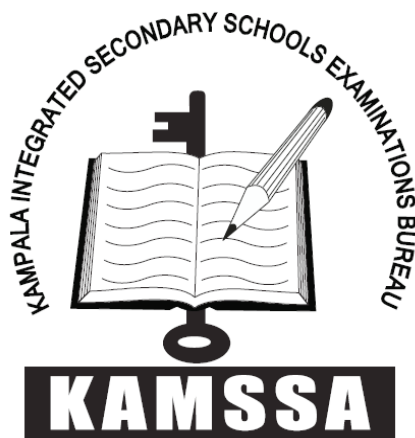


**P425/1**  
**PURE**  
**MATHEMATICS**  
*Paper 1*  
**JAN./FEB. 2021**

*3hours*



## **KAMSSA JOINT MOCK EXAMINATIONS**

**Uganda Advanced Certificate Of Education**

**PURE MATHEMATICS**

**Paper 1**

**3hours**

### **Instructions to candidates:**

- Answer **All** the **eight** questions in section **A** and **five** questions from section **B**.
- Any additional question (s) answered will not be marked.
- All** working **must be** shown clearly.
- Begin each answer on a fresh page.
- Graph paper is provided.
- Silent non-programmeable, scientific calculators and mathematical tables with a list of formulae may be used.
- State the degree of accuracy at the end of each answer given. If a calculator or a mathematical table is used, indicate **Cal** for calculator or **Tab** for mathematical tables.

## SECTION A

1. The quadratic polynomial  $P(x)$  leaves a remainder of  $-6$  on division by  $(x + 1)$ , a remainder  $-5$  on division by  $(x + 2)$  and no remainder on division by  $(x + 3)$ . Find  $P(x)$  and solve the equation  $P(x) = 0$  (5marks)
2. Find all values of  $x$ . to the nearest degree, between  $0^\circ$  and  $360^\circ$  for which  $2\tan 3x + 1 = 0$  (5marks)
3. Evaluate  $\int x^2 \ln(x^2) dx$  (5marks)
4. Prove by induction that  $11^{n+2} + 12^{2n+1}$  is divisible by 133 for  $n \geq 0$  (5marks)
5. A line  $L$ , passes through the point  $(1,0,2)$  and has direction given by the vector  $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$  find the equation of the line through the point  $A(2,3,1)$  and perpendicular to line  $L$ . (5marks)
6. Water is poured into an inverted cone, axis vertical. If the depth of the water increases from 5cm to 7cm in 20 seconds, find the rate at which the depth is increasing when the depth is 9cm. (5marks)
7. Prove that circles  $x^2 + y^2 - 10x - 7y + 31 = 0$  and  $x^2 + y^2 + 2x + 2y = 23$  touch externally and calculate the coordinates of the point of contact. (5marks)
8. Evaluate  $\int_0^{\frac{\pi}{3}} \frac{\cos\theta d\theta}{\cos\theta + \sin\theta}$  (5marks)

## SECTION B

9. Show that, if the chord joining the points P ( $ap^2, 2aq$ ) and Q ( $aq^2, 2aq$ ) on the parabola  $y^2 = 4ax$  passes through  $(a,0)$ , then  $p.q = -1$ . Further, the tangent at P meets the line through Q parallel to the axis of the parabola at R, prove that the line  $x + a = 0$  bisects PR (12marks)

10.(i) Differentiate with respect to x

(a)  $\sqrt{(1 + 4x^2)}$  (4marks)

(b)  $\sin^2 5x$  (4marks)

(ii) if  $y = \sqrt{\left(\frac{x}{2x+1}\right)}$  find the value of  $\frac{dy}{dx}$  when  $x = 4$  (4marks)

11. Sketch the curve  $y = \frac{(x+2)(x-4)}{(x+1)(x-3)}$  (12marks)

12.(a) Prove that  $\sqrt{\frac{\tan \theta + \sin \theta}{\cot \theta - \cos \theta}} = \tan^2 \theta \sqrt{\frac{1 + \sin \theta}{1 - \cos \theta}}$  (07marks)

(b) Given that  $a = 2 \cos x + 3 \cos 2x$  and  $b = 2 \sin 2x$ . Show that  $1 \leq a^2 + b^2 \leq 25$  (05marks)

13. (a) if  $y = e^x \cos 3x$  show that

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 10y = 0 \quad (06marks)$$

(b) Find the area of the region between the x – axis and the graph  $y = x^2 \cos x$  from

$$\frac{-\pi}{2} \text{ to } x = \frac{\pi}{2} \quad (06marks)$$

14.(a) Express the complex numbers  $Z = 4i$  and  $z_2 = 2 - 2i$  in trigonometric form hence

evaluate  $\frac{z_1}{z_2^2}$  (06marks)

(a) Find the value of x and y in  $\frac{x}{2+3i} - \frac{y}{3-2i} = \frac{6+2i}{1+8i}$  (06marks)

15. A hemispherical bowl of radius  $a$  cm is initially full of water. The water runs out of a small hole at the bottom of the bowl at a constant rate which is such that it would empty the bowl in 24s. Given that, when the depth of the water is  $x$  cm, the volume is  $\frac{1}{3}\pi x^2 (3a - x)$  cm<sup>3</sup>.

Prove that the depth is decreasing at a rate  $\frac{a^3}{[36x(2a-x)]}$  cm/s. find after what time the depth is  $\frac{1}{2}a$  cm, and the rate at which the water level is then decreasing. *(12marks)*

16.(a) Given that  $k$  is a constant, find the solution of the differential equation  $\frac{dy}{dt} + ky = 2$ .

For which  $y = 3$  when  $t = 0$  *(05marks)*

(b) The rate at which a chemical compound burns is proportional to the un burnt portion.

Initially when the observation was made  $\frac{1}{5}$  of the chemical compound had burnt. Three hours later,  $\frac{1}{3}$  of the chemical compound had burnt. Find the fraction of the chemical compound which remained after 7.5 hours. *(07 marks)*

**END**