HOLIDAY ASSESSMENTS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

Principal Subject

Time 3 Hours

Instructions:

Answer all questions in section "A" and not more than five from section "B"

SECTION "A"

1. From first principals, show that $\frac{d}{dx}(tan\theta) = 1 + tan^2\theta$ (7 marks)
2. By recognizing the function and its derivative, find $\int sin^2\theta cos\theta\ d\theta$. $[Hint; Use\ u = sin\theta]$ (3 marks)
3. By evaluation, show that $\int_0^{\frac{\pi}{2}} sin2xcosx\ dx = \frac{2}{3}$. (5 marks)
4. The distance, S (m) of a particle from a fixed point is given by the expression; $S = t^2(t^2 + 6) - 4t(t-1)(t+1)$ where t, is the time. Find the velocity and acceleration of the particle at $t = 1\ second$. (5 marks)
5. The pints P(4,-6,1), Q(2,8,4) and R(3,7,4) lie in the same plane. Find the angle between PQ and PR. (5 marks)
6. Show that cos4$\theta = 8cos^4\theta - 8cos^2\theta + 1 = \frac{tan^4\theta - 6tan^2\theta + 1}{tan^4\theta + 2tan^2\theta + 1}$ $[Hint; Use\ sec^2\theta = 1 + tan^2\theta]$ (5 marks)
7. Given that$y = asec\theta\ and\ x = btan\theta$; deduce that; $y\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^2 = \frac{y}{x}\frac{dy}{dx}$ (5 marks)
8. Given that;$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\ and\ B = \begin{pmatrix} 0 & 1 \\ 5 & 0 \end{pmatrix}$, show that$\det(A).\det(B) = \det(AB)$ (5 marks)

SECTION"B"

9. (a). By reducing to echelon form, solve the simultaneous equations below; $x + y + z = 0, x + 2y + 2z = 2\ and\ 2x + y + 3z = 4$ (5 marks)

   (b). Find the equation of a tangent and the normal to the curve $y = 4x^3 - 6x^2 + 3x$ at a point P(1,1) (7 marks)

10. (a). Evaluate $\int tan^6x\ dx$ (5 marks)

    (b). Evaluate$\int \frac{3}{4+9x^2}\ dx$ (4 marks)

    (c). Evaluate$\int cos^5\theta\ d\theta$ (3 marks)

11. (a). Find the Cartesian of the plane containing the following points; A(1,2,5), B(1,0,4) and C(5,2,1) (6 marks)

    (b). Find the angle between the line $\frac{x+4}{8} = \frac{y-2}{2} = \frac{z+1}{-4}$ and $4x + 3y - 3z + 1 = 0$ (6 marks)

12. (a). Solve the equation$3cosx + 4sinx = 2\ for\ 0^0 \leq x \leq 360^0$ (5 marks)

(b). Form the equation of a circle through A(-1,4), B(2,5) and C(0,1) and hence find the gradient of the tangent at (2,5) (7 marks)

13. (a). Given that $\int_0^a (x^2 + 2x - 6)\, dx = 0$, find the value of a. (6 marks)

(b). Solve the equation; $\log_2 x - \log_x 8 = 2$ (6 marks)

14. (a). The function $f(x) = x^3 + px^2 - 5x + q$ a factor of $(x - 2)$ and a value of 5 when $x = -3$. Find the value of p and q (4 marks)

(b). The roots of the equation $ax^2 + bx + c = 0$ are α *and* β, find the equation whose roots are $\frac{\alpha}{\beta}$ *and* $\frac{\beta}{\alpha}$. (5 marks)

(c). Simplify; $\frac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form of $p + q\sqrt{3}$. (3 marks)

15. (a). Solve $2\sqrt{(x-1)} - \sqrt{(x-4)} = 1$. (6 marks)

(b). Solve the simultaneous equations; $x + y + z = 2$ and $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5}$ (6 marks)

16. (a). Given that $sinx + siny = p$ and $cosx + cosy = q$. Show that;

(i). $\tan\left(\frac{x+y}{2}\right) = \frac{p}{q}$.

(ii). $\cos(x + y) = \frac{p^2-q^2}{p^2-q^2}$. (6 marks)

(b). Solve the following simultaneous equations for $0^0 \leq x{:}\, y \leq 360^0$;

$cosx + 4siny = 1$ *and* $4siny - 3cosx = cosx\ siny$ (6 marks)