Dr. Bbosa Science

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UACE MATHEMATICS PAPER 2 2015 guide

SECTION A (40 marks)

Answer all questions in this section

1. Find the magnitude and direction of the resultant of forces $\begin{pmatrix}-3\\1\end{pmatrix}N$, $\begin{pmatrix}4\\2\end{pmatrix}N$ and $\begin{pmatrix}1\\2\end{pmatrix}N$ (05marks)
2. Use the trapezium rule with five subintervals to estimate $\int_2^4 \frac{5}{(x+1)}dx$. Give your answer correct to 3 decimal places (05marks)
3. The table below shows the mass of boys in a certain school

Mass (kg)	15	20	25	30	35
Number of boys	5	6	10	20	9

Calculate the mean mass (05marks)

4. Two cyclist A and B are 36m apart on a straight road. Cyclist B starts from rest with an acceleration of $6ms^{-2}$ while A is in pursuit of B with velocity of $20ms^{-1}$ and acceleration of $4ms^{-2}$. Find the time when A overtakes B.(05marks)
5. Events A and B are independent. P(A) = x, P(B) = x + 0.2 and $P(A\cup B) = 0.65$. Find the value of x (05marks)
6. The table below shows the values of a function f(x) for given values of x.

x	f(x)
9	2.66
10	2.42
11	2.18
12	1.92

Use linear interpolation or extrapolation to find

(a) f(10.4)

(b) the value of x, corresponding to f(x) = 1.46 (05marks)

7. the marks in an examination were found to be normally distributed with mean 53.9 and standard deviation 16.5. 20% of the candidate who sat this examination failed. Find the pass mark for the examination. (05marks)
8. A fixed hollow hemisphere has center O and is fixed so that the plane of the rim is horizontal. A particle A of weight $30\sqrt{2}$ N can move on the inside surface of the hemisphere. The particle is acted up on by a horizontal force P, whose line of action is in a vertical plane through O and A. OA makes an angle 45^0 with the vertical. If the coefficient of friction between the particle and hemisphere is 0.5 and the particle is just about to slip downwards, find the

(a) Normal reaction
(b) Value of P (05marks)

SECTION B

Answer any five questions from this section. All questions carry equal marks.

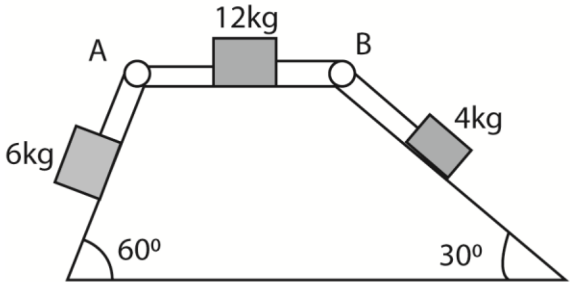
9. The probability density function (p.d.f) of a random variable Y is given by

$$f(x) = \begin{cases} \frac{(y+1)}{4} & 0 \leq y \leq k \\ 0, & \text{elsewhere} \end{cases}$$

Find
(a) The value of k (06marks)
(b) The expectation of Y (03marks)
(c) P($1 \leq Y \leq 1.5$) (03marks)

10. The numbers A = 6.341 and B = 2.6 have been rounded to the given number of decimal places
(a) Find the maximum possible error in AB (05marks)
(b) Determine the interval within which $\frac{A^2}{B}$ is expected to lie.
Give your answer correct to 3 decimal places (07marks)

11. The diagram below shows a 12kg mass on a horizontal rough plane. The 6kg and 4kg masses are on rough planes inclined at angles 600 and 300 respectively. The masses are connected to each other by light inextensible strings over light smooth pulleys A and B.



The planes are equally rough with coefficient of friction $\frac{1}{12}$. If the system is released from rest find the;
(a) Acceleration of the system (08marks)
(b) Tensions in the strings. (04marks)

12. The table below gives the points awarded to eight schools by the judges J_1, J_2 and J_3 during a music competition. J_1 was the chief judge.

J_1	72	50	50	55	35	38	82	72
J_2	60	55	70	50	50	50	73	70
J_3	50	40	62	70	40	48	67	67

(a) Determine the rank correlation coefficient between the judges of
(i) J_1 and J_2
(ii) J_1 and J_3 (10marks)
(b) Who of the two other judges had a better correlation with the chief judge? Give a reason. (02marks)

13. A ball is projected from point A and falls at point B which is in level with A at a distance of 160m from A. The greatest height of the ball attained is 40m. find the;
(a) angle and velocity at which the ball is projected (10marks)
(b) time taken for the ball to attain the greatest height (02marks)

14. (a) Show that the iterative formula based on Newton Raphson's method for approximating the root of the equation 2Inx – x + 1 = 0 is given by

$x_{n+1} = x_n \left(\frac{2Inx_n - 1}{x_n - 2}\right)$, n = 0, 1,2 ……………………… (03marks)

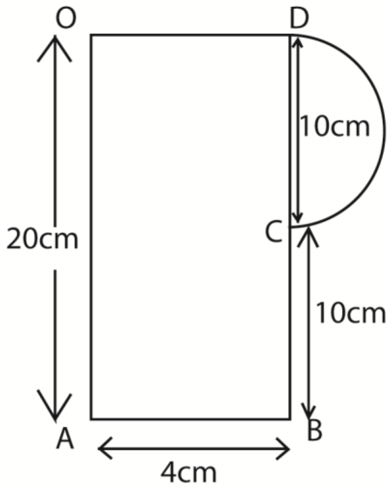
(b) Draw a flow chart that:

(i) reads the initial approximation x0 of the root

(ii) computes and prints the root correct to two decimal places, using the formula in (a) (05marks)

(c) Taking x_0 = 3.4, perform a dry run to find the root of the equation (04marks)

15. The figure below represents a lamina formed by welding together a rectangular metal sheet and a semi-circular metal sheet. Find the position of the centre of gravity of the lamina from the side OA.



16. A box A contains 4 white and 2 red balls. Another box B contains 3 white and 3 red balls. A box is selected at random and two balls are picked one after the other without replacement.
    (a) Find the probability that the two balls picked are red. (07marks)
    (b) Given that two white balls are picked, what is the probability that they are from box B? (05marks)

Solution

SECTION A (40 marks)

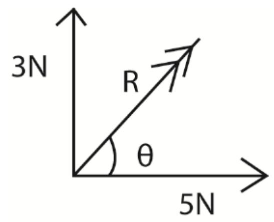
Answer all questions in this section

1. Find the magnitude and direction of the resultant of forces $\binom{-3}{1} N, \binom{4}{2} N$ and $\binom{1}{2} N$ (05marks)

$R = \binom{-3}{1} + \binom{4}{2} + \binom{1}{2} = \binom{5}{3}$

$|R| = \sqrt{5^2 + 3^2} = \sqrt{34} = 5.83N$

Direction



tanθ = $\left(\frac{3}{5}\right)$

θ = $\tan^{-1}\left(\frac{3}{5}\right) = 30.96^0$

∴ the resultant force is 5.83N and the direction 30.960 as shown in the diagram above

2. Use the trapezium rule with five subintervals to estimate $\int_2^4 \frac{5}{(x+1)}dx$. Give your answer correct to 3 decimal places (05marks)

$h = \frac{4-2}{4} - 0.4$

x	y	
2.0	1.6667	
2.4		1.4706
2.8		1.3158
3.2		1.1905
3.6		1.0870
4.0	1.0000	
	2.6667	5.0639

$\int_2^4 \frac{5}{(x+1)}dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})]$

= $\frac{h}{2}(2.6667 + 2\ x\ 5.0639)$

= 2.5589

= 2.558(3D)

3. The table below shows the mass of boys in a certain school

Mass (kg)	15	20	25	30	35
Number of boys	5	6	10	20	9

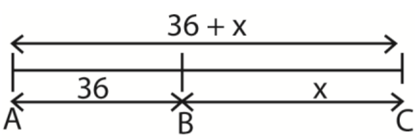
Calculate the mean mass (05marks)

x	f	Fx
15	5	75
20	6	120
25	10	250
30	20	600
35	9	315
	$\sum f = 50$	$\sum fx = 1360$

Mean mass = $\frac{\sum fx}{\sum f} = \frac{1360}{50} = 27.2kg$

4. Two cyclist A and B are 36m apart on a straight road. Cyclist B starts from rest with an acceleration of 6ms^{-2} while A is in pursuit of B with velocity of 20ms^{-1} and acceleration of 4ms^{-2}. Find the time when A overtakes B.(05marks)

Let x be the distance from B where A overtakes B



Using s = ut + ½ at^2

For A

$36 + x = 20t + ½ x 4 x t^2$
$36 + x = 20t + 2t^2$ ……………………….. (i)
For B
$x = 0 + ½ x 6 x t^2$
$x = 3t^2$ ……………………………………. (ii)
Substituting for x in equation (i)
$36 + 3t^2 = 20t + 2t^2$
$t^2 – 20t + 36 = 0$
$(t – 18)(t – 2) = 0$
Either
t – 18 = 0; t = 18s
Or
t -2 = 0; t =2s

5. Events A and B are independent. P(A) = x, P(B) = x + 0.2 and $P(A\cup B) = 0.65$. Find the value of x (05marks)
$P(A\cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A).P(B)$
0.65 = x + (0.2 + x) - x(x + 0.2)
$x^2 – 1.8x + 0.45 = 0$
(x – 0.3)(x – 1.5) = 0
Either
x – 0.3 = 0
x = 0.3
Or
(x – 1.5) = 0
x = 1.5
∴ x = 0.3

6. The table below shows the values of a function f(x) for given values of x.

x	f(x)
9	2.66
10	2.42
11	2.18
12	1.92

Use linear interpolation or extrapolation to find
(a) f(10.4)
Extract

10	10.4	11
2.42	f(x)	2.1

Using gradient approach
$$\frac{2.18 - f(x)}{11 - 10.4} = \frac{2.18 - 2.42}{11 - 10}$$
$$\frac{2.18 - f(x)}{0.6} = \frac{-0.24}{1}$$
f(x) = 2.18 + 0.24 x 0.6 = 2.324

(b) the value of x, corresponding to f(x) = 1.46 (05marks)

Extract

x	12	11
1.46	1.92	2.18

Using gradient approach

$\frac{2.18-1.46}{11-x} = \frac{2.18-1.92}{11-12}$

$\frac{0.72}{11-x} = \frac{0.26}{-1}$

X = 11 - $\frac{-1\ x\ 0.72}{0.26}$ = 13.769

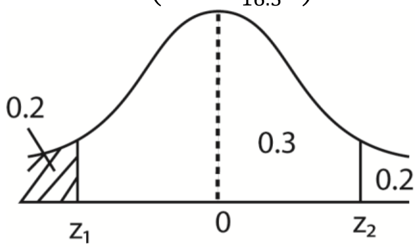
7. The marks in an examination were found to be normally distributed with mean 53.9 and standard deviation 16.5. 20% of the candidate who sat this examination failed. Find the pass mark for the examination. (05marks)

Let the pass mark be x1

P(X< x_1) = 0.2

By standardisation

P(P<x1) = $P\left(z < \frac{x_1-53.9}{16.5}\right) = 0.2$



P(Z < z_1) = P(Z > z_2) = 0.2

⇨ P(0 < Z < z_2) = 0.3

z_2 = 0.882

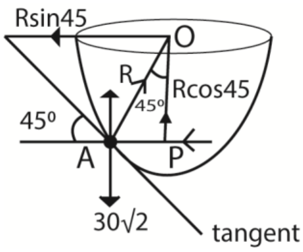
z_1 = -0.842

⇨ $\frac{x_1-53.9}{16.5} = -0.842$

$x_1 = 53.9 - 16.5\ x\ 0.842$ = 40

Hence the pass mark was 40%$\sqrt{2}$

8. A fixed hollow hemisphere has center O and is fixed so that the plane of the rim is horizontal. A particle A of weight 30$\sqrt{2}$ N can move on the inside surface of the hemisphere. The particle is acted up on by a horizontal force P, whose line of action is in a vertical plane through O and A. OA makes an angle 45^0 with the vertical. If the coefficient of friction between the particle and hemisphere is 0.5 and the particle is just about to slip downwards, find the



(a) Normal reaction

µRcos45 + Rcos45 = 30$\sqrt{2}$

$R\left(\frac{1}{2}.\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 30\sqrt{2}$

$\frac{3R}{4} = 30$
$R = 40N$

(b) Value of P (05marks)
P = R(sin45 – μsin45)
= $40\left(\left(\frac{\sqrt{2}}{2} - \frac{1}{2}.\frac{\sqrt{2}}{2}\right)\right)$
= $10\sqrt{2}$
= 14.1421N

SECTION B

Answer any five questions from this section. All questions carry equal marks.

9. The probability density function (p.d.f) of a random variable Y is given by

f(x) = $\begin{cases} \frac{(y+1)}{4} & 0 \leq y \leq k \\ 0, & \text{elsewhere} \end{cases}$

Find

(a) The value of k (06marks)

$\int_0^k \frac{(y+1)}{4} dy = \frac{1}{4}\left[\frac{y^2}{2} + y\right]_0^k = 1$

$\frac{1}{4}\left[\left(\frac{k^2}{2} + k\right) - 0\right] = 1$

$k^2 + 2k - 8 = 0$

(k + 4)(k-2) = 0

Either

k + 4 = 0; k = -4

Or

k-2 = 0; k = 2

$\therefore k = 2$ (since k is greater than zero)

(b) The expectation of Y (03marks)

E(Y) = $\int_0^2 y dy$

=$\int_0^2 y\left[\frac{y+1}{4}\right] dy$

=$\int_0^2 \left(\frac{y^2+y}{4}\right) dy$

= $\frac{1}{4}\left[\frac{y^3}{3} - \frac{y^2}{2}\right]_0^2$

=$\frac{1}{4}\left[\left(\frac{8}{3} - \frac{4}{2}\right) - 0\right]$ = $\frac{7}{6} = 1.166$

(c) P($1 \leq Y \leq 1.5$) (03marks)

P($1 \leq Y \leq 1.5) = \int_1^{1.5}\left[\frac{y+1}{4}\right] dy$

=$\frac{1}{4}\left[\frac{y^2}{2} + y\right]_1^{1.5}$

=$\frac{1}{4}\left[\left(\frac{(1.5)^2}{2} + 1.5\right) - \left(\frac{1}{2} + 1\right)\right]$

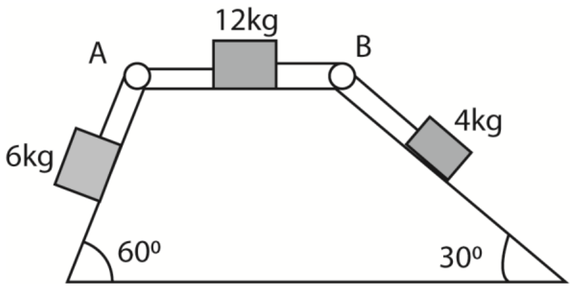
=$\frac{1}{4}(2.625 - 1.5)$

= 0.28125

10. The numbers A = 6.341 and B = 2.6 have been rounded to the given number of decimal places
   (a) Find the maximum possible error in AB (05marks)
   Let Z = AB
   $Z + \Delta Z = (A + \Delta A)(B + \Delta B)$
   $= AB + A\Delta B + B\Delta A + \Delta A\Delta B$
   $\Delta Z = AB + A\Delta B + B\Delta A + \Delta A\Delta B - AB$
   $= A\Delta B + B\Delta A + \Delta A\Delta B$
   But $\Delta A\Delta B \cong 0$
   ⇨ $|\Delta Z| = |A\Delta B||B\Delta A| = A|\Delta B| + B|\Delta A|$
   = 6.341 x 0.05 + 2.6 x 0.0005
   = 0.31835

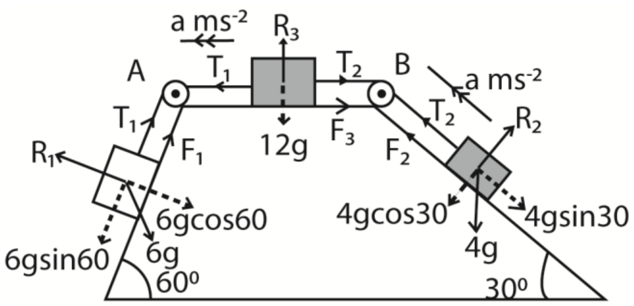
   (b) Determine the interval within which $\frac{A^2}{B}$ is expected to lie.
   Give your answer correct to 3 decimal places (07marks)
   $\left(\frac{A^2}{B}\right)_{max} = \frac{(6.341+0.0005)^2}{(2.6-0.05)} = \frac{(6.3415)^2}{2.55}$ = 15.77044
   $\left(\frac{A^2}{B}\right)_{min} = \frac{(6.341-0.0005)^2}{(2.6+0.05)} = \frac{(6.3405)^2}{2.65}$ = 15.17054
   ∴ the interval is (15.17054 - 15.77044)

11. The diagram below shows a 12kg mass on a horizontal rough plane. The 6kg and 4kg masses are on rough planes inclined at angles 600 and 300 respectively. The masses are connected to each other by light inextensible strings over light smooth pulleys A and B.



The planes are equally rough with coefficient of friction $\frac{1}{12}$. If the system is released from rest find the;

   (a) Acceleration of the system (08marks)



   For 6kg mass
   6gsin60-($T_1 + \frac{1}{12}\ x\ 6gcos60) = 6a$
   6gsin60 – T_1 - $\frac{1}{2}cos60 = 6a$ ……………………….. (i)
   For 4kg mass

$T_2 - (\frac{1}{12}\ x\ 4gcos30 + 4gsin30) = 4a$

$T_2 - \frac{1}{3}gcos30 - 4gsin30 = 4a$ …………………. (ii)

For 12kg mass

$T_1 - (T_2 + \frac{1}{12}\ R_3) = 12a$

$T_1 - (T_2 + \frac{1}{12}\ x\ 12g) = 12a$

$T_1 - T_2 - g = 12a$……………………………………… (iii)

Eqn. (i) + Eqn. (ii) + Eqn. (iii)

$6gsin60 - \frac{1}{2}gcos60 - \frac{1}{3}gcos30 - 4gsin\ 30 - g = 22a$

16.24327742 = 22a

$a = \frac{16.24327742}{22} = 0.73833ms^{-2}$

(b) Tensions in the strings. (04marks)

From equation (i)

$T_1 = 6gsin60 - \frac{1}{2}cos60 - 6a$

$= 6gsin60 - \frac{1}{2}cos60 - 6\ x\ 0.73833$

= 44.0423N

From eqn. (ii)

$T_2 = \frac{1}{3}gcos30 + 4gsin30 + 4a$

$= \frac{1}{3}gcos30 + 4gsin30 + 4\ x\ 0.73833$

= 25.3823N

12. The table below gives the points awarded to eight schools by the judges J_1, J_2 and J_3 during a music competition. J_1 was the chief judge.

J_1	72	50	50	55	35	38	82	72
J_2	60	55	70	50	50	50	73	70
J_3	50	40	62	70	40	48	67	67

(a) Determine the rank correlation coefficient between the judges of

(i) J_1 and J_2

J_1	J_2	R_{J_1}	R_{J_2}	D_1	D_1^2
72	60	2.5	4	-1.5	2.25
50	55	5.5	5	0.5	0.25
50	70	5.5	2.5	3	9
55	50	4	7	-3	9
35	50	8	7	1	1
38	50	7	7	0	0
82	73	1	1	0	0
72	70	2.5	2.5	0	0
					$\sum D_1^2$ = 21.5

$\rho_1 = 1 - \frac{6\sum D_1^2}{n(n^2-1)}$

$= 1 - \frac{6\ x\ 21.5}{8(8^2-1)}$

$= \frac{125}{168} = 0.7440$

(ii) J_1 and J_3 (10marks)

J_1	J_3	R_{J_1}	R_{J_3}	D_2	D_2^2
72	50	2.5	5	-2.5	6.25
50	40	5.5	7.5	-4	4
50	62	5.5	4	0.5	2.25
55	70	4	1	3	9
35	40	8	7.5	0.5	0.25
38	48	7	6	1	1
82	67	1	2.5	-1.5	2.25
72	67	2.5	2.5	0	0
					$\sum D_1^2$ = 25

$\rho_2 = 1 - \frac{6\sum D_2^2}{n(n^2-1)}$

$= 1 - \frac{6 \, x \, 25}{8(8^2-1)}$

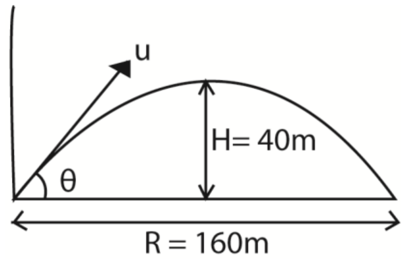
$= 0.7023$

(b) Who of the two other judges had a better correlation with the chief judge? Give a reason. (02marks)

J_2 has a better correlation with the Chief Judge because the coefficient of correlation is smaller showing a stronger mutual relationship

13. A ball is projected from point A and falls at point B which is in level with A at a distance of 160m from A. The greatest height of the ball attained is 40m. find the;

(a) angle and velocity at which the ball is projected (10marks)



Using $v^2 = u^2 + 2as$

$v^2 = u^2\sin^2\theta - 2gh$

At maximum height $v_y = 0$

$0 = u^2\sin^2\theta - 2gH$

$u^2 = \frac{2gH}{\sin^2\theta}$ …………………………………………… (i)

Range, R = $\frac{2u^2\sin\theta\cos\theta}{g}$ ……………………………… (ii)

Eqn. (i) and eqn. (ii)

R = $2 \, x \, \frac{2gH}{\sin^2\theta} . \frac{\sin\theta\cos\theta}{g}$ = 4Hcotθ …………….. (iii)

Substituting for R and H in eqn. (iii)

160 = 4 x 40cotθ

tan θ = 1

θ = $\tan^{-1}$ 1 = 45^0

Substituting for θ

$u^2 = \frac{2gH}{\sin^2\theta} = \frac{2 \, x \, 40 \, x \, 9.8}{\sin^2 45^0}$

$u = \sqrt{\frac{2\ x\ 40\ x\ 9.8}{sin^2\ 45^0}}$ = 39.60ms^{-1}

(b) time taken for the ball to attain the greatest height (02marks)

$t = \frac{usin\ \theta}{g} = \frac{39.60\ x\ \sin 45^0}{9.8} = 2.8573s$

14. (a) Show that the iterative formula based on Newton Raphson's method for approximating the root of the equation 2Inx – x + 1 = 0 is given by

$x_{n+1} = x_n\left(\frac{2Inx_n-1}{x_n-2}\right)$, n = 0, 1,2 ……………………….. (03marks)

f(x) = 2Inx – x + 1

f'(x) = $\frac{2}{x} - 1$

also

f(x_n) = 2Inx_n – x_n + 1

f'(x_n)= $\frac{2}{x_n} - 1$

Using $x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)}$

By substitution, we get

$$x_{n+1} = x_n - \frac{2Inx_n - x_n+1}{\left(\frac{2}{x_n}-1\right)}$$

$$= \frac{x_n\left(\frac{2}{x_n}-1\right) - 2Inx_n - x_n+1}{\left(\frac{2}{x_n}-1\right)}$$

$$= \frac{x_n(2-x_n) - x_n(2Inx_n - x_n+1)}{(2-x_n)}$$

$$= \frac{x_n(2-x_n-\ 2Inx_n + x_n-1)}{(2-x_n)}$$

$$= \frac{x_n(1-\ 2Inx_n)}{(2-x_n)}$$

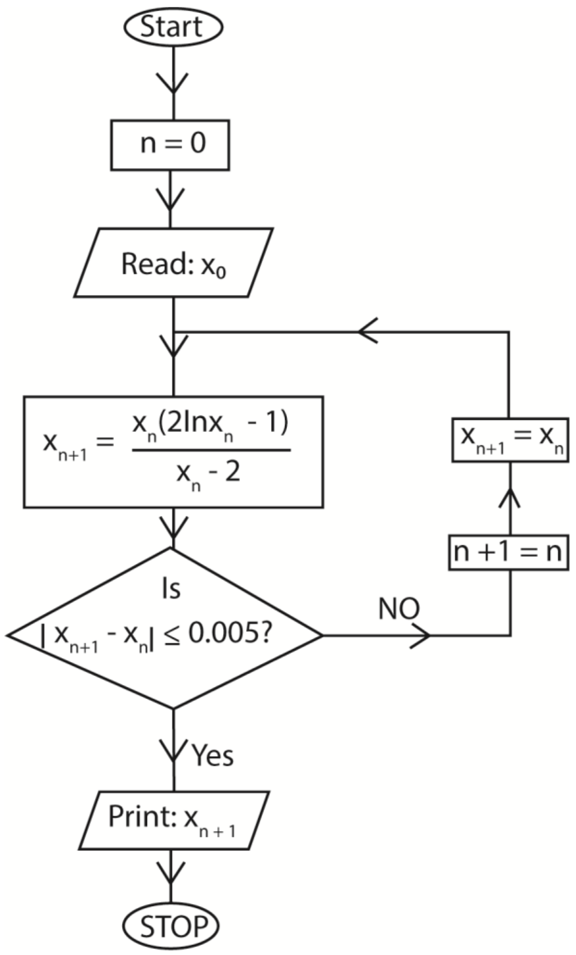
$$= \frac{-x_n(\ 2Inx_n-1)}{-(\ x_n-2)}$$

$$= \frac{x_n(\ 2Inx_n-1)}{(\ x_n-2)}$$

(b) Draw a flow chart that:

(i) reads the initial approximation x_0 of the root

(ii) computes and prints the root correct to two decimal places, using the formula in (a) (05marks)



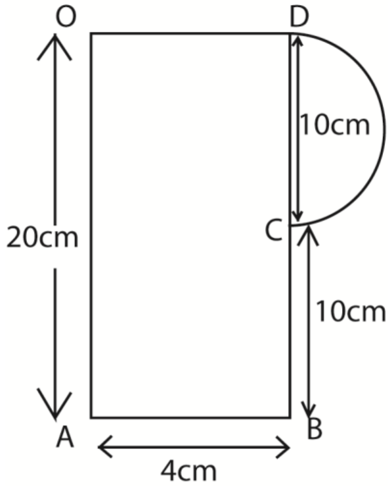
(c) Taking $x_0 = 3.4$, perform a dry run to find the root of the equation (04marks)

Dry run

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	3.4	3.51548	0.11548
1	3.51548	3.51286	0.00262
2	3.51286	3.51286	0.0000

Hence the root = 3.51(2D)

15. The figure below represents a lamina formed by welding together a rectangular metal sheet and a semi-circular metal sheet. Find the position of the centre of gravity of the lamina from the side OA.



Let w = weight per unit area

Area of the rectangle ABDO = 20 x 4 = 80cm^2

Weight of ABDO = 80w

Area of semicircle = $\frac{1}{2}\pi r^2$ = $\frac{1}{2}\pi(5)^2$ = 39.2857cm^2

Weight of semicircle = 39.2857w

Portion	Weight	Distance of C.O.G from OA
Rectangle ABDO	80w	2cm
Semicircle	39.2857w	$\left(4 + \frac{20}{3\pi}\right)$ = 6.1221cm
Whole lamina	119.2857w	$\bar{x}$

Equating moments along OA

Moment of sum = moment of whole body

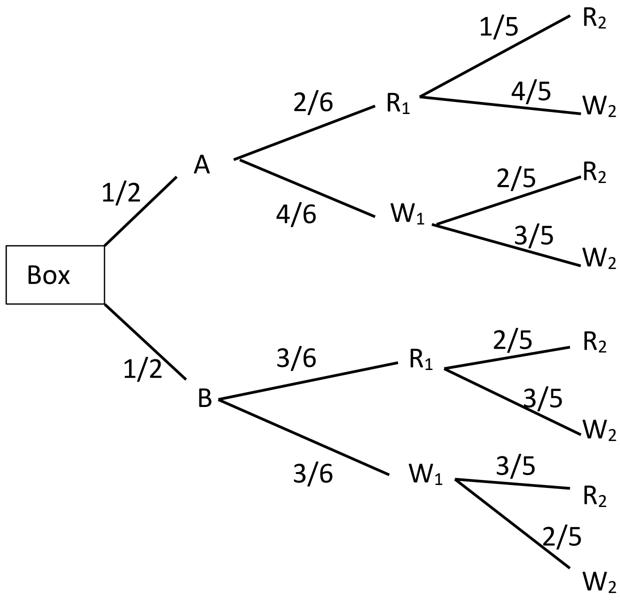
80w x 2 + 39.2857w x 6.1221= 119.2857w$\bar{x}$

400.510984 = 119.2857$\bar{x}$

$\bar{x} = \frac{400.510984}{119.2857}$ = 3.3576cm

Hence the centre of gravity of lamina from OA = 3.3576cm

16. A box A contains 4 white and 2 red balls. Another box B contains 3 white and 3 red balls. A box is selected at random and two balls are picked one after the other without replacement.



(a) Find the probability that the two balls picked are red. (07marks)

$P(R_1 \cap R_2) = P(A \cap R_1 \cap R_2) + P(B \cap R_1 \cap R_2)$

$= \frac{1}{2}\ x\ \frac{2}{6}\ x \frac{1}{5} + \frac{1}{2}\ x\ \frac{3}{6}\ x\ \frac{2}{5}$

$= \frac{1}{30} + \frac{1}{10} = \frac{2}{15} = 0.1333$

(b) Given that two white balls are picked, what is the probability that they are from box B? (05marks)

Let W be event that both white balls are picked

$P(W) = P(A \cap W_1 \cap W_2) + P(B \cap W_1 \cap W_2)$

$= \frac{1}{2}\ x\ \frac{4}{6}\ x \frac{3}{5} + \frac{1}{2}\ x\ \frac{3}{6}\ x\ \frac{2}{5}$

$= \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

But $P(^B/_W) = \frac{P(B \cap W)}{P(W)} = \frac{P(B \cap W_1 \cap W_2)}{P(W)}$

$= \frac{1}{10} \div \frac{3}{10}$

$= \frac{1}{10}\ x\ \frac{10}{3} = \frac{1}{3} = 0.333$

Thank you

Dr. Bbosa Science