PROPOSED MARKING GUIDE UACE 2024
PURE MATHEMATICS UMTA
P425/1

NO	SOLUTION	MKS	COMMENT
1	Let $u = \ln x, \frac{dv}{dx} = x^4$ $\frac{du}{dx} = \frac{1}{x}, v = \frac{x^5}{5}$ $\int x^4 \ln x \, dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^5 \cdot \frac{1}{x} dx$ $= \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + c$		
		05	
2	For $2x + 3y = 7$ $y = -\frac{2}{3}x + \frac{7}{3}, m_1 = -\frac{2}{3}$ For $x = 6y + 5$ $y = \frac{1}{6}x - \frac{5}{6}, m_2 = \frac{1}{6}$ Using $\theta = \tan^{-1}\left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $\theta = \tan^{-1}\left\|\frac{-\frac{2}{3} - \frac{1}{6}}{1 + -\frac{2}{3} \times \frac{1}{6}}\right\|$ $\theta = \tan^{-1}\left(\frac{15}{16}\right)$ $\theta = 43.15^0$		
		05	
3	$y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ Rationalizing		

4	$y = \sqrt{\frac{(1-\cos 2x)(1-\cos 2x)}{(1+\cos 2x)(1-\cos 2x)}}$ $y = \sqrt{\frac{(1-\cos 2x)^2}{1-cos^2 2x}}$ $y = \frac{1-\cos 2x}{\sin 2x}$ $\frac{dy}{dx} = \frac{\sin 2x \cdot 2\sin 2x-(1-\cos 2x)\cdot 2\cos 2x}{sin^2 2x}$ $\frac{dy}{dx} = \frac{2sin^2 2x-2\cos 2x+2cos^2 2x}{sin^2 2x}$ $\frac{dy}{dx} = \frac{2(1-\cos 2x)}{sin^2 2x}$ $\frac{dy}{dx} = \frac{2(1-\cos 2x)}{1-cos^2 2x}$ $\frac{dy}{dx} = \frac{2}{1+\cos 2x}$ $\frac{dy}{dx} = \frac{2}{2cos^2 x}$ $\therefore \frac{dy}{dx} = sec^2 x$		
		05	
4	Let $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}, \boldsymbol{b} = \boldsymbol{i} - 3\boldsymbol{j} - 5\boldsymbol{k}, \boldsymbol{c} = 3\boldsymbol{i} - 4\boldsymbol{j} - 4\boldsymbol{k}$ For coplanar vectors, $\boldsymbol{c} = \mu\boldsymbol{a} + \lambda\boldsymbol{b}$ $\begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix} = \mu\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ $2\mu + \lambda = 3$ ………………………..(i) $-\mu - 3\lambda = -4$ $\mu + 3\lambda = 4$ ………………………...(ii) $\mu - 5\lambda = -4$ ………………………(iii) (ii)−(iii); $8\lambda = 8$		

	$\lambda = 1$ From (ii); $\mu + 3(1) = 4$ $\mu = 1$ Substituting for μ and λ in (i); $2(1) + 1 = 3$ $3 = 3$ $\therefore$ since the values of μ and λ are consistent, then the vectors are coplanar.		
		05	
5	$\tan x + \tan 2x + \tan x \tan 2x = 1$ $\tan x + \frac{2\tan x}{1-tan^2x} + \tan x \cdot \frac{2\tan x}{1-tan^2x} = 1$ $\tan x - tan^3x + 2\tan x + 2tan^2x = 1 - tan^2x$ $tan^3x - 3tan^2x - 3\tan x + 1 = 0$ Let $\tan x = m$ $m^3 - 3m^2 - 3m + 1 = 0$ Put $m = -1$; $-1 - 3 + 3 + 1 = 0$ $0 = 0$ If $m = -1$ is a root, then $m + 1$ is a factor.		

$$
\begin{array}{r|l}
 & m^2 - 4m + 1 \\
\hline
m + 1 & m^3 - 3m^2 - 3m \mp 1 \\
 & \underline{-\ m^3 + m^2} \\
 & \quad -4m^2 - 3m + 1 \\
 & \quad \underline{-\ -4m^2 - 4m} \\
 & \qquad\qquad m + 1 \\
 & \qquad\quad \underline{-\ m + 1} \\
 & \qquad\qquad - \quad -
\end{array}
$$

⇨ $m^3 - 3m^2 - 3m + 1 = 0$

$(m + 1)(m^2 - 4m + 1) = 0$

$m = -1$ or $m^2 - 4m + 1 = 0$

ALT:

Let $t = \tan x$

$t + \frac{2t}{1-t^2} + t \cdot \frac{2t}{1-t^2} = 1$

$t - t^3 + 2t + 2t^2 = 1 - t^2$

$t^3 - 3t^2 - 3t + 1 = 0$

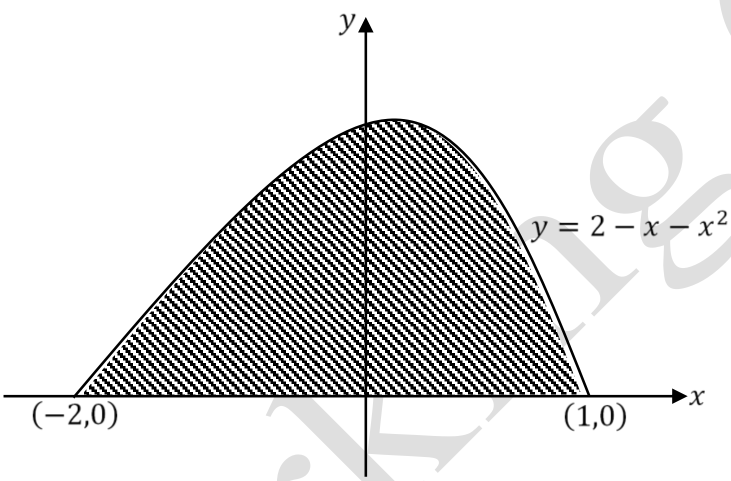
Put $t = -1$;

$-1 - 3 + 3 + 1 = 0$

$0 = 0$

If $t = -1$ is a root, then $t + 1$ is a factor.

	$t+1 \overline{)\, t^3-3t^2-3t\mp1}$ quotient t^2-4t+1 $-\ t^3+t^2$ $\quad -4t^2-3t+1$ $-\ -4t^2-4t$ $\qquad t+1$ $-\ t+1$ $\qquad -\quad -$ $\Rightarrow t^3-3t^2-3t+1=0$ $(t+1)(t^2-4t+1)=0$ $t=-1$ or $t^2-4t+1=0$		
		05	
6	$9\log_x 5=\log_5 x$ $\frac{9}{\log_5 x}=\log_5 x$ $(\log_5 x)^2=9$ $\log_5 x=\pm3$ When $\log_5 x=3$ $x=5^3$ $x=125$ When $\log_5 x=-3$ $x=5^{-3}$		

	$x = \frac{1}{125}$		
		05	
7	$y = (1 - x)(x + 2)$ Intercepts, $x, y = 0$ $(1 - x)(x + 2) = 0$ $x = 1, x = -2, (1,0), (-2,0)$ $y, x = 0$ $y = (1 - 0)(0 + 2) = 2, (0, 2)$  $\mathrm{A} = \int_{-2}^{1} (2 - x - x^2)\, dx$ $\mathrm{A} = \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1}$ $\mathrm{A} = \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-6 - 2 + \frac{8}{3}\right)$ $\mathrm{A} = \frac{7}{6} + \frac{16}{3}$ $\mathrm{A} = \frac{13}{2}$ or 6.5 or $6\frac{1}{2}$ sq. units		
		05	

8	$3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$ $3^{2x} \cdot 3^1 - 3^x \cdot 3^1 - 3^x + 1 = 0$ $3 \cdot (3^x)^2 - 3 \cdot 3^x - 3^x + 1 = 0$ Let $m = 3^x$ $3m^2 - 3m - m + 1 = 0$ $3m^2 - 4m + 1 = 0$ $3m^2 - 3m - m + 1 = 0$ $3m(m-1) - (m-1) = 0$ $(3m-1)(m-1) = 0$ $m = \frac{1}{3}$ or $m = 1$ When $m = 1;\ 3^x = 1$ $3^x = 3^0$ $x = 0$ When $m = \frac{1}{3};\ 3^x = 3^{-1}$ $x = -1$		
		05	
9	$y = \frac{3x+3}{x(3-x)}$ a) $y(3x - x^2) = 3x + 3$ $yx^2 + (3-3y)x + 3 = 0$ For non-existence, $b^2 - 4ac < 0$ $9(1-y)^2 - 4 \times y \times 3 < 0$ $3(1 - 2y + y^2) - 4y < 0$		

$3y^2 - 10y + 3 < 0$

$(y-3)(3y-1) < 0$

Critical values ; $y = 3, y = \frac{1}{3}$

y	$y < \frac{1}{3}$	$\frac{1}{3} < y < 3$	$y > 3$
$(y-3)(3y-1)$	$+$	$-$	$+$

∴For non-existence $\frac{1}{3} < y < 3$

Turning points

For $y = 3$;

$3x^2 - 6x + 3 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$, $(1,3)_{min}$

For $y = \frac{1}{3}$;

$\frac{1}{3}x^2 + 2x + 3 = 0$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$x = -3$; $\left(-3, \frac{1}{3}\right)_{max}$

b) ***Intercepts***

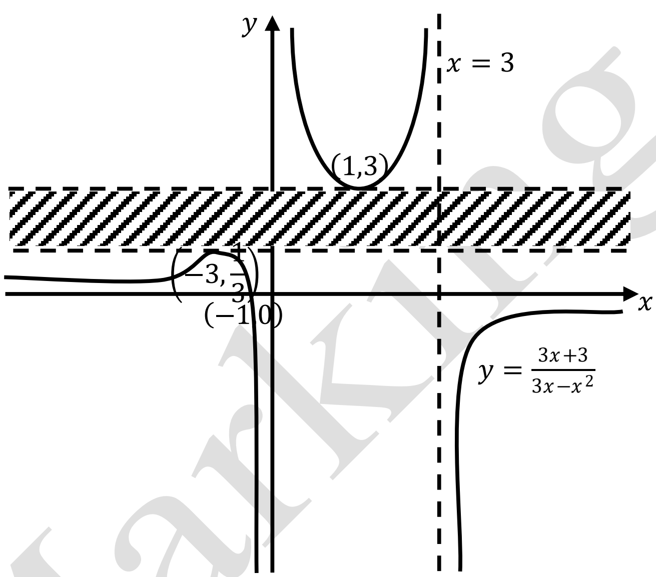
$x; y = 0$

$3x + 3 = 0$

$x = -1, \quad (-1,0)$

$y; x = 0$

$y = \frac{3(0)+3}{0}$, y is undefined

	Asymptotes Vertical, $x(3-x)=0$ $x=0, x=3$ Horizontal, $y=\frac{\frac{3}{x}+\frac{3}{x^2}}{\frac{3}{x}-1}$ As $x \to \pm\infty; y \to 0$ i.e $y=0$ c) 		
		12	
10	a) $\sqrt{3-x}-\sqrt{7+x}=\sqrt{16+2x}$ Squaring both sides; $3-x-2\sqrt{(21-4x-x^2)}+7+x=16+2x$ $-2\sqrt{(21-4x-x^2)}=6+2x$		

$-\sqrt{(21-4x-x^2)} = 3 + x$

Squaring both sides again,

$21 - 4x - x^2 = 9 + 6x + x^2$

$2x^2 + 10x - 12 = 0$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6, x = 1$

For $x = -6$;

L.H.S $= \sqrt{9} - \sqrt{1}$

$= 2$

R.H.S $= \sqrt{4} = 2$

For $x = 1$;

L.H.S $= \sqrt{2} - \sqrt{8}$

$= -\sqrt{2}$

R.H.S $= \sqrt{18}$

$= 3\sqrt{2}$

$\therefore x = -6$

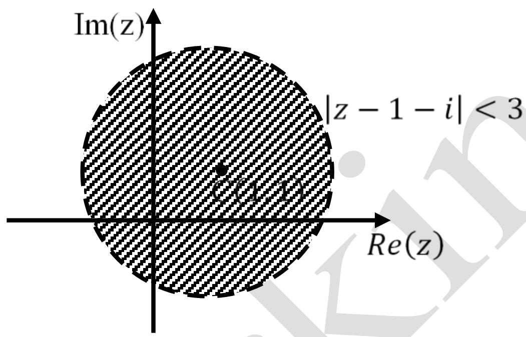
b) Let $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5} = k$

$\frac{3x+3y+3z}{6} = k$

$\frac{x+y+z}{2} = k$

$\frac{2}{2} = k$

	$k = 1$ $x + 2y = -3$ …………….(i) $y + 2z = 4$ ………………(ii) $2x + z = 5$ ………………(iii) From (i); $x = -3 - 2y$ Then in (iii); $2(-3 - 2y) + z = 5$ $-6 - 4y + z = 5$ $-4y + z = 11$……………….(iv) 4(ii)+(iv); $9z = 27$ $z = 3$ From (ii); $y + 2(3) = 4$ $y = -2$ From $x = -3 - 2y$ $x = -3 + 4 = 1$ $\therefore x = 1, y = -2, z = 3$		
		12	
11	a) L.H.S $= \frac{(\cos 4\theta + i \sin 4\theta)^3 (\cos 2\theta - i \sin 2\theta)^5}{(\cos 3\theta + i \sin 3\theta)^4 (\cos 4\theta - i \sin 4\theta)^6}$ $= \frac{(\cos\theta + i\sin\theta)^{12}(\cos\theta + i\sin\theta)^{-10}}{(\cos\theta + i\sin\theta)^{12}(\cos\theta + i\sin\theta)^{-24}}$ $= \frac{(\cos\theta + i\sin\theta)^{12-10}}{(\cos\theta + i\sin\theta)^{12-24}}$ $= \frac{(\cos\theta + i\sin\theta)^{2}}{(\cos\theta + i\sin\theta)^{-12}}$ $= (\cos\theta + i\sin\theta)^{14}$		

	$= \cos 14\theta + i \sin 14\theta$ b) $\lvert z - 1 - i \rvert < 3$ Centre, C$(1, 1)$ and radius, $r = 3$ units Let $z = x + yi$ $\lvert x + yi - 1 - i \rvert < 3$ $\lvert (x-1) + i(y-1) \rvert < 3$ $\sqrt{(x-1)^2 + (y-1)^2} < 3$ $(x-1)^2 + (y-1)^2 < 9$ $x^2 + y^2 - 2x - 2y + 2 < 9$ $x^2 + y^2 - 2x - 2y - 7 < 0$ 		
		12	
12	$y = 2x^2, y = 10x - x^2$ For $y = 10x - x^2$ Intercepts, $x, y = 0$ $0 = x(10 - x)$ $x = 0, x = 10$ $(0,0), (10,0)$		

$y, x = 0$

$y = 0(10 - 0) = 0,\ (0, 0)$

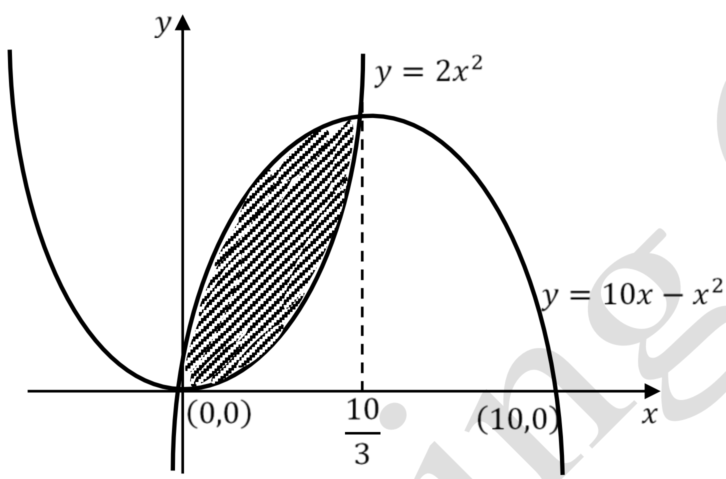
Points of intersection

$2x^2 = 10x - x^2$

$3x^2 - 10x = 0$

$x(3x - 10) = 0$

$x = 0, x = \frac{10}{3}$



$\mathrm{V} = \pi \int_a^b (y_1^2 - y_2^2)\, dx$

$\mathrm{V} = \pi \int_0^{8/3} [(10x - x^2)^2 - (2x^2)^2]\, dx$

$\mathrm{V} = \pi \int_0^{8/3} (100x^2 - 20x^3 + x^4 - 4x^4)\, dx$

$\mathrm{V} = \pi \int_0^{8/3} (100x^2 - 20x^3 - 3x^4)\, dx$

$\mathrm{V} = \pi \left[\frac{100}{3}x^3 - 5x^4 - \frac{3}{5}x^5\right]_0^{8/3}$

$\mathrm{V} = \pi \left(\frac{100}{3}\left(\frac{8}{3}\right)^3 - 5\left(\frac{8}{3}\right)^4 - \frac{3}{5}\left(\frac{8}{3}\right)^5 - 0\right)$

V = 298.3506π or 937.2961 cubic unit

		12	
13	a) L.H.S $= \frac{\sin 5x - \sin 7x + \sin 8x - \sin 4x}{\cos 4x - \cos 5x - \cos 8x + \cos 7x}$ $= \frac{\sin 5x - \sin 7x + \sin 8x - \sin 4x}{\cos 4x - \cos 8x + \cos 7x - \cos 5x}$ $= \frac{2\cos 6x \sin(-x) + 2\cos 6x \sin 2x}{-2\sin 6x \sin(-2x) - 2\sin 6x \sin x}$ $= \frac{2\cos 6x(\sin 2x - \sin x)}{2\sin 6x(\sin 2x - \sin x)}$ $= \cot 6x$ b) $4\left(\frac{1-t^2}{1+t^2}\right) - 6\left(\frac{2t}{1+t^2}\right) = 5$,where $t = \tan\left(\frac{x}{2}\right)$ $4 - 4t^2 - 12t = 5 + 5t^2$ $9t^2 + 12t + 1 = 0$ $t = \frac{-12 \pm \sqrt{12^2 - 4\times 9\times 1}}{2\times 9}$ $t =$ or $t =$		
		12	
14	Let $\frac{3x^3+x+1}{(x-2)(x+1)^3} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$ $3x^3 + x + 1 \equiv A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2)$ Put $x = 2; 27 = 27A$ $A = 1$ Put $x = -1;\ -3 = -3D$ $D = 1$ Comparing coefficient of; $x^3; 3 = A + B$ $3 = 1 + B$ B= 2		

	Put $x = 0; 1 = A - 2B - 2C - 2D$ $1 = 1 - 2(2) - 2C - 2(1)$ $2\text{C} = -6$ $\text{C} = -3$ $\therefore \frac{3x^3+x+1}{(x-2)(x+1)^3} \equiv \frac{1}{x-2} + \frac{2}{x+1} - \frac{3}{(x+1)^2} + \frac{1}{(x+1)^3}$ Hence; $\int_3^4 \frac{3x^3+x+1}{(x-2)(x+1)^3}dx = \int_3^4 \frac{1}{x-2}dx + \int_3^4 \frac{2}{x+1}dx - 3\int_3^4 \frac{1}{(x+1)^2}dx + \int_3^4 \frac{1}{(x+1)^3}dx$ $= \Big[\ln(x-2)\Big]_3^4 + 2\Big[\ln(x+1)\Big]_3^4 + \left[\frac{3}{x+1}\right]_3^4 - \frac{1}{2}\left[\frac{1}{(x+1)^2}\right]_3^4$ $= (\ln 2 - \ln 1) + 2(\ln 5 - \ln 4) + \left(\frac{3}{5} - \frac{3}{4}\right) - \frac{1}{2}\left(\frac{1}{25} - \frac{1}{16}\right)$ $= \ln 2 + 2\ln\left(\frac{5}{4}\right) - \frac{3}{20} + \frac{9}{800}$ $= 1.00068428318836$ ≈ 1.001		
		12	
15	a) $\frac{dy}{dx} = x - \frac{2y}{x}$ $\frac{dy}{dx} + \frac{2y}{x} = x$ $\text{I.F} = e^{\int \frac{2}{x}dx}$ $\text{I.F} = e^{2\ln x}$ $\text{I.F} = e^{\ln x^2}$ $\text{I.F} = x^2$ Multiplying through by x^2 $x^2\frac{dy}{dx} + 2xy = x^3$		

	$\int \frac{d}{dx}(x^2 \cdot y) = \int x^3 dx$ $x^2 y = \frac{x^4}{4} + c$ At point $(2, 4)$; $x = 2$ and $y = 4$ $4 \times 4 = 4 + c$ $c = 16 - 4 = 12$ $\therefore y = \frac{x^2}{4} + \frac{12}{x^2}$ b) $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow x^2\left(v + x\frac{dv}{dx}\right) = x^2 + (vx)^2 + x(vx)$ $v + x\frac{dv}{dx} = 1 + v^2 + v$ $x\frac{dv}{dx} = 1 + v^2$ Separating variables; $\int \frac{1}{1+v^2} dv = \int \frac{1}{x} dx$ $\tan^{-1}(v) = \ln x + c$ $\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$ $\frac{y}{x} = \tan(\ln x + c)$ $\therefore y = x\tan(\ln x + c)$		
		12	
16	a) $\boldsymbol{r} = \begin{pmatrix} 1 + 2t \\ 1 + 2t \\ -3 + t \end{pmatrix}$		

$$\Rightarrow \begin{pmatrix} 1+2t \\ 1+2t \\ -3+t \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix} = 13$$

$6 + 12t - 3 - 6t - 6 + 2t = 13$

$8t = 16$

$t = 2$

Let $r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$x = 1 + 2 \times 2 = 5$

$y = 1 + 2 \times 2 = 5$

$z = -3 + 2 = -1$

(5,5,1) is the point of intersection.

Let θ be the required angle.

$\boldsymbol{d} \cdot \boldsymbol{n} = |\boldsymbol{d}||\boldsymbol{n}| \sin\theta$

$$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix} = \sqrt{2^2 + 2^2 + 1^2}\,\sqrt{6^2 + (-3)^2 + 2^2}\,\sin\theta$$

$12 - 6 + 2 = 3 \times 7 \times \sin\theta$

$8 = 21 \sin\theta$

$\theta = \sin^{-1}\left(\frac{8}{21}\right)$

$\theta = 22.39^0$

b)

		12	