7. UNEB 1996/2/16

The following table gives the marks obtained in calculus, physics and statistics by seven students

Calculus	72	50	60	55	35	48	82
Physics	61	55	70	50	30	50	73
Statistics	50	40	62	70	40	40	60

Determine the rank correlation coefficient for the performance of students in

(i) Calculus and physics

Spearman's correlation coefficient

C	P	Rc	Rp	d	d^2
72	61	2	3	-1	1
50	55	5	4	1	1
60	70	3	2	1	1
55	50	4	5.5	-1.5	2.25
35	30	7	7	0	0
48	50	6	5.5	0.5	0.25
82	73	1	1	0	0
					$\sum d^2 = 5.5$

$\rho = 1 - \left[\frac{6\sum d^2}{n(n^2-1)}\right]$

$= 1 - \left[\frac{6 \times 5.5}{7(7^2-1)}\right] = 0.902$

There is high positive relationship between calculus and physics

Kendall' correlation coefficient

Rc	Rp	C	D
1	1	6	0
2	3	4	1
3	2	4	0
4	5.5	1	1
5	4	2	0
6	5.5	1	0
7	7	=18	=2

$\tau = \frac{C-D}{C+D} = \frac{18-2}{18+2} = 0.8$

There is high positive relationship between calculus and physics

(ii) Calculus and statistics ($\rho = 0.64$)

Spearman's correlation

C	S	Rc	Rs	d	d^2
72	50	2	4	-2	4
50	40	5	6	-1	1
60	62	3	2	1	1
55	70	4	1	3	9
35	40	7	6	1	1
48	40	6	6	0	0
82	60	1	3	-2	4
					$\sum d^2 = 20$

$\rho = 1 - \left[\frac{6\sum d^2}{n(n^2-1)}\right]$

$= 1 - \left[\frac{6 \times 20}{7(7^2-1)}\right] = 0.643$

Kendall's rank correlation coefficient

Rc	Rs	C	D
1	3	4	2
2	4	3	2
3	2	3	1
4	1	3	0
5	6	0	0
6	6	0	0
7	6	=13	=5

τ = $\frac{C-D}{C+D}$ = $\frac{13-5}{13+5}$ = 0.444

8. UNEB/1999/2/8

Given the table below

x	80	75	86	60	75	92	86	50	64	75
y	62	58	60	45	68	68	81	48	50	70

Determine the rank correlation coefficient between the variable x and y, comment on your results (ρ = 0.715)

Spearman's correlation

x	y	Rx	Ry	d	d^2
80	62	4	5	-1	1
75	58	6	7	-1	1
86	60	2.5	6	-3.5	12.25
60	45	9	10	-1	1
75	68	6	3.5	2.5	6.25
92	68	1	3.5	-2.5	6.25
86	81	2.5	1	1.5	2.25
50	48	10	9	1	1
64	50	8	8	0	0
75	70	6	2	4	16
					47

$$\rho = 1- \left[\frac{6\sum d^2}{n(n^2-1)}\right]$$

$$= 1- \left[\frac{6\ x\ 47}{10(10^2-1)}\right] = 0.715$$

Kendall's correlation coefficient

Rx	Ry	C	D
1	3.5	6	2
2.5	6	4	4
2.5	1	7	0
4	5	4	2
6	7	3	2
6	3.5	3	2
6	2	3	0
8	8	2	0
9	10	0	1
10	9	=32	=13

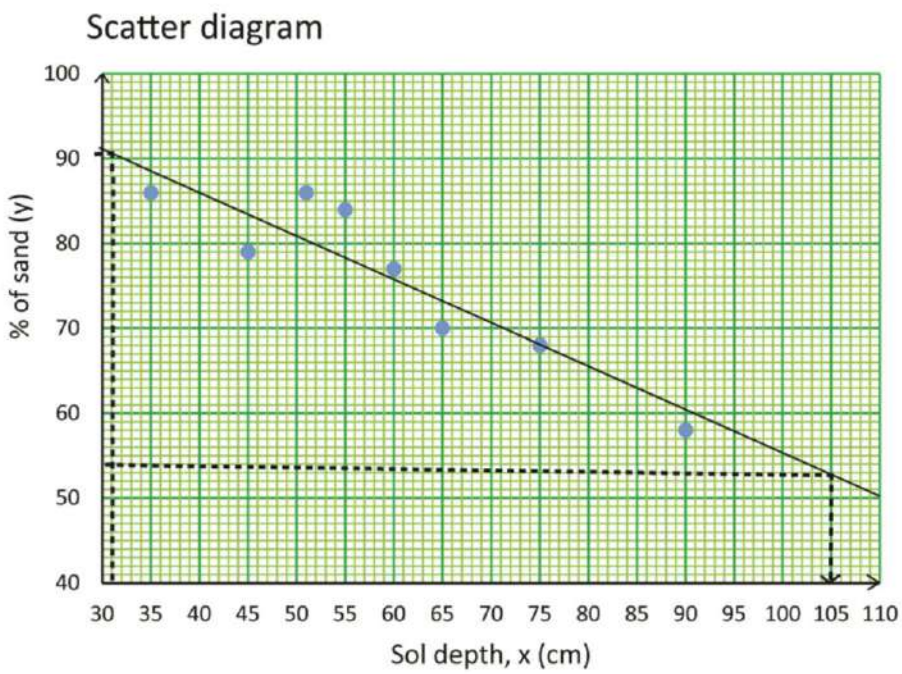
τ = $\frac{C-D}{C+D}$ = $\frac{32-13}{32+13}$ = 0.422

9. UNEB2003/2/15

The table below shows the percentage of sand y in the soils at different depth x (in cm)

Soil depth (x)(cm)	35	65	55	25	45	75	20	90	51	60
% of sand, y	86	70	84	92	79	68	96	58	86	77

(a) Plot the results on a scatter diagram. Comment on the relationship between the depth of the soil and the percentage of sand in the soil



(b) Draw the line of the best fit on you graph and use it to estimate

(i) The percentage of sand in the soil at a depth of 31cm (91%)

(ii) Depth of the soil with 54% sand (105cm)

(iii) Calculate the rank correlation coefficient

Spearman's rank correlation coefficient

soil depth (x)	%of sand (y)	Rx	Ry	d	d^2
35	86	8	3.5	4.5	20.25
65	70	3	8	-5	25
55	84	5	5	0	0
25	92	9	2	7	49
45	79	7	6	1	1
75	68	2	9	-7	49
20	96	10	1	9	81
90	58	1	10	-9	81
51	86	6	3.5	2.5	6.25
60	77	4	7	-3	9
					$\sum d^2 = 321.5$

$\rho = 1 - \left[\frac{6\sum d^2}{n(n^2-1)}\right]$

$= 1 - \left[\frac{6 \times 321.5}{10(10^2-1)}\right] = -0.948$

Kendall's correlation coefficient

Rx	Ry	C	D
1	10	0	9
2	9	0	8
3	8	0	7
4	7	0	6
5	5	1	4
6	3.5	1	2
7	6	0	3
8	3.5	0	2
9	2	0	1
10	1	2	42

$\tau = \frac{C-D}{C+D} = \frac{2-42}{2+42} =$ -0.91

10. UNEB 2004/2/7

Eight applicants for a certain job obtained the following marks in aptitude and written test

Applicants	A	B	C	D	E	F	G	H
Aptitude test	33	45	16	42	45	35	40	48
Written test	57	60	40	75	68	48	54	68

(i) Calculate the coefficient of rank correlation of applicant's performance in the two tests

Aptitude (A)	Written (W)	R_A	R_W	d	d^2
33	57	7	5	2	4
45	60	2	4	-2	4
16	40	8	8	0	0
42	75	4	1	3	9
45	68	3	2.5	0.5	0.25
35	48	6	7	-1	1
40	54	5	6	-1	1
48	68	1	2.5	-1.5	2.25
					21.5

$\rho = 1- \left[\frac{6\sum d^2}{n(n^2-1)}\right]$

$= 1- \left[\frac{6 \: x \: 21.5}{8(8^2-1)}\right] = 0.744$

(ii) There is high positive relationship between Aptitude and written

Kendall's correlation coefficient

Rx	Ry	C	D
1	2.5	5	1
2	4	4	2
3	2.5	4	1
4	1	4	0
5	6	2	1
6	7	1	1
7	5	1	0
8	8	=21	=6

$\tau = \frac{C-D}{C+D} = \frac{21-6}{21+6} = 0.555$

(ii) There is moderate positive relationship between Aptitude and written

(ii) Comment on your results

11. UNEB 2005/2/7

The table below shows the marks scored by students in mathematics and fine art tests

students	A	B	C	D	E	F	G	H	I	J
Mathematics,	40	48	79	26	55	35	37	70	60	40
Fine art	59	62	68	47	46	39	63	29	55	67

Calculate the coefficient of rank correlation for the students' performance in the two subjects and comment on your results.

Math	Fine Art	R_M	R_F	d	d^2
40	59	6.5	5	1.5	2.25
48	62	5	4	1	1
79	68	1	1	0	0
26	47	10	7	3	9
55	46	4	8	-4	16
35	39	9	9	0	0
37	63	8	3	5	25
70	29	2	10	-8	64
60	55	3	6	-3	9
40	67	6.5	2	4.5	20.25
					$\sum d^2 = 146.5$

$\rho = 1 - \left[\frac{6\sum d^2}{n(n^2-1)}\right]$

$= 1 - \left[\frac{6 \times 146.5}{10(10^2-1)}\right]$

$= 0.112$

Kendall's correlation coefficient

RM	RF	C	D
1	1	9	0
2	10	0	8
3	6	3	4
4	8	1	5
5	4	3	2
6.5	5	2	2
6.5	2	3	0
8	3	2	0
9	9	0	1
10	7	23	22

$\tau = \frac{C-D}{C+D} = \frac{23-22}{23+22} = 0.022$

12. UNEB 2007/2/12

Below are marks scored by 8 students A, B, C … H in Mathematics, Economics and geography in the end of term examination.

	A	B	C	D	E	F	G	H
Math	52	75	41	60	81	31	65	52
Economics	50	60	35	65	66	45	69	48
Geography	35	40	60	54	63	40	65	72

Determine the rank correlation coefficient for the performance of students in

(i) Math and economics

Spearman's correlation coefficient

Math, M	Economic C	R_M	R_E	d	d^2
52	50	5.5	5	0.5	0.25
75	60	2	4	-2	4
41	35	7	8	-1	1
60	65	4	3	1	1
81	66	1	2	-1	1
31	45	8	7	1	1
65	69	3	1	2	4
52	48	5.5	6	-0.5	0.25
					$\sum d^2$ =12.5

ρ = 1- $\left[\frac{6\sum d^2}{n(n^2-1)}\right]$

= 1- $\left[\frac{6 \, x \, 12.5}{10(10^2-1)}\right]$

= 0.851

Kendall's correlation coefficient

RM	RE	C	D
1	2	6	1
2	4	4	2
3	1	5	0
4	3	4	0
5.5	5	3	0
5.5	6	2	0
7	8	1	1
8	7	=25	=4

$\tau = \frac{C-D}{C+D} = \frac{25-4}{25+4}$ = 0.724

(ii) Geography and math

Spearman's rank correlation coefficient

Geog, G	Math, M	R_G	R_M	d	d^2
35	52	8	5.5	2.5	6.25
40	75	6.5	2	4.5	20.25
60	41	4	7	-3	9
54	60	5	4	1	1
63	81	3	1	2	4
40	31	6.5	8	-1.5	2.25
65	65	2	3	-1	1
72	52	1	5.5	-4.5	20.25
					64

ρ = 1- $\left[\frac{6\sum d^2}{n(n^2-1)}\right]$

= 1- $\left[\frac{6 \, x \, 64}{8(8^2-1)}\right]$

= 0.238

Kendall's rank coefficient

RG	RM	C	D
1	5.5	2	4
2	3	4	2
3	1	5	0
4	7	1	3
5	4	2	1
6.5	2	2	0
6.5	8	0	1
8	5.5	16	11

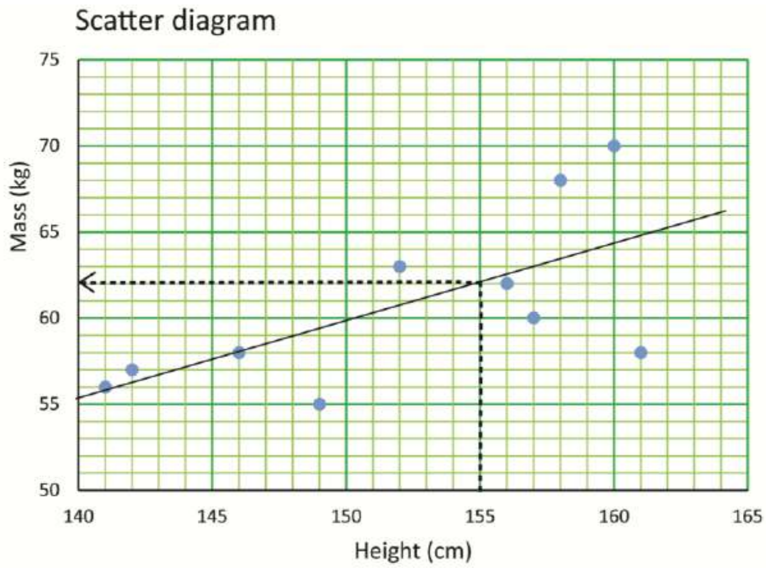
$\tau = \frac{C-D}{C+D} = \frac{16-11}{16+11}$ = 0.185

(iii) Comment on the significance of the math in performance of economics and geography. (ρ = 0.86, τ = 0.79 based on 8 observations at a 1% level of significance)
Since$|\rho_C(0.851)| < |\rho_T(0.86)|$ and $|\tau_C(0.724)| < |\tau\rho_T(0.79)|$; there is no significant relationship between math and economics at 1% significance level.

13. UNEB 2011/2/12
The heights and ages of ten students are given in the table below

Height, cm	156	151	152	146	160	157	149	142	158	140
Mass, kg	62	58	63	58	70	60	55	57	68	56

(a) Plot the data on a scatter diagram



(b) Draw the line of best fit on you graph and use it to estimate the mass corresponding to a height of 155cm(63kg)

(c) Calculate the rank correlation coefficient for the data. Comment on the significance of the height on masses of students (ρ = 0.79, τ =0.64 based on 10 observations at 1% level of significance.)

Method I: Using Spearman's rank correlation coefficient

Height (x)	Mass (y)	Rx	Ry	Rx – Ry = d	d^2
156	62	4	4	0	0
151	58	6	6.5	-0.5	0.25
152	63	5	3	2	4
146	58	8	6.5	1.5	2.25
160	70	1	1	0	0
157	60	3	5	-2	4
149	55	7	10	-3	9
142	57	9	8	1	1
158	68	2	2	0	0
140	56	10	9	1	1
					$\sum d^2 = 21.5$

$$\rho = 1- \left[\frac{6\sum d^2}{n(n^2-1)}\right] = 1- \left[\frac{6\,x\,21.5}{10(10^2-1)}\right] \quad = 0.87$$

Since $|\rho_C(0.0.87)| > |\rho_T(0.79)|$ there is no significant relationship between height and age at 1% significance level.

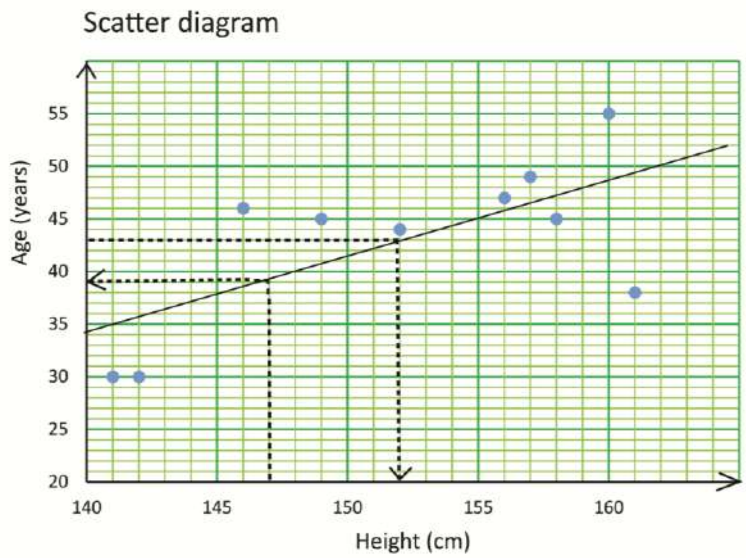
Since $|\tau_C(0.6\,)| < |\tau\rho_T(0.0.64\,)|$;

14. UNEB 2013/2/9

The heights and ages of ten farmers are given in the table below

Height, cm	156	151	152	160	146	157	149	142	158	140
Age, years	47	38	44	55	46	49	45	30	45	20

(a) Plot the data on a scatter diagram



(b) Draw the line of best fit on your diagram and use it to estimate

(i) Age when height = 147 (39)

(ii) Height when the age is 43 (152)

(c) Calculate the rank correlation coefficient for the data. Comment on your results

Method I: Using Spearman's rank correlation coefficient

Height (x)	Age (y)	Rx	Ry	Rx – Ry = d	d^2
156	47	4	3	1	1
151	38	6	8	-2	4
152	44	5	7	-2	4
160	55	1	1	0	0
146	46	8	4	4	16
157	49	3	2	1	1
149	45	7	5.5	1.5	2.25
142	30	9	9.5	-0.5	0.25
158	45	2	5.5	-3.5	12.25
140	30	10	9.5	0.5	0.25
					$\sum d^2 = 41$

p =1 - $\frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \ x \ 41}{10(10^2-1)} = 0.7515(4D)$

Method II: using Kendall's rank correlation coefficient

Let the farmers be A, B, C, D, E, F, G, H, I, J

Farmers	A	B	C	D	E	F	G	H	I	J
Height	156	151	152	160	146	157	149	142	158	140
Age	47	38	44	55	46	49	45	30	45	30

By re-arranging the findings we have

Farmers	D	I	F	A	C	B	G	E	H	J
Height	1	2	3	4	5	6	7	8	9	10
Age	1	5.5	3	3	7	8	5.5	4	9.5	9.5
agreements	9	4	7	6	3	2	2	2	0	=35
Disagreements	0	3	0	0	2	2	1	0	0	=8

s = total agreements – total disagreements

= 35 – 8 = 27

$\tau = \frac{2s}{n(n-1)} = \frac{2 \ x \ 27}{10(10-1)} = \frac{54}{90} = 0.6$ Or τ = $\frac{35-8}{35+8} = 0.63$

Comment

Since$|\rho_C(0.7515)| < |\rho_T(0.79)|$ and $|\tau_C(0.6\ )| < |\tau\rho_T(0.0.64\ )|$; there is no significant relationship between height and age at 1% significance level.

Kendall's rank correlation coefficient

By naming the pairs we have

A(156, 62), B(151, 58), C(152, 63), D(146, 58), E(160, 70), F(157, 60), G149, 55), H(142, 57), I(158, 68), J(141, 56)

	E	I	F	A	C	B	G	D	H	J
x	1	2	3	4	5	6	7	8	9	10
y	1	2	5	4	3	6.5	10	6.5	8	9
C	9	8	5	5	5	3	0	2	1	=38
D	0	0	2	1	0	0	3	0	0	=6

τ (tau) = $\frac{C-D}{C+D}$

τ (tau) = $\frac{38-6}{38+6}$ = 0.73

Since $\tau_C(0.73) > \tau_T\ (0.64)$, a significant relationship exist between the heights and masses of student.

15. UNEB 2015/2/12

The table gives the points awarded to eight schools by three judges, J_1, J_2 and J_3 during a music competition. J_1 was the chief judge.

J_1	72	50	50	55	35	38	82	72
J_2	60	55	70	50	50	50	73	70
J_3	50	40	62	70	40	48	67	67

(a) Determine the rank correlation coefficients between the judges of

(i) J_1 and J_2
(ii) J_1 and J_3
(b) Who of the two judges had a better correlation with the chief judge? Give a reason.

Solution
(i) J_1 and J_2

J_1	J_2	R_{J_1}	R_{J_2}	D_1	D_1^2
72	60	2.5	4	-1.5	2.25
50	55	5.5	5	0.5	0.25
50	70	5.5	2.5	3	9
55	50	4	7	-3	9
35	50	8	7	1	1
38	50	7	7	0	0
82	73	1	1	0	0
72	70	2.5	2.5	0	0
					$\sum D_1^2$ = 21.5

$\rho_1 = 1 - \frac{6\sum D_1^2}{n(n^2-1)}$
$= 1 - \frac{6\ x\ 21.5}{8(8^2-1)}$
$= \frac{125}{168} = 0.7440$

(ii) J_1 and J_3 (10marks)

J_1	J_3	R_{J_1}	R_{J_3}	D_2	D_2^2
72	50	2.5	5	-2.5	6.25
50	40	5.5	7.5	-4	4
50	62	5.5	4	0.5	2.25
55	70	4	1	3	9
35	40	8	7.5	0.5	0.25
38	48	7	6	1	1
82	67	1	2.5	-1.5	2.25
72	67	2.5	2.5	0	0
					$\sum D_1^2$ = 25

$\rho_2 = 1 - \frac{6\sum D_2^2}{n(n^2-1)}$
$= 1 - \frac{6\ x\ 25}{8(8^2-1)}$
$= 0.7023$

(a) Who of the two other judges had a better correlation with the chief judge? Give a reason. (02marks)
J_2 has a better correlation with the Chief Judge because the coefficient of correlation is smaller showing a stronger mutual relationship

Upper Critical Values for Kendall's Rank Correlation Coefficient $\hat{\tau}$

Note: In the table below, the critical values give significance levels as close as possible to but not exceeding the nominal α.

	Nominal α					
n	0.10	0.05	0.025	0.01	0.005	0.001
4	1.000	1.000	-	-	-	-
5	0.800	0.800	1.000	1.000	-	-
6	0.600	0.733	0.867	0.867	1.000	-
7	0.524	0.619	0.714	0.810	0.905	1.000
8	0.429	0.571	0.643	0.714	0.786	0.857
9	0.389	0.500	0.556	0.667	0.722	0.833
10	0.378	0.467	0.511	0.600	0.644	0.778
11	0.345	0.418	0.491	0.564	0.600	0.709
12	0.303	0.394	0.455	0.545	0.576	0.667
13	0.308	0.359	0.436	0.513	0.564	0.641
14	0.275	0.363	0.407	0.473	0.516	0.604
15	0.276	0.333	0.390	0.467	0.505	0.581
16	0.250	0.317	0.383	0.433	0.483	0.567
17	0.250	0.309	0.368	0.426	0.471	0.544
18	0.242	0.294	0.346	0.412	0.451	0.529
19	0.228	0.287	0.333	0.392	0.439	0.509
20	0.221	0.274	0.326	0.379	0.421	0.495
21	0.210	0.267	0.314	0.371	0.410	0.486
22	0.203	0.264	0.307	0.359	0.394	0.472
23	0.202	0.257	0.296	0.352	0.391	0.455
24	0.196	0.246	0.290	0.341	0.377	0.449
25	0.193	0.240	0.287	0.333	0.367	0.440
26	0.188	0.237	0.280	0.329	0.360	0.428
27	0.179	0.231	0.271	0.322	0.356	0.419
28	0.180	0.228	0.265	0.312	0.344	0.413
29	0.172	0.222	0.261	0.310	0.340	0.404

8.INDEX NUMBERS

Index numbers

This the percentage ratio of one quantity to the other, e.g. price index

Index number is a technique of measuring changes in a variable or group of variables with respect to time, geographical location or other characteristics. It is a statistical measure of change in a representative group of individual data points.

For example, if the price of a certain commodity rises from shs. 100 in the year 2007 to shs. 150 in the year 2017, the price index number will be 150 showing that there is a 50% increase in the prices over this period

Simple index numbers
The simple index numbers include;
(i) Price index or price relative
Price relative = $\frac{P_1}{P_o}\ x\ 100$ where P_1= price the current year and P_0 – price in the base year
(ii) Wage index
Wage index = $\frac{W_1}{W_o}\ x\ 100$
(iii) Quantity index (quantum) index
Quantum index = $\frac{Q_1}{Q_o}\ x\ 100$

Example 1
A loaf of bread costed shs. 1200/= in 2008 and shs. 1800/= in 2014. Taking 2008 as the base year, find the price relative in 2014
Solution
Price relative = $\frac{P_1}{P_o}\ x\ 100 = \frac{1800}{1200}\ x\ 100 = 150$

Example 2
In 2020, the price index of a commodity using 2019 as the base was 180. In 2021, the price index using 2020 as the base year was 150. What is the price index in 2021 using 2019 as the base year?
Solution
$\frac{P_{2020}}{P_{2019}}\ x\ 100 = 180$
$\frac{P_{2020}}{P_{2019}} = 1.80$ ……….. (i)
$\frac{P_{2021}}{P_{2020}}\ x\ 100 = 150$
$\frac{P_{2021}}{P_{2020}} = 1.50$ ………. (ii)
Eqn. (i) x Eqn. (ii)

$\frac{P_{2020}}{P_{2019}} \cdot \frac{P_{2021}}{P_{2020}}$ = 1.8 x 1.5

$\frac{P_{2021}}{P_{2019}} = 2.7$

$\frac{P_{2021}}{P_{2019}} x\ 100 = 2.7x\ 100 = 270$

∴ the price index in 2021 using 2019 as the base year = 270

Example 3

The wage of a nurse in Uganda in 2010 was 350,000/=, the wage of the nurse in 2015 was increase by shs. 150,000/=. Using 2010 as the a base year calculate the nurses wage index in 2015.

Wage index = $\frac{W_1}{W_o}\ x\ 100$ = $\frac{500,000}{350,000}\ x\ 100 = 142.9$

Price indices

Price indices are divided into

(a) Simple price index
(b) Simple Aggregate price index
(c) Weighed price index

(a) Simple price index
This is the average of the price relative
It is given by simple price index = $\frac{\sum \frac{P_1}{P_0}}{n}$ x100
Where n = number of items.

(b) Simple aggregate price index
It is given by simple aggregate price index = $\left(\frac{\sum P_1}{\sum P_0}\ x\ 100\right)$

Example 4

The table shows the prices 0f bread and meat per kg in 2000 and 2008

	Year	
item	2000	2008
Beans	700	1200
Meat	2500	4500

Using 2000 as the base year, find

(a) Price relatives of each commodity
(b) Simple price index
(c) Simple aggregate price index

Solution

(a) Price index = $\frac{P_1}{P_o}\ x\ 100$
For beans: P.R = $\frac{1200}{700}\ x\ 100 =\ 171.43$
For meat: P.R = $\frac{4500}{2500}\ x\ 100 =\ 180$

(b) Simple price index (S.P.I) = $\frac{171.43+180}{2} = 175.72$

(c) Simple aggregate price index (S.A.P.I) = $\frac{1200+450}{700+2500}\ x\ 100 = 178.13$

Example 5

In 2014 the price of a shirt, a dress and a pair of shoes were shs. 20,000; shs. 35,000 and shs. 45, 000 respectively. Given that in 2017 the prices were shs. 25,000, shs. 50,000 and shs. y respectively. Find the value of y if the aggregate price index was 130 while taking 2014 as the base year.

Solution

Simple aggregate price index = $\left(\frac{\sum P_1}{\sum P_0} x\ 100\right)$

$\left(\frac{25{,}000+50{,}000+y}{20{,}000+35{,}000+45{,}000}\right) x\ 100 = 130$

$\left(\frac{75.000+y}{1000}\right) = 130$

$y = shs.55{,}000$

Weighted price index (composite index)

If the weight or quantity in the base year and current year are the same, we use

(i) Weighted aggregate price index

Weighted aggregate price index = $\left(\frac{\sum P_1 w}{\sum P_0 w}\right) x\ 100$

Example 6

The table below shows the prices (shs.) and amounts of items bought for assembling a phone in 2012 and 2015

Items	Prices (shs		Quantity
	2012	2015	
Transistor	12,000	18,000	8
Resistor	16,500	21,000	22
Capacitor	15,000	17,000	9
Diode	16,000	18,000	2
Circuit	20,000	25,000	1

Calculate the composite price index for a phone taking 2012 as the base year

W.A.P.I = $\left(\frac{18{,}000\ x\ 8\ +21{,}000\ x\ 22+17{,}000\ x\ 9\ +18{,}000\ x\ 2+25{,}000\ x\ 1}{12{,}000\ x\ 8+16{,}500\ x\ 22+15{,}000\ x\ 9+16{,}000\ x\ 2+20{,}000\ x\ 1}\right)\ x100 = 126.94$

Example 7

The table below shows the prices (shs) and amount of items bought for making a cake in 2008 and 2009.

Items	Prices (shs		Quantity
	2008	2009	
Flour per kg	6,000	7,800	3
Sugar per kg	5,000	4,000	1
Milk per litre	1,000	1,500	2
Eggs per egg	200	300	8

(a) Calculate the weighted aggregate price index taking 2008 as the base year

(b) In 2009, the cost of making a cake was shs. 80,000/=. Using the weighted aggregate price index above, find the cost of the coke in 2008.

Solution

(a) W.A.P.I = $\left(\frac{7800\ x\ 3+4000\ x\ 1+1500\ x\ 2+3\ x\ 8}{6000\ x\ 3+5000\ x\ 1+1000\ x\ 2+200\ x\ 8}\right) x\ 100 = 123.3083$

(b) $\frac{P_1}{P_o}\ x\ 100 = 123.3083$

$\frac{80{,}000}{P_o}\ x\ 100 = 123.3083$

$P_0 = shs.\ 64{,}878.0333$

(ii) Average weighted price index

Average weighted price index = $\frac{\sum\frac{P_1}{P_0}w}{\sum w}\ x\ 100$

When the price relative (P.R) is given then

Average weighted price index =$\frac{\sum(P.R\ x\ w)}{\sum w}$

Example 8

The table shows the expenditure (Ug. Shs.) of a student during the first and second terms

Items	Expenditure (shs.)		Amount
	1st term	2nd term	
clothing	46,500	49,350	5
Pocket money	55,200	37,500	3
Books	80,000	97,500	8

Using the first term expenditure as the base, find the average weighted price index

Average weighted price index = $\frac{\sum\frac{P_1}{P_0}w}{\sum w}\ x\ 100$

$= \left(\frac{\frac{49{,}350}{46{,}500}\ x\ 5+\frac{37{,}500}{55{,}200}\ x\ 3+\frac{97{,}500}{80{,}000}\ x\ 8}{5+3+8}\right) x\ 100$

= 106.841

Example 9

The table below shows the price relatives together with their weights for a certain family

Item	Weight	Price relative
Food	172	120
Water	160	124
Housing	170	125
Electricity	210	135
Clothing	140	104

Find the:

(i) simple price index

(ii) cost of living

Solution

(i) S.P.I = $\frac{\sum\left(\frac{P_1}{P_0}\right)}{n} = \frac{120+124+125+135+104}{5} = 121.6$

(ii) Cost of living = $\frac{\sum(P.R\ x\ w)}{\sum w} = \frac{120\ x\ 172\ +124x\ 160\ +125\ x\ 170\ +135\ x\ 210+104\ x\ 140}{172+160+170+210+140}$ = 122.82

Weighted aggregate price indices/ Paache's theory/value index

If the weight or quantity in the base year and current year are different, we use

Weighted aggregate price index = $\frac{\sum P_1W_1}{\sum P_0W_0}\ x\ 100$

Example 10

The table below shows the prices of items per kg in the year 2001 and 2002

Item	2001 = 100		2002	
	Price (shs.)	Quantity (kg)	Price (shs.)	Quantity (kg)
Rice	2800	20	3200	30
Millet	1500	10	1900	10
Beans	2000	5	2500	70

Calculate for 2002

(i) Price index

(ii) Simple aggregate price index

(iii) Simple aggregate quantity index

(iv) Weighted aggregate price index

Solution

(i) Price index = $\frac{P_{2002}}{P_{2001}} x\ 100$

For rice, price index = $\frac{3200}{2800} x\ 100 = 114.29$

For millet, price index = $\frac{1900}{1500} x\ 100 = 126.67$

For beans, price index = $\frac{2500}{2000} x\ 100 = 125$

(ii) S.A.P.I = $\left[\frac{3200+1900+250}{2800+1500+2000}\right] x\ 100 = 120.63$

(iii) S.A.Q.I = $\frac{\sum Q_{2002}}{\sum Q_{2001}}\ x\ 100$

= $\left[\frac{30+10+70}{20+20+5}\right] x\ 100 = 314.29$

(iv) W.A.P.I = $\frac{\sum P_1W_1}{\sum P_0W_0}\ x\ 100$

= $\left[\frac{3200\ x\ 30\ +1900\ x\ 10+2500\ x\ 70}{2800\ x\ 20+1500\ x\ 10+2000\ x\ 5}\right] x\ 100$

= 358.02

Revision exercise

1. UNEB 2020/2/5

The table below shows the price indices of beans, maize, rice and meat with corresponding weights

Item	Price index 2008 (2007 = 100%)	Weight
Beans	105	4
Maize	x	7
Rice	104	2
Meat	113	5

Calculate the;

(a) Value of x given that the price indices of maize in 2007 and 2008 using 2006 as the base year are 112 and 130 respectively. (**116.0714**)

(b) Weighted price index for 2008 using 2007 as the base year (**111.4167**)

2. UNEB 2018/2/5

The price index of an article in 2000 based on 1998 was 130. The price index for the article in 2005 based on 2000 was 80. Calculate

(a) Price index of the article in 2005 based on 1998. (104)

(b) Price of the article in 1998 if the price of the article was 45,000 in 2005.

UNEB 2017/2/7

The table below shows the price (shs.) and amount of items bought weekly by a restaurant in 2002 and 2003.

Items	Price (shs.)		Amount
	2002	2003	
Mil k per litre	400	500	200
Eggs per tray	2500	3000	18
Cooking oil per litre	2400	2100	2
Flour per packet	2000	2200	15

Calculate

(a) the weighted aggregate price index taking 2002 as the base year. (**119.63**)

(b) In 2003, the restaurant spent shs. 450,000/=. Using the weighted aggregate price index, find how the restaurant could have spent in 2002. (**376,096.95**)

3. The table below shows the prices (shs.) an amount of items bought in 2006 and 2007

Items	Price (shs.)		Amount
	2006	2007	
Mil k per litre	300	400	1
Eggs per tray	2500	3000	3
Cooking oil per litre	3000	8000	4
Flour per packet	1500	1800	15

Taking 2006 as the base year

(a) Calculate the simple aggregate price index (**180.92**)

(b) Calculate the weighted aggregate price index(**161.702**)

4. The table below shows the prices in US dollars and weights of five components of an engine, in 1998 and 2005

components	weight	Prices (US D)	
		1998	2005
A	6	35	60
B	5	70	135
C	3	43	105
D	2	180	290
E	1	480	800

(a) Taking 1998 as the base year, calculate the

(i) Simple aggregate price index (**172.03**)

(ii) Price relative of each component (**171.4, 192.9, 244.2, 161.1, 166.7**)

(iii) Weighted price index (**178.55**)

(b) Using the price index in (a)(i) estimate the cost of the engine in 1998 if the cost of the engine in 2005 was 1600 USD (**896.11**)

5. The table below shows the prices and amounts of items bought in 2004 and 2005

Item	Prices (shs)		Amount
	2004	2005	
A	635	887.5	6
B	720	815	4
C	730	1045	3
D	362	503	7

(a) Calculate the simple aggregate price index (**132.836**)
(b) Calculate the weighted aggregate price index (**133.52**)
(c) Calculate the price of an item costing 500 in 2004 using weighted aggregate price index above. (**667.64**)

6. The table below shows the prices of items per kg in the year 2005 and 2007

Item	Posho	Beans	Rice	Beef	Chicken
Price in 2005	1200	2000	1200	4000	8000
Price in 2007	1600	2500	1600	6000	9500

Calculate for 2007 using 2005a as the base year
(a) Simple price index (**132.0833**)
(b) Simple aggregate price index (**129.2683**)

7. The table below shows the prices in the year 2010 and 2018

Item	Price in 2010	Price in 2018
Flour per kg	3000	5400
Eggs per dozen	5,000	7800

Calculate for 2018 using 2010 as the base year
(i) Simple price index (**168**)
(ii) Simple aggregate price index (**165**)

8. The table below shows the prices and quantities of four items in the year 2020 and 2021

Item	Price per unit		Quantities	
	2020	2021	2020	2021
A	100	120	36	42
B	110	100	96	88
C	50	65	10	12
D	80	85	11	10

(a) Calculate the price index (120, 90.91, 130, 106.25)
(b) Simple aggregate price index (108.82)
(c) Weighted aggregate price index (99.55)
(d) Cost in 2021, A, B, C and Dare ingredients to make chapatti and in 2020a price of chapatti costed shilling 600 using index in (iii) above. (**shs. 597.3**)

9. MOTION IN A STRAIHT LINE

Motion in straight line

Distance and displacement

Distance is a length between 2 fixe points

Displacement is the distance covered in a specific direction

Speed and velocity

Speed is the rate of change of distance with time

Velocity is the rate of change of displacement with time

Average speed = $\frac{total\ distance}{total\ time\ taken}$

Average velocity = $\frac{total\ displacement}{total\ time\ taken}$

Example 1

Find the distance travelled in 5s by a body moving with a constant speed of $3.2ms^{-1}$

Solution

Average speed = $\frac{total\ distance}{total\ time\ taken}$ | $3.2 = \frac{total\ distance}{5}$ | distance = 16m

Example 2

John ran 1500m in 3minutes and 33s, find his average speed.

Average speed = $\frac{total\ distance}{total\ time\ taken}$ | speed = $\frac{1500}{(3\ x\ 60+33)}$ = $7.04ms^{-1}$

Acceleration

It is the rate of change of velocity

Acceleration =$\frac{change\ in\ velocity}{total\ time\ taken}$ | a = $\frac{v-u}{t}$ where v = final velocity, u = initial velocity, t = time

Uniform acceleration

This is the constant rate of change of velocity with time

Equations of uniform acceleration

1st equation

Suppose a body moving in a straight line with uniform acceleration a, increases its velocity from u to v in time t, then from the definition of acceleration

a = $\frac{v-u}{t}$

at = v -u

v = u + at 1

2nd equation

Suppose an object with velocity u moves with uniform acceleration a time t and attains a velocity v, the distance s travelled by the object is given by: s = average velocity x time

s = $\left(\frac{v+u}{2}\right)t$ but v = u + at

s = $\left(\frac{u+u+at}{2}\right)t$

s = $\left(\frac{2ut+at^2}{2}\right)$

s = ut + $\frac{1}{2}at^2$ 2

3rd equation

s = average velocity x time

s = $\left(\frac{v+u}{2}\right)t$ but t = $\frac{v-u}{a}$

s = $\left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$ =

s = $\left(\frac{v^2-u^2}{2a}\right)$

$\boldsymbol{v^2 = u^2 - 2as}$ 3

Example 3

A car is initially at rest at a point O. The car moves from O in a straight line with an acceleration of $4ms^{-2}$. find how far the car

(i) is from O after 2s

From **s = ut + $\frac{1}{2}at^2$;** **s =** 0 x 2 + $\frac{1}{2}x\ 4\ x\ 2^2$ = 8m

(ii) is from O after 3s

s = 0 x 2 + $\frac{1}{2}x\ 4\ x\ 2^2$ **=** **18**m

(iii) distance travelled in the third second = 18 – 8 = 10m

Example 4

A body at O moving with a velocity $10ms^{-2}$ decelerates at $2ms^{-2}$.

(a) find the displacement of the body from O after 7s

From s = ut + $\frac{1}{2}at^2$

s = 10 x 7 + $\frac{1}{2}x - 2\ x\ 7^2$ = 21m

(b) how far from O does the body come to rest and how long does it take

s = $\left(\frac{v^2-u^2}{2a}\right) = \frac{0^2-10^2}{2\ x-2}$ = 25m

t = $\frac{v-u}{a} = \frac{0-10}{-2}$ = 5s

Example 5

A taxi approaching a stage runs two successive half kilometres in 16s and 20s respectively. Assuming the retardation is uniform, find

(i) Initial speed of the taxi

$s = ut + \frac{1}{2}at^2$

For the first half kilometre or 500m

$500 = 16u + \frac{1}{2}a\,(16)^2$ ……… (i)

for the kilometre or 1000m

$1000 = 36u + \frac{1}{2}a\,(36)^2$…….. (ii)

from eqn. (i) and eqn. (ii)

$a = \frac{25}{72}$ and $u = 34.028ms^{-1}$

(ii) the further distance, the taxi runs before stopping

$s = \left(\frac{v^2 - u^2}{2a}\right) = s = \left(\frac{0^2 - (34.028)^2}{2\left(\frac{25}{72}\right)}\right) = 1667.3\text{m}$

Extra distance = 1667.3 – 1000 = 667.3m

Example 6

An overloaded taxi travelling at constant velocity of 90km/h overtakes a stationary traffic police car. 2s later, the police car sets in pursuit, accelerating at a uniform rate of $6ms^{-2}$. How far does the traffic car travel before catching up with the taxi?

Solution

t_1 = time taken by the taxi

t_2 = time taken by the police car

$t_1 = 2 + t_2$

speed of the taxi in m/s

$90km/h = \frac{90 \times 1000}{3600} = 25ms^{-1}$

$s = ut + \frac{1}{2}at^2$

$s_T = 25t_1$

$s_C = 0 \times t_2 + \frac{1}{2} \times 6 \times {t_2}^2 = 3\,{t_2}^2$

For the car to catch taxi; $s_T = s_C$

$25t_1 = 3\,{t_2}^2$

$25(2 + t_2) = 3\,{t_2}^2$

$t = 10s$ or $t = \frac{4}{3}s$

the car leaves 2s later then 10s is the correct time since it gives positive distance

$s_C = 3\,{t_2}^2 = 3 \times 10^2 = 300m$

Example 7

A lorry starts from a point A and moves along a straight horizontal road with a constant acceleration of $2ms^{-2}$. At the same time a car moving with a speed of $20ms^{-1}$ and a constant acceleration of $3ms^{-1}$ is 400m behind the point A and moving in the same direction as the lorry. find:

(a) how far from A the car overtakes the lorry.
a car over takes the lorry; both move in the same time, t

s = ut + $\frac{1}{2}at^2$

distance moved by the car = 400 + distance moved by the lorry

20t + $\frac{1}{2}$ x $3x$ t^2 = 400 + $\frac{1}{2}$ x 2 x t^2

t^2 + 40t – 800 = 0; t = 14.64s or t = -54.64s

Hence t = 14.64s

sL = $\frac{1}{2}$ x 2 x $(14.64)^2$= 214.33m

(b) the speed of the lorry when it is being overtaken

v = u + at

= 0 + 2 x 14.64 = 29.28ms^{-1}

Example 8

The seed of a taxi decreases from 90kmh^{-1} to 18kmh^{-1} in a distance of 120 metres. Find the speed of the taxi when it had covered a distance of 50metres. (05marks)

Given u = 90kmh^{-1}, v = 18kmh^{-1}, s = 120m = 0.12km
Using $v^2 = u^2 + 2as$
$18^2 = 90^2 + 2a(0.12)$
a = -32400kmh^{-2}
When s = 50m = 0.05km, u = 90kmh^{-1}, a = -32400kmh^{-2}
Using $v^2 = u^2 + 2as$
$v^2 = 90^2 – 2 \times 32400 \times 0.05 = 4860$
v = $\sqrt{4860} = 69.71kmh^{-1}$

Example 9

(a) Show that the final velocity v of a body which starts with an initial velocity u and moves with uniform acceleration a consequently covering a distance x, is given by v = $[u^2 + 2ax]^{\frac{1}{2}}$

x = average velocity x time

x = $\left(\frac{v+u}{2}\right)t$ but t = $\frac{v-u}{a}$

x = $\left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$ = $\left(\frac{v^2-u^2}{2a}\right)$

$\boldsymbol{v^2 = u^2 + 2ax}$

v = $[u^2 + 2ax]^{\frac{1}{2}}$

(b) Find the value of x in (a) if v = 300m/s, u = 10m/s and a = 5m/s

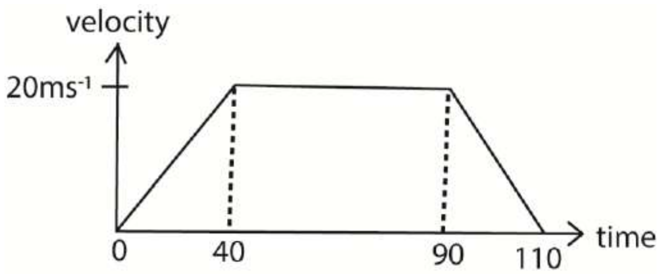
30 = $[10^2 + 2x\ 5x]^{\frac{1}{2}}$
900 = 100 + 10x
x =80m

Velocity-time graphs

Example 10

A car started from rest and attained a velocity of 20m/s in 40s. It then maintained the velocity attained for 50s. After that it was brought to rest by a constant breaking force in 20s.

(i) Draw a velocity-time graph for the motion



(ii) using the graph, find the total distance travelled by the car

Total distance = total area under the graph

$=\frac{1}{2}bh + lw + \frac{1}{2}bh$

$=\frac{1}{2}x\ 40\ x\ 20 + 50x\ 20 + \frac{1}{2}x\ 20\ x\ 20$ = 1600m

Method II (area of a trapezium)

A = $\frac{1}{2}h(a+b) = \frac{1}{2}x\ 20(50+110)$ = 1600m

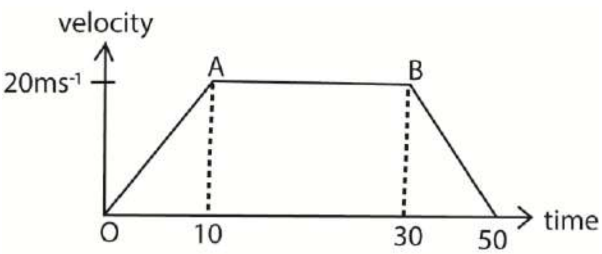
(iii) what is the acceleration of the car?

a= $\frac{v-u}{t} = \frac{20-0}{40} = 0.5ms^{-2}$

Example 11

A car from rest accelerates steadily to 10s up to a velocity f 20ms. It continues with uniform velocity for further 20s and then decelerates so that it stops in 20s.

(a) Draw a velocity-time graph to represent the motion



(b) Calculate

(i) acceleration

a= $\frac{v-u}{t} = \frac{20-0}{10} = 2ms^{-2}$

(ii) deceleration

a= $\frac{v-u}{t} = \frac{0-20}{20} = -1ms^{-2}$

(i) Distance = area under the graph

A = $\frac{1}{2}x\ 10\ x\ 20 + 20x\ 20 + \frac{1}{2}x\ 20\ x\ 20$

= 700m

Method II (area of a trapezium)

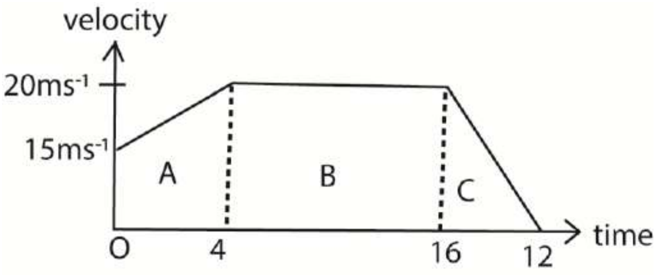
A = $\frac{1}{2}\ x\ 20(50+20)$= 700m

Average speed = $\frac{distance}{time}$

$=\frac{700}{50} = 14m/s$

Example 12

The graph below shows the motion in the body.



(a) Describe the motion of the body
A body with initial velocity of 15m/s accelerates steadily to a velocity of 20m/s in 4s, it then continues with a uniform velocity for 6s and brought to rest in 2s.

(b) Calculate the total distance travelled
Distance = 4 x 15 + $\frac{1}{2}x\ 4\ \ x\ 5 + 20\ x\ 6 + \ \frac{1}{2}x\ 20\ \ x\ 2$ = 210m

Revision exercise

1. P, Q and R are points on a straight road such that PQ = 20m and QR = 55m. A cyclist moving with uniform acceleration passes O and then notices that it takes him 10s and 15s to travel between P and Q and Q and R respectively. find the acceleration [a = $\frac{2}{15}ms^{-2}$]
2. A car travels from Kampala to Jinja and back. It takes average speed on the return journey is 4km/h greater than that on the outward journey and it takes 12 minutes less. Given that Kampala and Jinja are 80km apart, find the average speed on the outward journey.[30.05kmh]
3. Car A traveling at 35ms^{-1} along a straight horizontal road, accelerates uniformly at 0, 4ms^{-2}. At the same time, another car B moving at 44ms^{-1} and accelerating uniformly at 0.5ms^{-2} is 200m behind A
   (i) Find the time taken before car B over takes car A. [20s]
   (ii) speed with which B over takes A. [55m/s]
4. A car is being driven along a road at 72kmh^{-1} notices a fallen tree on the road 800m ahead and suddenly reduces the speed to 36kh^{-1} by applying brakes. For how long were the brakes applied [53.33s]
5. A train starts from station a with a uniform acceleration of 0.2ms^{-2} for 2 minutes and attains a maximum speed and moves uniformly for 15 minutes. it is then brought to rest at constant retardation of 5/3ms^{-2} at station B. find the distance between A and B. [23212.8m]
6. A motorcycle decelerated uniformly from 20kmh^{-1} to 8kmh^{-1} in travelling 896m. find the rate of deceleration in ms2 [0.0145ms^{-2}]
7. A body moves with a uniform acceleration and covers a distance of 27m in 3s; it then moves with a uniform velocity and covers a distance of 60m in 5s. Find the initial velocity and acceleration of the body. [6ms^{-1}, 2ms^{-2}]
8. A particle is projected away from an origin O with initial velocity of 0.25ms^{-1}. The particle travels in a straight line and accelerates at 1.5ms^{-2}. find
   (i) how far the particle is from O after 4s [7.5m]
   (ii) the distance travelled by the particle during the fourth second after projection. [5.5m]
9. A taxi which is moving with a uniform acceleration is observed to take 20s and 30s to travel successive 400m. find
   (i) initial speed of the taxi. [$\frac{68}{3}ms^{-1}$]

(ii) the further distance it covers before stopping [163.3m]

10. Two cyclist A and B are 36m apart on a straight road. Cyclist B starts from rest with an acceleration of $6ms^{-2}$ while A is in pursuit of B with velocity of $20ms^{-1}$ and acceleration of $4ms^{-1}$. Find the time taken when A overtakes B [13466s]

10.VERTICAL MOTION

Vertical motion under gravity

When a body is projected **vertically downwards**, it is subjected to an acceleration of $9.8ms^{-2}$. i.e. $a = g = 9.8ms^{-2}$

Equations of motion become

$v = u + gt$; **$h = ut + \frac{1}{2}gt^2$;** **$v^2 = u^2 + 2gh$**

When a body is projected **vertically upwards**, it is subjected to a retardation of $9.8ms^{-2}$. i.e. $a = g = 9.8ms^{-2}$

Equations of motion become

$v = u - gt$; **$h = ut - \frac{1}{2}gt^2$;** **$v^2 = u^2 - 2gh$**

Maximum /greatest height

When a particle is projected vertically upwards, the final velocity is 0ms-1 at its maximum height

$v^2 = u^2 - 2gh$

$0 = u^2 - 2gh_{max}$

$h_{max} = \frac{u^2}{2g}$

Time to reach maximum height

$v = u – gt$

$0 = u – gt$

$t = \frac{u}{g}$

Time of flight

$T = \frac{2u}{g}$

Example 1

A stone is dropped from a point which is 40m above the ground. Find the time taen for the stone to reach the ground

$h = ut + \frac{1}{2}gt^2$

$40 = 0 \text{ x } t + \frac{1}{2} \text{ x } 9.8 \text{ x } t^2$

$t = \sqrt{\frac{40}{9.8}} = 2.857s$

Example 2

A ball is thrown vertically upwards with an initial speed of $30ms^{-1}$. Calculate

(i) Time taken to reach thrower

$T = \frac{2u}{g} = \frac{2 \times 30}{9.8} = 6.12s$

(ii) maximum height reached

$h_{max} = \frac{u^2}{2g} = \frac{30^2}{2 \times 9.8} = 45.92m$

Example 3

A particle is projected from the ground level vertically upwards with velocity of $19.6ms^{-1}$. Find

(i) greatest height reached

$h_{max} = \frac{u^2}{2g} = \frac{19.6^2}{2 \times 9.8} = 19.6m$

(ii) time taken by the particle to reach maximum height

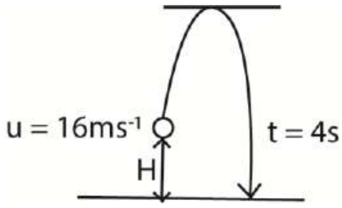
$t = \frac{u}{g} = \frac{19.6}{9.8}$ 2s

(iii) Time of flight

T = 2t = 2 x 2 = 4s

Example 4

1. A stone is thrown vertically upwards with velocity $16ms^{-1}$ from a point H meters above the ground level. The stone hits the ground 4 seconds later.
   Calculate the
   (a) Value of H (03marks)



Using $s = ut + ½ at^2$; s = -H(below point of projection), u = $16ms^{-1}$, a = -g, t = 4s

$-H = 16 \times 4 - \frac{1}{2} \times 9.8 \times 4^2$

H = 14.4m

(b) Velocity of the stone as it hits the ground (02marks)

Using v = u + at; v = -v(below point of projection), a = -g, t = 4s

-v = 16 -9.8 x 4

v = $23.2ms^{-1}$

∴ the velocity of the stone as it hits the ground is $23.2ms^{-1}$

Example 6

A stone is thrown vertically upwards with a velocity of $21ms^{-1}$. Calculate the

(a) Maximum height attained by the stone (03marks)

$H = \frac{u^2}{2g} = \frac{21^2}{2 \times 9.8} = 225m$

(b) Time the stone takes to reach the maximum height. (02marks)

$t = \frac{u}{g} = \frac{21}{9.8} = 2.143s$

Example 7

A particle is projected vertically upwards with velocity ums-1. After t seconds another particle is projected vertically upwards from the same point of projection and with the same initial velocity. Prove that the particles collide after$\left(\frac{2}{2}+\frac{u}{g}\right)s$. Hence show that they meet at a height of $\frac{u^2-(gt)^2}{8g}$.

Solution

t_1 = time taken by 1st particle

t_2 = time taken by 2nd particle

$t_1 - t_2$ = t .... (i)

t_1 and t_2 are roots of the equation

h = ut - $\frac{1}{2}gt^2$ or gt^2 – 2ut + 2h = 0

$t_1 = \frac{2u+\sqrt{4u^2-8gh}}{2g}$

$t_2 = \frac{2u-\sqrt{4u^2-8gh}}{2g}$

$\frac{2u+\sqrt{4u^2-8gh}}{2g} - \frac{2u-\sqrt{4u^2-8gh}}{2g}$ = t

$\sqrt{4u^2-8gh} = gt$ ...... (ii)

From eqn (ii)

$h = \frac{4u^2-(gt)^2}{8g}$

$t_1 = \frac{2u+\sqrt{4u^2-8gh}}{2g}$ putting eqn. (ii)

$$t_1 = \frac{2u+gt}{2g} = \left(\frac{2}{2}+\frac{u}{g}\right)s$$

Example 8

A particle is projected upwards with velocity of 10ms**-1**. After 2s another particle is projected vertically upwards from the same point of projection with the same initial velocity. Find the height above the level of projection where the particle meet and time taken by the first particle before they meet.

t_1 = time taken by 1st particle

t_2 = time taken by 2nd particle

$t_1 - t_2$ = t .... (i)

t_1 and t_2 are roots of the equation

h = ut - $\frac{1}{2}gt^2$ or gt^2 – 2ut + 2h = 0

$t_1 = \frac{20+\sqrt{400-8gh}}{2g}$

$t_2 = \frac{20-\sqrt{400-8gh}}{2g}$

$\frac{20+\sqrt{400-8gh}}{2g} - \frac{20-\sqrt{400-8gh}}{2g}$ = t

$\sqrt{400-8gh} = gt$ ...... (ii)

From eqn (ii)

$h = \frac{400-(2\,x\,9.8)^2}{8\,x\,9.8} = 0.202m$

$t_1 = \frac{2u+\sqrt{4u^2-8gh}}{2g}$ putting eqn. (ii)

$t_1 = \frac{2u+gt}{2g} = \left(\frac{2}{2}+\frac{u}{g}\right) = \frac{2\,x10+9.8\,x\,2}{2\,x\,9.8} = 2.02\,s$

Revision exercise

1. A particle is projected vertically upwards with a velocity of 21ms^{-1}. How long it takes to reach a point 280m below the point of projection. [10s]
2. A particle is projected vertically upwards with a velocity of 17.5ms^{-1}. Find
   (i) how high the particle goes. [15.6m]

(ii) what time elapse before it's at a height of 10m [$\frac{5}{7}s$; $\frac{22}{7}s$

3. A particle is projected vertically upwards with velocity of 24.5ms^{-1}. Find
   (a) when its velocity is 4.9ms^{-1} [2s]
   (b) how long it takes to return to the point of projection.[5s]
   (c) at what time it will be 19.6m above the point of projection. [1s and 4s]
4. A particle is projected vertically upwards with a velocity of 35ms^{-1}. find
   (a) how long it takes to reach the greatest height.[3.57s]
   (b) distance it ascends during the 3rd second of motion. [10.5m]
5. Two objects are dropped from a cliff of height H. the second is dropped when the first has travelled a distanc e d. Prove that the instant when the first object reaches the bottom, the second is a distance $2\sqrt{DH} - D$ from the top of the cliff.
6. A particle is projected vertically upwards from point O with a speed of $\frac{4}{3}v\ ms^{-1}$. After it has travelled a distance of $\frac{2}{5}X\ m$ above O on its upward motion, another particle is projected vertically upwards from the same point with the same initial speed. Given that the particles collide at a height $\frac{2}{5}X\ m$ above O, prove that
   (i) the maximum height, H is given by $8v^2 = 9gH$
   (ii) when the particle collide 9X = 20H.
7. A particle is projected vertically upwards with velocity um/s. After t seconds another particle is projected vertically upwards from the same point of projection and with the same initial velocity. Prove that the particles collide each other having a velocity of $\frac{1}{2}gt$.
8. A particle is projected vertically upwards with velocity 28m/s. After 2s another particle is projected vertically upwards from the same point of projection and with an initial velocity of 21m/s. Find when the two particles are at the same height and the velocity of each body at that instant. [4.9s after the first particle is projected, 20m/s, 7.4m/s]
9. A particle is projected vertically upwards with velocity 25m/s. After 4s another particle is projected vertically upwards from the same point of projection and with the same initial velocity. Find the time and height when the two particles meet.[4.55s after the first particle is projected, 12.288m]
10. A stone is dropped from the top of a tower. In the last second of its motion, it falls through a distance which is a fifth of the height of the tower. Find the height of the tower. [439.6m]

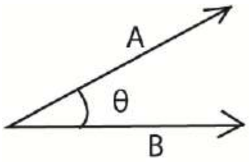
11.RESULTANT FORCES

Resultant of forces

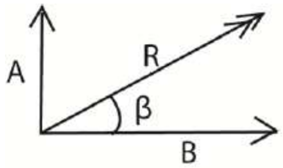
A force is anything which change a body's state of rest or uniform motion in a straight line. Examples are weight, tension, reaction, friction, resistance force.

Resultant of two forces

Consider two forces A and B inclined to each other at an angle θ



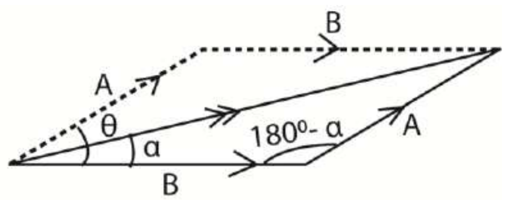
(i) θ is right angle (θ =90^0)



Resultant, R = $\sqrt{A^2 + B^2}$

Direction of resultant, β =$\tan^{-1}\left(\frac{A}{B}\right)$

(ii) **θ is acute (0^0 ≤ θ ≤ 90^0)**

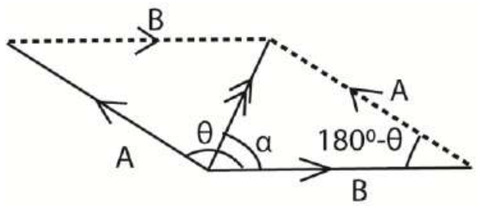


Direction of resultant, $\frac{\sin\alpha}{A} = \frac{\sin(180-\theta)}{R}$

$\alpha = \sin^{-1}\left(\frac{\text{Asin}(180-\theta)}{R}\right)$

Resultant, R =$\sqrt{[A^2 + B^2 - 2ABcos\ (180-\theta)]}$

(iii) **θ is obtuse (90^0 ≤ θ ≤ 180^0)**



Direction of resultant, $\frac{\sin\alpha}{A} = \frac{\sin(180-\theta)}{R}$

$\alpha = \sin^{-1}\left(\frac{\text{Asin}(180-\theta)}{R}\right)$

Resultant, R =$\sqrt{[A^2 + B^2 - 2ABcos\ (180-\theta)]}$

Example 1

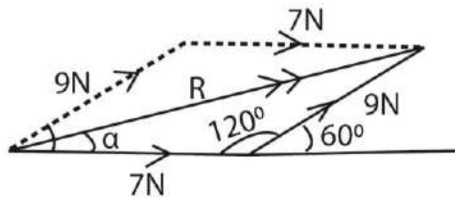
Two forces of magnitude 5N and 12N act on a particle with their direction inclined at 90^0. Find the magnitude and direction of the resultant

$R = \sqrt{5^2 + 12^2}$ = 13N   α $= \tan^{-1}\left(\frac{5}{12}\right)$ = 22.6^0

The resultant = 13N at 22.6^0 to 12N force

Example 2

Forces of magnitude 7N and 9N act on a particle at an angle of 60^0 between them. Find the magnitude and direction of the resultant.



Direction of resultant, $\frac{\sin\alpha}{9} = \frac{\sin(180-\theta)}{13.89}$

$\alpha = \sin^{-1}\left(\frac{9\sin(180-60)}{13.89}\right)$ = 34.13^0

Resultant, R $=\sqrt{[A^2 + B^2 - 2ABcos\,(180-\theta)]}$

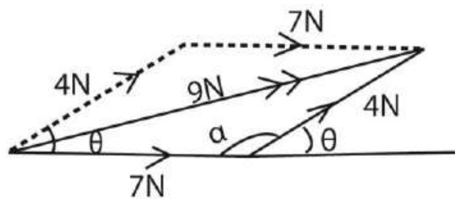
$=\sqrt{[7^2 + 9^2 - 2x7\ x\ 9cos\,(180-60)]}$

= 13.89N

Example 3

Find the angle between a force of 7N and 4N their resultant has a magnitude of 9N

Solution



$9^2 = 7^2 + 4^2 - 2 \times 7 \times 4 \times \cos\alpha$

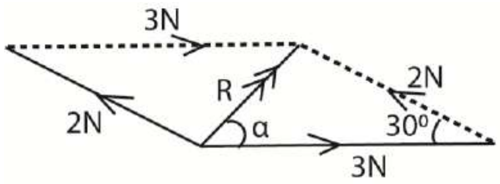
$\alpha = \cos^{-1}\left(-\frac{2}{7}\right)$ = 106.6^0

the angle θ between the forces = 180 – 106.6

= 73.4^0

Example 4

Forces of 3N and 2N act on a particle at an angle of 150^0 between them. Find the magnitude and direction of the resultant.



$R^2 = 2^2 + 3^2 - 2 \times 2 \times 2 \times \cos(30)$

R = 1.61N

Direction of resultant, $\frac{\sin\alpha}{2} = \frac{\sin(30)}{1.61}$

$\alpha = \sin^{-1}\left(\frac{2\sin(180-60)}{1.61}\right)$ = 38.3^0

Revision exercise

1. Two forces of magnitude 7N and 24N act on a particle with their direction at 90^0. Find the magnitude and direction of the resultant. [25N, 16.26^0 with 24N force]
2. Forces of 5N and 8N act on a particle at an angle of 50^0 between them. Find the magnitude and direction of the resultant. [11.9N at 19^0 with 8N force]
3. Forces of 4N and 6N act on a particle at angle 60^0 between them. Find the magnitude and the direction of the resultant. [5.29N, at 40.9^0 with 6N force]
4. Forces of 9N and 10N act on a particle at angle 40^0 between them. Find the magnitude and the direction of the resultant. [17.9N, at 18.9^0 with 10N force]
5. Forces of 12N and 10N act on a particle at angle 105^0 between them. Find the magnitude and the direction of the resultant. [13.5N, at 45.7^0 with 12N force]
6. Forces of 8N and 3N act on a particle at angle 160^0 between them. Find the magnitude and the direction of the resultant. [5.28N, at 11.2^0 with 8N force]
7. Find the angle between a force of 10N and 4N their resultant has a magnitude of 8N. [130.5^0]
8. The angle between a force αN and a force of 3N is 120^0. If the resultant of the two forces has magnitude 7N, find the value of α. [8N]
9. The angle between a force βN and a force of 8N is 45^0. If the resultant of the two forces has a magnitude 15N, find the value of β. [8.24N]

12. FORCE AND NEWTON'S LAWS

Force and Newton's laws of motion

Law I: A body continues in its state of rest or uniform motion in a straight line unless acted upon by an external force

Law II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

F = m$\left(\frac{v-u}{t}\right)$ but $\left(\frac{v-u}{t}\right)$ =a

= ma

NB: F must be the resultant force

Example 1

Find the acceleration produced when a body of mass 5kg experiences a resultant force of 10N

F = ma | 10 = 5a | a = 2ms^{-2}

Example 2

A car of mass 600kg travels a distance of 24m while uniformly accelerated from rest to 12ms^{-1}

(i) Find the acceleration of the car
$v^2 = u^2 + 2as$
$12^2 = 0^2 + 2a \times 24$
a = 3ms^{-2}

(ii) determine the accelerating force
F = ma = 600 x 3 = 1800N

Example 3

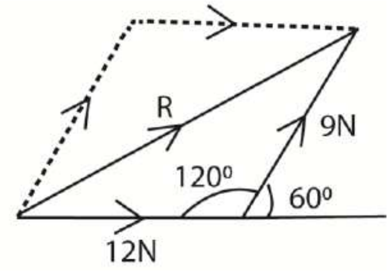
A body of mass 500g experiences a resultant force 3N. Find

(i) Acceleration produced
F = ma | 3 = 0.5 x a | a = 6ms^{-2}

(ii) Distance travelled by the body while increasing speed from 1ms^{-1} to 7ms^{-1}
$v^2 = u^2 + 2as$
$7^2 = 1^2 + 2\times 6 \times s$
s = 4m

Example 4

Two forces of magnitude 12N and 9N act on a particle producing an acceleration of 3.65ms^{-2}. The two forces act at an angle of 600 to each other. Find the mass of the particle.



$R^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \cos 120^0$

R = 18.25N

F = ma

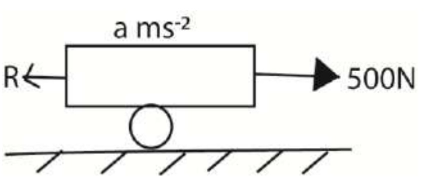
18.25N = 3.65m

m = 5kg

When resistance or friction is involved

Example 5

A car moves along a level road at constant velocity of 22ms^{-2}. If its engine is exerting a forward force of 500N, what resistance is the car experiencing.



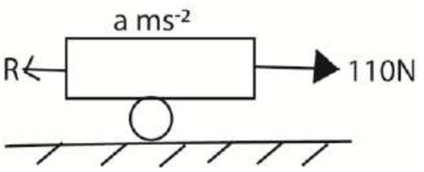
F = ma

500 – R = m x 0

R = 500N

Example 6

A car of mass 500kg moves along a level road with acceleration of 2ms^{-2}. Its Engine is exerting a forward force of 110N. What is the resistance a car is experiencing?



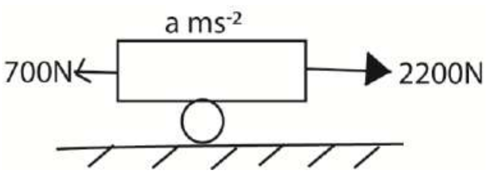
F = ma

500 – R = 500 x 2

R = 100N

Example 7

A van of mass 2tonnes moves along a level road against resistance of 700N. If its engine is exerting a forward force of 2200N. Find the acceleration of the van



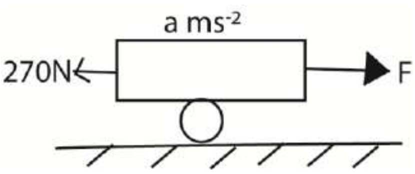
F = ma

2200 – 700 = 2000 x a

a = 0.75ms^{-2}

Example 8

Find the constant force necessary to accelerate a car of mass 1000kg from 15ms^{-1} to 20ms^{-1} in 10s against a resistance of 270N



v = u + at

20 = 15 + 10a

a = 0.5ms^{-2}

F = ma

F – 270 = 1000 x 0.5

F = 770N

Calculations involving vector form

Find the resultant force required to make a body of mass 2kg at (5i + 2j)ms^{-2}.

F = ma

F = $2\binom{5}{2} = \binom{10}{4} N$

Example 9

F = ma $\qquad \binom{4}{2} + \binom{1}{1} = 0.5a \qquad$ a = $\binom{6}{6} ms^2$

Find the acceleration produced in a body of mass 500N is subjected to forces of (4i + 2j)N and (-i + j)N

F = ma $\qquad \binom{4}{2} + \binom{1}{1} = 0.5a \qquad$ a = $\binom{6}{6} ms^2$

Example 10

Find the magnitude of the acceleration produced in a body of mass 2kg subjected to forces of (2i – 3j +4k)N and (i + 5j + 2k)N

F = ma $\qquad \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} = 2a \qquad$ a = $\begin{pmatrix} 1.5 \\ 1 \\ 3 \end{pmatrix} ms^2 \qquad$ $|a| = \sqrt{1.5^2 + 1^2 + 3^2} = 2.3ms^{-2}$

Example 10

A particle of mass 2.5kg is acted on by a resultant force of 15N acting in the direction (2i – j – 2k). find the magnitude of the acceleration

F = 15 x $\frac{2i-j-2k}{\sqrt{2^2+(-1)^2+(-2)^2}}$

= 15 x $\frac{2i-j-2k}{3}$

F = 10i – 5j -10k

F = ma

$\begin{pmatrix} 10 \\ -3 \\ -10 \end{pmatrix} = 2a$

a = $\begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix}$

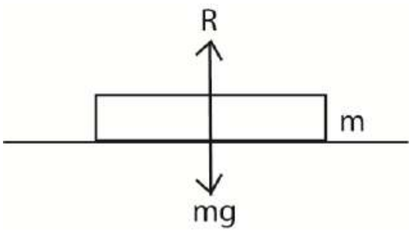
$|a| = \sqrt{10^2 + (-3)^2 + (-10)^2}$

=6ms-2

Law III: To every action there is an equal but opposite reaction

Consider

1. a body of mass m place on a smooth horizontal surface

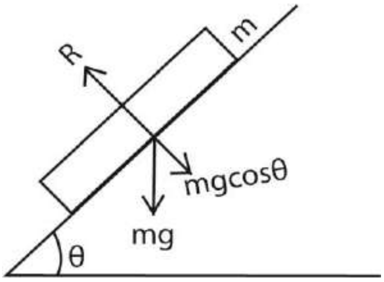


R = mg

R = normal reaction

mg gravitational pull (weight)

2. Mass m laced on a smooth inclined lane of angle of inclination θ



R = mgcosθ

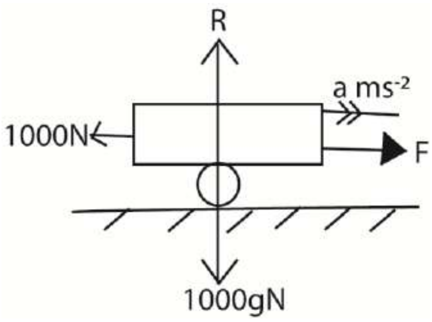
- All objects placed on or moving on an inclined plane experience a force mgsinθ **down** the plane no matter the direction of movement.
- If the plane is rough, the body experience a frictional force whose direction is opposite of the direction of motion

Motion on horizontal plane

Example 11

A car of 1000kg is accelerating at 2ms^{-2}. If the resistance to motion is 100N

(i) Find the normal reaction of the car on the road surface



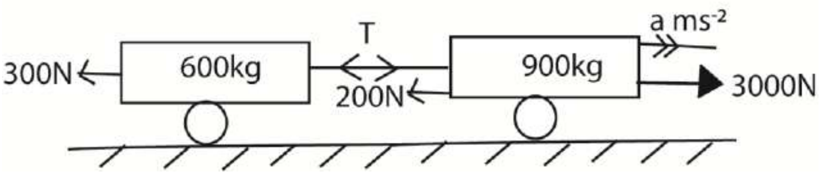
R = 1000gN

= 1000 x 9.8 = 9800N

(ii) What accelerating force acts on the car?
F = ma
F – 1000 = 1000 x 2; F = 3000N

Example 12

A car of mass 900kg tows a trailer of mass 600kg along a level road by means of a rigid bar. The car experiences a resistance of 200N and the trailer a resistance of 300N, if the car engine exerts a force of 3kN, find the acceleration produced and the tension in the tow bar



For 900kg: 3000 – (T + 200) = 900a … (i)

For 600kg: T – 300 = 600a ……………. (ii)

(i) and (ii) a= 1.6667ms^{-2}

Alternatively

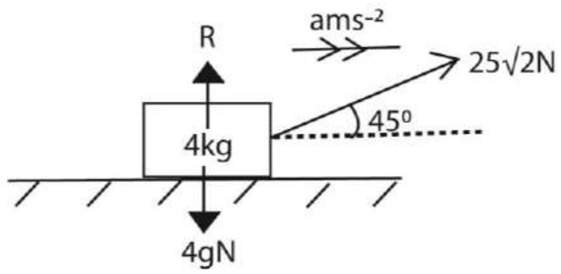
3000 – (200 + 300) = (900 + 600)a

a = 1.6667ms^{-2}

Force inclined at an angle to the horizontal

Example 13

A body of mass 4kg is acted on by force of 25$\sqrt{2}$N which is inclined at 450 to a smooth horizontal surface. Find the acceleration of the body and the normal reaction between the body and the surface.



(→) 25$\sqrt{2}$Ncos45 = 4a

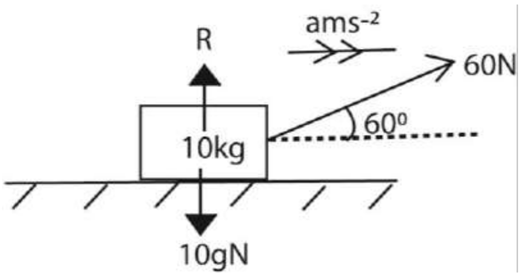
a = 6.25ms^{-2}

(↑) R + 25$\sqrt{2}$Nsin 45 - = 0

R = 14.2N

Example 14

A body of mass 10kg is initially at rest on a rough horizontal surface. It is pulled along the surface by constant force of 60N inclined at 600 above the horizontal. If the resistance to motion totals 10n, find the acceleration of the body and the distance travelled in the first 3s.



(→) 60sos60 - 10 = 10a

a = 2ms^{-2}

s = ut +$\frac{1}{2}at^2$

s = 0 x 3 + $\frac{1}{2}$ x 2 x 3^2 =9m

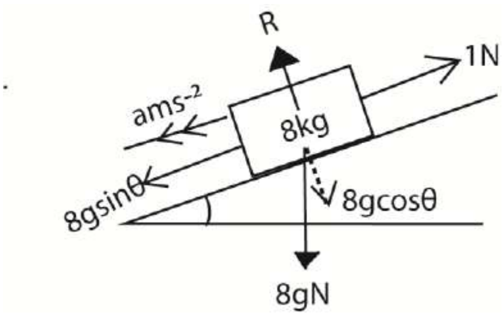
Revision exercise 1

1. A railway engine of mass 100tonnes is attached to a line of trucks of total mass 80 tonnes. Assuming there is no resistance to motion, find the tension in the coupling between the engine and the leading truck when the train has acceleration of $0.020ms^{-2}$ [25.6kN]
2. A body of mass 5kg, initially at rest on a smooth horizontal surface is pulled along the surface by a constant force P inclined at 450 above the horizontal. In the first 5 seconds of motion, the body moves a distance of 10m along the surface. Find the
   (i) acceleration of the body [$0.8ms^{-2}$]
   (ii) magnitude of P [$4\sqrt{2}$N]
   (iii) normal reaction between the body and the surface. [45N]
3. A body of mass m kg, initially at rest on a smooth horizontal surface is pulled along the surface by a constant force P inclined at θ above the horizontal. Show that the body moves a distance in time t along the surface given by $\frac{Pt^2cos\theta}{2m}$.
4. A body of mass m kg, initially at rest on a rough horizontal surface is pulled along the surface by a constant force P inclined at θ above the horizontal. If the mass acquire velocity v in a distance d. Show that the resistance to motion is given by Pcosθ = $\frac{mv^2}{2d}$

Motion on an inclined plane

Example 15

A body of mass 8kg is released from on the surface of a plane at 1 in 40. If the resistance to motion is 1N, find the acceleration of the body and the speed it acquired after 6s.



$\sin\theta = \frac{1}{40}$

F = ma

8gsinθ – 1 = 8a

a = $0.12ms^{-2}$

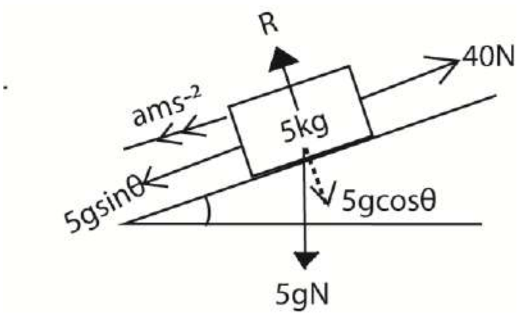
v = u + at

= 0 + 0.12 x 6 =$0.72ms^{-2}$

Example 16

A body of mass 5kg is pulled u a smooth plane inclined at 30^0 to the horizontal by a force of 40N acting parallel to the plane. Find

(i) acceleration of the body
(ii) force exerted on the body by the plane R



F = ma

40 -5gsin30^0 = 5a

a = $3.095ms^{-2}$

(ii) R = 5gcos30

= 5 x 9.8cos30 = 42.4N

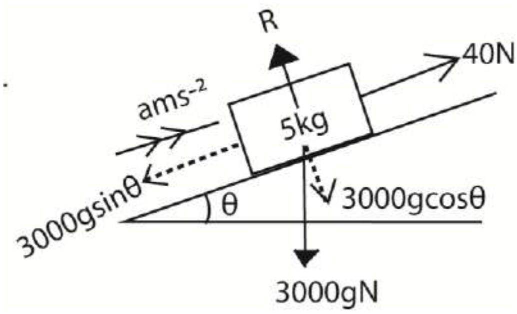
Example 17

A lorry of mass 3tonnes travelling at 90k/h starts to climb an incline of 1 in 5. Assuming the attractive pull between its tyres and the road remains constant and that its velocity reduces to 54kmh in a distance of 500m. Find the attractive pull.

u = 90kmh = $\frac{90 \, x \, 1000}{3600}$ = 25ms^{-1}

V = 54kmh = $\frac{54 \, x \, 1000}{3600}$ = 15ms^{-1}

a = $\frac{v^2 - u^2}{2s} = \frac{15^2 - 25^2}{2 \, x \, 500}$ = -0.4ms^{-2}



sinθ = $\frac{1}{40}$

F = ma

F - 3000gsinθ = 3000a

F – 3000 x 9.8 x $\frac{1}{5}$ = 3000 x 0.4

F = 4686N

Revision exercise 2

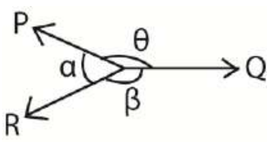
1. A particle of mass 5kg resting on smooth plane inclined at $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$to the horizontal. Find the magnitude of the horizontal force required to keep the particle in equilibrium and the normal reaction. [28.29N, 56.58N]
2. The engine of a train exerts a force of 35,000N on a train of mass 240 tonnes and draws up a slope of 1 in 120 against resistance totalling to 60N per tonne. Find the acceleration of the train. [0.004167ms^{-2}]
3. A car of mass 2.5 metric tonnes is drawn up a slope of 1 in 10 from rest with acceleration of 1.2ms^{-2} against a constant frictional force of $\frac{1}{100}$ of the weight of the vehicle using a cable. Find the tension in the cable. [5695N]
4. A mass 5kg is initially at the bottom of a smooth slope which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. The mass is pushed up the slope by horizontal force 50N, find
   (i) the normal reaction between the mass and the plane [69.2N]
   (ii) calculate the acceleration up the slope [2.12ms^{-2}]
   (iii) how far up the slope the mass travels in the first 4s [16.96m]
5. A body of mass 100kg is released from rest at the top of a smooth slope which is inclined at 30^0 to the horizontal. Find
   (i) velocity of the body when it has travelled 20m down the slope. [14ms^{-1}]
   (ii) velocity, if the mass of the body was 50kg. [14ms^{-1}]
6. A body of mass 20kg is released from rest at the top of a smooth slope which is inclined at 30^0 to the horizontal. If the body accelerates down the slope at 3ms^{-2}, find the constant resistance to motion experienced by the body. [38N]
7. A body of mass 20kg is released from rest at the top of a rough slope which is inclined at 30^0 to the horizontal. 6s later the body has a velocity of 21ms^{-1} down the slope, find the constant resistance to motion experienced by the body. [28N]

8. A car of 1 tonne accelerated from $36kmh^{-1}$ to $72kmh^{-1}$ while moving 0.5km up a road inclined at an angle of α to the horizontal where sinα = $\frac{1}{20}$. If the total resistive force to its motion is 0,3kN, find the driving force of the car engine. [1090N]
9. A railway truck of mass 6.0 tonnes moves with an acceleration of $0.050ms^{-2}$ down a track which is inclined to the horizontal at an angle α where sinα = $\frac{1}{120}$. find the resistance to motion. [190N]
10. A body of mass 5.0kg is pulled along a smooth horizontal ground by means of 40N acting at 60^0 above the horizontal. find
    (i) Accelerating force [$4ms^{-2}$]
    (ii) Force the body exerts on the ground [14.4N]
11. A body of mass 3.0kgslides down a plane which is inclined at 300 to the horizontal. Find the acceleration of the body if
    (i) the plane is smooth [$4.9ms^{-2}$]
    (ii) there is frictional resistance of 9.0N [$1.9ms^{-2}$]
12. A car of mass 1000kg tows a caravan of mass 600kg u a road which rises 1m vertically for every 20m of its length. There is a constant frictional resistance of 200N and 100N to the motion of the car and caravan respectively. The combination has an acceleration of $1.2ms^{-2}$ with the engine exerting a constant driving force. (Take g = $10ms^{-2}$). Find
    (a) driving force[3020N]
    (b) Tension in the tow-bar [1120N]

13. EQUILIBRIUM OF THREE FORCES.

Equilibrium of three forces L ami's theorem

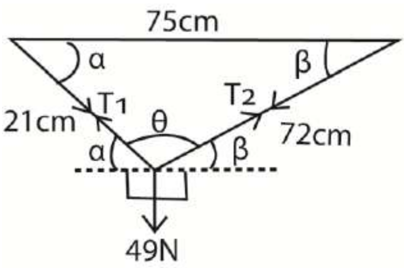
For any three forces acting on a particle in equilibrium where none of them is parallel to each other, Lami's theorem is applicable



$$\frac{P}{sin\beta} = \frac{Q}{sin\alpha} = \frac{R}{sin\theta}$$

Example 1

A weight of 49N is suspended by two strings of length 21 cm and 72cm attached to 2 points in a horizontal line a distance of 75cm apart. Find the tension in the strings so that the particle remain in equilibrium



By cosine rule:

$75^2 = 21^2 + 72^2 - 2 \times 21 \times 72\cos\theta$

$\theta = 90^0$

Similarly, $\beta = 16.26^0$ and $\alpha = 73.74^0$

$$\frac{T_1}{\sin(16.26+90)} = \frac{49}{\sin 90};$$

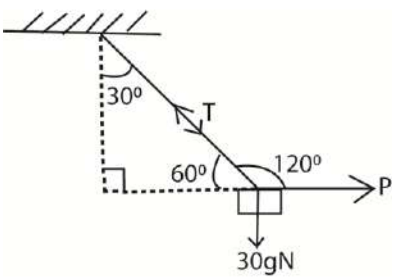
$$\therefore T_1 = 47N$$

$$\frac{T_2}{\sin(73.74+90)} = \frac{49}{\sin 90};$$

$$\therefore T_2 = 13.72N$$

Example 2

Mass of 30kg hangs vertically at the end of a light string. If the mass is pulled by a horizontal force P so that the string makes 30^0 with the vertical. Find the magnitude of the force and the tension in the string so that the particle remain in equilibrium.

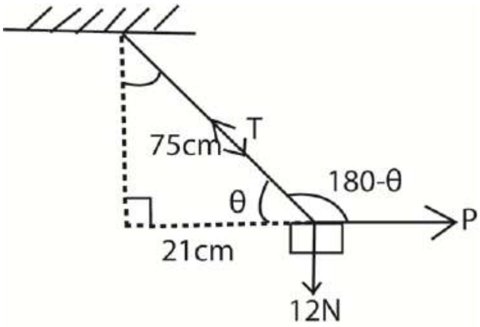


$\frac{T}{\sin 90} = \frac{30 \times 9.8}{\sin 120}$; T = 339.48N

$\frac{P}{\sin(60+90)} = \frac{30 \times 9.8}{\sin 120}$; P = 16974N

Example 3

One end of a light inextensible string of length 75cm is fixed to a point on a rigid pole. The particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force, P. Find the magnitude of the force, P and the tension of the string so that the particle remain in equilibrium



$\theta = \cos^{-1}\left(\frac{21}{75}\right) = 73.74^0$

$\frac{T}{\sin 90} = \frac{12}{\sin(180-73.74)}$

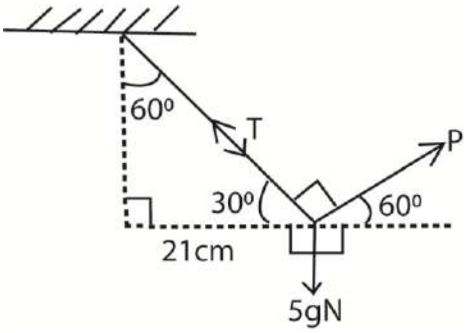
T = 12.5N

$\frac{P}{\sin(90+73.4)} = \frac{12}{\sin(180-73.74)}$

P = 3.5N

Example 4

A light inextensible string AB whose end A is fixed has end B attached to a particle of mass 5kg. A force P acting perpendicular to the string is applied on the particle keeping it in equilibrium with the string inclined at 60^0 to the vertical. Find the value of P and the tension in the string



$\frac{T}{\sin(90+60)} = \frac{5\ x\ 98}{\sin 90}$

T = 24.5N

$\frac{5\ x\ 98}{\sin 90} = \frac{P}{\sin\ (90+30)}$

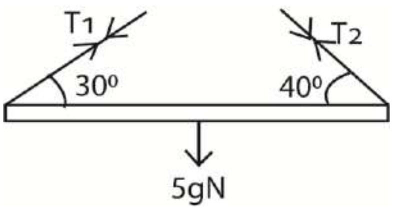
P = 42.44N

Example 5

A non-uniform beam of mass 5kg rests horizontally in equilibrium supported by two strings attached to the ends of the beam.

The strings makes 300 and 400 with the horizontal beam as shown above. Find the tension in the strings.

Solution



(→) $T_1\cos30 = T_2\cos40$; $T_1 = 0.8846T_2$

(↑)$T_1\sin30 + T_2\sin40 = 5g$

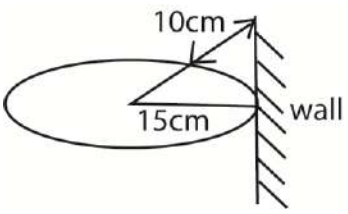
$0.8846T_2\sin30 + T_2\sin40 = 5 \times 9.8$

$T_2 = 45.159$N

$T_1 = 0.8846 \times 45.159 = 39.94$N

Example 6

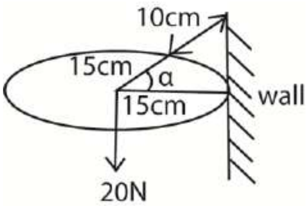
A sphere of weight 20N and radius 15cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown.



(i) calculate the reaction on the sphere due to the wall

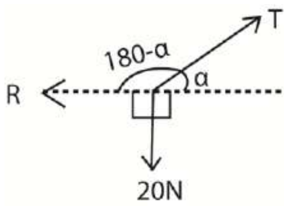
(ii) Find the tension in the string

Solution



α = $\cos^{-1}\left(\frac{15}{25}\right)$ = 53.13^0

Using Lami's theory



$\frac{T}{sin90} = \frac{20}{\sin(180-53.13)}$; T =25N

$\frac{R}{\sin(90+53.13)} = \frac{20}{\sin(180-53.13)}$; R = 15N

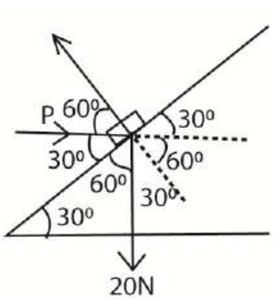
Example 7

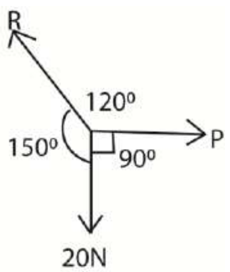
A particle of weight 20N is held at equilibrium on a smooth plane inclined at 30^0 to the horizontal by a horizontal force P.

(i) Find the value of P and the reaction between the particle and the plane.

(ii) If the force P is removed and a string parallel to the plane is used to hold the particle, find the tension in the string and the new value of the reaction.

Solution

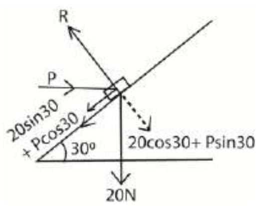




$\frac{P}{sin150} = \frac{R}{sin90} = \frac{20}{sin120}$

R = 23.09N and P = 11.55N

Alternatively: by resolving forces



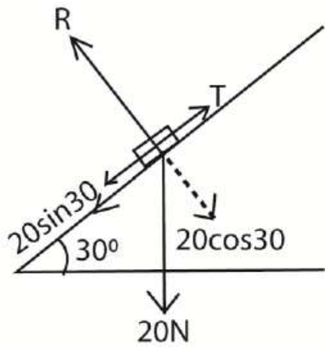
At equilibrium parallel to plane forces = 0

Pcos30 + 20sin30 = 0; P = 11.55N

R = 20cos30 + Psin30

R = 20cos30 + 11.55sin30 = 23.09N

(ii)



Parallel to the plane T = 20sin30 = 10N

Perpendicular to the plane R = 20cos30 = 13N

Alternatively by Lami's theory

$\frac{T}{sin150} = \frac{R}{sin120} = \frac{20}{sin90}$

T = 10N

R = 1.3N

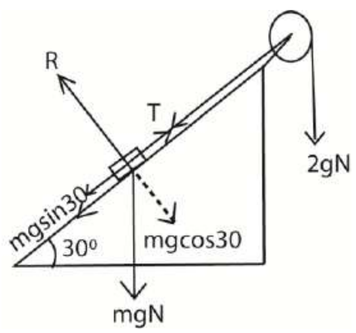
Example 8

A light inextensible string passes over a smooth fixed pulley at the top of a smooth plane inclined at 300 to the horizontal. A particle of mass 2kg is attached to one end of the string and rests vertically in equilibrium when the particle of ma resting on the surface of the plane is attached to the other end of the string. Find

(i) the normal reaction between m and the plane
(ii) tension in the string and the value of m.

Solution

By resolving forces



For 2kg mass: T – 2 x 9.8 = 0; T = 19.62N

Parallel to the plane

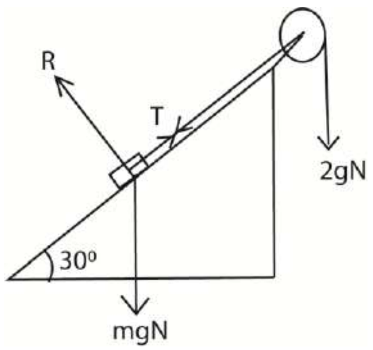
T – mgsin30 = 0; m = 4kg

Perpendicular to the plane

R = mgco30

R = 4 x 98cos30 = 33.98

Alternatively by using Lami's theorem



For 2kg mass: T – 2 x 9.8 = 0; T = 19.62N

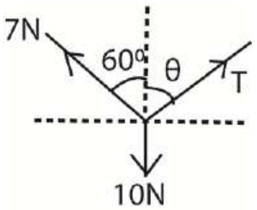
$\frac{T}{sin150} = \frac{mg}{sin90} = \frac{R}{sin120}$

$\frac{19.62}{sin150} = \frac{mg}{sin90} = \frac{R}{sin120}$

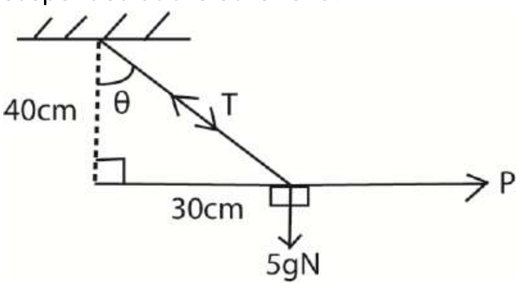
m = 4kg and R = 33.98N

Revision exercise

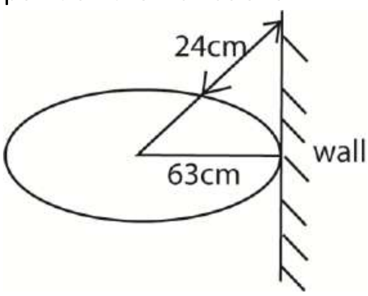
1. A particle P of mass 2kg is suspended from a fixed point O by means of a light inextensible string. The string is taut and makes an angle of 300 with the downward vertical through O and a particle is held in equilibrium by means of a horizontal force of magnitude F acting on the particle. Find the value of F and the tension in the string [F = 11.3161, T = 22.6321N]
2. A particle of mass 3kg lies on a smooth plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$. The particle is held in equilibrium by horizontal force of magnitude FN. The line of action of this force is the same vertical plane as a line of greatest slope of inclined plane. Find the value of F. [22.05N]
3. The diagram below shows a body of weight 10N supported in equilibrium by two light inextensible strings. The tension in the strings are 7N and T and the angle the string makes with the upward vertical are 60^0 and θ respectively.



Find T and θ. [T = 8.9N, θ = 43^0]
4. A particle of weight 8N is attached to a point B by a light inextensible string AB. It hangs in equilibrium with point A fixed and AB at an angle of 30^0 to the downward vertical. A force F at B acting at right angles to AB, keeps the particle in equilibrium. Find the magnitude of force F and the tension in the string. [4N, $4\sqrt{3}N$]
5. The diagram shows a light inextensible string with one end fixed at A and a mass of 5kg suspended at the other end.

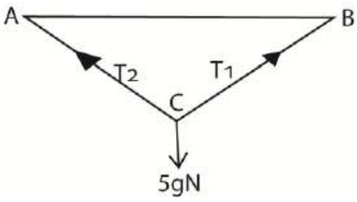


The mass is held in equilibrium at an angle θ to the downward vertical by a horizontal force P. Find the value of θ, P and the tension in the string [θ = 36.9^0, P = 36.75N, T = 61.25N)
6. A sphere of mass 5kg and radius 63cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 24cm attached to a point on the sphere an to a point on the wall as shown.



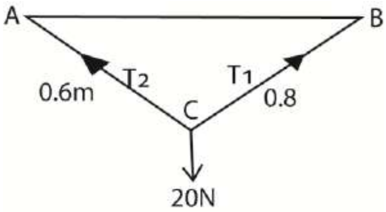
Find the tension in the string. [71.05N]
7. A particle whose weight is 50N is suspended by a light string which is 35^0 to the vertical under the action of a horizontal force F. Find the force F and the tension in the string. [35.0N, 61.0N]

8. A particle of weight w rests on a smooth plane which inclined at 40^0 to horizontal. The particle is prevented from slipping by a force of 50.0N acting parallel to the plane and up a line of greatest slope. Calculate w and reaction due to the plane. [77.8N, 59.6N]
9. A mass of 2kg is suspended by two light inextensible strings. One making an angle of 600 with the upward vertical and the other 300with the upward vertical. Find the tension in each string. [9.8N, 17.0N]
10. A heavy uniform rod of weight W is hung from a point by two equal strings, one attached to each end of the rod. A body of weight w is hang half-way between A and the center of the rod. Prove that the ratio of tension in the string is$\frac{2W+3w}{2W+w}$.
11. A non-uniform beam AB of length 8m and its weight 10N acts from a point G between A and B such that AG = 6m. The beam is supported horizontally by strings attached to A and B. The string attached to A makes an angle of 30^0 with AB. Find the angle that the string attached to B makes with AB and find the tension in the strings. [60^0, 5N, 8.66N]
12. A light inextensible string of length 40cm has its upper end fixed to a point A and carries a mass of 2kg at its lower end. A horizontal force applied to the mass keeps it in equilibrium, 20cm from the vertical through A. Find the magnitude of this horizontal force and the tension in the string. [11.3N, 22.6N]
13. The diagram shows a body of mass 5kg supported by two light inextensible strings, the other ends of which are attached to two points A and B on same level as each other end 7m apart.



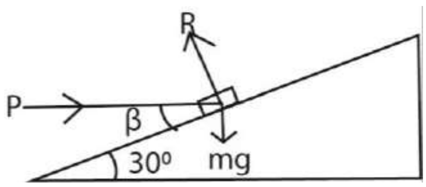
The body rests in equilibrium at 3m vertically below AB. If angle CBA = 45^0, find T_1 and T_2 the tensions in the strings. [35N, $28\sqrt{2}N$]

14. The diagram shows a body of weight 20N supported by two light inextensible strings of length 0.6m and 0.8m from two points 1m apart on a horizontal beam.



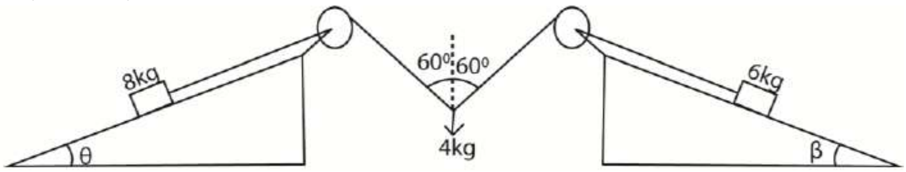
The body rests in equilibrium, find T_1 and T_2 the tensions in the strings. [16N, 12N]

15. A light inextensible string of length 50cm has its upper end fixed at point A and carries a particle of 8kg at its lower end. A horizontal force P applied to the particle in equilibrium 30cm from the vertical through A, find the magnitude of P and the tension in the string. [58.8N, 98N}
16. A article is in equilibrium under the action of forces 4N due north, 8N due west, $5\sqrt{2}$N south east and P, find the magnitude and direction of P. [3.16N, N71.6^{0}E]
17. A force P holds a particle of mass mkg in equilibrium on a smooth plane which is inclined at 300 to the horizontal.



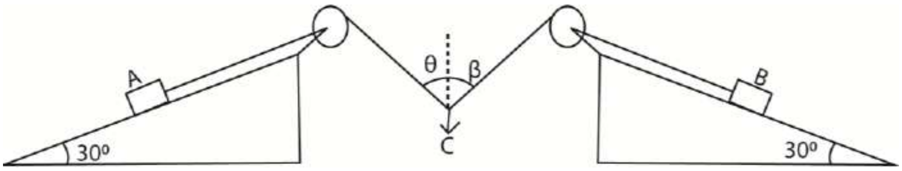
If P makes an angle β with the plane, find β when R the normal reaction between the particle and the plane is 15mg [51.7^0]

18. The diagram below shows masses of 8kg and 6kg lying on smooth planes of inclination θ and β respectively



Light inextensible strings attached to these masses pass along the line of greatest slopes over smooth pulleys and are connected to 4kg mass hanging freely. The strings both make an angle of 600 with the upward vertical as shown above. If the system rest in equilibrium find θ and β. [θ = 30^0 and β 41.8^0]

19. The diagram below shows masses A and B each lying on smooth planes of inclination30^0.



Light inextensible strings attached to A and B pass along the lines of greatest slopes, over smooth pulleys and are connected to a third mass C hanging freely. The strings make angles of θ and β with the upward vertical as shown above. If A, B and C have masses 2m, m, and m respectively and the system rests in equilibrium show that sinθ = 2sinβ and cosβ +2cosθ =2. Hence find θ and β. [29.0^0, 75.5^0]

14.RESOLUTION OF FORCES ACTING ON A POLYGON

Resolutions of forces acting on a polygon

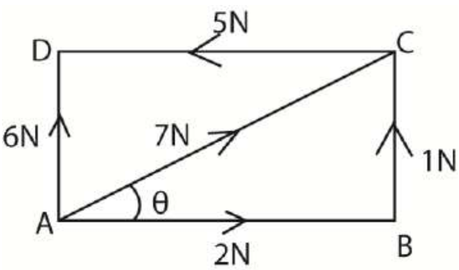
For any regular polygon

- all sides are equal
- all angles are equal
- an exterior angle $=\frac{360}{n}$ where n is the number of sides

Example 1

ABCD is a rectangle with AB= 4cm and BC = 3cm. Forces of magnitude 2N, 1N, 5N,6N and 7N act along AB, BC, CD, AD and AC respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine

(i) the magnitude of the resultant force

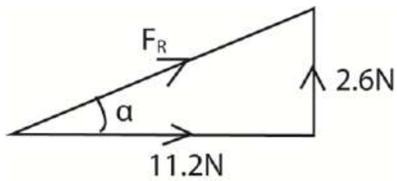


$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87$

R = $\begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}-5\\0\end{pmatrix} + \begin{pmatrix}0\\6\end{pmatrix} + \begin{pmatrix}7cos36.87\\7sin36.87\end{pmatrix} = \begin{pmatrix}2.6\\11.2\end{pmatrix}$

$R = \sqrt{2.6^2 + 11.2^2}$ = 11.498N

(ii) direction of the resultant with AB



Direction,$\alpha = \tan^{-1}\left(\frac{2.6}{11.2}\right)$

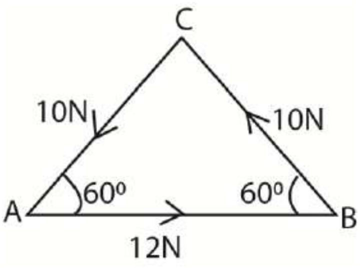
$=13.069^0$

Direction is 13.069^0 above AB

Example 2

ABC is an equilateral triangle. Forces of magnitude 12N, 10N and 10N act along AB, BC and CA respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine

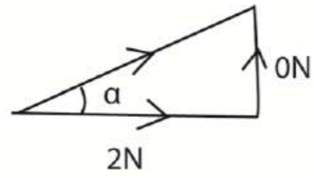
(i) the magnitude of the resultant force



$$R = \binom{12}{0} + \binom{-10cos60}{10sin60} + \binom{-10cos60}{-10sin60} = \binom{2}{0}$$

$$R = \sqrt{2^2 + 0^2} = 2\text{N}$$

(ii) Direction of the resultant with AB

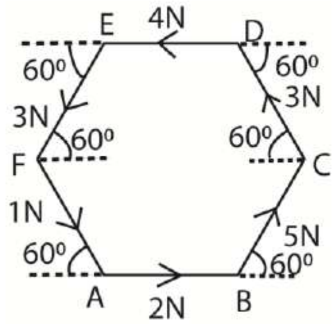


Direction $\alpha = \tan^{-1}\left(\frac{0}{2}\right) = 0^2$

Example 3

ABCDEF is a regular hexagon. Force of magnitude 2N, 5N, 3N, 4N, 3N and 1N act along the line AB, BC, CD, DE, EF and FA respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine

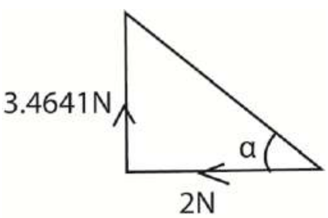
(i) the magnitude of the resultant force and



$$R = \binom{2}{0} + \binom{5cos60}{5sin60} + \binom{-3cos60}{3sin60} + \binom{-4}{0} + \binom{-3cos60}{-3sin60} + \binom{1\cos 60}{-1sin60} = \binom{-2}{3.4641}$$

$$R = \sqrt{(-2)^2 + 3.4641^2} = 4N$$

(ii) direction of the resultant with AB.

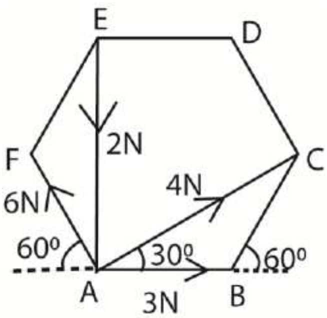


$\alpha = \tan^{-1}\left(\frac{3.461}{2}\right) = 60^0$ to AB

Example 4

ABCDEF is a regular hexagon. Forces of magnitude 3N, 4N, 2N and 6N act along the line AB, AC, EA and AF respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine

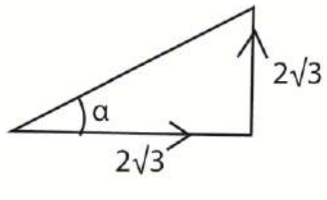
(i) the magnitude of the resultant force



$$R = \binom{3}{0} + \binom{4cos30}{4sin30} + \binom{-6cos60}{6sin60} + \binom{0}{-2} = \binom{2\sqrt{3}}{3\sqrt{3}}$$

$$R = \sqrt{(2\sqrt{3})^2 + (3\sqrt{3})^2} = 6.245N$$

(ii) direction of the resultant force



$$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{2\sqrt{3}}\right) = 56.3^0$$

1. ABCD is a square. Forces of magnitude 6N, 4N and $2\sqrt{2}N$ act along AD, AB and AC respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force.
   [10N at 53.1^0 with AB]
2. ABCD is a square. Forces of magnitude 2N, 1N, $\sqrt{2}N$ and 4N act along AB, BC and AC and DA respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force.
   [5.13N at 33.7^0 with AB]
3. ABCD is a square. Three forces of magnitude 4N, 10N and $7N$ act along AB, AD and CA respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude [5.1388N]
4. In equilateral triangle PQR, three forces of magnitude 5N, 10N and 8N act along the side PQ, QR and PR respectively. Their direction are the order the letters. Find the magnitude of the resultant force. [16.1N]
5. ABCD is a square. Forces of magnitude $6\sqrt{3}N$ N, 2N and $4\sqrt{3}N$ act along AB, CB and CD respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force.
   [4N at 30^0 to AB]
6. ABCD is a rectangle with AB= 4cm and BC = 3cm. Forces of magnitude 3N, 1N, and 10N act along AB, DC and AC respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine the magnitude and direction of the resultant force. [13.4N at 26.6^0 with AB]
7. ABCD is a rectangle. Forces of magnitude 8N, 4N, 10N and 2N act along AB, CB, CD and AD respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine the magnitude and direction of the resultant force.
   [283N at 45^0 at AB]
8. In equilateral triangle ABC, forces of magnitude 10N each act along the side AB, BC and AC respectively. Their direction are the order the letters. Find the magnitude of the resultant force and the angle it makes with AB. [20N at $60^0$to AB]
9. In equilateral triangle ABC, forces of magnitude 5N, 9N and 7N act along the side AB, BC and CA respectively. Their direction are the order the letters. Find the magnitude of the resultant force and the angle it makes with AB. [$2\sqrt{3}$N at $30^0$to AB]

10. In equilateral triangle ABC, forces of magnitude 4N, 4N and 6N act along the side AB, BC and AC respectively. Their direction are the order the letters. Find the magnitude of the resultant force and the angle it makes with AB. [10N at 60^0 to AB]
11. ABCDEF is a regular hexagon. Forces of magnitude 2N, 5N, 3N, 4N, 3N and 1N act along the line AB, BC, CD,DE, EF and AF respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [6N at 60^0 to AB]
12. ABCDEF is a regular hexagon. Forces of magnitude 8N, 7N, 6N, 4N, 7N, and 6N act along the line AB, BC, CD, DE, EF and FA respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [12.5N at 76^0 to AB]
13. PQRSTU is a regular hexagon. Forces of magnitude 4N, 5N, 2N, and 6N act along the line PQ, PR, PT and PU respectively, in each case the direction of the force being given by the order of the letters. Given that PQ is horizontal, determine the magnitude and direction of the resultant force. [11.065N at 61.2^0 to PQ]
14. ABCD is a square. Forces of magnitude $10N$ N, 9N, 8N and $5N$ act along AB, BC, CD and AD respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force.
[$2\sqrt{5}$N at 63.43^0 to AB]
15. ABCD is a rectangle with AB= 4cm and BC = 3cm. Forces of magnitude 3N, 10N, 4N, 6N and 5N act along AB, BC, CD, DA, and AC respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine the magnitude and direction of the resultant force. [7.62N at 66.8^0 with AB]

15. CONNECTED PARTICLES

Connected particles

Simple connections

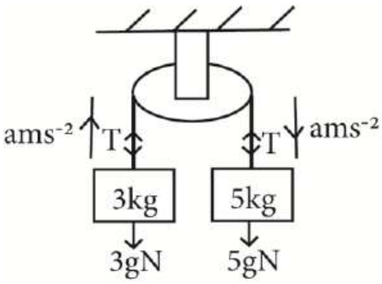
When two particles are connected by a light inextensible string passing over a smooth pulley and allowed to move freely, then as long as the string is tout, the following must be observed.

- acceleration of the particles is the same
- tension in the uninterrupted string is constant
- tensions in interrupted strings are different.

Example 1

Two particles of masses 5kg and 3kg are connected by a light inextensible string passing over a smooth fixed pulley. Find

(a) acceleration of the particle
(b) the tension in the string



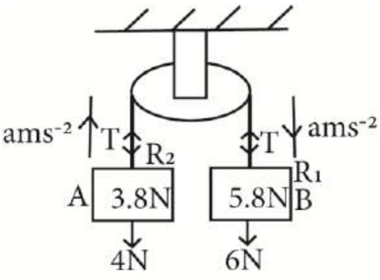
For 5kg mass: 5g – T = 5a .... (i)
For 3kg mass: T – 3g = 3a .... (ii)

(i) and (ii)
2g = 8a
a = $\frac{2 \times 9.8}{8}$ = 2.45ms^{-2}
(ii) tension in the string
T -3g = 3a
T = 3 x 2.45 + 3 x 9.8 = 36.78N
(iii) Force on the pulley
R = 2T = 3 x 36.78 = 73.56N

Example 2

An inextensible string attached to two scale A and B each of weight 20g passes over a smooth fixed pulley. Particles of weight 3.8N and 5.8N are placed in pans A and B respectively. If the system s released from rest (take g = 10ms^{-2}). Find the

(a) Tension in the string
(b) Reaction of the scale pan holding the 3.8N weight



Weight of the scale pan = $\frac{20}{1000} x\ 10 = 0.2N$

Total weight of A = 3.8 + 0.2 = 4N

Total weight of B = 5.8 + 0.2 = 6N

For 6N: 6 – T = 0.6a .... (i)

For 4N: T – 4 = 0.4a .... (i)

Adding (i) and (ii)

a = 2ms^{-2}

T = 4 + 0.4 x 2 = 4.8N

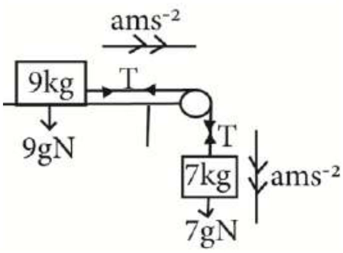
For scale pan A R_2 – 3.8 = 0.38a

R_2 = 3.8 + 2 x 0.38 x 2 = 4.56N

Example 3

A mass of 9kg resting on a smooth horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table to the pulley is a 7kg mass hanging freely 1.5m above the ground. Find

(a) common acceleration
(b) tension in the string
(c) force on the pulley when the system is allowed to move freely
(d) time taken for the 7kg mass to hit the ground



F = ma

For 7kg mass: 7g – T = 7a

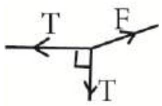
For 9kg mass: T = 9a

(i) + (ii): 7g = 16a

a = $\frac{7\ x\ 9.8}{16} = 4.29ms^{-2}$

(b) Tension: T = 9a = 9 x 4.29 = 38.61N

(c) The force on the pulley, F:



F =$\sqrt{T^2 + T^2} = T\sqrt{2}$ = 38.61$\sqrt{2}$ = 54.603N

(d) s = ut + $\frac{1}{2}at^2$

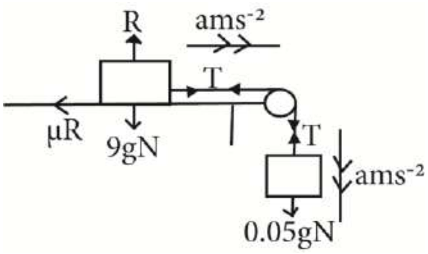
1.5 = 0 x t + $\frac{1}{2} x\ 4.29\ x\ t^2$

t = 0.84s

Example 4

A mass of 90g resting on a rough horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table attached to a 50g mass hanging freely. The coefficient of friction between the 90g mass and the table is $\frac{1}{3}$ and the system is released from rest, find

(a) common acceleration (b) the tension in the string



For 50g mass: 0.05g – T = 0.05a ….(i)

For 90g mass: T – μR = 0.09a

$T - \frac{1}{3} x\ 0.09\ x\ 9.8 = 0.09a$ …. (ii)

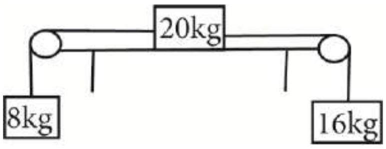
(i) + (ii): $0.05g - \frac{1}{3} x\ 0.09\ x\ 9.8 = 0.14a$

$a = \frac{0.02g}{0.14} = 1.4ms^{-2}$

(b) 0.05g – T = 0.05a

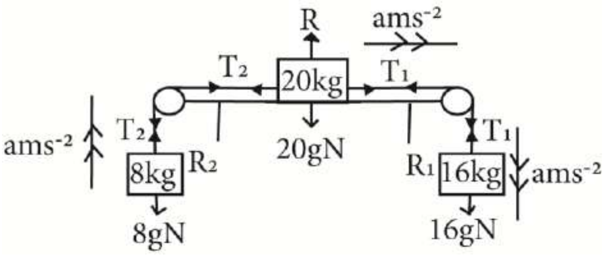
T = 0.05 x 9.8 – 0.05 x 1.4 = 0.42N

Example 5



The figure shows a block of mass 20kg resting on a smooth horizontal table. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) Acceleration of 16kg mass
(b) tension in the string
(c) reaction on each pulley



For 16kg mass: $16g - T_1 = 16a$ …… (i)

For 20kg mass: $T_1 - T_2 = 20a$ ……. (ii)

For 8kg mass: $T_2 - 8g = 8a$ ……… (iii)

(i) + (ii) + (iii): 8g = 44a

$a = \frac{8\ x\ 9.8}{44} = 1.782ms^{-2}$

(b) Tension in the string

$16g - T_1 = 16a$

$T_1 = 16 \times 9.8 - 16 \times 1.782 = 128.288N$

$T_2 - 8g = 8a$

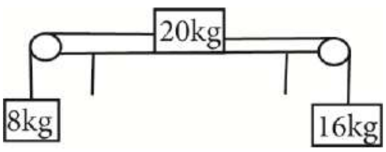
$T_2 = 8 \times 9.8 + 8 \times 1.782 = 92.656N$

(c) Reaction on each pulley

$R_1 = \sqrt{T_1^{\ 2} + T_1^{\ 2}} = T_1\sqrt{2} = \sqrt{2} \times 128.288 = 181.427N$

$R_2 = \sqrt{T_2^{\ 2} + T_2^{\ 2}} = T_2\sqrt{2} = \sqrt{2} \times 92.626 = 131\ N$

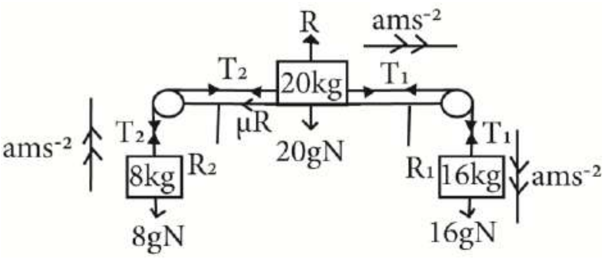
Example 6



The figure shows a block of mass 20kg resting on a rough horizontal table of coefficient of friction 0.21. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) acceleration of the 16kg mass
(b) Tension in each string
(c) reaction on each pulley

Solution



For 16kg mass: $16g - T_1 = 16a$ …… (i)

For 20kg mass: $T_1 - T_2 - 20g\mu = 20a$ ……. (ii)

For 8kg mass: $T_2 - 8g = 8a$ ……… (iii)

(i) + (ii) + (iii): $8g - 20g\mu = 44a$

a = $\frac{8 \times 9.8 - 20 \times 9.8 \times 0.21}{44} = 0.846ms^{-2}$

(b) Tension in the string

$16g - T_1 = 16a$

$T_1 = 16 \times 9.8 - 16 \times 0.846 = 143.264N$

$T_2 - 8g = 8a$

$T_2 = 8 \times 9.8 + 8 \times 0.846 = 85.168N$

(c) Reaction on each pulley

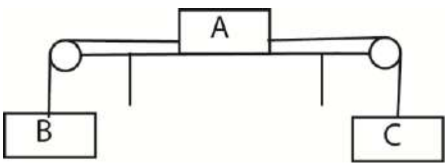
$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 2 \times 128.291 = 202.606N$

$R_2 = \sqrt{T_2^2 + T_2^2} = T_2\sqrt{2} = \sqrt{2} \times 85.168 = 120.446N$

Revision exercise 1

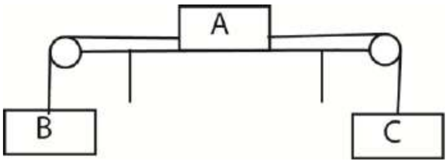
1. Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find
   (a) acceleration of the particles [3.92ms^{-2}]
   (b) the tension in the string [41.16N]
   (c) the force on the pulley [82.32N]
2. Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find
   (a) acceleration of the particles [4.9ms^{-2}]
   (b) the tension in the string [29.4N]
   (c) distance moved by the 6kg mass in the first 2s of motion [9.8m]
3. A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a rope which passes over a frictionless pulley so that they hang vertically downwards.
   (a) what is the tension in the section of the rope supporting the man [807.06N]
   (b) what is the acceleration of the bucket [1.73ms^{-2}]
4. Two particles of masses 200g and 300g are connected to a light inelastic string passing over a smooth pulley; when released freely find
   (i) common acceleration [1.96ms-2]
   (ii) the tension in the string [2.352N]
   (iii) the force on the pulley [4.704N]

5. The diagram below shows a particles of mass 8kg connected to a light scale pan by a light inextensible string which passes over a smooth fixed pulley. The scale pan holds two blocks A and B of mass 3kg and 4kg, with B resting on top of A. If the system is released from rest find
   (a) acceleration of the system [$0.653ms^{-2}$]
   (b) the reaction between A and B[41.813N]
6. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate
   (a) the common acceleration of the masses [$3.675ms^{-2}$]
   (b) the tension in the string [18.375N]
   (c) the force acting on the pulley[26N]
7. A mass of 3kg on a smooth horizontal table is attached by a light inextensible sting passing over a smooth pulley at the edge of the table, to another mass of 2kg hanging freely 2.1m above the ground; find
   (a) common acceleration[$3.92ms^{-2}$]
   (b) the tension in the string [11.76N]
   (c) The force on the pulley in the system if it's allowed to move freely. [16.63N]
   (d) the velocity with which the 2kg mass hits the ground [$4.06ms^{-1}$]
8. A mass of 5kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a 3kg mass hanging freely. the coefficient of friction between the 5kg mass and the table is 0.25 and the system is released from rest find
   (a) common acceleration[$2.144ms^{-2}$]
   (b) tension in the string [22.97N]
9. A mass of 11kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to 500g mass hanging freely. The coefficient of friction between the 1kg mass and the table is 0.1 and the system is released from rest find
   (a) common acceleration [$2.61ms^{-2}$]
   (b) the tension in the string [3.593N]
10. The objects of mass 3kg and 5kg are attached to ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3kg mass touching the floor and the 5kg mass at 4m above the floor and then release, what is
   (a) the acceleration of the system[$2.45ms^{-2}$]
   (b) tension in the chord[36.75N]
   (c) the time that will elapse before the 5kg object hits the floor [1.81s]
11.



The diagram shows a particle A of mass 2kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 5kg and C of mass 3kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the
   (a) common acceleration [$0.98ms^{-2}$]
   (b) the tension of each string [12.37N, 44.15N]

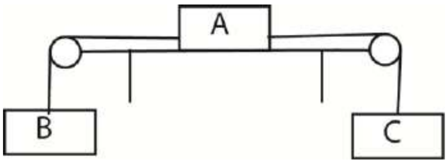
12.



The diagram shows a particle A of mass 3kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 4kg and C of mass 6kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

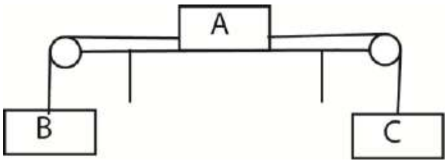
(a) common acceleration [$0.75ms^{-2}$]
(b) the tension of each string [54.277N, 31.662N]

13.



The diagram shows a particle A of mass 5kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 3kg and C of mass 2kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest body B descend with an acceleration of 0.28ms-2, find the coefficient of friction between the body A and the surface of the table. [0.143]

14.



The diagram shows a particle A of mass 10kg resting on a smooth horizontal table. It is attached to the particle B of mass 4kg and C of mass 7kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

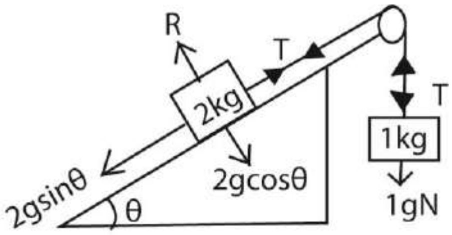
(c) common acceleration [$1.4ms^{-2}$]
(d) the tension of each string [44.8N, 58.8N]

Connected particles on inclined planes

Example 7

A mass of 2kg lies on a smooth plane of inclination 1 in3. One end of a light inextensible sting is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its end. If the system is released from rest , find the

(i) acceleration of the masses
(ii) tension in the string
(iii) distance each particle travels in the first 2s.



sinθ =$\frac{1}{3}$ F =ma

For 2kg mass: T – 2gsinθ = 2a….. (i)

For 1kg mass: 1g – T = 1a ………. (ii)

(ii) + (i): = 1g – 2gsinθ = 3a

a = $\frac{9.8-2\ x\ 9.8\ x\frac{1}{3}}{3}$ = 1.089ms^{-2}

(ii) Tension: 1g – T = 1a

T = 9.8 – 1.089 = 8.71N

(iii) s = ut + $\frac{1}{2}at^2$

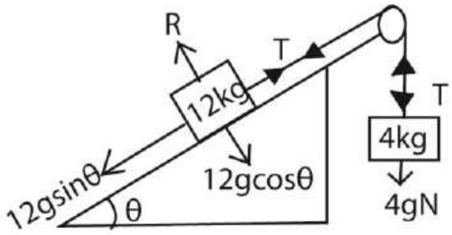
s = 0 x 2 + $\frac{1}{2}x\ 1.089\ x\ 2^2\ = 2.178m$

Example 8

A mass of 12kg lies over a smooth incline plane 6m long and 1m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane to freely suspended mass of 4kg at its other end. If the system is released from rest, find the

(a) acceleration of the system
(b) velocity with which the 4kg mass hits the ground
(c) time the 4kg mass takes to hit the ground.

Solution



sinθ =$\frac{1}{6}$ F =ma

For 12kg mass: T – 12gsinθ = 2a….. (i)

For 4kg mass: 4g – T = 4a ………. (ii)

(ii) + (i): = 4g – 12gsinθ = 16a

a = $\frac{4\ x 9.8-12\ x\ 9.8\ x\frac{1}{6}}{16}$ = 1.225ms^{-2}

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 1.225 = 34.3N

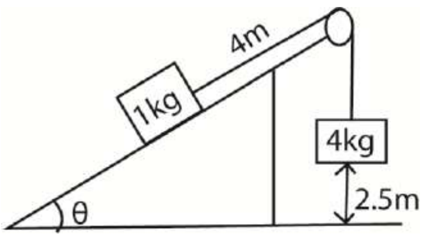
(iii) s = ut + $\frac{1}{2}at^2$

1 = 0 x t + $\frac{1}{2}x\ 1.225\ x\ t^2$

t = 1.28s

Example 9

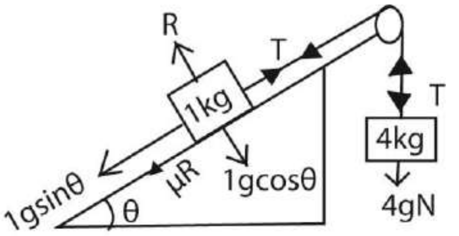
A mass of 1kg lies on a rough plane with coefficient of friction 0.25. One end of a light inextensible string is attached to 1kgmass and passes up the line of greatest slope over a smooth fixed pulley at the top of the plane and the other end of a string is tied to a mass of 4kg hanging freely.



The plane makes an angle θ with the horizontal where sinθ = $\frac{3}{5}$. When the system is released from rest, find:

(i) the acceleration of the system
(ii) tension in the string
(iii) velocity with which the 4kg mass hits the floor
(iv) velocity with which the 1kg mass hits the pulley

Solution



sinθ = $\frac{3}{5}$; cosθ = $\frac{4}{5}$ F =ma

For 12kg mass: T – 1gsinθ- 0.25R = 1a..... (i)

For 4kg mass: 4g – T = 4a .... (ii)

(ii) + (i): = 4g – 1gsinθ - 0.25R = 5a

a = $\frac{4\,x 9.8 - 1\,x\,9.8\,x\,\frac{3}{5} - 0.25\,x\,1\,x\,9.8\,\,x\,\frac{4}{5}}{5}$ =6.272ms^{-2}

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 6.272 = 14 .112N

(iii) $v^2 = u^2 + 2as$ but u = 0

v = $\sqrt{2\,x\,6.272\,x\,2.5}$ = 5.6ms^{-1}

(iv) When a 4kg mass hits the floor, the 1kg mass has still to move 4 – 2.5 = 1.5m before hitting the pulley It will experience a retarding force due to gravity and friction

F = 1a = 1gsinθ + 0.25R

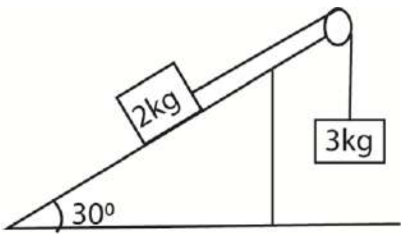
=$(1\,x\,9.8\,x\,\frac{3}{5} + 0.25\,x\,1\,x\,9.8\,x\,\frac{4}{5})$

a = - 7.84ms^{-2}

$v^2 = u^2 + 2as$

v = $\sqrt{5.6^2 - 2\,x\,7.84\,x\,1.5}$ =2.8ms^{-1}

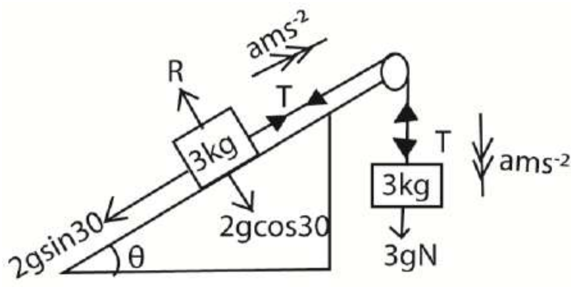
Example 10



A particle of mass 2kg on a rough plane inclined at 300 to the horizontal is attached by means of light inextensible string passing over a smooth pulley at the top edge of the plane to a particle of mass 3kg which hangs freely. If the system is released from rest with above parts of the strings taut, the 3kg mass travels a distance of 0.75m before attains a speed of 2.25ms^{-1}. Calculate

(a) acceleration
(b) coefficient of friction between the plane and 2kgmass
(c) reaction of the pulley on the string

Solution



(i) $v^2 = u^2 + 2as$

$a = \frac{2.25^2 - 0^2}{2 \times 0.75} = 3.375ms^{-2}$

(ii) F = ma

For 2kg mass: $T - 2g\sin\theta - \mu R = 2x\ a$ ..... (i)

For 4kg mass: $3g - T = 3\ a$ .... (ii)

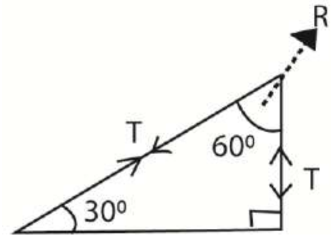
(ii) + (i): $3g - 2g\sin\theta - \mu(2g\cos\theta) = 5a$

$$\mu = \frac{(3 \times 9.8) - (2 \times 9.8\sin 30 + 5 \times 3.375)}{2 \times 9.8 \times \cos 30} = 0.161$$

(iii)

Tension: $3g - T = 3a$

$T = 3 \times 9.8 - 3 \times 3.375 = 19.275N$



Using parallelogram law of force

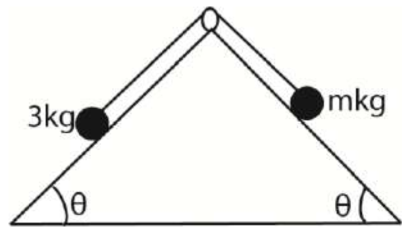
$R^2 = T^2 + T^2 + 2 \times T \cos 60 = 3T^2$

$R = 19.275\sqrt{3} = 33.4N$

Double inclined plane

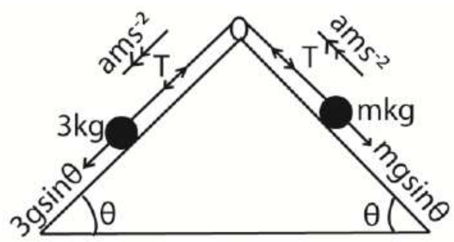
Example 11

The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where sinθ = 0.6. Two particles of mass 3kg and mkg, where m< 3kg are connected by a light inextensible string passing over a smooth fixed pulley.



The particles are released from rest with a string taut. After travelling a distance of 1.08m, the speed of the particle is 1.8ms-1. Calculate

(i) acceleration
(ii) tension in the string
(iii) value of m



(i) $v^2 = u^2 + 2as$

$1.8^2 = 0^2 + 2 \times a \times 1.08$

$a = 1.5ms^{-2}$

(ii) F = ma

For 3kg mass: $3g\sin\theta - T = 3a$

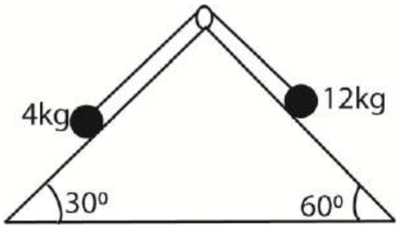
$T = 3 \times 9.8 \times 0.6 - 3 \times 1.5 = 13.14N$

(iii) For mkg mass; $T - mg\sin\theta = ma$

$13.14 = m(9.8 + 1.5)$; $m = 1.78kg$

Example 12

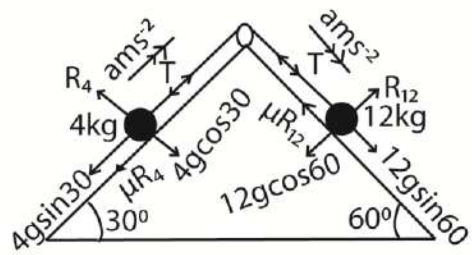
Two rough planes inclined at 300 and 600 to the horizontal and of the same height are placed back to back. Masses of 4kg and 12 kg are placed on the faces and connected by a light string passing over smooth pulley on the top of the planes.



If the coefficient of friction is 0.5 on both faces, find

(a) acceleration
(b) Tension in the strings

Solution



For 4kg mass: T -4gsin30 – 0.5 x 4gcos30 = 4a ……………. (i)

For 12kg mass: 12gsin60 –T – 0.5 x 12gcos 60 = 12a ….. (ii)

(i) + (ii)

12gsin60 – 4gsin30 - 0.5(4gcos30 + 12gcos 60) = 16a

a = 2.25ms^{-2}

(b) For 4kgmass

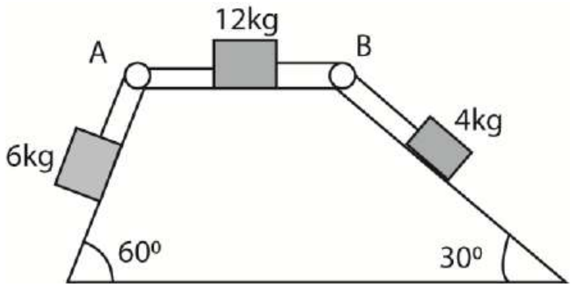
T -4gsin30 – 0.5 x 4gcos30 = 4a

T = 4gsin30 + 0.5 x 4gcos30 + 4x2.25

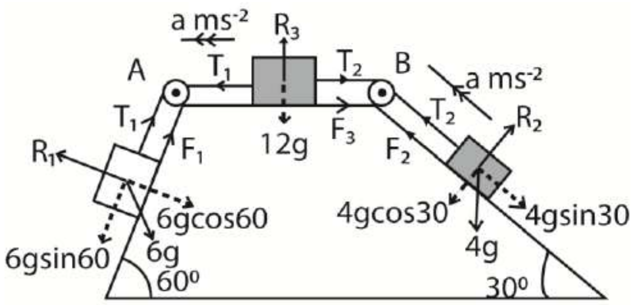
T = 45.54

Example 13

1. The diagram below shows a 12kg mass on a horizontal rough plane. The 6kg and 4kg masses are on rough planes inclined at angles 600 and 300 respectively. The masses are connected to each other by light inextensible strings over light smooth pulleys A and B.



The planes are equally rough with coefficient of friction $\frac{1}{12}$. If the system is released from rest find the;
(a) Acceleration of the system (08marks)



For 6kg mass

6gsin60-($T_1 + \frac{1}{12}\ x\ 6gcos60) = 6a$

6gsin60 – T_1 - $\frac{1}{2}cos60 = 6a$ .... (i)

For 4kg mass

T_2 – ($\frac{1}{12}\ x\ 4gcos30 + 4gsin30) = 4a$

T_2 - $\frac{1}{3}gcos30 - 4gsin30$ = 4a .... (ii)

For 12kg mass

T_1 – (T_2 +$\frac{1}{12}\ R_3$) = 12a

T_1 – (T_2 +$\frac{1}{12}\ x\ 12g$) = 12a

T_1 –T_2 - g = 12a.. (iii)

Eqn. (i) + Eqn. (ii) + Eqn. (iii)

6gsin60 - $\frac{1}{2}gcos60 - \frac{1}{3}gcos30 - 4gsin\ 30 - g = 22a$

16.24327742 = 22a

a = $\frac{16.24327742}{22} = 0.73833ms^{-2}$

(b) Tensions in the strings. (04marks)

From equation (i)

$T_1 = 6gsin60 - \frac{1}{2}cos60 - 6a$

$= 6gsin60 - \frac{1}{2}cos60 - 6\ x\ 0.73833$

= 44.0423N

From eqn. (ii)

T_2 = $\frac{1}{3}gcos30 + 4gsin30 + 4a$

= $\frac{1}{3}gcos30 + 4gsin30 + 4\ x\ 0.73833$

= 25.3823N

Revision exercise 2

1. A mass of 2kg lies on a smooth inclined plane 9m long and 3m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope

over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its other end. If the system is released from rest, find

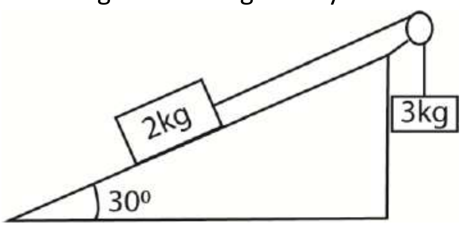
(i) acceleration of the system[1.089ms^{-2}]

(ii) tension in the string. [8.711N]

(iii) velocity with which the 1kg mass will hit the ground[2.556ms^{-1}]

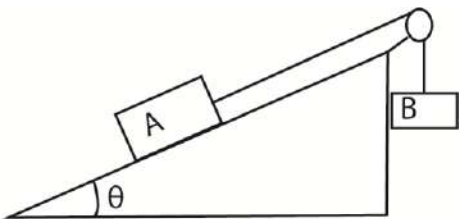
(iv) time the 1kg mass will hit the ground[2.347s]

2. A mass of 15kg lies on a smooth plane of inclination in 49. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 10kg at its other end. If the system is released from rest, find the acceleration of the masses and the distance each travel in the first 2s. [3.8ms^{-2}, 7.6m]
3. A mass of 2kg lies on a rough plane which is inclined at 300 to the horizontal. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 5kg at its other end. The system is released from rest as the 2kg mass accelerates up the slope, it experiences a constant resistance to motion of 14N down the slope due to friction. Find the tension of the string. [31N]
4. A mass of 10kg lies on a smooth plane which is inclined at θ to the horizontal. The mass is 5m from the top, measured along the plane. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 15kg at its other end. The 15kg mass is 4m above the floor. The system is released from rest and the string first goes slack $1\frac{3}{7}s$ latter. Find the value of θ. [30^0]
5. One of two identical masses lies on a smooth plane, which is inclines at $\sin^{-1}\left(\frac{1}{4}\right)$ to the horizontal and is 2m from the top. A light inextensible string attached to this mass passes along the line of greatest slope over a smooth pulley fixed at the top of the incline, the other end carries the other mass hanging freely 1m above the floor. If the system is released from rest, find the time taken for the hanging mass to reach the floor. [0.663s]
6. A particle of mass 2kg on a smooth plane inclined at 300 to the horizontal is attached by means of a light inextensible string passing over a smooth pulley at the edge of the plane to a particle of mass 4kg which hangs freely.



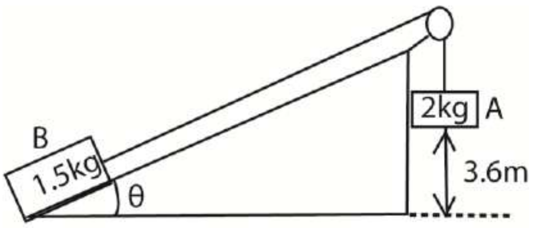
If the system is released from rest with above parts of the string taut, find the speed acquired by the particles when both have moved a distance of 1m [2.8ms^{-1}}

7. A body A of mass 13kg lying on a rough inclined plane, coefficient of friction, μ. From A, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley to a body B of mass mkg hanging freely, the plane makes an angle θ with the horizontal where sinθ = $\frac{5}{13}$.



When m = 1kgand the system is released from rest, B has upward acceleration of a ms^{-2}. When m = 11kg and the system released from rest, B has downward acceleration of ams^{-2}. Find a and μ. [1.96ms^{-2}, 0.1]

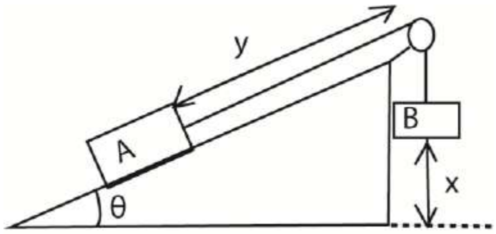
8. A particle A of mass 2kg and B of mass 1.5kg are connected by light inextensible string passing over a smooth pulley. The system is released from rest with A at height of 3.6m above the horizontal ground and B at the foot of a smooth slope inclined at an angle θ to the horizontal where $\sin\theta = \frac{1}{6}$. Take g = 10ms^{-2}.



Calculate

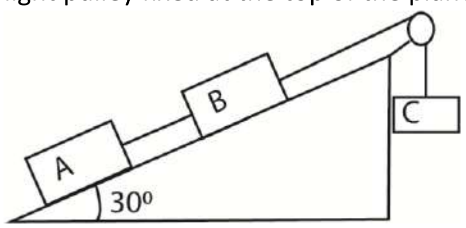
(i) the magnitude of the acceleration of particles [6ms^{-1}]
(ii) the speed with which A reaches the ground[5ms^{-2}]
(iii) the distance B moves up the slope before coming to instantaneous rest. [14.4m]

9. A mass A of 4kg and a mass B of 3kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest and mass accelerates up along a smooth slope inclined at an angle θ to horizontal where θ = 30^0.



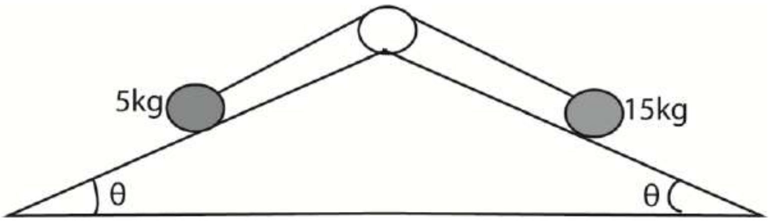
If y = 3m and x = 2.8m, calculate the velocity with which A hits the pulley [2.42ms^{-1}]

10. The diagram below shows particles A, B and C of masses 10kg, 8kg and 2kg respectively connected by a light inextensible strings. The string connecting B and C passes over a smooth light pulley fixed at the top of the plane.



If the coefficient of friction between the plane and particles A and B are 0.22 and 0.25 respectively, calculate
(i) acceleration of the system [1.6477ms^{-2}]
(ii) tension in the strings[22.89N, 13.851N]

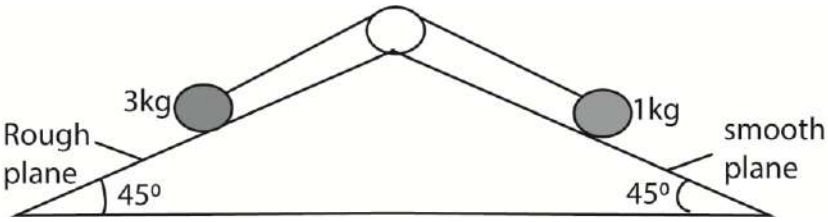
11. The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where $\sin\theta = \frac{3}{5}$. Two particles of mass 5kg and 15kg are connected by a light inextensible string over a smooth fixed pulley.



The particles are releases from rest with a string taut calculate

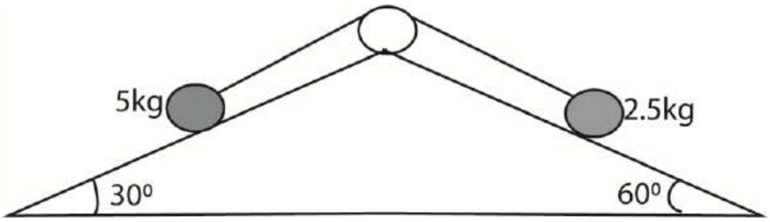
(i) Acceleration of the particles
(ii) Tension in the string

12. The diagram below shows two smooth plane and a rough plane both inclined at 450 to the horizontal. Two particles of mass of mass 1kg and 3kg are connected by light inextensible string passing over a smooth fixed pulley.



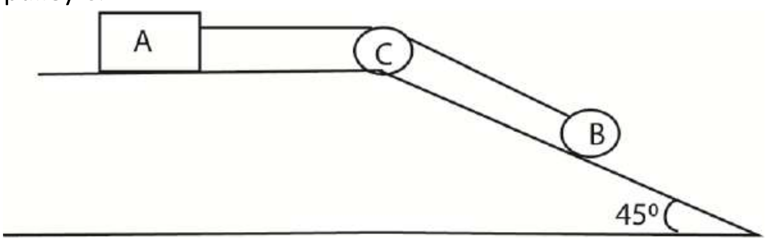
The particle are released from rest with a string taut. Calculate
(i) acceleration of the particle [$1.4ms^{-2}$]
(ii) tension in the string [.48N
(iii) coefficient of friction [0.4]

13. Two equally rough planes inclined at 30^0 and 60^0 to the horizontal and of the same height are placed back to back. Masses of 5kg and 2.5kg are placed on the faces and connected by a light string passing over a smooth pulley at the top of the planes.



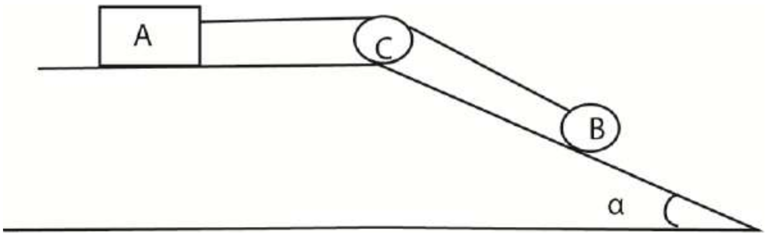
If the string is taut and 5kg is just about to slip downwards find the
(i) coefficient of friction[0.06]
(ii) tension in the string [21.9538N]

14. In the diagram, particle A and particle B are masses of 10kg and 8kg respectively and rest on planes as shown below. They are connected by a light inextensible string passing over a smooth pulley C.



Find the acceleration in the system and the tension in the string if
(i) the particles are in contact with smooth planes [$3.08ms^{-2}$, 30.N]
(ii) the particles are in contact with rough planes with coefficient of friction 0.25. [$0.95ms^{-2}$, 33.98N]

15. In the diagram particles A and B are of masses mkg and 5mkg respectively and rest on the planes as shown below. They are connected by a light inextensible string passing over a smooth fixed pulley at C

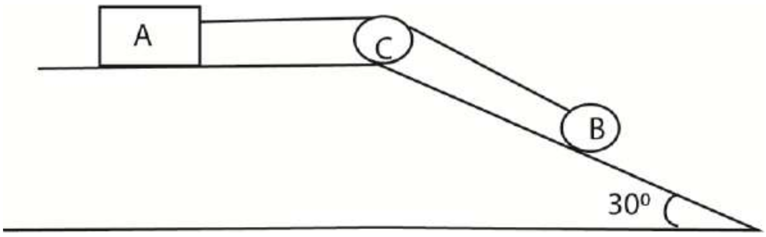


Find the acceleration of the system and the tension in the string if sinα = $\frac{4}{5}$ when;

(i) the particles are in contact with smooth plane[6.533ms-2, 6.533N]

(ii) the particles are in contact with rough plane of coefficient of friction $\frac{1}{3}$. [4.356ms^{-2}, 7.622N]

16. In the diagram particles A and B of masses 2.4kg and 3.6kg respectively. A rests on a rough horizontal plane (coefficient of friction 0.5), it is connected by a light inextensible string passing over a smooth pulley C to particle B resting on smooth plane inclined at 30^0 to the horizontal.



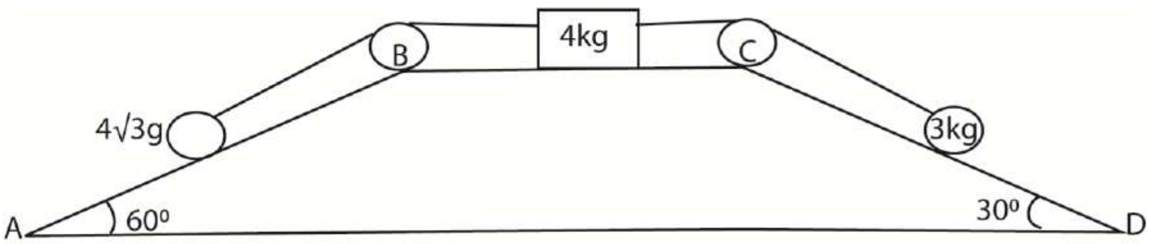
When the system is released from rest find

(i) acceleration of the system and tension in the string [0.98ms^{-2}, 14.112N]

(ii) the force on the pulley C [7.3049N]

(iii) the velocity of A mass after 2 seconds[1.96ms^{-2}]

17. The diagram below shows a 4kg mass on a horizontal rough plane with coefficient of friction 0.25. The $4\sqrt{3}kg$ mass rests on a smooth plane inclined at angle 600 to the horizontal while the 3kg mass rests on a rough plane inclined at an angle 300 to the horizontal and coefficient of friction $\frac{1}{\sqrt{3}}$. the masses are connected to each other by a light inextensible strings over light smooth fixed pulleys B and C.



Find the

(i) acceleration of the system[1.407ms-2]

(ii) tension in the string [49.051N, 33.622N]

(iii) work done against frictional force when the particles each moved 0.5m [12.25J]

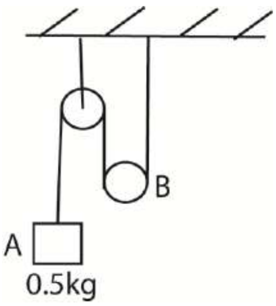
Multiple connections

- Acceleration of a particle moving between to portions of the string is equal to half the net acceleration of the particle (s) attached to the end of the string
- The tension in uninterrupted string is constant
- The tensions in interrupted strings are different.

Case I: A pulley moving between two portions of a string

Example 15

The diagram below shows particle A of mass 0.5kg attached to one end of alight inextensible string passing over a fixed pulley and under a movable light pulley B. The other end of the string is fixed as shown

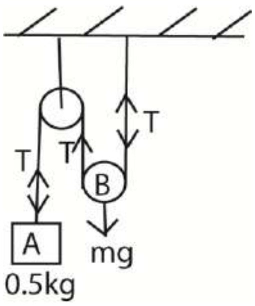


(i) What mass should be attached at B for the system to be in equilibrium
(ii) If B is 0.8kg what are the accelerations of particles A and pulley B?
(iii) find the tension in the string in (ii)

Solution

(i) Let T = tension in string

m = mass at B



For the system to be in equilibrium upward forces are equal to downward force. by resolving vertically

For mass A: T= 0.5g .... (i)

For pulley B: 2T = mg

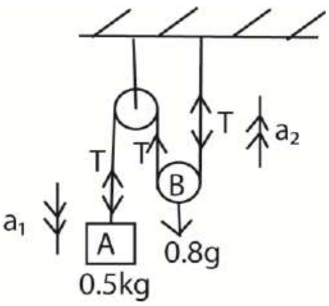
$T = \frac{mg}{2}$ .... (i)

equating (i) to (ii)

$\frac{mg}{2} = 0.5g$; m = 1kg

(ii) Let a_1 = acceleration of A

a_2 = acceleration of B



For mass A: $0.5g - T = 0.5a_1$......... (i)

For pulley B: $2T - 0.8g = 0.8a_2$ but $a_2 = \frac{1}{2} a_1$

⇨ $2T - 0.8g = 0.4a_1$
$T - 0.4g = 0.2a_1$ .... (i)

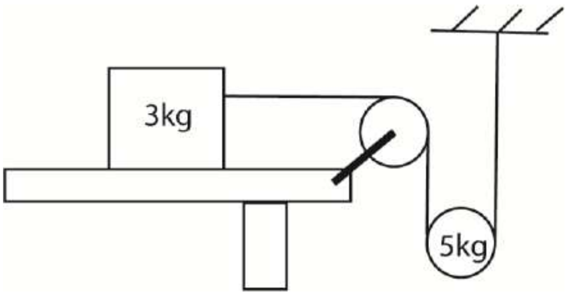
(i) + (ii) $a1 = \frac{9.9}{7} = 1.4ms^{-2}$ and $a2 = 0.7\ ms^{-2}$

From eqn. (i)

T = 0.5 x 9.8 - 0.5 x 1.4 = 4.2N

Example 16

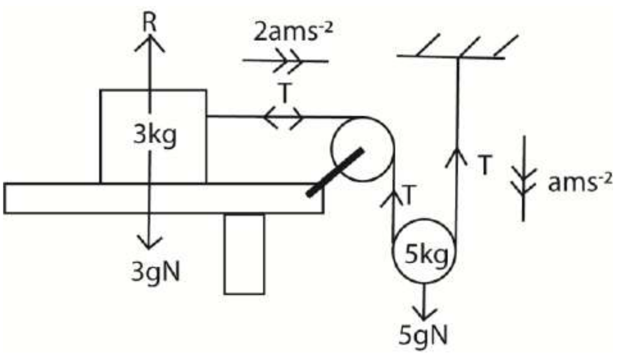
A particle of mass 3kg on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass 5kg, its other end being fixed so that the string beyond the table are vertical.



Find

(i) acceleration of 3kg and 5g
(ii) Tension in the string

Solution



F = ma

For 3kg: T = 3 x 2a ……… (i)

For 5kg: 5g – 2T = 5a …… (ii)

(ii) + 2 x (i)

5x 9.8 = 17a

a = $\frac{49}{17}$ = 2.8824ms^{-2}

Acceleration of 5kg: = 2.8824ms^{-2}

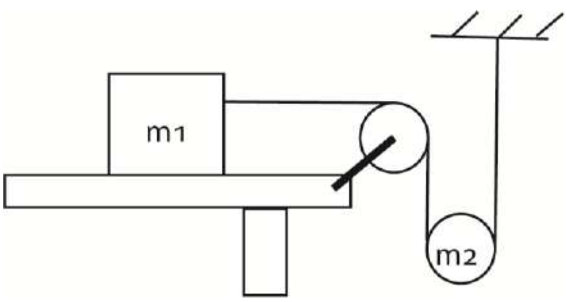
Acceleration of 3kg: = 2.8824 x 2ms^{-2}

= 5.7648ms^{-2}

T = 6a = 2.8824 x 6 = 17.2944N

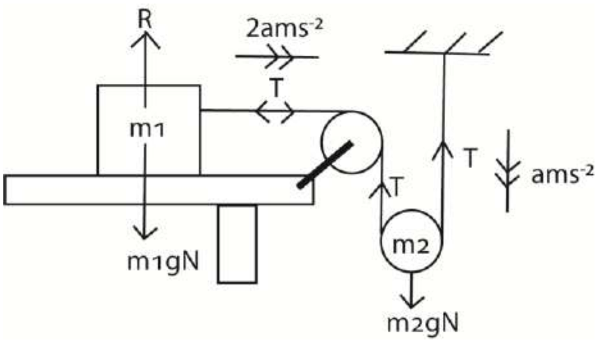
Example 17

A particle of mass m_1 on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass m_2, its other end being fixed so that the parts of the string beyond the table is vertical.



Show that m_2 descends with acceleration $\frac{m_2 g}{4m_1+m_2}$

Solution



F = ma

For m_1 kg mass: $T = m_1 \times 2a$ …… (i)

For m_2 kg mass: $m_2g - 2T = m_2a$ … (ii)

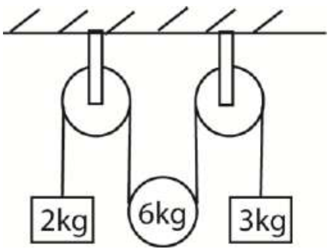
(ii) + 2 x (i)

$m_2g = 4m_1a + m_2a$

$a = \frac{m_2 g}{4m_1 + m_2}$

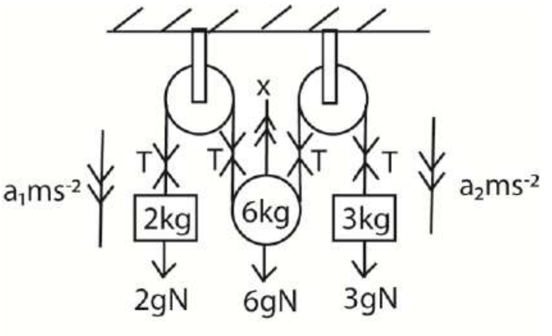
Example 18

A string has a load of mass 2kg attached at one end. The string passes over a smooth fixed pulley then under a movable pulley of mass 6kg and over another fixed pulley and has a load of mass 3kg attached to its end.



Find the acceleration of the movable pulley and the tension in the string

Solution



$$x = \frac{a_1 + a_2}{2}$$

For 2kg mass: $2g - T = 2a_1$ …………………….. (i)

For 3kg mass: $3g - T = 3a_2$ …………………….. (ii)

For 6kg mass: $2T - 6g = 6 x \frac{1}{2}(a_1 + a_2)$…. (iii)

(ii) – (i): $g = (3a_2 - 2a_1)$ …………………………… (iv)

2 x (ii) + (iii): $0 = 9a_2 + 3a_1$ …………………….. (v)

3 x (iv)- (v): $3g = -9a_1$

$a1 = \frac{-g}{3} = -3.267ms^{-2}$

From (v): $0 = 9a_2 + 3a_1$

$0 = 9a_2 + 3(-3.267)$

$a_2 = 1.089ms^{-2}$

Acceleration of pulley $= \frac{1}{2}(a_1 + a_2)$

$= \frac{1}{2}(-3.267 + 1.089)$

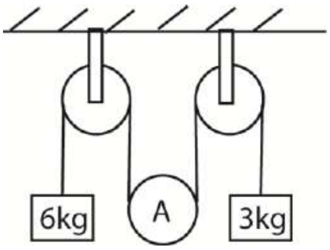
$= -1.089ms^{-2}$

Tension: $T = 2g - 2a_1$

$T = 2 \times 9.8 - 2 \times -3.267 = 26.134N$

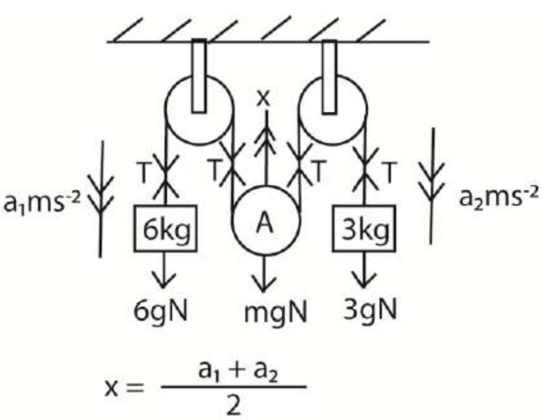
Example 19

In the pulley system below, A is a heavy pulley which is free to move



Find the mass of pulley A if it does not move upwards of downwards when the system is released from rest.

Solution



For 2kg mass: $6g - T = 6a_1$ ……………………. (i)

For 3kg mass: $3g - T = 3a_2$ ……………………. (ii)

For mkg mass: $2T - mg = 0$……………………. (iii)

$a_1 = -a_2$

$6g - T = -6a_2$ ………. (iv)

$3g - T = 3a_2$ ………… (v)

(iv) – (v)

$3g = 9a_2$

$a_2 = \frac{-g}{3} = -3.267ms^{-2}$

$3g - T = 3a2$

$T = 3 \times 9.8 - 3(-3.267) = 39.201$

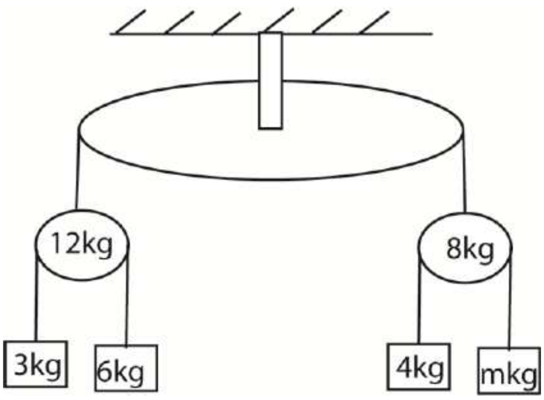
$2T - mg = 0$

$m = \frac{2 \times 39.201}{9.8} = 8kg$

Case 2: A pulley moving on one portion of a string

Example 20

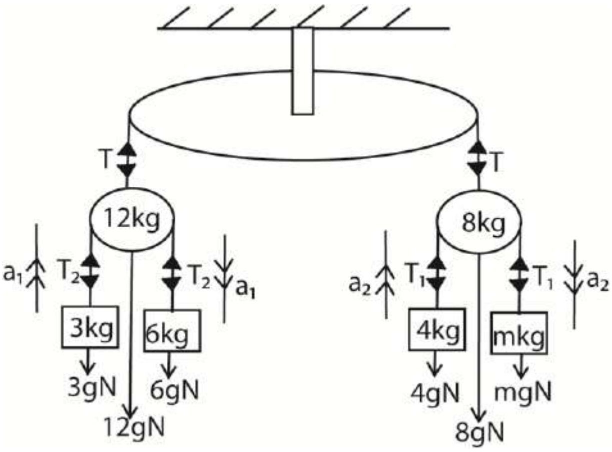
The diagram below shows two pulley to pulleys of masses 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.



The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the

(i) tensions in the string
(ii) value of m

Solution.



For 3kg mass: $T_2 - 3g = 3a_2$ .... (i)

For 6kg mass: $6g - T_2 = 6a_2$ .... (ii)

For 4kg mass: $T_1 - 4g = 4a_1$ .... (iii)

For mkg mass: $mg - T_1 = ma_1$ ....(iv)

For 8kg mass: $2T_1 + 8g = T$ .... (v)

For 12kg mass: $2T_2 + 12g = T$ .... (vi)

eqn. (i) + eqn. (ii): $3g = 3a_2$

$a_2 = \frac{3 \times 9.8}{9} = 3.2667\text{ms}^{-2}$

eqn. (i): $T_2 - 3g = 3a_2$

$T_2 = 3 \times 9.8 + 3 \times 3.2667 = 39.2001\text{N}$

eqn. (vi): $2T_2 + 12g = T$

$T = 2 \times 39.2001 + 12 \times 9.8 = 196.0002\text{N}$

eqn. (v): $2T_1 + 8g = T$

$T_1 = \frac{196.0002 - 8 \times 9.8}{2} = 58.8001\text{N}$

eqn. (iii): $T_1 - 4g = 4a_1$

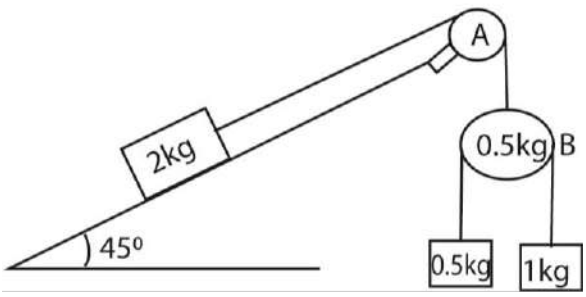
$a_1 = \frac{58.8001 - 4 \times 9.8}{4} = 4.9\text{ms}^{-2}$

eqn. $mg - T_1 = ma_1$

$m = \frac{58.8001}{9.8 - 4.9} = 12\text{kg}$

Example 21

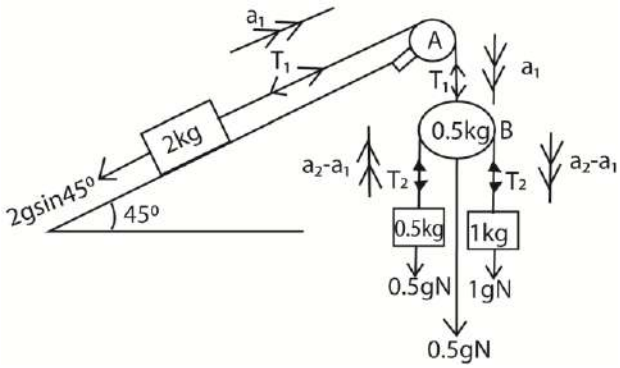
The diagram shows a particle of mass 2kg on a smooth plane inclined at 450 to the horizontal and attached by means of a light inextensible string over a smooth pulley, A at the top of the plane to pulley B of mass 0.5kg which hangs freely. Pulley B carries to particles of mass 0.5kg and 1kg on either side



Find

(a) acceleration of 2kg, 0.5kg and 1kg mass
(b) the tension in the strings

Solution



For 2kg mass: T_1 - 2gsin45 = $2a_1$ ………. (i)

For 0.5kg mass: $T_2 - 0.5g = (a_2 - a_1)$ …… (ii)

For 1kg mass: $gN - T_2 = 1(a_1 + a_2)$ ……… (iii)

For pulley B: $2T_2 + 0.5g - T_1 = 0.5a_1$ ……. (iv)

eqn. (ii) + eqn (iii): 0.5g = 1.5a2 + 0.5a1

$9.8 = 3a_2 + a_1$ ………. (v)

eqn. (i) + eqn. (iv): $2T_2$ – 2gsin45 + 0.5g =$2.5a_1$

$2T_2$ -8.9593 = $2.5a_1$ ……….. (vi)

2 x eqn. (iii) + eqn. (vi): 10.6407 = $4.5a_1 + 2a_2$

5.3204 = $2.5a_1$ + a2……………. (vii)

eqn. (vii) – eqn. (v): $5.75a_1 = 6.1612$

$a_1 = \frac{6.1612}{5.75} = 1.0715ms^{-2}$

from eqn. (v): $9.8 = 3a_2 + a_1$

$a_2 = \frac{9.8-1.0715}{3} = 2.9095ms^{-2}$

Acceleration of 2kg mass = $1.0715ms^{-2}$

Acceleration of 0.5kg mass = $2.9095ms^{-2}$

Acceleration of 1kg = 2.9095 + 1.0715

= $3.981ms^{-2}$

From eqn. (i): T_1 - 2gsin45 = $2a_1$

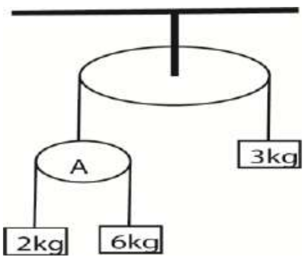
T1 = 2 x 1.0715 + 2 x 9.8sin45 =16.0023N

from eqn. (iv): $2T_2 + 0.5g - T_1 = 0.5a_1$

$T_2 = \frac{0.5 \times 1.0715 + 16.0023 - 4.9}{2}$ =5.8190N

Example 22

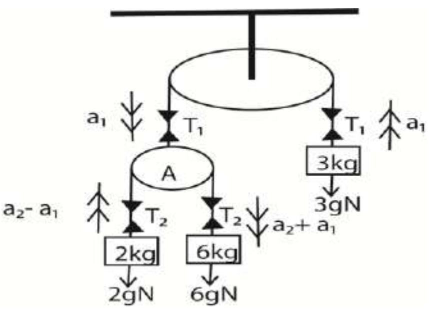
The diagram shows a fixed pulley carrying a string which has a mass of 3kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 6kg at one and a mass of 2kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 2kg, 6kg and 3kg masses
(c) tension in the string

solution



For 3kg mass: $T_1 - 3g = 3a_1$ ……………… (i)

For 6kg mass: $6g - T_2 = 6(a_2 + a_1)$ ……… (ii)

For 2kg mass: $T_2 - 2g = 2(a_2 - a_1)$ ……… (iii)

For pulley A: $2T_2 - T_1 = 0 \times a_1$ ………….. (iv)

eqn. (ii) and eqn. (iii): $4g = 8a_2 + 4a_1$ ….. (v) eqn. (i) + eqn. (iv): $2T_2 - 3g = 3a_1$...... (vi)

2 x eqn. (iii) - eqn. (vi)

$-g = 4a_2 - 7a_1$ .... (vii)

2eqn. (vii) – eqn. (v)

$-18a_1 = -6g$

$a_1 = \frac{6 \times 9.8}{18} = 3.27ms^{-2}$

$4g = 8a_2 + 4a_1$

$a_2 = \frac{9.8 - 3.27}{2} = 3.27ms^{-2}$

Acceleration of pulley A $= 3.27ms^{-2}$

Acceleration of 2kg $= 3.27ms^{-2} - 3.27ms^{-2} = 0$

Acceleration of 6kg $= 3.27ms^{-2} + 3.27ms^{-2}$

$= 6.54ms^{-2}$

Acceleration of 3kg $= 3.27ms^{-2}$

$T_1 - 3g = 3a_1$

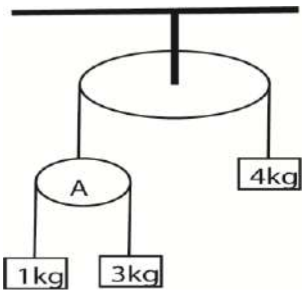
$T_1 = 3 \times 3.27 + 3 \times 9.8 = 39.21N$

$2T_2 - T_1 = 0 \times a_1$

$T_2 = \frac{39.21}{2} = 19.61N$

Example 23

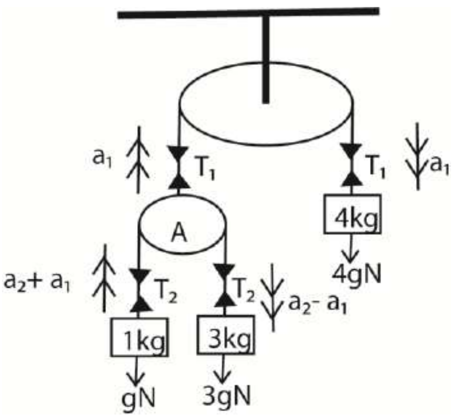
The diagram shows a fixed pulley carrying a string which has mass of 4kg attached at one end and a light pulley A at the other end. Another string passes over pulley A and carries a mass of 3kg at one end and a mass of 1kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 1kg, 3kg and 4kg masses
(c) tension in the string

Solution



For 4kg mass: $4g - T_1 = 4a_1$ .... (i)

For 3kg mass: $3g - T_2 = 3(a_2 - a_1)$ .... (ii)

For 1kg mass: $T_2 - g = (a_2 + a_1)$ .... (iii)

For pulley A: $T_1 - 2T_2 = 0 \times a_1$ .... (iv)

eqn. (ii) and eqn. (iii): $g = 2a_2 - a_1$ ..... (v)

eqn. (i) + eqn. (iv): $4g - 2T_2 = 4a1$ .... (vi)

2 x eqn.(iii) + eqn. (v): $2g = 2a_2 + 6a_1$ ..... (vii)

eqn. (vii) – eqn. (i): $7a_1 = g$

$a_1 = \frac{9.8}{7} = 1.4ms^{-2}$

$g = 2a_2 - a_1$

$a_2 = \frac{9.8 + 1.4}{2} = 5.6ms^{-2}$

Acceleration of pulley A $= 1.4ms^{-2}$

Acceleration of1kg mass $= 5.6 + 1.4 = 7ms^{-2}$

Acceleration of 3kg mass $= 5.6 - 1.4 = 4.2ms^{-2}$

Acceleration of 4kg mass $= 1.4ms^{-2}$

$4g - T_1 = 4a_1$

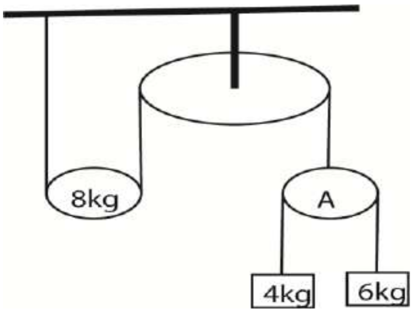
$T_1 = 4 \times 9.8 - 4 \times 1.4 = 33.6N$

$T_1 - 2T_2 = 0 \times a_1$

$T_2 = \frac{33.6}{2} = 16.8N$

Example 24

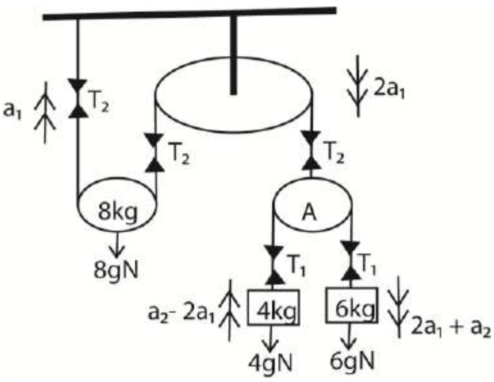
The diagram below shows a fixed pulley carrying a movable pulley of mass 8kg at one end and a light pulley A attached at the other end. A string passes over pulley A and carries a mass of 4kg at one end and a mass of 6kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 8kg, 6kg and 4kg masses
(c) tension in the string

Solution



For 8kg mass: $2T_2 - 8g = 8a_1$ .... (i)

For 4kg mass: $T_1 - 4g = 4(a_2 - 2a_1)$ .... (ii)

For 6kg mass: $6g - T_1 = 6(2a_1 + a_2)$ .... (iii)

For pulley A: $2T_1 - T_2 = 0 \times a_1$ .... (iv)

eqn. (ii) and eqn. (iii): $2g = 10a_2 + 4a_1$

$4.9 = 2.5a_2 + a_1$ ..... (v)

eqn. (i) + 2 x eqn. (iv): $4T_1 - 8g = 8a_1$.... (vi)

4 x eqn. (iii) + eqn. (vi): $16g = 56a_1 + 24a_2$

$2g = 7a_1 + 3a2$ .... (vii)

7 x eqn. (v) – eqn. (vii): $14.5a_2 = 14.7$

$a_2 = \frac{14.7}{14.5} = 1.0138ms^{-2}$

eqn. (v); $4.9 = 2.5a_2 + a_1$

$a_1 = 4.9 - 2.5 \times 1.0138 = 2.3655ms^{-2}$

Acceleration of pulley = $2a_1 = 2 \times 2.3655$

$= 4.731ms^{-2}$

Acceleration of 6kg = 4.731 + 1.0138

$= 5.7448ms^{-2}$

Acceleration of 3kg = $a_2 - 2a_1$

$1.0138 - 4.731 = - 3.7172ms^{-2}$

From eqn. (i): $2T_2 - 8g = 8a_1$

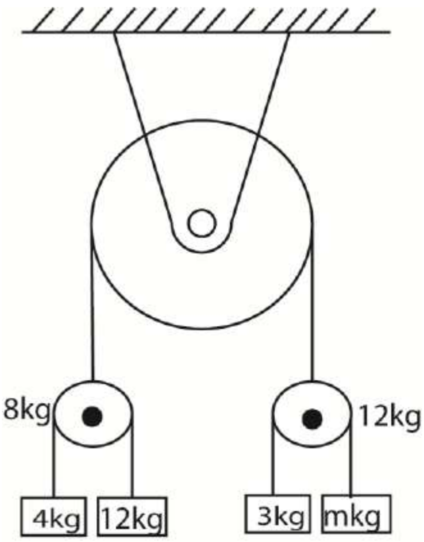
$T_2 = \frac{8 \times 4.731 + 8 \times 9.8}{2} = 58.124N$

From eqn. (iv)

$T_1 = \frac{T_2}{2} = \frac{58.124}{2} = 29.062N$

Example 25

The diagram below shows two pulleys of mass 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.



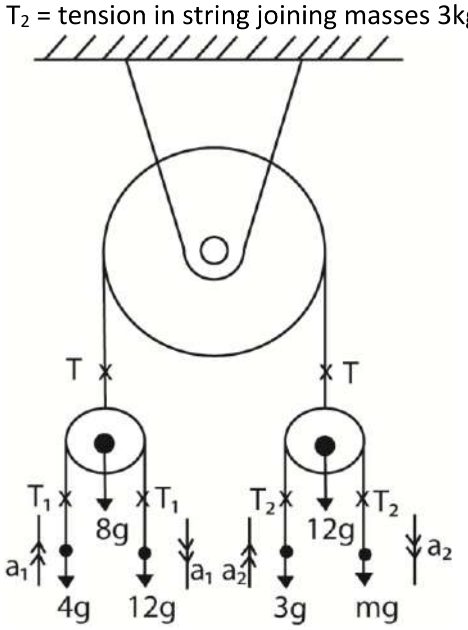
The acceleration of 4kg and 12kg masses are $\frac{g}{2}$ upward and $\frac{g}{2}$ downward respectively. The acceleration of the 3kg and m masses are $\frac{g}{3}$ upwards and $\frac{g}{3}$ downwards respectively. The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the

(a) tensions in the strings (09marks)

Let T = tension n the string joining masses 8kg and 12kg

T_1 = tension in the string joining masses 4kg and 12kg

T_2 = tension in string joining masses 3kg and mkg



Since the string of the fixed pulley remains stationary, this means the pulleys of the 8kg and 12kg are stationary or fixed

Resolving vertically

2T = 8g + 12g

T = 10g = 10 x 9.8 = 98N

For 4kg mass; resultant force = T_1- 4g

$4a_1 = T_1 - 4g$

$T_1 = 4g + 4a_1 = 4g + 4 \times \frac{g}{2} = 6g$

= 6 x 9.8 = 58.8N

For 3kg mass; resultant force = T_2- 3g

⇨ $3a_2 = T_2 - 3g$

$T_2 = 3g + 3a_2 = 3g + 3 \times \frac{g}{3} = 4g$

= 4 x 9.8 = 39.2N

Hence the tensions in the strings are 98N, 58.8N and 39.2N

(b) value of m. (03marks)

For the m kg mass

Resultant force = mg $-T_2$

$ma_2 = mg - T_2$

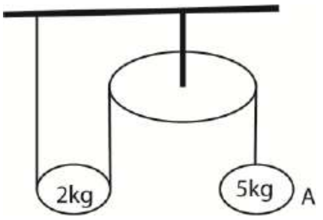
$m(\frac{g}{3}) = mg - 4g$

$\frac{2}{3}mg = 4g$

$m = \frac{12}{2} = 6kg$

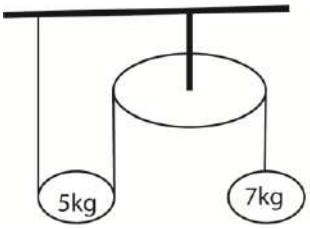
Revision exercise 3

1. A string with one end fixed passes under a movable pulley of mass 2kg, over a fixed pulley and carries a 5kg mass at its end



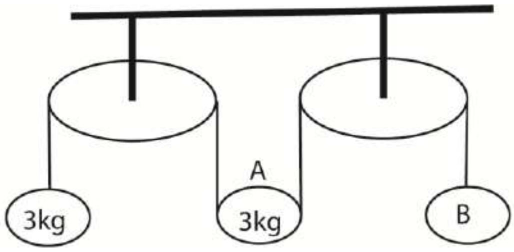
Find the acceleration of the movable pulley and the tension in the string. [3.56ms^{-2}, 13.36N]

2. a string with one end fixed passes under a movable pulley of mass 5kg, over a fixed pulley and carries a mass of7kg at its other end.



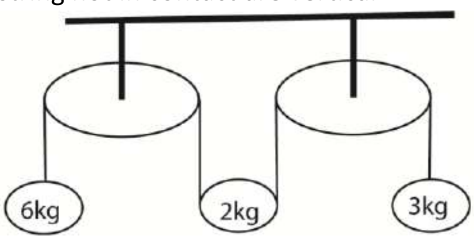
Find the acceleration of the movable pulley and the particle [2.673ms^{-2}, 5.146ms^{-2}]

3. In the pulley system below, A is a heavy pulley which is free to move.



Find the mass B, if it does not move upwards or downwards when the system is released from rest. [1kg]

4. Two particles of mass 3kg and 6kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 2kg, the portions of the string not in contact are vertical

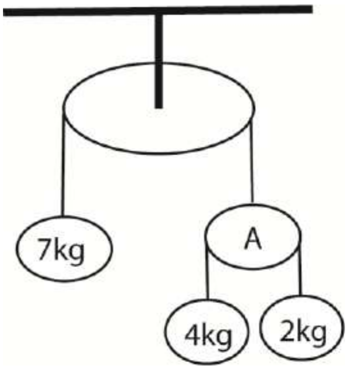


If the system is released from rest, find

(a) acceleration of movable pulley [5.88ms^{-2}]

(b) tension in the string [15.6N]

5. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over the pulley A and caries a mass of 4kg at one end and a mass of 2kg at the other end.

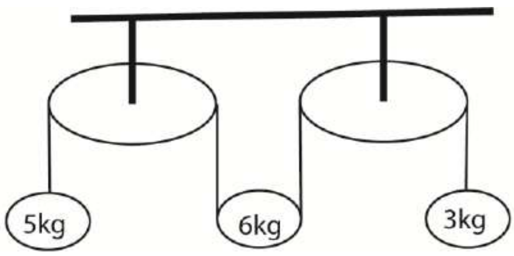


If the system is released from rest, find

(a) acceleration of 4kg mass [2.38ms^{-2}]

(b) tension in the strings [59.33N, 29.66N]

6. Two particles of mass 3kg and 5kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 6kg, the portions of the string not in contact are vertical

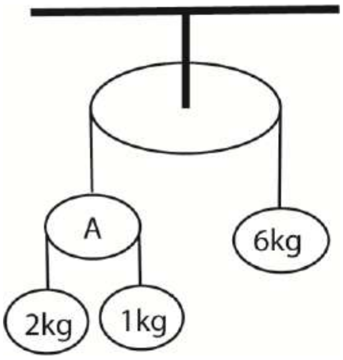


If the system is released from rest, find

(a) acceleration of movable pulley [$1.089ms^{-2}$]

(b) tension in the string [32.667N]

7. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 4kg at one end and a mass of 2kg at the other end.

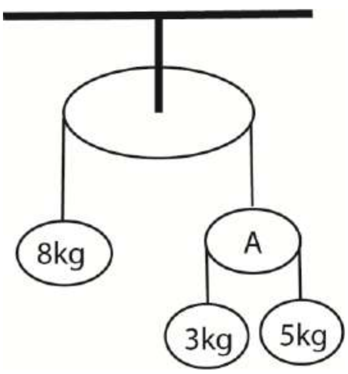


If the system is released from rest, find

(a) acceleration of 1kg mass [$8.2923ms^{-2}$]

(b) tension in the strings
[18.0923N, 36.1846N]

8. The diagram shows a system of masses and pulleys.

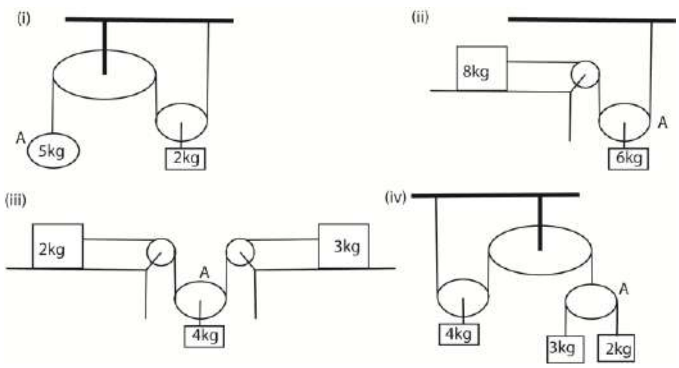


If the system is released from rest, find

(a) acceleration of 5kg mass [$2.8451ms^{-2}$]

(b) tension in the strings
[75.8712N, 37.9356N]

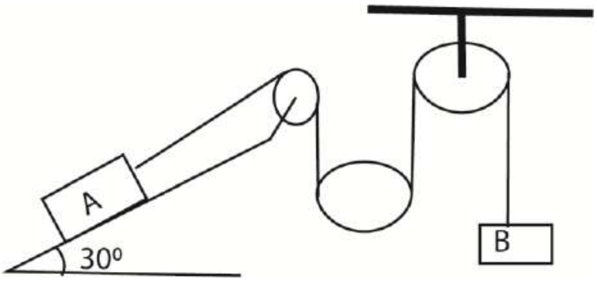
9. For each of the systems below: all the strings are light and inextensible, all pulleys are light and smooth and all surface are smooth. In each case find the acceleration of A and the tension in the string.



(i) [7.127ms^{-2}, 13.364N] (ii) [1.547ms^{-2}, 24.758N]

(iii) [3.564ms^{-2}, 10.691N]   (iv) [4.731ms^{-2}, 12.166N, 24.331N]

10. Two particles A and B of mass 4kg and 2kg respectively and a movable pulley c of mass 6kg are connected by a light inextensible string as shown below



Given that the coefficient of friction between A and the plane is 0.2 and the system is released from rest, find the acceleration of A, B, C and the tension in the string.
[A = -0.25ms^{-2}, B = 2.9ms^{-2}, C = 1.325ms^{-2}]

16. FRICTION

Friction

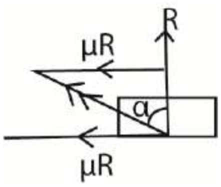
Friction is a force that opposes relative motion or attempted motion between two bodies in contact.

Friction force F = μR where R = normal reaction and μ = coefficient of friction

At limiting equilibrium, the body is at the point of moving (slip or slide) and friction force is maximum.

Angle of friction

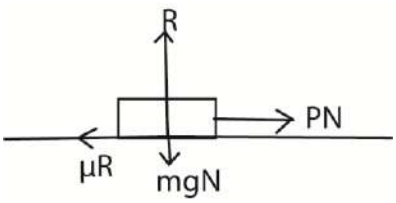
This is the angle between the resultant force and the normal reaction



tanα = $\frac{\mu R}{R} = \mu$

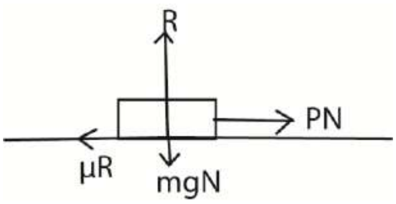
A horizontal plane

(i) at limiting equilibrium (about to slip or slid)



P = μR

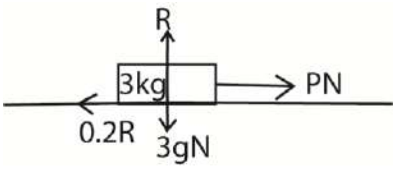
(ii) In motion



P – μR = ma

Example 1

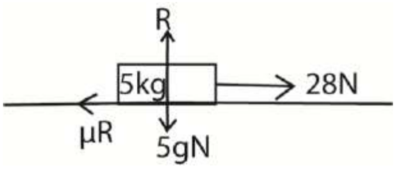
Calculate the maximum frictional force which can act when a block of mass 3kg rests on a rough horizontal surface, the coefficient of friction between the surface being 0.2



F = μR = 0.2 x 3 x 9.8 = 5.88N

Example 2

When a horizontal force of 28N is applied to a body of mass 5kg which is resting on a rough horizontal surface, the body is found to in limiting equilibrium. Find the coefficient of friction between the body and the plane



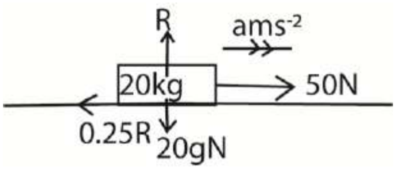
28 = μR

28 = μ x 5 x 9.8

$\mu = 0.57$

Example 3

A block of mass 20kg rests on a rough horizontal plane. The coefficient of friction between the block and the plane is 0.25. If a horizontal force of 50N acts no the body, find the acceleration of the body.



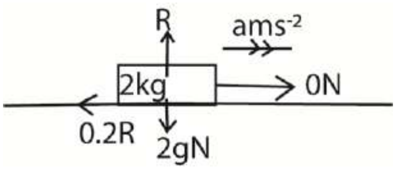
50 - μR = 20a

50 - $(0.25\ x\ 20\ x\ 9.8)$ = 20a

a = 0.05ms^{-2}

Example 4

A block of mass 2kg sliding along a smooth surface at a constant speed of 2ms-1. When the mass encounter a rough surface of coefficient of friction 0.2, it comes to rest. Find the distance the body will move across the rough surface before it comes to rest.



F = ma

0 – μR = 20a

-0.2 x 2 x 9.8 = 20a

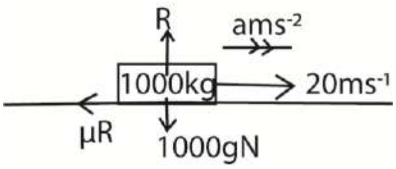
a = -1.96ms^{-2}

s = $\frac{v^2 - u^2}{2a} = \frac{0^2 - 20^2}{2x\ (-1.96)}$ = 1.02m

Example 5

A car of mass 1000kg moving along a straight road with speed of 72kmh^{-1} is brought to rest by a speedy application of brakes in a distance of 5m. Find the coefficient of kinetic friction between the tyres and the road.

u = $\frac{72\ x\ 1000}{2600}$ = 20ms^{-1}



a = $\frac{v^2 - u^2}{2s} = \frac{0^2 - 20^2}{2x\ 5}$ = -4ms^{-2}

ma = μR

4 x 1000 = 1000 x 9.8 x μ

μ = 0.41

Alternatively

Work done against friction = loss in kinetic energy

μ(mg)x s = $\frac{1}{2}mv^2$

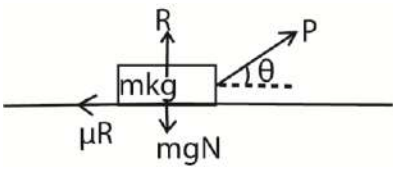
μ x 9.8 x 50 = $\frac{1}{2}x\ 20^2$

μ = 0.408

Revision exercise 1

1. When a horizontal force of 0.245N is applied to a body of mass 250g which is resting on a rough horizontal plane, the body is found to be in limiting equilibrium. Find the coefficient of friction between the body and the plane. [0.1]
2. A body of mass 40kg is resting on a rough horizontal plane and can just move by a force of 98N acting horizontally. Find the coefficient of friction. [0.25]
3. A block of mass 0.5kg rests on a rough horizontal plane. The coefficient of friction between the block and the table is 0.1. When a horizontal force of 1N acts on the block, find
   (i) friction force experienced by the block. [0.49N]
   (ii) acceleration with which the block will move. [1.02ms^{-2}]
4. When a horizontal force of 37N is applied to the body of mass 10kg which is resting on a rough horizontal surface, the body moves along the surface with acceleration 1.25ms^{-2}. Find the coefficient of friction between the body and the surface. [0.25]

5. A force inclined at an angle θ to the horizontal



At limiting equilibrium

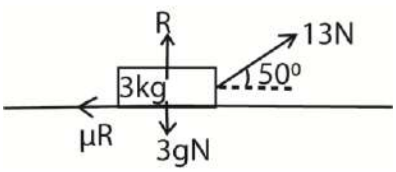
(→): Pcosθ = μR

(↑): R + Psinθ = mg

Example 6

A particle of mass 3kg resting on a rough horizontal plane is pulled by a force of magnitude 13N inclined at an angle 50^0 to the horizontal, if the particle does not move find the

(i) Normal reaction (ii) coefficient of friction

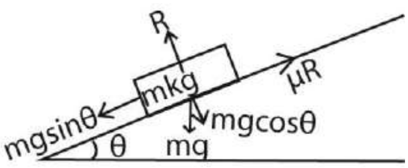


(↑):R = 3 x 9.8 – 13sin 50^0 = 19.4414N

(→): 13cos50^0 = μ x 19.4414

μ = 0.4298

Friction and inclined planes



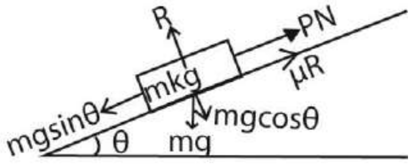
At limiting equilibrium

mgsinθ = μR

mgsinθ = μmgcosθ

μ = tanθ

(ii) A force P applied parallel to and up the plane to just move the particle upwards



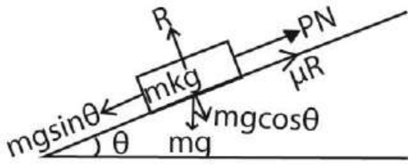
Normal to the plane: mgcosθ = R

Parallel to the plane; mgsinθ + μR = P

P = mgsinθ + μmgcosθ

(iii)

(iv) A force P applied parallel to and up the plane so that the particle is on the point of moving downwards (prevent moving downwards)



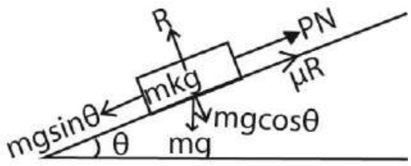
Normal to the plane: mgcosθ = R

Parallel to the plane; mgsinθ = P+ μR

P = mgsinθ - μmgcosθ

(v)

(vi) A force P applied parallel to and up the plane to move the particle upwards



Normal to the plane: mgcosθ = R

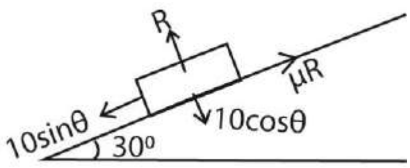
Parallel to the plane; P- (mgsinθ + μR) = ma

P – (mgsinθ + μmgcosθ) = ma

(vii)

Example 7

A particle of weight 10N rests on a rough plane inclined at $30^0$to the horizontal and is just about to slip. Find the value of coefficient of friction between the plane and the particle.



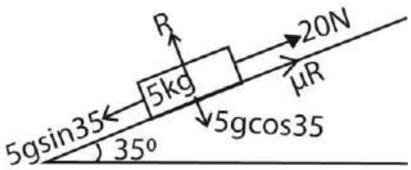
R = 10cos 30 and μR = 10sin30

μ(10cos30) = 10sin30

μ = 0.5774

Example 8

A body of mass 5kg lies on a rough plane which is inclined at 35^0 to the horizontal. When a force of 20N is applied to the body parallel to and up the plane, the body is on the point of moving down the plane. Find the coefficient of friction between the body and the plane.



At limiting equilibrium

R = 5gcos 35

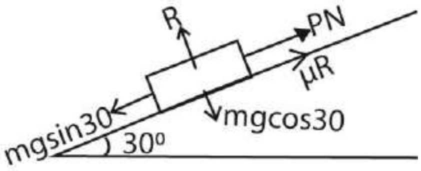
20 +μR = 5gsin35

20 +μ(5gcos35) =5gsin35

μ = 0.2

Example 9

A block of wood of mass 150g rest on an inclined plane. If the coefficient of friction between the surface of contact is 0.3. Find the force parallel to the plane necessary to prevent slipping when the angle of the plane to the horizontal is 30^0.



At limiting equilibrium

R =0.15 x 9.8cos 30

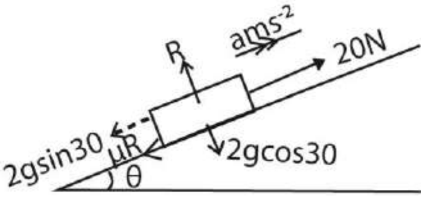
P +μR = 0.15 x 9.8sin30

P +0.3(0.15 x 9.8cos 30) =5gsin30;

P= 0.353N

Example 10

A body of mass 2kg lies on a rough plane which is inclined at $\sin^{-1}\left(\frac{5}{13}\right)$ to the horizontal. A force of 20N is applied to the body, parallel to and up the plane. If the body accelerates up the plane at $1.5ms^{-2}$, find the coefficient of friction between the body and the plane.



Sin θ =$\frac{5}{13}$, cos θ = $\frac{12}{13}$

R = 2 x 9.8 cosθ = 2 x 9.8 x $\frac{12}{13}$ =18.09N

F = ma

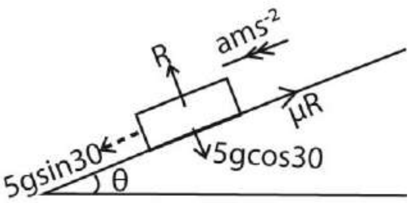
20 – (2gsinθ + μR) = 2a

20 –(2 x 9.8 x $\frac{5}{13}$ + 2 x 9.8 x $\frac{12}{13}$ x μ) = 2 x 1.5

μ =0.523

Example 11

A body of mass 5kg is released from rest on a rough surface of a plane inclined at 30^0 to the horizontal. If the body takes2.5s to acquire a speed of 4ms-2 from rest, find the frictional force and coefficient of friction.



v = u + at

4 = 0 + 2.5a

a = $1.6ms^{-2}$

F = ma

5 x 9.8sin30 – μR = 5 x 1.6;

Frictional force, μR = 16.5N

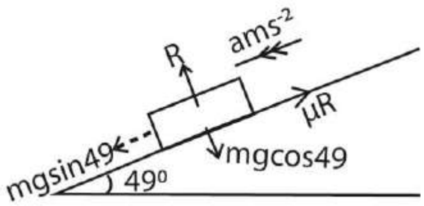
μR = 16.5N

μ = $\frac{16.6}{5\ x\ 9.8cos30}$ = 0.243

Example 12

A car of mass 500kg moves from rest with engine switched off down a road which is inclined at an angle 490 to the horizontal.

(a) calculate the normal reaction
(b) if the coefficient of friction between the tyres and the surface of the road is 0.32. Find the acceleration of the car.



(a) R =mgcos49 = 500 x 9.8cos49 = 3217.97N

(b) F = ma

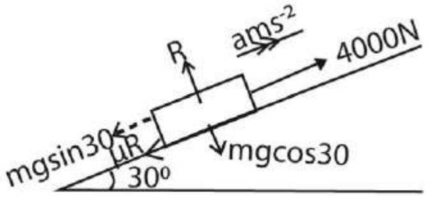
mgsin49 – μR = 500a

500 x 9.8sin49 – 0.32 x 3217.97 = 500a

a = 5.34ms^{-2}

Example 13

A car of mass 1000kg climbs a plane which is inclined at 30^0 to the horizontal. The speed of the car at the bottom of the incline is 36kmh^{-1}. If the coefficient of friction between the plane and the car tyres is 0.3 and the engine exerts a force of 4000N, how far up the incline does the car move in 5s



u = 36kh^{-1} = $\frac{36 \; x \; 1000}{3600}$ = 10ms^{-1}

F = ma

4000 – (mgsin30 + μR) = ma

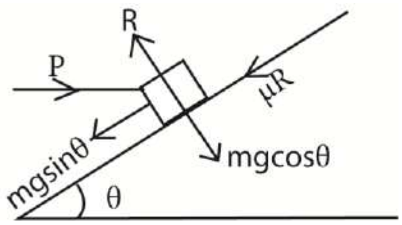
4000 – (1000 x 9.8sin30 + 0.3 x 1000 x 9.8cos30) = 1000a

a = -3.45ms^{-2}

s = ut + $\frac{1}{2}at^2$ = 10 x 5 + $\frac{1}{2}\; x \;(-3.45) x \; 5^2$ = 6.9m

Horizontal force on inclined planes

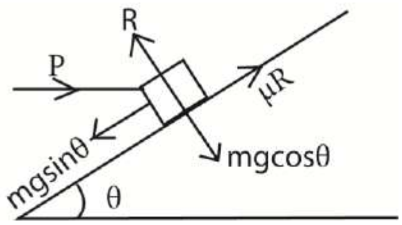
(i) A horizontal force P required to just move the particle upwards



Normal to the plane: mgcosθ + Psinθ = R

Parallel to the plane: mgsinθ + μR = Pcosθ

(ii) a horizontal force O required to prevent the particle from moving downwards

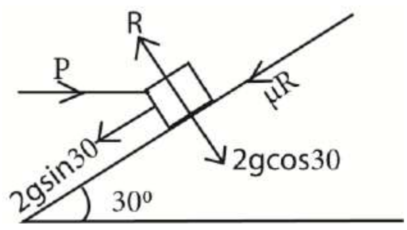


Normal to the plane: mgcosθ + Psinθ = R

Parallel to the plane: mgsinθ - μR = Pcosθ

Example 14

A body of mass 2kg lies on a rough plane inclined at 300 to the horizontal. When a horizontal force of 20N is applied to the body in an attempt to push it up the plane, the body is found to be on the point of moving up the plane. Find the coefficient of friction between the body and the plane.



At limiting
Normal to the plane: 2gcos30 + 20sin30 = R……(i)

Parallel to the plane: 2gsin30 + μR = 20cos30 …… (ii)

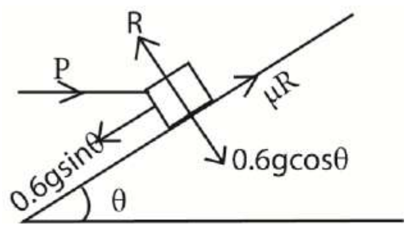
(i) and (ii)

2gsin30 +μ(2gcos30 + 20sin30) = 20cos30

μ =0.279

Example 15

A horizontal force of 1N is just sufficient to prevent a brick of mass 600g sliding down a rough plane which is inclined at $\sin^{-1}\left(\frac{5}{13}\right)$to horizontal. Find the coefficient of friction between the brick and the plane.



$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

At limiting
Normal to the plane: 0.6gcosθ + 1xsinθ = R……(i)

Parallel to the plane: 0.6gsinθ - μR = 1 x cosθ …… (ii)

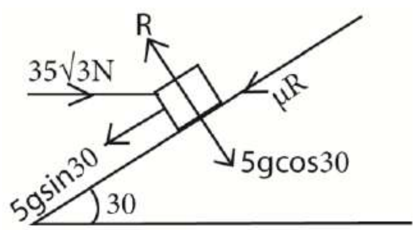
(i) and (ii)

$0.6 \times 9.8 \times \frac{5}{13} - \mu(0.6 \times 9.8 \times \frac{12}{13} + 1 \times \frac{5}{13}) = 1 \times \frac{12}{13}$

μ =0.23

Example 16

A body of mass 5kg is initially at the bottom of a rough inclined plane of length 6.3m. The plane is inclined to the horizontal and the coefficient of friction between the body and the plane is $\frac{1}{2}\sqrt{3}$. A constant horizontal force of $35\sqrt{3}N$is applied to the body causing it to accelerate up the plane. Find the time taken for the body to reach the top and its speed on arrival.



R = 5gcos 30 + $35\sqrt{3}$sin30

$35\sqrt{3}$cos 30 – (μR + 5gsin30) = 5a

a = 1.39ms^{-2}

s = ut + $\frac{1}{2}at^2$

6.3 = 0 x t + $\frac{1}{2}x1.39\ x\ t^2$; t = 3s

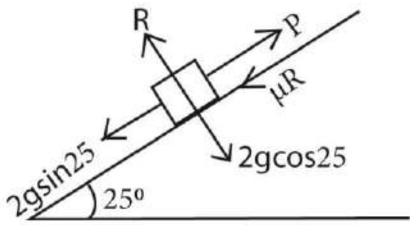
v = u + at

v = 0 + 1.39 x 3 =4.17ms^{-1}

Example 17

A box of mass 2kg at rest on a plane inclined at 25^0 to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane?

Let the minimum force required be P



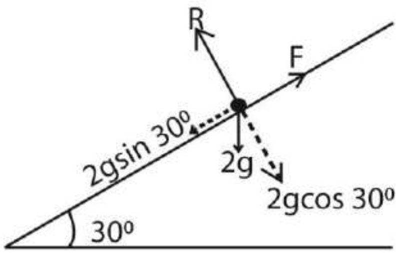
Perpendicular to the plane: R = 2gcos25 .... (i)
Along the plane: P = 2gsin25 + μR ……………(ii)

(i) and (i)

P = 2gsin25 + 0.4(2gcos25) = 8.3 + 7.1 = 15.388N

Example 18

A particle of mass 2kg rests in limiting equilibrium on a rough plane inclined at 30^0 to the horizontal. Find the value of coefficient of friction.

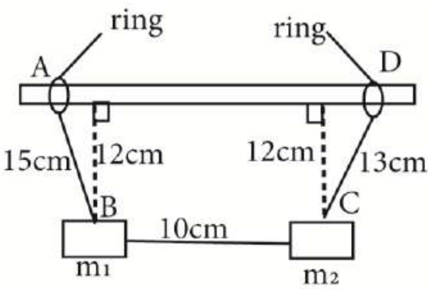


R = 2gcos 30^0
F = 2gsin 3^0
μR = 2gsin30^0
$\mu[2gcos\ 30^0]$ = 2gsin30^0
$\mu\ =\ \frac{2gsin\ 30^0}{2gcos\ 30^0}$ = tan 30^0 = 0.57735

Revision exercise

1. The resistance to motion of a lorry of mass mkg is 1/200 of its weight. When travelling at 108kmh^{-1} on a level road and ascends a hill inclined at 1 in 100. Its engine fails to work. Find how far up the hill (in km) the lorry moves before it comes to rest. [36.12m]
2. A vehicle of mass 2.5metric tonnes is drawn up on a slope of 1 in 10 from rest with an acceleration of 1.2 ms-2 against a constant frictional resistance of $\frac{1}{100}$ of the weight of the vehicle, using a cable. Find the tension in the cable. [T = 5695N]
3. (a) A particle of mass, m kg is projected with a velocity of 10ms-1 up a rough plane of inclination 300 to the horizontal. If the coefficient of friction between the particle and the plane is $\frac{1}{4}$. Calculate how far up the plane the particle travels.[s = 7.121m]
   (b) A car is working at 5kW and is travelling at a constant speed of 72knh^{-1}. Find the resistance to motion. [250N]

4. A body of mass 8kg rests on a rough plane inclined at θ to horizontal. If the coefficient of friction is μ, find the least horizontal force in terms of μ, θ and g which will hold the body in equilibrium. [$\frac{8g(sin\theta-\mu cos\theta)}{(cos\theta+\mu sin\theta)}$]
5. A carton of 3kg rests on a rough plane inclined at an angle 30^0 to the horizontal. The coefficient of friction between the carton and the plane is $\frac{1}{3}$ > find a horizontal force that should be applied to make the carton just about to move up the plane. [33.155N]
6. A particle of weight 20N is placed on a rough plane inclined at an angle of 400 to the horizontal. the coefficient of friction between the plane and the particle is 0.25. When a horizontal force P is applied on the particle it rests in equilibrium. Calculate the value of P. [9.739N]
7. The diagram below shows the three strings AB = 15cm, BC = 10cm and CD = 13cm, A and D are fixed to small rings each of mass 2kg which can slide on a rough horizontal rail AD. Masses m_1 and m_2 are attached at B and C respectively. The system rests in equilibrium with BC at a distance 12cm below AD.



   (a) Show that $9m_1 = 5m_2$.
   (b) If the coefficient of friction between each ring and the rail is 0.25 and the ring A is on the pint of slipping, determine the value of m_1. [m_1 = 1kg]
8. A 2kg body lies on a plane of inclination 600. The coefficient of friction between the body and the plane is 0.25. Find the least horizontal force which prevents the body from sliding down the plane. [20.27N]
9. A particle od mass 12kg slides from rest down a plane inclined at 50^0 to the horizontal. If the coefficient of friction between the particle and the plane is 0.4, calculate the acceleration of the particle. [4.99ms^{-2}]
10. A body of mass 3kg is released from a rough surface which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. If after 2.5s the body has acquired a velocity of 4.9ms-1 down the surface. Find the coefficient of friction between the body and the surface.

17. LINEAR MOMENTUM

Linear momentum

This is the product of mass of a body and its velocity.
Momentum = mass x velocity

Collisions

Case 1: bodies separate after collision

Consider two bodies A and B with body A having a mass of M_A, initial velocity U_A, and body B having a mass f M_B, initial velocity U_B, after collision body A has a final velocity V_A and body B has a final velocity V_B.

$M_AU_A + M_BU_B = M_AV_A + M_BV_B$

Loss in k.e = $\left(\frac{1}{2}M_AU_A{}^2 + \frac{1}{2}M_BU_B{}^2\right) - \left(\frac{1}{2}M_AV_A{}^2 + \frac{1}{2}M_BV_B{}^2\right)$

%loss in k.e = $\frac{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)-\left(\frac{1}{2}M_AV_A{}^2+\frac{1}{2}M_BV_B{}^2\right)}{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)}x100\%$

Case2: bodies stick together and move with a common velocity after collision

Consider two bodies A and B with body A having a mass of M_A, initial velocity U_A, and body B having a mass f M_B, initial velocity U_B, after collision body A and body B stick together and move with common velocity V.

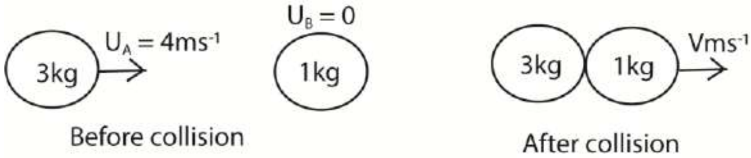
$M_AU_A + M_BU_B = (M_A + M_B)V$

Loss in k.e = $\left(\frac{1}{2}M_AU_A{}^2 + \frac{1}{2}M_BU_B{}^2\right) - \left(\frac{1}{2}(M_A + M_B)V^2\right)$

%loss in k.e = $\frac{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)-\left(\frac{1}{2}(M_A+M_B)V^2\right)}{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)}x100\%$

Example 1

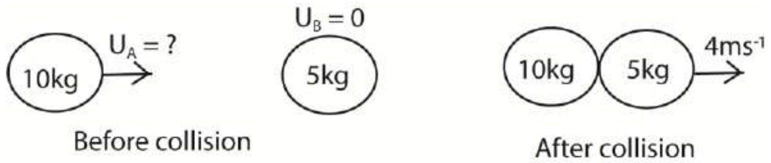
A trolley of mass 3kg travelling at a velocity of $4ms^{-1}$ collide with another trolley of mass 1kg which is at rest. At what velocity do the two bodies move together after collision?



$M_AU_A + M_BU_B = (M_A + M_B)V$
(3 x 4) + (1 x 0) = (3 + 1)V
12 = 4V
V = $3ms^{-1}$

Example 2

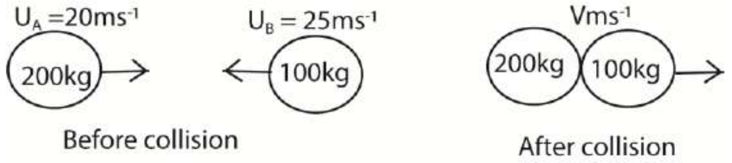
An object of mass 10kg collides with a stationary object of mass 5kg. If the objects stick together and move forward with a velocity of $4ms^{-1}$. What was original velocity of the moving objects?



$M_AU_A + M_BU_B = (M_A + M_B)V$
$(10 \times U) + (5 \times 0) = (10 + 5) \times 4$
$U = 60ms^{-1}$

Example 3

Two bodies of masses 200kg and 100kg travel towards each other with velocities of $20ms^{-1}$ and $25ms^{-1}$ respectively and join to form one body on collision. Find the common velocity.



$M_AU_A + M_BU_B = (M_A + M_B)V$
$(200 \times 20) + (100 \times -25) = (200 + 100) \times v$
$V = 5ms^{-1}$

Example 4

A particle of mass 2kg moving with a speed $10ms^{-1}$ collides with a stationary particle of mass 7kg. Immediately after impact, the particle moves with the same speed but in opposite directions. Find the loss in kinetic energy

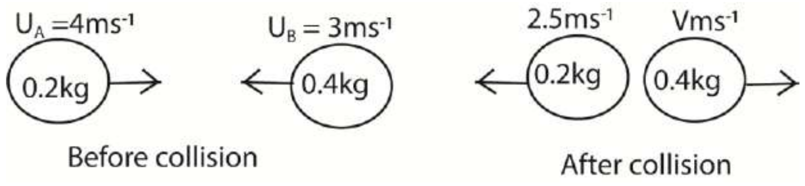
$M_AU_A + M_BU_B = M_AV_A + M_BV_B$
$2 \times 10 + 7 \times 0 = 2 \times -v + 7 \times v$
$v = 4ms^{-1}$
k.e before = $\frac{1}{2}\ x\ 2\ x\ 10^2 + \frac{1}{2}\ x\ 7\ x\ 0^2 = 100J$

k.e after = $\frac{1}{2}\ x\ 2\ x\ 4^2 + \frac{1}{2}\ x\ 7\ x\ 4^2$ = 72J

lossin k.e = 100 – 72 = 28J

Example 5

Two particles are moving towards each other along a straight line. The first particle has mass of 0.2kg and moving with velocity $4ms^{-1}$ and then the second has a mass of 0.4kg moving with a velocity of $3ms^{-1}$. On collision, the first particle reverses its direction and moves with a velocity of $2.5ms^{-1}$. Find the percentage loss in kinetic energy.



$M_AU_A + M_BU_B = M_AV_A + M_BV_B$
$0.2 \times 4 + 0.4 \times -3 = 0.2 \times -2.5 + 0.4V$
$V = 0.25ms^{-1}$
k.e before = $\frac{1}{2}\ x\ 0.2\ x\ 4^2 + \frac{1}{2}\ x\ 0.4\ x\ 3^2$
= 3.4J

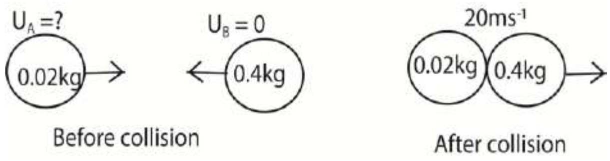
k.e after = $\frac{1}{2}\ x\ 0.2\ x\ -2.5^2 + \frac{1}{2}\ x\ 0.4\ x\ 0.25^2$
= 0.6375J

% loss in k.e = $\frac{(3.4-0.6375)}{3.4}\ x\ 100\%$ = 81.25%

Example 6

A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of $20ms^{-1}$. Calculate

(a) the speed with which the bullet hits the wood



$M_AU_A + M_BU_B = (M_A + M_B)V$
0.02 x UA + 0.4 x 0 = 0.42 x 20
UA = 420ms^{-1}

(b) The kinetic energy loss

k.e energy before = $\frac{1}{2}\ x\ 0.02\ x\ 420^2 = 1764J$

k.e after = $\frac{1}{2}\ x\ 0.42\ x\ 20^2 = 84J$

Loss in k.e = 1764 - 84 = 1680J

Example 7

Two bodies A and B of mass 4kg and 3kg moving with velocities (2i + 3j)ms^{-1} and (5i - 6j)ms^{-1} respectively collide. After collision A moves with a velocity (5i)ms^{-1}, Find the

(i) velocity of B after collision
$M_AU_A + M_BU_B = M_AV_A + M_BV_B$
4(2i + 3j) + 3(5i – 6j) = 4 (5i) + 3V_B
V_B = (i -2j) ms^{-1}

(ii) loss in kinetic energy
k.e before = $\frac{1}{2}\ x\ 4(2^2 + 3^2) + \frac{1}{2}\ x\ 3(5^2 + (-6)^2)$ = 117.5J
k.e after = $\frac{1}{2}\ x\ 4(5^2) + \frac{1}{2}\ x\ 3(1^2 + (-2)^2)$= 57.5J
Loss in k.e = 117.5 – 57.5 = 60J

Example 8

Two bodies A and B of mass 7.5kg and 5.0kg moving with velocities of (–i – 2j)ms^{-1} and (9i + 8j)ms^{-1} respectively. After collision the bodies stick together and move with a common velocity, find the

(i) common velocity
$M_AU_A + M_BU_B = (M_A + M_B)V$
7.5(–i – 2j) + 5.0(9i + 8j) = 12.5V
V = (3i + 2j)ms^{-1}

(ii) percentage loss in kinetic energy
k.e before = $\frac{1}{2}\ x\ 7.5((-1)^2 + (-2)^2) + \frac{1}{2}\ x\ 5(9^2 + 8^2)$ = 381.25J
k.e after = $\frac{1}{2}\ x\ 12.5(3^2 + 2^2)$= 81.25J
Loss in k.e = 381.25 – 81.25 = 300J
%loss in k.e = $\frac{300}{381.25}\ x\ 100\% = 78.69\%$

Example 9

Two bodies A and B of masses 3kg and 2kg respectively are 7m a part on a smooth horizontal surface. A moving directly towards B with a speed of 2ms^{-1} and acceleration of 0.3ms^{-2}. B is moving in the in the same direction as A with a speed of 5ms^{-1} and retardation of 0.2ms^{-2}. If the bodies collide and coalesce, calculate

(i) Time taken before collision occurs
let the distance travelled by B before collision = x
s = ut + ½ at^2

For A: 7 + x = 2t + $\frac{1}{2}$ x $0.3t^2$ ……… (i)

For B: x = 5t – $\frac{1}{2}$ x $0.2t^2$ ………………….(ii)

Subtract (ii) from (i)

7 = -3t + $0.25t^2$

$t^2 - 12t - 28 = 0$

$t = 14s$

(ii) Common velocity immediately after collision

Initial velocity of A before collision V_A = 2 + 0.3 x 14 = 6.2ms^{-1}

Initial velocity of B before collision VB = 5 – 0.2 x 14 = 2.2ms^{-1}

Let the common velocity be V

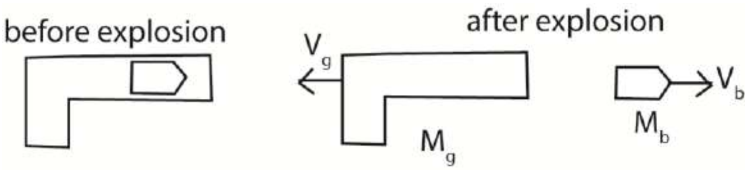
$M_AU_A + M_BU_B = (M_A + M_B)V$

3 x 6.2 + 2 x 2.2 = 5V

V = 4.6ms^{-1}

Recoil velocity of a gun and muzzle velocity of a bullet

When a bullet of mass Mb is fired with a muzzle vel0city of Vb, the gun of mass Mg jerks backward with a recoil velocity of Vg.



M_g x 0 + M_b x 0 = M_g x $-V_g$ + M_bV_b

M_g x V_g = M_bV_b

Example 10

A bullet of mass 60g is fired from a gun of mass 3kg. The bullet leaves the gun with velocity of 400ms^{-1}. Find the initial speed of recoil of the gun and gain in kinetic energy of the system.

0.06 x 400 = 3 x V

V = 8ms^{-1}

Gain in k.e = k.e after = $\frac{1}{2}\ x\ 0.06\ x\ 400^2 + \frac{1}{2}\ x\ 3\ x\ 8^2$ =4896J

Example 11

A gun of mass 3000kg fires horizontally a shell at initial velocity of 300ms^{-1}. If the recoil of the gun is brought to rest by a constant opposing force of 9000N in 2 seconds, find the

(a)(i) Initial velocity of the recoil gun

F = ma

-9000 = 3000a

a = -3ms^{-2}

v = u + at

0 = u – 3 x 2

u = 6ms^{-1}

(ii) Gain in kinetic energy of the shell just after firing

$M_gV_g = M_bV_b$

$3000 \times 6 = M_b \times 300$

$M_b = 60kg$

(b)(i) displacement of the gun

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 6^2}{2\ x{-3}} = 6m$$

(ii) work done against the opposing force

W = Fs = 9000 x 6 = 54,000J

Revision exercise

1. A bullet of mass 50g travelling horizontally at 80ms-1 hits a block of wood of mass 10kg resting on a smooth horizontal plane. If the bullet emerges with a speed of $50ms^{-1}$, find the speed with which the block moves. [$0.15ms^{-1}$]
2. A bullet of mass 0.1kg travelling at 420ms-1 hits a block of wood of mass 2kg resting on a smooth horizontal plane. If the bullet becomes embedded on the block, find the speed with which the block moves after impact. [20ms-1]
3. A 2kg object moving with a velocity of $8ms^{-1}$ collides with a 3kg object moving with a velocity $6ms^{-1}$ along the same direction. If the collision is completely inelastic, calculate the decrease in kinetic energy.[2.4J]
4. A particle A of mass 150g lies at rest in a smooth horizontal surface. A second particle B of mass 100g is projected along the surface with the speed ums^{-1} and collides directly with A. On collision the masses coalesce and move with speed $4ms^{-1}$. Find the value of u and loss in the kinetic energy of the system during impact. [$10ms^{-1}$, 3J]
5. Two bodies A and B of masses 2kg and 4kg moving with velocities of $8ms^{-1}$ and $5ms^{-1}$ respectively collide and move in the same direction. Object A's new velocity is $6ms^{-1}$.
   (i) find the velocity of B after collision [$6ms^{-1}$]
   (ii) calculate the percentage loss in kinetic energy [5.26%]
6. Two bodies A and B of mass 2kg and 3kg moving with velocities of $4ms^{-1}$ and $3ms^{-1}$ respectively in the same direction collide and coalesce, find he
   (i) common speed after collision [$3.4ms^{-1}$]
   (ii) loss in kinetic energy [0.6J]
   (iii) percentage loss in kinetic energy [2.03%]
7. A bullet of mass 30g is fired horizontally at $200ms^{-1}$ and hits a block of wood of mass 2kg resting on smooth horizontal plane. If the bullet becomes embedded on the block, find the
   (i) common velocity of bullet and wood. [$2.96ms^{-1}$]
   (ii) percentage loss in kinetic energy. [98.52%]
8. Two smooth sphere A and B of equal radii and mass 3kg and 1.5kg respectively are travelling along the same horizontal line in opposite direction. The speeds of A and B are $6ms^{-1}$ and $2ms^{-1}$ respectively. The sphere A collides and after collision B reverses its direction and moves with speed of $4ms^{-1}$. Find the velocity of A after collision. [$3ms^{-1}$]
9. Two smooth spheres A and B of equal radii and masses 180g and 100g respectively travelling along the same horizontal line. The initial speeds of A and B are $2ms^{-1}$ and $6ms^{-1}$ respectively. The spheres collide and after collision both spheres reverse their directions and B moves with speed of $3ms^{-1}$. Find the speed of A after collision and loss in kinetic energy of the system. [$3ms^{-1}$, 0.9J]

10. A particle of mass 2kg moving with speed $10ms^{-1}$ collides with stationary particles of mass 7kg. Immediately after impact the particles move with the same speeds but in opposite direction. Find the loss in kinetic energy during collision. [28J].
11. Two identical rail way truck are travelling in the same direction along the same straight piece of track with constant speed of $6ms^{-1}$ and $2ms^{-1}$. The faster truck catches up with the other one on collision, the two trucks couple together. Find the common speed of the trucks after collision. [$4ms^{-1}$]
12. A 2kg object moving with a velocity of $6ms^{-1}$ collides with a stationary object of mass 1kg. If the collision is perfectly elastic, calculate the velocity of each object after collision. [$2ms^{-1}$, $8ms^{-1}$]
13. A van of mass of mass 1200kg and a lorry of mass 3200kg collide. Just before the crash they are moving directly towards each other and each has a speed of $12ms^{-1}$. Immediately after the crash they move with the same velocity. Find the loss in kinetic energy. [251kJ]
14. A body of mass 6kg moving with velocity $(8i – 4j)ms^{-1}$ collides with a body of mass 2kg which is at rest. On collision the two bodies coalesce, find the
    (i) common velocity after collision [$(6i – 3j)ms^{-1}$]
    (ii) loss in kinetic energy [960J]
15. A body of mass 2kg moving with velocity $(-2i + 4j)ms^{-1}$ collides with a body of mass 3kg moving with velocity $(3i + 4j)ms^{-1}$. On collision the two bodies coalesce, find the
    (i) common velocity after collision [$(i + 4j)\ ms^{-1}$]
    (ii) loss in kinetic energy [15J]
16. A body of mass 500g moving with velocity $(2i – 4j)ms^{-1}$ collides with a body of mass 1500g moving with velocity $(6i + 8j)ms^{-1}$. On collision the two bodies coalesce, find the
    (i) common velocity after collision [$(5i + 5j)ms^{-1}$]
    (ii) loss in kinetic energy [30J]
17. Two bodies A and B of mass 2kg and 5kg moving with velocities of $(-2i + 3j)ms^{-1}$ and $(6i – 10j)ms^{-1}$ respectively collide. After collision A moves with velocity $(3i – 2j)ms^{-1}$, find the
    (i) velocity of B after collision [$(4i – 8j)ms^{-1}$]
    (ii) loss in kinetic energy [140J]
18. Two bodies A and B of mass 5kg and 2kg moving with velocities of $(-4i + 3j)ms^{-1}$ and $(3i – 10j)ms^{-1}$ respectively collide. After collision A moves with a velocity $(-2i + j)ms^{-1}$, find the
    (i) velocity of B after collision [$(-2i + 4j)ms^{-1}$]
    (ii) loss in kinetic energy [40J]
19. A body X, of mass 250g moving with velocity $(-2i + 3j)ms^{-1}$ collide with a body Y of mass 750g moving with velocity $(5i + 8j)ms^{-1}$. After collision X moves with a velocity (-2i + j)ms-1, find the
    (i) velocity of Y after collision [$(4i + 6j)ms^{-1}$]
    (ii) loss in kinetic energy [5.25J]
20. A shell of mass 5kg is fired from a gun of mass 2000kg. The shell leaves the gun with a velocity of $400ms^{-1}$. find the initial speed of recoil of the rifle and gain in kinetic energy of the system [$1ms^{-1}$, 2520J)
21. A bullet of mass 20g is fired from a rifle of mass 2.5kg. The bullet leaves the gun with a velocity of $500ms^{-1}$. Find the initial of the recoil of rifle and gain in kinetic energy of the system. [$4ms^{-1}$, 2520J]
22. A bullet of mass 5kg is fired from a rifle of mass 2000g. The bullet leaves the gun with a velocity of $400ms^{-1}$. Find the recoil velocity of the rifle. [$1ms^{-1}$]

18.PARALLEL FORCES IN EQUILIBRIUM

Parallel forces in equilibrium

Conditions for a body to be in equilibrium

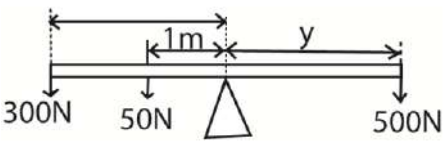
When a system of parallel forces act on a body then it will be in equilibrium when;

(i) the sum of forces acting in one direction are equal to the sum of forces acting in opposite direction.

(ii) sum of clockwise moments about a point are equal to the sum of anticlockwise moment about the same point

Example 1

Given the diagram below. Find the value of y

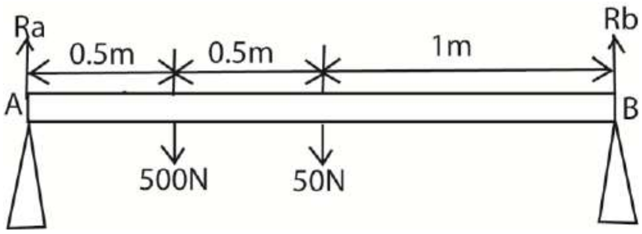


500 x y = (300 x 3) + (50 x 1)

y = $\frac{950}{500} = 1.9m$

Example 2

A uniform beam of weight 50N and length 2m rests horizontally on two supports pivoted at each end. A load of weight 500N is placed 0.5m from one end. Find the reaction on each support.



B: Ra x 2 =50 x 2 + 500 x 1.5

2Ra = 50 + 750

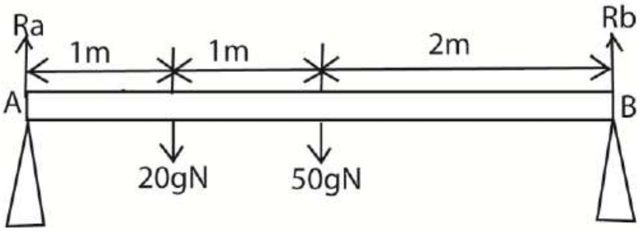
Ra = 400N

Also Ra + Rb = 500N + 50 N

Rb = 550N – 400N = 150N

Example 3

A uniform beam of mass 50kg and length 4m rests horizontally on two supports pivoted at each end. A load of 20kg is placed 1m from one end. Find the reaction on each support



B: Ra x 4 =50g x 2 + 20g x 3

4Ra = 100g + 60g = 160 x 9.8

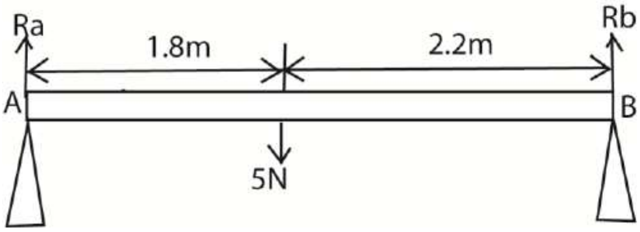
Ra = 392N

Also Ra + Rb = 500N + 50 N

Rb = 20gN + 50gN – 392N = 294N

Example 4

A non- uniform beam AB of length 4m has its weight 5N acting at a point 1.8m from end A. The beam rests horizontally on two supports pivoted at each end. Find the reaction on each support.



B: Ra x 4 =5 x 2.2 =11

Ra = 2.75N

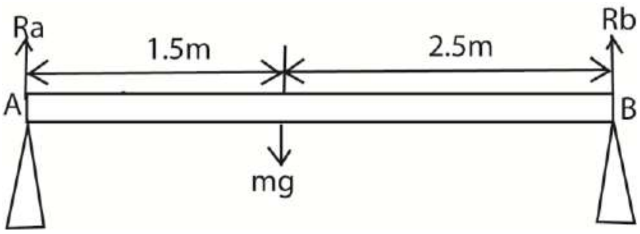
Also Ra + Rb = 5N

Rb = 5N – 2.75N = 2.25N

Example 5

A non- uniform beam AB of length 4m rests in horizontal position on vertical support at A and B. The centre of gravity is at 1.5m from end A. The reaction at B is 37.5N fine the

(a) mass of the beam (b) reaction at B



A: $37.5 \times 4 = mg \times 2.5$

$m = 10.2kg$

Also $Ra + 37.5 = 10.2 \times 9.8$

$Ra = 62.5N$

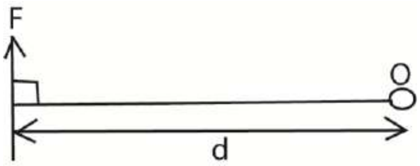
Revision exercise

1. A uniform beam AB of length 10m rests horizontally on two supports A and B. If the beam has a mass of 20g, find the reaction on each support. [98N, 9N]
2. A uniform beam of length 14m and mass 20kg rests horizontally on two supports, one at A and another at C which is 4m from B. find the reactions at each support [58.8N, 137.2N]
3. A uniform beam AB of length 10m and mass 20kg rests horizontally on two supports, one at A and another at C which is 2m from B. If a weight of mass 20kg is attached to the beam at a point 6m from A. Find the reaction on the supports. [392N, 196.2N]
4. A uniform beam AB of length 4m and mass 10kg rests horizontally on two supports at A and the other at C which is 1m from B. Where must a body of mass 50kg stand on the beam so that the reaction on each support is equal? [1.4m]
5. A uniform beam AB of length 12m and mass 12kg rests on two supports A and B. At what distance must a particle of mass 4g be tied so that the reaction of each support is equal. [9m from A]
6. A playground sea saw consists of a uniform beam of length 4m supported at its mid-point. If a girl of mass 25kg sits at one end of the sea saw, find where her brother of mass 40kg must sit if the sea saw is to balance horizontally. [75cm from other end]
7. A broom consists of a uniform broom stick of length 120cm and mass 4kg and a broom head of mass 6kg attached at the other end. Find where a support should be placed so that the broom balances horizontally. [24cm from the head]
8. A non-uniform beam AB of length 4m rests horizontally on two supports, one at A and the other at B. The reaction at the supports are 5gN and 3gN respectively. If instead the rod the rod were to rest horizontally on one support, find how far from A this support would have to be placed. [1.5m from A]
9. A uniform beam AB of mass 80g and of length 100cm is pivoted at 30cm from A, a force of 10N is placed on the beam at the 80cm from end A and a string is tied at the 40cm from end B so that the beam rests horizontally. Find the tension in the string. [17.2N]
10. A uniform beam AB of length 100cm is pivoted at 60cm from end B. The beam rests horizontally when a mass at A is 35g. Calculate the mass 9m) of the beam. [0.14kg]
11. A uniform meter rule pivoted at 10cm mark balances when a mass of 400N is suspended at the 0cm mark. If the system is in equilibrium. Find the mass of the ruler [10kg]
12. Two boys are carrying a uniform ladder of weight 800N, if the boys hold the ladder at 2m and 3m respectively from the centre of gravity, calculate the weight that each boy support. [480N, 320N]

19. MOMENT OF A FORCE

Moment of a force

This is the product of a force and perpendicular distance from the pivot to the line of action of the force. The unit of moments is Nm.



The moment of force about point O is F x d

Matrix approach of finding sum of moments about the origin

If forces (a_1i + b_1j)N, (a_2i + b_2j)N, ……….(a_ni + b_nj) act on the body at point (x_1 + y_1), (x_2 + y_2), ….. ((x_n + y_n). The sum of the moments about the origin is

$$G = \begin{vmatrix} x_1 & a_1 \\ y_1 & b_1 \end{vmatrix} + \begin{vmatrix} x_2 & a_2 \\ y_2 & b_2 \end{vmatrix} + \cdots + \begin{vmatrix} x_n & a_n \\ y_n & b_n \end{vmatrix}$$

$$G = (b_1x_1 - a_1y_1) + (b_2x_2 - a_2y_2) + \cdots + (b_nx_n - a_ny_n)$$

Note

If G is positive, the sum of moments will be anticlockwise and if G is negative the sum of moments will be clockwise.

Example 1

Find the moment about the origin of a force of 4jN acting at a point which has position vector -5iN

Solution

$G = \begin{vmatrix} -5 & 0 \\ 0 & 4 \end{vmatrix}$= -5x4 – 0x0 = -20Nm clockwise

Example 2

Find the moment about the origin of a force of 4jN acting at a point which has position vector 5iN

$G = \begin{vmatrix} 5 & 0 \\ 0 & 4 \end{vmatrix}$= 5x4 – 0x0 = 20Nm anticlockwise

Example 3

Forces of (2i-3j)N, (4i + j)N and (5i -3j)N act on a body at points with Cartesian co-ordinates (1,1), (2, 4), and (-1, 3) respectively. Find the sum of moments of the forces about the origin.

Solution

$G = \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ 4 & 1 \end{vmatrix} + \begin{vmatrix} -1 & 5 \\ 3 & -3 \end{vmatrix}$= (1 x -3 – 2 x 1)+(2 x 1 – 4x 4)+(-1 x -3 – 3 x 5) = -31Nm

= 31Nm clockwise

Example 4

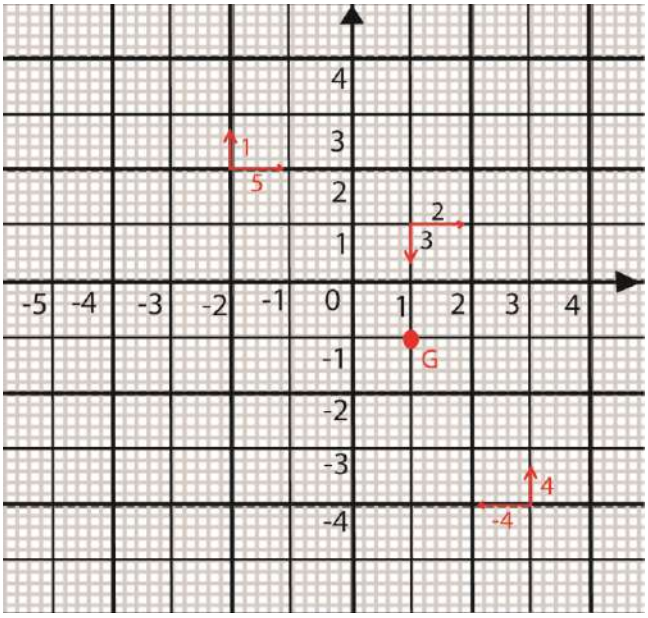
Forces (2i -3j)N, (5i + j)N and (-4i+4j) act on a body at points with position vector (i + j), (-2i +2j) and (3i -4j) respectively. Find the sum of moments of forces about the

(i) origin

$G = \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} -2 & 5 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 3 & -4 \\ -4 & 4 \end{vmatrix}$= (1 x -3 -2 x 1) + (-2 x 1 – 2 x 5) + (3 x 4 - –4x -4)

= -21Nm = 21Nm clockwise

(ii) point with position vector (i –j)



G = (5 x 3) + (1 x 3) +(2 x 0) + (2 x 2) + (4 x 3) – (4 x 2) = 26Nm clockwise

Revision exercise

1. Find the moment about the origin of a force of 3i acting at a point which has position vector (2i + 3j)m. [9Nm clockwise]
2. Find the moment about the origin of force (4i + 2j)N acting at a point which has position vector (3i + 2j)m. [2Nm clockwise]
3. A force of (3i -2j)N act at a point which has position vector (5i + j)m. Find the moment about the point which has a position vector (i +2j)m. [5Nm clockwise]
4. A force of (2i + j)N act at a point which has position vector (2i + 2j)m and a force of 5iN at a point which has position vector (-2i + j)m. Find the sum of moments of these forces about the origin. [7Nm clockwise]
5. A force of (3i + 2j)N act at a point which has position vector (5i + j)m and a force of (I + j)N act at a point which has position vector (2i + j)m. Find the sum of moments of these forces about the point which has position vector (i + 3j)m. [17Nm anticlockwise]

Couple of forces

These are equal forces acting in opposite direction

Conditions for forces to form a couple

Forces reduce to a couple if;

- resultant force is zero
- the sum of moments about a point is not zero

Example 1

Forces of (-5i – j)N, -3j and (5i + 4j) act on a body a point with position vectors (i – j)m, (2i + j)m and (4i – 5j)m respectively. Show that these forces reduce to a couple

Solution

$$R = \binom{-5}{-1} + \binom{0}{-1} + \binom{5}{4} = \binom{0}{0}$$

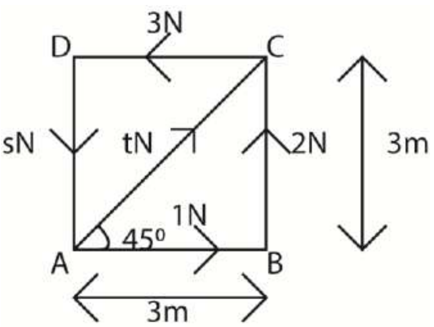
$$G = \begin{vmatrix} 1 & -5 \\ -1 & -1 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} 4 & 5 \\ -5 & 4 \end{vmatrix}$$

= [(1 x -1) – (-5 x -1)]+[(3 x -3) – (1 x 0)] + [(4 x 4) – (5 x -5)] = 29Nm

Since the resultant force is zero an the sum of moment G is not zero, the forces reduce to a couple.

Example 2

ABCD is a square of side 3m. Forces of magnitude 1N, 2N, 3N, sN and tN act along the line AB, BC, CD, DA and AC respectively, in each case the direction of the force being given by the order of letters. Taking AB as horizontal and BC as vertical, find the values of s and t so that the resultant of the forces is a couple.



$$R = \binom{1}{0} + \binom{0}{2} + \binom{-3}{0} + \binom{0}{-s} + \binom{tcos45}{tsin45} = \binom{0}{0}$$

(→) tcos45 = 2; t = $\frac{2}{cos45}$ = $2\sqrt{2}$

(↑) 2 – s + tsin45 = 0; s = 2 + $2\sqrt{2}$ sin45 = 4N

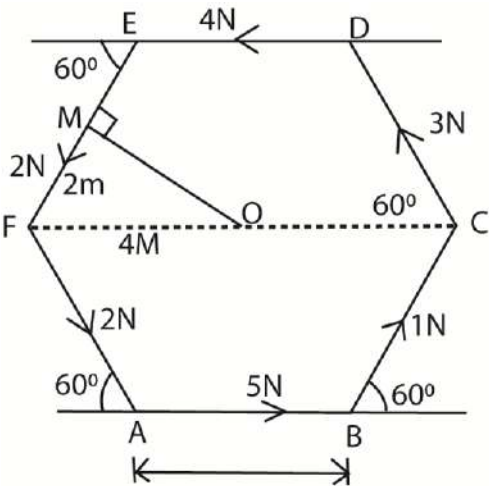
It must also be shown the G≠0

A G = 2 x 3 + 3 x 3 = 15Nm

Example 3

ABCDEF is a regular hexagon of side 4m. Forces of magnitude 5N, 1N, 3N, 4N, 2N and 2N act along the lines AB, BC, CD, DE, EF and FA respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal, show that these forces reduce to a couple.

Solution



OM = $\sqrt{4^2 - 2^2} = 2\sqrt{3}m$

($\rightarrow$) 5 – 4 + (1 – 3 – 2 +2)cos60 = 0

($\uparrow$) (1 + 3 – 2 – 2) sin60 = 0

O G = $2\sqrt{3}$ (5 + 1 + 3 + 4 + 2)

= $34\sqrt{3}$N anticlockwise

Since R = 0 and G$\neq$ 0 then is a couple.

Revision exercise

1. Forces of 6N, 8N, 6N and 8N act along sides of a rectangle ABCD where AB =8m and BC = 6m in the direction AB, BC, CD and DA respectively.
   (a) show that the forces reduce to couple
   (b) find the moment of the couple about A. (100Nm)
2. Forces of 5N, 3N, 5N and 3N act along the side of a square ABCD of side 4m in the directions AB, BC, CD and DA respectively.
   (a) show that the forces reduce to couple
   (b) find the moment of the couple about A. (100Nm
3. ABCD is a rectangle with AB = 6m and BC = 2m. A force of 3N acts along each of the four sides AB, BC, CD and DA in the directions indicated by the order of the letters. Show that the forces form a couple and find its moment. [21Nm]
4. ABCD is a rectangle with AB = 6m and BC = 2m. A forces of 5N, 5N, xN, and xN acts along each of the four sides CB, AD, AB and CD in the directions indicated by the order of the letters. If the system is in equilibrium, find the value of x. [15N]
5. ABCD is a square of side 40cm. Forces of magnitude 20N, 15N, 20N and Y act along the line AB, BC, CD and DA respectively in each case the direction of the force being given by the order of letters. If the system is equivalent to a couple, find the magnitude of Y and the moment of the couple. [15N, 14Nm]
6. A force of (3i – 5j)N acts at a point which has position vector (6i + j)m and a force of (-3i + 5j)N acts at the point which has position vector (4i + j)m. Show that the forces reduce to a couple and find the moment of the couple [10Nm]
7. A force of (4i + 3j)N acts at a point which has position vector (6i + 3j)m and a force of (-4i -3j)N acts at the point which has position vector (3i – j)m. Show that the forces reduce to a couple and find the moment of the couple [7Nm]