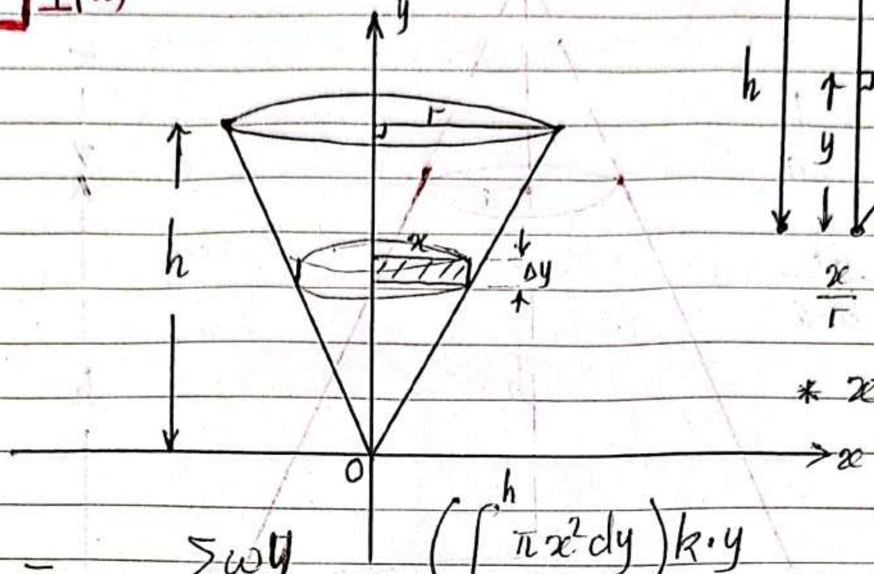
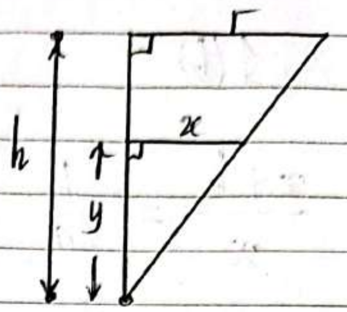
"(MECHANICS)"

SOLUTIONS.

"STATICS" 1(a)





$$\frac{x}{r} = \frac{y}{h}$$

$$* \; x = \frac{yr}{h} \; *$$

$$\bar{y} = \frac{\Sigma w y}{\Sigma w} \Rightarrow \frac{\left(\int_0^h \pi x^2 dy\right) k \cdot y}{\left(\frac{1}{3}\pi r^2 h\right) k}$$

$$\bar{y} = \frac{3\int_0^h x^2 y \, dy}{r^2 h}$$

$$\bar{y} = \frac{3\int_0^h \left(\frac{yr}{h}\right)^2 \cdot y \, dy}{r^2 h}$$

$$\bar{y} = \frac{3}{h^3}\left[\frac{y^4}{4}\right]_0^h$$

$$\bar{y} = \frac{3}{h^3}\left\{\frac{h^4}{4} - 0\right\} \Rightarrow \frac{3}{4}h$$

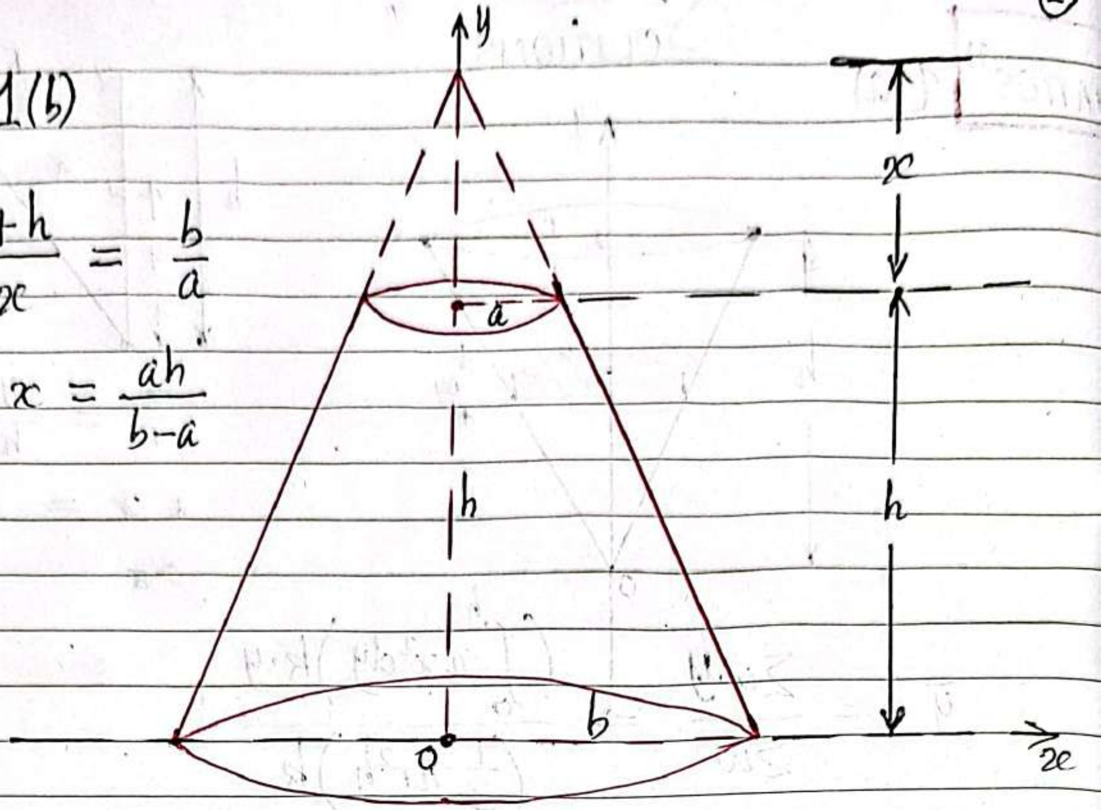
$\therefore$ Required distance $= h - \frac{3}{4}h$

$= \frac{1}{4}h.$

1(b)

$$\frac{x+h}{x} = \frac{b}{a}$$

$$\Rightarrow x = \frac{ah}{b-a}$$



Cut off cone height ; $x = \frac{ah}{b-a}$.

Original cone height ; $x+h = \left(\frac{ah}{b-a} + h\right) \Rightarrow \frac{bh}{b-a}$

$$\bar{y} = \frac{\Sigma Wy}{\Sigma W}$$

$$= \frac{\frac{1}{3}\pi b^2 \cdot \left(\frac{bh}{b-a}\right) \cdot k \cdot \frac{1}{4} \cdot \frac{bh}{b-a} - \frac{1}{3}\pi a^2 \cdot \left(\frac{ah}{b-a}\right) \cdot k \cdot \left(h + \frac{1}{4}\frac{ah}{(b-a)}\right)}{\frac{1}{3}\pi b^2 \cdot \left(\frac{bh}{b-a}\right) \cdot k - \frac{1}{3}\pi a^2 \cdot \left(\frac{ah}{b-a}\right) \cdot k}$$

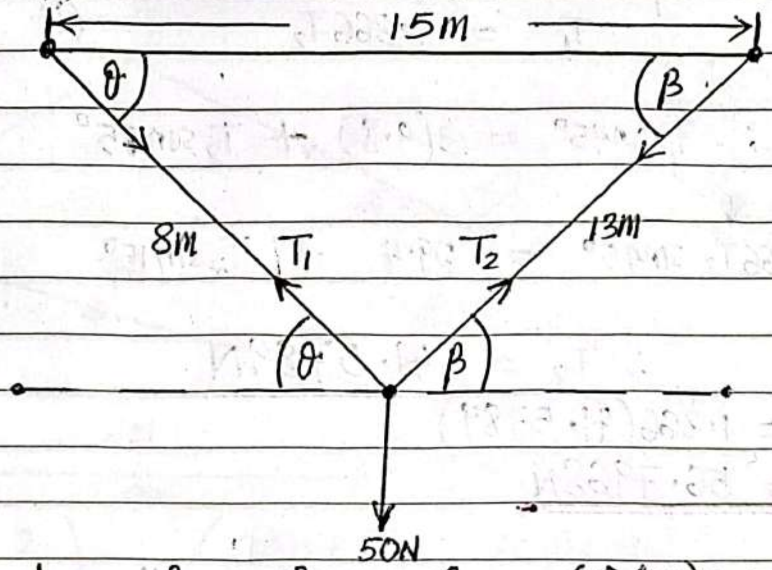
$$= \frac{\frac{b^4 h}{4(b-a)^2} - \frac{ha^3(4b-3a)}{4(b-a)^2 \cdot 1}}{\frac{b^3 - a^3}{(b-a)}}$$

$$= \frac{h}{4}\left\{\frac{b^4 - 4a^3b + 3a^4}{(b-a)(b^3 - a^3)}\right\} \Rightarrow \frac{h}{4}\left(\frac{b^4 - 4a^3b + 3a^4}{(b-a)^2(b^2 + ba + a^2)}\right)$$

On long division;

$$\therefore \bar{y} = \frac{h}{4}\left[\frac{\cancel{(b-a)^2} \cdot (b^2 + 2ab + 3a^2)}{\cancel{(b-a)^2} \cdot (b^2 + ba + a^2)}\right] \#$$

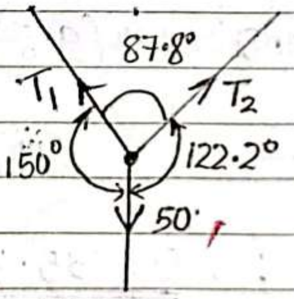
2(a)



From cosine rule; $13^2 = 8^2 + 15^2 - 2(8)(15)\cos\theta$ ∴ $\theta = 60°$

From sine rule; $\frac{\sin\beta}{8} = \frac{\sin 60°}{13}$; ∴ $\beta = 32.2°$

Now;

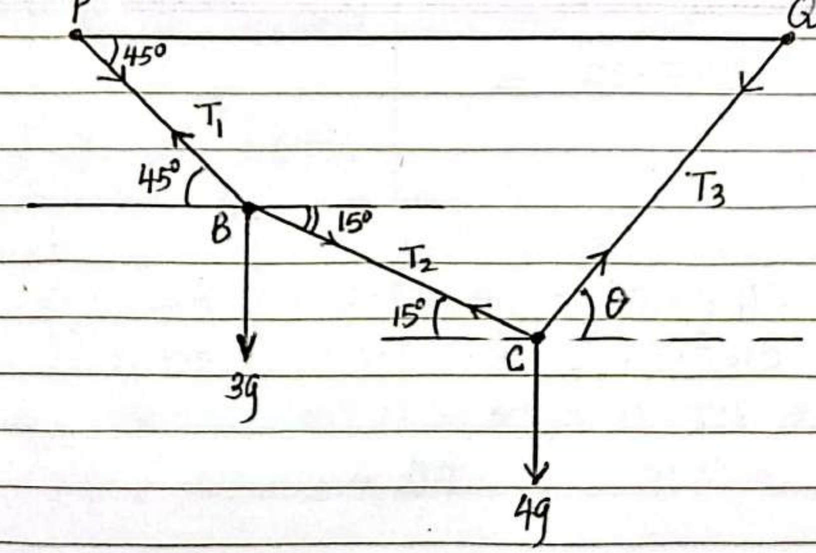


$\frac{T_1}{\sin 122.2°} = \frac{50}{\sin 87.8°}$

$T_1 = 42.3409N$

Also; $\frac{T_2}{\sin 150°} = \frac{50}{\sin 87.8}$; $T_2 = 25.0184N$

2(b)



At B; $(\rightarrow)$: $T_1 \cos 45^\circ = T_2 \cos 15^\circ$

$T_1 = 1.366 T_2$ --- (i)

$(\uparrow)$: $T_1 \sin 45^\circ = 3(9.8) + T_2 \sin 15^\circ$

$1.366 T_2 \sin 45^\circ = 29.4 + T_2 \sin 15^\circ$

$\therefore T_2 = 41.5789 N$

$\Rightarrow T_1 = 1.366(41.5789)$
$= 56.7968 N$

At C; $(\rightarrow)$: $T_3 \cos\theta = 41.58789 \cos 15^\circ$

$T_3 \cos\theta = 40.1621$ --- (ii)

$(\uparrow)$: $41.5789 \sin 15^\circ + T_3 \sin\theta = 4(9.8)$

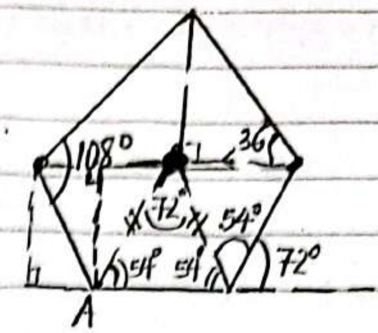
$T_3 \sin\theta = 28.4386$ --- (iii)

(iii) $\div$ (ii) $\Rightarrow$ $\tan\theta = 0.7081$; $\therefore \theta = 35.3^\circ$

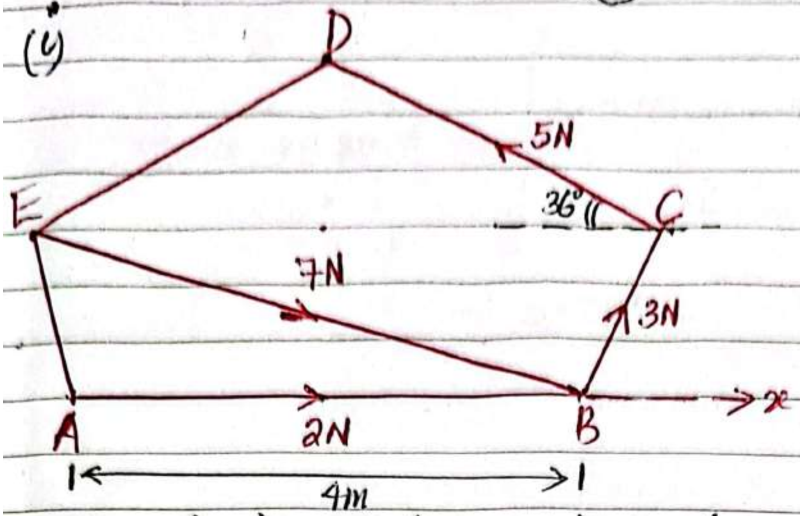
$$T_3 = \frac{28.4386}{\sin(35.3^\circ)}$$

$= 49.2139 N$

3



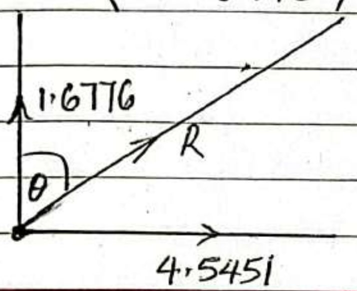
(i)



$$\underset{\sim}{R} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 3\cos 72^\circ \\ 3\sin 72^\circ \end{pmatrix} + \begin{pmatrix} -5\cos 36^\circ \\ 5\sin 36^\circ \end{pmatrix} + \begin{pmatrix} 7\cos 36^\circ \\ -7\sin 36^\circ \end{pmatrix}$$

$$\underset{\sim}{R} = \begin{pmatrix} 4.5451 \\ 1.6776 \end{pmatrix} N \qquad \therefore |\underset{\sim}{R}| = \sqrt{4.5451^2 + 1.6776^2}$$

$$= 4.8448N$$



$$\tan\theta = \frac{4.5451}{1.6776} \; ; \; \theta = 69.74^\circ$$

(ii) $\curvearrowleft A$; $G = (3\sin 72^\circ)\cdot 4 + (5\cos 36^\circ)(4\sin 72^\circ) + (5\sin 36^\circ)(4 + 4\cos 72^\circ) - 7(4\sin 36^\circ)$

$$G = 25.7315 \text{ Nm}$$

$$\begin{vmatrix} x & 4.5451 \\ y & 1.6776 \end{vmatrix} = 25.7315$$

$$1.6776x - 4.5451y = 25.7315$$

At H; $y = 0$; $1.6776x = 25.7315$

$$x = 15.3382m$$

$$\therefore AH = 15.34m.$$

(iii) $A(0,0)$, Line $1.6776x - 4.5451y - 25.7315 = 0$

$$\text{Distance required} = \left| \frac{1.6776(0) - 4.5451(0) - 25.7315}{\sqrt{(1.6776)^2 + (-4.5451)^2}} \right|$$

$$\simeq 5.3112 \, m.$$

4.

(i) $\underset{\sim}{R} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} N$

$\circlearrowleft O : G = \begin{vmatrix} 3 & 3 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 4 \\ 3 & -5 \end{vmatrix} + \begin{vmatrix} -2 & -5 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ -2 & 3 \end{vmatrix}$

$= 6 + -17 + 1 + -2$

$= -12 \text{ Nm}$

Line; $\begin{vmatrix} x & 4 \\ y & 3 \end{vmatrix} = -12$; $3x - 4y = -12$

Along the x-axis ; $y = 0, \Rightarrow 3x = -12$

$x = -4$

$\therefore$ required position is $(-4\underset{\sim}{i})$ m. or $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ m.

(ii) For equilibrium: ✓ resultant force $= 0$, and ✓ resultant moment $= 0$

$\Rightarrow \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$p = -4, \quad q = -3$

$-12 + b + \begin{vmatrix} 2 & -4 \\ 1 & -3 \end{vmatrix} = 0$

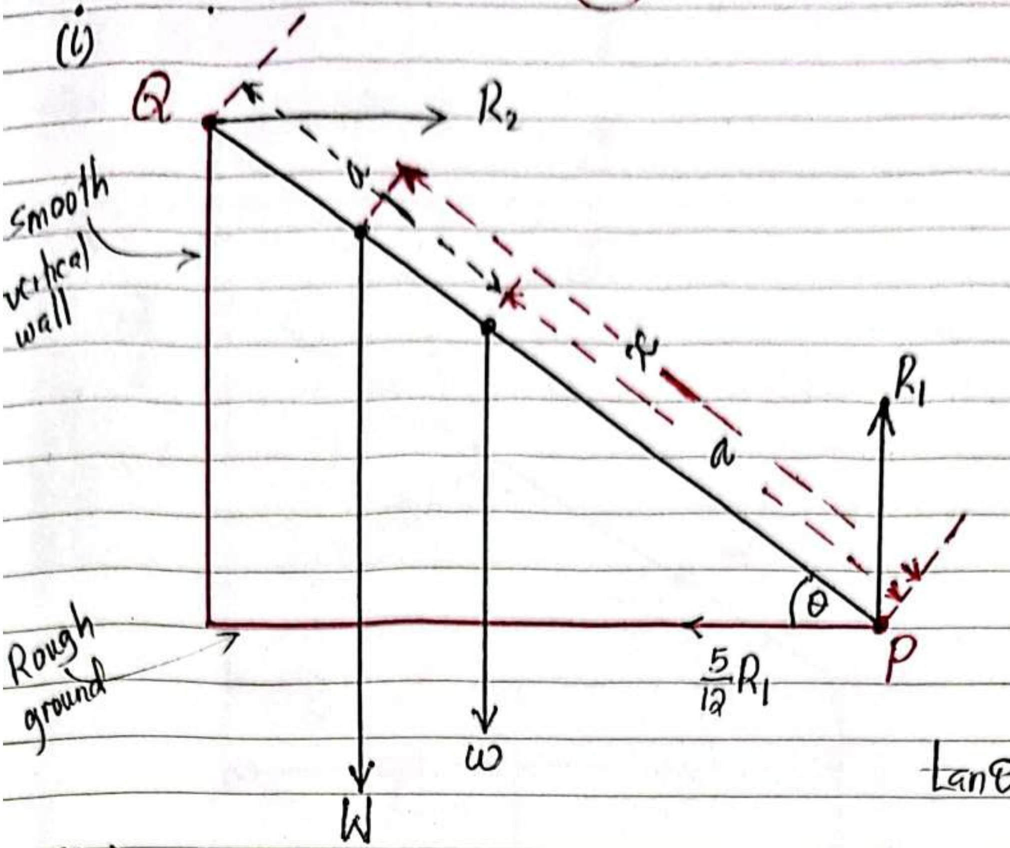
$b - 12 - 2 = 0$

$b = 14$

$\therefore p = -4, \quad q = -3$ and $b = 14$.

(5.)

(i)



$\tan\theta = 2$

$(\uparrow)$: $R_1 = W + w$ ——— (i)

$(\rightarrow)$: $R_2 = \frac{5}{12}R_1 \Rightarrow R_2 = \frac{5}{12}(W+w)$ ——— (ii)

$\circlearrowleft P$: $w\cdot(a\cos\theta) + W\cdot(x\cos\theta) = R_2\cdot(2a\sin\theta)$

$wa + Wx = 2aR_2\tan\theta.$

$\Rightarrow wa + Wx = 2a\cdot\frac{5}{12}\cdot(W+w)\cdot 2$

$3Wx = \frac{1}{3}\cdot 5\cdot a(W+w) - wa.$

$x = \frac{a}{3W}\cdot(5W+5w-3w).$

(ii) $x = \frac{a(2w+5W)}{3W}.$ #

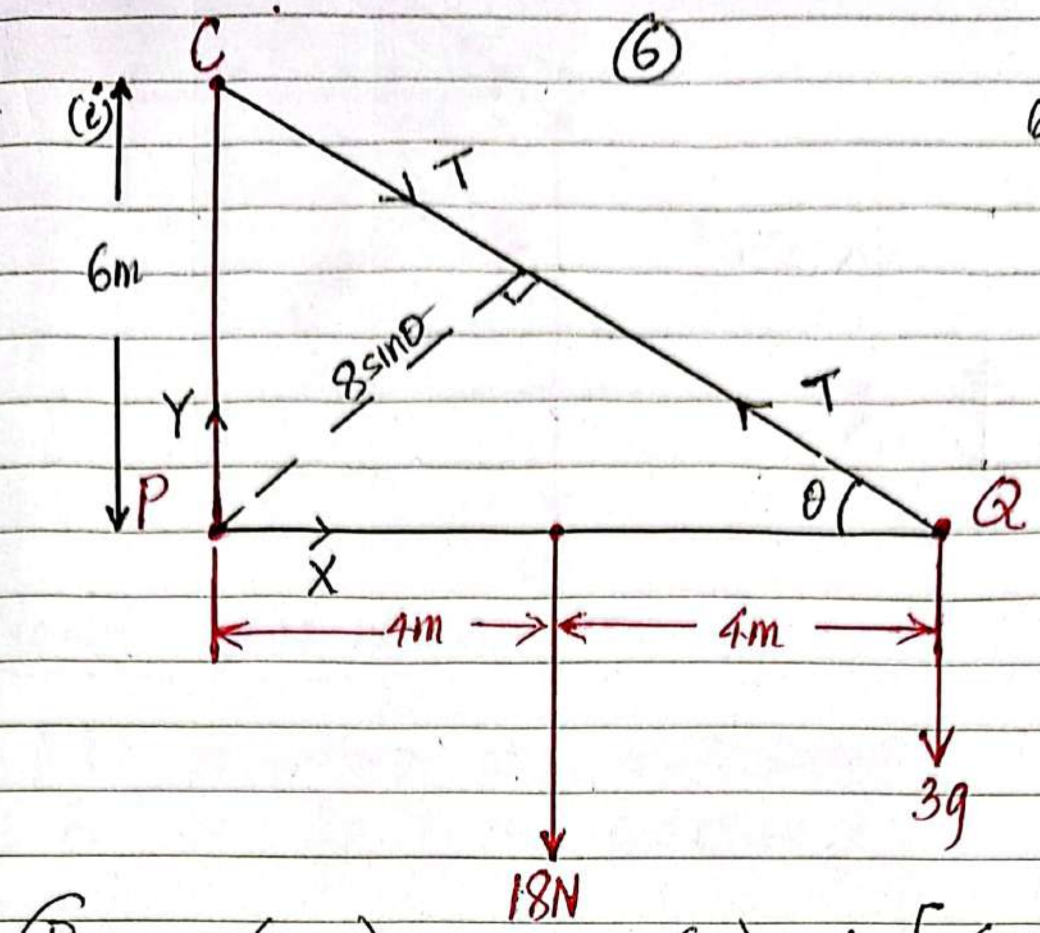
Hence; At the top;
$x = 2a$

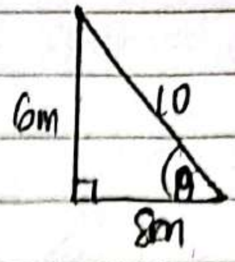
$2a = \frac{a(2w+5W)}{3W}$

$6W = 2w + 5W$

$\boxed{W = 2w.}$ #

(6)





$\cos\theta = \frac{8}{10}$

$\sin\theta = \frac{6}{10}$

$\circlearrowleft P: \quad T\cdot(8\sin\theta) = 18\cdot(4) + [3\cdot(9\cdot8)](8)$

$T \times 8 \times \frac{6}{10} = 72 + 235\cdot2$

$T = 64N$

(ii) $(\rightarrow): \quad X = T\cos\theta$

$= 64 \times \frac{8}{10}$

$= 51\cdot2N$

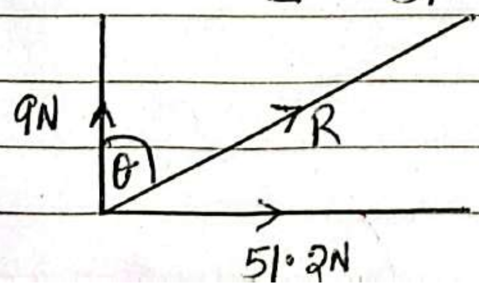
$(\uparrow): \quad Y + T\sin\theta = 18 + 3(9\cdot8)$

$Y + 64 \times \frac{6}{10} = 18 + 29\cdot4$

$Y = 9N$

$R = \sqrt{(51\cdot2)^2 + 9^2}$

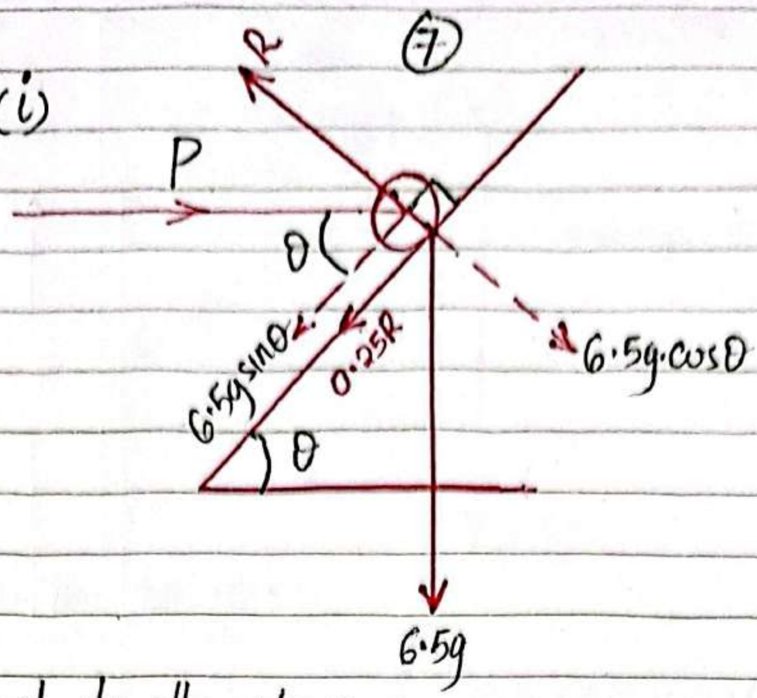
$= 51\cdot985N$



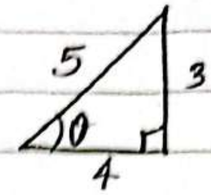
$\theta = \tan^{-1}\left(\frac{51\cdot2}{9}\right) \Rightarrow 80\cdot03^\circ$

7

(i)



$\tan\theta = \frac{3}{4}$



$\Rightarrow \sin\theta = 3/5$

$\cos\theta = 4/5$

Normal to the plane;

$R = P\sin\theta + 6{\cdot}5 \times 9{\cdot}8 \times \cos\theta$

$R = \frac{3}{5}P + 6{\cdot}5 \times 9{\cdot}8 \times \frac{4}{5}$

$R = 0{\cdot}6P + 50{\cdot}96$ — — — — (i)

Along the plane;

$P\cos\theta = 6{\cdot}5 \times 9{\cdot}8 \times \sin\theta + 0{\cdot}25R$

$\frac{4P}{5} = 6{\cdot}5 \times 9{\cdot}8 \times \frac{3}{5} + 0{\cdot}25(0{\cdot}6P + 50{\cdot}96)$

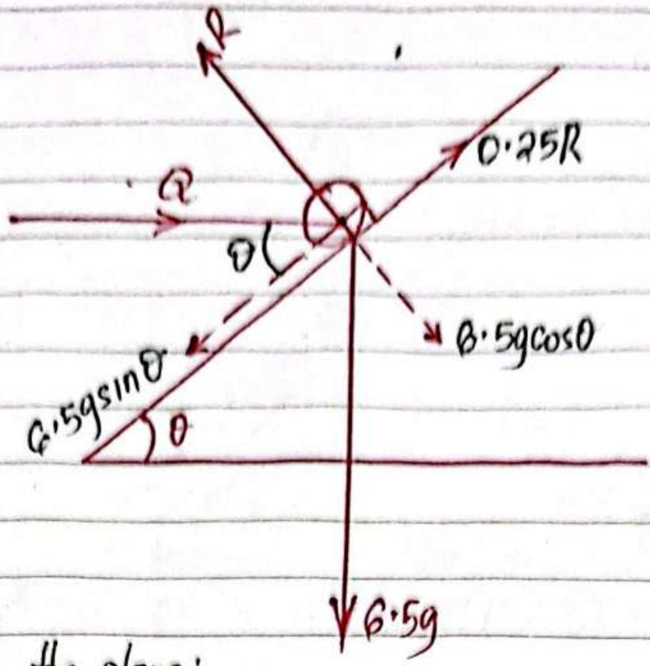
$0{\cdot}65P = 38{\cdot}22 + \cancel{89{\cdot}18}\ 12{\cdot}74$

$P = \cancel{196N}\ 78{\cdot}4N$

$\therefore$ Force (P) to move the box up the plane is

$78{\cdot}4N.$

(ii)



$\tan\theta = 3/4$
$\sin\theta = 3/5$
$\cos\theta = 4/5$

Normal to the plane;

$$R = Q\sin\theta + 6{\cdot}5 \times 9{\cdot}8 \times \cos\theta$$

$$R = \frac{3}{5}\cdot Q + 6{\cdot}5 \times 9{\cdot}8 \times \frac{4}{5}$$

$$R = 0{\cdot}6Q + 50{\cdot}96$$

Along the plane;

$$Q\cos\theta + 0{\cdot}25R = 6{\cdot}5 \times 9{\cdot}8 \times \sin\theta$$

$$\frac{4}{5}\cdot Q + 0{\cdot}25(0{\cdot}6Q + 50{\cdot}96) = 6{\cdot}5 \times 9{\cdot}8 \times \frac{3}{5}$$

$$0{\cdot}95Q = 38{\cdot}22 - 12{\cdot}74$$

$$Q = 26{\cdot}8211\,N$$

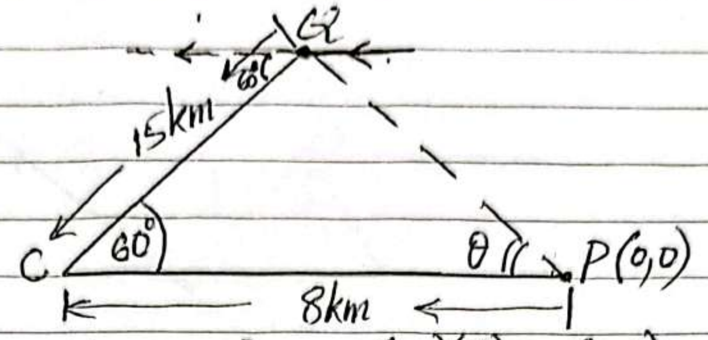
$\therefore$ Force (Q) to prevent the box from sliding down the plane is $\underline{26{\cdot}8211\,N}$

"KINEMATICS"

(1.)

(i) Hint: Obtain the initial distance between P and Q.

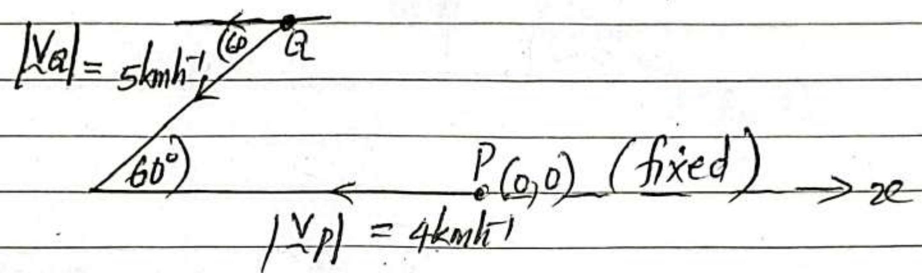
Fig:1



$$PQ^2 = 8^2 + 15^2 - 2(15)(8)\cos(60^\circ)$$

$$PQ = 13\text{km}.$$

Fig:2



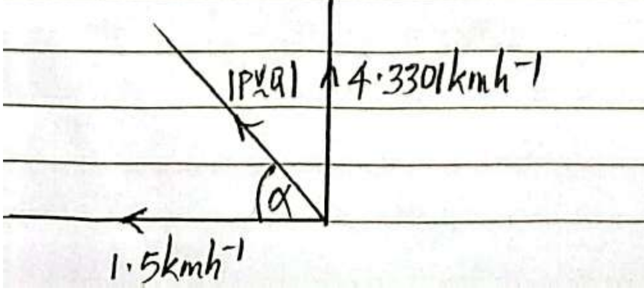
$${}_P\underset{\sim}{V}_Q = \underset{\sim}{V}_P - \underset{\sim}{V}_Q$$

$$= \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} -5\cos 60^\circ \\ -5\sin 60^\circ \end{pmatrix}$$

$$= \begin{pmatrix} -1.5 \\ 4.3301 \end{pmatrix} \text{kmh}^{-1}$$

$$|{}_P\underset{\sim}{V}_Q| = \sqrt{(-1.5)^2 + (4.3301)^2}$$

$$|{}_P\underset{\sim}{V}_Q| = 4.5826\text{kmh}^{-1}$$



$$\alpha = \tan^{-1}\left(\frac{4.3301}{1.5}\right)$$

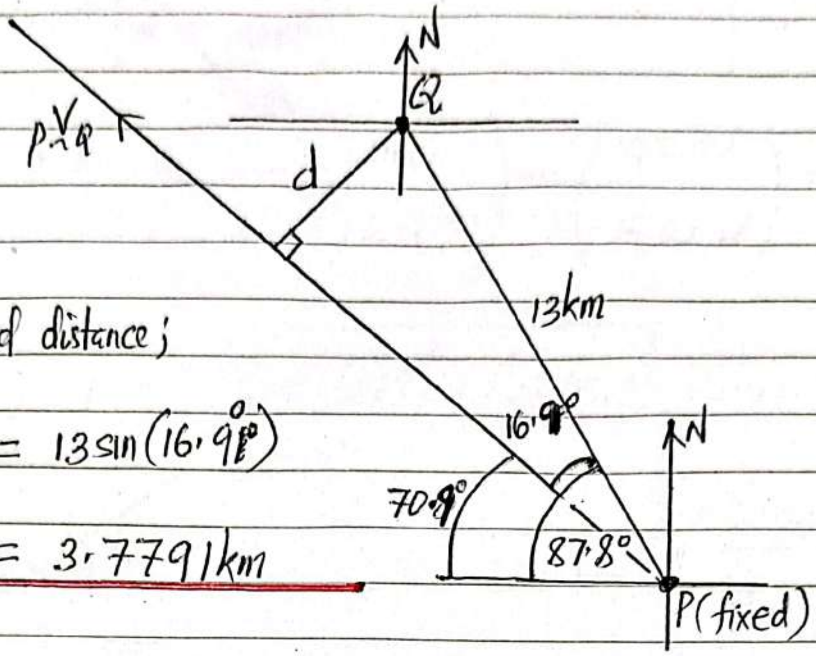
$$\alpha = 70.89^\circ$$

$$\alpha = 70.89^\circ \simeq \underline{70.9^\circ}$$

From fig: 1

$$\sin\theta = \frac{\sin 60^\circ}{13} \times 15$$

$$\theta = 87.8^\circ$$



⇒ Required distance;

$$d = 13\sin(16.9^\circ)$$

$$= 3.7791\,km$$

(ii) Time $= \dfrac{13\cos(16.9^\circ)}{4.5826}$

$$= 2.7143\,h$$

(iii) When; $t = 2.7143h$

Distance travelled by P towards C is (4×2.7143)

$= 10.8572\,km$

⇒ Distance beyond C $= 10.8572 - 8$

$= 2.8572\,km.$

Also Distance travelled by Q towards C is (5×2.7143)

$= 13.5715\,km$

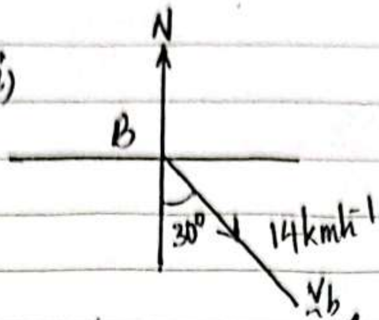
⇒ Distance ~~beyond~~ before C $= 15 - 13.5715$

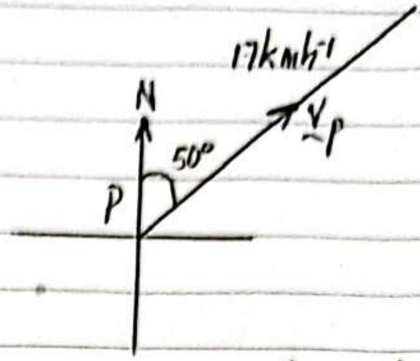
$= 1.4285\,km.$

2.

(a)

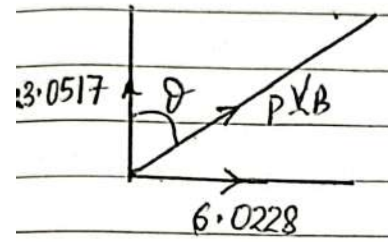
(i)





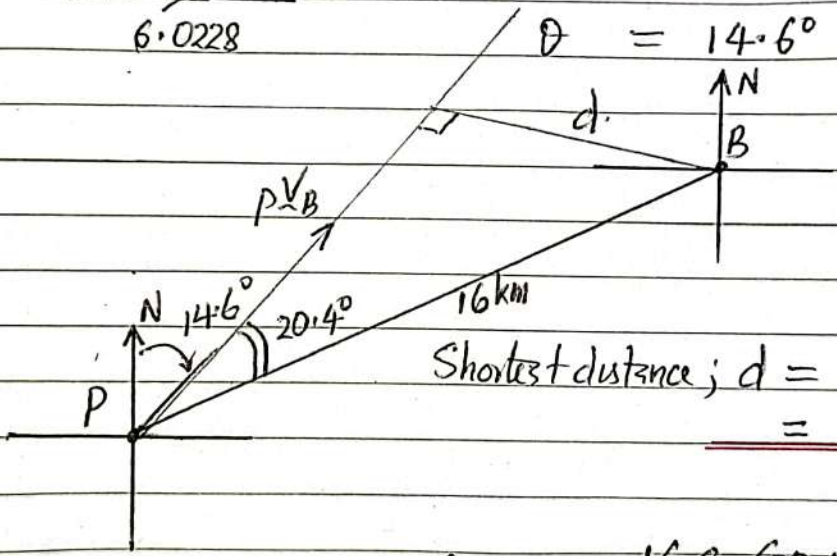
$$_P\underset{\sim}{V}_B = \begin{pmatrix} 17\sin(50^\circ) \\ 17\cos(50^\circ) \end{pmatrix} - \begin{pmatrix} 14\sin 30^\circ \\ -14\cos 30^\circ \end{pmatrix} \Rightarrow \begin{pmatrix} 6.0228 \\ 23.0517 \end{pmatrix} \text{kmh}^{-1}$$

$$|_P\underset{\sim}{V}_B| = \sqrt{(6.0228)^2 + (23.0517)^2} \Rightarrow 23.8255\,\text{kmh}^{-1}$$



$$\theta = \tan^{-1}\left(\frac{6.0228}{23.0517}\right)$$

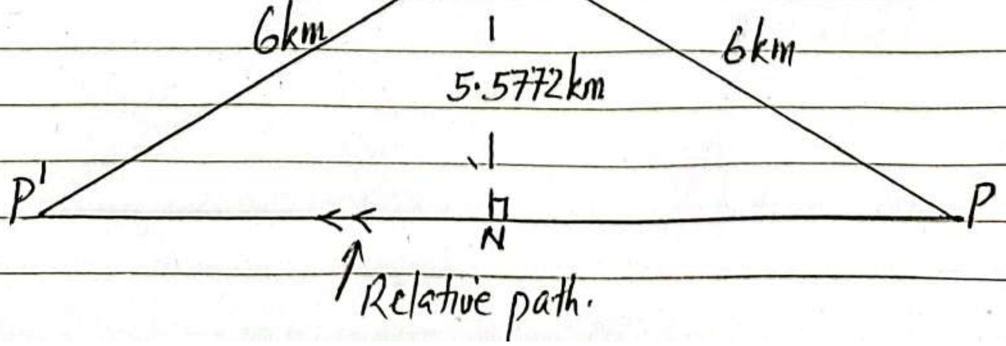
$$\theta = 14.6^\circ$$



Shortest distance; $d = 16 \times \sin 20.4^\circ$
$= 5.5772\,\text{km}$

(ii) Time required $= \dfrac{16\cos(20.4^\circ)}{23.8255} \Rightarrow 0.6294\,\text{h}$

(b)



$$\text{Required time} = \frac{\text{distance } PP'}{|{}_P\underline{V}_B|}$$

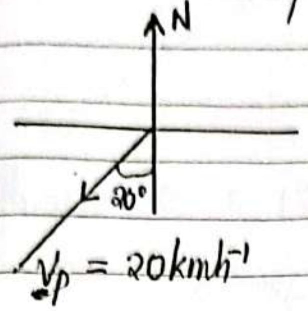
$$= \frac{2NP}{{}_PV_B}$$

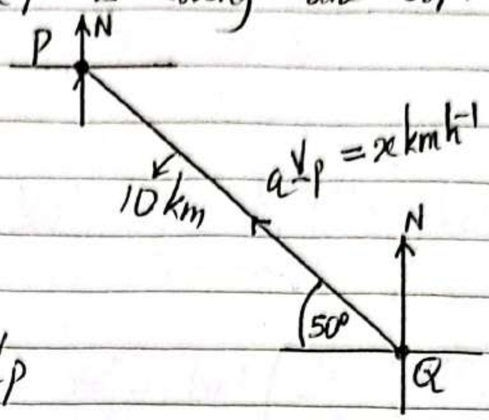
$$= \frac{2\sqrt{6^2 - 5.5772^2}}{23.8255}$$

$$= 0.1857h.$$

3.

(i) For interception, ${}_q\underset{\sim}{V}_p$ is along line QP.





Now; ${}_q\underset{\sim}{V}_p = \underset{\sim}{V}_q - \underset{\sim}{V}_p$

$\Rightarrow \quad \underset{\sim}{V}_q = \underset{\sim}{V}_p + {}_q\underset{\sim}{V}_p.$

$$\underset{\sim}{V}_q = \begin{pmatrix} -20\sin 20° \\ -20\cos 20° \end{pmatrix} + \begin{pmatrix} -x\cos 50° \\ x\sin 50° \end{pmatrix}$$

$$\therefore \quad \underset{\sim}{V}_q = \begin{pmatrix} -6.8404 - 0.6428x \\ -18.7939 + 0.766x \end{pmatrix}$$

but; $|\underset{\sim}{V}_q| = 19 kmh^{-1}$

$$\Rightarrow \sqrt{(-6.8404 - 0.6428x)^2 + (-18.7939 + 0.766x)^2} = 19$$

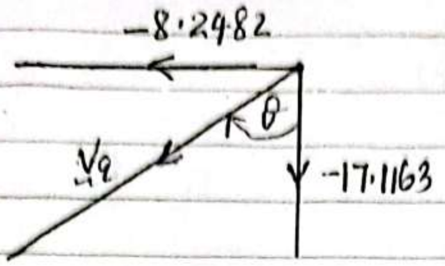
$$400.0017 - 19.9982x + 0.9999x^2 = 361$$

$$x^2 - 20x + 39.0056 = 0$$

$$x = \frac{20 \pm \sqrt{400 - 156.0224}}{2}$$

$$x = 2.1901 \quad \text{or} \quad 17.8099$$

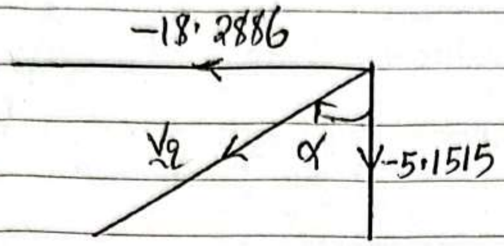
$\therefore$ When $x = 2.1901$; $\underset{\sim}{V}_q = \begin{pmatrix} -8.2482 \\ -17.1163 \end{pmatrix} kmh^{-1}$



$$\theta = \tan^{-1}\left(\frac{8.2482}{17.1163}\right)$$

$$\theta = 25.73^\circ$$

When $x = 17.8099$; $\quad \underline{V}_q = \begin{pmatrix} -18.2886 \\ -5.1515 \end{pmatrix} \text{kmh}^{-1}$



$$\alpha = \tan^{-1}\left(\frac{18.2886}{5.1515}\right)$$

$$\alpha = 74.27^\circ$$

Courses required are: S 25.73° W and S 74.27° W.

3.

(ii) When $x = 2.1901$

Time taken to intercept $= \dfrac{10}{2.1901} \Rightarrow 4.566h.$

When $x = 17.8099$

Time taken to intercept $= \dfrac{10}{17.8099} \Rightarrow 0.5615h.$

4.

(a)

(i) $12:00 - 11:45 = 0.25h.$

$$\underset{\sim}{r}_A = \underset{\sim}{r}_0 + \underset{\sim}{u}\cdot t \Rightarrow \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}12\\-4\end{pmatrix}\cdot(0.25) = \begin{pmatrix}1\\2\end{pmatrix}km$$

(ii) At closest position ${}_A\underset{\sim}{V}_B \cdot {}_A\underset{\sim}{r}_B = 0$

$$ {}_A\underset{\sim}{V}_B = \underset{\sim}{V}_A - \underset{\sim}{V}_B \Rightarrow \begin{pmatrix}12\\-4\end{pmatrix} - \begin{pmatrix}2\\-14\end{pmatrix}$$

$$= \begin{pmatrix}10\\10\end{pmatrix}kmh^{-1}$$

From noon; $\underset{\sim}{r}_A = \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}12\\-4\end{pmatrix}\cdot t \Rightarrow \begin{pmatrix}1+12t\\2-4t\end{pmatrix}$

$$\underset{\sim}{r}_B = \begin{pmatrix}8\\7\end{pmatrix} + \begin{pmatrix}2\\-14\end{pmatrix}\cdot t \Rightarrow \begin{pmatrix}8+2t\\7-14t\end{pmatrix}$$

$$ {}_A\underset{\sim}{r}_B = \begin{pmatrix}1+12t\\2-4t\end{pmatrix} - \begin{pmatrix}8+2t\\7-14t\end{pmatrix}$$

$$= \begin{pmatrix}10t-7\\10t-5\end{pmatrix}km$$

Now; $\begin{pmatrix}10\\10\end{pmatrix}\cdot\begin{pmatrix}10t-7\\10t-5\end{pmatrix} = 0$

$100t - 70 + 100t - 50 = 0$; $t = 0.6h$

$= 36$ minutes

Required time $= 12:00$ noon $+ 36$min

$= 12:36$pm.

(iii) When $t = 0.6h$

$$ {}_A\underset{\sim}{r}_B = \begin{pmatrix} 10(0.6) - 7 \\ 10(0.6) - 5 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} km $$

$\therefore$ Shortest distance $= |{}_A\underset{\sim}{r}_B| \Rightarrow \sqrt{(-1)^2 + 1^2}$

$= 1.4142km$

(iv) When $t = 0.6h$

$$ \underset{\sim}{r}_A = \begin{pmatrix} 1 + 12(0.6) \\ 2 - 4(0.6) \end{pmatrix} \Rightarrow \begin{pmatrix} 8.2 \\ -0.4 \end{pmatrix} km $$

$\therefore$ Required distance $= |\underset{\sim}{r}_A|$

$= \sqrt{(8.2)^2 + (-0.4)^2}$

$= 8.2098km$

(b) At collision $\underset{\sim}{r}_A = \underset{\sim}{r}_B$

but; $\underset{\sim}{r}_B = \begin{pmatrix} 8 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ -14 \end{pmatrix} t \Rightarrow \begin{pmatrix} 8 - 2t \\ 7 - 14t \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 1 + 12t \\ 2 - 4t \end{pmatrix} = \begin{pmatrix} 8 - 2t \\ 7 - 14t \end{pmatrix}$; Along OX : $1 + 12t = 8 - 2t$

$t = 0.5h$

Along OY: $2 - 4t = 7 - 14t$

$t = 0.5h$

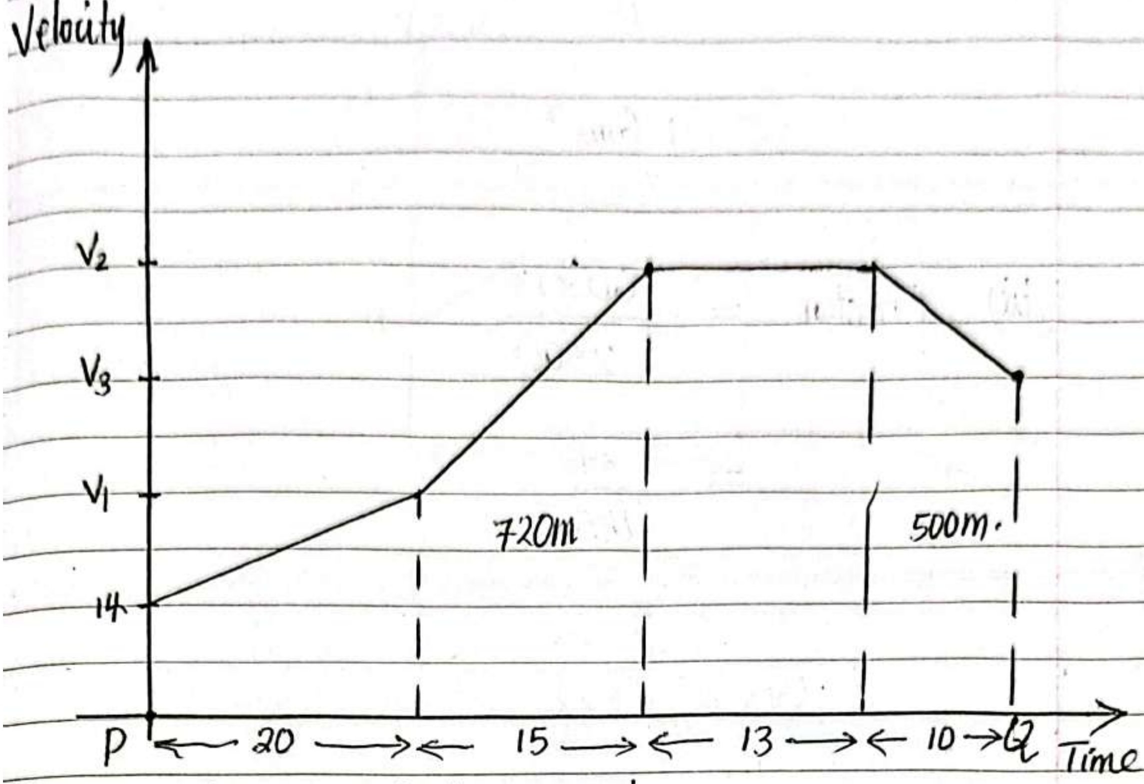
Since t is the same, then the ships will collide.

Collision time $=$ 12:00 noon $+$ 30 min $\Rightarrow$ 12:30pm

Collision position ; $\underset{\sim}{r} = \begin{pmatrix} 1 + 12(0.5) \\ 2 - 4(0.5) \end{pmatrix} \Rightarrow \begin{pmatrix} 7 \\ 0 \end{pmatrix} km.$

5.

Distance $PQ = 2.5 \times 1000 \Rightarrow 2500m$



During the second phase;

$$\frac{1}{2}(V_1 + V_2) \times 15 = 720$$

$$V_1 + V_2 = 96 \text{ ——— (i)}$$

During the last phase;

$$\frac{1}{2}(V_2 + V_3) \times 10 = 500$$

$$V_2 + V_3 = 100 \text{ ——— (ii)}$$

If Total distance $= 2500$

$$\frac{1}{2}(14 + V_1) \times 20 + 720 + 13V_2 + 500 = 2500$$

$$V_1 + 1.3V_2 = 114 \text{ ——— (iii)}$$

On solving (i) and (iii); $V_1 = 36 kmh^{-1}$

$V_2 = 60 kmh^{-1}$

$\Rightarrow V_3 = 40 kmh^{-1}$

(ii) $a = \dfrac{60 - 36}{15}$

$= 1 \cdot 6ms^{-2}$

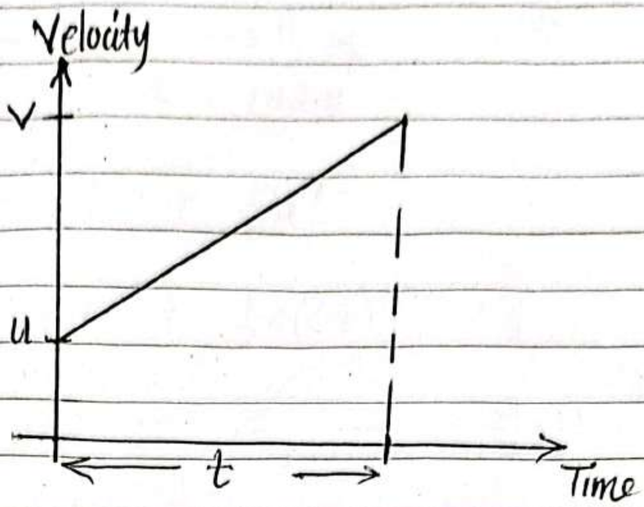
(iii) Fraction $= \dfrac{60 \times 13}{2500}$

$= \dfrac{39}{125}$

6.

(a)

(i)



(ii) $a = \dfrac{v-u}{t}$; $t = \dfrac{v-u}{a}$

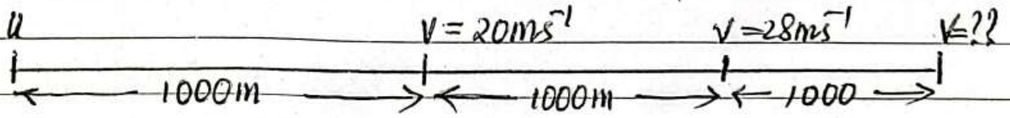
$\Rightarrow \quad s = \frac{1}{2} \cdot t \cdot (u+v)$.

$2s = \dfrac{v-u}{a} \cdot \dfrac{v+u}{1}$

$2as = v^2 - u^2$

$\therefore \quad v^2 = u^2 + 2as \quad$ #

(b)



When $v = 20$, $s = 1000m$

$\Rightarrow \quad 20^2 = u^2 + 2a(1000)$

$u^2 = 400 - 2000a \quad \text{———} \quad (1)$

When $v = 28$, $s = 2000m$

$\Rightarrow \quad 28^2 = u^2 + 2a(2000)$

$u^2 = 784 - 4000a \quad \text{———} \quad (2)$

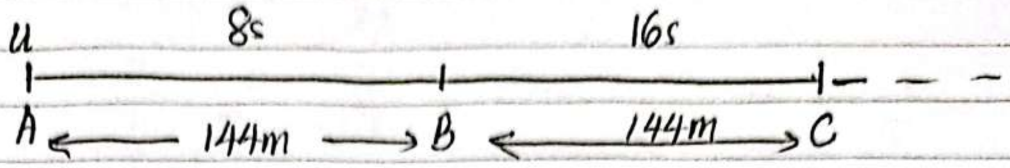
On solving one & two ; $a = 0.192\,ms^{-2}$

$u = 4ms^{-1}$

When $s = 3000m$, $v^2 = 4^2 + 2(0.192)(3000)$

$v = \sqrt{1168} \Rightarrow \underline{34.176ms^{-1}}$

6(c)



When $t = 8s$, $s = 144m$

Using; $s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad 144 = 8u + \frac{1}{2}a(64)$

$u + 4a = 18 \quad \text{———} \quad (1)$

When $t = (8+16)s \Rightarrow 24s$, $s = 288m$

$\Rightarrow \quad 288 = 24u + \frac{1}{2}a(576)$

$u + 12a = 12 \quad \text{———} \quad (2)$

On solving (1) and (2); $a = -0.75ms^{-2}$

$u = 21ms^{-1}$

To come to rest, $v = 0$

$\Rightarrow \quad 0^2 = 21^2 + 2(-0.75)(s)$

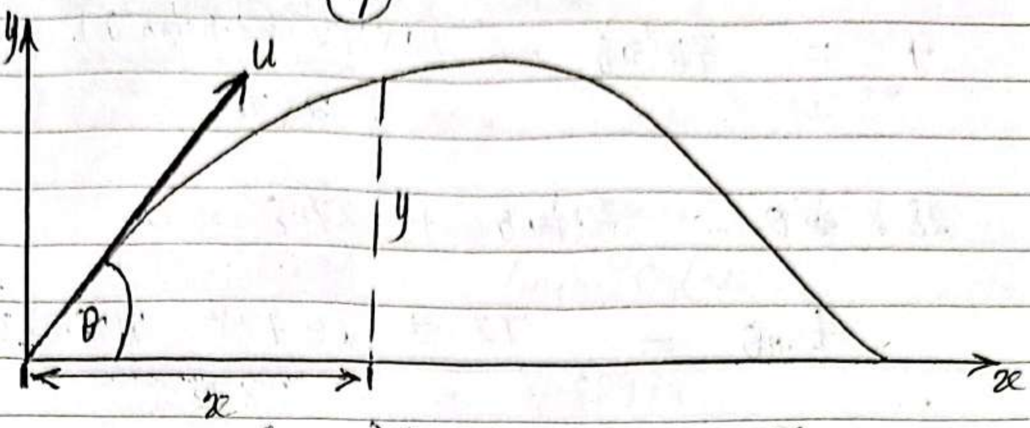
$s = 294m$ (From the start)

Further distance $= 294 - 288$

$= 6m.$

7

(a)



$(\rightarrow)$: $x = (u\cos\theta)t$; $t = \dfrac{x}{u\cos\theta}$ —— (i)

$(\uparrow)$: $y = (u\sin\theta)t - \frac{1}{2}gt^2$ —— (ii)

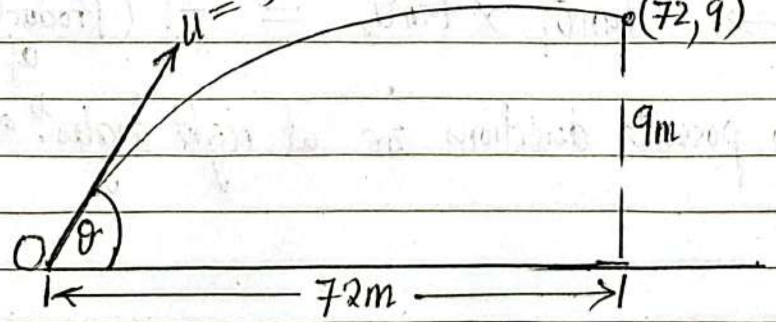
$\Rightarrow y = u\sin\theta \cdot \dfrac{x}{u\cos\theta} - \frac{1}{2}g \cdot \left(\dfrac{x}{u\cos\theta}\right)^2$

$y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow y = x\tan\theta - \dfrac{gx^2}{2u^2}(\sec^2\theta)$; $1+\tan^2\theta = \sec^2\theta$

$\therefore y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}$ #

(b)



$$9 = 72\tan\theta - \frac{10 \times 72^2 \cdot (1 + \tan^2\theta)}{2 \times 30^2}$$

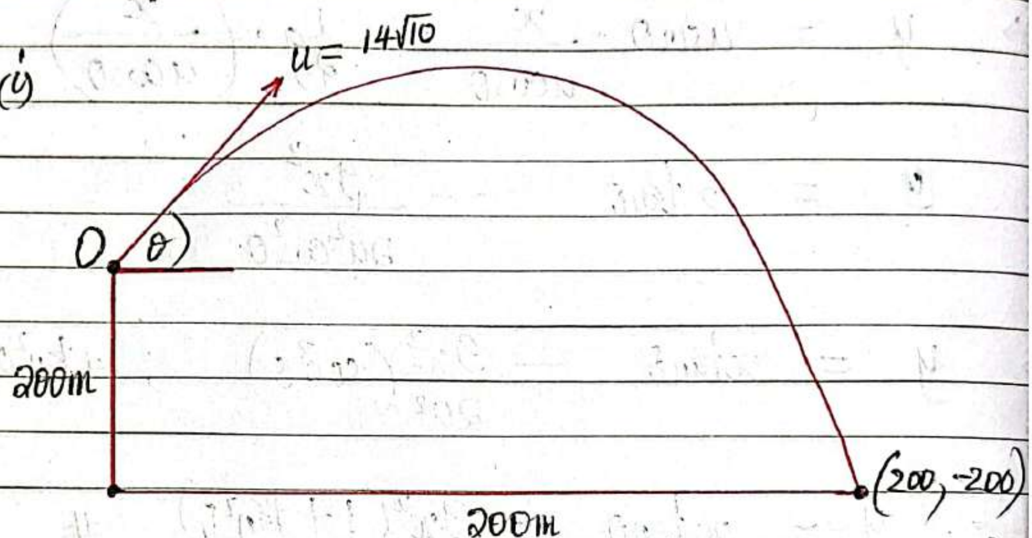
$$28.8\tan^2\theta - 72\tan\theta + 37.8 = 0$$

$$\tan\theta = \frac{72 \pm \sqrt{(-72)^2 - 4(28.8)(37.8)}}{2 \times 28.8}$$

$$\tan\theta = 0.75 \quad \text{or} \quad \tan\theta = 1.75$$

$$\therefore \quad \theta = 36.87^\circ \quad \text{or} \quad 60.26^\circ$$

(c) (i)



$$\Rightarrow -200 = 200\tan\theta - \frac{9.8 \times 200^2 \times [1 + \tan^2\theta]}{2 \times (14\sqrt{10})^2}$$

$$(\tan\theta)^2 - 2(\tan\theta) - 1 = 0$$

if θ_1 and θ_2 are the angles;

$$\Rightarrow \tan\theta_1 \times \tan\theta_2 = -1 \text{ (Product of roots of the eqn)}$$

" $\therefore$ the two possible directions are at right angles" above

(ii) On solving ; $(\tan\theta)^2 - 2(\tan\theta) - 1 = 0$

$$\Rightarrow \tan\theta_1 = 1-\sqrt{2}, \quad \theta_1 = -22.5^\circ$$

$$t = \frac{x}{u\cos\theta} \Rightarrow \frac{200}{(14\sqrt{10})(\cos(-22.5^\circ))}$$

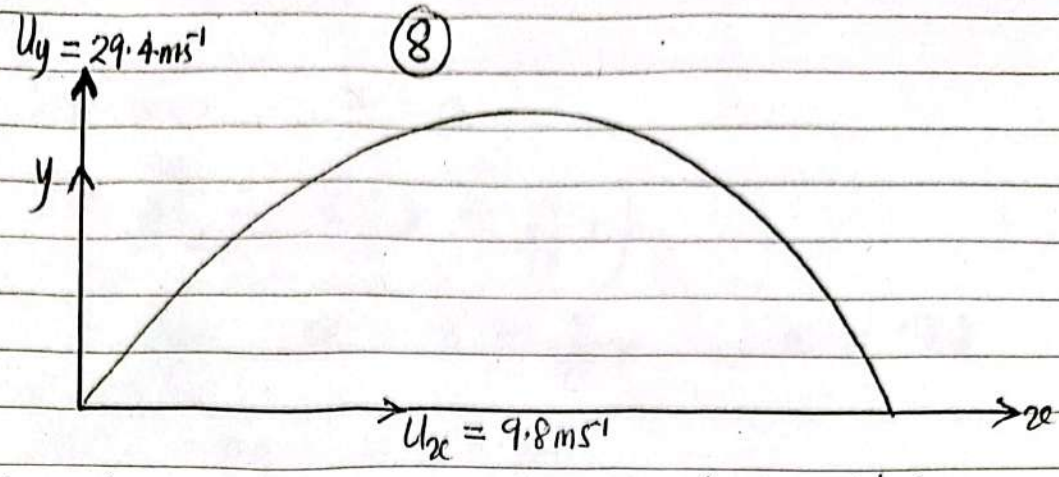
$$= 4.8897s$$

Also, $\Rightarrow \tan\theta_2 = 1+\sqrt{2}, \quad \theta_2 = 67.5^\circ$

$$t = \frac{200}{(14\sqrt{10})(\cos(67.5^\circ))} \Rightarrow 11.8049s$$

(8)

(a)



Velocity at any time; $\underset{\sim}{V} = V_x \underset{\sim}{i} + V_y \underset{\sim}{j}$

$V_x = U_x + a_x \cdot t = 9.8 \quad ; \quad (a_x = 0)$

$V_y = U_y + a_y \cdot t \Rightarrow 29.4 - 9.8t$

$\underset{\sim}{V} = 9.8\underset{\sim}{i} + (29.4 - 9.8t)\underset{\sim}{j}$

Position vector at any time; $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$

$x = U_x \times t = 9.8t$

$y = U_y t - \frac{1}{2}(9.8)(t^2) = (29.4t - 4.9t^2)$

$\underset{\sim}{r} = (9.8t)\underset{\sim}{i} + (29.4t - 4.9t^2)\underset{\sim}{j}$

(b) If $x = 9.8t \quad ; \quad t = \frac{x}{9.8}$ ——— (i)

$$y = 29.4\left(\frac{x}{9.8}\right) - 4.9\left(\frac{x}{9.8}\right)^2$$

$$y = 3x - \frac{5}{98}x^2$$

Hence;

For range, $y = 0$

$$0 = x\left(3 - \frac{5}{98}x\right)$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad 3 - \frac{5}{98}x \quad ; \; x = 58.8$$

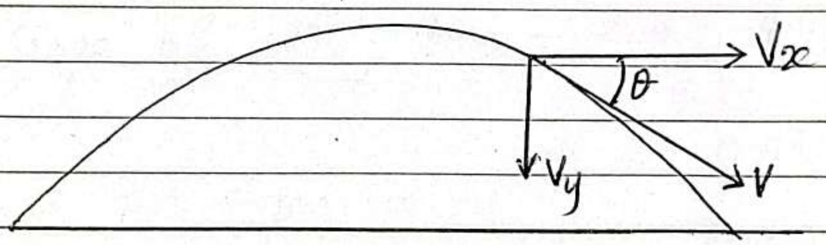
$\therefore$ Range $= 58.8m$

At maximum Height; $x = \frac{\text{Range}}{2} \Rightarrow \frac{58.8}{2} = 29.4m$

$$y = 3 \times 29.4 - \frac{5}{98} \times 29.4^2 \Rightarrow 44.1m$$

$\therefore$ Maximum Height $= 44.1m$

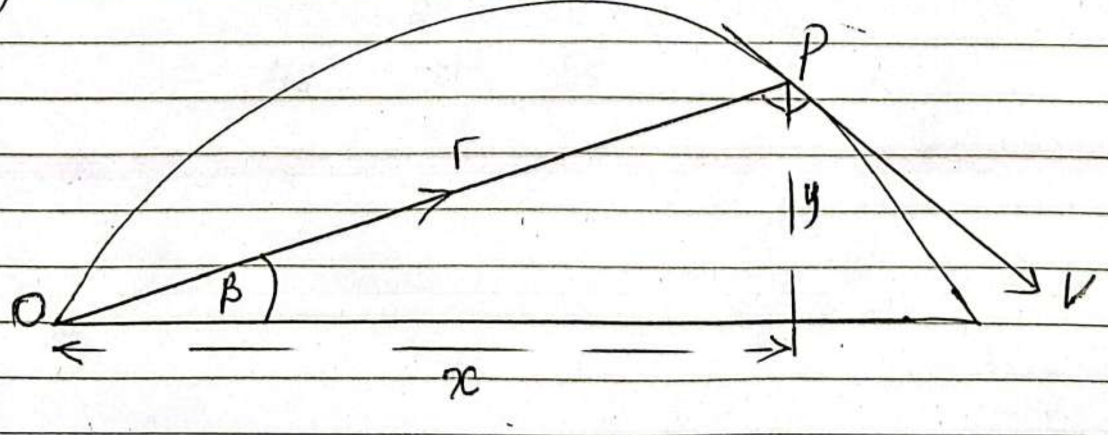
(c)



$$\tan\theta = \frac{V_y}{V_x} \Rightarrow \frac{29.3 - 9.8(t)}{9.8} = (3 - t)$$

$\therefore$ Direction $\Rightarrow \theta = \tan^{-1}(3 - t)$

(d)



$\Rightarrow$ The velocity $\underset{\sim}{v}$ and position $\underset{\sim}{r}$ are at right angles;

$$\underset{\sim}{v} \cdot \underset{\sim}{r} = 0$$

$$\begin{pmatrix} 9.8 \\ 29.4 - 9.8t \end{pmatrix} \cdot \begin{pmatrix} 9.8t \\ 29.4t - 4.9t^2 \end{pmatrix} = 0$$

$$\Rightarrow \quad t^2 - 9t + 20 = 0$$

$$t = \frac{9 \pm \sqrt{9^2 - 80}}{2}$$

$$\underline{t = 4s \quad \text{or} \quad 5s}$$

<u>OR:</u> Slope of $\underset{\sim}{v}$; $\tan\theta = \frac{v_y}{v_x} \Rightarrow (3-t)$

Slope of $\underset{\sim}{r}$; $\tan\beta = \frac{29.4t - 4.9t^2}{9.8t}$

$$\Rightarrow 3 - 0.5t$$

Product of the slopes $= -1$

$$(3-t)(3-0.5t) = -1$$

$$0.5t^2 - 4.5t + 10 = 0$$

$$t^2 - 9t + 20 = 0$$

$$t = \frac{9 \pm \sqrt{81-80}}{2}$$

$$\underline{= 4s \quad \text{or} \quad 5s}$$

"NUMERICAL" METHODS

(1) Let $m = \dfrac{x}{y-z}$

$$m + \Delta m = \frac{x + \Delta x}{(y+\Delta y) - (z+\Delta z)}$$

$$\Delta m = \frac{x+\Delta x}{y+\Delta y - z - \Delta z} - \frac{x}{y-z}$$

$$\Delta m = \frac{(x+\Delta x)(y-z) - x(x+\Delta y - z - \Delta z)}{(y-z)(y+\Delta y - z - \Delta z)}$$

$$\Delta m = \frac{(y-z)\Delta x - x\Delta y + x\Delta z}{(y-z)^2 + (y-z)\Delta y - (y-z)\Delta z}$$

$$(y-z)^2 \cdot \Delta m + (y-z)\Delta y \Delta m - (y-z)\Delta z \Delta m = (y-z)\Delta x - x\Delta y + x\Delta z$$

Since Δy, Δz and Δm are very small, then $\Delta y \Delta m \approx 0$ and $\Delta z \Delta m \approx 0$.

$$(y-z)^2 \Delta m = (y-z)\Delta x - x\Delta y + x\Delta z$$

$$\Delta m = \frac{(y-z)\Delta x - x\Delta y + x\Delta z}{(y-z)^2}$$

$$|\text{Relative error}| = \left| \frac{(y-z)\Delta x - x\Delta y + x\Delta z}{(y-z)^2} \div \frac{x}{y-z} \right|$$

$$= \left| \frac{(y-z)\Delta x - x\Delta y + x\Delta z}{x(y-z)} \right|$$

$$= \left| \frac{\Delta x}{x} - \frac{\Delta y}{y-z} + \frac{\Delta z}{y-z} \right|$$

$$\leq \left| \frac{\Delta x}{x} \right| + \left| \frac{\Delta y}{y-z} \right| + \left| \frac{\Delta z}{y-z} \right| \quad ; \left| \frac{-\Delta y}{y-z} \right| = \left| \frac{\Delta y}{y-z} \right|$$

$\therefore$ Maximum Relative error $= \left| \frac{\Delta x}{x} \right| + \left| \frac{\Delta y}{y-z} \right| + \left| \frac{\Delta z}{y-z} \right|$

Hence;

Absolute error $= \dfrac{(y-z)\Delta x + x\Delta y + x\Delta z}{(y-z)^2}$

$$= \frac{(2.15-1.9)\times 0.05 + 1.6\times 0.005 + 1.6\times 0.05}{(2.15-1.9)^2}$$

$= 1.608$

Percentage error $= \left\{ \dfrac{0.05}{1.6} + \dfrac{0.005}{2.15-1.9} + \dfrac{0.05}{2.15-1.9} \right\} \times 100$

$= 25.125$

Limits = Working value $\pm$ Absolute error

Working value $= \dfrac{1.6}{2.15-1.9} = 64$

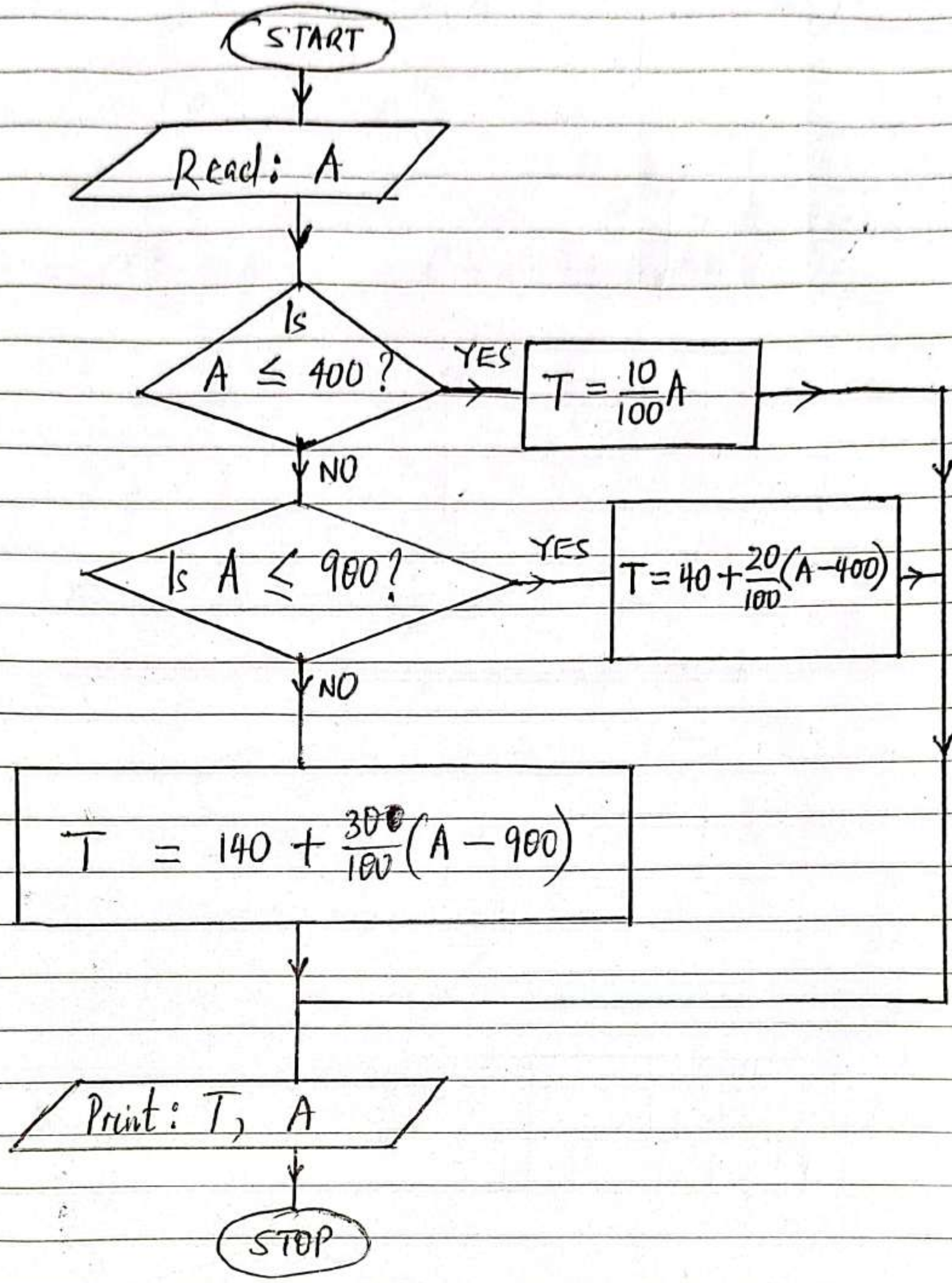
Lower limit $= 64 - 1.608 = 62.392$

Upper limit $= 64 + 1.608 = 65.608$

$\therefore$ Required interval is $[62.392, 65.608]$

(2.)

(i)



(ii) To compute and Print the Incometax of an employee.

(iii)

A	T
1500	320
750	110

3

(a) Let; $x = \ln\left(A^{1/k}\right) \Rightarrow \ln\left(\sqrt[k]{A}\right)$

$$e^{x} = A^{1/k}$$

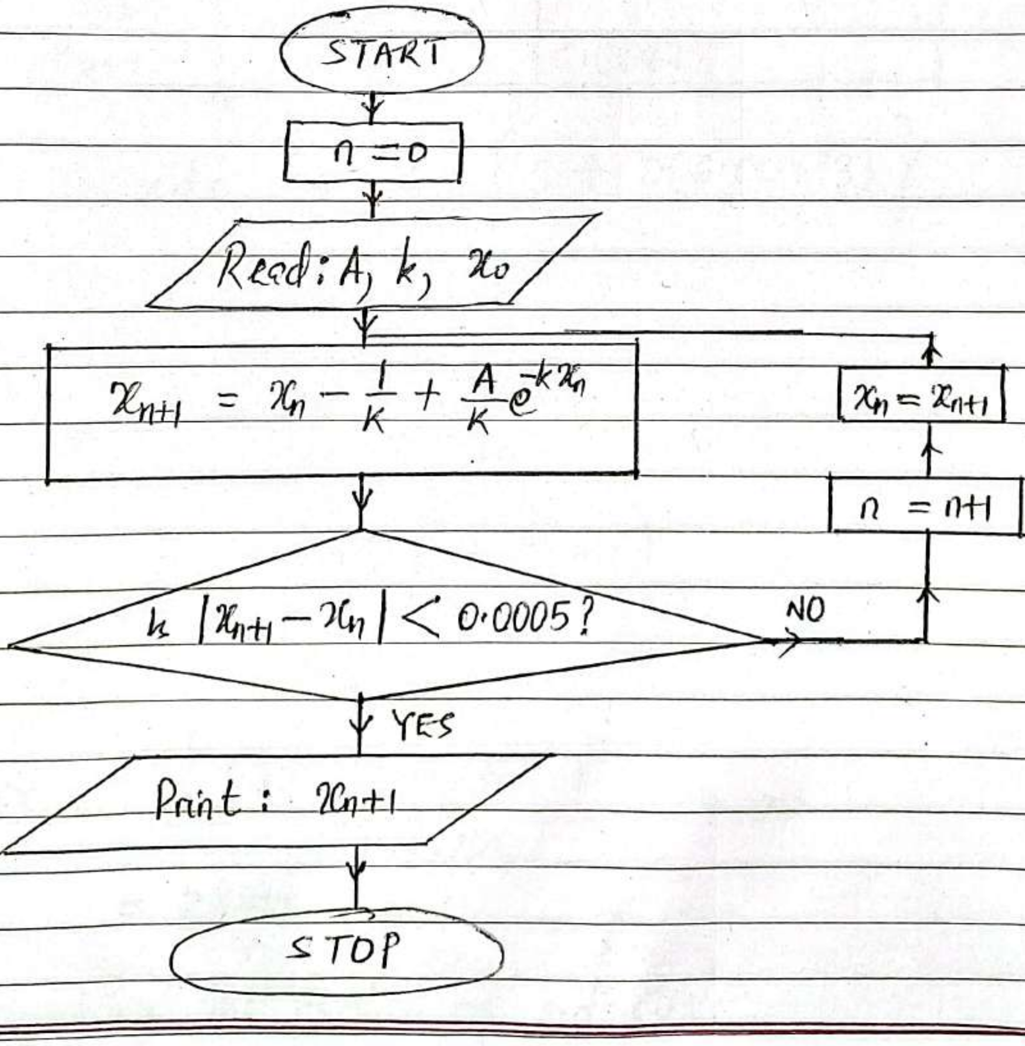
$$e^{kx} = A \Rightarrow e^{kx} - A = 0$$

$$f(x) = e^{kx} - A \qquad f'(x) = ke^{kx}$$

$$x_{n+1} = x_n - \left(\frac{e^{kx_n} - A}{ke^{kx_n}}\right)$$

$$= x_n - \frac{1}{k} + \frac{A}{k}e^{-kx_n} \quad \#$$

(b)



(c)

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.25	1.2476	0.0024
1	1.2476	1.2476	0

$\therefore$ Root $= 1.248$

4

$$h = \frac{\pi/3 - 0}{6-1} \Rightarrow \frac{\pi}{15} \; ; \text{Let } y = \sec^2 x$$

x	y	
0	1	
$\pi/15$		1.04518
$2\pi/15$		1.19823
$\pi/5$		1.52786
$4\pi/15$		2.23346
$\pi/3$	4	
SUM	5	6.00473

$$\int_0^{\pi/3} \sec^2 x \, dx \approx \frac{1}{2} \times \frac{\pi}{15} \left\{ 5 + 2(6.00473) \right\}$$

$$\approx 1.78123$$

$$\int_0^{\pi/3} \sec^2 x \, dx = \left[\tan x \right]_0^{\pi/3}$$

$$= (\tan \pi/3) - (\tan 0)$$

$$= 1.73205$$

$$\% \text{ error} = \frac{|1.73205 - 1.78123|}{1.73205} \times 100$$

$$= 2.8394$$

By increasing the number of ordinates.

5.

(i) For the first formula

$x_0 = 0.4$

$x_1 = 2(0.4)^2 \cdot e^{0.4} = 0.4774$

$x_2 = 2(0.4774)^2 \cdot e^{0.4774} = 0.7347$

For the second formula.

$x_0 = 0.4$

$x_1 = \frac{1}{2}e^{-0.4} = 0.3352$

$x_2 = \frac{1}{2}e^{-0.3352} = 0.3576$

(iii) The second formula is suitable since it converges hence;

Root $= 0.358$

(ii) for $f(x) = 2x^2e^x$

$f'(x) = 2x^2e^x + 4xe^x$

$f'(0.4) = 2(0.4)^2e^{0.4} + 4(0.4)e^{0.4} = 2.8643$

for $g(x) = \frac{1}{2}e^{-x}$

$g'(x) = -\frac{1}{2}e^{-x} \Rightarrow g'(0.4) = -\frac{1}{2}e^{-0.4}$
$= -0.3352$

Since $-1 < g'(0.4) < 1$, then the second formula converges and thus suitable.

Hence;

$x_0 = 0.4 \Rightarrow x_1 = \frac{1}{2}e^{-0.4} = 0.3352$

Root $= 0.358$ $\quad x_1 = \frac{1}{2}e^{-0.3352} = 0.3576$

(iii) From; $x_{n+1} = \frac{1}{2}e^{-x_n}$

$$\Rightarrow 2x = e^{-x}$$

$$(2x)\cdot e^{x} = e^{-x}\cdot e^{x}$$

$$2xe^{x} = 1$$

$$\Rightarrow 2xe^{x} - 1 = 0 \quad \#$$

6.

(a)

$$f(x) = \cos(x^2) - x + 3$$

x	2.5	3
$f(x)$	1.4994	−0.9111

Sign Change

$\therefore\ 2.5 < \text{root} < 3$

Hence;

2.5	x	3
1.4994	0	−0.9111

$$\frac{x - 2.5}{0 - 1.4994} = \frac{3 - 2.5}{-0.9111 - 1.4994}$$

$$x = 2.8117$$

$$f(2.8117) = \cos(2.8117)^2 - 2.8117 + 3 \Rightarrow 0.1366$$

2.8117	y	3
0.1366	0	−0.9111

$$\frac{y - 2.8117}{0 - 0.1366} = \frac{3 - 2.8117}{-0.9111 - 0.1366}\ ;\ y = 2.8356$$

$$f(2.8356) = \cos(2.8356)^2 - 2.8356 + 3 \Rightarrow -0.0212$$

2.8117	r	2.8356
0.1366	0	−0.0212

$$\frac{r - 2.8117}{0 - 0.1366} = \frac{2.8356 - 2.8117}{-0.0212 - 0.1366}\ ;\ r = 2.8323891$$

$\therefore$ Root $= 2.832$

6 (b)

P	350	400
Q	240	300

(i)

x	350	400
0	240	300

$$\frac{x - 350}{0 - 240} = \frac{350 - 400}{240 - 300}$$

$$x = 150m$$

(ii)

350	400	y
240	300	y

$$\frac{y - 400}{y - 300} = \frac{400 - 350}{300 - 240}$$

$$y = 900m$$

6(c)

x	1·15	1·25	k
$f(x)$	1·32	1·26	1·22

$$\frac{k - 1{\cdot}15}{1{\cdot}22 - 1{\cdot}32} = \frac{1{\cdot}25 - 1{\cdot}15}{1{\cdot}26 - 1{\cdot}32}$$

$$k = 1{\cdot}31667$$

$$k = 1{\cdot}317$$

7.

$f(x) = x^4 - x - 10$

x	-2	-1	0	1	2
$f(x)$	8	-8	-10	-10	4

Signchange Signchange

$-2 < \text{root} < -1$ $\quad 1 < \text{root} < 2$

Hence; $f(x) = x^4 - x - 10$

$f'(x) = 4x^3 - 1$

$$x_{n+1} = x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$= \frac{3x_n^4 + 10}{4x_n^3 - 1}$$

set $x_0 = -1.7$; $\quad x_1 = \dfrac{3(-1.7)^4 + 10}{4(-1.7)^3 - 1}$

$= -1.6975$, $e_1 = 0.0025$

$$x_2 = \frac{3(-1.6975)^4 + 10}{4(-1.6975)^3 - 1}$$

$= -1.6975$, $e_2 = 0$

$\therefore$ Root $= -1.698$