\sin 2\theta\,(1 + 2\sin 4\theta \cos 4\theta) = 0$ $\sin 2\theta\,(1 + \sin 8\theta) = 0$ $\sin 2\theta = 0$, or, $(1 + \sin 8\theta) = 0$ for, $\sin 2\theta = 0$, $2\theta = 0°$, $\Rightarrow \theta = 0°$ for, $(1 + \sin 8\theta) = 0$, $8\theta = 270°$, $\Rightarrow \theta = 33.75°$ $\therefore \theta = 0°, 33.75°$ $\theta = 0, \frac{33.75\pi}{180}$ $\theta = 0, \frac{3\pi}{16}$	B1 B1 B1 A1
		05
7	let, $y + \delta y \approx (x + \delta x)^{\frac{1}{5}} = (243 + 1)^{\frac{1}{5}}$ $y = x^{\frac{1}{5}} = (243)^{\frac{1}{5}} = 3$, $\delta x = 1$ $\frac{dy}{dx} = \frac{1}{5}x^{-\frac{4}{5}} = \frac{1}{5}(243)^{-\frac{4}{5}} = \frac{1}{5} \times \frac{1}{81} = \frac{1}{405}$ $\delta y \approx \frac{dy}{dx} \times \delta x = \frac{1}{405} \times 1 = \frac{1}{405}$ $(243)^{\frac{1}{5}} \approx y + \delta y = 3 + \frac{1}{405} = 3\frac{1}{405}$	B1-both y and δx values B1 B1 M1 A1
		05
8	$3\boldsymbol{AB} = 2\boldsymbol{AC}$ $3\left[\begin{pmatrix}-2\\5\\-4\end{pmatrix} - \begin{pmatrix}2\\-1\\0\end{pmatrix}\right] = 2\left[\overrightarrow{OC} - \begin{pmatrix}2\\-1\\0\end{pmatrix}\right]$ $\begin{pmatrix}-12\\18\\-12\end{pmatrix} = 2\overrightarrow{OC} - \begin{pmatrix}4\\-2\\0\end{pmatrix}$ $\begin{pmatrix}-8\\16\\-12\end{pmatrix} = 2\overrightarrow{OC}$ $\overrightarrow{OC} = \frac{1}{2}\begin{pmatrix}-8\\16\\-12\end{pmatrix} = \begin{pmatrix}-4\\8\\-6\end{pmatrix}$ $\therefore C(-4, 8, -6)$	M1 M1 M1 B1 A1
		05
9	(a). $Z_1 - Z_2 = \frac{2i}{1+3i} - \frac{3+2i}{5}$ $= \frac{2i(1-3i)}{(1+3i)(1-3i)} - \frac{3+2i}{5}$ $= \frac{2i+6}{1+9} - \frac{3+2i}{5}$ $= \frac{2i+6}{10} - \frac{3+2i}{5}$ $= \frac{i+3}{5} - \frac{3+2i}{5}$	M1 B1

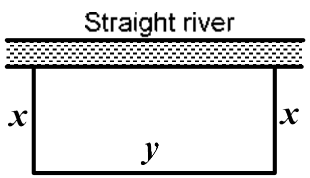
	$= \frac{i+3-3-2i}{5}$ $= -\frac{1}{5}i$ $\lvert Z_1 - Z_2 \rvert = \left\lvert -\frac{1}{5}i \right\rvert = \frac{1}{5}$ (b). (i). $\frac{Z+i}{Z+2} = \frac{x+yi+i}{x+yi+2}$ $= \frac{[x+(y+1)i][(x+2)-yi]}{[(x+2)+yi][(x+2)-yi]}$ $= \frac{(x^2+2x) - xyi + (xy+2y+x+2)i + (y^2+y)}{(x+2)^2+y^2}$ $= \frac{(x^2+y^2+2x+y) + (2y+x+2)i}{(x+2)^2+y^2}$ (ii). Imaginary part $= \frac{2y+x+2}{(x+2)^2+y^2} = 0$ $2y + x + 2 = 0$ $y = -\frac{1}{2}x - 1$ The locus is in the form $y = mx + c$ hence a straight line. Gradient $= -\frac{1}{2}$	M1 B1 M1 A1 M1 M1 B1 M1 A1 A1
		12
10	(a). $\cos 2x = 4\cos^2 x - 2\sin^2 x$ $\cos 2x = 2(1 + \cos 2x) - (1 - \cos 2x)$ $\cos 2x = 2 + 2\cos 2x - 1 + \cos 2x$ $0 = 1 + 2\cos 2x$ $\cos 2x = -0.5$ $2x = 120°, 240°$ $x = 60°, 120°$ (b). $\sin(x+\alpha) = P\sin(x-\alpha)$ $\sin x\cos\alpha + \cos x\sin\alpha = P(\sin x\cos\alpha - \cos x\sin\alpha)$ Dividing throughout by $\cos x\cos\alpha$ gives: $\tan x + \tan\alpha = P\tan x - P\tan\alpha$ $\tan\alpha + P\tan\alpha = P\tan x - \tan x$ $\tan\alpha\,(1+P) = \tan x\,(P-1)$ $\tan\alpha\left(\frac{1+P}{P-1}\right) = \tan x$ $\Rightarrow \tan x = \left(\frac{P+1}{P-1}\right)\tan\alpha$, hence shown **For the hence part,** $\sin(x+20°) = 2\sin(x-20°)$ By comparison, $\alpha = 20°$, $P = 2$.	B1 B1 B1 B1 B1 A1 B1 M1 B1

	$\tan x = \left(\frac{P+1}{P-1}\right)\tan\alpha$	
	$\tan x = \left(\frac{2+1}{2-1}\right)\tan 20°$	M1
	$\tan x = 1.0919$	M1 A1
	$x = \tan^{-1}1.0919 = 47.52°, 227.52°$	
		12
11	$\frac{dx}{dt} = \frac{(1+t^3)(2t) - t^2(3t^2)}{(1+t^3)^2}$	M1 M1
	$= \frac{2t + 2t^4 - 3t^4}{(1+t^3)^2} = \frac{2t - t^4}{(1+t^3)^2} = \frac{t(2-t^3)}{(1+t^3)^2}$	B1
	$\frac{dy}{dt} = \frac{(1+t^3)(3t^2) - t^3(3t^2)}{(1+t^3)^2}$	M1 M1
	$= \frac{3t^2 + 3t^5 - 3t^5}{(1+t^3)^2} = \frac{3t^2}{(1+t^3)^2}$	B1
	$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx} = \frac{3t^2}{(1+t^3)^2}\times\frac{(1+t^3)^2}{t(2-t^3)} = \frac{3t}{(2-t^3)}$	M1 B1
	$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{3t}{(2-t^3)}\right]\times\frac{dt}{dx}$	
	$= \frac{(2-t^3)(3) - (3t)(-3t^2)}{(2-t^3)^2}\times\frac{(1+t^3)^2}{t(2-t^3)}$	M1 M1
	$= \frac{6 - 3t^3 + 9t^3}{(2-t^3)^2}\times\frac{(1+t^3)^2}{t(2-t^3)}$	
	$= \frac{(6+6t^3)(1+t^3)^2}{t(2-t^3)^3}$	
	$= \frac{6(1+t^3)(1+t^3)^2}{t(2-t^3)^3}$	B1
	$= \frac{6(1+t^3)^3}{t(2-t^3)^3}$	A1
		12
12	(a).	
	$3x - y + z = 2 \quad \longrightarrow (1)$	
	$x + 5y + 2z = 6 \quad \longrightarrow (2)$	
	$2\times(1) - (2)$ gives:	
	$\begin{array}{r} 6x - 2y + 2z = 4 \\ -\quad x + 5y + 2z = 6 \\ \hline 5x - 7y = -2 \end{array}$	M1
	$x = \frac{7y-2}{5} \quad \longrightarrow (3)$	B1
	$5\times(1) + (2)$ gives:	
	$\begin{array}{r} 15x - 5y + 5z = 10 \\ +\quad x + 5y + 2z = 6 \\ \hline 16x + 7z = 16 \end{array}$	M1

	$x = \frac{16-7z}{16} \quad \longrightarrow (4)$	B1
	The Cartesian equation of line A is	
	$x = \frac{7y-2}{5} = \frac{16-7z}{16}$	B1
	(b). (i).	
	Direction vector, $\underset{\sim}{\boldsymbol{d}} = \begin{pmatrix}3\\-1\\1\end{pmatrix}$	B1
	Position vector $= \begin{pmatrix}1\\1\\0\end{pmatrix}$	B1
	Cartesian equation of line B is	
	$\frac{x-1}{3} = \frac{y-1}{-1} = z$	B1
	(ii).	
	For line A,	
	$x = \frac{7y-2}{5} = \frac{16-7z}{16}$	
	$x = \frac{7y-2}{5} = \frac{7z-16}{-16}$	
	$\underset{\sim}{\boldsymbol{d_B}} = \begin{pmatrix}1\\5/7\\-16/7\end{pmatrix} = \frac{1}{7}\begin{pmatrix}7\\5\\-16\end{pmatrix}$	B1
	$\underset{\sim}{\boldsymbol{d}} = \begin{pmatrix}3\\-1\\1\end{pmatrix}$	
	$\underset{\sim}{\boldsymbol{d_A}}.\underset{\sim}{\boldsymbol{d_B}} = \begin{pmatrix}7\\5\\-16\end{pmatrix}.\begin{pmatrix}3\\-1\\1\end{pmatrix} = 21 - 5 - 16 = 0$	M1 B1
	$\therefore \theta = 90°$	A1
		12
13	(a).	
	$\int \frac{1+\sqrt{x}}{2\sqrt{x}}dx = \int\left(\frac{1}{2\sqrt{x}} + \frac{\sqrt{x}}{2\sqrt{x}}\right)dx = \int\left(\frac{1}{2}x^{\frac{-1}{2}} + \frac{1}{2}\right)dx$	B1
	$= x^{\frac{1}{2}} + \frac{1}{2}x + c$	M1
	$= \sqrt{x} + \frac{1}{2}x + c$	A1
	ALT:	
	let, $\quad u = \sqrt{x}$	
	$u^2 = x$	
	$2u\frac{du}{dx} = 1$	
	$dx = 2u\,du$	
	$\int\frac{1+\sqrt{x}}{2\sqrt{x}}dx = \int\frac{1+u}{2u}\times 2u\,du$	

	$\int (1+u)\,du = u + \frac{1}{2}u^2 + c$ $= \sqrt{x} + \frac{1}{2}x + c$ (b). $\frac{dy}{dx} = -\frac{2y}{x}$ $\int \frac{dy}{2y} = -\int \frac{dx}{x}$ $\frac{1}{2}\ln y = -\ln x + c$ At point $(2, 4)$, $\frac{1}{2}\ln 4 = -\ln 2 + c$ $\ln 2 = -\ln 2 + c$ $c = 2\ln 2$ $\frac{1}{2}\ln y = -\ln x + 2\ln 2$ $\ln y = -2\ln x + 4\ln 2$ $\ln y = \ln(16x^{-2})$ $y = \frac{16}{x^2}$ $x^2 y = 16$	M1 B1 B1 M1 B1 M1 B1 M1 B1
		12
14	(a). For perpendicular lines, $m_{OP} \times m_{OQ} = -1$ $\frac{2at_1 - 0}{at_1{}^2 - 0} \times \frac{2at_2 - 0}{at_2{}^2 - 0} = -1$ $\frac{2at_1}{at_1{}^2} \times \frac{2at_2}{at_2{}^2} = -1$ $\frac{4}{t_1 t_2} = -1$ $4 = -t_1 t_2$ $t_1 t_2 + 4 = 0$, as required (b). $\frac{d}{dx}(xy) = \frac{d}{dx}(8)$ $y + x\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-y}{x}$ At point $(2, 4)$, Gradient of tangent $= \frac{-4}{2} = -2$ Gradient of normal $= \frac{-1}{-2} = \frac{1}{2}$ Equation of the normal at point $(2, 4)$ is,	M1 M1 B1 M1 B1

	$\frac{y-4}{x-2} = \frac{1}{2}$ $y - 4 = \frac{1}{2}x - 1$ assymptotes are, $y = \pm x$ For the asymptote $y = x$ $x - 4 = \frac{1}{2}x - 1$ $\frac{1}{2}x = 3$ $x = 6$ $\therefore M(6, 6)$ For the asymptote $y = -x$ $-x - 4 = \frac{1}{2}x - 1$ $\frac{-3}{2}x = 3$ $x = -2$ $\therefore N(-2, 2)$ length $MN = \sqrt{(-2-6)^2 + (2-6)^2} = \sqrt{80} \approx 8.9443$ units	B1 B1 M1 B1 B1 M1 A1
		12
15	(a). $1 \times 3 + 2 \times 4 + \cdots + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$ For $n = 1$, $L.H.S = 1 \times 3 = 3$ $R.H.S = \frac{1}{6} \times 1 \times 2 \times 9 = 3$ True for $n = 1$. For $n = 2$, $L.H.S = 1 \times 3 + 2 \times 4 = 11$ $R.H.S = \frac{1}{6} \times 2 \times 3 \times 11 = 11$ True for $n = 2$. Suppose it's true for $n = k$, the series becomes: $1 \times 3 + 2 \times 4 + \cdots + k(k+2) = \frac{1}{6}k(k+1)(2k+7)$ For $n = (k+1)$, $R.H.S = \frac{1}{6}(k+1)(k+2)(2k+9)$ $L.H.S = \{1 \times 3 + 2 \times 4 + \cdots + k(k+2)\} + (k+1)(k+3)$ $= \frac{1}{6}k(k+1)(2k+7) + (k+1)(k+3)$ $= \frac{k(k+1)(2k+7) + 6(k+1)(k+3)}{6}$ $= \frac{1}{6}(k+1)(2k^2 + 7k + 6k + 18)$	B1 B1 B1-assumption M1

	$=\frac{1}{6}(k+1)(2k^2+13k+18)$	
	$=\frac{1}{6}(k+1)(2k^2+4k+9k+18)$	**M1**
	$=\frac{1}{6}(k+1)(2k(k+2)+9(k+2))$	
	$=\frac{1}{6}(k+1)(2k+9)(k+2)$	**B1**
	True for $n=(k+1)$.	
	Since its true for $n=1, n=2, n=k$ and $n=(k+1)$, then it's true for all positive integers of n.	
	(b).	
	$R=5, P=150{,}000, n=7$ years	
	Total amount, $A_{\text{total}}=\sum_{1}^{7} A_n$, where $A_n=P\left(1+\frac{R}{100}\right)^n$	**B1**
	$\Rightarrow A_{\text{total}}=A_1+A_2+\cdots+A_7$	
	$=P[(1+0.05)^1+(1+0.05)^2+\cdots+(1+0.05)^7]$	**M1**
	$=P[1.05+1.05^2+\cdots+1.05^7]$	
	$=P\left[\frac{a(r^n-1)}{r-1}\right]$, where $a=r=1.05$	**B1**
	$\therefore A_{\text{total}}=150{,}000\left[\frac{1.05(1.05^7-1)}{1.05-1}\right]=1{,}282{,}366.331$	**M1 M1 A1**
		12
16	(a).	
	$x^2+3y^2=k$	
	$2x+6y\frac{dy}{dx}=0$	**M1**
	$\frac{dy}{dx}=\frac{-2x}{6y}=-\frac{x}{3y}$	**B1**
	At a point $(1,2)$,	
	$\frac{dy}{dx}=-\frac{1}{3\times 2}=-\frac{1}{6}$	**M1 A1**
	(b).	
	Let x and y be the dimensions.	
		
	Area, $A=xy=7200$	
	$y=\frac{7200}{x}$	**B1**
	length of wire, $l=2x+y$	

	$l=2x+\frac{7200}{x}$	**M1**
	$\frac{dl}{dx}=2-\frac{7200}{x^2}$	**M1**
	For minimum length of wire, $\frac{dl}{dx}=0$	
	$2-\frac{7200}{x^2}=0$	
	$2=\frac{7200}{x^2}$	**M1**
	$x^2=3600$	
	$x=60$ m	
	$y=\frac{7200}{60}=120$ m	**A1**
	The dimensions are 60 m by 120 m.	**M1 A1** **B1**
		12

END

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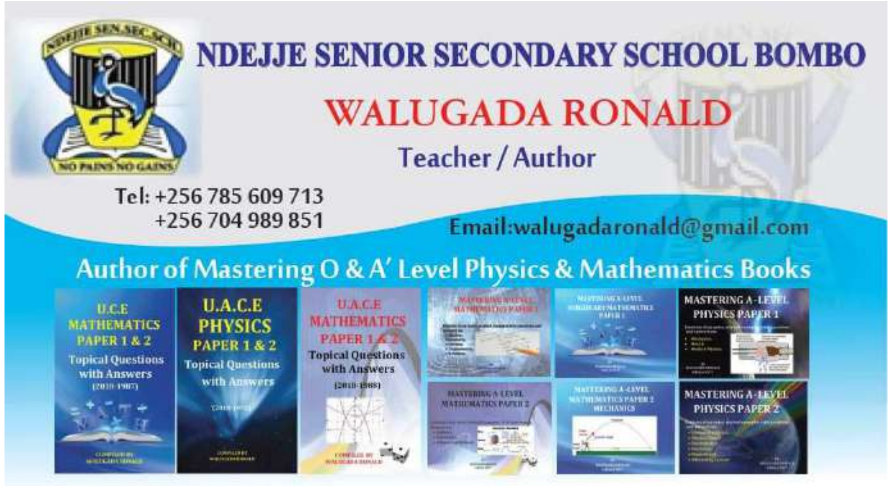

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