

P425/1
PURE
MATHEMATICS
Paper 1
July /Aug. 2023
3 hours



UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all questions in section A and any five from section B.

All necessary working must be shown clearly.

Silent non - programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section B will not be marked.

SECTION A (40 MARKS)
Answer ALL questions in this section

1. Use the Echelon method to solve the simultaneous equations

$$2x - y + 3z = 14$$

$$x + 4y - z = -5$$

$$3x + y + 4z = 17$$

2. Prove the identity:

$$\sin 5A \cos 3A - \cos 7A \sin A = \sin 4A \cos 2A$$

(05 marks)

3. Calculate the total area bounded by the curve $y = 3x^2 - 6x$, the x -axis and the lines $x = -1$ and $x = 2$.

(05 marks)

4. Find a unit vector perpendicular to the vectors;

$$a = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \text{ and } b = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

(05 marks)

5. A circle whose centre lies in the first quadrant touches the positive x -axis at $(4, 0)$, and touches the line $3y = 4x$. Find the radius of the circle, and state the coordinates of its centre.

(05 marks)

6. Given that x and y are real numbers such that:
 $xz + y^2 = 7i - 2$, where $z = 2 + i$, find the modulus of $x + iy$.

(05 marks)

7. Differentiate the function $x \sin x$ from first principles.

(05 marks)

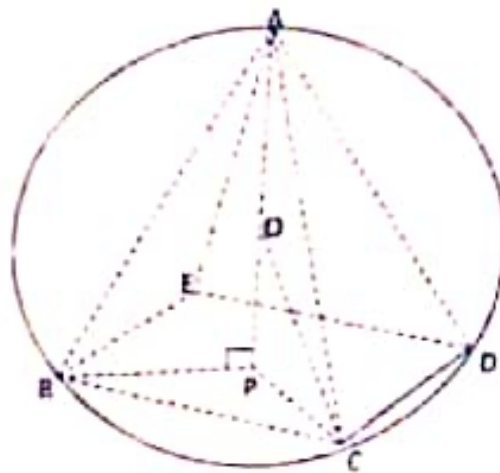
8. A curve is represented by the parametric equations;
 $x = t^2$, $y = 5t - 7$, find the equation of the tangent to the curve at the point $(4, 3)$.

(05 marks)

SECTION B (60 MARKS)

9. Given the lines $r_1 = \begin{pmatrix} 6 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and $r_2 = \begin{pmatrix} 7 \\ 3 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$
- Find the coordinates of their point of intersection. (04 marks)
 - Calculate the acute angle between the lines. (04 marks)
 - Find the Cartesian equation of the plane containing the lines. (04 marks)
10. (a) The roots of the equation $x^2 + px + (p + 9) = 0$ differ by 3, find the possible values of p . (05 marks)
- (b) Use the remainder theorem to find the remainder when the polynomial $P(x) = x^3 - 3x^2 + 2x - 5$ is divided by $(x - 2)^2$. (07 marks)
11. (a) Given that $\cos(\theta + 60^\circ) = \sin\theta$, show that $\tan\theta = 2 - \sqrt{3}$; hence or otherwise solve for θ in the interval $[0^\circ, 360^\circ]$. (06 marks)
- (b) Given that A, B and C are angles of a triangle. Prove that: $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A \sin B \cos C$. (06 marks)
12. (a) Use small changes to evaluate $\tan 46^\circ$ to 4 dps. (05 marks)
- (b) Evaluate: $\int_4^5 \frac{x^2}{x^2 - 9} dx$ to 4 dps. (07 marks)
13. (a) The n^{th} term of a series is $3^n + 4n$. Calculate the sum of the first 20 terms of the series. (05 marks)
- (b) Expand $\sqrt{1 - 4x}$ up to the term in x^4 . State the range of values of x within which the expansion is convergent. Hence evaluate: $\sqrt{15}$ to 4 dps. (07 marks)

14.



ABCDE is right pyramid with a square base. The pyramid is completely inscribed in a sphere of radius $OC = 6\text{ cm}$, where O is the centre of the sphere. P is the centre of the square base BCDE as shown.

Given that $OP = x$.

- (a) Show that the volume of the pyramid,
 $V = \frac{2}{3}(6+x)^2(6-x)\text{ cm}^3$ (07 marks)
- (b) Calculate the maximum volume of the pyramid. (05 marks)
15. (a) Show that the equation of the chord joining the point $P(p^2, 2p)$ and $Q(q^2, 2q)$ on the parabola $y^2 = 4x$ is
 $2x - (p+q)y + 2pq = 0$ (04 marks)
- (b) If the chord in (a) above passes through the point $R(4, 0)$ show that
 $pq = -4$, hence:
- (i) show that the chord PQ makes a right angle at the origin $O(0,0)$.
- (ii) find the locus of the mid-point of PQ . (08 marks)
16. In a certain game reserve, there are 80 elephants. Poachers start killing the elephants at a rate which is directly proportional to the number of elephants remaining in the forest. After one month 40 elephants have been killed. Let x be the number of elephants killed after t months.
- (a) Show that: $\ln\left(\frac{80}{80-x}\right) = t \ln 2$ (07 marks)
- (b) Calculate the:
- (i) number of elephants killed after 2 months.
- (ii) time taken to kill 75 elephants, and in this case state the average number of elephants killed per day. (05 marks)

END