P425/1
PURE MATHEMATICS
Paper 1
3 hours
S.5 Sciences 2024
Term II 2024, Test 1

INSTRUCTIONS:
Answer **all** *questions in section* **A** *and any* **five** *from section* **B**.
Any additional question(s) answered will **not** *be marked.*
All necessary working **must** *be shown clearly.*
Silent, non-programmable scientific calculators and mathematical tables may be used.

SECTION A; (40 MARKS)

Answer **all** *questions from this section.*

1. Solve the equation $4\,sin^2\,2\theta = 1$ for $0° \leq \theta \leq 360°$ (*05 marks*)

2. Find the greatest value of $\boldsymbol{\mu}$ for which the equation $(\boldsymbol{\mu} - 1)x^2 - 2x + (\boldsymbol{\mu} - 1) = 0$ has real roots. (*05 marks*)

3. If $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are roots of the equation $5x^2 - 3x - 1 = 0$, form an equation with integral coefficients whose roots are $\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\beta}}$ and $\frac{\boldsymbol{\beta}^2}{\boldsymbol{\alpha}}$. (*05 marks*)

4. Evaluate S_5 for the series $3 + 9 + 27 + 81 + \cdots$ (*05 marks*)

5. The coefficient of x^5 in the expansion of $(1 + 5x)^8$ is equal to the coefficient of x^4 in the expansion of $\frac{1}{(\boldsymbol{K}+5x)^{-7}}$.
Find the value of $\boldsymbol{K}$. (*05 marks*)

6. Solve for $\boldsymbol{x}$ if $0.7 = 1.2^{2\boldsymbol{x}-1}$ (*05 marks*)

7. Prove by induction that for all positive integral values of $\boldsymbol{n}$, $10^{\boldsymbol{n}} - 1$ is a multiple of 9. (*05 marks*)

8. Find the possible values of $\boldsymbol{P}$ in the equation $\boldsymbol{P} + \sqrt{\boldsymbol{P}} = \frac{6}{25}$. (*05 marks*)

SECTION B; (60 MARKS)

Answer any **five** *questions from this section.*
All questions carry equal marks

9. (a) Given that $f(x) = x^3 + \boldsymbol{k}x^2 - 2x + 1$; When $f(x)$ is divided by $(x - \boldsymbol{k})$, the remainder is $\boldsymbol{k}$. Find the possible values of k. (*05 marks*)

(b) Express the polynomial $2x^4 + x^3 - x^2 + 8x - 4$ as a product of two linear factors and a quadratic factor $P(x)$. Prove that these are no real values of x for which $P(x) = 0$ (*07 marks*)

10.(a) Eighty coins are placed in a line on the ground. The distance between any two consecutive coins is 10 metres. How far must a person travel to bring them one by one to a basket placed 10 metres behind the first coin? (*06 marks*)

(b) Find three numbers in a G.P such that their sum is 19 and their product is 216. (*06 marks*)

11.(a) Expand $(4 + x)^{-\frac{1}{2}}$ in ascending power of x stating the first four terms. Hence deduce the approximate value of $\frac{1}{\sqrt{\left(\frac{416}{100}\right)}}$. (*07 marks*)

(b) In how many ways can the letters of the word MOBILE be arranged so that the consonants always occupy the odd places. (*05 marks*)

12.(a) Express $\frac{11x-1}{(1-x)^2(2+3x)}$ in partial fractions. (*07 marks*)

(b) Given that $\frac{3x-4}{x^2+x-12} = \frac{A}{x-3} + \frac{B}{x+4}$. Find the values of A and B. (*05 marks*)

13. Solve the Equations:

(a) $\log_x 32 - \log_{256} x = 1$ (*06 marks*)

(b) Given that the line with parametric equations $x = 2t^2 - 1$ and $y = 3(t + 1)$ intersects with the line $3x - 4y = 3$. Find the possible values of t. Hence the coordinates of the points of intersection. (*06 marks*)

14.(a) Solve the equation: $9x^4 - 45x^2 = 324$

(b) Find the square root of $6 + 14\sqrt{5}$ (*12 marks*)

END