Digital Teachers
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Dr. Bbosa Science

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UACE MATHEMATICS PAPER 1 2014 guide

SECTION A (40 marks)

Answer all questions in this section

1. A class performed on an experiment to estimate the diameter of a circular object. A sample of five students had the following results in centimetre. 3.13, 3.16, 2.94, 3.33 and 3.0. Determine the sample;
   (i) Mean
   (ii) Standard deviation (05marks)
2. The table below shows the values of a function f(x)

x	1.8	2.0	2.2	2.4
f(x)	0.532	0.484	0.436	0.384

   Use linear interpolation to find the value of
   (i) F(2.08)
   (ii) x corresponding to f(x) = 0.5 (05marks)

3. The seed of a taxi decreases from 90kmh^{-1} to 18kmh^{-1} in a distance of 120 metres. Find the speed of the taxi when it had covered a distance of 50metres. (05marks)
4. Events A and B are such that $P(A \cap B) = \frac{1}{12}, and\ P(^{A}/_{B}) = \frac{1}{3}$. Find P(B∩ A') (05marks)
5. Find the approximate value of $\int_0^2 \frac{1}{1+x^2} dx$ using trapezium rule with 6 ordinates. Give your answer to 3 decimal places (05marks)
6. Forces of 7N and 4N act away from a common point and make an angle of θ0 with each other. Given the magnitude of their resultant is 10.75N, find the;
   (i) the value of θ.
   (ii) the direction of the resultant 05marks)
7. An industry manufactures iron sheets of mean length 3.0m and standard deviation of 0.05m. given the lengths are normally distributed, find the probability that the length of any iron sheet picked at random will be between 2.95 and 3.15m. (05marks)
8. A particle of mass m kg is released at rest from the highest point of a solid spherical object of radius a metres. Fin the angle to the vertical at which the article leaves the sphere. (05marks)

SECTION B

Answer any five questions. All questions carry equal marks

9. The height (cm) an ages (years) of random sample of ten farmers are given in the table below

Height (cm)	156	151	152	160	146	157	149	142	158	140
Ages (years)	47	38	44	55	46	49	45	30	45	30

(a)(i) Calculate the rank correlation coefficient

(ii) comment on your result (06marks)

(b) Plot a scatter diagram for the data

Hence draw a line of best fit(02marks)

(c) Use your diagram in (b) to find

(i) y when x = 147

(ii) x when y = 43 (04marks)

10. A mass of 12kg rests on a smooth inclined lane which is 6m long and 1m high. The mass is connected by a light inextensible string which passes over a smooth pulley fixed at the top of the plan to a mass of 4kg which is hanging freely. Which the string taut, the system is releases from rest.
    (a) Find the
        (i) acceleration of the system
        (ii) tension in the string. (08marks)
    (b) Determine the;
        (i) Velocity with which the 4kg mass hits the ground
        (ii) Time the 4kg mass takes to hit the ground. (04marks)

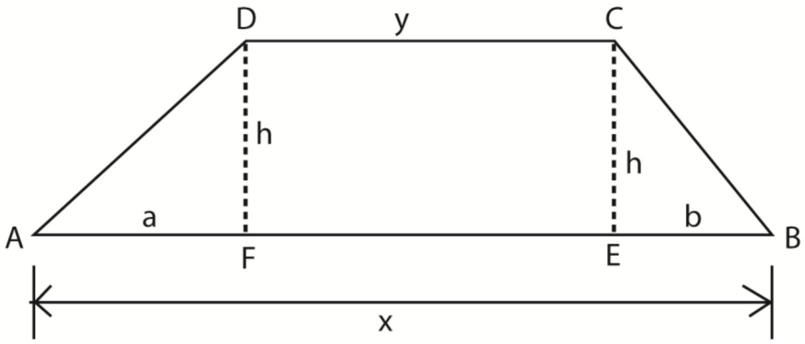
11. The probability density function (p.d.f) of a continuous random variable x is given by

$$f(x) = \begin{cases} kx\,(16 - x2), & 0 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

Where k is a constant

Find the;
(a) value of k (04marks)
(b) mode of X (04mrks)
(c) mean of X (04marks)

12. (a) Particles of masses, 5kg, 2kg 3kg and 3kg act at points with position vectors 3i – j, 2i + 3j, -2i + 5j and –I – 2j respectively. Find the position vector of their centre of gravity. (06marks)
    (b) The figure ABCD below shows a metal sheet of uniform material cut in the shape of a trapezium. $\overline{AB} = x, \overline{CD} = y, \overline{AF} = a, \overline{EB} = b$ and h is the distance between AB and CD



Show that the centre of gravity of the sheet is at a distance $\frac{h}{3}\left[\frac{3y+a+b}{x+y}\right]$ from side AB. (06marks)

13. The number x and y are measured with possible errors of Δx and Δy.

(a) Show that the maximum absolute error in the quotient $\frac{x}{y}$ is given by

$\frac{|y||\Delta y|+|x||\Delta x|}{y^2}$ (06marks)

(b) Find the interval within which the exact value of $\frac{2.58}{3.4}$ is expected to lie. (06marks)

14. A particle I projected with a speed of 36ms^{-1} at an angle of 40^0 to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70 metres on the horizontal plane from a point of projection. Find the;
    (a) (i) time taken for the particle to reach the wall.
    (ii) height of the wall (08marks)
    (b) Maximum height reached by the particle from the point of projection. (04marks)
15. (a) Show that the iterative formula based on Newton Raphson's method for solving the equation Inx + x – 2 = 0 is given by

    $X_{n+1} = \frac{x_n(3-Inx_n)}{1+x_n}, n = 0, 1,2 ....$ (04marks)

    (b)(i) Construct a flow chart that;

    - reads the initial approximation as r

    - computes using the interactive formula in (a), and prints the root of equation Inx +x -2 = 0, when the error is less than 1.0 x 10^{-4}.

    (ii) Perform a dry run of the flow chart when r = 1.6. (08marks)

16. A research station supplies three varieties of seeds S_1, S_2 and S_3 in the ratio 4:2:1. The probabilities of germination of S_1 S_2 and S_3 are 50%, 60% and 80% respectively
    (a) Find the probability that a seed selected at random will germinate.
    (b) Given that 150 seeds are selected at random, find the probability that less than 90 of the seeds will germinate. Give your answer to two decimal places.

Solutions

SECTION A (40 marks)

Answer all questions in this section

1. A class performed on an experiment to estimate the diameter of a circular object. A sample of five students had the following results in centimetre. 3.13, 3.16, 2.94, 3.33 and 3.0. Determine the sample;

Diameter, x (cm)	x^2
2.94	8.6436
3.00	9.0000
3.12	9.7344
3.16	9.9856
3.33	11.0889
$\sum x = 15.55$	$\sum x^2 = 48.4525$

(i) Mean

$\bar{x} = \frac{\sum x}{n} = \frac{15.55}{5} = 3.11$

(ii) Standard deviation (05marks)

S.D = $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$ = $\sqrt{\frac{45.4525}{5} - (3.11)^2} = 0.1356\ (4D)$

2. The table below shows the values of a function f(x)

x	1.8	2.0	2.2	2.4
f(x)	0.532	0.484	0.436	0.384

Use linear interpolation to find the value of

(i) F(2.08)

1.8	2.08	2.0
0.532	f(x)	0.484

$\frac{f(x)-0.436}{2.08-2.0} = \frac{0.436-0.484}{2.2-2.0}$

$\frac{f(x)-0.436}{0.08} = \frac{-0048}{0.2}$

f(x) = 0.4648 or 0.465 (3D)

(ii) x corresponding to f(x) = 0.5 (05marks)

1.8	x	2.0
0.532	0.5	0.484

$\frac{0.5-0.532}{x-1.8} = \frac{0.484-0.532}{2.0-18}$

$\frac{-0.032}{x-1.8} = \frac{-0.048}{0.2}$

$x =$ 1.9333 or 1.9 (1D)

3. The seed of a taxi decreases from 90kmh^{-1} to 18kmh^{-1} in a distance of 120 metres. Find the speed of the taxi when it had covered a distance of 50metres. (05marks)

Given u = 90kmh^{-1}, v = 18kmh^{-1}, s = 120m = 0.12km

Using $v^2 = u^2 + 2as$

$18^2 = 90^2 + 2a(0.12)$

a = -32400kmh^{-2}

When s = 50m = 0.05km, u = 90kmh^{-1}, a = -32400kmh^{-2}

Using $v^2 = u^2 + 2as$

$v^2 = 90^2 - 2 \times 32400 \times 0.05 = 4860$

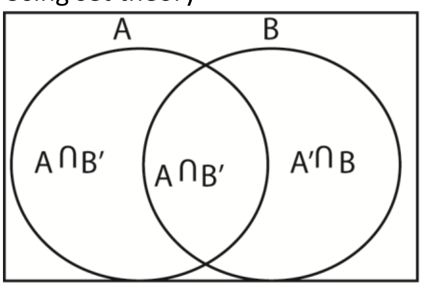
v = $\sqrt{4860} = 69.71 kmh^{-1}$

4. Events A and B are such that $P(A \cap B) = \frac{1}{12}, and\ P(A/B) = \frac{1}{3}$. Find P(B∩ A') (05marks)

$P(A/B) = \frac{P(A\cap B)}{P(B)}$

$P(B) = \frac{P(A\cap B)}{P(A/B)} = \frac{1}{12} \div \frac{1}{3} = \frac{1}{12}\ x\ 3 = \frac{3}{12} = \frac{1}{4}$

Using set theory



P(B) = P(∩ $B) + P(B \cap A')$

$\frac{1}{4} = \frac{1}{12} + P(B \cap A')$

$P(B \cap A') = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$

Or

Using contingency table

1	B	B'	1
A	A∩B	A∩B'	A
A'	A∩B	A∩B'	A'
1	B	B'	1

P(B) = P($\cap B) + P(B \cap A')$

$\frac{1}{4} = \frac{1}{12} + P(B \cap A')$

$P(B \cap A') = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$

Or

P($^{A}/_{B}) + P\left(^{A'}/_{B}\right) = 1$

$P\left(^{A'}/_{B}\right) = 1 - \frac{1}{3} = \frac{2}{3}$

$\frac{P(A' \cap B)}{P(B)} = \frac{2}{3}$

$P(A' \cap B) = \frac{2}{3}$ x $\frac{1}{4} = \frac{1}{6}$

5. Find the approximate value of $\int_0^2 \frac{1}{1+x^2} dx$ using trapezium rule with 6 ordinates. Give your answer to 3 decimal places (05marks)

h = $\frac{2-0}{5} = \frac{2}{5} = 0.4$

Let y = $\frac{1}{1+x^2}$

x	y	
0	1	
0.4		0.86207
0.8		0.60976
1.2		0.40984
1.6		0.28090
2.0	0.2	
Sum	1.2	2.16257

$$\int_0^2 \frac{1}{1+x^2} dx = \frac{h}{2}[(y_0 + y_n) + (y_1 + y_2 + \cdots + y_{n-1})]$$

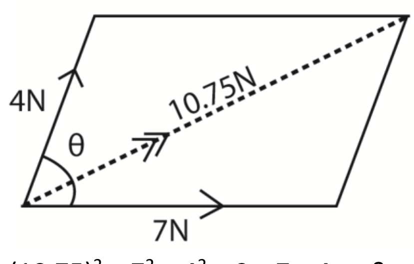
$$= \frac{0.4}{2}(1.2 + 2.16257)$$

$$= 1.105\ (3D)$$

6. Forces of 7N and 4N act away from a common point and make an angle of θ^0 with each other. Given the magnitude of their resultant is 10.75N, find the;

(i) the value of θ.

Using parallelogram of forces and cosine rule

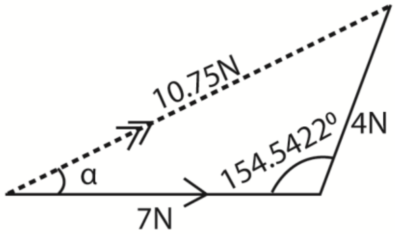


(10.75)^2 = 7^2 + 4^2 + 2 x 7 x 4cosθ

56 cosθ = 50.5625

θ = $\cos^{-1}\frac{50.5626}{50}$ =25. 4578^0 (4D)

(ii) the direction of the resultant 05marks)



Using sine rule

$\frac{sin\alpha}{4} = \frac{\sin 154.5422}{10.75}$

$\alpha = \sin^{-1}\left(\frac{4\sin 154.5422}{10.75}\right)$ = 9.2^0

Using cosine rule

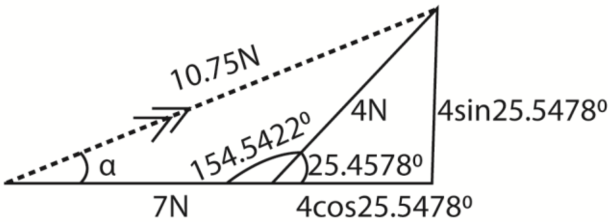
4^2 = $(10.75)^2 + 7^2$ - 2 x 7 x 10.75cosα

$\alpha = \cos^{-1}\left(\frac{148.5625}{150.5}\right) = 9.2^0$

∴ the direction of resultant force is 9.2^0 with the 7N force as shown in the diagram

Alternatively

The triangle of forces above may be expressed as below

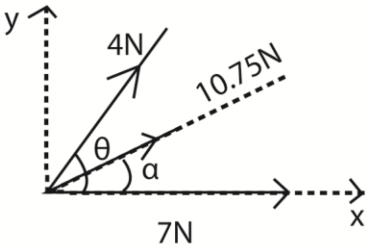


From tringle ACD

tanα = $\frac{\overline{CD}}{\overline{AC}} = \frac{4\sin 25.4578}{7+4\cos 25.4578}$ =9.2^0

Using resolution of forces

NB. This is not restrictive question, either 7N or 4N force is taken as horizontal force



Resolving force

$\begin{pmatrix}7\\0\end{pmatrix} + \begin{pmatrix}4cos\theta\\4sin\theta\end{pmatrix} = \begin{pmatrix}10.75cos\alpha\\10.75sin\alpha\end{pmatrix}$

7 + 4cosθ = 10.75cosα ……… (i)

4sinθ = 10.75sinα ……………. (ii)

Eqn. $(i)^2$ + Eqn. $(ii)^2$

$(7 + 4\cos\theta)^2 + (4\sin\theta)^2 = (10.75\cos\alpha)^2 + (10.75\sin\alpha)^2$

$49 + 56\cos\theta + 16\cos^2\theta + 16\sin^2\theta = (10.75)^2(\cos^2\alpha + \sin^2\alpha)$

$49 + 56\cos\theta + 16(\cos^2\theta + \sin^2\theta) = (10.75)^2(\cos^2\alpha + \sin^2\alpha)$

65 +56cosθ = 115.5625

$\theta \cos^{-1}\left(\frac{50.5625}{56}\right) = 25.4578^0$

Alternatively

(→), x = 7 + 4cosθ and (↑), y = 4sinθ

$x^2 + y^2 = (10.75)^2$

$(7 + 4\cos\theta)^2 + (4\sin\theta)^2 = 115.5625$

$65 + 56\cos\theta = 115.5625$

$\theta \cos^{-1}\left(\frac{50.5625}{56}\right) = 25.4578^0$

From eqn. (ii)

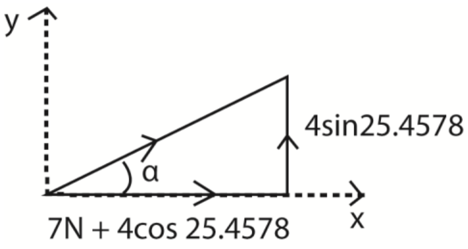
$4\sin 25.4578 = 10.75\sin\alpha$

$\alpha = \sin^{-1}\left(\frac{4\sin 25.4578}{10.75}\right) = 9.2^0$

Or

X = 7 + 4cos 25.4578

Y =4sin25.4578



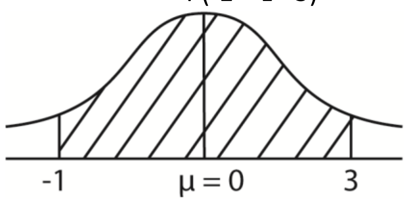
$\alpha = \tan^{-1}\left[\frac{4\sin 25.4578}{(7 + 4\cos 25.4578)}\right] = 9.2^0$

7. An industry manufactures iron sheets of mean length 3.0m and standard deviation of 0.05m. given the lengths are normally distributed, find the probability that the length of any iron sheet picked at random will be between 2.95 and 3.15m. (05marks)

   Let x = length of iron sheets

   $P(2.95 < x < 3 = P\left(\frac{2.95-3.0}{0.05} < z < \frac{3.15-3.0}{0.05}\right)$

   $= P(-1 < z < 3)$

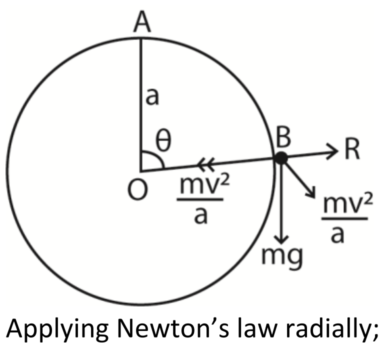
   

   $P(-1 < z < 3) = P(0 < z < 1) + (0 < z < 3)$

   $= 0.3413 + 0.4987$

   $= 0.8400$

8. A particle of mass m kg is released at rest from the highest point of a solid spherical object of radius a metres. Fin the angle to the vertical at which the article leaves the sphere. (05marks)

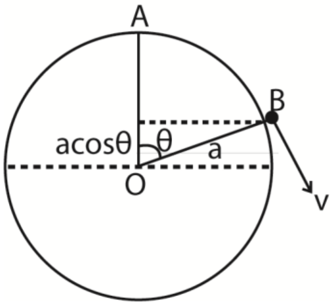
   

   Applying Newton's law radially;

   Let v = velocity of article at B

mgcosθ – R = $\frac{mv^2}{a}$ ……….(i)

Applying the conservation of mechanical energy



Measuring mechanical energy from level O

At point A

P.E = mga

K.E = $\frac{1}{2}mv^2 = \frac{1}{2}\ x\ m\ x\ 0 = 0$

M.E at A = mga + 0 = mga

At point B

P.E = mgacosθ

K.E = $\frac{1}{2}mv^2$

M. E = mgacosθ +$\frac{1}{2}mv^2$

But M.E at A = M.E at B

mga = mgacosθ +$\frac{1}{2}mv^2$

ga =gacosθ + $\frac{1}{2}v^2$

$v^2 = 2ga - 2gacos\theta$

Substituting v^2 into eqn. (i)

mgcosθ – R = $\frac{m(2ga-2gacos\theta}{a}$

mgcosθ – R =2mg – 2mgcosθ

At point B, when the particle leaves the sphere, R = 0

mgcosθ =2mg – 2mgcosθ

3mgcosθ = 2mg

θ = $\cos^{-1}\left(\frac{2}{3}\right)$ = 48.19^0

SECTION B

Answer any five questions. All questions carry equal marks

9. The height (cm) an ages (years) of random sample of ten farmers are given in the table below

Height (cm)	156	151	152	160	146	157	149	142	158	140
Ages (years)	47	38	44	55	46	49	45	30	45	30

(a)(i) Calculate the rank correlation coefficient

(ii) comment on your result (06marks)

Method I: Using Spearman's rank correlation coefficient

Height (x)	Age (y)	Rx	Ry	Rx – Ry = d	d^2
156	47	4	3	1	1
151	38	6	8	-2	4
152	44	5	7	-2	4
160	55	1	1	0	0
146	46	8	4	4	16
157	49	3	2	1	1
149	45	7	5.5	1.5	2.25
142	30	9	9.5	-0.5	0.25
158	45	2	5.5	-3.5	12.25
140	30	10	9.5	0.5	0.25
					$\sum d^2 = 41$

p =1 - $\frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\ x\ 41}{10(10^2-1)} = 0.7515(4D)$

(ii) there is a high positive correlation between heights and ages of farmers

Method II: using Kendall's rank correlation coefficient

Let the farmers be A, B, C, D, E, F, G, H, I, J

Farmers	A	B	C	D	E	F	G	H	I	J
Height	156	151	152	160	146	157	149	142	158	140
Age	47	38	44	55	46	49	45	30	45	30

By re-arranging the findings we have

Farmers	D	I	F	A	C	B	G	E	H	J
Height	1	2	3	4	5	6	7	8	9	10
Age	1	5.5	3	3	7	8	5.5	4	9.5	9.5
agreements	9	4	7	6	3	2	2	2	0	=35
Disagreements	0	3	0	0	2	2	1	0	0	=8

s = total agreements – total disagreements

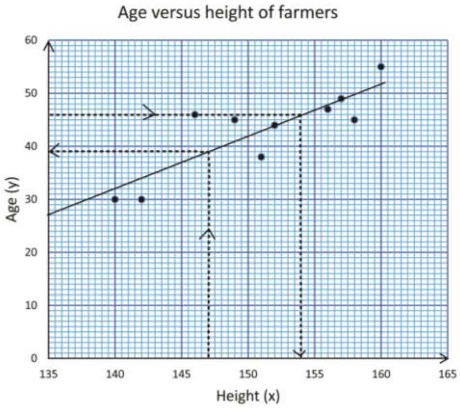
= 35 – 8 = 27

$\tau = \frac{2s}{n(n-1)} = \frac{2\ x\ 27}{10(10-1)} = \frac{54}{90} = 0.6$

(ii) there is **substantial positive** correlation between the heights and ages of farmers.

(b) Plot a scatter diagram for the data

Hence draw a line of best fit (02marks)



(c) Use your diagram in (b) to find

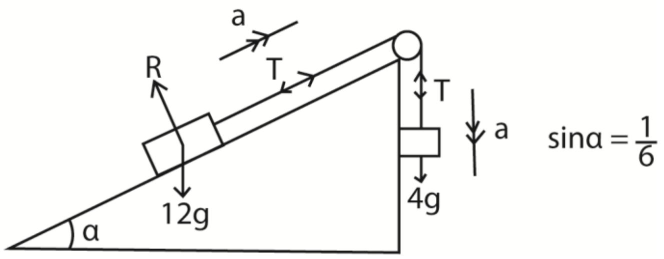
(i) y when x = 147; y = 39.5

(ii) x when y = 43; x =154 (04marks)

10. A mass of 12kg rests on a smooth inclined lane which is 6m long and 1m high. The mass is connected by a light inextensible string which passes over a smooth pulley fixed at the top of the plan to a mass of 4kg which is hanging freely. Which the string taut, the system is releases from rest.

(a) Find the

(i) acceleration of the system



For 4kg mass, resultant force = 4g – T

4a = 4g – T ………………. (i)

For 12kg mass, resultant force = T – 12gsinα

12a = T – 12g $(\frac{1}{6})$ …….. (ii)

Eqn. (i) + eqn. (ii)

16a = 2g

a = $\frac{2 \, x \, 9.8}{16}$ = 1.225ms^{-2}

(ii) tension in the string. (08marks)

From eqn. (i)

4(1.225) = 4 x 9.8 – T

T = 34.2N

(b) Determine the;

(i) Velocity with which the 4kg mass hits the ground

Since the 4kg mass starts to move from rest, u = 0, s = 1m

Using $v^2 = u^2 + 2as$

v^2 = 0 + 2 x 1.225 x 1

v = $\sqrt{2.45} = 1.5652 \ (4D)$

(ii) Time the 4kg mass takes to hit the ground. (04marks)

Using v = u + at

$\sqrt{2.45} = \ 1.225t$

t = 1.3s

11. The probability density function (p.d.f) of a continuous random variable x is given by

$$f(x) = \begin{cases} kx\,(16 - x2), & 0 \leq x \leq 4 \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Where k is a constant

Find the;

(a) value of k (04marks)

For continuous pdf, $\int_a^b f(x)dx = 1$ where a and b are lower and upper limits respectively

⇨ $k\int_0^4 x(16-\ x^2)dx = 1$

$k\int_0^4 (16x-\ x^3)dx = 1$

$k\left(8x^2-\frac{x^4}{4}\right)_0^4 = 1$

k[(128 – 64)-(0)] = 1

k(128 – 64) = 1

64k = 1

k = $\frac{1}{64}$

(b) mode of X (04mrks)

Mode is the value of x for which f'(x) = 0

f(x) = $\frac{x}{64}(16-\ x^2)$ = $\frac{16x}{64}-\ \frac{x^3}{64}$

f'(x) = $\frac{16}{64}-\ \frac{3x^2}{64}$

⇨ $\frac{16}{64}-\ \frac{3x^2}{64} = 0$

$3x^2\ = 16$

$x = \sqrt{\frac{16}{3} = 2.3094}$

(c) mean of X (04marks)

E(X) = $\frac{1}{64}\int_0^4 x^2(16-\ x^2)dx$

= $\frac{1}{64}\int_0^4 (16x^2-\ x^4)dx$

= $\frac{1}{4}\int_0^4 x^2dx - \frac{1}{64}\int_0^4 x^4dx$

=$\frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 - \frac{1}{64}\left[\frac{x^5}{5}\right]_0^4$

= $\frac{1}{12}(64-0) - \frac{1}{320}(1024-0)$ =$\frac{32}{15} = 2.1333\ (4D)$

∴E(X) = 2.1333(4D)

12. (a) Particles of masses, 5kg, 2kg 3kg and 2kg act at points with position vectors 3i – j, 2i + 3j, -2i + 5j and –I – 2j respectively. Find the position vector of their centre of gravity. (06marks)

Total mass = 5 + 2 +3 + 2 = 12kg

Weight = 12g

Let $(\bar{x},\bar{y})$ be the centre of gravity

Equating moments along x-axis

5g x -1 + 2g x 3 + 3g x 5 + 2g x -2= 12g x $\bar{y}$

12g =12g x $\bar{y}$

$\bar{y} = 1$

Equating moments along y-axis

5g x 3 + 2g x2+ 3g x -2 + 2g x -1 = 12g x $\bar{x}$

15g + 4g +15g – 6g -2g = 12g$\bar{x}$

11g = 12g$\bar{x}$

$\bar{x} = \frac{11}{12} = 0.917$

Hence the position of their center of gravity =(0.917i, j)

Or

Sum of moments = sum of moments

$5g\begin{pmatrix}3\\-1\end{pmatrix} + 2g\begin{pmatrix}2\\3\end{pmatrix} + 3g\begin{pmatrix}-2\\5\end{pmatrix} + 2g\begin{pmatrix}-1\\-2\end{pmatrix} = 12g\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$

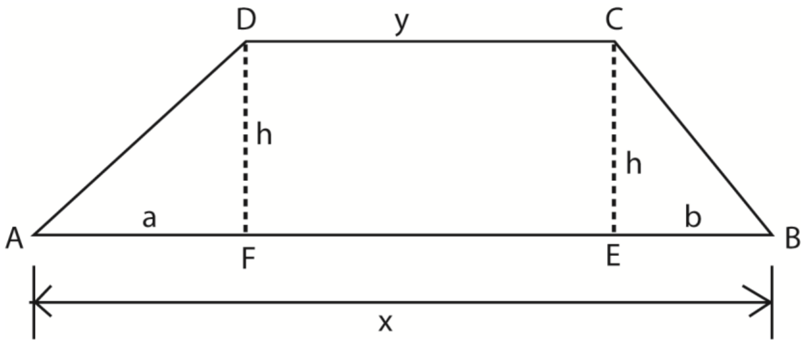
$\begin{pmatrix}15g\\-5g\end{pmatrix} + \begin{pmatrix}4g\\6g\end{pmatrix} + \begin{pmatrix}-6g\\15g\end{pmatrix} + \begin{pmatrix}-2g\\-4g\end{pmatrix} = 12g\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$

$\begin{pmatrix}11g\\12g\end{pmatrix} = \begin{pmatrix}12g\bar{x}\\12g\bar{y}\end{pmatrix}$

$\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} = \begin{pmatrix}\frac{11}{12}\\1\end{pmatrix}$

Hence the position vector of the centre of gravity is $\begin{pmatrix}\frac{11}{12}\\1\end{pmatrix}$ or (0.917i, j)

(b) The figure ABCD below shows a metal sheet of uniform material cut in the shape of a trapezium. $\overline{AB} = x, \overline{CD} = y, \overline{AF} = a, \overline{EB} = b$ and h is the distance between AB and CD



Show that the centre of gravity of the sheet is at a distance $\frac{h}{3}\left[\frac{3y+a+b}{x+y}\right]$ from side AB. (06marks)

Let w = weight per unit area and y =distance of C.O.G of ABCD from AB

Figure	Area	Weight	CO.G from AB
AFD	$\frac{1}{2}ah$	$\frac{1}{2}ahw$	$\frac{1}{3}h$
FECD	yh	yhw	$\frac{1}{2}h$
EBC	$\frac{1}{2}bh$	$\frac{1}{2}bhw$	$\frac{1}{3}h$
ABCD	$\frac{1}{2}h(x+y)$	$\frac{1}{2}h(x+y)w$	$\bar{y}$

Equating moments along AB

$\frac{1}{2}ahw\ x\ \frac{1}{3}h + \text{ yhw x } \frac{1}{2}h + \frac{1}{2}bhw\ x\ \frac{1}{3}h = \frac{1}{2}h(x+y)w\bar{y}$

$\frac{1}{6}ah^2w + \frac{1}{2}\ yh^2w\ + \frac{1}{6}bh^2w = \frac{1}{2}h(x+y)w\bar{y}$

$h(a + 3\ y +\ b) = 3(x+y)\bar{y}$

$\bar{y} = \frac{h}{3}\left[\frac{3y+a+b}{x+y}\right]$

13. The number x and y are measured with possible errors of Δx and Δy.

(a) Show that the maximum absolute error in the quotient $\frac{x}{y}$ is given by

$\frac{|y||\Delta y|+|x||\Delta x|}{y^2}$ (06marks)

$Let\ z = \frac{x}{y}\ and\ \Delta z\ be\ the\ possible\ error\ in\ z$

z + Δz = $\frac{x+\Delta x}{y+\Delta y}$ = $\frac{(x+\Delta x)(y-\Delta y)}{(y+\Delta y)(y-\Delta y)}$ = $\frac{xy-x\Delta y+y\Delta x-\Delta x\Delta y}{y^2-(\Delta y)^2}$

But $|\Delta x\Delta y| \cong |\Delta y|^2 = 0$

z + Δz == $\frac{xy-x\Delta y+y\Delta x}{y^2}$

Δz == $\frac{-x\Delta y+y\Delta x}{y^2}$

$|\Delta z| = \left|\frac{-x\Delta y+y\Delta x}{y^2}\right|$

$|\Delta z| \leq \frac{|-x\Delta y|+|y\Delta x|}{y^2}$

$\leq \frac{|x\Delta y|+|y\Delta x|}{y^2}$

$\leq \frac{|x||\Delta y|+|y||\Delta x|}{y^2}$

Or

z + Δz = $\frac{x+\Delta x}{y+\Delta y}$

= (x +Δx)(y + Δy)$^{)-1}$

= $(x+\Delta x)\left[y^{-1}+\frac{y^{-2}\Delta y}{1!}+\frac{y^{-2}(\Delta y)^2}{2!}+\cdots\right]$

=$(x+\Delta x)\left[\frac{1}{y}+\frac{\Delta y}{y^2}+\cdots\right]$

=$(x+\Delta x)\left[\frac{y+\Delta y}{y^2}+\cdots\right]$

=$\left(\frac{(x+\Delta x)(y+\Delta y)}{y^2}\right)$

z + Δz = $\frac{xy-x\Delta y+y\Delta x-\Delta x\Delta y}{y^2}$

But $|\Delta x\Delta y| \cong 0$

z + Δz == $\frac{xy-x\Delta y+y\Delta x}{y^2}$

Δz == $\frac{-x\Delta y+y\Delta x}{y^2}$

$|\Delta z| = \left|\frac{-x\Delta y+y\Delta x}{y^2}\right|$

$|\Delta z| \leq \frac{|-x\Delta y|+|y\Delta x|}{y^2}$

$\leq \frac{|x\Delta y|+|y\Delta x|}{y^2}$

$\leq \frac{|x||\Delta y|+|y||\Delta x|}{y^2}$

Or

Z = $\frac{x}{y}$

$z_{max} = \frac{x_{max}}{y_{min}} = \frac{x+\Delta x}{y-\Delta y}$

$z_{min} = \frac{x_{min}}{y_{max}} = \frac{x-\Delta x}{y+\Delta y}$

Error bound = $\frac{1}{2}(z_{max} - z_{min})$

=$\frac{1}{2}\left(\frac{x+\Delta x}{y-\Delta y} - \frac{x-\Delta x}{y+\Delta y}\right)$

=$\frac{1}{2}\left(\frac{(x+\Delta x)(y+\Delta y)-(x-\Delta x)(y-\Delta y)}{(y-\Delta y)(y+\Delta y)}\right)$

=$\frac{1}{2}\left(\frac{(xy+x\Delta y+y\Delta x+\Delta x\Delta y)-(xy-x\Delta y-y\Delta x+\Delta x\Delta y)}{y^2-(\Delta y)^2}\right)$

$$\Delta z = = \frac{1}{2}\left[\frac{2x\Delta y + 2y\Delta x}{y^2 - (\Delta y)^2}\right] = \frac{x\Delta y + y\Delta x}{y^2 - (\Delta y)^2}$$

But $(\Delta y)^2 = 0$

$$|\Delta z| = \left|\frac{-x\Delta y + y\Delta x}{y^2}\right|$$
$$|\Delta z| \le \frac{|-x\Delta y| + |y\Delta x|}{y^2}$$
$$\le \frac{|x\Delta y| + |y\Delta x|}{y^2}$$
$$\le \frac{|x||\Delta y| + |y||\Delta x|}{y^2}$$

(b) Find the interval within which the exact value of $\frac{2.58}{3.4}$ is expected to lie. (06marks)

Let $Z = \frac{2.58}{3.4}$, x = 2.58 and y = 3.4; $\Delta x = 0.005$ and $\Delta y = 0.05$

$$z_{max} = \frac{x_{max}}{y_{min}} = \frac{x + \Delta x}{y - \Delta y} = \frac{2.58 + 0.005}{34 - 0.05} = \frac{2585}{3.35} = 0.7716(4D)$$
$$z_{min} = \frac{x_{min}}{y_{max}} = \frac{x - \Delta x}{y + \Delta y} = \frac{2.58 - 0.005}{34 + 0.05} = \frac{2575}{3.45} = 0.7464\ (4D)$$

Hence the interval is (0.7464, 0.7716)

Or

$$|\Delta z| = \frac{|x||\Delta y| + |y||\Delta x|}{y^2} = \frac{|2.58||0.05| + |3.4||0.005|}{|3.4|^2} = \frac{0.129 + 0.017}{11.56} = \frac{0.146}{11.56} = 0.01263\ (5D)$$

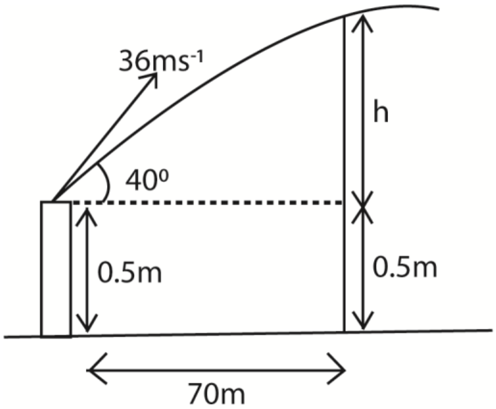
Working value = z = $\frac{2.58}{3.4}$ = 0.75882 (5D)

Minimum value = 0.75882 -0.01263 = 0.7462(4D)

Maximum value = 0.75882+ 0.01263 = 0.7715(4D)

Hence the interval is (0.7462, 0.7715)

14. A particle I projected with a speed of $36ms^{-1}$ at an angle of 40^0 to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70 metres on the horizontal plane from a point of projection. Find the;



Vx = $36\cos 40^0$

Vy = $36\sin 40^0$

(a) (i) time taken for the particle to reach the wall.

Time take to clear the wall = time taken to cover a horizontal distance of 70m

Usin X= Vxt

$t = \frac{70}{36\sin 40^0} = 2.5384s$

(ii) height of the wall (08marks)

Using h = $u\sin\theta t - \frac{1}{2}gt^2$

$= 36\sin 400 \times 2.5384 - \frac{1}{2}\ x\ 9.8\ x\ (2.5384)^2 = 27.1664 + 0.5 = 27.6664m$

(b) Maximum height reached by the particle from the point of projection. (04marks)

From $v^2 = u^2 + 2as$

At maximum height vertical component of velocity is zero

⇨ 0 = $(36 \sin 40^2)^2 - 2\ x\ 9.8H$

H = $\frac{(36 \sin 40^2)^2}{2\ x\ 9.8}$ =27.32m

15. (a) Show that the iterative formula based on Newton Raphson's method for solving the equation Inx + x – 2 = 0 is given by

$X_{n+1} = \frac{x_n(3-Inx_n)}{1+x_n}, n = 0, 1,2 ....$ (04marks)

$let\ f(x) = Inx + x - 2$

$f(x_n) = Inx_n + x_n - 2$

$f'(x) = \frac{1}{x_n} + 1 = \frac{1+x_n}{x_n}$

Using N.R.M

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \left(\frac{Inx_n + x_n - 2}{\frac{1+x_n}{x_n}}\right)$

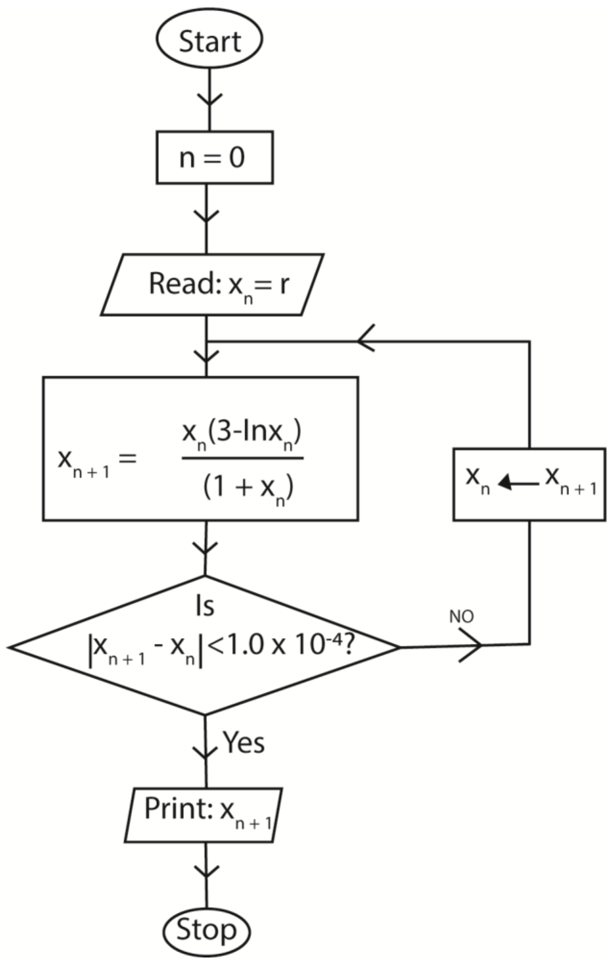
$= \frac{x_n}{1} - \frac{x_n(Inx_n + x_n - 2)}{1+x_n} = \frac{x_n(1+x_n - Inx_n - x_n + 2)}{1+x_n}$

$= \frac{x_n(3-Inx_n)}{1+x_n}$; n = 0, 1, 2, ……………

(b)(i) Construct a flow chart that;

- reads the initial approximation as r

- computes using the interactive formula in (a), and prints the root of equation Inx +x -2 = 0, when the error is less than 1.0×10^{-4}.



(ii) Perform a dry run of the flow chart when r = 1.6. (08marks)

n	x_n	X_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.6	1.5569	0.0431
1	1.5569	1.5571	0.0002
2	1.5571	1.5571	0.0000

Hence the root = 1.557(3D)

16. A research station supplies three varieties of seeds S_1, S_2 and S_3 in the ratio 4:2:1. The probabilities of germination of S_1 S_2 and S_3 are 50%, 60% and 80% respectively

4:2:1

4 + 2+ 1 = 7

$P(S_1) = \frac{4}{7}$; $P(S_2) = \frac{2}{7}$; $P(S_3) = \frac{1}{7}$

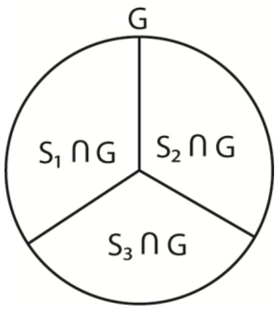
Let G = germination of seeds

$P\left({}^{G}/_{S_1}\right) = 50\% = 0.5$

$P\left({}^{G}/_{S_2}\right) = 60\% = 0.6$

$P\left({}^{G}/_{S_3}\right) = 80\% = 0.8$

(a) Find the probability that a seed selected at random will germinate.



P(G) = $P(s_1 \cap G) + P(s_2 \cap G) + P(s_3 \cap G)$

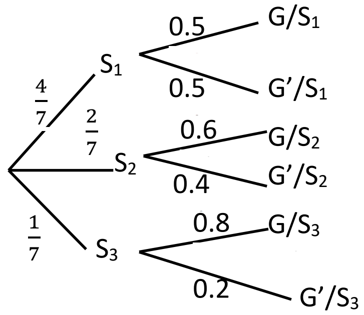
= $P(S_1).P\left({}^{G}/_{S_1}\right) + P(S_2).P\left({}^{G}/_{S_2}\right) + P(S_3).P\left({}^{G}/_{S_3}\right)$

$= \frac{4}{7}\ x\ 0.5 + \frac{2}{7}\ x\ 0.6 + \frac{1}{7}\ x\ 0.8$

$= \frac{2}{7} + \frac{1.2}{7} + \frac{0.8}{7}$

$= \frac{4}{7}$

Or Using factor tree diagram



P(G) $= \frac{4}{7}\ x\ 0.5 + \frac{2}{7}\ x\ 0.6 + \frac{1}{7}\ x\ 0.8$

$= \frac{2}{7} + \frac{1.2}{7} + \frac{0.8}{7}$

$= \frac{4}{7}$

(b) Given that 150 seeds are selected at random, find the probability that less than 90 of the seeds will germinate. Give your answer to two decimal places.

n = 150; P $= \frac{4}{7}$; q = $\frac{3}{7}$

since n = large (= 150), we use the normal approximate this binomial

μ = np = $\frac{4}{7}\ x 150 = \frac{600}{7}$

σ = $\sqrt{npq} = \sqrt{\frac{600}{7}\ x\ \frac{3}{7}} = \frac{30\sqrt{2}}{7}$

Let X = number of seeds that will germinate

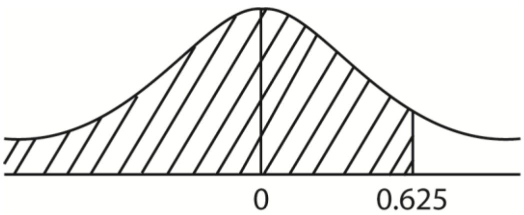
P(x < 90) = P(x ≤ 89)

$= P\left(z\ \leq\ \frac{89.5 - \frac{600}{7}}{\frac{30\sqrt{2}}{7}}\right)$

$= P\left(z\ \leq\ \frac{7\left(89.5 - \frac{600}{7}\right)}{30\sqrt{2}}\right)$

$= P\left(z\ \leq\ \frac{628.5 - 600}{30\sqrt{2}}\right)$

$= P(z\ \leq 0.6250)$



= 0.5 + ($0 \leq z \leq 0.625$)

= 0.5 + 0.2340

= 0.7340

= 0.73 (2D)

Thank you

Dr. Bbosa Science