FORTPORTAL CITY MATHEMATICS SEMINAR SERIES

1ST EDITION - 2024

UACE - PURE MATHEMATICS

INTRODUCTION

- These series, are a brainchild of teachers within Fort portal city that Are enthusiastic about producing good results in UACE mathematics through boosting the confidence of the candidates ahead of UACE examinations through leaner centered seminars.
- The teachers accompanying these learners will play a role of guiding, emphasizing key points and procedure as well as maintaining discipline.
- Teachers are advised against doing these questions for the learners ahead of the seminars as this will defeat the aim and objective of the seminar. A teacher can guide and assist the learners through exposition especially on the specific questions they are to present.

MODE OF OPERATION

- A candidate or candidates from each of the participating schools is to present a solution.
- Between the receipt of a copy of these questions and the day of the seminar, each school is expected to try all of the questions. This will help to ensure that all participating members participate and benefit from the presentation on all questions.
- The confirmed participating schools will know the specific questions they are to present at most two days to the seminar day to help them polish on them for effective presentation. They will also be sent with the final answers to the questions they are to present but without the working.
- Candidates are encouraged to discuss and consult on all the questions before the seminar. Besides, the assigned questions, there shall be swing questions which shall be assigned randomly. Not all the 30 questions are assigned to specific schools.

ALGEBRA

Question 1

a) Solve the equation $2\left(9^{x}\right)+3^{x-1}+4=2\left(3^{x+1}\right)$ ***(06 marks)***

b) If α and β are the roots of the equation $x^2-x-3=0$. Find a quadratic equation with integral coefficients whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. ***(06 marks)***

Question 2

a) Solve the equation $\sqrt{3-x}-\sqrt{7+x}=\sqrt{16+2x}$ ***(06 marks)***

b) Solve for x, y and z given $\frac{x+2y}{-3}=\frac{y+2z}{4}=\frac{2x+z}{5}$ and $x+y+z=2$. ***(06 marks)***

Question 3

a) If α and β are roots of the equation $2x^2-7x+1=0$. Show that;

$$\left(\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}\right)=\frac{41}{2}$$

(05 marks)

b) Given that $(x-2)^2$ is a factor of the polynomial $f(x)=x^4+ax^3+bx^2+cx+4$ and f(x) leaves a remainder of 2 when divided by $(x-1)$. Find the values of a, b and c.

(07 marks)

VECTORS

Question 4

a) Given that points A(1,3,2), B(2,-1,1) , C(-1,2,3) and D(-2,6,4) are vertices of a parallelogram. Find the area of the parallelogram ABCD. ***(04 marks)***

b) Find the position vector of the point of intersection of the lines;

$$r_1=\begin{pmatrix}2\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\2\end{pmatrix} \text{ and } r_2=\begin{pmatrix}0\\-1\\4\end{pmatrix}+\mu\begin{pmatrix}-3\\3\\0\end{pmatrix}$$

Write down the vector equation of the plane containing lines r_1 and r_2 hence or otherwise find the Cartesian equation of the plane containing the lines r_1 and r_2.

(08 marks)

Question 5

a) Determine the equation of the plane equidistant from the points A(1,3,5) and B(2,-4,4)

(04 marks)

b) i) Find the coordinates of the point, P, in which the plane $4x+5y+6z=87$ intersects the line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z+4}{2}$

ii) Calculate the angle between the line and the plane in (b) (i) above. ***(08 marks)***

Question 6

a) Find the point of intersection between the line $r=i+j-3k+t(2i+2j+k)$ and the plane $r.(6i-3j+2k)=13$ and find the angle between the two. ***(06 marks)***

b) Show that the following vectors form a right angled triangle $a=(3i-2j+k)$, $b=(i-3j+5k), c=(2i+j-4k)$. Hence, find the area of the triangle. ***(06 marks)***

COMPLEX NUMBERS

Question 7

a) Find the square roots of $-2+2\sqrt{3}i$ using De Moivre's theorem. ***(05 marks)***

b) Given that w and z are two complex numbers, solve the simultaneous equations;

$3z+w=9+11i$, $iw-z=8-2i$. Hence express $\frac{w}{z}$ in polar-argument form. ***(07 marks)***

Question 8

a) Given the complex number Z and it's conjugate $\overline{Z}$, satisfy the equation $Z\overline{Z}+3\overline{Z}=37-9i$, find the values of Z. ***(07 marks)***

b) Find the Cartesian equation of the locus of a point represented by the equation

$\left|\frac{z-3}{z-4+2i}\right|=1$ **.** Describe the locus fully ***(05 marks)***

Question 9

a) Given that $z=-1+i\sqrt{3}$, find the value of the real number p such that $\arg(z^2+pz)=\frac{5\pi}{6}$. ***(04 marks)***

b) Given that $z=(-1+i\sqrt{3})^8$. Express z in the form $x+iy$. Hence or otherwise find the cube roots of z. ***(08 marks)***

TRIGONOMETRY

Question 10

a) By expressing in the appropriate R-form,

i. Solve the equation $3\cos\theta-4\sin\theta=5$.

ii. Find the maximum value of $\frac{7}{18+3\cos\theta-4\sin\theta}$ and the smallest value of θ for which that occurs. ***(08 marks)***

b) Show that $\cos\theta+\cos3\theta+\cos7\theta=4\cos\theta\cos2\theta\cos4\theta$. ***(04 marks)***

Question 11

a) P is an obtuse angle and Q is a reflex angle less than 2π radians. Given that $\sin A=\frac{4}{5}$ and $\sec B=\frac{1}{2}$, without using tables or a calculator. Find the values of ;

i. $\cos(B-A)$

ii. $\operatorname{cosec}2B$ ***(07 marks)***

b) Solve $\tan^{-1}(x)-\tan^{-1}(x-1)=\tan^{-1}3$ ***(05 marks)***

Question 12

a) Solve the equation: $\frac{4\sin^2\theta}{\operatorname{cosec}\theta}+\frac{3}{\operatorname{cosec}^2\theta\sec\theta}=\sin^2\theta$ for $0^\circ\le\theta\le360^\circ$. ***(06 marks)***

b) Solve $\cos\theta-\cos3\theta-\cos7\theta+\cos9\theta=0$ for $0^\circ\le\theta\le90^\circ$ ***(06 marks)***

CURVE SKETCHING AND INEQUALITIES

Question 13

Solve the inequalities;

a) $\frac{x-2}{x+1}\ge\frac{x+1}{x+3}$ ***(06 marks)***

b) $\left|\frac{2x-4}{x+1}\right|<4$ ***(06 marks)***

Question 14

Show that the curve $y=\frac{2(x-2)}{(x-1)(x-3)}$,has no real turning points. Hence sketch the curve.

(12 marks)

Question 15

A curve is given by the parametric equations; $x=4t$ and $x=\frac{3t^2}{t-1}$. Find the Cartesian equation of the curve and hence sketch it. ***(12 marks)***

INTEGRATION

Question 16

a) Integrate $\int x\tan^2 x\,dx$ ***(06 marks)***

b) Show that $\int x\sin^{-1}x\,dx=\frac{1}{4}(2x^2-1)\sin^{-1}x+\frac{1}{4}x\sqrt{(1-x^2)}+c$ ***(06 marks)***

Question 17

Two airplanes M and N were flown in the sky. Plane M described a path of $y = 20x - 2x^2$ and N describe a path of $y = 4x + 14$ where (x,y) is the grid reference of the planes in the sky.

a) Using differentiation, sketch the path traced by the two planes. ***(05 marks)***

b) At what points were the two planes at the same level. ***(03 marks)***

c) Find the area enclosed by the path of the two planes. ***(04 marks)***

Question 18

a) Express $\frac{x^3 + 9x^2 + 28x + 28}{(x+3)^2}$ into partial fractions.

b) Hence or other show that;

$$\int_0^1 \frac{x^3 + 9x^2 + 28x + 28}{(x+3)^2}\,dx = \frac{10}{3} + \ln\frac{4}{3}$$ ***(12 marks)***

DIFFERENTIATION

Question 19

a) Differentiate $x^{2x}\tan x$ with respect to x. ***(05 marks)***

b) Find the equation of the normal to the curve; $x = \frac{t}{1+t}$, $y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ ***(07 marks)***

Question 20

a) Given that $x^2 + y^2 - 2xy + 3y - 2x = 7$. Find $\frac{dy}{dx}$ in terms of x and y. ***(02 marks)***

b) If $x = a(t^2 - 1)$, $y = 2a(t+1)$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t. ***(05 marks)***

c) An error of $2\frac{1}{2}\%$ is made in the measurement of the area of a circle. What percentage error results in:

   i. the radius. ***(03 marks)***

   ii. the circumference. ***(02 marks)***

Question 21

a) The sum of the height and radius of a right circular cone is 9cm. Show that the maximum volume of the cone will be $36\pi cm^3$. ***(06 marks)***

b) If $y = \frac{\cos x}{x^2}$, prove that ; $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (2 + x^2)y = 0$ ***(06 marks)***

DIFFERENTIAL EQUATIONS

Question 22

The number, x of reported cases of an infectious disease, *t* months after it+-- was reported is now dropping. The rate at which it's dropping is proportional to the square of the reported cases. Initially there were 2500 reported cases and one month later they had dropped to 1600 cases.

a) Form a differential equation to model the information above. ***(02 marks)***

b) By solving the differential equation, show that $x = \frac{40{,}000}{9t + 6}$ ***(07 marks)***

c) Find after how many months there will be 250 reported cases. ***(03 marks)***

Question 23

a) The gradient of a curve satisfies $\frac{dy}{dx} = \frac{1}{3y^2(x-1)}$, $x > 1$. Given the curve passes through (2,-1). Determine the equation of the curve. ***(05 marks)***

b) In a rat breeding centre, the population of the rats grows in such a way that at time t months, the rate at which the population is growing is proportional to the number of rats, R present at that time. If the rat population doubles in the first 2 months. When will the population of the rats quadruple the initial population. ***(07 marks)***

Question 24

a) Use the substitutions $y = vx$ to solve the differential equation $x^2 \frac{dy}{dx} = x^2 + y^2 + xy$

(06 marks)

b) The gradient of the tangent at any point (x, y) of the curve is $x - \frac{2y}{x}$. Given that the curve passes through (2,4). Find the equation of the curve. ***(06 marks)***

COORDINATE GEOMETRY

Question 25

a) Show that the equation of the circle passing through the points (-2,-4), (3,1) and (-2,0) is $(x-1)^2 + (y+2)^2 = 13$ ***(07 marks)***

b) With reference to the circle in (a) above, show that the tangent at point (3,1) is parallel to the diameter that passes through the point (-2,0). ***(05 marks)***

Question 26

a) A curve is represented by parametric equations $x = 4\cos\theta$, $y = 3\cos\theta$; show that the parametric equations represent an ellipse with Cartesian equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

(03 marks)

b) The normal to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at $P(4\cos\theta, 3\sin\theta)$ meets the x- and y- axes at A and B respectively. Find the equation of the normal. If M is the midpoint of AB, show that the locus of point M is also an ellipse. ***(09 marks)***

Question 27

a) Find the equation of the tangent to the hyperbola whose points are of the parametric form $\left(2t, \frac{2}{t}\right)$. ***(05 marks)***

b) Find the equations of the tangents in (a), which are parallel to $y - \sqrt{3}x = 0$.

(04 marks)

c) Determine the distance between the tangents in (b). ***(03 marks)***

SERIES, INDUCTION AND BINOMIAL THEOREM

Question 28

a) Find the term independent of x in the expansion $\left(\frac{1}{x^2} - x\right)^{18}$ ***(04 marks)***

b) Show that, if x is small enough for its cube and higher powers to be neglected.

$\sqrt{\left[\frac{1+x}{1-x}\right]} = 1 + x + \frac{1}{2}x^2$. By putting $x = \frac{1}{7}$, show that $\sqrt{3} \approx \frac{196}{113}$. ***(08 marks)***

Question 29

a) Prove by mathematical induction that $9^n - 5^n$ is divisible by 4 for all positive for all positive integral values of n. ***(06 marks)***

b) The first three terms in the expansion of $(1 + ax)^n$ are $1 + 12x + 81x^2$.

(i) Find the possible values of *a* and *n*.

(ii) State the validity of the expansion. ***(06 marks)***

Question 30

a) Given the geometrical progression (G.P) 2,6,18,54, ……

Find the sum of the first ten terms of the G.P. ***(03 marks)***

b) In an Arithmetical Progression (A.P), the sum of the fifth and sixteenth terms is 44. The sum of the first 18 terms is three times the sum of the first ten terms.

Determine the;

(i) value of the first term

(ii) common difference of the A.P

(iii) sum of the first 30 terms of the A.P ***(09 marks)***

END