P425/1
PURE MATHEMATICS
Paper 1
July /Aug. 2023
3 hours

UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in* ***section A*** *and any* **five** ***from section B.***

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section ***B*** *will* **not** *be marked.*

SECTION A (40 MARKS)

Answer ALL questions in this section

1. Use the Echelon method to solve the simultaneous equations:

$2x - y + 3z = 14.$

$x + 4y - z = -5$

$3x + y + 4z = 17$

(05 marks)

2. Prove the identify:

$Sin5ACos3A - Cos7AsinA = Sin4ACos2A$

(05 marks)

3. Calculate the total area bounded by the curve $y = 3x^2 - 6x,$ the $x - axis$ and the lines $x = -1$ and $x = 2.$ ***(05 marks)***

4. Find a unit vector perpendicular to the vectors;

$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***(05 marks)***

5. A circle whose centre lies in the first quadrant touches the positive $x -$ axis at +4, and touches the line $3y = 4x.$ Find the radius of the circle, and state the coordinates of its centre. ***(05 marks)***

6. Given that x and y are real numbers such that:

$xz + y\bar{z} = 7i - 2,$ where $z = 2 + i,$ find the modulus of $x + iy.$

(05 marks)

7. Differentiate the function $xsinx$ from first principles. ***(05 marks)***

8. A curve is represented by the parametric equations;

$x = t^2;$ $y = 5t - 7,$ find the equation of the tangent to the curve at the point $(4, 3).$ ***(05 marks)***

SECTION B (60 MARKS)

9. Given the lines $r_1 = \begin{pmatrix} 6 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and $r_2 = \begin{pmatrix} 7 \\ 3 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.

(a) Find the coordinates of their point of intersection. (04 marks)

(b) Calculate the acute angle between the lines. (04 marks)

(c) Find the Cartesian equation of the plane containing the lines. (04 marks)

10. (a) The roots of the equation $x^2 + px + (p + 9) = 0$ differ by 3, find the possible values of p. (05 marks)

(b) Use the remainder theorem to find the remainder when the polynomial $P(x) = x^3 - 3x^2 + 2x - 5$ is divided by $(x - 2)^2$. (07 marks)

11. (a) Given that $Cos(\theta + 60^o) = Sin\theta$, show that $tan\theta = 2 - \sqrt{3}$; hence or otherwise solve for θ in the interval $[0^o, 360^o]$. (06 marks)

(b) Given that A, B and C are angles of a triangle. Prove that;
$Sin^2A + Sin^2B - Sin^2C = 2SinASinBCosC$. (06 marks)

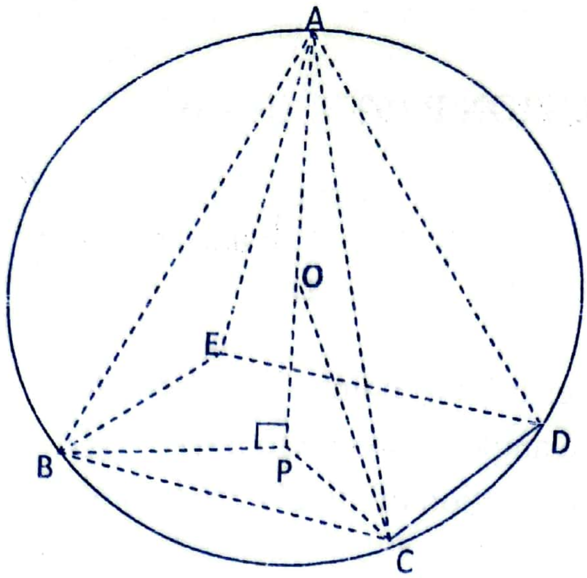
12. (a) Use small changes to evaluate $tan46^o$ $to\ 4\ dps$. (05 marks)

(b) Evaluate: $\int_4^5 \frac{x^3}{x^2 - 9}dx$ $to\ 2dps$. (07 marks)

13. (a) The nth term of a series is $3^n + 4n$. Calculate the sum of the first 20 terms of the series. (05 marks)

(b) Expand $\sqrt{1 - 4x}$ up to the term in x^4. State the range of values of x within which the expansion is convergent. Hence evaluate; $\sqrt{15}$ to 4dps. (07 marks)

14.



ABCDE is right pyramid with a square base. The pyramid is completely inscribed in a sphere of radius $\overline{OC} = 6cm$, where O is the centre of the sphere. **P** is the centre of the square base BCDE as shown.
Given that $\overline{OP} = x$.

(a) Show that the volume of the pyramid;
$V = {}^{2}/_{3}(6 + x)^2(6 - x)\ cm^3$ (07 marks)

(b) Calculate the maximum volume of the pyramid. (05 marks)

15. (a) Show that the equation of the chord joining the point $P(p^2, 2p)$ and $Q(q^2, 2q)$ on the parabola $y^2 = 4x$ is

$$2x - (p + q)y + 2pq = 0$$

(04 marks)

(b) If the chord in (a) above passes through the point $R(4, 0)$ show that $pq = -4$, hence:

(i) show that the chord $\overline{PQ}$ makes a right angle at the origin $0(0,0)$.

(ii) find the locus of the mid-point of $\overline{PQ}$. (08 marks)

16. In a certain game reserve, there are 80 elephants. Poachers start killing the elephants at a rate which is directly proportional to the number of elephants remaining in the forest. After one month 40 elephants have been killed. Let x be the number of elephants killed after t months.

(a) Show that; $ln\left(\frac{80}{80-x}\right) = tln2$ (07 marks)

(b) Calculate the:

(i) number of elephants killed after 2 months.

(ii) time taken to kill 75 elephants, and in this case state the average number of elephants killed per day. (05 marks)

END