MARKING SCHEME

1: (i) 0.7859

(ii) −1.0191

(iii) 2.6345

2. (i) $\frac{1+\tanh x}{1-\tanh x} = \cosh 2x + \sinh 2x$

Consider L.H.S

$\frac{1+\tanh x}{1-\tanh x}$

$$\frac{1+\frac{\sinh x}{\cosh x}}{1-\frac{\sinh x}{\cosh x}} = \frac{\cosh x+\sinh x}{\cosh x-\sinh x} \quad \text{(1 Mark)}$$

$$= \frac{(\cosh x+\sinh x)(\cosh x+\sinh x)}{(\cosh x+\sinh x)(\cosh x-\sinh x)}$$

$$= \frac{\cosh^2 x+\sinh^2 x+2\sinh x\cosh x}{\cosh^2 x-\sinh^2 x} \quad \text{(1 mark)}$$

But $\cosh^2 x+\sinh^2 x = \cosh 2x$, $2\sinh x\cosh x = \sinh 2x$ and $\cosh^2 x - \sinh^2 x = 1$

$$\therefore \text{LHS} = \frac{\cosh 2x+\sinh 2x}{1} = \cosh 2x + \sinh 2x \quad \text{(0.5 marks)}$$

(ii) Given $a\cosh x + b\sinh x = c$

Required to prove $a^2 = b^2 + c^2$

Proof: $a\cosh x + b\sinh x = c$

By definition $\cosh x = \frac{1}{2}(e^x+e^{-x})$ and $\sinh x = \frac{1}{2}(e^x-e^{-x})$ (0.5 mark)

$\frac{a}{2}(e^x+e^{-x})+\frac{b}{2}(e^x-e^{-x}) = c$ (0.5 marks)

$(a+b)e^x + (a-b)e^{-x} = 2c$

Multiply by e^x throughout gives

$(a+b)e^{2x} - 2ce^x + (a-b) = 0$ (0.5 marks)

For equal roots

$(2c)^2 = 4(a+b)(a-b)$

$4c^2 = 4(a^2-b^2)$

$c^2 = a^2-b^2$

$a^2 = b^2+c^2$ (0.5 marks)

(b) (i) Given: $5\cosh x + 3\sinh x$

Required its minimum value.

Let $y = 5\cosh x + 3\sinh x$ (0.5 mark)

$dy/dx = 5\sinh x + 3\cosh x$

For minimum value, $dy/dx = 0$

$0 = 5\sinh x + 3\cosh x$

$5\sinh x = -3\cosh x$

$\frac{\sinh x}{\cosh x} = -3/5$

$\tanh x = -3/5$ (0.5 mark)

$x = \tanh^{-1}(-3/5)$

$x = \frac{1}{2}\ln\left[\frac{1+(-3/5)}{1-(-3/5)}\right] = \frac{1}{2}\ln(2/8)$

$x = \frac{1}{2}\ln(1/4) = -\ln 2 = \ln 1/2$ (0.5 marks)

From $\frac{dy}{dx} = 5\sinh x + 3\cosh x$

$\frac{d^2y}{dx^2} = 5\cosh x + 3\sinh x$

where $x = \ln(1/2)$

$\frac{d^2y}{dx^2} = 5\cosh(\ln 0.5) + 3\sinh(\ln 0.5) = 4.$ (0.5 mark)

Hence x gives minimum value

Minimum value is

$\therefore y = 5\cosh(\ln 1/2) + 3\sinh(\ln 1/2) = 4.$ (0.5 marks)

2 (b) (ii) $\int \frac{\sinh 3x + \sinh 5x}{\cosh 3x - \cosh 5x} dx$

By factor formulae

$$\int \frac{\sinh 3x + \sinh 5x}{\cosh 3x - \cosh 5x} dx = \int \frac{2\sinh\left(\frac{3x+5x}{2}\right)\cosh\left(\frac{3x-5x}{2}\right)}{2\sinh\left(\frac{3x+5x}{2}\right)\sinh\left(\frac{3x-5x}{2}\right)} \quad \text{(1 marks)}$$

$$= -\int \frac{\cosh x}{\sinh x} dx \quad \text{(0.5 marks)}$$

$$= -\ln(\sinh x) + C \quad \text{(0.5 marks)}$$

$$\therefore \int \frac{\sinh 3x + \sinh 5x}{\cosh 3x - \cosh 5x} dx = \ln|\operatorname{cosech} x| + C \quad \text{(0.5 marks)}$$

3.

Chemical type	Nitrogen	Phosphorus acid	Cost	
				(1 marks)
A	0.1	0.06	2000	
B	0.05	0.1	3000	
Total	14	14		

Let: x be kilograms of fertilizer type A used and y be that of type B. (1 marks)

Constraints:

$0.1x + 0.05y \geq 14$ or $10x + 5y \geq 1400$ (1.5 marks)

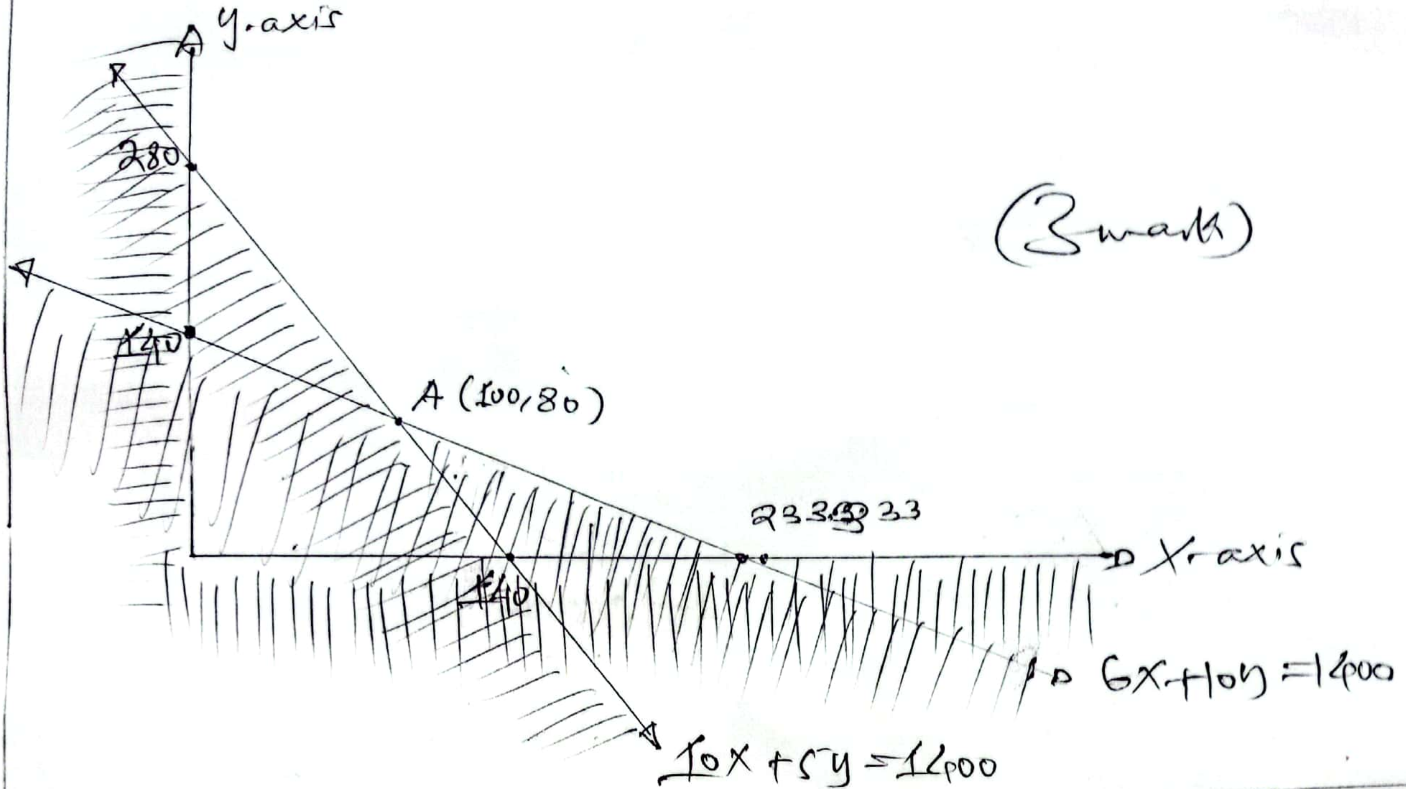
$0.06x + 0.1y \geq 14$ or $6x + 10y \geq 1400$

$x \geq 0, y \geq 0$

objective function:

$f(x,y) = 2000x + 3000y$ (0.5 marks)

3.



(3 marks)

Corner points	Value of objective function $f(x,y)=2000x+3000y$
A (100, 80)	440,000
B (233.33, 0)	466,660
C (0, 280)	840,000

(2 marks)

To meet the requirement of minimum cost, the farmer should use 100kg of fertilizer type A and 80 kg of fertilizer type B (1 mark)

4. (a) i/ Distribution table.

Class interval	Frequency	Class mark (x)
3-7	1	5
8-12	5	10
13-17	8	15
18-22	12	20
23-27	7	25
28-32	3	30
33-37	4	35
	40	

4 (b) Given: Lower Quartile = 40 and number of children $N = 680$

Fraction of students measured less or equal to $40 = \frac{1}{4}$

Fraction of students measured more than $40 = \frac{3}{4}$

$\therefore$ Number of students measured more than 40 inches $= \frac{3}{4} \times 680 = \underline{510}$.

5 (a) (i) $[(A \cup B) \cup (A - B)]'$

$= [(A \cup B) \cup (A \cap B')]'$

$= [(A \cup (A \cap B'))] \cup [B \cup (A \cap B')]'$ Distributive law

$= [A \cup (A \cup B)]'$ Absorption law

$= (A \cup B)'$ Associative and Idempotent laws

$= A' \cap B'$ ~~Dem~~ De Morgans law (02 mark)

(ii) $B - (A' - B)$

$= B - (A' \cap B')$ Definition

$= B - (A \cup B)'$ De morgan's law

$= B \cap [(A \cup B)']'$ Definition

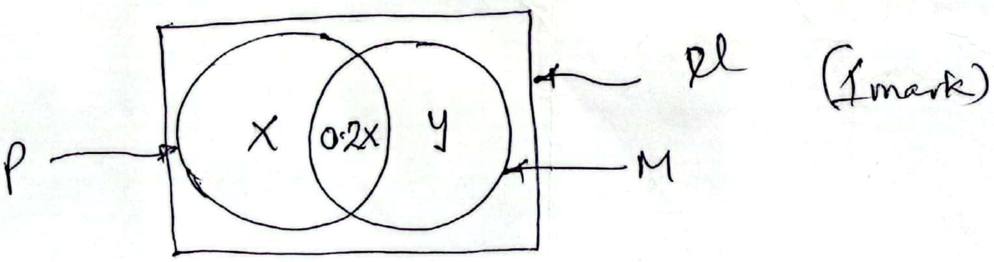
$= B \cap (A \cup B)$ Compliment law

$= B \cap (B \cup A)$ Commutative law (02 mark)

$= B$ Absorption law.

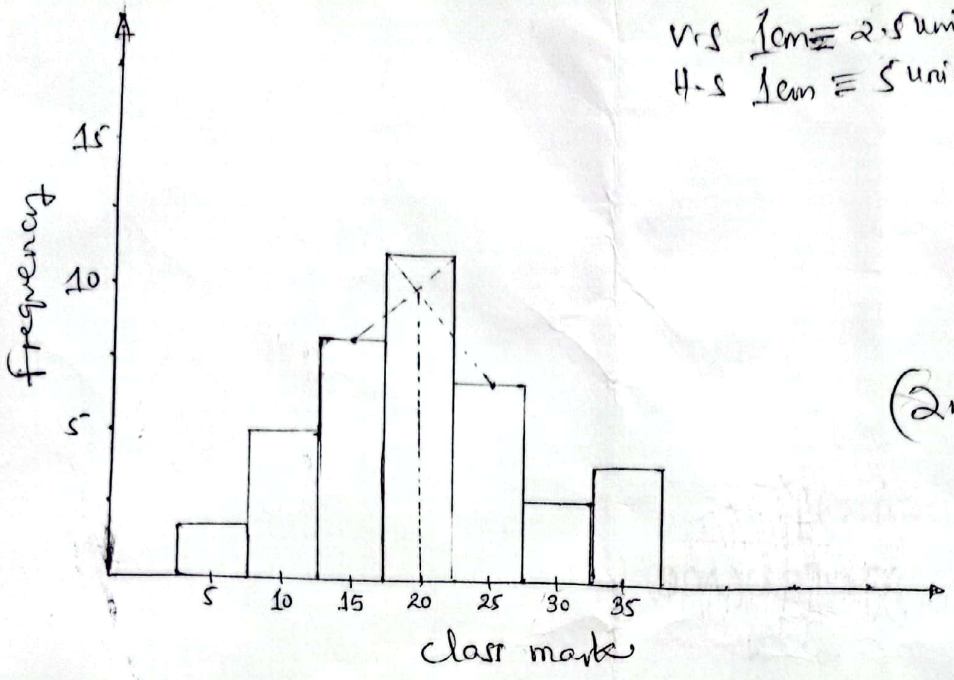
(b) Let: P be a set ~~of~~ for students taking Physics
M be a set of students participating in mathematics

Let x be the number of students taking physics only. (1 mark)



4. HISTOGRAM

V.S 1cm ≡ 2.5 units
H.S 1cm ≡ 5 units.



(2 marks)

From the histogram the mode = 15 + (0.95 × 5) = 19.75 (1 mark)

(ii) Percentage of scores less or equal to 7

$= \frac{8+5+11}{40} \times 100\% = 55\%$ (2 marks)

(iii) Variance and standard deviation.

class interval	frequency	class mark(x)	$x-\bar{x}$	$f(x-\bar{x})^2$	$f(x)$
3-7	1	5	15.5	240.25	5
8-12	5	10	10.5	551.25	50
13-17	8	15	-5.5	242.00	120
18-22	12	20	-0.5	3.00	240
23-27	7	25	4.5	141.75	175
28-32	3	30	9.5	270.75	90
33-37	4	35	14.5	841.00	140
	40			2290.00	820

Mean $\bar{x} = \frac{820}{40} = 20.5$ (0.5 marks)

Variance $= \frac{\Sigma f(x-\bar{x})^2}{\Sigma f} = \frac{2290}{40} = 57.25$ (0.5 marks)

Standard deviation $= \sqrt{\text{Variance}} = \sqrt{57.25} = 7.57$ (0.5 marks)

5 Given $n = (\xi) = 72$, $y = 5(x + 0.2x)$

Now $x + 0.2x + y = 72$

$x = 10$ (1 mark)

(i) 12 Students are taking physics (1 mark)

(ii) 62 students are taking mathematics (1 mark)

(iii) 2 students are taking both papers (1 mark)

6 (a) (i) Given $f(x) = ax + b$, $g(x) = cx + d$

Required to find conditions c and d in terms of a and b for $f \circ g = g \circ f$

First: $f \circ g = f[g(x)] = a(cx + d) + b = acx + ad + b$ (0.5 mark)

Second: $g \circ f = g\{f(x)\} = c(ax + b) + d = acx + cb + d$ (0.5 mark)

But: $f \circ g = g \circ f$

$acx + ad + b = acx + cb + d$

$ad + b = cb + d$

$\frac{d}{c-1} = \frac{b}{a-1}$ (1 mark)

(ii) Given: $x^4 - 2x^3 + 3x^2 + ax + b$

Required value of a and b for the equation is a perfect square

For perfect square

$[x^2 + nx + m]^2 = x^4 - 2x^3 + 3x^2 + ax + b$

$x^4 + 2nx^3 + (2m + n^2)x^2 + 2mnx + m^2 = x^4 - 2x^3 + 3x^2 + ax + b$ (0.5 marks)

Comparing we have

$2n = -2$ and so $n = -1$ (0.5 marks)

$2m + n^2 = 3$

$2m + 1 = 3$

$2m = 2$

$m = 1$

$2mn = a$

$a = 2(1)(-1) = -2$ (0.5 marks)

$b = m^2 = 1$

Hence, values of $a = -2$ and $b = 1$ (0.5 marks)

6 (b) Given $f(x) = \frac{x^3}{x^2-5}$

(i) Vertical asymptotes; $x^2 - 5 = 0$
$x = \pm\sqrt{5}$ (1 mark)

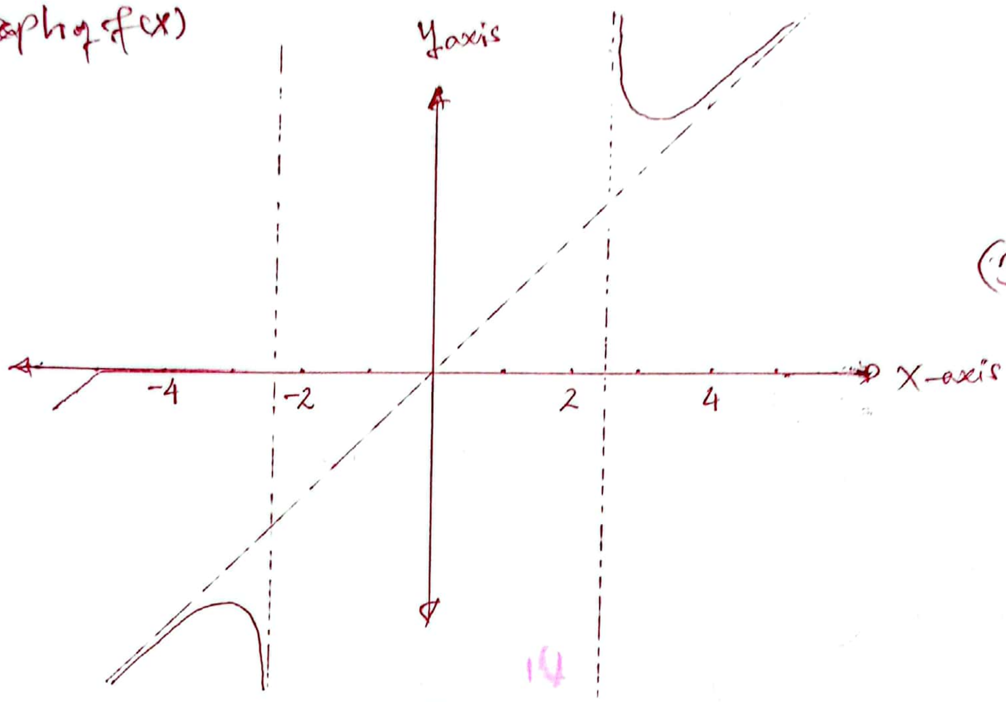
Oblique asymptotes

$$y = \frac{x^3}{x^2-5} = x + \frac{5}{x^2-5} = x + \frac{5/x^2}{1 + 5/x^2}$$ (1 mark)

as $x \to \infty$

$y = x$ (1 mark)

Graph of f(x)



(3 marks)

(ii) Domain $= \{x : x \in R; x \neq \pm\sqrt{5}\}$ (0.5 marks)

Range $= \{y : y \in R\}$ (0.5 marks)

7. Given $\frac{x^2}{1} + \frac{y^2}{(0.622)^2} = 1$

Compare with : $\cos^2\theta + \sin^2\theta = 1$
$\cos\theta = x$ and $y = 0.622\sin\theta$

Length of the arc is given by

$$L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{(-\sin\theta)^2 + (0.622\cos\theta)^2}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{1 - \cos^2\theta + 0.386884\cos^2\theta}\, d\theta$$

7. Hence, $f(\theta) = \sqrt{1 - 0.613116\cos^2\theta}$, $a = 0$, $b = \pi/2$, $n = 7 - 1 = 6$

Width of interval, $h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \pi/12$ (0.5 marks)

Table of values.

n	θ_n	$f(\theta_n)$	$f(\theta_0)+f(\theta_6)$	odd ordinates	Even ordinates
0	0	0.6220	0.6220		
1	$\pi/12$	0.65418		0.65420	
2	$2\pi/12$	0.73496			0.73496
3	$3\pi/12$	0.83273		0.83273	
4	$4\pi/12$	0.92017			0.92017
5	$5\pi/12$	0.97925		0.97925	
6	$6\pi/12$	1.00000	1.00000		
	TOTAL		1.620	2.46618	1.65513

(0.5 marks)

By Simpson's rule (0.5 marks)

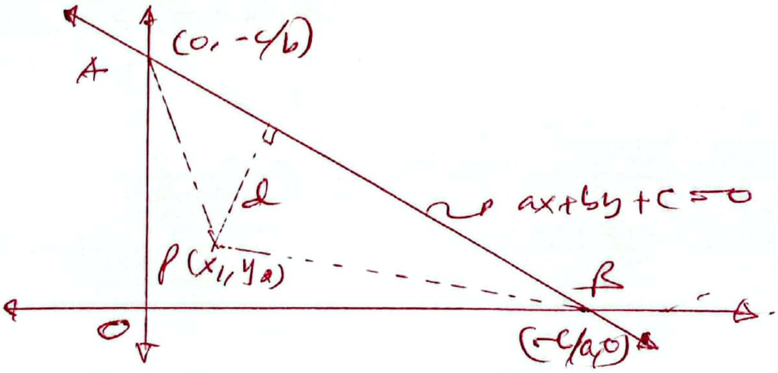
$$\int_0^{\pi/2} f(\theta)\,d\theta = \frac{h}{3}\left[f(\theta_0) + f(\theta_6) + 4\sum \text{odd ordinates} + 2\sum \text{even ordinates}\right]$$

$$= \frac{\pi}{36}\left[1.622 + (4 \times 2.46618) + (2 \times 1.65573)\right] \quad 0.5\text{ marks}$$

$$= 1.2913 \text{ Units.}$$

8 (a) (i) Required to prove, $d = \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}$

Refer the sketch below



(1 marks)

Area of triangle APB

$$A = \frac{1}{2} b d = \frac{1}{2}\sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}} \times d$$

$$A = \frac{c}{2ab}\sqrt{a^2+b^2} \times d \quad \text{--- (i)} \quad (1\text{ mark})$$

Also $A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} -c/a & 0 & 1 \\ 0 & -c/b & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{c}{ab} + \frac{y_1 c}{a} + \frac{x_1 c}{b}\right]$$

$$= \frac{c}{2ab}\left[c + by_1 + ax_1\right] \quad \text{--- (ii)} \quad (2\text{ marks})$$

Equating eqns (i) and (ii) we have

$$\sqrt{a^2+b^2} \times d = c + by_1 + ax_1$$

$$d = \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \quad (1\text{ mark})$$

(ii) Given: $(x_1, y_1) = (3, 2)$ and line $3x - 4y + 4 = 0$

shortest distance is

$$d = \left|\frac{3x - 4y + 4}{\sqrt{3^2 + 4^2}}\right|_{(x,y) = (3,2)}$$

$$d = \left|\frac{3(3) - 4(2) + 4}{5}\right|$$

$$d = 1 \text{ unit} \quad (1.5\text{ marks})$$

(b)(i) Given: $A(3,1)$, $B(0,6)$, $C(-5,3)$

Distance $AB = \sqrt{(3-0)^2 + (1-6)^2} = \sqrt{34}$ units

Distance $BC = \sqrt{(0+5)^2 + (6-3)^2} = \sqrt{34}$ units

Distance $AC = \sqrt{(3+5)^2 + (1-3)^2} = \sqrt{68}$ units

Since $AB = BC$, the triangle is isosceles.

8. (b) (ii) Given points $A(2,3)$ $(-1,6)$ and $P(x,y)$

$AP = PB$

$$\sqrt{(x-2)^2+(y-3)^2} = \sqrt{(x+1)^2+(y-6)^2}$$

$x^2-4x+4+y^2-6y+9 = x^2+2x+1+y^2-12y+36$ (2marks)

Hence the locus of point P is $6x-6y+23=0$.

9. (a) (i) Required to show that $\int \sec x\,dx = \ln|\sec x+\tan x|+C$

Consider LHS

$$=\int \sec x\,dx = \int \frac{\sec x[\sec x+\tan x]}{[\sec x+\tan x]}\,dx \quad (0.5\text{marks})$$

$$= \int \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\,dx \quad (0.5\text{mark})$$

Let $t = \sec x+\tan x$ so that $dt = (\sec^2 x+\sec x\tan x)dx$ (0.5m

$$= \int \frac{1}{t}\,dt = \ln t + C$$

$$\therefore \int \sec x\,dx = \ln|\sec x+\tan x|+C \quad (0.5\text{mark})$$

(ii) $\int \tan^2 x\,dx$

From $\sec^2 x = 1+\tan^2 x \Rightarrow \tan^2 x = \sec^2 x - 1$ (0.5mark)

$$\int \tan^2 x\,dx = \int(\sec^2 x - 1)\,dx \quad 0.5\text{mark})$$

$$= \int \sec^2 x\,dx - \int dx$$

$$= \tan x - x + C \quad (01\text{mark})$$

(b) (i) Given $\int \frac{4x+2}{x^2+x+5}\,dx$

$$\int \frac{4x+2}{x^2+x+5}\,dx = 2\int\frac{2x+1}{x^2+x+5}\,dx \quad (01\text{mark})$$

Let $t = x^2+x+5$ so that $dt = (2x+1)dx$

$$\int \frac{4x+2}{x^2+x+5}\,dx = 2\int \frac{1}{t}\,dt = 2\ln t + C$$

$$= 2\ln(x^2+x+5)+C \quad (01\text{mark})$$

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9 (b) (i) $\int \frac{4x+2}{x^2+x+5}dx = 2\ln(x^2+x+5)+C$ (01 marks)

(ii) Given $\int \frac{2x^2+x+1}{(x+1)(x^2+1)}dx$

Partialize the function (01 marks)

$$\frac{2x^2+x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$2x^2+x+1 = A(x^2+1)+(Bx+C)(x-1)$$

If $x=1$, $4=2A$, so $A=\frac{1}{2}$

If $x=0$, $1=A-C$, so $C=-\frac{1}{2}$

If $x=-1$, $2=2A+2B-2C$, so, $B=0$ (0.5 marks)

$$\therefore \frac{2x^2+x+1}{(x-1)(x^2+1)} = \frac{1}{2}\left(\frac{1}{x-1}\right) - \frac{1}{2}\left(\frac{1}{x^2+1}\right)$$

$$\int \frac{2x^2+x+1}{(x-1)(x^2+1)}dx = \frac{1}{2}\int\frac{1}{x-1}dx - \frac{1}{2}\int\frac{1}{x^2+1}dx \quad (0.5\text{ marks})$$

$$= \frac{1}{2}\ln(x-1) - \frac{1}{2}\tan^{-1}x + C$$

$$\therefore \int \frac{2x^2+x+1}{(x-1)(x^2+1)}dx = \frac{1}{2}\ln(x-1) - \frac{1}{2}\tan^{-1}x + C \quad (1\text{ marks})$$

10 (a) Given $z=\sqrt{x^2+y^2}$

(i) $\frac{dz}{dx} = \frac{d}{dx}(x^2+y^2)^{1/2} = \frac{1}{2}(2x)(x^2+y^2)^{-1/2}$ (3 marks)

$$\frac{dz}{dx} = \frac{x}{\sqrt{x^2+y^2}}.$$

(ii) $\frac{dz}{dy} = \frac{d(x^2+y^2)^{1/2}}{dy} = \frac{1}{2}(2y)(x^2+y^2)^{-1/2}$ (2 marks)

$$\therefore \frac{dz}{dy} = \frac{y}{\sqrt{x^2+y^2}}$$

10 (iii) Total derivative of z

$$dz = \frac{dz}{dx} \cdot dx + \frac{dz}{dy} \cdot dy$$ (4 marks)

$$\therefore dz = \frac{x}{\sqrt{x^2+y^2}} dx + \frac{y}{\sqrt{x^2+y^2}} dy$$

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