P425/2
APPLIED
MATHEMATICS
Paper 2
July /Aug. 2023
3 hours

UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** questions in *section A* and any **five** from *section B*.

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section **B** will **not** be marked.

Turn Over

SECTION A (40 MARKS)

1. Given that A and B are independent events such that:

   $P(A) = {}^5/_8$ and $P(A^1 U B) = {}^1/_2$; find;

   a) $P(A U B^1)$

   b) $P(A^1 U B^1)$ **(05 marks)**

2.

x	0.8	1.2	1.6
lnx	-0.24	0.18	0.48

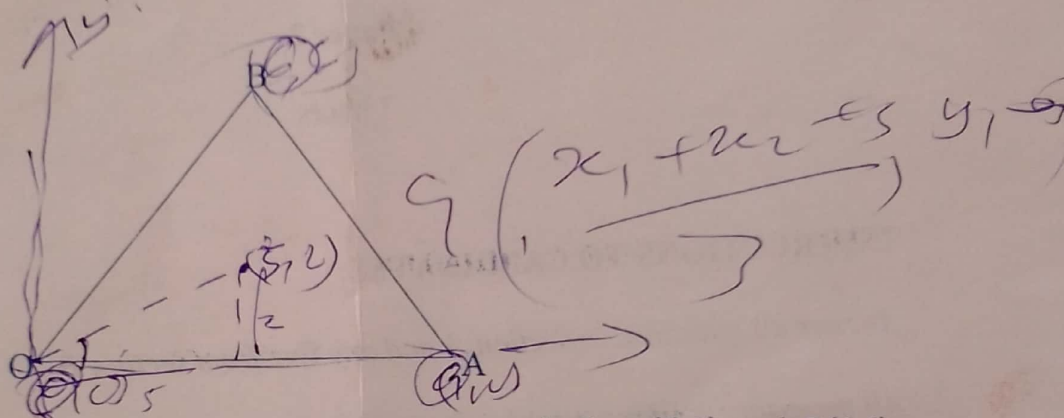   Use linear estimation to find;
   (i) ln (0.5)
   (ii) x when $lnx = -0.12$ **(05 marks)**

3.

   OAB is a uniform triangular lamina with vertices $O(0,0)$, $A(9,0)$ and $B(6,6)$

   (a) Find the coordinates of the centre of gravity of the lamina. **(02 marks)**

   (b) If the lamina is freely suspended at point $O(0,0)$, calculate the angle the side OA makes with the vertical. **(03 marks)**

4.

				Ranks			
Height	1	2	3	4	5	6	7
Mass	2	1	4	3	7	5	6

   The heights and corresponding masses of 7 tourists were taken, and ranked as shown.
   Calculate the spearman's rank correlation coefficient for this data. Comment on your result. **(05 marks)**

2

5. The velocity of a particle after t seconds is;
$12t^2 i + (8t + 23)j \, ms^{-1}$. Calculate the average speed of particle in the time interval $t = 1$ to $t = 3 \, s$. **(05 marks)**

6. Given that $x = 1.25 \, (2dps)$; $y = 1.600 \, (3 \, dps)$, calculate the interval within which the exact value of xy lies. Deduce the maximum error in xy. **(05 marks)**

7. In any trial, the probability that a head occurs when a coin is tossed is three times the probability that a tail occurs. The coin is tossed 15 times, calculate the probability that a head will occur at least 7 times. **(05 marks)**

8. A particle projected with a speed of $12ms^{-2}$ to move in a straight line on a rough horizontal surface comes to rest in 5 seconds. Calculate the distance it covers in its last second of motion. **(05 marks)**

SECTION B (60 MARKS)

9. Study the frequency table below;

Marks	0 – 10	10 -	15 -	25 -	40 -	50 - 60
No. of candidates	8	10	25	15	4	2

(a) Calculate the;
   (i) mean mark
   (ii) modal mark **(07 marks)**

(b) Plot a cumulative frequency curve and use it to estimate the 10th to 80th percentile deviation. **(05 marks)**

10. (a) Use the trapezium rule with 7 ordinates to evaluate;
$$\int_0^{\pi/2} (2x + cosx)dx$$

(06 marks)

(b) Calculate the percentage error made in the evaluation in (a) above. Suggest how this error can be minimized. **(06 marks)**
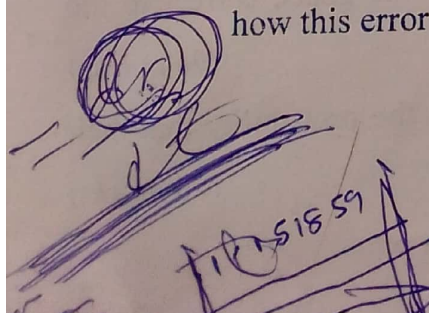
11. A particle executes simple harmonic motion. If it has speeds of $8ms^{-1}$ and $6ms^{-1}$ at points at respective distances of 3m and 4m from the centre of motion; calculate the;

    (a)   amplitude and period of motion.   (08 marks)

    (b)   time the particle takes to move directly from A to B.   (04 marks)

12. X is a continuous random variable whose distribution function is given by;

$$F(x) = \begin{cases} a(x^2 - 1) & ; \quad 1 \le x \le 2 \\ a + bx & ; \quad 2 \le x \le 3 \\ 1 & ; \quad x \ge 3 \end{cases}$$

    (a)   Find the constants a and b; hence sketch the graph of $F(x)$.   (04 marks)

    (b)   Compute; $P(X < 2.5 \; / \; X > 1.5)$ (04 marks)

    (c)   Calculate, $E(X)$ the mean of X (04 marks)

13. A uniform ladder AB of mass 10kg stands on a rough horizontal surface at A, and leans against a rough vertical wall at B, the coefficients of friction at A and B being $\frac{1}{2}$ and $\frac{1}{3}$ respectively. The angle of inclination of the ladder to ground is $tan^{-1}\frac{3}{4}$. A boy of mass 40kg starts to climb the ladder.

    Calculate the;

    a) distance he climbs before the ladder starts to slide.   (06 marks)

    b) minimum horizontal force that should be applied at A so that the boy just reaches the top of the ladder.   (06 marks)

14. The weights of goats sold at a certain market are normally distributed with a mean of 16kg. given that 8 of every 12 goats picked at random weighed more than 20kg.
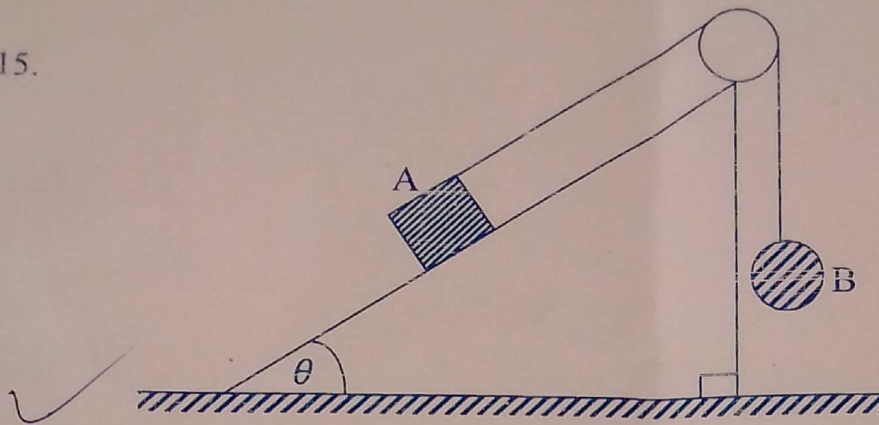
    (a)   Calculate the standard deviation of the masses of the goats, correct to the nearest whole number.   (06 marks)

    (b)   A random sample of 25 goats is picked, calculate the probability that their mean weight exceeds 15kg.   (06 marks)

15.

(a)   A particle B of mass **m** kg keeps particle A of mass 10kg at rest on a rough inclined plane of angle $\theta = tan^{-1} \frac{4}{3}$. If the coefficient of friction between A and the inclined plane is 0.5, calculate the minimum and maximum values of **m**.

(b)   If the mass of A is doubled, calculate the magnitude of the accelerations of the particles.   *(04 marks)*

16. (a)   Show that the equation $x\sin x = 1$ has a root lying between 1 and 1.5.

(03 marks)

(b)   Use linear interpolation once to find the first approximation, x_0 of the equation. Hence use the Newton – Raphson Method to compute the root correct to 4dps.

(09 marks)

END

$P(2.5 \times < 1.5)$ 0.4

$1.11 41157142$