UACE PHYSICS PAPER 2001 GUIDE

Instructions to the candidates:

Answer **five** questions taking at least one from each of the sections **A, B, C** and **D**, but not more than one question should be chosen from either section **A** or **B**

Any additional question (s) will not be marked.

Mathematical tables and squared paper will be provided

Non programmable calculators may be used.

Assume where necessary

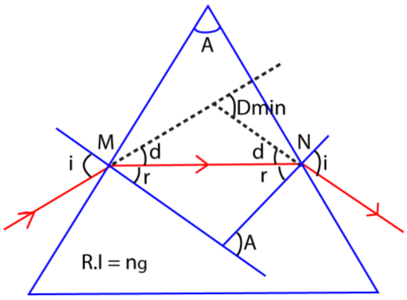
Acceleration due to gravity, g	$9.81ms^{-2}$
Electron charge, e	$1.6 x10^{-19}C$
Electron mass	$9.11 \times 10^{-31}kg$
Plank's constant, h	$6.6 \times 10^{-34}Js$
Speed of light in the vacuum, c	$3.0 \times 108ms^{-1}$
Specific heat capacity of water	$4.200Jkg^{-1}K^{-1}$
Avogadro's number, N_A	$6.02 \times 10^{23}mol^{-1}$
The constant, $\frac{1}{4\pi\varepsilon_0}$	$9.0 \times 10^{9}F^{-1}m$
Permittivity of free space, μ_0	$4.0\pi \times 10^{-7}Hm^{-1}$
Permittivity of free space, ε_0	$8.85 \times 10^{-12}Fm^{-1}$
One electron volt	$1.6 \times 10^{-19}J$
Resistivity of Nichrome wire at 25^0C	$1.2 \times 10^{-6}\Omega m$

SECTION A

1. (a) State the laws of refraction of light (02marks)
   - the incident ray, refracted ray and the normal at the point of incidence all lie in the same plane
   - for given frequency of light, the ratio of the angle of incidence to sine of angle of refraction is constant for a given pair of media.

(b) (i) Derive an expression for the refractive index of a prism in terms of refracting angle A and the angle of minimum deviation, D. (05marks)

At minimum deviation, light passes **symmetrically** through the prism. That is to say, the angle of incidence is equal to the angle of emergence.



Consider a ray on one face of the prism at an angle **i1**and leaves it at an angle **i2** to the normal as shown

For minimum deviation, $i_1 = i_2 =$ **i** and $r_1 = r_2 =$ **r**.

From the diagram, Dmin = d + d

Dmin = 2d where d = i - r

Dmin = 2i – 2r---------------------------- (a)

But r + r = A

⇨ 2r = A **OR** $r = \frac{A}{2}$ -------------------------------------- (b)

Combining equation (a) and (b) gives

Dmin = 2i – A

$i = \frac{D_{mim}+A}{2}$ ---------------------------------(c)

At M Snell's law becomes

$n_a \sin i = n_g \sin r$

$$n_g = \frac{n_a \sin i}{\sin r}$$-------------------------------(d)

Substituting equation (b) and (c) in (d) gives

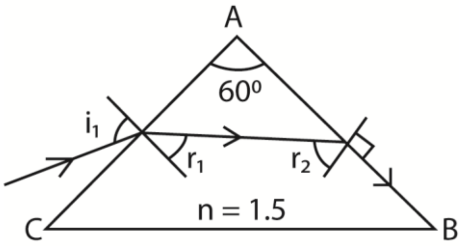
$$n_g = n_a \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin A/2}$$

Since $n_a = 1$,

$$n_g = \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin{}^{A}/_{2}}$$

(ii) Monochromatic light is incident on one refracting face of a prism of refracting angle 60^0, made of glass of refractive index 1.50.

Calculate the least angle of incidence for the ray to emerge through the second refracting face. (05marks)



Using Snell's law on face AB

$1.5\sin r_2 = \sin 90^0$

$r_2 = 41.8$

Also applying Snell's law on face AC

$r_1 = 60 - 41.8 = 18.2^0$

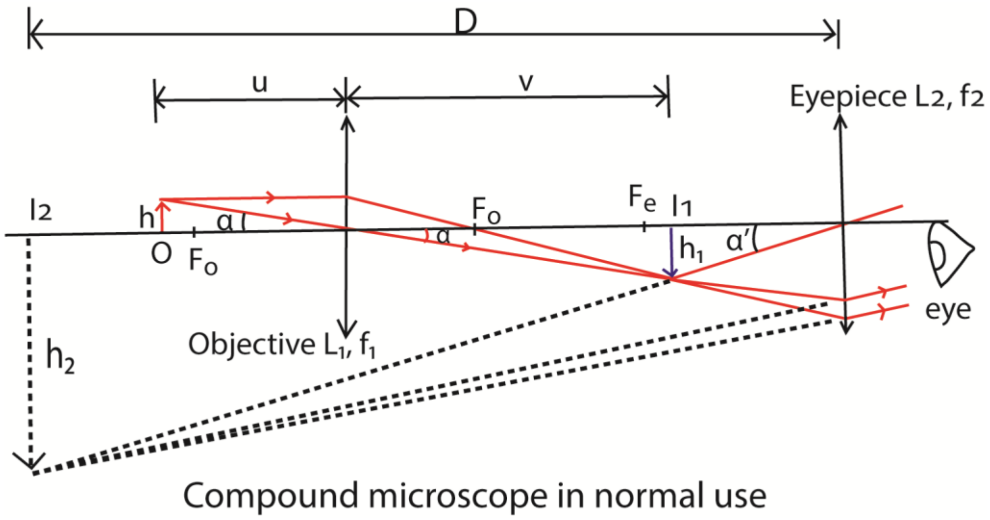
$\sin i = 1.5\sin 18.2$

$i = 27.9^0$

(c) (i) State three differences between compound microscopes and telescopes. (03marks)

Compound microscope	Astronomical telescope
- View near objects	- View distant object
- Objective lens has smaller focal length	- Objective lens has longer focal length
- In normal adjustment the final image is at near point	- In normal adjustment, final image at infinity
- Has greater resolving power	- Has lower resolving power

(ii) Describe, with the aid of a ray diagram, how a compound microscope forms a final image at near point.(05marks)



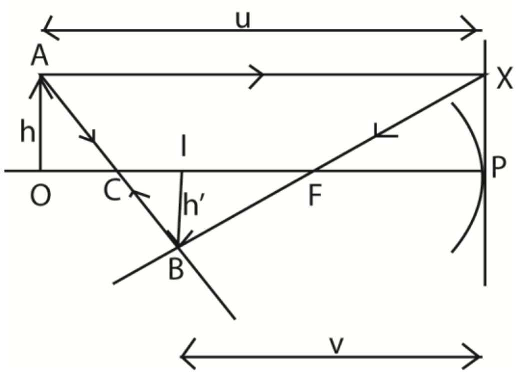
Compound microscope in normal use

The objective forms a real magnified image of the object at a distance less than f_e from the eye piece.

The eye piece form a virtual magnified image of the intermediate image at nearest distance of distinct vision (D=25 cm)

2. (a) Define the terms radius of curvature and focal length of a converging mirror. (02marks)
Radius of curvature of a converging mirror is the distance from the pole of the converging mirror to the center of curvature.
Focal length of converging mirror is the distance from the pole of the mirror to the principal focus.

(b) (i) Draw a ray diagram to show the formation of a real image of a real object in a converging mirror (02marks)



(ii) Use the ray diagram in (b)(i) to derive the expression, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$; where u. v and f are the object distance, image distance and focal length respectively. (05marks)
PC = r and PF = f

Triangle OAC and triangle ICB are similar

$$\frac{OA}{IB} = \frac{u-2f}{2f-v}$$

Also triangle

Triangle

XRF and PQF are similar

$\frac{XP}{IB} = \frac{PF}{IF} = \frac{PF}{IP-PF}$

$= \frac{f}{v-f}$

But XP = OA

$\therefore \frac{u-2f}{2f-v} = \frac{f}{v-f}$

$uv - uf - 2vf + 2f^2 = 2f^2 - vf$

$uv - uf - vf = 0$

after dividing by uvf

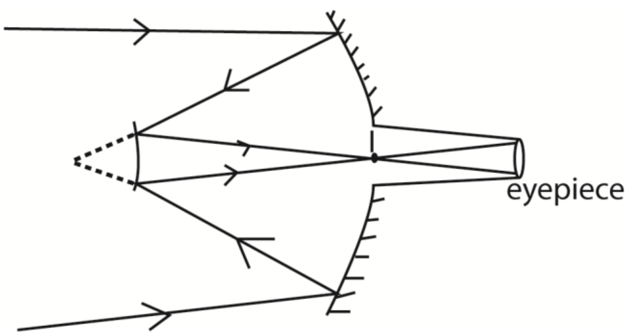
$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

(c) (i) With the aid of ray diagram, describe the structure and action of a reflecting telescope in normal adjustment. (05marks)

The objective is a large parabolic mirror with a circular aperture of long focal length. A beam of from a distant object is reflected and intercepted by a convex mirror.

The convex mirror forms a real and inverted image at I, the pole of the objective mirror and principal focus of the eyepiece

The eyepiece forms a virtual, magnified image at infinity.



(ii) State two advantages of reflecting telescope over an astronomical telescope. (02marks)

- has high resolving power
- has no chromatic aberration
- produce brighter images
- cheap
- spherical aberration is minimized

-

(d) An astronomical telescope has objective of focal length 100cm and eyepiece of focal length 10cm. Calculate the separation of the objective if the lenses are arranged in such a way that the final image is formed at 25cm from the eye. (04marks)

Action of the eyepiece

f = 10cm, v = -25cm, u= ?

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{10} = \frac{1}{u} - \frac{1}{25}$, u = 7.1cm

Separation of lenses = f_0 + u

=100 + 7.1 = 107.1cm

3. (a) (i) Distinguish between free and damped oscillation (02marks)

In free oscillation, the energy and amplitude are constant while in damped oscillation, the total energy and amplitude decrease with time.

(ii) Describe how the amplitude of a forced oscillation builds up to a constant value.

A constant amplitude is maintained by a periodic input of force to an oscillating system. The periodic force compensate the energy lost by oscillation to the system.

(b) The displacement in meters of a plane progressive wave is given by the equation $y = 0.2\sin\pi(200t - \frac{20x}{17})$. Find

(i) Wavelength and (02marks)

Compare $y = 0.2\sin\pi(200t - \frac{20x}{17})$ with $y = a\sin(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda})$.

$\frac{20\pi x}{17} = \frac{2\pi x}{\lambda}$; λ = 1.7m

(ii) Speed of the wave (02marks)

$\frac{2\pi t}{T}$ = 200πt but T = $\frac{\lambda}{V}$

$\frac{2}{\frac{\lambda}{v}} = 200$

$v = \frac{200\lambda}{2} = \frac{200 \ x \ 1.7}{2} = 170ms^{-1}$

(c)(i) Explain the occurrence of beats in sound (03marks)

Beats are the periodic variation in intensity of sound heard when two notes of nearly equal frequency and amplitude moving with the same velocity overlap. When notes arrive in phase to an observer, they reinforce and produce louder notes. Where the notes arrive to the observer out of phase, no sound is heard.

(ii) Two tuning forks X and Y are sounded together to produce beats of frequency 8HZ. Fork X has a known frequency of 512Hz.

When Y is loaded with a small plasticine, beats at frequency of 2Hz are heard when the two tuning forks are sounded together.

Calculate the frequency of Y when unloaded. (03marks)

$f_x - f_y = 8$ or $f_y - f_x = 8$

Given $f_x = 512Hz$

$f_y = 512 - 8 = 504Hz$ or $f_y = 520Hz$

loading y reduces the frequency of y

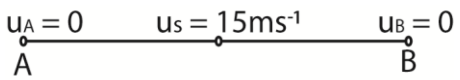
∴ Frequency, $f_y = 520Hz$

(d)(i) What is Doppler's Effect? (01mark)

Doppler Effect is the apparent change in the frequency of radiant energy or sound that occurs as a result of the relative motion of the source and the observer.

(ii) A car sounds its horn as it travels at a steady speed of $15ms^{-1}$ along a straight load between two stationary observers A and B. The observer A hears a frequency of 538Hz while B hears a lower frequency.

Calculate the frequency heard by B, assuming the speed of sound in air is $340ms^{-1}$.



$\lambda'_A = \frac{v - u_s}{f}$ and $f'_A = \frac{vf}{v - u_s}$

Substituting $v = 340ms^{-1}$, $us = 15ms^{-1}$ and $f'_A = 538Hz$

$f = \frac{f'_A \, x \, (v - u_s)}{v} = \frac{538\,(340-15)}{340} = 514Hz$

also, $\lambda'_B = \frac{v + u_s}{f}$ and $f'_B = \frac{vf}{v + u_s} = \frac{340 \, x \, 514}{(340+15)} = 492Hz$

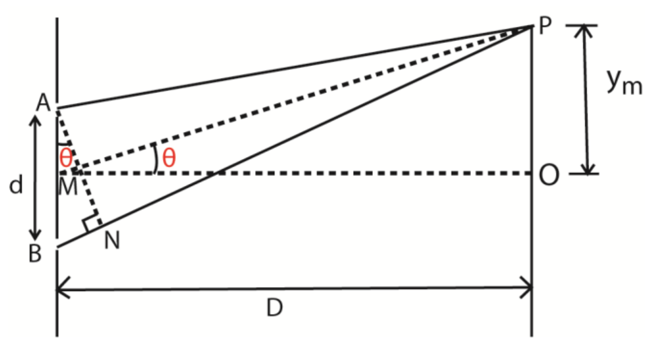
4. (a) (i) What is meant by interference and diffraction of light waves? (02marks)

Diffraction is the spreading of light beyond the geometrical shadow leading to interference pattern just beyond geometrical shadow.

(ii) State the conditions of necessary for observing diffraction (01marks)

The dimensions of aperture should be comparable with wavelength of light.

(c) (i) Derive the expression for fringe separation in Young's interference pattern in terms of the silt-separation, d, the distance, D, of the screen from the double slits and the wavelength, λ, of the light.



A and B are coherent sources.

Suppose waves from A and B superpose at P to form bright fringe

Path difference, BN = BP – AP = dsinθ

For D>>d, θ is small that sinθ = dtanθ

⇨ BN = dtanθ = $\frac{dy_m}{D}$

For the mth bright fringe, BN = mλ, where λ is the wavelength

⇨ $\frac{dy_m}{D}$ = mλ

$y_m = \frac{m\lambda D}{d}$

For (m + 1)th bright fringe, $\frac{dy_{m+1}}{D}$ = (m + 1)λ

$y_{m+1} = \frac{(m+1)\lambda D}{d}$

Fringe separation, y = $y_{(m+1)} - y_m = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d} = \frac{\lambda D}{d}$

(d) Two slits of 0.5mm apart are placed at a distance of 1.0m from the screen. The slits are illuminated with light of wavelength 550nm. Calculate the distance between the fourth and second bright fringes of the interference pattern. (05marks)

$x_2 = \frac{2\lambda D}{d}$ and $x_4 = \frac{4\lambda D}{d}$

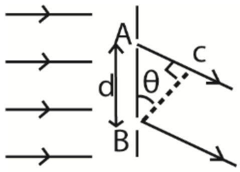
$x_4 - x_2 = \frac{2\lambda D}{d}$

Substituting λ = 550 x 10^{-9}m, d = 0.5 x 10^{-3}m and D = 1.0m

$x_4 - x_2 = \frac{2 \times 550 \times 10^{-9} \times 1}{0.5 \times 10^{-3}}$ = 2.2 x 10^{-3}m

(e) A transmission diffraction grating of spacing, d, is illuminated normally with light of wavelength, λ.

(i) Derive the condition for occurrence of diffraction maxima. (03marks)



Path difference for between waves A and B = distance AC = dsin θ

For reinforcement (diffraction maxima) path difference = nλ where n = 1, 2, 3…

Therefore dsinθ = nλ

(ii) Describe briefly the intensity distribution on a screen placed beyond the grating (02marks)

A central principal maxima is the most intense and the intensity decreases outwards on either side.

(iii) What is the effect on the diffraction pattern of using a grating with a larger number of lines? (02marks)

Using a grating with a large number of lines reduces the slit separation, d, and the peaks become sharper.

SECTION B

5. (a)(i) Define the ampere (2marks)

An ampere is a current when maintained in two parallel conductors of negligible cross section area and separation of 1m cause a force of 2×10^{-7}N per meter between them.

(ii) Write down the expression for the force on a conductor carrying current which is inclined at an angle θ to a uniform magnetic field (02mark)

Force, $F = BIL \sin\theta$

Where B = magnetic flux density
L = length of the conductor
I = current

(b)

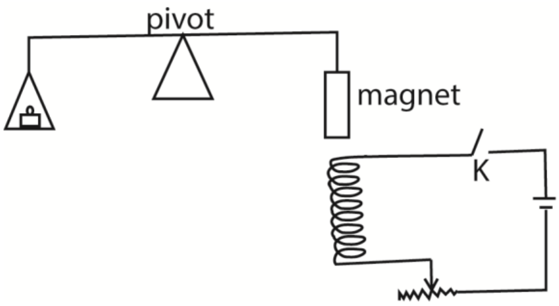


Figure above represents a current balance. When switch, K, is open the force required to balance the magnet is 0.2N. When switch, K, is closed and a current of 0.5A flows, a force of 0.22N is required for balance.

(i) Determine the polarity at the end of the magnet closest to the coil (03marks)

Solution

When K is closed, current observed from end of the coil near the magnet will be flowing in an anticlockwise direction implying that it becomes a north pole.
Since a large force is needed to balance the magnet on closing K, there attraction between the magnet and the coil hence the end of the magnet near the coil is a South Pole.

(ii) Calculate the weight required for balance when a current of 2A flows through the coil (03marks)

If W_m is the of the magnet, F_m is the magnetic force due to the coil, and W is the weight required, then

$W = W_m + F_m$ …………………… ((i)

But $W_m = 0.2$ and $F_m = kI^2$

When I = 0.5, F_m = (0.22 – 0.2 = 0.02

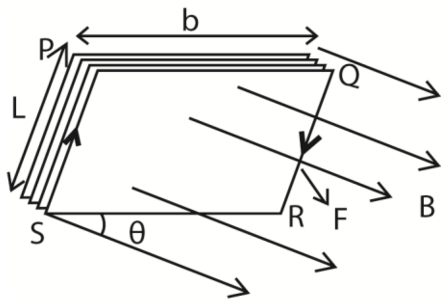
Hence k $=\frac{0.02}{0.5^2}$ = 0.08

When I = 2, k = 0.08, F_m = 0.08 x 2^2 = 0.32

Substituting F_m in eqn.

W = 0.2 +0.32 = 0.52N

(c) A rectangular coil of N turns each of dimensions L x b is inclined at an angle θ to uniform magnetic field of flux density, B. derive an expression for torque on the coil if a current I is passed through it.



- When current flows through the coil, the conductor experiences a magnetic force.
- Force on side PQ = NBIbsinθ (downwards) while Force on side RS = NBINsinθ (upwards). The two forces cancel out due to rigidity of the coil.
- Side PS experiences force NBILcosθ perpendicularly into the page while RQ experiences force NBIL cosθ perpendicularly out of page. The two forces constitute a couple whose moment of force

  τ = F x b

  = NBILbcosθ

  = NBIAcosθ (where A is the area = L x b)

(d) A ballistic galvanometer of sensitivity 2 divisions per μC is connected across a coil of 10 turns wound tightly round the middle of a solenoid of 10^3 turns per meter and diameter 5.0cm. When the current in the solenoid is reversed, the ballistic galvanometer deflects through 8 divisions. If the total resistance of the coil and galvanometer is 20Ω, find the current in the coil.

Solution

Q = $\frac{8}{2}\ x\ 10^{-6}$ = 4 x 10^{-6}C; r = 20Ω, n = 10^3

A = $\frac{\pi d^2}{4}$ = $\frac{\pi(5\ x\ 10^{-2})^2}{4}$ = 1.96 x 10^{-2}m^2

B in the solenoid = μ_0nL

Magnetic flux linking the coil = ϕ_1 = μ_0nI x NA

When current is reversed, magnetic flux linking the coil, ϕ_2 = -μ_0nI x NA

Change in magnetic flux linking the coil, $\phi_1 - \phi_2 = 2\mu_0 nINA$

Charge, Q = $\frac{\Delta\phi}{R}$

I = $\frac{QR}{2\mu_0 nNA} = \frac{4\,x10^{-6}\,x\,20}{2\,x\,4\pi\,x\,10^{-7}\,x\,10\,x\,10^3\,x\,1.96\,x\,10^{-3}}$ = 1.62A

6. (a) State the laws of electromagnetic induction. (02marks)

Faraday's law of electromagnetic induction states that the magnitude of induced e.m.f in a circuit is directly proportional to the rate of change in magnetic flux linking the circuit.
Lenz's law of electromagnetic induction states that the induced e.m.f is in such a direction as to oppose the change causing it.

(b) A circular coil of 100turns and cross sectional area $0.2m^2$ is placed with its plane perpendicular to horizontal magnetic field of flux density 1.0×10^{-2}T.
The coil is rotated about a vertical axis so that it turns through 60^0 in 2s.
Calculate:

(i) The initial flux linkage through the coil (02mark)

Initial flux linkage, ϕ_1 = NBA = $100 \times 0.2 \times 1 \times 10^{-2}$ = 0.2Wb

(ii) The e.m.f induced in the coil (03marks)

Induced e.m.f = - rate of change in magnetic flux
ϕ_1 = NBAcos60^0 = $100 \times 0.2 \times 1 \times 10^{-2} \times 0.5$ = 0.1Wb
e.m.f = $\frac{\Delta Q}{\Delta t} = \frac{-(\phi_1 - \phi_2)}{\Delta t} = \frac{-(0.1-0.2)}{2}$ = 0.05V

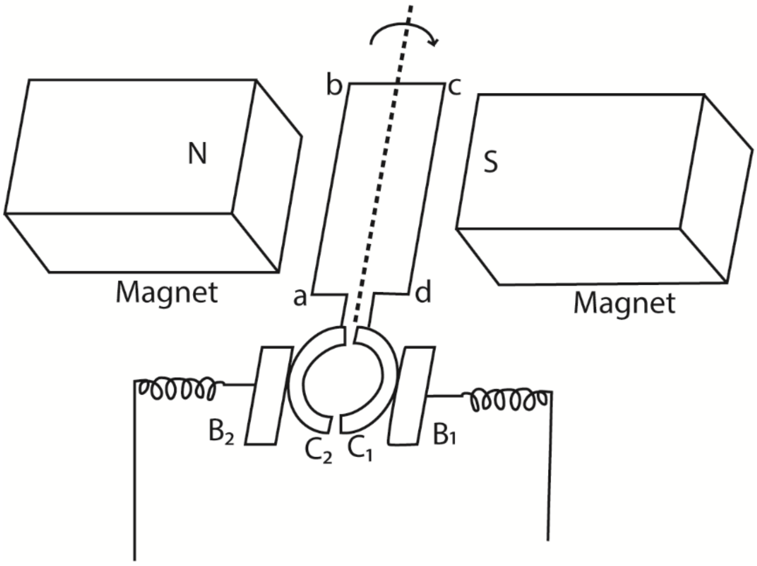
(c)(i) Explain the origin of the back e.m.f in an electric motor (02mark)

When the armature of an electric motor rotating, the wires are cutting the lines of force and an e.m.f is induced in the armature. The direction of the induced e.m.f is such that it opposes the applied e.m.f and hence back e.m.f.

(ii) A motor whose armature resistance is 2Ω is operated on 240V mains supply. If the back e.m.f in the motor is 220V, calculate the armature current. (03marks)

I = $\frac{V-E}{r} = \frac{240-220}{2}$ = 10A

(d) (i) Describe with the aid of a diagram the mode of action of a simple d.c. generator. (06marks)



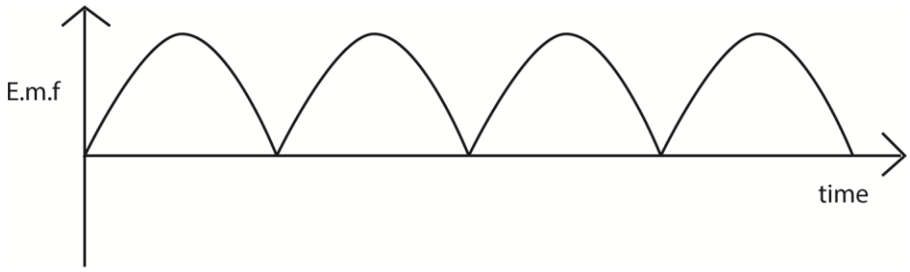
B_1 and B_2 are carbon brushes, C_1 and C_2 are split ring commutators and abcd is a rectangular coil

When the coil rotates at uniform velocity in magnetic field, e.m.f is induced in it.

When the coil is in vertical position, the commutators change brushes C_1 to B_2 and C_2 to B_1.

E.m.f reverses direction but the current does not change direction. Hence current flows in the same direction in a resistor.

(ii) Sketch the output of a d.c. generator. (01mark)



(iii) What is the major difference between a d.c. motor and a.c. generator? (01mark)

In a d.c. motor, electrical energy is converted into mechanical energy while for a d.c. generator mechanical energy is converted into electrical energy.

7. (a) Define root mean square value (r.m.s) of an alternating current. (01mark)

Root mean square value of an a.c is the value of steady current which dissipates heat at the same rate in a given resistor as a.c.

(b) A sinusoidal alternating voltage V = 170sin120πt, voltage, is applied across a resistor of resistance 100Ω

Determine

(i) The r.m.s value of current which flows. (03marks)

Ir.m.s = $\frac{V_{r.m.s}}{R}$ but $\frac{V_0}{\sqrt{2}}$

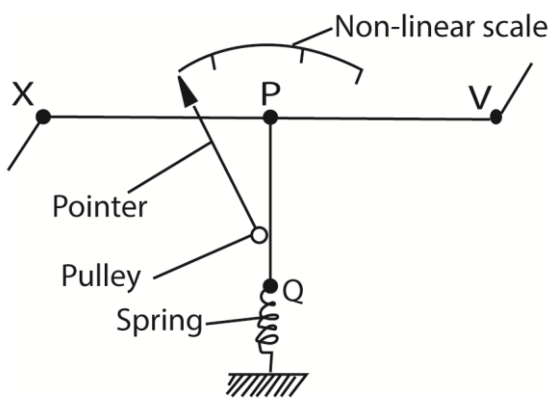
$\therefore\ I_{r.m.s} = \frac{V_0}{R\sqrt{2}} = \frac{170}{100\sqrt{2}} = 1.2\ A$

(ii) The frequency of the current through the resistor. (02marks)

$\omega = 2\pi f$; $\omega = 120\pi$

$\therefore f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60Hz$

(c) With the aid of a labelled diagram describe the structure and action of hot wire ammeter. (06mark)



- The current flows through a fine resistance-wire XY, which it heats.
- The wire warms up to such a temperature that it loses heat-mainly by convection-at a rate equal to the average rate at which heat is developed in the wire.
- The rise in temperature of the wire makes it expand and sag; the sag is taken up by a second fine wire PQ, which is held taut by a spring.
- The wire PQ passes round a pulley attached to the pointer of the instrument, which rotates as the wire XY sags.
- The deflection of the pointer is roughly proportional to the average rate at which heat is developed in the wire XY; it is therefore roughly proportional to the average value of the square of the alternating current, and the scale is a square-law one.

(d) Explain the term self-induction and mutual induction. (03 marks)

Self-induction is the process by which e.m.f is induced in a circuit due to changing current in the same circuit.

Mutual induction is the process by which e.m.f is induced in a circuit due change in current in the nearby circuit.

(e) A coil of self-inductance, L and negligible resistance is connected across a source of alternating voltage, $V = V_0\cos\omega t$.

(i) Find the expression for the current which flows in the coil. (03marks)

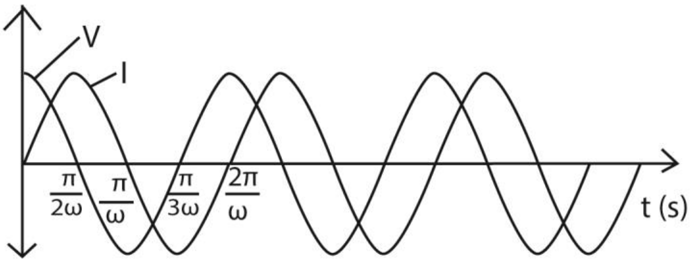
If E is induced e.m.f and V is the applied voltage, then V = -E

$E = \frac{LdI}{dt}$; since $V = V_m$ then V_0sincos$\omega t = \frac{LdI}{dt}$

$\int \cos \omega t \, dt = \frac{L}{V_0} \int dI$

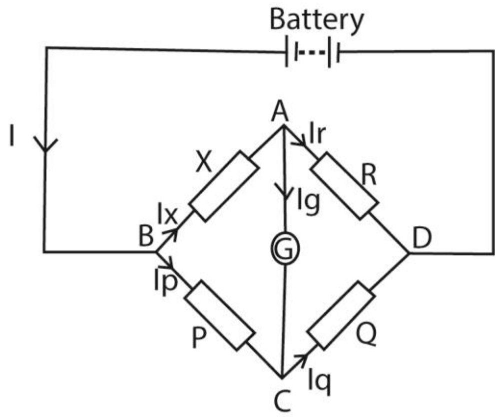
$\frac{1}{\omega} \sin \omega t = \frac{V_0}{L\omega} \sin \omega t$

(ii) Sketch, using the same axes, the time variation of the applied voltage and the current which flows in the coil (02marks)



SECTION C

8. (a) Derive the condition for a Wheatstone bridge to be balanced. (04marks)



When resistances P, Q, R and X are such that no current flows through the galvanometer, the bridge is said to be balanced, i.e., Ig = 0

⇨ Potential at A = potential at C
$V_{AB} = V_{BC}$ and $V_{AD} = V_{DC}$
IxX = IpP and IrR = IqQ
$\frac{I_x X}{I_r R} = \frac{I_p P}{I_Q Q}$
Since Ig = 0, then Ix = Ir and Ip = Iq
Hence $\frac{X}{R} = \frac{P}{Q}$

(b) (i) Define temperature coefficient of resistance. (01mark)
Temperature coefficient of resistance is the fractional change in resistance at 00C per degree Celsius rise in temperature.

(ii) When a coil X connected across the left hand gap of a meter bridge is heated to a temperature of 30^0C, the balance point is found to be 51.5cm from the left-hand end of the slide wire. When the temperature is raised to 100^0C, the balance point is 54.6cm from the left end. Find the temperature coefficient of resistance of X. (06marks)

Solution

Resistance at 30^0C is given by

$R_{30} = R_S\frac{51.5}{48.5} = 1.062R_s$

Resistance at 100^0C is given by

$R_{30} = R_S\frac{54.6}{45.4} = 1.203R_s$

From $R_\theta = R_0(1 + \alpha\theta)$

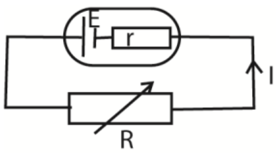
$1.062R_s = R_0(1 + 30\alpha$ …………………… (i)

$1.203R_s = R_0(1 + 100\alpha$ …………………… (ii)

From (i) and (ii) $\alpha = 2 \times 10^{-3}K^{-1}$

(c) (i) A battery of e.m.f E and internal resistance, r, is connected to a resistor of variable resistance, R. Obtain the expression for maximum power dissipated in a resistor.

Circuit diagram



E = I(R +r)

$EI = I^2(R + r)$

Power dissipated in a resistor = I^2R

For maximum power, $\frac{dP}{dR} = 0$

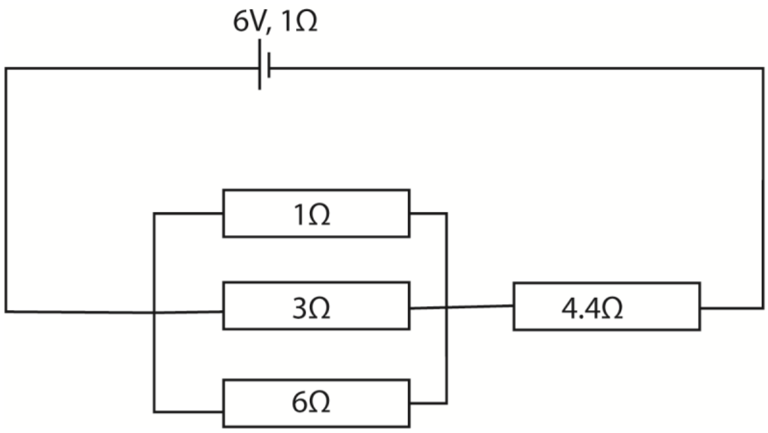
$\frac{dP}{dR} = E\frac{dI}{dR} - 2Ir\frac{dI}{dR} = 0$

$> E = 2Ir$

$substituting\ \ for\ E\ in\ E = IR + Ir\ and\ siplifying$

R= r, hence $P_{max} = I^2R = \frac{E^2}{4r}$. Since $I = \frac{E}{2r}$

(i) A battery of e.m.f 6V and internal resistance 1Ω is connected across a network of resistor as shown in the diagram below



Find the current supplied by the battery. (04marks)

Effective resistance R_1 for resistors in parallel

$\frac{1}{R_1} = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} = \frac{6+2+1}{6} = \frac{9}{6}$

$R_1 = \frac{6}{9}$

Effective resistance, R of $R_{1,}$ 1Ω and 4.4Ω resistor in series = $4.4 + \frac{6}{9} + 1 = 6.07Ω$

Current = $\frac{V}{R} = \frac{6}{6.07} = 0.99A$

9. (a) State coulomb's law of electrostatics (01marks)

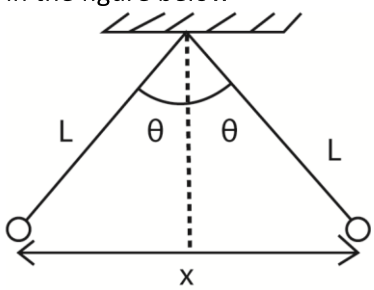
The force between two point charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance of separation of the charges.

(b) (i) Define electric field intensity and electric potential. (02marks)

Electric potential is a point in electric field is work done to transfer 1C charge from infinity to that point.

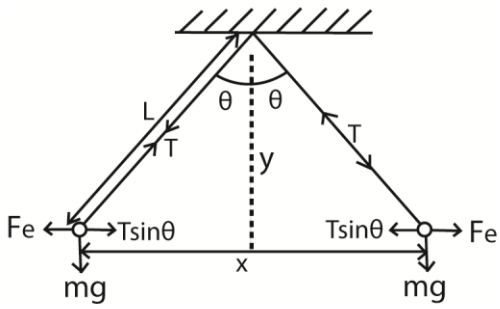
Electric field intensity at a point is the force experienced by 1C of positive charge placed at that point in electric field.

(ii) Two identical conducting balls of mass, m, are each suspended in air from a silk thread of length L. when the two balls are each given identical charge, q, they move apart as shown in the figure below



If at equilibrium each thread makes a small angle θ with the vertical, show that the separation, x, is given by

$x = \left[\frac{q^2L}{2\pi\varepsilon mg}\right]^{\frac{1}{3}}$; where ε is permittivity of air



Horizontally, Fe = Tsinθ ………………. (i)

Vertically mg = Tcosθ …………………. (ii)

Dividing Eqn. (i) by Eqn (ii)

$\frac{F_e}{mg} = \tan\theta$, but $F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$

Hence tanθ = $\frac{q^2}{4\pi\varepsilon_0 x^2 mg}$ and sin θ = $\frac{x}{2L}$

For small angles, tanθ ≈ sinθ ≈ θ

⇨ $\frac{q^2}{4\pi\varepsilon_0 x^2 mg} = \frac{x}{2L}$

x =$\frac{q^2 L}{4\pi\varepsilon_0 mg}$

(c) (i) Define the term capacitance of a capacitor. (01mark)

Capacitance of a capacitor is the ratio of the magnitude of charge on one of the plates of the capacitor, to the potential difference between the plates.

(ii) State the factors that affect capacitance of a capacitor (03marks)

- Plate separation
- Area of overlap
- Permittivity of the material between the plates

(iii) Show that the energy stored n a capacitor of capacitance, C charged to a p.d V is equal to $\frac{1}{2}CV^2$. (03marks)

Suppose the p.d between the plates at some instant is V. When a small charge of δq is transferred from the negative plate to the positive plat, the p.d increases by δV.

Work done to transfer the charge; δw = (V + δV)δq

≈ Vδq

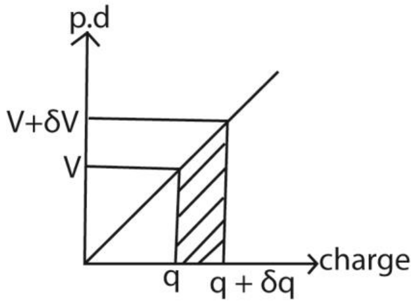
But V = $\frac{q}{C}$

∴ $\frac{q}{C}\delta q$

Total work done=$\int_0^q \frac{q}{C}\delta q$ = $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

Or

From q = CV, V is proportional to q, this gives the graph of V against q below



Shaded area = $\frac{1}{2}(V + V + \delta V)\ x\ dV$

= work done to increase charge on the capacitor by δq

∴ to charge a capacitor from q = 0 to q = Q

Work done, w = average voltage x charge

= $\frac{1}{2}(0 + V)x\ Q$

=$\frac{1}{2}QV$

But Q = CV

∴w = $\frac{1}{2}CV^2$

(d) The plates of parallel plate capacitor each of area $2.0cm^2$are 5mm apart. The plates are in vacuum and potential difference of 10,000V is applied across the capacitor.

Find the magnitude of the charge on the capacitor (04marks)

Q = $\frac{\varepsilon_0 A}{d}V = \frac{8.85 \, x \, 10^{-12} \, x \, 2.0 \, x \, 10^{-4} \, x \, 10000}{5 \, x \, 10^{-3}} = 3.54NC$

10. (a)Define temperature efficiency of resistance and electrical resistivity. (02marks)

Temperature coefficient of resistance is the fractional change in resistance at 00C per degree Celsius rise in temperature.

(b) A nichrome wire of length 1.0m and diameter 0.72mm at 25^0C, is made into a coil. The coil is immersed in $200cm^3$ of water at the same temperature and current of 5.0A is passed through the coil for 8minutes until when the water starts to boil at 100^0C.

Find

(i) The resistance of the coil at 25^0C. (02marks)

R = $\frac{\rho L}{A}$ but A = $\frac{\pi d^2}{4}$

$\therefore R = \frac{4\rho L}{4\pi d^2} = \frac{4 \, x \, 1.2 \, x \, 10^{-6} x \, 1}{4\pi \, x \, (0.72 \, x \, 10^{-3})^2} = 2.95\Omega$

(ii) The electrical energy expended assuming all of it goes into heating the water (02marks)

W = mc.Δθ = $200 \times 10^{-6} \times 10^3 \times 4.2 \times 10^3 \times (100 - 25) = 6.3 \times 10^4$J

(iii) The mean temperature coefficient of resistance of nichrome wire between 0^0C and 100^0C. (06marks)

Electrical energy dissipated, = $I^2R_{mean}t$

⇨ $6.3 \times 10^4 = 5^2 \times R_{mean} \times 8 \times 60$

R_{mean} = 5.25Ω

$R_{mean} = \frac{R_{25} + R_{100}}{2}$

From R =

$R_{100} = 2R_{mean} - R_{25}$

= 2 x 5.25 – 2.95

= 7.55Ω

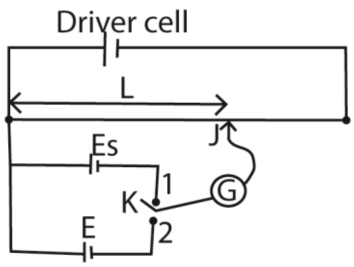
From Rθ = R0(1 + αθ)

$2.95 = R_0(1 + 25\alpha$ …………………… (i)

$7.55 = R_0(1 + 100\alpha$ …………………… (ii)

From (i) and (ii) $\alpha = 4.3 \times 10^{-2}K^{-1}$

(c) Describe, with the aid of circuit diagram how a slide wire potentiometer can be used to measure e.m.f of a cell. (04marks)



Es is the cell with known e.m.f and
E is the cell whose e.m.f is to be determined
AB = 1m

Switch K in position 1. Balance length Ls where the galvanometer shows no deflection is determined
If k is the calibration constant, Es = kLs ……………. (i)
Switch K in position 2. Balance length L where the galvanometer shows no deflection is determined
If k is the calibration constant, E = kL ……………. (ii)
From eqn. (i) and (ii)
$E = \frac{LE_S}{L_S}$

(d) An accumulator of e.m.f 2.0V is connected across a uniform wire of length 1.0m and resistance 8.0Ω. A cell of e.m.f 1.50V is connected in series with a galvanometer and connected across a length L of slide wire. The galvanometer shows no deflection when L is 90.0cm. Find the internal resistance of an accumulator. (04marks)

Resistance of AC = $\frac{8\,x90}{100} = 7.2\Omega$

Current supplied by the driver cell = $\frac{2}{8+r}$ = $\frac{V}{R}$

⇨ $\frac{2}{8+r} = \frac{1.5}{7.2}$

r = 1.5Ω

END