WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
UACE August
Mathematics P425/2
July/August 2023

1. $P(B)={}^{1}/_{6}$, $P(AnB)={}^{1}/_{12}$, $P(B/A)={}^{1}/_{3}$

From $P(B/A)=\dfrac{P(BnA)}{P(A)}$

(a) (i) $P(A)=\dfrac{P(BnA)}{P(B/A)}$

$=\dfrac{1}{12}\div\dfrac{1}{3}$ — M_1

$=\dfrac{1}{4}$ — A_1

(ii) $P(A/B^{1})=\dfrac{P(AnB^{1})}{PCB^{1})}$

$=\dfrac{P(A)-P(AnB)}{PCB^{1})}$

$=\dfrac{\frac{1}{4}-\frac{1}{12}}{\frac{5}{6}}$ — M_1

$=\dfrac{1}{5}$ — A_1

(iii) For independence $\dfrac{1}{12}=\dfrac{1}{4}\cdot\dfrac{1}{6}$

$\dfrac{1}{12}\neq\dfrac{1}{24}$ — B_1

A and B are not independent.

05 marks

2. (i) — B_1

T(s)	240	300	360
θ^{0}c	75	θ_1	69^{0}

$\dfrac{69-75}{360-240}=\dfrac{\theta_1-75}{300-240}$ — M_1

$\dfrac{-6}{120}=\dfrac{\theta_1-75}{60}$

$\theta_1=75+\left(\dfrac{-6\times60}{120}\right)$ — A_1

$=72^{0}c$

(ii)

T(s)	450	600	T_1
θ^0c	54	46	42

$$\frac{42-54}{T_1-450}=\frac{45-54}{600-450}$$ M₁

$$\frac{-12}{T_1-450}=\frac{-8}{120}$$

$$T_1=450+\frac{14\times150}{8}$$

$$=675s$$ A₁

5 marks

3.

$$u=72\times\frac{1000ms^{-1}}{3600}$$

$$=20ms^{-1}$$ B₁ — for both $20ms^{-1}$ and $10ms^{-1}$

$$v=36\times\frac{1000}{3600}$$

$$=10ms^{-1}$$

From $v^2=u^2+2as$

$$10^2=20^2+2a\times800$$ M₁

$$100=400+1600a$$

$$a=\tfrac{-3}{16}ms^{-2} or -0.1875ms^{-2}$$ A₁

From $v=u+at$

$$10=20\ -\frac{3}{16}t$$ M₁

$t=\frac{160}{3}$ second or 53.3 seconds A₁ — Atleast 1.dp

5 marks

4.

$$\frac{P2021}{P2000}=\frac{90}{100},\frac{P2022}{P2021}=\frac{120}{100}$$ B₁

$$\frac{P2022}{P2000}=\frac{P2022}{P2021}\times\frac{P2021}{P2000}$$

$$=\frac{120}{100}\times\frac{90}{100}$$ M₁

$$=\frac{27}{25}$$ A₁

$$\therefore P2022=\frac{27}{25}\times200{,}000$$ M₁

$$=216000/-$$ A₁

5 marks

5. $d = \frac{2-1}{5} = 0.2$ B$_1$

x	y_n	y_{n-1}
1.0	0.8415	
1.2		1.1184
1.4		1.3796
1.6		1.5993
1.8		1.7529
2.0	1.8186	
sum	2.6601	5.8502

B$_1$ – All values of x **B$_1$ all values y**

Using the trapezium rule for 6 ordinates.

$= \frac{1}{2} \times 0.2 \times [2.6601 + 2 \times 5.8502]$ M$_1$

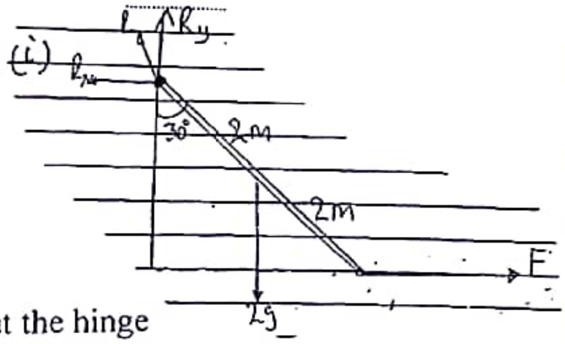
$= 0.1 \times 14.3605$

$= 1.43605$

$= 1.436 (3dp)$ A$_1$

5 marks

6. (i)



B$_1$ Forces drawn with straight edge.

Taking moments about the hinge M$_1$

$4F\cos 30^0 = 2\sin 30 \times 2g$

$F = \frac{2 \times \sin 30 \times 2 \times 9.8}{4 \times \cos 30}$

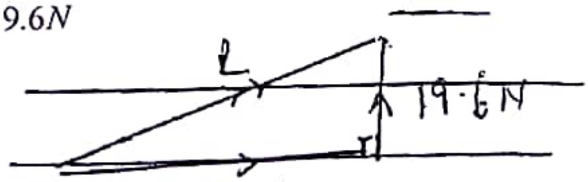
$= 5.6580N$ A$_1$

(ii) Resolving forces

$= Ry = 2g$ $R_x = F$

$= 2 \times 9.8$ $R_x = 5.6580N$

$= 19.6N$



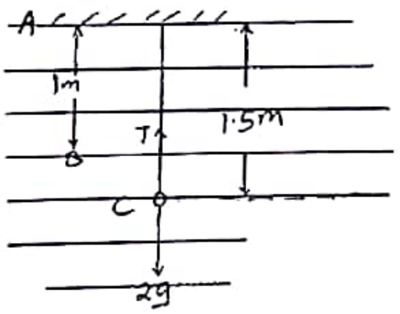
5.6580 M$_1$

5.6580

$R = \sqrt{5.6580^2 + 19.6^2}$ A$_1$

$\therefore |R| = 20.4003N$

5 marks

7.	(a)	Let J: Jane, M: Mary, A: Alice $P(J) = P(J^1 n M^1 n A) = P(J^1) \cdot P(M^1) \cdot P(A)$ $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$ $= \frac{25}{216}$ or 0.1157 (4dp)	M_1	
	(b)	1^{st} time $P(A) = \frac{1}{6}$ 2^{nd} time $P(A) = \frac{1}{6} \times \left(\frac{5}{6}\right)^3$ 3^{rd} time $P(A) = \frac{1}{6} \times \left(\frac{5}{6}\right)^6$ $P(wining) = \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^3 + \frac{1}{6}\left(\frac{5}{6}\right)^6 + \ldots\ldots\ldots$ $a = \frac{1}{6}$, $r = \left(\frac{5}{6}\right)^3$ $Sx = \frac{a}{1-r}$ $= \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^3}$ $= \frac{36}{91}$ or 0.3956 (4dp)	M_1 M_1 A_1	
				5 marks
8.		 From point A to B Loss in P.E = gain in K. E = 2 x 9.8 x 1 = 19.6 J From B to C Loss in P.E and K.E = elastic P.E stored $19.6 + 2x9.8\,x = \frac{\lambda x^2}{2l}$ $19.6 + 2x9.8(1.5-1) = \frac{\lambda(1.5-1)^2}{2\times 1}$ $\therefore \lambda = 235.2N$	B_1 A_1 M_1 M_1 A_1	
				5 marks

9.								
Height	Freq	x	fx	fx^2	$C.f$		cb	
120 - 124	5	122	610	74420	5			
125 – 129	17	127	2159	274193	22			
130 – 134	20	132	2640	348480	42			
135 – 139	25	137	3425	469225	67			
140 – 144	15	142	2130	302460	82			
145 – 149	6	147	882	129654	88			
150 - 154	2	152	304	46208	90			
	$\Sigma f = 90$		$\Sigma fx = 12150$ B_1	$\Sigma fx^2 = 1644640$ B_1				

(a) Mean $= \dfrac{\Sigma fx}{\Sigma f}$ $\qquad SD = \sqrt{\dfrac{1644640}{90} - (135)^2}$ $\qquad M_1$

$= \dfrac{12150}{90}$ $\quad M_1$ $\qquad = 6.9841$

$= 135cm$ $\quad A_1$ $\qquad A_1$

(b) On graph paper at the back.

(c) (i) Median $= \dfrac{50}{100} \times 90$

$= 45$

$= 135cm \pm 0.5$ $\qquad B_1$ — The location should be seen from graph

(ii) 20^{th} percentile $= \dfrac{20}{100} \times 90$

$= 18$

$= 128.5$cm (graph)

80^{th} percentile $= \dfrac{80}{100} \times 90$

$= 72$

$= 141$ cm (graph) $\qquad A_1$ — the location of 20^{th} of 80^{th} percentile seen on the graph for both values

Range $= 141 - 128.5$

$= 12.5cm \pm 0.5$

10 (a) $x = \sqrt[4]{N}$

$x^4 - N = 0$ $\qquad B_1$

$f(x) = x^4 - N$

$f^1(x) = 4x^3$

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^4 - N}{4x_n^3}$$

$$= \frac{4x_n^4 - x_n^4 + N}{4x_n^3}$$

$$= \frac{3x_n^4 + N}{4x_n^3}$$ M$_1$

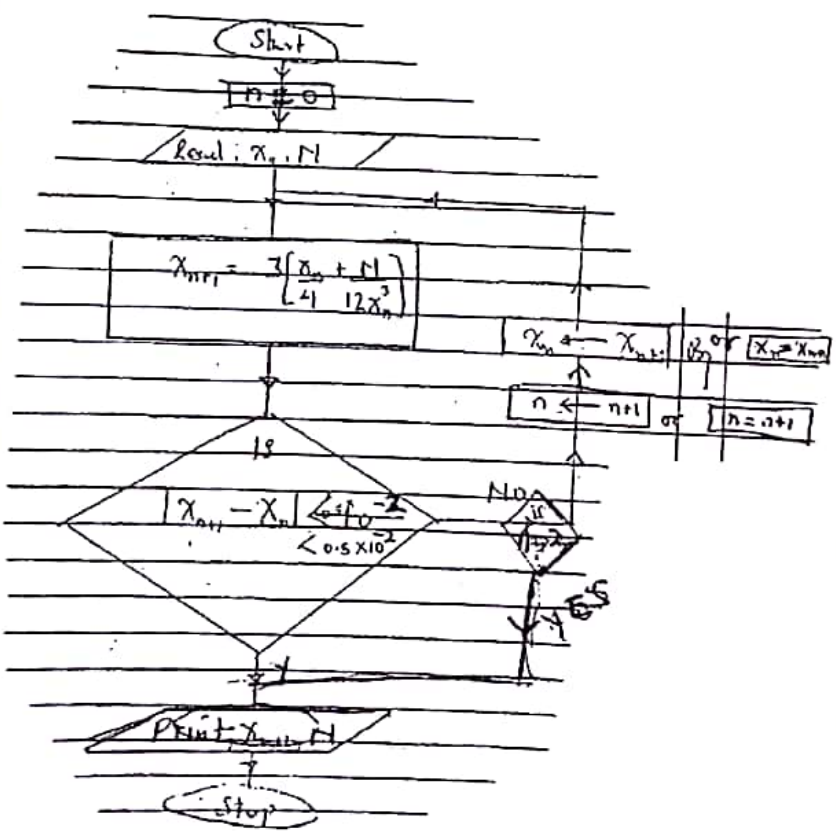
$$= \frac{3x^4}{4x_n^3} + \frac{N}{4x_n^3}$$

$$= 3\left[\frac{x_n}{4} + \frac{N}{3.4x_n^3}\right]$$

$$= 3\left[\frac{x_n}{4} + \frac{N}{12x_n^3}\right]$$ as required

(b) (i) & (ii)

A$_1$



B$_1$

B$_1$

B$_1$

B$_1$

B$_1$

(c)

n	x_4	x_{n11}	$\lvert x_{n11}x_4 \rvert$
0	3	3.1667	0.1667
1	3.1667	3.1544	0.0123
2	3.1544	3.1543	0.0001

B₁ For correct $x_n + 1$

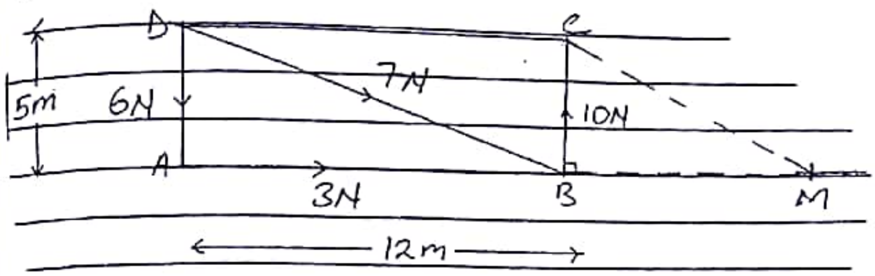
B₁ For correct $|x_{n+1} - x_n|$

$x = 3.15,\ N = 99$

B₁ Correct answer

12 marks

11 (a)



B₁ All forces indicated with straight edges and arrows.

$$\underset{\sim}{R} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}0\\10\end{pmatrix} + \begin{pmatrix}0\\-6\end{pmatrix} + \begin{pmatrix}+7\times\frac{12}{13}\\ -7\times\frac{5}{13}\end{pmatrix}$$

M₁

$$= \begin{pmatrix}-9.4615\\1.3077\end{pmatrix} N$$

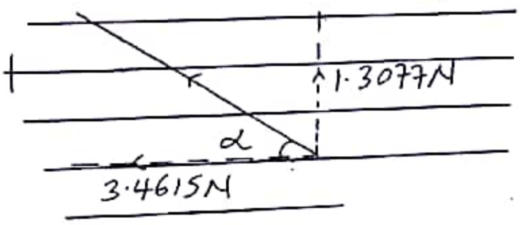
B₁

$$\left|\underset{\sim}{R}\right| = \sqrt{(+9.4615)^2 + (1.3077)^2}$$

M₁

$$= 9.5514N$$

A₁



$$\alpha = \tan^{-1}\left(\frac{1.3077}{9.5514}\right)$$

$$= 7.87^0$$

The resultant is 9.5514N in the direction N82.13^{0}W

A₁

(b) $G = 10 \times 12 - \left(7\times12\times\frac{5}{13}\right)$

$= 87.6923$Nm

B₁ With units

$$\begin{vmatrix}x & 9.4615\\ y & 1.3077\end{vmatrix} = 87.6923$$

$$1.3077x - 9.4615y = 87.6923$$

B₁

When $y = 0, x = \dfrac{87.6923}{1.3077}$

$= 67.0584m$

B₁ For correct value (output)

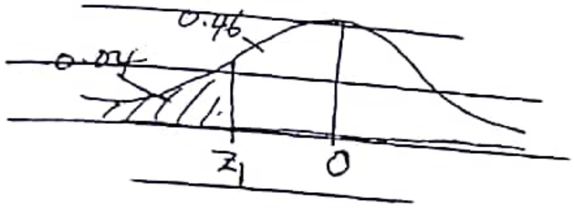
B₁ Output

	$\overline{BM} = 67.0584 - 12$		
	$= 55.0584$	M_1	
	$\overline{MC} = \sqrt{5^2 + 550584^2}$	A_1	
	$= 55.285m$		
12	(i) Area, $\frac{1}{2}k\left(2+\frac{3}{2}\right)=1$	M_1	
	$\frac{7}{4}k = 1$		
	$k = \frac{4}{7}$	A_1	
	(ii) for $0 \le x \le \frac{3}{2}$, $f(x) = \frac{4}{7}$	B_1	
	$for\ \frac{3}{2} \le x \le 2$, $\frac{0-\frac{4}{7}}{2-\frac{3}{2}} = \frac{y-\frac{4}{7}}{x-\frac{3}{2}}$	M_1	
	$\frac{\frac{-4}{7}}{\frac{1}{2}} = \frac{y-\frac{4}{7}}{x-\frac{3}{2}}$		
	$y - \frac{4}{7} = \frac{-8}{7}\left(x-\frac{3}{2}\right)$		
	$y = \frac{-8x}{7} + \frac{16}{7}$	A_1	
	$f(x) = \frac{8}{7}(2-x)$		
	$f(x) = \begin{cases} \frac{4}{7}, & 0 \le x \le \frac{3}{2} \\ \frac{8}{7}(2-x), & \frac{3}{2} \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$	B_1	
	(iii) $P\left(\frac{1}{2} \le x \le \frac{7}{2}\right) = \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{4}{7}dx + \int_{\frac{3}{2}}^{\frac{7}{4}} \frac{8}{7}(2-x)dx$		
	$= \left[\frac{4x}{7}\right]_{\frac{1}{2}}^{\frac{3}{2}} + \frac{8}{7}\left[2x - \frac{1}{2}x^2\right]_{\frac{3}{2}}^{\frac{7}{4}}$	M_1	Correct Integration with limits
	$= \frac{4}{7} + \frac{8}{7}\left[\left(\frac{7}{2} - \frac{49}{32}\right) - \left(3 - \frac{9}{8}\right)\right]$		
	$= \frac{4}{7} + \frac{8}{7}\left(\frac{63}{32} - \frac{15}{8}\right)$	M_1	Substitution of Limits.
	$= \frac{4}{7} + \frac{3}{28}$		
	$= \frac{19}{28}$	A_1	(atleast 4dp)

	(iv) $E(x)=\int_0^{\frac{3}{2}} x.\frac{4}{7}dx+\int_{\frac{3}{2}}^{2} x.\frac{8}{7}(2-x)dx$		
	$=\frac{4}{7}\left(\frac{1}{2}x^2\right)\Big\|_0^{\frac{3}{2}}+\frac{8}{7}\left[x^2-\frac{1}{3}x^3\right]_{\frac{3}{2}}^{2}$	M_1	Correct integration with limits
	$=\frac{2}{7}.\frac{9}{4}+\frac{8}{7}\left(\frac{4}{3}-\frac{9}{8}\right)$	M_1	Correct integration with limits
	$=\frac{9}{14}+\frac{8}{7}\left(\frac{5}{24}\right)$		
	$=\frac{37}{42}$	A_1	(at least 2dp)
			12 marks
13	(a) $y_1=x_1+e_1,\ y_2=x_2+e_2$ $ey_1y_2=y_1y_2-x_1x_2$ $=(x_1+e_2)(x_2+e_2)-x_1x_2$	M_1	
	$=x_1x_2+x_1e_2+x_2e_2+e_1e_2-x_1x_2$ $=x_1e_2+x_2e_1+e_1e_2$	B_1	
	As $e_1 e_2$ becomes too small, then $e_1e_2\approx 0$	B_1	
	$ey_1y_2=x_1e_2+x_2e_1$ $\left\|\frac{ey_1y_2}{y_1y_2}\right\|=\left\|\frac{x_1e_2+x_2e_1}{x_1x_2}\right\|$ $=\left\|\frac{e_2}{x_2}+\frac{e_1}{x_1}\right\|$	M_1	
	$\leq\left\|\frac{e_2}{x_2}\right\|+\left\|\frac{e_1}{x_1}\right\|$ From triangular inequality	B_1	
	$\left\|\frac{ey_1y_2}{y_1y_2}\right\|_{Max}=\left\|\frac{e_1}{x_1}\right\|+\left\|\frac{e_2}{x_2}\right\|$	A_1	
	(b) Let a = 2.675, b=4.800, c= 15.2 $e_a=0.5\times 10=^{-}3$, $e_b=0.5\times 10^{-3}$ $e_c=0.5\times 10^{-1}$ d = 0.92 $e_d=0.5\times 10^{-2}$ $Max\left(2.675\left(4.800-\frac{15.2}{0.92}\right)\right)$		
	$=2.6755\left[4.8005-\frac{15.15}{0.925}\right]$ $=-30.9766$	M_1 A_1	
	$Min\left[2.675\left(4.800-\frac{15.2}{0.92}\right)\right]$ $=2.6745\left[4.7995-\frac{15.25}{0.915}\right]$	M_1	
	$=-31.7389$ $Range[-31.7508,-30.977]$	A_1 B_1	

14	(a) $\underset{\sim}{r}(t) = 4\sin 3t\underset{\sim}{i} + 8\cos 3t\underset{\sim}{j}$ at $t = 0$, $r(o) = 4\sin o\underset{\sim}{i} + 8\cos o\underset{\sim}{j}$ $= 8\underset{\sim}{j}m$ $v = \frac{d(r(t))}{dt} = 12\cos 3t\underset{\sim}{i} - 24\sin 3t\underset{\sim}{j}$ at $t = 0$, $v(o) = 12\cos oi + 8\cos oi - 24\sin o\underset{\sim}{j}$ $= 12\underset{\sim}{i} \quad ms^{-1}$ $\underset{\sim}{a} = \frac{dv}{dt} = -36\sin 3t\underset{\sim}{i} - 72\cos 3t\underset{\sim}{j}$ but $\underset{\sim}{F} = ma$ $\underset{\sim}{F} = 3(-36\sin 3t\underset{\sim}{i} - 72\cos 3t\underset{\sim}{j})$ $= 3(-9)(4\sin 3t\underset{\sim}{i} - 8\cos 3t\underset{\sim}{j})$ $= -27(4\sin 3t\underset{\sim}{i} + 8\cos 3t\underset{\sim}{j})$ $= -27\underset{\sim}{r}$	 B_1 M_1 M_1 A_1 B_1 M_1 M_1 A_1
	(b) Speed = $10ms^{-1}$ cross section area = $5cm^2$ h = 4m Volume of water = $10 \times \frac{5}{100^2}m^3$ $= \frac{50}{100^2}m^3$ Mass of water raised and issued per second. $= \frac{50}{100^2} \times 1000kg(density)$ $= 5kg$ $PE = mgh$ $= 5(9.8)(4)J$ $= 196J$ $KE = \frac{1}{2}mv^2$ $= \frac{1}{2}(s)(10)^2$ $= 250J$ Work done per second by the pump. = PE + K.E = (96 + 250) J = 446J = 446W	 B_1 B_1 M_1 A_1
15	(a) Let x be a .r.v for "the marks scored" n = 350 $P(X < 40) = \frac{14}{350}$ $P(X > 60) = \frac{21}{350}$	

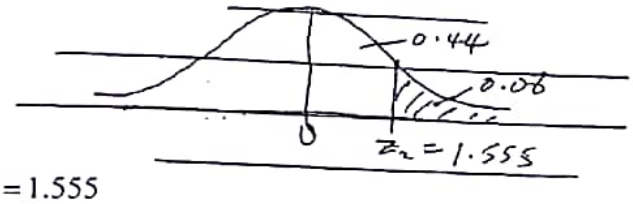
$$P(X<40)=P\left(z<\frac{40-\mu}{\sigma}\right)$$



$$\frac{40-\mu}{\sigma}=-1.751$$ B₁

$40-\mu=-1.751\,\sigma$ ....(i)

$$P(X>60)=P(z<)\frac{60-\mu}{\sigma}$$



$$\frac{60-\mu}{\sigma}=1.555$$

$60-\mu=1.555\,\sigma$ ......(ii) B₁

$\mu-1.751\,\sigma=40$

$$\frac{\mu+1.555\sigma=60}{-3.30688=-20}$$ M₁

$\sigma=6.050$ (3dp) A₁

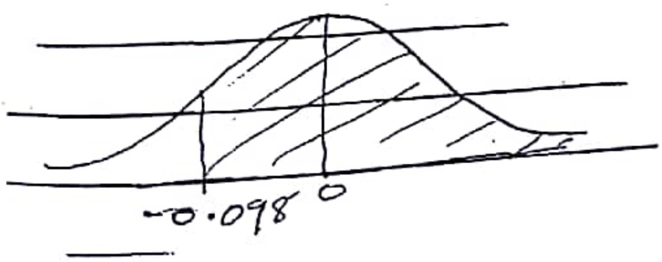
From $\mu=40+1.751\,\sigma$ M₁

$= 40 + 1.751\ (6.050)$ A₁

$= 50.594$ (3dp)

(b) $P(X>50)=P\left(Z>\dfrac{50-50.594}{6.050}\right)$ M₁

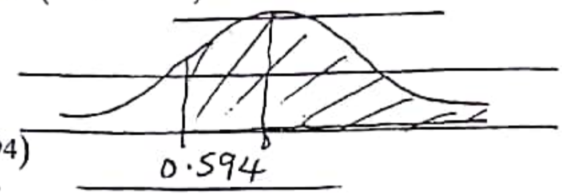
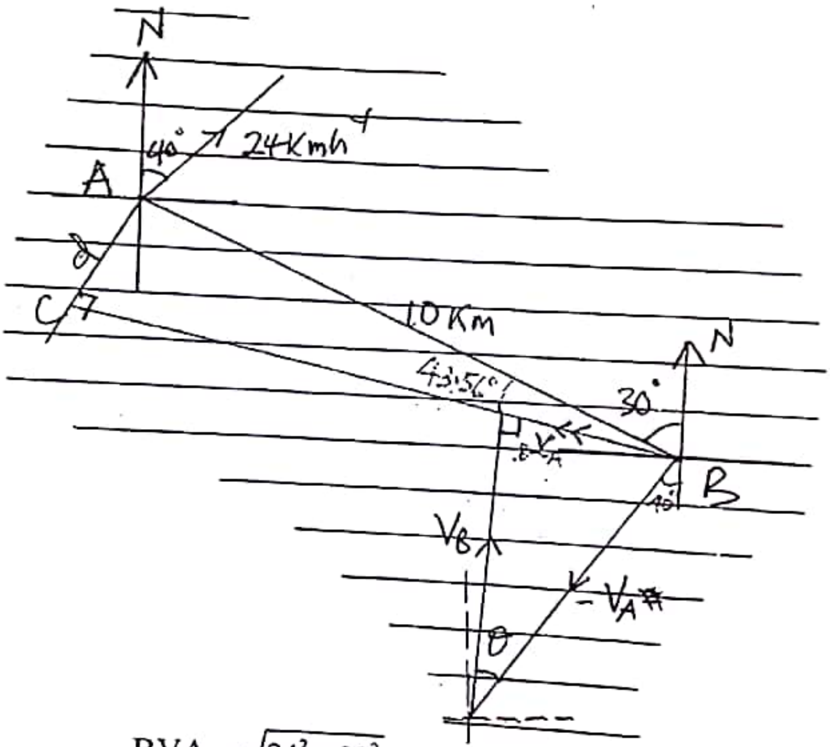
$=P(Z>-0.098)$

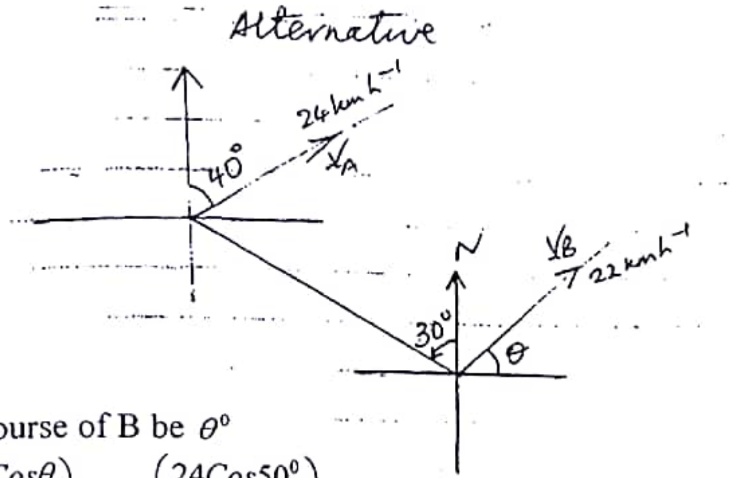


$=0.5+\phi(0.098)$ A₁

$=0.5+0.0391$

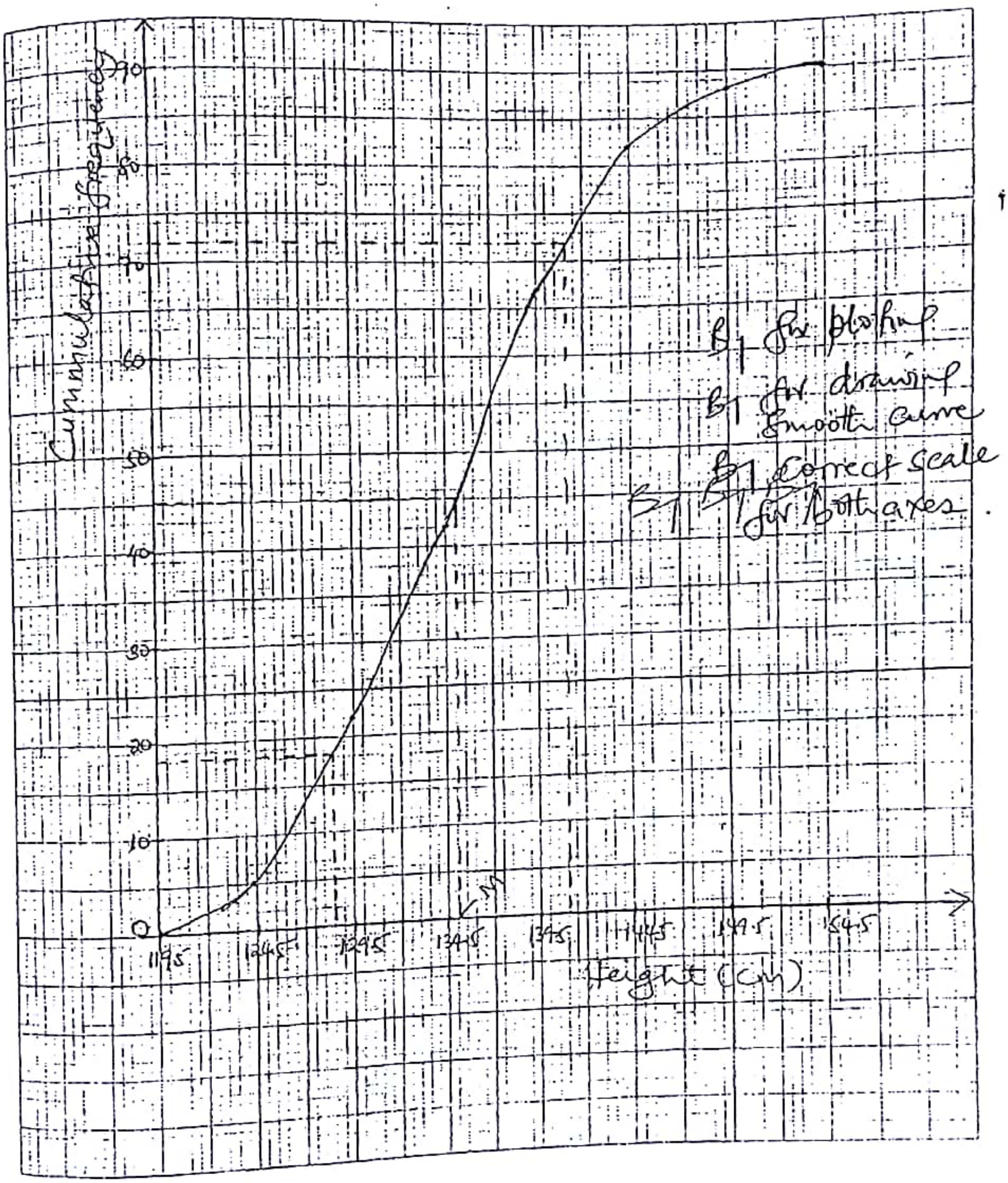
$=0.5391$

	(c) $p(X>47) = P\left(Z > \frac{47-50.594}{6.050}\right)$ $= P(Z > -0.594)$	M_1	
	 $= 0.5 + \phi(0.594)$ $= 0.5 + 0.2238$ $= 0.7238$	B_1	
	Number of students = 350 (0.7238 – 0.5391)	M_1	
	= 350 (0.1847) = 64.6 $\approx 65 students$	A_1	
16	(a) 	B_1	Location of position
		B_1	correct vector diagrams
	BVA $= \sqrt{24^2 - 22^2}$ $= 9.5917 \text{kmh}^{-1}$	M_1 B_1	
	Cos $\theta = \frac{22}{24}, \theta = Cos^{-1}\left(\frac{22}{24}\right), \theta = 23.56^0$	M_1B_1	
	Course is N16.44^{0}E or Bearing is 016.44^0	A_1	
	(b) d = 10 sin 43.56^0 = 6.8911km	M_1 A_1	
	(c) Time $= \frac{BC}{\lvert BVA \rvert} = \frac{10Cos43.56^0}{9.5917}$ $= 0.7555 hrs \times 60$ $= 45 \min utes.$	M_1 B_1 A_1	
			12marks

(a) 	B_1	Correct vector diagrams
Let the course of B be θ^0 $V_B=\begin{pmatrix}22Cos\theta\\22Sin\theta\end{pmatrix}, V_A=\begin{pmatrix}24Cos50^0\\24Sin50^0\end{pmatrix}$	B_1	for both
${}_BV_A=\begin{pmatrix}22Cos\theta\\22Sin\theta\end{pmatrix}, V_A=\begin{pmatrix}24Cos50^0\\24Sin50^0\end{pmatrix}$ $=\begin{pmatrix}22Cos\theta-15.4269\\22Sin\theta-18.3851\end{pmatrix}$	B_1	
$VB.{}_BV_A=0, \begin{pmatrix}22Cos\theta\\22Sin\theta\end{pmatrix}\cdot\begin{pmatrix}22Cos\theta-15.4269\\22Sin\theta-183851\end{pmatrix}=0$	M_1	
$339.3918Cos\theta+404.4722Sin\theta=484$ $\sqrt{339.3918^2+404.4722^2}Cos(\theta-50^0)=484$	M_1	
$Cos(\theta-50^0)=\frac{484}{528.0005}$ $\theta-50^0=23.56^0$ $\theta=73.56^0$	B_1	
The course must be on bearing 016.44^0 or N 16.44^{0}E	A_1	

	(b) $r_B(o)=\begin{pmatrix}0\\0\end{pmatrix}km \quad r_A(o)=\begin{pmatrix}-10Cos60^0\\+10Sin60^0\end{pmatrix}$ $r_B(t)=\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}22Cos73.56^0\\22Sin73.56^0\end{pmatrix}t$ $r_A(t)=\begin{pmatrix}-10Cos60^0\\10\sin 60^0\end{pmatrix}+\begin{pmatrix}24Cos50^0\\24Sin50^0\end{pmatrix}t$ $B^r A^{(\theta)}=\begin{pmatrix}6.2262t\\21.8006t\end{pmatrix}+\begin{pmatrix}-5+15.4269t\\8.6603+18.3851t\end{pmatrix}$ $B^r A^{(t)}=\begin{pmatrix}5-9.227t\\-8.6603+2.7155t\end{pmatrix}$..................... $\otimes$ $B^r.B^vA=0$ $\begin{pmatrix}5-9.227t\\-8.6603+2.7155t\end{pmatrix}\cdot\begin{pmatrix}-9.2007\\2.7155\end{pmatrix}=0$ -46.0035 + 84.6529t – 23.5170 + 7.3739t =0 92.0268t = 69.5205 T = 0.7554 hours x·60 Or t = 45 minutes Distance $B^rA.B^VA=0(0.7554)=\begin{pmatrix}-1.9502\\-6.6090\end{pmatrix}km$ $\lvert B^rA(0.7554)\rvert=\sqrt{(-1.9502)^2+(-6.6090)^2}$ $=6.8907km$	B_1 B_1 M_1 A_1 A_1	for both
	alternative ii (from) $B^rA(t)=\begin{pmatrix}5-9.2007t\\-8.6603+2.7155t\end{pmatrix}$ $\frac{d}{dt}\lvert B^rA(t)\rvert^2=0$ $\frac{d}{dt}\left((5-9.2007t)^2+(-8.6603+2.7155)^2\right)=0$ t = 0.7554 x 60 = 45 minutes $\lvert B^rA(t)=0.7554\rvert=\left\lvert\begin{pmatrix}-1.9502\\-6.6090\end{pmatrix}\right\rvert$ $=6.8907km$	B_1 M_1 A_1 M_1 A_1	

9. (b)



Cummulative frequency
90
80
70
60
50
40
30
20
10
0
119.5
124.5
129.5
134.5
139.5
144.5
149.5
154.5
Height (cm)
B1 for plotting
B1 for drawing smooth curve
B1 correct scale for both axes

END