PROPOSED MARKING GUIDE
UTES MTC 2 UACE 2024 0782857003
SECTION B. By Grant.K

9. for $f(x) = c(x+3)$

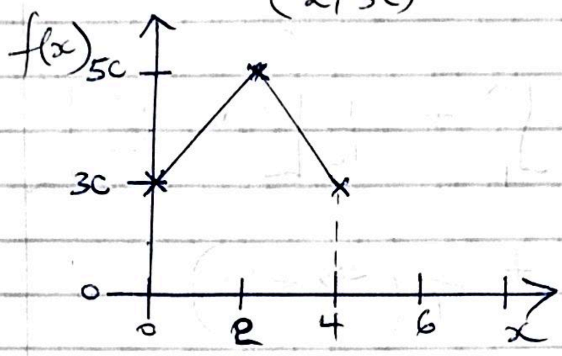
$f(0) = 3c \qquad f(2) = 5c$

i) $(0, 3c) \qquad (2, 5c)$

for $f(x) = c(7-x)$

$f(2) = 5c \qquad f(4) = 3c.$

$(2, 5c) \qquad (4, 3c)$



$$\left(\frac{1}{2}\times 4\times 2c\right) + (3c\times 4) = 1$$

$$(2\times 2c) + 12c = 1$$

$$16c = 1$$

$$c = \frac{1}{16}.$$

ii) $E(x) = \sum x f(x).$

$$= \frac{1}{16}\int_0^2 x(x+3)\,dx + \frac{1}{16}\int_2^4 x(7-x)\,dx$$

$$= \frac{1}{16}\int_0^2 (x^2+3x)\,dx + \frac{1}{16}\int_2^4 (7x-x^2)\,dx$$

$$= \frac{1}{16}\left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^2 + \frac{1}{16}\left[\frac{7x^2}{2} - \frac{x^3}{3}\right]_2^4$$

$$= \frac{1}{16}\left[\frac{26}{3} - 0\right] + \frac{1}{16}\left[\frac{104}{3} - \frac{34}{3}\right]$$

$$= \frac{13}{24} + \frac{35}{24}$$

$$= 2.$$

(iii)

$P(1 \leq x < 3)$

$$= \frac{1}{16}\int_1^2 (x+3)\,dx + \frac{1}{16}\int_2^3 (7-x)\,dx$$

$$= \frac{1}{16}\left[\frac{x^2}{2} + 3x\right]_1^2 + \frac{1}{16}\left[7x - \frac{x^2}{2}\right]_2^3$$

$$= \frac{1}{16}\left(8 - \frac{7}{2}\right) + \frac{1}{16}\left(\frac{33}{2} - 12\right)$$

$$= \frac{9}{32} + \frac{9}{32}$$

$$= 0.5625 \text{ or } \frac{9}{16}.$$

11. a) $F = ma.$

$a = F/m$

$= \frac{1}{4}(4\underset{\sim}{i} + 12t\underset{\sim}{j} - 3\underset{\sim}{k})\ ms^{-2}.$

b)

$v = \int a\, dt.$

$v = \int(\underset{\sim}{i} + 3t\underset{\sim}{j} - \frac{3}{4}\underset{\sim}{k})\, dt$

$v = t\underset{\sim}{i} + \frac{3}{2}t^2\underset{\sim}{j} - \frac{3}{4}t\underset{\sim}{k} + c$

when $t = 0$ $v = 0$

$\Rightarrow c = 0.$

$v_{(t=t)} = (t\underset{\sim}{i} + \frac{3}{2}t^2\underset{\sim}{j} - \frac{3}{4}t\underset{\sim}{k})\ ms^{-1}.$

c) $W = F \cdot s.$

$s = \int v\, dt.$

$s_{t=t} = \int(t\underset{\sim}{i} + \frac{3}{2}t^2\underset{\sim}{j} - \frac{3}{4}t\underset{\sim}{k})\, dt$

$s = \frac{t^2}{2}\underset{\sim}{i} + \frac{t^3}{2}\underset{\sim}{j} - \frac{3}{8}t^2\underset{\sim}{k} + c$

when $t = 0$, $s = (2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k})\,m.$

$2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k} = c.$

$s(t=t) = (\frac{t^2}{2} + 2)\underset{\sim}{i} + (\frac{t^3}{2} - 3)\underset{\sim}{j} + (1 - \frac{3}{8}t^2)\underset{\sim}{k}.$

$s(t=2) = (4\underset{\sim}{i} + \underset{\sim}{j} - \frac{1}{2}\underset{\sim}{k})\,m.$

Displacement after 2s $= (4\underset{\sim}{i} + \underset{\sim}{j} - \frac{1}{2}\underset{\sim}{k}) - (2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k})$

$= (2\underset{\sim}{i} + 4\underset{\sim}{j} - \frac{3}{2}\underset{\sim}{k})\,m.$

Work done $= F \cdot S$

$$= (4\underset{\sim}{i} + 24\underset{\sim}{j} - 3\underset{\sim}{k})(2\underset{\sim}{i} + 4\underset{\sim}{j} - \tfrac{3}{2}\underset{\sim}{k})$$

$$= 8 + 96 + 4.5$$

$$= 108.5J.$$

15. a) $x = \cos x$.

$f(x) = x - \cos x$.

$f(0.2) = -0.78007$

$f(1.5) = 1.42926$.

Since there is a sign change, then there is a root between 0.2 and 1.5.

b) i)

$$X_{n+1} = X_n - \frac{f(x)}{f'(x)}$$

$f(x) = x - \cos x$

$f'(x) = 1 + \sin x$.

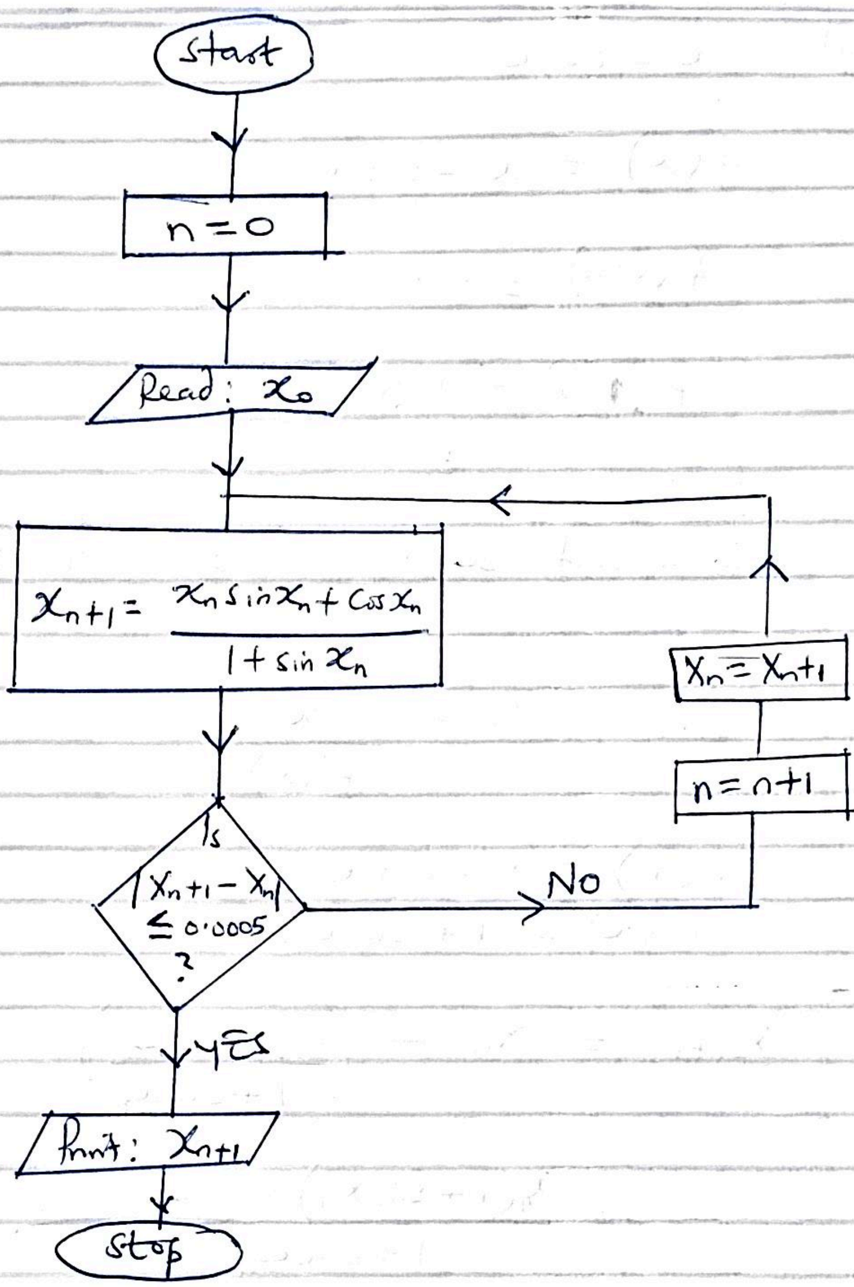
$$X_{n+1} = X_n - \left(\frac{x_n - \cos x_n}{1 + \sin x_n}\right)$$

$$= \frac{x_n(1 + \sin x_n) - x_n + \cos x_n}{1 + \sin x_n}.$$

$$= \frac{x_n + x_n \sin x_n - x_n + \cos x_n}{1 + \sin x_n}.$$

$$x_{n+1} = \frac{x_n \sin x_n + \cos x_n}{1 + \sin x_n}.$$

15 b) (ii)



iii) Dry run

n	x_0	x_{n+1}	$\lvert x_{n+1} - x_0 \rvert$
0	0.85	0.741498	0.108502
1	0.741498	0.739086	0.002412
2	0.739086	0.739085	0.00001
3	0.739085	0.739085	0

Root = 0.739 (3 dps).

16.

a) Distance ~~for~~ in fourth second $= 25.6m$

$= S_4 - S_3 = 25.6m.$

Distance ~~for~~ in the eigth second $= 32m$

$= S_8 - S_7 = 32.$

from $S = ut + \frac{1}{2}at^2.$

$S_4 = 4u + \frac{1}{2}a(4^2)$

$= 4u + 8a$

$S_3 = 3u + \frac{1}{2}a(3^2)$

$= 3u + 4.5a$

$\Rightarrow (4u + 8a) - (3u + 4.5a) = 25.6$

$u + 3.5a = 25.6$ —— ①

$S_8 = 8u + \frac{1}{2}a(8^2)$

$= 8u + 32a$

$S_7 = 7u + \frac{1}{2}a(7^2)$

$7u + 24.5a$

$\Rightarrow (8u + 32a) - (7u + 24.5a) = 32.$

$u + 7.5a = 32$ —— ②

Solving ① and ②

$$- \left| \begin{array}{l} u + 3.5a = 25.6 \\ u + 7.5a = 32 \end{array} \right.$$

$-4a = -6.4$

$a = 1.6ms^{-2}$

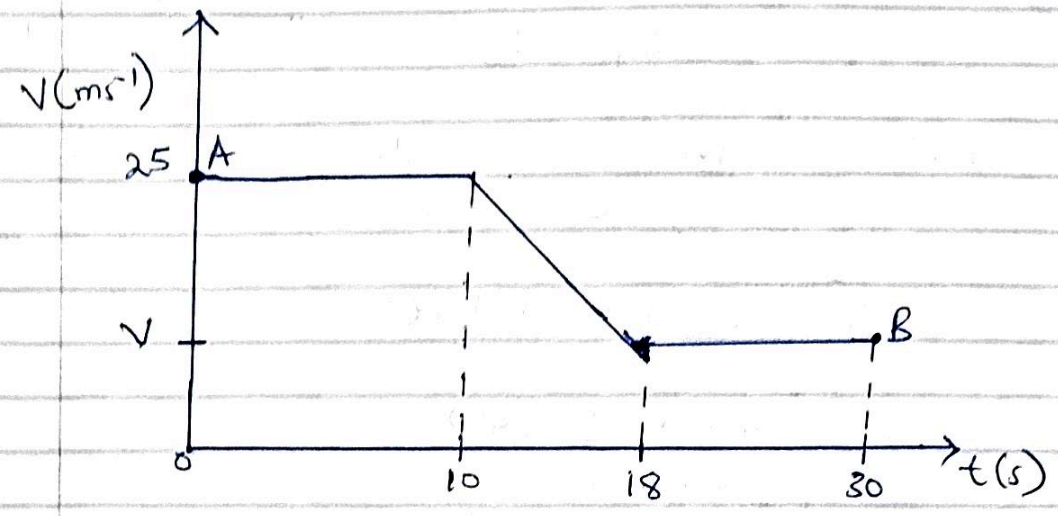
from ①

$u = 25.6 - (3.5 \times 1.6)$

$u = 20ms^{-1}.$

∴ The initial speed of the lorry was $20ms^{-1}$.

16 b)



Total distance = 526 m.

$(25 \times 10) + \frac{1}{2} \times 8 (v + 25) + (v \times 12) = 526$

$250 + 4(v+25) + 12v = 526$

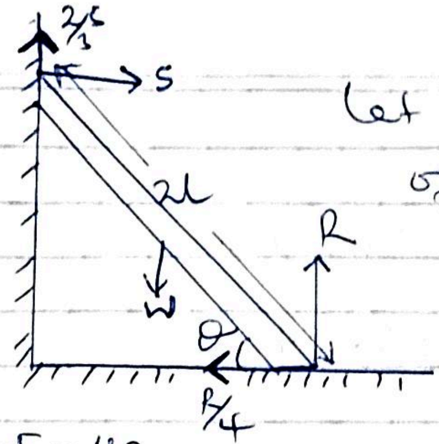
$250 + 4v + 100 + 12v = 526$

$16v = 526 - 350$

$16v = 176$

$v = 11 ms^{-1}$

14.



Let the ladder be of length $2l$.

from $F = \mu R$

$F = \frac{1}{4}R$ [at the lower end]

$F = \frac{2}{3}S$ [at the top end]

Resolving vertically; $\frac{2}{3}S + R = W$.

$R = \frac{3W - 2S}{3}$ —①

Resolving horizontally; $S = R/4$

$R = 4S$ —②

① = ②

$\frac{3W - 2S}{3} = 4S$

$3W - 2S = 12S$

$14S = 3W$

$S = \frac{3}{14}W$ —③

Taking moments about the foot of the ladder.

$W \times l\cos\theta = S \times 2l\sin\theta + \frac{2}{3}S \times 2l\cos\theta$ —④

Put ③ in ④

$W \times l\cos\theta = \frac{3}{14}W \times 2l\sin\theta + \frac{2}{3}\cdot\frac{3}{14}W \times 2l\cos\theta$

Dividing thru by Wl.

$\cos\theta = \frac{3}{7}\sin\theta + \frac{2}{7}\cos\theta$.

$\frac{5}{7}\cos\theta = \frac{3}{7}\sin\theta$

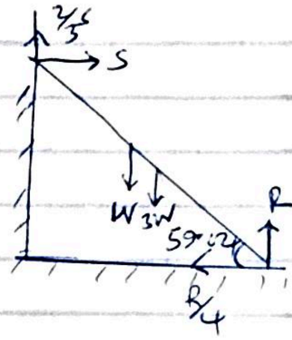
$5\cos\theta = 3\sin\theta$

$\tan\theta = 5/3.$

$\theta = \tan^{-1}(5/3)$

$\theta = 59.04^\circ$

* for a man of weight 3W ascending the ladder above,



Resolving vertically now.

$$\frac{2}{3}S + R = W + 3W \quad (5)$$

Resolving horizontally, $S = R/4 \quad (6)$

from (5) $R = \dfrac{12W - 2S}{3}$

from (6) $R = 4S$

$$\frac{12W - 2S}{3} = 4S$$

$$12W - 2S = 12S.$$

$$14S = 12W$$

$$S = \frac{6}{7}W.$$

Taking moments about the foot of the ladder.

$W \times l\cos\theta + 3W \times x\cos\theta = S \times 2l\sin\theta + \frac{2}{3}S \times 2l\cos\theta$

from $\tan\theta = 5/3$, $\sin\theta =$

$$(Wl + 3Wx)\cos\theta = 2Sl\sin\theta + \frac{4lS}{3}\cos\theta.$$

$$\cancel{\left(Wl + 3Wx - \frac{4lS}{3}\right)\cos\theta = 2Sl\sin\theta}.$$

$$Wl\cos\theta + 3Wx\cos\theta = \frac{6}{7}W \times 2l\sin\theta + \frac{4}{7}W \times 2l\cos\theta$$

Dividing through $W\cos\theta$.

$$l + 3x = \frac{12}{7}l\tan\theta + \frac{8}{7}l$$

$$\frac{12l\tan\theta}{7} + \frac{l}{7} = 3x \quad \Bigg\| \quad \frac{12}{7}l \times \frac{5}{3} + \frac{l}{7} = 3x$$

~~5x~~

$$\frac{20l}{7} + \frac{l}{7} = 3x.$$

$$\frac{21l}{7} = 3x$$

$$3l = 3x$$

$$l = x.$$

The man can ascend half-way up the ladder since the ladder was $2l$.

12.

Age	f	x	fx	c.b	i	f.d
18-<19	24	18.5	444	18 - 19	1	24
19-<20	70	19.5	1365	19 - 20	1	70
20-<24	76	22.0	1672	20 - 24	4	19
24-<26	48	25.0	1200	24 - 26	2	24
26-<30	16	28.0	448	26 - 30	4	4
30-<32	6	31.0	186	30 - 32	2	3
	$\Sigma f =$ 240		$\Sigma fx =$ 5315			

a) $\text{Mean} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{5315}{240}$

$= 22.15$

$\simeq 22$ years.

b) ii)