P425/1

Pure

Mathematics

Paper 1

July/August 2017

3 hours

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS 2017

PURE MATHEMATICS

PAPER 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

- Attempt **all the eight** questions in Section **A** and **five** from Section **B**.
- All working must be shown clearly.
- Begin each answer on a fresh sheet of paper.
- Mathematical tables with a list of formulae and squared papers are provided.
- Silent non-programmable calculators may be used.
- State the degree of accuracy at the end of the answer to each question attempted using a calculator or tables and indicate **"Cal"** for Calculator or **"Tab"** for Mathematical tables.

SECTION A

1. The sum of the second and third terms of a Geometric Progression (GP) is 48. The sum of the fifth and sixth terms is 1296. Find the common ratio, the first term and the sum of the first 12 terms of the Geometric Progression. (05 marks)

2. Use De Moivre's theorem to prove that:
$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. (05 marks)

3. When a polynomial $\mathrm{P}(x)$ is divided by $x^2 - 5x - 14$, the remainder is $2x + 5$. Find the remainder when $\mathrm{P}(x)$ is divided by:
   (i) $x - 7$
   (ii) $x + 2$ (05 marks)

4. OAB is a triangle in which $\mathrm{OA} = a$, $\mathrm{OB} = b$. C is a point on AB such that $\mathrm{AC}:\mathrm{CB} = 3:1$. D is midpoint of OA, DC and OB, both produced meet in point T. Find vector OT in terms of a and b. (05 marks)

5. Find the integral $\int x\cos^2 x \ dx$. (05 marks)

6. Given that $y = x + a$ is a tangent to the curve $y = ax^2 + bx + c$ at the point $(2, 4)$. Find the values of the constants a, b and c. (05 marks)

7. Find the volume of the solid of revolution generated when the area under $y = \frac{1}{x-2}$ from $x = 3$ to $x = 4$ is rotated through four right angles about the $x -$ axis.
(05 marks)

8. In triangle ABC, AB= X-Y BC= X and CA= X+Y, Show that $\cos B = \frac{X-4Y}{2(X-Y)}$
(05 marks)

SECTION B (60 MARKS)

9. (a) Find the centroid of the triangle whose sides are given by the equations $x + y = 11$, $y = x - 1$ and $3y = x - 3$. (05 marks)

(b) ABCD is a rhombus such that the coordinates A$(-3, -4)$ and C$(5, 4)$. Find the equation of the diagonal BD of the rhombus. If the gradient of side BC is 2. Obtain the coordinates of B and D.

Prove that the area of the rhombus is $21\frac{1}{3}$ square units. (07 marks)

10. Show that:

$$\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)}dx = \frac{\pi}{20}$$ (12 marks)

11. (a) Using Maclaurin's theorem, expand $e^{-x}\sin x$ up to the term in x^3. Hence evaluate $e^{-\frac{\pi}{3}}\sin\frac{\pi}{3}$.To four decimal places.

(05 marks)

(b) The curve $y = x^3 + 8$ cuts the x and y axes at the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B and C hence find the area enclosed between the curve and the line. (07 marks)

12. (a) The position vectors of the points P and Q are $4\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{i} + 2\boldsymbol{k}$ respectively. Find the coordinates of the point R such that PQ:PR $= 2:1$.

(04 marks)

(b) If the vector $5\boldsymbol{i} - \lambda\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to the line $r = \boldsymbol{i} - 4\boldsymbol{j} + t(2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k})$. Find the value of λ.

(03 marks)

(c) Obtain the equation of the plane that passes through (1,-2,2) and its perpendicular to the line $\frac{x-9}{4} = \frac{y-6}{-1} = \frac{z-8}{1}$ (05 marks)

13. The parametric equations $x = \frac{1+t}{1-t}$ and Y= $\frac{2t^2}{1-t}$ represent a curve.

(i) Find the cartesian equation of the curve. (04 marks)

(ii) Determine the turning points of the curve and their nature. (03 marks)

(iii) State the intercepts and asymptotes of the curve. (03 marks)

(iv) Hence sketch the curve. (02 marks)

14. (a) Determine the maximum value of the expression $6\sin x - 3\cos x$. (03 marks)

(b) Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$

(03 marks)

(c) In a triangle ABC, prove that:
$\sin \mathrm{B} + \sin \mathrm{C} - \sin \mathrm{A} = 4\cos\frac{1}{2}\mathrm{A}\sin\frac{1}{2}\mathrm{B}\sin\frac{1}{2}\mathrm{C}$ (06 marks)

15. (a) Simplify $(2+5i)^2 + 5\frac{(7+2i)}{3-4i} - i(4-6i)$ expressing your answer in terms form $a + b\boldsymbol{i}$. (05 marks)

(b) If $Z = x + y\boldsymbol{i}$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Show that the locus of Arg $\left(\frac{z-1}{z-i}\right) = \frac{\pi}{3}$ is a circle, find its centre and radius. (07 marks)

16. (a) Using the substitution $y = ux$, solve the differential equation.

$x^2\frac{dy}{dx} = x^2 + xy + y^2$. (04 marks)

(b) The rate at which a liquid runs from a container is proportional to the square root of the depth of the opening below the surface of the liquid. A cylindrical petrol storage tank is sunk in the ground with its axis vertical. There is a leak in the tank at an unknown depth. The level of the petrol in the tank originally full is found to drop by 20 cm in 1 hour and by 19 cm in the next hour. Find the depth at which the leak is located. (08 marks)

End