BE TRUE

TRINITY COLLEGE NABBINGO
SOLUTIONS FOR 'A' LEVEL MATHEMATICS SEMINAR QUESTIONS ON SATURDAY
5th OCTOBER 2024

ALGEBRA

1. (a) (i)

$$x = \sqrt[3]{P} + \frac{1}{\sqrt[3]{P}}$$
$$x^3 = \left(\sqrt[3]{P} + \frac{1}{\sqrt[3]{P}}\right)^3$$
$$= \left(\sqrt[3]{P}\right)^3 + 3\left(\sqrt[3]{P}\right)^2\left(\frac{1}{\sqrt[3]{P}}\right) + 3\left(\sqrt[3]{P}\right)\left(\frac{1}{\sqrt[3]{P}}\right)^2 + \left(\frac{1}{\sqrt[3]{P}}\right)^3$$
$$= P + 3\sqrt[3]{P} + 3\left(\frac{1}{\sqrt[3]{P}}\right) + \frac{1}{P}$$

$$y^2 = \left(\sqrt{P} + \frac{1}{\sqrt{P}}\right)^2$$
$$= P + 2 + \frac{1}{P} + 3\left(\sqrt[3]{P} + \frac{1}{\sqrt[3]{P}}\right) - 3\left(\sqrt[3]{P} + \frac{1}{\sqrt[3]{P}}\right)$$
$$= P + 3\left(\sqrt[3]{P} + \frac{1}{\sqrt[3]{P}}\right) + \frac{1}{P} - 3\left(\sqrt[3]{P} + \frac{1}{\sqrt[3]{P}}\right) + 2$$
$$y^2 = x^3 - 3x + 2$$
$$y^2 = x(x^2 - 3) + 2$$

(ii)

$\log_5 21 = m$
$21 = 5^m$
$7 + 3 = 5^m$ ---- *

$\log_9 75 = n$

$75 = 9^n$

$25 \times 3 = 3^{2n}$

$25 = 3^{2n-1}$

$5^2 = 3^{2n-1}$

$5 = 3^{\frac{2n-1}{2}}$

Or

$3 = 5^{\frac{2}{2n-1}}$ substituting for 3 in *
$7\ 5^{\frac{2}{2n-1}} = 5^m$

$= \qquad m - \left(\frac{2}{2n-1}\right)$

$7 = \frac{5^m}{5^{\frac{2}{2n-1}}} = 5$

$7 \quad = \quad 5^{\frac{2mn-m-2}{2n-1}}$

Introducing $\log_5 5$ on both side

$\log_5 7 = \log_5 5^{\frac{2mn-m-2}{2n-1}}$

$\log_5 7 = \frac{2mn-m-2}{2n-1}$

Or $\log_5 7 = \frac{1}{2n-1}(2mn - m - 2)$

(c) $p^3 + q^3 = 4,\ pq = \frac{1}{2}(p^3 + q^3) + 1 = \frac{1}{2}(4) + 1$=3

Roots, $\frac{1}{p^6}\ and\ \frac{1}{q^6}$.

Sum of roots $\frac{1}{p^6} + \frac{1}{q^6} = \frac{p^6+q^6}{(qp)^6} = \frac{(p^3+q^3)^2-2(pq)^3}{(pq)^3}$

= $\frac{(4)^2-2(3)^2}{(3)^6} = \frac{-38}{729}$

Product of roots $\frac{1}{p^6}.\frac{1}{q^6} = \frac{1}{(pq)^6} = \frac{1}{3^6} = \frac{1}{729}$

Equation: $x^2 - (sun\ of\ roots)\,x + pd += 0$

$x^2 + \frac{38}{729}x + \frac{1}{729} = 0$

$\Longrightarrow 729x^2 + 38x + 1 = 0$

2. (a) Coefficients of term in the expression of;

$(1+x)^{n,}$ coefficients of ;

$T_5 = {}^nC_4,\ T_6 = {}^nC_5,\ \ T_7 = {}^nC_6$

If they are in AP,

Then $T_6 - T_5 = T_7 - T_6$

${}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$

$= \frac{n?}{5?\,(n-5)} - \frac{n?}{4(n-4)} = \frac{n}{6(n-6)} - \frac{n}{5?\,(n-5)}$

$= \frac{1}{120(n-5)} - \frac{1}{24(n-4(n-5))} = \frac{1}{720(n-6)} - \frac{1}{120(n-5)(n-6)}$

$= \frac{1}{24(n-5)}\left(\frac{1}{5} - \frac{1}{n-4}\right) = \frac{1}{120(n-6)}\left(\frac{1}{6} - \frac{1}{n-5}\right)$

$\frac{n-9}{n-4} = \frac{n-11}{6} \Longrightarrow n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0, n = 7,\ \ n = 14$

(b) $(1+x+x^2) = 1 + n(x+x^2) + \frac{n(n-1)}{2?0}(x+x^2)^2 + \ldots\ldots\ldots$

$= 1 + nx + nx^2 + \frac{1}{2}n(n-1)x^2 + \cdots$

For $\left(\frac{1+9x}{1-3ax}\right)^3 = (1+9x)^3(1-3ax)^{-3}$

$= (1 + 3ax + 3a^2x^2 + \cdots)(1 + 9ax + 18a^2x^2 + \cdots)$

$= 1 + 9ax + 549^2x^2 + 3ac + 27a^2x^2 + 3a^2x^2 + \cdots.$

$= 1 + 12ax + 48a^2x^2 + \cdots.$

Comparing the first three terms;

$1 + nx + nx^2 + \frac{1}{2}n(n-1)x^2 = 1 + 12az + 48a^2x^2 + \cdots$

Equating coefficients

$n = 12a, \ldots \ldots (i)$

$84a^2 = n + \frac{n(n-1)}{2} \ldots \ldots \ldots (ii)$

Solving (i) and (ii)

$84a^2 = 12a + 12a\frac{(12a)-1}{2}$

$12a^2 - 6a = 0, \quad a\#0, \ a = {}^1/_2, \quad n = 6.$

3. (a) $\frac{1}{x} = a, \frac{1}{y} = b \ and \frac{1}{z} = c$

$2\left(a + 2b + 2c = \frac{1}{2}\right) \ldots \ldots \ldots \ldots \ldots \ldots (i)$

$4b + 2b + 3c = \frac{2}{3} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (ii)$

$3a + 4b + 4c = \frac{1}{3} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (iii)$

(ii)-(i) $3a + c = \frac{1}{6}$

$2(i) - (iii), a = \frac{-2}{3}, -2 + c = \frac{1}{6}, c = \frac{13}{6} \ sub. in \ eqn(i)$

$\frac{-2}{3} + 2b + \frac{13}{3} = \frac{1}{2}, -2 + 6b + 13 = \frac{3}{2}, 6b + 11 = \frac{-19}{2}$

$$b = \frac{-19}{12}, x = \frac{-3}{2}, y = \frac{-12}{19}, z = \frac{6}{13}$$

(b) $x^2 + 2xy + y^2 = 25, \ (x+y)^2 = 25, \ x + y = 5, x = 5 - y$

$sub. for \ x \ in x^2 - xy + y^2 = 9$

$(5-y)^2 - y(5-y) + y^2 = 9$

$3y^2 - 15y + 16 = 0$

$solving \ for \ x \ and \ y, x = 1.5426, 3.4574 \ and \ y = 3.4574, 1.5426.$

4. (a) Let $a_n = 8^n - 7n + 6$

For $n = 1, a = 8 - 7 + 6 = 7, divisable \ by \ 7$

For $n = 2$

$a_2 = 64 - 14 + 6 = 56\ divisble\ by\ 7$

For $n = k$ $\quad a_k = 8^k - 7k + 6$

When $n = k + 1,\ a_{k+1} = 8^{k+1} - 7(k + 1) + 6$

Thus a_{k+1} __ $a_k = 8^{k+1} - 7(k + 1) + 6 - (8^k + 7k + 6)$

$= 8^k.8 - 8^k - 7k + 7k - 7 + 6 - 6$
$= 7.8^k - 7 = 7(8^k - 1)$

Which is divisible by 7. If it is true for $n = 1, 2, \ldots \ldots k, k + 1$, then $8^n - 7n + 6$ is divisible by 7 for all $n = 1$.

(b) For $n = 1$

$LHS = sin\theta$

$RHS = \frac{sin^2\theta}{sin\theta} = sin\theta$

Since LHS = RHS, hence it holds

For $n = 1$

For $n = 2$

LHS = $sin\theta + sin3\theta = 2sin2\theta cos\theta$

$= 4sin\theta cos^2\theta$

RHS= $\frac{sin^2\theta cos^2\theta}{sin\theta}$

$= 4sin\theta cos^2\theta$

RHS = LHS. Hence it holds for $n = 2$

Assume it holds for $n = k$

$sin\theta + sin3\theta + sin5\theta + \cdots sin(2k)\theta = \frac{sin^2 k\theta}{sin\theta}$

For $n = 2k + 1,$

LHS $sin^2 \frac{k\theta}{sin\theta} + sin(2(k + 1) - 1)\theta$

$\frac{sin^2 k\theta}{sin\theta} + sin(2k + 1)\theta$

$$\frac{sin^2 k\theta + sin(2k+1)\theta sin\theta}{sin\theta}$$

$$\frac{\frac{1}{2}(1-cos^2 k\theta)-\frac{1}{2}(cos2(k+1)\theta - cos2k\theta)}{sin\theta}$$

$$= \frac{\frac{1}{2}(1-cos2(k+1)\theta)}{sin\theta}$$

$$= \frac{\frac{1}{2}(2sin^2(k+1)\theta)}{sin\theta}$$

$= \frac{sin^2(k+1)\theta}{sin\theta}$ Hence holds for $n = k+1$

4. (a) $l0g_{30}^{5} = q, log_{30}^{\left(\frac{30}{6}\right)} = q, l0g_{30}^{30} - log_{30}^{6} = q, 1 - log_{30}^{6} = q, log_{30}^{6} = 1 - q$

$l0g_{30}^{(2x3)}$=1-q, $log_{30}^{2} + log_{30}^{3} = 1 - q, log_{30}^{2} + p = 1 - q$, $log_{30}^{2} = 1 - q - p$

$log_{30}^{32} = log_{30}^{2^5}$

=$5log_{30}^{2}$=$5(1-q-p)$

(b) $P(x) = (x^2 - a)Qx + Rx$

$P(x) = (x+3)(x-3)Qx + ax + b$

$Rx \quad = ax + b$ $\qquad x = 3$

$R(3) = 3a + b = 0$ $\qquad 3a + b = 0 \ldots\ldots(i)$

$R(-3) = -3a + b = 12$ $\qquad -3a + b = 12 \ldots.(ii)$

$6a = -12 \qquad a = -6$

$18 + b = 12 \qquad b = 6$

$R(x) = -6 + 6$

6. (a) $\frac{(2x+y)^{2/3}(2x-y)^{-1/2}}{(4x^2-y)^{1/6}} = (2x+y)^p(2x-y)^q$

$(2x+y)^{1/2}(2x-y)^{-2/3} = (2x+y)^p(2x-y)^q$

$\Rightarrow p = 1/2, \quad q = -2/3$

(b) $\log_2 x + \log_2 y = 3 \quad \Rightarrow \log_2 xy = 3$

$xy = 8 \ldots \ldots \ldots .(i)$

$\log_4 x - \log_4 y = {}^{-1}/_2 \quad {}^{x}/_{y} = 4^{\,-1}/_2$

$\frac{x}{y} = \frac{1}{2} \qquad y = 2x \ldots \ldots \ldots (ii)$

Solving (i) and (ii) $\qquad x + 2, \quad y + 4$

(c) $\log_{10} 2 = a, \qquad 2 = 10^a$

$\log_8 2 = a \log_8 10 \implies \log_8 2 \, \frac{(1-a)}{a}$

$\log_8 5 = \frac{\log_3 2}{3 \log_3 2}\left(\frac{1-a}{a}\right) = \frac{1-a}{3a}$

8 (a) αβ=q , α+β=-p

(i) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= -p^3 + 3p = 3pq - p^3$

$(ii) \quad (\alpha - \beta^2)(\beta - \alpha^2) = \alpha\beta - (\alpha^3 + \beta^3) + \alpha^2\beta^2$

$= \alpha\beta - \{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)\} + (\alpha\beta)^2$

$= q - \{-p^3 - 3q(-p)\} + q^2$

$= p^3 + q^2 - 3pq + q$

$When\ \alpha = \beta^2 or\ \alpha^2 = \beta\ then;$

$(\beta^2 - \beta^2)(\alpha^2 - \alpha^2) = p^3 - 3pq + q^2$

$\Rightarrow p^3 - 3pq + q^2 = 0$

(b) $Let\ P = 120\ (starting\ number\ of\ goats).$

At the start of 2024, No of goats=P-N

At the end of 2024,No of goats=(P-N)1.2

At the start of 2025,No of goats =(P-N)1.2-N

At the end of 2025 No of goats $=\{(P-N)1.2 - N\}1.2$

$= P(1.2)^2 - N(1.2)^2 - N(1.2)$

After n years the number of goats will be;

$p(1.2)^n - N(1.2)^n - N(1.2)^{n-1} + \cdots \ldots \ldots \ldots + N(1.2)$

After 5 years

$120 = 120(1.2)^5 - N(1.2)^5 - N(1.2)^4 + \cdots \ldots \ldots \ldots \ldots . + N(1.2)$

$= 120(1.2)^5 - N(1.2^5 + 1.2^4 + \cdots \ldots + 1.2)$

$120 = 120(1.2)^5 - N(1.2)\left(\frac{1.2^5 - 1}{1.2 - 1}\right)$

120=298.5984-N (8.92992)

$N = \frac{178.5984}{8.92992}$ =20 goats.

TRIGONOMETRY

21. (a) $cosA = \frac{3}{5}$ $\quad cosA = 2cos^2\frac{A}{=2} - 1 = 2.\frac{9}{25} - 1 = \frac{-7}{25}$

$sinA = 2sinAcosA = 2 \times \frac{3}{5} \times \frac{4}{5} = {}^{24}/{}_{25}$

$cosB = tan\frac{B}{2} = \frac{12}{5}, cosB = 1 - tan\frac{B}{2} = \frac{1 - {}^{144}/{}_{25}}{{}^{169}/{}_{25}} = \frac{-119}{169}$

$sinB = \frac{2tan\frac{B}{2}}{1 + tan^2{}^{B}/{}_{2}} = \frac{\frac{2\times 12}{5}}{1 + \frac{144}{25}} = \frac{24}{25}.\frac{25}{169} = \frac{24}{169}$

Sec $(A + B) = \frac{1}{cos(A+B)}$

$cos(A + B) = cosAcosB - sinAsinB$

$= \left(\frac{-7}{25} \times \frac{-119}{169}\right) - \left(\frac{24}{25} \times \frac{24}{169}\right)$

$= \frac{833}{25\times 169} - \frac{576}{25\times 169} = \frac{257}{25\times 169}$

$sec(A + B) = \frac{25\times 169}{257}$

(b) $10sin^2x + 10sinxcosx - cos^2x = 2$

Divide through cos^2x

$$\frac{10sin^2x+10sin\times cosx-cos^2x=2}{cos^2x}=\frac{2}{cos^2x}$$

$$10tan^2x+10tanx-1=2sec^2x$$

$$10tan^2x+10tanx-1=2(1+tan^2x)$$

$$8tan^2x+10tanx-3=0$$

$$tanx=\frac{10\pm\sqrt{10^2-4(8)(-3)}}{2\times 8}$$

$$=\frac{-10\pm 14}{16}$$

$$=\frac{1}{4}\ ,\quad \frac{-3}{2}$$

$$tanx=\frac{1}{4}\qquad \therefore x=tan^{-1}(0.25)=14.0^0$$

$$tanx=-1.5\quad x=tan^{-1}(-1.5)=56.3^0$$

22. (a) $tan2\theta=tan(\theta+\theta)=\frac{tan\theta+tan\theta}{1-tan\theta tan\theta}=\frac{2tan\theta}{1-tan^2\theta}$

$$tan4\theta=tan2(2\theta)=tan4\theta$$

$$=\frac{tan2\theta+tan2\theta}{1-tan^2 2\theta}$$

$$=\frac{\left(\frac{2tan\theta}{1-tan^2 2\theta}\right)+\frac{2tan\theta}{(1-tan^2\theta)}}{1-\left(\frac{tan^2\theta}{1-tan^2\theta}\right)^2}$$

$$=\left(\frac{4tan\theta}{1-tan^2\theta}\right)\frac{\left(1-tan2^{\theta}\right)^2}{(1-tan^2\theta)^2-4tan^2 2\theta}$$

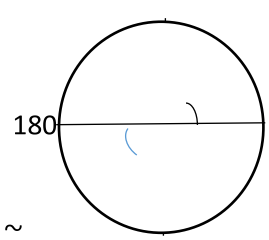
$$=\frac{4tan\theta\left(1-tan^2\theta\right)}{(1-tan^2\theta)^2-4tan^2\theta}$$

$$tan4\theta=\frac{4tan\theta-4tan^3\theta}{1-6tan^2\theta+tan^4\theta}$$

For $t=tan\theta,\ \ and\ \ tan4\theta=1$

We obtain

$$4^4+4t^3-6t^2-4t+1=0$$



$\therefore 4\theta = tan^{-1}(1)$

$4\theta = 45, 225, 405, 585, 765, 945, 112, 1305.$

$\theta = 11.25, 101.25, 146.25, 191.25, 236.25, 281.25, 326.25$

(b) $2tan^{-1}x + tan^{-1}(1/7) = \pi/4$

Let $A = tan^{-1}x,\ tanA = x,\ tan2A = \frac{2x}{1-x^2}$

$B = tan^{-1}(1/7)$, $tanB = 1/7$

$\therefore 2A + B = \pi/4$

$tan(2A + B) = tan(\pi/4) = 1$

$\frac{tan2A+tanB}{1-tan2AtanB=1}$

$tan2A + tanB = 1 - tan2AtanB$

$\frac{2x}{1-x^2} + \frac{1}{7} = 1 - \left(\frac{2x}{1-x^2}\right)$

$\left(\frac{2x}{1-x^2}\right)\left(1 + \frac{1}{7}\right) = 1 - \frac{1}{7}$

$\left(\frac{2x}{1-x^2}\right).\frac{8}{7} = \frac{6}{7}$

$\frac{2x}{1-x^2} = \frac{3}{4}$

$8x = 3 - 3x^2$

$3x^2 + 8 - 3 = 0$

$x = \frac{-8\pm\sqrt{64}-4(3)(-3)}{2\times3} = \frac{-8\pm10}{6} = \frac{1}{3}, -2$

23(a) $2cot\frac{3x}{2} - sin3x = sin3xcot^2\frac{3x}{2}$

$\frac{2cos\frac{3x}{2}}{sin\frac{bx}{2}} - sin3x$

$2\frac{cosx}{2} - 2sin\frac{3x}{2}cos\frac{3x}{2}$

$2cos\frac{3x}{2}\left(\frac{1}{sin\frac{3x}{2}} - sin\frac{3x}{2}\right)$

$2cos\frac{3x}{2}\left(\frac{1-sin^2\frac{3x}{2}}{sin\frac{3x}{2}}\right)$

$2cot\frac{3x}{2}.cos^2\frac{3x}{2}.$

$2cot\frac{3x}{2}.cos\frac{3x}{2}.cos\frac{3x}{2}.\frac{sin\frac{3x}{2}}{sin\frac{3x}{2}}$

$cot\left(\frac{3x}{2}\right).cot\left(\frac{3x}{2}\right).2cos\frac{3x}{2}sin\frac{3x}{2}$

$cot^2\frac{3x}{2}.sin3x$

24. $8cos3\theta cos2\theta cos\theta - 1 = \frac{sin7\theta}{sin\theta}$

Soln:

LHS. $cos3\theta cos2\theta cos\theta - \frac{sin\theta}{sin\theta}$

$\frac{8cos3\theta cos2\theta cos\theta sin\theta - sin\theta}{sin\theta}$

$\frac{4cos3\theta cos2\theta . sin2\theta - sin\theta}{sin\theta}$

$\frac{2cos3\theta . 2sin2\theta cos2\theta}{sin\theta}$

$\frac{2cos3\theta sin4\theta - sin\theta}{sin\theta}$

Using factor $P + Q = 6\theta \ldots\ldots\ldots (i)$

$P - Q = 8\theta \ldots\ldots\ldots (ii)$

$2P = 14\theta$

$P = 7\theta$

Subtracting

$2Q = -2\theta$

$Q = -\theta$

$\therefore 2cos3\theta sin4\theta = sin7\theta - sin(-\theta) = sin7\theta + sin\theta$

$\therefore \frac{sin7\theta + sin\theta - sin\theta}{sin\theta} = \frac{sin7\theta}{sin\theta}$

24.

(b) $sec\theta - tan\theta = cos\frac{\theta}{2}$. Applying double angle

$\frac{1}{cos\theta} - \frac{sin\theta}{cos\theta}$

$\frac{1 - sin\theta}{cos\theta} = \frac{cos^2\frac{\theta}{2} + sin^2\frac{\theta}{2} - 2\frac{\theta}{2}cos\frac{\theta}{2}}{cos^2\frac{\theta}{2} - sin^2\frac{\theta}{2}}$

$= \frac{COS^2\frac{\theta}{2} - 2sin\frac{\theta}{2}cos\frac{\theta}{2} + sin^2\frac{\theta}{2}}{cos^2\frac{\theta}{2} - sin^2\frac{\theta}{2}}$

$= \frac{\left(cos\frac{\theta}{2} - sin\frac{\theta}{2}\right)^2}{\left(cos\frac{\theta}{2} + sin\frac{\theta}{2}\right)\left(cos\frac{\theta}{2} - sin\frac{\theta}{2}\right)}$

$= \frac{COS\frac{\theta}{2} - sin\ \frac{\theta}{2}}{cos\frac{\theta}{2} + sin\frac{\theta}{2}}$

25. (a) $42sin\theta + 40cos\theta$ in form $psin(\theta + \alpha)$

Solution

$42sin\theta + 40cos\theta \equiv Pcos\alpha + Pcos\alpha sin$

$\equiv Pcos\alpha sin\theta + Psin\alpha cos\theta$

$Pcos\alpha + 42 \ldots \ldots \ldots (i)$

$Psin\alpha = 40 \ldots \ldots (ii)$

Equation 1 and 2 squared and added;

$P^2cos^2\alpha + P^2sin\alpha = 42^2 + 40^2$

$P^2(cos^2\alpha + sin^2\alpha) = 3364$

$P = 58$

$Equation\ (2) \div Equation(1)$

$\frac{Psin\alpha}{Pcos^2} = \frac{40}{4^2}$

$\therefore 42sin\theta + 40cos\theta \equiv 58sin\theta(\theta + 43.6^0)$

Maximum:$\frac{1}{58sin(\theta \times 43.6)+8^2} \quad = \frac{1}{82-58} = \frac{1}{24} = 0.042$

$sin(\theta + 43.6) = 1$

$\theta + 43.6 = sin^{-1} = 270$

$\theta = 226.4^0 \Longrightarrow (0.042\ at\ 226.4^0)_{max}$

Maximum:$\frac{1}{58sin(\theta+43.6)+8^2} = 0.0071$

$sin(\theta + 43.6) = 1$

$\theta + 43.6 = sin^{-1}(1) = 90$

$\theta = 46.4^0 \qquad \Longrightarrow (0.0071\ at\ 46.4^0)min$

(b) $sin^2A - sin^2B = sin(A + B)sin(A - B)$

$\Longrightarrow Difference\ of\ two\ squares\ \&factor\ formula$

$\big(sin(A) + sin(B)\big)(sinA - sinB)$

$2sin\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right).2cos\left(\frac{A+B}{2}\right)sin\frac{(A-B)}{2}$

$2sin\frac{(A+B)}{2}cos\left(\frac{A+B}{2}\right).2sin\left(\frac{A-B}{2}\right)cos\left(\frac{A-B}{2}\right)$

$sin(A+B).sin(A-B)$

26. (a) If $tax = ktan(A-x)$ then $sin(2x-A) = \left(\frac{k-A}{k+1}\right)sinA$

$\frac{sinx}{cosx} = \frac{ksin(A-x)}{cos(A-x)}$

$\therefore sinxcos(A-x) = ksin(A-x)cosx$

$sinxcos(A-x) = kcosx\ sin(A-x)$

Applying factorise formula

$P+\theta = 2x$

$P-Q = 2(A-x)$

Adding: $P = A$

Subtracting $2Q = 2x - 2(A-x)$

$$Q = \frac{2}{2}(2x-A) = (2x-A)$$

LHS

$\frac{1}{2}sin(2x-A) + sin(2x-A) = \frac{k}{2}sinA - \frac{1}{2}sinA$

$\frac{1}{2}sin(2x-A) + \frac{k}{2}sin(2x-A) = \frac{k}{2}sinA - \frac{1}{2}sinA$

$(k+1)sin(2x-A) = (k-1)sinA$

$\therefore sin(2x-A) = \left(\frac{k-1}{k+1}\right)sinA$

(ii) For $2tanx - tan(30-x)$

$\therefore A = 30, \quad k = \frac{1}{2}$

$\therefore sin(2x-30) = \frac{-1/2}{3/2}sin30 = \frac{1}{3}\times\frac{1}{2} = \frac{-1}{6}$

$\therefore sin(2x-30) = {}^{-1}/_{6}$

(b) $\Delta|e\ PQR$

$\frac{1}{P}cos^2\frac{P}{2} + \frac{1}{q}cos^2\frac{Q}{2} + \frac{1}{r} + cos^2\frac{R}{2} = \frac{(p+q+r)^2}{4pqr}$

Soln.

$cos^2\frac{p}{2} = \frac{s(s-p)}{qr}, \qquad cos\frac{Q}{2} = \sqrt{\frac{s(s-q)}{pr}}, \quad cos^2\frac{R}{2} = \sqrt{\frac{s(s-r)}{qr}}$

$\frac{1}{p}\frac{(s(s-p))}{qr} + \frac{s(s-q)}{pqr} + \frac{s(s-r)}{pqr}$

$= \frac{s^2}{pqr}(s-p+s-q+s-r) = \frac{s}{pqr}\left(3s-(p+q+r)\right)$

$= \frac{s^2}{pqr}$

$where\ \ S = \frac{(p+q+r)}{2}$

$\therefore \frac{s^2}{pqr} = \frac{(p+q+r)^2}{4pqr}$

27. (a) $tan(\theta+60^0)tan(\theta-60^0) = \frac{tan^2\theta-3}{1-3tan^2\theta}$

$tan(\theta-60^0) = \frac{tan\theta+tan60}{1-tan\theta tan60}$

$tan(\theta-60) = \frac{tan\theta-tan60}{1+tan\theta tan60}$

But $tan60 = \sqrt{3}$

Then

$tan(\theta+60).tan(\theta-60) = \frac{(tan\theta+\sqrt{3})}{(1-tan\theta tan60)}.\frac{tan\theta-\sqrt{3}}{\left(1+tan\theta\sqrt{3}tan\theta\right)}$

$= \frac{tan\theta+\sqrt{3}}{1-\sqrt{3}tan\theta}. \qquad \frac{tan\theta-\sqrt{3}}{1+\sqrt{3}tan\theta}$

$= \frac{tan^2-3}{1-3tan^2\theta}$

$\Longrightarrow \frac{tan^2\theta-3}{1-3tan^2\theta} = 4sec^2-3$

$\frac{tan^2\theta-3}{1-3tan^2\theta} = 4(1+tan^2\theta)-3$

$tan^2(\theta-3) = 1+4tan^2\theta-3tan^2\theta-tan^4\theta$

$-12tan^4\theta = -4$

$tan^4\theta = \frac{1}{3} \qquad tan\theta = \sqrt[4]{\frac{1}{3}}$

$$tan^2 = \frac{+b}{-\sqrt{3}}$$

$= -2$

$$s_2 = \frac{2(-2) + 3(10) + 6(60) - 20}{\sqrt{(2^2 + 3^2 + 2)}} = \frac{12}{7}$$

Since s_1 ands_2 have different signs, hence points A and B are on opposite sides of the plane.

28. (a) $\boldsymbol{AB} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$

$\boldsymbol{AC} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -2 \\ 4 \end{pmatrix}$

$Normal, n = \boldsymbol{AB} \times \boldsymbol{AC}$

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -3 & \boldsymbol{0} & 2 \\ \boldsymbol{-7} & -2 & 4 \end{vmatrix}$$

$= 4\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$

$= 2(2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k})$

$\therefore n = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$

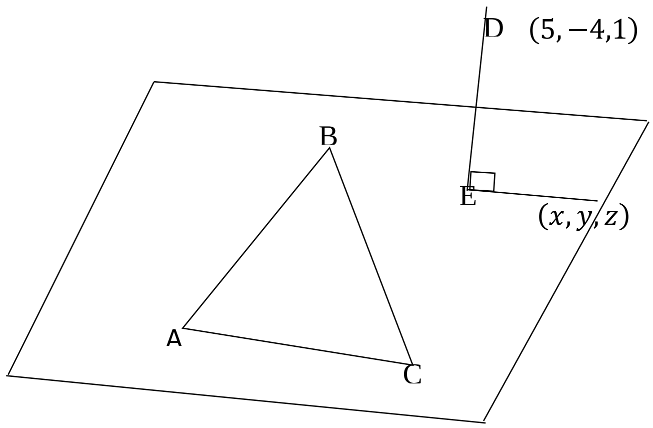
Equation of the plane $= r.n = n.a$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

$2x - y + 3z = 8 - 2 - 3$

$2x - y + 3z = 3$

(b)



Equation of the line DE

$\boldsymbol{r} = \boldsymbol{a} + \boldsymbol{d}$ $\qquad\qquad x = 5 + 2\lambda$

$= \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ $\qquad y = -4 - \lambda$

$z = 1 + 3\lambda$

Points of intersection of plane and line

$2(5 + 2\lambda) - (-4 - \lambda) + 3(1 + 3\lambda) = 3$

$10 + 4\lambda + 4 + \lambda + 3 + a\lambda = 3$

$14\lambda = -14$

$\lambda = -1$

$x = 3$

$y = -3$

$z = -2$

$\therefore E = (3\boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k})$

Alternatively

$ED = \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} - \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$= \begin{pmatrix} 5 & -x \\ -4 & -y \\ 1 & -z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

Where $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = n$

$5 - x = 2 \qquad -y = -1 + 4 \qquad -z = 2$

$x = 3 \qquad y = -3 \qquad z = -2$

(c) Vector equation of the straight line D and E

$DE = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$\boldsymbol{c} = (5\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}) + \lambda(2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k})$

OR

$\boldsymbol{r} = 3\boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k} + \lambda(2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k})$

29. (a) $\boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \qquad \boldsymbol{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

$\lambda a + \mu b$

$= \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

$c = \begin{pmatrix} \lambda & +2\mu \\ 2\lambda & +\mu \\ \lambda & -\mu \end{pmatrix}$

$c.a = 0$

$\begin{pmatrix} \lambda & +2\mu \\ 2\lambda & +\mu \\ \lambda & -\mu \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 0$

$\lambda + 2\mu + 4\lambda + 2\mu + \lambda - \mu = 0$

$6\lambda = -3\mu$

$\frac{\lambda}{\mu} = \frac{-1}{2}$

$-1:2$

(b) $r_1 = r_2$

$$r_1 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad r_2 = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Equating $r_1 = r_2$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 3+\lambda \\ 1+2\lambda \\ -1+3\lambda \end{pmatrix} \qquad 1 \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 2+\mu \\ 5-\mu \\ \mu \end{pmatrix}$$

$$\begin{pmatrix} 3+\lambda \\ 1+2\lambda \\ -1+3\lambda \end{pmatrix} = \begin{pmatrix} 2+\mu \\ 5-\mu \\ \mu \end{pmatrix}$$

$3 + 2\lambda + \mu \ldots\ldots\ldots..(i)$

$1 + 2\lambda = 5 - \mu \ldots\ldots..(ii)$

$-1 + 3\lambda = \mu \ldots\ldots\ldots\ldots.(iii)$

Substitute (iii) in (ii)

$1 + 2\lambda = 5 - (3\lambda - 1)$ $\qquad$ $\mu = 3\lambda - 1$

$1 + 2\lambda = 5 - 3\lambda + 1$ $\qquad$ $\mu = 3 - 1$

$5\lambda = 5$ $\qquad$ $\mu = 2$

$\lambda = 1$

$$\begin{pmatrix} 3+1 \\ 1+2 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 2+2 \\ 5-2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$$

Since $x_1 = x_2,\ y_1 = y_2,\ z_1 = z_2$ then the two lines r_1 and r_2 intersect.

(b) (ii)

$n = d_1 \times d_2$

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & k \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix}$$

$n = 5\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$

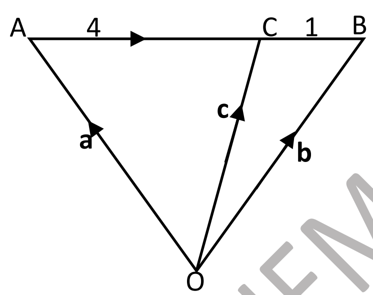
$r.n = n.a$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} . \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$5x + 2y - 3z = 15 + 2 + 3$

$5x + 2y - 3z = 20$

30. (a)



$OC = OA + AC$

$$= \boldsymbol{a} + \frac{4}{5}AB$$

$$= \boldsymbol{a} + \frac{4}{5}(OB - OA)$$

$$= \boldsymbol{a} + \frac{4}{5}(\boldsymbol{b} - \boldsymbol{a})$$

$$= \frac{1}{5}\boldsymbol{a} + \frac{4}{5}\boldsymbol{b}$$

$$= OC = \frac{a + 4b}{5}$$

Alternatively

Using ratio theorem;

$$OC = \frac{\lambda\boldsymbol{b} + \mu\boldsymbol{a}}{\lambda + \mu}$$

$$= \frac{4\boldsymbol{b} + \boldsymbol{a}}{4 + 1}$$

$$= \frac{4\boldsymbol{b} + \boldsymbol{a}}{5}$$

Substituting for $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$OC = \frac{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + 4\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}}{5}$$

$$= \frac{\begin{pmatrix} 6 \\ -4 \\ 13 \end{pmatrix}}{5}$$

$= \frac{6}{5}\boldsymbol{i} - \frac{4}{5}\boldsymbol{j} + \frac{13}{5}\boldsymbol{k}$

(b) $r = \begin{pmatrix}1\\2\\3\end{pmatrix} + \mu\begin{pmatrix}3\\-1\\2\end{pmatrix}$

$\vec{R} = \begin{pmatrix}1+3\mu\\2-\mu\\3+2\mu\end{pmatrix}$

$\overrightarrow{AR} = \overrightarrow{OR} - \overrightarrow{OA}$

$= \begin{pmatrix}1+3\mu\\2-\mu\\3+2\mu\end{pmatrix} - \begin{pmatrix}4\\-3\\10\end{pmatrix}$

$\overrightarrow{AR} = \begin{pmatrix}3\mu-3\\5-\mu\\2\mu-7\end{pmatrix}$

$\overrightarrow{AR}.\boldsymbol{d} = 0$

$\begin{pmatrix}3\mu-3\\5-\mu\\2\mu-7\end{pmatrix}.\begin{pmatrix}3\\-1\\2\end{pmatrix} = 0$

$9\mu - 9 - 5 + \mu + 4\mu - 14 = 0$

$14\mu - 28 = 0$

$\mu = 2$

$\overrightarrow{AR} = \begin{pmatrix}6-3\\5-2\\4-7\end{pmatrix}$

$= \begin{pmatrix}3\\3\\-3\end{pmatrix}$

$|\overrightarrow{AR}| = \sqrt{3^2 + 3^2 + (-3)^2}$

$= 5.1962\ units$

(c) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

$\overrightarrow{AB} = \begin{pmatrix}3\\3\\4\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}2\\1\\1\end{pmatrix}$

$$\overrightarrow{BC} = \begin{pmatrix}4\\6\\6\end{pmatrix} - \begin{pmatrix}3\\3\\4\end{pmatrix} = \begin{pmatrix}1\\3\\2\end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}4\\6\\6\end{pmatrix} = \begin{pmatrix}-3\\-4\\-3\end{pmatrix}$$

$$\begin{pmatrix}2\\1\\1\end{pmatrix} + \begin{pmatrix}1\\3\\2\end{pmatrix} + \begin{pmatrix}-3\\-4\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$< CAB$
$\overrightarrow{AB}\ .\ \overrightarrow{AC} = |\overrightarrow{AB}||\overrightarrow{AC}| \cos \boldsymbol{a}$
$6 + 4 + 3 = (16)(134) \cos \boldsymbol{a}$

$$a = cos^{-1}\left(\frac{13}{\sqrt{(6\times 34)}}\right)$$

$$a = 24.47^0$$

Since there exists an angle and the total displacement is O, the points A, B and C are vertices of a triangle.

31. (a) The vectors on plane r_1 are $\begin{pmatrix}1\\-2\\1\end{pmatrix}$ and $\begin{pmatrix}0\\3\\2\end{pmatrix}$ normal to plane r_1 ;

$$\left|\begin{pmatrix} i & -j & k \\ 1 & -2 & 1 \\ 0 & 3 & 2 \end{pmatrix}\right|$$

$n_1 = 7\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$
Cartesian equation to the plane r_1;
$r.n = n.a$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}-7\\-2\\3\end{pmatrix} = \begin{pmatrix}-7\\-2\\3\end{pmatrix}.\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$-7x - 2y + 3z = -7 + 3$
$7x + 2y - 3z = 4$

The vectors on plane r_2 are; $\begin{vmatrix} i & -j & k \\ 0 & 0 & 5 \\ -2 & 4 & 3 \end{vmatrix}$

$n_2 = -20\boldsymbol{i} - 10\boldsymbol{j}$
$n_2 = 10(-2\boldsymbol{i} - \boldsymbol{j})$
$n_2 = -10(2\boldsymbol{i} + \boldsymbol{j})$
$n_2 = 2\boldsymbol{i} + \boldsymbol{j}$

Cartesian equation to the plane $r_{2;}$

$r.n = n.a$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$2x + y = 5$

Angle between the two planes;

$n_1 . n_2 = In_1IIn_2Icos\theta$

$$\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \sqrt{7^2 + 2^2 + (-3)^2} \ . \quad \sqrt{2^2 + 1^2} cos\theta$$

$16 = \sqrt{62} \ . \sqrt{5}\, cos\theta$

$\theta = cos^{-1} \left(\frac{16}{\sqrt{62 \times 5}}\right)$

$\theta = 24.67^0$

$\therefore cos 24.67^0 = 0.9087$

(b) Line of intersection of the panes

$7x + 2y - 3z = 4 \dots \dots \dots \dots \dots . (1)$

$2x + y = 5 \dots \dots \dots \dots \dots \dots \dots \dots (2)$

Eliminating y

$(1) \times 1 \qquad 14x + 4y - 6z = 8$

$(2) \times 7 \qquad \underline{14x + 7y - 0 = 35}$

$-3y - 6z = -27$

$\frac{6x}{6} = \frac{27 - 3y}{6}$

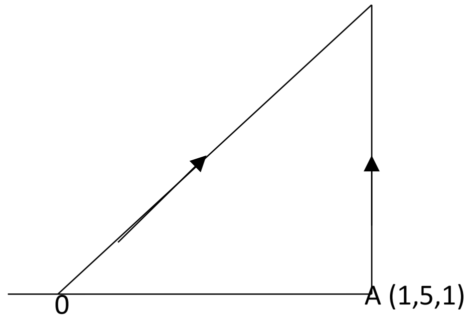
$z = \frac{9 - y}{2}$

$\frac{x+1}{1} = \frac{9-y}{2} = \frac{z}{1}$

$r = -2i + 9j + \lambda(2j + k)$

(c) $r = -2i + 9 + \lambda(i + 2j + k)$

$R(-2 + \lambda, 9 - 2x, \lambda)$



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$$AR = \begin{pmatrix} -2+\lambda \\ 9-2\lambda \\ \lambda \end{pmatrix} - \begin{pmatrix} - \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -3+\lambda \\ 4-2\lambda \\ \lambda-1 \end{pmatrix}$$

$AR.(\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}) = 0 \qquad -3 + \lambda 8 + 4\lambda + \lambda - 1 = 0$

$$\boldsymbol{AR} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \quad |\boldsymbol{AR}| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}\, units$$

32. (a) $\boldsymbol{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$$AC = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$$

Normal $n = \begin{vmatrix} \boldsymbol{i} & -\boldsymbol{j} & k \\ 1 & -2 & 1 \\ -2 & 1 & -4 \end{vmatrix}$

$= 7\boldsymbol{i} + 6\boldsymbol{j} + 5\boldsymbol{k}$

Equation $r.n = n.a$

$(x\boldsymbol{i} + y\boldsymbol{j} + 2\boldsymbol{k}).(> \boldsymbol{i} + 6\boldsymbol{j} + 5\boldsymbol{k}) = (7\boldsymbol{i} + 6\boldsymbol{j} + 5\boldsymbol{k}).(\boldsymbol{i} + 5 + 2\boldsymbol{k})$

$7x + 6y + 2z = 7 + 6 + 10$

$7x + 6y + 2z = 23$

(b) Equation of line $r_1 = -13i + j + 2k + \lambda(12i + 6j + 3k)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -13 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 6 \\ 3 \end{pmatrix}$$

$x = -13 + 12\lambda$

$y = 1 + 6\lambda$

$z = 2 + 3\lambda$

Substituting for x, y, z in the plane.

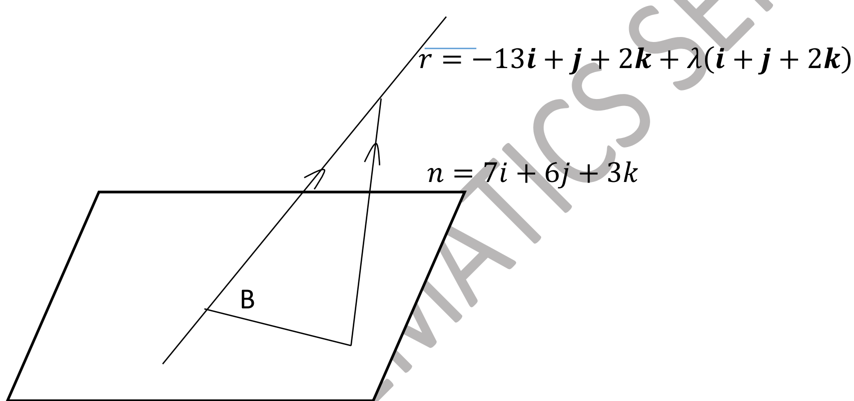
$7(-13+12\lambda)+6(1+6\lambda)+2(2-3\lambda)=23$
$-91+84\lambda+6+36\lambda+4+6\lambda=23$
$-81+126\lambda=23$
$126\lambda=104\ ,\quad \lambda=\frac{104}{126}=\frac{52}{63}$

$x=-13+12\times\frac{52}{63}=\,^{65}/_{21},\ \ y=1+6\times\frac{52}{63}=\frac{125}{21}.$

$z=2\times 3.\frac{52}{63}=\frac{146}{21}$

Coordinate of $E\left(\frac{65}{21}\ ,\ \ \frac{125}{21}\ ,\ \ \frac{146}{21}\right)$

(ii)



$(i+j+2k),(7i+6j+3k)=\sqrt{(1)^2+(1)^2+(2)^2}.\sqrt{7^2}+6^2+3^2 sinB$
$7+6+6=\sqrt{1+1+4}.\sqrt{49+36+9}sinB$
$19=\sqrt{26}.\sqrt{94}sinB$
$sinB\ =\frac{19}{\sqrt{6}\sqrt{94}}=\frac{19}{23.7487}$
$B\ =sin^{-1}\left(\frac{19}{23.7487}\right)=53.13^0$

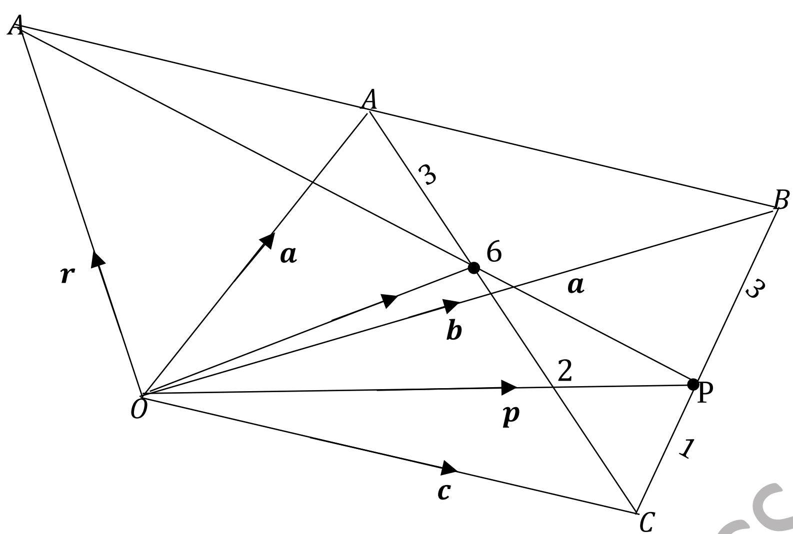
33. (a) Let the displacement of A from the plane be s_1 and B from the plane be s_2.

$S_1=\frac{2(2)+3(7)+6(-5)-20}{\sqrt{(2^2+3^2+6^2)}}=\frac{12}{7}$

$=-2$

$$s_2=\frac{2(-2)+3(10)+6(60)-20}{\sqrt{(2^2+3^2+2)}}=\frac{12}{7}$$

Since s_1 ands_2 have different signs, hence points A and B are on opposite sides of the plane.



$$\boldsymbol{QP} = \boldsymbol{OC} + \boldsymbol{CP}$$

$$= \boldsymbol{OC} + \frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{CB}$$

$$= \boldsymbol{OC} + \frac{\mathbf{1}}{\mathbf{4}}(\boldsymbol{OB} - \boldsymbol{OC})$$

$$=\boldsymbol{c} + \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{b} - \boldsymbol{c}) = \frac{\mathbf{1}}{\mathbf{4}}(\boldsymbol{b} + 3\boldsymbol{c})$$

$$\boldsymbol{OQ} = \boldsymbol{OA} + \boldsymbol{AO}$$

$$= \boldsymbol{OA} + {}^{3}/_{5}\, AC$$

$$= \boldsymbol{OA} + \frac{3}{5}\boldsymbol{OC} - {}^{-3}/_{5}\, \boldsymbol{a}$$

$$= \frac{\mathbf{1}}{\mathbf{5}}(2\boldsymbol{a} + 3\boldsymbol{c})$$

Let $\quad \boldsymbol{RB} = t\ \ \boldsymbol{AB}$

$$PR = kPQ$$

$$\boldsymbol{PQ} = \boldsymbol{PC} = \boldsymbol{CO} + \boldsymbol{OQ}$$

$$= -\boldsymbol{OP} + \boldsymbol{OQ} = \boldsymbol{OQ} - \boldsymbol{OP}$$

$$= \frac{1}{5}(2\boldsymbol{a} + 3\boldsymbol{c}) - \frac{1}{4}(\boldsymbol{b} + 3\boldsymbol{c})$$

$= \frac{8\boldsymbol{a}+12\boldsymbol{c}-5\boldsymbol{b}+15c}{20}$

$= \frac{8}{20}\boldsymbol{a} - \frac{3}{20}\boldsymbol{c} - \frac{5}{20}\boldsymbol{b}$

$\boldsymbol{AB} = \boldsymbol{OB} - \boldsymbol{OA} = \boldsymbol{b} - \boldsymbol{a}$

$\boldsymbol{PR} + \boldsymbol{RB} = \boldsymbol{PB}$

$\frac{8}{20}\boldsymbol{ka} - \frac{-3}{20}k\boldsymbol{a} - \frac{5}{20}k\boldsymbol{b} + t\boldsymbol{b} - t\boldsymbol{a} = \frac{3}{4}(\boldsymbol{b} - \boldsymbol{c})$

$\left(\frac{8}{20}k - \frac{20}{20}t\right)\boldsymbol{a} + \left(\frac{20t-5k}{20}\right)\boldsymbol{b} - \frac{3}{20}k\boldsymbol{c} = \frac{3}{4}\boldsymbol{b} - \frac{3}{4}\boldsymbol{c} + \boldsymbol{a}$

Equating coefficients

$8k - 20t = 0 \Longrightarrow 2k = 5t \dots\dots\dots\dots *$

$20t - 5k = 15 \Longrightarrow 4t - k - 3 \dots\dots..*$

$\frac{3k}{20} = \frac{3}{4}$ $\qquad \frac{k}{5} = 1$ $\qquad k = 5$

From $* \; 2 + 5 = 5t$

$5t = 10$ $\qquad t = 2$

OR$= \boldsymbol{OB} + \boldsymbol{BR}$

$= \boldsymbol{OB} + \boldsymbol{BA}$

$= \boldsymbol{OB} + t(\boldsymbol{OA} - \boldsymbol{OB})$

$= \boldsymbol{b} + 2(\boldsymbol{a} - \boldsymbol{b})$

$= 2\boldsymbol{a} - \boldsymbol{b}$

Alternative

$\boldsymbol{OR} = \boldsymbol{OP} + \boldsymbol{PR}$

$= \boldsymbol{OP} + \boldsymbol{PQ} = \frac{1}{4}(\boldsymbol{b} + 3\boldsymbol{c}) + 5\left(\frac{8\boldsymbol{a}}{20} - \frac{3}{20}\boldsymbol{c} - \frac{5}{20}\boldsymbol{b}\right)$

$= 2\boldsymbol{a} - \boldsymbol{b}$

34. (a) (i) $let\ y = 2^{cosx^2}$

$Iny = cosx^2 In2$

$\frac{1}{y}\frac{dy}{dx} = In2(-2xsinx^2) + cosx^2.0$

$\frac{dy}{dx} = y(-2xsin^2)In2$
$= -2^{cosx^2(2xsinx^2)In2}$

(ii) $y = \log_e\left(\frac{(1+x)e^{-2x}}{1-x}\right)$

$y = \frac{1}{2}In\left(\frac{(1+x)e^{-2x}}{1-x}\right)$
$y = \frac{1}{2}\{In(1+x) - 2x - In(1-x)\}$
$\frac{dy}{dx} = \frac{1}{2}\left\{\frac{1}{1+x} - 2 + \frac{1}{1-x}\right\}$
$\frac{dy}{dx} = \frac{1}{(1+x)} - 1 + \frac{1}{2(1-x)}$
$= \frac{1}{2}\left(\frac{1+x^2}{1-x^2}\right)$

(b) (i) $y = \frac{1}{x}$, $\frac{dy}{dx} = \frac{-1}{x^2}$ $at\ x = 2$
$\frac{dy}{dx} = \frac{-1}{(2)^2} = \frac{-1}{4}$ gradient of normal
$= 4$

$when\ x = 2$
$y = {}^1/_2$

Equation of normal $\frac{y-\frac{1}{2}}{x-2} = 4$
$y - {}^1/_2 = 4x - 8$
$y = 4x - 8 + {}^1/_2$
$y = 4x - {}^{15}/_2$ equation of normal.

(ii) Points of contact with the curve again solve equations of the normal and the curve.
$4x - {}^{15}/_2 = \frac{1}{x}$
$4x^2 - {}^{15}/_2\, x = 1$
$8x^2 - 15x - 2 = 0$
$8x^2 - 16x + x - 2 = 0$
$8x(x-2) + (x-2) = 0$
$(8x-2) + (x-2) = 0$
$x = {}^{-1}/_8 \quad x = 2$

When $x = 2$ $\quad y = {}^1/_2 \quad where = {}^1/_8,\ y = 8$
Points of contact $(2, {}^1/_2),\ ({}^1/_8, 8)$

35. (a) (i) $2cosy - x^2 = 1$

$-2siny\frac{dy}{dx} - 2x = 0$

$\frac{dy}{dx} = \frac{2x}{-2siny} = \frac{-x}{siny}$

(ii) $Let\ u = -x \quad \frac{du}{dx} = -1$

$v = siny \quad \frac{dv}{dx} = cosy\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{siny(-1)--x\left(cosy\frac{dy}{dx}\right)}{sin^2y}$

$\frac{-siny+xcosy\left(\frac{-x}{siny}\right)}{sin^2y}$

$= -\frac{sin^2y+x^2cosy}{sin^2y}.\frac{1}{siny}$

$\frac{d^2y}{dx^2} = -\frac{\left(x^2cosy+sin^2y\right)}{sin^3y}$

40.a)

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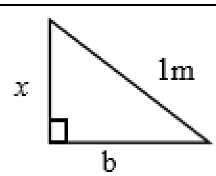
<td>

$1^2 = b^2 + x^2$
$x = \sqrt{1 - b^2}$</td>
</tr>
</table>

$h = 1 + \sqrt{1 - b^2}$
$A = \frac{1}{2} 2b\left(1 + \sqrt{1 - b^2}\right)$
$A = b\left(1 + \sqrt{1 - b^2}\right)$

$\frac{dA}{db} = \left(1 + \sqrt{1 - b^2}\right) + b \times -2b\frac{1}{2}(1 - b^2)^{-\frac{1}{2}}$
$= \frac{\sqrt{1-b^2} + (1-b^2) - b^2}{\sqrt{1-b^2}} = 0$
$\sqrt{1 - b^2} + (1 - b^2) - b^2 = 0$
$\sqrt{1 - b^2} = 2b^2 - 1$
$1 - b^2 = 4b^4 - 4b^2 + 1$
$4b^4 - 3b^2 = 0$
$b^2(4b^2 - 3) = 0$
$b = 0, \frac{\sqrt{3}}{2}m$

$\text{Base} = \sqrt{3}\text{m}$

$h = 1 + \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$
$= 1 + \frac{1}{2}$
$= \frac{3}{2}m$

36. (a) $y = \frac{x^2 - 11x + 28}{3x - 3}$

Intercepts when $x = 0$ $y = \frac{28}{3}$ $\left(0, {}^{-28}/_{3}\right)$

When $y = 0$
$x^2 - 11x + 28 = 0$ $x^2 - 7x - 4x + 28 = 0$
$x(x - 7) - 4(x - 7) = 0$
$(x - 4)(x - 7) = 0$ $x = 4,$ $x = 7$
$(4, 0), (7, 0)$

Equations of asymptotes
Vertically $3x - 3 = 0$ $x = 1$
Slanting $3x - 3\sqrt{x^2 - 11x + 28}$
$x^2 - x$
$= 10x + 28$
$-10x - 10$

Slanting asymptotes $y = \frac{x}{3} - \frac{10}{3}$

x	y
0	$\frac{-10}{3}$
10	0

Or $3y = x - 10$

$Point\ (0,\ 0) \qquad \left(0\ ,\ \frac{-10}{3}\right)$

$$\frac{dy}{dx} = \frac{(2x-11)(3x-3)-3\left(x^2-11x+28\right)}{(3x-3)^2}$$

At turning point $\frac{dy}{dx} = 0$

$3x^2 - 6x - 51 = 0$

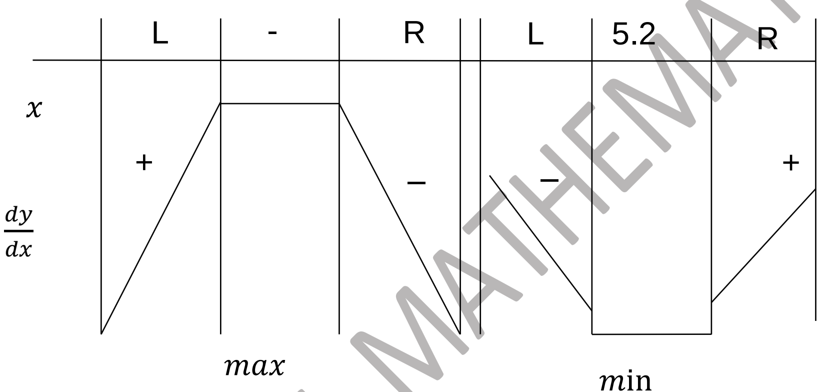
$x^2 - 2x - 17 = 0$

$x = 2 \pm \frac{\sqrt{4+68}}{2}$

$x = 5.2, \quad or \ \ -3.2$

$y(5.2) \approx -0.2$

$y(-3.2) = -5.8$



Hence $(-3.2, -5.8) \quad is\ maximum$

$(5.2,\ \ -0.2) \quad minimum$

Region of Restrictions.

$x^2 - 11x + 28 - 3yx + 3y = 0$

$x^2 - (11 + 3y)x + (28 + 3y) = 0$

For real $x \qquad b^2 - 4ac \geq 0$

$[-(11 + 3y)^2]^2 - 4(28 + 3y) \geq 0$

$121 + 66y + 9y^2 - 12y - 112 \geq 0$

$y^2 + 6y + 1 \geq 0$

$y = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 1}}{2}$

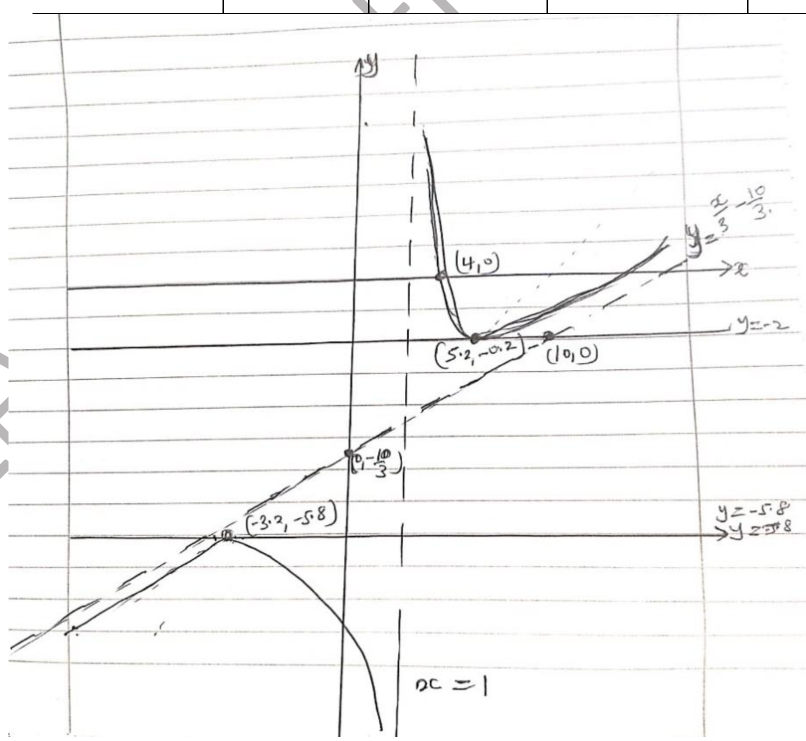
$y = \sqrt{-6 \pm \sqrt{3^2}}$ $\quad$ $y = -5.8$

$y = -0.2$

y	$y < -5.8$	$-5.8 < y < -.02$	$y > 0.2$
$y + 5.8$	$-$	$+$	$+$
$y + 0.2$	$-$	$-$	$+$
z	$+$	$-$	$+$

Hence the cure does not exist between $-5.8 < y < -0.2$

	$x < -1$	$1 < y < 4$	$4 < x > 7$	$x > 7$
$x - 4$	$-$	$-$	$+$	$+$
$x - 7$	$-$	$-$	$-$	$+$
$3x - 3$	$-$	$+$	$+$	$+$
y	$-$	$+$	$-$	$+$



37. (a) $y = \frac{x^2-4x+3}{1+x^2}$

$y(1+x^2) = x^2 - 4x + 3$

$y + yx^2 = x^2 - 4x + 3$

$yx^2 - x^2 + 4x + y - 3 = 0$

$(y-1)x^2 + 4x + 4x + y - 3 = 0$

For real x $\quad b^2 - 4ac \geq 0$

$(4)^2 - 4(y-1)(y-3) \geq 0$

$16 - 4y^2 + 16y - 16 \geq 0$

$16y - 4y^2 \geq 0$

$4y(4-y) \geq 0 \qquad y = 0, \ \ y = 4$

y	$y < 0$	$0 < y < 4$	$y < 4$
$4y$	$+ve$	$+ve$	$+ve$
$4-y$	$+ve$	$+ve$	$-ve$
z	$+ve$	$+ve$	$-ve$

Soln set: $\{y < 0, \qquad 0 < y < 4\}$

(b) $y = e^{4x}cos3x$

$\frac{dy}{dx} = 4e^{4x}cos3x - 3e^{4x}sin3x$

$\frac{dy}{dx} = 4y - 3e^{4x}sin3x$

$\Longrightarrow 3e^{4x}cos3x.3 + 4e^{4x}sin3x$

$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 3(e^{4x}cos3x.3 + 4e^{4x}sin3x)$

$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 3(3y) = 12e^{4x}sin3x$

$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$

38. (a) $Let\ x = 30^0 \qquad 180 = \pi$

Convert radian $\delta x = \frac{-0.8\pi}{180^0}$

$Let\ y = cosec^2x \quad , \quad y = \frac{1}{sin^2 30} = \left(\frac{1}{\frac{1}{2}}\right)^2 = 4$

$\frac{dy}{dx} = -2cosecx\ cotxcosecx$

$= -2cosec^2xcotx$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx} \ , \quad \delta y = \frac{dy}{dx}\delta x$

$\delta y = (-2cosec^2 30\ cot30).\left(\frac{-0.8\pi}{180}\right)$

$\delta y = -2\left(\frac{1}{sin^2 30}\right).\left(\frac{1}{tan30}\right).\left(\frac{-0.8\pi}{180^0}\right)$

$\delta y = 2(4 \times 1.732)\left(\frac{0.8 \times 3.14}{180^0}\right)$

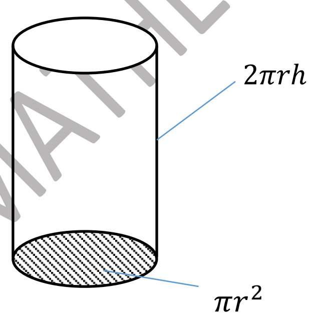
$\delta y = 0.1934$

Approximate $= y + \delta y$

$4 + 0.1934$

$4.1934 \quad 4dp$

(b)



Total area $2\pi rh + \pi r^2 = 12$

$h = \frac{12 - \pi r^2}{2\pi r}$

Volume $v = \pi r^2 h = \pi r^2 \left(\frac{12 - \pi r^2}{2\pi r}\right)$

$v = 6r - {}^1\!/_2\, \pi r^3$

$\frac{dv}{dr} = 6 - \frac{3}{2}\pi r^2 \qquad At\ maximum\ \ \frac{dv}{dr} = 0$

$\Longrightarrow 6 = {}^{3}/_{2}\,\pi r^2, \quad r^2 = \frac{4}{\pi} \quad r = \frac{2}{\sqrt{\pi}}$

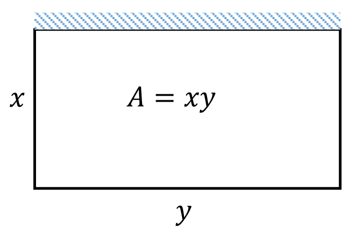
Volume $v_{max} = 6\left(\frac{2}{\sqrt{\pi}}\right) - \frac{1}{2}\pi\left(\frac{2}{\sqrt{\pi}}\right)^3$

$= \frac{12}{\sqrt{\pi}} - \frac{\pi}{2}.\frac{8}{\pi\sqrt{\pi}}$

$\frac{12}{\sqrt{\pi}} - \frac{4}{\sqrt{\pi}}$

$\frac{12-4}{\sqrt{\pi}} = \frac{8}{\sqrt{\pi}}$

39 (b)



$2000(2x) + y(3000) = 1{,}200{,}000$

$4x + 3y = 1200$

$y = \left(400 - \frac{4x}{3}\right)$

$A = 400x - \frac{4}{3}x^2$

$\frac{dA}{dx} = 400 - \frac{8}{3}x$

At $max. \frac{dA}{dx} = 0$

$400 = \frac{8x}{3} \qquad x = \frac{3+400}{8} = 150$

$A\,\text{max} = 200 \times 150 = 30{,}0000m^2$

41 (a) $x^2 + y^2 = 3xy$

$2x + 2y\frac{dy}{dx} = 3\left(x\frac{dy}{dx} + y\right)$

$2x + 2y\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$

$3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - 3y$

$\frac{dy}{dx}(3x - 2y) = 2x - 3y$

$\frac{dy}{dx} = \frac{2x-3y}{3x-2y}$

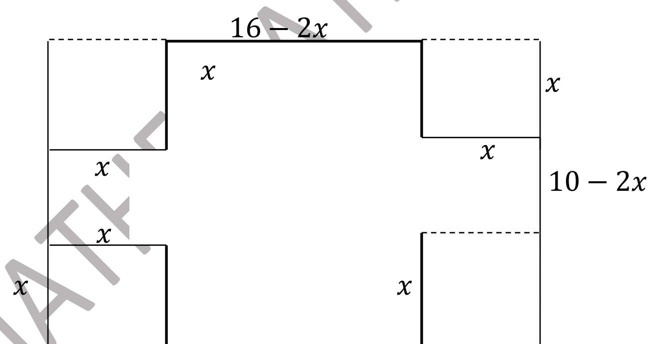
$Al\ {}^{3}/_{2}\ ,{}^{3}/_{2}$

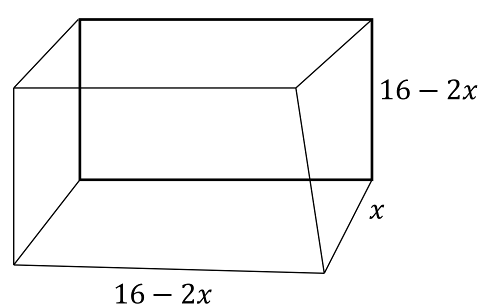
$\frac{dy}{dx} = \frac{2({}^{3}/_{2})-3x{}^{3}/_{2}}{3({}^{3}/_{2})-2({}^{3}/_{2})}$

$= \frac{3-{}^{9}/_{2}}{{}^{9}/_{2}-3} = \frac{6-9}{2} \div \frac{9-6}{2}$

$= \frac{-3}{2} \times \frac{2}{3} = -1$

(b)





$Vol. = (16 - 2x)(10 - 2x)x$

$V = 4x^2 - 52x + 160 \quad or \quad x^2 - 13x + 40$

$\frac{dv}{dx} = 8x - 5^2$

$At\ \max \frac{dv}{dx} = 0$

$8x = 5^2$ $\qquad x = 6.5$

$V_{max} = 4(6.5)^2 - 52(6.5) + 160$

$= 169 - 338 + 160$

41. (a) $y = \frac{4Inx-3}{4Inx+3}$

$u = 4Inx - 3 \qquad \frac{dy}{dx} = 4.\frac{1}{x} = \frac{4}{x}$

$v = 4Inx + 3$

$\frac{dv}{dx} = \frac{4}{x}$

$\frac{dy}{dx} = \frac{(4Inx+3).\frac{4}{x}-(4Inx-3)\frac{4}{x}}{(4Inx+3)^2}$

$\frac{16Inx+12-16Inx+12}{x(4Inx+3)^2}$

$\frac{24}{x(4Inx+3)^2}$

(b) $\frac{dy}{dx} = \left(x - \frac{1}{x}\right)$

$\int dy = \int\left(x - {}^{1}/_{x}\right)dx$

$y = \frac{x^2}{2} - Inx + c \qquad at\ (1, 2)$

$2 = \frac{1}{2} - 0 + c \qquad c = {}^{3}/_{2}$

The curve is $y = \frac{x^2}{2} - Inx + \frac{3}{2}$

Area $A = \int_{11}^{2} y\, dx$

$= \int_1^2 \frac{x^2}{2} dx - \int_1^2 Inx dx + \int_1^2 \frac{3}{2} dx$

$\frac{x^3}{6}\int_1^2 - (xInx - x)\int_1^2 + \frac{3}{2}x\int_1^2$

$\frac{8}{6} - \frac{1}{6} - (2In2 - 2) - (In1 - 1) + \frac{6}{2} - \frac{3}{2}$

$\frac{7}{6} - 2In2 + 2 - 1 + \frac{3}{2}$

$\frac{7}{6} + 1 + {}^{3}/_{2} - 2\ In2$

$\frac{7+6+9}{6} - 2\ In\ 2$

$\frac{22}{6} - 2\ In\ 2$

$\frac{11}{3} - 2In2\ As\ required$

43. (a) $\int \frac{\sqrt{16-}x^2}{x^2}$

Let $x = 4sin\theta$

$dx = 4cos\theta d\theta$

$\int \sqrt{\frac{16-4sin\theta}{4sin\theta}}.4cos\theta d\theta$

$\int \frac{4cos\theta.4cos\theta d\theta}{16sin^2\theta}$

$\int cot^2\theta d\theta$

$\int (cosec^2\theta - 1)d\theta$

$-cot\theta - \theta + c$

$-\frac{1}{x}\sqrt{(16 - x^2)} - sin^{-1}\left(\frac{x}{4}\right) + c$

(b) $\int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2dx}} dx = \frac{4}{2} + \sqrt{3} - 1$

Let $x = 2sin\theta$.

x	$\sqrt{3}$	1
θ	${}^{\pi}/_{3}$	${}^{\pi}/_{6}$

$dx = 2cos\theta d\theta$

$\int_{\pi/6}^{\pi/3} \frac{(2sin\theta+3)}{\sqrt{(4-4sin^2\theta)}}.2cos\theta d\theta$

$\int_{\pi/6}^{\pi/3} \frac{(2sin\theta+3)}{2cos\theta}.2cosd\theta = \oint$

$\int_{\pi/6}^{\pi/2} (2sin\theta + 3)d\theta$

$(-2 + 3\theta)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$\left[-2\left(\frac{1}{2}\right) + 3\left(\frac{\pi}{3}\right)\right] - \left[-2\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{\pi}{6}\right)\right]$

$-1 + \pi/2 + \sqrt{3} - \pi/2$

$\underline{\underline{\pi/2 + \sqrt{3} - 1}}$

44. (a) $\int_1^e \left(\frac{2-Inx}{x^2}\right) dx$

$\int_1^e \left(\frac{2}{x^2} - \frac{1}{x^2}Inx\right) dx$

$\int_1^e \frac{2}{x^2} dx - \int_1^e \frac{1}{x^2} Inx\ dx$

$\left[-2\left(\frac{1}{x}\right)\right]_1^e - \int_1^e \frac{1}{x^2} Inx\ dx$

For $\int \frac{1}{x^2}\ Indx$. $Let\ u = Inx,\ \frac{du}{dx} = \frac{1}{x}$

$\frac{dv}{dx} = \frac{1}{x^2}. \therefore v = \frac{-1}{x}$

$\int \frac{1}{x^2}\ Inx\ \ dx = \frac{-1}{x} Inx - \int \left(\frac{-1}{x}\right).\frac{1}{x} dx$

$= \frac{-1}{x} Inx + \left(\frac{-1}{x}\right) + c$

$\left[-2\left(\frac{1}{x}\right)\right]_1^e - \left[\frac{-1}{x} Inx - \frac{1}{x}\right]_1^e$

$\left[\frac{-2}{e} - (-2)\right] - \left[\left(\frac{1}{e} Ine + \frac{1}{e}\right) - (0 - 1)\right]$

$\left(2 - \frac{-2}{e}\right) - \left(\frac{-2}{e} + 1\right) = 1$

(b) $\int_{\pi}^{\frac{\pi}{4}} \frac{1}{3sin^2 \times cos^2 x} dx$

Divide through $cos^2 x$

$\int_{\pi}^{\pi/4} \frac{sec^2 x}{3tan^2 \times +1} dx$

Let $t = tanx$, $dt = sec^2 x\, dy$

$$\begin{array}{c|c} x & t \\ \frac{\pi}{4} & 1 \\ \pi & 0 \end{array}$$

$\int_0^1 \frac{sec^2 x}{3t^2+1} - \frac{dt}{sec^2 x}$

$\int_0^1 \frac{1}{3t^2+1} dt$

Let $t = \frac{1}{\sqrt{3}} \cdot \frac{sec^2 udu}{(1+tan^2 u)}$

$\int \frac{1}{\sqrt{3}} \cdot du = \frac{1}{\sqrt{3}} u = \frac{1}{\sqrt{3}} tan^{-1}\left(\sqrt{3t}\right)$

$= \frac{1}{\sqrt{3}} \cdot \frac{\pi}{3} - 0 = \frac{\pi}{3\sqrt{3}}$

45. (a) $\int x\, logx\, dx$

(i) $x \frac{Inx}{In10} dx$

$\frac{1}{In10} \int x\, Inx\, dx$

Let $u = Inx$, $du = \frac{1}{x}.$, $\frac{dv}{dx} = x$, $v = \frac{1}{2} x^2$

$\frac{1}{In10} \int x\, Inx\, dx = \frac{1}{In10}\left[\frac{1}{2} x^2 Inx - \int \frac{1}{2} x^2 . \frac{1}{x}\, dx\right]$

$= \frac{1}{In10}\left[\frac{1}{2} x^2 Inx - \frac{1}{4} x^2\right] + c$

(ii) $\int sec^3 2x tan2x\, dx$

$u = sec2x$

$du = 2sec2xtan2xdx$

$\int sec^3 2xtan2x . \frac{du}{2sec2xtan2x}$

$\frac{1}{2} \int u^2 . du = \frac{1}{6} u^3 + c = \frac{1}{6} sec^3 2x + c.$

(b) $\int_0^{\frac{\sqrt{3}}{3}} 6x tan^{-1}(3x)dx$

$u = tan^{-1}(3x)$; $\frac{du}{dx} = \frac{3}{1+9x^2}dx$, $v = 3x^2$

$\int 6x\ tan^{-1}(3x)dx = 3x^2\ tan^{-1}(3x) - \int 3x^2 \cdot \frac{3}{1+9x^2}dx$

$= 3x^2 tan^{-1}(3x) - \int \left(1 - \frac{1}{1-9x^2}\right)dx$

$= \left[3x^2 tan^{-1}(3x) - x + \frac{1}{3}tan^{-1}(3x)\right]_0^{\frac{\sqrt{3}}{3}}$

$= \left(3.\frac{3}{9}tan^{-1}(\sqrt{3}) - \frac{\sqrt{3}}{3} + \frac{1}{3}tan^{-1}\sqrt{3}\right) - 0 - 0 - 0$

$= \frac{\pi}{3} - \frac{\sqrt{2}}{3} + \frac{1}{3}(\pi/3)$

$= \frac{1}{9}(4\pi - 3\sqrt{3})$

46. (a) $f(x) = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, $\int fx\ dx$

$\int \frac{sin^{-1}x}{\sqrt{1-x^2}}dx$

Let $u = sin^{-1}x$, $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{u}{\sqrt{1-x^2}} \cdot \sqrt{(1-x^2)}du = \int udu$

$= \frac{1}{2}u^2 + c = \frac{1}{2}(sin^{-1}x)^2 + c$

(b) $\frac{5x^3+2x^2+5x}{1-x^4} = \frac{5x^3+2x^2+5x}{(1+x^2)(1-x^2)}$

$= \frac{5x^3+2x^2+5x}{(1+x)(1-x)(1+x^2)}$

$\frac{5x^3+2x^2+5x}{1-x^4} = \frac{A}{1+x} + \frac{B}{1-x} + \frac{Cx+D}{1+x^2}$

$5x^3 + 2x^2 + 5x = A(1-x)(1+x^2) + B(1+x)(1+x^2) + (Cx+D)$

$For\ x = 1\ ; 12B(2)(2) + C + D$

$12 = 4B + C + D \ldots\ldots\ldots (1)$

$For\ x = -1, -8 = A(2)(2) + D - C$

$-8 = 4A - C + D \ldots\ldots\ldots. (2)$

$For\ \ v = 0 \qquad 0 = A + B + D \ldots\ldots\ldots\ldots\ldots(3)$

Coef. $x^3 \qquad 5 = -A + B \ldots\ldots\ldots\ldots\ldots\ldots(4)$

$x^2 \qquad 2 = A + B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$

Using (4) as 5 $\ B = {}^{7}/_{2} \quad and$

$$3 = -2A \qquad\qquad A = {}^{-3}/_{2}$$

From (3) $\qquad D = -A - B$

$$-\left(\frac{-3}{2}\right) - \left(\frac{7}{2}\right)$$

$$= \frac{3-7}{2} = -2$$

Using (2) $\qquad -8 = 4A - C + D$

$$C = 4A + 8 + D$$

$$= 4\left({}^{-3}/_{2}\right) + 8 - 2$$

$$= -6 + 8 - 2 = 0$$

$$\therefore \frac{5x^3+2x^2+5x}{1-x^4} = \frac{-2}{1+x} + \frac{{}^{7}/_{2}}{1-x} + \frac{-2}{1+x^2}$$

$$\therefore \int \frac{5x^3+2x^2+5x}{1-x^4}\,dx = \int \left(\frac{-2}{1+x} + \frac{{}^{7}/_{2}}{1-x} - \frac{2}{1+x^2}\right) dx$$

$$= -2In(1+x) - \frac{7}{2}In(1-x) - 2tan^{-1}x + c$$

47. (a) $\int_{\pi}^{4\pi/3} cosec\left(\frac{1}{2}x\right) dx$

$$\int \frac{1}{sin\frac{x}{2}}\,dx = \int \frac{1}{2sin\frac{x}{4}cos\frac{x}{4}}\,dx$$

$$= \int \frac{sec^{2x}/_4}{2tan^{x}/_4}\,dx$$

Let $\ u = tan\,{}^{x}/_{4}, \quad du = \frac{1}{4}sec^2\frac{x}{4}\,dx$

$$\int \frac{sec^2 \cancel{{}^{x}/_{4}}}{2u} \cdot \frac{4du}{sec^2 \cancel{{}^{x}/_{4}}}$$

$$2\int \frac{1}{u}\,du = 2Inu$$

$$= \left[2Intan\left(\frac{x}{4}\right)\right]_{\pi}^{4\pi/_3} = \left[2.1n\left(tan\,^{\pi}/_3\right) - 2Intan\left(^{\pi}/_4\right)\right]$$

$$= 2In\left(\sqrt{3}\right) - 2In1$$

$$2.\frac{1}{2}In3 = In3$$

(b) (i) $\int \frac{1}{x}\frac{1}{\sqrt{x^4-1}}dx$

$$(x^4-1)^{\frac{1}{2}} = \left(x^4\left(1 - {}^{1}/_{x^4}\right)\right)^{\frac{1}{2}} = x^2\left(1 - \frac{1}{x^4}\right)^{\frac{1}{2}}$$

$$\int \frac{1}{x}.\frac{1}{x^2\sqrt{1-{}^{1}/_{x^4}}}dx$$

Let $\frac{1}{x^2} = sin\theta$

$$x^{-2} = sin\theta$$

$$-2x^{-3}dx = cos\theta d\theta$$

$$\int \frac{1}{x^3}.\frac{1}{\sqrt{(1-sin^2\theta)}}.\frac{-cos\theta d\theta}{2x^{-3}}$$

$$\int \frac{-cos\theta d\theta}{2cos\theta} = \frac{-1}{2}\theta + c$$

$$= \frac{-1}{2}sin^{-1}\left(\frac{1}{x^2}\right) + c$$

(ii) $\int \frac{secxtan}{9+4sec^2x}$

Let $u = secx$

$$du = secxtanxdv$$

$$\int \frac{secxtan-x}{9+4u^2}.\frac{du}{secxtanv}$$

$$\int \frac{du}{9+4u^2}$$

$$u = {}^{3}/_{2}\,tan\theta$$

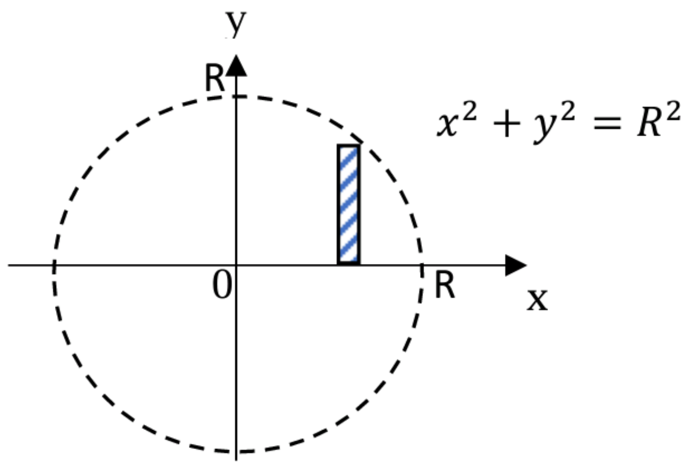
$$du = \frac{3}{2}sec2\theta d\theta$$

$$\int \frac{{}^{3}/_{2}sec^2\theta d\theta}{9(1+tan^2\theta)} = \int \frac{\frac{3}{2}}{9}d\theta$$

$$\frac{1}{6}\theta + C$$

$$\int \frac{secxtanx}{9+4sec^2x}dx = \frac{1}{6}tan^{-1}\left(\frac{2}{3}secx\right) + c$$

(b)



Quarter of the area is $\int_0^R y dx$

$= \int_0^R \left(\sqrt{R^2 - x^2}\right) dx$

Let $x = Rsinu$

$\frac{dx}{du} = Rcosu$

x	sinu	u
0	0	0
R	1	$\frac{\pi}{2}$

$= \int_0^{\frac{\pi}{2}} \left(\sqrt{R^2 - R^2 sin^2 u}\right) Rcosudu$

$= \int_0^{\frac{\pi}{2}} R^2 cos^2 udu$

$= \int_0^{\frac{\pi}{2}} R^2 \frac{1}{2}(1 + cos2u)du$

$= R^2 \left[\frac{1}{2}u + \frac{1}{4}sin2u\right]_0^{\frac{\pi}{2}}$

$= R^2 \left(\left(\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4}sin2\left(\frac{\pi}{2}\right)\right) - \left(\frac{1}{2}(0) + \frac{1}{4}sin2(0)\right)\right)$

$= \frac{\pi}{4}R^2$

49 Total area is $= 4 \times \frac{\pi}{4}R^2 = \pi R^2$

54. (a) $\frac{dy}{dx} + ycotx = 2$

$R = e^{\int cotx\, dx}$

$R = e^{Insinx} = sinx$

$nx\frac{dy}{dx} + y\frac{cosx}{sinx} = xsinx$

$\frac{d}{dx}(ysinx) = xsinx$

$\int d(ysinx) = (xsinx)dx$

$ysinx = \int xsinxdx$

$ysinx = -xcosx + sinx + c$

DIt	I
$+x$	$sinx$
-1	$-cosx$
0	$-sinx$

When $x = \frac{\pi}{2}$, $y = 1$

1.sin$\frac{\pi}{2} = -1cos\frac{\pi}{2} + sin\frac{\pi}{2} + c$

$\Longrightarrow ysinx = -xcosx + sinx$

$y = -x10 + x + 1$

$y = 1 - xcotx$

(b) $\frac{dN}{dt}\alpha N$

$\frac{dN}{dt} = KN,$ $\qquad \int \frac{1}{N}dN = \int kdt$

$In\, N = kt + c$ $\qquad t = 0 \quad N = 3$

$In\, N = kt + In3$ $\qquad t = 5, \ N = 9$

$In9 = 5k + In3$

$5k = In\left(\frac{9}{3}\right) = In3$ $\qquad 11 = {}^{In3}/_{5}$

$\Longrightarrow In\, N = \frac{t}{5}\ In3 + In3$, $\qquad N = \frac{3t}{5} + 3$

$N = \frac{3t+15}{5} \Longrightarrow N = 3\left(\frac{t+5}{5}\right)$

$t = 25$

$N = 3\frac{(25+5)}{5}$

$N = 18$

$N = 18{,}000{,}000$

55. (a) $\log \quad y = \log_2 x - \log_e x^2$

$y = \log_2 x - 2\log_e x$

$\frac{dy}{dx} = \frac{d}{dx}(\log_2 x) - \frac{d}{dx}(2Inx)$

$\frac{dy}{dx} = \frac{1}{xIn2} - 2.\frac{1}{x} + 0$

$Let\ P = \log_2 x$

$x = 2p$

$Inx = pIn2$

$\frac{1}{x} = In2\frac{dp}{dx} - 0$

$\frac{dp}{dx} = \frac{1}{xIn2}$

$\frac{d}{dx}(\log_2 x) = \frac{1}{xIn2}$

$\frac{dy}{dx} = \frac{1}{xIn2} - \frac{2}{x}$

(b) $x\frac{dy}{dx} - y = \frac{x}{x+1}$

$\frac{dy}{dx} - \frac{1}{x}y = \frac{1}{x+1}$

$\frac{d}{dx}\left({}^{y}/_{x}\right) = \frac{1}{x(x+1)}dx$

$\int d\left({}^{y}/_{x}\right) = \int \frac{1}{x(x+1)}$

${}^{y}/_{x} = \int \frac{1}{x}dx - \int \frac{1}{x+1}dx$

${}^{y}/_{x} = Inx - In(x+1) + c$

When 2, $y = 3$

$R = e^{\int \frac{1}{x}dx} = e^{In\,{}^{1}/_{x}}$

$R = \frac{1}{x}$

$$\boxed{y = xInx - xIn(x+1) + {}^{3}/_{2}\,x - xIn\,{}^{2}/_{3}}$$

56. (a) $\frac{dy}{dx} = e^{-x}$

$\int dy = \int xe^{-x}dx$

$y = \int xe^{-x}dx$

$u = \frac{du}{dx} = 1$

$\frac{dv}{dx} = e^{-x}$

$v = -e^{-x}$

$y = xe^{-x} - \int -e^{-x}.dx$

$y = -xe^{-x} + \int e^{-x}dx$

$y = -xe^{-x} - e^{-x} + c$ when $x = 0,\quad y = 2$

$2 = 0e^{0} - e^{0} + c$

$2 = -1 + c$ $c = 3$

$\Rightarrow y = -e^{-x} + 3 - e^{-x}$ or $y = 3 - e^{-x}(x+1)$

(b) $\frac{dp}{dt}\alpha\ P(P_1 - P_0)$

$\frac{dp}{dt} = kp = (P_0 - P)$ $\frac{1}{P(P_0-P)}dp = \int kdt$

$\int \frac{1}{P(P_0-P)}dp = \int kdt$ $\frac{1}{P(\beta-P)} = \frac{A}{P} + \frac{B}{P_0-P}$

$1 = A(P_0 - P) + B(P)$

$$A = \frac{1}{P_0}, B = \frac{1}{P_0}$$

$\frac{1}{B}\int \frac{1}{P}dp + \frac{1}{P_0 - P}dp = \int kdt$

$\frac{1}{P_0}InP - \frac{1}{P_0}\ In(P_0 - P) = kt + c$ when $t = 0$

$\frac{1}{P_0}In\ 10{,}000 - \frac{1}{P_0}In(P_0 10{,}000) = 1c + 0 + c$ $P = 10{,}000$

$c = \frac{1}{P_0}In\left(\frac{10{,}000}{P_0 - 10{,}000}\right)$

$\Longrightarrow \frac{1}{P_0}In\left(\frac{P}{P_0 - P}\right) = kt + \frac{1}{P_0}In\left(\frac{10{,}000}{P_0 - 10{,}000}\right)$

Given $\frac{dp}{dt} = kp$ $\frac{dp}{dt} = 0.0000005$

$\Longrightarrow 0.0000005 = kx10000$

$k = 5 \times 10^{-11}$

$\Longrightarrow \frac{1}{P_0}In\left(\frac{P}{P_0 - P}\right) = 5 \times 10^{-11}t + \frac{1}{P_0}In\left(\frac{10{,}000}{P_0 - 10{,}000}\right)$

$P_0 = 200{,}000$

$When\ P = 51\%\ of\ P_0,\ i.e. P = \frac{51}{100} \times 200{,}000$

$P = 102{,}000$

$\Longrightarrow \frac{1}{200{,}000}In\left(\frac{102{,}000}{200{,}000 - 102{,}000}\right) = 5 \times 10^{-11}t + \frac{1}{200{,}000}In\left(\frac{10{,}000}{190{,}000}\right)$

$2.00 \times 10^{-7} = 5 \times 10^{-11}t - 1.4722 \times 10^{-5}$

$1.4922 \times 10^{-5} = 5 \times 10^{-11}t$

$t = \frac{1.492 \times 10^{-5}}{5 \times 10^{-11}} = 298{,}440days$

57. (a) $\frac{dy}{dx} = \frac{y}{x} + tan\, {}^{y}/_{x}$ Let $y = vx$

$V + x\frac{dv}{dx} = \frac{Vx}{x} + tan\left(\frac{vx}{x}\right)$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$V + x\frac{dv}{dx} = v + tanv$

$x\frac{dv}{de} = tanv$

$xdv = tanvdx$

$\frac{1}{tan}dv = \frac{1}{x}dx$

$\int cotvdv = \int \frac{1}{x}dx$

$\int \frac{cosv}{sinv}dv = \int \frac{1}{x}dx$

$Insiv - Inx = c$

$In\frac{sinv}{x} = c$

$\frac{sinV}{x} = e^c$

$sinv = xe^c$ $\quad\quad$ $sin\, {}^{y}/_{x} = xA$

${}^{y}/_{x} = sin^{-1}xA$

$y = xsin^{-1}(xA)$

(b) Let the distance he still has to cover = 5

$\frac{ds}{dt}\alpha\sqrt{S}$

$\frac{ds}{dt} = -1 < \sqrt{s}$

$\int \frac{1}{\sqrt{s}}ds = \int -kdt$

$\int s^{\frac{-1}{2}}ds = kt + c$ $\quad\quad$ when $t = 0$

$2(1600)^{\frac{1}{2}} = c$ $\quad\quad$ $S = 1600$

$2 \times 40 = c$ $\quad\quad$ $c = 80$

$d = 2S^{\frac{1}{2}} = -kt + 80$

$\frac{ds}{dt} = 10m/s$

$\Rightarrow 10 = k\sqrt{s}$, $\quad t = 0,\quad s = 1600$

$10 = k\sqrt{1600}\quad 10 = 40k,\quad k = {}^{1}/_{4} = 0.25$

$\Rightarrow 2s^{\frac{1}{2}} = -0.25t + 80$

When he covered the distance, the remaining distance $s = 0$

. $\Rightarrow 0 = -0.25t + 80$

$0.25t = 80$

$t = \frac{80}{0.25} = 320sec$

$= 5.33min$

58. (a) $3y + (x-2)\frac{dy}{dx} = \frac{2}{x-2}$

$\frac{dy}{dx} + \frac{3}{x-2}y = \frac{2}{(x-2)}2$

$R = e^{\int\frac{3}{x-2}}dx = {}_{e}^{3}\int\frac{3}{x-2}dx = e^{3}In(x-2)$

$R = e^{In(x-2)3} = (x-2)^3$

$(x-2)^3\frac{dy}{dx} + \frac{3}{x-2}(x-2)^2y = 2(x-2)$

$\frac{d}{dx}((x-2)^3y) = 2x-4$

$d = ((x-2)^3y) = \int(2x-4)dx$

$y(x-2)^3 = x^2 - 4x + c$

$y = \frac{x^2-4x}{(x-2)^3} = +\frac{c}{(x-2)^2}$

$y = \frac{x(x-4)}{(x-2)} + A$

(b) Let x be the fraction of the community infected.

$1 - x = fraction\ not\ yet\ infected$

$\frac{dx}{dt}\alpha\frac{x}{1-x}$

$\frac{dx}{dt} = \frac{kx}{1-x}$

$\int\frac{1-x}{x}dx = \int kdt$

$\int\frac{1}{x} - 1 = k\int dt$

$Inx - x = kt + c$

$\log_e x = x + kt + c$

$x = e^{x}.e^{kt}.e^{c}$

$e^{kt} = \frac{x}{e^x e^c}$ $\qquad$ $Let\ e^{-c} = A$

$e^{kt} = Ae^{-x}x.$

When $t = 0$ $\qquad x = \frac{1}{2}$

$e^0 = A(1/_2)e^{-1/_2}$

$A = \frac{2}{e^{-1/_2}}$ $\qquad A = 2e^{1/_2}$

$\Longrightarrow e^{kt} = 2xe^{1/_2}e^{-x}$

From $\frac{dx}{dt} = \frac{kx}{1-x}$

When first noticed $x = \frac{1}{2}$, $\frac{dx}{dt} = \frac{1}{4}$

$\Longrightarrow \frac{1}{4} = \frac{K(1/_2)}{1-1/_2} \Longrightarrow \frac{1}{4} = \frac{k}{2} \div \frac{1}{2}$

$\frac{1}{4} = \frac{k}{2} \times \frac{2}{1}$, $k = \frac{1}{4}$

$\Longrightarrow e^{1/_4 t} = 2e^{1/_2}xe^{-x}$

$$\left(e^{\frac{1}{4}t}\right)^4 = \left(2e^{1/_2}xe^{-x}\right)^4$$

$e^t = 16e^2x^4e^{-x}$

$e^t = 16x^4e^{2-4x}$ $\qquad$ As required

$x = 1$ $\qquad$ Using $e^t = 16x^4e^{2-4x}$

$e^t = 16(1)^4(2^{2-4})$

$e^t = 16e^{-2}$

$Ine^t = In16 + Ine^{-2}$

$t = \text{n}16 - 2$

$t = o.775month$

$t = 0.775 \times 30 = 23.18days.$

With you all the best in your forthcoming examinations. GOD BLESS YOU.

STATISTICS AND PROBABILITY:

59. Solution

Weight (x)	Packets (f)	fx	fx^2
999	1	999	998001
1000	7	7000	7000000
1001	12	12012	12024012
1002	8	8016	8032032
1003	3	3009	3018027
1004	1	1004	1008016
Total	32	32040	32080088

Mean $= \frac{32040}{32} = 1001.25$

The amount by which the mean exceeds 1000g is $(1001.25 - 1000) = 1.25$g

Standard deviation $\sqrt{\frac{32080088}{32} - \left(\frac{32040}{32}\right)^2} = 1.089725$

Standard Error of the mean $\frac{\sigma}{\sqrt{n}} = \frac{1.089725}{\sqrt{32}} = 0.19263798$

$p = 0.499 \quad z_{\frac{\alpha}{2}} = 3.090 \quad \overset{\wedge}{\sigma} = \frac{32}{31}(1.089725) = 1.124877$

Lower limit $= 1001.25 - 3.090 \times \frac{1.124877}{\sqrt{32}} = 1000.635547$

Upper limit $= 1001.25 + 3.090 \times \frac{1.124877}{\sqrt{32}} = 1001.864453$

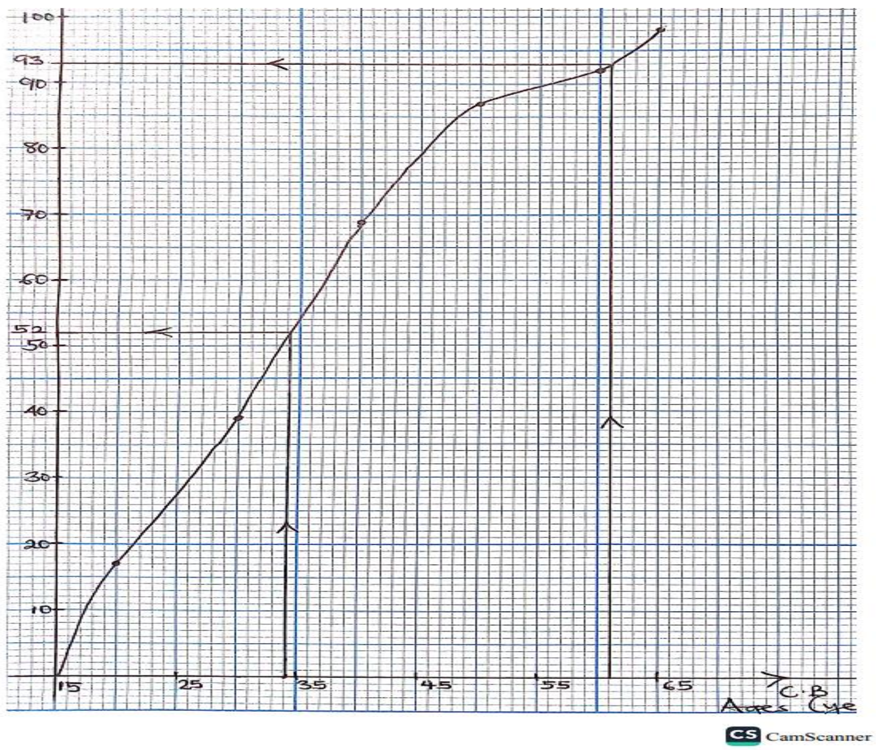
60.

Age(years)	x	f	fx	C f	cw
$15 - 20$	17.5	17	297.5	17	5
$20 - 30$	25	22	550	39	10
$30 - 40$	35	30	1050	69	10
$40 - 50$	45	18	810	87	10
$50 - 60$	55	5	275	92	10
$60 - 65$	62.5	6	375	98	5
Total		98	$\sum fx = 3357.5$		

(a) Average age $= \frac{3357.5}{98} = 34.2602$

(b) Employees above the average age $= (98 - 52) = 46$ employees. [45,47]

(c) Amount spent $= 20{,}000{,}000 \times (98 - 93) = shs\ 100{,}000{,}000$



class boundaries

61. The frequency distribution table below shows the mass in kilograms of unsealed bags of maize flour in a certain maize store.

Mass (kg)	51	53	54	55	56
Frequency	2	3	5	8	11

Find the mean and standard deviation of the mass of maize flour in the unsealed bags. Hence find the mean and standard deviation of the mass of maize in the sealed bags, if the quantity of maize flour in each bag is increased by 10% and then bags are sealed off.

Mass(kg)	f	fx	fx^2
51	2	102	5202
53	3	159	8427
54	5	270	14580
55	8	440	24200
56	11	616	34496
Total		$\sum fx = 1587$	$\sum fx^2 = 86905$

(a) Mean $= \frac{1587}{29} = 54.7241$ or $54\frac{21}{29}$

(b) Standard deviation$=\sqrt{\frac{86905}{29}-\left(\frac{1587}{29}\right)^2}=1.4117$

(c) $\frac{\sum f(\frac{110}{100}x)}{\sum f}=\frac{110\frac{\sum fx}{\sum f}}{100}=1.1\times 54.7241=60.1965$

(d) Standard deviation$=\sqrt{\frac{\sum f(\frac{110}{100}x)^2}{\sum f}-\left(\frac{\sum f(\frac{110}{100}x)}{\sum f}\right)^2}=1.1\times 1.4117=1.55287$

62. The table below shows the order in which ten Parishes where ranked in Sports and Music tournaments at the Diocese level.

Calculate the rank correlation coefficient between Sports and Music. Hence comment on your result at 5% level.

Parish	A	B	C	D	E	F	H	J	K	L
R_{sports}	1	8	4	3	9	2	5	6	7	10
R_{music}	4	8	3	1	10	2	7	5	6	9
d	-3	0	1	2	-1	0	-2	1	1	1
d^2	9	0	1	4	1	0	4	1	1	1
	$\sum d^2=22$									

$$\rho=1-\frac{6(22)}{10(100-1)}=0.8667$$

Since $0.8667>0.65$ it is significant at 5% level.

63. Two events A and B are independent such that their chance of occurring together is $\frac{1}{5}$ and the chance that either A or B occurs is $\frac{7}{8}$.

(a) Show that $A^I$and B^I are also independent

(b) Find the $P(A)$ and $P(B)$

(a) $P(A^I\cap B^I)=P(A^I)-P(A^I\cap B)$
$=P(A^I)-[P(B)-P(A\cap B)]$
$=P(A^I)-[P(B)-P(A).P(B)]$
$=P(A^I)-P(B)[1-P(A)]$
$=P(A^I)-P(B).P(A^I)$
$=P(A^I)[1-P(B)]$
$=P(A^I).P(B^I)$

(b) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$\frac{7}{8}=P(A)+P(B)-\frac{1}{5}$

$P(A) + P(B) = \frac{43}{40}$ but $P(A).P(B) = \frac{1}{5}$

$P(A) + \frac{1}{5P(A)} = \frac{43}{40}$ $\quad 5(P(A))^2 + 1 = \frac{43P(A)}{8}$

$40(P(A))^2 - 43P(A) + 8 = 0$ (show the working)

$P(A) = 0.8357$ or $P(A) = 0.2393$ the corresponding probabilities of B are

$P(B) = 0.2393$ or $P(B) = 0.8357$

64. In a group of East African Referee there are three from Uganda, Four from Tanzania and Five from Kenya. To officiate a tournament, three referees are chosen at random from the group. Calculate the probability that;

(a) A referee is chosen from each representative country

(b) exactly two referees are chosen from Tanzania

(c) the three referees are chosen from the same country

(a) $\frac{\binom{3}{1}\times\binom{4}{1}\times\binom{5}{1}}{\binom{12}{3}} = \frac{3}{11}$

(b) $\frac{\binom{3}{1}\times\binom{4}{2}\times\binom{5}{0}}{\binom{12}{3}} + \frac{\binom{3}{0}\times\binom{4}{2}\times\binom{5}{1}}{\binom{12}{3}} = \frac{12}{55}$

(c) $\frac{\binom{3}{3}}{\binom{12}{3}} + \frac{\binom{4}{3}}{\binom{12}{3}} + \frac{\binom{5}{3}}{\binom{12}{3}} = \frac{3}{44}$

66. Events K and L are such that $P(K^I \cap L) = 0.3$, $P(K \cup L) = 0.8$ and $P(K \cap L) = 0.2$. Find;

(i) $P(K^I \cup L)$

(ii) $P(K^I/L^I)$

(iii) Probability of K or L but not both.

(i) $P(K^I \cup L) = P(K^I) + P(L) - P(K^I \cap L)$

$P(K^I \cap L) = P(L) - P(K \cap L)$

$0.3 = P(L) - 0.2,$ $\quad P(L) = 0.5$

$P(K \cup L) = P(K) + P(L) - P(K \cap L)$

$0.8 = P(K) + 0.5 - 0.2$ $\quad P(K) = 0.5$

$P(K^I \cup L) = 0.5 + 0.5 - 0.3$

$P(K^I \cup L) = 0.7$

(ii) $P(K^I/L^I) = \frac{P(K^I \cap L^I)}{P(L^I)} = \frac{P(K \cup L)^I}{P(L^I)} = \frac{0.2}{0.5} = \frac{2}{5}$

(iii) $P(K^I \cap L) + P(L^I \cap K) = 0.3 + 0.3 = 0.6$

67. Given that a bag contains 6 Red pens, 3 Green pens and 7 Blue pens. Three pens are selected at random without replacement from the bag, find the probability that;

(i) all the three colours are represented.

(ii) the first is blue, the second is red and the third is green.

(a) $\frac{\binom{6}{1}\times\binom{3}{1}\times\binom{7}{1}}{\binom{16}{3}} = \frac{9}{40}$

(b) $\frac{7}{16}\times\frac{6}{15}\times\frac{3}{14} = \frac{3}{80}$

78. A discrete random variable Y has p.d.f given as;

$$f(y) = \begin{cases} ky & ; y = 1, 2 \\ k(6-y) & ; y = 3,4 \\ k & ; y = 5,6 \end{cases}$$

where k is a constant. Determine the;

(i) Value of k

(ii) $P(Y < 5/Y \geq 3)$

(iii) Probability distribution for a random variable X where $X = 2Y - 1$,Hence find E(X)

y	1	2	3	4	5	6
$f(y)$	k	$2k$	$3k$	$2k$	k	k
$f(y)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
X	1	3	5	7	9	11
$f(x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$xf(x)$	$\frac{1}{10}$	$\frac{6}{10}$	$\frac{15}{10}$	$\frac{14}{10}$	$\frac{9}{10}$	$\frac{11}{10}$

(i) $k + 2k + 3k + 2k + k + k = 1$ $\quad k = \frac{1}{10}$

(ii) $P(Y < 5/Y \geq 3) = \frac{P(Y<5)\cap(Y\geq3)}{P(Y\geq3)} = \frac{P(Y=3,4)}{P(Y=3,4,5,6)}$

$$\frac{\frac{3}{10}+\frac{2}{10}}{\frac{3}{10}+\frac{2}{10}+\frac{1}{10}+\frac{1}{10}} = \frac{5}{7}$$

(iii) $\sum xf(x) = \frac{1}{10}+\frac{6}{10}+\frac{15}{10}+\frac{14}{10}+\frac{9}{10}+\frac{11}{10} = \frac{56}{10}$

85. (i) Round off 8.00243 to 3 significant figures

$8.00243 \approx 8.00$

(ii) truncate 976800 to 3 significant figures.

$976800 \approx 976000$

87. The table below shows numbers and their squares

X	0.42	0.57	0.84	1.02
X^2	0.18	0.32	0.71	1.04

Use linear interpolation/ extrapolation to estimate;

(i) X when X^2 is 1.46

(ii) $\sqrt{(0.52)}$

(i)

0.84	1.02	y
0.71	1.04	1.46

$$\frac{y-1.02}{1.46-1.04} = \frac{1.02-0.84}{1.04-0.71}$$

$$y = 1.2491$$

0.57	x	0.84
0.32	0.52	0.71

$$\frac{x-0.57}{0.52-0.32} = \frac{0.84-0.57}{0.71-0.32} \quad x = 0.7086$$

90. (a) The dimensions of a rectangular plot of land are 30.26m and 14.45m. if the length and width have 5% and 3.2% errors respectively in estimates. Calculate the limits with in which the area of the plot lies, correct to two significant figures.

(b) the numbers $\boldsymbol{x}$ and $\boldsymbol{y}$ are approximations to X and Y respectively with errors $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ respectively. Show that the absolute relative error in the quotient $\sqrt{\frac{x}{y}}$ is given by $\frac{1}{2}\left(\left|\frac{e_1}{x}\right| + \left|\frac{e_2}{y}\right|\right)$.

(a) Area $l \times w$ $\quad e_l = \frac{5}{100} \times 30.26 = 1.513 \quad e_w = \frac{3.2}{100} \times 14.45 = 0.4624$

$A_{max} = (30.26 + 1.513) \times (14.45 + 0.4624) = 473.811685 \approx 470$

$A_{min} = (30.26 - 1.513) \times (14.45 - 0.4624) = 402.10153 \approx 400$

$[400, 470]$

$X = x + e_1, \qquad Y = y + e_2$

Let $z = \sqrt{\frac{x}{y}}$

$(z + e_z)^2 = \left(\sqrt{\frac{x+e_1}{y+e_2}}\right)^2$

$z^2 + 2ze_z + {e_z}^2 = \frac{x+e_1}{y+e_2}$

$$z^2 + 2ze_z + {e_z}^2 = \frac{(x + e_1)(y - e_2)}{(y + e_2)(y - e_2)}$$

$$z^2 + 2ze_z + {e_z}^2 = \frac{(xy - xe_2 + ye_1 - e_1e_2)}{(y^2 - {e_2}^2)}$$

But e_1, e_2 and e_z are very small, there fore, ${e_z}^2 \approx 0, {e_2}^2 \approx 0$ and $e_1e_2 \approx 0$ (assumption)

$$z^2 + 2ze_z = \frac{(xy - xe_2 + ye_1)}{(y^2)}$$

$$2ze_z = \frac{(-xe_2 + ye_1)}{(y^2)}$$

$$e_z = \frac{(-xe_2 + ye_1)}{(y^2)} \div 2\sqrt{\frac{x}{y}}$$

Absolute error $\quad |e_z| = \left|\frac{(-xe_2+ye_1)}{(y^2)} \times \frac{\sqrt{y}}{2\sqrt{x}}\right|$

Relative error $\quad \left|\frac{e_z}{z}\right| = \left|\frac{(-xe_2+ye_1)}{(y^2)} \times \frac{\sqrt{y}}{2\sqrt{x}} \div \sqrt{\frac{x}{y}}\right|$

$$\left|\frac{e_z}{z}\right| = \left|\frac{(-xe_2 + ye_1)}{(y^2)} \times \frac{y}{2x}\right|$$

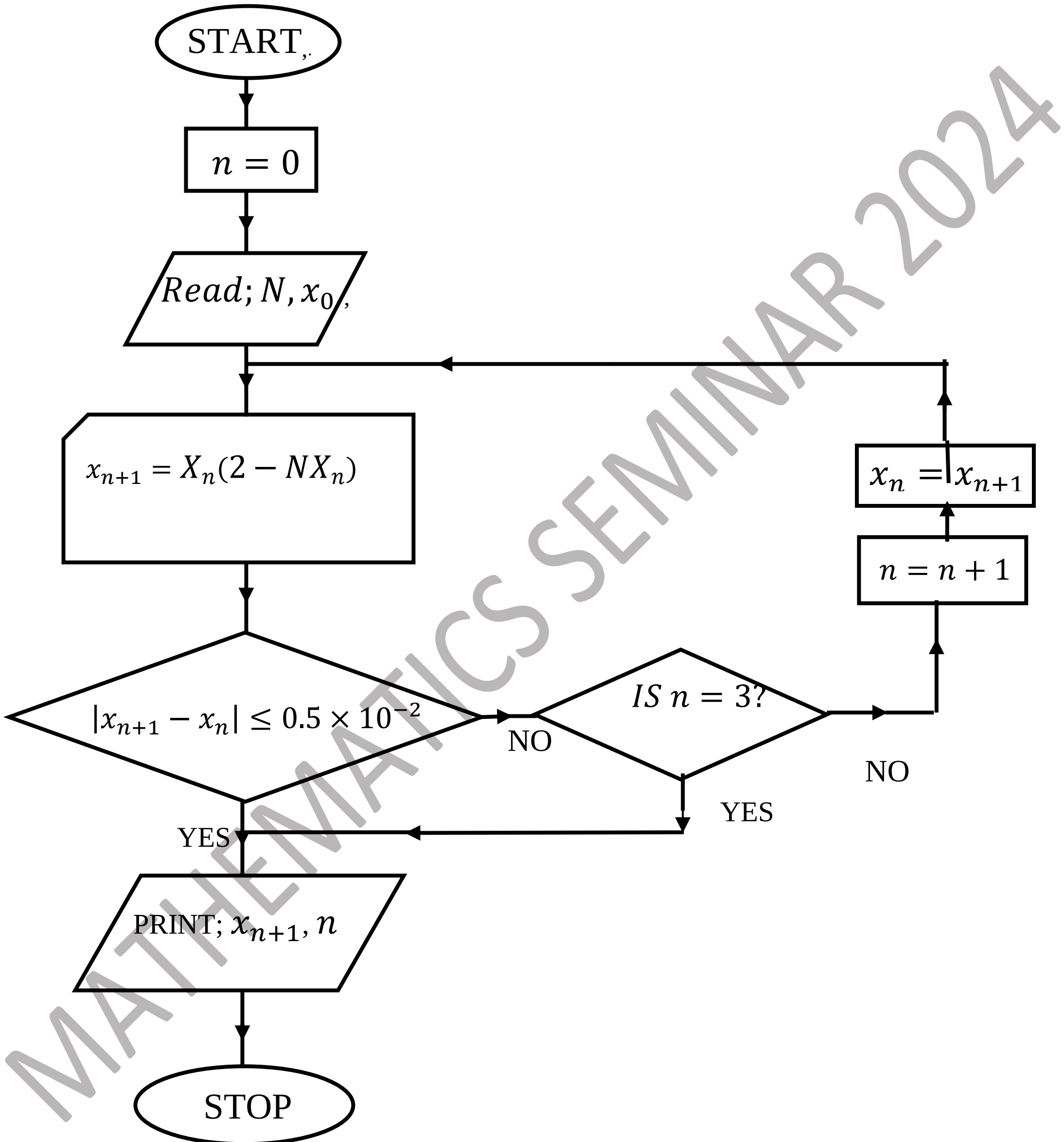
$$\left|\frac{e_z}{z}\right| = \left|\frac{(-xe_2 + ye_1)}{2(xy)}\right| \le \frac{1}{2}\left(\left|\frac{-e_2}{y}\right| + \left|\frac{e_1}{x}\right|\right)$$

$$= \frac{1}{2}\left(\left|\frac{e_1}{x}\right| + \left|\frac{e_2}{y}\right|\right)$$

91. (a) Show that the Newton Raphson formula for finding the reciprocal of a number N is given by: $X_{n+1} = X_n(2 - NX_n)\ ;\ n = 0,1,2,\ldots$

(b) Draw a flow chat that reads N and initial approximation X_0
Records the number of iterations n
Computes and prints N and its reciprocal to 2 decimal places after three iterations

(c) perform a dry run for $N = \frac{3}{2}$ and $X_0 = 0.5$

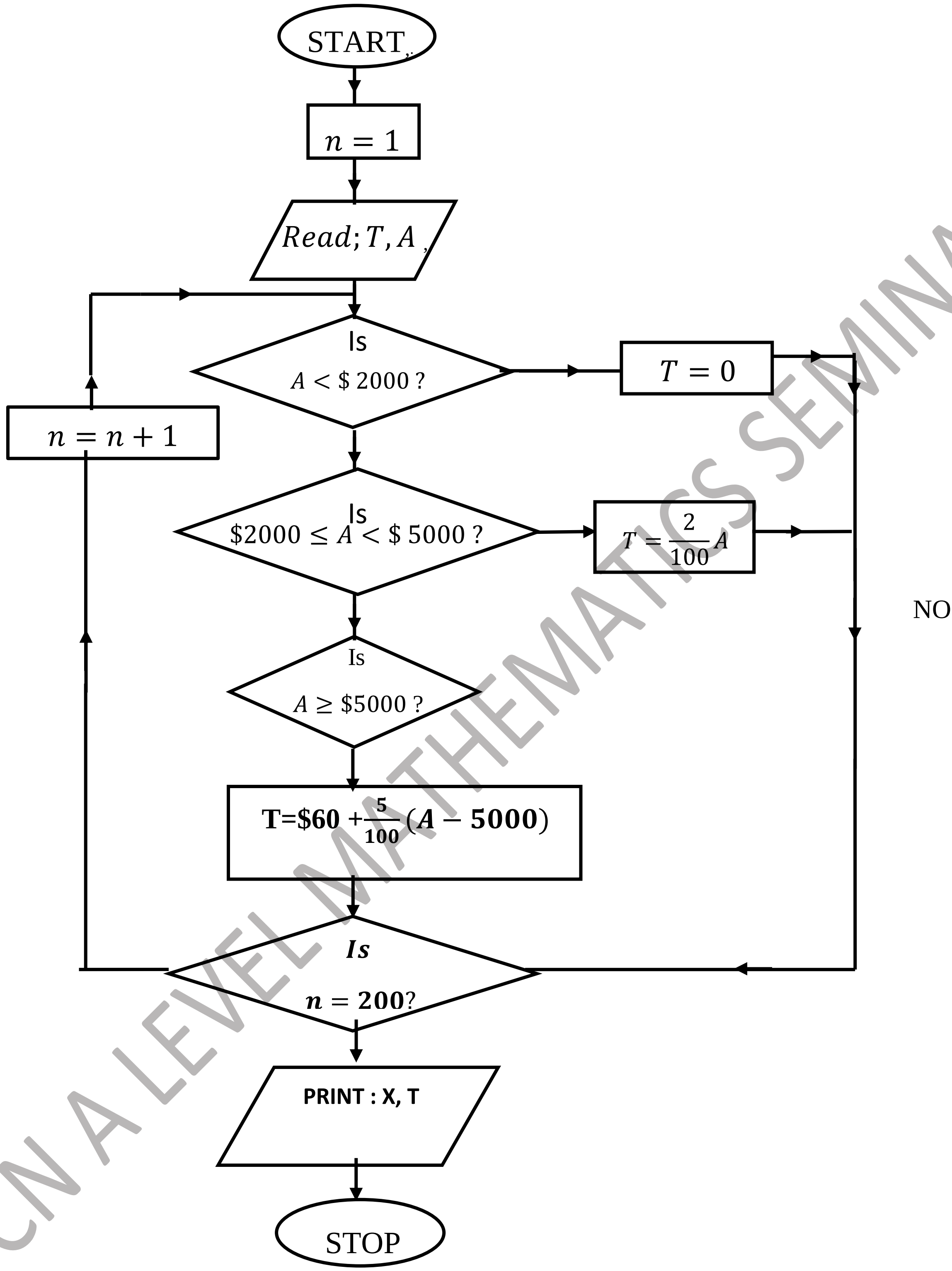


92. The information below gives the system of Tax (T) calculation for the amount of money (A) earned monthly by employees of a certain international company.

Monthly earnings (A)	**Tax(T)**
$A < \$\,2000$	zero
$\$\,2000 \leq A < \$\,5000$	2% of A
$A \geq \$\,5000$	\$ 60 plus 5% of the amount over \$ 5000

(a) draw a flow chart using the above data, given that the Algorithm stops when 200 counts (as) are made.

Calculate the Tax for an employee who earns \$ 6000 monthly.



Dry run

n	A	T
1	$ 6000	$ 110

$$T = \$60 + \frac{5}{100}(\$6000 - \$5000)$$

MECHANICS

101. A particle projected from a point O at angle of 50^o above the horizontal passed through the point P, with position vector $70\mathbf{i} + 28\mathbf{j}$. Find the

(i) initial velocity

(ii) time taken to reach P.

$$28 = 70\tan 50 - \frac{9.8 \times 70^2(1 + (\tan 50)^2)}{2(u)^2}$$

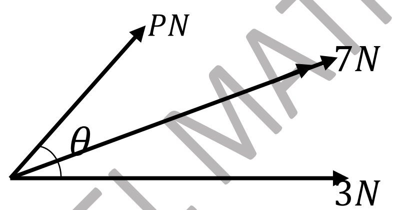
$u = 32.3806ms^{-1}$

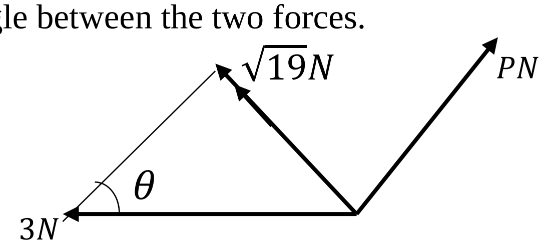
Time taken;

$70 = 32.3806\cos 50\, t$

$t = 3.3631s$

99. The resultant of two forces P N and 3N is 7N. If the 3N force is reversed, the resultant is $\sqrt{19}$N. Find the value of P and the angle between the two forces.





$7^2 = P^2 + 3^2 + 2 \times 3 \times P\cos\theta$

$\sqrt{19}^2 = P^2 + 3^2 - 2 \times 3 \times P\cos\theta$

$40 = P^2 + 6P\cos\theta$……..(i)

$10 = P^2 - 6P\cos\theta$……..(ii)

On adding (i)+(ii) $2P^2 = 50$

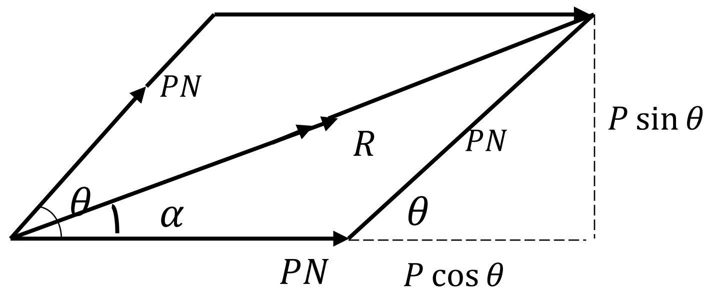
$P = 5N$

By subtracting (i)-(ii) $\cos\theta = \frac{1}{2}$ $\theta = 60°$

100. Two equal forces of magnitude PN act on a particle separated by angle θ.

(a) Show that their resultant force bisects the angle between them.

(b) Prove that their resultant is $2P\cos\frac{\theta}{2}$



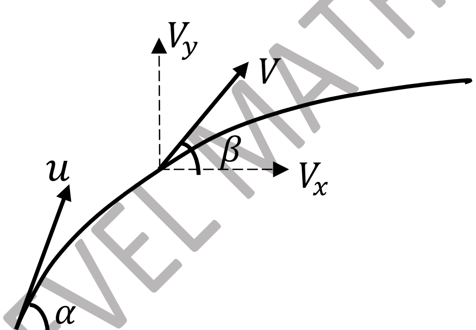
$\tan\alpha = \frac{P\sin\theta}{P + P\cos\theta}$

$\tan\alpha = \frac{P\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{P(1+\cos\theta)}$ $\quad\tan\alpha = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{(1+2\cos^2\frac{\theta}{2}-1)}$ $\quad\tan\alpha = \tan\frac{\theta}{2}$ $\quad\alpha = \frac{\theta}{2}$

(a) By cosine rule $R^2 = (P + P\cos\theta)^2 + (P\sin\theta)^2$

$$R^2 = P^2 + 2PP\cos\theta + (P\cos\theta)^2 + (P\sin\theta)^2$$
$$R^2 = 2P^2 + 2P^2\cos\theta$$
$$R^2 = 2P^2(1+\cos\theta)$$
$$R^2 = 2P^2(1 + 2\cos^2\frac{\theta}{2} - 1)$$
$$R = 2P\cos\frac{\theta}{2}$$

102. A particle is projected from a point O on level ground at an angle of elevation $\boldsymbol{\alpha}$ and while still raising it passes through Point P with speed V at which point its elevation is $\boldsymbol{\beta}$. Prove that the time taken to reach point P is; $\frac{\boldsymbol{V\sin(\alpha-\beta)}}{\mathbf{g}\boldsymbol{\cos\alpha}}$



$V_x = V\cos\beta \qquad V_y = V\sin\beta$

$V_x = u\cos\alpha \qquad V_y = u\sin\alpha - gt$

$u = \frac{V_x}{\cos\alpha} \qquad t = \frac{u\sin\alpha - V_y}{g}$

$u = \frac{V\cos\beta}{\cos\alpha} \qquad t = \frac{\frac{V\cos\beta}{\cos\alpha}\sin\alpha - V\sin\beta}{g} \qquad = \frac{\boldsymbol{V\sin(\alpha-\beta)}}{\mathbf{g}\boldsymbol{\cos\alpha}}$

103. Find the position vector of the centre of gravity of particles of masses 5kg, 2kg, 4kg and 3kg, situated at points (3,1), (4,3), (5,2) and (-3,1) respectively.

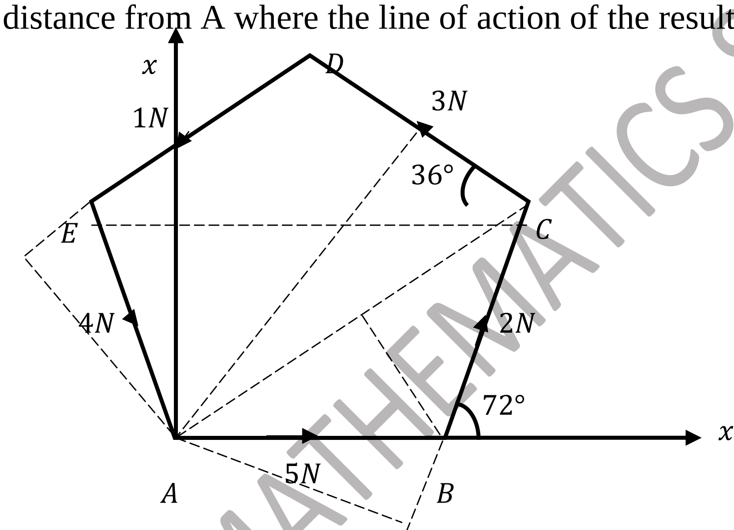
$$14g\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 5g\begin{pmatrix}3\\1\end{pmatrix} + 2g\begin{pmatrix}4\\3\end{pmatrix} + 4g\begin{pmatrix}5\\2\end{pmatrix} + 3g\begin{pmatrix}-3\\1\end{pmatrix}$$
$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}2.4286\\1.5714\end{pmatrix}$$

104. A regular pentagon ABCDE of side 2m is subjected to forces of magnitude 5N, 2N, 3N, 1N, and 4N acting along AB, BC, CD, DE, and EA respectively. The directions are indicated by the order of the letters. Taking AB as the reference positive x-axis,

(a) Determine the magnitude and direction of the resultant force.

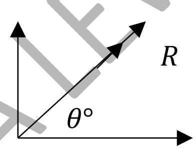
(b) Find the equation of the line of action of the resultant force.

(c) distance from A where the line of action of the resultant cuts AB.



$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}5\\0\end{pmatrix} + \begin{pmatrix}2\cos 72\\2\sin 72\end{pmatrix} + \begin{pmatrix}-3\cos 36\\3\sin 36\end{pmatrix} + \begin{pmatrix}-1\cos 36\\-1\sin 36\end{pmatrix} + \begin{pmatrix}4\cos 72\\4\sin 72\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3.6180\\6.8819\end{pmatrix}$$



$$R = \sqrt{3.6180^2 + 6.8819^2}$$
$$R = 7.7750N$$

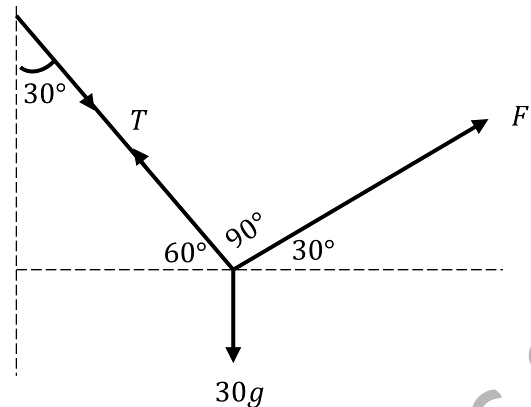
$\theta = \tan^{-1}\left(\frac{6.8819}{3.618}\right) = 62.27\,°$ $\quad N\ 27.73°E$

$$G = 2\times 2\sin 72 + 3\times 2\sin 54\times 2\times \sin 72 + 1\times 2\sin 72 = 14.9394Nm$$

$G = \begin{vmatrix}x & y\\ X & Y\end{vmatrix}$ $\quad 14.9394 = 6.8819x - 3.618y$ or $y = 1.9021x - 4.1292$

It meets line AB when $y = 0$ $\quad\quad$ $x = 2.1708$ AB produced.

105. A particle of mass 30kg is attached to one end of a light inextensible string whose other end is fixed. The particle is pulled a side by a force F, which is at right angles to the string so that at the position of equilibrium, the string makes an angle of $30°$ with the vertical. Find the magnitude of the force F and the tension in the string.

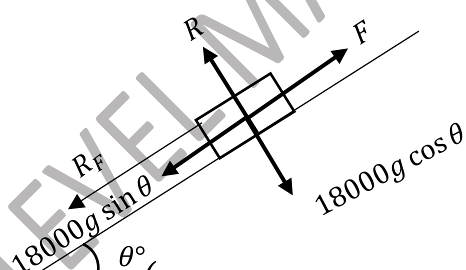


By Lami's theorem

$\frac{T}{\sin 120°} = \frac{30(9.8)}{\sin 90°}$ $\quad\quad$ $T = 254.6115N$

$\frac{F}{\sin 150°} = \frac{30(9.8)}{\sin 90°}$ $\quad\quad$ $T = 147N$

106. A bus of mass 18tonnes travels up a slope inclined at $\sin^{-1}\left(\frac{1}{50}\right)$ against a resistance of 0.1N per kilogram. Find the tractive force required to produce an acceleration of $0.05ms^{-2}$ and the power which is developed when the speed is $10ms^{-1}$.



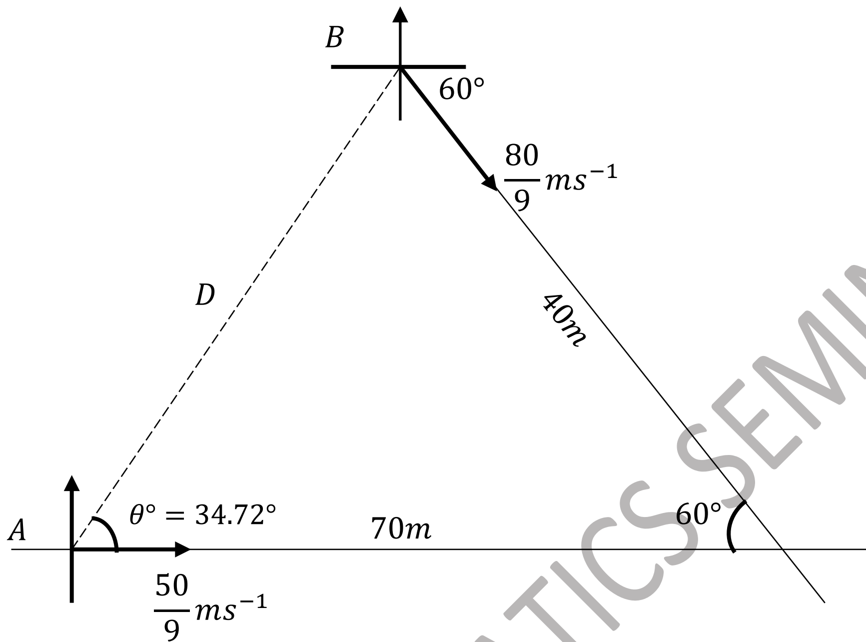
$T - \left(0.1 \times 18000 + 18000(9.8) \times \frac{1}{50}\right) = 18000(0.05)$

$T = 6228N$

$P = FV = 6228 \times 10 = 62280W$

108. Two cars A and B are proceeding one on each road, towards the point of intersection of two roads which meet at an angle of 60^0. If the speeds of A and B are $20kmh^{-1}$and $32kmh^{-1}$ and are 70m and 40m respectively from the cross road, and the cars maintain their speeds, determine the;

(i) speed of B relative to A.
(ii) time when they are nearest to each other.
(iii) the distance of B from the cross road when they are nearest to each other.



$D^2 = 40^2 + 70^2 - 2(40)(70)\cos 60°$
$D = 60.8276m$

$$\frac{40}{\sin\theta} = \frac{60.8276}{\sin 60°}$$

$\theta = 34.72°$

$$V_A = \begin{pmatrix} \frac{50}{9} \\ 0 \end{pmatrix} ms^{-1} \qquad V_B = \begin{pmatrix} \frac{80}{9}\cos 60° \\ -\frac{80}{9}\sin 60° \end{pmatrix} ms^{-1}$$

(i) $\quad {}_{B}V_{A} = V_B - V_A$

$$= \begin{pmatrix} \frac{80}{9}\cos 60° \\ -\frac{80}{9}\sin 60° \end{pmatrix} - \begin{pmatrix} \frac{50}{9} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{-10}{9} \\ -7.6980 \end{pmatrix}$$

Speed = $|relative\ velocity| = \sqrt{\left(\frac{-10}{9}\right)^2 + (-7.698)^2} = 7.7778ms^{-1}$

$r_{OA} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad r_{OB} = \begin{pmatrix} 60.8276\cos 34.72 \\ 60.8276\sin 34.72 \end{pmatrix} = \begin{pmatrix} 49.9970 \\ 34.6454 \end{pmatrix}$

$${}_{B}r_{A} = \begin{pmatrix} 49.9970 \\ 34.6454 \end{pmatrix} + \begin{pmatrix} \frac{-10}{9} \\ -7.6980 \end{pmatrix} t$$

$$= \begin{pmatrix} 49.9970 - \frac{10}{9}t \\ 34.6454 - 7.6980t \end{pmatrix}$$

(ii) From ${}_B\mathbf{r}_A \bullet {}_B\mathbf{V}_A = 0$

$$\begin{pmatrix} 49.9970-\frac{10}{9}t \\ 34.6454-7.6980t \end{pmatrix} \cdot \begin{pmatrix} \frac{-10}{9} \\ -7.6980 \end{pmatrix} = 0$$

$-55.5522 + 1.2346t - 266.7003 + 59.2592t = 0$

$60.4938t = 322.2525$

$t = 5.3270$ seconds

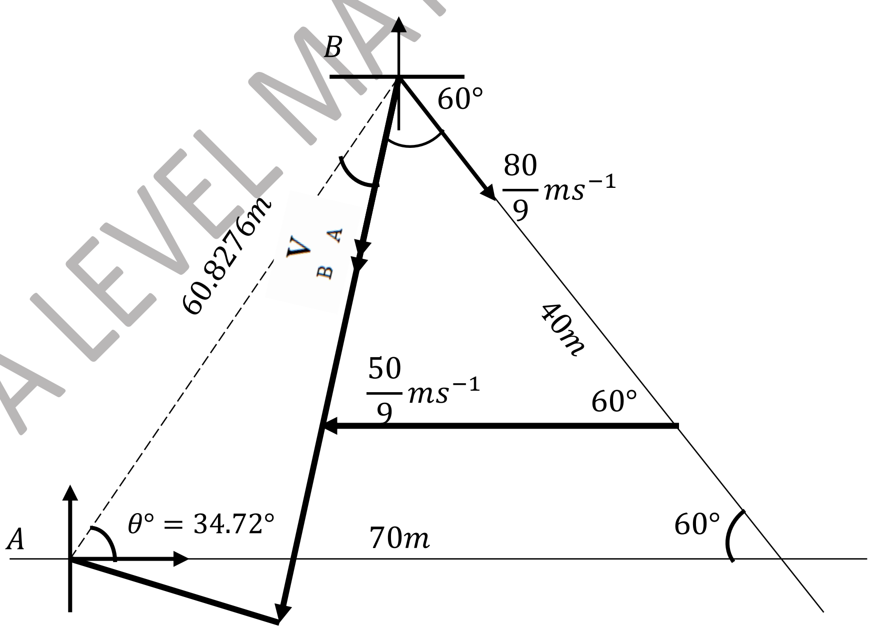
Least distance ${}_B\mathbf{r}_A = \begin{pmatrix} 49.9970-\frac{10}{9}(5.327) \\ 34.6454-7.6980(5.327) \end{pmatrix} = \begin{pmatrix} 44.0781 \\ -6.3618 \end{pmatrix}$

Least distance $= \sqrt{44.0781^2 + (-6.3618)^2} = 44.5348$m

$$r_{B} = \begin{pmatrix} 49.9970 \\ 34.6454 \end{pmatrix} + \begin{pmatrix} \frac{80}{9}\cos 60° \\ -\frac{80}{9}\sin 60° \end{pmatrix} \times 5.327 = \begin{pmatrix} 73.6726 \\ -6.3619 \end{pmatrix}$$

(iii) Distance of B $= |r_B| = \sqrt{73.6726^2 + (-6.3619)^2} = 73.9468$m
Distance of B from cross road $= 73.9468 - 40 = 33.9468$m.

ALTERNATIVE



$${}_{B}V_{A}^{\,2} = \left(\frac{50}{9}\right)^2 + \left(\frac{80}{9}\right)^2 - 2\left(\frac{80}{9}\right)\left(\frac{50}{9}\right)\cos 60°$$

$${}_{B}V_{A} = 7.7778ms^{-1}$$

$$D^2 = 40^2 + 70^2 - 2(40)(70)\cos 60°$$

$$D = 60.8276m$$

$$\frac{40}{\sin\theta} = \frac{60.8276}{\sin 60°} \qquad \theta = 34.72°$$

$$\frac{\frac{50}{9}}{\sin\alpha} = \frac{7.7778}{\sin 60°}$$

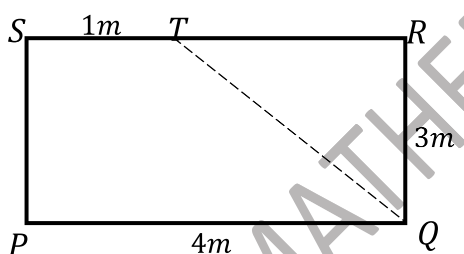
$$\alpha = 38.21°$$

$$(85.28 - 38.21) = 47.07°$$

Least distance$= 60.8276\sin 47.07° = 44.5371m$

Time $\frac{60.8275\cos 47.07}{7.7778} = 5.3267s$

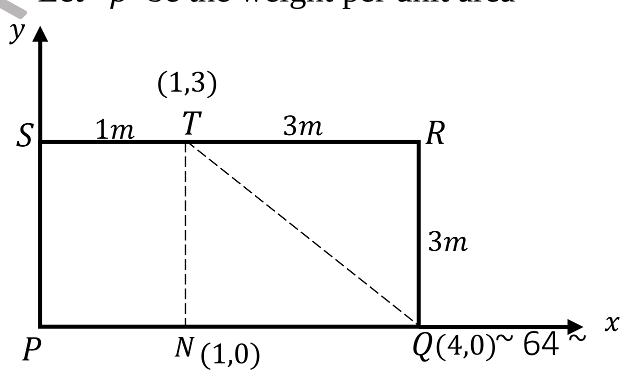
114. The diagram below shows a rectangular sheet PQRS. Point T is on the line RS such that $ST = 1m$



(a) If the sheet is folded a long QT until RQ lies along PQ. Find the centre of gravity of the folded sheet, hence find the angle of inclination PS makes with the verticle if the sheet is freely suspended at S by means of a thread and hangs in equilibrium.

(b) If the sheet is placed in a vertical plane with ST resting on a horizontal table, determine whether it topples.

(a) Let ρ be the weight per unit area



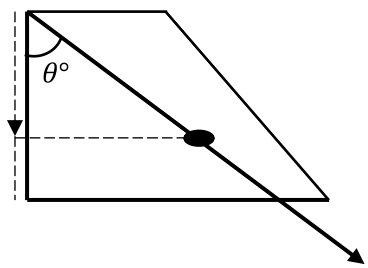
C.O.G of NQT $\left[\frac{1}{3}(1+1+4), \frac{1}{3}(0+0+3)\right]$

Body	Area(m^2)	Weight	c.o.g
PNTS	3	3ρ	(0.5,1.5)
NQT (folded)	4.5× 2	9ρ	(2,1)
PQTS(Total)	12	12ρ	$(\bar{x}, \bar{y})$

taking moments about the coordinate axes; $12\rho\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 3\rho\begin{pmatrix}0.5\\ 1.5\end{pmatrix} + 9\rho\begin{pmatrix}2\\ 1\end{pmatrix}$

$\bar{x} = 1.625 \bar{y} = 1.125$

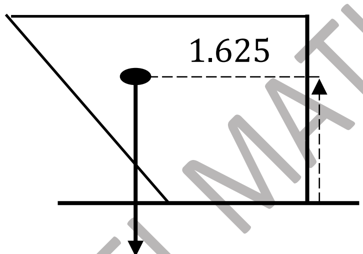
(b)



$$tan\theta = \frac{\bar{x}}{(3-\bar{y})}$$

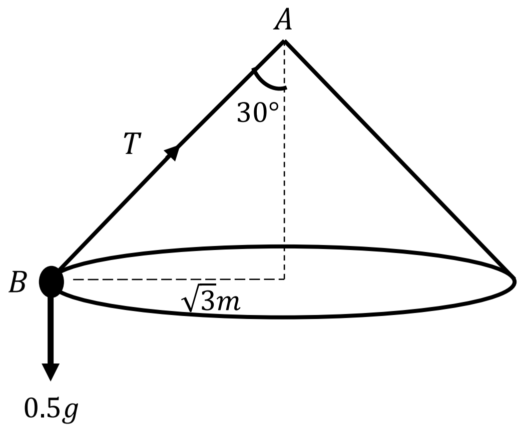
$$\theta = \tan^{-1}\left(\frac{1.625}{(3-1.125)}\right) = 40.91°$$

(c)



Since $\bar{x} = 1.625m > ST = 1m$ The sheet topples.

116. A conical pendulum consists of a light inextensible string AB carrying a particle of mass 0.5kg at the end B. The particle moves in a horizontal circle of radius $\sqrt{3}m$ with the centre vertically bellow A. if the angle between the string and the vertical is 30°, find the tension in the string and the angular speed of the particle.



$T\cos 30° = 0.5(9.8)$

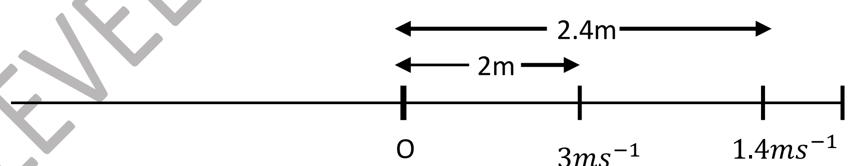
$T = 5.6580N$

$T\sin 30° = m\frac{V^2}{r}$

$T\sin 30° = 0.5\frac{V^2}{\sqrt{3}}$ $\qquad V = wr$

$V = 3.1305\text{ms}^{-1}$ $\qquad w = 1.8074\text{rads}^{-1}$

117. A particle moves with Simple Harmonic Motion about the mid position O. when passing two points which are 2m and 2.4m from O, the particle has speed $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the Amplitude of the motion and the greatest speed attained by the particle.



From $\quad V^2 = w^2(r^2 - x^2)$ where r is amplitude , x is displacement, V is velocity

$1.4^2 = w^2(r^2 - 2.4^2)$ ....(i) $\qquad 3^2 = w^2(r^2 - 2^2)$............(ii)

Dividing (ii) by (i) $\quad \frac{9}{1.4^2} = \frac{(r^2-4)}{(r^2-2.4^2)}$

$9r^2 - 9(2.4^2) = 1.4^2(r^2) - 4(1.4^2)$

$r = 2.5$m

Greatest speed $V = wr$

$3^2 = w^2(2.5^2 - 2^2)$
$w = 2 \text{ rad}s^{-1}$
$V_{max} = 2 \times 2.5 = 5ms^{-1}$

118. A car with uniform velocity of $20ms^{-1}$ is accelerated to attain velocity of $33.6ms^{-1}$. It maintained this velocity for a period which is 4 times the time taken to bring it to rest. If it is brought to rest in a period which is seven-tenth of the time the car is in acceleration, and the total time taken by the car in motion is 11 ¼ minutes;

(i) determine the time to accelerate from $20ms^{-1}$ to $33.6ms^{-1}$ and the time for constant velocity.

(ii) draw a velocity-time graph for the motion of the car

Determine the total distance covered.

$u = 20ms^{-1},\ V = 33.6ms^{-1},\ T = 11\frac{1}{4} minutes\ = 675s$

Let the time under acceleration be t

The time under deceleration is $= \frac{7}{10}t$

The time under constant velocity $= 4 \times \frac{7}{10}t = 2.8t$

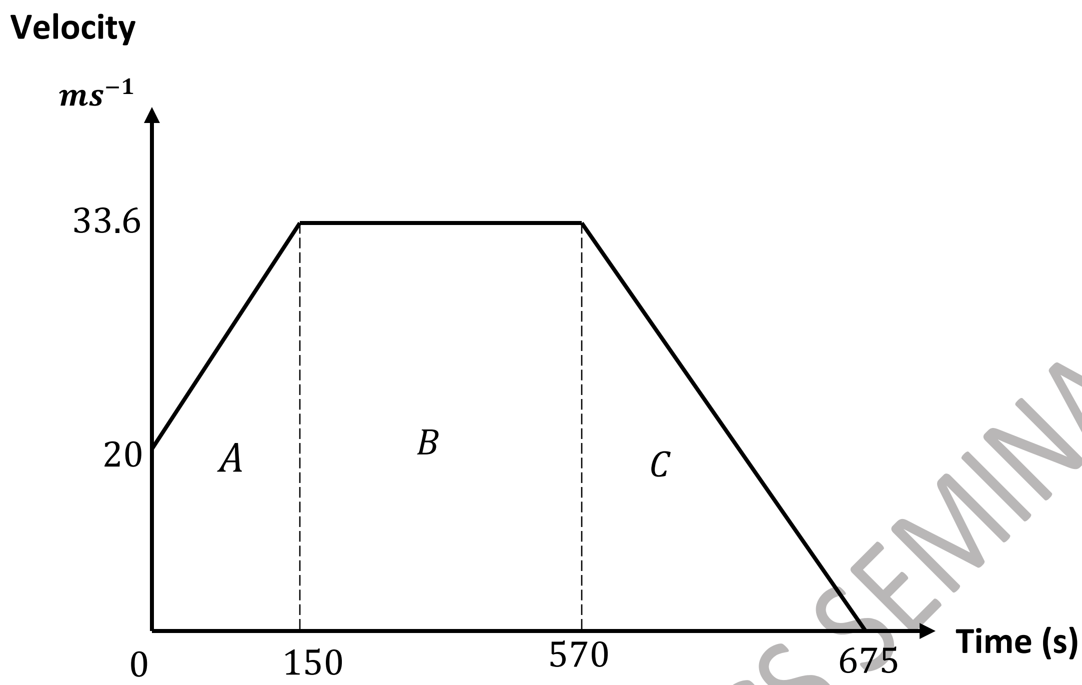
Total time $= 675s$

$t + \frac{7}{10}t + 2.8t = 675$

$t = 150s$

Time under constant velocity $= 2.8 \times 150 = 420s$

Velocity-time graph



Area A $= \frac{1}{2}(150)(20 + 33.6) = 4020m$

Area B $= 420 \times 33.6 = 14112m$

Area C $= \frac{1}{2}(105)(33.6) = 1764m$

Total distance covered$= A + B + C = 4020 + 14112 + 1764 = 19896m$

END

Practice makes perfect.