Sample Guide for Section A [illegible]
UA[illegible] 2024 Maths Paper 1

Qn 1

$\frac{x}{y} + \frac{6y}{x} = 5$ and $x - 2y - 2 = 0$

$x^2 + 6y^2 = 5xy$

From the lines, we have $x = 2y + 2$

$(2y+2)^2 + 6y^2 = 5(2y+2)y$

$4y^2 + 8y + 4 + 6y^2 = 10y^2 + 10y$

$4 = 2y$

$y = 2$

$\Rightarrow x = 2(2) + 2$

$= 6$

Coordinates $(6, 2)$

Qn 2. $\int_0^1 \frac{x}{\sqrt{x+1}}\,dx$

let $u = \sqrt{x+1}$

$u^2 = x + 1$

$x = u^2 - 1$

$dx = 2u\,du$

x	u
0	1
1	$\sqrt{2}$

$\Rightarrow \int_0^1 \frac{x}{\sqrt{x+1}}\,dx = \int_1^{\sqrt{2}} \frac{u^2-1}{u} \cdot 2u\,du$

$= 2\int_1^{\sqrt{2}} (u^2 - 1)\,du$

$= 2\left(\frac{u^3}{3} - u\right)\Big|_1^{\sqrt{2}}$

$= 2\left[\left(\frac{(\sqrt{2})^3}{3} - \sqrt{2}\right) - \left(\frac{1}{3} - 1\right)\right]$

$= 2\left(\frac{-\sqrt{2}}{3} + \frac{2}{3}\right)$

$= \frac{2}{3}\left(2 - \sqrt{2}\right)$

$\therefore \int_0^1 \frac{x}{\sqrt{1+x}}\,dx = \frac{2}{3}\left(2 - \sqrt{2}\right)$

Qn3.

$\sin x + \sin y = \beta_1$ — (1)

$\cos x + \cos y = \beta_2$ — (2)

(i) $\tan\left(\frac{x+y}{2}\right) = \frac{\beta_1}{\beta_2}$.

from (1)

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_1$

$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_1$ — (3)

from (2)

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_2$

$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \beta_2$ — (4)

(3) ÷ (4)

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{\beta_1}{\beta_2}.$$

$$\tan\left(\frac{x+y}{2}\right) = \frac{\beta_1}{\beta_2} \quad \boxed{i}$$

(ii) $\cos(x+y) = \frac{\beta_2^2 - \beta_1^2}{\beta_2^2 + \beta_1^2}$.

$(1)^2 + (2)^2$

$\beta_1^2 + \beta_2^2 = (\sin x + \sin y)^2 + (\cos x + \cos y)^2$

$\beta_1^2 + \beta_2^2 = \sin^2 x + \sin^2 y + \cos^2 x + \cos^2 y + 2(\sin x \sin y + \cos x \cos y)$

$\beta_1^2 + \beta_2^2 = 2 + 2(\cos x \cos y + \sin x \sin y)$

$\beta_1^2 + \beta_2^2 = 2(1 + \cos(x-y)) \Rightarrow 1 + \cos(x-y) = \frac{\beta_2^2 + \beta_1^2}{2}$ — (5)

$(3)^2 - (1)^2$

$\beta_2^2 - \beta_1^2 = (\cos x + \cos y)^2 - (\sin x + \sin y)^2$

$\beta_2^2 - \beta_1^2 = \cos^2 x + \cos^2 y - \sin^2 x - \sin^2 y + 2[\cos x \cos y - \sin x \sin y)$

$\beta_2^2 - \beta_1^2 = \cos 2x + \cos 2y + 2\cos(x+y)$

$\beta_2^2 - \beta_1^2 = 2\cos(x+y)\cos(x-y) + 2\cos(x+y)$

$\beta_2^2 - \beta_1^2 = 2\cos(x+y)[\cos(x-y)+1]$ — $(**)$

$(*)$ in to $(**)$

$$\beta_2^2 - \beta_1^2 = 2\cos(x+y) \cdot \frac{\beta_2^2 + \beta_1^2}{2}$$

$$\cos(x+y) = \frac{\beta_2^2 - \beta_1^2}{\beta_2^2 + \beta_1^2}$$

Qn.4.

let $y = \sqrt[3]{x}$

$x = 27$

$\partial x = 0.15$

The $\sqrt[3]{27.15} = y + \partial y$

$y = (x)^{1/3}$

$$\frac{dy}{dx} = \frac{1}{3x^{2/3}}$$

$$\delta y = \frac{1}{3x^{2/3}} \delta x$$

$$\delta y = \frac{1}{3(27)^{2/3}} \cdot 0.15$$

$$= \frac{1}{180}$$

$\Rightarrow \sqrt[3]{27.15} = \sqrt[3]{(27)} + \frac{1}{180}$

$= 3\frac{1}{180}$

Qn5. $A(3,0)$.

$P(x,y)$

line $(x+3)=0$

The distance AP = distance from the line

$$\sqrt{(x-3)^2+(y-0)^2} = \sqrt{(x+3)^2}$$

$$x^2-6x+9+y^2 = x^2+6x+9$$

$$y^2 = 12x$$

The locus is a parabola in the form $y^2 = 4ax$

$\Rightarrow a = 3$.

Directrix $x = -3$.

and the vertex at $(0,0)$.

Qn6.

$A(-2,0\ 6)$

$B(3,-4,5)$

$2x-y+3z-21=0$.

For the points to lie on both sides, then one pt must be >0 and another must <0.

for $A(-2,0,6)$

$$2(-2)-(0)+3(6)-21 = -7 < 0.$$

for $B(3,-4,5)$.

$$2(3)-(-4)+3(5)-21 = 4 > 0$$

At the origin $(0,0,0)$

$$2(0)-(0)+3(0)-21 = -21 < 0.$$

Thus the points lie on both sides of the plane.

Qn 7:

Let, the first term be a.
the common ratio be r.

from $U_n = ar^{n-1}$

$U_2 = ar = 24$

$a = \frac{24}{r}$

$U_3 = ar^2 = 12(b+1)$

$r(ar) = 12(b+1)$

but $ar = 24$

$24r = 12(b+1)$

$r = \left(\frac{b+1}{2}\right)$,

Then $U_1 + U_2 + U_3 = 76$

$a + ar + ar^2 = 76$

$\frac{24}{r} + 24 + 12(b+1) = 76$

but $r = \frac{b+1}{2}$

$(24) \div \left(\frac{b+1}{2}\right) + 24 + 12(b+1) = 76)$

$48 + 24(b+1) + 12(b+1)^2 = 76(b+1)$

$12 + 3(b^2 + 2b + 1) - 13(b+1) = 0$

$3b^2 - 7b + 2 = 0$

$3b^2 - 6b - b + 2 = 0$

$3b(b-2) - 1(b-2) = 0$

$(3b-1)(b-2) = 0$

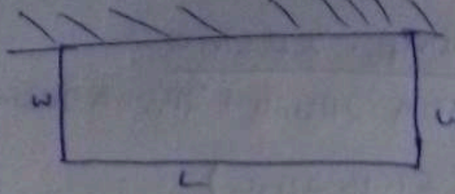
either $3b - 1 = 0 \Rightarrow b = \frac{1}{3}$

or $b - 2 = 0 \Rightarrow b = 2$.

~~for~~ The value of $b = \frac{1}{3}$

For the exact value of b, you need to test for both values and check which of the two values of b gives the sum of 76. After checking, $b = \frac{1}{3}$.

Ans.



$P = L + w + w$

but $P = 200$

$200 = L + 2w$

$L = 200 - 2w$

$A = L \cdot w$

$A = (200 - 2w)w$

$A = 200w - 2w^2$

$\frac{dA}{dw} = 200 - 4w$

for maximum area $\frac{dA}{dw} = 0$.

$200 - 4w = 0$.

$4w = 200$

$w = 50$.

$L = 200 - 2 \times 50$

$= 100$

The maximum/largest area: $A = Lw$

$= 100 \times 50$

$= 5000 m^2$

SECTION B:

9(a) From L.H.S

$$\frac{bc}{ab+ac} = \frac{2R\sin B \cdot 2R\sin C}{2R\sin A \cdot 2R\sin B + 2R\sin A\, 2R\sin C}$$

$$= \frac{4R^2(\sin B \sin C)}{4R^2 \sin A(\sin B + \sin C)}$$

$$= \frac{\sin B \sin C}{\sin(A)(\sin B + \sin C)}$$

$\div \sin B \sin C$

$$\Rightarrow \frac{bc}{ab+ac} = \frac{1}{\sin A\left(\frac{\sin B}{\sin B \sin C} + \frac{\sin C}{\sin B \sin C}\right)}$$

$$= \frac{1}{\sin A(\operatorname{cosec} C + \operatorname{cosec} B)}$$

for Δ,

$$A + B + C = 180$$

$$A = 180 - (B+C)$$

$$\sin A = \sin(180 - (B+C))$$

$$= \sin(B+C).$$

$$\therefore \frac{bc}{ab+ac} = \frac{1}{\sin(B+C)(\operatorname{cosec} C + \operatorname{cosec} B)}$$

$$= \frac{\operatorname{cosec}(B+C)}{\operatorname{cosec} C + \operatorname{cosec} B}$$

Q(b) $3\cot\theta + \csc\theta = 2$

Using t-substitution, $t = \tan\theta/_2$

$$\cot\theta = \frac{1-t^2}{2t}.$$

$$\csc\theta = \frac{1+t^2}{2t}.$$

$$3\left(\frac{1-t^2}{2t}\right) + \frac{1+t^2}{2t} = 2$$

$$3 - 3t^2 + 1 + t^2 = 4t.$$

$$t^2 + 2t - 2 = 0.$$

Solving for t:

Either $t = 0.732$

or $t = -2.732$

For $t = 0.732$.

$\tan\theta/_2 = 0.732 \Leftrightarrow \theta/_2 = \tan^{-1}(0.732)$

$\theta/_2 = 36.2°. 216.2°, ---$

$\theta = 72.4°, ---$

for $t = -2.732$

$\tan\theta/_2 = -2.732 \Leftrightarrow \theta/_2 = \tan^{-1}(-2.732)$

$\theta/_2 = -69.9, 290.1°, 110.1°$

$\theta = 220.2°, ---$

$\therefore \theta = 72.4°, 220.2°$

10)

a) $$\frac{(\sqrt{3}\cos\theta + i\sin\theta)^8}{(3\cos 2\theta + 3i\sin 2\theta)^3}$$

From $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$$\Rightarrow \frac{(\sqrt{3}(\cos\theta + i\sin\theta))^8}{(3\cos 2\theta + 3i\sin 2\theta)^3} = \frac{(\sqrt{3}(\cos\theta + i\sin\theta))^{2\cdot 4}}{(3\cos 2\theta + 3i\sin 2\theta)^3}$$

$$= \frac{(3\cos 2\theta + 3i\sin 2\theta)^4}{(3\cos 2\theta + 3i\sin 2\theta)^3}$$

$$= (3\cos 2\theta + 3i\sin 2\theta)^{4-3}$$

$$= 3\cos 2\theta + 3i\sin 2\theta$$

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b) $(1+3i)z_1 = 5(1+i)$

$$z_1 = \frac{5+5i}{1+3i}$$

$$= \frac{(5+5i)(1-3i)}{(1)^2 - (3i)^2}$$

$$= \frac{20 - 10i}{10}$$

$$z_1 = 2 - i$$

$$|z - z_1| = |z_1|$$

let $z = x + iy$.

$$|x + iy - 2 + i| = |2 - i|$$

$$|(x+2) + i(y+1)| = |2 - i|$$

$$\sqrt{(x-2)^2+(y+1)^2} = \sqrt{2^2+(-1)^2}$$

Squaring both sides,

$$(x-2)^2+(y+1)^2 = 4+1$$

$$x^2-4x+4+y^2+2y+1=5$$

$$x^2-4x+y^2+2y=0$$

$$(x-2)^2+(y+1)^2 = 2^2+1^2$$

$$(x-2)^2+(y+1)^2 = 5$$

c/p $(x+a)^2+(y-b)^2 = r^2$

$\Rightarrow (x-2)^2+(y+1)^2 = 5$ is a equation of the circle with;

Centre at $(2,-1)$

radius $r = \sqrt{5}$

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11)

a) Let the first term be a.

A·P	G·P
a	a
d=2	r=1/3

$$S_\infty = \frac{a}{1-r} \quad \text{for a G·p}$$

$$9 = \frac{a}{1-\frac{1}{3}}$$

$$a = 9(1-\tfrac{1}{3})$$

$$= 6.$$

$$S_n = \frac{n}{2}(2a+(n-1)d) \quad \text{for A·p}$$

$$S_{10} = \frac{10}{2}(2\cdot 6+(10-1)\cdot 2)$$
$$= 5(12+18)$$
$$= 150$$

b) Using the method of inclusive - exclusive

$\Rightarrow \begin{pmatrix}\text{No of ways with} \\ \text{3 Es seperated}\end{pmatrix} = \begin{pmatrix}\text{No of ways} \\ \text{without restriction}\end{pmatrix} - \begin{pmatrix}\text{No of ways} \\ \text{with 3Es toge} \\ \text{ther}\end{pmatrix} - \begin{pmatrix}\text{No of ways} \\ \text{with 2Es together}\end{pmatrix}$,

Without restriction $= \frac{8!}{2!\,3!}$ $= 3360$

$n = 8$
2 - for Ds
3 - for Es

With 3Es together $= \frac{6!}{2!}$ $= 360$

$n = 6$.
2 - for Ds

With 2Es together $= \frac{7!}{2!}$ $= 2520$

$n = 7$
2 - for Ds

No of ways with 3Es seperated.
$= 3360 - 360 - 2520$
$= 480$

12.

a) $A(1,1,1)$
$B(1,0,1)$
$C(3,2,-1)$

$\vec{AB} = \begin{pmatrix}1\\0\\1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix}$
$= \begin{pmatrix}0\\-1\\0\end{pmatrix}$

$\vec{AC} = \begin{pmatrix}3\\2\\-1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix}$
$= \begin{pmatrix}2\\1\\-2\end{pmatrix}$

mtel 1 (cross pdt)

$n = \vec{AB} \times \vec{AC}$
$= \begin{vmatrix}\hat{i} & -\hat{j} & \hat{k}\\ 0 & -1 & 0\\ 2 & 1 & -2\end{vmatrix}$

$n = 2\hat{i} + 2\hat{k}$

using $\hat{r}.n = a.n$.

$\begin{pmatrix}x\\y\\z\end{pmatrix} . \begin{pmatrix}2\\0\\2\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}2\\0\\2\end{pmatrix}$

$2x + 2y = 4$

$x + y = 2$

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mtd 2

using $\hat{r} = \vec{OA} + \lambda\vec{AB} + \mu\vec{AC}$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}0\\-1\\0\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-2\end{pmatrix}$

$\Rightarrow x = 1 + 2\mu$

$\mu = \frac{x-1}{2}$ — ①

$y = 1 - \lambda + \mu$

$z = 1 - 2\mu$

but from ①

$z = 1 - 2\left(\frac{x-1}{2}\right)$

$z = 1 - x + 1$

$x + z = 2$ As be4

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mtd 3. let $n = \begin{pmatrix}a\\b\\c\end{pmatrix}$.

$\vec{AB}.n = 0$

$\begin{pmatrix}0\\-1\\0\end{pmatrix} . \begin{pmatrix}a\\b\\c\end{pmatrix} = 0$

$b = 0$

$\vec{AC}.n = 0$

$\begin{pmatrix}2\\1\\-2\end{pmatrix} . \begin{pmatrix}a\\b\\c\end{pmatrix} = 0$

$2a + b - 2c = 0$
but $b = 0$

$2a = 2c$

$\frac{a}{c} = \frac{1}{1}$

$\Rightarrow 1:0:1 = a:b:c$.

$n = \begin{pmatrix}1\\0\\1\end{pmatrix}$

$r.n = a.n$

$\begin{pmatrix}x\\y\\z\end{pmatrix} . \begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}$

$x + y = 2$ As be4

12 b)

$$A = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

Let $B = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$$n = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}.$$

$$BA = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

mtd 1.

$$BA \times n = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix}.$$

$$\text{distance} = \left| \frac{BA \times n}{n} \right|$$

$$= \frac{\sqrt{(-4)^2 + (2)^2 + (3)^2}}{\sqrt{2^2 + 1^2 + 2^2}}$$

$$= \sqrt{\frac{29}{9}} \text{ units}$$

mtd 2.

Let the line be B.

$$B = \begin{pmatrix} 1 + 2\lambda \\ \lambda \\ 2 + 2\lambda \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 + 2\lambda \\ \lambda \\ 2 + 2\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2\lambda - 1 \\ \lambda + 1 \\ 2\lambda - 2 \end{pmatrix}$$

$$AB \cdot n = 0.$$

$$\begin{pmatrix} 2\lambda - 1 \\ \lambda + 1 \\ 2\lambda - 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 0$$

$$4\lambda - 2 + \lambda + 1 + 4\lambda - 4 = 0$$

$$9\lambda = 5$$

$$\lambda = 5/9.$$

$$AB = \begin{pmatrix} 2(5/9) - 1 \\ 5/9 + 1 \\ 2(5/9) - 2 \end{pmatrix}$$

$$= \frac{1}{9} \begin{pmatrix} 1 \\ 14 \\ -8 \end{pmatrix}$$

$$|AB| = \sqrt{\left(\frac{1}{9}\right)^2 + \left(\frac{14}{9}\right)^2 + \left(\frac{-8}{9}\right)^2}$$

$$= \sqrt{\frac{29}{9}} \text{ units}$$

Abbey

13)

i) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$P(5\cos\theta, 4\sin\theta)$

Equation of the normal is given by

check $\Rightarrow$ $a^2 y_1 (x - x_1) = b^2 x_1 (y - y_1)$

$a^2 = 25,\ b^2 = 16,\ x_1 = 5\cos\theta,\ y_1 = 4\sin\theta.$

$25 \cdot 4\sin\theta (x - 5\cos\theta) = 16 \cdot 5\cos\theta (y - 4\sin\theta)$

$5\sin\theta (x - 5\cos\theta) = 4\cos\theta (y - 4\sin\theta)$

$5x\sin\theta - 25\sin\theta\cos\theta = 4y\cos\theta - 16\sin\theta\cos\theta$

$5x\sin\theta - 4y\cos\theta = 9\cos\theta\sin\theta.$

At A, $y = 0$

$5x\sin\theta = 9\cos\theta\sin\theta$

$x = \frac{9}{5}\cos\theta.$

$A\left(\frac{9}{5}\cos\theta, 0\right)$

At B, $x = 0$

$-4y\cos\theta = 9\cos\theta\sin\theta$

$y = -\frac{9}{4}\sin\theta.$

$B\left(0, -\frac{9}{4}\sin\theta\right)$

mid point of $\overline{AB} = \left(\frac{\frac{9}{5}\cos\theta + 0}{2}, \frac{0 - \frac{9}{4}\sin\theta}{2}\right)$

$= \left(\frac{9}{10}\cos\theta, -\frac{9\sin\theta}{8}\right)$

B Q

For $x^2+y^2-2ax+c^2=0$

Completing squares,

$$x^2-2ax+y^2=-c^2$$

$$(x-a)^2+y^2=-c^2+a^2$$

$$\Rightarrow C_1(a,0),\ r_1^2=-c^2+a^2$$

For $x^2+y^2-2by-c^2=0$

completing squares also;

$$x^2+(y-b)^2=c^2+b^2$$

$$\Rightarrow C_2(0,b),\ r_2=c^2+b^2$$

For orthogonal, $(C_1C_2)^2=r_1^2+r_2^2$

$$(C_1C_2)^2=\underline{a^2+b^2}$$

$$r_1^2+r_2^2=-c^2+a^2+c^2+b^2$$

$$=a^2+b^2$$

since $(C_1C_2)^2=r_1^2+r_2^2$, then the two circles are orthogonal.

14)

a) $$\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}dx = \int_1^3 \frac{x^2+1}{x(x^2+4x+3)}dx$$

$$= \int_1^3 \frac{(x^2+1)}{x(x^2+x+3x+3)}dx$$

$$= \int_1^3 \frac{x^2+1}{x(x(x+1)+3(x+1))}dx$$

$$= \int_1^3 \frac{x^2+1}{x(x+1)(x+3)}dx$$

Using partial fraction,

$$\frac{x^2+1}{x(x+1)(x+3)} \equiv \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+3}$$

Solving it, $A = \frac{1}{3}$, $B = \frac{-7}{6}$, and $C = \frac{11}{6}$.

$$\Rightarrow \int_1^3 \frac{x^2+1}{x(x+1)(x+3)}dx = \int_1^3 \left(\frac{1}{3x} + \frac{-7}{6}\frac{1}{(x+1)} + \frac{11}{6}\cdot\frac{1}{x+3}\right)dx$$

$$= \frac{1}{3}\ln x - \frac{7}{6}\ln(x+1) + \frac{11}{6}\ln(x+3)\Big|_1^3$$

$$= \frac{1}{3}[\ln 3 - \ln 1] - \frac{7}{6}[\ln 4 - \ln 2] + \frac{11}{6}[\ln 6 - \ln 4]$$

$$= \frac{1}{3}\ln 3 - \frac{7}{6}\ln 4 + \frac{7}{6}\ln 2 + \frac{11}{6}\ln 6 - \frac{11}{6}\ln 4$$

$$= \frac{1}{3}\ln 3 + \frac{11}{6}\ln 6 + \frac{7}{6}\ln 2 - 3\ln 4$$

you can simplify into the simpliest form.

1(4)

b) $\displaystyle\int \frac{1}{3x^2+5x+4}\,dx = \int \frac{1}{3\left(x^2+\frac{5}{3}x+\frac{4}{3}\right)}\,dx$

$$= \frac{1}{3}\int \frac{dx}{\left(x^2+\frac{5}{3}x+\left(\frac{5}{6}\right)^2-\left(\frac{5}{6}\right)^2+\left(\frac{4}{3}\right)\right)}.$$

$$= \frac{1}{3}\int \frac{dx}{\left(x+\frac{5}{6}\right)^2+\frac{23}{36}}$$

$$= \frac{1}{3}\int \frac{dx}{\frac{23}{36}\left(\frac{36}{23}\left(x+\frac{5}{6}\right)^2+1\right)}$$

$$= \frac{12}{23}\int \frac{dx}{\left(\frac{6}{\sqrt{23}}x+\frac{5}{\sqrt{23}}\right)^2+1}$$

let $\tan u = \frac{6}{\sqrt{23}}x + \frac{5}{\sqrt{23}}$

$\sec^2 u\,du = \frac{6}{\sqrt{23}}\,dx \Rightarrow dx = \frac{\sqrt{23}}{6}\sec^2 u\,du.$

$$\Rightarrow \frac{12}{23}\int \frac{dx}{\left(\frac{6}{\sqrt{23}}x+\frac{5}{\sqrt{23}}\right)^2+1} = \frac{12}{23}\int \frac{\frac{\sqrt{23}}{6}\sec^2 u\,du}{\tan^2 u+1}$$

$$= \frac{2\cdot\sqrt{23}}{23}\int du.$$

$$= \frac{2\sqrt{23}}{23}u + c$$

$$= \frac{2\sqrt{23}}{23}\tan^{-1}\left(\frac{6}{\sqrt{23}}x+\frac{5}{\sqrt{23}}\right) + c$$

16b) $y = x - \frac{-8}{x^2}$

$y = x + \frac{8}{x^2}$

$y = \frac{x^3+8}{x^2}$.

a)(i) Intercepts, for y, $x=0$

$y = \frac{0+8}{0}$ (undefined)

There is no y-intercepts.

for x, $y=0$.

$0 = \frac{x^3+8}{x^2}$

$x^3 = -8$

$x = -2$.

$(-2, 0)$

(ii) Turning point

$\frac{dy}{dx} = 1 - 16/x^3$

At $\frac{dy}{dx} = 0$.

$1 - \frac{16}{x^3} = 0$

$x^3 = 16$

$x = \sqrt[3]{16}$

$= 2\sqrt[3]{2}$

$\simeq 2.52$ (2dps)

$y = (2.52) + \frac{8}{(2.52)^2}$

$= 3.78$ (2dps)

$(2.52, 3.78)$

Nature of the turning pt.

$\frac{d^2y}{dx^2} = \frac{48}{x^4}$.

$\left.\frac{d^2y}{dx^2}\right|_{(2.52,3.78)} = \frac{48}{(2.52)^4}$

$= 1.19 > 0$ min

Asymptotes -

vertical; $x^2 = 0$

$\Rightarrow x = 0$.

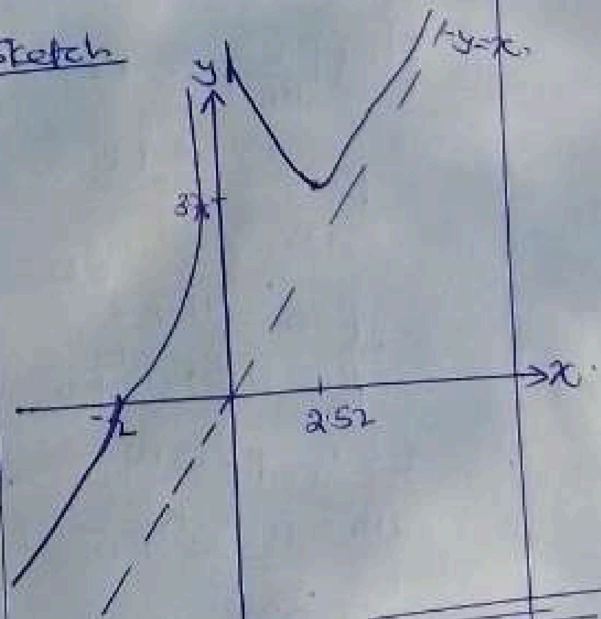
Horizontal $y = \frac{x^3+8}{x^2}$

$y = \frac{1 + 8/x^3}{\frac{1}{x}}$

As $x \to \infty$, $y \to \infty$

$\therefore$ There is no horizontal asymptote but the curve is oblique to the line $y = x$.

Sketch



END