A LEVEL

P425/1
MATHEMATICS
Paper I
Nov / Dec 1999
3 hours

SECTION A. (40 marks)

1 . Given that the equation $2x^2 + 5x - 8 = 0$, has roots α and β , find the equation whose roots are

$\frac{1}{(\alpha+2)^2}$ and $\frac{1}{(\beta+2)^2}$ *(04 marks)*

2. The vector equations of two lines are

$r_1 = \begin{pmatrix} 5 \\ -6 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and

$r_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \gamma\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Determine the point where $\mathbf{r_1}$ meets $\mathbf{r_2}$. ***(5 marks)***

3. Solve $\cos(\theta + 35^0) = \sin(\theta + 25^o)$

$\text{for } 0 \le \theta \le 360^0$ ***(5 marks)***

4. The population of a country increases by 2.75 % per annum . How long will it take for the population to tripple ? ***(5 marks)***

5. Differentiate with respect to x :

(i) $3x \ In\ x^2$,

(ii) $\cot 2x$ ***(5 marks)***

6. Evaluate $\int_0^1 \frac{\tan^{-1}(x)}{1 - x^2} dx.$ ***(5 marks)***

7. A curve is defined by the parametric equations

$x = t^2 - t$,

$y = 3t + 4.$

Find the equation of the tangent to the curve at (2 , 10). ***(5 marks)***

8. A cliff which is 100m high , runs in S.E - N.W direction along the coast . From the top of the cliff the angle of depression of a ship moving at a steady

speed of 24kmh^{-1} towards the coast is 08^0. Calculate the distance of the ship from the coast at that instant . What is the angle of elevation of the cliff from the ship one minute later ?

(5 marks)

SECTION B.(60 marks)

9. The locus of p is such that the distance OP is half the distance PR . where O is the origin and R is the point (-3 , 6) .

(i) Show that the locus of P describes a circle in the x- y plane.

(ii) Determine the radius and centre of the circle.

(iii) Where does P cut the line x = 3 ?

10. (a) Solve the equation

$2(3^{2x}) - 5(3^x) + 2 = 0$

(b) The equation of three planes P_1 ,P_2 and P_3 are

$2x - y + 3z = 3$,

$3x + y + 2z = 7$ and

$x + 7y - 5z = 13$ respectively.

Determine where the three planes intersect.

11. If z is a complex number , describe and illustrate on the Argand diagram the locus given by each of the following :

(i) $\left|\frac{z+i}{z-2}\right| = 3$, (ii) Arg $(z+3) = \frac{\pi}{6}$

12. (a) Solve $\sin 3x + 1/2 = \cos^2 x$

for $0 \le x \le 360^0$.

(b) Given that in any triangle ABC

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right),$$

Solve the triangle with two sides 5 and 7 and the included angle 45^0 .

13. A research to investigate the effect of a certain chemicals on a virus infection crops revealed that the rate at which the virus population is destroyed is directly proportional to the population at that time . Initially the population was P_o at months later it was found to be P.

(a) Form a differential equation connecting P and t

(b) Given that the virus population reduced to one third of the initial population in 14 months , solve the equation in (a) above.

(c) Find

(i) how long it will take for only 5 % of the original population to remain .

(ii) What percentage of the original virus population will be left after 2½ months.

14. (i) Find $\int \frac{x^2}{(x^4-1)}dx.$

(ii) Evaluate $\int_0^1 \frac{x}{\sqrt{1+x}}dx$

15. A hemispherical bowl of radius a cm is initially full of water . The water runs out through a small hole at the bottom of the bowl at constant rate such that it empties the in 24 s. Given the depth of water is x cm and the volume of the water is $\frac{1}{3}\pi x^2(3a-x)cm^3$, show that the depth of water at that instant is decreasing at a rate $a^2\lfloor 36x(2a-x)\rfloor^{-1}cms^{-1}$

Find how long it will take for the depth of the water to be at $\frac{1}{3}a$ cm and the rate at which the depth is decreasing at that instant.

16. (a) Find , in cartesian form the equation of the line that passes through the points A (1,2,5) B((1,0,4) and C (5,2,1)

(b) Find the angle between the line

$$\frac{x+4}{8}=\frac{y-2}{2}=\frac{z+1}{-4}$$ and the plane

$$4x + 3y - 3z + 1 = 0$$

P425/2
MATHEMATICS
Paper 2.
Nov / Dec 1999
3 hours.

SECTION A. (40 marks)

1. Given that A and B are mutually exclusive events and

P (A) = $\frac{2}{5}$ and P (B) = $\frac{1}{2}$, find:

(i) P(AUB)

(ii) $P(A\cap\overline{B})$

(iii) $P(\overline{A}\cap\overline{B})$ ***(5 marks)***

2. Four forces a**i** + (a -1) **j** , 3**i** +2a**j** , 5**i**-6j and **-i** - 2**j** act on a particle .

The resultant of the forces makes an angle of 45^o with the horizontal .

Find the value of a .

Hence determine the magnitude of the resultant force. ***(5 marks)***

3. Show by means of a graph that the equation $x + \log_e x = 0.5$ has only one real root that lies between $\frac{1}{2}$ and 1. ***(5 marks)***

4. An overloaded taxi travelling at a constant speed of $90kmh^{-1}$ overtakes a stationary traffic police car . Two seconds later the police car sets off in pursuit of the taxi before , accelerating at a rate of $6ms^{-2}$. How far does the traffic travel before catching up with the taxi? (5 marks)

5. The table below shows the variation of temperature with time in a certain experiment.

Time (s)	0	120	240	360	480	600
Temp. (^{0}C)	100	80	75	65	56	48

Use linear interpolation to find the

(i) value in ^{0}C corresponding to 400s,

(ii) time at which the temperature is 77^{0}C. ***(5 marks)***

6. A box of 4.9kg rests on a rough horizontal plane inclined at an angle of 600 to the horizontal. If the coefficient of friction between the box and the plane is 0.35 , determine the force acting parallel to the plane which will move the box up the plane.

(5 marks)

7.At a bus park, 60% of the buses are Isuzu make , 25% are Styre type and the rest are of Tata make. Of the Isuzu type , 50 % have radios , while for the Styer and Tata types only 5 % and 1% have radios, respectively . If a bus is selected at random from the park , determine the probability that :

(i) it has a radio

(ii) a styer is selected given that it has a radio. ***(5***

marks)

8. Given the variables x and y below,

x	80	75	86	60	75	92	86	50	64	75
y	62	58	60	45	68	68	81	48	50	70

Obtain a rank correlation coefficient between the variable x and y. Comment on your result.

(5 marks)

SECTION B.

9. (a) The area A of a parallelogramme formed by vectors a and b is given by $A|a||b|\sin\theta$, where θ is the angle between the vectors . Find the percentage error made in the area if $|a|$ and $|b|$ are measured with errors of $\pm 0.5^0$,and the angle with an error of $\pm 0.5^0$, given that $|\,\mathbf{a}\,| = 2.5$ cm,

$|\mathbf{b}| = 3.4$ cm and $\theta = 30^o$.

(b) Use the trapezium rule with sub - intervals to estimate $\int_0^{\pi} x \sin x dx$ correct to 3 decimal places. Determine the error in your estimation and suggest how this error may be reduced.

10. (a) A man buys 10 tickets from a total of 200 tickets in a lottery . There is only one prize ticket of shs 10,000.

(i) Find the probability that one of the tickets is a prize ticket.

(ii) If the price of each ticket is Shs 100 and assuming that all tickets were sold , find the expected loss.

(b) A man lives at a point which is 20 minutes walk from the taxi stage. Taxis arrive at the stage punctually. If the probability distribution function for getting a taxi is given by

$$f(x) = \left\{\begin{array}{l} \frac{1}{20}, 0 \le x \le 20 \\ 0, \text{Elsewhere} \end{array}\right\}$$

Determine the :

(i) expected time it takes to wait for a taxi ,

(ii)variance of the time it takes to wait for the taxi.

11.A particle is describing a simple harmonic motion in a straight line direction towards a fixed point O. When its distance from O is 3m its velocity is $25ms^{-1}$ and its acceleration $75ms^{-2}$. Determine the

(i) period and amplitude of oscillation,

(ii) time taken by the particle to reach O.

(iii) velocity of the particle as it passes through O.

12. (i) Show that the iterative formula for approximating the root of f(x) = 0 by the Newton - Raphson process for the equation $x_e^x + 5x - 10 = 0$ is

$$x_{n+1} = \frac{x_n^2 e^{x_n} + 10}{e^{xn}\left(x_n + 1\right) + 5} .$$

(ii) Show that the root of the equation in (i) above lies between 1 and 2. Hence find the root of the equation. Correct your answer to 2 decimal places.

13. A factory produces two types of bars of soap ,A and B. Their lengths are normally distributed with type A having average length of 115cm and standard deviation 3cm . Type B have an average length 190 cm and standard deviation 5 cm.

(a) Determine the percentage of type
(i) A bars of that have a length of more than 120 cm,
(ii) B bars of soap that have a length of more than 180cm.
(b) Find the 95% confidence limits for the mean of length of type A bars of soap.

14.A rod AB of length 0.6 m long and mass 10kg is hinged at A . Its centre of mass is 0.5m from A . Alight inextensible string attached at B passes over a fixed smooth pulley o.8m above A and supports a mass M hanging freely. If a mass of 5kg is attached at B so as to keep the rod in a horizontal position, find the

(i) value of M

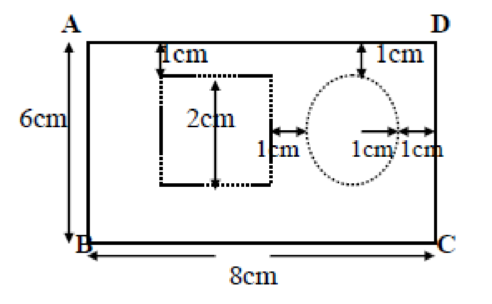
(ii) reaction at the hinge.

15. When a biased tetrahedron is tossed the probability that any of its face shows up is proportional to the square of the number on the face that shows up.

(i) Find the probability with which each of the numbers 1,2,3 and 4 on the faces of the tetrahedron appear.

(ii) If there independent tosses of the tetrahedron are made , what is the probability that the sum of the numbers on the faces that show up is a 3 or a 5.

16.



ABCD is a uniform rectangular sheet of cardboard of length 8cm and width 6cm. A square and circular hole are cut off from the cardboard as shown above. Calculate the position of the centre of gravity of the remaining sheet.