Examination Questions

1. (a) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

   Hence solve the equation $4x^3 - 3x - \frac{\sqrt{3}}{2} = 0$.

   (b) Find all the solutions of the equation $4\cos x - 5\sin x = 6$ in the range $0° \leq x \leq 360°$.

2. Given that $\sin x + \sin y = \alpha_1$ and $\cos x + \cos y = \alpha_2$, show that:

   (a) $\tan\left(\frac{x+y}{2}\right) = \frac{\alpha_1}{\alpha_2}$

   (b) $\cos(x+y) = \dfrac{\alpha_2^2 - \alpha_1^2}{\alpha_2^2 + \alpha_1^2}$.

   (c) Solve the simultaneous equations for values of x and y between $0°$ and $360°$:

$$\cos x + 4\sin y = 1$$

$$4\sec x - 3\csc y = 5$$

3. (a) Given that $7\tan\theta + \cot\theta = 5\sec\theta$, derive a quadratic equation for $\sin\theta$. Hence or otherwise, find all values of θ in the interval $0° \le \theta \le 180°$ which satisfy the given equation, giving your answers to the nearest $0.1°$ where necessary.

(b) The acute angle A and B are such that $\cos A = \frac{1}{2}$, $\sin B = \frac{1}{3}$. Show without using tables or calculator that;

$$\tan(A+B) = \frac{9\sqrt{3}+8\sqrt{2}}{5}.$$

4. (a) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(b) Find all the solutions to $2\sin 3\theta = 1$ for θ between $0°$ and $360°$. Hence find the solutions of $8x^3 - 6x + 1 = 0$.

5. (a) Show that $x = 1$ is a solution of the equation $x^3 - x^2 - 3x + 3 = 0$, and find the other two values of x which satisfy the equation.

(b) Use part (a) to show that $\tan\theta = 1$ is a solution of the equation $\tan^3\theta - 3\tan\theta + 4 = \sec^2\theta$. And hence find all the values of θ satisfying the equation $(0° \le \theta \le 360°)$.

6. (a) Using the formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$, show that;

$$\frac{\cos(A-B) - \cos(A+B)}{\sin(A+B) - \sin(A-B)} = \tan A\,.$$

(b) Using the result of (a) and the exact values of $\sin 60°$ and $\cos 60°$, find an exact value of $\tan 75°$ in its simplest form.

7. (a) Prove that $\sin x + \cot x \cos x = \csc x$.

(b) Hence or otherwise, find the values of x, $0° < x < 180°$, which satisfy the equation $\cot x \cos x = 3$, giving your answer to 1 decimal place.

8. (a) Express $f(x) = \sqrt{3}\sin x + \cos x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Hence solve the equation $\sqrt{3}\sin x + \cos x = \sqrt{2}$ where $0° < x < 180°$.

(b) Sketch the graph of $y = f(x)$ for $0° \le x \le 360°$.

(c) You are given that $y = 2f(x) + 1$. State the maximum and minimum values of y and the values of x when they occur.

9. (a) Prove that;

$$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}.$$

(b) Use the identity to find the values of θ, for $0° < \theta < 360°$, which satisfy the equation $\cot^2\theta - 2\cot\theta - 1 = 0$.

10. (a) Express $7\sin x + 24\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Hence solve the equation $7\sin x + 24\cos x = 15$, where $0° < x < 360°$.

(b) Prove that these values satisfy the equation $15\sec x - 7\tan x = 24$.

(c) Find the maximum value of the function $7\sin x + 24\cos x$ and give the smallest positive value of x for which this maximum value occurs.

11. (a) Express $2.5\sin 2x + 6\cos 2x$ in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your values of R and α to 2 decimal places.

(b) Express $5\sin x\cos x - 12\sin^2 x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b and c are constants to be found.

(c) Hence using your answer to part (a), deduce the maximum value of

$5\sin x\cos x - 12\sin^2 x$ and the value of x when it occurs.

12. (a) Given that $\cos(2x - 60) = 2\sin(2x + 30)$, prove that $\tan 2x = -\frac{1}{\sqrt{3}}$.

(b) Using the result from part (a), find the two values of x, $0° < x < 180°$, which satisfy the equation $2\sin(2x + 30) - \cos(2x - 60) = 0$.

13. (a) Express $9\cos\theta - 40\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$.

Hence solve the equation $9\cos\theta - 40\sin\theta = 6$, for $0° < \theta < 90°$, giving your answer to 1 decimal place.

(b) Solve the equation $13 + 10\cot\theta = 3\tan\theta$, for $0° < \theta < 180°$, giving your answer to 1 decimal place.

14. (a) Letting $A + B = P$, and $A - B = Q$ and using the expansion for $\sin(A \pm B)$, prove that

$$\sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

(b) Hence or otherwise solve the equation;

$\sin 4\theta - \sin 2\theta + \cos 3\theta = 0$ for $0° < \theta < 360°$.

15. (a) Prove that;

$$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}.$$

(b) Hence solve the equation $\tan\theta\,(4 - \tan\theta) = 1$, $0° < \theta < 360°$.

16. (a) Using the identity for $\cos(A + B)$, prove that $\cos\theta = 2\cos^2\left(\frac{1}{2}\theta\right) - 1$.

(b) Prove that $1 + \sin\theta + \cos\theta = 2\cos\left(\frac{1}{2}\theta\right)\left[\sin\left(\frac{1}{2}\theta\right) + \cos\left(\frac{1}{2}\theta\right)\right]$

(c) Hence, or otherwise, solve the equation $1 + \sin\theta + \cos\theta = 0$, $0° \le \theta \le 360°$.

17. (a) Find the solution of the equation $\tan x + \sec x = 3\cos x$ for $0° \le x \le 360°$.

(b) Express $5\sin^2 x - 3\sin x\cos x + \cos^2 x$ in the form $a + b\cos(2x - \alpha)$ where a, b and α are independent of x. Hence or otherwise, find the maximum and minimum values of $5\sin^2 x - 3\sin x\cos x + \cos^2 x$ as x varies.

18. (a) Express $\frac{\sin 2\theta - \cos 2\theta - 1}{2 - 2\sin 2\theta}$ in terms of $\tan\theta$.

(b) Solve $\sin 3x + \frac{1}{2} = 2\cos^2 x$ for $0° \le x \le 360°$.

19. (a) Prove that:

$$(\sin 2\theta - \sin\theta)(1 + 2\cos\theta) = \sin 3\theta.$$

(b) A vertical pole BAO stands with its base O on a horizontal plane, where $BA = c$ and $AO = b$. A point P is situated on a horizontal plane x *units* from O and the angle $APB = \theta$.

Prove that $\tan\theta = \frac{cx}{x^2+b^2+bc}$.

20. (a) Prove that $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$.

(b) Solve for x in:

(i) $\tan x + 3\cot x = 4$

(ii) $4\cos x - 3\sin x = 2;\ \ 0° \le x \le 360°$.

21. (a) Given that X, Y, and Z are angles of a triangle XYZ. Prove that $\tan\left(\frac{X-Y}{2}\right) = \frac{x-y}{x+y}\cot\frac{Z}{2}$. Hence solve the triangle if $x = 9\ cm$, $y = 5.7\ cm$, and $Z = 57°$.

(b) Use the substitute $t = \tan\frac{\theta}{2}$ to solve the equation $3\cos\theta - 5\sin\theta = -1$ for $0° \le \theta \le 360°$.

22. (a) Show that $\cos 4\theta = \frac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1}$.

(b) Given that in any triangle ABC

$$\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\left(\frac{A}{2}\right);$$

solve the triangle with two sides $5\ cm$ and $7\ cm$ and the included angle $45°$.

23. (a) Solve the equation $3\cos x + 4\sin x = 2$ for $0° \le x \le 36°$.

(b) If A, B, C are angles of the triangle, show that $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$.

24. (a) Show that $\frac{\sin\theta - 2\sin 2\theta + \sin 3\theta}{\sin\theta + 2\sin 2\theta + \sin 3\theta} = -\tan^2\frac{\theta}{2}$.

(b) Express $4\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \beta)$, where R is a constant and β an acute angle.

(c) Determine the maximum value of the expression and the value of θ for which it occurs.

(d) Solve the equation $4\cos\theta - 5\sin\theta = 2.2$, for $0° < \theta < 360°$.

25. (a) Find all the values of θ, $0° \le \theta \le 360°$ which satisfy the equation $\sin^2\theta - \sin 2\theta - 3\cos^2\theta = 0$.

(b) Show that $\frac{\cos A}{1+\sin A} = \cot\left(\frac{A}{2} + 45°\right)$. Hence or otherwise solve $\frac{\cos A}{1+\sin A} = \frac{1}{2}$; $0° \le A \le 360°$.

26. (a) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(b) Find the maximum and minimum values of $\cos\theta + 2\sin\theta$ and the smallest possible value for θ for which the maximum occurs.

The depth d *metres*, of water in a lake is modeled using the equation where t *hours* is the number of hours after 1200

$$d = 15 + \cos\left(\frac{\pi t}{12}\right) + 2\sin\left(\frac{\pi t}{12}\right),\ \ 0 \le t \le 24.$$

(c) Calculate the maximum depth of the water predicted by this model and the value of t when this maximum occurs.

(d) Calculate the depth of the water at 1200.

(e) Calculate to the nearest half hour the time in the evening when the depth of the water is 15 metres.

Examination Questions

1. (a) $x = -0.643, -0.342, 0.985$
   (b) $x = 288.2, 329.0$

2. (c) $x = 78.5, 281.5$; $y = 11.5, 168.5$

3. (a) $\theta = 19.5, 30, 150, 160.5$

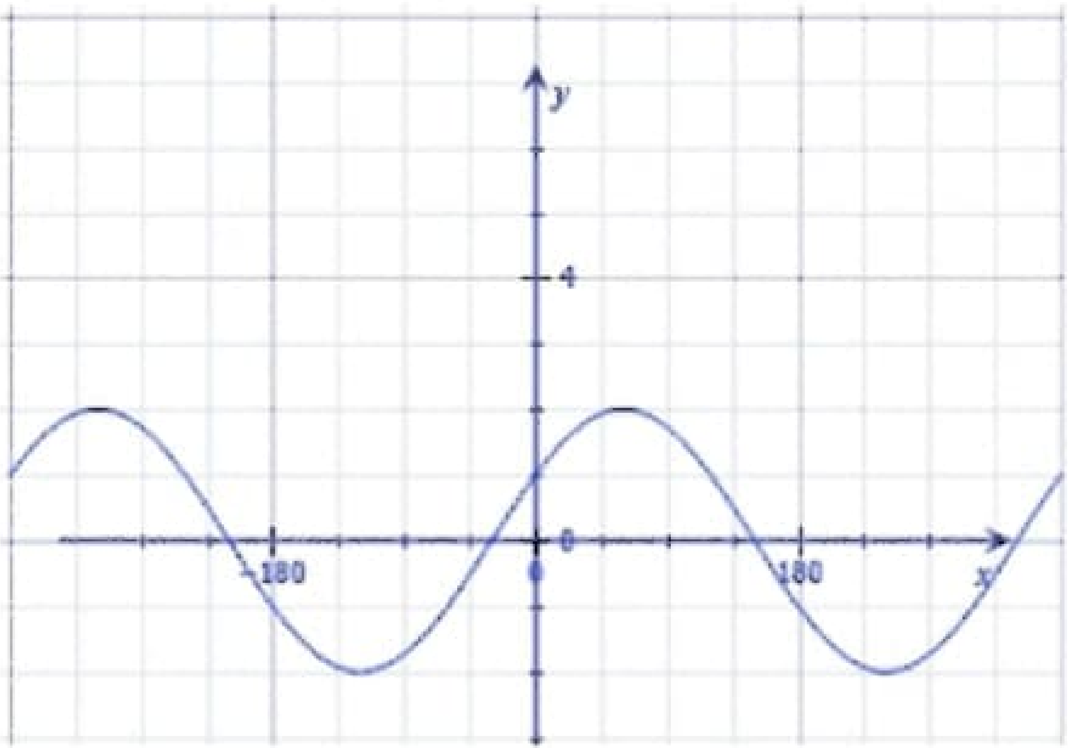
4. (b) $\theta = 10, 50, 130, 170, 250, 290$; $x = 0.1736,\ 0.7660,\ -0.9397$

5. (a) $x = \sqrt{3}$ or $x = -\sqrt{3}$
   (b) $\theta = 45, 60, 120, 135, 240, 300$

6. (b) $2 + \sqrt{3}$

7. (b) $x = 17.6, 162.4$

8. (a) $2\cos(x - 60)$; $x = 105$
   (b)



(c) $maxi = 5$ when $x = 60$; $mini = -3$ when $x = 240$

9. (b) $\theta = 22.5, 112.5, 202.5, 292.5$
10. (a) $25\sin(x + 73.7)$; $x = 69.4, 323.4$
(c) $maxi = 25$ when $x = 16.3$
11. (a) $6.5\sin(2x + 67.38)$
(b) $2.5\sin 2x + 6\cos 2x - 6$
(c) $maxi = 0.5$ when $x = 11.13$
12. (b) $x = 75, 165$
13. (a) $41\cos(\theta + 77.3)$; $\theta = 4.3$
(b) $\theta = 78.7, 146.3$
14. (b) $\theta = 30, 90, 150, 210, 270, 330$
15. (b) $\theta = 15, 75, 195, 255$
16. (c) $\theta = 180, 270$
17. (a) $x = 41.8, 138.2, 270$
(b) $mini = 0.5$; $max = 5.5$
18. (a) $\frac{1}{\tan\theta - 1}$
(b) $x = 30, 60, 120, 150, 240, 300$
19. (b) $\tan\theta = \frac{cx}{x^2 + b^2 + bc}$
20. (b) $x = 45, 71.6, 225, 251.6$; $x = 29.6, 256.6$
21. (a) $X = 83.9°$, $Y = 39.1°$, $z = 7.6\,cm$
(b) $\theta = 40.8, 201.0$
22. (b) $45.6°$, $89.4°$, $4.95cm$
23. (a) $x = 119.5, 299.5$
24. (b) $\sqrt{41}\cos(\theta + 51.3)$
(c) $maxi = \sqrt{41}$ when $\theta = 308.7$
(d) $\theta = 18.6, 238.8$
25. (a) $\theta = 71.6, 135, 251.6, 315$
(b) $A = 36.8$
26. (a) $R = 2.24$, $\alpha = 1.11\,rad$
(b) $\begin{pmatrix} maxi = \sqrt{5} \\ mini = -\sqrt{5} \end{pmatrix}, \theta = 1.11$
(c) $maxi\ depth = 15 + \sqrt{5}$
(17.24 m to 2dp); $t = 4.24$ (to 2dp)
(d) $d = 16m$
(e) $(t = 10.24)$,
2230 to nearest half hr