

NAMILYANGO COLLEGE

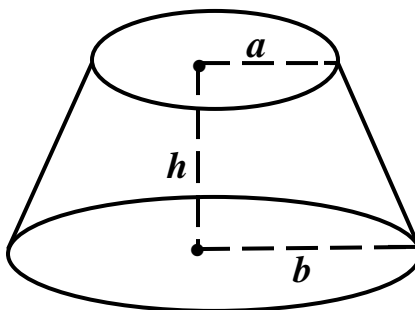
A-LEVEL APPLIED MATHEMATICS SEMINAR 2024

PAPER STRUCTURE	SECTION A	SECTION B
<i>Statistics and probability</i>	3	3
<i>Numerical methods</i>	2	2
<i>Static Mechanics</i>	1	1
<i>Dynamic Mechanics</i>	1	1
<i>Kinematic Mechanics</i>	1	1

STATIC MECHANICS

1(a) Show that the centre of gravity of a solid cone of radius r and height h lies along its axis at a distance $\frac{1}{4}h$ from the base

(b) The figure below shows a solid conical frustum of height h and whose top and bottom radii being a and b respectively



Show that the centre of gravity of this frustum lies along its axis at a distance

$$\frac{h(b^2 + 2ab + 3a^2)}{4(b^2 + ab + a^2)} \text{ from the bottom}$$

2. (a) A particle of weight $50N$ is supported by two light inextensible strings of lengths $8m$ and $13m$ attached to two fixed points $15m$ apart on a horizontal beam. Find the tension in each string

(b) The ends **P** and **Q** of a light inextensible string **PBCQ** are fastened to two fixed points on a horizontal beam. Particles of mass **3kg** and **4kg** are attached to the string at the points **B** and **C** respectively. If **PB** is inclined at 45° to the horizontal and $\angle PBC = 150^\circ$, find the:

- (i) tension in each portion of the string
- (ii) angle **CQ** makes with the horizontal

3. **ABCDE** is a regular pentagon of side **4m**. Forces of magnitude **2N**, **3N**, **5N** and **7N** act along \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} and \overrightarrow{EB} respectively. The resultant of this system of

forces cuts **AB** produced at **H**. Taking **A** as the origin and **AB** as the **x-axis**,

- (i) find the magnitude and direction of the resultant force
- (ii) show that length **AH** = **15.34m** correct to 4 S.f
- (iii) find the perpendicular distance from **A** to the line of action of the resultant force

4. Coplanar forces $(3\mathbf{i} + 3\mathbf{j})\text{N}$, $(4\mathbf{i} - 5\mathbf{j})\text{N}$, $(-5\mathbf{i} + 2\mathbf{j})\text{N}$ and $(2\mathbf{i} + 3\mathbf{j})\text{N}$ act at points with position vectors $(3\mathbf{i} + \mathbf{j})\text{m}$, $(\mathbf{i} + 3\mathbf{j})\text{m}$, $(-2\mathbf{i} + \mathbf{j})\text{m}$ and $(-2\mathbf{i} - 2\mathbf{j})\text{m}$ respectively.

- (i) Find the resultant force and find where its line of action cuts the **x-axis**
- (ii) A couple of moment **bNm** acting anticlockwise and a force $(p\mathbf{i} + q\mathbf{j})\text{N}$ acting at a point with position vector $(2\mathbf{i} + \mathbf{j})\text{m}$ are now added to the above system. If these reduce the system to equilibrium, find the values of **p**, **q** and **b**

5. A uniform ladder PQ of length $2a$ and weight w is inclined at an angle of $\tan^{-1} 2$ to the horizontal with its end Q resting against a smooth vertical wall and end P on a rough horizontal ground with which the coefficient of friction is $\frac{5}{12}$. If a boy of weight W can safely ascend a distance x up this ladder before it slips,

(i) show that $x = \frac{a(2w + 5W)}{3W}$

(ii) deduce that the boy can only reach the top of the ladder if $W = 2w$

6. A uniform rod PQ of length $8m$ and weight $18N$ is freely hinged at P and carries a mass of $3kg$ at Q . The rod is kept horizontally by a string attached at Q and to a point C distant $6m$ vertically above P . Find the:

(i) tension in the string

(ii) magnitude and direction of the reaction at the hinge

7. A box of mass $6.5kg$ is placed on a rough plane inclined at $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. The coefficient of friction between the box and the plane is 0.25 . Find the least horizontal force required:

(i) to move the box up the plane

(ii) to prevent the box from sliding down the plane

KINEMATIC MECHANICS

1. Two cyclists **P** and **Q** are travelling along straight roads which cross at an angle of 60° at point **C**. If their riding speeds towards **C** are 4kmh^{-1} and 5kmh^{-1} and they are respectively **8km** and **15km** from **C**, find the:

- (i) least distance between the cyclists
- (ii) time that elapses before the cyclists are closest
- (iii) distances of **P** and **Q** from **C** when they are nearest

2. A battleship and a patrol ship are initially **16km** apart with the battleship on the bearing of 035° from the patrol. The battleship sails at 14kmh^{-1} in the direction **S30°E** and the patrol ship at 17kmh^{-1} in the direction **N50°E**.

(a) Find the:

- (i) shortest distance between the ships
- (ii) time that elapses before the ships are closest

(b) If the guns on the battleship have a range of up to **6km**, find the time that elapses when the patrol ship is within range of these guns

3. Two boats **P** and **Q** are sailing with respective speeds of 20kmh^{-1} and 19kmh^{-1} .

Initially **P** is **10km** from **Q** on a bearing of 320° and is on a course of 200° .

Find the:

- (i) two possible courses **Q** can take in order to intercept **P**
- (ii) time taken for interception to occur in each case

4. (a) At certain times, the position vector \mathbf{r} and velocity vector \mathbf{v} of two ships

\mathbf{A} and \mathbf{B} are as follows:

$$\mathbf{r}_A = (-2\mathbf{i} + 3\mathbf{j})\text{km} \quad \mathbf{V}_A = (12\mathbf{i} - 4\mathbf{j})\text{kmh}^{-1} \text{ at } 11:45\text{am}$$

$$\mathbf{r}_B = (8\mathbf{i} + 7\mathbf{j})\text{km} \quad \mathbf{V}_B = (2\mathbf{i} - 14\mathbf{j})\text{kmh}^{-1} \text{ at } 12:00\text{noon}$$

If the ships maintain these velocities, find the:

(i) position vector of ship \mathbf{A} at noon

(ii) time when the ships are closest

(iii) shortest distance between the ships

(iv) distance of ship \mathbf{A} from the origin when the two ships are closest

(b) If instead ship \mathbf{B} had a velocity $\mathbf{V}_B = (-2\mathbf{i} - 14\mathbf{j})\text{kmh}^{-1}$, show that the ships

will collide and find when and where the collision occurs

5. Two stations \mathbf{P} and \mathbf{Q} are 2.5km apart. A train passes \mathbf{P} at a speed of 14ms^{-1} and accelerates uniformly for 20s to a speed \mathbf{v}_1 . Over the next 720m covered in 15s , its acceleration alters to a speed \mathbf{v}_2 . It travels at this speed for 13s and then over the next 500m covered in 10s with uniform deceleration its speed at \mathbf{Q} is \mathbf{v}_3 . Find the:

(i) values of \mathbf{v}_1 , \mathbf{v}_2 and \mathbf{v}_3

(ii) acceleration for the second part of the motion

(iii) fraction of the whole distance covered with constant speed

6. (a) The velocity of a uniformly accelerating train changes from u to v in time t .

(i) Sketch its velocity time graph

(ii) Derive the equation for its motion $v^2 = u^2 + 2as$, where a is its acceleration

(b) A uniformly accelerating train passes successive kilometer marks with

velocities 20ms^{-1} and 28ms^{-1} respectively. Find its velocity when passing the next kilometer mark

(c) A uniformly retarding car takes 8s and 16s to travel between successive

points A, B and C each 144m apart. Find the further distance it travels to come to rest

7. (a) A particle is projected from level ground at an angle of elevation θ with

initial speed $u\text{ms}^{-1}$. Show that the equation of its path is given by

$$y = x \tan \theta - \frac{gx^2(1 + \tan^2 \theta)}{2u^2}$$

(b) A ball kicked from level ground with a speed of 30ms^{-1} just clears a

vertical wall 9m high and 72m away. Calculate the possible angles of

projection (use $g = 10\text{ms}^{-2}$)

(c) A ball projected at an angle with a speed of $14\sqrt{10}\text{ms}^{-1}$ from the top of a

tower 200m high hits the ground at a point 200m away from the foot of the tower.

(i) Show that the two possible directions of projection are at right angles to each other

(ii) Find the two possible times of flight

8. A particle is projected from the origin O with velocity $\mathbf{u} = (9 \cdot 8\mathbf{i} + 29 \cdot 4\mathbf{j})\text{ms}^{-1}$ and moves freely under gravity.

(a) Find the particle's velocity and position vector after t seconds

(b) Show that the particle's equation of path is given by $y = 3x - \frac{5x^2}{98}$.

Hence find the particle's horizontal range and maximum height reached

(c) Find the direction in which the particle is moving after t seconds

(d) Find the two times when the direction in which the particle is moving is at right angles with the line joining the position of the particle to O

DYNAMIC MECHANICS

1. A force $(24t\mathbf{i} - 12t\mathbf{j})\text{N}$ acts on a particle of mass 2kg initially at rest at a point with position vector $(-4\mathbf{i} + 3\mathbf{j})\text{m}$. Find the:

(i) velocity of the particle after t seconds

(ii) distance from the origin after 2s

(iii) power exerted by the force at $t = 2\text{s}$

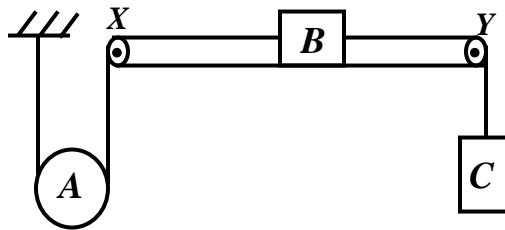
(iv) work done by the force between $t = 1\text{s}$ and $t = 2\text{s}$

2. A pile-driver of mass $m_1\text{kg}$ falls through a height $h\text{m}$ onto a pile of mass $m_2\text{kg}$ without rebounding. If the pile is driven into the ground a depth $d\text{m}$, show that

the resistance of the ground to penetration $R = \frac{m_1^2 gh}{(m_1 + m_2)d} + (m_1 + m_2)g$ and

the time for which the pile is in motion $T = \frac{(m_1 + m_2)d\sqrt{2gh}}{m_1 gh}$

3. (a) A car of mass **500kg** is moving up a hill inclined at $\sin^{-1}\left(\frac{1}{7}\right)$ to the horizontal. The resistance to motion of the car is **300N**. If the power output of the car is **84kW**, find the acceleration of the car when its speed is 35ms^{-1}
- (b) A car of mass **800kg** is moving at a constant speed of 20ms^{-1} down a hill inclined at an angle θ to the horizontal. The resistance to motion of the car is **1300N**. If the power output of the car is **10kW**, show that $\sin\theta = \frac{5}{49}$
4. A light inelastic string is fixed at one end and passes under a moveable pulley **A** of mass **4kg** which hangs vertically. The other end of the string is attached to particle **B** of mass **4kg** which lies on a rough horizontal table. A second inelastic string connects **B** to a freely hanging particle **C** of mass **10kg**. The strings are passing over smooth fixed pulleys **X** and **Y** as shown



If the system is released from rest and the coefficient of friction between **B** and the table is **0.5**, find the:

- (i) accelerations of **A**, **B** and **C**
- (ii) tension in the strings
- (iii) reaction of pulley **Y** on the string

5. A particle moving with S.H.M has velocities of 7.5ms^{-1} and 4ms^{-1} as it passes through points **P** and **Q** which are 0.9m and 0.2m respectively from the end points of its path. Find the:
- (i) length of its path and the period of the motion
 - (ii) maximum velocity and maximum acceleration
 - (iii) time it takes to travel directly from **P** to **Q**
 - (iv) time which elapses before it next passes through **Q**
 - (v) mean velocity during its motion from one extreme position to the other
6. A particle of mass **m** is suspended from a fixed point **O** by a light elastic string of natural length **l**. When the particle hangs in equilibrium, the extension of the string is **d**. The particle is then slightly vertically displaced from its equilibrium position and then released. Show that it moves with SHM of period $2\pi\sqrt{\frac{d}{g}}$

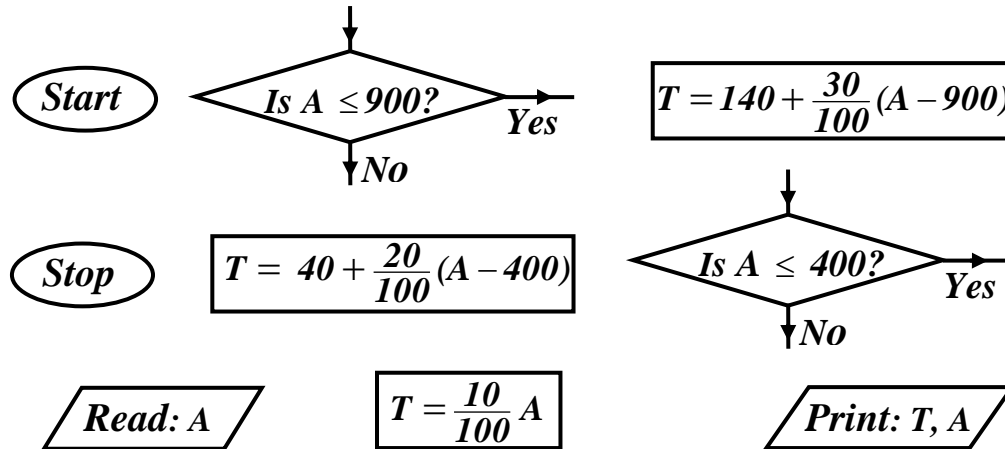
NUMERICAL METHODS

1. Three decimal numbers **X**, **Y** and **Z** were rounded off to give **x**, **y** and **z** with errors Δx , Δy and Δz respectively. Show that the maximum relative error in the approximation of $\frac{X}{Y-Z}$ by $\frac{x}{y-z}$ is $\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y-z}\right| + \left|\frac{\Delta z}{y-z}\right|$. Hence find the absolute error and percentage error in $\frac{1.6}{2.15 - 1.9}$ and the interval within which its exact value is expected to lie

2. The income tax of an employee is calculated as follows:

<i>Taxable Income, A (£)</i>	<i>Tax rate (%)</i>
01–400	10
401–900	20
Above 900	30

The taxation system is described by the following parts of the flowchart:



(i) Arrange the given parts to form a complete logical flowchart

(ii) State the purpose of the flowchart

(iii) Performing a dry run for the flow chart and complete the table below

<i>A</i>	<i>T</i>
1500	----
750	----

3. (i) Show that the Newton Raphson formula for finding the natural logarithm

of the k^{th} root of a number A is given by $x_{n+1} = x_n - \frac{1}{k} + \frac{A}{k} e^{-kx_n}$

(ii) Draw a flowchart that computes and prints the root in (a) above correct to 3 decimal places

(iii) Perform a dry run for your flowchart using $x_0 = 1.25$, $A = 147$ and $k = 4$

4. Find the percentage error in estimating $\int_0^{\frac{\pi}{3}} \sec^2 x \, dx$ using trapezium rule

with six ordinates correct to 4 decimal places and state how it can be reduced

5. An equation has two iterative formulae $x_{n+1} = 2x_n^2 e^{x_n}$ and $x_{n+1} = \frac{1}{2} e^{-x_n}$.

(i) Use each formula twice to deduce with a reason the most suitable one when

$x_0 = 0.4$. Hence state the root correct to 3 d.p

(ii) Without iterating deduce with a reason the most suitable formula when

$x_0 = 0.4$. Hence use it twice to find the root correct to 3 d.p.

(iii) Show that the equation for the two iterative formulae is $2xe^x - 1 = 0$

6.(a) Show that the equation $\cos(x^2) - x + 3 = 0$ has a root between 2.5 and 3.

Hence use linear interpolation thrice to find the root correct to 3 d.p

(b) In a motor rally, car **P** was observed to be at distances of 350m and 400m from the starting line when the chasing car **Q** was at distances of 240m and 300m respectively. How far was car **P** from the starting line when car **Q**:

(i) started chasing it

(ii) caught up with it

(c) Use the fact that $f(1.15) = 1.32$ and $f^{-1}(1.26) = 1.25$ to find the value of $f^{-1}(1.22)$ by linear interpolation correct to 3 d.p

7. Locate the ranges where the two real roots of the equation $x^4 - x - 10 = 0$ lie. Hence use Newton Raphson method to find the least root correct to 3 d.p

STATISTICS

1. The table below shows the prices of three items for the years **2023** and **2024**

Item	PRICE (£)		Weights
	IN 2023	IN 2024	
A	150	153	5
B	250	261	2
C	525	588	3

Taking **2023** as the base year, calculate the:

- (i) simple aggregate price index for **2024**. Comment on your result
 - (ii) weighted mean price index for **2024**. Comment on your result
 - (iii) weighted aggregate price index for **2024**. Comment on your result
 - (iv) cost of items in **2023**, similar to the items in **2024** whose cost was **£540** using the result in (iii) above
2. The grades of 8 students in **UNEB**, **pre mock** and **post mock** were as follows:

UNEB	B	A	O	C	B	E	O	D
Pre Mock	D	B	D	C	A	F	O	E
Post Mock	C	B	E	O	A	D	F	E

- (a) Calculate the rank correlation coefficient between the grades of:
 - (i) **UNEB** and **Pre Mock**
 - (ii) **UNEB** and **Post Mock**
- (b) Which of the two mocks had a better correlation with **UNEB**? Give a reason

3. The table below shows the weights in kg of **100** babies:

Weights	2	2.5	4.5	6	7	8
No of babies	35	20	20	10	10	5

(i) Calculate the mean, variance and median for the above data

(ii) Assuming this was a sample taken from a normal population, find the **90%** confidence interval for the mean weight of all babies

4. The table below shows the weights in kg of **40** boys:

Weights	30 – < 35	35 – < 40	40 – < 55	55 – < 60	60 – < 65
Frequency	8	5	12	9	6

(a) Calculate the mean, mode and percentage of boys heavier than **45 kg**

(b) Draw an ogive for the data and use it to estimate the:

(i) median weight

(ii) quartile deviation

(iii) range of the weights of the middle **70%** of the boys

(c) Draw a histogram for the data and use it to estimate the modal weight

5. The price index of an item in **2022** based on **2023** was **88**. Its price index in **2024** based on **2023** was **132**. Find its:

(i) price index in **2024** based on **2022**

(ii) price in **2022** if its price in **2024** was **£600**

PROBABILITY

1. A box contains **150** red and **50** blue pens. **48** pens are drawn in succession at random from the box with replacement. Find the probability of picking:

(i) exactly **30** red pens

(ii) at least **29** red pens

(iii) at least **8** but less than **20** blue pens

2. A continuous r.v $X \sim R(3, 15)$.

(a) Write down the p.d.f of X and sketch it

(b) Find: (i) $E(X)$ (ii) $Var(X)$ (iii) the upper quartile of X

(iv) $P(4 < X < 10)$ (v) $P(|X - 7| < 2 / X \geq 6)$

(vi) the distribution function of X and sketch it

3. Box **A** contains **4** red and **3** green pens, box **B** contains **3** red and **4** green pens, while box **C** contains **5** red and **2** green pens. Boxes **A**, **B** and **C** are in the ratio **2 : 3 : 5** respectively as likely to be picked. If a box is selected at random and two pens are picked from it without replacement,

(a) find the probability of picking:

(i) pens of different colours

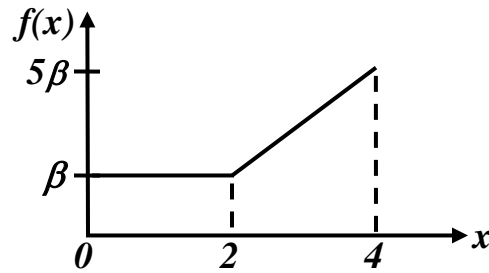
(ii) box **B** given that the pens drawn are of the same colour

(b) If X is the number of red pens drawn, find the:

(i) probability distribution of X

(ii) median, mean and variance of X

4. The p.d.f of a continuous r.v X is distributed as follows:



Find:

- (i) the value of β
- (ii) the equations of the p.d.f
- (iii) the mean and median of X
- (iv) the cumulative distribution function of X and sketch it
- (v) $P(X > 1/X < 3)$

5. A continuous r.v X has the following distribution function:

$$F(x) = \begin{cases} 0 & , \quad x \leq 0 \\ \frac{\sqrt{2}}{2} \sin x & , \quad 0 \leq x \leq \lambda \\ 1 - \frac{\sqrt{2}}{2} \cos x & , \quad \lambda \leq x \leq \frac{\pi}{2} \\ 1 & , \quad x \geq \frac{\pi}{2} \end{cases}$$

(a) Show that $\lambda = \frac{\pi}{4}$

(b) Find:

- (i) $P\left(\left|X - \frac{\pi}{4}\right| \leq \frac{\pi}{12}\right)$
- (ii) the equations of the p.d.f and sketch it, hence deduce the mean of X
- (iii) $E\left(3X - \frac{\pi}{3}\right)$

6. A discrete r.v X has the following p.d.f:

$$P(X = x) = \begin{cases} \frac{1}{60}(ax + b) & , \quad x = 1, 2, 3, 4 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Given that $F(3) = \frac{13}{20}$, find the:

(i) values of a and b , hence sketch the p.d.f of X

(ii) $P(X > 1/X < 3)$

(iii) mean and variance of X

(iv) $E(3X - 4)$ and $\text{Var}(3X - 4)$

7. (a) A random variable X is binomially distributed with mean 4.8 and variance

2.88 . Find $P(X < 6)$

(b) A student answers 12 questions. The chance of passing each question is $\frac{1}{3}$.

Find the probability of passing:

(i) exactly 7 questions

(ii) at least 2 questions

8. A random variable X is normally distributed such that $P(X < 76) = 0.9772$ and

$P(72 < X < 76) = 0.044$. Find:

(i) the mean and standard deviation of X

(ii) $P(X > 45)$

(iii) the interval which contains the middle 95% of distribution

9. A random sample of **100** nails taken from a normal population had the following

lengths x in cm: $\sum x = 380$ and $\sum x^2 = 1840$. Find the:

(i) unbiased estimate for the population variance

(ii) **90.8%** confidence interval for the population mean

10. A random sample of **36** items drawn from a normal population is such that the

95% confidence interval for the mean of all the items is **[67.9, 77.7]**. Find the

90% confidence limits for the mean of all the items

11. Given that $P(A \cup B) = \frac{9}{10}$, $P(A/B) = \frac{1}{3}$ and $P(B/A) = \frac{2}{5}$, find:

(i) $P(A)$

(ii) $P(\bar{A}/\bar{B})$

(iii) $P(A \text{ or } B \text{ but not both } A \text{ and } B)$

12. At a certain party, **25%** of the guests are women. Nile beer and Bell beer are

the only drinks available for the guests. **40%** of the women and **70%** of the men

take Nile beer. Of the men taking Nile beer, **80%** got drunk and of the men

taking Bell beer, **60%** got drunk. Of the women taking Nile beer, **50%** got drunk

and of the women taking Bell beer, **40%** got drunk. Find the probability that a

randomly selected guest:

(i) takes Nile beer

(ii) got drunk

(iii) got drunk given that is a woman

(iv) is a man given that he got drunk for taking Nile beer

13. (a) Events **A** and **B** are such that $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{17}{24}$.

Find: (i) $P(A \cap B)$ (ii) $P(\bar{A} \cap B)$ (iii) $P(\bar{A} \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$

(b) If \bar{A} and \bar{B} are independent events,

(i) Show that the events **A** and **B** are also independent

(ii) find $P(B)$ and $P(\bar{A} \cup \bar{B})$, if $P(A) = 0.375$ and $P(A \cup B) = 0.75$

(c) Find the possible values of $P(A)$, if **A** and **B** are independent events such that $P(A \cap B) = 0.3$ and $P(A \cup B) = 0.875$

14. A task in mathematics is given to three students whose chances of solving it are

$\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. Find the probability that: (i) the task is solved

(ii) only one student solves it (iii) at least two of them solved it

15. Two soldiers **A** and **B** in that order take turns shooting a bullet at a target. The

first one to hit the target wins the game. If their chances of hitting the target on

each occasion they shoot are $\frac{1}{3}$ and $\frac{1}{4}$ respectively, find the chance that:

(i) **A** wins the game on his third shot (ii) **A** wins the game.

16. Mutually exclusive events **A** and **B** are such that $P(A \cup B) = 0.75$ and

$P(A) = 0.27$, find: (i) $P(\bar{A} \cup B)$ (ii) $P(\bar{A} \cap \bar{B})$ (iii) $P(A \cap B)'$

17. Exhaustive events **A** and **B** are such that $5P(A) = 4P(B)$ and $P(A \cap B) = \frac{1}{5}$.

Find: (i) $P(A)$ (ii) $P(A/B)$ **END**