PROPOSED GUIDE LMA 0706518083

No. 1

Let P(T) be the probability of the tail
P(H) be the probability of the head

$P(H) = 2P(T)$

$2(P(T)) + P(T) = 1$

$P(T) = \frac{1}{3}, \quad P(H) = \frac{2}{3}$

Let X_T be the number of tails.

$P(X_T = 2) = {}^{7}C_2 \cdot (\frac{1}{3})^2 (\frac{2}{3})^5$

$= 0.3073$

B1 for determining prob P(T)
B1 for determining P(H)
B1 for letting X as number of tails
B1 for correct substition into formula
A1 for correct answer to atleast 4 dps.

No. 2(a)

Let V_B be the velocity of B after collision

$6 \times 4 + 2 \times 2 = 6 \times 2.6 + 2 \times V_B$

$V_B = 6.2 ms^{-1}$

B moves with velocity $6.2 ms^{-1}$ after collision.

M1 correct substition in the method.
A1 for correct answer of velocity of B

2(b)

Loss in kinetic energy $= \left(\frac{1}{2} \times 6 \times 2.6^2 + \frac{1}{2} \times 2 \times 6.2^2\right) - \left[\left(\frac{1}{2} \times 6 \times 4^2\right) + \left(\frac{1}{2} \times 2 \times 2^2\right)\right]$

$= 6.72 J$

B1 for the calculation of kinetic energy after collision
B1 for kinetic energy after collision
A1 for correct answer with units

3

x	f(a)	
0	0.1003	
½		0.0391
1		0.0801
1½		0.0602
2		0.0649
2½		0.0380
3	0.0327	
Total	0.133	0.2823

$h = \frac{1}{2} - 0$

$= \frac{1}{2}$

B1 for correct h.
B1 for the second column (y_0, y_n)
B1 for the column $(y_1 - - - y_{n-1})$

$\int_0^3 f(a)\,da \approx \frac{1}{2}\left(\frac{1}{2}\right)\left(0.133 + 2(0.2823)\right)$

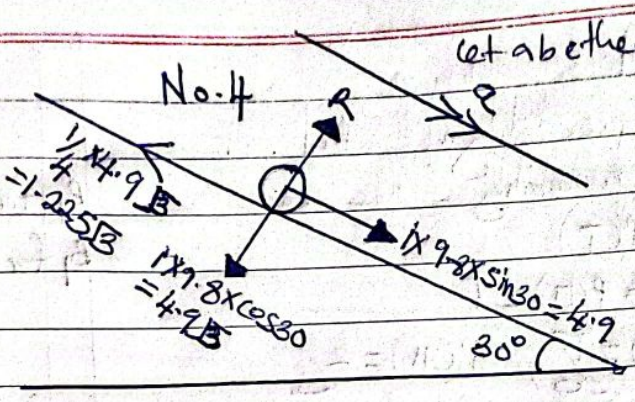
M1 for using correct Method and substion.

≈ 0.1744

≈ 0.174 (3 dps)

A1 for correct answer with 3 dps.

No.4



B1 for the horizontal and vertical components

B1 for the Frictional force

$4.9 - 1.2255\sqrt{3} = 1 \times a$

$a = 2.7782\ ms^{-2}$

B1 for the acceleration

$V^2 = 0^2 + 2 \times 2.7782 \times 4$

$V = 4.7144\ ms^{-1}$

M1 for correct substitution in the velocity method

A1 for correct velocity to at least 4 dps

The ball reaches the ground with velocity $4.7144\ ms^{-1}$

No.6

$\Delta x = 0.005$

$\Delta y = 0.000005$

$\Delta z = 0.05$

B1 for correct maximum absolute errors in x, y and z

$W_{max} = \frac{6.455 + 2.75^3}{\sqrt{0.002145}}$

$= 588.41365$

M1 for correct method of maximum value method

B1 for correct maximum value to at least 4 dps.

$W_{min} = \frac{6.445 + 2.65^3}{\sqrt{0.002155}}$

$= 537.71473$

M1 for correct minimum value method

Interval $= [537.71473, 588.41365]$

A1 for the interval

No.7(a)

$P(R') = P(S' \cap R') = 1 - P(S \cup R)$

$= 1 - [P(S) + P(R) - P(S) \cdot P(R)]$

$= P(S') - P(R)(1 - P(S))$

$= P(S')[1 - P(R)]$

B1 for correct independent expression.

$P(S) = P(S') \cdot P(R')$

$= (1 - P(S))P(R')$

$P(S) = \frac{P(R')}{1 + P(R')}$

B1 for

$$P(S') = \frac{1 - P(R)}{2 - P(R)}$$

$$= \frac{1 - 3/4}{2 - 3/4}$$

$$= 1/5$$

B1 for correct method

A1 for correct P(S')

7(b)

$$P(S' \cap R') = P(S') - P(S' \cap R')$$

$$= \left(1 - \frac{1}{5}\right) - \frac{1}{5}$$

$$= \frac{3}{5}$$

B1 for using the correct method

A1 for correct $P(S' \cap R')$

No. 8

Let W be the weight per unit area

$\bar{x}$ be the position of centre of the lamina from AB

REGION	AREA	WEIGHT	DISTANCE OF COG FROM AD
ABCD	$3600cm^2$	3600W	30cm
Circle	$1256.6371cm^2$	1256.6371W	40cm
Remainder	$2343.3629cm^2$	2343.3629W	$\bar{x}$

B1 for area column.

B1 for weight column.

$3600W \times 30 + 1256.6371W \times 40 = 2343.3629W\bar{x}$

$3600 \times 30 - 1256.6371W \times 40 = 2343.3629W\bar{x}$

B1 for calculation of sum of moments

B1 for calculation of moment about sum

$\bar{x} = 24.6375cm$

A1 for correct centre of gravity

The centre of gravity is 24.6375cm from AD.

Candidate's Name .....

Signature .....

Subject Name ..... Paper code ...../.....

Random No.

Personal Number

5(a)

AGE	CW	f	fd
0-10	10	15	1·5
10-20	10	19	1·9
20-30	10	16	1·6
30-40	10	18	1·8
40-50	20	30	1·5
60-80	20	6	0·3
80-90	10	1	0·1

B1 for column of fd

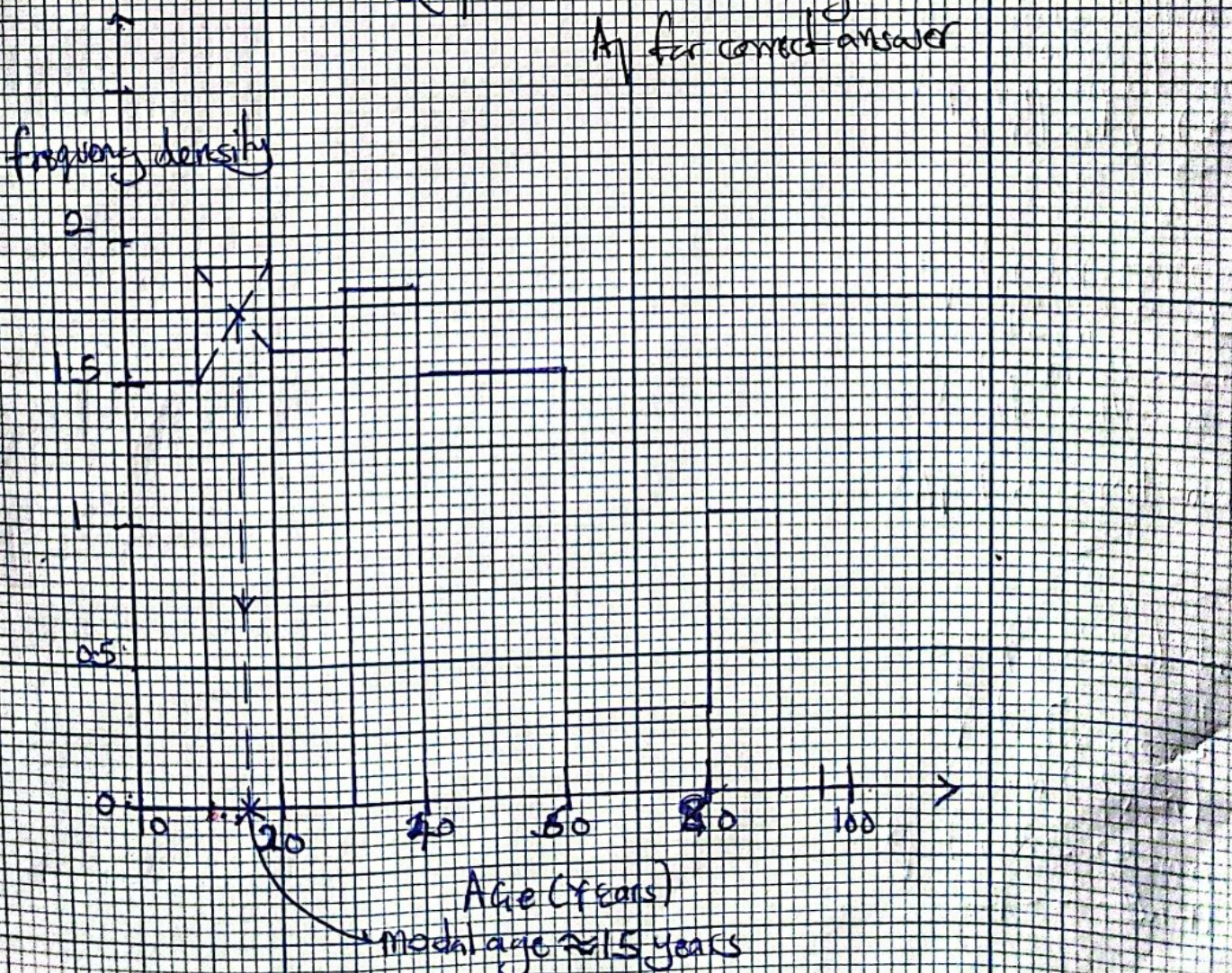
B1 for the Axis axis

B1 for shape

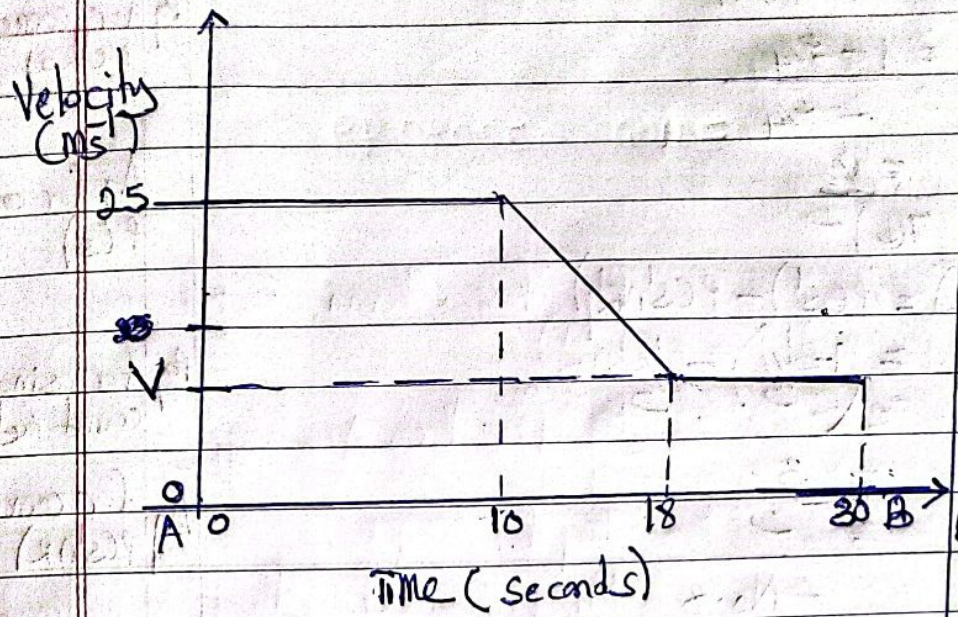
M1 for correct method of finding modal age

A1 for correct answer

5(b)



No. 10 (a)



B for correct shape of constant velocity

B for the correct deceleration graph

B for the time axis

B for velocity axis

10b(i)

distance = total Area

$526 = (25 \times 10) + \frac{1}{2} \times 8(25+V) + V \times (30-18)$ B for using the correct area method

M1

$526 = 250 + 100 + 4V + 12V$

B1 for correct answer of V

$V = 11 ms^{-1}$

B

10b(ii)

~~$H = 25 - a$~~ $V^2 = U^2 - 2as$ B for correct substitution in the formula of ~~distance~~ velocity.

$11^2 = 25^2 - 2as$

distance moved during deceleration

$= \frac{1}{2} \times (25 - V) \times 8$ B for the distance

$= \frac{1}{2} \times (25 - 11) \times 8$

$= 56m$

$11^2 = 25^2 - 2 \times a \times 56$ B for acceleration

$a = 4.5 ms^{-2}$

The deceleration of the car is $4.5 ms^{-2}$

(To be fastened together with other answers to paper) UCE

Candidate's Name ....

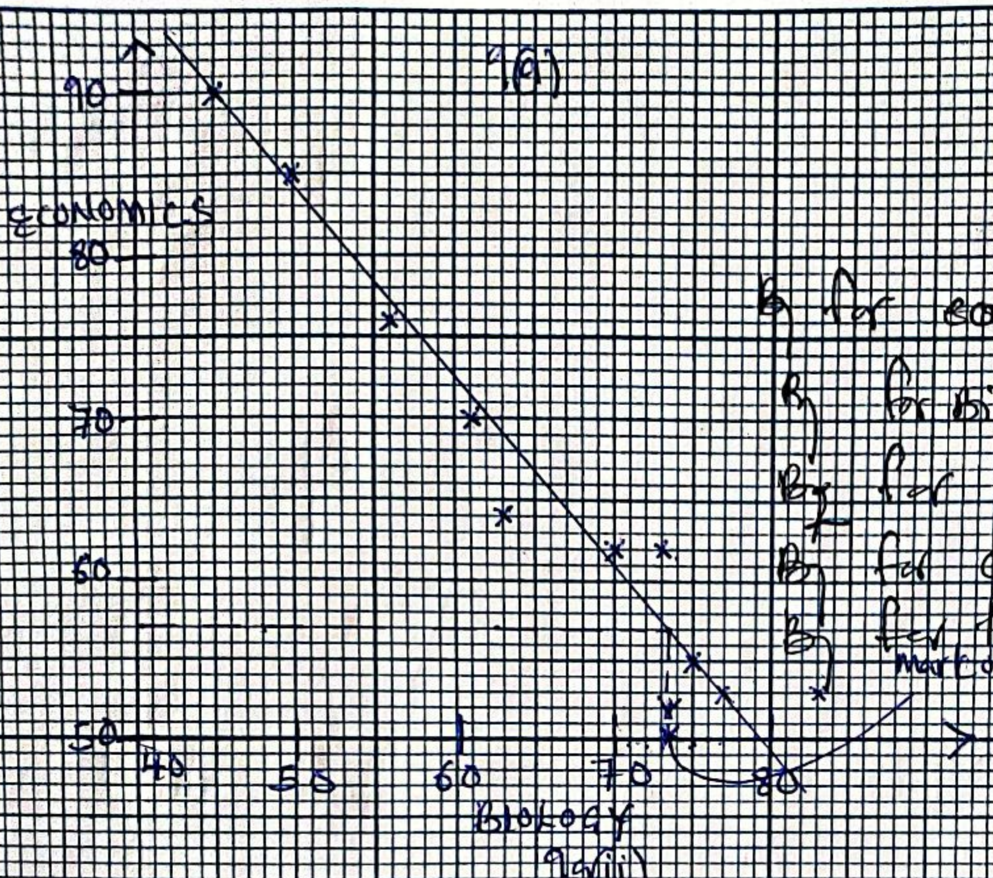
Signature ....

Subject Name .... Paper code ..../.......

Random No.

Personal Number

9(a)



B1 for ecomics axis

B1 for biology axis

B1 for correct plotting

B1 for clear line of best fit.

B1 for the correct biology mark

9a(ii)

R_B	R_E	$d(R_B-R_E)$	d^2
10	1	9	81
6	5	1	1
8	3	5	25
7	4	3	9
3	8	-5	25
1	9.5	-8.5	72.25
4	6.5	-2.5	6.25
9	2	7	49
2	9.5	-7.5	56.25
5	6.5	-1.5	2.25
			$\Sigma d^2 = 327$

B1 for d^2 column

B1 for ranking of biology

B1 for ranking of economics

$\rho = 1 - \frac{6 \times 327}{10(10^2-1)}$ B1 for correct method

$\rho = -0.9818$ B1 for correct coefficient ✓

Since 0.9818 is greater than 0.76 from the table the correlation is significant } B1 for comment

Total 10

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No. 11 a(i)

$f(1) = (1)e^{1} + 5(1) - 10$

$= -2.2817$

$f(2) = 2e^{(2)} + 5(2) - 10$

$= 14.7781$

B1 for f(1)

B1 for f(2) and substitution

B1 for substitution in the formula

11 a(ii)

Since $f(1) \cdot f(2) < 0$ then there is a root between 1 and 2

A1 for correct coment

11(b)

let $f(x) = xe^{x} + 5x - 10$

let the first root be x_0

1	x_0	2
-2.2817	0	14.7781

B1 for the first table

$\frac{x_0 - 1}{0 + 2.2817} = \frac{2 - 1}{14.7781 + 2.2817}$

B1 method

B1 for x_0

$x_0 = 1 + 1.337$

$f(1.1651) = 1.1651 e^{1.1651} + 5(1.1651) - 10$

$= -0.4389 - 0.8089$

B1 for f(1.1651)

let the second approximation be x_1

1.337	x_1	2
-0.8089	0	14.7781
-0.4389		

B1 for correct location of the next root

B1 for table.

$\frac{x_1 - 1.1651}{0 + 0.4389} = \frac{2 - 1.1651}{14.7781 + 0.8089}$

B1 correct method

$x_1 = 1.1892$

$root = 1.189$ 1.179 (3dps)

A1 for the root to 3dps

Total 12

No. 12 (a)

for $-1 \leq x \leq 0$

$F(0) = \frac{1+0}{6}$

$= \frac{1}{6}$

By for finding

Median does not lie in the first range

for $0 \leq x \leq 2$

$F(2) = \frac{1+2\times 2}{6}$

$= \frac{5}{6} > \frac{1}{2}$

By for method

Let m be the median

$F(m) = \frac{1+2m}{6}$

$\frac{1}{2} = \frac{1+2m}{6}$

$m = 1$

By for correct median

Median = 1

12(b)

for $-1 < x \leq 0$

$f(x) = \frac{d}{dx}\left(\frac{1+x}{6}\right)$

$= \frac{1}{6}$

By for the derivative of the first function.

for $0 \leq x \leq 2$

$f(x) = \frac{d}{dx}\left(\frac{1+2x}{6}\right)$

$= \frac{2}{6}$

By for derivative of second and third functions

for $2 \leq x \leq 8/3$

$f(x) = \frac{d}{dx}\left(\frac{4+3x}{12}\right)$

$= \frac{3}{12}$

for $x \geq 8/3$

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$$f(a) = \begin{cases} 1/6 & , -1 \le x \le 0 \\ 2/6 & , 0 \le x \le 2 \\ 3/12 & , 2 \le x \le 8/3 \\ 0 & , \text{elsewhere.} \end{cases}$$

A1 for correct f(a)

12 (c)

$P(1 \le x \le 2.5) = F(2.5) - F(1)$

B1 for the expression

$= \frac{11 + 8 \times 2.5}{12} - \frac{11 + 2 \times 1}{6}$

B1 for substitution

$= \frac{11}{24}$

A1 for correct probability

12 (d)

$$E(X) = \int_{-1}^{0} \frac{x}{6}\,dx + \int_{0}^{2} \frac{x}{3}\,dx + \int_{2}^{8/3} \frac{x}{4}\,dx$$

B1 for correct method of mean

$$= \frac{x^2}{12}\Big|_{-1}^{0} + \frac{x^2}{6}\Big|_{0}^{2} + \frac{x^2}{8}\Big|_{2}^{8/3}$$

B1 for the integration

$$= \frac{(0)^2}{12} - \frac{(-1)^2}{12} + \frac{(2)^2}{6} + \frac{(8/3)^2}{8} - \frac{(2)^2}{8}$$

$$= -\frac{1}{12} + \frac{4}{6} + \frac{8}{9} - \frac{1}{2}$$

$$= \frac{35}{36}$$

$= 0.97222$

A1 for correct fraction ensures seam to 4th and above

12(a)

Resultant force = $\underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3}$

$= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -11 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -12 \end{pmatrix} N$

B1 for correct method of resultant force

B1 for correct magnitude method

Magnitude = $\sqrt{(5)^2 + (-12)^2}$

$= 13N$

B1 for correct magnitude and units

12(b)

$G = \begin{vmatrix} 2 & 2 \\ -3 & 3 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} -2 & 3 \\ -11 & -2 \end{vmatrix}$

$= 12 + 19 + 37$

$= 68 Nm$ clockwise

$= 68 Nm$ clockwise

B1 for correct method of G

B1 for units of G.

B1 for correct value of G plus units

$68 = \begin{vmatrix} 5 & x \\ -12 & y \end{vmatrix}$

B1 for correct substitution in the equation

$68 = 5y + 12x$

B1 for the equation

12(c)

at the x axis $y = 0$

B1 for letting $y = 0$

$68 = 5(0) + 12x$

$x = 5.6667 m$

A1 for value of distance to more than 4 dps.

The distance is 5.6667m from the origin.

12(d)

$\underset{\sim}{F} + \begin{pmatrix} 5 \\ -12 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

B1 for correct method.

$\underset{\sim}{F} = \begin{pmatrix} -5 \\ 12 \end{pmatrix} N$

The force is $(-5\underset{\sim}{i} + 12\underset{\sim}{j})$ N

A1 for the force

14(a)

Let $x = N^{1/k}$

$x^k = N$

$x^k - N = 0$

Let $f(x) = x^k - N$

$f'(x) = kx^{k-1}$

$$x_{n+1} = x_n - \frac{x_n^k - N}{kx_n^{k-1}}$$

$$= \frac{kx_n^k - x_n^k + N}{kx_n^{k-1}}$$

$$x_{n+1} = \frac{(k-1)x_n^k + N}{kx_n^{k-1}} \quad \text{for } n = 0, 1, 2, 3 \ldots$$

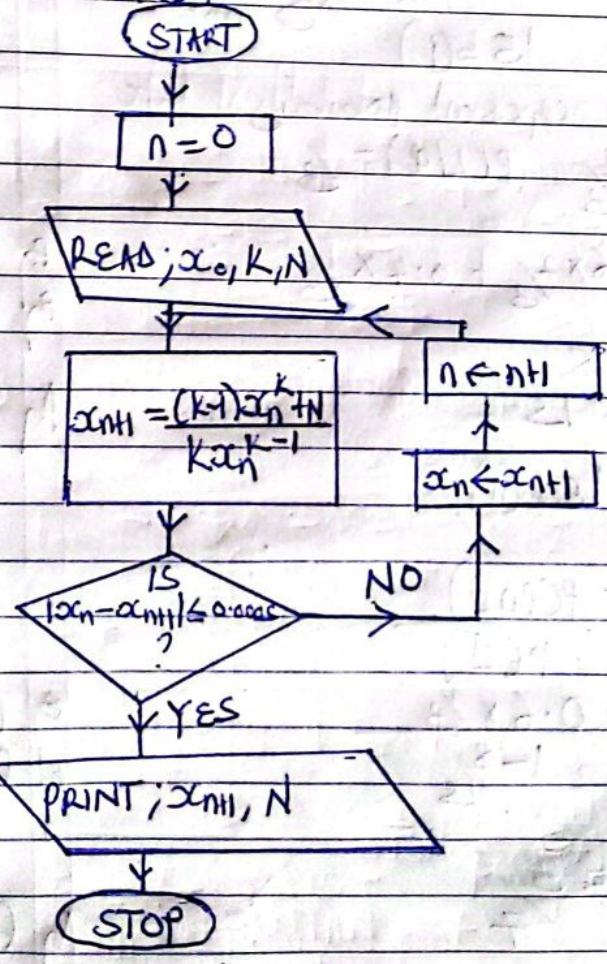
By for letting $x = N^{1/k}$

By for differentiation of $f(x)$

By correct substitution in the NRM method

By for correct derivation

14(b)



By for START, Read and initialization.

By for computation box and filling

By for the loop

By for the question box

By for printing and stopping.

14(c)

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.6	1.9935	0.3935
1	1.9935	1.9054	0.0881
2	1.9054	1.8989	0.0065
3	1.8989	1.8988	0.001

By for x_{n+1} column

By for $|x_{n+1} - x_n|$ column

By for x_n column.

The fourth root is 1.899 (3dps) when N=13

15 a(i)

P(only one check point) = P(A∩B'∩C') + P(A'∩B∩C') + P(A'∩B'∩C)

= 0.3×0.5×0.3 + 0.7×0.5×0.3 + 0.7×0.5×0.7

= 0.395

B1 for correct method

B1 for correct substitution

A1 for correct answer of probability

15 a(ii)

P(two or more points) = 1 − [P(only one point) + P(no check point)]

= 1 − [0.395 + 0.7×0.5×0.3]

= 0.5

B1 for correct method

B1 for correct substitution.

A1 for correct answer of probability

Accept any other alternative.

15 b(i)

Let P(L) represent probability of late.

P(L|P) = 2/3 P(L|Q) = 1/3

P(L) = 0.6×2/3 + 0.4×1/3

= 8/15

B1 for L∩P

B1 for L∩Q

A1 for correct answer

15 b(ii)

P(P|L') = P(P∩L') / P(L')

= (0.6×1/3) / (1 − 8/15)

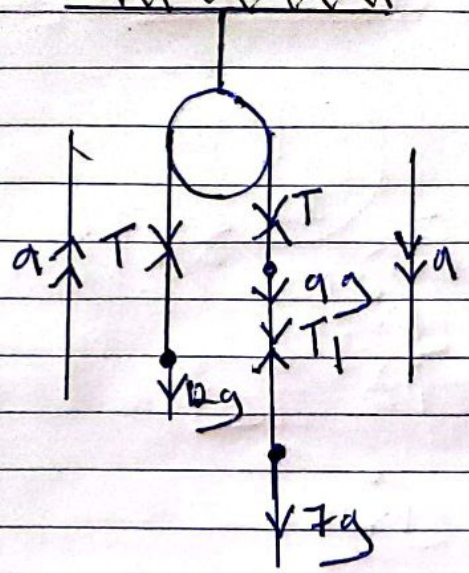
= 3/7

B1 for correct method

B1 for correct (P∩L')

A1 for correct answer

16 a(i)



Let T be the tension of the first string
Let T_1 be the tension of the second string
Let a be the acceleration

B1 for correct diagram
B1 for the future tensions

for the 12kg mass
$T - 12g = 12a$ ——— .
$T = 12a + 12g$ ——— (1)

B1 for the first expression of 12kg mass

for the 9kg mass.
$9g + T_1 - T = 9a$ ——— (2)
but $T = 12a + 12g$.
$9g + T_1 - (12a + 12g) = 9a$

B1 for the second expression of 9kg mass

B1 for substitution

$-3g - 12a + T_1 = 9a$
$T_1 = 21a + 3g$ ——— (3)

B1 for the expression of 7kg mass

for the 7kg mass.
$7g - T_1 = 7a$ ——— (4)
but $T_1 = 21a + 3g$.
$7g - 21a - 3g = 7a$
$4g = 22a$
$a = \frac{4 \times 9.8}{}$
$= 1.7818\ ms^{-2}$

B1 for the substitution

M1 for the acceleration

16 a(ii)

from (1) $T = 12 \times 1.7818 + 12 \times 9.8$
$= 138.9816\ N$

B1 for correct substitution
A1 for correct tension.

16 a(ii)

$V = 0 + 1.7818 \times 1.5$
$= 2.6727\ ms^{-1}$

B1 for correct substitution

P425/2
APPLIED MATHEMATICS
Paper 2
Nov./Dec. 2023
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g, to be 9.8 ms^{-2}.

Turn Over

SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

1. A coin is biased such that when it is tossed the head is twice as likely to occur as the tail. Find the probability that in seven tosses, there will be exactly two tails. *(05 marks)*

2. Two bodies A and B of masses 6 kg and 2 kg moving along a straight line with velocities 4 ms^{-1} and 2 ms^{-1} respectively, collide head on. After collision, A moves with a velocity of 2.6 ms^{-1} in the same direction.

Calculate the;

(a) velocity of B after collision. *(02 marks)*

(b) loss in kinetic energy. *(03 marks)*

3. The values of a function $f(x)$ are given in the table below.

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$f(x)$	0.1003	0.0391	0.0801	0.0602	0.0649	0.0380	0.0327

Use the trapezium rule to estimate the value of

$$\int_0^3 f(x)\,dx$$

correct to **three** decimal places. *(05 marks)*

4. A ball of mass 1 kg rolls from rest down a rough plane inclined at 30° to a horizontal ground. The ball rolls for 4 m before it reaches the ground. The coefficient of friction between the ball and the plane is $\frac{1}{4}$. Find the velocity with which the ball reaches the ground. *(05 marks)*

5. The table below shows the age distribution of a population of a certain town in a census.

AGE (years)	NUMBER ('000)
Under 10	15
10 and under 20	19
20 and under 30	16
30 and under 40	18
40 and under 60	30
60 and under 80	6
80 and under 90	1

(a) Draw a histogram for the data. *(03 marks)*

(b) Use the histogram to estimate the modal age of the population.

(02 marks)

6. The numbers $x = 6.45$, $y = 0.00215$ and $z = 2.7$ are each rounded off to the given number of decimal places.

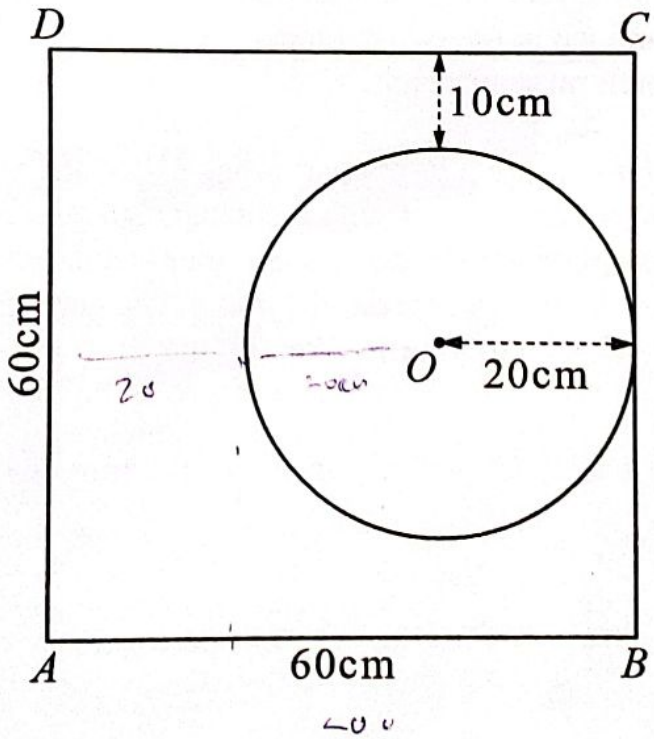
Determine the interval in which $w = \frac{x + z^3}{\sqrt{y}}$ lies. *(05 marks)*

7. Two independent events R and S are such that $P(R) = {}^3/_4$ and $P(S) = P(S' \cap R')$. Find;

(a) $P(S)$. *(03 marks)*

(b) $P(S' \cap R)$. *(02 marks)*

8. A uniform lamina in form of a square with side 60 cm has a circular hole of radius 20 cm made in it as shown in the diagram below.



Find the position of the centre of gravity of the lamina from side AD.

(05 marks)

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. The table below shows the scores of 10 candidates in Biology and Economics.

CANDIDATE	A	B	C	D	E	F	G	H	I	J
BIOLOGY	45	63	56	61	75	83	73	50	77	70
ECONOMICS	90	64	76	70	55	53	62	85	53	62

(a) (i) Plot a scatter diagram for the data.

(ii) Draw a line of best fit on the scatter diagram.

(iii) Use your line of best fit to estimate the Biology mark for a candidate who scored 57 in Economics.

(*06 marks*)

(b) Calculate a rank correlation coefficient between the candidates' performance in the two subjects.
Comment on your result. (*06 marks*)

10. Two points A and B are 526 m apart along a straight road. A car moving along the road passes point A with a constant speed of 25 ms^{-1}. The car maintains this speed for 10 seconds and then decelerates uniformly for 8 seconds until it attains a speed of V ms^{-1}. The car maintains this speed until it passes point B. The total time taken by the car to move from point A to B is 30 seconds.

(a) Sketch a Velocity – Time graph for the motion of the car. (*04 marks*)

(b) Determine the;

(i) value of V. (*05 marks*)
(ii) deceleration of the car. (*03 marks*)

11. Given that $f(x) = xe^x + 5x - 10$;

(a) (i) Evaluate $f(1)$ and $f(2)$, correct to **four** decimal places.
(ii) Deduce that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$. (*04 marks*)

(b) Use linear interpolation **twice** to obtain the root of the equation

$$xe^x + 5x - 10 = 0,$$

correct to **three** decimal places. (*08 marks*)

12. A continuous random variable X has a cumulative distribution function given by

$$F(x)=\begin{cases}\frac{1+x}{6}, & -1\le x\le 0\\ \frac{1+2x}{6}, & 0\le x\le 2\\ \frac{4+3x}{12}, & 2\le x\le \frac{8}{3}\\ 1, & x\ge \frac{8}{3}\end{cases}$$

Find;

(a) the median. *(03 marks)*

(b) the probability density function $f(x)$. *(03 marks)*

(c) $P(1\le X\le 2.5)$. *(03 marks)*

(d) the mean of X. *(03 marks)*

13. Three forces $\boldsymbol{F}_1=(2\boldsymbol{i}-3\boldsymbol{j})$ N, $\boldsymbol{F}_2=(5\boldsymbol{i}+2\boldsymbol{j})$ N and $\boldsymbol{F}_3=(-2\boldsymbol{i}-11\boldsymbol{j})$ N act at points (2, 3), (–2, 3) and (3, –2) respectively.

Determine the;

(a) magnitude of their resultant force. *(03 marks)*

(b) equation of the line of action of the resultant force. *(05 marks)*

(c) distance from the origin at which the resultant cuts the x-axis. *(02 marks)*

(d) force that should be added to form a couple. *(02 marks)*

14. (a) Show that the formula based on Newton Raphson method for approximating the k^{th} root of a number N is given by

$$x_{n+1}=\frac{(k-1)x_n^{\,k}+N}{kx_n^{\,k-1}} \qquad n=0,\,1,\,2,\,3,\,\ldots$$

(04 marks)

(b) Construct a flow chart that;

(i) reads in the initial approximation x_0, k and N,

(ii) computes and prints N and its k^{th} root correct to **three** decimal places. *(05 marks)*

(c) Perform a dry run for your flow chart when $N=13$, $x_0=1.6$ and $k=4$. *(03 marks)*

15. (a) A woman travelling to work by a car goes through three police check points A, B and C. The probabilities that she is delayed at A, at B and at C are 0.3, 0.5 and 0.7 respectively.
Determine the probability that she is delayed at;

(i) only one check point. (03 marks)

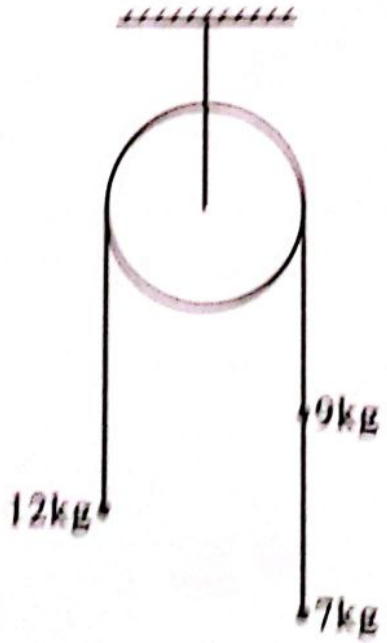
(ii) two or more check points. (03 marks)

(b) A man goes to work by route P or route Q. The probability that he takes route P is 0.6. The probability that he is late given that he goes through P is $^2/_3$ and through Q is $^1/_4$.

(i) Find the probability that he is late for work on a certain day. (03 marks)

(ii) Given that he is not late, determine the probability that he went through P. (03 marks)

16. The diagram below shows three masses of 12 kg, 9 kg and 7 kg connected by light inelastic strings. The string connecting the 12 kg and 9 kg masses passes over a smooth fixed pulley. The other string connects the 9 kg and 7 kg masses.



The system is released from rest and the 12 kg mass accelerates upwards.

(a) Calculate the;

(i) acceleration of the system.

(ii) tensions in the strings. (10 marks)

(b) Determine the velocity of the 12 kg mass after 1.5 seconds. (03 marks)