BE TRUE

MATHEMATICS SEMINAR QUESTIONS SCHEDULED ON 5TH OCTOBER 2024

PURE MATHEMATICS P425/1

ALGEBRA

1. (a) (i) Given that $x = \sqrt[3]{p} + \frac{1}{\sqrt[3]{p}}$ and $y = \sqrt[3]{p} + \frac{1}{\sqrt[3]{p}}$,show that $y^2 = x(x^2\text{-}3)$.

(ii) Given that $\log_5 21 = M$ and $\log_9 75 = n$, show that $\log_5 7 = \frac{1}{2n-1}(2mn - m - 2)$.

(b) The roots p and q of a quadratic equation are such that $p^3 + q^3 = 4$ and $pq = \frac{1}{2}(p^3 + q^3) + 1$. Find a quadratic equation with integral coefficient whose roots are p^{-6} and q^{-6}. **(Trinity College Nabbingo)**

2. (a) The coefficients of the 5th, 6th and 7th terms in the expansion of $(1 + x)^n$ are in A.P. Find n.

(b) Given that the first three terms in the expansion in ascending powers of x of $(1 + x + x^2)^n$ are the same as the first three terms in the expansion of $\left(\frac{1+ax}{1-3ax}\right)^3$.
Find the value of a and n. **(St. Theresa Bwanda)**

3. (a) Solve the system of equations
$$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} = \frac{1}{2}, \quad \frac{4}{x} + \frac{2}{y} + \frac{3}{z} = \frac{2}{3} \text{ and } \frac{3}{x} - \frac{4}{y} + \frac{4}{z} = \frac{1}{3}.$$

(b) Determine the values of x and y which satisfy the equations $x^2 + 2xy + y^2 = 25$ and $x^2 - xy + y^2 = 9$. **(Kinaawa SS)**

4. (a) Prove by mathematical induction that $11^{2n} - 1$ is divisible by 120.

(b) Prove by mathematical induction that $\sin\theta + \sin3\theta + \sin5\theta + \ldots + \sin(2n - 1)\theta = \frac{\sin^2 n\theta}{\sin\theta}$.
(Trinity College Nabbingo)

TRINITY COLLEGE NABBINGO "A"LEVEL MATHEMATICS SEMINAR 2024

5. (a) If $log_{30}^{3} = P$ and $log_{30}^{5} = Q$.Show that $log_{30}^{32} = 5(1 - P - Q)$

(b) A polynomial $p(x)$ is a multiple of $(x - 3)$ and the remainder when divided $(x + 3)$ is 12. Find the remainder when $p(x)$ is divided by $(x^2 - 9)$. **(Wits College)**

6. Given that; $\frac{(2x-y)^{\frac{-1}{2}}}{(4x^2-y^2)^{\frac{1}{6}}} = (2x + y)^p(2x - y)^q$, evaluate p and q. Hence find the value of the expression when $x = 7$ and $y = 8$.

7. A geometric progression has first term a where $a \neq 0$,and common ratio r where $r \neq 0$ The difference between the fourth term and first term is equal to four times the difference between The third and the second term.
(a) Show that $r^3 - 4r^2 + 4r - 1 = 0$
(b) Show that $r - 1$ is a factor of $r^3 - 4r^2 + 4r - 1 = 0$. Hence, factorize $r^3 - 4r^2 + 4r - 1 = 0$
(c) Hence find the possible values for the common ratio of the Gp.Give your answer in an exact

(b) Expand $(2 + x - 2x^2)^7$ as far as the term in x^3.

8. (a) If α and β are the roots of the equation $x^2 + px + q = 0$, express;
(i) $\alpha^3 + \beta^3$ and
(ii) $(\alpha - \beta^2)(\beta - d^2)$, in terms of p and q. Deduce that the condition for one root of the equation to be the square of the other is $p^3 - 3pq + q^2 = 0$.

(b) A farmer had 120 goats by the end of 2023.The goats increase by 20% after every year as she sells N every year celebration. She plans to change the number sold every year at the end of 2028 when the total number of goats will be at 120.Find the value Of N. **(Hanna International School)**

9. (a) Determine the square root of the complex number $15 + 8i$.

(b) Given a complex number $z = \frac{(1+3i)(i-2)^2}{i-3}$.
Determine;
(i) z in the form $a + bi$ where a and b are constants.
(ii) $Arg\,(z)$

(c) Evaluate $\frac{\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^4}{\left(\sin\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3}$ and give the solution in modulus argument form.

TCN A LEVEL MATHEMATICS SEMINAR 2024

10. (a) Show that $\frac{(\sqrt{3}-i)^5}{\sqrt{3}+i} = -16$.

(b) The complex number v and w satisfy the equations;
v+iw=5 and (1+2i) v-w=3i.Solve for v and w giving your answers in the form x+iy.

(c) Write the complex numbers $z_1=-1-i\sqrt{3}$ and z_1=2-2i in polar form. Hence find $z_1^2\ z_2^3$.

(Munyonyo HS Naluvule)

11. (a) Find z such that $\frac{3z}{z+1} = 2+3i$.

(b) Prove that $\frac{(cos\theta+isin\theta)^2}{(sin\alpha+icos\alpha)^5} = sin(2\theta+5\alpha) - icos(2\theta+5\alpha)$.

(c) Find the principal argument of a complex number$W = \frac{(3+4i)^4}{(3+i)^6}$

12. (a) Simplify $\frac{(cos5\theta+isin5\theta)(cos\theta-isin\theta)^4}{(cos\theta+isin\theta)^3}$.

(b) (i) Express $\frac{2i}{1+5i}$ in modulus argument form, hence represent $\frac{2i}{1+5i}$ on an argand diagram.

(ii) Find the principal argument of $\left(\frac{1}{2}+i\sqrt{3}\right)^{10}$.

(c) Given that $\sqrt{3}$ –i and 1+i$\sqrt{3}$ are roots of the equation $z^4+az^3+bz^2+cz+d=0$. Find the values a,b,c and d.

13. (a) Given that $\left|\frac{z-1}{z+1}\right| = 2$, find the cartesian equation of the locus of the complex number, z and represent the locus by sketch in the Argand diagram. Shade the region for which the inequalities $\left|\frac{z-1}{z+1}\right| > 2$ and $0 < argZ < \frac{3\pi}{4}$ are both satisfied.

(b) Given that $z = x+iy$ where x and y are real, show that, when $IM\left(\frac{z+1}{z+2}\right) = 0$, the point (x,y) lies on a straight line and that when $R_e\left(\frac{z+1}{z+2}\right) = 0$, the point (x,y) lies on a circle. Hence deduce the centre and radius of the circle.

(Our lady of Good Counsel)

COORDINATE GEOMETRY

14. (a) The perpendicular bisector of a straight line joining the points (3,2) and (5,6) meets the x-axis at A and the y-axis at B. Prove that the distance AB is equal to $6\sqrt{5}$.

(b) The point A has coordinates$(2,-5)$. The straight line $3x+4y-36=0$ cuts the x-axis at B. and the y axis at C. Find the;

(i) The equation of the line through A which is perpendicular to the BC.
(ii) The perpendicular distance from A to the line BC.
(iii) The area of the triangle.

(St Noah Girls)

15. (a) The line $y = mx$ and the curve $y = x^3 - 2x$ intersect at the origin and meet again, at a point A. If P is mid-point of OA, find the locus of P.

(b) A circle with Centre P and radius r touches externally the circle $x^2+y^2=4$ and $x^2+y^2-6x+8=0$. Prove that the x-coordinates of P is $\frac{r}{3}+2$.

16. The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lies on the parabolay$^2 = 4ax$. Prove that if PQ is a focal chord then the tangents to the curve at P and Q intersect at right angles at a point on the directrix.

17. Prove that the chord joining the points $(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ on the parabola $y^2 = 4ax$ has the equation$(p+q)y = 2x+2apq$. A variable chord PQ of the parabola is such that the lines OP and OQ are perpendicular, where O is the origin.

(i) Prove that the chord PQ cuts the x-axis at a fixed point and find the coordinates of this point.

(ii) Find the equation of the locus of the mid-point of PQ.

18. Find the coordinates of the Centre and foci of given ellipses. Determine the length of the major and minor axes. Also determine the equation of the directrices.

(a) $\frac{x^2}{9}+\frac{x^2}{4} = 1$
(b) $9x^2+16y^2 = 25$
(c) $25x^2+9y^2-100x-54y = 44$

19. (a) Show that $x = 1+4cos\theta$ and $y = 3+5sin\theta$ are parametric equatons of an ellipse. Find the coordinates of the centre and foci coordinates of the centre and foci.

(b) Given that $P(acos\theta, bsin\theta)$ and $Q(acos\phi, bsin\phi)$ are ends of the chord on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. Show that the equation of PQ is given by $aysin\left(\frac{\theta+Q}{2}\right) + bxcos\left(\frac{\theta+Q}{2}\right) = abcos\left(\frac{\theta-Q}{2}\right)$. If PQ is a focal chord, show that $e = \frac{cos\left(\frac{\theta-Q}{2}\right)}{cos\left(\frac{\theta+Q}{2}\right)}$

(Trinity College Nabbingo)

20. (a) Given that $r^2 = cosec2\theta$ is an equation of the rectangular hyperbola, find its Cartesian form and sketch it.

(b) The tangent at $P\left(Cp, \frac{c}{p}\right)$ and the normal at $Q\left(cq, \frac{c}{q}\right)$ to the rectangular hyperbola $xy = c^2$ meets on the y-axis, show that $2q = p(1-q^4)$.

TRIGONOMETRY

21. (a) Given that $\cos\frac{A}{2}=\frac{3}{5}$ and $\tan\frac{B}{2}=\frac{12}{5}$ where $\frac{A}{2}$ and $\frac{B}{2}$ are reflex angles, evaluate $\sec(A+B)$.

(b) Find the values of x that satisfy the equations;

(i) $10\sin^2x+10\sin x\cos x-\cos^2x=2$ for $0^0\le x\le 360^0$.

(ii) $\frac{5}{2}\cos(x-10^0)=1+2\sin^2(x-10^0)$, for $0^0\le x\le 360^0$ **(Lubaga Girls)**

22. (a) Prove that $\tan2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$, and $n4\theta=\frac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$, hence solve the equation $t^4+4t^3-6t^2-4t+1=0$.

(b) Without using tables or calculator, solve for x in the equation.

$2\tan^{-1}x+\tan^{-1}\left(\frac{1}{7}\right)=\frac{\pi}{4}$.

23. (a) In triangle ABC, AB=x-y ,BC=x+y and AC=x .Show that $\cos A=\frac{x-4y}{2(x-y)}$.

(b) Prove that $2\cot\frac{3x}{2}-\sin3x=\sin3x\cot^2\frac{3x}{2}$ **(Nsangi SS)**

24. (a) Prove that $8\cos\theta3\theta\cos2\theta\cos\theta-1=\frac{\sin7\theta}{\sin\theta}$.

(b) Prove that in any triangle PQR,

$\frac{1}{p}\cos^2\frac{1}{2}P+\frac{1}{q}\cos^2\frac{1}{2}Q+\frac{1}{r}\cos^2\frac{1}{2}R=\frac{(p+q+r)^2}{4pqr}$. **(KIIRA COLLEGE BUTIKI)**

25. (a) Express $42\sin\theta+40\cos\theta$ in the form of $P\sin(\theta+\alpha)$. Hence find the maximum and minimum value of $\frac{3}{42\sin\theta+40\cos\theta+82}$ stating the values of $0^0\le 0\le 360^0$ for which they occur.

(b) Prove that $\sin^2A-\sin^2B=\sin(A+B)\sin(A-B)$. **(KAWANDA SS)**

26. (a) Prove that if $\tan x=K\tan(A-x)$ then $\sin(2x-A)=\left(\frac{K-1}{K+1}\right)\sin A$. Find all the angles for $0^0\le x\le 360^0$ which satisfy the equation $2\tan x-\tan(30^0-x)=0$.

(b) Show that $\sec\theta-\tan\theta=\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$ **(St. Aloysius Nabbingo)**

27. (a) Prove that $\tan(\theta+60^0)\tan(\theta-60^0)=\frac{\tan^2\theta-3}{1-3\tan^2\theta}$. Hence solve the equation; $\tan(\theta+60^0)\tan(\theta-60^0)=4\sec^2\theta-3$, for $0^0\le\theta\le 360^0$.

VECTORS

28. The points A, B, C and D have position vectors $\begin{pmatrix}4\\2\\-1\end{pmatrix}$, $\begin{pmatrix}1\\2\\1\end{pmatrix}$, $\begin{pmatrix}-3\\0\\3\end{pmatrix}$ and $\begin{pmatrix}5\\-4\\1\end{pmatrix}$ respectively. The perpendicular from D to the plane containing A, B and C meets the plane at E. find;

(a) The Cartesian equation of the plane containing A, B and C.

(b) The position vector of the point E.

(c) The vector equation of the straight-line D and E.

29. (a) If $\boldsymbol{a}=\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{b}=2\boldsymbol{i}+\boldsymbol{j}-\boldsymbol{k}$. Given that $\boldsymbol{c}=\lambda\boldsymbol{a}+\mu\boldsymbol{b}$ and that $\boldsymbol{c}$ and $\boldsymbol{a}$ are perpendicular. Fin the ratio of λ to μ.

(b) the line r_1 and r_2 have equations $r_1=\begin{pmatrix}3\\1\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $r_2=\begin{pmatrix}2\\5\\0\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ respectively.

(i) Prove that r_1 and r_2 intersect and hence find the point of intersection.

(ii) Determine the cartesian equation of a plane containing r_1 and r_2.

30. (a) A and B are points where whose position vectors are $\boldsymbol{q}=2\boldsymbol{i}+\boldsymbol{k}$, $\boldsymbol{b}=\boldsymbol{i}-\boldsymbol{j}+3\boldsymbol{k}$ respectively. Determine the position vector of the point c that divides AB in the ration 4:1.

(b) Find the perpendicular distance from the point A with position vector $4\boldsymbol{i}-3\boldsymbol{j}+10\boldsymbol{k}$ to the line with vector equation given by

$\boldsymbol{r}=\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}+\mu(3\boldsymbol{i}-\boldsymbol{j}+2\boldsymbol{k})$

(c) Show that the points $A(1,2,3), B(3,3,4)$ and $C(4,6,6)$ are vertices of a triangle ABC. **(Aswa Muslim sch)**

31. The parametric equation of two planes are;

$r_1=\begin{pmatrix}1\\0\\1\end{pmatrix}+t\begin{pmatrix}1\\-2\\1\end{pmatrix}+s\begin{pmatrix}0\\3\\2\end{pmatrix}$ and $r_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}+\mu\begin{pmatrix}0\\0\\5\end{pmatrix}+\lambda\begin{pmatrix}-2\\4\\3\end{pmatrix}$.

(a) Find the cosine of the acute angle between the planes.

(b) The lines of intersection is l. Find in the form $r=\boldsymbol{a}+\lambda\boldsymbol{b}$, the equation of l.

(c) Show that the length of the perpendicular from the point (1,5,1) to the line l is $\sqrt{2}$. **(Buddo SS)**

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32. (a) Determine the equation of the plane through the points $A(1,1,2), B(2,-1,3) and\ C(-1,2,-2)$.

(b) A line through the point $D(-13,1,2)$ and parallel to the vector $12\boldsymbol{i}+6\boldsymbol{j}+3\boldsymbol{k}$ meets the plane in (a) at E. find;

(i) the coordinate of E

(ii) the angle between the line and the plane.

33. (a) Show that $A(2,7,-5)$ and $B(-2,0,6)$ are on opposite sides of the plane $2x+3y+6z=20$.

(b) A, B and C are non-collinear points with position vectors $\boldsymbol{a}, \boldsymbol{b}$ *and* $\boldsymbol{c}$ respectively. Point P is on BC such that $BP:PC=3:1, Q$ is on CA such that $CQ:QA=2:3$. If the point R is on BA produced such that P, Q and R are collinear, find in terms of **a**, **b** and **c** the position vector of P, Q and R. **(Savio HS)**

ANALYSIS

(a) DIFFERENTIATION

34. (a) (i) Given that In2=0.69315 and $y=2^x$, use small changes to find the approximate Value of $2^{3.8}$ correct to four decimal places.

(ii) Differetiate $\log_e\left(\frac{(1+x)e^{-2x}}{1-x}\right)^{\frac{1}{2}}$

(b) (i) Determine the equation of the normal to the curve $y=\frac{1}{x}$ at the point $x=2$.

(ii) Find the coordinates of the other point where the normal meets the Curve again. **(Trinity College Nabbingo)**

35. (a) Given that $2cosy-x^2=1$, show that

(i) $\frac{dy}{dx}=\frac{-x}{siny}$

(ii) $\frac{d^2y}{dx^2}=-\left(\frac{x^2cosy+sin^2y}{sin^3y}\right)$ **(Trinity College Nabbingo)**

36. Sketch the curve $y=\frac{x^2-11x+28}{3x-3}$. **(Mpigi Mixed SS)**

37. (a) Find the range of possible value of y for real x if $y=\frac{x^2-4x+3}{1+x^2}$

(b) Prove that if $y=e^{4x}cos3x$, then $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+25y=0$.

38. (a) Use small changes to approximate the value of $cosec^2 29.2^0$ (to $4.sfg$)

(b) An open cylindrical container is made from $12cm^2$ of metal sheet. Show that the maximum volume of the container is $\frac{8}{\sqrt{\pi}}$

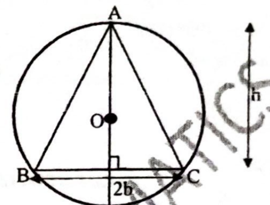
39. (a) The area of an isosceles triangle increases uniformly at a rate of $0.5cm^2s^{-1}$ while Keeping the unequal side at 10cm. Find the rate of increase of each of the equal Sides when each is 9cm long.

(b) A man is to fence his rectangular farm at shs. 1,200,000 using two types of Wires that cost shs. 2,000 and shs. 3000 per meter each. If the same type of wire must be on opposite sides of the farm, find the maximum area of the farm

40. The figure below shows a circular plate of Centre O and radius 1m. a symmetrical triangular plate ABC of height, h and base, 2b is to be cut out of the circular plate such that the line of symmetry passes through A and O.

(a) Show that the area of the triangle is given by $b\left(1+\sqrt{1-b^2}\right)$

(b) Hence, find the base and the height of the maximum area of the triangle



(St Henrys College Kitovu)

41. (a) The equation of the curve is given by $x^2+y^3=3xy$. Find the equation of the Normal to the curve at $\left(\frac{3}{2},\frac{3}{2}\right)$.

(b) An open box is to be made from a rectangular sheet measuring 16cm by 10cm by cutting squares of side xcm from each corner and turning up the edges. Calculate the value of x, so that the volume of the box is maximum. **(St Paul and Pauline)**

42. (a) Given that $y=\frac{4lnx-3}{4lnx+3}$. Show that $\frac{dy}{dx}=\frac{24}{x(4lnx+2)^2}$.

(b) The gradient of the curve at (x,y) is $\left(x-\frac{1}{x}\right)$ and the curve passes through the Point(1,2). Show that the area enclosed by the curve, the x-axis and $x=1,\ x=2$ is given by $\frac{11}{3}-2\,ln\,2$.

(b) INTEGRATION

43. (a) Integrate with respect to x.

$\frac{\sqrt{16-x^2}}{x^2}$

(b) By using substitution $x = 2sint$, show that;

$\int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}} dx = \frac{\pi}{2} + \sqrt{3} - 1.$

44. (a) Evaluate $\int_1^e \left(\frac{2-Inx}{x^2}\right) dx.$

(b) Using the substitution $t = tanx$, show that $\int_{\pi}^{\pi/4} \frac{1}{3sin^2x+cos^2x} dx = \frac{\pi}{3\sqrt{3}}$.

45. (a) Integrate each of the following;

(i) $x \log x$

(ii) $sec^3 2xtan2x$

(b) Show that $\int_0^{\frac{\sqrt{3}}{3}} 6xtan^{-1}(3x)dx = \frac{1}{9}(4\pi - 3\sqrt{3}).$

46. (a) Given $f(x) = \frac{sin^{-1}x}{1-x^2}$. Find $\int f(x)$.

(b) Express $f(x) = \frac{5x^3+2x^2+5x}{1-x^4}$ into partial fractions. Hence find $\int f(x)$.

47. (a) Prove that $\int_{\pi}^{4\pi/3} cosec\frac{1}{2}xdx = In3.$

(b) Find the following integrals

(i) $\int \frac{1}{x\sqrt{x^4-1}} dx.$

(ii) $\int \frac{secxtanx}{9+4sec^2x} dx$ **(Buloba High)**

48. (a) Evaluate $\int_{\frac{4}{5}}^{\frac{12}{5}} \frac{1}{x(1+x^2)^{\frac{3}{2}}} dx.$

(b) Find $\int cosx\sqrt{cos2x}.dx$

49. (a) Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec^2xtan^3x\, dx.$

(b) By integration, prove that the area of the circle of radius R is given by πR^2.

50. (a) Evaluate $\int_0^1 e^{\sqrt{x}} dx$.

(b) Show that $\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)} dx = \frac{\pi}{20}$

51. Find the coordinates of the points of intersection of the curves;
$y = \frac{x}{x+3}$ and $= \frac{x}{x^2+1}$. Sketch, the curves on the same diagram, showing any asymptotes or turning points. Show that the finite region in the first quadrant enclosed by the two curves is $\frac{7}{2}In\, 5 - 3In3 - 2.$

52. (a) Evaluate $\int_1^2 \frac{1}{\sqrt{12+8x-4x^2}} dx.$

(b) Show that $\int_0^{\frac{a}{2}} x^2\sqrt{(a^2-x^2)}\, dx = \frac{a^4}{8}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right)$. **(St Michael Mukono)**

53. Show that;

(a) $\int_0^{\frac{\pi}{4}} \theta sec^2\theta d\theta = \frac{\pi}{4} - \frac{1}{2}\log_e 2.$

(b) $\int_1^2 (x-1)^2 \log_e x\, dx = \frac{2}{3}\log_e 2 - \frac{5}{18}$. **(TOP TIMES HS)**

(c) DIFFERENTIAL EQUATION

54. (a) Solve the differential equation

$\frac{dy}{dx} + ycotx = x,$ given that $y(\pi/2) = 1.$

(b) The population of a certain village in Kenya increases at a rate proportional to the population present at any time. the population triples after every five years and the population was 3 million in 2000.

Show that $N = 3\left(\frac{t+5}{5}\right)$. Hence determine the population in 2025.

(St. Balikuddembe SS)

55. (a) Given that $y = \log_2 x - \log_e x^2$. Find; $\frac{dy}{dx}$.

(b) Solve the differential equation;

$x\frac{dy}{dx} - y = \frac{x}{x+1},$ if $y = 3$ when $x = 2$.

56. (a) The gradient function of a certain curve is equal to xe^{-x} and the curve passes through $(0,2)$. Find the equation of the curve.

(b) An election survey revealed that during the general campaign, a certain candidate is gaining support at a rate proportional to the product of those already supporting him and those not yet supporting him. Given P_0 is the total population of his constituency and P is those already supporting him at anytime t.

(i) Write down differential equation for the above situation.

(ii) If initially 10,000 voters are supporting him and he is gaining at rate of

0.0000005 voters per day. How many days does he need to get 51% voters supporting him if his constituency has 200,000 voters? **(Austine SS)**

57. (a) Solve the differential equation;

$\frac{dy}{dx} = \frac{y}{x} + \tan {}^{y}/{}_{x}$.

(b) An athlete runs at a speed proportional to the square root of the distance he still has to cover. If the athlete starts running at 10m/s and has a distance of 1600m to cover. Find how long he will take to cover that distance. **(Kisubi Seminary)**

58. (a) Solve $3y + (x-2)\frac{dy}{dx} = \frac{2}{x-2}$.

(b) The rate at which a disease spread through a certain community is found to be directly proportional to the fraction x of the community infected after time t but inversely proportion to the fraction not yet infected. Write a differential equation connecting x and t. Show that $e^{kt} = Ax^{-x}e$, where k and A are constant.
It was first noticed that half of the community was infected and by this instant the disease is spreading at a fraction $\frac{1}{4}$ per month. Show that $e^{t} = 16x^4 e^{2-4x}$. Find how long (in days) from the instant it was first noticed, it takes the community to be completely infected given that a month has 30 days.

PURE MATHEMATICS P425/1 LAY OUT

	SECTION	TOPICS	NUMBER OF QNS IN SEC A	NUMBER OF QNS IN SEC B
1.	Coordinate Geometry	Basic concepts, equation of a lines, Cartesian plane and coordinates, locus, circles, conic sections	01	01
2.	Algebra	Surds, logarithm, indices, quadratics equations, polynomials, series, remainder theory, complex numbers	02	02
3.	Trigonometry	Basic concepts, angles of a triangle	01	01
4.	Vectors	Basic concepts, lines and planes	01	01
5.	Analysis/ Calculus	Differentiation, integration, applications, inequalities, further curve sketching, differential equations	03	03

PREPARED BY MR. OJERA ALEX AMOS
(HOD MATHEMATICS)

APPLIED MATHEMATICS:

STATISTICS AND PROBABILITY

59. Packets of soap powder are filled by a machine. The weights of powder (to the nearest gram) in 32 packets chosen at random are summarized below;

Weight	999	1000	1001	1002	1003	1004
Packets	1	7	12	8	3	1

(a) Find the;
(i) amount by which the mean exceeds 1000g
(ii) standard deviation
(iii) standard error of the mean

(b) assuming this sample comes from normal distribution, find the 99.8% symmetrical confidence limits for the population mean. **(Kinaawa SS)**

60. The table below shows the frequency density of employees of a company in the year 2023

Age(years)	$15-<20$	$20-<30$	$30-<40$	$40-<50$	$50-<60$	$60-<65$
Frequency density	3.4	2.2	3.0	1.8	0.5	1.2

(a) Calculate the average age of the employees
(b) Construct a cumulative frequency distribution curve and use it to estimate the number of employees whose ages are above the average age.
(c) All employees above the age of 61 are to be retired and each paid a retirement package of 20,000,000. Estimate how much the company will spend in paying the employees when they retire. **(Lubaga Girls)**

61. The frequency distribution table below shows the mass in kilograms of unsealed bags of maize flour in a certain maize store.

Mass (kg)	51	53	54	55	56
Frequency	2	3	5	8	11

Find the mean and standard deviation of the mass of maize flour in the unsealed bags. Hence find the mean and standard deviation of the mass of maize in the sealed bags, if the quantity of maize flour in each bag is increased by 10% and then bags are sealed off.
(St Paul and Pauline)

62. The table below shows the order in which ten Catholic Parishes; A, B, C, D, E, F, G, H, I, J where ranked in Sports and Music tournaments at the Diocese level.

Position	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Sports	A	F	D	C	H	J	K	B	E	L
Music	D	F	C	A	J	K	H	B	L	E

Calculate the rank correlation coefficient between Sports and Music. Hence comment on your result at 5% level. **(Wits College Namulanda)**

63. Two events A and B are independent such that their chance of occurring together is $\frac{1}{5}$ and the chance that either A or B occurs is $\frac{7}{8}$.
(a) Show that $A'$and B' are also independent
(b) Find the $P(A)$ and $P(B)$

64. In a group of East African Referee there are three from Uganda, Four from Tanzania and Five from Kenya. To officiate a tournament, three referees are chosen at random from the group. Calculate the probability that;
(a) A referee is chosen from each representative country
(b) exactly two referees are chosen from Tanzania
(c) the three referees are chosen from the same country. **(St. Balikuddembe SS)**

65. At a certain fuel station, 30% of the customers buy V-power (V), 60% buy Un-leaded extra (U) and the remainder Diesel (D). of those who buy V, 25% fill the tank, 20% fill the tank with D while 30% do not fill their tank with U.
(i) Find the probability that when the vehicle leaves the station, it has a full tank
(ii) Given that a vehicle has a full tank, what is the probability that the tank contains Diesel? **(St. Balikuddembe SS)**

66. Events K and L are such that $P(K' \cap L) = 0.3$, $P(K \cup L) = 0.8$ and $P(K \cap L) = 0.2$. Find;
(i) $P(K' \cup L)$
(ii) $P(K'/L')$
(iii) Probability of K or L but not both.

67. Given that a bag contains 6 Red pens, 3 Green pens and 7 Blue pens. Three pens are selected at random without replacement from the bag, find the probability that;
(i) all the three colours are represented.
(ii) the first is blue, the second is red and the third is green.

68. The table below shows the prices in UG shs, and weights of five commodities in the month of July and September 2022

commodity	Meat	Sugar	Posho	Beans	Soap
July	16000	4000	3000	3800	8000
September	15000	5200	3600	5500	6500
weight	1	2	4	3	5

(a) Taking July as the base month, calculate;
the simple Aggregate price index for September
(i) price relative for each commodity
(ii) the cost of living index
(b) If the cost of an item in September is 50000/=, calculate its cost in July using the answer in (a) (iii) above. **(Austine SS)**

69. The probability that a seed chosen at random form the bag will germinate is $\frac{4}{7}$. If 150 seeds are chosen at random from the bag, calculate the probability that less than 90 seeds will germinate. **(Nsangi SS)**

70. Two fair tetrahedral dice whose faces are numbered; 1,2,3 and 4, are thrown at the same time. If the score is the sum of the numbers which show up on the faces of the dice, calculate the;
(i) the expected score for a throw
(ii) the most likely score for a throw **(Nsangi SS)**

71. A continuous random variable X is uniformly distributed over the interval $\alpha \leq x \leq \beta$. Given that $E(X) = 2$ and $P(X \leq 3) = \frac{5}{8}$. Find the;
(a) values of α and β
(b) p.d.f of X.

72. Given that $X \sim B(20, p)$ and that the mean is 4. Find the;
(i) Standard deviation
(ii) $P(X \geq 9)$

73. Three people Alice, Bob and Clare are rolling a Die. The winner is the first person to roll a six. If the die is unbiased and they roll a die in the order Alice, Bob and Clare, find the probability that;
(i) Clare wins on the first attempt
(ii) Bob wins on the second attempt
(iii) Alice wins the game **(St. Aloysius Nabbingo)**

74. In the year 2022, the price index of an item using 2020 as the base year was 80. In the year 2023, the index using 2022 as the base year was 140. Calculate the price of an item in 2023, given that the item costed Shs. 500,000 in 2020.

75. A game is played with three coins, A, B and C. coins A and B are biased so that the probability of obtaining a head is 0.4 for Coin A and 0.75 for Coin B. Coin C is not biased. The three coins are thrown once,
(a) Construct a probability distribution table for the number of heads obtained and use it to find the mean and variance of the number of heads obtained.
(b) Given that events A and B are independent,
(c) If $P(A) = x$, $P(B) = x + \frac{1}{5}$ and $P(A \cap B) = \frac{3}{20}$ find the value of x.

76. A continuous random variable X has a p.d.f given by;

$$f(x) = \begin{cases} \alpha x(3-x): & 0 \leq x \leq 2 \\ \alpha(4-x): & 2 \leq x \leq 4 \\ 0: & elsewhere \end{cases}$$

Where α is a constant,
(a) Find the value of α and sketch the p.d.f
(b) Obtain the $F(x)$, the cumulative distribution function, hence find the $P(1 \leq x \leq 3)$ **(KIIRA COLLEGE BUTIKI)**

77. A random variable X has a cumulative distribution function as below;

$$F(x) = \begin{cases} 0; & x < 0 \\ \frac{x^2}{16}; & 0 \le x \le 2 \\ ax - b; & 2 \le x \le 4 \\ \frac{1}{16}(12x - x^2 - 20); & 4 \le x \le 6 \\ 1; & x > 6 \end{cases}$$

(a) Find;

(i) The values of the constants a and b.

(ii) The $P(1 \le x \le 5/x \ge 2)$

(b) Find the median and the interquartile range.

(c) Sketch the graph of $f(x)$ and use it to deduce the mean. **(Mpigi Mixed SS)**

78. A discrete random variable Y has p.d.f given as;

$$f(y) = \begin{cases} ky & ; y = 1,2 \\ k(6-y) & ; y = 3,4 \\ k & ; y = 5,6 \end{cases}$$

where k is a constant. Determine the;

(i) Value of k

(ii) $P(Y < 5/Y \ge 3)$

(iii) Probability distribution for a random variable X where $X = 2Y - 1$,Hence find E(X) **(St Noah Girls)**

79. The speed of vehicles approaching a traffic road block is taken to be normally distributed. Traffic officers show that 95% of the vehicles are travelling at less than $85kmh^{-1}$and 10% are travelling at less than $55kmh^{-1}$

(a) Find the average speed of the vehicles and standard deviation

(b) Find the proportion of the vehicles travelling at more than $70kmh^{-1}$

(c) If a sample of 16 vehicles is selected, find the probability that the average speed will be between $60kmh^{-1}$ and $70kmh^{-1}$ **(Aswa Muslim sch)**

80. (a) Given that $X \sim N(\mu, \delta^2)$, $P(X < 35) = 0.2$ and $P(35 < X < 45) = 0.65$. Find μ and δ.

(b) A random variable $X \sim B(n, 0.6)$ and $P(T < 1) = 0.0256$. Find the value of n.

81. A bag contains 5 black pens and 3 Red pens. A second bag contains 3 black pens and 5 Red pens. A pen is picked at random from the first bag and placed in the second bag. A pen is then drawn at random from the second bag and placed in the first. Find the probability that each bag now contains ;

(i) 4 black and 4 Red pens.

(ii) Exactly the same number of each colour as it was initially.

82. A random variable T is defined as

$$f(t) = \begin{cases} \beta(0.5)^t; & t = 1,2,3,... \\ 0; & otherwise \end{cases}$$

(i) Find the value of β

(ii) $P(T \ge 2/T \le 6)$

83. A continuous random variable X is given by a probability density function below

$$f(x) = \begin{cases} \frac{1}{\pi}(1 + cosx); & 0 \le x \le \pi \\ 0; & elsewhere \end{cases}$$

Show that $(x) = \frac{\pi}{2} - \frac{2}{\pi}$.

84. (a) A coin is biased such that a head is thrice as likely to occur as a tail. It is tossed 120 times, find the probability that there will be;

(i) Between 70 to 90 heads.

(ii) Not more than 34 tails.

(b) The weights of a sample of 36 chicken from BULAMU poultry farm where recorded as follows;

Weight (kg)	3.50	3.90	4.10	4.60	4.90	5.20
Frequency (f)	4	6	10	9	5	2

Calculate the 97.5% confidence interval for the mean weight of chicken from a poultry farm.

NUMERICAL ANALYSIS

85. An error of 2 ½ % is made in the measurement of the area of a circle. Calculate the percentage error made in the radius of the circle.

86. (i) Round off 8.00243 to 3 significant figures

(ii) truncate 976800 to 3 significant figures.

87. The table below shows numbers and their squares

X	0.42	0.57	0.84	1.02
X^2	0.18	0.32	0.71	1.04

Use linear interpolation/ extrapolation to estimate;

(i) X when X^2 is 1.46

(ii) $\sqrt{(0.52)}$

88. Find the range with in which the exact value of $\frac{13.25}{0.6} - \frac{12.45}{3.8}$ lies, giving your answer correct to three significant figures.

89. Given two iterative formulae I and II for calculating the positive root of the quadratic equation $f(x) = 0$, as

I: $x_{n+1} = \frac{1}{2}(x_n{}^2 - 1)$ for $n = 1,2,3 \ldots$

II: $x_{n+1} = \frac{1}{2}\left(\frac{x_n{}^2+1}{x_n-1}\right)$ for $n = 1,2,3 \ldots$

Taking $x_0 = 2.5$, use each formula thrice to two decimal places to decide which is the most suitable formula. Give a reason for your answer.
Find the equation whose roots are being sought. **(Munyonyo HS Naluvule)**

90. (a) The dimensions of a rectangular plot of land are 30.26m and 14.45m. if the length and width have 5% and 3.2% errors respectively in estimates.
Calculate the limits with in which the area of the plot lies, correct to two significant figures.

(b) the numbers x and y are approximations to X and Y respectively with errors e_1 and e_2 respectively. Show that the absolute relative error in the quotient $\sqrt{\frac{x}{y}}$ is given by $\frac{1}{2}\left(\left|\frac{e_1}{x}\right| + \left|\frac{e_2}{y}\right|\right)$. **(Hanna International School)**

91. (a) Show that the Newton Raphson formula for finding the reciprocal of a number N is given by: $X_{n+1} = X_n(2 - NX_n)$; $n = 0,1,2, \ldots$

(b) Draw a flow chat that reads N and initial approximation X_0
Records the number of iterations n, Computes and prints N and its reciprocal to 2 decimal places after three iterations

(c) perform a dry run for $N = \frac{3}{2}$ and $X_0 = 0.5$. **(St. Theresa Bwanda)**

92. The information below gives the system of Tax (T) calculation for the amount of money (A) earned monthly by employees of a certain international company.

Monthly earnings (A)	Tax(T)
$A < \$\,2000$	zero
$\$\,2000 \leq A < \$\,5000$	2% of A
$A \geq \$\,5000$	\$ 60 plus 5% of the amount over \$ 5000

(a) draw a flow chart using the above data, given that the Algorithm stops when 200 counts (as) are made.

(b) Calculate the Tax for an employee who earns \$ 6000 monthly.
(Trinity College Nabbingo)

93. Find the range with in which the exact value of $\frac{x-y}{xy}$ lies to 3dps, given that $x = 5.32$ and y= 7.80 , find also the range for $\frac{y}{x} - \frac{x}{y}$ to 4 s.f. **(TOP TIMES HS)**

94. The table below shows extracts of tangents of angles

x	45.0	45.1	45.2	45.3
tanx	1.0000	1.0035	1.0070	1.0105

Using linear interpolation or extrapolation, find

(i) $\tan^{-1}(1.0052)$ (ii) $\tan(45.32)$

95. The volume V of a cylindrical object of radius r and height h is $V = \pi r^2 h$, where $r = 28.5 \pm 0.05$ and $h = 12.3 \pm 0.05$. Given the respective errors in measurement of the radius and height of the cylinder are Δr and Δh, Find the expression for relative error in the volume stating all the assumptions made. Hence find the relative error.
(Our lady of Good Counsel)

96. Use the trapezium rule with six ordinates to estimate the following to three decimal places.

(i) $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1+\cos x}}\,dx$, (ii) $\int_0^2 10^{2x}\,dx$ (iii) $\int_0^{\frac{\pi}{4}} \sec x\,dx$ (iv) $\int_0^{\frac{\pi}{3}} e^x \cos x\,dx$

Determine the percentage error made in each one of them and state how it can be reduced

(b) Use trapezium rule with six strips to estimate; $\int_1^2 x^{-2} \log x\,dx$, correct to two significant figures.

(c) Using trapezium rule with six ordinates, evaluate $\int_0^2 \log_e(x^2 + 3)\,dx$, correct to 4 significant figures. **(KAWANDA SS)**

97. Use trapezium rule with 9 ordinates to estimate the area bounded by the curve: $f(x) = xe^{-x}$, $x-$ and $y-axes$, and the line $x = 2$, giving your answer correct to three significant figures. Find the exact value, hence calculate the percentage error made in approximating the area under curve in (a) above. How can the error be minimized?

98. (a) Using graphical approach, show that the equation $xlnx + x - 3 = 0$ has a root between 1 and 2 and approximate the root using the graph to 1 decimal place.

(b) Derive a simple newton Raphson formula for finding the root of the equation above. Construct a flow chart that reads initial approximation x_0 in (a) above, computes and prints the root to 3 significant figures or performs three iterations.

(c) Perform a dry run for the flow chart above.

MECHANICS

99. The resultant of two forces P N and 3N is 7N. If the 3N force is reversed, the resultant is $\sqrt{19}$N. Find the value of P and the angle between the two forces.

100. Two equal forces of magnitude PN act on a particle separated by angle θ.
(a) Show that their resultant force bisects the angle between them.
(b) Prove that their resultant is $2P\cos\frac{\theta}{2}$

101. A particle projected from a point O at angle of $50°$ above the horizontal passed through the point P, with position vector $70\mathbf{i}+28\mathbf{j}$. Find the
(i) initial velocity
(ii) time taken to reach P. **(Savio HS)**

102. A particle is projected from a point O on level ground at an angle of elevation α and while still raising it passes through Point P with speed V at which point its elevation is β. Prove that the time taken to reach point P is; $\frac{V\sin(\alpha-\beta)}{g\cos\alpha}$.

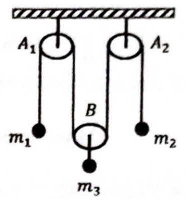
103. Find the position vector of the center of gravity of particles of masses 5kg, 2kg, 4kg and 3kg, situated at points (3,1), (4,3), (5,2) and (-3,1) respectively.

104. A regular pentagon ABCDE of side 2m is subjected to forces of magnitude 5N, 2N, 3N, 1N, and 4N acting along AB, BC, CD, DE, and EA respectively. The directions are indicated by the order of the letters. Taking AB as the reference positive x-axis,
(a) Determine the magnitude and direction of the resultant force.
(b) Find the equation of the line of action of the resultant force.
(c) distance from A where the line of action of the resultant cuts AB. **(Buloba High)**

105. A particle of mass 30kg is attached to one end of a light inextensible string whose other end is fixed. The particle is pulled a side by a force F, which is at right angles to the string so that at the position of equilibrium, the string makes an angle of 30° with the vertical. Find the magnitude of the force F and the tension in the string.

106. A bus of mass 18tonnes travels up a slope inclined at $\sin^{-1}\left(\frac{1}{50}\right)$ against a resistance of 0.1N per kilogram. Find the tractive force required to produce an acceleration of $0.05ms^{-2}$ and the power which is developed when the speed is $10ms^{-1}$.

107. A_1 and A_2 are two fixed pulleys in the same horizontal line. A light inextensible string is placed over A_1 and A_2 and carries weights w_1 and w_2 at its free ends. Another pulley B carrying a weight w_3 is placed on the part of the string between A_1 and A_2. If all the portions of the string not in contact with the pulleys are vertical as shown below,

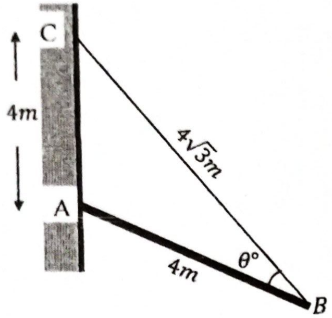


(a) prove that when all the weights are in motion the tension in the string is $\frac{4}{w_1^{-1}+w_2^{-1}+4w_3^{-1}}$ where $w_i = m_i g$, for $i = 1,2,3$.
(b) prove also that the condition that w_3 shall remain at rest while w_1 and w_2 are in motion is $4w_1w_2 = w_3(w_1+w_2)$. **(St Henrys College Kitovu)**

108. Two cars A and B are proceeding one on each road, towards the point of intersection of two roads which meet at an angle of 60^0. If the speeds of A and B are $20kmh^{-1}$and $32kmh^{-1}$ and are 70m and 40m respectively from the cross road, and the cars maintain their speeds, determine the;
(i) speed of B relative to A.
(ii) time when they are nearest to each other.
(iii) the distance of B from the cross road when they are nearest to each other.

109. A boy can swim at $\frac{5}{6}ms^{-1}$ in still water. He swims across a river 125m wide. The river flows at $\frac{25}{18}ms^{-1}$ parallel to the straight banks. How long will it take him if he swims so as to reach the opposite bank;
(i) as quickly as possible
(ii) as minimum down stream as possible?

110. A uniform rod AB of mass 5kg and length 4m, is hinged with its end A on a vertical wall. The other end B is attached to a string of length $4\sqrt{3}$m which is attached to the wall at C. length AC is 4m. The string makes an angle of $\theta°$ to BA as shown in the diagram below.



Determine the;

(a) The value of $\theta°$

(b) tension in the string and the reaction at the hinge. **(Buddo SS)**

111. (a) A body of mass 10kg is in limiting equilibrium on its own when placed on an inclined plane. If the angle of friction is $\tan^{-1}\left(\frac{3}{4}\right)$. Find the

(i) Inclination of the plane to the horizontal

(ii) Minimum force acting parallel to the incline that will just move the body up the incline.

(b) If a force F, inclined at an angle β to the surface of the plane is applied to the body, find the minimum value of F with the corresponding value of β when the body is on the point of moving up the plane.

112. A lorry of mass 4 tonne moving with velocity $54kmh^{-1}$ makes a head-on collision with a pick-up of mass 1 tonne moving with velocity of $36kmh^{-1}$. If the lorry moves in the same direction with the pick-up embedded in it after collision, find the

(i) Common velocity after collision

(ii) Loss in kinetic energy.

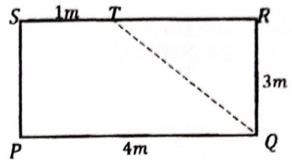
113. A ball is kicked with a velocity of $12ms^{-1}$at an angle of 45°to the horizontal towards a wall which is 8m away.

(a) Find how far up the wall the ball hits.

(b) Calculate the speed of the ball when it hit the wall

(c) Determine the direction the ball is moving when it hits the wall. **(St Michael Mukono)**

114. The diagram below shows a rectangular sheet PQRS. Point T is on the line RS such that $ST = 1m$



(a) If the sheet is folded a long QT until RQ lies along PQ. Find the centre of gravity of the folded sheet, hence find the angle of inclination PS makes with the vertical if the sheet is freely suspended at S by means of a thread and hangs in equilibrium.

(b) If the sheet is placed in a vertical plane with ST resting on a horizontal table, determine whether it topples. **(Trinity College Nabbingo)**

115. A body moving with acceleration $(e^{2t}\boldsymbol{i} - 4\sin 2t\,\boldsymbol{j} + 6\cos 2t\,\boldsymbol{k})ms^{-2}$ is initially located at the point $(2, -1, 1)m$ with a velocity of $(6\boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k})\ ms^{-1}$.

(a) find the speed of the body when $t = \frac{\pi}{4}$ s

(b) find the distance of the body from the origin at $t = \frac{\pi}{4}$ s

116. A conical pendulum consists of a light inextensible string AB carrying a particle of mass 0.5kg at the end B. The particle moves in a horizontal circle of radius $\sqrt{3}m$ with the centre vertically bellow A. if the angle between the string and the vertical is 30°, find the tensionin the string and the angular speed of the particle.

117. A particle moves with Simple Harmonic Motion about the mid position O. when passing two points which are 2m and 2.4m from O, the particle has speed $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the Amplitude of the motion and the greatest speed attained by the particle.

118. A car with uniform velocity of $20ms^{-1}$ is accelerated to attain velocity of $33.6ms^{-1}$. It maintained this velocity for a period which is 4 times the time taken to bring it to rest. If it is brought to rest in a period which is seven-tenth of the time the car is in acceleration, and the total time taken by the car in motion is 11 ¼ minutes;

(i) determine the time to accelerate from $20ms^{-1}$ to $33.6ms^{-1}$ and the time for constant velocity.

(ii) draw a velocity-time graph for the motion of the car
Determine the total distance covered. **(Kisubi Seminary)**

TRINITY COLLEGE NABBINGO "A"LEVEL MATHEMATICS SEMINAR 2024

APPLIED MATHEMATICS P425/2 LAY OUT

	SECTION	TOPICS	NUMBER OF QNS IN SEC A	NUMBER OF QNS IN SEC B
1.	Statistics	Descriptive stat Correlation and scatter diagrams Index numbers	01	01
2.	Probability	Probability theory Random variables, binomial, continuous, uniform, normal distribution, sample mean, population parameters and estimation	02	02
3.	Numerical Methods	Errors, interpolation/extrapolation, trapezium rule, roots of equations, iterative methods, Newton Raphson method, Flow charts	02	02
4.	Mechanics	Kinematics,(uniform motion, vertical, horizontal, projectiles), statics,(resultant force, equilibrium, moments, coplanar forces, hinge and jointed rods, center of gravity) Dynamics;(newton's laws, connected particles, work, energy and power, use of calculus, momentum and impulse, elasticity, circular motion and simple harmonic motion.	03	03

COMPILED BY MR. KAKEETO DAVID AND MR NSIMBI STEPHEN

END

Practice makes perfect.