PART ONE: PURE MATHEMATICS.

1. Three traders A, B and C bought stationary from a certain outlet in town. Trader A bought 10 dozen of Pencils, 8 dozen of Counter books and 2 dozen of rulers,
   trader B bought 10 dozen of counter books and 5 dozen of rulers and trader C bought 9 dozen of pencils and 6 dozen of rulers. The price of a dozen of pencils, Counter books and Rulers is UGX. 8,500, UGX. 96,000 and UGX. 24,000 respectively at the outlet. The traders sell a dozen of pencils, counter books and rulers at UGX.12,000, UGX. 108,000 and UGX. 30,000 respectively.
   a) Write down a,
      (i) 3 x 3 matrix for the items.
      (ii) 3 x 2 matrix for the prices.
   b) Use matrix multiplication to determine the profit made by each trader.

2. (a) Given that $M = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}$, $N = \begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix}$ and $H = \begin{pmatrix} 3 & 6 \\ 2 & 1 \end{pmatrix}$, find the determinant of MNH.

   (b) Determine the inverse of the matrix $\begin{pmatrix} -5 & 3 \\ 1 & -1 \end{pmatrix}$. Hence, solve the simultaneous equations below;

$$-5x + 3y = 2 \text{------}(1)$$
$$x - y = 4 \text{------}(2)$$

   (c) Given that $A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$ and $C = \begin{pmatrix} b & c \\ 0 & d \end{pmatrix}$. Find the value of a, b, c and d such that $A = BC$.

3. If $\boldsymbol{OA} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$, $\boldsymbol{OB} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$ and $\boldsymbol{OC} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$.

(a) Find the vectors;

(i) BC.

(ii) AB.

(b) Show that the vectors $\boldsymbol{AB}$ and $\boldsymbol{BC}$ are perpendicular.

(c) Determine the magnitude of the vector $2\boldsymbol{BC} - 3\boldsymbol{AB}$.

4. Given the points, P (2, -4), Q (-1, 0) and R (-6, 12). Determine the;

(i) Vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$.

(ii) Angle PQR.

(iii) Value of s and t for which $s(\overrightarrow{QP}) + t(\overrightarrow{QR}) = \boldsymbol{i} + 4\boldsymbol{j}$.

5. (a) Without using tables or calculators, evaluate $\log_2 32 \times \log_5 625$.

(b) Given that, $u = \log\left(\frac{5}{y}\right)$ and $v = \log\ (8y^3)$. Find the value of $3u + v$.

(c) Solve for x given that $1 + \log_2 x = 12 \log_x 2$.

6. (a) Solve the simultaneous equations;

$$2^x + 5^{y-1} = 13 \ldots\ldots\ldots\ldots \text{(i)}$$

$$2^{x+1} + 5^y = 41 \ldots\ldots\ldots\ldots \text{(ii)}$$

(b) Solve for x in the equation, $4^{x-1} - 2^{x+1} + 3 = 0$

7. (a) Without using a calculator or tables, evaluate;

$$\frac{6\sqrt{10} + 2\sqrt{40}}{\sqrt{2} \times \sqrt{20}}$$

(b) Express $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$ in the form $\boldsymbol{a} + \boldsymbol{b}\sqrt{\boldsymbol{c}}$. State the value of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.

(c) (i) Express $\sqrt{147} - \sqrt{75}$ in the simplest surd form.

(ii) Rationalise, $\frac{5+2\sqrt{3}}{2+\sqrt{3}}$.

8. The roots of a quadratic equation, $2x^2 - 5x + 3 = 0$ is α and β.

(a) Determine the value of;

(i) $\alpha^2 + \beta^2$

(ii) $\alpha - \beta$

(b) Find the equation whose roots are $(\alpha^2 + \beta^2)$ and $(\alpha - \beta)$.

9. (a) Find the equation whose roots are -3 and 4.

(b) Solve the equation $x^2 + 4x - 5 = 0$.

(c) The roots of the equation $4x^2 + 9x - k = 0$ are α and 2. Find the value of α and k.

10. (a) In an arithmetic progression (AP), the ninth term is seven times the first term and the sum of the first four terms is 34. Calculate the first term and the common difference.

(b) The sum of the first 15 terms of an arithmetic progression (A.P) is 615. The 13th term is six terms the 2nd term. Determine the;

(i) First term of the A.P.

(ii) Common difference of the A.P.

(c) An employee decided to make monthly savings of his salary by starting with Ugx. 60,000 from January 2017. He constantly increased the savings every month by Ugx. 5,000. Find the;

(i) Amount saved in August, 2018.

(ii) Total of all the savings at the end of August, 2018.

11. (a) Determine the three numbers in a geometric progression such that their sum is 14 and their product is 64.

(b) Determine the sum of the multiples of 9 between 40 and 200.

(c) The sum to infinity of a geometrical progression (GP) is $\frac{25}{4}$ and the first term is 5. Find the;

(i) Common ratio of the G.P.

(ii) Sum of the first ten terms of the G.P.

12. (a) The letters of the word COMMITTEE were arranged to form a password. Find the;

    (i) Total number of possible passwords formed.

    (ii) Number of passwords with O and I put together.

    (b) The letters of the word E.D.U.C.A.T.I.O.N. were arranged to form passcodes. Determine the number of passcodes generated that started with a vowel.

13. (a) A team of 10 members is to be selected from a group of 5 boys and 7 girls to represent the school at the national school quiz championship. Determine the;

    (i) Total number of possible teams formed.

    (ii) Number of teams where the number of boys is equal to that of the girls.

    (iii) Number of teams with at least a girl.

    (b) Given that a panel of four judges is to be selected from the subject heads of department of Mathematics, Physics, Chemistry, Biology, History, Geography, Religious studies, Languages, Business studies and Fine Art to facilitate the national school quiz championship. Find the;

    (i) Total number of panels formed.

    (ii) Number of panels with more judges from sciences than Arts.

    (iii) Number of panels formed where a Judge from languages must be included.

14. A bakery produces x – chocolate cakes and y – vanilla cakes for sale at a unit price of UGX 3500 and UGX 2000, respectively. A chocolate cake requires 2 cups of sugar and 1 cup of flour, while a vanilla cake requires 1 cup of sugar and 1.5 cups of flour. The bakery has 40 cups of sugar and 30 cups of flour available. Additionally, the oven used at the bakery cannot bake more than 20 cakes in total.

(a) Write down the inequalities from this information.

(b) On the same axes, show the region that satisfies the inequalities in (a) above.

(c) Determine the quantity of each type of cake the bakery must produce to maximize its sales, and state the maximum amount of money earned from these sales.

15. A company produces two types of devices, A and B. Each device requires a certain amount of raw material and labour. The company has a limited supply of raw material and labour hours available. The company also has a fixed cost of UGX 5,000,000 per month. The unit costs and revenues for each type of device are given in the table below.

Device	Raw material (kg)	Labour(hours)	Unit cost (UGX)
A	2	3	8,000
B	4	2	10,000

The maximum allowable raw material usage is 120 kg. Labor hours must not exceed 100 hours in total. There is a minimum monthly demand for 10 units of device A and 5 units of device B.

(a) Write down the four inequalities for this data.

(b) On the same axes, show the region that satisfies the inequalities in (a) above.

(c) Find the quantity of each type of device that can be produced at the lowest cost and specify the minimum cost.

16. Residents of an area need x – buses and y – taxis to start up a park in their neighbourhood with not more than 30 vehicles altogether. Initially, they need at least five buses and ten taxis in their park. A bus requires 3 units of parking space while a taxi requires 1 unit. The park has 54 units of parking space available.

(a) Write down four inequalities representing the given information.

(b) On the same axes, draw a graph showing the region satisfying the inequalities formed in (a) above.

(c) Given that a bus is charged UGX 15,000 and a taxi UGX 8,000 determine the number of vehicles of each type that gives the highest revenue. State the maximum amount earned.

17. The rate at which the number, x, of mice increases in a habitat is inversely proportional to their number at any time, t (in months). Initially, there were only two mice in the habitat. After one month, there were eight mice.
    (a) Write down the differential equation for this data.
    (b) Solve the differential equation formed in (a) above.
    (c) Determine the number of mice after 3 months.

18. The Area, $\boldsymbol{A}$, mm^2 of iron sheets rusts at a rate that is directly proportional to the rusted area at any time $\boldsymbol{t}$ days. Initially, $\boldsymbol{A} = 50\ mm^2$ and after 2 days, $\boldsymbol{A} = 75$ mm^2.
    (a) Write down the differential equation for this information.
    (b) Solve the differential equation in (a) above.
    (c) Determine the rusted Area when $\mathrm{t} = 5$ days.

19. (a) Sketch the curve $y = x^2 + 4x - 5$.
    (b) Determine the area bound by the curve and the $X -$ axis.

20. Given that the curve, $y = a - bx - x^2$ has a turning point at $T(1, 16)$ and intersects the X - axis at points P and Q.
    (a) Determine the;
        (i) Value of a and b.
        (ii) Coordinates of the points P and Q.
        (iii) Nature of the turning point at $T(1, 16)$.
    (b) Sketch the curve.
    (c) Find the area enclosed by the curve and the $X -$ axis.

PART II: STATISTICS

1. The age of students in a class was recorded as follows; 16, 18, 17, 19, 16, 17, 20, 18, 19, 17, 20, 17, 18, 21, 19, 17, 18, 16, 20 and 17.

   (a) Determine the;

   (i) Mode and its frequency. 18 appeared many times

   (ii) Median.

   (b) Draw an ungrouped frequency table for the data and use it to find;

   (i) Mean.

   (ii) Variance.

   (iii) Standard deviation.

2. The weight (kg) of candidates was recorded as shown in the table below.

Weight(kg)	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	3	4	8	7	9	5	4

   (a) Calculate the;

   (i) Mean.

   (ii) Variance.

   (b) Draw an Ogive for the data and use it to estimate;

   (i) Median.

   (ii) Number of candidates weighing at most 65 kg.

3. The table below shows the shows the number of units of electricity used by a household per month from January to September, 2022.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Units used	x	y	63	69	72	66	75	78	81

   (a) Given that the first and the second three-monthly moving average for the data were 60 kwh and 63 kwh respectively, find the value of x and y. Determine the other three-monthly moving averages for the data.

$M_1 = 60$
$M_2 = 63$

$x + y + 63 = 60$

(b) On the same axes, plot the graphs for the original data and the three-monthly moving averages.

(c) Use the trend to estimate the units used in October, 2022.

4. The table below shows the output of maize (in tonnes) produced by a farmer per year between 2010 – 2018.

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Output	6	9	12	9	15	12	18	15	12

(a) Calculate the 4 point moving averages for the data.

(b) On the same axes, plot the graph of the 4 point moving averages and the original data.

(c) Use your graph to estimate the output produced by the farmer in 2019.

5. The table below shows the unit price of items used by a household in 2020 and 2022 including their weights.

Item	Price (Ugx)		Weight
	2020	2022	
A	1,000	1,700	5
B	1,250	1,500	6
C	2,750	3,500	7
D	3,000	3,300	2

Taking 2020 as the base year, determine for the items in 2022, the;

(i) Price relatives. $\frac{P_1}{P_0} \times 100$

(ii) Simple aggregate price index. $\frac{\sum P_1}{\sum P_0} \times 100$

(iii) Weighted average price index. Comment on your result.

6. The table below shows the marks obtained in Sub-Math(X) and Physics(Y) by nine students.

X	51	62	64	47	54	44	68	61	56
Y	45	54	58	46	49	43	59	56	53

(a) (i) Draw a scatter diagram for the data.

(ii) On your scatter diagram, draw a line of best fit.

(iii) Use the line of best fit to estimate the value of X when $Y = 55$.

(b) Calculate the spearman's rank correlation coefficient and comment on the result.

7. (a) The events A and B are such that $P(A) = 0.45$, $P(A \cap B') = 0.28$ and $P(A' \cap B) = 0.25$. Find;

(i) $P(A \cap B)$.

(ii) $P(A \cup B)$.

(iii) $P(A' / B)$

(b) The events A and B are such that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.7$. Determine the value of $P(A \cap B)$. Hence show that A and B are independent.

(c) A box contains cards labelled with the letters of the word C.O.U.N.S.E.L. Two cards were selected at random from the box, calculate the probability that the cards selected contained,

(i) a card labelled with a vowel.

(ii) at least a card labelled with a consonant.

8. (a) In a certain school, 40% of the students offer Arts. Given that 70% of the Arts students offer Submaths and 90% of the Science students offer ICT, determine the probability that a student selected at random from the school offers;

(i) Submaths.

(ii) Arts given that she also offers submaths.

(b) A box contains 3 black and 5 white beads. A bead is selected at random from a box and not returned. After thorough mixing, another bead is selected at random from the box. Find the probability that;

(i) A white bead is selected.

(ii) At least one black bead is selected.

9. A random variable X has a probability distribution given by the following table.

x	0	1	2	3	4
$P(X = x)$	0.08	0.12	0.25	k	0.16

Determine the;

(i) Value of k.

(ii) $P(X \geq 1/X \leq 3)$.   Continuous Random

(iii) Median.

(iv) Variance of X, $Var(X)$.

10. A random variable X has a probability density function given below.

$$f(x) = \begin{cases} cx; & x = 1, 2, 3, 4. \\ c(8 - x); & x = 5, 6, 7. \\ 0; & \text{Otherwise} \end{cases}$$

Find the;

(i) Value of c.

(ii) P(X<6/X>3).

(iii) E(X).

(iv) Var(X).

11. (a) In a survey, 40% of the candidates offered submaths. If a sample of 10 candidates was considered, determine the;

(i) Probability that there was at least a candidate who offered submaths.

(ii) Expected number of candidates who offered submaths.

(b) Given that $X \sim B(n, p)$, where the expectation of X is 4 and its variance 2.4. Determine the value of n and p, hence find $P(X > 2)$.

12. A random variable X has a probability density function given below.

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$$f(x) = \begin{cases} kx(4-x), & 0 < x < 3 \\ 0, & \text{Otherwise} \end{cases}$$

$4x^2 - x^2$

Determine the;

(i) Value of k.

(ii) P(X>1).

(iii) E(X).

(iv) Var(X).

$\frac{4x^3}{3} - x^4$

(i) $\int_0^3 kx(4-x)dx$

$\frac{1}{k}\left[\left(2x^2 - \frac{x^3}{3}\right)\right]_0^3 = 1$

(ii) $\int_1^3 \frac{1}{k}\left(2x^2 - \frac{x^3}{3}\right)$

13. The height of 60 candidates in a certain School is normally distributed with a mean of 120cm and variance of 96cm. Determine the;

(a) Probability that a candidate selected at random is less than 100cm tall.

(b) Percentage of the candidates whose height lies between 125cm and 138cm.

(c) Number of candidates whose height is more than 105cm.

14. The marks scored by 120 candidates in a certain exam was found to be normally distributed with mean 60 marks and standard deviation 10.

Determine the;

(i) Percentage of the candidates who scored at most than 45 marks.

(ii) Number of distinction scores obtained if a distinction was awarded to a score of at least 80 marks.

(iii) Pass mark if 12 candidates failed to get a point.