1

$\frac{x}{y} + \frac{6y}{x} = 5$ —— ① and $x - 2y - 2 = 0$ —— ②

from ② $x = 2y + 2$ —— ⊗

Put ⊗ in ①

$\frac{(2y+2)}{y} + \frac{6y}{(2y+2)} = 5$

$\frac{(2y+2)^2 + 6y^2}{y(2y+2)} = 5$

$(2y+2)^2 + 6y^2 = 5y(2y+2)$

$4y^2 + 4 + 8y + 6y^2 = 10y^2 + 10y$

$10y^2 + 8y + 4 = 10y^2 + 10y$

$4 = 10y - 8y$

$4 = 2y$

$y = 2$, $x = 6$

Point of Intersection is $(6, 2)$.

2

$$\int \frac{x}{\sqrt{1+x}}\,dx$$

$$\text{let } u = \sqrt{1+x}$$

$$u^2 = 1+x$$

$$2u\,du = dx$$

$$\int \frac{x}{\sqrt{1+x}}\,dx = \int \frac{(u^2-1)\cdot 2u\,du}{u}$$

$$= 2\int (u^2-1)\,du$$

$$= 2\int u^2\,du - 2\int 1\,du$$

$$= \frac{2u^3}{3} - 2u + C$$

$$= \frac{2}{3}(1+x)^{3/2} - 2(1+x)^{1/2} + C$$

$$= \frac{2}{3}\sqrt{1+x}\left[(1+x)^{1} - 6\right] + C$$

$$= \frac{2}{3}\sqrt{1+x}\,(x-5) + C$$

$\sin x + \sin y = \beta_1$ ---- (i)

$\cos x + \cos y = \beta_2$ ---- (ii)

i) $\tan\left(\frac{x+y}{2}\right) = ?$

$\sin x + \sin y = \beta_1$

Use factor formula:

$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_1$ ---- (iii)

$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_2$ ---- (iv)

Eqn iii ÷ Eqn iv

$\tan\left(\frac{x+y}{2}\right) = \frac{\beta_1}{\beta_2}$ ———— $\boxed{\cdot}$

ii) $\cos(x+y) = \frac{\beta_2^2 - \beta_2^2}{\beta_2^2 + \beta_1^2}$

$\cancel{\cos(x+y)} \cancel{= \cos x \cos y}$

$\cos x + \cos y = \beta_2$ —— ① $\sin x + \sin y = \beta_1$ —— ②

Square both sides of eqn ① and ② and then add them to form eqn ③

$\cos^2 x + \cos^2 y + 2\cos x\cos y = \beta_2^2$

$\sin^2 x + \sin^2 y + 2\sin x \sin y = \beta_1^2$

Adding;

$(\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x\cos y + \sin x \sin y) = \beta_2^2 + \beta_1^2$ —— ③

Square both sides of eqn ① and ② and then subtract them to form eqn ④

$(\cos^2 x - \sin^2 x) + (\cos^2 y - \sin^2 y) + 2(\cos x \cos y - \sin x \sin y) = \beta_2^2 - \beta_1^2$ —— ④

Eqn ① ÷ Eqn ②

$$\Longleftrightarrow \frac{(\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y + \sin x \sin y)}{(\cos^2 x - \sin^2 x) + (\cos^2 y - \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)} = \frac{\beta_2^2 + \beta_1^2}{\beta_2^2 - \beta_1^2}$$

$$\frac{1 + 1 + 2(\cos x \cos y + \sin x \sin y)}{\cos 2x + \cos 2y + 2(\cos x \cos y - \sin x \sin y)} = \frac{\beta_2^2 + \beta_1^2}{\beta_2^2 - \beta_1^2}$$

$$\frac{2 + 2\cos(x-y)}{\cos 2x + \cos 2y + 2\cos(x+y)} = \frac{\beta_2^2 + \beta_1^2}{\beta_2^2 - \beta_1^2}$$

$$\frac{2 + 2\cos(x-y)}{2\cos\left(\frac{2x+2y}{2}\right)\cos\left(\frac{2x-2y}{2}\right) + 2\cos(x+y)} = \frac{\beta_2^2 + \beta_1^2}{\beta_2^2 - \beta_1^2}$$

$$\frac{2(1 + \cos(x-y))}{2\cos(x+y)(1 + \cos(x-y))} = \frac{\beta_2^2 + \beta_1^2}{\beta_2^2 - \beta_1^2}$$

$$\frac{1}{\cos(x+y)} = \frac{\beta_2^2 + \beta_1^2}{\beta_2^2 - \beta_1^2}$$

$$\cos(x+y) = \frac{\beta_2^2 - \beta_1^2}{\beta_2^2 + \beta_1^2} \quad \boxed{\cdot}$$

4.

$\sqrt{27.17} = ??$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$

$$= \frac{1}{2\sqrt{x}}$$

$$y + \partial y = \sqrt{x + \partial x}$$

$x = 27$, $\delta x = 0.15$, $y = \sqrt{x}$

$$\frac{\partial y}{\partial x} \simeq \frac{dy}{dx}$$

$$\partial y \approx \frac{dy}{dx} \cdot \partial x$$

$$\approx \frac{1}{2\sqrt{x}} \cdot \partial x$$

$$\approx \frac{1}{2\sqrt{27}} \cdot (0.15)$$

$$\approx 0.0144$$

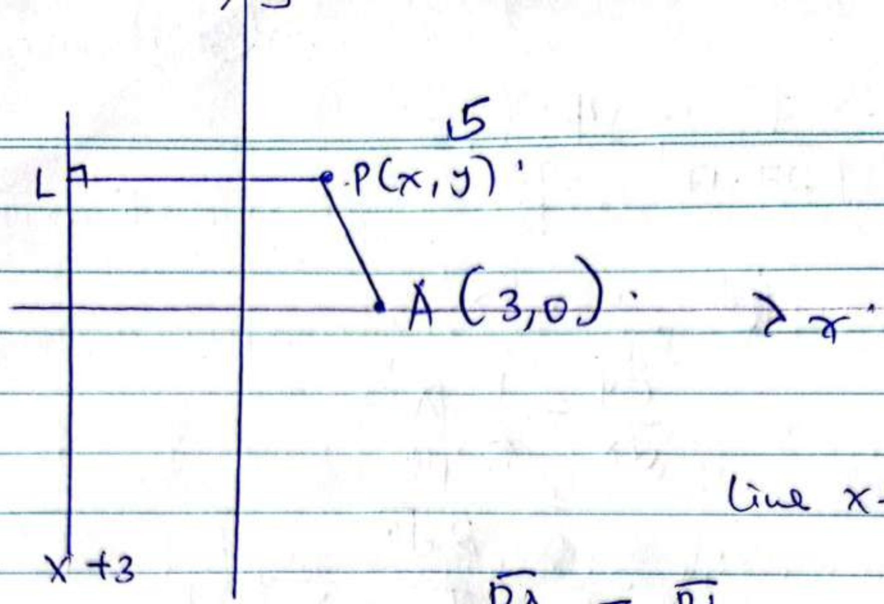
$$y + \delta y = \sqrt{x} + \delta y$$

$$\cancel{= \sqrt{5/1}}$$

$$= \sqrt{27} + \delta y$$

$$= 5.1962 + 0.0144$$

$\sqrt{27.15} \simeq 5.2106$ 4dp.



Line $x+3=0$.

$$\overline{PA} = \overline{PL}$$

$$\sqrt{(x-3)^2+(y-0)^2} = \left|\frac{ax+by+c}{\sqrt{a^2+b^2}}\right|$$

$$\sqrt{(x-3)^2+y^2} = \left|\frac{1(x)+0(y)+1}{\sqrt{1^2+0^2}}\right|$$

$$\left(\sqrt{(x-3)^2+y^2}\right)^2 = (x+1)^2$$

$$x^2-6x+9+y^2 = x^2+2x+1$$

$$-6x+9+y^2 = 2x+1$$

$$-8x+9+y^2 = 1$$

$$y^2 = 8x+1-9$$

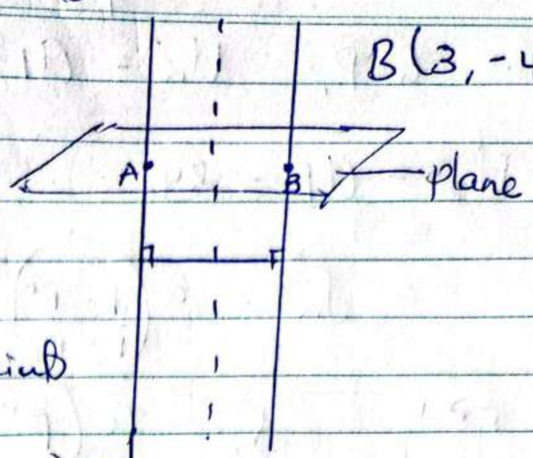
$$y^2 = 8x-8$$

The Locus is a parabola.

6.

A(-2, 0, 6)

B(3, -4, 5)



1. A and B are points on ∥ lines
2. ∠ btn these lines is zero.
3. They have a plane $\overline{AB}$ btn them.

If we divide the plane into 2 regions;
⇒ One point gives a value less and the other a value more.

i.e $2x - y + 3z = 21$

$2x - y + 3z - 21 = 0$ eqn of plane

for A(-2, 0, 6)

$2x - y + 3z - 21$

$2(-2) - (0) + 3(6) - 21$

$-7 < 0$

for B(3, -4, 5)

$2x - y + 3z - 21$

$2(3) - (-4) + 3(5) - 21$

$4 > 0$

Here.

If $A(-2, 0, 6) = A(x_1, y_1, z_1)$

$B(3, -4, 5) = B(x_2, y_2, z_2)$

$2x_1 - y_1 + 3z_1 - 21 < 0$

and $2x_2 - y_2 + 3z_2 - 21 > 0$

then the two points A and B lie on the opp side of the plane $2x - y + 3z = 21$

for G.P $\quad U_n = ar^{n-1}$

$$S_n = \frac{a(r^n - 1)}{(r-1)} \text{ for } r > 1$$

$U_2 = ar \qquad U_3 = ar^2 \qquad U_4 = ar^3$

But $\frac{U_4}{U_3} = \frac{U_3}{U_2} = \frac{U_2}{U_1} = r$ ——— ①

$U_2 = 24 = ar \qquad U_3 = ar^2 = 12(b+1)$

from ①; $\frac{ar^3}{ar^2} = \frac{ar^2}{ar} = \frac{ar}{a} = r$

$$\frac{ar^3}{12(b+1)} = \frac{12(b+1)}{24} = \frac{24}{a} = r$$

for 3 terms; $S_3 = \frac{a(r^3 - 1)}{(r-1)} = \frac{ar^3 - a}{(r-1)}$

$$76 = \frac{ar^3 - a}{(r-1)} \text{ ——— ①}$$

But if $\frac{ar^2}{ar} = \frac{ar}{a}$ then $\frac{12(b+1)}{24} = \frac{24}{a}$

$$a\left(\frac{12(b+1)}{24}\right) = 24$$

$$a(12(b+1)) = 24^2$$

$$12a(b+1) = 576 \text{ ——— ②}$$

also $\frac{ar^3}{12(b+1)} = \frac{12(b+1)}{24}$; $ar^3 = \frac{144(b+1)^2}{24}$.

$$\therefore 76 = \frac{\frac{144(b+1)^2}{24} - a}{(r-1)}$$

$$76(r-1) = 6(b+1)^2 - a \text{ ——— ③}$$

from eqn ②; $12a(b+1) = 576$

$$12(b+1) = \frac{576}{a}$$

$$12(b+1) = \frac{24}{a} \times 24$$

$$12(b+1) = r \times 24$$

$$(b+1) = 2r \quad \text{——— ④}$$

also $\frac{24}{a} = r$; $a = \frac{24}{r}$ ——— ⑤

put ⑤ into ③

$$76(r-1) = 6(b+1)^2 - a$$

$$(76r - 76) = 6(b+1) - \frac{24}{r}$$

$$76r^2 - 76r = 6r(b+1) - 24 \quad \text{——— ⑥}$$

Put ④ in ⑥.

$$76r^2 - 76r = 6r(2r) - 24$$

$$76r^2 - 76r = 12r - 24$$

$$76r^2 - 76r - 12r + 24 = 0$$

$$76r^2 - 64r + 24 = 0$$

$r_1 = 0.42$ $\qquad$ $r_2 = 0.42$.

$$\therefore r = \frac{8}{19}$$

$$a = 57$$

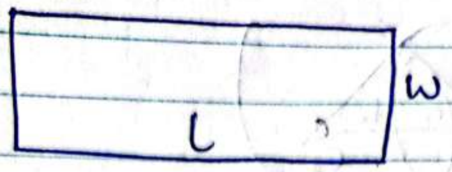
$$b+1 = 2r$$

$$b = 2r - 1 = 2\left(\frac{8}{19}\right) - 1$$

$$\therefore b = \frac{-3}{19}$$

8.

Rectangle



let the rectangle be with length l and width w and Area A.

$$\text{Area of rectangle} = (\text{length}) \cdot (\text{width})$$

$$A = lw \quad \text{——— (2)}$$

If 200m of wire make this rectangle

$$\Rightarrow P = 200m$$

$$200 = 2l + 2w$$

$$100 = l + w$$

$$l = (100 - w) \quad \text{——— (1)}$$

Put (1) into (2)

$$A = (100 - w)w$$

$$A = 100w - w^2$$

Differentiating both sides w.r.t w.

$$\frac{dA}{dw} = 100 - 2w$$

at Max, for largest Area; $\frac{dA}{dw} = 0$

$$100 - 2w = 0$$

$$w = 50m$$

$$\Rightarrow l = 50m$$

$$A = lw = 50 \times 50 = 50^2$$

$$\text{Area of the largest piece} = \underline{\underline{2500 \, m^2}}$$

9

9) $$\frac{\cos(B+C)}{\operatorname{cosec} B \operatorname{cosec} C} = \frac{bc}{ab+ac}$$

R·H·S

Using sine Rule; $b = 2R\sin B$
$a = 2R\sin A$
$c = 2R\sin C$

$$\frac{bc}{ab+ac} = \frac{2R\sin B \cdot 2R\sin C}{(2R\sin A\, 2R\sin B) + (2R\sin A\, 2R\sin C)}$$

$$= \frac{4R^2(\sin B \sin C)}{4R^2(\sin A\sin B + \sin A\sin C)}$$

$$= \frac{\sin B\sin C}{\sin A\sin B + \sin A\sin C}$$

divide the numerator and denominator by $\sin B\sin C$

$$= \frac{\dfrac{\sin B\sin C}{\sin B\sin C}}{\dfrac{\sin A\sin B}{\sin B\sin C} + \dfrac{\sin A\sin C}{\sin B\sin C}}$$

$$= \frac{1}{\sin A\left(\dfrac{1}{\sin C} + \dfrac{1}{\sin B}\right)}$$

$$= \frac{1}{\sin A(\operatorname{cosec} C + \operatorname{cosec} B)}.$$

$$= \frac{\dfrac{1}{\sin A}}{(\operatorname{cosec} C + \operatorname{cosec} B)}$$

for a triangle $A+B+C = 180$

$\sin A = \sin(180-(B+C)) = \sin(B+C)$

$$\frac{\frac{1}{\sin(B+C)}}{}$$

$$= \frac{}{\operatorname{Cosec} C + \operatorname{cosec} B}$$

$$= \frac{\operatorname{Cose}(B+C)}{\operatorname{Cosec} B + \operatorname{cosec} C} \text{ ——— } \boxed{\cdot}$$

b) $3\cot\theta + \operatorname{Cosec}\theta = 2$ $\qquad 0° \leq \theta \leq 360°$

$$3\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = 2$$

$$3\cos\theta + 1 = 2\sin\theta.$$

$$3\cos\theta - 2\sin\theta = 1$$

L-HS:

$$3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$$

$$3\cos\theta - 2\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

Equate corresponding term:

$R\cos\theta\cos\alpha = 3\cos\theta$; $\quad R\cos\alpha = 3$ ——— (1)

$-R\sin\theta\sin\alpha = -2\sin\theta$; $\quad R\sin\alpha = 2$ ——— (2)

Eqn (2) $\div$ Eqn (1) $\quad \alpha = \tan^{-1}(2/3)$

$$\alpha = 33.7°$$

$$(\text{Eqn}(2))^2 + (\text{Eqn}(1))^2$$

$$R^2(\sin^2\alpha + \cos^2\alpha) = 4 + 9$$

$$R = \sqrt{13}$$

$\Rightarrow \quad 3\cos\theta - 2\sin\theta = 1$

Becomes $\quad R\cos(\theta + \alpha) = 1$

$$\sqrt{3}\cos(\theta + 33.7) = 1$$

$$\theta + 33.7 = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°, 305.3°$$

$$\theta = (54.7 - 33.7)°, (305.3 - 33.7)°$$

$\therefore \theta = 21°$, and 271.6

a) $$\frac{\left[\sqrt{3}\,(\cos\theta + i\sin\theta)\right]^{8}}{\left[3(\cos 2\theta + 3i\sin 2\theta)\right]^{3}}$$ 10

But by Demoivre,

$$\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^{n}$$

$$\Rightarrow \frac{(\sqrt{3})^{8}\,(\cos 8\theta + i\sin 8\theta)}{(3^{3})\,(\cos 2\theta + \cancel{3}i\sin 2\theta)^{3}}$$

$$= \frac{81}{27}\,\frac{(\cos 8\theta + i\sin 8\theta)}{(\cos 6\theta + i\sin 6\theta)}$$

$$= 3\,\frac{(\cos\theta + i\sin\theta)^{8}}{(\cos\theta + i\sin\theta)^{6}}$$

$$= 3\,(\cos\theta + i\sin\theta)^{8-6}$$

$$= 3\,(\cos\theta + i\sin\theta)^{2}$$

$$= 3\,(\cos 2\theta + i\sin 2\theta)$$ done.

b) Let $z = x + iy$.

from $(1+3i)\,z_1 = 5(1+i)$

$$z_1 = \frac{5(1+i)}{(1+3i)} \cdot \frac{(1-3i)}{(1-3i)}$$

$$= \frac{5\left[1 - 3i + i - 3(i)^{2}\right]}{\left[1 - 3i + 3i - 9(i)^{2}\right]}$$

$$= \frac{5\left[1 - 2i + 3\right]}{\left[1+9\right]}$$

$$= \frac{5}{10}(4 - 2i) = \frac{20}{10} - \frac{10}{10} i$$

$$z_1 = 2 - i$$

if $z = x + iy$

then; $|z - z_1| = |z_1|$

$$|x+iy - (2-i)| = |2-i|$$

$$|(x-2) + i(y+1)| = |2-i|$$

$$\left(\sqrt{(x-2)^2 + (y+1)^2}\right)^2 = \left(\sqrt{(2)^2 + (-1)^2}\right)^2$$

$$x^2 + 4 - 4x + y^2 + 2y + 1 = 5$$

$$x^2 + y^2 - 4x + 2y + 5 = 5$$

$$x^2 + y^2 - 4x + 2y = 0 \quad \text{---} (1)$$

Hence its a circle.

$C(-g, -f)$

Compare eqn (1) with $x^2 + y^2 + 2gx + 2fy + c = 0$

$2gx = -4x$ $\qquad$ $2fy = 2y$

$g = -2$ $\qquad$ $f = 1$

$$C(-g, -f) = C(2, -1)$$

radius $r = \sqrt{g^2 + f^2 - c}$

$$= \sqrt{(2)^2 + (-1)^2 - (0)}$$

$$r = \sqrt{5}$$

$\therefore$ the locus of $|z - z_1| = |z_1|$ where z and z_1 are complex is a circle with centre $C(2, -1)$ and radius $r = \sqrt{5}$

11

a) Let A.P

$U_1 = x$

$d = ?$

$d = 2$

G.P.

$U_1 = x$

$r = \frac{1}{3}$

$S_\infty = \frac{a}{(1-r)} = 9$

$S_{10} = ?$ Note $S_n = \frac{n}{2}[2a + (n-1)d]$

for an A.P

$U_n = a + (n-1)d$

$U_1 = a = x$

for G.P $U_n = ar^{n-1}$, $U_1 = x$.

$x = a$

for $S_\infty = \frac{x}{1-\frac{1}{3}} = 9$

$x = 6$

$\Rightarrow a = x = 6$

$d = 2$.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 6 + (10-1)2]$

$= 150$

b)

"DEFEATED"

If the E's are together, they are counted as 1 letter

$$= \frac{(6+1)!}{3!}$$

$= 840$ ways

Total no of ways to be arrange

$= 8!$

$= 40320$ ways

No ways for the E's are seperated = Total − (No when they r together)

$= 40320 - 840$

$= 39,480$ ways. possible.

12.

a) $A(1,1,1)$ $B(1,0,1)$ $C(3,2,-1)$

$$AB = \begin{pmatrix}1\\0\\1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\-1\\0\end{pmatrix}$$

$$AC = \begin{pmatrix}3\\2\\-1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$$

$$\begin{pmatrix}p\\q\\r\end{pmatrix} \cdot \begin{pmatrix}0\\-1\\0\end{pmatrix} = \quad -q = 0$$

$$\begin{pmatrix}p\\q\\r\end{pmatrix} \cancel{x} \begin{pmatrix}2\\1\\-2\end{pmatrix} = 0 \quad ; \quad 2p + q - 2r = 0$$

$$2p - 2r = 0$$

$$p = r$$

$$\begin{pmatrix}p\\q\\r\end{pmatrix} = \begin{pmatrix}r\\0\\r\end{pmatrix} = r\begin{pmatrix}1\\0\\1\end{pmatrix}$$

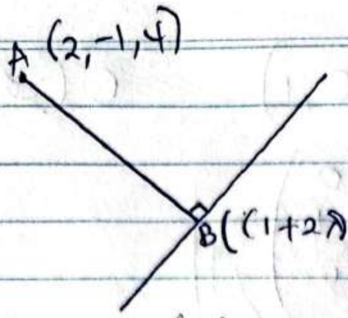
$$n = \begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$n \cdot r = n \cdot a$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} \cdot \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$x + 0y + z = 1 + 0 + 1$$

$x + z - 2 = 0$ is the Required plane



b) $r = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1+2\lambda \\ -\lambda \\ 2+2\lambda \end{pmatrix}$

$B((1+2\lambda), -\lambda, (2+2\lambda))$.

$$AB \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 0.$$

$$AB = OB - OA$$

$$= \begin{pmatrix} 1+2\lambda-2 \\ -\lambda - -1 \\ 2+2\lambda-4 \end{pmatrix} = \begin{pmatrix} -1+2\lambda \\ 1-\lambda \\ -2+2\lambda \end{pmatrix}.$$

$$AB \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 0.$$

$$\begin{pmatrix} -1+2\lambda \\ 1-\lambda \\ -2+2\lambda \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 0.$$

$$2(-1+2\lambda) + 1(1-\lambda) + 2(-2+2\lambda) = 0$$

$$-2+4\lambda+1-\lambda-4+4\lambda = 0 \quad ; \lambda = \cancel{4}\ 5/7.$$

$$AB = \begin{pmatrix} -1+2\times 5/7 \\ 1-5/7 \\ -2+2\times 5/7 \end{pmatrix} = \begin{pmatrix} 3/7 \\ 2/7 \\ -4/7 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

eqn of it $\Rightarrow \underset{\sim}{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3/7 \\ 2/7 \\ -4/7 \end{pmatrix}$

length of GAr $= \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{-4}{7}\right)^2}$

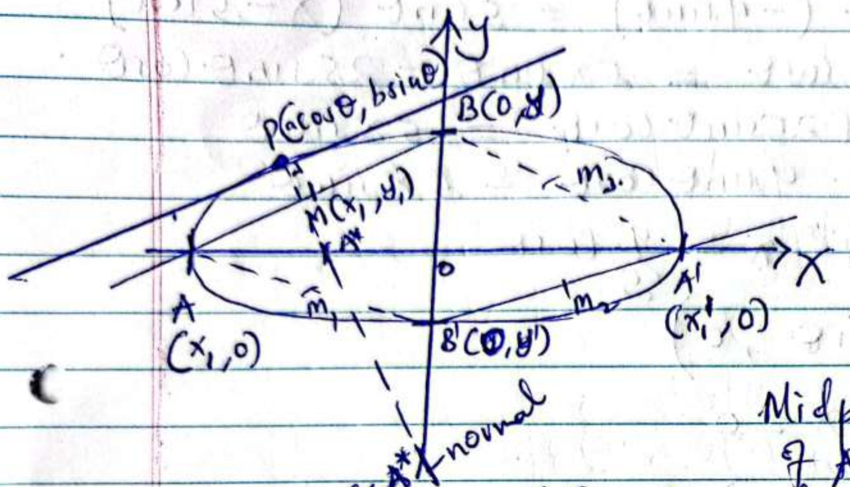
$$= \frac{\sqrt{29}}{7} = 0.7693 \text{ Units}.$$

13

a) $P(5\cos\theta, 4\sin\theta)$ Ellipse is; $\frac{x^2}{25}+\frac{y^2}{16}=1$

from $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Compare with $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$

$b=4$ $a=5$ or $b=5$; $a=4$



Midpoint of AB = (x_1+ can either be M, M_1, M_2, M_3

First obtain $A^*(x,y)$ and $B^*(x,y)$.
$A^*(x,0)$ and $B^*(0,y)$.

It's the normal that cuts the $x-y$ axes,
at $P(5\cos\theta, 4\sin\theta)$

$x = 5\cos\theta$ $\qquad$ $y = 4\sin\theta$

$\frac{dx}{d\theta} = -5\sin\theta$ $\qquad$ $\frac{dy}{d\theta} = 4\cos\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx}$$

grad; $M = 4\cos\theta \cdot \frac{-1}{5\sin\theta} = \frac{-4\cos\theta}{5\sin\theta}$

grad of normal, $M_1 = \frac{-1}{M} = \frac{-1}{\left(-\frac{4\cos\theta}{5\sin\theta}\right)}$

$\therefore$ grad, $M_1 = \frac{5\sin\theta}{4\cos\theta}$

Equation of normal
$(y-y_1) = m_1(x-x_1)$.

for $P(a\cos\theta, b\sin\theta) = P(5\cos\theta, 4\sin\theta)$

$$(y - 4\sin\theta) = \frac{5\sin\theta}{4\cos\theta}(x - 5\cos\theta)$$

$$4\cos\theta(y - 4\sin\theta) = 5\sin\theta(x - 5\cos\theta)$$

at $A^*(x, 0)$; $y = 0$

$$\therefore 4\cos\theta(-4\sin\theta) = 5\sin\theta(x - 5\cos\theta)$$

$$-16\sin\theta\cos\theta = 5x\sin\theta - 25\sin\theta\cos\theta.$$

$$-16\sin\theta\cos\theta + 25\sin\theta\cos\theta = 5x\sin\theta$$

$$9\sin\theta\cos\theta = 5x\sin\theta$$

$$x = \frac{9}{5}\cos\theta.$$

$$A^*\left(\tfrac{9}{5}\cos\theta, 0\right)$$

at $B^*(0, y)$; $x = 0$

$$4\cos\theta(y - 4\sin\theta) = 5\sin\theta(-5\cos\theta)$$

$$4y\cos\theta - 16\sin\theta\cos\theta = -25\sin\theta\cos\theta$$

$$4y\cos\theta = -25\sin\theta\cos\theta + 16\sin\theta\cos\theta$$

$$4y\cos\theta = -9\sin\theta\cos\theta$$

$$y = -\frac{9}{4}\sin\theta$$

$$B^*\left(0, -\tfrac{9}{4}\sin\theta\right).$$

$$\text{Mid point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{\frac{9}{5}\cos\theta + 0}{2}, \frac{0 + -\frac{9}{4}\sin\theta}{2}\right)$$

$$= \left(\frac{9\cos\theta}{10}, -\frac{9}{8}\sin\theta\right)$$ is the Mid point Required.

14. a)

$$\int \frac{x^2+1}{x^3+4x^2+3x}\,dx \;;$$

But $\frac{x^2+1}{x^3+4x^2+3x} = \frac{x^2+1}{x(x^2+4x+3)} = \frac{x^2+1}{(x)(x+3)(x+1)}$

Expres / Putting partial fraction.

$$\frac{x^2+1}{x(x+3)(x+1)} \equiv \frac{A}{x} + \frac{B}{(x+3)} + \frac{C}{(x+1)}$$

$$\frac{(x^2+1)}{x(x+3)(x+1)} \equiv \frac{A(x+3)(x+1) + B(x)(x+1) + C(x)(x+3)}{(x)(x+3)(x+1)}$$

$x^2+1 \equiv A(x+3)(x+1) + B(x)(x+1) + Cx(x+3).$

Put $x=-3$;

$10 = 6B \;; \quad B = 5/3$

Put $x=-1$;

$2 = -2C \;; \quad C = -1$

Put $x=0$;

$1 = 3A \;; \quad A = 1/3$

$$\Rightarrow \frac{x^2+1}{x(x+3)(x+1)} = \frac{1}{3x} + \frac{5}{3(x+3)} - \frac{1}{(x+1)}$$

$$\int \frac{x^2+1}{x(x+3)(x+1)}\,dx = \frac{1}{3}\int \frac{1}{x}\,dx + \frac{5}{3}\int \frac{1}{x+3}\,dx - \int \frac{1}{(x+1)}\,dx.$$

$$= \frac{1}{3}\ln x\Big|_1^3 + \frac{5}{3}\ln(x+3)\Big|_1^3 - \ln(x+1)\Big|_1^3$$

$$= \frac{1}{3}\Big[\ln x + \ln(x+3)^5 - \ln(x+1)^3\Big]_1^3$$

$$= \frac{1}{3}\left[\ln\left[\frac{x(x+3)^5}{(x+1)^3}\right]\right]_1^3$$

$$= \frac{1}{3}\left[\left(\ln\left(\frac{3(6)^5}{4^3}\right)\right) - \left(\ln\left(\frac{1(4)^5}{2^3}\right)\right)\right]$$

$$= \frac{1}{3}(5.89853 - 4.85203)$$

$$= 0.34883 \text{ 5 dp}$$

(b)

$$\int \frac{1}{3x^2+5x+4}\,dx$$

$$\cancel{\text{But } 3x^2+5x+4 \equiv \left(x+\tfrac{5}{3}\right)^2 + 4 - \tfrac{25}{9}}$$

$$\cancel{\equiv \left(x+\tfrac{5}{3}\right)^2 + \tfrac{11}{9}}$$

$$\Rightarrow \int \frac{1}{3x^2+5x+4}\,dx \;;$$

But;

$$3x^2+5x+4 \equiv x^2 + \tfrac{5}{3}x + \tfrac{4}{3}$$

$$\equiv \left(x+\tfrac{5}{6}\right)^2 + \tfrac{4}{3} - \tfrac{25}{36}$$

$$\equiv \left(x+\tfrac{5}{6}\right)^2 + \frac{23}{36}$$

$$\Rightarrow \int \frac{1}{3x^2+5x+4}\,dx = \int \frac{1}{\frac{23}{36} + \left(x+\frac{5}{6}\right)^2}\,dx.$$

$$= \int \frac{1}{\frac{23}{36}\left(1 + \frac{\left(x+\frac{5}{6}\right)^2}{\frac{23}{36}}\right)}$$

$$= \frac{36}{23}\int \frac{1}{\left(1 + \left(\frac{x+5/6}{\sqrt{23}/6}\right)^2\right.} dx$$

$$\text{let } \tan\theta = \frac{(x+5/6)}{\sqrt{23}/6}$$

$$\tan\theta = \frac{6}{\sqrt{23}}(x+5/6)$$

$$\sec^2\theta \, d\theta = \frac{6}{\sqrt{23}} dx.$$

$$\frac{\sqrt{23}}{6}\sec^2\theta \, d\theta = dx.$$

$$\frac{36}{23}\int \frac{1}{\left(1 + \left(\frac{x+5/6}{\sqrt{23}/6}\right)^2\right.} dx = \frac{36}{23}\int \frac{1}{\cancel{1+\tan^2\theta}} \cdot \frac{\sqrt{23}}{6}\cancel{\sec^2\theta} \, d\theta$$

$$= \frac{36}{23}\int \frac{\sqrt{23}}{6} \cdot 1 \, d\theta$$

$$= \frac{6\sqrt{23}}{23}\theta + c$$

$$= \frac{6\sqrt{23}}{23}\tan^{-1}\left[\frac{\left(x+5/6\right)}{\sqrt{23}/6}\right] + c$$

15 a)

each year $\longrightarrow$ 15 people die.

Popn ↑se is prop people prese.

let N be the no of people present.

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -KN$$

$$\int \frac{1}{N}\, dN = \int -K\, dt$$

$$\ln N = -Kt + C$$

$$\boxed{N = e^{-Kt} + e^{C} \quad \text{let } e^{C} = N_0 - \text{constant}}$$

Initially, $t = 0$ $\quad N = 120$ people

$$\ln(120) = C$$

$$\Rightarrow \ln N = -Kt + \ln 120$$

$$\ln N - \ln 120 = -Kt$$

$$\ln\left(\frac{N}{120}\right) = -Kt$$

$$\frac{N}{120} = e^{-Kt}$$

$$N = 120\, e^{-Kt} \text{ ---------- } (1)$$

at $t = 1$ year, $\quad N = 210$

from eqn (1) $\Rightarrow$ $N = 120\, e^{-Kt}$

$$210 = 120\, e^{-K\times 1}$$

$$\left(\frac{210}{120}\right) = e^{-K}$$

$$K = -0.5596$$

$$\therefore N = 120\, e^{0.5596t} \text{ ---------- } (2)$$

$\Leftrightarrow \quad N = 120e^{0.5596t}$

After 5 years; $t = 5$ yrs , $N = ?$

$N = 120e^{0.5596 \times 5}$

$N = 1969.4 \approx 1969$ people.

b)

$t = ?$ $\quad N = 37,275$.

from

$N = 120e^{0.5596t}$

$37275 = 120e^{0.5596t}$

$$t = \frac{\ln\left(\frac{37275}{120}\right)}{0.5596}$$

It will take; $t = 10.255$ years for the no of people in that village to be 37,275.

$$y = x - \frac{-8}{x^2} \quad 16$$

a) i) Intercepts

When $x = 0$, $y \to +\infty$; $(0, +\infty)$

When $y = 0$; $0 = x^3 - 8$

$x^3 = 8$

$x^3 = 2^3$ $\quad x = 2$

$(2, 0)$.

Intercepts ar $(0, +\infty)$ and $(2, 0)$

ii) Turning point ;

$$y = \frac{x^3 + 8}{x^2}$$

$$\frac{dy}{dx} = \frac{vdu - udv}{v^2}$$

$$= \frac{x^2(3x^2) - (x^3+8) \cdot 2x}{x^4}$$

$$= \frac{3x^4 - 2x^4 + 16x}{x^4}$$

$$= \frac{3x^3 - 2x^3 - 16}{x^3}$$

$$\frac{dy}{dx} = \frac{x^3 - 16}{x^3}$$

at t.p ; $\frac{dy}{dx} = 0$, $\quad x^3 - 16 = 0$

$x^3 = 16$

$x_1 = 2.52$

$x_2 = -1.26$

$x_3 = -1.26$

If $x = -1.26$

$$y = \frac{x^3 + 8}{x^2} = \frac{(-1.26)^3 + 8}{(-1.26)^2} = 3.78$$

If $x = 2.52$

$$y = \frac{(2.52)^3 + 8}{(2.52)^2} = 3.78$$

Turning points $(-1.26, 3.78)$ and $(2.52, 3.78)$

Nature;

most likely ans.

x-value	L	$x = -1.26$	R	L	$x = 2.52$	R
$\frac{dy}{dx}$	+	0	+	−	0	+
	/	—	/	\	—	/

(iii) Asymptotes

Vertical: $x^2 = 0$ $\quad x = 0.$

Horizontal: $y = \dfrac{x^3 + 8}{x^2}.$

$$
\begin{array}{r|l}
 & x \\
\hline
x^2 & x^3 + 8 \\
 & -x^3 \\
\hline
 & 8
\end{array}
$$

$y = x$

SKETCH!