P425/1
Pure Mathematics
Paper 1
July - August, 2024
3 hours

UGANDA MUSLIM TEACHERS' ASSOCIATION

UMTA JOINT MOCK EXAMINATIONS - 2024

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Pure Mathematics

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

- *Attempt **all** the **eight** questions in section A and **five** questions from section **B**.*
- *Any additional question(s) answered will **not** be marked.*
- ***All** working **must** be shown clearly. Begin each question on a fresh sheet of paper.*
- *Silent, nonprogrammable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A

1. $\int x^4 \ln x dx$ (05 marks)

2. Find the acute angle between the following lines, $2x + 3y = 7$, $x = 6y + 5$ (05 marks)

3. If $y = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ show that $\frac{dy}{dx} = sec^2 x$ (05 marks)

4. Show that the vectors $2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{i} - 3\boldsymbol{j} - 5\boldsymbol{k}$ *and* $3\boldsymbol{i} - 4\boldsymbol{j} - 4\boldsymbol{k}$ are coplanar. (05 marks)

5. Solve for **X** from 0^0 to 360^0 Given that tan x + tan 2x + tan x tan 2x = 1 (05 marks)

6. Solve for **X** given $9 \log_x 5 = \log_5 \boldsymbol{x}$. (05 marks)

7. Find the area bounded by the curve $y = (1-x)(x+2)$ and the x-axis. (05 marks)

8. Solve for **X** Given $3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$ (05 marks)

$x+2-x^2-2x$
$2-x^2-x$
$x+2-x^2-2x$
$2-x^2-x$

SECTION B

9. Given the curve $y = \frac{3x+3}{x(3-x)}$;

 (a) Find the region where the curve does not lie, hence determine the turning points and their nature.

 (b) State the asymptotes and find the intercepts.

 (c) Sketch the curve. (12 marks)

10. (a) Solve the equation $\sqrt{3-x} - \sqrt{7+x} = \sqrt{16+2x}$. (06 marks)

 (b) Solve for **x, y,** and **z** given $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5}$ and x + y + z = 2. (06 marks)

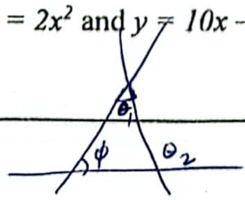
11. (a) Show that $\frac{(cos4\theta + isin4\theta)^3 (cos2\theta - isin2\theta)^5}{(cos3\theta + isin3\theta)^4 (cos5\theta - isin4\theta)^6} = cos20\theta + isin20\theta$. (06 marks)

 (b) Shade the region on Argand diagram of $|z - 1 - i| < 3$. (06 marks)

12. Sketch the curves $\mathbf{y} = 2x^2$ and $y = 10x - x^2$ on the same graph.

$5x+10y = -6x-3z$
$11x+10y+3z=0$

$\phi_1 + \phi = \phi_2$
$\phi_1 = \phi_2 - \phi$



$4x+8y = -3y-6z$
$4x+11y+6z=0$

$(-3-x)(-3-x)$
$9+3x+3x+x^2$

$\tan\phi_1 = \tan(\theta_1 - \phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi}$

Find the volume generated when the area enclosed between the curves is rotated through 360^0 (12 marks)

N. 13. (a) Prove that $\frac{sin5x - sin7x + sin8x - sin4x}{cos4x - cos5x - cos8x + cos7x} = \cot 6x$. (06 marks)

(b) Find all the possible values of **x** from 0^0 to 360^0 of the equation

$4cosx - 6sinx = 5$. (06 marks)

14. Partialise $f(x) = \frac{3x^3+x+1}{(x-2)(x+1)^3}$ Hence evaluate $\int_3^4 f(x)dx$. (12 marks)

✓15. (a) The gradient of the tangent at any point *(x,y)* of the curve is $x - \frac{2y}{x}$

Given that the curve passes through (2,4). Find the equation of the curve. (06 marks)

(b) Use substitutions $y = vx$ to solve the differential equation

$$x^2\frac{dy}{dx} = x^2 + y^2 + xy$$ (06 marks)

(10,0)
(0,0)

✓16. (a) Find the point of intersection between the line $\boldsymbol{r} = \boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k} + t\,(2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$

and the plane $\boldsymbol{r}.(6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}) = 13$ and find the angle between the two. (06 marks)

(5,25)

(b) Show that the following vectors form a right angled triangle

$\boldsymbol{a} = (3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}), \boldsymbol{b} = (\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{k}),\ \boldsymbol{c} = (2\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k})$. (06 marks)

$2 - \frac{8}{2}$

$\frac{x^2+2xy}{x}$

$2\sin 6x$

$2\sin 6x\cos 2x + 2\sin 2x\cos 6x$

$-4-5$

$x - \frac{2y}{x}.\ 1 -$

END

$\frac{x-4}{x-2} =$

$\sqrt{21} + \sqrt{35}$

(5,25)

(10,0)

(0,0)

3, 18

$81R_3 + 7R_2 = R_3$

$$\begin{pmatrix} 4 & 11 & 6 \\ 0 & 81 & 54 \\ 0 & 0 & 216 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$\sqrt{98}$

(0,0)
(1,2)
(2,8)
(3,18)

$-2+1+1$

$1+4-3$

$-4-9$

a^2+b^2

$z = 3$

1774.892

0	1	2
0	2	4