THREE DIMENSIONAL FIGURES

A three dimensional figures it the geometrical figures which occupies the spaces. Its also known as the solid figure or more simplified the solid

Example - Rectangular
- Cube
- prism
- Triangular prism
- Phyramid
- Cone
- cylinder

IDENTIFICATION OF THREE - DIMENSIONAL FIGURES.

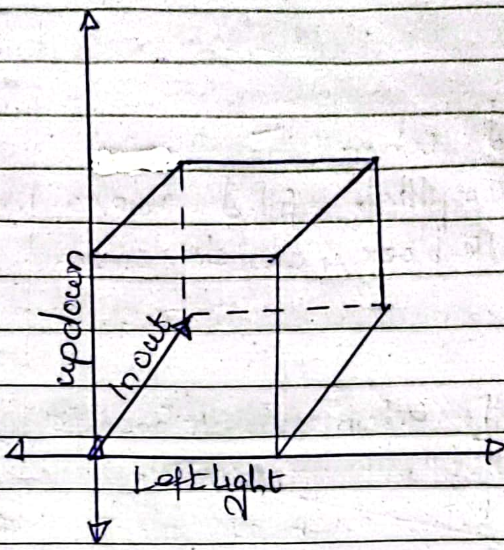
So for you have learnt about lines, cycles and plygons shapes in geometry

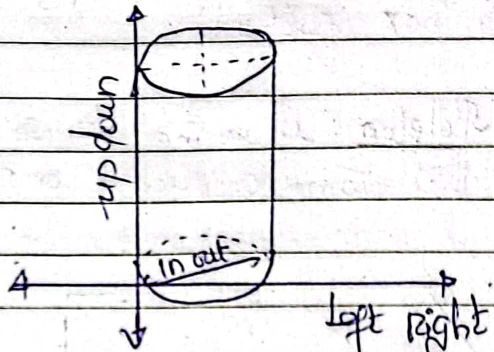
A line has only one direction to move along (left or right) thus its one dimensional figure

Plane shaped figure such as polygons and cycles have two dimension section to move across left or right) and up-down) those there are two dimensional figures

The figure such as cylinders have three directions to move through: left - light, up-down and in-out these are three-dimensional figures

Example





Note that all real-life objects are three dimensional even avery thin objects such as threads, a hair, or a flat pieces of paper have the thickness.
There fore all objects are in three dimensions

Characteristics of three dimension figures

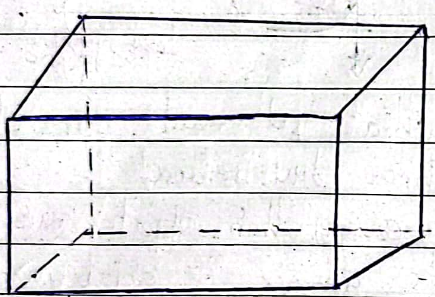
→ We classify these figures by shapes. some are regular shape but others are irregular shaped figure. For example, a chalk box is regular shaped figure

A three dimensional figures, whether regular or irregular shape can be inform of solid, ashell or skeleton

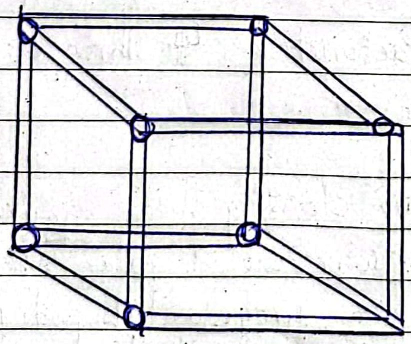
Its **solid** when the interior part is completely filled. For example an ice block, abrick and astone.

Its **ashell** when the interior part is not filled forexample empty or hollow such as empty shell egg box, football

Its **skeleton** when the its only shows the frame or model of the figure



solid Ice box



Skeleton.

The classification are either regul.. shaped three dimensional figure were described their characteri.. and properties

Polyhedron figures.

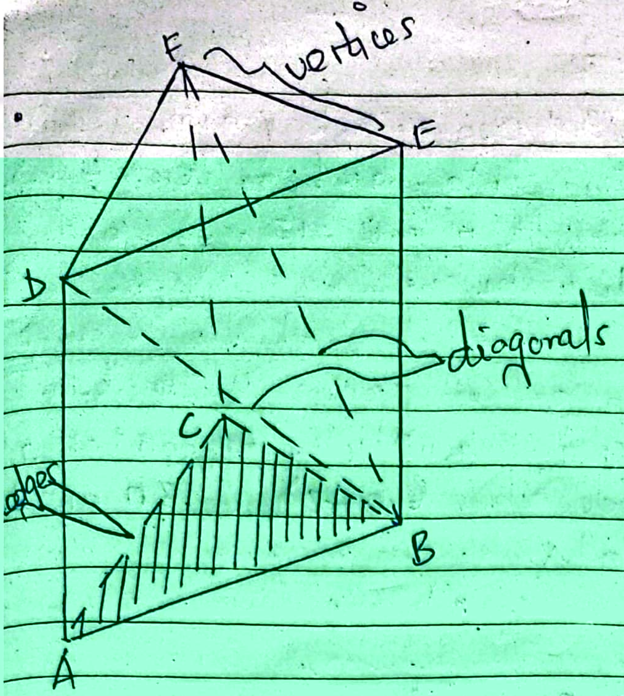
Those are the three dimensiona.. figures with polygon flat sur.. Forexample prism phyramid polyhedron its consist face, edge and vertex.

- **Aface**: is flat surface of polygon
- **Anedge**: is the line where t.. face are meet
- **Avertex**: is apoint where th.. two edge meet

However aline drawn on any fa.. (not edge) to join two vertices is diagonal

Note some diagonals do not lie on faces, they go through apolyhedron to join the vertices of different face

Consider the figure bellow Atetrahedron has no diagonal.



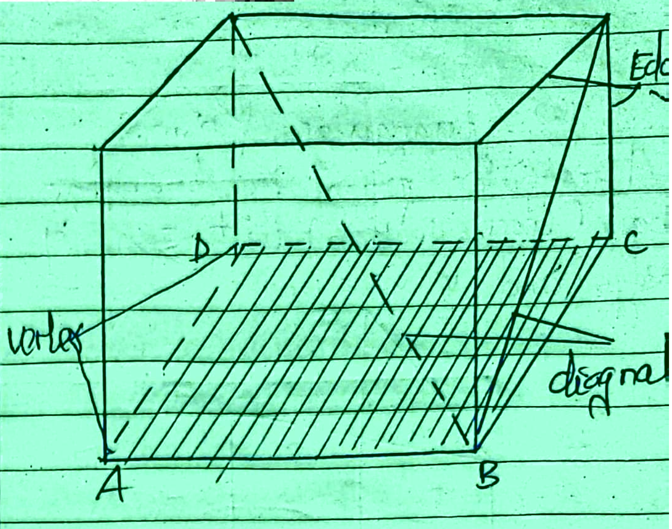
- Faces: ABC and DEF are triangular faces but ABED ACFD and BCFE are rectangular faces

- Edges: $\overline{AB}$, $\overline{AD}$, $\overline{DE}$, $\overline{BE}$, $\overline{CF}$, $\overline{CA}$, $\overline{BC}$, $\overline{DF}$ and $\overline{EF}$ are edge.

- Vertex A, B, C, D, E, F are vertex

- Diagonals: Each face of the rectangular block or solid has two diagonals. $\overline{BD}$ is diagonal on the face ABED. $\overline{BF}$ is a diagonal on the face from vertex B to Vertex B

Also Consider the figure below it indicate the vertex edges and Faces.



- Faces: ABCD, EFGH, ABFE, DCGH, ADHE and BCGF

- Edge: $\overline{AB}$, $\overline{AD}$, $\overline{AE}$, $\overline{BC}$, $\overline{BF}$, $\overline{CD}$, $\overline{CG}$, $\overline{DH}$, $\overline{EF}$, $\overline{EH}$, $\overline{FG}$ and

- Vertices: A, B, C, D, E, F, G and H

- Diagonals: $\overline{BG}$ is a diagonal through the polyhedron from Vertex B to vertex H which are not on the sa mple.

Classification of Polyhedrons

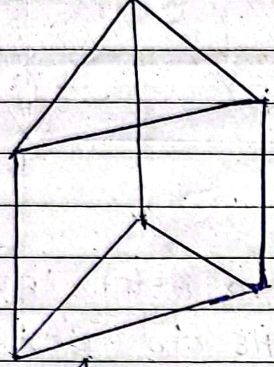
The polyhedrons are classified as follow

1: Prism

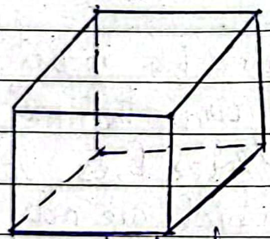
a prism is a polyhedron which has the same cross-section throughout.

are triangular prism, cube or sequare prism, cuboid and rectangular prism.

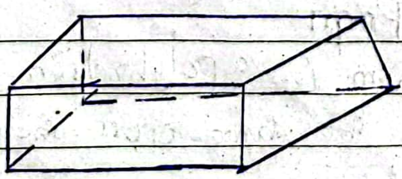
(a) A triangular Prism.
A triangular prism has trianglar base (cross-section) with 5 face, 9 edge and 6 vertices.



(b) A cube or sequare prism
A square prism (cube) has all edges of the same size. Thus all face are sequares, A cube has 6 face, 12 edges and 8 vertices



(c) A cuboid or rectangular Prism
A rectangular prism (cuboid) has all faces rectangular: which are either parallel or perpendicular to each other. A cuboid has 6 face 12 edge and 8 vertices



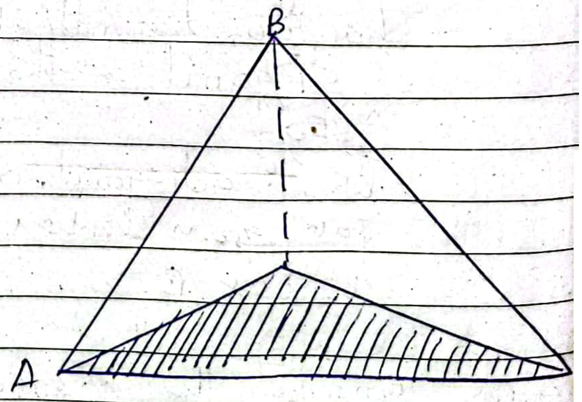
(d) pentagonal prism
has pentagonal base (cross-section) with 7 face, 15 edges and 10 vertices.

In general a prism whose base (cross-section) is a polygon of n sides has (n+2) face, 3n and 2n vertex.

2 Pyramid
is polyhedron with a polygon shaped base and all its other face being slanted triangles whose edges meet at one vertex.

When the vertex of pyramid is above the centre of the base the pyramid is called right pyramid.

The Name of pyramid is determined by the polygon it has as its base.

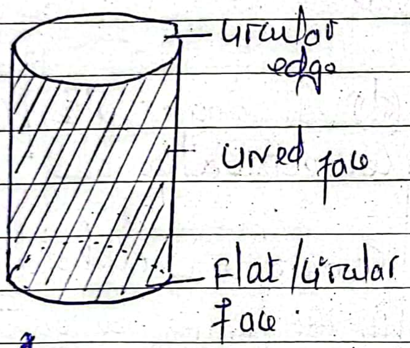


Classification of Non-poly hedrons

Those are the three dimension als figures with curved or flat circular face such as cylinder acone and sphere.
A curved face either meets a circular face to make a circular edge or meet to itself at vertex point

1 A cylinder

It has 2 flat circular face and 1 curved face, 2 circular edge and has no vertex



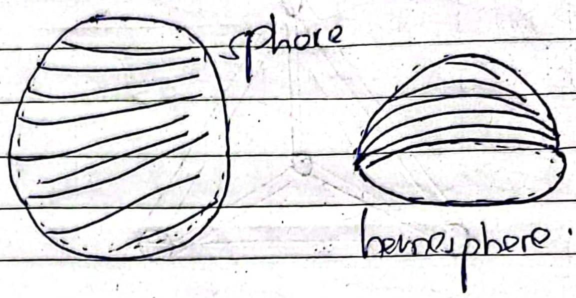
2 A cone

It looks like pyramid but has circular base and slanted height with curved surfaces. Thus it has 1 flat circular face, 1 slanted curved face 1 flat circular edge and 1 vertex

— vertex

03 A sphere

It's a ball-shaped figure with a curved face throughout. A half cut sphere is called a hemisphere

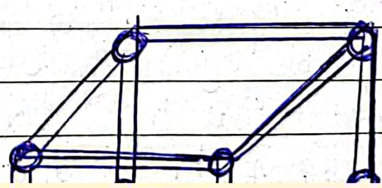


Construction and sketching of the dimensional figures

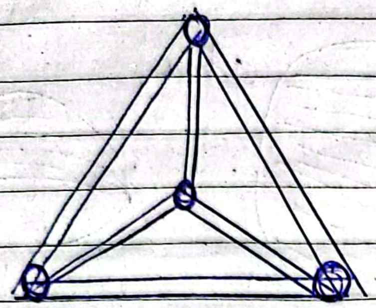
• Construction of skeletons

Polyhedral skeletons can be constructed using straws and pins or wires. Three straws are joined together by pins or wire to make the vertex of the figure and each straw represented the edge of the figure.

Example the construction of the rectangular prism and triangular pyramid



called anet
For example below are the nets of a cube and triangular Pyramid (tetrahedron)



To construct a three-dimensional figures we draw its net on a paper, leaving flaps for gluing the sides then cut and fold it. Note that one figure may have more than one different net. For example, figures below represent three different nets of the same triangular prism.

Sketching three-dimensional figures.

It's posible to represent a three dimensional figure on a two dimensional plane by using oblique projections. That is by drawing three or more face of three-dimensional figure on the same plane, the way they look when viewed from one of its vertical edge. The joined faces looks like the flapped page of an open book.

The oblique projection sketchings are governed by the following rules

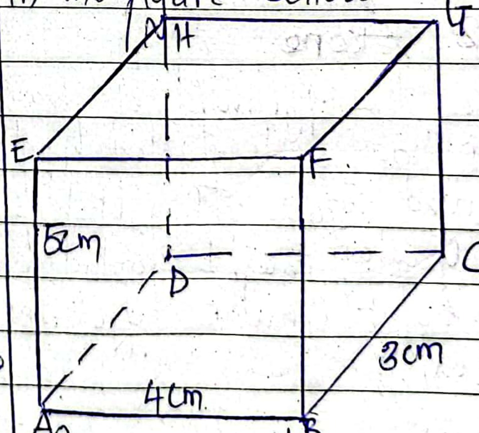
(a) the two base edges of the ~~vertex~~ vertical face viewed are drawn at angle 30° with the horizontal

(b) All vertical edges are represented by vertical lines drawn parallel to each other

(c) All other parallel edges are represented by non-vertical parallel lines

(d) All angles are marked with correct value.

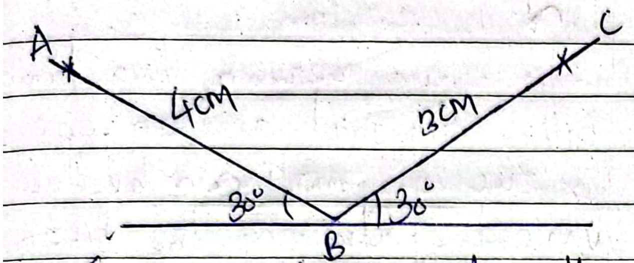
Consider a cuboid ABCDEFGH in the figure bellow



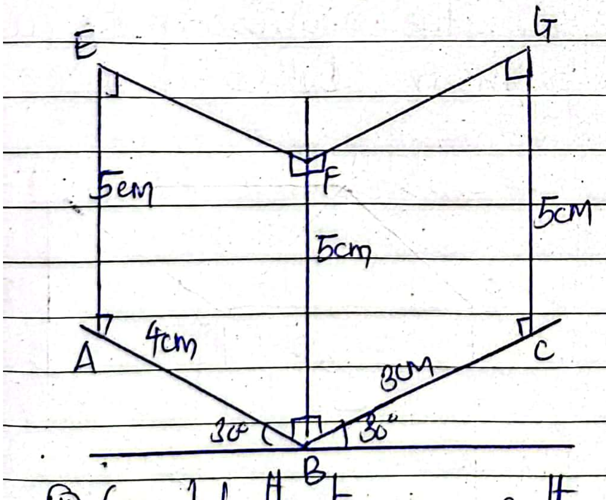
On viewing directly stright to the vertical edge BF, we see two vertical faces ABFE and BCGF

Thus, to sketch it using oblique projection we proceed step-
by step

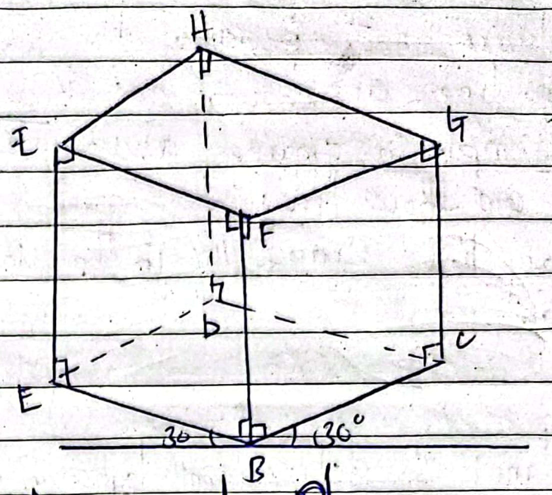
(i) At point B on horizontal line, Draw to base line each Making an angle of 30° with horizontal to represent the base edges AB and BC. Then, Measure their size 4cm and 3cM accordingly and mark the Point A and C



(ii) At point A, B, C draw the vertical line of length 5cM each Mark the Points E, F and G respectively. Then draw the lines to join the edge EF and FG, Making all right angles.

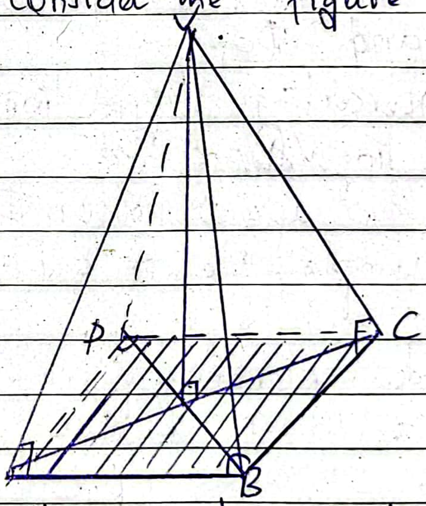


(iii) Complete the top face of the Cuboid by drawing the line EH and HG equal but parallel to the line EF. Then show the hidden edges, AD, DC and HD by doted line, Making all angles right angle. Hence these the final diagram is completely drawn.



Lines and Planes

The determination of line and plane is used to describe and draw three dimensional figures. Consider the figure bellow

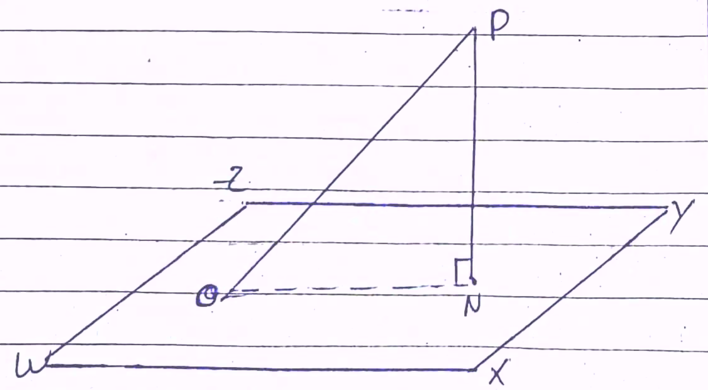


A line is determined either by two points or by two intersecting plane. For example point V and A determined the line segment VA. Also VAB intersect with plane ABCD to determine the line segment AB, which is one of the base edges. An example of this is the intersection of a wall and the floor or room

There are aline which do not intersectr even when produced and are not parallel.
Those line are known as skow ling
Forexample in figure above $\overline{VA}$ and $\overline{BC}$ are skow liner.
These line cannot lie in the same plane.

Aplane: Is determined by any one of the following
Consider the figure above.
(i) three point not in the same stright. Forexample V, C and A.
(ii) two parallel lines. Forexample lines $\overline{AB}$ and DC
(iii) two intersecting stright line forexample line $\overline{VA}$ and $\overline{VC}$
(iv) Aline and apoint not on the line Forexample, line BC and Point V.

Consider aplane and line segment as shown below.

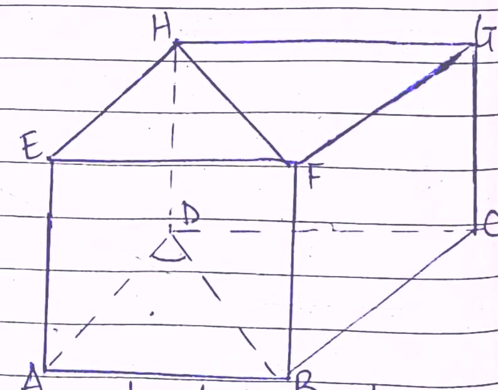


If aline segment PO intersects agiven plane at O and PN is the perpendicular from P to the plane, then line segment ON is called the projection of $\overline{PO}$ on the plane
Thus the projection of $\overline{OP}$ on WXYZ is ON where PN is perpendicular to the plane WXY

Angle between the lines

An angle between two line either diagonal and an edge or two diagonals can be determined by drawing the triangle diagram of the plane involved.

The angle between two skew lines is equal to the angle between lines which intersect and which are parallel to the skew lines. Consider the two lines HF and AD of the cube shown bellow



The angle between the skew lines is found by translating $\overline{HF}$ to $\overline{DB}$ and the angle required is the angle between $\overline{AD}$ and $\overline{DB}$ which is $A\hat{D}B$

note that $\overline{DA}$ can also be translated to $\overline{HE}$ to form $E\hat{H}F$ or translated to $\overline{GF}$ to form the same angle $G\hat{F}H$

Angle between line and plane

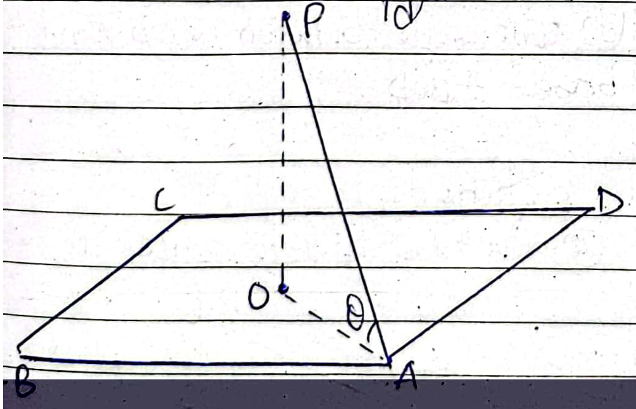
To find the angle between the line and plane, proceed as follows

(i) Draw the perpendicular from the free end point of a line to the plane to get its projection

(ii) Construct a true shape diagram showing the angle between the line and its projection

(iii) Then apply the appropriate trigonometric ratio to find the angle

Consider the figure bellow



θ is the angle between the line AP and the plane $ABCD$. $\overline{PO}$ is the perpendicular drawn from P to the plane $ABCD$. $\overline{AO}$ is the projection of $\overline{AP}$ on the plane $ABCD$ such that $\theta = P\hat{A}O$

Example 01

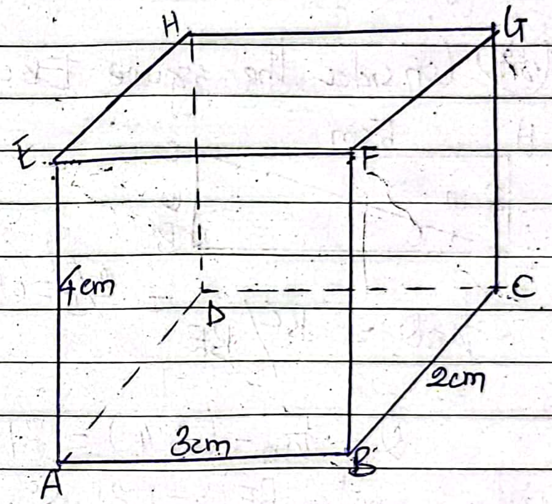
Consider the figure bellow then determine the angle of

(i) $G\hat{C}F$

(ii) $A\hat{B}E$

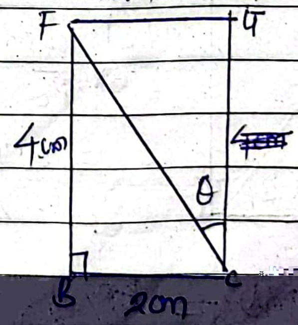
(iii) $B\hat{E}C$

(iv) $E\hat{C}F$



Soln

(i) Consider the figure BCGF

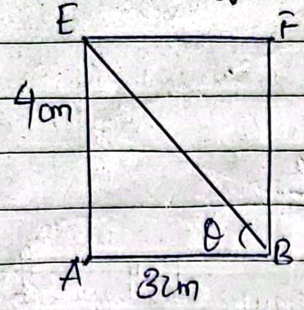


applying the tangent ration to ΔCGF

$\tan\theta = \frac{GF}{CG} = 2/4 = 0.5$

$G\hat{C}F = \tan^{-1}(0.5) = 26°.33$

(ii) Consider the figure ABFE

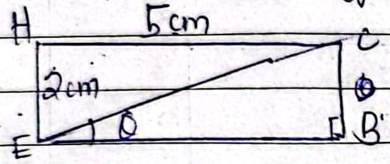


$\tan\theta = \frac{AE}{AB} = 4/3 = 1.33$

$\theta = \tan^{-1}(4/3) = 53.13^\circ$

$\therefore$ Angle $ABE = 53.13^\circ$.

(iii) Consider the figure EBCH

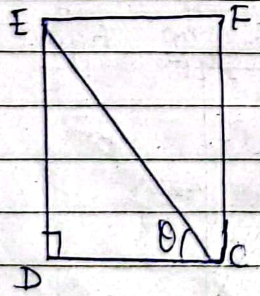


$\tan\theta = BC/BE = 2/5 = 0.4$

$\theta = \tan^{-1}(0.4) = 21.8^\circ$

$\therefore$ the angle $BEC = 21.8^\circ$

(iv) Consider the figure EFDC



but from above
(ii) $\overline{FC}$
$= 4.47cm$
$EF = AB = 3cm$

$\tan\theta = EF/CF$

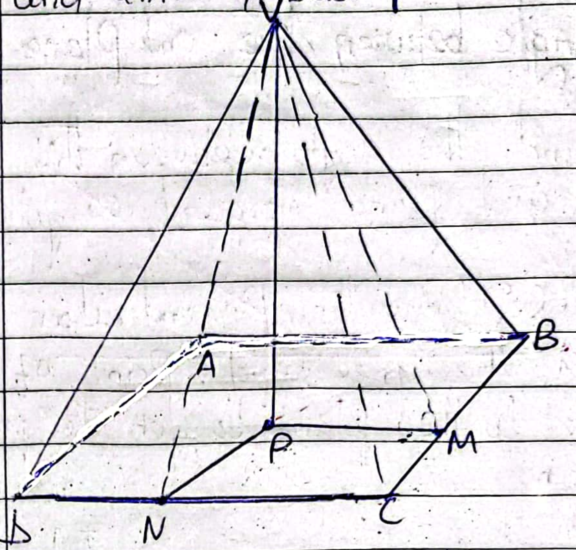
$\tan\theta = 3/4.47 = 0.671$

$\theta = \tan^{-1}(0.671) = 33.86^\circ$

$\therefore$ the angle $E\hat{C}F = 33.86^\circ$

Example 02

From the figure below rectangular Pyramid of a base ABCD, vertex V and Centre of base P



If $AB = CD = 8cm$ and $AD = BC = 6cm$ and if $VA = VB = VC = VD = 13cm$ calculate

(a) The height of VP

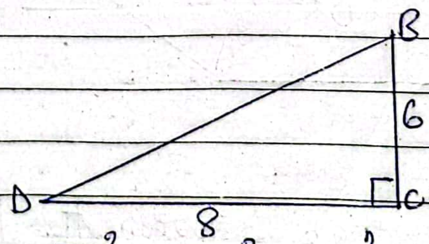
(b) The angle between $\overline{VB}$ and the base ABCD

(c) The angle between triangle VBC and the base ABCD

(d) The angle between VCD and the base ABCD

Solutions.

(a) Consider triangle DBC as



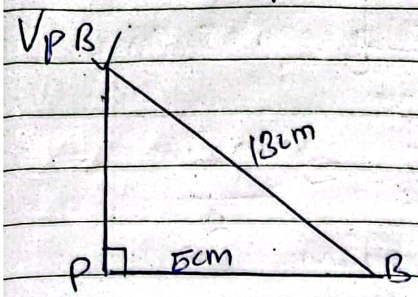
$\overline{BD}^2 = \overline{BC}^2 + \overline{DC}^2$

$= 6^2 + 8^2 = 100$

$= \sqrt{100}$

$\overline{BD} = 10$ cm

To calculate VP consider the triangle VPB



$BP = \frac{1}{2} BD = 5$ cm

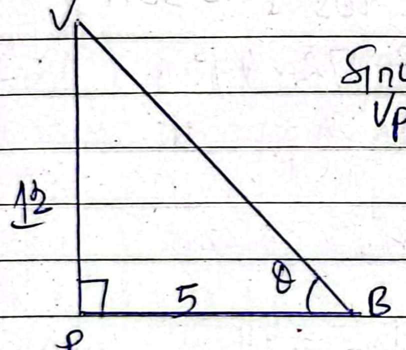
$VP^2 = 13^2 - 5^2 = 144$

$VP = \sqrt{144}$

$VP = 12$ cm

$\therefore$ The height VP is 12cm

(b) Consider the figure below



Since VP = 12 and BP = 5cm

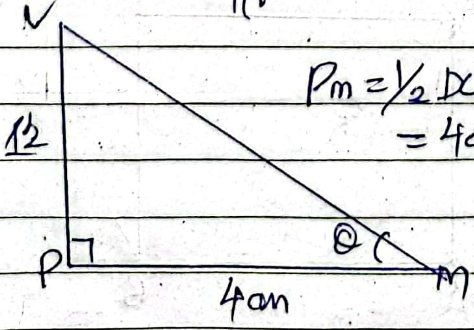
$\tan\theta = \frac{12}{5}$

$\tan\theta = 2.4$

$\theta = \tan^{-1}(2.4)$

$\theta = 67^{\circ}23'$

(c) Consider the figure below



$PM = \frac{1}{2} DC$
$= 4$cm

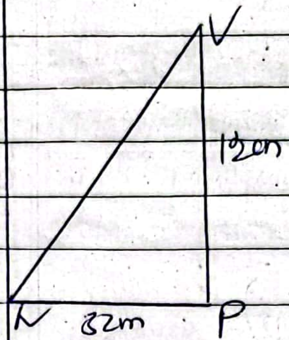
$\tan\theta = \frac{12}{4} = 3$

$\tan\theta = 3$

$\theta = \tan^{-1}(3)$

$\theta = 71^{\circ}.31$

(d) Consider the figure below



$PN = \frac{1}{2} BC$
$= 3$cm

$\tan\theta = \frac{12}{3} = 4$

$\tan\theta = 4$

$\theta = \tan^{-1}(4)$

$\theta = 75.58^{\circ}$

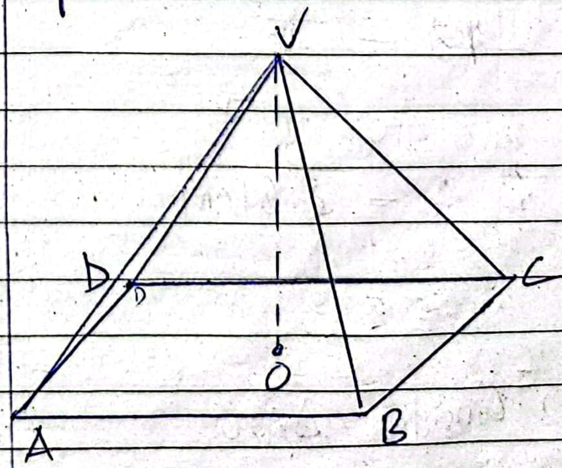
Example 03

In the figure below ABCD is rectangle in which AB = 3cm and BC = 2cm. V is apoint such that VA = VB = VC = VD = 6cm and AO = OC. Find

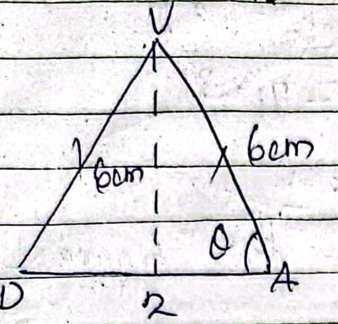
(a) The angle VAD

(b) The length of AC

(c) The angle between VA and plane ABCD



(a) Consider ΔVAD



From Cosine rules

$$\cos\theta = \frac{adj}{hyp}$$

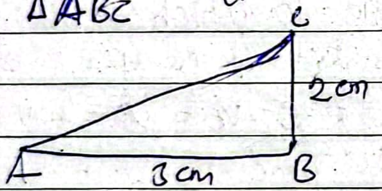
$$\cos\theta = (1/6cm)$$

$$\theta = \cos^{-1}(1/6)$$

$$\theta = 80.41^\circ$$

(b) Consider the figure below

ΔABC



$$a^2 + b^2 = c^2$$

$$(BC)^2 + (AB)^2 = (AC)^2$$

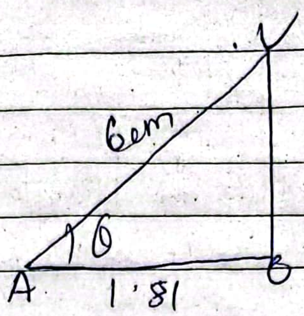
$$2^2 + 3^2 = AC^2$$

$$13 = AC^2$$

$$AC = \sqrt{13}$$

$$\therefore AC = 3.61cm$$

(c) Consider ΔAVO



$$AO = AC/2 = \frac{3.61}{2}$$

$$AO = 1.81cm$$

Let θ = an angle

$$\cos\theta = Adj/hyp$$

$$\cos\theta = \frac{1.81}{6cm}$$

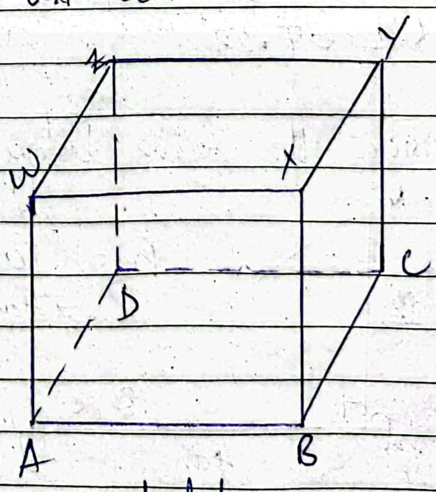
$$\cos\theta = 0.3017$$

$$\theta = \cos^{-1}(0.3017)$$

$$\theta = 72.44^\circ$$

Example 04.

A rectangular box with top WXYZ and base ABCD has AB = 6cm, BC = 8cm and WA = 8cm



calculate

(a) length of AC

"HAND WRITTING MATERIAL"

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- chemistry
- Biology

Pia tunauza solved (how to forge) practical za Masomo yote matatu

Aharsante sana kwa kuwa nasi

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