P425/1
PURE
MATHEMATICS
Paper 1
31 July 2024
3 hours

ENTEBBE JOINT EXAMINATION BUREAU

Uganda Advanced Certificate of Education

MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Attempt **ALL** *the eight questions in* Section **A** *and any* **five** *from* Section **B.**

Begin every answer on a fresh page.

Any additional questions answered will **not** *be marked.*

Mathematical tables and squared paper shall be provided

Silent, non – programmable calculators may be used.

State the degree of accuracy at the end of each answer attempted using a calculator or table and indicate **cal** *for calculator or* **tab** *for mathematical table.*

SECTION A: 40 MARKS

Attempt **all** *questions in this* Section.

1. If α and β are roots of the equation $x^2 - x - 2 = 0$.
Find a quadratic equation whose roots are $\beta - \frac{1}{\alpha^2}$ and $\alpha - \frac{1}{\beta^2}$ *(05 marks)*

2. A, B and C are angles of a triangle $Cos\ A = \frac{3}{5}$ $Cos\ B = \frac{5}{13}$
Without using tables or a calculator, show that $Cos\ C = \frac{33}{65}$ *(05 marks)*

3. Use Maclaurin's theorem to expand $ln\sqrt{1 - 2x}$ up to the term in x^3. *(05 marks)*

4. Solve for x: $e^x = 1 + 6e^{-x}$ *(05 marks)*

5. Find the perpendicular distance from the point $P\ (1, -1, 4)$ to the line $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + \lambda\ (2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k})$ *(05 marks)*

6. Evaluate $\int_0^{\pi/2} x \sin^2 3x\, dx$ *(05 marks)*

7. A line with a variable gradient is passing through the point $A\ (2, 3)$ and cuts the y – axis and x – axis at P and Q respectively. Find the locus of midpoint of PQ. *(05 marks)*

8. Find the volume of the solid generated when the region bounded by the curve $y = sin2x$ and the x – axis from $x = 0$ to $x = \frac{\pi}{2}$ is rotated about the x – axis. *(05 marks)*

SECTION B

9. (a) Show that $z = -1 + i$ is a root of the equation $z^4 - 2z^3 - z^2 + 2z + 10 = 0$. Find the remaining roots. *(06 marks)*

(b) If $z_1 = 4\left(\text{Cos}\,\frac{13}{24}\pi + i\,\text{Sin}\,\frac{13}{24}\pi\right)$ and $z_2 = 2\left(\text{Cos}\,\frac{5}{24}\pi + i\,\text{Sin}\,\frac{5}{24}\pi\right)$

Find z_1z_2 and $\frac{z_1}{z_2}$ in the form a + *i*b

(06 marks)

10. By substituting $u = e^x$, show that

$$\int_0^{ln_4} \frac{e^{2x}}{e^{2x} + 3e^x + 2}\, dx = ln\left(\frac{8}{5}\right)$$ *(12 marks)*

11. (a) Express $\sqrt{6}\, Cos\, \theta + \sqrt{10}\, Sin\, \theta$ in the form $R\, Cos\, (\theta - \alpha)$ where $R > 0$ and $0 < \alpha < 90^0$. Hence solve the equation $\sqrt{6}\, Cos\, \theta + \sqrt{10}\, Sin\theta = 3$ for $0 \leq \theta \leq 180^0$. *(06 marks)*

(b) If $t = tan\, \frac{\theta}{2}$; state expressions for $sec\, \theta$ and $tan\, \theta$ in terms of t.
Hence show that: $\sec\theta + \tan\theta = \tan\left(45^0 + \frac{\theta}{2}\right)$ *(06 marks)*

12. The line L_1 passes through the points $A\ (8, -1, 3)$ and $B\ (4, 0, 3)$ and line L_2 has vector equation $\boldsymbol{r} = -2\boldsymbol{i} + 8\boldsymbol{j} - \boldsymbol{k} + \mu\ (\boldsymbol{i} + 3\boldsymbol{j} + a\boldsymbol{k})$ and plane M has equation $4x - 2y - z + 5 = 0$.

(a) Find in Cartesian form the equation of the line L_1. *(05 marks)*

(b) Find the point of intersection of line L_1 and the plane M. *(04 marks)*

(c) Given that line L_2 and plane M are parallel, find the value of a. *(03 marks)*

13. Show that the curve $y = \dfrac{12x}{x^2 + 2x + 4}$ entirely lies in the range $-6 \leq y \leq 2$.

Hence, find the turning points and their nature. Sketch the curve. *(12 marks)*

14. (a) Solve the simultaneous equations $7x + 2y - 3z = 8$ and
$\dfrac{3x - y}{3} = \dfrac{4x - z}{4} = 3y - 2z$ *(06 marks)*

(b) Find the ranges of values of k for which the equation $2x^2 + 3x = kx - k - 3$ has two distinct roots. *(06 marks)*

15. (a) $ABCD$ is a square inscribed in a circle $x^2 + y^2 - 6x - 4y - 12 = 0$. Find the area of the square. *(05 marks)*

(b) Show that the curve $16x^2 + 9y^2 - 64x - 54y + 1 = 0$ represents an ellipse. Find the foci and equations of directrices. *(07 marks)*

Turn Over

16. (a) Solve $(x^2 + 4)\dfrac{dy}{dx} = 6xy$ given that $y(0) = 32$. *(04 marks)*

(b) Mr. Lubega starts to sip a bottle of soda of 1000 cm^3 at a rate of 10 cm^3 per minute. Given that the rate of consumption is inversely proportional to that of the volume of soda remaining at anytime. t. Find the time he takes to empty the bottle.

(08 marks)

END