

P425/1

PURE

MATHEMATICS

Paper 1

July/ August 2022

3hours



## **KAMSSA JOINT MOCK EXAMINATIONS**

### **Uganda Advanced Certificate Of Education**

#### **PURE MATHEMATICS**

##### **Paper 1**

3hours

#### **Instructions to candidates:**

- Answer *All the eight* questions in section A and *five* questions from section B.
- Any additional question (s) answered will not be marked.
- All working *must be shown* clearly.
- Begin each answer on a fresh page.
- Graph paper is provided.
- *Silent non-programmeable, scientific calculators and mathematical tables with at least of formulae* may be used.
- State the degree of accuracy at the end of each answer given. If a calculator or a mathematical table is used, indicate *Cal* for calculator or *Tab* for mathematical tables.

## SECTION A ( 40 MARKS)

*Answer all questions in this section*

1. Solve for  $y$ :  $4\cos y = 3\tan y + 3\sec y$  for  $0^\circ \leq y \leq 360^\circ$  (5 marks)
2. Evaluate:  $\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$  (5 marks)
3. Solve for  $t$  in :  $5^{2t} = 5^{t+1} - 6$  (5 marks)
4. Show that the locus of a point  $P(x,y)$  which moves such that it divides the line joining  $A(2, -3)$  and  $B(3,4)$  in the ratio 1:2 is a circle. State its radius and centre.(5 marks)

5. Given that  $y = \sqrt{4 + 3\sin x}$ , show that :

$$2y \frac{d^2y}{dx^2} + 2 \left( \frac{dy}{dx} \right)^2 + y^2 = 4 \quad (5 \text{ marks})$$

6. Find the perpendicular distance of the point  $P(3, -1, 2)$  from the line

$$r = i + j + 3k + \mu(2i + 4j - k). \quad (4 \text{ marks})$$

7. Find the values of  $k$  for which the quadratic equations  $x^2 + kx - 6k = 0$  and

$$x^2 - 2x - k = 0 \text{ have a common root.} \quad (6 \text{ marks})$$

8. Air is pumped into a spherical balloon at a rate of  $200\text{cm}^3\text{s}^{-1}$ . When the radius of the balloon is 80mm, find the rate at which the surface area is increasing. (5 marks)

## SECTION B: (60 MARKS)

*Answer any five questions from this section. All questions carry equal marks*

9. Evaluate the following:

a.  $\int \frac{1}{e^{2x}-1} dx$

b.  $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos t} dt$  (12 marks)

10. a. Find the coefficient of  $x^3$  in the expansion of  $\left(\frac{1}{x^2} - x\right)^{18}$  (5 marks)

b. Show that  $\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2 + \dots$  and hence using  $x = \frac{1}{7}$ , show that  $\sqrt{3} \approx \frac{196}{113}$ . (7 marks)

11. a. Given that  $2A + B = 45^\circ$ , Show that

$$\tan B = \frac{1-2\tan A - \tan^2 A}{1+2\tan A - \tan^2 A}$$
 (6 marks)

b. Find the value of  $x$  in:  $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$  (6 marks)

12. a. Use small changes to evaluate  $\tan 61^\circ$  to 2dps. (5 marks)

b. Show that  $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$  from first principles. (7 marks)

13. a. Given the equation of curve as  $x^2 + 4x - 8y - 4 = 0$ .

i. Show that the curve is a parabola.

ii. Find the coordinates of the vertex. (4 marks)

b. Show that if the line  $y = mx + c$  touches the curve  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then  $c^2 = a^2m^2 + b^2$ .

Find the equations of the tangents to the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ , which are parallel to  $y = x + 1$ .

(8 marks)

14. Describe the locus of a complex number  $\hat{z} = x + yi$  which moves in the complex

plane such that  $\arg\left(\frac{z-3}{z-2i}\right) = \frac{\pi}{4}$ . (12 marks)

15. a. Find the cartesian equation of the plane  $r = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}$ , given that

$R(x, y, z)$  is a general coordinate in the plane.

**(7 marks)**

- b. Find the cartesian equation of the line which passes through the point  $A(4, 4, -1)$

and is perpendicular to the plane in (a) above.

**(5 marks)**

16. The rate at which the temperature of a liquid in an un-covered pan falls is directly proportional to the difference between the temperature of the liquid and that of the surrounding. The temperature of the liquid is initially  $50^{\circ}\text{C}$ . After 20 minutes, the temperature of the liquid is  $35^{\circ}\text{C}$ . Given that the temperature of the surrounding is  $15^{\circ}\text{C}$ . What will be the temperature of the liquid after 26 minutes? (give your answer to 1 decimal place)

**(12 marks)**

**END**