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P425/1 UACE MARKING GUIDE 2023

Qn 1

$2\log_{10}^{y} = \log_{10}^{2} + \log_{10}^{x}$ --- ①		
$2^{y} = 4^{x}$ or $2^{y} = 2^{2x}$		
$\Rightarrow y = 2x$ ——— ⊛		
From ① $\log_{10} y^2 = \log_{10} 2x$	B_1	Either ⊛ or ⊛⊛
$\Rightarrow y^2 = 2x$ ——— ⊛⊛		
Combining ⊛ and ⊛⊛		
$\cancel{2x}$ $(2x)^2 = 2x$	M_1	One Variable
$4x^2 - 2x = 0$		
$2x(2x-1) = 0$		
$x = 0$ or $x = \frac{1}{2}$	A_1	both Values of x
$y = 0$ or $y = 1$	A_1	both Values of y
$\therefore x = \frac{1}{2}$ $y = 1$	B_1	Discarding
ALT I	05	
$2\log_{10}^{y} = \log_{10}^{2} + \log_{10}^{x}$ ---- ①		
$2^{y} = 4^{x}$, $2^{y} = 2^{2x}$	B_1	Moving from logs to Indices
$\Rightarrow y = 2x$ ——— ②		
$y^2 = 2x$ ——— ③		either ① or ② or 3
Subtract ② − ①		
$y^2 - y = 2x - 2x$	M_1	Solving
$y^2 - y = 0$		
$y(y-1) = 0$		
$y = 0$ or $y = 1$	A_1	set of solution
When $y = 0$, $x = 0$	A_1	set of solutions
when $x = 1$, $x = \frac{1}{2}$		
$\therefore x = \frac{1}{2}$, $y = 1$	B_1	Discarding

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ALTERNATIVE 2		
$y = 2x$ or $y^2 = 2x$	B_1	Either
$2\log_{10} 2x = \log_{10} 2x$		Ignoring Indices or logarithm
$2\log_{10} 2x - \log_{10} 2x = 0$	M_1	solving
$\log_{10} 2x = 0$		
$2x = 10^0 = 1$	B_1	moving from Indices
$x = \frac{1}{2}$	A_1	Value of x
$y = 1$		
$\therefore x = \frac{1}{2}$ and $y = 1$	A_1	Solution
Qn 2	05	
$5\tan^2 A - 5\tan A = 2\sec^2 A$		
$5\tan^2 A - 5\tan A = 2[1 + \tan^2 A]$	M_1	Replacing $\sec^2 A$
$3\tan^2 A - 5\tan A - 2 = 0$		
$3\tan^2 A - 6\tan A + \tan A - 2 = 0$		
$3\tan A[\tan A - 2] + (\tan A - 2) = 0$		
$(3\tan A + 1)(\tan A - 2) = 0$	M_1	Factorisation / Quadratic formula
$\tan A = -\frac{1}{3}$ or $\tan A = 2$	A_1	both values of $\tan A$
$A = \tan^{-1}(-\frac{1}{3})$ or $A = \tan^{-1}(2)$	M_1	Either $\tan^{-1}(-\frac{1}{3})$ or $\tan^{-1}(2)$
$A = 161.57^\circ, 341.57^\circ, 63.43^\circ, 243.43^\circ$	A_1	All 4 values of A
ALT I	05	Accept 1d.P and above
$5\tan^2 A - 5\tan A = 2\sec^2 A$		
$5\sin^2 A - 5\sin A\cos A = 2[\cos^2 A + \sin^2 A]$	M_1	Pythogras theorem
$5\sin^2 A - 2\sin^2 A - 5\sin A\cos A - 2\cos^2 A = 0$		
$3\sin^2 A - 5\sin A\cos A - 2\cos^2 A = 0$		
$3\sin^2 A - 6\sin A\cos A + \sin A\cos A - 2\cos^2 A = 0$		
$3\sin A(\sin A - 2\cos A) + \cos A(\sin A - 2\cos A) = 0$		
$(3\sin A + \cos A)(\sin A - 2\cos A) = 0$	M1	factorisation
$\tan A = -\frac{1}{3}$ or $\tan A = 2$	A1	both values of A
$A = \tan^{-1}(-\frac{1}{3})$ or $A = \tan^{-1}(2)$	M1	for either $\tan^{-1}(-\frac{1}{3})$ or $\tan^{-1}(2)$
$A = 161.57^\circ, 341.57^\circ, 63.43^\circ, 243.43^\circ$	A1	for all 4 Correct angles

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Paper Code. .... Paper Name................

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ALTERNATIVE 3

$5\tan^2 A - 5\tan A = 2\sec^2 A$

$5\sin^2 A - 5\sin A\cos A = 2$ — M_1 Double angle formula

$\frac{5}{2}[1-\cos 2A] - \frac{5}{2}\sin 2A = 2$

$1-\cos 2A - \sin 2A = 4/5$

$\cos 2A + \sin 2A = 1/5$

$\cos 2A + \sin 2A = \sqrt{2}\cos[2A-45^\circ]$ — M_1 Expression of $\cos(A-45^\circ)$

$\sqrt{2}\cos[2A-45^\circ] = 1/5$

$\cos(2A-45^\circ] = \frac{1}{5\sqrt{2}}$ — A_1

$2A-45^\circ = 81.87^\circ, 278.13^\circ, 441.87^\circ, 638.13^\circ$ — M_1 Reading of Angles

$A = 63.43^\circ, 243.43^\circ, 161.57^\circ, 341.57^\circ$ — A_1 for All 4 correct angles

05

Qn 3

$\overrightarrow{OR} = \frac{-3(\overrightarrow{OP})+2(\overrightarrow{OQ})}{-3+2}$ — B_1 Using ratio theorem correctly

$\overrightarrow{OR} = \frac{-3\begin{pmatrix}1\\-2\\1\end{pmatrix}+2\begin{pmatrix}3\\-4\\6\end{pmatrix}}{-1}$ — M_1 substituting $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

$= -\begin{pmatrix}-3+6\\6-8\\-3+12\end{pmatrix} = -\begin{pmatrix}3\\-2\\9\end{pmatrix}$ — M_1 PreMultiplying

$\overrightarrow{OR} = \begin{pmatrix}-3\\2\\-9\end{pmatrix}$ — A_1 correct Position vectors

$R(-3, 2, -9)$ — B_1 correct cordinates

05

ALTERNATIVE 1

$\frac{\overline{PR}}{\overline{RQ}} = \frac{2}{-3}$

$\overrightarrow{PR} = -\frac{2}{3}\overrightarrow{RQ}$

$3[\overrightarrow{OR}-\overrightarrow{OP}] = -2[\overrightarrow{OQ}-\overrightarrow{OR}]$ — B_1 Introducing $\overrightarrow{OR}$

$3\overrightarrow{OR} - 2\overrightarrow{OR} = -2\overrightarrow{OQ} + 3\overrightarrow{OP}$

$\overrightarrow{OR} = -2\begin{pmatrix}3\\-4\\6\end{pmatrix} + 3\begin{pmatrix}1\\-2\\1\end{pmatrix}$ — M_1 Substitution

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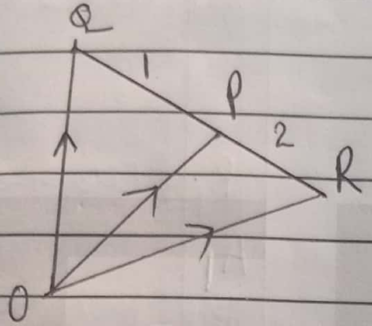
$$\overrightarrow{OR} = \begin{pmatrix}-6\\8\\-12\end{pmatrix} + \begin{pmatrix}3\\-6\\3\end{pmatrix}$$ M1 Pre Multiplication

$$\overrightarrow{OR} = \begin{pmatrix}-3\\2\\-9\end{pmatrix}$$ A1 Correct Position Vector

$R(-3, 2, -9)$ B1 coordinates

05

ALTERNATIVE 2



$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$$

$$= \overrightarrow{OP} + 2\overrightarrow{QP}$$ B1

$$\overrightarrow{OR} = \begin{pmatrix}1\\-2\\1\end{pmatrix} + 2\left[\begin{pmatrix}1\\-2\\1\end{pmatrix} - \begin{pmatrix}3\\-4\\6\end{pmatrix}\right]$$ M1

$$= \begin{pmatrix}-3\\2\\-9\end{pmatrix}$$ M1A

$R(-3, 2, -9)$ B1

05

Qn 4

$$x^3 + 2y^3 + 3xy = 0$$

$$3x^2 + 6y^2\frac{dy}{dx} + 3\left[x\frac{dy}{dx} + y\right] = 0$$ M1A1 ~~A~~ derivative of any implicit function either $2y^3$ or $3xy$; All parts differentiated correctly

$$(3x^2 + 3y) + (6y^2 + 3x)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{(3x^2+3y)}{6y^2+3x} = -\frac{(x^2+y)}{2y^2+x}$$

At $(2, -1) \Rightarrow x = 2, y = -1$

$$\frac{dy}{dx} = \frac{-(2^2+-1)}{2(-1)^2+2} = \frac{-3}{4}$$ A1 Or It's equivalent $-9/12$

Tangent $\frac{y+1}{x-2} = \frac{-3}{4}$ M1 Process of getting equation

$$4y + 4 = -3x + 6$$

$y = -\frac{3}{4}x + \frac{1}{2}$ OR $3x + 4y = 2$ A1 Any of the equation

OR $3x + 4y - 2 = 0$

ALTERNATIVE 1

$x^3 + 2y^3 + 3xy = 0$

$3x^2 + 6y^2\frac{dy}{dx} + 3\left[x\frac{dy}{dx} + y\right] = 0$ — $M_1 A_1$

At $(2,-1) \Rightarrow x = 2,\ y = -1$

$3(2)^2 + 6(-1)^2\frac{dy}{dx} + 3\left[2\frac{dy}{dx} + -1\right] = 0$

$12 + 6\frac{dy}{dx} + 6\frac{dy}{dx} - 3 = 0$

$12\frac{dy}{dx} = -9$

$\frac{dy}{dx} = -\frac{9}{12} = -\frac{3}{4}$ — A_1

Tangent $\cancel{\frac{y+1}{x-3}}$ $\frac{y+1}{x-2} = \frac{-3}{4}$ — M_1

$4y + 4 = -3x + 6$

$3x + 4y - 2$ OR $y = \frac{1}{2} - \frac{3}{4}x$ — A_1

05

Qn 5

$\frac{5-4x}{1-x} - 3 < 0$ — M_1 Either side equal to zero

$\frac{5-4x-3+3x}{1-x} < 0$ — M_1 common denominator

$\frac{2-x}{1-x} < 0$

Critical values $x = 1,\ x = 2$ — B_1 Critical values

	$x<1$	$1<x<2$	$x>2$
$2-x$	+	+	−
$1-x$	+	−	−
$\left(\frac{2-x}{1-x}\right)$	+	−	+

M_1 Analysis table

Valid range for which $\frac{2-x}{1-x} < 0$ is $1 < x < 2$ — A_1 Correct range

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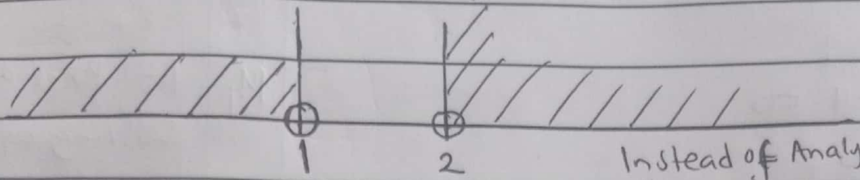
ALTERNATIVE

from critical vales $x=1$, $x=2$



Instead of Analysis table

$1 < x < 2$ — A_1

Qn 6

$\frac{dV}{dt} = 2\,cm^3 s^{-1}$

$V = \frac{1}{3}\pi x^3\,cm^3$

$\frac{dV}{dx} = \pi x^2$ — M_1 for obtaining $\frac{dV}{dx}$

$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$

$2 = (\pi x^2) \cdot \frac{dx}{dt}$ — M_1 Substitution

$\frac{dx}{dt} = \frac{2}{\pi x^2}$ — A_1 for $\frac{dx}{dt}$ made the subject

When $x = 5cm$

$\frac{dx}{dt} = \frac{2}{\pi(5)^2}$ — M_1 for substitution

$= \frac{2}{25\pi}$ or $0.0255\,cm\,s^{-1}$ — A_1 Correct Value 4 d.ps

05

Accept $\frac{dV}{dt} = -2\,cm^3 s^{-1}$ $\Rightarrow \frac{-2}{25\pi}$

$\frac{dV}{dx} = \pi x^2$ — M_1

$= \pi(5)^2$ — M_1

$= 25\pi$

$\frac{dx}{dt} = \frac{dx}{dV} \cdot \frac{dV}{dt}$

$= \frac{1}{25\pi} \times 2$ — A_1 $\frac{dx}{dt}$; M_1 For substitution

$= \frac{2}{25\pi}$ — A_1 correct

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Qn 7

$x-3y-4=0$ and $y+3x-2=0$

or $y=-3x+2$

$x-3(-3x+2)-4=0$ M1 At point of solving equation

$10x-10=0$

$x=1$ Substitution/elimination

$\Rightarrow y=-3(1)+2=-1$ A1 both x and y

L passes through $(1,-1)$

The line $y=-\frac{3}{4}x$

Gradient of L is $\frac{-1}{(-3/4)} = 4/3$ B1 Gradient of L

Equation L $\frac{y+1}{x-1} = \frac{4}{3}$ M1 Process of equation of the line

$3y+3=4x-4$

$3y-4x+7=0$ OR $-3y+4x-7=0$ A1 Equation of the line

ALTERNATIVE 1 05

$x=1, y=1$

$\boxed{4y+3x=0}$ change to $4x-3y$

Interchange coefficient

$4x-3y = 4(1)-3(1)$ B1 M1

$4x-3y=7$ A1

ALT 05

Required line L is of the form

$x-3y-4+\lambda(y+3x-2)=0$ ----(1) M1 General form of equation

$x-3y-4+3\lambda x+\lambda y-2\lambda=0$

$(1+3\lambda)x+(\lambda-3)y-2\lambda-4=0$

$(\lambda-3)y = -(1+3\lambda)x+(2\lambda+4)$

$y = -\frac{(1+3\lambda)x}{\lambda-3} + \frac{(2\lambda+4)}{\lambda-3}$

Gradient $= -\left(\frac{1+3\lambda}{\lambda-3}\right)$

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Paper Code. .... Paper Name. ....
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Gradient of the line $4y = -3x$ is $-3/4$

But $-\left(\frac{1+3\lambda}{\lambda-3}\right) = \frac{-1}{(-3/4)}$

$-\left(\frac{1+3\lambda}{\lambda-3}\right) = \frac{4}{3}$ B_1 Equating gradients

$3+9\lambda = 12-4\lambda$

$13\lambda = 9$

$\lambda = 9/13$ A_1 Value of λ

Put λ into (1)

$x-3y-4+\frac{9}{13}(y+3x-2) = 0$ M_1 Substituting

$13x-39y-52+9y+27x-18 = 0$

$40x-30y-70 = 0$ A_1 Eqn of required line

$4x-3y-7 = 0$

05

Qn 8 Solve $x\frac{dy}{dx} = 2y+x$

Use $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ B_1 For differentiating $y = vx$

$x\left(v + x\frac{dv}{dx}\right) = 2(vx)+x$ M_1 Substitution

$x\frac{dv}{dx} = v+1$

Seperating Variables

$\int\frac{dv}{v+1} = \int\frac{1}{x}dx$ M_1 Seperating Variables

$\ln(v+1) = \ln x + c$ A_1 Correct Integration

$\ln\left[\frac{v+1}{x}\right] = c$

$\frac{v+1}{x} = k$

$\frac{y}{x}+1 = kx$ OR $\ln\left(\frac{y}{x}+1\right) = \ln x + c$ A_1 Solution

OR $y+x = kx^2$

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9

OTHERWISE

$x\frac{dy}{dx} - 2y = x$		
$\frac{dy}{dx} - \frac{2}{x}y = 1$	M1	Form of $\frac{dy}{dx} + Py = Q$
Integrating Factor $R = e^{\int -\frac{2}{x}dx}$		
$R = e^{-2\ln x} = \frac{1}{x^2}$	B1	
$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = \frac{1}{x^2}$	B1	Integrating factor
$\frac{d}{dx}\left(\frac{y}{x^2}\right) = \frac{1}{x^2}$	M1	Exact D.E multiplying the integrating factor
$y/x^2 = \int \frac{1}{x^2}dx$	M1	solving
$\frac{y}{x^2} = -\frac{1}{x} + K$	A1	Solution
OR $y = -x + kx^2$ OR $y + x = kx^2$	05	

SECTION B

Qn 9(a) G.P 2, 6, 18, 54, - - - -		
1st term $a = 2$		
Common ratio, $r = 6/2 = 3$	B1	for r
$S_n = a\left\{\frac{r^n - 1}{r-1}\right\}$		
For $n = 10$, $S_{10} = 2\left[\frac{3^{10}-1}{3-1}\right] = 2\left[\frac{3^{10}-1}{2}\right]$	M1	For substitution for $a=2$ and $r=3$
$= 3^{10} - 1$		
$= 59048$	A1	
ALT $S_{10} = 2+6+18+54+162+486+1,458+4374+13,122+39,366$	B1	Generating terms
	M1	Adding terms
$= 59048$	A1	

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(b)	5th term $= a+4d$		Correct Expression of
	16th term $= a+15d$	M_1	Either 5th term or 16th term
	5th term + 16th term $=44$		
	$a+4d+a+15d = 44$	B_1	for adding two expressions and equating $=44$
	$2a+19d = 44$ ——— (1)		
	$S_{18} = 3S_{10}$		
	$\frac{18}{2}[2a+17d] = 3\left\{\frac{10}{2}(2a+9d)\right\}$	B_1M_1	For either S_{18} or S_{10}; equating $S_{18} = 3S_{10}$
	$9(2a+17d) = 15(2a+9d)$		
	$18a+153d = 30a+135d$		
	$12a = 18d$		
	$a = \frac{3}{2}d$ OR $d = \frac{2}{3}a$		
(i)	From (1) $2a+19(\frac{2}{3}a) = 44$	M_1	For substituting (solving in one variable)
	$6a+38a = 132$		
	$44a = 132$		
	$a = 3$	A_1	
(ii)	$d = \frac{2}{3}(3) = 2$	A_1	
(iii)	$S_{30} = \frac{30}{2}\{2(3)+29(2)\}$	M_1	Complete substitution
	$S_{30} = 960$	A_1	
	S.TOTAL	09	
		12	

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QN10

$$\frac{11x-1}{(1-x)^2(2+3x)} \equiv \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{(2+3x)}$$ M1 For $(1-x)^2$ as repeated factor

$$11x-1 \equiv A(1-x)(2+3x) + B(2+3x) + C(1-x)^2$$ M1 For equating numerators

When $x=1$, $10 = 5B$ M1 For solving any unknown

$\Rightarrow B = 2$ A1

When $x = -\frac{2}{3}$, $-\frac{22}{3} - 1 = C\left(1+\frac{2}{3}\right)^2$

$-\frac{25}{3} = \frac{25}{9}C$

$C = -3$ A1

Coefficients of x^2, $0 = -3A + C$

$C = 3A$

$-3 = 3A$

$A = -1$ A1

Hence $$\frac{11x-1}{(1-x)^2(2+3x)} = \frac{2}{(1-x)^2} - \frac{1}{(1-x)} - \frac{3}{(2+3x)}$$ B1

Hence evaluating

$$\int_0^{1/2} \frac{11x-1}{(1-x)^2(2+3x)}\,dx = \int_0^{1/2} \left(\frac{2}{(1-x)^2} - \frac{1}{(1-x)} - \frac{3}{(2+3x)}\right)dx$$ M1 Substitution Replacing e

$$= \int_0^{1/2} \left\{ 2(1-x)^{-2} - \frac{1}{(1-x)} - \frac{3}{(2+3x)} \right\} dx$$

$$= \left[\frac{2}{-1}(1-x)^{-1}\left(\frac{1}{-1}\right) - \left[-\ln(1-x)\right] - 3\left(\frac{1}{3}\right)\ln(2+3x) \right]_0^{1/2}$$ M1 If one of Parts is Correctly Integrated

$$= \left[\frac{2}{1-x} + \ln(1-x) - \ln(2+3x) \right]_0^{1/2}$$ A1 for ALL 3 parts

$$= \left[\frac{2}{1-\frac{1}{2}} + \ln\left(1-\tfrac{1}{2}\right) - \ln\left(2+\tfrac{3}{2}\right) \right] - \left[\tfrac{2}{1} + \ln 1 - \ln 2 \right]$$ M1 Substitution of Limits

$$= \left[\frac{2}{\frac{1}{2}} + \ln\left(\tfrac{1}{2}\right) - \ln\left(\tfrac{7}{2}\right) \right] - \left[2 - \ln 2 \right]$$

$$= 4 - \ln 2 - \ln \tfrac{7}{2} - 2 + \ln 2$$

$$= 2 + \ln\left(\tfrac{2}{7}\right) \text{ or } \ln\left(\tfrac{2}{7}\right) + 2$$ A1

12

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Paper Code. .... Paper Name............

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Qn 11 Had an error, Assumption is that a plane ~~is parallel to the direction vector $2i+3j+k$~~

$\vec{r} = \begin{pmatrix}-1\\2\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$	M_1	Rewriting the equation of the line
direction vector of line $\underset{\sim}{d} = \begin{pmatrix}1\\1\\2\end{pmatrix}$	B_1	Identifying d
Given vector $\underset{\sim}{b} = \begin{pmatrix}2\\3\\1\end{pmatrix}$	B_1	Identifying b
Position vector of common point of the line and plane $\underset{\sim}{a} = \begin{pmatrix}-1\\2\\4\end{pmatrix}$	B_1	Identifying a
Normal vector to the plane $\underset{\sim}{n} = \underset{\sim}{d} \times \underset{\sim}{b}$		
$\underset{\sim}{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$	M_1	Cross Product or any other method
$\underset{\sim}{n} = (1-6)\hat{i} - (1-4)\hat{j} + (3-2)\hat{k}$	M_1	
$\underset{\sim}{n} = -5\hat{i} + 3\hat{j} + \hat{k}$ OR $\underset{\sim}{n} = 5\hat{i} - 3\hat{j} - \hat{k}$	A_1	
Equation of the plane $\underset{\sim}{r} \cdot \underset{\sim}{n} = \underset{\sim}{n} \cdot \underset{\sim}{a}$		
$\begin{pmatrix}x\\y\\z\end{pmatrix} \cdot \begin{pmatrix}+5\\-3\\-1\end{pmatrix} = \begin{pmatrix}+5\\-3\\-1\end{pmatrix} \cdot \begin{pmatrix}-1\\2\\-4\end{pmatrix}$	M_1	For $\underset{\sim}{n} \cdot \underset{\sim}{r}$ and $\underset{\sim}{n} \cdot \underset{\sim}{a}$
$5x - 3y - z = 5(-1) + 3(2) - 1(-4)$	M_1	
$5x - 3y - z = -7$ OR ~~$5x-3y+z+7=0$~~ $5x - 3y + z + 7 = 0$	A_1	
$D = \left\lvert \frac{7}{\sqrt{(5)^2 + (-3)^2 + (-1)^2}} \right\rvert$	M_1	
$= \frac{7}{\sqrt{35}}$	A_1	
OR $= \frac{\sqrt{7}}{5}$		
OR $= 1.1832$	12	

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Paper Code. .... Paper Name........................
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ALTERNATIVE

$$\underline{r} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$ M1

$$\underline{r} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}_{B_1} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}_{B_1} + \mu \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}_{B_1}$$ M1

$x = -1 + \lambda + 2\mu$ —— ①
$y = 2 + \lambda + 3\mu$ —— ② M1 For all equations
$z = -4 + 2\lambda + \mu$ —— ③

① − ②; $x - y = -3 - \mu$ — ④ M1 eliminating one unknown

$2 \times (1) - ③$; $2x - z = 2 + 3\mu$ — ⑤ ~~M1~~ ~~eliminating one unknown~~

From eqn(4)

$\mu = -3 - x + y$

From ⑤ $2x - z = 2 + 3(-3 - x + y)$ M1 For substitution

$2x - z = 2 - 9 - 3x + 3y$

$5x - 3y - z + 7 = 0$ A1

(b) $D = \dfrac{7}{\sqrt{(5)^2 + (-3)^2 + (-1)^2}}$ M1

$= \dfrac{7}{\sqrt{35}}$

OR $= 1.1832$ A1

OR $= \dfrac{\sqrt{7}}{5}$

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Qn 12

$\left(\frac{1+3x}{1-x}\right)^{\frac{1}{2}} = (1+3x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	M_1	Expressing as product
$(1+3x)^{\frac{1}{2}} = 1+\frac{1}{2}(3x)+\frac{(\frac{1}{2})(-\frac{1}{2})(3x)^2}{2!}+\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(3x)^3}{3!}$	M_1	Binomial expansion
$= 1+\frac{3}{2}x-\frac{9}{8}x^2+\frac{27}{16}x^3$	A_1	correct expansion
Also		
$(1-x)^{-\frac{1}{2}} = 1-\frac{1}{2}(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})(-x)^2}{2!}+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-x)^3}{3!}$	M_1	Binomial Expansion
$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3$	A_1	correct expansion
$\Rightarrow \left(\frac{1+3x}{1-x}\right)^{\frac{1}{2}} = \left(1+\frac{3}{2}x-\frac{9}{8}x^2+\frac{27}{16}x^3\right)\left(1+\frac{x}{2}+\frac{3}{8}x^2+\frac{5}{16}x^3\right)$	M_1	Combining the products
$= 1+\frac{x}{2}+\frac{3}{8}x^2+\frac{5}{16}x^3+\frac{3}{2}x+\frac{3}{4}x^2+\cancel{\frac{9}{16}x^3}-\frac{9}{8}x^2-\cancel{\frac{9}{16}x^3}+\frac{27}{16}x^3$	M_1	
$\left(\frac{1+3x}{1-x}\right)^{\frac{1}{2}} = 1+2x+2x^3$	A_1	correct expansion
Hence substitute $x=\frac{1}{5}$		
$\left(\frac{1+3(\frac{1}{5})}{1-\frac{1}{5}}\right)^{\frac{1}{2}} = 1+2\left(\frac{1}{5}\right)+2\left(\frac{1}{5}\right)^3$	M_1	Substitution on both sides
$\left(\frac{8}{4}\right)^{\frac{1}{2}} = 1.416$		
$\frac{\sqrt{8}}{2} = 1.416$		
$\sqrt{8} = 2(1.416)$	M_1	Making $\sqrt{8}$ as subject
$\sqrt{8} = 2.832$	A_1	correct value
$\sqrt{8} = 2.83$ (2 d.p's)	B_1	CAO
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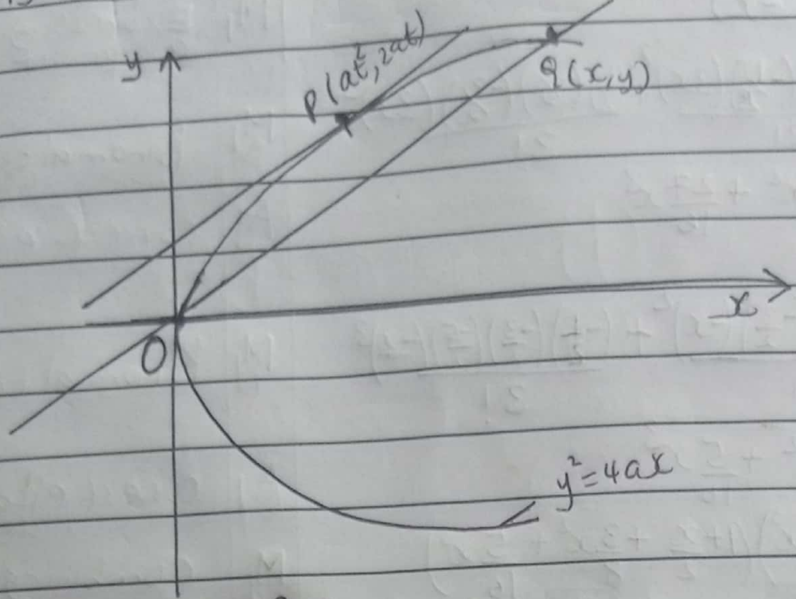
Qn 13



TANGENT AT P

$y^2 = 4ax$

$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$ — M1 Differentiation

$\frac{dy}{dx} = \frac{2a}{y}$

$= \frac{2a}{2at}$

$= \frac{1}{t}$ — A1 Gradient of tangent

Equation $\frac{y - 2at}{x - at^2} = \frac{1}{t}$ — M1 Finding equation

$ty - 2at^2 = x - at^2$

$ty - x = at^2$ — A1 Eqn Simplified

~~OR~~ Grad of chord OQ

$\frac{y - 0}{x - 0} = \frac{1}{t}$ — M1 Finding the equation

$ty = x$ — A1 equation of chord

$\therefore x = ty$

$y^2 = 4ax$

$y^2 = 4a(ty)$

$y(y - 4at) = 0$ — M1 factorisation

$y = 0$ or $y = 4at$

$x = ty = t(4at) = 4at^2$

$\therefore Q\ (4at^2, 4at)$ — A1 Coordinates of Q

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	Marks	Comments
Gradient of tangent at Q		
$M = \frac{2a}{4at} = \frac{1}{2t}$		
Tangent at Q		
$\frac{y-4at}{x-4at^2} = \frac{1}{2t}$	M_1	Process of getting equation
$2ty - 8at^2 = x - 4at^2$		
$2ty = x + 4at^2$	A_1	equation of tangent at Q {when at^2 have been collected together}
Solve $ty = x + at^2$		
and $2ty = x + 4at^2$		
subtract $ty = 3at^2$	M_1	solving
$y = 3at$		
$x = 2ty - 4at^2$		
$x = 2t(3at) - 4at^2$		
$x = 2at^2$		
$\therefore R(2at^2, 3at)$	A_1	Coordinates of Q
	12	
ALTERNATIVE		
Gradient of OQ, Let $x = aq^2, y = 2aq$		
$\frac{2aq-0}{aq^2-0} = \frac{1}{t}$	M_1	finding equation
$\frac{2}{q} = \frac{1}{t}$		
$q = 2t$	A_1	Correct q
$x = a(2t)^2 = 4at^2$	M_1	Substituting for all ~~coordinates~~
$y = 2a(2t) = 4at$		
$\therefore Q(4at^2, 4at)$	A_1	coordinates
$qy = x + aq^2$	M_1	substitution
$2ty = x + a(2t)^2$		
$2ty = x + 4at^2$	A_1	equation

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	Marks	Comments
Let $y = \dfrac{x^2+1}{(x+1)^3}$		
$\dfrac{dy}{dx} = \dfrac{(x+1)^3(2x) - (x^2+1)(3)(x+1)^2(1)}{(x+1)^6}$	M_1 M_1 M_1	$\dfrac{du}{dx}$ $\dfrac{dv}{dx}$ Substitution
$= \dfrac{(x+1)^2\{(x+1)(2x) - (x^2+1)(3)\}}{(x+1)^6}$		
$= \dfrac{2x^2+2x-3x^2-3}{(x+1)^4}$		
$\dfrac{dy}{dx} = \dfrac{-x^2+2x-3}{(x+1)^4}$	A_1 04	Simplified derivative
ALTERNATIVE1 $\ln y = \ln(x^2+1) - \ln(x+1)^3$	M_1	logs
$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2x}{x^2+1} - \dfrac{3}{(x+1)}$ or $\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2x}{x^2+1} = \dfrac{3(x+1)^2}{(x+1)^3}$	M_1	differentiated
$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2x(x+1)-3(x^2+1)}{(x^2+1)(x+1)}$		
$\dfrac{dy}{dx} = \dfrac{(x^2+1)}{(x+1)^3}\left\{\dfrac{2x(x+1)-3(x^2+1)}{(x^2+1)(x+1)}\right\}$	M_1	Replacing y with its value
$\dfrac{dy}{dx} = \dfrac{(x^2+1)}{(x+1)^3}\left\{\dfrac{2x^2+2x-3x^2-3}{(x^2+1)(x+1)}\right\}$		
$= \dfrac{(x^2+1)}{(x+1)^3}\left\{\dfrac{-x^2+2x-3}{(x^2+1)(x+1)}\right\}$		
$\dfrac{dy}{dx} = \dfrac{-x^2+2x-3}{(x+1)^4}$	A_1 04	
ALTERNATIVE2		
$y = (x^2+1)(x+1)^{-3}$		
$\dfrac{dy}{dx} = (x^2+1)(-3)(x+1)^{-4} + (x+1)^{-3}(2x)$ M_1 (for u) M_1 (for v)	M_1	Substitution
$= (x+1)^{-4}[-3(x^2+1) + (x+1)(2x)]$		
$= (x+1)^{-4}[-3x^2-3+2x^2+2x]$		
$= \dfrac{-x^2+2x-3}{(x+1)^4}$	A_1 04	

ALTERNATIVE

let $u = x^2+1$; $\frac{du}{dx} = 2x$ | M1 for dy/dx

$v = (x+1)^3$; $\frac{dV}{dx} = 3(x+1)^2$ | M1 for $\frac{dV}{dx}$

$$\frac{dy}{dx} = \frac{(x+1)^3(2x) - (x^2+1)(3)(x+1)^2}{(x+1)^6}$$ M1 substitution

$$= \frac{-x^2+2x-3}{(x+1)^4}$$ A1 For $\frac{dy}{dx}$

14b $x = \frac{3t}{t+3}$; $\frac{dx}{dt} = \frac{3(t+3)-3t(1)}{(t+3)^2}$ M1 Differentiation Product/quotient rule

$$\frac{dx}{dt} = \frac{9}{(t+3)^2}$$

$y = \frac{4t+1}{(t-2)}$; $\frac{dy}{dt} = \frac{4(t-2)-(4t+4)}{(t-2)^2}$ M1 Differentiation product/quotient rule

$$= \frac{-9}{(t-2)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{-9}{(t-2)^2} \times \frac{(t+3)^2}{9}$$ M1 Chain rule (A1) if the next answer is missing

$$= -\frac{(t+3)^2}{(t-2)^2}$$ A1 correct expression

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$ Process of $\frac{d}{dt}\left(\frac{dy}{dx}\right)$

$$= -\left[\frac{(t-2)^2 2(t+3) - (t+3)^2 2(t-2)}{(t-2)^4}\right] \times \frac{(t+3)^2}{9}$$ M1 M1 substitution of $\frac{dt}{dx}$

$$= -\frac{2(t+3)^2}{9}\left[\frac{(t-2)(t+3)[(t-2)-(t+3)]}{(t-2)^4}\right]$$ M1 Simplifying

$$= \frac{10}{9}\left[\frac{(t+3)^2}{(t-2)^2}\right] \text{ OR } \frac{10}{9}\left[\frac{t+3}{t-2}\right]^3$$ A1 Simplified expression

12

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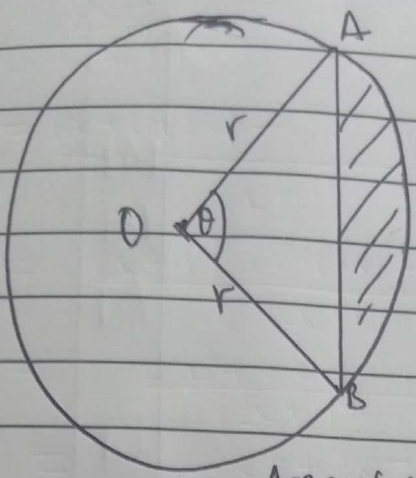
15(a)



Area of circle, $A_1 = \pi r^2$ — B_1

Area of Sector, $A_2 = \frac{1}{2} r^2 \theta$ — B_1

Area of ΔOAB, $A_3 = \frac{1}{2} r^2 \sin\theta$ — B_1

Area of minor segment $= A_2 - A_3$

$= \frac{1}{2} r^2 (\theta - \sin\theta)$ — M_1 Process of ~~Subtraction~~

But $3(A_2 - A_3) = A_1$

$3\left(\frac{1}{2} r^2\right)[\theta - \sin\theta] = \pi r^2$ — M_1 Using condition given

$2\pi = 3\theta - 3\sin\theta$

$\Rightarrow 3\theta = 3\sin\theta + 2\pi$ — B_1

(b) $\tan\alpha = \sec\alpha - \frac{1}{3}$

$\tan^2\alpha = \sec^2\alpha - \frac{2}{3}\sec\alpha + \frac{1}{9}$ — M_1 Squaring both sides

$\sec^2\alpha - 1 = \sec^2\alpha - \frac{2}{3}\sec\alpha + \frac{1}{9}$ — M_1 ~~Same variable~~ i.e $\sec^2\alpha$ or $\tan^2\alpha$

$\frac{2}{3}\sec\alpha = \frac{10}{9}$

$\sec\alpha = \left(\frac{10}{9}\right)\left(\frac{3}{2}\right) = \frac{5}{3} = \frac{30}{18}$ — M_1 correct $\sec\alpha$

$\Rightarrow \cos\alpha = 3/5$ — A_1 Correct $\cos\alpha$

$\tan\alpha = \frac{5}{3} - \frac{1}{3}$ — M_1 ~~Substitution~~ Subtraction

$\tan\alpha = 4/3$ — A_1 correct $\tan\alpha$

12

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ALTERNATIVE 15(b)

$\tan\alpha = \sec\alpha - \frac{1}{3}$

$\frac{2t}{1-t^2} = \frac{1+t^2}{1-t^2} - \frac{1}{3}$

$\frac{2t - 1 - t^2}{1-t^2} = -\frac{1}{3}$

$3(2t - 1 - t^2) = -1 + t^2$

$6t - 3 - 3t^2 = -1 + t^2$

$4t^2 - 6t + 2 = 0$

$2t^2 - 3t + 1 = 0$

$2t^2 - 2t - t + 1 = 0$

$2t(t-1) - 1(t-1) = 0$

$(2t-1)(t-1) = 0$

$t = \frac{1}{2}$ or $t \neq 1$ | M1 solving for t | A1 value of t

$\tan\frac{\alpha}{2} = \frac{1}{2}$

~~$\tan\alpha = \frac{1}{2}$~~

$\tan\alpha = \frac{2(\frac{1}{2})}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$ | M1 A1 Sub and correct tan α

$\cos\alpha = \frac{1-t^2}{1+t^2} = \frac{1-\frac{1}{4}}{1+\frac{1}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}}$ | M1 substitution

$\cos\alpha = \frac{3}{5}$ | A1 correct cos α

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Qn 16 $y = 5x(2-x) = 10x - 5x^2$

Intercepts:

When $y=0$, $x=0$, $x=2$

$(0,0)$ $(2,0)$ — B_1 Both intercepts

When $x=0, y=0$ $(0,0)$

$\therefore$ ~~(0,0) and (2,0) are~~ Turning points

$\frac{dy}{dx} = 10 - 10x = 0$ — M_1 Differentiation and equating to 0

$x = 1$ — A_1 value of x

~~Nd~~ Nature, $\frac{d^2y}{dx^2} = -10$ (maximum Point) — M1 process

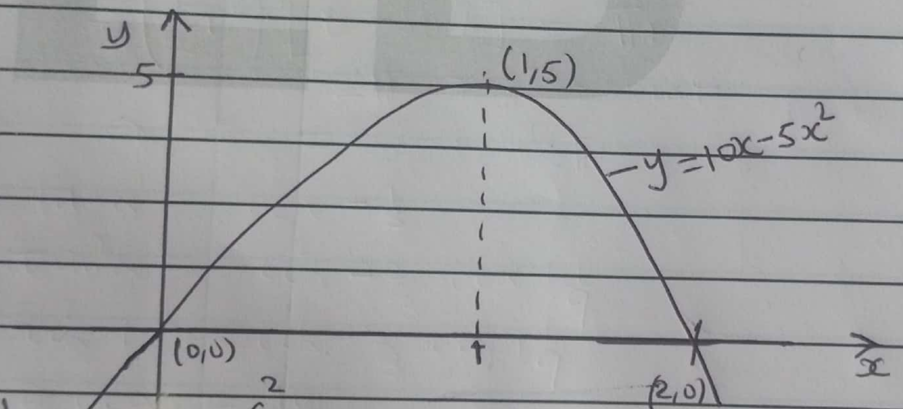
It can be implied on sketch

At $x = 1$, $y = 10(1) - 5(1)^2$

$\therefore$ $(1,5)$ is a maximum Point — B_1 maximum Point

Behaviour As $x \to \pm\infty$, $y \to -\infty$ — B_1

If Behaviour is missed above it is implied on sketch



~~B_1 for the points~~

B_1 for the shape

16(b) $V = \int_0^2 \pi y^2 dx$

$= \int_0^2 \pi(10x - 5x^2)^2 dx$ — M_1 Substitute into ~~the formula~~. If π misses Award 0

$= \pi \int_0^2 [100x^2 - 100x^3 + 25x^4] dx$

$= \pi\left[\frac{100x^3}{3} - 25x^4 + 5x^5\right]_0^2$ — M_1 Correct Integral

$= \pi\left[\left(\frac{800}{3} - 400 + 160\right) - 0\right]$ — M_1 Substituting the Limits

$= \frac{80\pi}{3}$ cubic units / 83.7758 cubic units — A_1

OR $\frac{80}{3}\left(\frac{22}{7}\right) = 83.81$ 2d.ps

12

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ALTERNATIVE

$y = -5(x^2 - 2x)$

$y = -5[x^2 - 2x + 1] + 5$ M1

$y = 5 - 5(x-1)^2$ M1

$y_{max} = 5$ when $x = 1$ A1

$(1,5)$ maximum point B1