S475/1
SUBSIDIARY MATHEMATICS
Paper 1
July 2024
$2\frac{2}{3}$ hours

Uganda Advanced Certificate of Education

SUBSIDIARY MATHEMATICS

Paper 1

2 hours 40 minutes

INSTRUCTIONS TO CANDIDATES:

Attempt all **eight** *questions in* Section **A** *and any* **four** *from* Section **B with** atleast one question from each part. *Any additional question(s) will not be marked.*

All necessary working must clearly be shown.

Graph papers are provided.

Silent, non-programmable Scientific Calculators and Mathematical tables with a list of formulae may be used.

SECTION A

Answer **all** *questions in this* Section.

1. Events A and B are independent such that $P(A) = {}^{3}/_{8}$ and $P(A^{1} \cup B) = {}^{3}/_{4}$ Find the

(i) P(B) *(03 marks)*

(ii) P(AUB) *(02 marks)*

2. Solve for n in ${}^{n}C_{4} = {}^{n}C_{2}$ *(05 marks)*

3. If $5\log_{x}(10 + 3x) = 10$, find the value of x. *(05 marks)*

4. In 2014, the unit price of salt, price and cooking oil was 1600, 4200 and 3600 respectively. Given that the unit price in 2015 was P, 6800 and 3200 respectively and the simple aggregate price index was 125, find the value of P. *(05 marks)*

5. The table below shows the number of text books owned by 10 students of a certain class and their total marks in an exam.

Student	A	B	C	D	E	F	G	H	I	J
Number of text books	5	8	2	9	7	5	3	10	1	4
Total marks in an exam	290	370	184	366	277	190	385	200	281	331

Calculate the rank correlation coefficient between the number of text books and the total marks. Comment on your result at 5% level of significance. *(05 marks)*

6. Given the vectors $\underset{\sim}{p} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$, $\underset{\sim}{q} = 4\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{r} = \underset{\sim}{i} + 2\underset{\sim}{j}$, find the length of the vector $\underset{\sim}{p} - 4\underset{\sim}{q} + 3\underset{\sim}{r}$. *(05 marks)*

7. Three bags X, Y and Z, each contain black, red and blue pens as follows:

	Black pens	Red pens	Blue pens
Bag X	3	1	3
Bag Y	2	3	3
Bag Z	5	6	4

A bag is chosen at random and then a pen is randomly picked from the selected bag. Determine the probability that the pen picked is:

(i). a blue pen,

(ii). Not a blue pen. *(05 marks)*

8. Solve the equation; $3^{2x} - 3^{x} - 6 = 0$. *(05 marks)*

SECTION B (60 MARKS)

Attempt four questions with at least one question from each part

PART ONE: PURE MATHEMATICS

9. (a) Given that $= \frac{1-\sin\theta}{1+\sin\theta}$, show that $P = (\sec\theta - \tan\theta)^2$. Hence deduce that if $\theta = 60°$, then $P = 7 - 4\sqrt{3}$. *(08 marks)*

(b). Solve the equation $3\cos^2 x - 2\sin^2 x - \sin x + 1 = 0$ for $0° \leq x \leq 360°$. *(07 marks)*

10. Given that curve $y = x^2 + 2x - 8$.

(a) Find the intercepts.

(b) Find the turning point and distinguish it.

(c) Sketch the curve and hence find the area enclosed by the curve $y = x^2 + 2x - 8$and the x – axis. *(15 marks)*

11. A hot body at a temperature of 100^0C is placed in a room of temperature of 20^0C. Ten minutes later, its temperature is 60^0C.

(i) Write down a differential equation to represent the rate of change of temperature, θ of the body with time, t.

(ii) Determine the temperature of the body after another 10 minutes.

(15 marks)

12. A club has x Regular members and y Gold members. Regular membership costs $shs25,000$ per year and Gold membership costs $shs\ 75,000$ per year. The club needs to collect at least $shs\ 1,500,000$ in membership fees each year. The club has a maximum of 50 members, and must have at least 15 regular members.

(a) Write down five inequalities which the club must satisfy.

(b) Represent the inequalities on the graph paper by shading the unwanted regions. (Use a scale of *1cm to represent 5 units* on both axes)

(c) Find the maximum amount the club can expect to receive in membership fees each year, and the number of Regular and Gold members that are needed to achieve this amount. *(15 marks)*

PART TWO: STATISTICS

13. The heights of 1500 students are normally distributed with mean 175cm and variance of $400cm^2$. Determine the number of students whose heights were

(i) between 140 cm and 160cm *(06 marks)*

(ii) greater than 205cm *(03 marks)*

(iii) less than 215 cm *(03 marks)*

14. A market stall holder sells clothes on three days a week – Tuesday (T), Friday (F), and Saturday (S) . Her takings over a five week period were as follows;

Week	1			2			3			4			5		
Day	T	F	S	T	F	S	T	F	S	T	F	S	T	F	S
Takings £	196	210	343	267	274	336	168	279	315	160	258	310	154	240	312

(a) On the same axes represent the data and the three days moving averages

(b) Closing the graph, suggest the day when a nearby clothes stall was closed.

(c) Predict the takings on Tuesday of week 6.

15. A certain aptitude test has 10 statements that require a candidate to respond by writing true or false. A candidate passes if he or she scores at least eight questions correct;

(a) Find the probability that;

(i) A candidate gets exactly 5 questions correct

(ii) A candidate passes the test *(09 marks)*

(b) Calculate the expected number and standard deviation of the correctly answered questions. *(06 marks)*

16. The marks scored by candidates in a submaths exam were as follows;

64	74	78	59	67	55	61	54
80	58	76	58	74	65	63	83
72	60	71	52	61	57	68	69
62	73	64	59	62	53	81	68
50	75	67	53	80	77	60	71

(a) Construct a grouped frequency table for the data using equal classes of width 4 marks starting with 50 – 53 as the first class. *(02 marks)*

(b) State the;
   (i) Median class
   (ii) Modal class and its frequency. *(03 marks)*

(c) Calculate the;
   (i) Mean mark
   (ii) Standard deviation *(10 marks)*

END

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Question 1

$P(A' \cap B) = P(A') \times P(B)$

$\frac{3}{4} = \left(1 - \frac{3}{8}\right) \times P(B)$

$P(B) = \frac{3}{4} \div \frac{5}{8}$

$P(B) =$

Question 1

(i) $P(A') = 1 - P(A)$

$= 1 - \frac{3}{8}$

$= \frac{5}{8}$

$P(A' \cup B) = P(A') + P(B) - P(A')P(B)$ M1

$\frac{3}{4} = \frac{5}{8} + P(B) - \frac{5}{8}P(B)$ M1

$\frac{1}{8} = \frac{3}{8}P(B)$

$P(B) = \frac{1}{3}$ A1

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{8} + \frac{1}{3} - \left(\frac{3}{8} \times \frac{1}{3}\right)$ M1

$= \frac{17}{24} - \frac{1}{8}$

$= \frac{7}{12}$ A1

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Question 2

$${}^nC_4 = {}^nC_2$$

$$\frac{n!}{(n-4)!\,4!} = \frac{n!}{(n-2)!\,2!} \quad M_1$$

$$\frac{1}{(n-4)!\times 4\times 3\times 2!} = \frac{1}{(n-2)(n-3)(n-4)!\times 2!}$$

$$\frac{1}{12} = \frac{1}{(n-2)(n-3)} \quad M_1$$

$$(n-2)(n-3) = 12 \quad M_1$$

$$n^2 - 5n + 6 - 12 = 0.$$

$$n^2 - 5n - 6 = 0$$

$$n = 6, \; n = -1 \quad B_1$$

$$\therefore n = 6 \quad A_1$$

Question 3

$$\log_x (10+3x) = 2$$

$$x^2 = 10 + 3x. \quad M_1$$

$$x^2 - 3x - 10 = 0. \quad M_1$$

$$x^2 + 2x - 5x - 10 = 0$$

$$x(x+2) - 5(x+2) = 0.$$

$$(x-5)(x+2) = 0. \quad M_1$$

$$x - 5 = 0, \quad x = 5 \quad B_1$$

$$x + 2 = 0 \quad x = -2$$

$$\therefore x = 5 \quad A_1$$

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Question 4

Simple Aggregate Price Index: $\frac{\sum P_{2015}}{\sum P_{2014}} \times 100$

$125 = \frac{P + 6800 + 3200}{1600 + 4000 + 3600} \times 100$ M1

M1

$\frac{125}{100} = \frac{P + 10,000}{9400}$ M1

$11,750 = P + 10,000$

$P = 11,750 - 10,000$ M1

$P = 1,750$ A1.

Question 5.

Student	Number of text books(N)	Total Marks	R_N	R_T	$d=(R_N-R_T)$	d^2
A	5	290	5.5	6	-0.5	0.25
B	8	370	8	9	-1	1
C	2	184	2	1	1	1
D	9	366	9	8	1	1
E	7	277	7	4	3	9
F	5	190	5.5	2	3.5	12.25
G	3	385	3	10	-7	49
H	10	200	10	3	7	49
I	1	281	1	5	-4	16
J	4	331	4	7	-3	9

B1 $\sum d^2 = 147.5$ B1

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$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 147.5}{10(10^2-1)} \quad M_1$$

$$= 1 - \frac{885}{990}$$

$$= 1 - 0.89$$

$$\rho_0 = 0.11 \quad A_1$$

At 5% significance $\rho_1 =$

Since ρ_0 ρ_1 ∴ there B_1

Question 6

$$\overrightarrow{OP} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad \overrightarrow{OQ} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad \overrightarrow{OR} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\underset{\sim}{p} - 4\underset{\sim}{q} + 3\underset{\sim}{r} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + 4\begin{pmatrix} 4 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad M_1$$

$$= \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 16 \\ 8 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad M_1$$

$$= \begin{pmatrix} -10 \\ -4 \end{pmatrix} \quad B_1$$

length $= \sqrt{(-10)^2 + (-4)^2}$ M_1

$= \sqrt{116}$

$= 10.770$ units A_1

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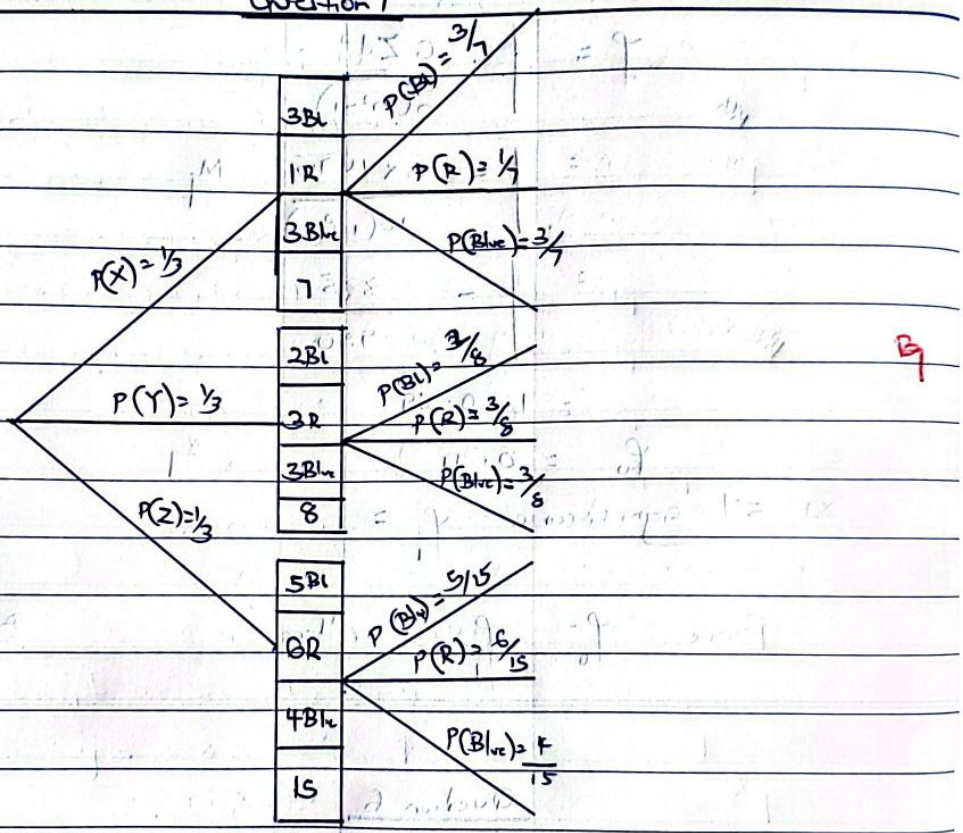
Question 7



B1

$P(\text{Blue}) = \left(\frac{3}{7} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{3}{8}\right) + \left(\frac{1}{3} \times \frac{4}{15}\right)$ M1

$= \frac{1}{7} + \frac{1}{8} + \frac{4}{45}$

$= \frac{899}{2520}$ A1

P (Not a blue pen) $= 1 - \frac{899}{2520}$ M1

$= \frac{1621}{2520}$ A1

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Question 8

Let $3^x = m$

$3^x \cdot 3^x - 3^x - 6 = 0$

$m \cdot m - m - 6 = 0$

$m^2 - m - 6 = 0$ M_1

$m^2 + 2m - 3m - 6 = 0$

$m(m+2) - 3(m+2) = 0$

$(m-3)(m+2) = 0$ M_1

$m - 3 = 0$ $\quad m + 2 = 0$ for 3

$m = 3$ $\quad m = -2$ B_1 B_1

$3^x = 3^1$ M_1

$x = 1$ A_1

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Question 10

(a) x - intercept occurs when $y = 0$.

$x^2 + 2x - 8 = 0$. M1

$x^2 - 2x + 4x - 8 = 0$.

$x(x-2) + 4(x-2) = 0$

$(x+4)(x-2) = 0$

$(x+4) = 0$

$x = -4$

or $x - 2 = 0$

$x = 2$

$\therefore (-4, 0)$ and $(2, 0)$ A1

y - intercept occurs when $x = 0$.

$y = -8$

$\therefore (0, -8)$ A1

(b) $\frac{dy}{dx} = 2x + 2$ M1

At turning point.

$\frac{dy}{dx} = 0$. M1 B1

$2x + 2 = 0$. M1

$2x = -2$

$x = -1$.

$y = (-1)^2 + 2(-1) - 8$

$= -9$ B1

$\therefore (-1, -9)$ B1

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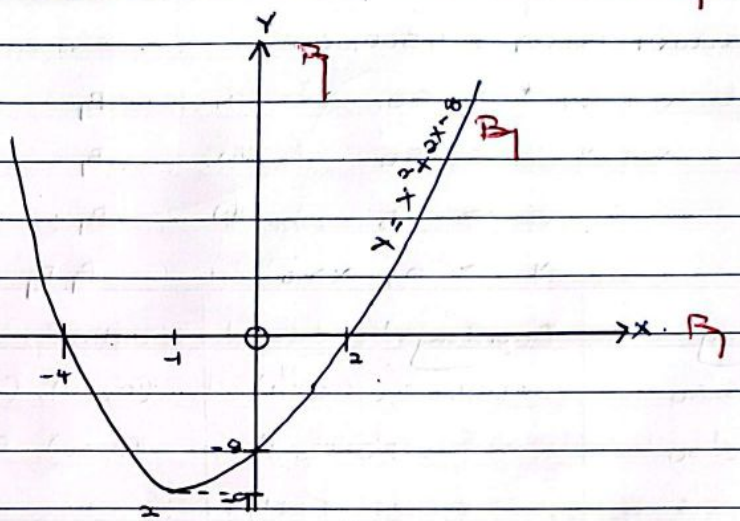
$\frac{d^2y}{dx^2} = 2$ (positive

$\therefore (-1, -9)$ is a minimum. A1



Area $= \int_{-4}^{2} (x^2 + 2x - 8)\,dx$. M1

$= \left[\frac{x^3}{3} + x^2 - 8x\right]_{-4}^{2}$ A1 M1

$= \left[\left(\frac{2^3}{3} - 2^2 - 8(2)\right) - \left(\frac{(-4)^3}{3} - (-4)^2 - 8(-4)\right)\right]$ M1

$= -36$ square units.

$\therefore$ the area is 36 square units below the x-axis. A1

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Question 12

(a) Let the number of Regular members be x.

Let the number of Gold members be Y.

$25{,}000x + 75{,}000y \geq 1{,}500{,}000$ ✓

$x + 3y \geq 60$ --- (i). B_1

$x + y \leq 50$ --- (ii). B_1

$x \geq 15$ --- (iii) B_1

$y \geq 0, \quad x \geq 0$ $B_1 B_1$

(b)

Inequality	Boundary line	Coordinates.
$x+3y \geq 60$	$x+3y = 60$ (solid)	(0, 20), (60, 0)
$x+y \leq 50$	$x+y = 50$ (solid)	(0, 50), (50, 0).
$x \geq 15$	$x = 15$ (solid)	

B_1 B_1

(c)

(x, y)	25,000x + 75,000y
(15, 35)	$25{,}000(15) + 75{,}000(35) =$ M_1 3,000,000 B_1
(20, 30)	$25{,}000(20) + 75{,}000(30) =$ 2,750,000
(25, 25)	$25{,}000(25) + 75{,}000(25) =$ 2,500,000.
(30, 20)	$25{,}000(30) + 75{,}000(20) =$ 2,250,000

B_1

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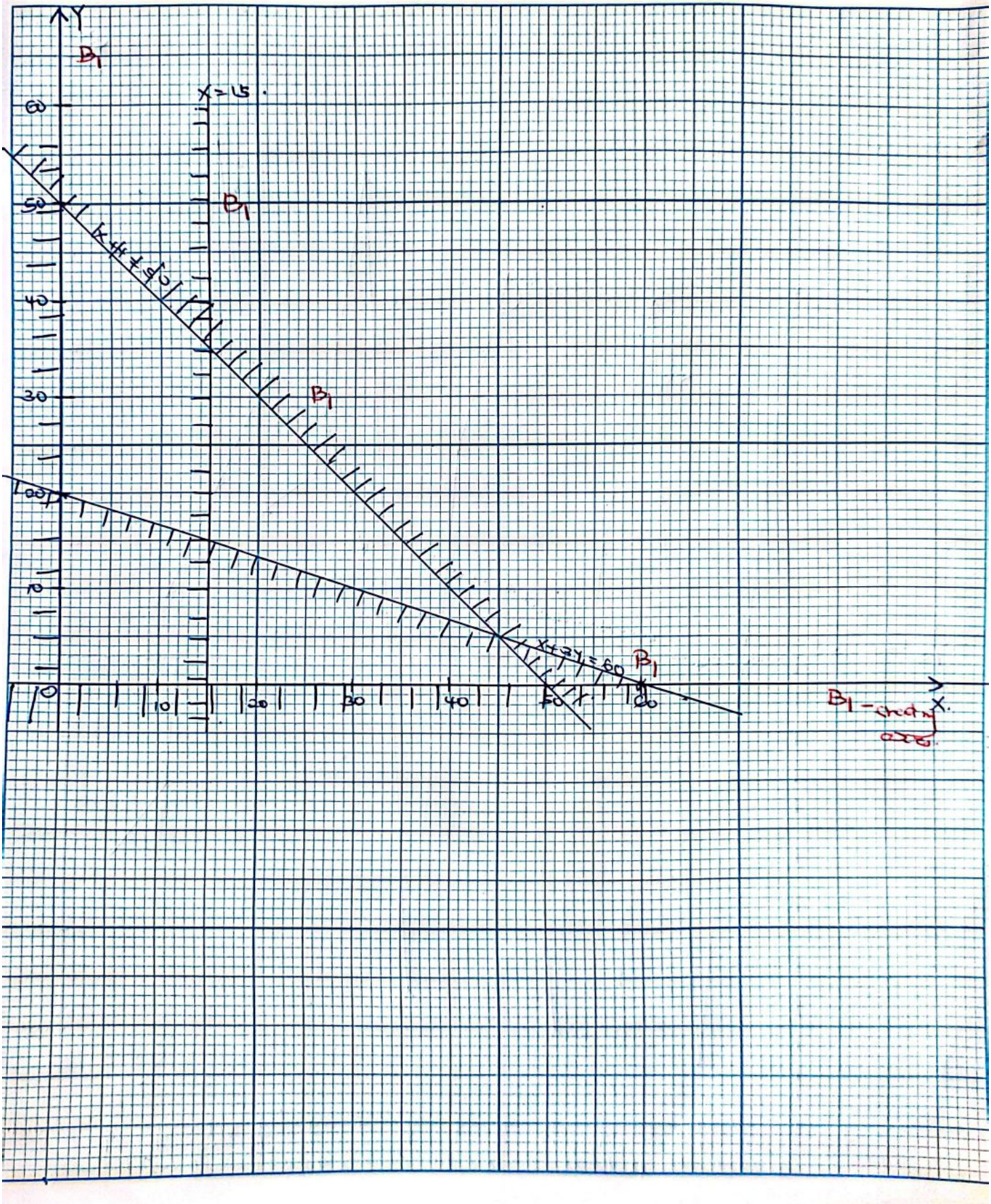

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Contact: 0787662456

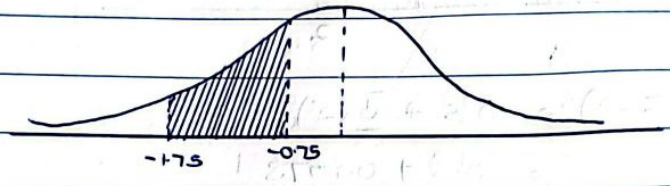
Question 13

(i) $\mu = 175cm \quad \sigma^2 = 400$

$\sigma = 20$

$P(140 < X < 160) = P\left(\frac{140-175}{20} < Z < \frac{160-175}{20}\right)$ M_1

$= P(-1.75 < Z < -0.75)$



M_1

$P(-1.75 < Z < -0.75) = \Phi(1.75) - \Phi(0.75)$ M_1

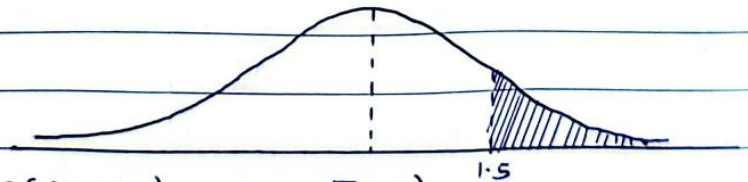
$= 0.4599 - 0.2734$ M_1

$= 0.1865$ B_1

Number of students $= 0.1865 \times 1500 = 280$ students. A_1

(ii) $P(X > 205) = P\left(Z > \frac{205-175}{20}\right)$ M_1

$= P(Z > 1.5)$



M_1

$P(Z > 1.5) = 0.5 - \Phi(1.5)$

$= 0.5 - 0.43319$ M_1

$= 0.0668$ B_1

Number of students $= 0.0668 \times 1500$

$= 101$ students. A_1

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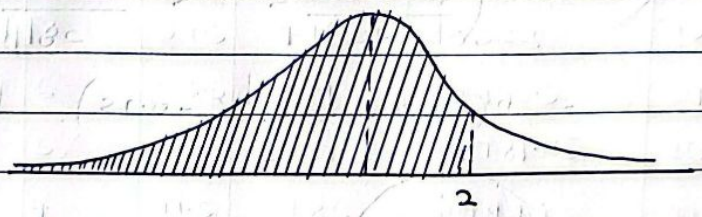
$P(X < 215) = P\left(Z < \frac{215-175}{20}\right)$ M1

$= P(Z < 2)$



B1

$P(Z<2) = 0.5 + \Phi(2)$

$= 0.5 + 0.4773$ M1

$= 0.9773$ B1

Number of student $= 0.9773 \times 1500$

$= 1,466$ student A1

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Question 14.

Week	Day	Takings	3-Pt-M.T	3-Pt-M.A.
1	T	196		
	F	210	749	249.7
	S	343	820	273.3
2	T	267	884	294.7
	F	274	877	292.3
	S	336	778	259.3
3	T	168	783	261
	F	279	762	254
	S	315	754	251.3
4	T	160	733	244.3
	F	258	728	242.7
	S	310	722	240.7
5	T	154	704	234.7
	F	240	706	235.3
	S	312	$x+552$	$\frac{x+552}{3}$
6	T	x.		

B_3 B_3

(b) On saturday of Week 1 B_1

(c) $\frac{x+552}{3} = 236$ M_1

$x = 708 - 552$

$x = 156$ A_1

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Question 16

(a)

Marks.	Tally	f	x	fx	fx^2	c.f	class boundaries
50-53	////	4	51.5	206	10609	4	49.5 - 53.5
54-57	///	3	55.5	1665	9240.75	7	53.5 - 57.5
58-61	### ///	8	59.5	478	28322	15	57.5 - 61.5
62-65	### /	6	83.5	381	24193.5	21	61.5 - 65.5
66-69	###	5	67.5	337.5	22781.25	26	65.5 - 69.5
70-73	////	4	71.5	286	20449	30	69.5 - 73.5
74-77	###	5	75.5	377.5	28501.25	35	73.5 - 77.5
78-81	////	4	79.5	318	25281	39	77.6 - 81.5
82-85	/	1 B_1	83.5	83.5 B_1	6972.25 B_1	40	81.5 - 85.5
B_1		$\Sigma f = 40$ B_1	B_1	$\Sigma fx = 2632$ B_1	$\Sigma fx^2 = 176,350$ B_1	B_1	

(b) Median class - 62 - 65 B_1

Modal class - 58 - 61 B_1

Modal frequency - 8 B_1

(c) $\text{Mean} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{2632}{40}$ M_1

$= 65.8$ A_1

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Standard deviation = $\sqrt{\frac{176,350}{40} - (65.8)^2}$

$= \sqrt{79.11}$ M1

$= 8.8944$ A1