NDEJJE SENIOR SECONDARY SCHOOL

Uganda Advanced Certificate of Education

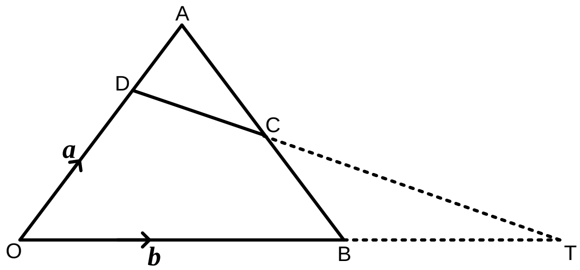
MARKING GUIDE FOR MOCK SET 4 EXAMINATIONS 2017

PURE MATHEMATICS

Paper 1

SNo.	Working	Marks
1	$ar + ar^2 = 48, \quad \Rightarrow ar(1+r) = 48 \quad \to (1)$ $ar^4 + ar^5 = 1296, \quad \Rightarrow ar^4(1+r) = 1296 \quad \to (2)$ $(2) \div (1)$ gives; $\dfrac{ar^4(1+r)}{ar(1+r)} = \dfrac{1296}{48}, \quad \Rightarrow r^3 = 27, \quad \Rightarrow r = 3$ From (1); $a = \dfrac{48}{r(1+r)} = \dfrac{48}{3(1+3)} = 4$ $\Rightarrow S_{12} = 4\left(\dfrac{3^{12}-1}{3-1}\right) = 1062880$	**B1** –for eqns 1 & 2 **M1** -solving **A1** –for r. **M1** **A1**
2	$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ $\cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta$ $+ 5\cos\theta\sin^4\theta + i\sin^5\theta = \cos 5\theta + i\sin 5\theta$ By comparison, $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta$ $+ 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$ $= -10\cos^3\theta + 11\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	**B1** -equating **M1** -expanding **M1** –equating real parts **M1** -simplification **A1**
3	(i). $g(x) = x^2 - 5x - 14 = (x-7)(x+2)$ let, $\quad R(x) = 2x + 5$ for, $\quad (x-7) = 0, x = 7, \quad \Rightarrow R(7) = 2(7) + 5 = 19$ (ii). for, $\quad (x+2) = 0, x = -2, \quad \Rightarrow R(-2) = 2(-2) + 5 = 1$	 **B1** **M1 A1** **M1 A1**

4



B1 –vector diagram

$$OD = DA = \frac{1}{2}\underset{\sim}{\boldsymbol{a}}, \qquad AB = OB - OA = \underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}$$

$$AC: CB = 3:1, \qquad \Rightarrow AC = \frac{3}{4}AB = \frac{3}{4}\underset{\sim}{\boldsymbol{b}} - \frac{3}{4}\underset{\sim}{\boldsymbol{a}}$$

B1 –for AC

$$DT = \mu DC = \mu(DA + AC) = \mu\left[\frac{1}{2}\underset{\sim}{\boldsymbol{a}} + \frac{3}{4}\underset{\sim}{\boldsymbol{b}} - \frac{3}{4}\underset{\sim}{\boldsymbol{a}}\right] = \frac{3}{4}\mu\underset{\sim}{\boldsymbol{b}} - \frac{1}{4}\mu\underset{\sim}{\boldsymbol{a}}$$

B1 –for DT

$$OT = \lambda OB = \lambda\underset{\sim}{\boldsymbol{b}}$$

$$OT = OD + DT$$

$$\lambda\underset{\sim}{\boldsymbol{b}} = \frac{1}{2}\underset{\sim}{\boldsymbol{a}} + \frac{3}{4}\mu\underset{\sim}{\boldsymbol{b}} - \frac{1}{4}\mu\underset{\sim}{\boldsymbol{a}}$$

Comparing coefficients of $\underset{\sim}{\boldsymbol{a}}$ gives:

$$0 = \frac{1}{2} - \frac{1}{4}\mu, \qquad \Rightarrow \mu = 2$$

B1 –for μ

Comparing coefficients of $\underset{\sim}{\boldsymbol{b}}$ gives:

$$\lambda = \frac{3}{4}\mu = \frac{3}{4}\times 2 = \frac{3}{2}, \qquad \Rightarrow OT = \lambda\underset{\sim}{\boldsymbol{b}} = \frac{3}{2}\underset{\sim}{\boldsymbol{b}}$$

B1 –for OT

5

$$\int x\cos^2 x\,dx = \int x\left(\frac{1+\cos 2x}{2}\right)dx$$

$$= \frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos 2x\,dx$$

B1

Sign	Differentiation	Integration
$+$	x	$\cos 2x$
$-$	1	$\frac{1}{2}\sin 2x$
$+$	0	$-\frac{1}{4}\cos 2x$

B1
B1

B1

$$\int x\cos^2 x\,dx = \frac{1}{2}x^2 + \frac{1}{2}\left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right] + c$$

M1 M1 – substitution & simplification

$$= \frac{1}{2}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + c$$

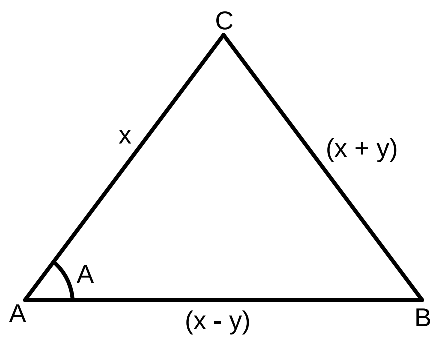
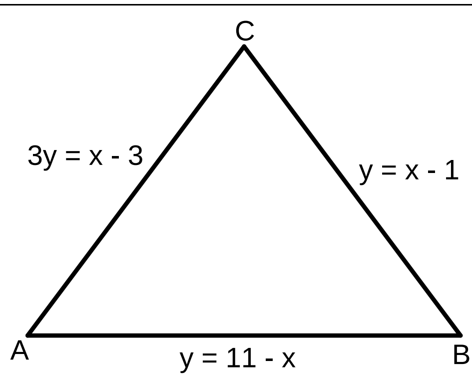
A1

6

$$y = ax^2 + bx + c, \qquad \Rightarrow \frac{dy}{dx} = 2ax + b$$

B1 – for dy/dx

At point $(2, 4)$,

	$y = x + a, \quad \Rightarrow 4 = 2 + a, \quad \Rightarrow a = 2$ gradient, $\frac{dy}{dx} = 2 \times 2 \times 2 + b = 1, \quad \Rightarrow b = -7$ $y = ax^2 + bx + c, \quad \Rightarrow 4 = 2(2)^2 + (-7)(2) + c$ $\Rightarrow 4 = 8 - 14 + c, \quad \Rightarrow c = 10$ $\therefore a = 2, \quad b = -7, \quad c = 10$	**M1** –solving **A1** –for a **A1** –for b **A1** –for c
7	$\text{Volume} = \pi \int_3^4 y^2\,dx = \pi \int_3^4 (x-2)^{-2}\,dx = \pi \left[\frac{(x-2)^{-1}}{-1}\right]_3^4$ $= \pi \left[\frac{1}{2-x}\right]_3^4 = \pi\left(-\frac{1}{2} + 1\right) = \frac{1}{2}\pi$ cubic units	**M1 M1** **M1 M1 A1**
8	 By cosine rule, $(x+y)^2 = x^2 + (x-y)^2 - 2x(x-y)\cos A$ $x^2 + 2xy + y^2 = x^2 + x^2 - 2xy + y^2 - 2x(x-y)\cos A$ $4xy - x^2 = -2x(x-y)\cos A$ $x - 4y = 2(x-y)\cos A$ $\cos A = \frac{x-4y}{2(x-y)}$	**B1** **M1** – substitution **M1 M1** – simplification **A1**
9	 At point A, $3(x - 11) = x - 3, \quad \Rightarrow x = 15$ when, $x = 15, \quad y = 15 - 11 = 4, \quad \Rightarrow A(15, 4)$ At point B, $11 - x = x - 1, \quad \Rightarrow x = 6$	**B1** **M1 B1**

$$\text{when,} \quad x = 6, \quad y = 6 - 1 = 5, \quad \Rightarrow B(6, 5)$$

At point C,

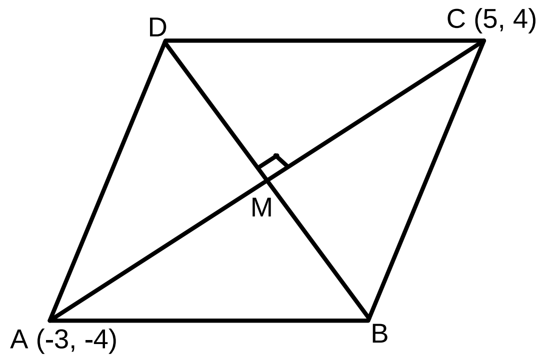
$$3(x-1) = x - 3, \quad \Rightarrow x = 0$$

$$\text{when,} \quad x = 0, \quad y = 0 - 1 = -1, \quad \Rightarrow C(0, -1)$$

$$\text{centroid} = \left(\frac{15+6+0}{3}, \frac{4+5-1}{3}\right) = \left(7, \frac{8}{3}\right)$$

M1 A1

(b).



$$\text{Gradient of AC} = \frac{-4-4}{-3-5} = 1, \quad \Rightarrow \text{Gradient of BD} = -1$$

B1 –graident AC

$$\text{Midpoint of AC,} \quad M\left(\frac{-3+5}{2}, \frac{-4+4}{2}\right) = (1, 0)$$

The equation of line BD is given by:

$$\frac{y-0}{x-1} = -1, \quad \Rightarrow y = -x + 1$$

B1 –equation BD

The equation of line BC is given by:

$$\frac{y-4}{x-5} = 2, \quad \Rightarrow y = 2x - 6$$

B1 –equation BC

At point B,

$$-x + 1 = 2x - 6, \quad \Rightarrow x = \frac{7}{3}$$

$$x = \frac{7}{3}, \quad y = -\frac{7}{3} + 1 = -\frac{4}{3}, \quad \Rightarrow B\left(\frac{7}{3}, -\frac{4}{3}\right)$$

$$\text{Midpoint of AC} = \left(\frac{\frac{7}{3} + x}{2}, \frac{-\frac{4}{3} + y}{2}\right) = (1, 0)$$

B1

$$\frac{7}{3} + x = 2, \quad \Rightarrow x = \frac{1}{3}$$

$$-\frac{4}{3} + y = 0, \quad \Rightarrow y = \frac{4}{3}$$

$$\Rightarrow D\left(\frac{1}{3}, \frac{4}{3}\right)$$

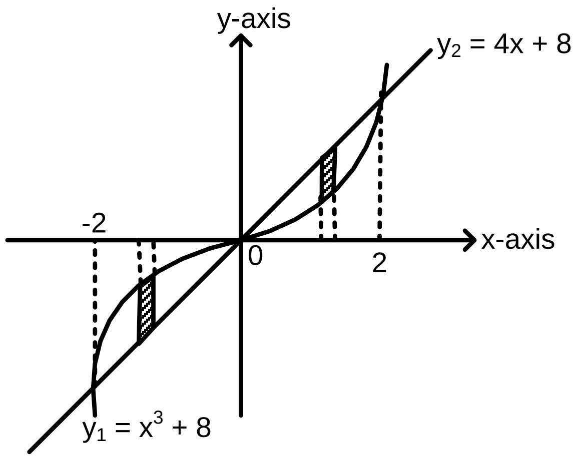
B1 –for D

The coordinates of B and D are $B\left(\frac{7}{3}, -\frac{4}{3}\right)$ and $D\left(\frac{1}{3}, \frac{4}{3}\right)$.

$$AC = OC - OA = \begin{pmatrix} 5 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

	$$MB = OB - OM = \frac{1}{3}\begin{pmatrix}7\\-4\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{3}\begin{pmatrix}4\\-4\end{pmatrix}$$ $$\text{Area} = \lvert AC\rvert\lvert MB\rvert = \sqrt{8^2+8^2} \times \frac{1}{3}\sqrt{4^2+4^2} = \frac{64}{3}$$ $$= 21\frac{1}{3}\ \text{sq. units}$$	**M1** **A1**
10	$$\frac{x^2+6}{(x^2+4)(x^2+9)} \equiv \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+9)}$$ $$x^2+6 \equiv (Ax+B)(x^2+9) + (Cx+D)(x^2+4)$$ Comparing coefficients of; x^0, $9B + 4D = 6 \longrightarrow (1a)$ x^1, $9A + 4C = 0 \longrightarrow (1b)$ x^2, $B + D = 1 \longrightarrow (1c)$ x^3, $A + C = 0 \longrightarrow (1d)$ Equation $(1a) - (1c)$ gives: $$5B = 2, \qquad \Longrightarrow B = \frac{2}{5}$$ From equation $(1c)$; $$D = 1 - B = 1 - \frac{2}{5} = \frac{3}{5}$$ Equation $(1b) - (1d)$ gives: $$5A = 0, \qquad \Longrightarrow A = 0$$ From equation $(1c)$; $$C = -A = 0$$ $$\frac{x^2+6}{(x^2+4)(x^2+9)} \equiv \frac{2}{5(x^2+4)} + \frac{3}{5(x^2+9)}$$ $$\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)}dx = \frac{2}{5}\int_0^1 \frac{1}{(x^2+4)}dx + \frac{3}{5}\int_0^1 \frac{1}{(x^2+9)}dx$$ $$= \frac{2}{5}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^1 + \frac{3}{5}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_0^1$$ $$= \frac{2}{5}\left[\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) - 0\right] + \frac{3}{5}\left[\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\right) - 0\right]$$ $$= \frac{1}{5}\tan^{-1}\left(\frac{1}{2}\right) + \frac{1}{5}\tan^{-1}\left(\frac{1}{3}\right) = \frac{1}{5}\left[\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)\right]$$ $$\text{let,} \quad \alpha = \tan^{-1}\left(\frac{1}{2}\right), \quad \Longrightarrow \tan\alpha = \frac{1}{2}$$ $$\text{let,} \quad \beta = \tan^{-1}\left(\frac{1}{2}\right), \quad \Longrightarrow \tan\beta = \frac{1}{3}$$ $$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \left(\frac{1}{2}+\frac{1}{3}\right)\Big/\left(1 - \frac{1}{2}\times\frac{1}{3}\right) = \frac{5}{6} \div \frac{5}{6}$$ $$= 1$$ $$\Longrightarrow (\alpha+\beta) = \tan^{-1} 1 = \frac{\pi}{4}$$	**M1** **M1** **M1** **A1** **A1** **A1** **B1** **M1** **M1** **M1** **M1**

	$\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)}\,dx = \frac{1}{5}\left[\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)\right] = \frac{1}{5}(\alpha+\beta)$	**B1**
	$= \frac{1}{5} \times \frac{\pi}{4} = \frac{\pi}{20}$	
11	(a).	
	Let, $f(x) = e^{-x}\sin x, \quad \Rightarrow f(0) = e^0 \sin 0 = 0$	
	$f'(x) = e^{-x}\cos x - e^{-x}\sin x = e^{-x}(\cos x - \sin x),$	
	$\Rightarrow f'(0) = 1$	**B1**
	$f''(x) = e^{-x}(-\sin x - \cos x) - e^{-x}(\cos x - \sin x)$	
	$= -2e^{-x}\cos x, \quad \Rightarrow f''(0) = -2$	**B1**
	$f'''(x) = 2e^{-x}\sin x + 2e^{-x}\cos x = 2e^{-x}(\sin x + \cos x),$	
	$\Rightarrow f'''(0) = 2$	
	By Maclaurin's theorem,	
	$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$	
	$= 0 + x \times 1 + \frac{x^2}{2!} \times (-2) + \frac{x^3}{3!} \times 2 + \cdots$	**M1**
	$\therefore e^x \sin x = x - x^2 + \frac{1}{3}x^3 + \cdots$	**A1**
	For the hence part,	
	$e^{-\frac{\pi}{3}} \sin\frac{\pi}{3} = \frac{\pi}{3} - \left(\frac{\pi}{3}\right)^2 + \frac{2}{3}\left(\frac{\pi}{3}\right)^3 \approx 0.3334 \text{ (4 s.f)}$	**M1 A1**
	(b).	
	$y = x^3 + 8$	
	when, $y = 0, \quad 0 = x^3 + 8, \quad x = -2, \quad \Rightarrow A(-2, 0)$	
	when, $x = 0, \quad y = 0 + 8 = 8, \quad \Rightarrow B(0, 8)$	**B1** –for A & B
	The equation of line AB is given by:	
	$\frac{y-8}{x-0} = \frac{0-8}{-2-0}, \quad \Rightarrow y = 4x + 8$	
	When the line AB meets the curve,	
	$4x + 8 = x^3 + 8, \quad \Rightarrow x(4 - x^2) = 0$	
	$x = 0, \quad \text{or}, \quad x = \pm 2$	
	when, $x = 2, \quad y = 8 + 8 = 16, \quad \Rightarrow C(2, 16)$	**B1**
	(ii).	

		**B1**
	$$\text{Area} = \int_{-2}^{0}(y_1 - y_2)\,dx + \left\|\int_{0}^{2}(y_2 - y_1)\,dx\right\|$$ $$\int_{-2}^{0}(y_1 - y_2)\,dx = \int_{-2}^{0}(x^3 - 4x)\,dx = \left[\frac{1}{4}x^4 - 2x^2\right]_{-2}^{0}$$ $$= 0 - (4 - 8) = 4$$ $$\int_{0}^{2}(y_2 - y_1)\,dx = \int_{0}^{2}(4x - x^3)\,dx = \left[2x^2 - \frac{1}{4}x^4\right]_{0}^{2}$$ $$= (4 - 8) - 0 = -4$$ $$\text{Area} = \int_{-2}^{0}(y_1 - y_2)\,dx + \left\|\int_{0}^{2}(y_2 - y_1)\,dx\right\| = 4 + \|-4\|$$ $$= 8 \text{ sq. units}$$	**M1** **A1**
12	(a). $$PQ = OQ - OP = \begin{pmatrix}1\\2\\0\end{pmatrix} - \begin{pmatrix}4\\-3\\5\end{pmatrix} = \begin{pmatrix}-3\\5\\5\end{pmatrix}$$ $$PQ:PR = 2:1, \quad \Longrightarrow \frac{PQ}{PR} = \frac{2}{1}, \quad \Longrightarrow PR = \frac{1}{2}PQ$$ $$OR = OP + \frac{1}{2}PQ = \begin{pmatrix}4\\-3\\5\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-3\\5\\5\end{pmatrix} = \begin{pmatrix}2.5\\-0.5\\7.5\end{pmatrix}$$ The coordinates are $R(2.5, -0.5, 7.5)$. (b). For perpendicular vectors, $$\begin{pmatrix}5\\-\lambda\\1\end{pmatrix}\cdot\begin{pmatrix}2\\3\\-4\end{pmatrix} = 0$$ $$10 - 3\lambda - 4 = 0, \quad \Longrightarrow \lambda = 2$$ (c).	**B1** –for PQ **B1** –for PR **B1** –for OR **B1** –for coordinate R **M1 M1** –dotting and equating to zero. **B1** –for λ

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Normal vector, $\underset{\sim}{\boldsymbol{n}} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$

Position vector, $\underset{\sim}{\boldsymbol{a}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$$\underset{\sim}{\boldsymbol{r}}.\underset{\sim}{\boldsymbol{n}} = \underset{\sim}{\boldsymbol{n}}.\underset{\sim}{\boldsymbol{a}}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}.\begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}.\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

$$4x - y + z = 4 + 2 + 2$$

$$4x - y + z = 8$$

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B1

B1

B1

M1

A1

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(i).

From, $x = \frac{1+t}{1-t}$, $x - tx = 1 + t$, $\Longrightarrow t = \frac{x-1}{x+1}$

$$t^2 = \frac{x^2 - 2x + 1}{x^2 + 2x + 1}$$

$$y = \frac{2t^2}{1-t} = \frac{2\left(\frac{x-1}{x+1}\right)^2}{\left(1 - \frac{x-1}{x+1}\right)} = \frac{2\left(\frac{x-1}{x+1}\right)^2}{\left(\frac{2}{x+1}\right)} = \frac{(x-1)^2}{(x+1)}$$

$$\Longrightarrow y = \frac{(x-1)^2}{(x+1)}$$

(ii).

$$\frac{dy}{dx} = \frac{(x+1) \times 2(x-1) - (x-1)^2}{(x+1)^2}$$

For turning points, $\frac{dy}{dx} = 0$

$$\frac{2(x+1)(x-1) - (x-1)^2}{(x+1)^2} = 0$$

$$(x-1)[2(x+1) - (x-1)] = 0$$

$$(x-1)(x+3) = 0$$

$$x = 1, \quad \text{or,} \quad x = -3$$

when, $x = 1$, $y = \frac{(1-1)^2}{(1+1)} = 0$

when, $x = -3$, $y = \frac{(-3-1)^2}{(-3+1)} = -8$

The turning points are: $(1, 0)$ and $(-3, -8)$.

<table>
<tr><td>x</td><td>L</td><td>-3</td><td>R</td><td></td><td>L</td><td>1</td><td>R</td></tr>
<tr><td>Sign of $\frac{dy}{dx}$</td><td>$+$</td><td>0</td><td>$-$</td><td></td><td>$-$</td><td>0</td><td>$+$</td></tr>
<tr><td>Nature</td><td></td><td>Max</td><td></td><td></td><td></td><td>Min</td><td></td></tr>
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(iii).

$$y = \frac{(x-1)^2}{(x+1)} = \frac{x^2 - 2x + 1}{x + 1}$$

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B1 –for t

M1 – substitution

A1

M1

A1

B1 –turning points

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By synthetic method

	1	-2	1
$x=-1$		-1	3
	1	-3	4

$y = x - 3 + \dfrac{4}{x+1}$, $\quad \Rightarrow y = x - 3$ is the slanting asymptote

B1 –slanting asymptote

Vertical asymptote

$$as\ y \to \infty, (x+1) \to 0$$

$$\Rightarrow x = -1 \text{ is the vetical asymptote}$$

B1 –vertical asymptote

Intercepts

$$y = \frac{(x-1)^2}{(x+1)}$$

when, $\quad x = 0, \quad y = 1$

when, $\quad y = 0, \quad (x-1)^2 = 0, \quad \Rightarrow x = 1$

The intercepts are $(0, 1)$ and $(1, 0)$.

B1 – intercepts

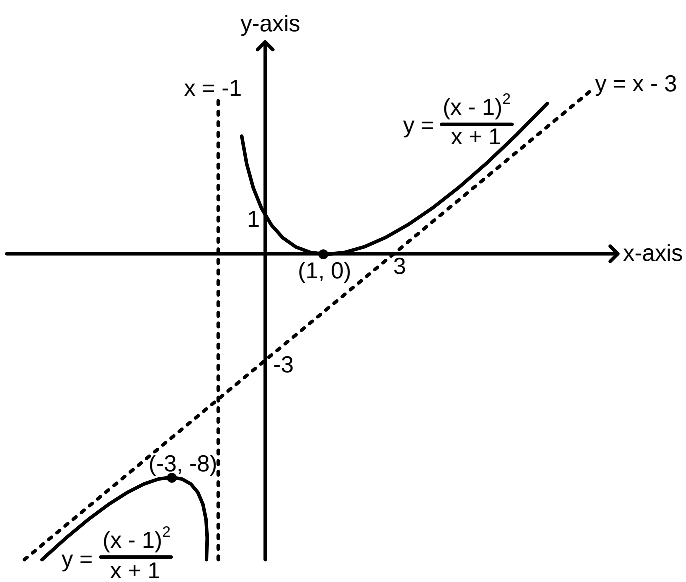
(iv). The Critical values include: $x = -1, x = 1$.

Region where the curve lies:

	$x < -1$	$-1 < x < 1$	$x > 1$
$(x-1)^2$ $(x+1)$	$+$ $-$	$+$ $+$	$+$ $+$
y	$-$	$+$	$+$

B1

Sketch of the curve



B1 B1

14 (a).

$$6\sin x - 3\cos x \equiv R\sin(x - \alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$$

B1

By comparison,

$$R\cos\alpha = 6 \quad \to (1a), \qquad R\sin\alpha = 3 \quad \to (1b)$$

	$(1b) \div (1a)$ gives:	
	$\frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{6}$, $\Longrightarrow \tan\alpha = 0.5,$ $\Longrightarrow \alpha = 26.57°$	**B1** –for α
	$R = \sqrt{3^2 + 6^2} = \sqrt{45}$	**B1** –for R
	$\Longrightarrow 6\sin x - 3\cos x \equiv \sqrt{45}\sin(x - 26.57°)$	
	Maximum value:	
	$\{6\sin x - 3\cos x\}_{max} = \sqrt{45} \times 1 = \sqrt{45} \approx 6.708$	**B1**
	(b).	
	$L.H.S = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \frac{1 + \tan 11°}{1 - \tan 11°} = \frac{\tan 45° + \tan 11°}{\tan 45° - \tan 11°}$	
	$= \tan(45 + 11)° = \tan 56°$	
	(c).	
	$L.H.S = \sin B + \sin C - \sin A$	
	$= \left[2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right] - 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$	**B1**
	For angles of a triangle, A, B, C,	
	$\sin\left(\frac{B+C}{2}\right) = \sin\left(90 - \frac{A}{2}\right) = \cos\left(\frac{A}{2}\right)$	**B1**
	$\cos\left(\frac{B+C}{2}\right) = \cos\left(90 - \frac{A}{2}\right) = \sin\left(\frac{A}{2}\right)$	**B1**
	$\Longrightarrow L.H.S = \left[2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)\right] - 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$	
	$= 2\cos\left(\frac{A}{2}\right)\left[\cos\left(\frac{B-C}{2}\right) - \sin\left(\frac{A}{2}\right)\right]$	**B1**
	$= 2\cos\left(\frac{A}{2}\right)\left[\cos\left(\frac{B-C}{2}\right) - \cos\left(\frac{B+C}{2}\right)\right]$	**B1**
	$= 2\cos\left(\frac{A}{2}\right)\left[-2\sin\left(\frac{B}{2}\right)\sin\left(-\frac{C}{2}\right)\right]$	
	$= 4\cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$	**B1**
15	(a).	
	$(2 + 5i)^2 + 5\frac{(7 + 2i)}{3 - 4i} - i(4 - 6i)$	**B1**
	$= 4 + 20i - 25 + \frac{(35 + 10i)(3 + 4i)}{9 + 16} - 4i - 6$	**B1**
	$= 16i - 27 + \frac{105 + 140i + 30i - 40}{25}$	**B1**
	$= \frac{(400i - 675) + (65 + 170i)}{25}$	**B1**
	$= \frac{570i - 610}{25} = \frac{114i}{5} - \frac{122}{5} = 22.8i - 24.4$	**B1**
	(b).	
	$\frac{z - 1}{z - i} = \frac{(x - 1) + yi}{x + (y - 1)i} = \frac{\{(x - 1) + yi\} \times \{x - (y - 1)i\}}{\{x + (y - 1)i\} \times \{x - (y - 1)i\}}$	**B1**

	$=\dfrac{(x-1)x-(x-1)(y-1)i+xyi+y(y-1)}{x^2+(y-1)^2}$ $=\dfrac{x^2-x-(xy-x-y+1)i+xyi+y^2-y}{x^2+(y-1)^2}$ $=\dfrac{(x^2+y^2-x-y)-(-x-y+1)i}{x^2+(y-1)^2}$ real part $=\dfrac{(x^2+y^2-x-y)}{x^2+(y-1)^2}$ imaginary part $=\dfrac{-(-x-y+1)}{x^2+(y-1)^2}$ $Arg\left(\dfrac{z-1}{z-i}\right)=\tan^{-1}\left(\dfrac{\text{imaginary part}}{\text{real part}}\right)=\dfrac{\pi}{3}$ $\tan^{-1}\left(\dfrac{x+y-1}{x^2+y^2-x-y}\right)=\dfrac{\pi}{3}$ $\dfrac{x+y-1}{x^2+y^2-x-y}=\tan\dfrac{\pi}{3}=\sqrt{3}$ $x+y-1=\sqrt{3}(x^2+y^2-x-y)$ $x^2\sqrt{3}+y^2\sqrt{3}-x(1+\sqrt{3})-y(1+\sqrt{3})+1=0$ The locus is a circle. By comparison with the general equation: $x^2+y^2+2gx+2fy+c=0$ $2g=-\left(\dfrac{1+\sqrt{3}}{\sqrt{3}}\right)=-\left(\dfrac{3+\sqrt{3}}{3}\right), \quad \Rightarrow g=-\left(\dfrac{3+\sqrt{3}}{6}\right)$ $f=g=-\left(\dfrac{3+\sqrt{3}}{6}\right)\approx -0.7887, \quad c=\dfrac{1}{\sqrt{3}}$ centre $=(-g,-f)=\left(\dfrac{3+\sqrt{3}}{6},\dfrac{3+\sqrt{3}}{6}\right)$ radius $=\sqrt{g^2+f^2-c}=\sqrt{\left(\dfrac{3+\sqrt{3}}{6}\right)^2+\left(\dfrac{3+\sqrt{3}}{6}\right)^2-\dfrac{1}{\sqrt{3}}}$ $=0.8165$ units	**B1** **M1** **A1** **B1** **M1** **A1**
16	(a). $x^2\dfrac{dy}{dx}=x^2+xy+y^2$ but, $\quad y=ux, \quad \Rightarrow \dfrac{dy}{dx}=\left(u+x\dfrac{du}{dx}\right)$ Substituting for y and $\frac{dy}{dx}$ gives: $x^2\left(u+x\dfrac{du}{dx}\right)=x^2+ux^2+u^2x^2$ $ux^2+x^3\dfrac{du}{dx}=(1+u+u^2)x^2$	**B1** **M1**

	$$u + x\frac{du}{dx} = 1 + u + u^2$$ $$x\frac{du}{dx} = 1 + u^2$$ $$\int \frac{du}{1+u^2} = \int \frac{1}{x}dx$$ $$\tan^{-1} u = \ln x + c$$ $$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$$	**M1** **A1**
	(b). Let h be the depth of the opening below the surface of the liquid at any time, t. Let h_0 be the initial depth of the opening below the surface of the liquid when the tank is full. $$\frac{dh}{dt} \propto \sqrt{h}$$ $$\frac{dh}{dt} = -kh^{\frac{1}{2}}$$ $$\int h^{-\frac{1}{2}}\,dh = -\int k\,dt$$ $$2\sqrt{h} = -kt + c$$ When $t = 0, h = h_0$ $$2\sqrt{h_0} = c$$ $$2\sqrt{h} = -kt + 2\sqrt{h_0}$$ When $t = 1, h = h_0 - 20$ $$2\sqrt{h_0 - 20} = -k + 2\sqrt{h_0}$$ $$-k = 2\sqrt{h_0 - 20} - 2\sqrt{h_0}$$ $$2\sqrt{h} = 2t\left(\sqrt{h_0 - 20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$$ When $t = 2, h = h_0 - 20 - 19 = h_0 - 39$ $$2\sqrt{h_0 - 39} = 4\left(\sqrt{h_0 - 20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$$ $$\sqrt{h_0 - 39} = 2\sqrt{h_0 - 20} - \sqrt{h_0}$$ $$h_0 - 39 = 4(h_0 - 20) - 4\sqrt{{h_0}^2 - 20h_0} + h_0$$ $$4\sqrt{{h_0}^2 - 20h_0} = 4h_0 - 41$$ $$16{h_0}^2 - 320h_0 = 16{h_0}^2 - 328h_0 + 1681$$ $$8h_0 = 1681$$ $$h_0 = 210.125 \text{ cm}$$	**B1** **B1** **B1** **B1** **M1** **M1** **M1** **A1**

END