PURE MATHEMATICS SEMINAR QUESTIONS 2024

ALGEBRA

1. Given that the equations $x^2 + px + q = 0$ and $x^2 + mx + k = 0$ have a common root. Show that $(q - k)^2 = (m - p)(pk - mq)$

2. Solve the inequality. $\frac{x+1}{2x-3} \leq \frac{1}{x-3}$

3. (a) Given that $(x - 1)^2$ is a factor of the function $f(x) = x^3 + 4ax^2 + bx + 3a$. Find the values of a and b.
   (b) The roots of the equation $x^2 + ax + b = 0$ are α and β. Given that $(\alpha + 2)(\beta + 2) = 14$ and $(\alpha - 2)(\beta - 2) = 2$. Find the values of a and b.

4. Express $\left(-1 - \sqrt{3}i\right)^6$ in the form $x + iy$.

5. Find the square root of $5 + 12i$.

6. Expand $\frac{1}{\sqrt{1+x}}$ up to the term x^2 and by letting $x = \frac{1}{4}$,

   show that $\sqrt{5} = \frac{256}{115}$.

7. Solve for n given that $n_{C_4} = 5(^{n-2}C_3)$.

8. Find the sum of the series $1^3 + 3^3 + 5^3 +$ ~~[illegible]~~ $---+ n^3$

9. Solve the equation $\sqrt{(3 - x)} - \sqrt{7 + x} = \sqrt{16 + 2x}$

10. Solve the equation $(x^2 + 2x) + \frac{12}{x^2+2x} = 7$.

TRIGONOMETRY

11. (a) Using the t – formulae prove that $1 + sec2\theta = tan2\theta cot\theta$.
   (b) express $2\sqrt{3}\, sin\theta cos\theta + 2cos^2\theta$ in the form $a\sin(2\theta + \alpha) + b$. Hence solve the equation $2\sqrt{3}\, sin\theta cos\theta + 2cos^2\theta = 3$ for $0^0 \leq \theta \leq 360^0$.
   (c) Solve for x in: $\tan^{-1} x + \tan^{-1}\left(\frac{2}{3}x\right) = \frac{\pi}{4}$.

12 (a) Prove that in any triangle ABC $\cot\left(\frac{A-B}{2}\right) = \frac{a+b}{a-b}\cot\frac{C}{2}$.
   (b) In triangle ABC, prove that $a^2 = (b - c)^2 + 4bc\sin^2\frac{A}{2}$

13 (a) Given that $y = (secx + tanx)^2$. Show that

$$cosx\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2ytanx$$

Matsiko wilberforce Tel: 0702-300600 / 0785-625292

(b) Solve the trigonometric equation

$cos3\theta - sin3\theta = sin\theta - cos\theta$ for $-180^0 \le \theta \le 180^0$

14. (a) Solve the equation.

$5sin\theta tan\theta - 10tan\theta + 3sin\theta - 6 = 0$ for $0^0 \le \theta \le 360^0$

(b) Prove that $tan3\theta = \frac{3t-t^3}{1-3t^2}$ where $t = tan\theta$.

Hence solve $t^3 - 3t^2 - 3t + 1 - 0$ correct to 3Sfs.

VECTORS

15. (a) Find the equation of the plane containing the lines $\frac{x-2}{-2} = \frac{y+3}{1} = \frac{z-1}{2}$

and $\underline{r} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 8 \\ 1 \\ 2 \end{pmatrix}$.

(b) (i) Find the point of intersection of the line $\frac{x-2}{2} = \frac{y+3}{3} = \frac{z-1}{5}$ and the line given by the equation $\underline{r} = (4 + 2t)\underline{i} + (8 - 5t)\underline{j} + (7 + 4t)\underline{k}$.

(ii) Determine the angle between the two lines in (b) above.

16. (a) A plane is perpendicular to each of the planes $x + y + 3z = 0$, $3x - 2y + 4z = 0$ and passes through the points $(1,1,1)$. Find the equation of the plane.

(b) Given that the point C divides the line $\overline{AB}$ in the ratio 1:2 and the position vectors of A and C are $-4i - 3j + 5k$ and $3i - 2j + 12k$ respectively. Find the co-ordinates of point B.

17. (a) A line passes through the midpoint of $A(4,3,2)$ and $B(6,2,1)$. This line is parallel to the plane $6x + 3y + 9z = 11$. Find the equation of the line.

(b) A line passes through the point $P(1, -3, -4)$and is perpendicular to the plane $-4x + 3y + 6z = 0$. If this line meets another plane $6x + 8y + 4z + 11$. Find the point of intersection.

18. (a) L is the $\frac{x}{2} = \frac{y-1}{1} = \frac{z+1}{-1}$ and P is the plane $x - 4y - 2z = 5$. Show that L is parallel to P and calculate the distance between L and P.

(b) Find the Cartesian equation of a second plane Q which contains L and is perpendicular to P.

GEOMETRY

19. (a) Prove that the line $2x - 3y + 26 = 0$ is a tangent to the circle $x^2 + y^2 - 4x + 6y - 104 = 0$ and hence find the coordinates of the point of intersection.

(b) Prove that the circles $x^2 + y^2 - 6x - 12y + 40 = 0$ and $x^2 + y^2 - 4y = 16$ are orthogonal.

20. (a) A point P on the curve is given parametrically by $x = 3 - \cos\theta$ and $y = 2 + \sec\theta$. Find the:

(i) Equation of the normal to the curve at the point $\theta = \frac{\pi}{3}$.

(ii) Cartesian equation of the curve.

(b) Show that the curve $x = 5 - 6y + y^2$ represents a parabola, find the direction of x and sketch the curve.

21. (a) The tangent to the parabola $y^2 = 4x$ at the point P(1,2) meets the hyperbola $xy = 12$ in points A and B.

(i) Find the co-ordinates A and B.

(ii) Show that the acute angle between the tangents to the hyperbola at A and B is $\tan^{-1}\left(\frac{7}{24}\right)$.

(b) Show that the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$. Hence find the equations of tangents to the ellipse $4x^2 + 3y^2 = 1$ which are perpendicular to the line $3x - 4y + 5 = 0$.

ANALYSIS

22. (a) Sketch the curve $y = \frac{4+3x-x^2}{x-8}$, clearly find the nature of the turning points and state their asymptotes.

(b) (i) On the same axes, sketch the curve $y = x(x + 2)$ and $y = x(4 - x)$

(ii) Find the area enclosed by the two curves in (b) (i) above.

(iii) Determine the volume generated when the area enclosed by the two curves in (a) (i) above is rotated about the *x-axis*.

23. (a) Use the method of small changes to find the value of $\frac{1}{\sqrt{0.97}}$ correct to 3 dps.

(b) Evaluate $\int_0^1 \frac{8x-8}{(x+1)^2(x-3)^2}\,dx$.

(c) (i) Given that $f(x) = \frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$. Express $f(x)$ into partial fractions

(ii) Hence evaluate $\int_4^5 f(x)\,dx$.

(d) Evaluate $\int_0^{\frac{\pi}{2}} \frac{4}{1+\cos 2x}\,dx$.

24. (a) Evaluate $\int_1^2 (x-1)^2 \ln x\,dx$.

(b) Find Machaurin's expansion of $y = \ln\left(\frac{2-x}{1+x}\right)^2$

Showing the first three non zero terms. Hence find the approximate value of $2\ln\left(\frac{1.99}{1.01}\right)$ correct to 3s.f.

25. (a) If $x = 3(1 - cos\theta)$ and $y = \theta - sin\theta$. Find $\frac{d^2y}{dx^2}$ in its simplest form.

(b) Water is emptied from a cylindrical tank of radius 20cm at the rate of 2.5 litres s^{-1} and fresh water is added at the rate of 2 litres s^{-1}. Find the rate which the water level is changing.

26. (a) Solve the differential equation $(x-2)\frac{dy}{dx} = y - 1$ given $y(3) = 2$.

(b) The population of Mbarara municipality is rising annually at a rate which is proportional to the population at that instant. Given that the population of it at the beginning of 1980 was 8,000,000 and at the beginning of 2010 is 24,000,000. Calculate the expected population of the municipality at the beginning of 2040. P

27. (a) Solve the equation $e^x\frac{dy}{dx} - y = e^x y$ given that $y = 0$ when $x = 0$.

(b) The gradient function of a curve at any point (x, y) is $K\left(\frac{x}{y}\right)$ where K is a constant. The points $(3, -2)$ and $(0, 2\sqrt{2})$ lie on the curve. Show that the curve is an ellipse and determine its eccentricity.

28. (a) Show that $\int_0^1 \tan^{-1}x \;\; dx = \frac{1}{4}(\pi - ln4)$.

(b) (i) Differentiate with respect to x, $y = ln\left[e^x\left(\frac{x-2}{x+2}\right)\right]^{\frac{3}{4}}$

(ii) If $y = e^{-2mx}sin4mx$, show that $\frac{d^2y}{dx^2} + 4m\frac{dy}{dx} + 20m^2y = 0$.

29. (a) Differentiate from first principals $y = \sqrt{cosx}$.

(b) The gradient of the curve at the point $P(x, y)$ is $\frac{4-2xy}{x^2}$. The point A(3,2) lies on the curve.

Find the equation of the

(i) Normal to the curve at the point A.

(ii) Curve

30. Evaluate the following

(a) $\int_0^1 (\sin^{-1}x)^2\, dx$. correct to 4 decimal points.

(b) $\int_0^1 \frac{3-x}{(x+1)(x^2+1)}dx$.

(c) $\int \frac{5lnx}{x-4x(lnx)^2}dx$

END