

P425/2

APPLIED MATHEMATICS

Paper 2

3 hours



MMM JOINT MOCK EXAMINATIONS BOARD

Uganda Advanced Certificate of Education.

APPLIED MATHEMATICS

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3 hours

INSTRUCTIONS TO CANDIDATES:

*Attempt **all** the **eight** questions in section **A** and **not** more than **five** from section **B**.*

*Any additional question(s) answered will **not** be marked.*

*All necessary working **must** be done on the same page as the rest of answers.*

Mathematical table with list of formulae and squared papers are provided.

Silent, non programmable scientific calculators may be used.

For numerical work assume $g = 9.8ms^{-2}$

SECTION A

1. Events A and B are in a sample space such that $p(A)=0.39$; $p(A \cup B)=\frac{3}{4}$ and

$$p\left(\frac{A}{B}\right)=\frac{17}{53}.$$

Find; (i) $p(B)$

ii) $p(\bar{A} \cup \bar{B})$

(5 marks)

2. A body of mass 4kg moves with a velocity of $V=e^t \hat{i} + 2e^{2t} \hat{j} - \sin t \hat{k}$. Find the power developed after 4 seconds. (5 marks)

3. Using the trapezoidal rule with eight ordinates approximate the integral.

$$\int_1^8 \log_{10}(e^{3x}) dx \text{ to 3 significant figures.} \quad (5 \text{ marks})$$

4. A random variable, X has a probability distribution

$$p(X = x) = \begin{cases} \binom{4}{x}/k_1 & x = 0, 1, 2, 3, 4 \\ 0 & \text{else where} \end{cases}$$

Find; i) Value of k

ii) $E(5X - 7)$

(5 marks)

5. Show that the equation $\pi \sin x - x = 0$ has a root between $\frac{\pi}{2}$ and π . Hence use linear interpolation once to estimate the root to three decimal places. (5 marks)

6. A body of mass 2kg and velocity $(5\hat{i} + 7\hat{j})\text{ms}^{-1}$ collides with another body of mass 3kg and velocity $(2\hat{i} - 3\hat{j})\text{ms}^{-1}$. If the bodies coalesce after impact,

Find; i) their speed after impact.

ii) The loss in kinetic energy.

(5 marks)

7. Points A, B and C lie on a straight road such that $\overline{AB} = 500\text{m}$ and $\overline{BC} = 600\text{m}$. A car travelling along the road takes 20 seconds and 35 seconds travel distances \overline{AB} and \overline{BC} respectively.

Determine; i) the initial velocity and retardation.

ii) the distance it covers before coming to rest.

(5 marks)

8. A random variable, X is binomially distributed such that $X \sim B(50, p)$ and has mean 20.

Find; i) Value of Var (x)

ii) $p(X \geq 25)$

(5 marks)

SECTION B

9. A particle P has a constant speed of 6ms^{-1} in the direction $\hat{i} + 2\hat{j} - 2\hat{k}$ and particle Q has a constant speed of 7ms^{-1} in the direction $2\hat{i} + 3\hat{j} - 6\hat{k}$. Initially the particles were at points with position vectors $(\hat{i} - \hat{j} + 2\hat{k})\text{m}$ and $(\hat{i} + 5\hat{j} + 4\hat{k})\text{m}$ respectively.

a) Find; i) The velocity of P relative to Q

ii) The position vector of P relative to Q at any time, t.

(5 marks)

b) i) Show that in the subsequent motion, the least distance between P and Q is $2\sqrt{5}$ m.

ii) Find the time at which the particles are closest together.

(7 marks)

10. a) The table below shows scores obtained in two mathematics test papers marked out of 40.

Paper 1, x	27	37	31	34	29	35	31	37
Paper 2, y	23	28	30	32	27	24	30	31

Calculate the correlation coefficient based on ranks and comment based at 5%.

(5 marks)

- b) The table below shows the time spent on phone each day by a group of people.

Time (min)	0-< 5	5-< 10	10-< 20	20-< 30	30-< 40	40-< 70
No. of people	11	12	32	18	10	6

i) Draw a histogram and estimate the mode

ii) Calculate the mean.

(7 marks)

11. a) Given the $x = 52.426$, $y = 37.3$ and $Z = 12.93$ are rounded off to the given decimal places, find the limits within which the exact value of $\frac{x}{z-y}$ lies. (5 marks)

b) Show that the maximum absolute relative error in the volume, V of a cone of radius r and height h given as.

$$\left| \frac{\delta v}{v} \right| = \left| \frac{\delta h}{h} \right| + 2 \left| \frac{\delta r}{r} \right|$$

Hence find the percentage error in volume given that $r = 28.5$ and $h = 12.31$

(7 marks)

12. a) An elastic string is fixed at one end horizontally. When a force of 4N is applied to the other end, it extends by 0.2m. When the string hangs vertically supporting a mass of 1kg at the free end; the string is of length 2.49 m.

Find its natural length and the modulus of elasticity.

(6 marks)

- b) A particle describing simple harmonic motion in a straight line has a velocity of 6 ms^{-1} an acceleration of 48 ms^{-2} , when 3m from point O, the centre of oscillation.

Find;

i) the period and amplitude of motion

ii) The time taken to reach point O.

(6 marks)

13. A continuous random variable, X has a probability distribution function given by

$$f(x) = \begin{cases} \frac{2}{13}(x+1) & 0 \leq x \leq a \\ \frac{2}{13}(5-x) & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

a) Determine the values of a and b

b) Find the value of E(X)

c) Obtain the expression for F(X) and sketch it.

(12 marks)

14. a) Show that the root of the equation $e^{2x} = 4x+2$ lies between 0 and 1.

(3 marks)

- b) Derive the iterative formula based on Newton-Raphson method for solving the equation in (a) above.

(3 marks)

- c) Construct a flow chart based on NRM in (b) above to find the root of the equation in (a) above that;

i) Reads the initial approximation X_0 .

ii) Computes and prints the root to 2 decimal places.

(3 marks)

- d) Perform a dry run of the flow chart.

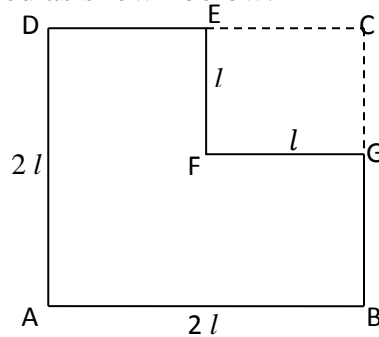
(3 marks)

15. a) Three particles of masses 3kg, 5kg and $\lambda \text{ kg}$ are placed at points with coordinates (4, 0), (0, -3) and (4, 2) respectively. If the centre of mass of the system is (2, β),

find the values of λ and β .

(4 marks)

- b) A square lamina ABCD of side $2l$ from a uniform thin material has a square CEFG of length l removed as shown below.



- i) Find the position of centre of gravity of the remainder from AB and AD.
ii) If the lamina is suspended from B, find the angle AB makes with the vertical.

(8 marks)

16. a) A random sample of size 120 is taken from a normally distributed population with mean 16.5 and variance 18. Determine the 96% confidence interval for the population mean.

(6 marks)

- b) On a highway, the speed of buses follows a normal distribution. It was observed by traffic police that 20% of buses travelled at a speed less than 100kmh^{-1} and 68% of the buses have speed between 100kmh^{-1} and 110kmh^{-1} .

Find the mean and standard deviation.

(6 marks)

END