A LEVEL

P425/1

MATHEMATICS

Paper 1

March 1987

3 hours

1.(a) Show that $\log_a b = \dfrac{\log_c b}{\log_c a}$

Hence or otherwise solve the equation

$\log_2 0.013 - \log_3 30 = \log_{10} x^2$

(b) The polynomial $5x^3 + px^2 + qx + r$ has a factor $x - 2$ and a remainder of $3x+1$ when divided by $x^2 - 1$. Find the values of p,q and r.

2(a) Prove by induction that

$$\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1)$$

(b) The roots of the equation

$ax^2 + bx + c = 0$ are α and β.

(i) show that if

$\alpha - \beta = 1$ then $a^2 = 4(b^2 - ac)$

3.(a) The sum of the first ten terms of an arithmetic progression is 250 . Given that the difference between the tenth and the first is 36, find the common difference.

(b) Find the binomial expression of

$$\sqrt{\frac{(1+x)^3}{2+3x}}$$

in ascending powers of x up to the term x^3 where $|x| < 1$

4. Differentiate

(i) $\sec^2(\cot x)$

(ii) $a^{\cos^2 x^2}$

where a is a constant.

(b) A curve is defined by the parametric equations

$x = at^2 \quad y = a\left(t - t^2\right)$

Find the stationary point on the curve and show whether it is a maximum or minimum .

5(a) The tangent to the curve $x^2+xy+2y^2 = 7$ at a point p is parallel to the x-axis. Find the coordinates of p.

(b) Two ships A and B start moving from the same point at the same time . Ship A moves $40kmh^{-1}$,N 30^0 E and ship B moves $48kmh^{-1}$ due east , find the rate at which they are separating from each other at the end of two hours.

6. Find

(i) $\int \frac{3x^3 + 2x^2 - 3x - 1}{x\left(x^2 - 1\right)}$

(ii) $\int e^{3x} \cos(2x+1)dx$

7.(a) Use the substitution $x = \frac{1}{u}$ to evaluate

$$\int_2^3 \frac{dx}{x\left(x^2 + x\right)^{1/2}}$$

(b) Find

(i) the centre of gravity of the area bounded by the curve $y = x^2 + 2$ and the line $y = 3$.

(ii) the volume of solid generated about the line $y = 3$

SECTION III TRIGONOMETRY AND GEOMETRY

8. (a) Prove that

$$\sin x + \tan^2 x = \frac{1 - \cos 2x + 2\sin x \cos^2 x}{1 + \cos 2x}$$

(b) Given that a,b and c are the lengths of the sides of a triangle ABC , show that

$$\frac{a+b-c}{a+b+c} = \tan\frac{1}{2}A \tan\frac{1}{2}B$$

9. (a) Show that if $t = \tan\frac{\theta}{2}$ then

$$\sin\theta = \frac{2t}{1+t^2}$$

Hence , or otherwise solve the equation

$$3\cos\theta - 5\sin\theta + 1 = 0$$

(b) Find the general solution of the equation

$$\cos\theta + \sin\theta + \sec\theta + \operatorname{cosec}\theta = 0$$

10. The point P (a secθ,b tanθ) lies on the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

show that

(i) the equation of the tangent to the hyperbola is

$bx - ay\sin\theta - ab\cos\theta = 0$

(ii) if the tangent at P cuts the asymptotes at Q and R then QR is bisected by P.

11. (a) The coordinates of three points A,B and C are (5,3) ,(-2,2) and (2,4) respectively. Find the equations of the perpendicular bisectors of AB and AC . Hence find the equation of the circle that passes through A,B and C.

(b) The tangents at a point Q on the circle $x^2 + y^2 = 4$ and a point R on the circle $x^2 + y^2 = 16$ meet at a apoint P , Given that $\overline{PQ}^2 + \overline{PR}^2 = 8$,find the locus of P.

SECTION IV.STATISTICS

12.(a) The table shows the retail price of three commodities together with the corresponding quantities consumed for the period 1980-82

	Prices			Quantities		
Year	1980	1981	1982	1980	1981	1982
A	39	38	41	96	98	102
B	61	62	60	11	12	14
C	50	55	48	70	85	80

Using 1980' as the base year , calculate

(I) the simple aggregate price index

(ii) the weighted aggregate price index

(b) The table shows the average monthly production of a certain commodity in thousands of tonnes.

Year	1980	1981	1982	1983	1984	1985	1986
Production	48	36	43	45	38	36	31

On the same coordinates axes represent the average monthly production and the four -year moving averages for the data.

(c) A student obtained 80% and 88% in mathematics and physics examination respectively. The mean mark in Mathematics was 74% and standard deviation 10 marks and the corresponding values for physics are 80% and 16 marks respectively. In which subject was his relative performance better.

13. (a) Given that c(n,r) means the number of combinations of n objects taken r at a time , solve the equation

$c(10,4)=c(x,3) + c(x,4)$

(b) A three digit integer is to be formed using the digits 1,2,3,4 and 5.

if no digit is to be used more than once find

(i) the total number of different integers that can be formed

(ii) the probability that the integer formed is greater than 500

(iii) the probability that the integer formed is a multiple of 3

How many different integers can be formed if one of the digits may be repeated?

SECTION V VECTORS.

14.(a) Show that the points A,B C with position vectors 2**i**+3**j**, 4**i**+5**j**, 6**i**+9**j** respectively are the vertices of a triangle . Find the area of the triangle

(b) A line L_1 passes through the point (-4,7,5) and parallel to the vector 3**i-2j+4k** . Find

(i) the point of intersection of L_1 and the line

$$\frac{x}{2}=\frac{y+5}{1}=\frac{z+9}{8}$$

(ii) the equation of the plane containing L_1 and the origin.

SECTION VI COMPLEX NUMBERS

15.(a) Evaluate

(b) Find the modulus and argument of the complex number $\omega = 1 + \cos 2\theta - i\sin 2\theta$

(c) Solve the equation

$$z^2 + 2z^2 - z - \bar{z} + 9 = 0$$

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MATHEMATICS
Paper2
March 1987
3 hours.

SECTION 1 NUMERICAL METHODS

1. Show that an interative method for finding the square root of a number N is given by

$$\frac{1}{2}\left(X_n + \frac{N}{X_n}\right) n = 0,1 - - - - -$$

Draw a flow chart

(i) reads N and the initial approximation.

(ii) computes and prints the square root of N correct to 3 decimal places.

Perform dry run of the flow chart

for $N = 28$ and $X_0 = 5$

2. (a) Given that X and Y are measured with possible errors ΔX and ΔY respectively, show that the relative error in the product XY is

$$\frac{|\Delta X|Y + |\Delta Y|X}{XY}$$

State clearly an assumptions made

Given that X =5.43 and Y =27.2 write down the maximum possible error in X and Y, Hence find the interval in which the product XY lies.

(b) The table shows the distance in centimetres travelled by a particle in five seconds of its motion

Time, t	0	1	2	3	4	5
Distance	0	15	37.5	68	104	135

Use linear interpolation to estimate

(i) the distance travelled at $t = 2.2$

(ii) the time when the distance travelled is 50cm.

3. (a) A particle moving with an acceleration given by

A = **3** $\mathbf{e^{-t}}$ **i + 5costj - 4sinti** is located at (2,-3,4) and has velocity v = **6i - 2j + 3k** at time t = 0.
Find
(i) the velocity,
(ii) the displacement at any time t.
(b) Forces of magnitude 3N, 4N, 4N, 3N and 5N act along the lines AB,BC CD, DA and AC respectively of a square ABCD of side a : the direction of the forces being given by the order of letters
Find
(i) the resultant force
(ii) the point where the line of action of the resultant force cuts AB.

4.(a) A sphere of mass m and radius r rests on a smooth plane inclined at an angle θ to the horizontal . it is supported by a string of length r fastened to the plane.
Find
(i) the tension in the string

(ii) the reaction at the point of contact of the sphere with the plane.

(b) A uniform rod of length 2l inclined at an angle θ to the horizontal rests in a vertical plane against a smooth horizontal bar at a height h above the ground and the rod is about to slip , show that the coefficient of friction between the rod and the ground is

$$\frac{\lambda\sin^2\theta\cos\theta}{h-\lambda\cos^2\theta\sin\theta}$$

5. The point P is 50km west of Q
Two aircraft A and B fly simultaneously from P and Q with velocities 400kmh^{-1} N 50^{0}E and 500 kmh^{-1} N 70^{0}W respectively.
Find
(i) the closest distance between the aircrafts.
(ii) the time of flight up to this point.

6. A particle is projected upwards with a velocity v

from a point on a plane inclined at an angle θ to the horizontal . if the angle of projection is α to the horizontal(θ < α).

(i) Find the range along the plane

(ii) show that the maximum range along the plane is

$$R_{max} = \frac{v^2}{g(1+\sin\theta)}$$

(iii) Find the time taken to cover R_{max} when $\theta = 30^0$ and $v = 20ms^{-1}$.

7(a) two particles A and B of mass m lie on a smooth table and are connected by an inextensible string . Particle A is given an impulse I in a direction making an angle θ with the line BA

Find

(i) the angle at which the particle A begins to move.

(ii) the impulse tension in the string,

(b) A bullet of mass 20g is fired with a velocity of $500ms^{-1}$ into a block of wood of mass 1kg resting on along smooth table.

find

(i) the common velocity of the bullet and the block when the bullet is embedded in the block.
(ii) the loss in kinetic energy.

8. (a) A mass of 10kg moving with simple harmonic motion in the horizontal direction is initially located at a distance 4m from the origin O and has a velocity of $20ms^{-1}$ and acceleration $100ms^{-2}$ directed towards O. Find the force on the mass when t = 2s

9(b) A sphere of mass 5kg attached at the end of a vertical spring of negligible mass stretches it 10 cm. Find the equation of motion and the amplitude if
(i) the sphere is pulled down 5cm and then released.
(ii) the sphere is pulled down 10cm and then given an initial velocity of vm/s downwards.

SECTION II DIFFERENTIAL EQUATIONS.

9. (a) Solve the equation

$$\frac{dy}{dx}+\frac{2xy}{x^2+1}-x=0$$

(b) A liquid cools in a room at constant temperature of 22^0C at a rate proportional to the excess temperature initially the temperature of the liquid was 100^0C and one minute later a was 92.2^0C ,Find the temperature of the liquid after 5 minutes.

SECTION III STATISTICS

10. (a) Given that A and B are mutually exclusive events and P(A) = 0.4 and P(B) = 0.5 , find

(i) $P(A\cup B)$

(ii) $P(A\cap\overline{B})$

(iii) $P(\overline{A}\cap\overline{B})$

(b) A box P contains 4 white and 6 red balls and a box Q contains 6 white and 2 red balls.

A box is chosen at random and a ball is drawn at random from it

(i) Find the probability that the ball drawn is red.
(ii) Given that the ball selected is white , find the probability that box P was selected.

11. (a) A game consists of drawing card at random without displacement from five cards consisting of three spades and two diamonds.
Given that the game is terminated when a diamond is drawn, find the expected number of draws
(b) A multiple choice test consists of ten questions. Each has five alternatives with only one correct answer. Five marks are awarded for each correct answer and one mark is subtracted for each incorrect answer or unattempted question.
if a candidate chooses the answer at random , find
(i) the expected number of correct answers
(ii) the expected overall marks
(iii) the probability that the candidate gets more than 8 marks.
12.A random variable x has the probability density function

$$f(x) = \begin{cases} kx & 0\langle x\langle 1 \\ (k/2)x & 1\le x\le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find

(i) the value of the constant k

(ii) E(x)

(iii) the median of x

(iv) $P(x<1.5 / 1\le x\le 2)$

(v) distribution function of x and sketch it.

13. (a) The error x made in determining a certain parameter in an experiment is a random variable having a uniform distribution over the interval(α,β) Given that the mean error is zero and the variance is 0.12, find the values of α and β
Hence , find the probability that the error will exceed 0.5

(b) A random sample of 16 tins of coffee selected from 95% confidence interval of the average weights in grammes

120 101 103 100 104 102 103 105
101 103 102 101 104 100 102 103

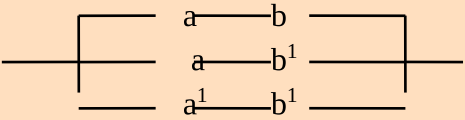
Given that the weights are normally distributed find a 95% confidence interval of the average weight of the tins.

14. The weights of bars of soap produced in a certain factory are normally distributed.
Given that 10% of the bars weigh less than 140g and 20% weigh more than 165g, find
(i) the mean and variance of the distribution.
(ii) the percentage of bars that would be expected to weigh less than 145g.
If the variance of the distribution is reduced by 30%, find the production of bars that would be expected to weigh less than 148g.

SECTION IV LOGIC,ELECTRICAL CIRCUITS AND ALGEBRAIC STRUCTURES.

15. (a) use a truth table to show whether or not the statement $p \wedge (p \rightarrow \sim q) \rightarrow q$

(b) Given the circuit



(i) Write down a Boolean function for the circuit

(ii) Simplify your function and hence draw the simplest equivalent circuit.

(iii) Verify that the two circuits are equivalent.

(c) Given the truth table

p	q	S	T	U
T	T	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	T	F	T

where S ,T and U are function of p and q, find S(p,q), T(p,q) and U (p,q).