

PERMUTATIONS AND COMBINATIONS

- Permutations are arrangements of things taken some or all at a time.
- In a permutation, order of the things is taken into consideration.
- ${}^n P_r$ represents the number of permutations (without repetitions) of n dissimilar things taken r at a time.
- **Fundamental Principle:** If an event is done in 'm' ways and another event is done in 'n' ways, then the two events can be together done in mn ways provided the events are independent.
- ${}^n P_r = \frac{n!}{(n-r)!} = n(n-1)(n-2) \dots (n-r+1)$
- ${}^n P_1 = n$, ${}^n P_2 = n(n-1)$, ${}^n P_3 = n(n-1)(n-2)$.
- ${}^n P_n = n!$
- $\frac{{}^n P_r}{{}^n P_{r-1}} = n - r + 1$
- ${}^{n+1} P_r = r \cdot {}^n P_{r-1} + {}^n P_r$
- ${}^n P_r = r \cdot {}^{n-1} P_{r-1} + {}^{n-1} P_r$
- The number of permutations of n things taken r at a time containing a particular thing is $r \times {}^{n-1} P_{r-1}$.
- The number of permutations of n things taken r at a time not containing a particular thing is ${}^{n-1} P_r$.
- i) The number of permutations of n things taken r at a time allowing repetitions is n^r .
- ii) The number of permutations of n things taken not more than r at a time allowing repetitions is $\frac{n(n^r - 1)}{(n - 1)}$
- The number of permutations of n things of which p things are of one kind and q things are of another kind etc., is $\frac{n!}{p!q! \dots}$.
- The sum of all possible numbers formed out of all the 'n' digits without zero is $(n-1)!$ (sum of all the digits) (111.....n times).
- The sum of all possible numbers formed out of all the 'n' digits which includes zero is

- $[(n-1)! (\text{sum of all the digits})(111\dots n \text{ times})] - [(n-2)! (\text{sum of all the digits})(111\dots (n-1) \text{ times})]$
- The sum of all possible numbers formed by taking r digits from the given n digits which do not include zero is ${}^{n-1}p_{r-1} (\text{sum of all the digits})(111\dots r \text{ times})$.
- The sum of all possible numbers formed by taking r digits from the given n digits which include zero is ${}^{n-1}p_{r-1} (\text{sum of all the digits})(111\dots r \text{ times}) - {}^{n-2}p_{r-2} (\text{sum of all the digits})(111\dots (r-1) \text{ times})$.
- i) The number of permutations of n things when arranged round a circle is $(n-1)!$
- ii) In case of necklace or garland number of circular permutations is $\frac{(n-1)!}{2}$
- Number of permutations of n things taken r at a time in which there is at least one repetition is $n^r - {}^np_r$.
- The number of circular permutations of ' n ' different things taken ' r ' at a time is $\frac{{}^np_r}{r}$.

Very Short Answer Questions

1. If ${}^nP_3 = 1320$, find n ,

Sol: ${}^nP_r = \frac{n!}{(n-r)!}$

$$= n(n-1)(n-2).....(n-r+1)$$

$$\therefore {}^nP_3 = 1320$$

$$= 10 \times 12 \times 11$$

$$= 12 \times 11 \times 10 = {}^{12}P_3$$

$$\therefore n = 12$$

2. If ${}^nP_7 = 42$, nP_5 , find n

Sol: ${}^nP_7 = 42$, nP_5

$$n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)$$

$$= 42 \cdot n(n-1)(n-2)(n-3)(n-4)$$

$$\Rightarrow (n-5)(n-6) = 42$$

$$\Rightarrow (n-5)(n-6) = 7 \times 6$$

$$\Rightarrow n-5 = 7 \text{ or } n-6 = 6$$

$$\therefore n = 12$$

3. If ${}^{(n+1)}P_5 : {}^nP_6 = 2:7$ find n

Sol: ${}^{(n+1)}P_5 : {}^nP_6 = 2:7 \Rightarrow \frac{{}^{(n+1)}P_5}{{}^nP_6} = \frac{2}{7}$

$$\Rightarrow \frac{(n+1)n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)(n-4)(n-5)} = \frac{2}{7}$$

$$\Rightarrow 7(n+1) = 2(n-4)(n-5)$$

$$\Rightarrow 7n + 7 = 2n^2 - 18n + 40$$

$$\Rightarrow 2n^2 - 25n + 33 = 0$$

$$\Rightarrow 2n(n-1) - 3(n-11) = 0$$

$$\Rightarrow (n-11)(2n-3) = 0 \Rightarrow n = 11 \text{ or } \frac{3}{2}$$

Since n is a positive integer, $n = 11$

4. If ${}^{12}P_5 + 5 \cdot {}^{12}P_4 = {}^{13}P_r$ and r

Sol: We have

$${}^{(n-1)}P_r + r \cdot {}^{(n-1)}P_{(r-1)} = {}^nP_r \text{ and } r \leq n$$

$${}^{12}P_5 + 5 \cdot {}^{12}P_4 = {}^{13}P_r \text{ (Given)}$$

$$\Rightarrow r = 5$$

5. If ${}^{18}P_{r-1} : {}^{17}P_{r-1} = 9:7$, find r

Sol: ${}^{18}P_{r-1} : {}^{17}P_{r-1} = 9:7 \Rightarrow \frac{{}^{18}P_{r-1}}{{}^{17}P_{r-1}} = \frac{9}{7}$

$$\Rightarrow \frac{18!}{[18-(r-1)]!} \times \frac{[17-(r-1)]!}{17!} = \frac{9}{7}$$

$$\Rightarrow \frac{18!}{(19-r)!} \frac{(18-r)!}{17!} = \frac{9}{7}$$

$$\Rightarrow \frac{18 \times 17! (18-r)!}{(19-r)(18-r)17!} = \frac{9}{7}$$

$$\Rightarrow 18 \times 7 = 9(19-r)$$

$$\Rightarrow 14 = 19 - r \therefore r = 19 - 14 = 5$$

6. A man has 4 sons and there are 5 schools within his reach. In how many ways can he admit his sons in the schools so that no two of them will be in the same school.

Sol: The number of ways of admitting 4 sons into 5 schools if no two of them will be in the same = ${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$

7. Find the number of 4 digit numbers that can be formed using the digits 1, 2, 4, 5, 7, 8 when repetition is allowed.

Sol: The number of 4 digit numbers that can be formed using the digits 1, 2, 4, 5, 7, 8 when repetition is allowed = $6^4 = 1296$

8. Find the number of 5 letter words that can be formed using the letters of the word RHYME if each letter can be used any number of times.

Sol: The number of 5 letter words that can be formed using the letters of the word RHYME if each letter can be used any number of times = $5^5 = 3125$

9. Find the number of functions from set A containing 5 elements into a set B containing 4 elements

Sol: Set A contains 5 elements, Set B contains 4 elements since The total number of functions from set A containing elements to set B containing n elements in n^m .

For the image of each of the 5 elements of the set A has 4 choices.

\therefore The number of functions from set a containing 5 elements into a set B containing 4 element = $4 \times 4 \times \dots \times 4 (5 \text{ times}) = 4^5 = 1024$

10. Find the number of ways of arranging 7 persons around a circle.

Sol: Number of persons, $n = 7$

\therefore The number of ways of arranging 7 persons around a circle $= (n - 1)! = 6! = 720$

11. Find the number of ways of arranging the chief minister and 10 cabinet ministers at a circular table so that the chief minister always sit in a particular seat.

Sol: Total number of persons $= 11$

The chief minister can be occupied a particular seat in one way and the remaining 10 seats can be occupied by the 10 cabinet ministers in $(10)!$ ways.

\therefore The number of required arrangements $= (10)! \times 1 = (10)! = 36,28,800$

The number of ways of preparing a chain with 6 different

$$\text{Colored beads.} = \frac{1}{2}(6-1)! = \frac{1}{2} \times 5! = \frac{1}{2} \times 120 = 60$$

12. Find the number of ways of preparing a chain with 6 different coloured beads.

Sol: We know that the number of circular permutations of hanging type that can be formed using

n things is $\frac{(n-1)!}{2}$.

Hence the number of different ways of preparing the chains with 6 different coloured beads

$$= \frac{(6-1)!}{2} = \frac{5!}{2} = 60.$$

13 Find the number of ways of arranging the letters of the word.

(i) **INDEPENDENCE** (ii) **MATHEMATICS** (iii) **SINGING**

(iv) **PERMUTATION** (iv) **COMBINATION** (vi) **INTERMEDIATE**

Sol: (i) The word **INDEPENDENCE** contains 12 letters in which there are 3 N's are alike, 2 D's are alike, 4 E's are alike and rest are different

$$\therefore \text{The number of required arrangements} = \frac{(12)!}{4!3!2!}$$

(ii) The word **MATHEMATICS** contains 11 letters in which there are 2 M's are alike, 2 A's are alike, 2 T's are alike and rest are different

$$\therefore \text{The number of required arrangements} = \frac{(11)!}{2!2!2!}$$

(iii) The word **SINGING** contains 7 letters in which there are 2 I's are alike, 2 N's are alike, 2 G's are alike and rest is different. \therefore The number of required arrangement = $\frac{7!}{2! \times 2! \times 2!}$

(iv) The word **PERMUTATION** contains 11 letters in which there are 2 T's are alike and rest are different

$$\therefore \text{The number of required arrangements} = \frac{(11)!}{2!}$$

(v) The word **COMBINATION** contains 11 letters in which there are 2 O's are alike, 2 I's are alike, 2 N's are alike and rest are different

$$\therefore \text{The number of required arrangements} = \frac{11!}{2!2!2!}$$

(vi) The word **INTERMEDIATE** contains 12 letters in which there are 2 I's are alike, 2 T's are alike, 3 E's are alike and rest are different,

$$\therefore \text{The number of required arrangements} = \frac{(12)!}{2!2!3!}$$

14. Find the number of 7 digit numbers that can be formed using 2,2,2,3,3,4,4.

Sol. In the given 7 digits, there are three 2's, two 3's and two 4's.

\therefore The number of 7 digit numbers that can be formed using the given digits $= \frac{7!}{3!2!2!}$

15. If ${}^nP_4 = 1680$, find n

Sol: Given ${}^nP_4 = 1680$

But ${}^nP_4 = n(n-1)(n-2)(n-3)$

Thus, we are given $n(n-1)(n-2)(n-3)$
 $= 1680 = 8 \times 7 \times 6 \times 5$

On comparing the largest integers on both sides. We get n = 8

16. If ${}^{12}P_r = 1320$, find r

Sol: $1320 = 12 \times 11 \times 10 = {}^{12}P_3$. **Hence** $r = 3$

17. If ${}^{(n+1)}P_5 : {}^nP_5 = 3:2$, find n

Sol: ${}^{(n+1)}P_5 : {}^nP_5 = 3:2 \Rightarrow \frac{n+1}{n-4} = \frac{3}{2}$

$\Rightarrow 2n + 2 = 3n - 12 \Rightarrow n = 14$

It can be verified that $n = 14$

Satisfies the given equation

18. If ${}^{56}P_{(r+6)} : {}^{54}P_{(r+3)} = 30800 : 1$, **find r**

Sol: ${}^{56}P_{(r+6)} : {}^{54}P_{(r+3)} = 30800 : 1$

$$\Rightarrow \frac{(56)!}{(56 - (r+6))!} \times \frac{((54) - (r+3))!}{(54)!} = \frac{30800}{1}$$

$$\Rightarrow \frac{(56)!}{(50-r)!} \times \frac{(51-r)!}{(54)!} = 30800$$

$$\Rightarrow (56)(55)(51-r) = 30800$$

$$\Rightarrow (51-r) = \frac{30800}{56 \times 55} = 10 \Rightarrow r = 41$$

It can be verified that $r = 41$ **satisfies the given equation**

19. Find the numbers of ways of arranging 6 boys and 6 girls in a row. In how many of these arrangements.

(i) All the girls are together (ii) No two girls are together (iii) Boys and girls come alternately?

Sol: 6 boys and 6 girls are altogether 12 persons. They can be arranged in a row in (12)! ways.

Treat the 6 girls as one unit. Then we have 6 boys and 1 unit of girls. They can be arranged in 7! ways. Now, the 6 girls can be arranged among themselves in 6! ways. Thus the number of ways in which all 6 girls are together is (7! x 6!).

(ii) First arrange the 6 boys in a row in 6! ways. Then we can find 7 gaps between them (including the beginning gap and the ending gap) as shown below by the letter x :

x B x B x B x B x B x B x

Thus we have 7 gaps and 6 girls. They can be arranged in 7P_6 ways.

Hence, the number of arrangements which no two girls sit together is $6! \times {}^7P_6 = 7! \cdot 6!$

(iii) The row may begin with either a boy or a girl, that is, 2 ways. If it begins with a boy, then odd places will be occupied by boys and even places by girls. The 6 boys can be arranged in 6

odd places in $6!$ ways and 6 girls in the 6 even places in $6!$ ways. Thus the number of arrangements in which boys and girls come alternately is $2 \times 6! \times 6!$,

20. Find the number of injections of a set with 'm' elements into a set with 'n' elements.

Sol: Let A be a set with m elements and B a set with n elements. If $m > n$, then we cannot define an injection from A into B. Hence the number of injections is '0'

Now, suppose $m \leq n$. Then the required number of the injections is ${}^n P_m$

21. Find the number of injections of set A with 5 elements to a set B with 7 elements.

Sol: If a set A has m elements and the set B has n elements ($m \leq n$), then the number of injections from A into B = ${}^n P_m$

\therefore The number of injections from set A with 5 elements into set B with 7 elements. = ${}^7 P_5 = 2,520$

22. a) Find the number of functions from a set A containing m elements to a set B containing n elements.

Sol: Let $A = \{a_1, a_2, \dots, a_m\}$ and $B = \{b_1, b_2, \dots, b_n\}$

First to define the image of a_1 we have n choices (any element of B). Then to define the image of a_2 we again have n choices (since a_1, a_2 can have the same image).

Thus we have n choices for the image of each of the m elements of the set A. Therefore, the number of different ways of defining the images of elements of A (with images in B) is

$$n \times n \times \dots \times n (m \text{ times}) = n^m$$

b) Find the number of surjection's from a set A with n elements to a set B with 2 elements.

Sol: Let $A = \{a_1, a_2, \dots, a_n\}$ and $B = \{x, y\}$. The total number of functions from A to B is 2^n

(by problem 2). For a surjection, both the elements x, y of B must be in the range. Therefore,

a function is not a surjection if the range contains only x (or y). There are only two such functions.

Hence, the number of surjection's from A to B is $2^n - 2$

23. Find the number of chains that can be prepared using 7 different coloured beads.

Sol: We know that the number of circular permutations of hanging type that can be formed using n things is $\frac{1}{2} \{(n-1)!\}$. Hence the number of chains is $\frac{1}{2} \{(7-1)!\} = \frac{1}{2} (6!) = 360$

24. Find the number of different ways of preparing a garland using 7 distinct red roses and 4 distinct yellow roses such that no two yellow roses come together.

Sol: First arrange 7 red roses in a circular form (garland form) in $(7-1)! = 6!$ ways. Now, there are 7 gaps and 4 yellow roses can be arranged in these 7 gaps in 7P_4 ways.

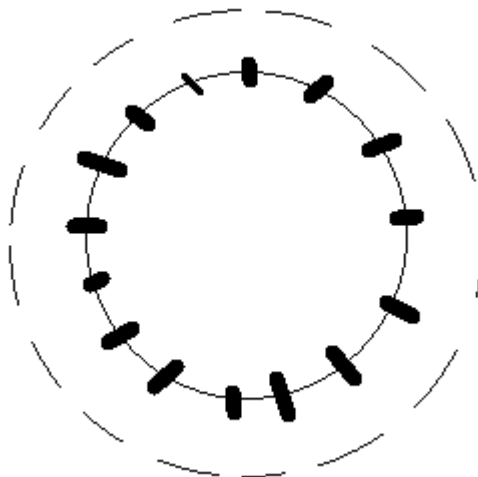
Thus, the total number of circular permutations is $6! \times {}^7P_4$.

But, in the case of garlands, clock-wise and anti clock-wise arrangements look alike.

Hence, the number of required ways is. $\frac{1}{2} (6! \times {}^7P_4)$

25. 14 persons are seated at a round table. Find the number of ways of selecting two persons out of them who are not seated adjacent to each other.

Sol. The seating arrangement of given 14 persons at the round table as shown below.



Number of ways of selecting 2 persons out of 14 persons = ${}^{14}C_2 = \frac{14 \times 13}{1 \times 2} = 91$

There are 14 pairs in which each pair contains two persons adjacent to each other and one among these 14 pairs can be selected in 14 ways and these are not allowed.

\therefore The required no. of ways = $91 - 14 = 77$

26. There are 4 copies (alike) each of 3 different books. Find the number of ways of arranging these 12 books in a shelf in a single row.

Sol. We are given 12 books in which 4 alike books are of one kind, 4 alike books are of the second kind and 4 alike books are of the third kind. Hence, the number of ways of

Arranging the given 12 books is $\frac{12!}{4!4!4!}$

27. How many four digit numbers can be formed using the digits 1, 2, 5, 7, 8, 9? How many of them begin with 9 and end with 2?

Sol: The number of four digit numbers that can be formed using the given digits 1, 2, 5, 7, 8, 9 is ${}^6P_4 = 360$. Now, the first place and last place can be filled with 9 and 2 in one way.

9				2
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The remaining 2 places can be filled by the remaining 4 digits 1, 5, 7, 8. Therefore these two places can be filled in 4P_2 ways. Hence, the required number of ways =

$$1 \cdot {}^4P_2 = 12.$$

28. Find the number of ways in which 4 letters can be put in 4 addressed envelopes so that no letter goes into the envelope meant for it.

Sol: Required number of ways is

$$4! \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right) = 12 - 4 + 1 = 9.$$

Note: If there are n things in a row, a permutation of these n things such that none of them occupies its original position is called a derangement of n things.

The number of derangements of n distinct things is :

$$n! \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + (-1)^n \frac{1}{n!} \right) = 9.$$

29. Find the number of ways of selecting 4 boys and 3 girls from a group of 8 boys and 5 girls.

Sol: 4 boys can be selected from the given 8 boys in 8C_4 ways and 3 girls can be selected from the given 5 girls in 5C_3 ways. Hence, by the fundamental principle the number of required selections is:

$${}^8C_4 \times {}^5C_3 = 70 \times 10 = 700.$$

Short Answer Questions

1. If there are 25 railway stations on a railway line, how many single second class tickets must be printed so as to enable a passenger to travel from one station to another. Number of stations on a railway line

Sol: Number of single second class tickets must be printed so as to enable a passenger to travel from one station to another $= {}^{25}P_2 = 25 \times 24 = 600$

2. In a class there are 30 students. On the New Year day, every student posts a greeting card to all his/her classmates. Find the total number of greeting cards posted by them.

Sol: Number of students in a class are 30,

\therefore Total number of greeting cards posted by every student to all his/her classmates

$$= {}^{30}P_2 = 30 \times 29 = 870$$

3. Find the number of ways of arranging the letters of the word TRIANGLE. So that the relative positions of the vowels and consonants are not distributed

Sol: Vowels : A, E, I, O, U

In a given, word

Number of vowels is 3

Number of consonants is 5

C C V V C C C V

Since the relative positions of the vowels and consonants are not disturbed. The 3 vowels can be arranged in their relative positions in $3!$ Ways and the 5 consonants can be arranged in their relative positions in $5!$ Ways.

\therefore The number of required arrangements = $(3!)(5!) = (6)(120) = 720$

4. Find the sum of all 4 digit numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition.

Sol: First method: The number of 4 digit numbers formed

by using the digits 0, 2, 4, 7, 8 without repetition

$$= {}^5P_4 - {}^4P_3 = 120 - 24 = 96$$

Out of these 96 numbers,

${}^4P_3 - {}^3P_2$ numbers contain 2 in units place

${}^4P_3 - {}^3P_2$ numbers contain 2 in tens place

${}^4P_3 - {}^3P_2$ numbers contain 2 in 100's place

4P_3 numbers contain 2 in 1000's place

∴ The value obtained by adding 2 in all the numbers

$$= ({}^4P_3 - {}^3P_2) 2 + ({}^4P_3 - {}^3P_2) 20 + ({}^4P_3 - {}^3P_2) 200 + {}^4P_3 \times 2000$$

$$= {}^4P_3(2+20+200+2000) - {}^3P_2(2+20+200) - 24 \times (2222) - 6(222)$$

$$= 24 \times 2 \times 1111 - 6 \times 2 \times 111$$

Similarly, the value obtained by adding 4 is $24 \times 4 \times 1111 - 6 \times 4 \times 111$

The value obtained by adding 7 is $24 \times 7 \times 1111 - 6 \times 7 \times 111$

The value obtained by adding 8 is $24 \times 8 \times 1111 - 6 \times 8 \times 111$

∴ The sum of all the numbers

$$= (24 \times 2 \times 1111 - 6 \times 2 \times 111) + (24 \times 4 \times 1111 - 6 \times 4 \times 111) + (24 \times 7 \times 1111 - 6 \times 7 \times 111) + (24 \times 8 \times 1111 - 6 \times 8 \times 111)$$

$$= 24 \times 111.1 \times (2 + 4 + 7 + 8) - 6 \times 111 \times (2 + 4 + 7 + 8)$$

$$= 26664(21) - 666(21) = 21(26664 - 666)$$

$$= 21(25998) = 5,45,958.$$

5. Find the number of numbers greater than 4000 which can be formed using the digits 0, 2, 4, 6, 8 without repetition.

Sol. While forming any digit number with the given digits, zero cannot be filled in the first place. We can fill the first place with the remaining 4 digits. The remaining places can be filled with the remaining 4 digits.

All the numbers of 5 digits are greater than 4000. In the 4 digit numbers, the number starting with 4 or 6 or 8 are greater than 4000. The number of 4 digit numbers which begin with 4 or 6 or 8 = $3 \times {}^4P_3 = 3 \times 24 = 72$

The number of 5 digit numbers = $4 \times 4! = 4 \times 24 = 96$

∴ The number of numbers greater than 4000 is $72 + 96 = 168$

6. Find the number of ways of arranging the letters of the word MONDAY so that no vowel occupies even place.

Sol: In the word MONDAY there are two vowels, 4 consonants and three even places, three odd places.

Since no vowel occupies even place, the two vowels can be filled in the three odd places in 3P_2 ways.

The 4 consonants can be filled in the remaining 4 places in $4!$ ways.

\therefore The number of required arrangement = ${}^3P_2 \times 4! = 6 \times 24 = 144$

7. Find the number of ways of arranging 5 different mathematics books, 4 different physics books and 3 different chemistry books such that the books of the same subject are together.

Sol: Given number Mathematics books = 5

Given number of Physics books = 4

Given number of Chemistry books = 4

Treat all Mathematics books as 1 unit, Physics books as 1 unit, Chemistry books as 1 unit.

The number of ways of arranging 3 units of books = $3!$

5 different Mathematics books can be arranged in $5!$ Ways.

4 different Physics books can be arranged in $4!$

3 different Chemistry books can be arranged in $3!$ Ways.

\therefore By fundamental principle of counting, required number of ways of arranging books

$$= 3! \times 5! \times 4! \times 3!$$

$$= 6 \times 120 \times 24 \times 6 = 103680$$

8. Find the number of palindromes with 6 digits that can be formed using the digits.

- (i) 0, 2, 4, 6, 8 (ii) 1, 3, 5, 7, 9

Sol: i) Given digits are 0, 2, 4, 6, 8.

The first place and last place (lakh's place and unit's place) of a 6-digit palindrome number is filled by same digit.

This can be done in 4 ways (using non-zero digit).

Similarly ten thousand's place and ten's place is filled by same digit.

As repetition is allowed this can be done in 5 ways.

Thousand's place and hundred's place is filled by same digit in 5 ways.

∴ Total number of 6 digital palindromes formed using given digits are :

$$4 \times 5 \times 5 = 100.$$

ii) Given digits are 1, 3, 5, 7, 9.

The first place and last place (lakh's place and unit's place) of a 6-digit palindrome number is filled by same digit in 5 ways.

As repetitions allowed.

Similarly ten thousand's place and ten's place is filled by same digit in 5 ways.

Thousand's place and hundred's place is filled by same digit in 5 ways.

∴ Total number of 6 digital palindromes formed using given digits are :

$$5 \times 5 \times 5 = 125.$$

9. Find the number of 4 digit telephone numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 with at least one digit repeated.

Sol. The number of 4 digit numbers formed using the digits 1, 2, 3, 4, 5, 6 when repetition is allowed = 6^4 .

The number of 4 digit numbers formed using the digits 1, 2, 3, 4, 5, 6 when repetition is not allowed = 6P_4

∴ The number of 4 digit telephone numbers in which at least one digit repeated

$$= 6^4 - {}^6P_4 = 1296 - 360 = 936$$

10. Find the number of bijections from a set A containing 7 elements onto itself.

Sol: The number of bijections from set A with n elements to set B with same number of elements in A is n !

$$\text{Let } A = [a_1, a_2, a_3, a_4, a_5, a_6, a_7]$$

For a bijection, the element a_1 has 7 choices

For its image, the element a_2 has 6 choices

For its image, the element a_3 has 5 choices

For its image, the element a_4 has 4 choices

For its image, the element a_5 has 3 choices

For its image, the element a_6 has 2 choices

For its image and the element a_7 has 1 choice

For its image.

∴ The number of bijections from set A with 7 elements onto itself

$$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \text{ or } 7! = 5040$$

11. Find the number of ways of arranging r things in a line using the given 'n' different things in which at least one thing is repeated.

Sol: The number of ways of arranging, r things in a line using the given n different things

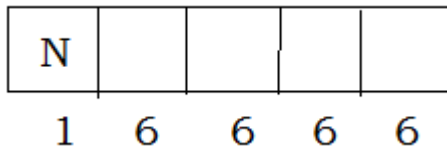
(i) When repetition is allowed is n^r

(ii) When repetition is not allowed is nP_r

∴ The number of ways of arranging 'r' things in a line using the 'n' different things in which at least one thing is repeated = $n^r - {}^nP_r$

12. Find the number of 5 letter words that can be formed using the letters of the word NATURE that begin with N, when repetition is allowed.

Sol: First we can fill up the first place with N in one way.



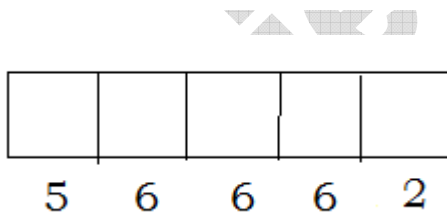
The remaining 4 places can be filled with anyone of the 6 letters in $6 \times 6 \times 6 \times 6 = 6^4$ ways.

\therefore The number of 5 letter words that can be formed using the letters of the word NATURE that begin with N when repetition is allowed $= 1 \times 6^4 = 1296$

13. Find the number of 5 - digit numbers divisible by 5 that can be formed using the digits 0, 1, 2, 3, 4, 5 when repetition is allowed.

Sol. The unit place of 5 digit number which can be divisible by 5 using the given digits can be filled by either 0 or 5 in two ways.

The first place can be filled any one of the given digits except '0' in 5 ways. The remaining 3 places can be filled by any one of the given digits in $6 \times 6 \times 6$ ways (\because repetition is allowed)



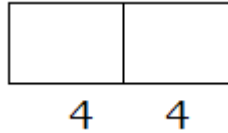
\therefore The number of 5 digit numbers divisible by 5 that can be formed using the given digits when repetition is allowed $= 2 \times 5 \times 6 \times 6 \times 6 = 2160$ ways.

14. Find the number of numbers less than 2000 that can be formed using the digits 1, 2, 3, 4, if repetition is allowed.

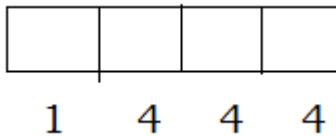
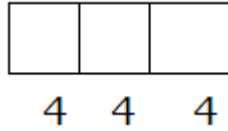
Sol. All the single digit numbers, two digit numbers, three digit numbers and the four digit numbers started with 1 are the numbers less than 2000 using the digits 1, 2, 3, 4.

The number of single digit numbers formed using the given digits - 4

The number of two digit numbers formed using the given digits when repetition is allowed = $4 \times 4 = 16$



The number of three digit numbers formed using the given digits = $4 \times 4 \times 4 = 64$



The number of 4 digit numbers started with 1 formed using the given digits = $4 \times 4 \times 4 = 64$

\therefore The total number of numbers less than 2000 that can be formed using the digits 1, 2, 3, 4 if repetition is allowed = $4 + 16 + 64 + 64 = 148$

15. Find the number of ways of arranging 4 boys and 3 girls around a circle so that all the girls sit together.

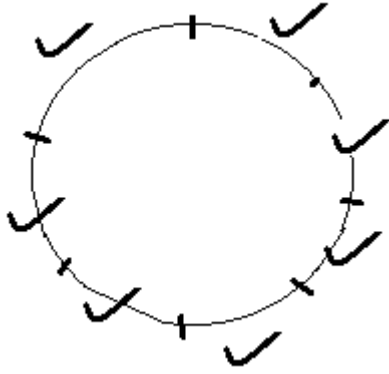
Sol: Treat all the 3 girls as one unit. Then we have 4 boys and 1 unit of girls. They can be arranged around a circle in $4!$ ways. Now, the 3 girls can be arranged among themselves in $3!$ ways.

\therefore The number of required arrangements = $4! \times 3! = 24 \times 6 = 144$

16. Find the number of ways of arranging 7 gents and 4 ladies around a circular table if no two ladies wish to sit together.

Sol: - First arrange the 7 gents around a circular table in $6!$ ways.

Then we can find 7 gaps between them. The 4 ladies can be arranged in these 7 gaps in 7P_4 ways.



∴ The number of required arrangements

$$= 6! \times {}^7P_4 = 720 \times 7 \times 6 \times 5 \times 4$$

$$= 6,04,800$$



17. Find the number of ways of arranging 7 guests and a host around a circle, if 2 particular guests wish to sit on either side of the host.

Sol. Number of guests = 7

Treat the two particular guests along the host as one unit.

Then we have 5 guests and one unit of 2 particular guests along the host. They can be arranged around a circle in 5! ways. The two particular guests can be arranged on either side of the host in 2! ways. ∴ Number of required arrangements = $5! \times 2! = 120 \times 2 = 240$

18. Find the number of ways of preparing a garland with 3 yellow, 4 white and 2 red roses of different sizes such that the two red roses come together.

Sol. Treat that 2nd roses of different sizes as one unit.

Then we have 3 yellow, 4 white and one unit of red roses.

Then they can be arranged in garland form in $\frac{1}{2}(8-1)! = \frac{1}{2}(7!)$ ways. Now 2 red roses in one unit can

be arranged among themselves in 2! ways.

∴ The number of ways of preparing a garland

$$= \frac{1}{2} (7!) \times 2! = \frac{1}{2} \times 5040 \times 2 = 5040$$

19. Find the number of 4 letter words that can be formed using the letters of the word RAMANA

Sol: The given word RAMANA has 6 letters in which there are 3 A's are alike and rest are different.

Using these 6 letters, 3 cases arises to form 4 letter words.

Case I: All different letters R, A, M, N Number of 4 letter words formed - $4! \sim 24$

Case II: Two like letters A, A and two out of R, M, N

The two different letters can be chosen from 3 letters in ${}^3C_2 = 3$ ways.

$$\therefore \text{Number of 4 letters word formed} = 3 \times \frac{4!}{2!}$$

$$= 3 \times 12 = 36$$

Case III: Three like letters A, A, A and one out of R, M, N.

One letter can be chosen from 3 different letters in ${}^3C_1 = 3$ ways.

∴ Number of 4 letter words formed

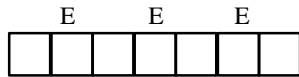
$$= 3 \times \frac{4!}{3!} = 3 \times 4 = 12$$

∴ Total number of 4 letter words formed from the word RAMANA = $24 + 36 + 12 = 72$

20. How many numbers can be formed using all the digits 1, 2, 3, 4, 3, 2, 1 such that even digits always occupy even places?

Sol. In the given 7 digits, there are two 1's, two 2's, two 3's and one 4.

The 3 even places can be occupied by the even digits 2, 4, 2, in $\frac{3!}{2!}$. (Even place = E)



The remained odd places can be occupied by the odd digits 1, 3, 3, 1 in $\frac{4!}{2!2!}$ ways.

∴ The number of required arrangements

$$= \frac{3!}{2!} \times \frac{4!}{2! \times 2!} = 3 \times 6 = 18$$

21. In a library, there are 6 copies of one book, 4 copies each of two different books 5 copies each of three different books and 3 copies each of two different books. Find the number of ways of arranging all the books in a shelf in a single row.

Sol. Total number of books in a library are $6 + (4 \times 2) + (5 \times 3) + (3 \times 2) = 35$

∴ The number of required arrangements

$$= \frac{(35)!}{6!(4!)^2 (5!)^3 (3!)^2}$$

22. A book store has 'm' copies each, 'n' different books. Find the number of ways of arranging the books in a shelf in a single row.

Sol. Total number of books in a book store are $= m \times n = mn$

$$\therefore \text{The number of required arrangements} = \frac{(mn)!}{(m!)^n}$$

23. Find the number of 5-digit numbers that can be formed using all the digits 0,1,1, 2,3.

Sol. '0' can also be taken as one digit, the number of 5 digit number formed $= \frac{5!}{2!} = 60$

(Among them, the number that starts with zero is only 4 digit number. The number of 4!

$$\text{Numbers start with zero} = \frac{4!}{2!} = 12$$

Hence the number of 5 digit numbers that can be formed by using all the given digits

$$= 60 - 12 = 48$$

24. In how many ways can the letters of the word CHEESE be arranged so that no two E's come together?

Sol. The given word contains 6 letters in which one C, one H, 3 E's and one S.

Since no two E's come together, first arrange the remaining 3 letters in 3! ways. Then we can

find 4 gaps between them. The 3 E's can be arranged in these 4 gaps in $\frac{{}^4P_3}{3!}$

∴ The number of required arrangements = $3! \times 4 = 24$

25. Find the number of 5 letter words that can be formed using the letters of the word MIXTURE which begin with a vowel when repetition is allowed.

Sol: The word MIXTURE has 7 letters 3 vowels {E, I, U}

and 4 consonants {M, R, X, T} we have to fill up 5 blanks

$\square \square \square \square \square$

Fill the first place with one of the 3 vowels in 3 ways

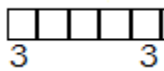
Each of the remaining 4 places can be filled in 7 ways (since repetition is allowed)

∴ The number of 5 letter words

$$= 3 \times 7 \times 7 \times 7 \times 7 = 3 \times 7^4$$

26. Find the number of 5 letter words that can be formed using the letters of the word EXPLAIN that begin and end with a vowel, when repetitions are allowed.

Sol: The word EXPLAIN has 7 letters, among them 3 vowels {A, E, I} and 4 consonants {L, P, N, X}



We can fill the first and last places with vowels each in 3 ways.

Now each of the remaining 3 places can be filled in 7 ways (using any letter of given 7 letters).

Hence the number of 5 letter words which begin and end with vowels

$$= 3 \times 7 \times 7 \times 7 \times 3 = 3,087$$

27. Find the number of ways of arranging the letters of the word SINGING so that

(i) They begin and end with I

(ii) The two G's come together

Sol: The word SINGING has 2 I's, 2 G's and 2 N's and one S. Total 7 letters.



(i) First, we fill the first and last places with I's in $\frac{2!}{2!} = 1$ way as shown below.

Now we fill the remaining 5 places with the remaining 5 letters in $\frac{5!}{2!2!} = 30$ ways.

Hence the number of required permutations = 30

- (ii) Treat two G's as one unit. Then we have 5 letters 2 I's, 2 N's and one S + one unit of 2G's
 $= 6$ can be arranged in $\frac{6!}{2!2!} = \frac{720}{2 \times 2} = 180$ ways.

Now the two G's among themselves can be arranged in one way. Hence the number of received permutations $= 180 \times 1 = 180$.

28. Find the number of ways of arranging a^4, b^3, C^5 in its expanded form

Sol. The expanded form of $a^4 b^3 C^5$ is aaaa bbb ccccc

There are $4 + 3 + 5 = 12$ letters

There can be arranged in $\frac{(12)!}{4!3!5!}$ ways

29. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order. Find the rank of the word EAMCET.

Sol: The dictionary order of the letters of the word EAMCET IS

A C EE M T

In the dictionary order first gives the words which begins with the letters A.

If we fill the first place with A, remaining 5 letters can be arranged in $\frac{5!}{2!}$ ways (since there are 2

E's) on proceeding like this, we get

A -----> $\frac{5!}{2!}$ words

C -----> $\frac{5!}{2!}$ words

E A C -----> 3! Words

E A E -----> 3! Words

E A M C E T -----> 1 word

Hence the rank of the word EAMCET is

$$= 2 \times \frac{5!}{2!} + 2 \times 3! + 1$$

$$= 120 + 12 + 1 = 133$$

30. Find the number of ways of arranging 8 men and 4 women around a circular table.

In how many of them

- (i) All the women come together
- (ii) No two women come together

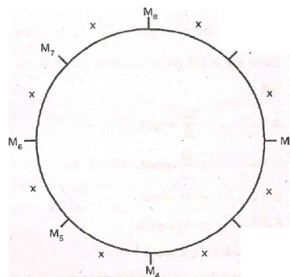
Sol. Total number of persons = 12 (8 men + 4 women)

Therefore, the number of circular permutations is (11) !

- (i) Treat the 4 women as one unit. Then we have 8 men + 1 unit of women = 9 entities.

Which can be arranged around a circle in 8! ways. Now, the 4 women among themselves can be arranged in 4! ways. Thus, the number of required arrangements is 8! x 4!.

- (ii) First arrange 8 men around a circle in 7! ways. Then there are 8 places in between them as shown in fig by the symbol x (one place in between any two consecutive men). Now, the 4 women can be arranged in these 8 places in 8P_4 ways.



Therefore, the number of circular arrangements in which no two women come together is 7! X 8P_4 .

31. Find the number of ways of seating 5 Indians, 4 Americans and 3 Russians at a round table so that (i) All Indians sit together (ii) No two Russians sit Together

(iii) Persons of the same nationality sit together

Sol. (i) Treat the 5 Indians as a single unit. Then we have 4 Americans, 3 Russians and 1 unit of Indians. That is, 8 entities in total. Which can be arranged at a round table in $(8 - 1)! = 7!$ ways.

Now, the 5 Indians among themselves can be arranged in $5!$ ways. Hence, the number of required arrangements is $7! \times 5!$

(ii) First we arrange the 5 Indians + 4 Americans around the table in $(9 - 1)! = 8!$ ways.

Now, we can find 9 gaps in between these 9 persons (one gap between any two consecutive persons).

The 3 Russians can be arranged in these 9 gaps in 9P_3 ways. Hence, the number of required arrangements is $8! \times {}^9P_3$.

(iii) Treat the 5 Indians as one unit, the 4 Americans as the second unit and the 3 Russians as the third unit.

These 3 units can be arranged at round table in $(3 - 1)! = 2!$ ways.

Now, the 5 Indians among themselves can be permuted in $5!$ ways. Similarly, the 4 Americans in $4!$ ways and the 3 Russians in $3!$ ways. Hence the number of required arrangements is

$$2! \times 5! \times 4! \times 3!$$

Long Answer Questions

1. Find the number of 5 letter words that can be formed using the letters of the word **CONSIDER**. How many of them begin with 'C', how many of them end with 'R' and how many of them begin with 'C' and end with 'R'?

Sol: Given word 'CONSIDER' contains 8 different letters.

The number of 5 letter words that can be formed using the letters of word 'CONSIDER' = 8P_5 .

Out of them,

- i) If first place is filled by 'C', the remaining 4 places by remaining 7 letters can be filled in 7P_4 ways.

∴ Number of words begin with 'C' are 7P_4 .

- ii) If last place is filled by 'R', the remaining first four places by remaining 7 letters can be filled 7P_4 ways.

∴ Number of words end with 'R' are 7P_4 .

- iii) If first place is filled with 'C' and last place is filled by 'R', the remaining 3 places between them by remaining 6 letters can be filled in 6P_3 ways.

∴ Number of words begin with 'C' and end with 'R' are 6P_3 .

2. Find the number of ways of seating 10 students A_1, A_2, \dots, A_{10} in a row such that

- i) A_1, A_2, A_3 sit together
- ii) A_1, A_2, A_3 sit in a specified order.
- iii) A_1, A_2, A_3 sit together in a specified order.

Sol: A_1, A_2, \dots, A_{10} are the 10 students.

- i) A_1, A_2, A_3 sit together:

Treat A_1, A_2, A_3 as 1 unit.

This unit with remaining 7 students can be arranged in $8!$ ways.

The students A_1, A_2, A_3 can be arranged in $3!$ ways.

∴ The number of ways of seating 10 students such that A_1, A_2, A_3 sit together = $8! \times 3!$.

ii) A_1, A_2, A_3 sit in a specified order :

In 10 positions remaining 7 students other than A_1, A_2, A_3 can be arranged in ${}^{10}P_7$ ways.

As A_1, A_2, A_3 sit in a specified order, they can be arranged in 3 gaps in only 1 way.

\therefore The number of ways of A_1, A_2, A_3 sit in a specified order = ${}^{10}P_7$.

iii) A_1, A_2, A_3 sit together in a specified order :

Treat A_1, A_2, A_3 as 1 unit and they are in a specified order.

This unit with remaining 7 students can be arranged in 8! Ways.

\therefore The number of ways A_1, A_2, A_3 sit together in specified order = 8!.

- 3. Find the number of ways in which 5 red balls, 4 black balls of different sizes can be arranged in a row so that (i) no two balls of the same colour come together, (ii) the balls of the same colour come together.**

Sol: Given 5 red balls and 4 black balls are of different sizes.

i) No two balls of the same colour come together:

First arrange 4 black balls in row, which can be done in 4! ways

$\times B \times B \times B \times B \times$

Then we find 5 gaps, to arrange 5 red balls. This arrangement can be done in 5! Ways.

\therefore By principle of counting total number of ways of arranging = $5! \times 4!$.

ii) The balls of the same colour come together :

Treat all red balls as one unit and all black balls as another unit.

The number of ways of arranging these two units = 2!

The 5 red balls can be arranged in 5! Ways while 4 black balls are arranged in 4! ways.

\therefore By fundamental principal of counting, the required number of ways = $2! \times 4! \times 5!$.

4. Find the number of 4 digit numbers that can be formed using the digits 1, 2, 5, 6, 7, How many of them are divisible by (i) 2 (ii) 3 (iii) 4 (iv) 5 (v) 25.

Sol. The number of 4 digit numbers that can be formed using the digits 1, 2, 5, 6, 7 is ${}^5P_4 = 120$.

(i) A number is divisible by 2 when its unit place must be filled with an even digit from among the given integers. This can be done in 2 ways.

			2 or 6
--	--	--	--------

Now, the remaining 3 places can be filled with remaining 4 digits in ${}^4P_3 = 4 \times 3 \times 2 = 24$ ways.

\therefore The number of 4 digit numbers divisible by 2 $= 2 \times 24 = 48$

(ii) A number is divisible by 3 if the sum of the digits is divisible by 3. Sum of the given 5 digits $= 1 + 2 + 5 + 6 + 7 = 21$.

The 4 digits such that their sum is a multiple of 3 from the given digits are 1, 2, 5, 7 (sum is 15)

They can be arranged in $4!$ Ways and all these 4 digit numbers are divisible by 3.

\therefore The number of 4 digit numbers divisible by 3 $= 4! = 24$

(iii) A number is divisible by 4 only when the last two places (tens and units places) of it is a multiple of 4.

		x	x
--	--	---	---

Hence the last two places with one of the following 12, 16, 52, 56, 72, 76. Thus the last two places can be filled in 6 ways.

The remaining two places can be filled by remaining 3 digits in ${}^3P_2 = 3 \times 2 = 6$ ways.

\therefore The number of 4 digit numbers divisible by 4 $= 6 \times 6 = 36$.

(iv) A number is divisible by 5 when its units place must be filled by 5 from the given integers 2, 5, 6, 7. This can be done in one way

			5
--	--	--	---

The remaining 3 places can be filled with remaining 4 digits in ${}^4P_3 = 4 \times 3 \times 2$ ways..

The number of 4 digit numbers divisible by 5 = $1 \times 24 = 24$

(v) A number is divisible by 25 when its last two places are filled with either 25 or 75.

		x	x
--	--	---	---

Thus the last two places can be filled in 2 ways.

The remaining 2 places from the remaining 3 digits can be filled in ${}^3P_2 = 6$ ways.

\therefore The number of 4 digit numbers divisible by 25 = $2 \times 6 = 12$

5. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the ranks of the words

(i) REMAST

(ii) MASTER.

Sol. (i) The alphabetical order of the letters of the given word is A, E, M, R, S, T

The number of words begin with A is $5! = 120$

The number of words begin with E is $5! = 120$

The number of words begin with M is $5! = 120$

The number of words begin with RA is $4! = 24$

The number of words begin with REA is $3! = 6$

The next word is REMAST.

\therefore Rank of the word REMAST =

$$3(120) + 24 + 6 + 1 = 360 + 31 = 391$$

- (ii) The alphabetical order of the letters of the given word is A, E, M, R, S, T The number of words begin with A is $5! = 120$

The number of words begin with E is $5! = 120$

The number of words begin with MAE is $3! = 6$

The number of words begin with MAR is $3! = 6$

The number of words begin with MASE is $2! = 2$

The number of words begin with MASR is $2! = 2$

The next word is MASTER.

∴ Rank of the word MASTER

$$= 2(120) + 2(6) + 2(2) + 1$$

$$= 240 + 12 + 4 + 1 = 257.$$

6. If the letters of the word BRING are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the 59th word.

Sol: Given word is BRING.

∴ The alphabetical order of the letter is :

B, G, I, N, R.

In the dictionary order, first we write all words beginning with B.

Clearly the number of words beginning with B are $4! = 24$.

Similarly the number of words begin with G are $4! = 24$.

Since the words begin with b and G sum to 48, the 59th word must start with I.

Number of words given by IB = $3! = 6$

Hence the 59th word must start with IG.

Number of words begin with IGB = $2! = 2$

Number of words begin with IGN = $2! = 2$

∴ Next word is 59th = IGRBN.

7. Find the sum of all 5 digit numbers that can be formed using the digits 1, 2, 4, 5, 6 without repetition.

Sol. The number of 4 digit numbers formed by using the digits 1, 2, 4, 5, 6

without repetition = ${}^5P_4 = 120$

Out of these 120 numbers,

4P_3 numbers contain 2 in units place

4P_3 numbers contain 2 in tens place

4P_3 numbers contain 2 in hundreds place

4P_3 numbers contain 2 in thousands place

∴ The value obtained by adding 2 in all the numbers.

$$= {}^4P_3 \times 2 + {}^4P_3 \times 20 + {}^4P_3 \times 200 + {}^4P_3 \times 2000 = {}^4P_3 (2 + 20 + 200 + 2000) = {}^4P_3 (2222)$$

Similarly, the value obtained by adding 1 is ${}^4P_3 \times 1 \times 1111$

The value obtained by adding 4 is ${}^4P_3 \times 4 \times 1111$

The value obtained by adding 5 is ${}^4P_3 \times 5 \times 1111$

The value obtained by adding 6 is ${}^4P_3 \times 6 \times 1111$

The sum of all the numbers

$$= {}^4P_3 \times 1 \times 1111 + {}^4P_3 \times 2 \times 1111 + {}^4P_3 \times 4 \times 1111 + {}^4P_3 \times 5 \times 1111 + {}^4P_3 \times 6 \times 1111$$

$$= {}^4P_3 (1111) (1 + 2 + 4 + 5 + 6)$$

$$= 24 (1111) (18)$$

$$= 4,79,952$$

9. There are 9 objects 9 boxes. Out of 9 objects, 5 cannot fit in three small boxes. How many arrangements can be made such that each object can be put in one box only.

Sol: Given that there are 9 objects and 9 boxes.

As 5 objects out of 9, cannot fit in 3 small boxes, these 5 objects should be arranged in remaining 6 boxes.

This can be done in 6P_5 ways.

\therefore The remaining 4 blanks are to be filled with remaining 4 objects.

This can be done in $4!$ Ways.

\therefore The required number of ways = ${}^6P_5 \times 4! = 17,280$.

10.9 different letters of an alphabet are given. Find the number of 4 letter words that can be formed using these 9 letters which have

- (i) No letter repeated (ii) At Least one letter repeated.

Sol. The number of 4 letter words can be formed using the 9 different letters of an alphabet when repetition is allowed = 9^4

(i) The number of 4 letter words can be formed using the 9 different letters of an alphabet in which no letter is repeated = ${}^9P_4 = 9 \times 8 \times 7 \times 6 = 3024$

(ii) The number of 4 letter words can be formed using the 9 different letters of an alphabet in which at least one letter repeated - $9^4 - {}^9P_4 = 6561 - 3024 = 3537$.

11. Find the number of 4-digit numbers which can be formed using the digits 0, 2, 5, 7, 8 that are divisible by (i) 2 (ii) 4 when repetition is allowed.

Sol: Given digits are 0, 2, 5, 7, 8.

i) Divisible by 2:

The thousand's place of 4 digit number when repetition is allowed can be filled in 4 ways.
(using non-zero digits)

The 4-diti number is divisible by 2, when the units place is an even digit. This can be done in 3 ways.

The remaining 2 places can be filled by 5 ways each i.e., $5^2 = 25$ ways.

\therefore Number of 4 digit numbers which are divisible by 2 is $4 \times 3 \times 25 = 300$.

ii) Divisible by 4:

A number is divisible by 4 only when the number in last two places (ten's and unit's) is a multiple of 4.

As repletion is allowed the last two places should be filled with one of the following:

00, 08, 20, 28, 52, 72, 80, 88

This can be done is 8 ways.

Thousand's place is filled in 4 ways.

(i.e., using non-zero digits)

Hundred's placed can be filled in 5 ways.

\therefore Total number of 4 digit numbers formed $= 8 \times 4 \times 5 = 160$.

12. **Find the number of 4 digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 which are divisible by 6 when repetition of the digits is allowed.**

Sol. The first place of the number can be filled by any one of the given digits except '0' in 5 ways. The 2nd and 3rd places can be filled by any one of the given 6 digits in 6^2 ways.

x	x	x	x
5	6	6	1

After filling up the first 3 places, if we fill the units place with the given 6 digits, we get 6 consecutive positive integers. Out of these 6 consecutive integers exactly one will be divisible by '6'. Hence the units place can be filled in one way.

\therefore The number of 4 digit numbers formed using the given digits which are divisible by 6 when repetition is allowed. $= 5 \times 6^2 \times 1 = 180$

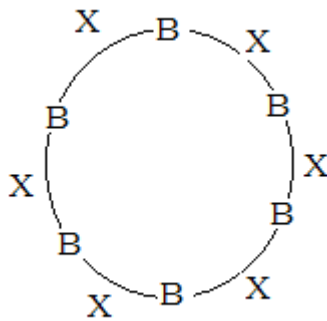
13. Find the number of ways of arranging 6 boys and 6 girls around a circular table so that

(i) all the girls sit together (ii) no two girls sit together (iii) boys and girls sit alternatively.

Sol. (i) Treat all the 6 girls as one unit. Then we have 6 boys and 1 unit of girls. They can be arranged around a circular table in $6!$ ways. Now, the 6 girls can be arranged among themselves in $6!$ ways.

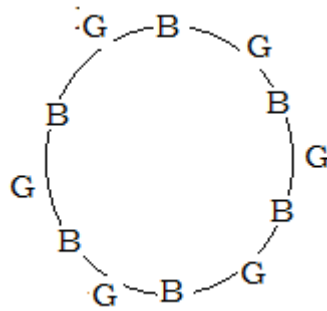
\therefore The number of required arrangements = $6! \times 6! = 720 \times 720 = 5,18,400$

(ii) First arrange the 6 boys around a circular table in $5!$ ways. Then we can find 6 gaps between them. The 6 girls can be arranged in these 6 gaps in $6!$ ways.



\therefore The number of required arrangements = $5! \times 6! = 120 \times 720 = 86,400$

(iii) Here the number of girls and number of boys are same.



Hence the arrangements of boys and girls sit alternatively in same as the arrangements of no two girls sit together or arrangements of no two boys sit together. First arrange the 6 girls around a circle table in $5!$ ways. Then we can find 6 gaps between them. The 6 boys can be arranged in these 6 gaps in $6!$ ways.

\therefore The number of required arrangements = $5! \times 6! = 120 \times 720 = 86,400$

- 14. Find the number of ways of arranging 6 red roses and 3 yellow roses of different sizes into a garland. In how many of them (i) all the yellow roses are together (ii) no two yellow roses are together**

The number of circular permutations like the garlands of flowers, chains of beads

etc., of n things $= \frac{1}{2}(n-1)!$

Total number of roses $= 6 + 3 = 9$

\therefore The number of ways of arranging 6 red roses and 3

yellow roses of different sizes into a garland $= \frac{1}{2} \times (9-1)! = \frac{1}{2} \times 8! = \frac{1}{2} \times 40320 = 20160$

- (i) Treat all the 3 yellow roses as one unit. Then we have 6 red roses and one unit of yellow roses. They can be arranged in garland form in $(7-1)! = 6!$ ways. Now, the 3 yellow roses can be arranged among themselves in $3!$ ways.

But in the case of garlands, clockwise arrangements look alike.

\therefore The number of required arrangements $= \frac{1}{2} \times 6! \times 3! = \frac{1}{2} \times 720 \times 6 = 2160$

- (ii) First arrange the 6 red roses in garland form in $5!$ ways. Then we can find 6 gaps between them. The 3 yellow roses can be arranged in these 6 gaps in 6P_3 ways.

But in the case of garlands, clock-wise and anti-clock wise arrangements look alike.

\therefore The number of required arrangements $= \frac{1}{2} \times 5! \times {}^6P_3 = \frac{1}{2} \times 120 \times 6 \times 5 \times 4 = 7200$

- 15. A round table conference is attended by 3 Indians, 3 Chinese, 3 Canadians and 2 Americans. Find the number of ways of arranging them at the round table so that the delegates belonging to same country sit together.**

Sol: Since the delegates belonging to the same country sit together, first arrange the 4 countries in a round table in $3!$ ways. Now, 3 Indians can be arranged among themselves in $3!$ ways, 3 Chinese can be arranged among themselves in $3!$ ways, 3 Canadians can be arranged

among themselves in $3!$ ways, and 2 Americans can be arranged among themselves in $2!$ ways.

\therefore The number of required arrangements = $3! \times 3! \times 3! \times 3! \times 2! = 2592$

16. A chain of beads is to be prepared using 6 different red coloured beads and 3 different blue coloured beads. In how many ways can this be done so that no two blue coloured beads come together.

Sol: First arrange the 6 red coloured beads in the form of chain of beads in $(6-1)! = 5!$ ways. Then there are 6 gaps between them, The 3 blue coloured beads can be arranged in these 6 gaps in 6P_3 ways.

Then the total number of circular permutations = $5! \times {}^6P_3$

But in case of chain of beads, clock-wise and anticlockwise arrangements look alike.

\therefore The number of required arrangements = $\frac{1}{2} \times 5! \times {}^6P_3 = \frac{1}{2} \times 120 \times 6 \times 5 \times 4 = 7200$

17. A family consists of a father, a mother, 2 daughters and 2 sons. In how many different way can they sit at a round table, if the 2 daughters wish to sit on either side of the father?

Sol: Total number of persons in a family = 6

Treat the 2 daughters along with a father as one unit.

Then we have a mother, 2 sons and one unit of daughters along with father in a family. They can be seated around a table in $(4-1)! = 3!$ ways. The 2 daughters can be arranged on either side of the father in $2!$ ways.

\therefore Number of required arrangements = $3! \times 2! = 6 \times 2 = 12$

18. Find the number of ways of arranging the letters of the word ASSOCIATIONS. In how many of them (i) All the three S's come together ii) The two A's do not come together.

The number of linear permutations of 'n' things in which 'p' alike things of one kind, 'q' alike things of 2nd kind, 'r' alike things of 3rd kind and the rest are different

$$\text{is } \frac{n!}{p!q!r!}$$

The given word ASSOCIATIONS has 12 letters in which there are 2 A's are alike, 3 S's are alike, 2 O's are alike 2 I's are alike and rest are different.

$$\therefore \text{They can be arranged} = \frac{(12)!}{2!3!2!2!}$$

(i) Treat the 3 S's as one unit. Then we have $9 + 1 = 10$ entities in which there are 2 A's are alike, 2 O's are alike, 2 I's are alike and rest are different.

$$\text{They can be arranged in } \frac{(10)!}{2!2!2!} \text{ ways}$$

The 3 S's among themselves can be arranged in $\frac{3!}{3!} = 1$ way.

$$\therefore \text{The number of required arrangements} = \frac{(10)!}{2!2!2!}$$

(ii) Since 2 A's do not come together, first arrange the remaining 10 letters in which there are 3 S's are alike, 2 O's are alike 2 I's are alike and rest are different in $\frac{(10)!}{3!2!2!}$ ways. Then we can find

11 gaps between them. The 2 A's can be arranged in these 11 gaps in $\frac{{}^{11}P_2}{2!}$

$$\text{ways} = \frac{(10)!}{3!2!2!} \times \frac{{}^{11}P_2}{2!}$$

$$\therefore \text{The number of required arrangements} = \frac{(10)!}{3!2!2!} \times \frac{{}^{11}P_2}{2!}$$

19. Find the number of ways of arranging letters of the word MISSING so that two S's are together and the two i's are together

Sol. In the given word MISSING contains 7 letters in which there are 2 I's are alike, 2 S's are alike and rest are different.

Treat the 2 S's as one unit and 2 I's as one unit. Then we have $3 + 1 + 1 = 5$ entities. These can be arranged in $5!$ ways. The 2 S' scan be arranged among themselves in $\frac{2!}{2!} = 1$ ways and the 2 I'

scan be arranged among themselves in $\frac{2!}{2!} = 1$ ways

\therefore The number of required arrangements $= 5! \times 1 \times 1 = 120$

20. If the letters of the word AJANTA are permitted in all possible ways and the words then formed are arranged in dictionary order. Find the rank of the words i) AJANTA ii) JANATA

Sol: - The dictionary order of the letters of the word AJANTA is A AA J NT

(i) In the dictionary order first comes that words which begin with the letter A. If we fill the first place with A, we may set the word AJANTA. Second place can be filled with A, the remaining 4 places can be filled in $4! = 24$ ways. On proceeding like this, we get

$$AA \quad = 4! = 24$$

$$AJAA---- = 2! = 02$$

$$AJ ANA----- = 1 = 01$$

$$AJANTA = 1 = 01$$

\therefore Rank of the word AJANTA $= 24 + 02 + 01 + 01 = 28$

- (ii) In the dictionary order first comes that words which begin with the letter A. If we fill the first place with A, remaining 5 letters can be arranged in $\frac{5!}{2!}$ ways (since there 2 A's remain)

On proceeding like this, we get

$$A = \frac{5!}{2!} = 60$$

$$JAA----- = 3! = 6$$

$$JANAA--=1 = 1$$

$$JANATA = 1 = 1$$

\therefore Rank of the word JANATA is $=60+6+1+1=68$.

COMBINATIONS

1. ${}^nC_r = \frac{n!}{r!(n-r)!}$
2. ${}^nC_r = {}^nC_{n-r}$
3. ${}^nC_0 = 1, {}^nC_1 = n, {}^nC_2 = \frac{n(n-1)}{1.2}, {}^nC_n = 1$
4. ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$
5. ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$
6. ${}^nP_r = {}^nC_r \times r!$
7. $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$
8. $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}$
9. ${}^nC_x = {}^nC_y \Rightarrow x = y \text{ or } x + y = n.$
10. The number of combinations of n things taken r at a time containing a particular thing is ${}^{n-1}C_{r-1}$ and not containing a particular thing is ${}^{n-1}C_r$.
11. The number of combinations of n things taken one or more at a time is $({}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n) = 2^n - 1.$
12. The number of diagonals in a polygon of n sides is $\frac{n(n-3)}{2}.$
13. (a) The number of ways of distributing $m+n$ things ($m \neq n$) to two persons, m things to one and n things to the other is $\frac{(m+n)!}{m!n!} \times 2$
 (b) The number of ways of dividing $m + n$ things ($m \neq n$) into two groups of m things and n things is $\frac{(m+n)!}{m!n!}$

14. The number of ways of distributing $2n$ things equally to two persons is $\frac{2n!}{(n!)^2}$.
15. The number of ways of dividing $2n$ things into two equal groups is $\frac{2n!}{(n!)^2 2!}$.
16. The number of ways of dividing kn things into k equal groups is $\frac{(kn)!}{(n!)^k k!}$.
17. The sum of all the divisors of the number is $\left(\frac{2^{a+1}-1}{2-1}\right) \cdot \left(\frac{3^{b+1}-1}{3-1}\right) \cdot \left(\frac{5^{c+1}-1}{5-1}\right) \dots$

Very Short Answer Questions

1. If ${}^nC_4 = 210$, find n

Sol: ${}^nC_4 = 210$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} = 10 \times 21$$

$$\Rightarrow n(n-1)(n-2)(n-3)$$

$$= 10 \times 21 \times 1 \times 2 \times 3 \times 4 = 10 \times 7 \times 3 \times 2 \times 3 \times 4$$

$$= 10 \times 9 \times 8 \times 7 \therefore n = 10$$

2. If ${}^{12}C_r = 495$, find the possible values of r

Hint ${}^nC_r = {}^nC_{n-r}$

Sol: ${}^{12}C_r = 495 = 5 \times 99$

$$= 11 \times 9 \times 5 = \frac{12 \times 11 \times 9 \times 5 \times 2}{12 \times 2}$$

$$= \frac{12 \times 11 \times 10 \times 9}{1 \cdot 2 \cdot 3 \cdot 4} = {}^{12}C_4 \text{ or } {}^{12}C_8$$

$$\therefore r = 4 \text{ or } 8$$

3. If ${}^{10}C_2 = 3 \cdot {}^{n+1}C_3$ find n

Sol: ${}^{10}C_2 = 3 \cdot {}^{n+1}C_3$

$$\Rightarrow 10 \times \frac{n(n+1)}{1 \cdot 2} = \frac{3(n+1)(n-1)}{1 \cdot 2 \cdot 3}$$

$$\Rightarrow 10 = n + 1 \therefore n = 9$$

4. If ${}^nP_r = 5040$ and ${}^nC_r = 210$ find n and r.

Hint: ${}^nP_r = r! \cdot {}^nC_r$ and

$${}^nP_r = n(n-1)(n-2) \dots [n-(r-1)]$$

Sol: ${}^nP_r = 5040, {}^nC_r = 210$

$$r! = \frac{{}^nP_r}{{}^nC_r} = \frac{5040}{210} = \frac{504}{21} = 24 = 4!$$

$$\therefore r = 4$$

$${}^nP_r = 5040 \Rightarrow {}^nP_4 = 5040 = 10 \times 504$$

$$= 10 \times 9 \times 56 = 10 \times 9 \times 8 \times 7 = {}^{10}P_4$$

$$\therefore n = 10 \therefore n = 10, r = 4$$

5. If ${}^nC_4 = {}^nC_6$, find n.

We know that ${}^nC_r = {}^nC_s \Rightarrow r = s$ or $r + s = n$

Sol: ${}^nC_4 = {}^nC_6$

$$\therefore n = 4 + 6 = 10, (\because 4 \neq 6)$$

6. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$ find r.

Sol: ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$

$$\Rightarrow 2r - 1 = 2r + 4 \text{ or } (2r - 1) + (2r + 4) = 15$$

$$\text{i.e., } 4r + 3 = 15 \Rightarrow 4r = 12 \Rightarrow r = 3$$

$$\therefore r = 3$$

$$\therefore 2r - 1 = 2r + 4 \Rightarrow -1 = 4 \text{ which is impossible}$$

$$\therefore r = 3$$

7. If ${}^{17}C_{2t+1} = {}^{17}C_{3t-5}$ find t

Sol: ${}^{17}C_{2t+1} = {}^{17}C_{3t-5}$

$$\Rightarrow 2t+1 = 3t-5 \text{ or } (2t+1) + (3t-5) = 17$$

$$\Rightarrow 1+5-t \text{ or } 5t-21$$

$$\Rightarrow 1+5-t \text{ or } 5t-21$$

$$\Rightarrow t = 6 \text{ or } t = \frac{21}{5} \text{ which is not an integer}$$

$$\therefore t = 6$$

8. If ${}^{12}C_{r+1} = {}^{12}C_{3r-5}$, find r

Sol: ${}^{12}C_{r+1} = {}^{12}C_{3r-5}$

$$\Rightarrow r+1 = 3r-5 \text{ or } (r+1) + (3r-5) = 12$$

$$\Rightarrow 1+5 = 2r \text{ or } 4r-4 = 12$$

$$\Rightarrow 2r = 6 \text{ or } 4r = 16$$

$$\Rightarrow r = 3 \text{ or } r = 4$$

$$\therefore r = 3 \text{ or } 4$$

9. If ${}^9C_3 + {}^9C_5 = {}^{10}C_r$ then find r

Hint: ${}^nC_r = {}^nC_{n-r}$

Sol: ${}^{10}C_r = {}^9C_3 + {}^9C_5$

$$\therefore {}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r \Rightarrow {}^9C_3 + {}^9C_5 = {}^9C_3 + {}^9C_5 = {}^{10}C_6 \text{ or } {}^{10}C_4 = {}^{10}C_r \text{ (Given)}$$

$$\Rightarrow r = 4 \text{ or } 6$$

10. Find the number of ways of forming a committee of 5 members from 6 men and 3 ladies.

Sol: Total number of persons = $6 + 3 = 9$

\therefore Number of ways of forming a committee of 5 members

$$\text{from 6 men and 3 ladies} = {}^9C_5 = \frac{9 \times 8 \times 7 \times 6 \times 5}{5 \times 4 \times 3 \times 2 \times 1} = 126$$

11. In questions no.10 how many committees contain at least 2 ladies

Sol: Since a committee contains atleast 2 ladies, the members of the committee may be of the following two types

(i) 3men, 2 ladies (ii) 2 men, 3 ladies

The number of selections in the first type = ${}^6C_3 \times {}^3C_2 = 20 \times 3 = 60$

The number of selections in the second type = ${}^6C_2 \times {}^3C_3 = 15 \times 1 = 15$

\therefore The required number of ways of selecting the committee containing at least 2 ladies = $60 + 15 = 75$

12. If ${}^nC_5 = {}^nC_6$ then find ${}^{13}C_n$

Sol: $\because {}^nC_5 = {}^nC_6 \Rightarrow n = 6 + 5 = 11$

$${}^{13}C_n = {}^{13}C_{11} = {}^{13}C_2 = \frac{13 \times 12}{1 \times 2} = 78$$

13. Find the number of ways of forming a committee of 4 members out of 5 boys and 5 girls such that there is at least one girl in the committee.

Sol. The number of ways of forming a committee of 4 members out of the given 10 members is ${}^{10}C_4$. Out of these, the number of committees having only boys is 5C_4

(That is, selecting all the 4 members from the 5 boys).

Thus, the number of committees with at least one girl is

$${}^{10}C_4 - {}^5C_4 = 210 - 5 = 205$$

14. If there are 5 alike pens, 6 alike pencils and 7 alike erasers, find the number of ways of selecting any number of (one or more) things out of them.

Sol: The required number of ways $= (5+1)(6+1)(7+1) - 1 = 335$

15. Find the number of positive division of 1080.

Sol: $1080 = 2^3 \times 3^3 \times 5^1$

\therefore The number of positive divisions of 1080

$$= (3+1)(3+1)(1+1)$$

$$= 4 \times 4 \times 2 = 32$$

16. In how many ways 9 mathematics papers can be arranged so that the best and the worst (i) may come together (ii) may not come together?

Sol: i) If the best and worst papers are treated as one unit, then we have

$$9 - 2 + 1 = 7 + 1 = 8 \text{ papers.}$$

Now these can be arranged in $(7+1)!$ ways and the best and worst papers between themselves can be permuted in $2!$ ways. Therefore the number of arrangements in which best and worst papers come together is $8!2!$.

ii) Total number of ways of arranging 9 mathematics papers is $9!$. The best and worst papers come together in $8!2!$ ways. Therefore the number of ways they may not come together is $9! - 8!2!$

$$= 8!(9 - 2) = 8! \times 7.$$

17. (i) If ${}^{12}C_{s+1} = {}^{12}C_{2s-5}$, then find s .

(ii) If ${}^nC_{21} = {}^nC_{27}$, then find ${}^{49}C_n$

Sol: (i) ${}^{12}C_{s+1} = {}^{12}C_{2s-5} \Rightarrow$ either $s+1 = 2s-5$ or $s+1+2s-5=12 \Rightarrow s=6$ or $3s=16$

$\therefore s=6$ (since 's' is non negative integer)

(ii) ${}^nC_{21} = {}^nC_{27} \Rightarrow n = 21 + 27 = 48$

Hence ${}^{49}C_n = {}^{49}C_{48} = {}^{49}C_1 = 49$

18. If a set of 'm' parallel lines intersect another set of 'n' parallel lines (not parallel to the lines in the first set), then find the number of parallelograms formed in this lattice structure.

Sol. To form a parallelogram, we have to select 2 lines from the first set which can be done in mC_2 ways and 2 lines from the second set which can be done in nC_2 ways. Thus, The number of parallelograms formed is ${}^mC_2 \times {}^nC_2$

Short Answer Questions

1. Prove that for $3 \leq r \leq n$

$${}^{(n-4)}C_r + 3{}^{(n-3)}C_{r-1} + 3{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} = {}^nC_r$$

Sol. LHS

$$\begin{aligned} &= \left[{}^{(n-3)}C_r + {}^{(n-3)}C_{r-1} \right] + 2 \left[{}^{(n-3)}C_{r-1} + {}^{(n-3)}C_{r-2} \right] + \left[{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} \right] \\ &= {}^{(n-3+1)}C_r + 2{}^{(n-3+1)}C_{r-1} + {}^{(n-3+1)}C_{r-2} \\ &= {}^{(n-2)}C_r + 2{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2} \\ &= \left[{}^{n-2}C_r + {}^{(n-2)}C_{r-1} \right] + \left[{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2} \right] \\ &= {}^{(n-2+1)}C_r + {}^{(n-2+1)}C_{r-1} \\ &= {}^{(n-1)}C_r + {}^{(n-1)}C_{r-1} \\ &= {}^{(n-1+1)}C_r = {}^nC_r = \text{RHS} \end{aligned}$$

2. Find the value of ${}^{10}C_5 + 2 \cdot {}^{10}C_4 + {}^{10}C_3$

Sol: ${}^{10}C_5 + 2 \cdot {}^{10}C_4 + {}^{10}C_3$

$$\begin{aligned} &\text{Hint : } {}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r \\ &= \left({}^{10}C_5 + {}^{10}C_4 \right) + \left({}^{10}C_4 + {}^{10}C_3 \right) = {}^{11}C_5 + {}^{11}C_4 \\ &= {}^{12}C_5 = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792 \end{aligned}$$

3. Simplify $\sum_{r=0}^4 (38-r)C_4$

$$\begin{aligned} &\text{Sol: since } {}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r \quad {}^{34}C_5 + \sum_{r=0}^4 (38-r)C_4 = {}^{34}C_5 + {}^{28}C_4 + {}^{27}C_4 + {}^{36}C_4 + {}^{35}C_4 + {}^{34}C_4 \\ &= \left({}^{34}C_5 + {}^{34}C_4 \right) + {}^{35}C_4 + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4 \\ &= \left({}^{35}C_5 + {}^{35}C_4 \right) + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4 \end{aligned}$$

$$\begin{aligned} &= \left({}^{37}C_5 + {}^{36}C_4 \right) + {}^{27}C_4 + {}^{38}C_4 \\ &= \left({}^{37}C_5 + {}^{37}C_4 \right) + {}^{38}C_4 = {}^{38}C_5 + {}^{38}C_4 = {}^{39}C_5 \end{aligned}$$

4. In a class there are 30 students. If each student plays a chess game with each of other students, then find the total number of chess games played by them.

Sol: Number of students in a class = 30

Since each student plays a chess game with each of the other students, the total number of chess games played by them = ${}^{30}C_2 = 435$

5. Find the number of ways of selecting 3 girls and 3 boys out of 7 girls and 6 boys

Sol: The number of ways of selecting 3 girls 3 boys out of 7 girls and 6 boys

$$= {}^7C_3 \times {}^6C_3 = 35 \times 20 = 700$$

6. Find the number of ways of selecting a committee of 6 members out of 10 members always including a specified member.

Sol: Since a specified member always includes in a committee, remaining 5 members can be selected from remaining 9 members in 9C_5 ways

\therefore Required number of ways selecting a committee = ${}^9C_5 = 126$

7. Find the number of ways of selecting 5 books from 9 different mathematics books that a particular book is not included.

Sol: Since a particular book is not included in the selection, the 5 books can be selected in the selection, the 5 books can be selected from remaining 8 books in 8C_5 ways.

\therefore The required number of ways of selecting 5 books = ${}^8C_5 = 56$

8. Find the number of ways of selecting 3 vowels and 2 consonants from the letters of the word equation.

Sol: The word EQUATION contains 5 vowels and 3 consonants.

The 3 vowels can be selected from 5 vowels in ${}^5C_3 = 10$ ways

The 2 consonants can be selected from 3 consonants in ${}^3C_2 = 3$ ways

∴ **The required number of ways of selecting 3 vowels and 2 consonants** $= 10 \times 3 = 30$

9. Find the number of diagonals of a polygon with 12 sides.

Sol: Number of sides of a polygon = 12

Number of diagonals of a n-sided polygon $= {}^nC_2 - n$

∴ Number of diagonals of 12 sided polygon $= {}^{12}C_2 - 12 = 54$.

10. If n persons are sitting in a row, find the number of ways of selecting two persons out of them who are sitting adjacent to each other

Sol: The number of ways of selecting 2 persons out of n persons sitting in a row, who are sitting adjacent to each other $= n - 1$.

11. Find the number of ways of giving away 4 similar coins to 5 boys if each boy can be given any number (less than or equal to 4) of coins.

Sol: The 4 similar coins can be divided into different number of groups as follows,

- (i) One group containing 4 coins
- (ii) Two groups containing 1, 3 coins reply
- (iii) Two groups containing 2, 2 coins reply
- (iv) Two groups containing 3, 1 coins reply
- (v) Three groups containing 1, 1, 2 coins reply
- (vi) Three groups containing 1, 2, 1 coins reply
- (vii) Three groups containing 2, 1, 1 coins reply
- (viii) Four groups containing 1, 1, 1, 1 coins reply

These groups can given away to 5 boys in

$$= {}^5C_1 + 2 \times {}^5C_2 + {}^5C_2 + {}^5C_3 \times \frac{3!}{2!} + {}^5C_4 = 5 + 20 + 10 + 30 + 5 = 70 \text{ ways}$$

12. i) If ${}^{12}C_{s+1} = {}^{12}C_{(2s-5)}$, then find s.

ii) If ${}^nC_{21} = {}^nC_{27}$, then find ${}^{50}C_n$.

Sol: i) By Theorem 5.6.16.

$${}^{12}C_{s+1} = {}^{12}C_{(2s-5)}$$

\Rightarrow Either $s + 1 = 2s - 5$ or

$$(s + 1) + (2s - 5) = 12$$

$$\Rightarrow s = 6 \text{ or } s = 16/3$$

$$\Rightarrow s = 6 \text{ (since 's' is a non negative integer)}$$

ii) By Theorem 5.6.16.

$${}^nC_{21} = {}^nC_{27} \Rightarrow n = 21 + 27 = 48$$

Therefore,

$${}^{50}C_n = {}^{50}C_{48} = {}^{50}C_2 = \frac{50 \times 49}{1 \times 2} = 1225.$$

Long Answer Questions

1. Prove that

$$\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5.....(4n-1)}{\{1.3.5.....(2n-1)^2\}}$$

Sol:

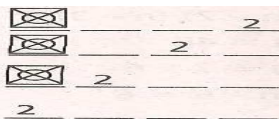
$$\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{\frac{(4n)!}{(2n)!(2n)!}}{\frac{(2n)!}{n!n!}}$$

$$= \frac{(4n)!}{(2n)!^2} \times \frac{n!n!}{(2n)!}$$

$$= \frac{(4n)(4n-1)(4n-2)(4n-3)(4n-4).....5.4.3.2.1}{[(2n)(2n-1)(2n-2)(2n-3)(2n-4).....5.4.3.2.1]^2} \times \frac{n!n!}{(2n)!}$$

$$= \frac{[(4n-1)(4n-3).....5.3.1][(2n)(2n-1)(2n-2).....2.1.2^{2n}]}{[\{(2n-1)(2n-3).....5.3.1\} \{n(n-1)(n-2).....2.1.2^n\}]^2} \times \frac{n!n!}{(2n)!}$$

$$= \frac{[(4n-1)(4n-3).....5.3.1](2n)!2^{2n}}{[(2n-1)(2n-3).....5.3.1]^2 [n!]^2 (2n)^2} \frac{(n!)^2}{(2n)!}$$



$$\therefore \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5.....(4n-1)}{[1.3.5.....(2n-1)^2]}$$

2. If a set A has 12 elements. Find the number of subsets of A having

(i) 4 elements (ii) at least 3 elements (iii) almost 3 elements

Sol. Number of elements in set A = 12

(i) Number of subsets of A with exactly 4 elements = ${}^{12}C_4 = 495$

(ii) The required subset contains atleast 3 elements.

Number of subsets of A with exactly 0 elements is ${}^{12}C_0$

Number of subsets of A with exactly 1 element is ${}^{12}C_1$

Number of subsets of A with exactly 2 element ${}^{12}C_2$

Total number of subsets of A formed = 2^{12} .

$$= 4096 - 79 = 4017$$

(iii) The required subset contains almost 3 elements i.e., it may contain 0 or 1 or 2 or 3 elements.

Number of subsets of A with exactly 0 elements is ${}^{12}C_0$

Number of subsets of A with exactly 1 element is ${}^{12}C_1$

Number of subsets of A with exactly 2 elements is ${}^{12}C_2$

Number of subsets of A with exactly 3 elements is ${}^{12}C_3$

\therefore Number of subsets of A with almost 3 elements

$$= {}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3$$

$$= 1 + 12 + 66 + 220 = 299$$

3. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be at least 5 bowlers in the team.

Sol. Since the team consists of at least 5 bowlers, the selection may be of the following types.

	Batsmen (7)	Bowlers (6)
Type 1	6	5
Type 2	5	6

The number of selections in first type = ${}^7C_6 \times {}^6C_5 = 7 \times 6 = 42$

The number of selections in second type = ${}^7C_5 \times {}^6C_6 = 21 \times 1 = 21$

\therefore The required number of ways selecting the cricket team = $42 + 21 = 63$.

4. If 5 vowels and 6 consonants are given, then how many 6 letter words can be formed with 3 vowels and 3 consonants.

Sol: Given 5 vowels and 6 consonants.

6 letter word is formed with 3 vowels and 3 consonants.

Number of ways of selecting 3 vowels from 5 vowels is 5C_3 .

Number of ways of selecting 3 consonants from 6 consonants is 6C_3 .

\therefore Total number of ways of selecting = ${}^5C_3 \times {}^6C_3$.

These letters can be arranged themselves in $6!$ ways.

\therefore Number of 6 letter words formed = ${}^5C_3 \times {}^6C_3 \times 6!$.

5. There are 8 railway stations along a railway line. In how many ways can a train be stopped at 3 of these stations such that no two of them are consecutive.

Sol: Number of ways of selecting 3 stations out of 8 = ${}^8C_3 = 56$

Number of ways of selecting 3 out of 8 stations such that 3 are consecutive = 6

Number of ways of selecting 3 out of 8 stations such that 2 of them are consecutive

$$= 2 \times 5 + 5 \times 4 = 10 + 20 = 30.$$

∴ Number of ways of a train to be stopped at 3 of 8

$$\text{Stations such that no two of the are consecutive} = 56 - (6 + 30) = 20$$

6. Find the number of ways of forming a committee of 5 members out of 6 Indian and 5 Americans so that always the Indians will be in majority in the committee

Sol: Since committee contains majority of Indians, the members of the committee may be of the following types.

	Indians (6).	Americans (5)
Type I	5	0
Type II	4	1
Type III	3	2

The number of selections in type I = ${}^6C_5 \times {}^5C_0 = 6 \times 1 = 6$

The number of selections in type II = ${}^6C_4 \times {}^5C_1 = 15 \times 5 = 75$

The number of selections in type III = ${}^6C_3 \times {}^5C_2 = 20 \times 10 = 200$

∴ The required number ways of selecting a committee = $6 + 75 + 200 = 281$.

7. A question paper is divided into 3 sections A, B, C containing 3, 4, 5 questions respectively. Find the number of ways of attempting 6 questions choosing at least one from each section.

Sol: A question paper contains 3 sections A, B, C containing 3, 4, 5 questions respectively.

Number of ways of selecting 6 questions out of these 12 questions = ${}^{12}C_6$.

Number of ways of selecting 6 questions from section B and C (i.e. from 9 questions) = 9C_6 .

Number of ways of selecting 6 questions from section A and C (i.e. from 8 questions) = 8C_6 .

Number of ways of selecting 6 questions from section A and B (i.e. from 7 questions) = 7C_6 .

∴ Number of ways of selecting 6 questions choosing at least one from each section

$$= {}^{12}C_6 - {}^7C_6 - {}^9C_6 - {}^8C_6 = 805.$$

8. Find the number of ways in which 12 things be

(i) Divided into 4 equal groups

(ii) distributed to 4 persons equally.

Sol: i) Dividing 12 things into 4 equal groups:

$$\text{Number of ways of dividing 12 things into 4 equal groups} = \frac{12!}{(3!)^4 4!}$$

ii) Distributing 12 things to 4 persons equally.

$$\text{Number of ways of distributing 12 things to 4 persons equally} = \frac{12!}{(3!)^4}.$$

9. A class contains 4 boys and g girls. Every Sunday, five students with at least 3 boys go for a picnic. A different group is being sent every week. During the picnic, the class teacher gives each girl in the group a doll. If the total numbers of dolls distributed is 85, find g.

Sol: A class contains 4 boys and 'g' girls.

In selecting 5 students with at least 3 boys for picnic two cases arises.

i) Selecting 3 boys and 2 girls:

$$\text{Number of ways of selecting 3 boys and 2 girls} = {}^4C_3 \times {}^gC_2 = 4({}^gC_2)$$

As each group contains 2 girls, number of dolls required = $8({}^gC_2)$.

ii) Selecting 4 boys and 1 girl:

$$\text{Number of ways of selecting 4 boys and 1 girl} = {}^4C_4 \times {}^gC_1 = g$$

\therefore As each group contains only 1 girl, number of dolls required = g

\therefore Total number of dolls = $8({}^gC_2) + g$

$$\text{i.e., } 85 = 8 \frac{g(g-1)}{2} + g$$

$$\Rightarrow 85 = 4g^2 - 3g$$

$$\Rightarrow 4g^2 - 3g - 85 = 0$$

$$\Rightarrow (4g + 17)(g - 5) = 0$$

$$\Rightarrow g = 5 (\because 'g' \text{ is non-negative integer}).$$

10. Find the number of 4 letter words that can be formed using the letters of the word, MIRACLE. How many of them

(i) Begin with an vowel (ii) Begin and end with vowels

(iii) End with a consonant?

Sol: The word MIRACLE has 7 letters. The number of 4 letter words that can be formed using these letters = ${}^7P_4 = 7 \times 6 \times 5 \times 4 = 840$

Now that 4 blanks

□□□□

(i) We can fill the first place with one of the 3 vowels {A, E, I} in ${}^3P_1 = 3$ ways.

Now the remaining 3 places can be filled using the remaining 6 letters in ${}^6P_3 = 6 \times 5 \times 4 = 120$ ways

∴ The number of 4 letter words that begin with an vowel = $3 \times 120 = 360$ ways.

Fill the first and last places with 2 vowels in ${}^3P_2 = 3 \times 2 = 6$ ways.

The remaining 2 places can be filled with the remaining 5 letters in ${}^5P_2 = 5 \times 4 = 20$ ways.

∴ The number of 4 letter words that begin and end with vowels = $6 \times 20 = 120$ ways.

(iii) We can fill the last place-with one of the 4 consonants {C, L, R, M} in ${}^4P_1 = 4$ ways.

The remaining 3 places can be filled with the remaining 6 letters in ${}^6P_3 = 6 \times 5 \times 4 = 120$ ways.

∴ The number of 4 letter words that end with a consonant is = $4 \times 120 = 480$ ways.

11. Find the number of ways of permuting all the letters of the word PICTURE, so that

(i) All vowels come together

(ii) No two vowels come together

Sol: The word PICTURE has 3 vowels {E, I, U} and 4 consonants {C, P, R, T}

(i) Treat the 3 vowels as one unit. Then we can arrange 4 consonants + 1 unit of vowels in $5!$ ways.

Now 3 vowels among themselves can be permuted in $3!$ ways. Hence

The number of permutations in which 3 vowels come together.

$$5! \times 3! = 120 \times 6 = 720 \text{ ways.}$$

(ii) No two vowels come together First arrange the 4 consonants in $4!$ ways. Then in between the vowels, in the beginning and in the ending, there are 5 gaps as

Shown below by the x letter

$x \square x \square x \square x \square x$

In these 5 places we can arrange 3 vowels in 5P_3 ways.

\therefore The number of words in which no two vowels come together $= 4! \times {}^5P_3$

$$= 24 \times 5 \times 4 \times 3 = 1440 \text{ ways.}$$

12. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order. Find the ranks of the word PRISON

Sol. The letters of the given word in dictionary order are

I N O P R S

In the dictionary order, first all the words that begin with I come. If I occupies the first place then the remaining 5 places can be filled with the remaining 5 letters in $5!$ ways.

Thus, there are $5!$ number of words that begin with I. On proceeding like this we get

I-----> 5!ways

N----->5! ways

O-----> 5!ways

PI----->4!ways

PN----->4!ways

PO----->4! ways

PRIN----->2! ways

PRIO----->2! ways

P R I S N O -----> 1 way

PRISON----->1 way

Hence the rank of PRISON is

$$3 \times 5! + 3 \times 4! + 2 \times 2! + 1 \times 1$$

$$= 360 + 72 + 4 + 1 + 1$$

$$= 438$$



13. Find the number of 4 digit numbers that can be formed using the digits 2, 4, 5, 7, 8. How many of them are divisible by (i) 2 (ii) 3 (iii) 4 (iv) 5 and (v) 25

Sol. Clearly, the number of 4 digit numbers that can be formed using the given 5 digits is

$${}^5P_4 = 120$$

(i) Divisible by 2 Units place must be filled with an even digit from among the given integers.

This can be done in 3 ways. {2 or 4 or 8}

Now, the remaining 3 places can be filled with the remaining 4 digits in 4P_3 ways. Therefore, the

$$\text{number of 4 digit numbers divisible by 2 is } 3 \times {}^4P_3 = 3 \times 24 = 72$$

(ii) Divisible by 3

We know that a number is divisible by 3 only when the sum of the digits in that number is a multiple of 3. Here, the sum of the given 5 digits is 26. We can select 4 digits

such that their sum is a multiple of 3 in 3 ways. They are

4, 5, 7, 8 (sum is 24) (leaving 2)

2, 4, 7, 8 (sum is 21) (leaving 5)

2, 4, 5, 7 (sum is 18) (leaving 8)

In each case, we can permute them in $4!$ Ways and all these 4 digit numbers are divisible by 3.

Thus, the number of the 4 digit numbers divisible by 3 is $3 \times (4!)$

$$= 3 \times 24 = 72.$$

(iii) Divisible by 4

A number is divisible by 4 only when the number formed by the digits in the last two places (tens and units places) is a multiple of 4.

$\square \square \square \square$

Therefore, the last two places should be filled with one of the following.

24, 28, 48, 52, 72, 84

Thus the last two places can be filled in 6 ways. Then we are left with 2 places and 3 digits.

They can be filled 3P_2 ways. Thus the number of 4 digit numbers divisible by 4 is $6 \times {}^3P_2 = 36$.

(iv) Divisible by 5

A number is divisible by 5, if the units place is '0' or 5. But '0' is not in the given digits. Hence we must fill the units place by 5 only.

$\square \square \square 5$

Now, the remaining 3 places can be filled with the remaining 4 digits in 4P_3 ways. Therefore, the number of 4 digit numbers divisible by 4 is ${}^4P_3 = 24$

(v) Divisible by 25

A 4 digit number formed by using the given digits is divisible by 25 if the last two places (tens and units places) are filled as either 25 or 75.

$\square\square\square\square$

Thus, the last two places can be filled in '2' ways. Now, the remaining 2 places with the remaining '3' digits can be filled in 3P_2 ways.

Therefore, the number of 4 digit numbers that are divisible by 25 is $2 \times {}^3P_2 = 12$.

14. Find the sum of all 4 digit numbers that can be formed using the digits 1, 3, 5, 7, 9.

Sol. We know that the number of 4 digit numbers that can be formed using the digits 1, 3, 5, 7, 9 is ${}^5P_4 = 120$.

We have to find their sum. We first find the sum of the digits in the units place of all the 120 numbers. Put 1 in the units place.

$\square\square\square\square$

The remaining 3 places can be filled with the remaining 4 digits in 4P_3 ways. Which means that there are 4P_3 number of 4 digit numbers with 1 in the units place. Similarly, each of the other digits 3, 5, 7, 9 appears in the units place 4P_3 times. Hence, by adding all these digits of the units place, we get the sum of the digits in the units place.

$${}^4P_3 \times 1 + {}^4P_3 \times 3 + {}^4P_3 \times 5 + {}^4P_3 \times 7 + {}^4P_3 \times 9$$

$$= {}^4P_3 (1 + 3 + 5 + 7 + 9)$$

$$= {}^4P_3 (25)$$

Similarly, we get the sum of all digits in 10's place also as ${}^4P_3 \times 25$. Since it is in 10's place, its value is ${}^4P_3 \times 25 \times 10$

Like this the values of the sum of the digit in 100's place and 1000's place are respectively

$${}^4P_3 \times 25 \times 100 \text{ and } {}^4P_3 \times 25 \times 1000$$

On adding all these sums, we get the sum of all the 4 digit numbers formed by using the digits

1, 3, 5, 7, 9.

Hence the required sum is ${}^4P_3 \times 25 \times 1 + {}^4P_3 \times 25 \times 10 + {}^4P_3 \times 25 \times 100 + {}^4P_3 \times 25 \times 1000$

$$= {}^4P_3 \times 25 \times 1111 = 24 \times 25 \times 1111 = 6,66,600$$

15. Find the number of 4 digit numbers using the digits 1, 2, 3, 4, 5, 6 that are divisible by

- (i) 2 (ii) 3 when repetitions are allowed.**

Sol: (i) Number divisible by 2

Take 4 blank places, First the units place

			x
--	--	--	---

Can be filled by an even digit in 3 ways (2 or 4 or 6). The remaining three places can be filled with the 6 digits in 6 ways each. Thus they can be filled in $6 \times 6 \times 6 = 6^3$ ways

. Therefore, the number of 4 digit numbers divisible by 2 is $3 \times 6^3 = 3 \times 216 = 648$

(ii) Numbers divisible by 3 First we fill up the first 3 places 6 digits in 6^3 ways

x	x	x	
---	---	---	--

After filling up the first 3 places, if we fill the units place with the given 6 digits, we get 6 consecutive positive integers. Out of these six consecutive integers exactly 2 will be divisible by '3'. Hence the units place can be filled in '2' ways. Therefore, the number of 4 digit numbers divisible by 3

$$= 6^3 \times 2 = 216 \times 2 = 432$$

16. Find the number of 4 letter words that can be formed using the letters of the word PISTON in which at least one letter is repeated.

Sol: The word PISTON has 6 letters the number of 4 letter words that can be formed using these 6 letters

(i) When repetition is allowed $= 6^4$

(ii) When repetition is not allowed $= {}^6P_4$

\therefore The number of 4 letter words in which at least one letter repeated is

$$= 6^4 - {}^6P_4 = 1296 - 360 = 936$$

17. Find the number of 5 digit numbers that can be formed using the digits. 1, 1, 2, 2, 3. How many of them are even?

Sol. In the given 5 digits, there are two 1's and two 2's.

Hence the number of 5 digit numbers that can be formed is $\frac{5!}{2!2!} = 30$

Now, for the number to be even, it should end with 2.

				2
--	--	--	--	---

After filling the units place with 2, the remaining 4 places can be filled with

the remaining 4 digits 1, 1, 2, 3 in $\frac{4!}{2!} = 12$ ways. Thus, the number of 5

digit even numbers that can be formed using the digits. 1, 1, 2, 2, 3, is 12.

- 18. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 5 bowlers and 3 wicket keepers with at least 3 bowlers and 2 wicket keepers.**

Sol.

Bowlers (5)-	Wicket Keepers	Bats men (7)	Number of ways of selecting them
3	2	6	${}^5C_3 \times {}^3C_2 \times {}^7C_6 = 210$
4	2	5	${}^5C_4 \times {}^3C_2 \times {}^7C_5 = 315$
5	2	4	${}^5C_5 \times {}^3C_2 \times {}^7C_4 = 105$
3	3	5	${}^5C_3 \times {}^3C_3 \times {}^7C_5 = 210$
4	3	4	${}^5C_4 \times {}^3C_3 \times {}^7C_4 = 175$
5	3	3	${}^5C_5 \times {}^3C_3 \times {}^7C_3 = 35$

The required teams can contain the following compositions

Therefore, the number of required ways = $210 + 315 + 105 + 210 + 175 + 35 = 1050$.

- 19. There are m points in a plane out of which p points are collinear and no three of the points are collinear unless all the three are from these p points. Find the number of different**

- (i) Straight lines passing through pairs of distinct points
(ii) Triangles formed by joining these points (by line segments).

Sol. (i) From the given m points, by drawing straight lines passing through 2 distinct points at a time, we are supposed to get mC_2 number of lines. But, since p out of these m points are collinear, by forming lines passing through these p points 2 at a time we get only one line instead of getting pC_2 . Therefore, the number of different lines passing through pairs of distinct points is

$${}^mC_2 - {}^pC_2 + 1.$$

- ii) From the given m points, by joining 3 points at a time, we are supposed to get mC_3 number of triangles. Since p out of these m points are collinear by joining these p points 3 at a time we do not get any triangle when as we are supposed to get pC_3 number of triangles. Hence the number of triangles formed by joining the given m points $= {}^mC_3 - {}^pC_3$

20. A teacher wants to take 10 students to a park. He can take exactly students at a time and will not take the same group more than once. Find the number of times (i) Each Student can go To the Park

(ii) The Teacher can go to the park.

Sol.(i) To find the number of times a specific student can go to the park, we have to select 2 more students from there mining 9 students. This can be done in 9C_2 ways.

Hence each student can go to the park 9C_2 times $= \frac{9 \times 8}{1 \times 2} = 36$ times

ii) The no. of times the teacher can go to park = The no. of different ways of selecting 3 students out of 10 $= {}^{10}C_3 = 120$

21. A double decker minibus has 8 seats in the lower and 10 seats in the upper deck. Find the number of ways of arranging 18 persons in the bus, if 3 children want to go to the upper deck and 4 old people cannot go to the upper deck.

Sol: Allowing 3 children to the upper deck and 4 old people to the lower deck, we are left with 11 people and 11 seats (7 seats in the upper deck and 4 in the lower deck).

We can Select 7 people for the upper deck out of 11 people in ${}^{11}C_7$ ways. The remaining 4 persons go to the lower deck.

Now we can arrange 10 persons (3 children and 7 others) in the upper deck and 8 persons (4 old people and 4 others) in the lower deck in $(10)!$ and $(8)!$ Ways

respectively. Hence the required number of arrangements $= {}^{11}C_7 \times 10! \times 8!$

22. Prove that

$$(i) {}^{10}C_3 + {}^{10}C_6 = {}^{11}C_4 \quad (ii) {}^{25}C_4 + \sum_{r=0}^4 {}^{(29-r)}C_3 = {}^{30}C_4$$

Sol: (i) ${}^{10}C_3 + {}^{10}C_6 = {}^{10}C_3 + {}^{10}C_4$ (since ${}^nC_r = {}^nC_{n-r}$) $= {}^{11}C_4$

$$(ii) {}^{25}C_4 + \sum_{r=0}^4 {}^{(29-r)}C_3$$

$$= {}^{25}C_4 + \{ {}^{25}C_3 + {}^{27}C_3 + {}^{28}C_3 + {}^{29}C_3 \}$$

$$= {}^{26}C_4 + {}^{26}C_3 + {}^{27}C_3 + {}^{28}C_3 + {}^{29}C_3$$

$$(\text{Since } {}^{25}C_3 + {}^{25}C_4 = {}^{26}C_4)$$

$$= {}^{27}C_4 + {}^{27}C_3 + {}^{28}C_3 + {}^{29}C_3 = {}^{28}C_3 + {}^{28}C_3 + {}^{29}C_3$$

$$= {}^{29}C_4 + {}^{29}C_3 = {}^{30}C_4$$

23. Find the number of ways of selecting 11 members cricket team from 7 batsman, 6 bowlers and 2 wickets keepers so that team contains 2 wicket keepers and at least 4 bowlers.

Sol: The required teams can contains the following compositions.

Bowlers	Wicket keepers	Batsmen	Number of ways of selecting team
4	2	5	${}^6C_4 \times {}^2C_2 \times {}^7C_5$ $= 15 \times 1 \times 21 = 315$
5	3	4	${}^6C_5 \times {}^2C_2 \times {}^7C_4$ $= 6 \times 1 \times 35 = 210$
6	2	3	${}^6C_6 \times {}^2C_2 \times {}^7C_3$ $= 1 \times 1 \times 35 = 35$

Therefore, the number of selecting the required cricket team $= 315 + 210 + 35 = 560$.

24. A double decker mini bus has 8 seats in the lower and 10 seats in the upper deck. Find the no. of ways of arranging 18 persons in the bus, if 3 children want to go the upper deck and 4 old people cannot go to the upper deck?

Sol: Allowing 3 children to the upper deck and 4 old people to the lower deck, we are left with 11 people and 11 seats (7 seats in the upper deck and 4 in the lower deck). we can select 7 people in ${}^{11}C_7$ ways. The remaining 4 persons go to the lower deck.

Now, we can arrange 10 persons (3 children and 7 others) in the upper deck and 8 persons (4 old people and 4 others) in the lower deck in $(10)!$ and $(8)!$ ways respectively. Hence the required number of arrangements $= {}^{11}C_7 \times 10! \times 8!$