P425/1
PURE MATHEMATICS
Paper 1
July/Aug. 2020
3 hours

ASK INTERGRATED TEACHERS' MOCK EXAMINATIONS BURREAU

AITEL JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**

Any additional question(s) answered will **not** *be marked*

Show **all** *the necessary workings clearly*

Begin each question on a fresh page of paper

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used

SECTION A (40 MARKS)

Attempt **all** *questions in this section*

1. Given that $a = \log_5 35$ and $b = \log_9 35$, show that $\log_5 21 = \frac{1}{2b}(2ab - 2b + a)$.

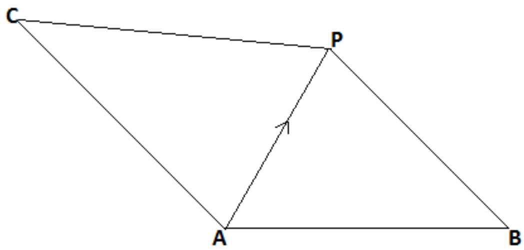
(05 marks)

2. Solve the inequality: $|x-2| > 3|2x+1|$. (05 marks)

3. Prove that: $\tan^{-1}\frac{1}{2} - \operatorname{cosec}^{-1}\frac{\sqrt{5}}{2} = \cos^{-1}\frac{4}{5}$. (05 marks)

4. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in x^2 and by letting $x = \frac{1}{4}$, show that $\sqrt{5} \approx \frac{256}{115}$. (05 marks)

5.



$A,\ B,\ C$ and P are four points such that $3\overrightarrow{\mathbf{AP}} = 2\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{AC}}$, show that $B,\ P$ and C are collinear and that P is the point of trisection of the line BC.

(05 marks)

6. Given that $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, show that $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$. (05 marks)

7. Find the volume of the solid generated by rotating the area bounded by the curve $y = \cos\frac{1}{2}x$ from $x = 0$ to $x = \pi$ about the $x-$ axis. (05 marks)

8. Solve the d.e given $\cos x\frac{dy}{dx} - 2y\sin x = 1$. (05 marks)

SECTION B (60 MARKS)

Attempt any **five** *questions in this section*

9 (a) Evaluate the coefficient of x in the expansion of $\left(x+\frac{2}{x^2}\right)^{10}$. (05 marks)

(b) Prove by Mathematical induction that: $\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{3}{4}-\frac{2n+1}{2n(n+1)}$. (07 marks)

10 (a) Prove the identity: $\cos^6 x+\sin^6 x=1-\frac{3}{4}\sin^2 2x$. (06 marks)

(b) Solve the equation: $4\sin^2 x+8\cot^2 x=5\operatorname{cos}ec^2 x$ for $0\le x\le 2\pi$. (06 marks)

11. Sketch the curve $y=\frac{4+3x-x^2}{x-8}$, clearly find the nature of the turning points and state their asymptotes. (12 marks)

12 (a) The points $P\left(5p,\ \frac{5}{p}\right)$ and $Q\left(5q,\ \frac{5}{q}\right)$ lie on the rectangular hyperbola $xy=25$. Find the equation of the tangent at P and hence deduce the equation of the tangent at Q. (05 marks)

(b) The tangents at P and Q meet at point N, show that the coordinates of N are $\left(\frac{10pq}{p+q},\ \frac{10}{p+q}\right)$, hence find the locus of N given that $pq=-1$. (07 marks)

13 (a) Given that $z(5+5i)=a(1+3i)+b(2-i)$ where a and b are real numbers and that $\arg z=\frac{\pi}{2}$ and $|z|=7$, find the values of a and b. (06 marks)

(b) Describe the locus of the complex number z when it moves in the argand diagram such that $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$. (06 marks)

14 (a) Evaluate: $\int_0^{\pi/3} x\sin 3x\,dx$ (06 marks)

Turn Over

(b) Prove that: $\int_{\pi}^{4\pi/3} \operatorname{cosec}\frac{1}{2}x \, dx = In3$ (06 marks)

15 (a) Find the point of intersection between the plane $\mathbf{r} \cdot (\mathbf{2i} - \mathbf{j} + \mathbf{k}) = \mathbf{4}$ and the line passing through the point $(3,\ 1,\ 2)$ and is perpendicular to this plane.

(05 marks)

(b) Find the perpendicular distance of the point $(4,\ -3,\ 10)$ to the line

$\frac{x-1}{3} = 2 - y = \frac{z-3}{2}$. (07 marks)

16. A liquid is being heated in an oven maintained at a constant temperature of $180^o C$. It is assumed that the rate of increase in the temperature of the liquid is proportional to $(180 - \theta)$, where $\theta^o C$ is the temperature of the liquid at time t minutes. If the temperature of the liquid rises from $0^o C$ to $120^o C$ in 5 minutes, find the temperature of the liquid after a further 5 minutes. (12marks)

END