

P425/1  
Pure mathematics  
**Paper 1**  
JULY/AUG 2023  
3 hours

**KAMSSA Mock Examinations**  
**Uganda Advanced Certificate of Education**  
**PURE MATHEMATICS**  
**GUIDE**  
3 hours

**INSTRUCTIONS:**

- Answer all the *eight* questions in section *A* and any *five* in section *B*.
- Any additional question(s) answered will not be marked.
- All necessary working must be clearly shown.
- Begin each answer on a fresh sheet of paper.
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

# SECTION A (40 MARKS)

Answer all questions in this section

1. Solve for  $x$  in:  $3 \cot x + \operatorname{cosec} x = 2$  for  $0^\circ \leq x \leq 360^\circ$ .

(05 marks)

$$3 \cot x \cos x = 2$$

$$0^\circ \leq x \leq 360^\circ$$

Soln

$$\frac{3 \cos x}{\sin x} + \frac{1}{\sin x} = 2$$

$$3 \cos x - 2 \sin x = -1$$

MTD 1

$$3 \cos x - 2 \sin x = R \cos (x + \alpha)$$

$$= R \cos x \cos \alpha - R \sin x \sin \alpha$$

$$R \cos \alpha = 3$$

$$R \sin \alpha = 2$$

$$\tan \alpha = \frac{2}{3}$$

$$\alpha = 33.69^\circ$$

$$(R \cos \alpha)^2 + (R \sin \alpha)^2 = (3)^2 + (2)^2$$

$$R^2 = 13$$

$$R = \sqrt{13}$$

$$3 \cos x - 2 \sin x = \sqrt{13} \cos (x + 33.69^\circ)$$

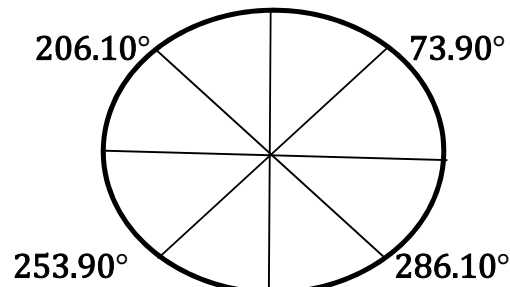
$$\text{So; } \sqrt{13} \cos (x + 33.69^\circ) = -1$$

$$x + 33.69^\circ = \cos^{-1} \left( \frac{-1}{\sqrt{13}} \right)$$

$$x + 33.69^\circ = 73.90^\circ$$

$$x + 33.69^\circ = 106.10^\circ, 253.90^\circ$$

$$x = 72.41^\circ, 220.21^\circ$$



$$\tan \frac{x}{2} = -2.7321$$

$$\frac{x}{2} = 69.90^\circ$$

$$\frac{x}{2} = 110.10^\circ$$

$$x = 220.20^\circ$$

$$\tan \frac{x}{2} = 0.7321$$

$$\frac{x}{2} = 36.21^\circ$$

$$x = 72.42^\circ$$

$$\therefore x = 72.42^\circ, 220.20^\circ$$

MTD II

$$3 \cos x - 2 \sin x = -1$$

$$3 \left( \frac{1-t^2}{1+t^2} \right) - 2 \left( \frac{2t}{1+t^2} \right) = -1$$

$$3-3t^2-4t=-1-t^2$$

$$2t^2 + 4t - 4 = 0$$

$$t^2 + 2t - 2 = 0$$

$$t = \frac{-2 \pm \sqrt{2^2 - 4(1)(-2)}}{2(1)}$$

$$t = -2.7321$$

$$t = 0.7321$$

2. Find the number of ways the word *ARSENAL* can be arranged when the A's are not close to each other. (05 marks)

ARSENAL

$$\text{Total number of ways} = \frac{7!}{2!}$$

$$= 2520 \text{ ways}$$

$$\text{When the A's are together} = 6!$$

$$= 720 \text{ ways}$$

$$\text{When the A's are not together} = 2520 - 720$$

$$= 1800 \text{ ways}$$

3. Express  $p(x) = 2x^2 - 4x + 1$  in the form  $a(x - b)^2 + c$  and hence state the least value of  $p(x)$ . (05 marks)

$$\begin{aligned}
 P(x) &= 2x^2 - 4x + 1 \\
 &= 2\left(x^2 - 2x + \frac{1}{2}\right) \\
 &= 2\left(x^2 - 2x + (-1)^2 - (-1^2) + \frac{1}{2}\right) \\
 &= 2\left((x - 1)^2 - \frac{1}{2}\right) \\
 &= 2(x - 1)^2
 \end{aligned}$$

When  $x=1$ ,

$$\begin{aligned}
 P(1) &= 2(1-1)^2 - 1 \\
 &= -1
 \end{aligned}$$

4. Differentiate and simplify the function:  $\tan^{-1}\left(\frac{1+2x}{1-2x}\right)$ . (05 marks)

$$\begin{aligned}
 \text{let } y &= \tan^{-1}\left(\frac{1+2x}{1-2x}\right) \\
 \tan y &= \frac{1+2x}{1-2x} \\
 \sec^2 y \cdot \frac{dy}{dx} &= \frac{(1-2x) \cdot 2 - (1+2x)(-2)}{(1-2x)^2} \\
 \sec^2 y \cdot \frac{dy}{dx} &= \frac{2-4x+2+4x}{(1-2x)^2} \\
 \frac{dy}{dx} &= \frac{4}{(1-2x)^2} \cdot \frac{1}{\sec^2 y} \\
 &= \frac{4}{(1-2x)^2} \cdot \frac{1}{1 + \left(\frac{1+2x}{1-2x}\right)^2} \\
 &= \frac{4}{(1-2x)^2} \cdot \frac{(1-2x)^2}{(1-2x)^2 + (1+2x)^2} \\
 &= \frac{4}{1-4x+4x^2 + 1+4x+4x^2} \\
 &= \frac{4}{2+8x^2} \\
 &= \frac{2}{1+4x^2}
 \end{aligned}$$

5. Given that the equation  $x^2 + 3x + 2 = 0$  has roots  $k$  and  $l$ , find the equation whose roots are  $\frac{k}{l^2}$  and  $\frac{l}{k^2}$ . (05 marks)

$$x^2 + 3x + 2 = 0$$

K, l

Sum of old roots,

$$K+l = -3$$

Product of old roots B (for both sum and product)

$$Kl = 2$$

Sum of new roots,

$$\begin{aligned}\frac{k}{l^2} + \frac{l}{k^2} &= \frac{k^3 + l^3}{(lk)^2} \\ &= \frac{(k+l)^3 - 3kl(k+l)}{(lk)^2} \\ &= \frac{(-3)^2 - 3(2)(-3)}{(2)^2} \\ &= \frac{9}{4}\end{aligned}$$

Product of new roots

$$\begin{aligned}\frac{k}{l^2} \cdot \frac{l}{k^2} &= \frac{1}{lk} \\ &= \frac{1}{2}\end{aligned}$$

The eqn is  $x^2 - \left(\frac{-9}{4}\right)x + \frac{1}{2} = 0$

$$4x^2 + 9x + 2 = 0$$

6. Given that the equation  $z^3 - 4z^2 + 6z - 4 = 0$  has one of the roots as  $i + 1$ , find the other roots. (05 marks)

$$z^3 - 4z^2 + 6z - 4 = 0$$

$$z = i + 1 = 1 + i$$

$$z = 1 - i \text{ also a root}$$

$$\text{sum of roots} = 2$$

$$\text{product of roots} = (1 + i)(1 - i)$$

$$= 2$$

$$\text{Divisor} = z^2 - 2z + 2$$

$$\begin{array}{r} z - 2 \\ z^2 - 2z + 2 \sqrt{z^3 - 4z^2 + 6z - 4} \\ - z^3 - 2z^2 + 2z \\ \hline -2z^2 + 4z - 4 \\ -2z^2 + 4z - 4 \\ \hline \hline \end{array}$$

$$z^3 - 4z^2 - 6z - 4 = (z^2 - 2z + 2)(z - 2)$$

$$\Rightarrow (z^2 - 4z^2 + 2)(z - 2) = 0$$

$$\Rightarrow z - 2 = 0$$

$$z = 2$$

7. Show that the points  $P(1,2,3)$ ,  $R(3,8,1)$ , and  $T(7,20,-3)$  are collinear. (05 marks)

$P(1,2,3)$ ,  $R(3,8,1)$ ,  $T(7,20,3)$

$$\vec{PR} = \vec{OR} - \vec{OP}, \quad \vec{PT} = \vec{OT}$$

$$= \begin{pmatrix} 3 \\ 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 18 \\ -6 \end{pmatrix}$$

$$\vec{PR} = K \vec{PT}$$

$$\begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} = K \begin{pmatrix} 6 \\ 18 \\ -6 \end{pmatrix}$$

$$2 = 6K, K = \frac{1}{3}$$

$$6 = 18K, k = \frac{1}{3}$$

$$-2 = -6, k = \frac{1}{3}$$

$$\Rightarrow \vec{PR} = \frac{1}{3} \vec{PT}, \therefore \text{PR and T are collinear.}$$

(for  $\vec{PR} = K \vec{PT}$  and conclusion)

8. Find the volume of solid generated when are area bounded by the curve  $y = 3 - x^2$  and the line  $y = 2$  is rotated about the line  $y = 2$  to 3 significant figures. (05 marks)

at intersection

$$2 = 3 - x^2$$

$$x^2 = \pm 1$$

$$V = \pi \int_{-1}^1 y^2 dx$$

$$V = \pi \int_{-1}^1 (3 - x^2 - 2) dx$$

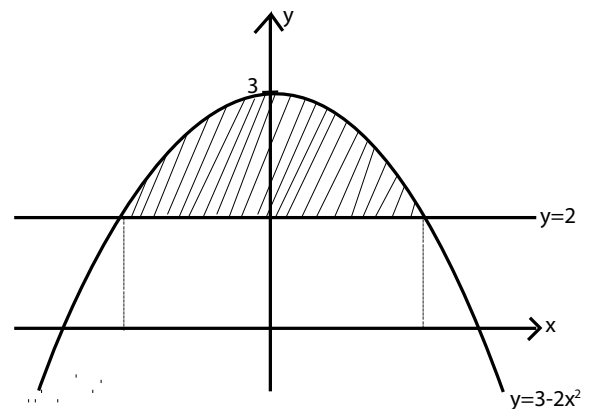
$$V = \pi \int_{-1}^1 1 - 2x^2 + x^2 dx$$

$$V = \pi \left[ x - \frac{2}{3} x^3 + \frac{x^5}{5} \right]!$$

$$V = \pi \left[ \left( 1 - \frac{2(+1)^3}{3} + \frac{(+1)^5}{5} \right) - \left( (-1) - \frac{2(-1)^3}{3} + \frac{(-1)^5}{5} \right) \right]$$

$$= \frac{16}{15} \pi \text{ cubic units}$$

$$= 1.0667\pi \text{ cubic}$$



## SECTION B: (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks

9. (a) In any triangle PQR, show that  $\cos 2P + \cos 2Q + \cos 2R = -1 - \cos P \cos Q \cos R$ .

$$\text{LHS} = \cos 2P + \cos 2Q + \cos 2R$$

$$= 2\cos(P+Q) \cos(P-Q) + 2\cos^2 R - 1$$

$$\text{But } P+Q+R=180^\circ$$

$$P+Q=180^\circ - R$$

$$\cos(P+Q) = \cos(180^\circ - R)$$

$$\cos(P+Q) = -\cos R$$

$$\text{LHS} = -2\cos R \cos(P-Q) + 2\cos^2 R - 1$$

$$= -2\cos R (\cos(P-Q) - \cos R) - 1$$

$$= -2\cos R (\cos(P-Q) + \cos(P+Q)) - 1$$

$$= -2\cos R (2\cos P \cos(-Q)) - 1$$

$$= -4\cos R \cos P \cos Q - 1$$

$$= -1 - 4\cos P \cos Q \cos R$$

(b) A and B are angles of such that  $\cos A = \frac{3}{5}$ , and  $\cos B = \frac{5}{13}$ , if A is acute and B is reflex, without using tables or a calculator, find the values of:

i.  $\tan 2A$

$$a^2 + (5)^2 = (13)^2$$

$$a = 12$$

$$(b)^2 + (3)^2 = (5)^2$$

$$b = 4$$

$$\tan A = \frac{4}{3}$$

$$(ii) \cos(A+B)$$

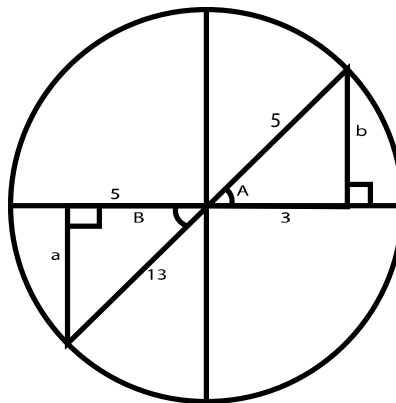
$$\sin A = \frac{4}{5}$$

$$\sin B = \frac{12}{13}$$

$$(i) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$= \frac{2 \left( \frac{4}{3} \right)}{1 - \left( \frac{4}{3} \right)^2}$$

$$= \frac{24}{7}$$



ii.  $\cos(A+B)$

(12 marks)

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \left( \frac{3}{5} \right) \cdot \left( \frac{5}{13} \right) - \left( \frac{4}{5} \right) \cdot \left( \frac{12}{13} \right)$$

$$= -\frac{33}{65}$$

10.(a) Differentiate the following with respect to x:

i.  $(\ln x)^x$

$$\text{let } y = (\ln x)^x$$

$$\ln y = x \ln(\ln x)$$

$$\frac{1}{y} \frac{dy}{dx} = \ln(\ln x) + 1 + x \cdot \frac{1}{\ln(\ln x)} \cdot \frac{1}{x}$$

$$\frac{1}{y} \frac{dy}{dx} = \ln(\ln x) + \frac{1}{\ln(\ln x)}$$

$$\frac{dy}{dx} = \left( \ln(\ln x) + \frac{1}{\ln(\ln x)} \right) (\ln x)^x$$

ii.  $5^{x^2-1}$

(05 marks)

$$5^{x^2-1}$$

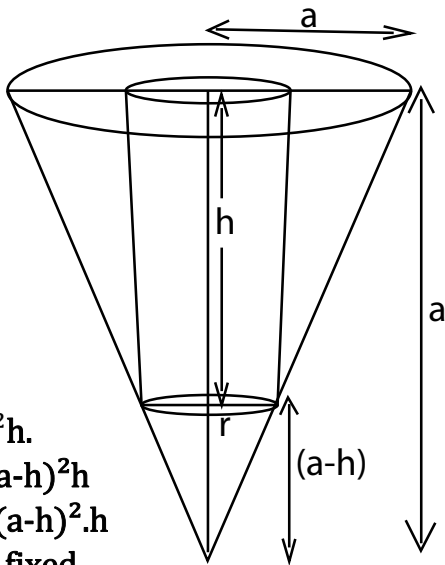
Let  $y = 5^{x^2-1}$

$$\ln y = (x^2 - 1) \ln 5$$

$$\frac{1}{y} \frac{dy}{dx} = 2x \ln 5$$

$$\frac{dy}{dx} = 2x 5^{x^2-1} \ln 5$$

- (b) A closed hollow right circular cone has internal height  $a$  and internal radius  $a$ .  
a. A solid circular cylinder of height  $h$  just fits inside the cone with the axis of the cylinder lying along the axis of the cone. Show that the volume of the cylinder is  $\pi h(a - h)^2$ . If  $a$  is fixed and  $h$  may vary, find  $h$  in terms of  $a$  when the volume of the cylinder is maximum. (07 marks)



$$\frac{a}{r} = \frac{a}{a-h}$$

$$r = a - h$$

$$V = \pi r^2 h$$

$$V = \pi (a-h)^2 h$$

$$V = \pi h(a-h)^2$$

$a$  is fixed,

for max, volume,  $\frac{dv}{dh} = 0$

$$\frac{dv}{dh} = \pi h \cdot 2(a-h)(-1) + (a-h)^2 \cdot \pi$$

$$= \pi(a-h)[-2h + a - h]$$

$$= \frac{\pi(a-h)(a-3h)}{\pi(a-h)(a-3h)=0}$$

$$a-h=0, \quad a-3h=0$$

$$h=a, \quad h=\frac{a}{3}$$

at  $h=a$

$$V = \pi a(a-a)^2$$

$$V = 0$$

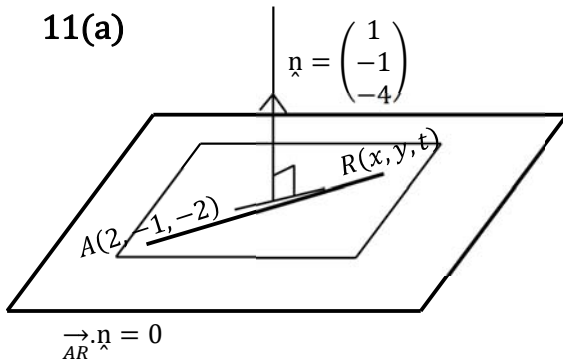
$$\text{At } h = \frac{a}{3}$$

$$V = \pi \frac{a}{3} \left( a - \frac{a}{3} \right)^2$$

$$V = \frac{2a^3}{3} \quad \therefore h = \frac{a}{3}$$

11. (a) Find the equation of a plane which contains a point A (2, 1, -2) and is parallel to the plane  $x - y - 4z = 3$ . (04 marks)

11(a)



$$\vec{AR} = \vec{OR} - \vec{OA}$$

$$= \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} x - 2 \\ y - 1 \\ z + 2 \end{pmatrix}$$

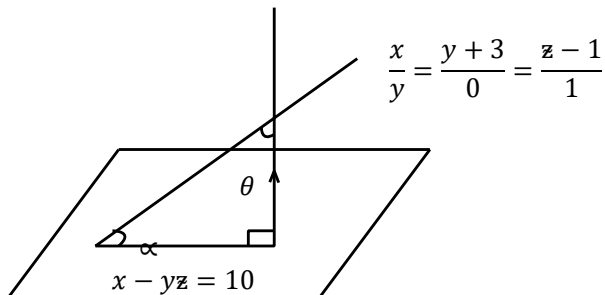
$$= \begin{pmatrix} x - 2 \\ y - 1 \\ z + 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix} = 0$$

$$x - 2 - y + 1 - 4z - 8 = 0$$

$$x - y - 4z = 10$$

- (b) If a line  $\frac{x}{2} = \frac{y+3}{0} = \frac{z-1}{1}$  intersects with the plane in (a) above, find the:

Point of intersect



$$\text{Let } \frac{x}{2} = \frac{y+3}{0} = \frac{z-1}{1} = x$$

$$x = 2x, y = -3, z = x + 1$$

$$2x - (-3) - 4(x + 1) = 10$$

$$2x + 3 - 4x - 4 = 10$$



$$-2x=11$$

$$x=-\frac{11}{2}$$

$$x=2\left(\frac{-11}{2}\right)$$

$$=-11$$

$$y=-3$$

$$z=-\frac{11}{2}+1$$

$$=-\frac{9}{2}$$

$$\text{the point is } \left(-11, -3, -\frac{9}{2}\right)$$

i. Angle made between the line and plane.

(08 marks)

$$\hat{n} \cdot \hat{b} = |\hat{n}| |\hat{b}| \cos \theta$$

$$\begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \sqrt{(1)^2 + (-1)^2 + (-4)^2} \sqrt{(2)^2 + (1)^2} \cos \theta$$

$$-2 = \sqrt{18} \sqrt{5} \cos \theta$$

$$\theta = \cos^{-1} \left( \frac{-2}{\sqrt{90}} \right)$$

$$\theta = 102.17^\circ$$

$$\alpha + \theta = 90^\circ$$

$$\alpha = 90^\circ - 102.17^\circ$$

$$\alpha = -12.17^\circ$$

12.(a) Find the length of the latus rectum of the parabola  $4x = t^2$  and  $2y = t$ .

$$4x=t^2, 2y=t$$

$$4x=(2y)^2$$

$$4x=4y^2$$

$$y^2=x$$

$$\text{from } y^2=4ax$$

$$4a=1 \text{ unit}$$

(b) The conic section below has eccentricity,  $e < 1$  and equation  $\frac{x^2}{9} + y^2 = 1$ . Find the value of  $e$

$$\frac{x^2}{9} + y^2 = 1$$

$$a^2 = 9$$

$$a = 3$$

$$b^2 = 1$$

$$b = 1$$

$$\text{for } b^2 = a^2(1 - e^2)$$

$$1^2 = (3)^2(1 - e^2)$$

$$\frac{1}{9} = 1 - e^2$$

$$e^2 = 1 - \frac{1}{9}$$

$$e^2 = \frac{8}{9}$$

$$e = \frac{2\sqrt{2}}{3}$$

(c) find the equation of tangent to the hyperbola  $x^2 - 9y^2 = 1$  at  $P(\sec \beta, \frac{1}{3} \tan \beta)$

$$x^2 - 9y^2 = 1$$

$$2x - 18y \frac{dy}{dx} = 0$$

$$18y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{x}{9y}$$

$$\frac{\sec \beta}{9 \left( \frac{1}{3} \tan \beta \right)}$$

$$\frac{1}{3\sin\beta}$$

Let (x,y)

$$\frac{y - \frac{1\tan\beta}{3}}{x - \sec\beta} = \frac{1}{3\sin\beta}$$

$$3y \sin\beta - \tan\beta = x - \sec\beta$$

$$x = 3y \sin\beta = x - \tan\beta \sin\beta + \sec\beta$$

(12 marks)

13. Express  $h(x) = \frac{2x^2+1}{(x-1)(x+2)}$  in to partial fractions and hence evaluate  $\int_2^3 h(x) dx$ , correct to three significant figures. (12 marks)

$$\frac{2x^2+1}{(x-1)(x+2)} = \frac{2x^2+1}{x^2+x-2}$$

$$\frac{2}{x^2+x-2} = \frac{2x^2+1}{x^2+x-2} - \frac{2x^2+2x-4}{x^2+x-2}$$

$$\frac{2x^2+1}{(x-1)(x+2)} = 2 + \frac{-2x+5}{(x-1)(x+2)}$$

$$\text{So, } \frac{2x^2+1}{(x-1)(x+2)} \equiv \frac{A}{x-1} + \frac{B}{x+2}$$

$$-2x+5 \equiv A(x+2) + B(x-1)$$

When  $x = -2$

$$9 = -3B$$

$$B = -3$$

$$\frac{-2x+5}{(x-1)(x+2)} = \frac{1}{x-1} - \frac{3}{x+2}$$

$$\begin{aligned} f(x) &= 2 + \frac{1}{x-1} = \frac{3}{x+2} \int_0^1 f(x) dx = \int_2^3 2 dx + \int_2^3 \frac{1}{x-1} dx - \int_2^3 \frac{3}{x+2} dx \\ &= [2x + \ln(x-1) - 3\ln(x+2)] \\ &= (6 + \ln 2 - 3\ln 5) - (4 + \ln 1 - 3\ln 4) \\ &= 2.0237 \end{aligned}$$

14.(a) The  $p^{th}$ ,  $q^{th}$  and  $r^{th}$  terms of an Arithmetic progression are in a geometric progression, show that the ratio is  $\frac{p-q}{q-r}$  or  $\frac{q-r}{p-q}$ . (06 marks)

$$Up = a + (p-1)d, Uq = a + (q-1)d, Ur = a + (r-1)d$$

$$\text{Ratio} = \frac{a+(q-1)d}{a+(p-1)d} = \frac{a+(r-1)d}{a+(q-1)d}$$

$$[a + (q-1)]d^2 = (a + (p-1)d)(a + (r-1)d)$$

$$a^2 + 2ad(q-1) + d^2(q-1)^2 = a^2 + ad(r-1) + ad(p-1) + d^2(p-1)(r-1)$$

$$2adq - 2ad + d^2q^2 - 2d^2q + d^2 = adr - ad + ad(r-1) + ad(p-1) + d^2(p-1)(r-1)$$

$$2adq + d^2q^2 - 2d^2q = adr + adq + d^2pr - d^2p - d^2r$$

$$d^2(q^2 - 2q - pr + p + r) = ad(r + p - 2q)$$

$$a = \frac{(q^2 - 2q - pr + p + r)}{(r + p - 2q)}$$

$$\text{Ratio} = \frac{(q^2 - 2q - pr + p + r)d + (q-1)d}{r + p - 2q}$$

$$\begin{aligned} &= \frac{(q^2 - 2q - pr + p + r)d + (q-1)d}{r + p - 2q} \\ &= \frac{q^2 - 2q - pr + p + r + (q-1)(r + p - 2q)}{q^2 - 2q - pr + p + r + (p-1)(r + p - 2q)} \\ &= \frac{q^2 - 2q - pr + p + r + qr + qp - 2q^2 - r - p + 2q}{q^2 - 2q - pr + p + r + pr + p^2 - 2pq - r - p + 2q} \\ &= \frac{-q^2 - pr + qr + qp}{q^2 + p^2 - 2pq} \\ &= \frac{qp - q^2 + qr}{(p-q)^2} \\ &= \frac{q(p-q) - r(p-q)}{(p-q)^2} \\ &= \frac{(p-q)(q-r)}{(p-q)^2} \\ &= \frac{q-r}{p-q} \end{aligned}$$

(b) The sum of n terms of a sequence is  $S_n = 2^{2n} - n$  where n is a natural number. Find the first three terms of the sequence. (06 marks)

$$S_n = 2^{2n} - n$$

For n=1,

$$S_1 = 2^2 - 1$$

$$S_1 = 3$$

$$U_1 = 3$$

n<sub>2</sub>=2

$$S_2 = 2^2 - 2$$

$$S_2 = 14$$

$$U_1 + U_2 = 14$$

$$3 + U_2 = 14$$

$$U_2 = 11$$

$$3, 11, 47, \dots$$

n= 3

$$S_3 = 2^3 - 3$$

$$= 61$$

$$U_1 + U_2 + U_3 = 61$$

$$14 + U_3 = 61$$

$$U_3 = 47$$

15. Show that the function  $y = \frac{x^2 - 1}{x^2 + 4x}$  has no real stationary points and hence sketch the curve. (12 marks)

$$Y = \frac{x^2 - 1}{x^2 + 4x}$$

$$\frac{dy}{dx} = \frac{(x^2 + 4x)(2x) - (x^2 - 1)(2x + 4)}{(x^2 + 4x)^2}$$

$$\Rightarrow \frac{x^2 + 4x(2x) - (x^2 - 1)(2x + 4)}{(x^2 + 4x)^2} = 0$$

$$2x^2 + 8x^2 - (2x^3 - 4x^2 - 2x - 4) = 0$$

$$12x^2 + 2x + 4 = 0$$

$$x = -2 \pm \frac{\sqrt{(2)^2 - 4(12)(4)}}{2(12)}$$

x has no real roots, no turning point.

For x-intercepts, y=0

$$0 = \frac{x^2 - 1}{x^2 + 4x}$$

$$x^2 - 1 = 0$$

$$x = \pm 1 \quad (1, 0), (-1, 0)$$

y-intercept, x=0;

$$y = \frac{-1}{0}$$

= no y- intercept

Vertical

Asymptote

$$x^2 + 4x = 0$$

$$x(x + 4) = 0$$

$$x = 0, x = -4$$

## Horizontal asymptote

$$y = \frac{x^2 - 1}{x^2 + 4x}$$

$$y = \frac{1 - \frac{1}{x^2}}{1 + \frac{4}{x}}$$

$$x \rightarrow \infty$$

	-5	-2	-0.5	0.5	2
	$x < -4$	$-4 < x < -1$	$-1 < x < 0$	$0 < x < 1$	$x < 2$
$x^2 - 1$	+	+	-	-	+
	+	-	-	+	+
	+	-	+	-	+

$$y = \frac{(x-1)(x+1)}{x(x+4)}$$

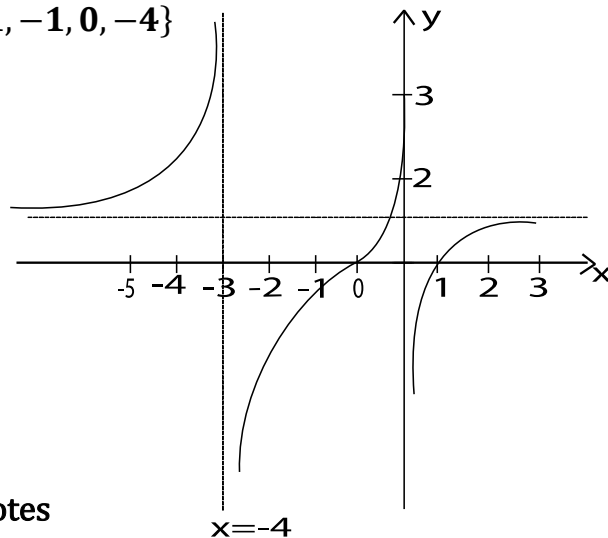
when  $y=1$

$$1 = \frac{x^2 - 1}{x^2 + 4x}$$

$$x^2 + 4 = x^2 - 1$$

$$x = -\frac{1}{4}$$

Uncritical values  $\{1, -1, 0, -4\}$



B<sub>1</sub> for only asymptotes

B<sub>2</sub> for all asymptotes and 2 curves plotted.

16. Solve the following differential equations:

a.  $x^2 + y^2 - xy \frac{dy}{dx} = 0$

b.  $x^3 y \frac{dy}{dx} = x + 1$ , for  $y(1) = -2$ .

(12 marks)

$$xy \frac{dy}{dx} = x^2 + y^2$$

let  $y = Vx$

$$\frac{dy}{dx} = V \cdot 1 + x \frac{dv}{dx}$$

$$= V + \frac{x dv}{dx}$$

$$x \cdot Vx \frac{dy}{dx} = x^2 + (Vx)^2$$

$$V \frac{dy}{dx} = 1 + V^2$$

$$\frac{dy}{dx} = \frac{1 + V^2}{V}$$

$$V + x \frac{dv}{dx} = \frac{1+V^2}{V}$$

$$\frac{x dv}{dx} = \frac{1+v^2}{V} - V$$

$$\frac{x dv}{dx} = \frac{1}{v}$$

$$\int V dV = \int \frac{dx}{x}$$

$$\frac{V^2}{2} = \ln x + c$$

(b)

$$x^3 y \frac{dy}{dx} = x + 1$$

$$y \frac{dy}{dx} = \frac{x+1}{x^3}$$

$$y dy = \frac{1}{x^2} = \frac{1}{x^3} dx$$

$$\int y dy = \int \frac{1}{x^2} + \frac{1}{x^3} dx$$

$$\frac{y^2}{2} = \frac{-1}{x} - \frac{1}{2x^2} + c$$

$$y(1) = -2$$

$$\frac{y^2}{2x^2} = \ln x + c$$

OR

$$\frac{V^2}{2} = \ln x + \ln A$$

$$\frac{y^2}{2x^2} = \ln Ax$$

$$y^2 = 2x^2 \ln Ax.$$

$$\frac{(-2)^2}{2} = \frac{-1}{1} - \frac{1}{2} + c$$

$$2 = \frac{-3}{2} + c$$

$$c = \frac{7}{2}$$

$$y^2 = \frac{-1}{x} - \frac{1}{2x^2} + \frac{7}{2}$$

$$x^2 y^2 = 2x - 1 + 7x^2$$

END