P425/1
PURE MATHEMATICS
Paper 1
July/Aug. 2024
3 hours

UGANDA TEACHERS' EXAMINATIONS SCHEME

Uganda Advanced Certificate of Education

JOINT MOCK EXAMINATIONS

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the eight questions in section* **A** *and any* **five** *from section* **B**.

Any additional question(s) attempted will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Squared paper is provided.

Silent non-programmable scientific calculator and mathematical tables with a list of formulae may be used.

SECTION A:(40 MARKS)

*Answer **all** questions in this section.*

1. Given that $\log_{10} x = \log_5(2x)$, wihout using a calculator or mathematical tables of logarithm, show that $x = \frac{1}{10}$ *(05 marks)*

2. Solve the equation $Z^3 - 5Z^2 + 7Z + 13 = 0$ where Z is a complex number *(05 marks)*

3. If $\cos A = \frac{3}{5}$ where A is reflex and $\tan B = \frac{-8}{15}$ where B is obtuse, find $\sin(A-B)$ in the form $\frac{p}{q}$ *(05 marks)*

4. Prove by induction that

$$2 + 6 + 12 + 20 + \ldots\ldots + n(n+1) = \frac{n}{3}(n+1)(n+2)$$ *(05 marks)*

5. Water is filled in a cone of radius 6cm and height 10cm at a rate of $\frac{2}{\pi} cm^3 s^{-1}$ Determine the rate of increase in depth of water in the cone when the cone is half way full. *(05 marks)*

6. Given three points A(5,1,-2) , B(7,x,-1) and C (12,2,-5), Find the values of x if the vectors **AB** and **BC** are perpendicular. *(05 marks)*

7. Find $\int \frac{3}{x\sqrt{(4-lnx)}} dx$ *(05 marks)*

8. Solve the inequality $\frac{x}{x-4} < 5$ *(05marks)*

SECTION B; (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks*

9. Find (a) $\int(\cos3x - \sin5x)^2 dx$ *(07marks)*

   (b) $\int \dfrac{\sqrt{\tan6x}}{\sin6x\cos6x} dx$ *(05marks)*

10. (a) Expand $\dfrac{1}{(1+2x-3x^2)^{10}}$ in ascending powers of x up to the third nonzero term. *(06 marks)*

    (b) The expansion $(1 + ax)^b$ up to the third term gives $1 + 3x - x^2$. Find the values of a and b *(06 marks)*

11. Sketch a graph of $y = \dfrac{x-1}{x^2-4}$ *(12 marks)*

12. (a) Solve; $\cos3x + \cos2x + \cos x = 0$ for $0 \le x \le 2\pi$ *(06 marks)*

    (b) Prove that in any triangle ABC,

    $\sin A + \sin B + \sin C = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$ *(06 marks)*

13. (a) Determine the Cartesian equation of a plane passing through a point A (5,-1,4) which is parallel to the vectors $5i - 3j + k$ and $2i + j - 3k$ *(04 marks)*

Turn Over

(b)   OAB is a triangle with **OA=a** and **OB=b.** M is the mid point of **OB** while N is on **OA** such that N divides OA internally in the ratio 3:2. If **MA** and **NB** intersect at point T, express the vector **NT** in terms of **a** and **b**

(08 marks)

14.   Express the expression $f(x) = \frac{3x-1}{(2-x)(1-x)^2}$ into partial fractions and hence find $\int f(x)dx$

(12 marks)

15.   (a)   Determine the coordinates of point S the focus to the parabola
$$y^2 - 6y - 8x + 25 = 0$$
(05 marks)

(b)   The normals to the parabola $y^2 = 4ax$ at points P(ap²,2ap) and Q(aq²,2aq) meet at point T. Find the coordinates of T   *(07 marks)*

16.   (a)   Solve the differential equation $\frac{dy}{dx} - 2xy = x^2 e^{5x+x^2}$

Given that $y = 2$ when $x = 0$ *(06 marks)*

(b)   The population P of the village increases at a rate proportional to available population. The population was 20,000 people in the year 2012 and it became 30,000 people in the year 2016.Estimate the population of the village in the next year.   *(06 marks)*

END