FURTHER DIFFERENTIATION

Implicit Differentiation

Up to the present we have dealt only with explicit functions of x, e.g. $y = x^2 - 5x + 4/x$. Here y is given as an expression in x. If, however, y is given implicitly by an equation such as $x = y^4 - y - 1$, we cannot express y in terms of x.

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}(1+x)^n = n(1+x)^{n-1} \cdot \frac{d(1+x)}{dx}$$

$$\frac{d}{dx}y^n = ny^{n-1} \cdot \frac{dy}{dx}$$

Example

Find the gradient of the curve $x^2 + 2xy - 2y^2 + x = 2$ at the point (-4,1).

Ans.=$-\frac{5}{12}$

$$\frac{d}{dx}\left(x^2 + 2xy - 2y^2 + x\right) = 2$$

$$2x + 2\frac{d}{dx}(xy) - 2\frac{d}{dx}(y^2) + \frac{d}{dx}(x) = 2$$

$$2x + 2\left(x\frac{dy}{dx} + y\right) - 2\cdot 2y\cdot\frac{dy}{dx} + 1 = 0$$

$$2x + 2x\frac{dy}{dx} + 2y - 4y\frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx} = \frac{-1 - 2x - 2y}{2x - 4y}$$

Substituting for $(-4,1)$

$$\left.\frac{dy}{dx}\right|_{x=-4} = \frac{-1 - 2(-4) - 2}{2 - 4(-4)}$$

$$= \frac{-5}{12}$$

Higher Derivatives (Second derivative)

Considering displacement ,s , velocity, v, and acceleration, a.

$v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$ $\therefore$ $a = \frac{d^2s}{dt^2}$

$f'(x) = \frac{dy}{dx}$, $f''(x) = \frac{d^2y}{dx^2}$

d two y by d x squared

Example

If $x = a(t^2 - 1)$, $y = 2a(t + 1)$, find $\frac{dy}{dx}$ and $\frac{d^2x}{dx^2}$ in terms of t.

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{dx}{dt} = \frac{d}{dt}\left(a(t^2 - 1)\right) = a\left(\frac{d}{dt}t^2 + \frac{d}{dt}1\right)$$

$$= 2at$$

$$\frac{dy}{dt} = 2a \qquad \therefore \frac{dy}{dx} = 2a \cdot \frac{1}{2at}$$

$$= \frac{1}{t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{1}{t}\right) \cdot \frac{1}{2at}$$

$$= \frac{-1}{t^2} \cdot \frac{1}{2at}$$

$$= -\frac{1}{2at^3}$$

SMALL CHANGES

$$\frac{\Delta y}{\Delta x} \simeq \frac{dy}{dx} \text{ as } \Delta x \to 0$$

$$\Delta y \simeq \frac{dy}{dx} \cdot \Delta x \text{ as } \Delta x \to 0$$

Examples

The side of a square is 5 cm. Find the increase in the area of the square when the side expands 0.01 cm.

Find the approximation for $\sqrt{9.01}$.

let the side be x

$A = x^2$

$\frac{dA}{dx} = 2x$

$x = 5\,\text{cm}$

$\Delta x = 0.01$

$\frac{\Delta A}{\Delta x} \simeq \frac{dA}{\Delta x}$ as $\Delta x \to 0$

$\frac{\Delta A}{\Delta x} \simeq 2x$

$\Delta A \simeq 2x \cdot \Delta x$

$\simeq 2(5)(0.01)$

$= 0.1\,\text{cm}^2$

let $y = \sqrt{x}$

$y + \Delta y = \sqrt{x + \Delta x}$

$x = 9$

$\Delta x = 0.01$

$\frac{dy}{dx} = \frac{d}{dx} x^{\frac{1}{2}}$

$= \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

$\frac{\Delta y}{\Delta x} \simeq \frac{dy}{dx}$ $\Delta x \to 0$

$\therefore \Delta y \simeq \frac{1}{2\sqrt{x}} \cdot \Delta x$

$\simeq \frac{1}{2\sqrt{9}} \cdot 0.01$

$= \frac{1}{600}$

$\sqrt{9.01} = y + \Delta y \simeq \sqrt{9} + \frac{1}{600}$

$= 3 + 0.00167$

THANKS FOR BEING WITH US