NAME: ……

SCHOOL: ……… RANDOM NO: …………….

P425/1
MATHEMATICS
Paper 1
July/Aug. 2024
3 **hours**

ASK INTEGRATED TEACHERS MOCK
EXAMINATIONS BURREAU

AITEL JOINT MOCK EXAMINATIONS 2024.

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section **A** and any **five** questions from section **B***

***All** necessary working **must** be shown clearly*

Begin each solution to a new question on a fresh page.

*Any additional question(s) answered will **not** be marked.*

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn over

SECTION A (40 marks)

*(Attempt **all** questions in this section)*

1. Find the coordinates of the points where the line $4x - 5y + 6a = 0$ cuts the curve given parametrically by $(at^2, 2at)$ in terms of a. (05 marks)

2. If $Z = 2 + i$ is a root of the equation, $2Z^3 - 9Z^2 + 14Z - 5 = 0$, find the other roots. (05 marks)

3. Show that
$$\frac{\sin\theta + 2\sin 2\theta + \sin 3\theta}{\sin\theta - 2\sin 2\theta + \sin 3\theta} = -Cot^2\frac{\theta}{2}$$ (05 marks)

4. Evaluate
$$\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx$$ (05 marks)

5. Given that $y = \frac{x+6}{\sqrt{(x+2)}}$, find $\frac{dy}{dx}$ when $x = 2$. (05 marks)

6. Expand $\sqrt{(1-x)}$ in ascending powers of x including the term x^4. Use your expansion to find $\sqrt{90}$ correct to four significant figures. (05 marks)

7. In a culture of bacteria, the rate of growth is proportional to the population present at a time t. The population doubles every day. Given that the initial population, p_o is one million. Determine the number of days when the population will be 100 million. (05 marks)

8. In a triangle PQR, **PQ** =**p** and **PR** = **r**. Given that M is the midpoint of **PQ** and X is a point on **QR** such that QX : QR is 3:5. Find **MX** in terms of **p** and **r**. (05 marks)

SECTION B (60 marks)

*(Attempt any **five** questions. **All** questions carry equal marks)*

9. (a) Show that $\cos 3\theta = 4cos^3\theta - 3\cos\theta$. Hence solve the equation $4x^3 - 3x - \frac{\sqrt{3}}{3} = 0$. For $0^0 \leq x \leq 90^0$ (06 marks)

   (b) Using the substitution $t = \tan\frac{\theta}{2}$, solve $3\sin\theta - \cos\theta = 3$ (06 marks)

10. (a) Express $\frac{x^3-3}{(x-2)(x^2+1)}$ as a partial fraction. Hence find $\int \frac{x^3-3}{(x-2)(x^2+1)}\,dx$ (08 marks)

    (b) Evaluate $\int_0^{\frac{2}{3}\pi} sin\,^3x\,dx$ (04 marks)

11. (a) Find n if ${}^{n}C_{14} = {}^{n}C_{16}$ (04 marks)

(b) Prove by induction that $5^n + 4n - 1$ is divisible by 8 for all positive integers.

(05 marks)

(c) What is the number of terms of a geometric progression (GP) 5, 10, 20, … that can give a sum greater than 800,000? (03 marks)

12. (a) Find the acute angle between the line $\frac{x+4}{8} = \frac{-y+2}{-2} = \frac{z+1}{-4}$ and the plane $4x + 3y - 3z = -1$ (06 marks)

(b) Show that the lines $\boldsymbol{r} = (-2\boldsymbol{i} + 5\boldsymbol{j} - 11\boldsymbol{k}) + \alpha(3\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$ and $\boldsymbol{r} = (8\boldsymbol{i} + 9\boldsymbol{j}) + t(4\boldsymbol{i} + 2\boldsymbol{j} + 5\boldsymbol{k})$ intersect and find the position vector of their point of intersection. (06 marks)

13. (a) If the roots of the equation $x^2 + 2px + q = 0$ differ by 2. Prove that $p^2 - q - 1 = 0$ (04 marks)

(b) The polynomial $f(x) = x^4 + 4x^3 + px^2 + qx + r$ is a perfect square of a second degree polynomial. Show that $q + 8 = 2p$ and $q^2 = 16r$. (08 marks)

14. (a) Determine the equation of a circle passing through the points A(-1,2), B(2,4) and C(0,4). (07 marks)

(b) If $y = mx - 5$ is a tangent to the circle $x^2 + y^2 = 9$, find the possible values of m.

(05 marks)

15. (a) Given that $y = \sqrt{\frac{1-\sin x}{1+\sin x}}$, show that $\frac{dy}{dx} = \frac{-1}{1+\sin x}$ (06 marks)

(b) A rectangular sheet of paper is of sides 8cm by 5cm. equal squares of side $x$cm are cut from each corner and the edges are then folded to make an open box of volume Vcm^3. Show that $V = 40x - 26x^2 + 4x^3$. Find the maximum possible volume. (06 marks)

16. Sketch the curve $y = \frac{x+1}{(x-1)(2x+1)}$. Showing clearly the nature of the turning points.

(12 marks)

END