SNo.	Working	Marks
1	(i).	

$$\sum X = 110 + 101 + 91 + 89 + 122 + 115 + 106 + 109 + 112 + 105 + 106 = 1166$$

B1-$\sum X$

$$\therefore \text{Mean value of } X = \frac{\sum X}{n} = \frac{1166}{11} = 106$$

M1 A1-division and output

(ii).

Median = 106

$$\underbrace{89 \quad 91 \quad 101 \quad 105 \quad 106}_{Q_1 = 101} \quad 106 \quad \underbrace{109 \quad 110 \quad 112 \quad 115 \quad 122}_{Q_3 = 112}$$

$$\text{Interquartile range} = Q_3 - Q_1 = 112 - 101 = 11$$

M1 A1- subtraction and output

05

SNo.	Working	Marks
2	(i).	

Actual weight	24	54	x
Recorded weight	35	60	x
Difference	11	6	0

B1-identifying its extrapolation

$$\frac{x-24}{54-24} = \frac{x-35}{60-35}$$

$$\frac{x-24}{30} = \frac{x-35}{25}$$

M1-equating quotients

$$25x - 600 = 30x - 1050$$

$$5x = 450$$

$$x = 90 \text{ g}$$

A1-output

(ii).

Actual weight	0	24	54
Recorded weight	y	35	60

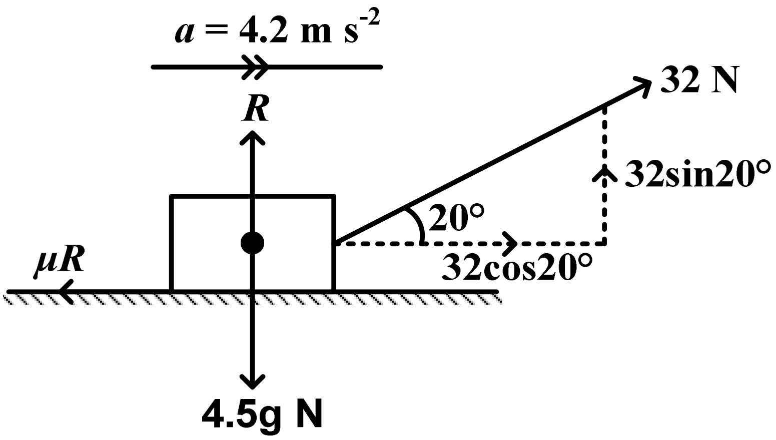
$$\frac{y-35}{60-35} = \frac{0-24}{54-24}$$

$$\frac{y-35}{25} = \frac{-24}{30}$$

M1-equating quotitents

$$y = 35 - \frac{24}{30} \times 25 = 15 \text{ g}$$

A1-output

		05
3	 Resolving vertically gives: $R + 32\sin 20° = 4.5g$ $R = 4.5 \times 9.8 - 32\sin 20°$ $= 33.1554$ N Resolving horizontally gives: $32\cos 20° - \mu R = ma$ $32\cos 20° - \mu \times 33.1554 = 4.5 \times 4.2$ $33.1554\mu = 11.1702$ $\mu = 0.3369$	**B1-**force diagram **M1-**resolving vertically **B1-**normal raction **M1-**resolving horizontally **A1-**value of μ
		05
4	$\sum W = x + 2x + y + y + 6 = 40$ $3x + 2y = 34 \quad \longrightarrow (1)$ Weighted average price index $= \dfrac{\sum(I_{2004} \times W)}{\sum W}$ $126.7 = \dfrac{(110 \times x) + (140 \times 2x) + (130 \times y) + [118 \times (y + 6)]}{40}$ $126.7 = \dfrac{110x + 280x + 130y + 118y + 708}{40}$ $5068 = 390x + 248y + 708$ $390x + 248y = 4360 \quad \longrightarrow (2)$ Equation $130 \times (1) - (2)$ gives, $390x + 260y = 4420$ $-\ 390x + 248y = 4360$ $12y = 60$ $y = 5$ From equation (1), $3x + 2 \times 5 = 34$ $3x = 24$ $x = 8$	**B1-**eqn 1 **M1-**substitution **B1-**eqn 2 **M1-**solving **A1-**both x and y correct

05

5

$$y_n = \frac{1}{2x_n + 1}\,, \qquad h = \frac{0.5 - 0.1}{6 - 1} = 0.08$$

B1-value of h

n	x_n	y_0, y_5	$y_1, \dots y_4$
0	0.1	0.83333	
1	0.18		0.73529
2	0.26		0.65789
3	0.34		0.59524
4	0.42		0.54348
5	0.5	0.5	
sums		1.33333	2.5319

B1-x_n values

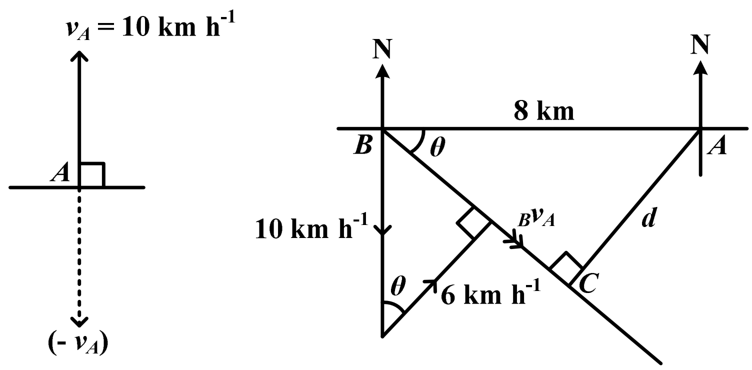
B1-y_n values (more than 3 d.p)

$$\int_{0.1}^{0.5} \frac{1}{2x+1}\,dx \approx \frac{1}{2}h[(y_0 + y_5) + 2(y_1 + \cdots + y_4)]$$
$$\approx \frac{1}{2} \times 0.08 \times [1.33333 + 2 \times 2.5319]$$
$$\approx 0.2558852 \approx 0.256 \text{ (3 s.f)}$$

M1- substitution
A1-output (strictly 3 s.f)

05

6 (i).



B1-velocity diagram

$$\cos\theta = \frac{6}{10}\,, \qquad \Rightarrow \theta = 53.13°$$
$$\therefore \text{Bearing} = 053.13°$$

M1-attempting to find bearing
A1-stating required bearing

(ii).

Shortest distance, $\quad d = AB\sin\theta$
$= 8\sin 53.13° = 6.40$ km

M1 A1- substitution and output

05

7 (i).

$$\frac{d}{dt}\left(\frac{2at - t^2}{a^2}\right) = \frac{2a - 2t}{a^2}$$

M1-derivative

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$$\frac{d}{dt}(1) = 0$$

$$\therefore f(t) = \begin{cases} \frac{2a-2t}{a^2} & ; \quad 0 \le t \le a \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

(ii).

$$P(20 \le t \le 40) = F(40) - F(20)$$
$$= \frac{2a \times 40 - 40^3}{a^2} - \frac{2a \times 40 - 40^3}{a^2}$$
$$= \frac{40a - 56000}{a^2}$$

ALT:

$$P(20 \le t \le 40) = \int_{20}^{40} \frac{2a-2t}{a^2} dt = \frac{1}{a^2}\left[2at - t^2 \quad\right]_{20}^{40}$$
$$= \frac{1}{a^2}[(2a \times 40 - 40^3) - (2a \times 20 - 20^3)]$$
$$= \frac{1}{a^2}(40a - 56000)$$

</td><td>

M1-derivative

B1-correct p.d.f

M1- subtraction
A1-simplified output

M1- substituting limits
A1-simplified output

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<tr><td>8</td><td>

(a).

$$\underset{\sim}{\boldsymbol{F}} = \underset{\sim}{\boldsymbol{F_1}} + \underset{\sim}{\boldsymbol{F_2}} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} \text{ N}$$

$$\text{Moment of } \underset{\sim}{\boldsymbol{F}} \text{ about the origin} = \left(\underset{\sim}{\boldsymbol{r}} \times \underset{\sim}{\boldsymbol{F}}\right) = \begin{vmatrix} \underset{\sim}{\boldsymbol{i}} & \underset{\sim}{\boldsymbol{j}} & \underset{\sim}{\boldsymbol{k}} \\ 1 & -1 & 1 \\ 5 & 3 & 1 \end{vmatrix}$$
$$= \underset{\sim}{\boldsymbol{i}}\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} - \underset{\sim}{\boldsymbol{j}}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + \underset{\sim}{\boldsymbol{k}}\begin{vmatrix} 1 & -1 \\ 5 & 3 \end{vmatrix}$$
$$= \underset{\sim}{\boldsymbol{i}}(-1-3) - \underset{\sim}{\boldsymbol{j}}(1-5) + \underset{\sim}{\boldsymbol{k}}(3+5)$$
$$= \left(-4\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}} - 8\underset{\sim}{\boldsymbol{k}}\right) \text{ N m}$$

(ii).

$$\text{Moment of the couple} = -\left(-4\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}} - 8\underset{\sim}{\boldsymbol{k}}\right)$$
$$= \left(4\underset{\sim}{\boldsymbol{i}} - 4\underset{\sim}{\boldsymbol{j}} + 8\underset{\sim}{\boldsymbol{k}}\right) \text{ N m}$$

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M1 B1- addition and output

M1- substitution

A1-output

B1-moment of couple

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<tr><td>9</td><td>

x	y	R_x	R_y	d	d^2
16	280	10	2.5	7.5	56.25
21	300	9	1	8	64
36	180	4	7	– 3	9
44	116	2	10	– 8	64
25	280	7	2.5	4.5	20.25
55	128	1	9	– 8	64

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B1-both ranking correct

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23	250	8	4	4	16
38	150	3	8	– 5	25
30	246	6	5	1	1
32	190	5	6	– 1	1
$\sum x =$ 320	$\sum y =$ 2120				$\sum d^2 =$ 320.5

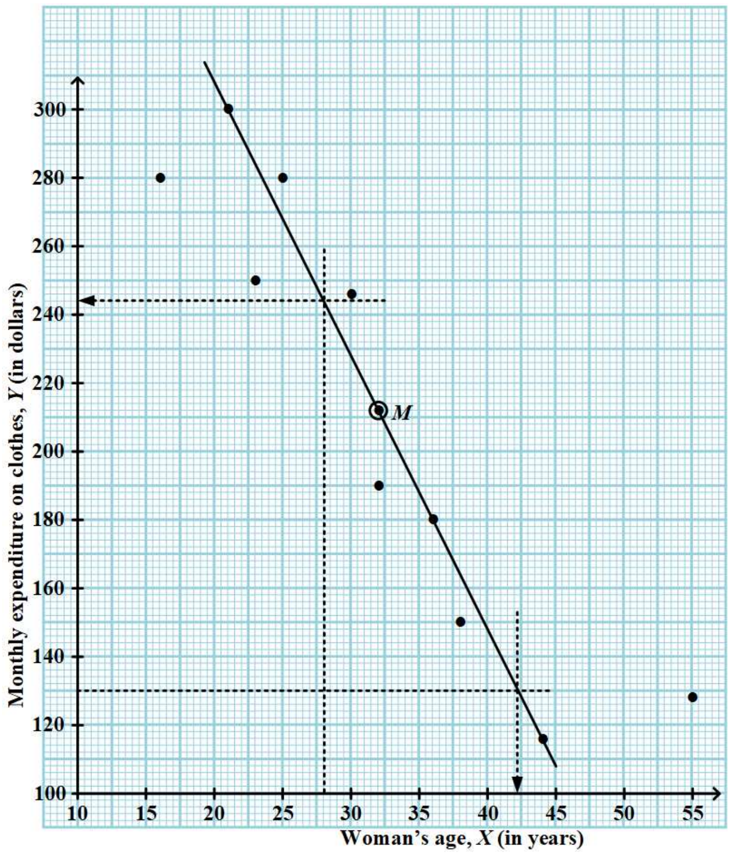
B1-$\sum d^2$ correct

(a).

$$\overline{x} = \frac{\sum x}{n} = \frac{320}{10} = 32, \qquad \overline{y} = \frac{\sum y}{n} = \frac{2120}{10} = 212$$

$\therefore$ Mean point is $M(32, 212)$

B1-mean point



B1-both axes labelled and with uniform scale

B1 B1-plotting

B1-line of best fit

(b). (i).
The age of a woman who spends 130 dollars monthly on clothes is 42.25 years.

B1-estimation

(ii).
The monthly expenditure on clothes for a 28-year-old woman is 244 dollars.

B1-estimation

(c).

$$\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 320.5}{10(10^2 - 1)} = -0.9424$$

Comment: Significant at 5%.

M1 A1-substitution and output
B1-comment

		12
10	(i). $$x = 26.23, \quad \Rightarrow e_x = \frac{4}{100} \times 26.23 = 1.0492$$ $$y = 13.18, \quad \Rightarrow e_y = \frac{3}{100} \times 13.18 = 0.3954$$ $$z = 5.1, \quad \Rightarrow e_y = \frac{2}{100} \times 5.1 = 0.102$$ (ii). Lower limit, $\left(xy - \frac{y}{z}\right)_{\min} = x_{\min} \times y_{\min} - \frac{y_{\max}}{x_{\min}}$ $$= (26.23 - 1.0492) \times (13.18 - 0.3954) - \frac{(13.18 + 0.3954)}{(26.23 - 1.0492)}$$ $$= 25.1808 \times 12.7846 - \frac{13.5754}{25.1808}$$ $$= 321.3873386 \approx 321.387 \ \text{(3 d.p)}$$ Upper limit, $\left(xy - \frac{y}{z}\right)_{\max} = x_{\max} \times y_{\max} - \frac{y_{\min}}{x_{\max}}$ $$= (26.23 + 1.0492) \times (13.18 + 0.3954) - \frac{(13.18 - 0.3954)}{(26.23 + 1.0492)}$$ $$= 27.2792 \times 13.5754 - \frac{12.7846}{27.2792}$$ $$= 369.8573942 \approx 369.857 \ \text{(3 d.p)}$$ (iii). Working value, $\left(xy - \frac{y}{z}\right) = 26.23 \times 13.18 - \frac{13.18}{26.23}$ $$= 345.2089219 \approx 345.209 \ \text{(3 d.p)}$$ $$\text{Absolute error} = \frac{\text{Max.} - \text{Min.}}{2}$$ $$= \frac{369.857 - 321.387}{2} = 24.235$$ $$\text{Percentage error} = \frac{\text{Absolute error}}{\text{Working value}} \times 100$$ $$= \frac{24.235}{345.209} \times 100 = 7.0204$$	**B1-**for e_x **B1-**for e_y **B1-**for e_z **M1-** substitution **A1-**output (strictly 3 d.p) **M1-** substitution **A1-**output (strictly 3 d.p) **B1-**working value (3 d.p or more) **M1 B1-** substitution and output **M1 A1-** substitution and output (2 d.p or more)
		12
11	(a). (i). $$\underset{\sim}{\boldsymbol{r}} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 30 \\ 40 \end{pmatrix} t - \begin{pmatrix} 0 \\ 5 \end{pmatrix} t^2$$ Initially, when $t = 0$ $$\underset{\sim}{\boldsymbol{r}}(t = 0) = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 30 \\ 40 \end{pmatrix} \times 0 - \begin{pmatrix} 0 \\ 5 \end{pmatrix} \times 0^2 = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$ $$\therefore \text{Height} = 5 \text{ m}$$ (ii). For vertical motion,	**B1-**$\underset{\sim}{\boldsymbol{r}}(t = 0)$ **B1-**height

	$$y = ut\sin\theta - \frac{1}{2}gt^2$$ By comparison, $$\frac{1}{2}gt^2 = 5t^2$$ $$g = 10 \text{ m s}^{-2}$$ (b). $$\underset{\sim}{\boldsymbol{r}}(t=3) = \binom{0}{5} + \binom{30}{40} \times 3 - \binom{0}{5} \times 3^2 = \binom{90}{80} \text{ m}$$ $$\underset{\sim}{\boldsymbol{r}}(t=5) = \binom{0}{5} + \binom{30}{40} \times 5 - \binom{0}{5} \times 5^2 = \binom{150}{80} \text{ m}$$ $$\text{Required displacement} = \binom{150}{80} - \binom{90}{80} = \binom{60}{0} \text{ m}$$ (c). $$\text{from,} \quad \underset{\sim}{\boldsymbol{r}} = \binom{0}{5} + \binom{30}{40}t - \binom{0}{5}t^2$$ $$x = 30t, \quad \Rightarrow t = \frac{x}{30}$$ $$y = 5 + 40t - 5t^2$$ $$y = 5 + 40 \times \frac{x}{30} - 5 \times \left(\frac{x}{30}\right)^2$$ $$y = 5 + \frac{4}{3}x + \frac{x^2}{180}$$	**M1-**comparison **B1-**value of g **B1-**$\underset{\sim}{\boldsymbol{r}}(t=3)$ **B1-**$\underset{\sim}{\boldsymbol{r}}(t=5)$ **M1 A1-**subtraction and output **B1-**for x **B1-**for y **M1-**substitution **B1-**required trajectory
		12
12	(i). $$P(X=0) = P(\text{no blue}) = \frac{^{50}C_3}{^{75}C_3} = \frac{784}{2701}$$ $$P(X=1) = P(\text{one blue and two others})$$ $$= \frac{^{25}C_1 \times {}^{50}C_2}{^{75}C_3} = \frac{1225}{2701}$$ $$P(X=2) = P(\text{two blue and one other})$$ $$= \frac{^{25}C_2 \times {}^{50}C_1}{^{75}C_3} = \frac{600}{2701}$$ $$P(X=3) = P(3\text{ blue}) = \frac{^{25}C_3}{^{75}C_3} = \frac{92}{2701}$$ x: 0, 1, 2, 3 $P(X=x)$: $\frac{784}{2701}$, $\frac{1225}{2701}$, $\frac{600}{2701}$, $\frac{92}{2701}$ (ii). $$E(X) = \sum_{\text{all } x} xP(X=x)$$ $$= \left(0 \times \frac{784}{2701}\right) + \left(1 \times \frac{1225}{2701}\right) + \left(2 \times \frac{600}{2701}\right) + \left(3 \times \frac{92}{2701}\right) = 1$$	**M1 B1-**correct combinations and output **M1 B1-**correct combinations and output **M1 B1-**correct combinations and output **M1 B1-**correct combinations and output **B1-**probability distribution **M1 M1 A1-**multiplication,

		addition and output
		12
13	(a). $f(x) = 20\cos x - x, \quad f'(x) = -20\sin x - 1$ $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}, \quad \text{for } n = 1, 2, 3, \ldots$ $x_{n+1} = x_n - \dfrac{20\cos x_n - x_n}{-20\sin x_n - 1} = x_n + \dfrac{20\cos x_n - x_n}{20\sin x_n + 1}$ Taking $x_0 = \frac{\pi}{2}$, $x_1 = \dfrac{\pi}{2} + \dfrac{20\cos\frac{\pi}{2} - \frac{\pi}{2}}{20\sin\frac{\pi}{2} + 1} = \dfrac{\pi}{2} + \dfrac{0 - \frac{\pi}{2}}{20 + 1}$ $= \dfrac{\pi}{2} - \dfrac{\pi}{42} = \dfrac{10\pi}{21}$, as required (b). let, $x = \sqrt[5]{N}, \quad \Rightarrow x^5 = N, \quad \Rightarrow x^5 - N = 0$ $f(x) = x^5 - N, \quad f'(x) = 5x^4$ $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}, \quad \text{for } n = 1, 2, 3, \ldots$ $x_{n+1} = x_n - \dfrac{{x_n}^5 - N}{5{x_n}^4} = \dfrac{5{x_n}^5 - ({x_n}^5 - N)}{5{x_n}^4} = \dfrac{4{x_n}^5 + N}{5{x_n}^4}$ $= \dfrac{4{x_n}^5}{5{x_n}^4} + \dfrac{N}{5{x_n}^4} = \dfrac{1}{5}\left(4x_n + \dfrac{N}{{x_n}^4}\right)$ $\therefore x_{n+1} = \dfrac{1}{5}\left(4x_n + \dfrac{N}{{x_n}^4}\right), \quad \text{for } n = 1, 2, 3, \ldots$ For the hence part: $N = 50, \quad x_0 = 2$ $x_1 = \dfrac{1}{5}\left(4 \times 2 + \dfrac{50}{2^4}\right) = 2.22500$ $x_2 = \dfrac{1}{5}\left(4 \times 2.22500 + \dfrac{50}{2.22500^4}\right) = 2.18802$ $x_3 = \dfrac{1}{5}\left(4 \times 2.18802 + \dfrac{50}{2.18802^4}\right) = 2.18673$ $x_4 = \dfrac{1}{5}\left(4 \times 2.18673 + \dfrac{50}{2.18673^4}\right) = 2.18672$ $\therefore \text{Root} = 2.187 \text{ (3 d.p)}$ or, $50^{\frac{1}{5}} = 2.187 \text{ (3 d.p)}$	**M1-**derivative **M1-**substitution **M1-**substitution **B1-**required expression **B1-**function and derivative **M1-**substitution **B1-**required expression **M1-**for x_1 (more than 3 dp) **M1-**for x_2 (more than 3 dp) **M1-**for x_3 (more than 3 dp) **M1-**for x_4 (more than 3 dp) **A1-**root (strictly 3 d.p)
		12
14	(a).	

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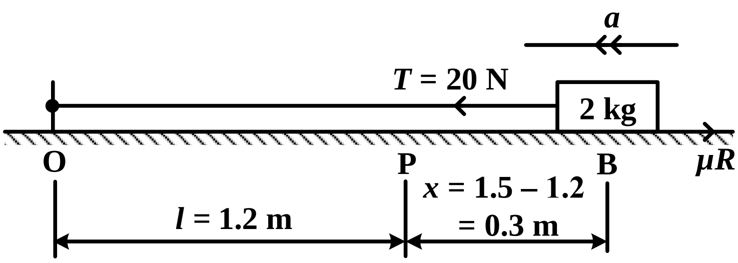

$$T = \frac{\lambda x}{l}$$
$$20 = \frac{\lambda \times 0.3}{1.2}$$
$$\lambda = 80 \text{ N}$$
For motion BP,
Work done by friction
$$= \text{Initial total energy} - \text{Final total energy}$$
$$\mu R \times x = (K.E_B + E.P.E_B) - (K.E_P + E.P.E_P)$$
$$\mu mg \times x = \left(0 + \frac{\lambda x^2}{2l}\right) - \left(\frac{1}{2}mv^2 + 0\right)$$
$$\frac{2}{5} \times 2 \times 9.8 \times 0.3 = \frac{80 \times 0.3^2}{2 \times 1.2} - \frac{1}{2} \times 2 \times v^2$$
$$2.352 = 3 - v^2$$
$$v^2 = 0.648$$
$$v = 0.8050 \text{ m s}^{-1}$$
(ii).
For motion PC,
$$\text{Work done by friction} = \text{Initial K.E} - \text{Final K.E}$$
$$\mu mg \times s = \frac{1}{2}mv^2 - 0$$
$$\frac{2}{5} \times 2 \times 9.8 \times s = \frac{1}{2} \times 2 \times 0.8050^2$$
$$7.84s = 0.648025$$
$$s = 0.0827 \text{ m}$$
$$\text{Distance } BC = BP + PC = 0.3 + 0.0827 = 0.3827 \text{ m}$$
</td><td>

M1- substitution

A1-output

M1 M1 M1- work done, loss in K.E, equating
A1-required velocity

M1 M1 M1- work done, loss in K.E, equating
B1-output
M1 A1- addition and output
</td></tr>
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12
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<tr><td>15</td><td>

(a).
$$398 + a = 615, \quad \Rightarrow a = 217$$
$$398 + b = 603, \quad \Rightarrow b = 205$$
$$a + c = 397, \quad \Rightarrow c = 397 - 217 = 180$$
$$615 + d = 1000, \quad \Rightarrow d = 385$$
(b). (i).
$$P(\text{male and speeding}) = P(M \cap S) = \frac{398}{1000}$$
(ii).
</td><td>

B1-value of a
B1-value of b
B1-value of c
B1-value of c

B1- $P(M \cap S)$
</td></tr>
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$$P(\text{female and not speeding}) = P(F \cap S') = \frac{c}{1000} = \frac{180}{1000}$$

$$P(\text{not speeding}) = P(S') = \frac{397}{1000}$$

$$P(\text{female given not speeding}) = P(F/S') = \frac{P(F \cap S')}{P(S')}$$

$$= \frac{180}{1000} \div \frac{397}{1000} = \frac{180}{397}$$

(b).

$$P(\text{male}) = P(M) = \frac{615}{1000}$$

$$P(\text{speeding}) = P(S) = \frac{603}{1000}$$

$$P(M).P(S) = \frac{615}{1000} \times \frac{603}{1000} = \frac{370845}{1000000} = 0.370845$$

$$\text{but,} \qquad P(M \cap S) = \frac{398}{1000} = 0.398$$

Since $P(M \cap S) \neq P(M).P(S)$, it implies that the events of being a male and speeding are not independent.

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B1-mobile $P(F \cap S')$ or $P(S')$ correct

M1 A1-division and output

B1-mobile $P(M)$ or $P(S)$ correct

M1 B1- multiplication and output

B1-conclusion

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<td>16</td>
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(a).

Taking moments about the y-axis,

$$\bar{x} = \frac{\int_0^2 xy\,dx}{\int_0^2 y\,dx}$$

$$\int_0^2 xy\,dx = \int_0^2 x(x^2+1)\,dx = \int_0^2 (x^3+x)\,dx$$

$$= \left[\frac{1}{4}x^4 + \frac{1}{2}x^2\right]_0^2 = \left(\frac{1}{4} \times 2^4 - \frac{1}{4} \times 2^2\right) - 0 = 6$$

$$\text{Area} = \int_0^2 y\,dx = \int_0^2 (x^2+1)\,dx$$

$$= \left[\frac{1}{3}x^3 + x\right]_0^2 = \left(\frac{1}{3} \times 2^3 - 2\right) - 0 = \frac{2}{3}$$

$$\bar{x} = \frac{\int_0^2 xy\,dx}{\int_0^2 y\,dx} = 6 \div \frac{2}{3} = 9$$

(b).

Let ρ be the mass per unit area.

<table>
<tr><td>Figure</td><td>Weight</td><td>Distance from side OA</td></tr>
<tr><td>ABEF</td><td>$= \frac{2}{3}\rho$</td><td>$= 9$</td></tr>
<tr><td>BCDE</td><td>$= (3 \times 5)\rho = 15\rho$</td><td>$= 2 + \frac{3}{2} = 3.5$</td></tr>
<tr><td>Whole lamina</td><td>$= \frac{2}{3}\rho + 15\rho = \frac{47}{3}\rho$</td><td>$\bar{x}$</td></tr>
</table>

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M1 B1- integration and output

M1 B1- integration and output

M1 A1-division and output

B1-correct distance for figure BCDE

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	By taking moments about the y −axis, $$\left(\frac{2}{3}\rho \times 9\right) + (15\rho \times 3.5) = \frac{47}{3}\rho \times \overline{x}$$ $$6 + 52.5 = \frac{47}{3} \times \overline{x}$$ $$58.5 = \frac{47}{3} \times \overline{x}$$ $$\overline{x} = \frac{351}{94} \approx 3.7340 \text{ units}$$ The centre of gravity of the lamina is 3.7340 units from the y-axis.	**M1 M1 M1 M1-**each moment, equating **A1-**output
		12

END