UMTA PURE MATHS 2024
Proposed Marking Guide

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1. $\int x^4 \ln x \, dx$.

let $u = \ln x$, let $\frac{dv}{dx} = x^4$

$\frac{du}{dx} = \frac{1}{x}$ $\quad$ $v = \frac{x^5}{5}$

$uv - \int v \frac{du}{dx} dx$

$\frac{x^5}{5} \ln x - \int \frac{1}{5} x^5 \cdot \frac{1}{x} dx$

$\frac{x^5}{5} \ln x - \int \frac{x^4}{5} dx$.

$\frac{x^5}{5} \ln x - \frac{x^5}{25} + C$

(2) $2x+3y=7$ — l_1
$x=6y+5$ — l_2

$3y=-2x+7$ $\quad m_1=-2/3$
$y=-2/3\,x+7/3$

$x=6y+5$
$y=1/6\,x+5/6$, $\quad m_2=1/6$.

angle btn 2 lines

$$\tan\theta=\frac{|m_1-m_2|}{1+m_1m_2}$$

$$\tan\theta=\frac{|-2/3-1/6|}{1+(-2/3\times 1/6)}$$

$$\theta=\underline{\underline{43.2^\circ}}$$

(3) $$y=\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}=\frac{1-(1-2\sin^2 x)}{1+2\cos^2 x-1}$$

$$y=\sqrt{\frac{2\sin^2 x}{2\cos^2 x}}=\tan^2 x=\tan x$$

$$y=\tan x$$

$$\frac{dy}{dx}=\sec^2 x$$ as required

$a = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix} \quad c = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$

vectors are coplanar if the scalar tripple pdt is 0.

$a \cdot (b \times c) = 0.$ $\qquad -4 - -15$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} 1 & -3 & -5 \\ 3 & -4 & -4 \end{vmatrix}$$

$-8\underset{\sim}{i} - 11\underset{\sim}{j} + 5\underset{\sim}{k}$

$$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -11 \\ 5 \end{pmatrix}$$

$-16 + 11 + 5 = 0$

hence the vectors are coplanar.

⑤ $\tan x + \tan 2x + \tan x \tan 2x = 1.$

$$\frac{\sin x}{\cos x} + \frac{\sin 2x}{\cos 2x} + \frac{\sin x \sin 2x}{\cos x \cos 2x} = 1.$$

$$\frac{\sin x \cos 2x + \cos x \sin 2x + \sin x \sin 2x}{\cos x \cos 2x} = 1$$

$\sin x \cos 2x + \cos x \sin 2x = \cos 2x \cos x - \sin 2x \sin x$

$\sin(x+2x) = \cos 2x \cos x$

$\sin(x+2x) = \cos(2x+x)$

$\sin 3x = \cos 3x.$

$\tan 3x = 1.$

$3x = 45°, 225°, \cancel{345}\ 405°, 585°, 765°, 945°$

$x = 15°, 75°, 135°, 195°, 255°, 315°$

(6) $9\log_x 5 = \log_5 x$

$$\frac{9}{\log_5 x} = \log_5 x$$

Let $\log_5 x = h$

$$\frac{9}{h} = h$$

$9 = h^2$

$h = \pm 3.$

$\log_5 x = 3$ or $\log_5 x = -3$

$5^3 = x$ $\quad$ $5^{-3} = x$

$x = 125$ $\quad$ $x = 1/125$

$\therefore$ $x = 125$ or 0.008.

QN 7.

$y = (1-x)(x+2)$

at $y=0$

$x=1,\ x=-2.$

$y = x + 2 - x^2 - 2x$

$y = -x^2 - x + 2.$

Scan with **Fast Scan**

2024

$$\int_{-2}^{1} -x^2 - x + 2 \, dx.$$

$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x \right]_{-2}^{1}$$

$$= \left[\frac{-1}{3} \frac{-1}{2} + 2 \right] - \left[-\left(\frac{-2}{3}\right)^3 - \frac{(2)^2}{2} + 2(-2) \right]$$

$= \underline{\underline{4.5 \text{ sq. units}}}$

QN. 8.

$$3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$$

$$3^{2x} \cdot 3 - 3(3^x) - 3^x + 1 = 0$$

Let $3^x = h$

$$3h^2 - 3h - h + 1 = 0.$$

$$3h(h-1) - 1(h-1) = 0$$

$$(h-1)((3h-1) = 0.$$

$h = 1$ or $1/3$.

$3^x = 3^0$ $\quad$ $3^x = 1/3$

$x = 0$ $\quad$ $x = -1.$

$\therefore \underline{\underline{x = 0 \text{ or } -1.}}$

Question 9.

9) $y = \frac{3x+3}{x(3-x)}$

a) $y = \frac{3x+3}{3x-x^2}$

$3xy - x^2y = 3x+3$

$x^2y + 3x - 3xy + 3 = 0.$

$x^2y + x(3-3y) + 3 = 0$

for no curve

$b^2 < 4ac$

$(3-3y)^2 < 4(3)y$

$9 - 18y + 9y^2 < 12y$

$9y^2 - 30y + 9 < 0.$

$3y^2 - 10y + 3 < 0.$

$3y^2 - 9y - y + 3 < 0.$

$(3y-1)(y-3) < 0$

$y = \frac{1}{3}$ or 3

$y < 1/3$	$1/3 < y < 3$	$y < 3.$
+	−	+

$\therefore$ the region is $1/3 < y < 3.$

hence turning points & nature.

for $y = 1/3$

$x^2\left(\frac{1}{3}\right) + x(3-3(1/3)) + 3 = 0$

$\frac{1}{3}x^2 + 2x + 3 = 0.$

$x^2 + 6x + 9 = 0.$

$(x+3)^2 = 0$

$x = -3$

$pt = (-3, 1/3)$ max

for $y = 3.$

$x^2(3) + x(3-3(3)) + 3 =$

$3x^2 - 6x + 3 = 0.$

$x^2 - 2x + 1 = 0.$

$(x-1)^2 = 0$

$x = 1$

Point is $(1, 3)$ minimum

(b) asymptotes.

vertical - $x(3-x) = 0$.

$x = 0, \quad x = 3$.

horizontal:

$$y = \frac{3x+3}{3x - 3x^2} \Rightarrow \frac{\frac{3x}{x^2} + \frac{3}{x^2}}{\frac{3x}{x^2} - \frac{x^2}{x^2}} \quad - \infty$$

$y = 0$.

intercepts at $y = 0$,

$3x + 3 = 0$

$x = -1$.

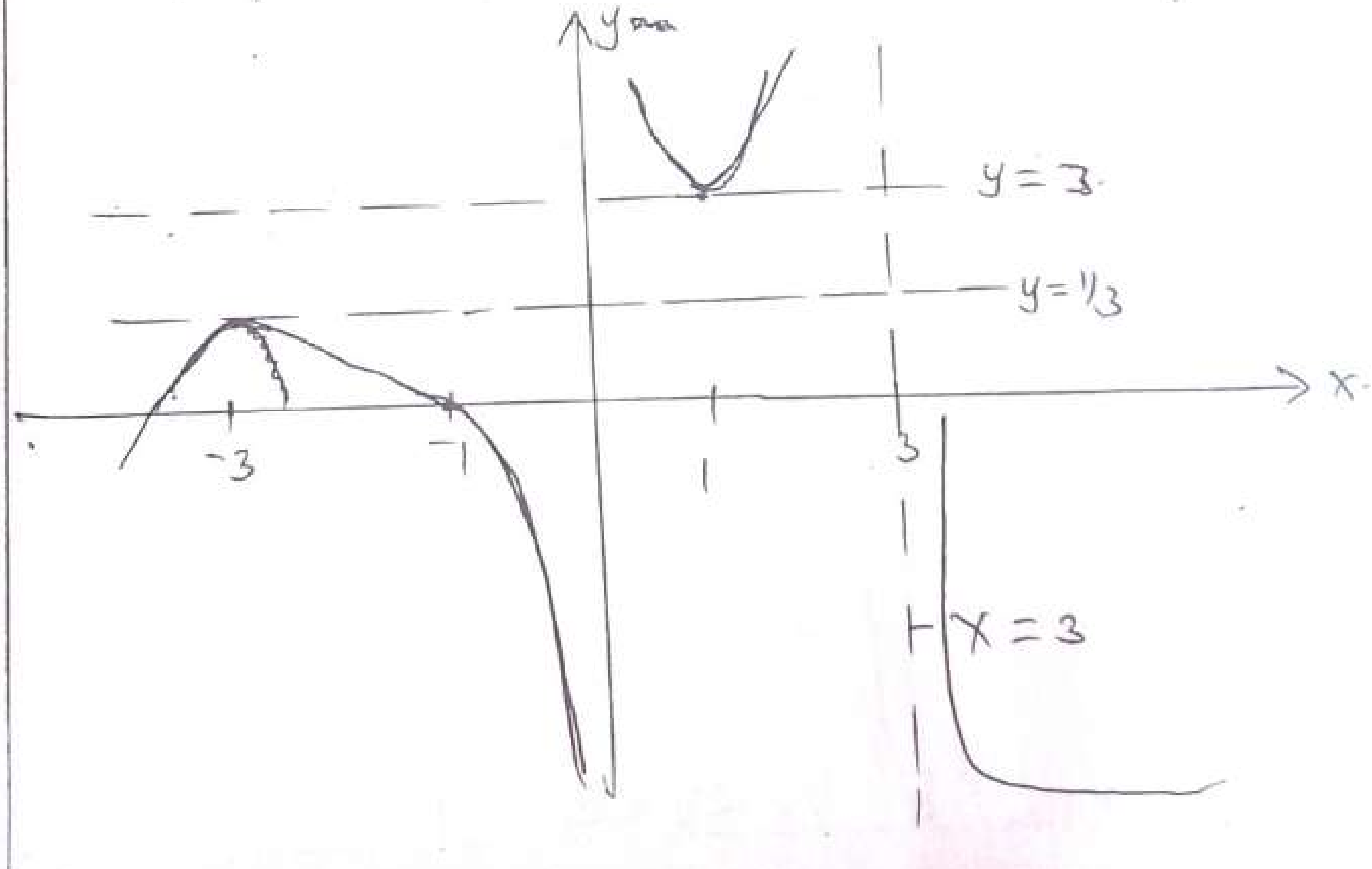
$(-1, 0)$

at $x = 0$

$y = \frac{0}{0(3)}$

$y =$ undefined.

$x < -1$	$-1 < x < 0$	$0 < x < 3$	$x > 3$
$-$	$+$	$+$	$+$
$-$	$-$	$+$	$-$
$+$	$-$	$+$	$-$



N 10

5)

$$(\sqrt{3-x} - \sqrt{7+x})^2 = (\sqrt{16+2x})^2$$

$$3-x - 2\sqrt{(3-x)(7+x)} + 7+x = 16+2x.$$

$$10 - 2\sqrt{21+3x-7x-x^2} = 16+2x.$$

$$-2\sqrt{-x^2-4x+21} = 6+2x.$$

$$(-\sqrt{-x^2-4x+21})^2 = (3+x)^2$$

$$-x^2-4x+21 = x^2+6x+9.$$

$$2x^2+10x-12 = 0$$

$$x^2+5x-6 = 0$$

$$(x+6)(x-1) = 0$$

$x = {}^-6$ or $x = +1.$

upon verifying:

for $x = {}^-6$

$\sqrt{3^-6} - \sqrt{7^-6} = \sqrt{16-12}$ x.

for $x = -1$

$\cancel{\sqrt{3+-1}}$

$$\sqrt{9} - \sqrt{1} = \sqrt{4}$$

$$3 - 1 = 2$$

$$\therefore \underline{\underline{x = -6}}$$

(b) $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5}$ and $x+y+z=2$.

$\frac{x+2y}{-3} = \frac{y+2z}{4}$

$4x+8y = -3y-6z$

$4x+11y+6z=0$

$\frac{y+2z}{4} = \frac{2x+z}{5}$

$5y+10z = 8x+4z$

$8x-5y-6z=0$

$x+y+z=2$ — (3)

$4x+11y+6z=0$ — (2)

$8x-5y-6z=0$ — (1)

(2) + (1)

$12x+6y=0$

$2x+y=0$ — (4)

$6x+6y+6z=12$

$4x+11y+6z=0$

$2x-5y=12$

but $y=-2x$.

$2x-5(-2x)=12$

$12x=12$

$x=1$.

$y=-2(x) = -2$.

from

$x+y+z=2$

$1+-2+z=2$

~~$-x+2=z$~~

$z-1=2$.

$z=3$

$\therefore x=1$

$y=-2$

$z=3$.

Scan with **Fast Scan**

QN 11

$$\cancel{(\cos 4\theta)^3}\ \frac{(\cos 4\theta + i\sin 4\theta)^3(\cos 2\theta - i\sin 2\theta)^5}{(\cos 3\theta + i\sin 3\theta)^4(\cos 5\theta - i\sin \cancel{4}\theta)^6}$$

$$\frac{\cancel{(\cos\theta + i\sin\theta)^{3\times 4}}(\cos\theta + i\sin\theta)^{(5\times -2)}}{\cancel{(\cos\theta + i\sin\theta)^{3\times 4}}(\cos\theta + i\sin\theta)^{6\times -5}}$$

$$\frac{(\cos\theta + i\sin\theta)^{-10}}{(\cos\theta + i\sin\theta)^{-30}} = (\cos\theta + i\sin\theta)^{-10 - -30}$$

$$= (\cos\theta + i\sin\theta)^{20}$$

$$= (\cos 20\theta + i\sin 20\theta)$$

The correct qn had to have $(\cos 5\theta - i\sin 5\theta)^6$

(b) :

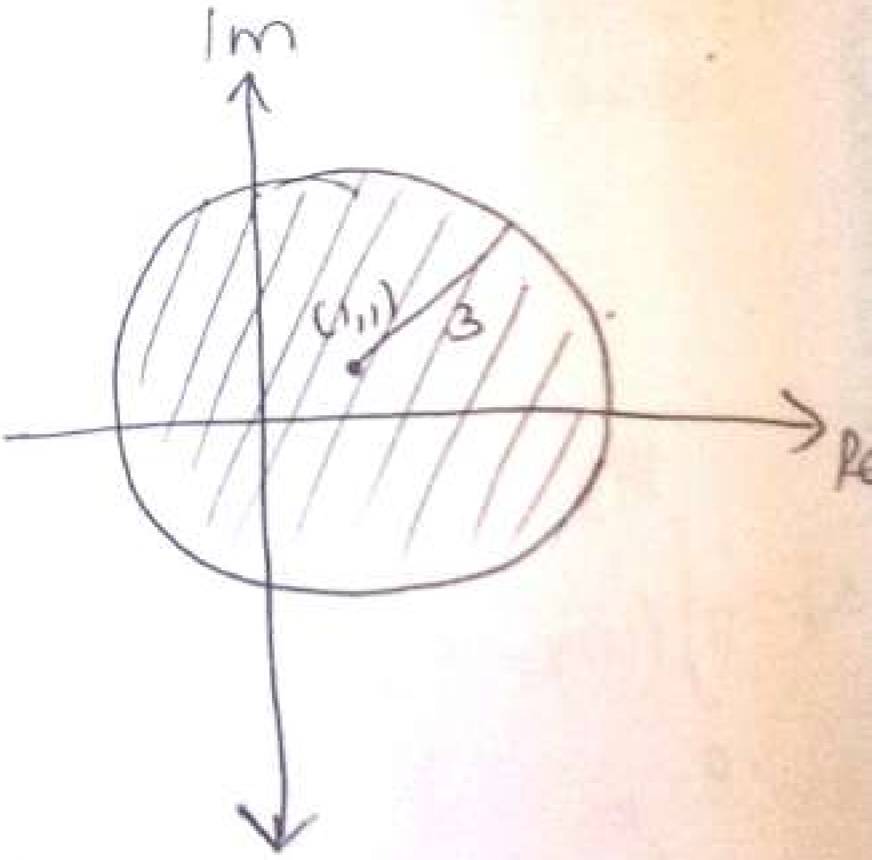
$|z-1-i| < 3$

let $z = x+iy$

$|x+iy-1-i| < 3$

$\sqrt{(x-1)^2 + (y-1)^2} < 3$

$(x-1)^2 + (y-1)^2 < 9$



it is a circle with center (1,1) and radius less than 9.

using (1,1) $(1-1)^2 + (1-1)^2 < 9$

$0 < 9$ ma... region is in the circ

QN 12

$y = 2x^2$ and $y = 10x - x^2$

$2x^2 = 10x - x^2$

$3x^2 = 10x$

$3x^2 - 10x = 0$

$x(3x - 10) = 0$

$x = 0,\ x = 10/3$

tp. $\frac{dy}{dx} = 4x$

nature = $\underline{y}$ min

$y = 10x - x^2$

$\frac{dy}{dx} = 10 - 2x$

at $\frac{dy}{dx} = 0$

$10 - 2x = 0$

$x = 5$

$y = 50 - 25 = 25$

$(5, 25)$

Intercepts:

for $y = 2x^2$

If $x = 0,\ y = 0$. $(0,0)$

for $y = 0,\ x = 0$ $(0,0)$

for $y = 10x - x^2$

for $x = 0$

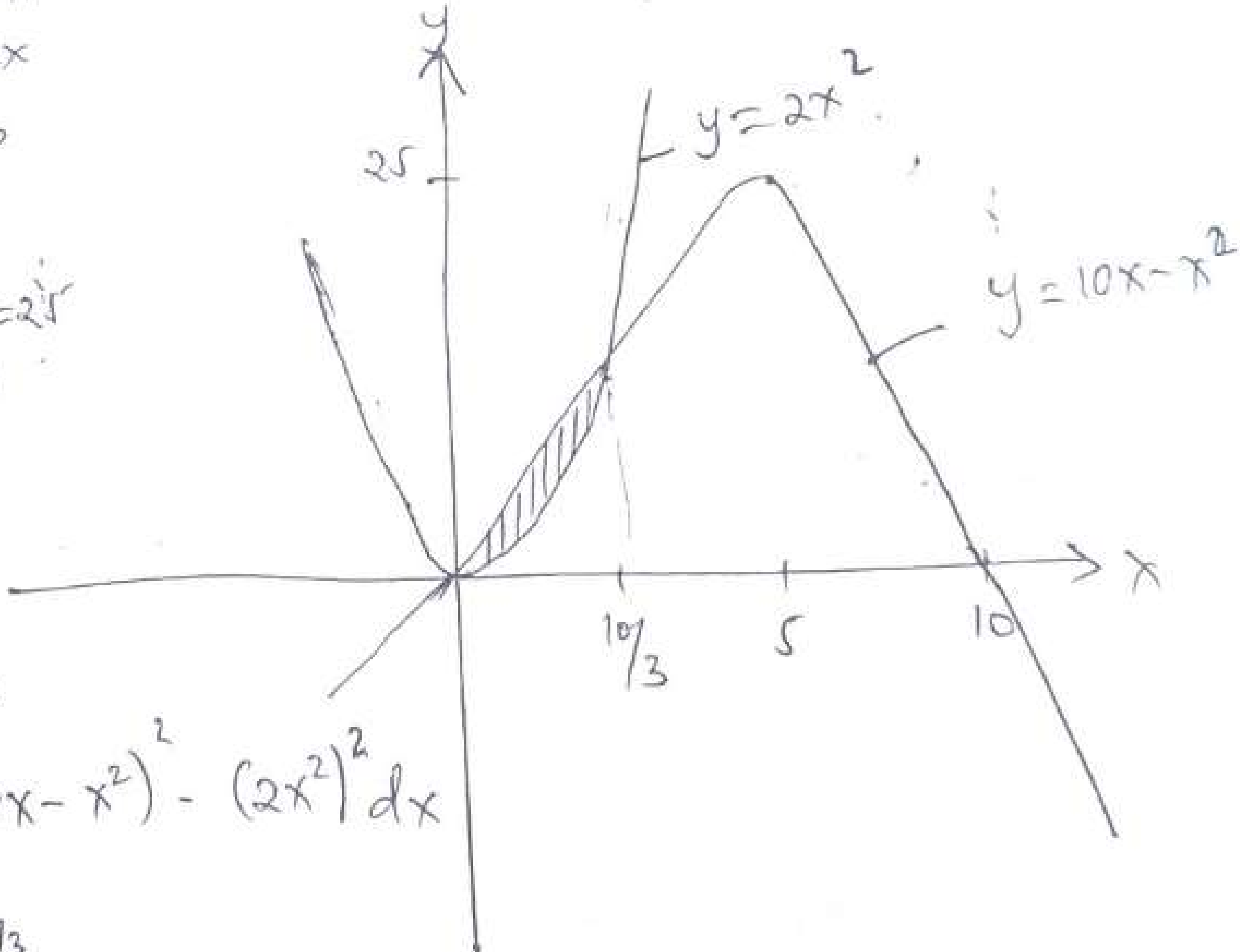
$y = 0$.

for $y = 0$

$x(10 - x) = 0$

$x = 0,\ x = 10$.

$(0,0)$ $(10,0)$



$$V = \pi \int_0^{10/3} (10x - x^2)^2 - (2x^2)^2\, dx$$

$$V = \pi \int_0^{10/3} 100x^2 - 20x^3 - x^4 - 4x^4\, dx$$

$V = 205.76\pi$ cubic units.

SN 13

(3)
a) $\dfrac{\sin 5x - \sin 7x + \sin 8x - \sin 4x}{\cos 4x - \cos 5x - \cos 8x + \cos 7x.}$

$\dfrac{\sin 5x - \sin 7x + \sin 8x - \sin 4x}{\cos 4x - \cos 8x + \cos 7x - \cos 5x.}$

$$\frac{2\cos\left(\frac{5x+7x}{2}\right)\sin\left(\frac{5x-7x}{2}\right) + 2\cos\left(\frac{8x+4x}{2}\right)\sin\left(\frac{8x-4x}{2}\right)}{-2\sin\left(\frac{4x+8x}{2}\right)\sin\left(\frac{4x-8x}{2}\right) + -2\sin\left(\frac{7x+5x}{2}\right)\sin\left(\frac{7x-5x}{2}\right)}$$

$$\frac{2\cos 6x \sin(-x) + 2\cos 6x \sin(+2x)}{-2\sin(6x)\sin(-2x) - 2\sin 6x \sin(x).}$$

$$\frac{\cancel{2}\cos 6x(-\sin x + \cancel{\sin 2x})}{\cancel{2}\sin 6x(\cancel{\sin 2x - \sin x})}$$

$\cot 6x$. as required.

(b) $4\cos x - 6\sin x = 5.$

let $4\cos x - 6\sin x = R\cos(x+\alpha).$

$4\cos x - 6\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha.$

$R\cos\alpha = 4$ — (1)

$R\sin\alpha = 6$ (2)

eqn (2) $\div$ eqn (1)

$\tan\alpha = 1.5$

$\alpha = 56.3°$.

eqn $(1)^2$ + eqn $(2)^2$

$R^2 = 6^2 + 4^2$

$R = \sqrt{52}$

$\sqrt{52}\cos(x + 56.3°) = 5$.

$\cos(x + 56.3°) = 5/\sqrt{52}$

$x + 56.3° = 46.1°, 313.9°, 406.1°$

$x = -10.2°, 257.6°, 349.8°$

$x = 257.6°, 349.8°$

QN. 14.

$$f(x) = \frac{3x^3 + x + 1}{(x-2)(x+1)^3} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$$

$$\frac{3x^3 + x + 1}{(x-2)(x+1)^3} \equiv \frac{A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2)}{(x-2)(x+1)^3}$$

$$3x^3 + x + 1 \equiv A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2)$$

for $x = 2.$

$4(8) + 2 + 1 = A(3)^3$

$27 = 27A.$

$A = 1.$

for $x = -1.$

$(-1) - 1 + 1 = D(-1-2)$

$-3 = -3D$

$D = 1.$

for $x = 0.$

$1 = A(1) + B(-2)(1) + C(-2)(1) + D(-2).$

$1 = A - 2B - 2C - 2D.$

$2B - 2C = -2$

$B - C = -1 \quad --- \quad (1)$

for $x = 1.$

$3(1) + 1 + 1 = A(-2)^3 + B(-1)(2)^2 + C(-1)(2) + D(-1)$

$5 = 8A - 4B - 2C - D.$

$4B + 2C = 8 - 5 - 1$

$4B + 2C = 2$

$2B + C = 1 \quad --- \quad (2)$

Solving

$B = 0, C = 1.$

$\therefore f(x) = \frac{1}{x-2} + \frac{1}{(x+1)^2} + \frac{1}{(x+1)^3}$

(b)

$$\int_3^4 f(x)\,dx = \int_3^4 \frac{dx}{x-2} + \int_3^4 \frac{dx}{(x+1)^2} + \int_3^4 \frac{dx}{(x+1)^3}$$

$$= \left[\ln(x-2) - \frac{1}{x+1} - \frac{1}{2(x+1)^2}\right]_3^4$$

$$= \left[\ln 2 - \frac{1}{5} - \frac{1}{2(25)}\right] - \left[\ln 1 - \frac{1}{4} - \frac{1}{2(16)}\right]$$

$0.473147 - -0.28125$

0.754397

$\underline{\underline{0.7544}}$ (4dp)

QN ~~#~~. 15

$$\frac{dy}{dx} = x - \frac{2y}{x}$$

$$\frac{dy}{dx} + \frac{2y}{x} = x.$$

$$IF = e^{\int \frac{2}{x} dx} = e^{\int 2\ln x} = x^2$$

$$x^2 \frac{dy}{dx} + x^2\left(\frac{2y}{x}\right) = x^3$$

$$\int \frac{d}{dx}(x^2 y)\,dx = \int x^3\,dx$$

$$yx^2 = \frac{x^4}{4} + c$$

$$4yx^2 = x^4 + c$$

at $(2, 4)$

$(4)(4) = (4)^4 + C$

$C = 48.$

$4x^2y = x^4 + 48.$

(b) $y = vx$

$y/x = v.$

$y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$$x^2\frac{dy}{dx} = x^2 + y^2 + xy.$$

$$\frac{dy}{dx} = 1 + \frac{y^2}{x^2} + \frac{y}{x}.$$

$$\cancel{v} + x\frac{dv}{dx} = 1 + v^2 + \cancel{v}.$$

$$\int \frac{dv}{1+v^2} = \int x\,dx$$

$$\tan^{-1} v = \frac{x^2}{2} + C$$

$$\tan^{-1}(y/x) = \frac{x^2}{2} + C.$$

QN 16.

$$r = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \text{ and } r\cdot\begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix} = 13.$$

$x = 1 + 2t$

$y = 1 + 2t$

$z = -3 + t.$

$$6x - 3y + 2z = 13$$

$$6(1+2t) - 3(1+2t) + 2(-3+t) = 13$$

$$\cancel{6} + 12t - 3 - 6t - \cancel{6} + 2t = 13$$

$$8t = 16$$

$$t = 2$$

$x = 1 + 2(2) = 5$

$y = 1 + 2(2) = 5$

$z = -3 + 2 = -1.$

$$Pt = (5, 5, -1)$$

angle

$$\sin\theta = \frac{b_1 \cdot b_2}{|b_1||b_2|}$$

$$= \frac{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}}{\sqrt{4+4+1}\sqrt{36+9+4}} = \frac{12 - 6 + 2}{\sqrt{9}\sqrt{49}} = \frac{8}{21}$$

$$\theta = 22.39^\circ$$

Scan with **Fast Scan**

MT 16(b)

$a = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$ $c = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

$ab = \begin{pmatrix} 1-3 \\ -3--2 \\ 5-1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix}$

$bc = \begin{pmatrix} 2-1 \\ 1--3 \\ -4-5 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -9 \end{pmatrix}$

$ca \quad \begin{pmatrix} 3-2 \\ -2-1 \\ 1--4 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$

$ab + bc + ca = 0$

$\begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ -9 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Choosing for one angle to be $90°$

angle btn a and c.

$\cos\theta = \dfrac{a \cdot c}{|a|\,|c|}$

$\cos\theta = \dfrac{\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}}{\sqrt{9+4+1}\sqrt{4+1+16}} = \dfrac{6-2-4}{\sqrt{13}\sqrt{21}} = \dfrac{0}{\sqrt{21}\sqrt{13}}$

$= 90°$.

$\therefore$ since one angle is 90 and $ab + bc + ca = 0$, they form a right angle triangle