UACE MATHEMATICS P425/2 MOCK MARKING GUIDE 2023

SECTION A

1. $$Resultant\ force, F = (i + j + k) + (i + 2j + 3k)\ \boldsymbol{m_1}$$

$$= (2i + 3j + 4k)\boldsymbol{B_1}$$

$$Displacement, S = (si + 4j + 3k) - (2i + 3j + 4k)$$

$$= (3i + j - k)m.\ \boldsymbol{m_1}$$

$$work\ done = force \times distance$$

$$= (2i + 3j + 4k).(3i + j - k)$$

$$= (6 + 3 - 4)$$

$$= 5J\ \boldsymbol{A_1}$$

Total = 5marks

2. $P(A) = 0.3, P(B) = 0.2$

$$\Leftrightarrow P(A') = 0.7, P(B') = 0.8$$

$$Required\ prbability = P(A')P(B) + P(A')P(B')P(A')P(B) + --$$

$$= (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + (0.7)(0.8)(0.7)(0.8)(0.7)(0.8)(0.7)(0.2) \pm --\ \boldsymbol{m_1}$$

$$= 0.14[1 + (0.56) + (0.56)^2 \pm ---]\ \boldsymbol{m_1}$$

$$= 0.14\left(\frac{1}{1 - 0.56}\right)\ \boldsymbol{m_1})$$

$$= \frac{0.14}{0.44}$$

$$= \frac{7}{22}$$

$$OR = 0.32\ \boldsymbol{A_1}$$

Total = 5marks

3. (a)

0.6	0.8
1.455	1.594
$\boldsymbol{B_1}$	$\boldsymbol{B_1}$

(b) $h = \frac{1-0}{5} = 0.2$ $\boldsymbol{B_1}$

$$\int_a^b y dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2((y_1 + y_2 + - - - +(y_{n-1})\}$$

$$\int_0^1 \sqrt{2^x + xdx} \approx \frac{1}{2}(0.2)\{(1 + 1.732) + (2(1.161 + 1.311 + 1.455 + 1.594)\}\ \boldsymbol{m_1}$$

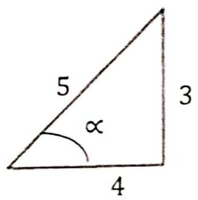
$$\approx 1.377\ \boldsymbol{A_1}$$

Total = 5marks

4. *Resolving vertically*

$$R + 40s\ in \propto = 20g.\ \boldsymbol{m_1}$$

Ressolving Horizontally



$$40cos \propto -FR = 20a.\ \boldsymbol{m_1}$$

$$FR = 0.1412$$

$$\Leftrightarrow 40cos \propto -0.14R = 20a\ \boldsymbol{m_1}$$

$$40cos \propto 0.14(20g - 40Sin \propto) = 20a.\ \boldsymbol{m_1}$$

$$40\left(\frac{4}{5}\right) - 0.14\left(20g - 40\left(\frac{3}{5}\right)\right) = 20a\ \boldsymbol{m_1}$$

$$\therefore a = 0.396ms^{-2}\ \boldsymbol{A_1}$$

Total = 5marks

5. $\sum P(X = x) = 1.$

$$all$$

$$\log_{36} a + \log_{36} b + \log_{36} c\ \boldsymbol{m_1}$$

$$\Leftrightarrow \log_{36}\ (a\ b\ c) = 1\ \boldsymbol{m_1}$$

$$abc = 36\ \boldsymbol{m_1}$$

$$abc = 2 \times 3 \times 6\ \boldsymbol{m_1}$$

$$\therefore a = 2$$

$$b = 3$$

$$c = 6\ \boldsymbol{A_1} - \boldsymbol{for\ all}$$

Total = 5marks

6. (a) *for* $x = 2.5$

$y = 1.499 > 0\ \boldsymbol{B_1}$

for $x = 3, y = -0.911 < 0\ \boldsymbol{B_1}$

since sign change, then there is a root in the range $2.5 < x > 3\ \boldsymbol{B_1}$

(b) *using linear interpolation*

x	2.5	x	3
y	1.499	$\propto$	-0.911

$$\frac{\propto -2-5}{3-\propto} = \frac{1.499}{0.911}\ m_1$$

$$\propto -2.5 = (1.645)(3-\propto)$$

$$\propto = 2.81\ A_1$$

$$\boldsymbol{Total = 5marks}$$

7.

$$v = \frac{(8i+11j)-(3i-4j)}{2.5}$$

$$v = 2i + 6j$$

$$using\ r_{t=}\ r_o + v_t$$

$$r(5) = (3i-4j) + (2i+6j)(5)\ m_1$$

$$= 3i - 4j + 10i + 30j\ m_1$$

$$= (13i + 26j)km\ A_1$$

$$\boldsymbol{Total = 5marks}$$

8. *let x denote IQ of the children. then x is a normal variate with parameters.*

$$\mu = 98 \text{ and } \sigma = 8$$

$$z = \frac{x-98}{8}$$

$$probability\ that\ IQ\ of\ a\ child\ is\ between\ 100\ and\ 120\ is$$

$$P(100 < \times\ 120) = P\left[\frac{100-98}{8} < \frac{X-98}{8} < \frac{120-98}{8}\right]\ m_1$$

$$= P\ (0.25 < Z < 2.75)$$

$$= P(0 < Z < 2.75) - P(0 < Z < 0.25)$$

$$= 0.4970 - 0.0987\ m_1$$

$$= 0.3983\ A_1$$

Among $N = 800$ *children, expected number of children having IQ between* 100 *and* 120 *is*

$$N \times P\ (100 < \times < 120) = 800 \times 0.3983\ m_1$$

$$\approx 319\ children.\ A_1$$

$$\boldsymbol{Total = 5marks}$$

SECTION B

9. (a)

$Class\ bound$	$Frequency$	x	fx
$0-20$	5	10	50
$20-40$	x	30	$30x$
$40-60$	10	50	500
$60-80$	y	70	$70y$
$80-100$	7	90	630
$100-120$	8	110	880
	$\varepsilon f = 50$ B_1		

$\varepsilon fx = 2060 + 30x + 70y$ $\quad B_1$

$\boldsymbol{Total\ Frequency}: 5 + x + 10 + y + 7 + 8 = 50$ $\boldsymbol{m_1}$

$$y = 20 - x \rightarrow (1)$$

$$Mean = \frac{\varepsilon fx}{\varepsilon f}$$

$$62.8 = \frac{2060 + 30x + 70y}{50} \quad m_1$$

$$2060 + 30x + 70y = 3140$$

$$3x + 7y = 108 \rightarrow (2)$$

$$eqn\ (1)\ into\ eqn\ (2)$$

$$3x + 7(20 - x) = 108$$

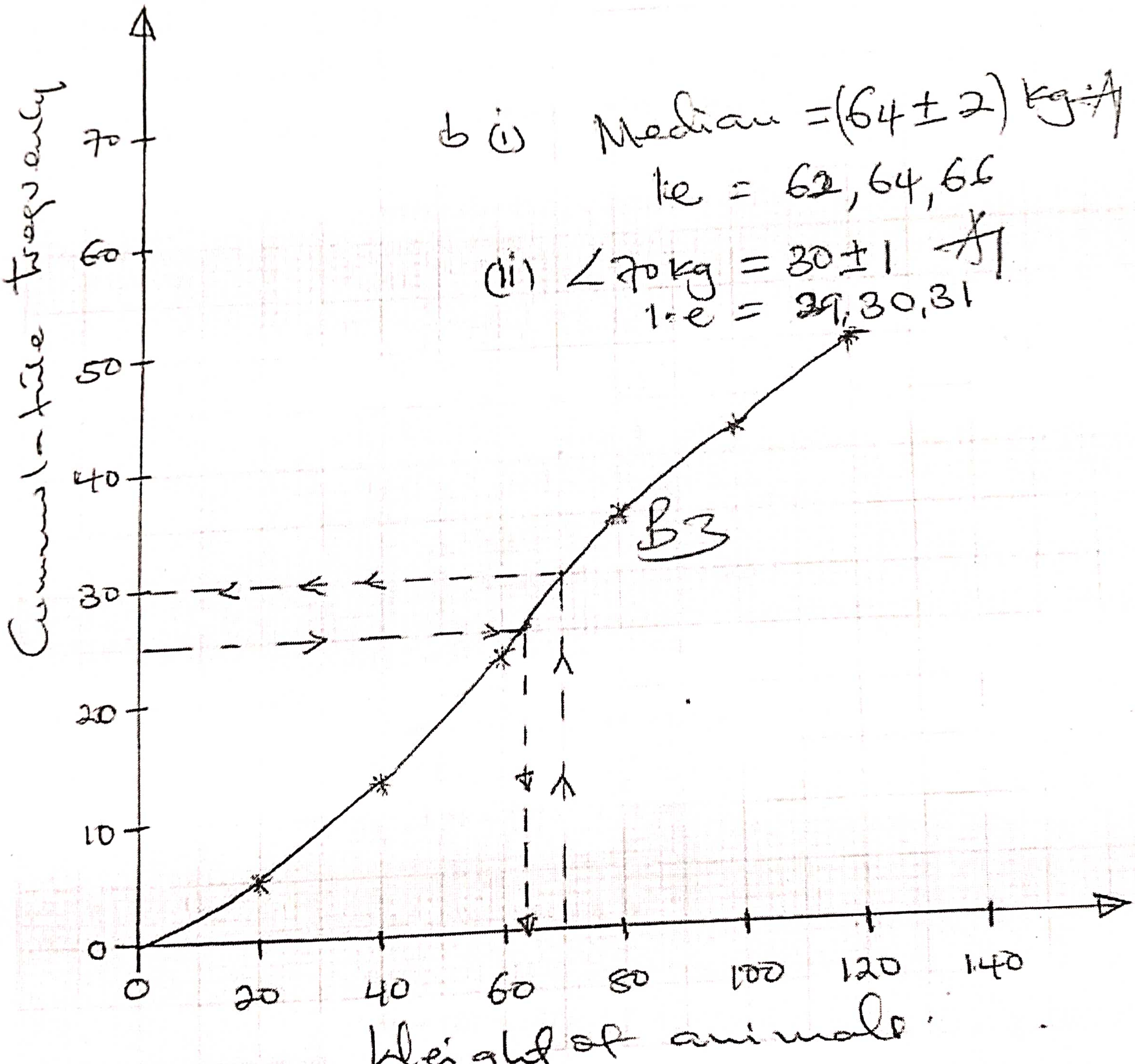
$$\therefore x = 8 \quad \boldsymbol{A_1}$$

$$From\ eqn\ (2)\ 3(8) + 7y = 108$$

$$\therefore y = 12 \quad \boldsymbol{A_1}$$

(b)

$Upper\ class\ boundaries$	$c.f$
-20	5
-40	13
-60	23
-80	35
-100	42
-120	50 $\boldsymbol{B_1}$



b (i) Median $= (64 \pm 2)$ kg A1

ie $= 62, 64, 66$

(ii) < 70 kg $= 30 \pm 1$ A1

i.e $= 29, 30, 31$

Total = 12 marks

10. (a) $V = 8t - \frac{3}{2}t^2 \ ; 0 \le \ + \le 4$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(8t - \frac{3}{2}t^2\right) \ m_1$$

$$= 8 - 3t$$

$$At\ greatest\ speed, a = 0$$

$$\Leftrightarrow 8 - 3t = 0$$

$$t = \frac{8}{3}\ s\ \boldsymbol{B_1}$$

$$\Leftrightarrow V = 8\left(\frac{8}{3}\right) - \frac{3}{2}\left(\frac{8}{3}\right)^2$$

$$V = \frac{32}{3}\ m/s\ \boldsymbol{A_1}$$

(b)

$$V = \frac{ds}{dt}$$

$$\int ds = \int \mathrm{vdt}$$

$$S = \int \left(8t - \frac{3}{2}t^2\right) dt$$

$$\Leftrightarrow S = 4t^2 - \frac{t^3}{2}\ \boldsymbol{m_1}$$

$$At\ t = 4, \qquad S = 4\,(4)^2 - \frac{4^3}{2}$$

$$\therefore S = 32m\ \boldsymbol{A_1}$$

(c)

$$for\ t > 4, S = \int (16 - 2t)\, dt$$

$$= 16t - t^2 + C\ \boldsymbol{m_1}$$

$$At\ t = 4, S = 32$$

$$\Leftrightarrow 32 = 16(4) - 4^2 + c\ \boldsymbol{m_1}$$

$$\Leftrightarrow c = 16\ \boldsymbol{B_1}$$

$$\Leftrightarrow S = 16t - t^2 - 16$$

$$\boldsymbol{For}\ t = 10s$$

$$S = 16\,(10) - 10^2 - 16$$

$$S = 44m$$

$$\boldsymbol{Since\ direction\ changed,}$$

$$For\ V = 0, 16 - 2t = 0, t = 8s$$

$$At\ t = 8s, S = 16\,(8) - 8^2 - 16$$

$$S = 48m \quad B_1$$

$$Change\ in\ distance = 48 - 44$$

$$= 4m \quad B_1$$

$$Now\ total\ distance\ travelled = 48 + 4$$

$$= 52m \quad A_1$$

$$\textbf{Total = 12 marks}$$

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11. (a) $Let\ f(x) = xe^x - 1$

$$f(0) = 0(e^{\circ}) - 1 = -1 \quad B_1$$

$$f(1) = 1(e^1) - 1 = 1.71828 \quad B_1$$

$since\ f(0) < 0\ and\ f(1) > 0, therefore\ there\ is\ a\ root\ between\ 0\ and\ I. \quad B_1$

(b) $Then\ f(x) = xe^x - 1$

$$f^1(x) = xe^x + e^x \quad M_1$$

$$(x+1)e^x$$

$$using\ Newton - Raplson\ Method;$$

$$X_{n+1=X_n} - \frac{f(X_n)}{f'(X_n)}, x_0 = \frac{0+1}{2} = 0.5 \quad M_1$$

$$\textbf{1st approximation}$$

$$x_{1=X_0} - \frac{x_o e^{x_{o-1}}}{(x_0+1)e^{x_o}}$$

$$= 0.5 - \frac{(0.5)e^{0.5} - 1 \quad M_1}{(0.5+1)e^{0.5}} = 0.571020 \quad B_1$$

$$|x_1 - x_0| = 0.571020 - 0.5$$

$$= 0.071020$$

$$|x_1 - x_0| > 0.0005$$

$$\textbf{2nd approximation}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= x_1 - \frac{x_1 e^{x_1} - 1}{(x_1+1)e^{x_1}}$$

$$0.5710 - \frac{(0.57102)e^{0.57102-1}}{(0.57102+1)e^{0.57102}} \quad M_1$$

$$= 0.56715 \quad B_1$$

$$|x_2 - x_1| = 0.56715 - 0.57102$$

$$= 0.00387$$

$$|x_2 - x_1| > 0.0005$$

$$3rd\ approximation$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\Leftrightarrow x_3 = x_2 - \frac{x_2e^{x_2-1}}{(x_2+1)e^{x_2}}$$

$$0.56715 - \frac{(0.56715)e^{0.56715-1}}{(0.56715+1)e^{0.56715}} \quad m_1$$

$$= 0.56714 \quad B_1$$

$$|x_3 - x_2| = 0.56714 - 0.56715$$

$$0.00001$$

$$|x_3 - x_2| < 0.0005$$

$$\therefore the\ root\ is \approx 0.567 \quad A_1$$

$$\boldsymbol{Total = 12\ marks}$$

12. (a) $\int_0^5 f(x)dx = 1$

$$\int_0^5 (a+bx)dx = 1$$

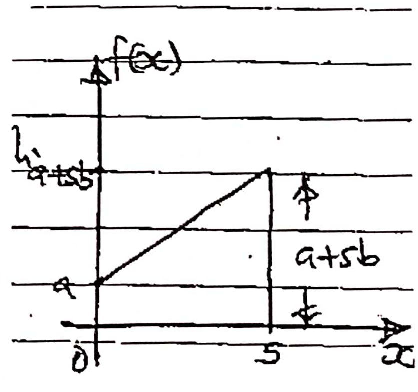
$$\left[ax + \frac{bx^2}{2}\right]_0^5 = 1 \quad m_1$$

$$a(5) + \frac{b(5)^2 - 0}{2} = 1 \quad m_1$$

$$10a + 25b = 2 \quad A_1$$

OR

$$Using\ Area\ under\ graph$$



$$Area = 1$$

$$\frac{(a+a+5b)5}{2} = 1$$

$$5(2a+5b) = 2$$

$$\therefore 10a + 25b = 2$$

(b). $Given\ E(x) = \int_0^5 xf(x)dx$

$$= \int_0^5 x(a+bx)dx$$

$$= \int_0^5 (ax+bx^2)dx$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^5 \quad m_1$$

$$= \frac{a(5)^2}{2} + \frac{b(5)^3}{3} - 0$$

$$= \frac{25a}{2} + \frac{125b}{3}$$

$$\Leftrightarrow \frac{25a}{2} + \frac{125b}{3} = \frac{35}{12}$$

$$\Leftrightarrow 30a + 100b = 7 \quad ----------- (1) \quad B_1$$

$$from\ 10a + 25b = 2$$

$$\Leftrightarrow 40a + 100b = 8 \quad ----------------- (2) \quad m_1$$

$$eqn\ (2) - eqn\ (1)\ gives$$

$$10a = 1$$

$$\therefore a = \frac{1}{10}$$

$$\therefore a = 0.1 \quad A_1$$

$$from\ eqn\ (1)$$

$$30\left(\frac{1}{10}\right) + 100b = 7 \quad m_1$$

$$\therefore b = \frac{1}{25}$$

$$\mathbf{OR}\ b = 0.04 \quad A_1$$

(c) $p(x < m) = \frac{1}{2}$

$$\Leftrightarrow \int_0^m \left(\frac{1}{10} + \frac{1x}{25}\right) dx = \frac{1}{2} \quad m_1$$

$$\left[\frac{1x}{10} + \frac{x^2}{50}\right]_0^m = \frac{1}{2} \quad m_1$$

$$\frac{m}{10} + \frac{m^2}{50} - 0 = \frac{1}{2}$$

$$\Leftrightarrow m^2 + 5m - 25 = 0$$

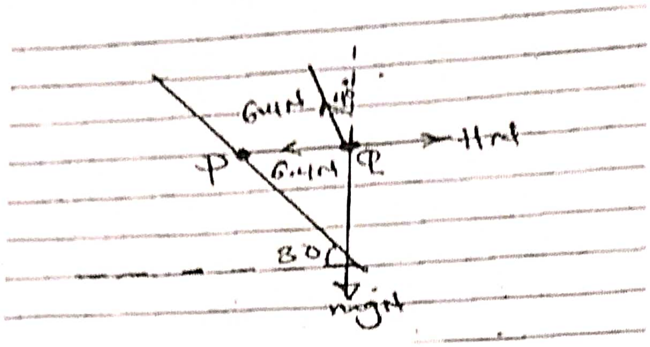
$$Either\ m = 3.0901$$

$$or\ m = -8.09(\ which\ is\ neglected)$$

$$\therefore median = 3.09(3sf) \quad A_1$$

$$\mathbf{Total = 12\ marks}$$

13.



(a) *resolving vertically.*

$$mg = 6.4\cos 40^\circ \quad m_1$$

$$m = \frac{6.4\cos 40^\circ}{9.8}$$

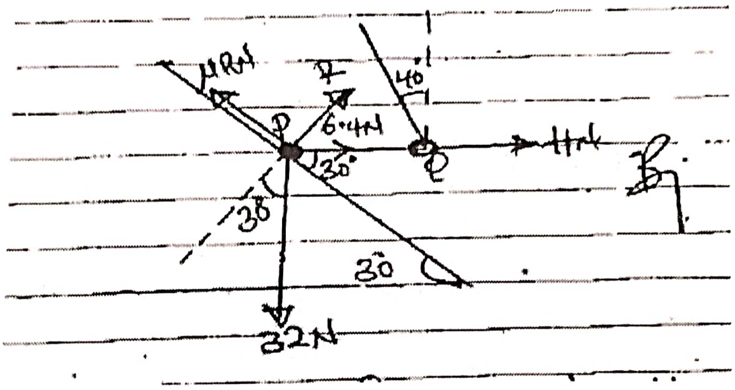
$$\therefore m = 0.5kg(1dpl) \quad A_1$$

Resolving Horizontally

$$H = 6.4 + 6.4\sin 40 \quad m_1$$

$$\therefore H = 10.5N(1dpl) \quad A_1$$

(b)



Ressolving perpendicular to the plane

$$R + 6.4\sin 30 = 32cos30 \quad m_1$$

$$R = 32cos30 - 6.4sin30$$

$$= 24.5128N \quad B_1$$

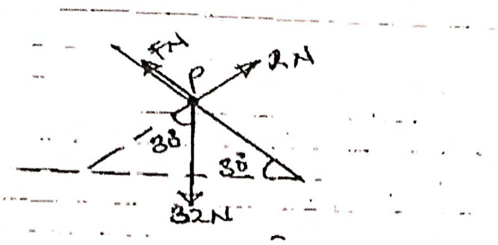
Resolving parallel to the plane

$$\mu(24.5128) = 6.4cos30^\circ + 32sin30^\circ \quad m_1$$

$$\mu = \frac{6.4cos30^\circ + 32sin30}{24.5128}$$

$$\therefore \mu \approx 0.879 \blacksquare \quad A_1$$

(c)



resolving perpendicular to the plane

$$R = 32\cos\ 30°$$

$$= 27.712N \quad \boldsymbol{B_1}$$

$$Maximum\ value\ of\ F = \mu R$$

$$= 0.879 \times 27.712$$

$$= 24.35N \quad \boldsymbol{B_1}$$

Component of weight down the plane

$$= 32sin30°$$

$$= 16\ N$$

$$\Leftrightarrow 16 < F$$

$$\therefore P\ is\ in\ equilibrium\ since\ 16N < NR \quad \boldsymbol{B_1}$$

$Total = 12\ marks$

14. (a) $Q = \frac{x^n}{y^m}$

$$InQ = In\left(\frac{x^n}{y^m}\right) \quad \boldsymbol{M_1}$$

$$InQ = Inx^n - InY^m$$

$$InQ = nInX - mInY \quad \boldsymbol{M_1}$$

$$\frac{d}{dx}InQ = \frac{d}{dx}\ (nInX - mInY)$$

$$\frac{de}{e} = \frac{ndx}{x} - \frac{mdY}{Y} \quad \boldsymbol{M_1}$$

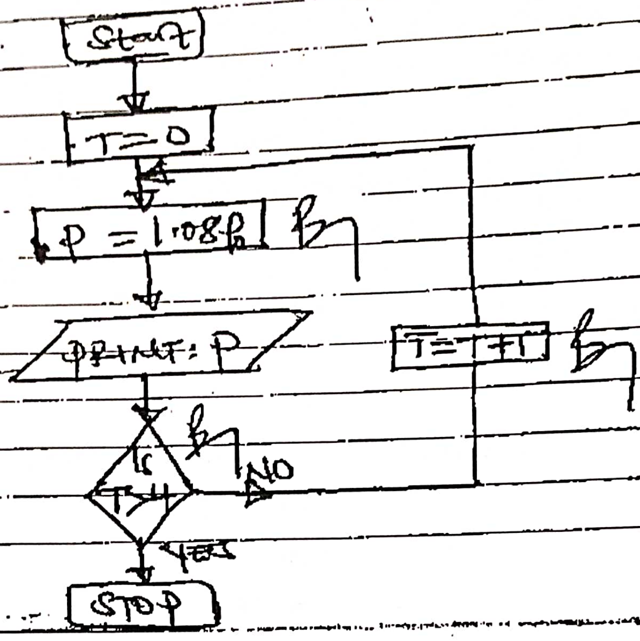
$$\frac{eq}{Q} = \frac{nex}{x} - \frac{meY}{Y} \quad \boldsymbol{M_1}$$

$$\left|\frac{eq}{Q}\right| = \left|\frac{nex}{x} - \frac{mey}{Y}\right|$$

$$|\ eq| = \pm Q\left[n\left|\frac{ex}{x}\right| + m\left|\frac{ey}{Y}\right|\right]$$

$$\therefore\ |eq| = \pm\left[\frac{x^n}{Y^n}\right]\left[n\left|\frac{ex}{x}\right| +\ m\left|\frac{ey}{Y}\right|\right] \quad \boldsymbol{B_1}$$

(b) (i)



(ii) ***Dry run***

T	P(Millions)
0	2
1	2.16 $\qquad M_1$
2	2.3328 $\qquad M_1$
3	2.5194 $\qquad M_1$
4	2.9390

$\therefore$ *On 1st 2027 January, his account will harvest* 2,939,000 $\quad A_1$

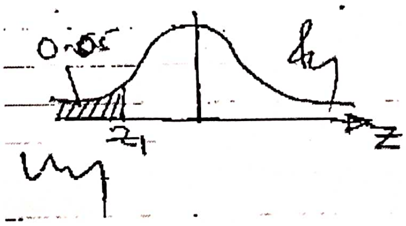
Total = 12 marks

15. *Let x be the Random Variable " height"*

(a)

$$x \sim N(\mu, \sigma^2)$$

$$\Leftrightarrow Z_1 = \frac{154 - \mu}{\sigma} \qquad M_1$$



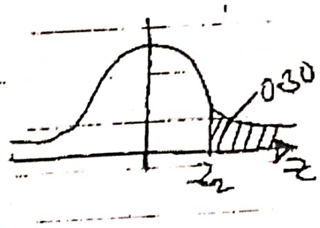
$$Z_1 = -1.6449$$

$$\Leftrightarrow -1.6449 = \frac{154 - \mu}{\sigma} \qquad M_1$$

$$\Leftrightarrow -1.6449\,\sigma = 154 - \mu \qquad A_1$$

$$\therefore \mu = 154 + 1.6449\,\sigma \qquad \blacksquare$$

(b)



$$z_2 = \frac{172 - \mu}{\sigma}$$

$$z_2 = 0.5244$$

$$0.5244 = \frac{172 - \mu}{\sigma} \qquad M_1$$

$$0.5244\sigma = 172 - \mu$$

$$\mu = 172 - 0.5244\sigma \ldots\ldots\ldots\ldots (1) \qquad B_1$$

$$\mu = 154 + 1.6449\sigma \ldots\ldots\ldots\ldots (2)$$

equality eqn(1) to eqn(2)

$$172 - 0.5244\sigma = 154 + 1.6449\sigma \qquad M_1$$

$$\Leftrightarrow \sigma = \frac{18}{2.1693}$$

$$\sigma = 8.2976$$

$$\therefore \sigma \approx 8.30 \qquad A_1$$

substituting value of σ in eqn(2)

$$\mu = 154 + 1.6449(8.2976)$$

$$= 167.648$$

$$\therefore \mu \approx 168 \qquad A_1$$

(c)

$$v \sim \mu\,(167.648,\ 8.2976^2)$$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{160 - 167.648}{8.2975} \qquad M_1$$

$$= -0.9217 \qquad B_1$$

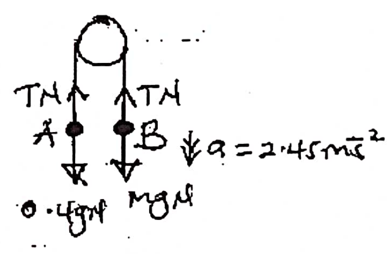
$$\Leftrightarrow p(x > 160) = p(z > -0.9217)$$

$$= p(z > -0.9217)$$

$$\therefore p(x > 160) = 0.8212 \qquad A_1$$

Total = 12 marks

16. (a)



Consider A

$$F = m\,a$$

$$T - 0.4g = 0.4(2.45) \qquad M_1$$

$$T = 0.4(2.45) + 0.4(9.8)$$

$$T = 4.9N \qquad A_1$$

(b)

$$\text{consider } B$$

$$\text{Using } F = m\,a$$

$$m\,g - T = m\,a$$

$$m(9.8) - 4.9 = m(2.45) \quad m_1$$

$$9.8m - 2.45m = 4.9$$

$$m = \frac{4.9}{7.35}$$

$$\Leftrightarrow m = \frac{2}{3}\ kg \quad B_1$$

$$\text{using } V = u + a\,t$$

$$v = 0 + 2.45(0.3)$$

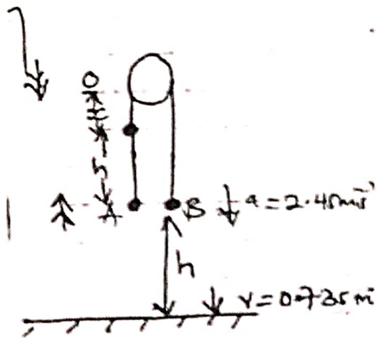
$$\Leftrightarrow v = 0.735ms^{-1} \quad B_1$$

$$\textbf{Momentum} = MV$$

$$\text{lost} \quad = \frac{2}{3} \times 0.735 \quad m_1$$

$$= 0.49Ns \quad A_1$$

(c)



$$\text{Considering motion of } B \downarrow$$

$$\text{using } S = \frac{(U+V)}{2}\,t$$

$$h = \frac{0.735(0.3)}{2} \quad m_1$$

$$\therefore h = 0.11025 \quad B_1$$

$$\text{Considering motion of } P \uparrow$$

$$\text{Using } V^2 = u^2 + 2as$$

$$S = \frac{v^2 - u^2}{2a}$$

$$x = \frac{-(0.735)^2}{2(-9.8)}$$

$$= 0.0275625m \quad B_1$$

$$\text{maximum height of } P = 2(0.11025) + 0.0275625 \quad m_1$$

$$= 0.248m. \quad A_1$$

$$\textbf{Total} = \textbf{12 marks}$$

*** **END** ***