P425/1
PURE MATHEMATICS
PAPER 1
July/August, 2022

NAALYA S.S.B. INTERNAL MOCK 2022
Uganda Advanced Certificate of Education
P425/1
Time: 3 Hours

INSTRUCTIONS TO CANDIDATES:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Additional question(s) answered will **not** be marked.
- **All** working **must** be shown clearly.
- Graph paper is provided
- Silent, non-programmable scientific calculators and mathematical table with a list of formulae may be used.

<u>Section A (40 Marks)</u>

Answer **ALL** *questions from this section.*

1. Solve the equations

$a-3b+6c=5$,
$a+6b+2c=4$.
$2a+b+c=0$.

(5 marks)

2. Differentiate $sinx$ from first principles (5 marks)

3. The first term of the GP is $\sqrt{3}-1$ and the sum of the first three terms is 3($\sqrt{3}-1$¿. Find the common ratio of the progression. (5 marks)

4. Find the points of intersection between the line $y=x+1$ and the circle $x^2+y^2+2x-3y-1=0$ (5 marks)

5. A point C divides the line AB in the ratios of $\propto:\beta$, show that the position vector of C, $OC=\frac{\beta 0A+\alpha OB}{\propto+\beta}$ (5 marks)

6. Show that $\int_0^1 \frac{4x+6}{(x+2)^2(x+1)^2}dx=\frac{2}{3}$ (5 marks)

7. Solve for x: $\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2=\frac{3}{2}$ for $0\le x\le 2\pi$ (5 marks)

8. A conical grass is filled with water such that the rate at which its radius is increasing is $0.4\,cm/s$, find the rate at which the volume is increasing at instant if $r=8\,cm$. (5 marks)

<u>Section B (60 Marks)</u>

Answer any **five** *questions from this section.*

Question9:

(a). Prove that $a^{\log_a b} = b$. Hence $solve\,for\,x\,if$ $25^{\log_5(x-2)} = 1$ (6 marks)

(b). Solve for x in: $3^{(x+3\sqrt{x})} = \frac{1}{9}$ (6 marks)

Question 10:

(a). Differentiate

(i) x^x

(ii) $\tan^{-1} x^x$

(b). Given that $siny + cosy = x$, show that $\frac{d^2y}{dx^2} = \frac{x}{(2-x^2)^{\frac{3}{2}}}$ (12 marks)

Question 11:

(a). Evaluate

(i) $\int_0^{\frac{\pi}{2}} \sin 3xcosxdx$.

(ii) $\int_0^1 \frac{xdx}{1+x^4}$

(b) find $\int \frac{12x}{(x-2)(2-3x)} dx$ (12 marks)

Question 12:

(a). Given that $Z_1 = 2\left[\cos\left(\frac{\pi}{3}\right) + isin\left(\frac{\pi}{3}\right)\right]$ and $Z_2 = 3\left[\cos\left(\frac{\pi}{4}\right) - isin\left(\frac{\pi}{4}\right)\right]$, find the polar form of a complex number $Z = Z_1 Z_2$ (6 marks)

(b). Given that $Z_1 = 3 + i$ and $Z_2 = x + i$ and $Arg(Z_1 Z_2) = \frac{\pi}{4}$, Find the value of x.

(6 marks)

Question 13:

(a) Given that $sinx+siny=a$ and $cosx+cosy=b$, show that $\cos(x+y)=\frac{(a+b)(a-b)}{a^2+b^2}$

(6 marks)

(b) Express $7\cos 2\theta+6\sin 2\theta$ in the form $\sqrt{r}\cos(2\theta-\alpha)$ where r is a constant and α is an acute angle hence solve the equation
$7\cos 2\theta+6\sin 2\theta=5$ for $0^o\leq\theta\leq 180^o$ (6 marks)

Question 14.

(a) A line $r=i+j-k+\propto(2i+j-2k)$ passes through the plane $2x-y+2z+3=0$ at point P, find coordinates of P.

(b) A line T that passes through P and parallel to a vector $i+2j-3k$ meets the line $r=\frac{x-7}{-3}=\frac{y+1}{-2}=\frac{z+4}{-2}$ at point M, find the coordinates of M

(12 marks)

Question 15

(a) Find the equation of the tangents and normal to the parabola $y^2=4ax$ at the point $P(ap^2,2ap)$.

(b) A point P is twice as far from the point $(3,0)$ as it is from line $x=5$. Find the Cartesian equation of the locus of P (12 marks)

Question 16.

The population of a certain type of fish in a reserved part of a lake is allowed to change at rate $\frac{dx}{dt}=10-2t$, where x is a population at time t years.

(a)If the population is 2000 initially, show that $x=2000+10t-t^2$.

(b) find how long the population takes to grow to its maximum population, hence the number of fish at that instant.

(c)calculate the population of fish at the instant when it's decreasing at 14 fish per day.