2.0 WAVES

A wave is any disturbance from an equilibrium position that travels with time from one region to another.

Wave motion is a means of transfer of energy from one point to another without the transfer of matter between the points

2.1 Waves are classified into two namely;

(i) Electromagnetic waves

(ii) Mechanical waves

Mechanical waves

These are waves that are produced by the disturbances of a material medium and are transmitted by the particles of the medium oscillating to and fro

Mechanical waves require a material medium for their transmission. They can be felt and seen

Examples of mechanical waves

- Sound waves
- Water waves
- Waves on compressed springs
- Waves on stretched string

Electromagnetic waves

These are waves produced by disturbances of varying electric and magnetic fields They do not require a material medium for their transmission and travel in a vacuum.

Examples of electromagnetic waves

- Light waves
- $\gamma - rays$
- Radio waves
- All other electromagnetic band waves

Note:

All electromagnetic waves travel at a speed of light $ie\ 3x10^8 m/s$

Properties of electromagnetic waves

(i) Electromagnetic waves travel in a vacuum and therefore do not require a material medium for their transmission.

(ii) Electromagnetic waves travel at a speed of light $ie\ 3x10^3 ms^{-1}$

(iii) They are made of varying electric and magnetic vibration.

(iv) They vibrate with a high frequency.

(v) They have no charge.

Increasing wavelength (decreasing frequency)

Ɣ-rays	X - rays	Ultra violet (U.V)	V i	si	le	s	e	tr	m	Infra -red	Radio waves Microwaves, T.V waves

V I B G Y O R

Differences between mechanical and electromagnetic waves

Mechanical waves	Electromagnetic waves
❖ Need a material medium for their transmission	❖ Can propagate in vacuum
❖ Propagate at relatively low speeds	❖ Propagate at high speeds
❖ Have longer wavelength	❖ Have shorter wavelength

❖ Are due of vibrations of particles in the transmitting medium	❖ Are due of vibrations of particles in the transmitting medium

2.2 Types of waves

There are two types of wave motion namely

- Transverse waves
- Longitudinal waves

Transverse waves

These are waves in which displacement of the particles in the medium is perpendicular to the direction of wave travel.

Transverse waves are characterized by crests and troughs

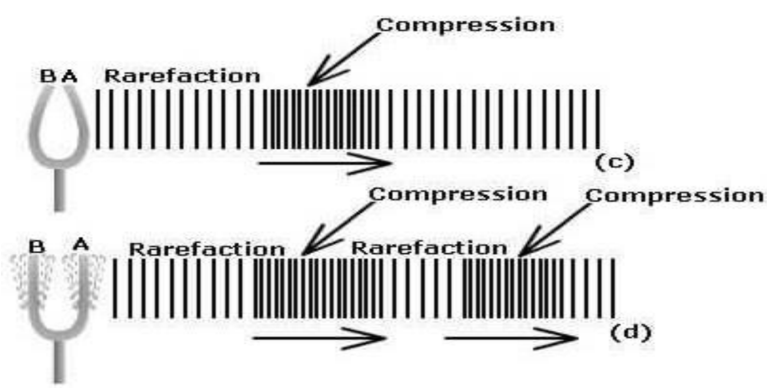
- Crest is the part of the wave above the line of zero disturbance
- Trough is the part of the wave below the line of zero disturbance

Examples

- Water waves
- Waves due plucked strings
- Light waves
- All electromagnetic waves ($eg\ \gamma - rays, X - rays$)

Longitudinal waves

These are waves in which the displacement of the particles is parallel to the direction of travel of the wave.



Longitudinal waves are characterized by compressions and rare factions

- Compressions are regions of high particle density in wave
- Rare factions are regions of low particle density in wave

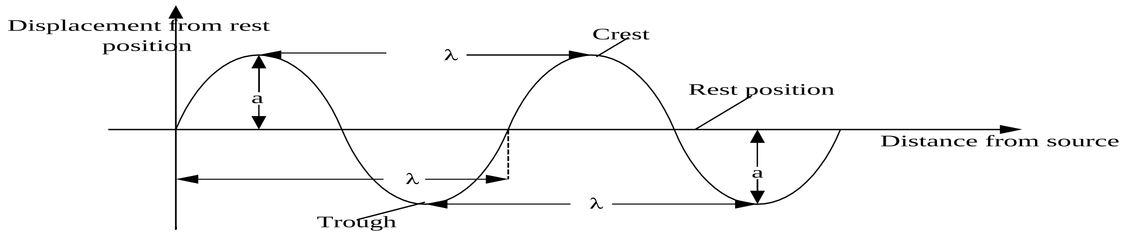
Examples

- Sound waves
- Waves on a compressed spring

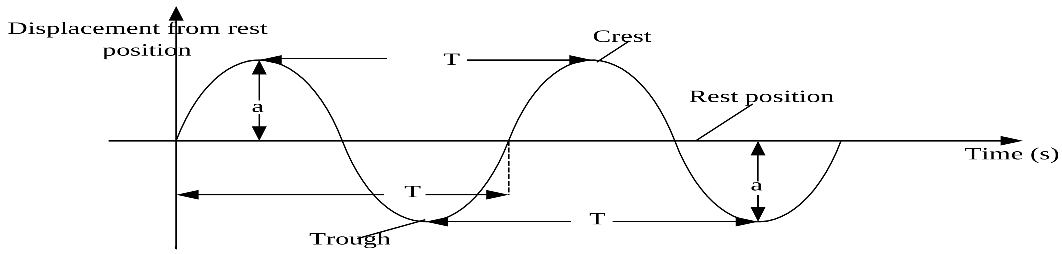
Differences between transverse and longitudinal waves

Transverse waves	Longitudinal waves
Particles vibrate at right angles to the direction of travel of the wave	Particles vibrate along the direction of travel of the wave
Transverse waves are represented by crests and troughs	longitudinal waves are represented by compression and rare faction regions

2.3 Representation of a wave



A displacement time graph can also be drawn



Terms used

(1) Amplitude (a)
This is the maximum displacement of a particle of a medium from its equilibrium position.

(2) Wave length (λ)
This is the distance between two successive particles in phase.
Wave length of **a transverse** wave is the distance between two successive crests or successive troughs.

(3) Oscillation or cycle
This is a complete to and fro movement of a wave particle in a medium

(4) Period (T)
This is the time taken for one particle to under one complete oscillation.
Or the time taken for a wave to travel a distance of one wavelength

$$\boxed{T = \frac{1}{f}}$$

Period T is measured in seconds

(5) Frequency (f)
The number of complete oscillations a wave particle makes in one second

$$\boxed{f = \frac{1}{T}}$$

The S.I unit of frequency is Hertz (Hz)

(6) Phase
Particles are in phase when they are exactly at the same point in their paths and are moving in the same direction

(7) Wave front
Is any section through an advancing wave in which all the particles are in the phase.

(8) A ray. This is the direction of an advancing wave

(9) Speed (V) of the wave
This is the linear distance travelled by a wave per unit time

$$V = \frac{linear\ distance}{time\ taken}$$

Since one complete wave is produced in time **T** and the length of one complete wave is $\boldsymbol{\lambda}$

$$V = \frac{\boldsymbol{\lambda}}{T}$$

$$V = \frac{\mathbf{1}}{T} x\, \boldsymbol{\lambda}$$

$$\text{But } f = \frac{\mathbf{1}}{T}$$

$$\boxed{V = f\,\boldsymbol{\lambda}}$$

EXAMPLES

1. Sanyu Fm broadcasts at a frequency of $88.2MHz$. Calculate the wavelength of the radio waves.

Solution

Note: All electromagnetic waves eg radio waves travel at a speed of light $3x10^8m/s$

$f = 88.2MHz = 88.2x10^6Hz,$
$v = 3x10^8m/s$
$v = f\,\lambda$
$3x10^8 = 88.2x10^6\,\lambda$

$\boldsymbol{\lambda} = \frac{3x10^8}{88.2x10^6}$
$\lambda = 3.4m$

2. A vibrator produces waves which travel a distance of $12m$ in $4s$. If the frequency of the vibrator is $2Hz$, what is the wavelength of the wave?

Solution

$f = 2Hz,\ t = 4s,\ distance = 12m$

$v = \frac{distance}{time} = \frac{12}{4}$

$v = 3m/s$

$v = f\,\lambda$
$3 = 2\,\lambda$
$\boldsymbol{\lambda} = \frac{3}{2}$
$\lambda = 1.5m$

3. A vibrator has a period of 0.02s and produces circular waves of water in a tank. If the distance between any two consecutive crests is 3cm, what is the speed of the wave?

Solution

$T = 0.02\ s$,
But $f = \frac{1}{T}$
$f = \frac{1}{0.02}$

$f = 50Hz$
$\lambda = 3cm$, $\lambda = 0.03m$
$v = f\,\lambda$
$v = 50x\ 0.03$
$v = 1.5m/s$

4. Water waves are produced at a frequency of 50Hz and the distance between 10 successive troughs is 18cm. Calculate the velocity of the waves.

Solution

$f = 50Hz$
$9\,\lambda = 18\ cm,\ \lambda = \frac{18}{9}cm$

$\lambda = 2cm,\ \lambda = 0.02\ m$
$v = f\,\lambda$
$v = 50x\ 0.02$
$v = 1m/s$

2.4 Properties of waves

All waves can be;

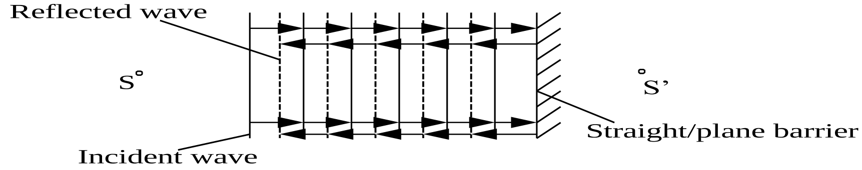
1. Reflected
2. Refracted
3. Diffracted
4. Interfered

1. REFLECTION OF WAVES

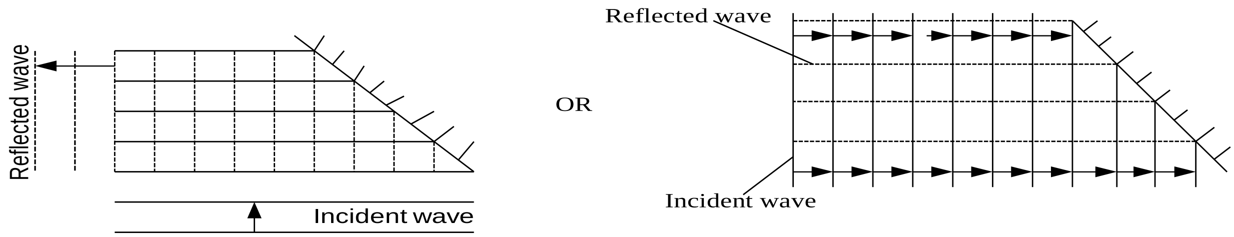
This is the bouncing back of waves when they meet a barrier

a) Plane reflector

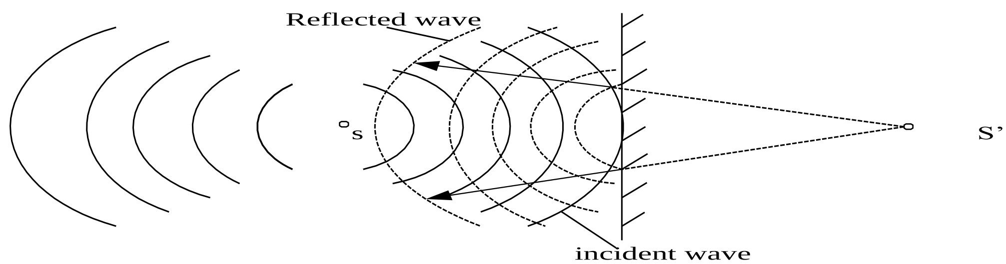
(i) Straight waves incident on a plane reflector



(ii) Straight waves incident on an inclined Plane surface

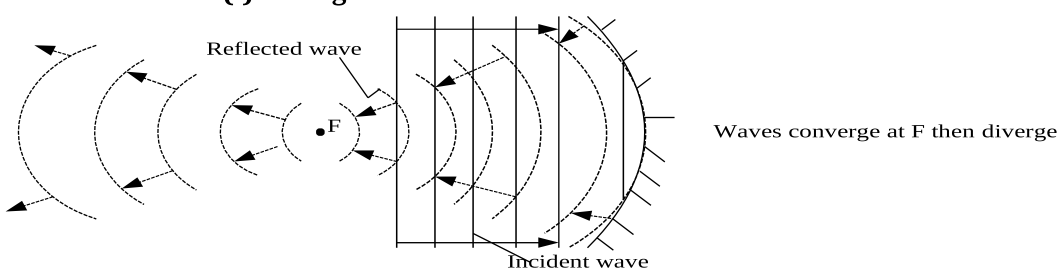


(iii) Circular waves incident on a plane reflector

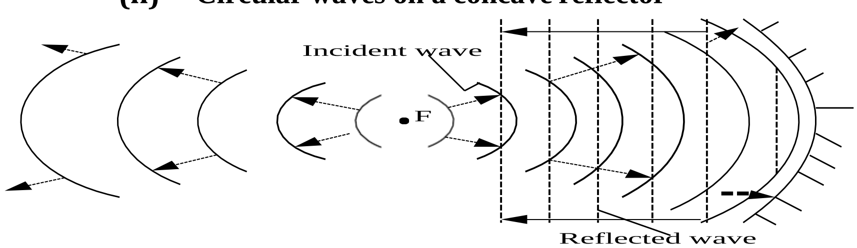


b) Concave reflector

(i) Straight waves incident on a concave reflector

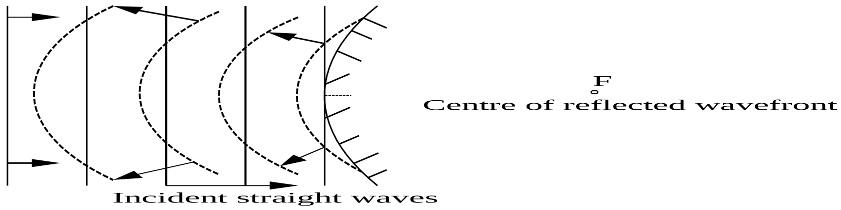


(ii) Circular waves on a concave reflector

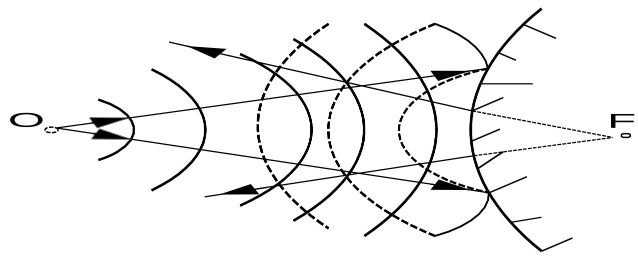


c) Convex reflector

(i) Plane waves incident on a convex reflector



(ii) Circular waves incident on a convex reflector



2.5 PROGRESSIVE WAVES

It is a wave in which the disturbance moves from the source to the surrounding places and energy is transferred from one point to another along the wave form

Examples

- Water waves
- All electromagnetic waves

Note: All transverse and longitudinal waves are progressive and the amplitude of a progressive wave is constant

Energy transmitted by a wave

In a progressive wave energy propagates through the medium in the direction in which the wave travels. So every particle in the medium possesses energy due to vibrations. This energy is passed on to the neighboring particles so for any system vibrating in form of simple harmonic motion, the energy of the vibrating particle changes from kinetic to potential energy and back but the total mechanical energy on the wave remains constant.

$$k.e = \frac{1}{2}mv^2$$

But $v = \omega A$

$$k.e = \frac{1}{2}m(\omega A)^2$$

$$\omega = 2\pi f$$

$$k.e = \frac{1}{2}m(2\pi f A)^2$$

$$\boxed{K.E = 2\pi^2 f^2 A^2 m}$$

Where f- frequency

A- Amplitude

M- mass of vibrating particle

$$Also\ energy\ per\ unit\ volume = \frac{E}{V}$$

$$\frac{E}{V} = \frac{2\pi^2 f^2 A^2 m}{\left(\frac{m}{\rho}\right)}$$

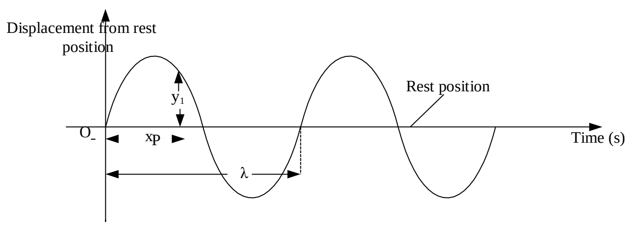
$$\boxed{Energy\ per\ unit\ volume = \mathbf{2\pi^2 f^2 A^2 \rho}}$$

Intensity of wave

This is the rate of flow of energy through an area of 1 m^2 perpendicular to the path of travel of wave

2.6 Equation of a progressive wave

Consider a wave form below



if the oscillation of the particle at O is simple harmonic with frequency f and angular velocity ω then its displacement y with time is given by

$$\boxed{y = a\sin\omega t} \dots\dots\dots\ (\mathbf{1})$$

Suppose the wave generated travels towards the right, the particle at P a distance x form O will lag behind by a phase angle φ

$$\boxed{y_1 = a\sin(\omega t - \varphi)} \dots\dots\dots\dots\ (2)$$

From the figure above, the phase angle of $2\pi = \lambda$ and phase angle $\varphi = x$

$2\pi = \lambda \dots\dots\dots\dots\dots\dots$ (1)

$\varphi = x \dots\dots\dots\dots\dots$ (2)

$$\frac{\varphi}{2\pi} = \frac{x}{\lambda}$$

$$\boxed{\varphi = \frac{2\pi x}{\lambda}}$$

Equation 2 will become

$$\boldsymbol{y_1} = \boldsymbol{a\sin}\left(\boldsymbol{\omega t} - \frac{2\pi x}{\lambda}\right)$$

But $\boldsymbol{\omega} = \boldsymbol{2\pi f} = \frac{\boldsymbol{2\pi}}{\boldsymbol{T}}$

$$\boldsymbol{y_1} = \boldsymbol{a\sin}\left(\frac{\boldsymbol{2\pi t}}{\boldsymbol{T}} - \frac{2\pi x}{\lambda}\right)$$

$$\boldsymbol{y_1} = \boldsymbol{a\sin 2\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} - \frac{x}{\lambda}\right)$$

Generally for a wave travelling to the right the equation of a progressive wave is $\boxed{y = \boldsymbol{a\sin 2\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} - \frac{x}{\lambda}\right)}$

Note;

If the wave is travelling to the left it arrives at P before O. This makes the vibration at P to lead the vibrations at O and its equation is given by $\boxed{y = \boldsymbol{a\sin 2\pi}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}} + \frac{x}{\lambda}\right)}$

Examples

2. A sound wave propagating in the x- direction is given by $y = \mathbf{0.4}\boldsymbol{sin}\ [\mathbf{10}\ (\mathbf{200}\boldsymbol{t} - x/100)]\ \boldsymbol{m}$

Find the speed of the wave

Solution

$y = \mathbf{0.4}\boldsymbol{sin}\ [\mathbf{10}\ (\mathbf{200}\boldsymbol{t} - \frac{x}{100})]$ Compare with

$y = \boldsymbol{a\sin 2\pi}\ (\boldsymbol{ft} - \frac{x}{\lambda})$

$\boldsymbol{2\pi ft} = \mathbf{10x200}\boldsymbol{t}$

$\boldsymbol{f} = \mathbf{318.5}\boldsymbol{Hz}$

$$\frac{10x}{100} = \frac{2\pi x}{\lambda}$$

$\lambda = 62.8m$

$\boldsymbol{v} = \boldsymbol{f\lambda}$

$\boldsymbol{v} = \mathbf{318.5}x62.8$

$\boldsymbol{v} = \mathbf{2.0x10^4}\boldsymbol{m/s}$

2.7 STATIONARY WAVE / STANDING WAVE

This a wave formed as a result of superposition of two progressive waves of equal amplitude and frequency but travelling at same speed in opposite direction.
Therefore in a stationary wave, energy does not move along with the wave.
Stationary waves are characterized by node (N) and antinodes (A)

Formation of a stationary wave

Stationary waves are formed when two waves of equal frequency and amplitude travelling at same speed in opposite direction are supposed resulting into formation of node and antinode
At antinodes, waves meet in phase and the amplitude is maximum. At nodes, the wave meet antiphase and amplitude is minimum.

Condition for stationary waves to be formed

- Waves must be moving in opposite direction.
- Waves must have the same speed, same frequency and equal amplitude.

Equation of a stationary wave

Consider a progressive wave travelling to the right. The displacement of any particle of the medium is given by

$$\boldsymbol{y_1 = a\sin(\omega t - \varphi)}$$

When this wave is reflected, it travels to the left. The displacement of any particle of medium will be

$$\boldsymbol{y_1 = a\sin(\omega t + \varphi)}$$

When the two waves superpose, the resultant displacement I given by $\boldsymbol{y = y_1 + y_2}$

$$\boldsymbol{y = a\sin(\omega t - \varphi) + a\sin(\omega t + \varphi)}$$

$$\boxed{\boldsymbol{y = 2a\cos\varphi\ \sin\omega t}}$$

Where amplitude of vibration is $2a\cos\varphi$

Where $\varphi = \frac{2\pi x}{\lambda}$

Note: amplitude of a stationary wave varies with x hence its not constant

Principle of super position of waves

It states that for two wave travelling in the same region, the total displacement at any point is equal to the vector sum of their displacement at that point when the two waves over lap

Examples

A plane progressive wave is given by

$y = a\ \sin\left(100\,\pi\, t - \frac{10\pi}{9}\,x\right)$ where x and y are millimetres and t is in seconds

(i) write the equation of the progressive wave which would give rise to a stationary wave if superimposed on the one above
(ii) find the equation of the stationary wave and hence determine its amplitude of vibration
(iii) determine the frequency and velocity of the stationary wave.

solution

(i) $y_2 = a \sin(100\pi t + \frac{10\pi}{9} x)$

(ii) $y = y_1 + y_2$

$$\boldsymbol{y} = a \sin(100\pi t - \frac{10}{9}\pi x) + a \sin(100\pi t + \frac{10}{9}\pi x)$$

$$\boldsymbol{y} = \mathbf{2}a \cos(\frac{10}{9}\pi x)\sin(100\pi t)$$

Amplitude of vibration $is\ \mathbf{2a}\ cos\ (\frac{10\pi}{9}x)$

(iii) $2\pi f = 100\pi$

$$f = 50Hz$$
$$v = 0ms^{-1}$$

Differences between stationary waves and progressive waves

Stationary waves	Progressive waves
1. Amplitude of the particles in the medium varies with position along the wave	1. All particles in the transmitting medium oscillate with the same amplitude.
2. Wave energy is not transferred but confined to a particular section of a wave	2. Wave energy is transferred form one point to another along the wave
3. Distance between any two successive nodes or antinodes is equal to $\lambda/2$	3. Distance between any two successive crests or troughs is equal to λ
4. Has nodes and antinodes	4. Doesn't have nodes and antinodes

2.8 MECHANICAL OSCILLATION

There are three types of oscillation i.e.

a) Free oscillation b) Damped oscillation c) Forced oscillation

a) Free oscillations

These are oscillations in which the energy of the system remains constant and is not lost to the surrounding. The amplitude of oscillation remains constant with time.

b) Damped oscillations

These are oscillations in which energy of oscillating system loses energy to the surrounding as a result of dissipative forces acting on it. Amplitude of oscillation decreases with time.

C) FORCED OSCILLATIONS

These are oscillations where the system is subjected to a periodic force which sets the system into oscillation. When the periodic force has the same frequency as the natural frequency of the oscillating system then resonance occurs.

2.9 SOUND WAVES

Sound is any mechanical vibration whose frequency lies within the audible range.
Sound waves propagate in air by series of compressions and rare factions.

Explain why sound propagates as an adiabatic process

Sound waves propagate in air by series of compressions and rare factions. In compressions the temperature of air rises unless heat is withdrawn. In rare factions, there is a decrease in temperature. The compressions and rare factions occur so fast that heat does not enter or leave the wave. Hence the process is adiabatic.

Characteristic of sound

a) Pitch

This is the characteristic of sound by which the ear assigns a place on a musical scale.
Pitch depends on the frequency of vibration of the sound waves ie it increases as the frequency of sound increases.

b) Loudness.

This is the magnitude of the auditory sensation produced by sound. Or Amount of sound energy entering the ear per second.

Factors that affect Loudness

- Sound intensity
- Amplitude of sound.

ECHOES

An echo is a reflected sound.

The time that elapses between hearing the original sound and hearing the echo depends on;

a) The distance away from the reflecting surface.
b) The speed of sound in the medium.

REVERBERATION

When sound is reflected from a hard surface close to the observer, the echo follows the incident sound so closely that the observer may not be able to distinguish between the two. Instead the observer gets an impression or hears a prolonged original sound. This effect is referred to as reverberation

Briefly explain why reverberation is necessary while making speeches

Too short a reverberation time makes a room sound dead but if it is too long, confusion results. For speeches half a second is acceptable. Reverberation time is made the same irrespective of the size of the audience b lining the walls with a soft material so that there is reduced reflection of sound

Refraction of sound

This is the change in the speed of sound waves as they move from one medium to another of different optical densities.

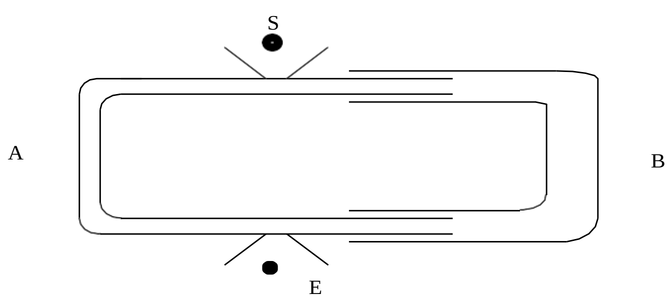
Explain why sound is easily heard at night than during day time

Distant sounds are more audible at night than day because the speed of sound in warm air exceeds that in the cold air and refraction occurs. At night the air is usually colder near the ground than it is higher up and refraction towards the earth occurs. During the day, the air is usually warmer near the ground than it is higher up

Interference of sound

Interference of waves is the superposition of waves from different two coherent sources resulting into alternate regions of maximum and minimum intensity.

Experiment to show interference of longitudinal waves



- Tube A is fixed while B is free to move.
- A note is sounded at S and detected at E.
- Tube B is then **pulled out slowly**. It is noted that the sound detected at E increases to a maximum and reduces to a minimumi intensity at **equal intervals of length** of the tube.
- The alternate maximum and minimum intensity of sound are interference patterns

Differences between sound and light waves

Sound waves	Light waves
- They cant travel through a vacuum	- They can travel through a vacuum
- They travel at a low speed i.e 330m/s	- They travel at a high speed i.e $3x10^{8}$m/s
- Require a material medium for their transmission	-Do not require a material medium for their transmission
- They cant eject electrons from a metal surface	- They can eject electrons from a metal surface by photo electric emission
- They are longitudinal waves	- They are transverse waves

2.10 BEATS

A beat is aperiodic rise and fall in the intensity of sound heard when two notes of nearly equal frequency but similar amplitude are sounded together.

Formation of beats

When two waves of nearly equal frequency and similar amplitude are sounded together they superpose.
When they meet in phase constructive interference takes place and a loud sound is heard. When they meet out of phase destructive interference takes place and a soft sound is heard.
A periodic rise and fall in intensity of sound is heard which is called beats

Beat frequency

Its defined as the number of intense sounds heard per second

Derivation of Beat frequency

Let f_1 and f_2 be frequencies of two sound notes.
Suppose a note of frequency $f_1$makes one cycle more than other in time T.
The number of cycles of frequency $f_1 = f_1T$
The number of cycles of frequency $f_2 = f_2T$
$f_1T - f_2T = 1$

$(f_1 - f_2)T = 1$

$1/T = (f_1 - f_2)$

But $\frac{1}{T} = f$

$\boxed{f = f_1 - f_2}$ This is called beat frequency

Uses of frequency

- Used in measurement of frequency of a note
- Determination of frequency of a musical note
- Tuning an instrument to a given note

Measurement of frequency of a note

- ➢ A note is sounded together with a tuning fork of known frequency, f_T
- ➢ The number of beats, n in t seconds are counted and the beat frequency, f_b

= n/t is calculated.

- ➢ One prong of the tuning fork is loaded with plasticine and then the experiment repeated. The new beat frequency $f_b{}^1$ is determined
- ➢ If $f_b{}^1 < f_b$then the frequency of the test note $f_n$is calculated from $f_n = f_T + f_b$
- ➢ If $f_b{}^1 > f_b{}^1$ then the frequency of the test note $f_n$is calculated from $f_n = f_T - f_b$

Example

1. Two tuning forks of frequency 256Hz and 250Hz respectively are sounded together in air. Find the number of beats per second produced

Solution

$f = f_1 - f_2$ | $f = 256 - 250$ | $\boldsymbol{f = 6\ Hz}$

2.11 Assignment

(Progressive, stationary waves in strings and pipes, beats)

1. The displacement y at any point and at any time t for a wave propagating in the positive x – direction is given by $y = 0.05\sin(100\pi t - 3\pi x)$, where x and y are metres. State for the wave motion (i) amplitude (ii) frequency (iii) wavelength (iv) the velocity

2. The displacement of a given wave traveling in the x – direction at the time t is $y = a\sin 2\pi\left(\frac{t}{10} - \frac{x}{2}\right)$ m. Find (i) velocity of the wave (ii) period of the wave.

3. The equation $y = A\sin(\omega t - kx)$ represents a plane wave traveling in a medium along the x – direction, y being the displacement at the point x at time t. (i) Deduce whether the wave is travelling in the positive x – direction or in the negative x – direction. (ii) If a =1.0x10^{-7} m, ω=6.6 x 10^{-3} rads^{-1} and k = 20 m^{-1} for the equation in (i) above, calculate (ii) the speed of the wave (ii) the wave frequency (iii) the maximum speed of a particle of the medium due to the wave.

4. A progressive wave is given by $a\sin\left(100\pi t - \frac{10\pi x}{9}\right)$, where x and y are in mm and t s.
(i) Write the equation of a progressive wave which would give rise to a stationary wave if superimposed on the above.
(ii) Find the equation of the stationary wave and hence determine the amplitude
(iii) Determine the velocity and frequency of the stationary wave.

5. A string vibrates according to the equation $y = 5\sin\frac{\pi x}{3}\cos 40\pi t,$ where x and y are in cm and t is in seconds. (i) What are the amplitudes of the component waves whose superposition gave rise to the above vibration. (ii) what is the distance between the nodes.

6. a) A progressive and stationary wave each have a frequency of 240Hz and a speed of 80ms^{-1} calculate the phase difference between two vibrating points in the progressive wave when they are 6 cm apart.
b) A wave of amplitude 0.2m, wave length 2m and frequency 50Hz , propagating in the X- direction . If the initial displacement is 0 at pt x = 0. (i) Write the expression of the displacement of the wave at any time. (ii) Find the speed of the wave.

7. Give the factors that affect the frequency of the transverse wave traveling along a string and how the frequency varies with each factor.
(b)A string of strength 31.6cm of fixed at both ends so that it is taut. The lowest frequency of the transverse were it can produce is 880Hz .Calculate the speed of wave.

8. a) A wire of length 50 cm, density 8.0 gcm-3 is stretched between two points. If the wire is set to vibrate at a fundamental frequency of 15 Hz, calculate (i) the velocity of the wave along the wire. (ii) the tension per unit area of cross section of the wire.
b) two open pipes of length 92 cm and 93 cm are found to give beat frequency of 3.0 Hz when each is sounding in its fundamental notes. If the end errors are 1.5 cm and 1.8 cm respectively, calculate the (i) velocity of sound in air (ii) frequency of each note.

9. A stretched wire of length 0.75m, radius 1.36mm and 1380kgm^{-3} is dumped to both sides and is plucked in the middle .The fundamental note is produced by the wire has the same frequency as the first overtone in the pipe of length 0.15m closed at one end (i) Sketch the standing wave pattern in the wire. (ii) Calculate the tension in the wire (Velocity of sound in air =330ms^{-1})

10. a) A glass tube open at the top is held vertically and filled with water. A tuning fork vibrating at 264 Hz is held above the table and water is allowed to flow out slowly .The first resonance occurs when the water level is 31.5cm from the top while the 2nd resonance occurs when the water level is 96.3cm from the top. Find (i) Speed of sound in the air column.(342.1ms^{-1})

(i) End correction. (0.009m)

(b) A long glass tube is filled with water. A tuning fork is held at the mouth of the tube and the tube is gradually emptied. Explain what happens.
c) A small speaker emitting a note of 250Hz is placed over the open upper end of a vertical tube when it is full of water. When the water is gradually run out of the tube, the air when it is 0.98m below the top column resonates, initially when the water surface is below 0.310 m below the top. Find velocity and end correction.

11. a) A steel wire of length 40.0 cm and diameter 0.0250 cm vibrates transversely in unison with the tube, open at each end and of effective length 60.0 cm, when each is sounding its fundamental note. The air temperature is 27˚C. Find the tension in the wire. (Assume that the velocity of sound in air at 0˚C is 331ms^{-1} and density of steel is 7800kgm^{-3}.)
b) A piano string 1.5m long is made of steel of density 7.7 x10^{3}kgm^{-3} and young's modulus 2.0x10^{11}Nm^{-2}. It is maintained at a tension which produces an elastic strain of 1% in the string. What is the frequency of the transverse vibrations of the strings?

12. A tuning fork of frequency 256 Hz is used to tune a sonometer wire of length 0.850m. The vibrating length of the wire is then shortened to 0.800m. (i) What would be the new frequency of the string when plucked? (ii) find the beat frequency heard when the tuning fork and the shortened wire are sounded simultaneously.

13. a) State the conditions that lead to the establishment of the standing wave.
b) A uniform tube 50cm long stands vertically with its lower end dipping into the water. The tube resonates to a tuning fork of frequency 256Hz when its length above water is 12cm and again when it is 39.6cm. Estimate the lowest frequency to which the tube resonates when it is open at both ends.
c) A wave of mass 1.0x 10^{-2}kg and decimeter 6.0x10^{-4}m is stretched between rigid supports 1.0m apart. The tension in the string is 60N. Find the change in the freq. of the fundamental rock when the temperature of the wire is lowered by 100K. (Young's modulus 2.0x10^{11}Pa, linear expansion 1.5x10^{-6}K^{-1})

14. Beats are produced by a plucked stretched wire and a resonance tube closed at one end each sounding at its fundamental note. The air column has a length of 0.168m the end correction being 0.012m. The wire has a vibrating length

of 0.27m and is under tension of 100N. The mass of the position of the wire is $4x10^{-4}$kg. (i) Calculate the frequency of the beats heard. If velocity of sound in air column is $350ms^{-1}$. **(ans. 5 Hz)**
(ii) Calculate the change in tension of the wire that would make the frequencies of the two notes the same. **(ans. 2N)**

15. The wire of a sonometer of mass per unit length $10^{-3}kgm^{-1}$ is stretched on the two bridges by a load of 40N. When the wire is struck at the center point so that it executes its fundamental vibration, and at the same time a tuning fork of 264Hz is sounded and beats are heard and found to have a frequency of 3Hz. If the load is slightly increased, the beat frequency is lowered. Calculate the separation of the standing wave.

16. The speed of sound waves in air is 330 ms^{-1}. A source of sound of frequency 300Hz radiates energy in all directions at a rate of 10 W. Find (i) the intensity of the sound at a distance of 20 m from the source (ii) the amplitude of the sound wave at this distance. density at s.t.p = $1.29kgm^{-3}$)

17. An organ pipe is sounded with a tuning fork of frequency 256 Hz. When the air in the pipe is at a temperature of 15°C, 23 beats in 10 seconds, when the tuning fork is loaded with a small piece of wax, the beat frequency is found to decrease. What change of temperature of the air in the pipe is necessary to bring the pipe and the unloaded fork into unison? (**ans: 5.3°C**)

18. A pianoforte wire having a diameter of 0.90 mm is replaced by another wire of the same material but with diameter 0.93 mm. If the tension of the wire is the same as before, what is the percentage change in the frequency of the fundamental note? What percentage change in the tension would be necessary to restore the original frequency?

2.11 DOPPLER EFFECT

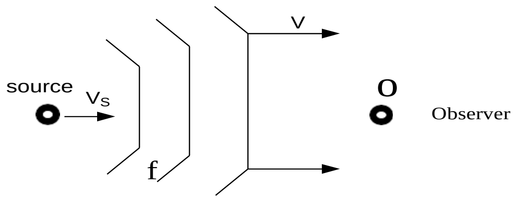
This is the apparent change in frequency and wave length of a wave when there is relative motion between the source of the waves and the observer

Doppler Effect takes place in both sound and light

Doppler Effect in sound

Case 1: source of sound in motion but observer fixed

(a) Source moving towards a stationary observer



$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source= $v - v_s$

Apparent change in wavelength, $\lambda_a = \frac{v - v_s}{f}$

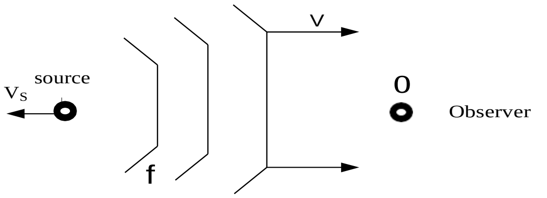
Velocity of wave relative to observer= $v - 0 = v$

Apparent change in frequency, $f_a = \frac{v}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v}{v - v_s}\right) f}$$

Since $v - v_s < v$ then the apparent frequency appears to increase when the source moves towards an observer

(b) Source moving away from a stationary observer



$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source= $v + v_s$

Apparent change in wavelength, $\lambda_a = \frac{v + v_s}{f}$

Velocity of wave relative to observer= $v - 0 = v$

Apparent change in frequency, $f_a = \frac{v}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v}{v + v_s}\right) f}$$

Since $v + v_s > v$ then the apparent frequency appears to decrease when the source moves away from an observer.

Note:

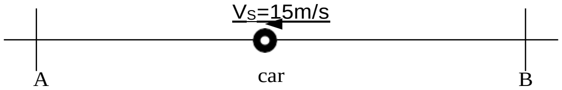
When the source is in motion, only wavelength and frequency change but the speed of the sound waves is not affected.

$$f_A = \left(\frac{v}{v - v_s}\right)f$$

Examples

1. A car sounds its horn as it travels at a steady speed of 15m/s along a straight road between two stationary observers A and B. Observer A hears a frequency of 538Hz while B hears a lower frequency. Calculate the frequency heard by B if the speed of sound in air is 340m/s.

Solution



Since B receives sound of lower frequency, the car is moves away from B

Towards A: $\boxed{f_A = \left(\frac{v}{v - v_s}\right) f}$

$$538 = \left(\frac{340}{340 - 15}\right) f$$

$$f_A = 514.265Hz$$

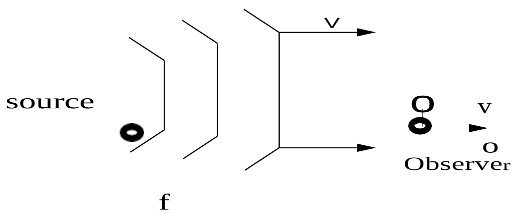
Away from B: $f_B = \left(\frac{v}{v+v_s}\right) f$

$$f_B = \left(\frac{340}{340 + 15}\right) x514.265$$

$$f_B = 492.54Hz$$

Case2: observer in motion while the source is stationary

(a) observer moving away from a stationary source



$v_o -$ *velocity of the observer*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source= $v - o = v$

Apparent change in wavelength $, \lambda a = \frac{v}{f}$

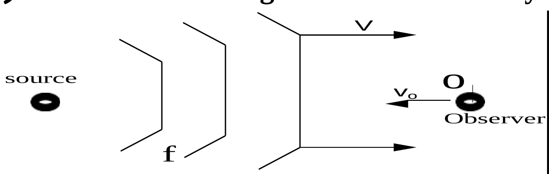
Velocity of wave relative to observer= $v - v_o$

Apparent change in frequency, $f_a = \frac{v - v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v - v_o}{v}\right)}$$

Since $v - v_o < v$ then the apparent frequency appears to decrease when the observer moves away from the source

(b) An observer moving towards a stationary source



$v_o -$ *velocity of the observer*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source= $v - o = v$

Apparent change in wavelength, $\lambda_a = \frac{v}{f}$

Velocity of wave relative to observer= $v + v_o$

Apparent change in frequency, $f_a = \frac{v+v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v+v_o}{v}\right) f}$$

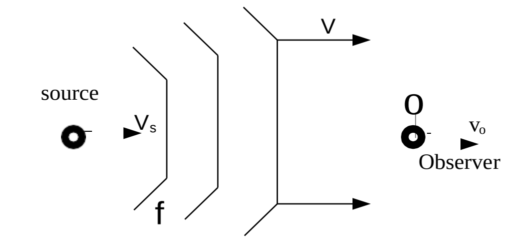
Since $v + v_s > v$ then the apparent frequency appears to increase when the observer approaches the source

Note:

The motion of the observer has no effect on the wavelength of the sound but it affects the relative velocity of sound

Case3: observer and source in motion

(a) Both in same direction(observer ahead of source)



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source $= v - v_s$

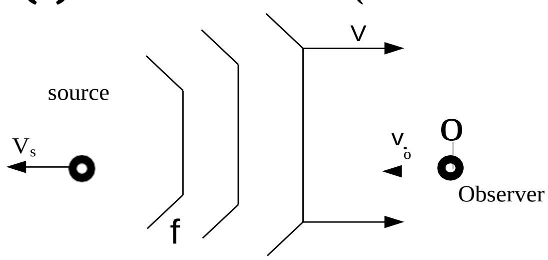
Apparent change in wavelength $\lambda_a = \frac{v-v_s}{f}$

Velocity of wave relative to observer $= v - v_o$

Apparent change in frequency, $f_a = \frac{v-v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v-v_o}{v-v_s}\right)}$$

(b) Both in same direction(source ahead of observer)



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

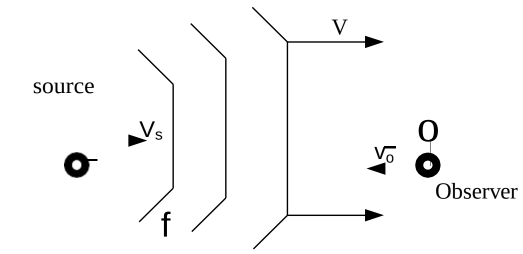
Apparent change in wavelength, $\lambda_a = \frac{v+v_s}{f}$

Velocity of wave relative to observer $= v + v_o$

Apparent change in frequency, $f_a = \frac{v+v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v+v_o}{v+v_s}\right)}$$

(c) Both moving towards each other



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

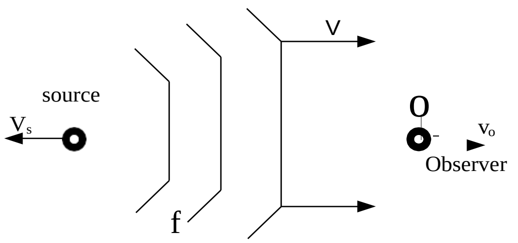
Apparent change in wavelength, $\lambda_a = \frac{v - v_s}{f}$

Velocity of wave relative to observer= $v + v_o$

Apparent change in frequency, $f_a = \frac{v + v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v + v_o}{v - v_s}\right)}$$

(d) Both moving away from each other



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source= $v + v_s$

Apparent change in wavelength, $\lambda_a = \frac{v + v_s}{f}$

Velocity of wave relative to observer= $v - v_o$

Apparent change in frequency, $f_a = \frac{v - v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v - v_o}{v + v_s}\right)}$$

Generally Waves reflected by the obstacle

$$\boxed{f_a = \left(\frac{v \pm v_o}{v \pm v_s}\right)}$$

Applications of Doppler Effect

(i) Used in radar speed traps
(ii) Measurement temperature of hot gases
(iii) Used in measurement of speed of the star

Speed traps

- Microwaves of frequency f_o from a stationary radar set are directed towards a motor vehicle moving with speed u
- Microwaves reflected from the moving car are detected at the radar
- The reflected signal mixes with the transmitted signals to obtain beats
- The beat frequency Δf which is equal to the difference between the frequency of the received and transmitted signal is determined
- The speed of the vehicle is $u = \frac{v\Delta f}{2f_o}$

Measurement of plasma temperature

- The broadening $\Delta\lambda$ of a spectral line emitted by the plasma is determined using a diffraction grating
- $\frac{\Delta\lambda}{\lambda_o} = \frac{2u}{c}$
- Assume $u = v_{rms}$
- $\frac{1}{2}\boldsymbol{mu^2} = \frac{3}{2}\boldsymbol{RT}$ where m= $\boldsymbol{molar\ mass}$

  $\boldsymbol{u} = \left(\frac{\boldsymbol{3RT}}{\boldsymbol{m}}\right)^{\frac{1}{2}}$
- $\boldsymbol{T} = \frac{\boldsymbol{m}}{\boldsymbol{12R}}\left(\frac{\Delta\lambda}{\lambda_o{}^2}\right)^{\boldsymbol{2}}\boldsymbol{C^2}$ T=

Speed of star

- The wavelength , λ of light emitted by the star is measured
- The absorption spectrum of an element known to be in the star is examined.
- The wavelength λ^1 of the missing line is measured
- Doppler shift = $|\lambda^1 - \lambda|$
  - $\left|\frac{\lambda^1-\lambda}{\lambda}\right| = \frac{u_s}{C}$
  - $u_s = \left|\frac{\lambda^1-\lambda}{\lambda}\right| C$

2.12 WAVES EXERCISE ON DOPLER EFFECT

1. A car sounds its horn as it travels at a steady speed of 15 ms^{-1} along a straight road between two stationary observers A and B. The observer A hears a frequency of 538 Hz while B hears a lower frequency. Calculate the frequency heard by B, assuming the speed of sound in air is 340 ms^{-1}.
2. A whistle of frequency 1000 Hz is sounded on a car travelling with a velocity of 18ms^{-1} towards a stationary source of sound of frequency 900Hz; calculate the frequency of the beats heard by the driver of the car.
3. A source of sound moving with velocity u_s approaches an observer moving with velocity u_o in the same direction. (i) Derive the expression for the frequency of sound heard by the observer. (ii) Explain what happens to the pitch of the sound heard by the observer in (i) above when the observer's source moves faster than the source.
4. An observer traveling with a constant velocity of 20ms^{-1}, passes close to a stationary source of sound and notices that there is a change of frequency of 50Hz as she passes the source. Find the frequency of the source. [Speed of sound in air =340ms^{-1}].
5. A locomotive train approaching a tunnel in a cliff at 90kmh^{-1} is sounding a whistle of frequency 100Hz. Calculate the change in the frequency of the echo that the driver hears as he approaches and as he moves away from the tunnel.
6. A train moving with uniform velocity, v, sounds a horn as it passes a stationary observer. (i) Derive the expression for the apparent frequency of the sound detected by the observer. (ii) If the frequency of the sound detected by the observer after the train passes is 1.2 times lower than frequency detected in c(ii), find the speed of the train. (Speed of sound in air = 330ms^{-1})
7. A tuning fork of frequency 256 Hz is used to tune a sonometer wire of length 0.850m. The vibrating length of the wire is then shortened to 0.800m. (i) What would be the new frequency of the string when plucked? (ii) find the beat frequency heard when the tuning fork and the shortened wire are sounded simultaneously.
8. The wire of a sonometer of mass per unit length $1x10^{-3}kgm^{-3}$ is stretched on the two bridges by a load of 40N. When the wire is struck at the centre point so that it executes its fundamental vibration and at the same time a tuning fork of 264 Hz is sounded, beats are heard and found to have a frequency of 3 Hz. If the load is slightly increased, the beat frequency is lowered. Calculate the separation of sonometer wire.
9. A source of sound, when stationary, generates waves of frequency of 500Hz. The speed of sound is 340 ms^{-1}. Find the wavelength of the waves detected by the observer and the frequency observed when (a) the source is stationary and observer moves towards it with speed 20 ms^{-1}, b) the source moves away from a stationary observer with speed 30 ms^{-1} c) the source moves with speed 30ms^{-1}in a direction away from the observer and the observer moves with speed 20 ms^{-1} towards the source.

{ a)0.680 m, 529Hz, b) 0.740 m, 459Hz, c) 0.740 m, 486 Hz)}

10. A train emerges from a tunnel at a speed of 20 ms-1 and sounds its whistle, which has a frequency of 450 Hz. Calculate the frequency of the echo from the tunnel entrance as observed by the train driver. (400Hz)

11. A hooter of frequency 360 Hz is sounded on a train approaching a tunnel in a cliff face at 25 ms^{-1}, normal to the cliff. Calculate the observed frequency of the echo from the cliff face as heard by the train driver. Assume the speed of sound in air is 330 ms^{-1} (419 Hz)

12. A vibrating sonometer wire emits a note which gives a beat frequency of 6.0 Hz when sounded in unison with a standard turning fork of frequency 256 Hz. When the fork is loaded the beat frequency decreases. Find the frequency of the note emitted by the sonometer? (262 Hz)
13. An observer moving at a speed of 10 ms^{-1} between two sources of sound A and B hears beats at 5 s^{-1}. If the frequency of waves produced by source A is 515 Hz and the observer is moving towards A, find the frequency of sound produced by B. (speed of sound in air is 340 ms^{-1})
14. (i) Derive an expression for the frequency of sound waves received by an observer in terms of' frequency of the sound, the speed of the source of the sound waves and the speed of sound when the source and observer are hurrying towards each other.
    ii) A stationary observer notices the pitch of a siren of a police car change in the ratio 4: 3 on passing him. If the speed of the sound is 350ms^{-1}, calculate the speed of the car.

2.13 RESONANCE

This is a condition obtained when a system is set to oscillate at its own natural frequency as a result of impulses received form another system vibrating at the same frequency

Other terms Fundamental frequency

This is the lowest possible frequency that an instrument can produce.

Overtones

These are note of higher frequencies than the fundamental frequency produced by an instrument.

Harmonic

These is one of the frequencies that can be produced by a particular instrument

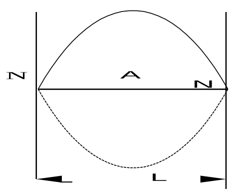
WAVES ON A STRETCHED STRING

When a stretched string is plucked, a progressive wave is formed and it travels to both ends which are fixed and these waves are reflected back to meet the incident wave. The incident and reflected waves both have the same speed, frequency and amplitude and therefore when they superimpose a stationary wave is formed.

Modes of vibration

When a string is plucked in the middle, the wave below is produced

(a) 1st harmonic (fundamental frequency)



$$l = \frac{\lambda}{2}$$
$$\lambda = 2l$$
$$v = f\lambda$$
$$\boxed{v = 2lf_o}$$

A is antinodes;

These are points on a stationary wave where particles have maximum displacement.

N is nodes;

This is a point on a stationary wave in which particles are always at rest (zero displacement)

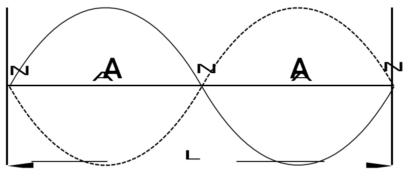
Note:

- The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$ where λ is wavelength.
- When a stationary wave is produced, the distance between the source and reflector is a multiple of $\frac{1}{2}\lambda$.

$$\boxed{distance = n\,\frac{\lambda}{2}}$$

Where n is the number of loops i.e. n is 1,2,3 … … … …

(b) 2nd harmonic (1st overtone)



$l = \lambda$

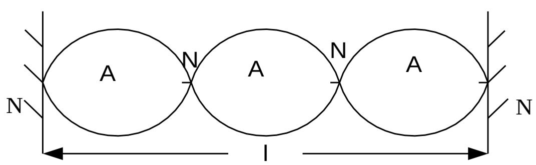
$v = f\lambda$

$v = lf_1$

$2lf_o = lf_1$

$\boxed{f_1 = 2f_o}$

(c) 3rd harmonic (2nd overtone)



$l = \frac{3}{2}\lambda$

$\lambda = \frac{2}{3}\,l$

$v = f\lambda$

$2lf_o = \frac{2}{3}lf_2$

$\boxed{f_1 = 3f_o}$

Generally $\boxed{f_n = nf_o}$

$\boxed{f_n = \frac{n\,v}{2\,l}}$ $n = 1,2,3,4,5,6\ n^{th} - harmonic$

Velocity of a transverse wave along a stretched string

The velocity of a wave on the sting depends on the following

(i) Tension T
(ii) Mass m
(iii) Length l

$V \propto T^x m^y l^z$

$V = k\,T^x m^y l^z$………. (x)

$[\,V\,] = [\,K\,][\,T]^x\,[m\,]^y\,[\,l^z]$

K is a dimensionless constant

$LT^{-1} = (MLT^{-2})^{\,x}\,(M)^y\,(L)^z$

For powers of T

$-2x = -1$………………..(1)

$x = \frac{1}{2}$

For powers of M,

$0 = x + y$……………….(2)

$0 = \frac{1}{2} + y$

$y = -\frac{1}{2}$

For powers of L,

$1 = x + z$

$z = \frac{1}{2}$

$V = k\, T^x m^y l^z$

$V = k\, T^{\frac{1}{2}} m^{\frac{-1}{2}} l^{\frac{1}{2}}$

$$\boxed{V = \sqrt{\frac{Tl}{m}}}$$

Examples

1. A string of length 0.5m has a mass of 5g. The string is stretched between two fixed points and plucked. If the tension is 100N, find the frequency of the second harmonic

Solution

$v = \sqrt{\frac{Tl}{m}}$

$v = \sqrt{\frac{100x0.5}{5x10^{-3}}}$

$v = 100m/s$

$v = f_0\lambda$

$v = 2lf_0$

$100 = 2x0.5f_0$

$f_0 = 100Hz$

$f_2 = 2\, f_0$

$f_2 = 2x100$

$f_2 = 200Hz$

Alternatively

$v = \sqrt{\frac{Tm}{l}}$

$v = \sqrt{100x5x10^{-3}\ 0.5}$

$v = 100m/s$

$f_n = \frac{n\, v}{2l}$

$f_2 = \frac{2x\ 100}{2x0.5}$

$f_2 = 200Hz$

2. A wire under a tension of 20N is plucked at the middle to produce a note of frequency 100Hz. Calculate the;

(i) diameter of the wire if its length is 1m and has a density of $600kgm^{-3}$

(ii) frequency of the first overtone **An(200Hz)**

solution

$v = \sqrt{\frac{Tl}{m}}$

$2lf_0 = \sqrt{\frac{Tl}{m}}$

$2x1x100 = \sqrt{\frac{20x1}{m}}$

$m = 0.0005kg$

$\rho = \frac{m}{volume}$

$volume = \frac{0.0005}{600} = 8.33x10^{-7}m^3$

$volume = \pi r^2 l$

$r = \sqrt{\left(\frac{8.33x10^{-7}}{\pi}\right)}$

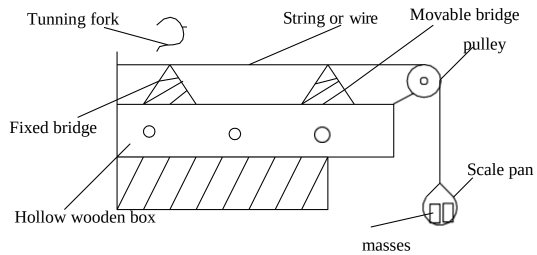
$r = 5.15x10^{-4}m$

$d = 2r = 2x5.15x10^{-4}$

$= 1.03x10^{-3}m$

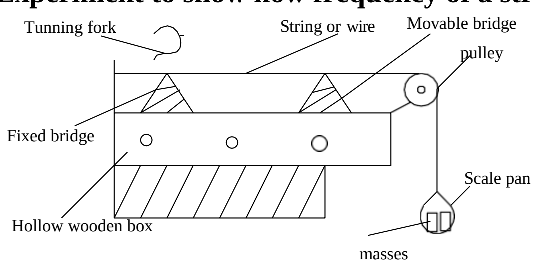
Factors on which frequency of a stretched string depends

Experiment to investigate the variation of frequency of a stretched string with length



- The experiment is set up as shown above
- Pluck the string in the middle and place a sounding tuning fork near it
- Move the bridge B towards A until when a loud sound is heard
- The distance l between the bridges is measured and recorded together with the frequency of the tuning fork
- Repeat the above procedures using different tuning forks of different frequency
- Tabulate your results including values of $\frac{1}{l}$
- Plot a graph of f against $\frac{1}{l}$
- It's a straight line passing through the origin
- implying that $f \propto 1/l$.

Experiment to show how frequency of a stretched string varies with tension



- The experiment is set up as shown above
- The length l between two bridges is kept constant
- A suitable mass m is attached to the free end of a string (scale pan)
- Pluck the string in the middle and a tuning fork of known frequency f is sounded near it
- Vary the mass on the scale pan until when a loud sound is heard
- Record the mass of the corresponding frequency f in a suitable table including values of f^2

- Repeat the above procedures using different tuning forks of different frequency
- Plot a graph of f^2against m
- It's a straight line passing through the origin implying that $f^2 \propto m$
- Since $T = mg$, it implies $f \propto \sqrt{T}$ hence frequency increases with increase in frequency

2.14 Resonance of air in pipes

When air is blown in a pipe, a longitudinal wave is formed. This wave travels along the pipe and if the pipe is closed the wave will be reflected back. The incident and reflected wave both have the same speed, same frequency and same amplitude. This results into formation of a stationary wave.

These are two type of pipe for air vibrations.

(i) Open pipes

This is one that has both ends open eg trumpet, a flute

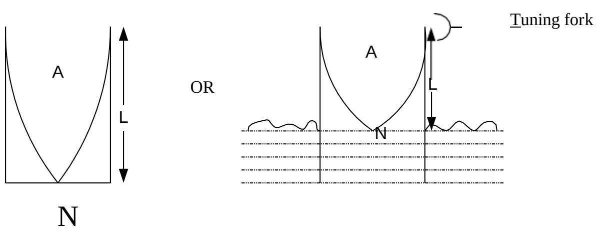
(ii) Closed pipes

It is one in which one end is open, while the other is closed eg a long drum.

a) Modes of vibration in closed pipes

For closed pipes, a node is formed at a closed end and an antinode at the open end

First harmonic / fundamental note



The length of the air column is L

$$L = \frac{1}{4}\lambda$$

$$\lambda = 4l$$

$$v = f_0\lambda$$

$$\boxed{v = 4lf_0}$$

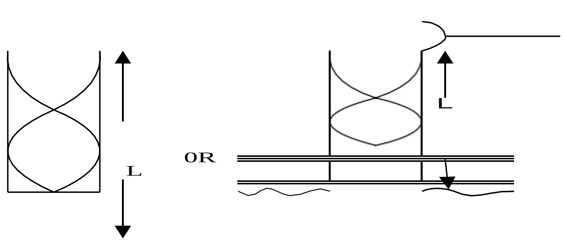
$$f_0 = \frac{v}{4l}$$

f_0 is the fundamental frequency

Describe the motion of air I a tube closed at one end and vibrating in its fundamental frequency

Air at end A vibrates with maximum amplitude. The amplitude of vibration decreases as end N is approached. Air at N is stationary. End N is node while end A is antinode

First overtones / third harmonic



The length of the air column is L

$$L = \frac{3}{4}\lambda$$

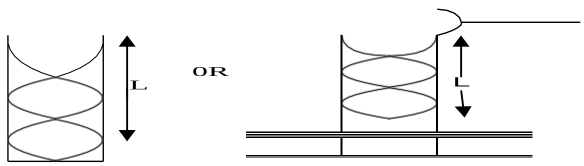
$$\lambda = \frac{4}{3}l$$

Second overtones/ fifth harmonic

$v = f_1 \lambda$

$4lf_0 = f_1 \frac{4}{3} l$

$f_1 = 3 f_0$



The length of the air column is L

$$L = \frac{5}{4}\lambda$$

$$\lambda = \frac{4}{5}l$$

$$v = f_2\lambda$$

$$4lf_0 = f_2\,(4/5)l$$

$$f_2 = 5\,f_0$$

Note: In closed pipes, harmonics produced must be with frequencies f_0, $3f_0$, $5f_0$, $7f_0 \ldots \ldots \ldots \ldots \ldots$

This implies that only <u>odd harmonics</u> are produced can be produced by closed pipes

$\boxed{f_n = \frac{n\,v}{4\,l}}$ $n = 1,3,5,7,9 \ldots \ldots$ $n^{th} - harmonic$

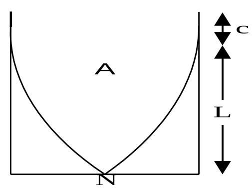
Variation of pressure with displacement of air in a closed pipe

At the mouth of the pipe, the air is <u>free to move</u> and therefore the displacement of air molecules is <u>large</u> and <u>pressure is low</u>. At the closed end the molecules are <u>less free</u> and the displacement is <u>minimal</u> and the <u>pressure is high</u>

END CORRECTIONS

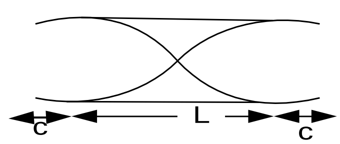
An antinode of stationary wave in a pipe is not formed exactly at the end of the pipe. Instead it is displaced by a distance, c. This distance is called the end correction

The effective length of a wave in the closed pipe of length l is $l + c$



$$l + c = \frac{\lambda}{4}$$

The effective length of a wave in an open pipe of length l is $l + 2c$



$$l + 2c = \frac{\lambda}{2}$$

Note:

C is related to the radius of the pipe by an equation $c = 0.6r$ implying that the end correction is more significant for wide pipes

Examples

1. A cylindrical pipe of length 30cm is closed at one end. The air in the pipe resonates with a tuning fork of frequency 825Hz sounded near the open end of the pipe. Determine the mode of vibration of air assuming there is no end correction. Take speed of sound in air as 330m/s.

Solution

$$f_n = \frac{nv}{4l}$$

$$825 = \frac{n\,x330}{4\,x0.3}$$

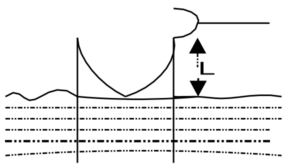
$$n = 3$$

But $n = 1,3,5,7,9 \ldots \ldots$

Mode of vibration is third harmonic

2. A long tube is partially immersed in water and a tuning fork of 425Hz is sounded and held above it. If the tube is gradually raised, find the length of the air column when resonance first occurs. [speed of sound in air is 340m/s]

Solution



$f = 425Hz,\ v = 340ms^{-1}$

$v = f\lambda$

$340 = 425x\lambda$

$\lambda = 0.8\ m$

$$L = \frac{1}{4}\lambda$$

$$L = \frac{1}{4}x0.8$$

$L = 0.2\ m$

3. A tube 100cm long closed at one end has its lowest frequency at 86.2Hz. With a tube of identical dimensions but open at both ends, the first harmonic occurs at 171Hz. Calculate

(i) The speed of sound

(ii) End correction

Solution

(i) For closed pipe: $v = 4(l + c)f_0$

$v = 4(1 + c)86.2$……….(1)

For open pipe: $v = 2(l + 2c)f_0$

$v = 2(1 + 2c)171$……….(2)

Equating (1)and (2)

$4(1 + c)86.2 = 2(1 + 2c)171$

$c = 8.25x10^{-3}m$

(ii) $v = 4(1 + c)86.2$

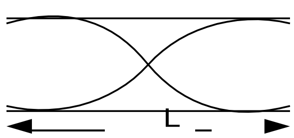
$v = 4(1 + 8.25x10^{-3})86.2$

$v = 347m/s$

b) Modes of vibration in open pipes

In open pipes, antinodes are found at the two open ends of the pipe

First harmonic / fundamental note



$$l = \frac{1}{2}\lambda$$

$\lambda = 2\,l$

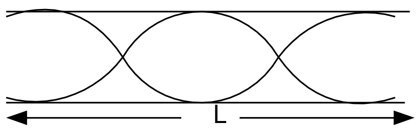
$v = f_0\lambda$

$v = 2lf_0$

$$\boxed{f_0 = \frac{v}{2\,l}}$$

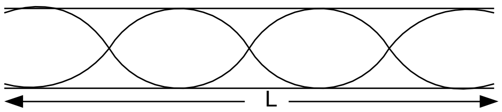
f_0 is the fundamental frequency

Second harmonic / first overtone



$$l = \lambda$$
$$v = f_1\lambda$$
$$2lf_0 = f_1 l$$
$$f_1 = 2\,f_0$$

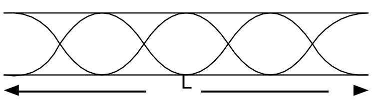
Third harmonic / Second overtone



$$l = \frac{3}{2}\lambda$$

$$\lambda = \frac{2}{3}\,l$$
$$v = f_2\,\lambda$$
$$2lf_0 = f_2 x \frac{2\,l}{3}$$
$$f_2 = 3\,f_0$$

Fourth harmonic / Third overtone



$$l = 2\,\lambda$$

$$\lambda = \frac{l}{2}$$
$$v = f_3\,\lambda$$
$$2lf_0 = f_3\,x\,\frac{l}{2}$$
$$f_3 = 4\,f_0$$

Note:

In open pipes, harmonics produced must be with frequencies $f_0, 2f_0, 3f_0, 4f_0, 5f_0 \ldots \ldots \ldots \ldots \ldots$ Open pipes produce both odd and even harmonics and this is why open pipes are preferred as musical instruments.

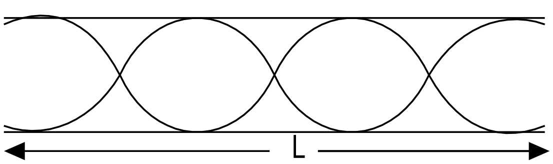
$$\boxed{f_n = \frac{n\,v}{2\,l}} \quad n = 1,2,3,4,5,6 \qquad n^{th} - harmonic$$

Example

The frequency of third harmonic in an open pipe is 660Hz, if the speed of sound in air is 330m/s. Find;

(i) the length of the air column
(ii) the fundamental frequency

Solution



i) $f = 660Hz, \quad v = 330ms^{-1}$

$$v = f\lambda$$
$$330 = 660x\lambda$$
$$\lambda = 0.5\,m$$

$$L = \frac{3}{2}\lambda$$
$$L = \frac{3}{2}x0.5$$
$$L = 0.75\,m$$

$ii)\quad f_2 = 3\,f_0$

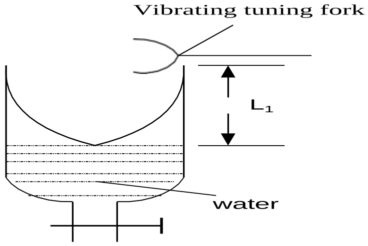
$$f_0 = \frac{660}{3}$$
$$f_0 = 220Hz$$

Note:

Different instruments produce different number of overtones. The numbers of overtones produced affect the quality of the note played. Hence the quality of the notes produced by different instruments are different

Qn: Explain why a musical note played on one instrument sounds different from the same note played on another instrument

Experiment: To measure velocity of sound in air by Resonance tube and a tuning fork of a known frequency



- A glass tube which can be drained from the bottom is filled with water.
- A sounding tuning fork of known frequency f is brought to the mouth of tube.
- The water is then slowly drained until a loud sound is heard.
- The tap is closed and the length of the air column l_1 is measured.
- The tuning fork is sounded again at the mouth of the tube and water is drained further until a loud sound is heard.
- The tap is closed and the length of the air column l_2 is measured.
- Velocity of sound in air is obtained from $v = 2f(l_2 - l_1)$

Theory

$$l_1 + c = \frac{1}{4}\lambda \text{.....................}(1)$$

$$l_2 + c = \frac{3}{4}\lambda \text{.....................}(2)$$

Equation (2) − Equation (1) 1

$(l_2 + c) - (l_1 + c) = 3/4\lambda - 1/4\lambda$

$$l_2 - l_1 = \frac{1}{2}\lambda$$

$$\lambda = 2(l_2 - l_1)$$

But $v = f\lambda$

$$\boxed{v = 2f(l_2 - l_1)}$$

Example…….

1. A tuning fork of frequency 256Hz produces resonance in a tube of length 32.5cm and also in one of length 95cm. Calculates the speed of sound in air column of the tube.

Solution

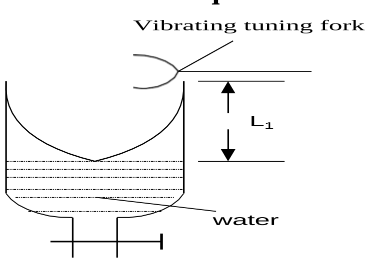
$v = 2f(l_2 - l_1)$

$v = 2 \times 256 \times \left(\frac{95 - 32.5}{100}\right)$

$v = 320ms^{-1}$

2. A uniform tube 50cm long is filled with water and a vibrating tuning fork of frequency 512Hz is sounded and held above the tube. When the level of water is gradually lowered, the air column resonates with the tuning fork when its length is 12cm and again when it is 43.3cm. Calculate
   (i) Speed of sound $\boldsymbol{An(322.56ms^{-1})}$
   (ii) The end corrections $\boldsymbol{An(3.67cm)}$
   (iii) Lowest frequency to which the air can resonate if the tube is empty **An(281.27Hz)**

Experiment: To measure velocity of sound in air using Resonance tube and different tuning forks of a known frequencies



- A glass tube which can be drained from the bottom is filled with water.
- A sounding tuning fork of known frequency f is brought to the mouth of tube.

Theory	
	❖ The water is then slowly drained until <u>a loud sound is heard</u>. ❖ The tap is closed and the length of the air column $\boldsymbol{l}$ is <u>measured</u>. ❖ The experiment is <u>repeated with other</u> tuning forks and the value of l and f is recorded including values of $\frac{1}{f}$ ❖ A graph of l against $\frac{1}{f}$ is plotted and slope s is obtained. ❖ Velocity of sound in air is obtained from $v = 4s$

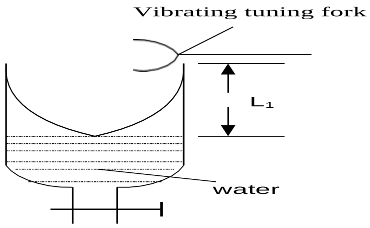
$$l + c = \frac{1}{4}\lambda$$

$$l = \frac{1}{4}\lambda - c$$

But $\lambda = \frac{v}{f}$

$$l = \frac{1}{4f}V - c$$

Experiment: To measure end corrections using Resonance tube and different tuning forks of a known frequencies



- A glass tube which can be drained from the bottom is filled with water.
- A sounding tuning fork of known frequency f is brought to the mouth of tube.
- The water is then slowly drained until a loud sound is heard.
- The tap is closed and the length of the air column l is measured.
- The experiment is repeated with other tuning forks and the value of l and f is recorded including values of $\frac{1}{f}$
- A graph of l against $\frac{1}{f}$ is plotted and the intercept c of the l axis determined from line graph
- The intercept c is the end corrections

2.15 VELOCITY OF SOUND IN GASES

Velocity of sound in gasses depends on the pressure and density of the gas

$$v \propto P\rho$$

$$v = kP\rho$$

Where k- constant

$[V] = [K][P]^x[\rho]^y$

$$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y$$

For L: $1 = -x - 3y$..............(1)

For M: $0 = x + y$.............(2)

For T: $-1 = -2x$...............(3)

$$x = \frac{1}{2}$$

$$y = -x$$

$$y = -\frac{1}{2}$$

$$v = k\sqrt{\frac{P}{\rho}}$$

But if $k = \sqrt{\gamma}$

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

γ-ratio of molar heat capacity at constant pressure to molar heat capacity at constant volume

Note: The speed of sound in air depends on pressure, density and temperature

Explanation

When temperature of air is increased, the pressure increases. If the air is not restricted in volume it expands leading to a reduction in density. From the above expression a reduction in density leads to increase in velocity. Hence increase in temperature leads to increase in velocity of sound in air

2.16 VELOCITY OF SOUND IN SOLIDS

Velocity of sound in solids depends on the young's modulus E and density ρ of the solid

$$v \propto E\rho$$

$$v = kE\rho$$

Where k- constant

$[V] = [K][E]^x[\rho]^y$

$$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y$$

For L: $1 = -x - 3y$.............. (1)

For M: $0 = x + y$............. (2)

For T: $-1 = -2x$ ....(3)

$$x = \frac{1}{2}$$

$y = -x$

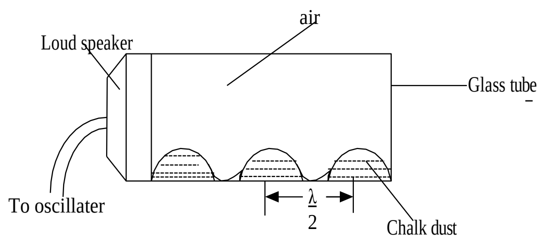
y=- $\frac{1}{2}$

$$v = k\sqrt{\frac{E}{\rho}}$$

But if $k = 1$

$$\boxed{v = \sqrt{\frac{E}{\rho}}}$$

Measurement of speed of sound in air using Kundt's dust tube

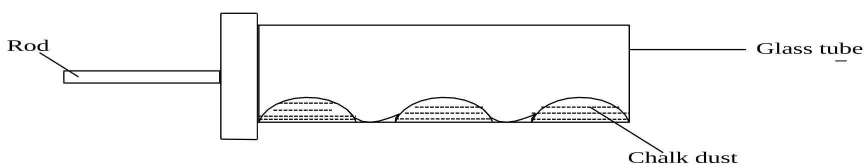


- A long glass tube is placed horizontally with chalk dust inside it
- The open end is fitted with a loud speaker which is connected to an oscillator of known frequency f
- When the oscillator is switched on, sound is produced and a stationary wave is formed in the glass tube which makes the power to settle into well-spaced heaps,
- Measure the distance l between the two consecutive heaps
- Wavelength of the wave generated is given by $\lambda = 2l$
- Speed of sound in air is got from $v = 2lf$

Note: Heaps are found at points where there are no vibrations (nodes)
Measurement of l from outside the tube may not be accurate hence a source of error

Examples

In an experiment to determine the speed of sound in air in a tube, chalk dust settled in heaps as shown in the diagram below;



If the frequency of the vibrating rod is 220Hz and the distance between three consecutive heaps is 1.50m, calculate the speed of sound in air

Solution

$\lambda = 1.50m$

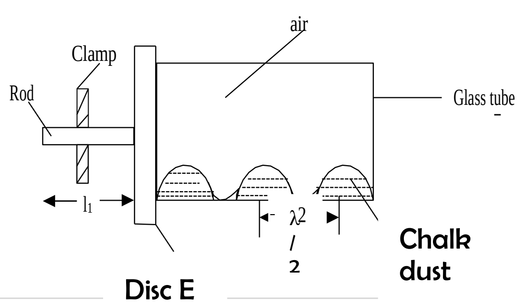
$v = \lambda f$

$v = 1.5x220$

$\boldsymbol{v = 330m/s}$

Measurement of speed of sound in a rod using Kundt's dust tube

- Sprinkle some chalk dust along the interior of the tube



- Sprinkle some chalk dust along the interior of the tube
- Clamp the rod at its mid-point with one end projecting into tube
- Connect disc E to the end of the tube such that it just covers the side of the tube
- Strike the rod using a piece of a cloth until when the [powder in the tube settles into heaps
- Measure the distance l_2 between two consecutive heaps and l_1 of the end

- Velocity of sound in the rod is obtained from $v_r = \frac{v_a l_1}{l_2}$

Where v_a is the velocity of sound in air which is in the tube

Theory

$v_r = 2f_r l_1$..(1) Where

f_r is frequency of the rod

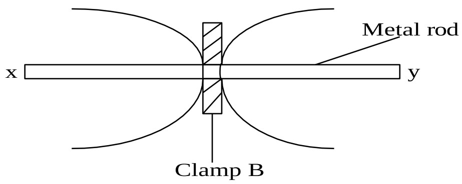
$v_a = 2f_a l_2 \ldots \ldots \ldots \ldots \ldots \ldots .$ (2)

But $f_r = f_a$

$$\frac{v_r}{v_a} = \frac{2f_r l_1}{2f_a l_2}$$

$$v_r = \frac{v_a l_1}{l_2}$$

Measurement of speed of sound in a rod



- Rod xy is fixed a clamp B at its a middle point
- The rod is then stroked and a stationary wave is formed due to the vibration of the rod
- A node is formed at the mid-point of the node and antinode ta the free ends x and y
- Measure length l of the rod
- Wavelength of the wave generated by the rod is given by $\lambda = 2l$
- Speed of sound in the rod is got from

$$v = 2lf$$

2.17 ASSIGNMENT

1. (a) (i) Distinguish between *progressive* and *stationary waves*.
(ii) Show that two waves of the same frequency and wavelength travelling in opposite directions in the same medium produce stationary wave when they meet.
(b) A plane progressive wave travelling in the x-direction is represented by the equation;
$y = 0.36\text{Sin } 7\pi\,(40t - \frac{x}{25})$ where t is the time in seconds, y is the displacement in metres.
Determine the
(i) periodic time,
(ii) speed of the wave.
(c) (i) Define the terms *fundamental note* and *overtone* as applied to a string instrument.
(ii) Two steel wires A, and B of equal length have diameters in the ratio 1:2, and are stretched so that the tensions in the wires are in the ratios 1:4. Find the ratio of the frequencies produced when the wires are plucked in the middle.
(d) Explain how beats are formed.

2. (i)State the principle of super position of waves
(ii)state the conditions for formation of stationary waves.
(b)state the difference between stationary and progressive waves
(c) A progressive as a stationary wave each have the same free period 0.0045 and same velocity of 30ms $^{-1}$. Calculate
(i) The phase difference between two vibrating points on the progressive which are 10cm apart
(ii) The equation of motion of the progressive wave if its amplitude is 0.03m.

(iii) The distance between nodes in the stationary wave
(iv) The equation of motion of the stationary wave if its amplitude is 0.01m.
(d) A wire of diameter 0.040cm and made of steel of density 8000kgm^{-3} is under constant tension of 80N. A fixed length of 50cm is set in transverse vibration. How do you cause the vibration of frequency about 840Hz to predominate in intensity?
(e) A sonometer wire of length 76cm is maintained under a tension of value 40N an an alternating current is passed through the wire. If the density of material of wire is 8800kgm^{-3} and diameter of the wire is 1mm, what is the frequency of the alternating current?

3. (a) (i) What is meant by the term **Resonance** and **damping**
(ii) How does the amount of damping affect resonance?
(b) Describe how you would determine the **speed of sound in air** using the resonance tube
(c) A source emitting a note of 250 HZ is placed over the open upper end of a vertical tube which is full of water. When the water is gradually run out of the tube the air column resonates, initially when the water surface is 30.0 cm below the top of the tube and next when it is 100.0 cm below the top. Find;
(i) the speed of sound in air
(ii) the end- correction
(d) (i) Explain how audible beats arise when two similar tuning forks of slightly

Different frequencies f_o and f are sounded together

(ii) Derive an expression for the number of beats heard per second

4a) Explain with the aid of sketches where appropriate the terms frequency, wavelength and velocity as applied to wave motion. Derive the expression relating these quantities.

b) a wave of amplitude 0.2m, wavelength 2.0m and period 0.02 s propagates in the positive x – direction. If the initial displacement is zero at a point x = 0,

(i) Write the expression for the displacement of the wave at any time, t.

(ii) find the speed of the wave

c) (i) What is meant by Doppler effect

(ii) A train moving with uniform velocity, v, sounds a horn as it passes a stationary observer. Derive the expression for the apparent frequency of the sound detected by the observer.

(iii) If the frequency of the sound detected by the observer after the train passes is 1.2 times lower than frequency detected in c(ii), find the speed of the train. (Speed of sound in air = $330ms^{-1}$)

d) State two uses of Doppler Effect.

5a) Distinguish between longitudinal and transverse waves, giving one example of each. (04)

b) State the conditions for obtaining a stationary wave.

c) Describe with the aid of a diagram, an experiment to investigate the variation of frequency of a stretched wire with length.

d) (i) Define beats and derive the expression for beat frequency

(ii) A tuning fork is sounded with a metal wire of length 0.5m vibrating at its fundamental mode. The diameter of material of wire is 0.5mm, density $7800kgm^{-3}$. The tension in the string is 100N. Beats of 3 s^{-1} are obtained. If the tuning fork is loaded with a small piece of wax, the beat frequency is found to decrease. Find the frequency of the turning fork.

6(a)(i) What is meant by 'beats'?

(ii) Describe how you can dethrone the frequency of a musical note using beats.

(iii) When a turning force of frequency 512HZ is sounded together with a sonometer wire emitting its fundamental frequency, 3 beats are heard every second. When the guitar string is tightened slightly, the beat frequency increases to 4HZ. If the length of the sonometer wire is 25cm and the mass per unit length of the material of the wire is $9 \times 10^{-3}kgm^{-2}$, calculate the tension in the sonometer wire.

b(i) What is a standing wave

(ii) A progressive wave y = a sin (wt-kx) is reflected at a barrier to interfere with the incoming wave, show that the resultant wave is a standing one.

(iii) Distinguish between modes and antinodes

(c)(i) Define the term reverberation

(ii) Why are echoes not formed in a small room when the speed of sound is $330ms^{-1}$?

(d) Why is the moon often referred to as a silent satellite

. (a)(i) Explain how beats are produced.

(ii) An observer moving between two identical stationary sources of sound, along the line joining them, hears beats at the rate of $3.0s^{-1}$. At what velocity is the observer moving if the frequencies of the sources are 480Hz and the velocity of sound when the observation was $340ms^{-1}$.

(b)(i) What is meant by *resonance*?

(ii) With the aid of a diagram, describe an experiment to investigate the variation of frequency of a wave in a stretched string with length of the string.

(c)(i) With the aid of a suitable diagram, explain the terms *fundamental note* and *overtone* as applied to a vibrating wire fixed at both ends

(ii)Explain the significance of overtone in production of music?

(iii) Distinguish between the terms nodes and antinodes with reference to stationary waves.

(iv) Steel wire of length 40.0 cm and diameter 0.0250 cm vibrates transversely in unison with the tube, closed at one end and of length 56.4 cm and end error 3.6 cm, when each is sounding its fundamental note. The air temperature is 27˚C. Find the tension in the wire. (Assume that the velocity of sound in air at 0˚C is $331ms^{-1}$ and density of steel is $7800kgm^{-3}$.)

(c) With the aid of a sonometer, describe an experiment to show how the frequency of a vibrating string is affected by changes of length. (5).

d) (i) Define the term Doppler Effect (1)

(ii) An observer moving at a speed of 10 ms^{-1} between two sources of sound A and B hears beats at 5 s^{-1}. If the frequency of waves produced by source A is 515 Hz and the observer is moving towards A, find the frequency of sound produced by B. (speed of sound in air is 340 ms^{-1})

(iii) Explain how Doppler Effect can be used to determine plasma temperature

7(a). Define the following terms as applied to wave motion:

(i) Wavelength
(ii) Frequency
(iii) Amplitude

(b) A progressive wave traveling along the positive x- direction is described by,

$$y = 0.025\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}x\right), \text{ metres.}$$

Find for the wave,

(i) the amplitude
(ii) the wavelength
(iii) the frequency.

(c) Explain, with the aid of suitable diagrams, the term fundamental mode as

(i) a pipe open at both ends.
(ii) a string stretched between two points.

(d) What is meant by the following terms,

(i) Doppler effect
(ii) Beats.

(e) An observer traveling with a constant velocity of 20ms^{-1}, passes close to a stationary source of sound and notices that there is a change of frequency of 50Hz as she passes the source. Find the frequency of the source. [Speed of sound in air =340ms^{-1}].