UACE MATHEMATICS PAPER 1 2018 guide

SECTION A (40 marks)

Answer all questions in this section

1. In triangle ABC a= 7cmm b = 4cm and c = 5cm. find the value of
   (a) cos A
   (b) Sin A (05marks)
2. Determine the angle between the lines $\frac{x+4}{8} = \frac{y-2}{2} = \frac{z+1}{-4}$ and the plane 4x + 3y – 3z + 1 = 0 (05marks)
3. Find $\int x^2 e^x dx$ (05marks)
4. Express the function f(x) = x^2 + 12x + 32, in form a$(x + b)^2$ + c.
   Hence find the minimum value of the function f(x) (05mars)
5. A point P moves such that its distance from two points A(-2, 0) and B (8,6) are in ratio AP: PB = 3:2. Show that the locus of P s a circle. (05marks)
6. Determine the equation of the tangent to the curve $y^3 + y^2 - x^4 = 1$ at the point (1, 1) (05marks)
7. Show that 2log4 + $\frac{1}{2}\log 25 - \log 20 = 2\log 2$. (05marks)
8. The region bounded by the curve y = x^2 – 2x and the x-axis from x = 0 to x = 2 is rotated about the x-axis. Calculate the volume of the solid formed. (05marks)

SECTON B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. The position vectors of the vertices of a triangle are O, r and s, where O is the origin. Show that its area (A) is given by $4A^2 = |r|^2|s|^2 - (r.s)^2$. (06marks)
   Hence, find the area of a triangle when r = $\binom{2}{3}$ and s = $\binom{1}{4}$ (06marks)

10. Express 5 + 12i in polar form
    Hence, evaluate $\sqrt[3]{5+12i}$, giving your answer in the form a + ib where a and b are corrected to two decimal places. (12marks)
11. (a) Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x. (06marks)
    (b) The period T of a swing of a simple pendulum of length of length l is given by
    $T^2 = \frac{4\pi^2 l}{g}$ where g is the acceleration due to gravity.

An error of 2% is made 9n measuring the length, l. determine the resulting percentage error in the period, T. (06marks)

12. (a) Show that tan4θ = $\frac{4t(1-t^2)}{t^4-6t^2+1}$, where t = tanθ (06marks)
    (a) Solve the equation sinx + sin5x = sin2x + sin4x for $0^0 < x < 90^0$. (06marks)
13. (a) The first three terms of a Geometric progression (G.P) are 4, 8 and 16. Determine the sum of the first ten terms of the G.P. (04marks)
    (b) An Arithmetic Progression (A.P) has a common difference of 3. A Geometric Progression (G.P) has a common ration of 2. A sequence is formed by subtracting the term of the A.P from the corresponding terms of the G.P. The third term of the sequence is 4. The sixth term of the sequence is 79. Find the first term of the
    (i) A.P (08 marks)
    (ii) G.P (06 mars)
14. Evaluate
    (a) $\int_0^{\frac{\pi}{2}} sin5xcos3xdx$ (06marks)
    (b) $\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2}$ (06marks)
15. The line y = mx + c is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (06mars)$
    (a) Obtain an expression for c in terms of a, b and m. (06mars)
    (b) Calculate the gradients of the tangents to the ellipse through the point $\left(\sqrt{(a^2 + b^2)},\ 0\right)$
16. The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding. The temperature of the body is initially 60^0C. After 15 minutes the temperature of the body is 50^0C. The temperature of the surrounding is 10^0C.
    (a) Form a differential equation for the temperature of the body. (09marks)
    (b) Determine the time it takes for the temperature of the body to reach 30^0C. (03marks)

Solutions

SECTION A (40 marks)

Answer all questions in this section

1. In triangle ABC a= 7cmm b = 4cm and c = 5cm. find the value of
   (b) cos A
   $a^2 = b^2 + c^2 - 2bc\cos A$
   cos A = $\frac{4^2 + 5^2 - 7^2}{2\,(4)(5)}$ = -0.2
   (c) Sin A (05marks)
   $\sin^2 A = 1 - \cos^2 A$
   sinA = $\sqrt{1-(-0.2)^2}$ = 0.9798

   or
   cosA = -0.2
   A = $\cos^{-1}(-0.2)$ = 101.54
   sin A = sin (101.54) =0.9798
2. Determine the angle between the lines $\frac{x+4}{8} = \frac{y-2}{2} = \frac{z+1}{-4}$ and the plane 4x + 3y – 3z + 1 = 0 (05marks)
   $$\begin{pmatrix} 8 \\ 2 \\ -4 \end{pmatrix} . \begin{pmatrix} 4 \\ 3 \\ -3 \end{pmatrix} = \sqrt{8^2 + 2^2 + (-4)^2} . \sqrt{4^2 + 3^2 + (-3)^2}\sin\theta$$
   32 + 6 + 12 = $\sqrt{84\ x\ 34}$sinθ
   sin θ = $\frac{50}{\sqrt{2856}}$
   θ = 69.33^0
3. Find $\int x^2 e^x dx\ (05marks)$
   Let u = x^2 and v' = e^x
   ⇨ u'= 2x and v = e^x
   $\int x^2 e^x dx = x^2 e^x - 2\int x e^x dx$
   Again let u = x and v' = e^x
   ⇨ u' = 1 and v = ex
   $$\int x^2 e^x dx = x^2 e^x - 2[\int x e^x - \int e^x\, dx]$$
   = $x^2e^x - 2xe^x + 2e^x + c$
4. Express the function f(x) = $x^2 + 12x + 32$, in form $a(x + b)^2 + c$.
   Hence find the minimum value of the function f(x) (05mars)
   f(x) = $x^2 + 12x + 32$
   f(x) = $(x + 6)^2 - 6^2 + 32$
   f(x) = $(x + 6)^2 - 4$
   For minimum value to occur, $(x + 6)^2 = 0$
   x = -6
   f'(x) = 2(x + 6)
   Hence the minimum value of f(x) = -4
5. A point P moves such that its distance from two points A(-2, 0) and B (8,6) are in ratio AP: PB = 3:2. Show that the locus of P s a circle. (05marks)
   $\frac{\overline{AP}}{\overline{PB}} = \frac{3}{2}$ => $2\overline{AP} = 3\overline{PB}$

$2\sqrt{(x+2)^2 + y^2} = 3\sqrt{(x-8)^2 + (y-6)^2}$

Squaring both sides

$4(x^2 + 4x + 4 + y^2)$ = 9(x^2 -16x + 64 + y^2 -12y + 36)

$4x^2$ + 16x + 16 + $4y^2$ = $9x^2$ – 144x + 900 + $9y^2$ – 108y

$5x^2$ + $5y^2$- 160x -108y + 884 = 0

6. Determine the equation of the tangent to the curve $y^3 + y^2 - x^4 = 1$ at the point (1, 1) (05marks)

$y^3 + y^2 - x^4 = 1$

$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} + -4x^3 = 0$

At (1, 1)

$3(1)^2\frac{dy}{dx} + 2(1)\frac{dy}{dx} + -4(1)^3 = 0$

$\frac{dy}{dx} = \frac{4}{5}$

$y - 1 = \frac{4}{5}(x-1)$

5y = 4x + 1

7. Show that $2\log 4 + \frac{1}{2}\log 25 - \log 20 = 2\log 2$. (05marks)

$2\log 4 + \frac{1}{2}\log 25 - \log 20$

$2\log 2^2 + \frac{1}{2}\log 5^2 - (\log 4 + \log 5)$

$2\log 2^2 + \frac{1}{2}\log 5^2 - \log 4 - \log 5$

$4\log 2 + \log 5 - 2\log 2 - \log 5$

2log 2

8. The region bounded by the curve $y = x^2 - 2x$ and the x-axis from x = 0 to x = 2 is rotated about the x-axis. Calculate the volume of the solid formed. (05marks)

$V = \pi\int_0^2 (x^2 - 2x)^2 dx$

$V = \pi\int_0^2 (x^4 - 4x^3 + 4x^2) dx$

$V = \pi\left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right]_0^2$

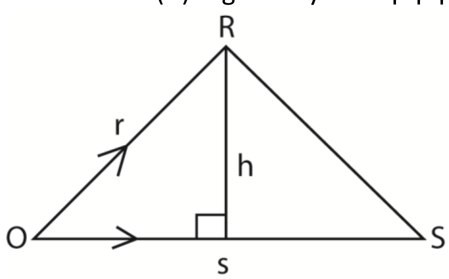
$V = \pi\left(\frac{2^5}{5} - 2^4 + \frac{4(2)^3}{3}\right)$

$= \frac{16\pi}{15}$

SECTON B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. The position vectors of the vertices of a triangle are O, r and s, where O is the origin. Show that its area (A) is given by $4A^2 = |r|^2|s|^2 - (r.s)^2$. (06marks)



r.s = $|r||s| \cos O$
$(r.s)^2 = |r|^2|s|^2cos^2 O$
$sin^2 O = 1 - \frac{(r.s)^2}{|r|^2|s|^2} = \frac{|r|^2|s|^2 - (r.s)^2}{|r|^2|s|^2}$
$A = \frac{1}{2}|r||s|sinO$
2A = $|r||s|sinO$
4A^2 = $|r|^2|s|^2sin^2 O$
4A^2 = $|r|^2|s|^2.\frac{|r|^2|s|^2 - (r.s)^2}{|r|^2|s|^2}$
4A^2 = $|r|^2|s|^2 - (r.s)^2$
Hence, find the area of a triangle when r = $\begin{pmatrix}2\\3\end{pmatrix}$ and s = $\begin{pmatrix}1\\4\end{pmatrix}$ (06marks)
$|r|^2 = 2^2 + 3^2 = 13$
$|s|^2 = 1^2 + 4^2 = 17$
$r.s = \begin{pmatrix}2\\3\end{pmatrix}.\begin{pmatrix}1\\4\end{pmatrix} = (2\ x\ 1) + (3\ x\ 4) = 14$
∴ 4A^2 = 13 x 17 – 14^2 = 25
$A = \sqrt{\frac{25}{4}}$ = 2.5units

10. Express 5 + 12i in polar form
Hence, evaluate $\sqrt[3]{5 + 12i}$, giving your answer in the form a + ib where a and b are corrected to two decimal places. (12marks)
$|5 + 12i| = \sqrt{5^2 + 12^2} = 13$
Arg(5 + 12i) = tan$^{-1}\left(\frac{12}{5}\right)$ = 67.38^0
5 + 12i = 13(cos 67.38^0 + isin67.38^0)
$\sqrt[3]{5 + 12i} = \sqrt[3]{13}\left[\cos\left(\frac{67.38+2\pi k}{3}\right) + isin\ \left(\frac{67.38+2\pi k}{3}\right)\right]$
$taking\ k = 0, 1,2$
Considering k = 0
$\sqrt[3]{5 + 12i} = \sqrt[3]{13}\left[\cos\left(\frac{67.38+0}{3}\right) + isin\ \left(\frac{67.38+0}{3}\right)\right]$

= 2.17 + 0.90i

Considering k = 1
$\sqrt[3]{5 + 12i} = \sqrt[3]{13}\left[\cos\left(\frac{67.38+2\pi}{3}\right) + isin\ \left(\frac{67.38+2\pi}{3}\right)\right]$

= -1.86 + 1.43i

Considering k = 2
$\sqrt[3]{5 + 12i} = \sqrt[3]{13}\left[\cos\left(\frac{67.38+4\pi}{3}\right) + isin\ \left(\frac{67.38+4\pi}{3}\right)\right]$

= -0.31 – 2.33i

11. (a) Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x. (06marks)
$y = \frac{x^3}{\sqrt{(1-2x^2)}}$
$\frac{dy}{dx} = \frac{3x^2\sqrt{1-2x^2} - \frac{-4x(x^3)}{2\sqrt{1-2x^2}}}{\left(\sqrt{1-2x^2}\right)^2}$

$= \frac{3x^2(1-2x^2)+2x^4}{(1-2x^2)\sqrt{1-2x^2}}$

$= \frac{3x^2-6x^4+2x^4}{(1-2x^2)\sqrt{1-2x^2}}$

$\frac{dy}{dx} = \frac{3x^2-4x^4}{(1-2x^2)^{\frac{3}{2}}}$

Alternatively

$$y = \frac{x^3}{\sqrt{(1-2x^2)}}$$

Introducing In on both sides

In y = $In\ x^3 - \frac{1}{2}In\ (1-2x^2)$

=3In x$- \frac{1}{2}In\ (1-2x^2)$

$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} - \frac{1}{2}.\frac{(-4x)}{(1-2x^2)}$

$\frac{dy}{dx} = \left(\frac{3}{x} + \frac{2x}{(1-2x^2)}\right).\frac{x^3}{\sqrt{(1-2x^2)}}$

$= \left(\frac{3(1-2x^2)+2x^2}{x(1-2x^2)}\right).\frac{x^3}{\sqrt{(1-2x^2)}}$

$=\left(\frac{3-4x^2}{1-2x^2}\right).\frac{x^2}{\sqrt{(1-2x^2)}}$

$=\frac{dy}{dx} = \frac{3x^2-4x^4}{(1-2x^2)^{\frac{3}{2}}}$

Alternatively

$y = \frac{x^3}{\sqrt{(1-2x^2)}}$

Squaring both sides

$y^2 = \frac{x^6}{(1-2x^2)}$

2Iny = 6Inx - In$(1-2x^2)$

$\frac{2}{y}\frac{dy}{dx} = \frac{6}{x} + \frac{4x}{(1-2x^2)}$

$\frac{dy}{dx} = \left(\frac{3}{x} + \frac{2x}{(1-2x^2)}\right).\frac{x^3}{\sqrt{(1-2x^2)}}$

$= \left(\frac{3(1-2x^2)+2x^2}{x(1-2x^2)}\right).\frac{x^3}{\sqrt{(1-2x^2)}}$

$=\left(\frac{3-4x^2}{1-2x^2}\right).\frac{x^2}{\sqrt{(1-2x^2)}}$

$=\frac{dy}{dx} = \frac{3x^2-4x^4}{(1-2x^2)^{\frac{3}{2}}}$

(c) The period T of a swing of a simple pendulum of length of length l is given by $T^2 = \frac{4\pi^2 l}{g}$ where g is the acceleration due to gravity.

An error of 2% is made 9n measuring the length, l. determine the resulting percentage error in the period, T. (06marks)

$T^2 = \frac{4\pi^2 l}{g}$ => $T = 2\pi\sqrt{\frac{l}{g}}$

$2T\frac{dT}{dl} = \frac{4\pi^2}{g}$

$\frac{dT}{dl} = \frac{2\pi^2}{gT} = \frac{2\pi^2}{g\left(2\pi\sqrt{\frac{l}{g}}\right)} = \frac{\pi}{\sqrt{gl}}$

$\frac{\delta T}{\delta l} = \frac{dT}{dl} \Rightarrow \delta T = \frac{dT}{dl}.\delta l$

$\frac{\delta T}{T}\ x\ 100\% = \frac{\frac{\pi\delta l}{\sqrt{gl}}x\ 100\%}{2\pi\sqrt{\frac{l}{g}}}$

$= \frac{\delta l}{2l}x\ 100\%$

$= \frac{1}{2}\ x\ \frac{2}{100}\ x\ 100\% = 1\%$

12. (a) Show that $\tan 4\theta = \frac{4t(1-t^2)}{t^4-6t^2+1}$, where $t = \tan\theta$ (06marks)

$\tan 4\theta = \frac{2\tan 2\theta}{1-\tan^2 2\theta}$ and $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{2t}{1-t^2}\right)}{1-\left(\frac{2t}{1-t^2}\right)^2}$

$= \frac{4t(1-t^2)}{t^4-6t^2+1}$

(b) Solve the equation sinx + sin5x = sin2x + sin4x for $0^0 < x < 90^0$. (06marks)

sinx + sin5x = sin2x + sin4x

sin5x –sin 4x-sin2x + sin x = 0

2sin3xcos2x -2sin3xcosx =0

2sin3x[cos2x-cos x] = 0

$-4\sin 3x \sin\frac{3x}{2}\sin\frac{x}{2} = 0$

Either sin3x =0, $\sin\frac{3x}{2} = 0$, $\sin\frac{x}{2} = 0$

$3x, \frac{3x}{2}, \frac{x}{2} = 0^0, 180^0$

$X = 60^0$

13. (a) The first three terms of a Geometric progression (G.P) are 4, 8 and 16. Determine the sum of the first ten terms of the G.P. (04marks)

(b) An Arithmetic Progression (A.P) has a common difference of 3. A Geometric Progression (G.P) has a common ration of 2. A sequence is formed by subtracting the term of the A.P from the corresponding terms of the G.P. The third term of the sequence is 4. The sixth term of the sequence is 79. Find the first term of the

(i) A.P (08 marks)

(a) a = 4, ar = 6

4r = 8

r = 2

$S_{10} = 4\left(\frac{2^{10}-1}{2-1}\right) = 4092$

(ii) G.P (06 mars)

A.P

x, x+3, x +6, x + 9, x + 12, x+ 15, ……

G.P

y, 2y, 4y, 8y, 16y, 32y, ….

4y – (x + 6) = 4

4y – x = 10 ……………..(i)

32y – (x + 15) = 79

32y – x = 94…………. (ii)

Eqn (ii) – Eqn (i)

28y = 84, => y = 3

Substituting for y into eqn (i)

12 – x = 10

x =2

(i) A.P, $U_1 = 2$

(ii) G.P, $U_1 = 3$

14. Evaluate

(c) $\int_0^{\frac{\pi}{2}} sin5xcos3xdx$ (06marks)

$$\int_0^{\frac{\pi}{2}} sin5xcos3xdx = \frac{1}{2}\int_0^{\frac{\pi}{2}} sin8x + sin2xdx$$

$$= \frac{1}{2}\left[-\frac{1}{8}cos8x - \frac{1}{2}cos2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[-\frac{1}{8}(cos4\pi - cos0) - \frac{1}{2}(cos\pi - cos0)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{8}(1-1) - \frac{1}{2}(-1-1)\right]$$

$$= \frac{1}{2}$$

(d) $\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2}$ (06marks)

Let X = $\frac{3}{2}\tan\theta$

dx = $\frac{3}{2}sec^2\theta$

x	θ
0	0
$\frac{\sqrt{3}}{2}$	$\frac{\pi}{6}$

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2} = \int_0^{\frac{\pi}{6}} \frac{\frac{3}{2}sec^2\theta}{9+4\left(\frac{9}{4}tan^2\theta\right)}$$

$$= \frac{1}{6}\int_0^{\frac{\pi}{6}} \frac{sec^2\theta\, d\theta}{1+tan^2\theta} = \frac{1}{6}\int_0^{\frac{\pi}{6}} d\theta$$

$$= \frac{1}{6}[\theta]_0^{\frac{\pi}{6}} = \frac{1}{6}\left(\frac{\pi}{6} - 0\right)$$

$$= \frac{\pi}{36} = 0.087266$$

15. The line y = mx + c is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(06mars)$

(c) Obtain an expression for c in terms of a, b and m. (06mars)

$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$

$b^2x^2 + a^2(m^2x^2 + 2mx + c^2) = a^2b^2$

$(b^2 + a^2m^2)x^2 + 2a^2mx + a^2c^2 - a^2b^2$

$(2a^2mc)^2 = 4(b^2 + a^2m^2)(a^2c^2 - a^2b^2)$

$4a^4m^2c^2 = 4a^2(b^2c^2 - b^4 + a^2m^2c^2 - a^2m^2b^2)$

$b^2c^2 = b^4 + a^2m^2b^2$

$c^2 = a^2m^2 + b^2$

(d) Calculate the gradients of the tangents to the ellipse through the point $\left(\sqrt{(a^2 + b^2)},\ 0\right)$

y = mx $\pm\sqrt{(a^2m^2 + b^2)}$

0 =m$\sqrt{(a^2 + b^2)} \pm \sqrt{(a^2m^2 + b^2)}$

$\left(\pm\sqrt{(a^2m^2 + b^2)}\right)^2 = \left(\mathrm{m}\sqrt{(a^2 + b^2)}\ \right)^2$

$a^2m^2 + b^2 = m^2(a^2 + b^2)$

$a^2m^2 + b^2 = m^2a^2 + m^2b^2$

$b^2 = m^2b^2$

$m^2 = 1$

m $= \pm 1$

16. The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding. The temperature of the body is initially 60^0C. After 15 minutes the temperature of the body is 50^0C. The temperature of the surrounding is 10^0C.

(c) Form a differential equation for the temperature of the body. (09marks)

$\frac{d\theta}{dt} \propto (\theta - 10)$

$\frac{d\theta}{dt} = k(\theta - 10)$

$\frac{d\theta}{(\theta-10)}$ = -kdt

$\int \frac{d\theta}{(\theta-10)} = -k\int dt$

$In\ (\theta - 10) = -kt + c$

When t = 0, θ = 60

c = $In\ (60 - 10)$ =In50

when t = 15, θ = 50

$In\ 40 = -k\ x\ 15 + In50$

15k = In$\left(\frac{50}{40}\right)$

k = $\frac{1}{15}In\left(\frac{5}{4}\right)$

Hence the differential equation is

$\frac{d\theta}{dt} = \frac{1}{15}In\left(\frac{5}{4}\right)(\theta - 10)$

(d) Determine the time it takes for the temperature of the body to reach 30⁰C. (03marks)

In (θ – 10) = -$\frac{1}{15}In\left(\frac{5}{4}\right)t$ + In 50

When t = T and θ = 30

-$\frac{1}{15}In\left(\frac{5}{4}\right)T = In\ 50 - In\ 20$

T = $\frac{15In\left(\frac{5}{2}\right)}{In\left(\frac{4}{4}\right)}$ = 61.5943minutes

Thank you

Yours Dr. Bbosa Science