P425/1
Pure mathematics
Paper 1
July/August 2024
3 hours

NAMIREMBE DIOCESE COUHEIA SECONDARY MOCK EXAMINATIONS 2024

Uganda Advanced certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *questions in section* **A** *and only five questions from section* **B**.
- *All necessary calculations* **MUST** *be done on the same page as the rest of the answers.*
- *Any extra question(s) attempted in section* **B** *will not be marked.*
- *Begin each other question on a fresh sheet of paper.*
- *All working must be shown clearly.*
- *Silent, non-programmable, scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Answer all the questions in this section.

1. Solve the equation $Z^2 - 3(1+i)z + 2(3+i) = 0$ *(05 marks)*

2. Prove that $\frac{\sin\theta \cos 2\theta + \cos 3\theta \cos 6\theta}{\sin\theta \sin 2\theta + \sin 3\theta \sin 6\theta} = \cot 5\theta$ *(05 marks)*

3. Differentiate $\frac{x^2+2}{(x^2-3x+2)^{\frac{1}{2}}}$ with respect to x, expressing your answer in the simplest form. *(05 marks)*

4. If $\left(\frac{1}{2}, 2\right)$ is one extremity of a focal chord of the parabola $y^2 = 8x$, find the coordinates of the other extremely. *(05 marks)*

5. Find the polynomial function $P(x)$ of degree 3, with a leading coefficient 2 such that $P(0) = 1, P(1) = 0$ *and* $P(2) = 1$. *(05 marks)*

6. The area of an equilateral triangle increases at a rate of $20cm^2s^{-1}$, find the rate of increase in length of the side of the triangle when the side is $5cm$. *(05 marks)*

7. Given the points $L(2,-1,0), M(4,7,6)$ *and* $N(8,5,-4)$, find the vector equation of the line which joins the midpoint of LM *and* MN. *(05 marks)*

8. Find the volume of the solid of revolution formed when the area enclosed by the curve $y = 5 + 4x - x^2$ and $x - axis$ is rotated through 4 right angles about the line $y = 0$ *(05 marks)*

SECTION B (60 MARKS)

Answer any five questions from this section.

9. AB is a diameter of a circle in which *A (1,3)* and *B (7, -1)*
   (a) Find the:
   (i) centre C of the circle; *(02 marks)*
   (ii) radius r of the circle; *(02 marks)*
   (iii) equation of the circle. *(02 marks)*

(b) The line $y + 5x = 8$ cuts the circle at A and again at a second point D. Calculate the coordinates of D. *(03 marks)*

(c) Prove that the line $\overline{AB}$ is perpendicular to the line $\overline{CD}$. *(03 marks)*

10. (a) Given the parametric equations $x = 3 + 4cos\alpha; y = 5 - 8sin\alpha$.
Find $\frac{d^2y}{dx^2}$ *(07 marks)*

(b) If $y = \frac{cosx}{x^2}$, prove that: $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (2 + x^2)y = 0$ *(05 marks)*

11. (a) Given that the second, fourth and ninth term of an Arithmetic Progression (AP) are consecutive terms of a Geometric Progression (GP), show that the common ratio of the GP is $r = 2.5$. *(06 marks)*

(b) A piece of land of area $50{,}100m^2$ is divided in such a way that the areas of the plots are in Arithmetic Progression (AP). If the area of the smallest and the largest plots are $2m^2$ and $1000m^2$ respectively, find the:

(i) number of plots in the piece of land;

(ii) total area of the first 13 plots to the nearest square metres. *(06 marks)*

12. (a) Given that $sin\ A = \frac{\sqrt{3}}{2}$ and $tanB = \frac{1}{\sqrt{3}}$, where A and B are acute angles, find the value of cos(A+B) *(04 marks)*

(b) Three points A, B and C are in a straight line on a horizontal ground with B between A and C. A vertical pole at A is supported by two wires attached to its top and the points B and C. The wire at C makes an angle of 30^o with the ground and the wire at B makes 50^o with the ground. If $\overline{BC}$ = 4cm, find the:

(i) length of the two wires;

(ii) height of the pole. *(08 marks)*

13. Evaluate: (a) $\int_0^{\frac{\pi}{4}} \frac{cosx}{(1-sinx)sinx}dx$ *(06 marks)*

(b) $\int_1^2 \frac{dx}{x^2\sqrt{4-x^2}}$ *(06 marks)*

14. (a) Find the vector parametric equations of the plane passing through the points $A(2,3,4), B(-2,4,-3)$ *and* $C(1,0,3)$ and deduce its cartesian equation. *(07 marks)*

(b) Determine the point of intersection of the plane in (a) above and the line $x+1=\frac{y+2}{5}=\frac{2-z}{2}$. *(05 marks)*

15. (a) Find the middle term in the expansion of $\left(3x^2+\frac{1}{2x}\right)^{10}$. *(04 marks)*

(b) Obtain the binomial expansion of $\sqrt{\left(\frac{1+x}{1-x}\right)}$ up to the term in x^2. Hence by putting $x=\frac{1}{5}$, find $\sqrt{6}$ correct to 3 significant figures. *(08 marks)*

16. (a) Given that $\frac{dy}{dx}=e^{-2y}$ and $y=0$ *when* $x=5$, find the value of x when $y=3$ *(05 marks)*

(b) The population of a community is known to increase at a rate proportional to the number of people present at time t. If the population has doubled in 6 years, how long will it take to triple? *(07 marks)*

END