P425/1

PURE MATHEMATICS

Paper 1

Jul/Aug 2024

3 HOURS

BUSOGA REGION JOINT EXAMINATION BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

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3 Hours

INSTRUCTIONS TO CANDIDATES

*Answer **all** the **eight** questions in section **A** and any **five** questions from section **B***

SECTION A (40 MARKS)

Answer all questions from this section.

1. Find the coordinates of the point on $y = (x^2 - 5)$ at which the gradient is 3. Hence find the values of K for which the line y = 3x + K is a tangent to $y = (x^2 - 5)$.

(05 Marks)

2. Given the equation $|z + 2 + 3i| = 3$. Find the maximum value of $|z - I - i|$.

(05 Marks)

3. If the roots of the equation $ax^2 + bx + c = 0$ differ by 4. Show that $\frac{b^2}{4a} = (4a + c)$.

(05 Marks)

4. Find the perpendicular distance from the point M(4, -3, 10) to the line with vector equation, $r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

(05 Marks)

5. Prove that $4cos3AcosA = \frac{\sin 5A}{\sin A} - 1$ **(05 Marks)**

6. Show that $\int_1^{10} x\, logx^2 dx = 2\left(50 - \frac{99}{4ln10}\right)$ **(05 Marks)**

7. Solve the differential equation, $(1 + x)\frac{dy}{dx} = xy + xe^x$ given that y(o) = 1

(05 Marks)

8. Show that parametric equation $x = 9\cos\theta$ and $y = 16\sin\theta$ represents an ellipse, Hence determine the foci and directrices.

(05 Marks)

SECTION B (60 MARKS)

Answer **five** questions from this section.

All questions carry equal marks.

9. (a) Given that, $p^{3-x}\, q^{5x} = p^{x+5}q^{3x}$, show that $x \log_r\left(\frac{q}{p}\right) - \log\frac{p}{r} = 0$

(05 Marks)

(b) Solve the simultaneous equations

$$\sqrt{a} + \sqrt{b} = 5$$
$$2a - b = 14$$ **(07 Marks)**

10.(a) Evaluate, $\int_0^1 \frac{y^2(1+y^3)}{(y^6+2y^3)^7}\,dy$ **(05 Marks)**

(b) Calculate the area enclosed by the curve $y = \sin x$ and the line y = 0.5, from $x = 0$ to $x = \pi$ and the x-axis. **(07 Marks)**

11.Two planes L_1 and L_2 are defined $3x - 4y + 2z - 5 = 0$ and $r = \begin{pmatrix}1\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\5\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ respectively.

(a) Cartesian equation of plane L_2 **(04 Marks)**

(b) Acute angle between the two planes **(04 Marks)**

(c) Vectors equation of the line of intersection of L_1 and L_2. **(04 Marks)**

12.(a) Solve the equation;

$$\tan 4x + \tan 2x = 0 \text{ for } 0^0 \leq x \leq 180^0$$ **(05 Marks)**

(b) Given that $\tan\theta = \frac{p}{q}$, Prove that $\frac{P}{P+Q} = \frac{\sin\theta}{\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)}$ **(05 Marks)**

13.(a) If $y = \frac{\cos\lambda x}{1+\sin\lambda x}$ where λ is a constant show that $\frac{dy}{dx} = \frac{-\lambda}{1+\sin\lambda x}$ **(05 Marks)**

(b) A hollow right circular cone has base radius 4cm and vertical height 20cm. it is held upside down with its axis vertical. It contains water which is being added at the constant rate of 1.5cm^3/second and which leaks away through a small hole in the vertex at the constant rate of 2cm^3/second. At what rate is the depth of water changing when the depth is 12cm. **(05 Marks)**

14.(a) Determine the vertex, focus and axis of the parabola,

$y^2 - 2y - 8x - 17 = 0$ **(05 Marks)**

(b) The tangents to the parabola $y^2 = 4ax$ at points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$meet at point R, find the coordinates of R. **(07 Marks)**

15.Given that $x^2y + 2xy = 3y + 8$.

Find the;

(i) Ranges of values of y for the real of x. hence determine the coordinates of the turning points and its nature. **(05 Marks)**

(ii) Equations of the asymptotes and hence sketch the curve. **(07 Marks)**

16. According to Newton's law of cooling, the rate of cooling of a body in air is proportional to the difference between the temperature of the body and that of the air. If the air temperature is kept at 25^{o}c and the body cools from 95^{0}c to 60^{o}c in 25 minutes

    a) Form a differential equation to represent the rate of change of temperature θ of a body with respect to time(t). **(07 Marks)**

    b) In what further time will the body cool to 32^{o}c. **(05 Marks)**

END