P425/1
PURE
MATHEMATICS
PAPER 1
June/July. 2023
3 hours

ACEITEKA JOINT MOCK EXAMINATIONS, 2023

Uganda Advanced Certificate of Education

Pure Mathematics

Paper 1

Time: 3 Hours

NAME:...**INDEX No:** ....

INSTRUCTIONS TO CANDIDATES:

Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.

Indicate the five questions attempted in section B in the table aside.

Additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

Qn 1: Solve the inequality $\frac{x+3}{x-2} \geq \frac{x+1}{x-2}$. [5 Marks]

Qn 2: Find the angle $\alpha = \angle BAC$ of the triangle ABC whose vertices are $A(1,0,1), B(2,-1,1)$ and $C(-2,1,0)$. [5 Marks]

Qn 3: The roots p and q of a quadratic equation are such that $p^3 + q^3 = 4$ and $pq = \frac{1}{2}(p^3 + q^3) + 1$. Find a quadratic equation with integral coefficients whose roots are p^{-6} and q^{-6}. [5Marks]

Qn 4: Use method of small changes to find the value of $\frac{1}{\sqrt{0.97}}$ correct to 3 decimal places. [5 Marks]

Qn 5: Points S and S' are the foci of the ellipse $\frac{x^2}{36} + \frac{y}{16} = 1$.

Find the coordinates of S and S'. [5 Marks]

Qn 6: Evaluate: $\int_0^1 \frac{8x-8}{(x+1)^3(x-3)^3}dx$. [5 Marks]

Qn 7: Given the function, $f(x) = \frac{3}{13 + 6\sin x - 5\cos x}$.

Use the substitution $t = \tan\left(\frac{x}{2}\right)$, to show that $f(x)$ can be written

in the form: $\frac{3(1+t^2)}{2(3t+1)^2 + 6}$. [5 Marks]

Qn 8: Given that $y = \frac{\sin x}{x}$, show that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$. [5 Marks]

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). Prove by induction that for all positive integer $\sum_{r=1}^{n}(3r+1)(r+2) = n(n+2)(n+3)$

[6 Marks]

(b). Prove by induction that for all positive **odd** integers, n, $f(n) = 4^n + 5^n + 6^n$ is divisible by 15. [6 Marks]

Question 10:

A circle that passes through the points $A(3,4)$ and $B(6,1)$ and the equation of the tangent to this circle at A is the line $2y = x + 5$. Find:

(i). the coordinates of the centre of circle. [9 Marks]

(ii). the radius of the circle. [2 Marks]

(ii). the equation of the circle. [1 Mark]

Question 11:

(a). Given that $f(x) = \dfrac{64x^4 - 148x + 78}{(4x-5)^3}$. Express $f(x)$ into partial fractions.

(b). Hence evaluate $\int_4^6 f(x)dx$. [12 Marks]

Question 12:

(a). Use de Moivre's theorem to prove that: $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$.

(b). Hence or otherwise, find the distinct roots of the equation $2 + 10x - 40x^3 + 32x^5 = 0$ giving your answer to 3 decimal places where appropriate.

[12 Marks]

Question 13:

The planes P_1 and P_2 are respectively given by the equations:

$r = 2i + 4j - k + \lambda(i + 2j - 3k) + \mu(-i + 2j + k)$ and

$r.(2i - j + 3k) = 5$; where λ and μ are scalar parameters. Find:

(i). the Cartesian equation for plane, P_1.

(ii). to the nearest degree, the acute angle between P_1 and P_2.

(iii). the coordinates of the point of intersection of the plane, P_1, and the line $\dfrac{x-1}{5} = \dfrac{y-3}{-3} = \dfrac{z+2}{4}$. [12 Marks]

Question 14:

(a). Show that the volume of the solid generated by rotating the area enclosed by the curve $y = 2^x$, the lines $x = 0$ and $y = 2$ about the x-axis is $\dfrac{\pi}{\ln 4}(4\ln 4 - 3)$. [8 Marks]

(b). Evaluate $\int_0^{\frac{\pi}{4}} \dfrac{4}{1 + \cos 2x} dx$. [4 Marks]

Question 15:

(a). Given that $\cot^2\theta + 3\operatorname{cosec}^2\theta = 7$, show that $\tan\theta = \pm 1$. **[4 Marks]**

(b). (i). Express the function $y = 3\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ where R is a constant and $0 \le \alpha \le 2\pi$.

Hence find the coordinates of the minimum point of y.

(ii). State the values of x at which the curve cuts the $x-$axis . **[8 Marks]**

Question 16:

A sample of bacteria in a sealed container is being studied.

The number of bacteria, p, in thousands, is given by the differential equation:

$$(1+t)\frac{dp}{dt} + p = (1+t)\sqrt{t}$$

where t is the time in hours after the start of the study.

Initially, there are exactly 5,000 bacteria in the container.

(a). Determine, according to the differential equation, the number of bacteria in the container 8 hours after the start of the study.

(b). Find, according to the differential equation, the rate of change of the number of bacteria in the container 4 hours after the start of the study.

[12 Marks]

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
June/July. 2023
3 hours

UGANDA ADVANCED CERTIFICATE OF EDUCATION
MOCK EXAMINATIONS 2023

Applied Mathematics

Paper 2

Time: 3 Hours

NAME: ....INDEX No:.............

INSTRUCTIONS TO CANDIDATES:

Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.

Indicate the five questions attempted in section B in the table aside.

Additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Graph paper is provided.

Where necessary, take acceleration due to gravity, $g = 9.8 \text{ m s}^{-2}$.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer **all** *the questions in this section.*

Qn 1: The discrete random variable, x, has the following probability distribution, where θ is an unknown parameter belonging to the interval $\left[0, \frac{1}{3}\right]$

Value of x	1	3	5
Probability	θ	$1-3\theta$	2θ

Obtain the expression for $E(X)$ in terms of θ and show that $Var(X) = 4\theta(3-\theta)$ **[5 Marks]**

Qn 2: At time $t = 0$, two particles A and B have position vectors $(2i + 3j - 4k)$m and $(8i + 6k)$m and respectively.

Particle A moves with constant velocity $(-i + 3j - 5k)ms^{-1}$ and B with constant velocity, Vms^{-1}. Given that when $t = 5$ seconds, B passes through the point that A passed through one second earlier, find V. **[5 Marks]**

Qn 3: The table below is an extract from the table of a certain function $f(x)$.

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	0.0998	0.1987	0.2955	0.3894	0.4794

Use linear interpolation to find:

(i). $f(0.15)$ (ii). $f^{-1}(0.35)$ **[5 Marks]**

Qn 4: A spinner can land on red or blue. When the spinner is spun, there is a probability of $\frac{1}{3}$ that it lands on blue. The spinner is spun repeatedly. Given that the random variable, X, represents the number of the spin when the spinner first lands on blue, find $p(X \leq 4)$.

[5 Marks]

Qn 5: Three boys are pulling a heavy trolley by means of three ropes.
The boy in the middle is exerting a pull of 100 N. The other two boys, whose ropes both make an angle of 30° with the centre rope, are pulling with forces of 80 N and 140 N. Determine the magnitude of the resultant pull on the trolley. **[5 Marks]**

Qn 6: Use the trapezium rule with six ordinates to estimate $\int_2^5 xe^{-x}dx$, correct to 3 decimal places.

[5 Marks]

Qn 7: A particle is describing simple harmonic motion in a straight line directed towards a fixed point, O. When its distance from O is 3m, its velocity is $27ms^{-1}$ and its acceleration is $81ms^{-2}$. Determine the amplitude of oscillation. **[5 Marks]**

Qn 8: Show that the variance of n one's, 6 two's and 7 threes is a factor of the reciprocal of $(n + 13)$

[5 Marks]

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

The awards to 8 schools by Judges A and B in a quiz contest were:

Judge A (x)	60	56	50	56	60	52	56	54
Judge B (y)	52	60	75	66	54	70	60	68

(a). (i). Plot a scatter diagram for the given data. Comment on your result.

(ii). Draw a line of best fit on the scatter diagram.

(ii). Estimate the marks awarded by Judge A if Judge B awarded 55. **[7 Marks]**

(b). Calculate the rank correlation coefficient between the two judges. Comment on your result. **[5 Marks]**

Question 10:

(a). The numbers x and y are approximated by X and Ywith error Δx and Δy respectively. Show that the maximum relative error in $\frac{x}{y}$ is given by: $\left(\left|\frac{\Delta x}{x}\right|+\left|\frac{\Delta y}{y}\right|\right)\cdot\left|\frac{x}{y}\right|$ **[6 Marks]**

(b). Given that $x = 2.45$ and $y = 5.250$ are rounded off to the given number of decimal places. Determine the interval within which the exact value of $\frac{y-x}{y+x}$ lies. Give your answer to 4 decimal places. **[6 Marks]**

Question 11:

A particle *A*, of mass, ***m*** kg, has position vector $(11i + 6j)$metres and a velocity $(2i + 7j)ms^{-1}$. At the same moment, a second particle B, of mass,2m kg, has position vector$(7i + 10j)$ metres and a velocity$(5i + 4j)ms^{-1}$.

(a). If the particles continue to move with these velocities, prove that the particles will collide. **[4 Marks]**

(b). Given that the particles coalesce after collision, find the common velocity of the particles after collision. **[4 Marks]**

(c). Calculate the loss of kinetic energy caused by the collision. **[4 Marks]**

Question 12:

Calculate the probability of arranging the letters of the word "**PARALLELOGRAM**" in a row such that:

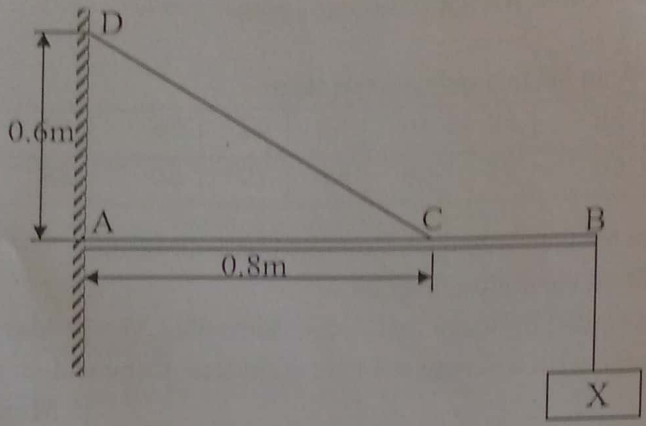
(i). the A's are separated. **[6 Marks]**

(ii). each word begins and ends with "**R**". **[3 Marks]**

(iii). "P" and "E" are always next to each other? **[3 Marks]**

Question 13:

The diagram below shows a uniform rod, AB of weight 10N, hinged to a vertical wall at A. The rod is held in a horizontal position by means of a light inextensible string. One end of the string is attached to a point C on the rod and the other end is attached to a point D on the wall. The point D is 0.6 m vertically above A and the length of AC is 0.8 m. A particle X, of weight 25N is attached to the rod at B and the tension in the string is 75N.



(a). Find the length of the rod AB.

(b). Calculate the magnitude and direction of the reaction at the hinge at A. **[12 Marks]**

Question 14:

(i). By plotting graphs of $y = \sin x$ and $y = \ln x$ on the same axes.

(ii). Show that the equation $\sin x = \ln x$ has a root between 2 and 3. Hence use Newton Raphson method to find the root, correct to three decimal places. **[12 Marks]**

Question 15:

The heights of the students at a university are assumed to follow a normal distribution. 1% of the students are over 200 cm tall and 76% are between 165 cm and 200 cm tall. Find:

(a). the mean and standard deviation of the distribution.

(b). the percentage of the students who are under 158 cm tall. **[12 Marks]**

Question 16:

(a). Village B is in a direction $N12^0 W$ from village A. When a man cycles from A to B at $12 kmh^{-1}$, the wind appears to be coming from $S50^0 W$. When he returns from B to A at the same speed, the wind appears to be from due south. Assuming that the velocity of the wind is the same throughout, find its true velocity. **[8 Marks]**

(b). Two points A and B on the banks of a river are directly opposite.

A boy capable of swimming at $1\frac{7}{18} ms^{-1}$ in still water wishes to swim directly from A to B.

Given that the river is flowing at a rate of $\frac{5}{6} ms^{-1}$, determine:

(i). the boy's speed along AB,

(ii). the width of the river if it takes 2 minutes to cross the river. **[4 Marks]**

END