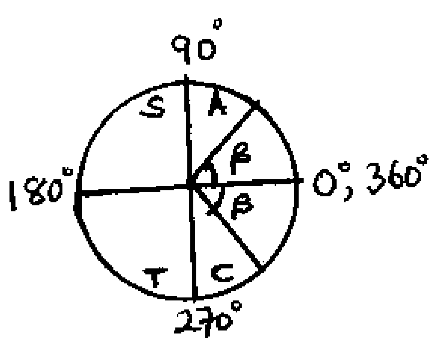
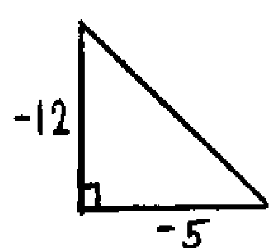
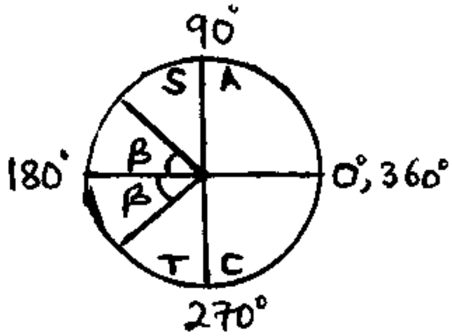
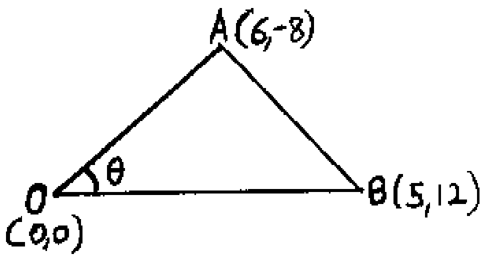
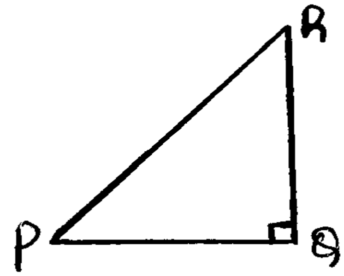
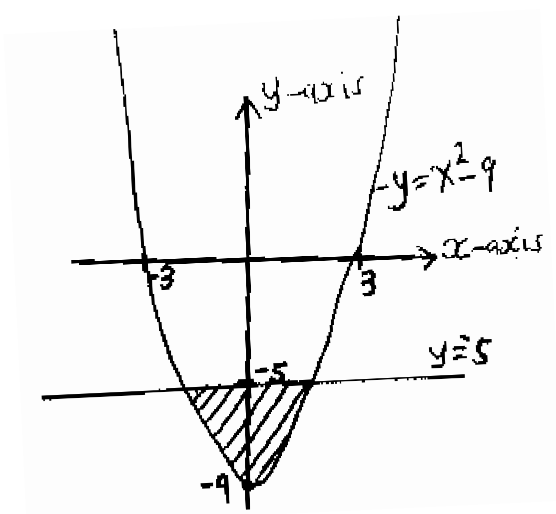
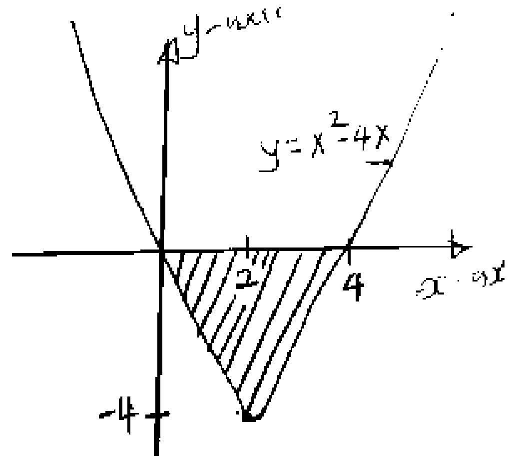
SUBSIDIARY MATHEMATICS SEMINAR SOLUTIONS 2023

1. (a)	$let\ f(x) = 2x^3 + 6x + qx - 5$ $x + 2 = 0 \quad => x = -2$ $f(-2) = 2(-2)^3 + 6(-2) + q(-2) - 5$ $0 = -16 - 12 - 2q - 5$ $q = -16.5$ $=> f(x) = 2x^3 + 6x - 16.5x - 5$ $for\ remainder\ when\ f(x) is\ divided\ by$ $2x - 1\ using\ remainder\ theorem\ then$ $2x - 1 = 0 \quad => x = \frac{1}{2}$ $2(\frac{1}{2})^3 + \left(6 \times \frac{1}{2}\right) - \left(16.5 \times \frac{1}{2}\right) - 5 = -10$ $hence\ the\ remainder\ is - 10$	(b)	$px^2 + qx + r = 0$ $let\ the\ rrots\ be\ \alpha\ and\ \beta\ \ then\ \alpha - \beta = 3$ $sum\ of\ roots\ ;\ \alpha + \beta = \frac{-q}{p}$ $product\ of\ roots\ ; \alpha\beta = \frac{r}{p}$ $from;\ \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta\ and$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ $=> 3^2 = \left(\frac{-q}{p}\right)^2 - 4\left(\frac{r}{p}\right)$ $9 = \frac{q^2}{p^2} - \frac{4r}{p}$ $9p^2 = q^2 - 4pr$ $q^2 = 9p^2 + 4pr$
(c)	$x^2 - 4x + 2 = 0$ $sum\ of\ roots;\ \alpha + \beta = \frac{-(-4)}{1} = 4$ $product\ of\ roots\ ; \alpha\beta = \frac{2}{1} = 2$ $sum\ of\ new\ roots = (\alpha + 2) + (\beta - 2)$ $= \alpha + \beta = 4$ $product\ of\ new\ roots = (\alpha + 2)(\beta - 2)$ $\alpha\beta - 2\alpha + 2\beta - 4 = 2 - 4 + 2(\alpha + \beta)$ $(\beta - \alpha)^2 = \beta^2 + \alpha^2 - 2\alpha\beta$ and $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\beta - \alpha = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$ $\sqrt{4^2 - 4(2)} \ = \sqrt{8}$ $=> product\ of\ new\ roots = -2 + 2\sqrt{8}$ $equation\ is\ x^2 - (sum\ of\ roots)x$ $+ product\ of\ roots = 0$ $x^2 - 4x + \left(-2 + 2\sqrt{8}\right) = 0$	2.	$\log_2 x + 2\log_4 y = 4\ ;$ $\log_2 x + \frac{2\log_2 y}{2\log_2 2} = 4$ $\log_2 x + \log_2 y = 4; => \log_2 xy = 4$ $xy = 2^4\ \ hence\ xy = 16$ $\log_{10}(x + y) = 1, then\ x + y = 10\ \ldots.\ eqn\ i$ $if\ xy = 16\ \ then\ \ x = \frac{16}{y}$ $substituting\ for\ x\ in \ldots\ldots\ldots eqn\ i$ $\frac{16}{y} + y = 10;\ => y^2 - 10y + 16 = 0$ $(y - 8)(y - 2) = 0\ either\ y = 8\ or\ y = 2$ $for\ y = 8\ then\ x = \frac{16}{8} = 2\ and\ also$ $for\ y = 2, then\ x = \frac{16}{2} = 8$
3.	$5^{2m+1} + 4 = 21 \times 5^m$ $5^{2m} \cdot 5^1 + 4 = 21 \times 5^m$ $5^{m^2} \cdot 5^1 + 4 = 21 \times 5^m \quad let\ 5^m\ be\ y$ $5y^2 + 4 = 21y\ \ => 5y^2 - 20y - y + 4 = 0$ $5y(y - 4) - 1(y - 4) = 0$ $(5y - 1)(y - 4) = 0\ either\ y = \frac{1}{5}\ or\ y = 4$ $for\ y = 5^{-1}, => 5^m = 5^{-1}\ hemce\ \ m = -1$ $y = 4, => 5^m = 4\ taking\ \log\ both\ sides$		$\log 5^m = \log 4\ , => m\log 5 = \log 4$ $m = \frac{\log 4}{\log 5}\ \ , => m = 0.86135311615$ $lnm\ \ for\ m = -1, \ln(-1)\ \ is\ not\ possible$ $but\ lm(0.86135311615) = -0.1492(4dp)$

4.	$grandient = 6x - 5, => \frac{dy}{dx} = 6x - 5$ $y = \int (6x-5)dx \ , => y = 3x^2 - 5x + c$ $at\ (0,2)\ => 2 = 3(0)^2 - 5(0) + c$ $=> c = 2, hence\ y = 3x^2 - 5x + 2$ $for\ \ y = 0\ means\ \ 3x^2 - 5x + 2 = 0$ $discriminant = b^2 - 4ac.\ from\ equation$ $a = 6, b = -5\ and\ c = 2$ $discriminant = (-5)^2 - 4(6)(2) = -23$ $since\ discriminant\ is\ negative\ the\ equation$ $will\ have\ no\ real\ roots.$	5. (a) (b) (i)	$s_\infty = \frac{25}{6}\ , a = 5, ..\ \ but\ \ s_\infty = \frac{a}{1-r}$ $\frac{25}{4} = \frac{5}{1-r}\ \ \ \ => 5 - 5r = 4\ \ then\ \ r = \frac{1}{5}$ $s_n = \frac{a(1-r^n)}{1-r}$ $=> s_{10} = \frac{5(1-(0.2)^{10})}{1-0.2}$ $s_{10} = 6.25\ \ and\ common\ ratio\ is\ 0.2$
5.(b) (i) (ii)	$a = 250, \qquad d = 30$ $n^{th} = a + (n-1)d$ $9^{th} = 250 + (9-1) \times 30$ $= 490\ tourits$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $S_{11} = \frac{11}{2}(2 \times 250 + (11-1) \times 30)$ $S_{11} = 4{,}400\ tourists$	5.(c)	$\{(45\ , 54\ , 63\ , 72\ , \dots \dots \dots 288)\}$ $=>\ \ a = 45, \qquad and\ \ \ d = 9$ $n^{th} = a + (n-1)d$ $288 = 45 + (n-1) \times 9$ $\frac{288-45}{9} = n - 1$ $27 + 1 = n\ \ , =>\ \ n = 28\ numbers$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $S_{11} = \frac{28}{2}(2 \times 45 + (28-1) \times 9)$ $sum\ of\ multiple\ is = 4{,}662$
6. (a)	$(2tan\beta)^2 - sec\beta = 1.5$ $4tan^2\beta - sec\beta = 1.5.$ $from\ 1 + tan^2\beta = sec^2\beta$ $4(sec^2\beta - 1) - sec\beta = 1.5, , , let\ \ m = sec\beta$ $4m^2 - m - 5.5 = 0$ $solving\ the\ equations, , , m = sec\beta = 1.3042$ $and\ m = sec\beta = -1.0542.$ $\beta = \cos^{-1}\left(\frac{1}{1.3042}\right) = \quad 39.99^0\ \ .$  $\beta = 39.99^0, 320.01^0.$	6 (b). (c).	$given\ \ x = asec\theta\ \ \ and\ \ y = b + Ccos\theta$ $x = \frac{a}{cos\theta} \dots .(i)\ and\ also\ cos\theta = \frac{y-b}{c}.$ $making\ cos\theta\ the\ subject\ in\ , \dots (i) yeilds$ $cos\theta = \frac{a}{x}\ \ equating\ both\ cos\theta\ yeilds$ $\frac{y-b}{c} = \frac{a}{x}\ \ hence\ x(y-b) = ca$ Given that $\tan\beta = \frac{-12}{5}$ 

	$\beta = \cos^{-1}\left(\frac{1}{1.0542}\right) = \quad 18.45^0$  $\beta = 161.55^0, 198.45^0.$ $=> required\ angle\ are\ 39.99^0\ and\ 161.55^0.$		$1 + tan^2\beta = sec^2\beta$ $sec^2\beta = 1 + (\frac{-12}{5})^2 => sec\beta = \sqrt{\frac{169}{25}}$ $sec\beta = \frac{-13}{5}$ $cosec^2\beta = 1 + cot^2\beta$ $cosec^2\beta = 1 + \left(\frac{5}{-12}\right)^2$ $cosec\beta = \sqrt{\frac{169}{144}}\ \ hence\ \ cosec\beta = \frac{13}{12}$ $7sec\beta + 12cosec\beta = 7\left(\frac{-13}{5}\right) + 12\left(\frac{13}{12}\right)$ $=> 7sec\beta + 12cosec\beta = -5.2$
8 (a)	 $\overrightarrow{OA} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}\ \ and\ \ \overrightarrow{OB} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ $\overrightarrow{OA} \cdot \overrightarrow{OB} = \lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert COS\theta$ $\overrightarrow{OA} \cdot \overrightarrow{OB} = \begin{pmatrix} 6 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix} = 30 - 96 = -66$ $\lvert\overrightarrow{OA}\rvert = \sqrt{(6)^2 + (-8)^2} = 10$ $\lvert\overrightarrow{OB}\rvert = \sqrt{(5)^2 + (12)^2} = 13$ $\overrightarrow{OA} \cdot \overrightarrow{OB} = \lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert COS\theta$ $-66 = 10 \times 13cos\theta$ $\theta = \cos^{-1}(\frac{-66}{130})\ \ => \theta = 120^0$	(c)	 $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = \begin{pmatrix} -7 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \end{pmatrix} = \begin{pmatrix} -8 \\ -2 \end{pmatrix}$ $\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$ $\overrightarrow{QR} \cdot \overrightarrow{QP} = \lvert\overrightarrow{QR}\rvert\lvert\overrightarrow{QP}\rvert COS\theta$ $\overrightarrow{QR} \cdot \overrightarrow{QP} = \begin{pmatrix} -8 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix} = -8 + 8 = 0$ $\lvert\overrightarrow{QR}\rvert = \sqrt{(-8)^2 + (-2)^2} = \sqrt{68}$ $\lvert QP\rvert = \sqrt{(1)^2 + (-4)^2} = 3$ $\overrightarrow{OA} \cdot \overrightarrow{OB} = \lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert COS\theta$ $0 = 3 \times \sqrt{68}cos\theta$ $\theta = \cos^{-1}(\frac{0}{3\sqrt{68}})\ \ => \theta = 90^0$ $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$ $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = \begin{pmatrix} -7 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -9 \\ 2 \end{pmatrix}$

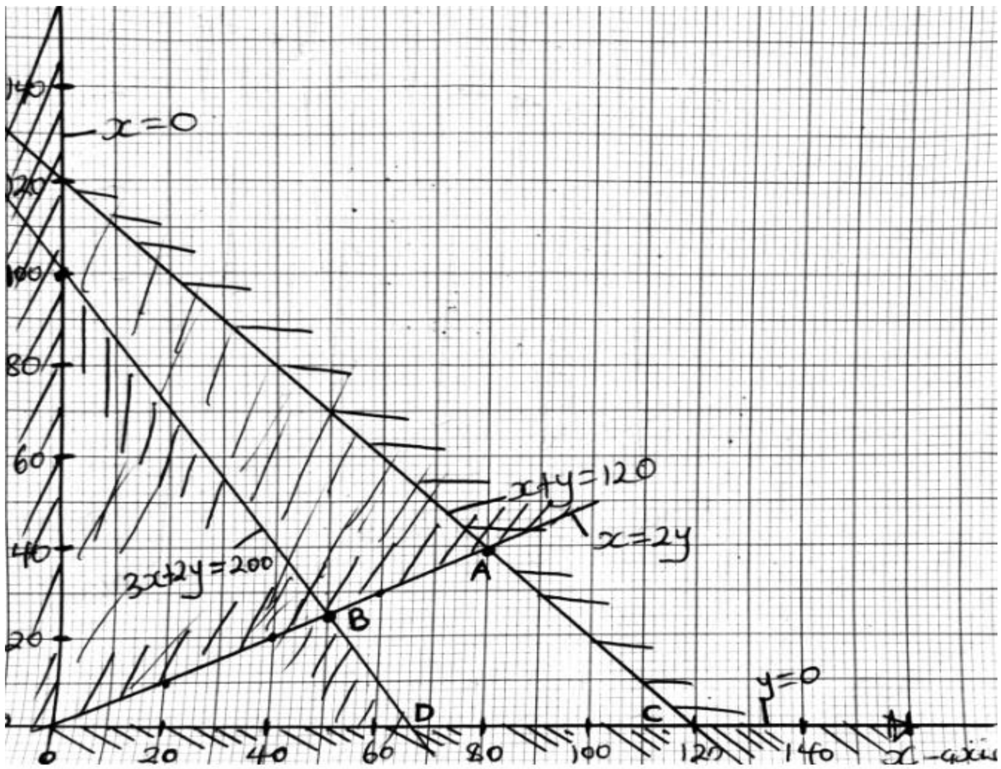
8(b)	$\binom{a}{8} = 2\binom{2}{4}$ *since they are parallel* => $m = 2n$ *hence* $a = 2 \times 2 = 4$ .. *so* $a = 4$		$\overrightarrow{PQ} = \binom{-1}{4}$ *and* $\overrightarrow{QR} = \binom{-8}{-2}$ $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$ $\overrightarrow{PQ} + \overrightarrow{QR} = \binom{-1}{4} + \binom{-8}{-2} = \binom{-9}{2}$ *since* $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$ *and angle PQR* $= 90^0$ *then PQR is a right − angled at Q*
8(d)	$\overrightarrow{OM} = x - y$, *and* $\overrightarrow{ON} = 2x + 3y$ $\overrightarrow{OM} \cdot \overrightarrow{ON} = 0$ *since are perpendicular* $(x - y) \cdot (2x + 3y) = 0$ $x \cdot (2x + 3y) - y \cdot (2x + 3y) = 0$ $x \cdot 2x + 3y \cdot x - 2x \cdot y - 3y \cdot y = 0$ $2x^2 + x \cdot y - 3y^2 = 0$ $x \cdot y = 3y^2 - 2x^2$ $\|x\| = 8$, *and* $\|y\| = 6$ $x \cdot y = 3(6)^2 - 2(8)^2 = -20$ $x \cdot y = \|x\|\|y\|cos\theta$ $\theta = \cos^{-1}(\frac{-20}{48})$ => $\theta = 114.62^0$	9. (a) (b)	$x + y \leq 120$ … … … … … … (i) $x \geq 2y$ … … … … … … … … (ii) $1500x + 1000y \geq 100{,}000$ *reduced to* $3x + 2y \geq 200$ … … . (iii) $x \geq 0$ … … . (iv) *and* $y \geq 0$………. (v) <table><tr><th>Seats (x, y)</th><th>$1500 + 1000y$</th><th>Amount</th></tr><tr><td></td><td></td><td></td></tr></table>
	$3A + 2C + B = I$ *where* $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $2C = I - 3A - B.$ $2C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 3\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} - \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix}$ $2C = \begin{pmatrix} 1 - 9 - 8 & 0 - 15 + 3 \\ 0 + 6 + 4 & 1 - 12 - 7 \end{pmatrix}$ $C = \begin{pmatrix} -8 & -6 \\ 5 & -9 \end{pmatrix}$ *Det of* $C = (-8 \times -9) - (5 \times -6) = 102$ $C^2 = \begin{pmatrix} -8 & -6 \\ 5 & -9 \end{pmatrix}\begin{pmatrix} -8 & -6 \\ 5 & -9 \end{pmatrix}$ $C^2 = \begin{pmatrix} 64 - 30 & 48 + 54 \\ -40 - 45 & -30 + 81 \end{pmatrix}$ $=> C^2 = \begin{pmatrix} 34 & 102 \\ -85 & 51 \end{pmatrix}$		$MN = K$ $cos^3\beta \times sec\beta + cosec\beta \times sin^3\beta = a$ $=> cos^2\beta + sin^2\beta = a$ *comparing with* $cos^2\theta + sin^2\theta = 1$ $a = 1$ $cos\beta \times sec\beta + 2cosec^3 \times sin^3\beta = b$ $1 + 2 = b$ *hence* $b = 3$ $cos^3\beta \times 0 + cosec\beta \times 1 = Ycosec\beta$ $Y = 1$ $2cosec^3\beta = 4pcosec^3\beta$ $2 = 4p$ => $p = \frac{1}{2}$ *for no unique solution means det* $\begin{pmatrix} 3 & m \\ 2 & 1 \end{pmatrix} = 0$ $(3 \times 1) - (2 \times m) = 0$ $m = \frac{3}{2}$

10.	$$y + 9 = x^2,,,,, => y = x^2 - 9$$ $$\frac{dy}{dx} = 2x$$ $$At\ turning\ points, \frac{dy}{dx} = 0$$ $$=> 2x = 0\ hence\ x = 0$$ $$y = (0)^2 - 9 = -9\ turning\ point\ (0,9)$$ $$\frac{d^2y}{dx^2} = 2$$ $$since \frac{d^2y}{dx^2}\ is\ positive\ then$$ $$(0,-9) is\ a\ minimum\ turning\ point$$ $$intercepts$$ $$x - intercept; y = 0\ hence\ x^2 - 9 = 0$$ $$x = \sqrt{9} => x = 3\ or\ x = -3$$ $$intercept\ points\ are\ (3,0)\ and\ (-3,0)$$ $$y - intercept; x = 0\ hence\ y = o^2 - 9 = -9$$ $$intercept\ point\ is\ (0,-9).$$ $$y + 9 = x^2 \ldots\ldots, y = -5$$ $$-5 + 9 = x^2\ \ hence\ x = 2\ or\ x = -2$$  $$Area = \int_{-2}^{2} (x^2 - 9) - (-5) dx$$ $$= \left\|\left(\frac{1}{3}x^3 - 4x\right)\right\|_{-2}^{2}$$ $$area = \|-10\| = 10\ sq\ units$$	10 (c) (e)	$$y = x^2 - 4x,,,, => \frac{dy}{dx} = 2x - 4$$ $$2x - 4 = 0,, hnce\ x = 2$$ $$y = (2)^2 - 4(2) = -4\ \ point\ is\ (2,-4)$$ $$\frac{d^2y}{dx^2} = 2, \quad (2,-4) is\ min -\ turning\ point$$ $$intercepts$$ $$y - intercept, x = 0$$ $$y = 0^2 - 4(0) = 0\ point\ is\ (0,0)$$ $$x - intercept\ ,, y = 0$$ $$0 = x^2 - 4x. =>, x = 0\ \ or\ x = 4$$ $$points\ are\ (0,0)\ and\ (4,0)$$  $$Area = \int_0^4 (x^2 - 4x) dx$$ $$Area = \left\|\left(\frac{1}{3}x^3 - 2x^2\right)\right\|_0^4$$ $$\left(\frac{1}{3}4^3 - 2(4)^2\right) - \left(\frac{1}{3}0^3 - 2(0)^2\right)$$ $$Area = \frac{32}{3}\quad or\ 10.6667\ square\ units$$ $$3x\frac{dy}{dx} - 4\frac{x^2}{y} = 0\ \ ; x = 3\ and\ y = 4$$ $$\frac{dy}{dx} = \frac{4x}{3y}$$ $$\int y dy = \int \frac{4}{3} x\ dx$$ $$\frac{1}{2}y^2 = \frac{2}{3}x^2 + c\ \ when\ x = 3\ and\ y = 4$$ $$\frac{1}{2}(4)^2 = \frac{2}{3}(3)^2 + c,, hnce\ c = 2$$

			$\frac{1}{2}y^2 = \frac{2}{3}x^2 + 2$ $3y^2 = 4x^2 + 12$
f. (i)	$let\ the\ ammount\ of\ chemical\ be\ m\ in\ grams$ $\frac{dm}{dt}\alpha\ m\ \ hence\ \frac{dm}{dt} = -km$ $\int \frac{1}{m}dm = \int -kdt$ $\ln m = -\ kt + c$ $At\ t = 0,,, m = m_0 = 100g$ $\ln 100 = -k(0) + c\ hence\ ....\ c = \ln 100$ $At\ t = 5, and\ m = 90g$ $\ln 90 = -k(5) + \ln 100$ $k = \frac{1}{5}\ln\left(\frac{100}{90}\right)\ \ hence\ k = \frac{1}{5}\ln\left(\frac{10}{9}\right)$ $\ln m = -\frac{1}{5}\ln\left(\frac{10}{9}\right)t + \ln 100$	f.(ii) (a) (b)	$At\ t = 20,$ $\ln m = -\frac{1}{5}\ln\left(\frac{10}{9}\right) \times 20 + \ln 100$ $\ln m = 4.183728$ $m = e^{(4.183728)}$ $m = 65.61g$ $m = 30g$ $ln30 = -\frac{1}{5}\ln\left(\frac{10}{9}\right)t + \ln 100$ $t = \frac{5\ln\left(\frac{100}{30}\right)}{\ln\left(\frac{10}{9}\right)} = 57.1359\ minutes$
11. (i) (ii)	$\boldsymbol{PART\ 2\ APPLIED\ MATHEMATICS}$ $simple\ aggregate\ price\ index = \frac{\sum p_1}{\sum p_0} \times 100$ $= \frac{60 + 135 + 105 + 290 + 800}{35 + 70 + 43 + 180 + 480} \times 100$ $= 172.0297$ $For\ A. price\ relative = \frac{60}{35} \times 100 = 171.4286$ $for\ B. price\ relaive = \frac{135}{70} \times 100 = 192.857$ $For\ C. price\ relative = \frac{105}{43} \times 100 = 244.186$ $For\ D. price\ relative = \frac{290}{180} \times 100 = 161.11$ $For\ E. price\ relative = \frac{800}{480} \times 100 = 166.67$	11.(iii) (iv)	$weighted\ aggregate\ price\ index$ $= \frac{\sum p_1\, w}{\sum p_0\, w} \times 100$ $= \frac{(60 \times 6) + (135 \times 5) + (105 \times 3) + (290 \times 2) + (800 \times 1)}{(35 \times 6) + (70 \times 5) + (43 \times 3) + (180 \times 2) + (480 \times 1)} \times 100$ $= \frac{2730}{1529} \times 100$ $= 178.5481$ $cost\ of\ an\ engine$ 1998 $172.0297 = \frac{1600}{p_{1998}} \times 100$ $p_{1998} = \frac{1600 \times 100}{172.0297} = 930.0720$ $cost\ of an\ engine\ in$ 1998 is 930.0720

9.

$x + y \leq 120 \ldots\ldots\ldots\ldots\ldots\ldots (i)$
$x \geq 2y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$
$1500x + 1000y \geq 100{,}000$
$reduced\ to\ 3x + 2y \geq 200 \ldots\ldots.(iii)$
$x \geq 0 \ldots\ldots.(iv)$ $\ and\ \ y \geq 0$………. (v)



$seats\ (x, y)$	$1500x + 1000y$	$Amount$
$A(80{,}40)$	$(1500 \times 80) + (1000 \times 40)$	160,000
$B(50{,}25)$	$(1500 \times 50) + (1000 \times 25)$	100,000
$C(120{,}0)$	$(1500 \times 120) + (1000 \times 0)$	180,000
$D(67{,}0)$	$(1500 \times 67) + (1000 \times 0)$	100,500

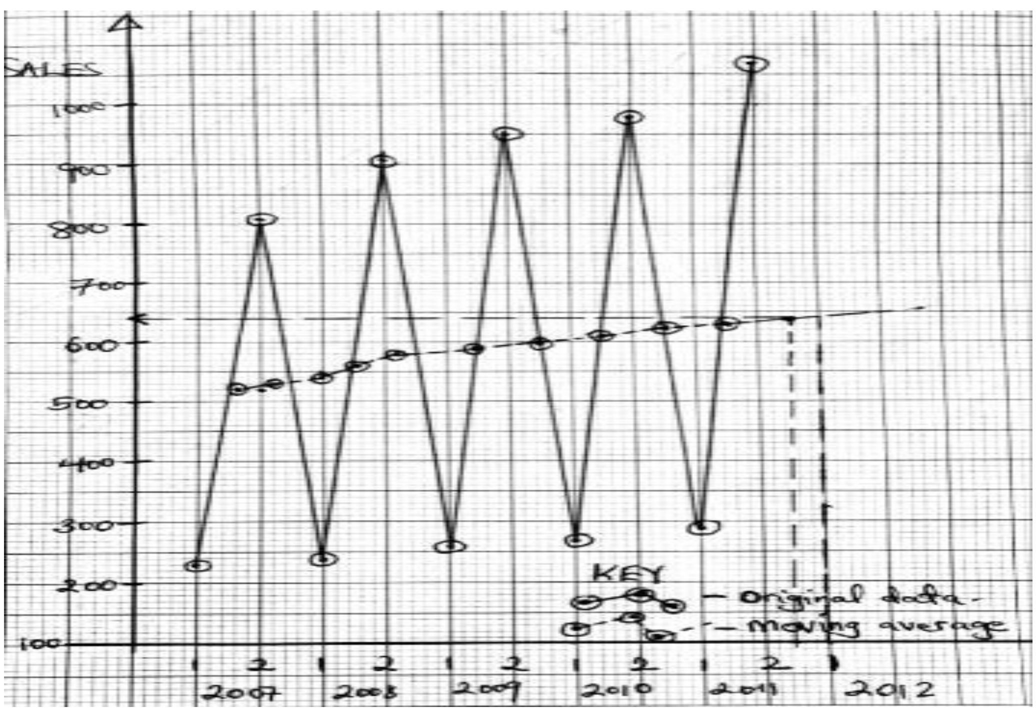
(i) $80\ seats\ for\ A\ and\ 40\ seats\ for\ B.$

(ii) $maximum\ profit\ is = shs\ 160{,}000$

(iii) $67\ for\ A\ and\ none\ for\ B$

12.

YEAR	HALF	SALES	MOVING TOTALS	MOVING AVERAGES
	1	230		
2007			1,040	520.0
	2	810		
			1,051	525.5
	1	241		
2088			1,093	546.5
	2	852		
			1,111	555.5
	1	259		
2009			1,161	580.5
	2	902		
			1,174	587.0
	1	272		
2010			1,206	603.0
	2	934		
			1,222	611.0
	1	288		
2011			1,254	627.0
	2	966		



$let\ the\ sales\ in\ 1^{st}\ half\ of\ 2012\ be\ x..=> \frac{x+966}{2} = 640,,=> x = 314$

13.

IQ	Frequency (f)	X	fX	fx^2	Cumulative frequency (F)
$45-<55$	1	50	50	2500	1
$55-<65$	1	60	60	3600	2
$65-<75$	2	70	140	9800	4
$75-<85$	6	80	480	38400	10
$85-<95$	21	90	1890	170100	31
$95-<105$	29	100	2900	290000	60
$105-<115$	24	110	2640	290400	84
$115-<125$	16	120	1920	230400	100
	$\sum f = 100$		$\sum fx = 10080$	$\sum fx^2$ =1035200	

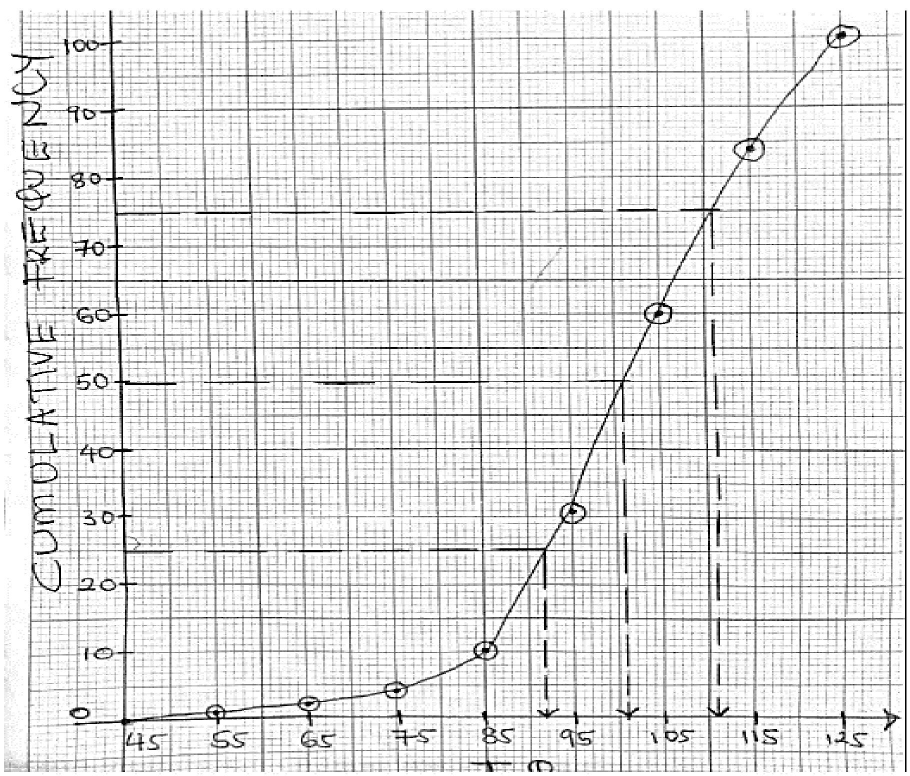
(a)

$$mean = \frac{\sum fx}{\sum x} = \frac{10080}{100} = 100.8$$

$$standard\ deviation = \sqrt{\frac{1035200}{100} - (100.8)^2}$$

$= 13.8333.$

(b)

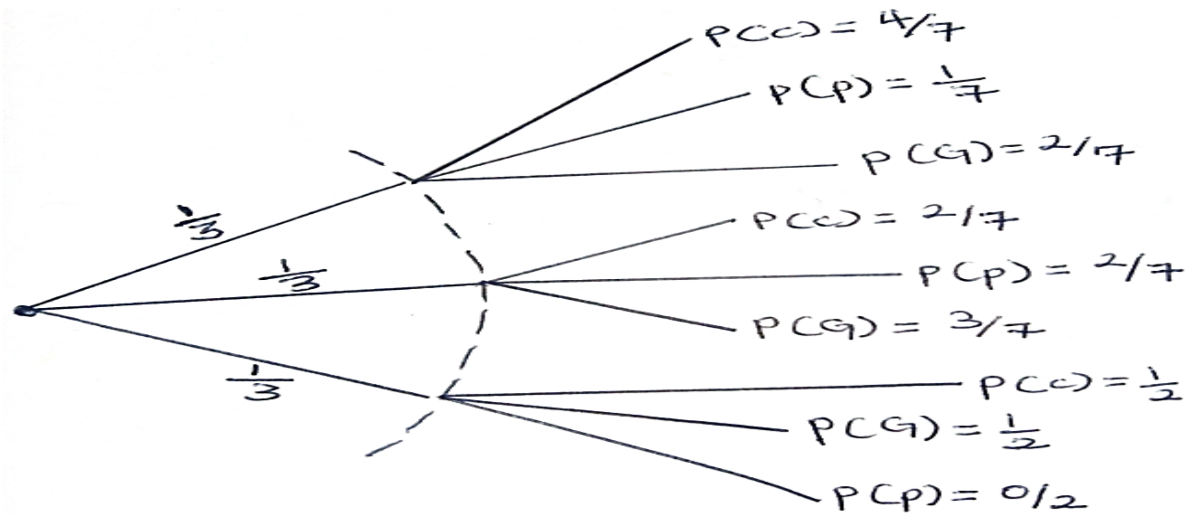


(c) (d)	$median = \left(\frac{100}{2}\right)th = 50^{th} = 101$ $upper\ quartile = (\frac{3}{4} \times 100)^{th} = 75^{th} = 110.5$ $lower\ quartile = (\frac{1}{4} \times 100)^{th} = 25^{th} = 92.5$ $semi-interquartile\ range = \frac{1}{2}(110.5 - 92.5) = 9$

14.

$$Total\ numbe\ of\ grand\ parent = 2 + 3 + 1 = 6$$
$$total\ number\ 0f\ people = 4 + 1 + 2 + 2 + 2 + 3 + 1 + 1 = 15people$$

$6P(Grand\ parent) = \frac{6}{15} = \frac{2}{5}$



$$Probability\ of\ picking\ grand\ parent = \frac{1}{3}(G_1 + G_2 + G_3)$$
$$= \frac{1}{3}\left(\frac{2}{6} + \frac{3}{7} + \frac{1}{2}\right) = \frac{1}{3}\left(\frac{28 + 36 + 42}{84}\right) = \frac{1}{3}\left(\frac{106}{84}\right)$$
$$= \frac{106}{252} = \frac{53}{126}$$

15. (i)	$P(B) + P(B^1) = 1$ $\frac{3}{2}P(A \cap B^1) + \frac{7}{3}P(A \cap B) = 1 \dots\dots (I)$ $P(A \cap B^1) + P(A \cap B) = P(A)$ $P(A \cap B^1) + P(A \cap B) = \frac{1}{2} \dots\dots (II)$ $(I) \times 6\ and\ (II) \times 9\ \ and\ solving$ $them\ simultaneously$ $9P(A \cap B^1) + 14P(A \cap B) = 6$	16.	$Arrangements\ in\ PROBABILITY$ $P(\ two\ Is\ are\ separate)$ $total\ number\ of\ ways = \frac{n!}{p!\,q!\,\dots}$ $without\ any\ restrictionsways\ are = \frac{11!}{2! \times 2!..}$ $number\ of\ ways\ with\ Is\ together = \frac{10!}{2!}$ $P(two\ I"s\ are\ separate =$

$$9P(A \cap B^1) + 9P(A \cap B) = \frac{9}{2}$$

$$on\ subtracting, 5P(A \cap B) = 6 - \frac{9}{2} \ldots\ldots$$

(ii)

$$=> P(A \cap B) = \frac{3}{10}$$

(iii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + \frac{7}{3}P(A \cap B) - P(A \cap B)$$
$$= \frac{1}{2} + \left(\frac{7}{3} \times \frac{3}{10}\right) - \frac{3}{10}$$
$$hence\ P(A \cup B) = \frac{9}{10}$$

(iv)

$$P(B) = \frac{7}{3}P(A \cap B)$$
$$= \frac{7}{3} \times \frac{3}{10} = \frac{7}{10}$$

$$P\left({}^{B}/_{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{\frac{3}{10}}{\frac{1}{2}}$$
$$= \frac{3}{5}$$

$$\left(\frac{\frac{11!}{2! \times 2!} - \frac{10!}{2!}}{\frac{11!}{2! \times 2!}}\right)$$
$$= \frac{9}{11}$$

17.

(i)

L	1	2	3	4	5
G	4	3	2	1	0

$$No\ of\ ways = {}^6_1C \times {}^5_4C + {}^6_2C \times {}^5_3C + {}^6_3C \times {}^5_2C + {}^6_4C \times {}^5_1C + {}^6_5C \times {}^5_0C$$
$$= 30 + 150 + 200 + 25 + 6$$

(ii)

$$= 461\ ways.$$

$$No\ restriction;$$

L	0	1	2	3	4	5
G	5	4	3	2	1	0

(ii)

$$number\ of\ ways = 461 + {}^6_0C \times {}^5_5C$$
$$= 462\ ways$$

$$not\ more\ than\ 3\ gentlemen$$

L	0	1	2	3
G	5	4	3	2

$$number\ of\ ways = {}^6_5C \times {}^5_0C + {}^6_4C \times {}^5_1C + {}^6_3C \times {}^5_2C + {}^6_2C \times {}^5_3C$$
$$= 6 + 75 + 200 + 150 = 431\ ways$$

18.

$$for\ binomial\ probabilitis,,, X \sim B(n, p)$$
$$hence\ X \sim B(10, 0.5)$$
$$Where\ p = 0.5\ \ from\ p + q = 1\ ,,, q = 0.5$$

(i)

$$P(X = 5) = 0.2461 \quad (tab)$$

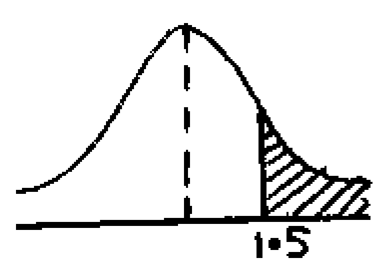
(ii)

$$atmost\ 7\ failed\ means\ atleast\ 8\ passed$$
$$P(X = 8) + P(X = 9) + P(X = 10)$$
$$+ \qquad +$$
$$probability\ for\ passing$$
$$= 0.0439 + 0.0098 + 0.0010$$
$$= 0.0547$$

19.(b)

(i)

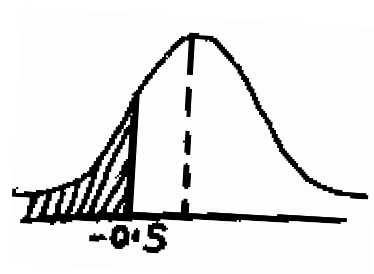
$$P(M > 65)$$
$$on\ standardising\ \ means\ P\left(Z > \frac{65 - 50}{10}\right)$$
$$= P(Z > 1.5)$$

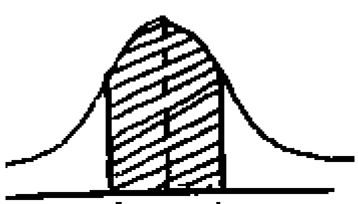


$$= 0.5 - P(0 < Z < 1.5)$$
$$= 0.5 - 0.43319$$
$$= 0.0668$$

<table>
<tr>
<td>(iii)</td>
<td>$E(x) = np$
$E(x) = 10 \times 0.5 = 5$

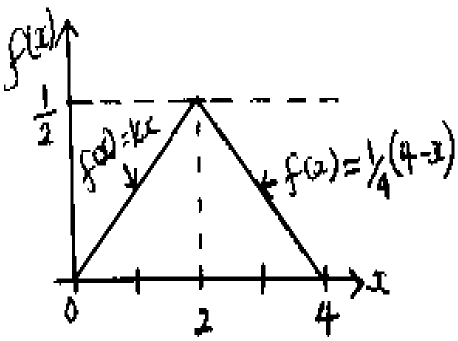
$standard\ deviation = \sqrt{variance}$
$= \sqrt{npq} \quad = \sqrt{10 \times 0.5 \times 0.5}$
$= \frac{1}{2}\sqrt{10}$</td>
<td></td>
<td>$number\ of\ candidates = 0.0668 \times 10{,}000$
$= 668.1 \approx 668\ candidates$</td>
</tr>
<tr>
<td>19 (a)
(i)</td>
<td>$Let\ M\ represent\ marks$
$M \sim N(50, 10^2)$
(I) $P(M > 70)$
$standardising\ we\ get.\ P\left(Z > \frac{70-50}{10}\right)$
$= P(Z > 2)$


$0.5 - P(0 < Z < 2)$
$0.5 - 0.47725$
$= 0.0228$</td>
<td>9(b)
(ii)</td>
<td>$P(M < 45)$
$On\ standardising,,P\left(Z < \frac{45-50}{10}\right)$
$P(Z < -0.5)$


$0.5 - P(0 < Z < 0.5)$
$0.5 - 0.19146$
0.30854
$Number\ of\ candidates = 0.30854 \times 10{,}000$
$= 3085.4 \approx 3085\ candidates$</td>
</tr>
<tr>
<td>19(a)
(ii)</td>
<td>$p(40 < M < 60)$
$standardises\ ,,P\left(\frac{40-50}{10} < Z < \frac{60-50}{10}\right)$
$= p(-1 < Z < 1)$


$2P(0 < Z < 1)$
$2 \times 0.34134 = 0.6827$</td>
<td>20.
(i)</td>
<td>$f(x) = \begin{cases} mx, & x = 1,2,3,4,5 \\ m(10-x),, & x = 6,7,8,9 \end{cases}$
<table><tr><th>x</th><th>$P(X = x)$</th></tr><tr><td>1</td><td>M</td></tr><tr><td>2</td><td>2m</td></tr><tr><td>3</td><td>3m</td></tr><tr><td>4</td><td>4m</td></tr><tr><td>5</td><td>5m</td></tr><tr><td>6</td><td>4m</td></tr><tr><td>7</td><td>3m</td></tr><tr><td>8</td><td>2m</td></tr><tr><td>9</td><td>m</td></tr></table>
$m + 2m + 3m + 4m + 5m + 4m + 3m + 2m + m = 1$
$25m = 1\ hence \quad => m = \frac{1}{25}$</td>
</tr>
</table>

20.(ii) (a)	$P\left(X>2/_{X\le 6}\right)=\frac{P(2<X\le 6)}{P(X\le 6)}$ $P(2<X\le 6)=P(X=3)+P(X=4)+P(X=5)+P(X=6)$ $=3m+4m+5m+4m$ $16\times\frac{1}{25}=\frac{16}{25}.$ $P(X\le 6)=P(2<X\le 6)+P(X=2)+P(X=1)$ $=\frac{16}{25}+2m+m$ $\frac{16}{25}+\frac{2}{25}+\frac{1}{25}=\frac{19}{25}$ $P\left(X>2/_{X\le 6}\right)=\frac{P(2<X\le 6)}{P(X\le 6)}$ $=\frac{16}{25}\Big/\frac{19}{25}=\frac{16}{19.}$	20.(ii) (b)	$E(3X-1)=3E(X)-1$ $E(X)=(1\times m)+(2\times 2m)+(3\times 3m)+(4\times 4m)+(5\times 5m)+(6\times 4m)+(7\times 3m)+(8\times 2m)+(9\times m)$ $=\frac{1}{25}+\frac{4}{25}+\frac{9}{25}+\frac{16}{25}+\frac{25}{25}+\frac{24}{25}+\frac{21}{25}+\frac{16}{25}+\frac{9}{25}$ $=\frac{125}{25}$ $=5$ $E(3X-1)=3E(X)-1$ $=(3\times 5)-1$ $=14$
21. (i)	$f(x)=\begin{cases}\frac{1}{4}x, & 0\le x\le a\\ \frac{1}{4}(4-x), & a\le x\le b\\ 0, elsewhere\end{cases}$ $f(a)=\frac{a}{4}\ldots\ldots(i)\ for\ \ 0\le x\le a$ $f(a)=\frac{1}{4}(4-a)\ldots\ldots(ii) for\ \ a\le x\le b$ $equating\ (i) and\ (ii) yeilds$ $\frac{1}{4}(4-a)=\frac{a}{4}=>1-\frac{a}{4}=\frac{a}{4}$ $\frac{a}{2}=1\ hence\ a=2$ $\int_0^2\frac{x}{4}dx+\int_2^b\frac{1}{4}(4-x)dx=1$ $\left.\frac{x^2}{8}\right\|_0^2+\frac{1}{4}\left.(4x-\frac{x^2}{2})\right\|_2^b=1$ $\frac{2^2}{8}-\frac{0^2}{8}+\frac{1}{4}\left[\left(4b-\frac{b^2}{2}\right)-\left(4(2)-\frac{4}{2}\right)\right]=1$	(iii) (iv)	$P\left(\frac{1}{2}\le X\le 2\frac{1}{2}\right)=\int_{0.5}^{2}\frac{1}{4}xdx+\int_{2}^{2.5}\frac{1}{4}(4-x)dx$ $\left.\frac{x^2}{8}\right\|_{0.5}^{2}+\frac{1}{4}\left.(4x-\frac{x^2}{2})\right\|_2^{2.5}$ $\frac{2^2}{8}-\frac{0.5^2}{8}+\frac{1}{4}\left[\left(4(2.5)-\frac{(2.5)^2}{2}\right)-\left(4(2)-\frac{4}{2}\right)\right]$ $\frac{15}{32}+\frac{7}{32}$ $=\frac{11}{16}$ $E(X)=x\int f(x)dx$ $\int_0^2\frac{x^2}{4}dx+\int_2^4\frac{1}{4}(4x-x^2)dx$ $\left.\frac{x^3}{12}\right\|_0^2+\frac{1}{4}\left.(2x^2-\frac{x^3}{3})\right\|_2^4$ $\frac{2^3}{12}-\frac{0^2}{12}+\frac{1}{4}\left[\left(2(4)^2-\frac{4^3}{2}\right)-\left(2(2^2)-\frac{2^3}{2}\right)\right]$ $=\frac{2}{3}+\frac{4}{3}$

	$\frac{1}{2}+\frac{1}{4}\left(4b-\frac{b^2}{2}\right)-6=1$ $b-\frac{b^2}{8}=2 \implies b^2-8b+16=0$ $(b-4)^2=0$ $hence\ b=4.$  		$E(X)=2$ $E(X^2)=x^2\int f(x)dx$ $=\int_0^2 \frac{x^3}{4}dx+\int_2^4 \frac{1}{4}(4x^2-x^3)dx$ $\frac{x^4}{16}\Big\|_0^2+\frac{1}{4}(\frac{4}{3}x^3-\frac{x^4}{4})\Big\|_2^4$ $\frac{2^4}{16}-\frac{0^4}{16}+\frac{1}{4}\left[\left(\frac{4}{3}(4)^3-\frac{4^4}{4}\right)-\left(\frac{4}{3}(2^3)-\frac{2^4}{4}\right)\right]$ $=1+\frac{11}{3}$ $=4\frac{2}{3}$ $var(x)=E(X^2)-(E(X))^2$ $=4\frac{2}{3}-2^2$ $=\frac{2}{3}$

END