Connected particles

Simple connections

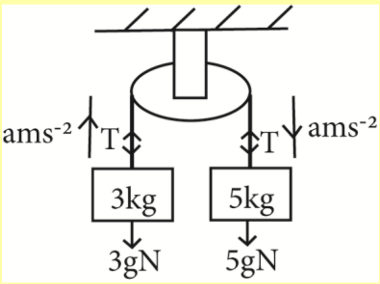
When two particles are connected by a light inextensible string passing over a smooth pulley and allowed to move freely, then as long as the string is tout, the following must be observed.

- acceleration of the particles is the same
- tension in the uninterrupted string is constant
- tensions in interrupted strings are different.

Example 1

Two particles of masses 5kg and 3kg are connected by a light inextensible string passing over a smooth fixed pulley. Find

(a) acceleration of the particle
(b) the tension in the string



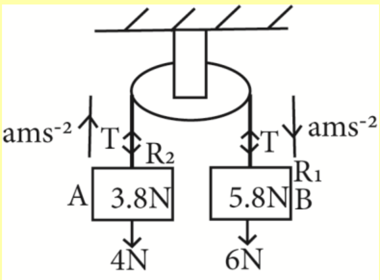
For 5kg mass: 5g – T = 5a …. (i)
For 3kg mass: T – 3g = 3a …. (ii)

(i) and (ii)
2g = 8a
a = $\frac{2 \, x \, 9.8}{8}$ = 2.45ms^{-2}
(ii) tension in the string
T -3g = 3a
T = 3 x 2.45 + 3 x 9.8 = 36.78N
(iii) Force on the pulley
R = 2T = 3 x 36.78 = 73.56N

Example 2

An inextensible string attached to two scale A and B each of weight 20g passes over a smooth fixed pulley. Particles of weight 3.8N and 5.8N are placed in pans A and B respectively. If the system s released from rest (take g = 10ms^{-2}). Find the

(a) Tension in the string
(b) Reaction of the scale pan holding the 3.8N weight



Weight of the scale pan = $\frac{20}{1000} x\ 10 = 0.2N$

Total weight of A = 3.8 + 0.2 = 4N

Total weight of B = 5.8 + 0.2 = 6N

For 6N: 6 – T = 0.6a …….. (i)

For 4N: T – 4 = 0.4a …….. (i)

Adding (i) and (ii)

a = 2ms^{-2}

T = 4 + 0.4 x 2 = 4.8N

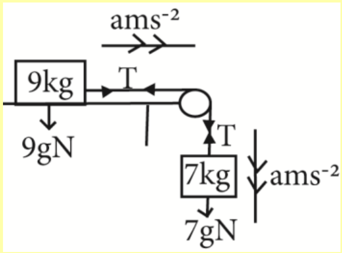
For scale pan A R_2 – 3.8 = 0.38a

R_2 = 3.8 + 2 x 0.38 x 2 = 4.56N

Example 3

A mass of 9kg resting on a smooth horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table to the pulley is a 7kg mass hanging freely 1.5m above the ground. Find

(a) common acceleration
(b) tension in the string
(c) force on the pulley when the system is allowed to move freely
(d) time taken for the 7kg mass to hit the ground



F = ma

For 7kg mass: 7g – T = 7a

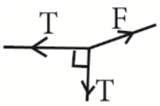
For 9kg mass: T = 9a

(i) + (ii): 7g = 16a

a = $\frac{7\ x\ 9.8}{16} = 4.29ms^{-2}$

(b) Tension: T = 9a = 9 x 4.29 = 38.61N

(c) The force on the pulley, F:



F =$\sqrt{T^2 + T^2} = T\sqrt{2}$ = 38.61$\sqrt{2}$ = 54.603N

(d) s = ut + $\frac{1}{2}at^2$

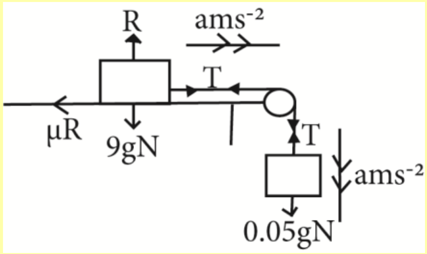
1.5 = 0 x t + $\frac{1}{2} x\ 4.29\ x\ t^2$

t = 0.84s

Example 4

A mass of 90g resting on a rough horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table attached to a 50g mass hanging freely. The coefficient of friction between the 90g mass and the table is $\frac{1}{3}$ and the system is released from rest, find

(a) common acceleration (b) the tension in the string



For 50g mass: $0.05g - T = 0.05a$ ....(i)

For 90g mass: $T - \mu R = 0.09a$

$T - \frac{1}{3} x\ 0.09\ x\ 9.8 = 0.09a$ .... (ii)

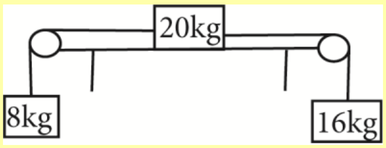
(i) + (ii): $0.05g - \frac{1}{3} x\ 0.09\ x\ 9.8 = 0.14a$

$a = \frac{0.02g}{0.14} = 1.4ms^{-2}$

(b) $0.05g - T = 0.05a$

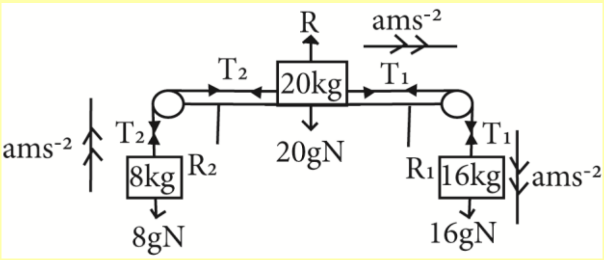
$T = 0.05 \times 9.8 - 0.05 \times 1.4 = 0.42N$

Example 5



The figure shows a block of mass 20kg resting on a smooth horizontal table. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) Acceleration of 16kg mass
(b) tension in the string
(c) reaction on each pulley



For 16kg mass: $16g - T_1 = 16a$ ...... (i)

For 20kg mass: $T_1 - T_2 = 20a$ .... (ii)

For 8kg mass: $T_2 - 8g = 8a$ .... (iii)

(i) + (ii) + (iii): $8g = 44a$

$a = \frac{8\ x\ 9.8}{44} = 1.782ms^{-2}$

(b) Tension in the string

$16g - T_1 = 16a$

$T_1 = 16 \times 9.8 - 16 \times 1.782 = 128.288N$

$T_2 - 8g = 8a$

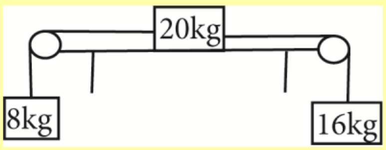
$T_2 = 8 \times 9.8 + 8 \times 1.782 = 92.656N$

(c) Reaction on each pulley

$R_1 = \sqrt{T_1^{\ 2} + T_1^{\ 2}} = T_1\sqrt{2} = \sqrt{2} \times 128.288 = 181.427N$

$R_2 = \sqrt{T_2^{\ 2} + T_2^{\ 2}} = T_2\sqrt{2} = \sqrt{2} \times 92.626 = 131\ N$

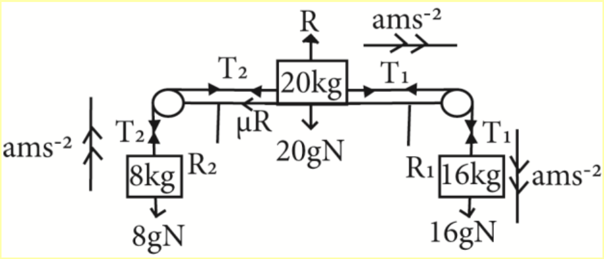
Example 6



The figure shows a block of mass 20kg resting on a rough horizontal table of coefficient of friction 0.21. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) acceleration of the 16kg mass
(b) Tension in each string
(c) reaction on each pulley

Solution



For 16kg mass: $16g - T_1 = 16a$ …… (i)

For 20kg mass: $T_1 - T_2 - 20g\mu = 20a$ ……. (ii)

For 8kg mass: $T_2 - 8g = 8a$ ……… (iii)

(i) + (ii) + (iii): $8g - 20g\mu = 44a$

$a = \frac{8 \times 9.8 - 20 \times 9.8 \times 0.21}{44} = 0.846ms^{-2}$

(b) Tension in the string

$16g - T_1 = 16a$

$T_1 = 16 \times 9.8 - 16 \times 0.846 = 143.264N$

$T_2 - 8g = 8a$

$T_2 = 8 \times 9.8 + 8 \times 0.846 = 85.168N$

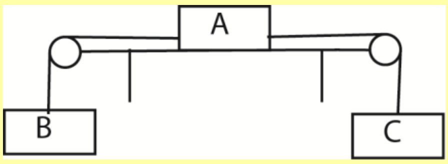
(c) Reaction on each pulley

$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 2 \times 128.291 = 202.606N$

$R_2 = \sqrt{T_2^2 + T_2^2} = T_2\sqrt{2} = \sqrt{2} \times 85.168 = 120.446N$

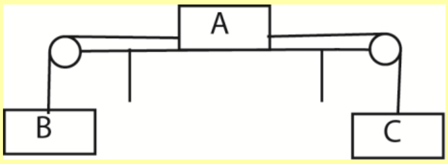
Revision exercise 1

1. Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find
   (a) acceleration of the particles [$3.92ms^{-2}$]
   (b) the tension in the string [41.16N]
   (c) the force on the pulley [82.32N]
2. Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find
   (a) acceleration of the particles [$4.9ms^{-2}$]
   (b) the tension in the string [29.4N]
   (c) distance moved by the 6kg mass in the first 2s of motion [9.8m]
3. A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a rope which passes over a frictionless pulley so that they hang vertically downwards.
   (a) what is the tension in the section of the rope supporting the man [807.06N]
   (b) what is the acceleration of the bucket [$1.73ms^{-2}$]
4. Two particles of masses 200g and 300g are connected to a light inelastic string passing over a smooth pulley; when released freely find
   (i) common acceleration [1.96ms-2]
   (ii) the tension in the string [2.352N]
   (iii) the force on the pulley [4.704N]

5. The diagram below shows a particles of mass 8kg connected to a light scale pan by a light inextensible string which passes over a smooth fixed pulley. The scale pan holds two blocks A and B of mass 3kg and 4kg, with B resting on top of A. If the system is released from rest find
   (a) acceleration of the system [$0.653ms^{-2}$]
   (b) the reaction between A and B[41.813N]
6. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate
   (a) the common acceleration of the masses [$3.675ms^{-2}$]
   (b) the tension in the string [18.375N]
   (c) the force acting on the pulley[26N]
7. A mass of 3kg on a smooth horizontal table is attached by a light inextensible sting passing over a smooth pulley at the edge of the table, to another mass of 2kg hanging freely 2.1m above the ground; find
   (a) common acceleration[$3.92ms^{-2}$]
   (b) the tension in the string [11.76N]
   (c) The force on the pulley in the system if it's allowed to move freely. [16.63N]
   (d) the velocity with which the 2kg mass hits the ground [$4.06ms^{-1}$]
8. A mass of 5kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a 3kg mass hanging freely. the coefficient of friction between the 5kg mass and the table is 0.25 and the system is released from rest find
   (a) common acceleration[$2.144ms^{-2}$]
   (b) tension in the string [22.97N]
9. A mass of 11kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to 500g mass hanging freely. The coefficient of friction between the 1kg mass and the table is 0.1 and the system is released from rest find
   (a) common acceleration [$2.61ms^{-2}$]
   (b) the tension in the string [3.593N]
10. The objects of mass 3kg and 5kg are attached to ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3kg mass touching the floor and the 5kg mass at 4m above the floor and then release, what is
    (a) the acceleration of the system[$2.45ms^{-2}$]
    (b) tension in the chord[36.75N]
    (c) the time that will elapse before the 5kg object hits the floor [1.81s]
11.
    

    The diagram shows a particle A of mass 2kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 5kg and C of mass 3kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the
    (a) common acceleration [$0.98ms^{-2}$]
    (b) the tension of each string [12.37N, 44.15N]

12.

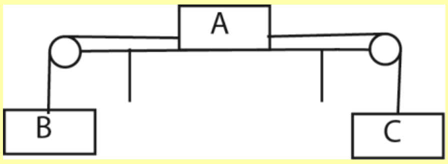


The diagram shows a particle A of mass 3kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 4kg and C of mass 6kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

(a) common acceleration [$0.75ms^{-2}$]

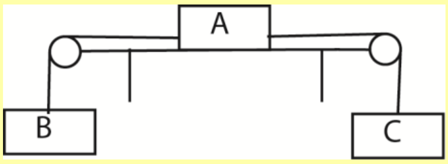
(b) the tension of each string [54.277N, 31.662N]

13.



The diagram shows a particle A of mass 5kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 3kg and C of mass 2kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest body B descend with an acceleration of 0.28ms-2, find the coefficient of friction between the body A and the surface of the table. [0.143]

14.



The diagram shows a particle A of mass 10kg resting on a smooth horizontal table. It is attached to the particle B of mass 4kg and C of mass 7kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

(c) common acceleration [$1.4ms^{-2}$]

(d) the tension of each string [44.8N, 58.8N]

Connected particles on inclined planes

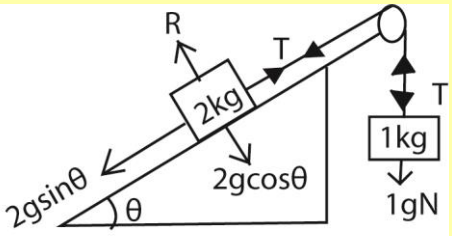
Example 7

A mass of 2kg lies on a smooth plane of inclination 1 in3. One end of a light inextensible sting is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its end. If the system is released from rest , find the

(i) acceleration of the masses

(ii) tension in the string

(iii) distance each particle travels in the first 2s.



$\sin\theta = \frac{1}{3}$ F =ma

For 2kg mass: T – 2gsinθ = 2a….. (i)

For 1kg mass: 1g – T = 1a ……….. (ii)

(ii) + (i): = 1g – 2gsinθ = 3a

a = $\frac{9.8 - 2 \, x \, 9.8 \, x \, \frac{1}{3}}{3}$ = 1.089ms^{-2}

(ii) Tension: 1g – T = 1a

T = 9.8 – 1.089 = 8.71N

(iii) s = ut + $\frac{1}{2}at^2$

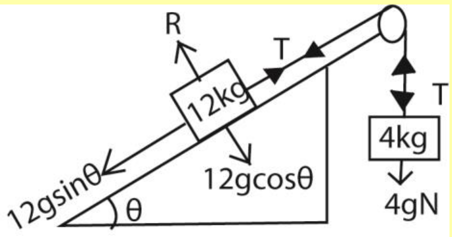
s = 0 x 2 + $\frac{1}{2} x \; 1.089 \; x \; 2^2 = 2.178m$

Example 8

A mass of 12kg lies over a smooth incline plane 6m long and 1m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane to freely suspended mass of 4kg at its other end. If the system is released from rest, find the

(a) acceleration of the system
(b) velocity with which the 4kg mass hits the ground
(c) time the 4kg mass takes to hit the ground.

Solution



$\sin\theta = \frac{1}{6}$ F =ma

For 12kg mass: T – 12gsinθ = 2a….. (i)

For 4kg mass: 4g – T = 4a ……….. (ii)

(ii) + (i): = 4g – 12gsinθ = 16a

a = $\frac{4 \, x 9.8 - 12 \, x \, 9.8 \, x \, \frac{1}{6}}{16}$ = 1.225ms^{-2}

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 1.225 = 34.3N

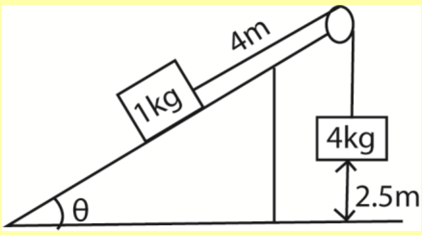
(iii) s = ut + $\frac{1}{2}at^2$

1 = 0 x t + $\frac{1}{2} x \; 1.225 \; x \; t^2$

t = 1.28s

Example 9

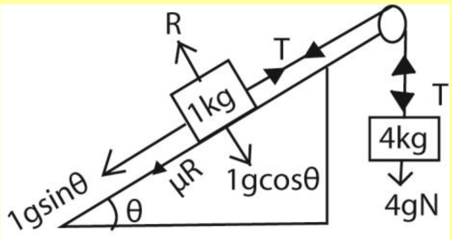
A mass of 1kg lies on a rough plane with coefficient of friction 0.25. One end of a light inextensible string is attached to 1kgmass and passes up the line of greatest slope over a smooth fixed pulley at the top of the plane and the other end of a string is tied to a mass of 4kg hanging freely.



The plane makes an angle θ with the horizontal where $\sin\theta = \frac{3}{5}$. When the system is released from rest, find:

(i) the acceleration of the system
(ii) tension in the string
(iii) velocity with which the 4kg mass hits the floor
(iv) velocity with which the 1kg mass hits the pulley

Solution



$\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$ F =ma

For 12kg mass: T – 1gsinθ- 0.25R = 1a….. (i)

For 4kg mass: 4g – T = 4a ……….. (ii)

(ii) + (i): = 4g – 1gsinθ - 0.25R = 5a

$a = \frac{4 \times 9.8 - 1 \times 9.8 \times \frac{3}{5} - 0.25 \times 1 \times 9.8 \times \frac{4}{5}}{5} = 6.272ms^{-2}$

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 6.272 = 14 .112N

(iii) $v^2 = u^2 + 2as$ but u = 0

$v = \sqrt{2 \times 6.272 \times 2.5} = 5.6ms^{-1}$

(iv) When a 4kg mass hits the floor, the 1kg mass has still to move 4 – 2.5 = 1.5m before hitting the pulley It will experience a retarding force due to gravity and friction

F = 1a = 1gsinθ + 0.25R

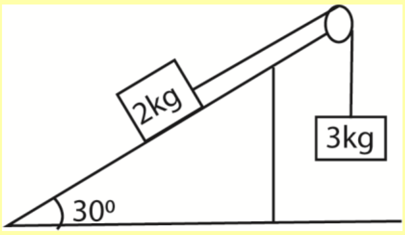
$= (1 \times 9.8 \times \frac{3}{5} + 0.25 \times 1 \times 9.8 \times \frac{4}{5})$

$a = -7.84ms^{-2}$

$v^2 = u^2 + 2as$

$v = \sqrt{5.6^2 - 2 \times 7.84 \times 1.5} = 2.8ms^{-1}$

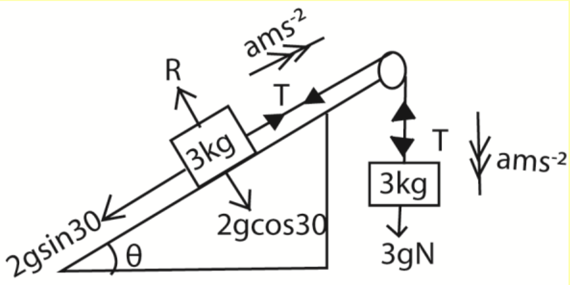
Example 10



A particle of mass 2kg on a rough plane inclined at 300 to the horizontal is attached by means of light inextensible string passing over a smooth pulley at the top edge of the plane to a particle of mass 3kg which hangs freely. If the system is released from rest with above parts of the strings taut, the 3kg mass travels a distance of 0.75m before attains a speed of $2.25ms^{-1}$. Calculate

(a) acceleration
(b) coefficient of friction between the plane and 2kgmass
(c) reaction of the pulley on the string

Solution



(i) $v^2 = u^2 + 2as$

a = $\frac{2.25^2 - 0^2}{2 \times 0.75}$ = $3.375ms^{-2}$

(ii) F = ma

For 2kg mass: T – 2gsinθ- μR = 2x a ….. (i)

For 4kg mass: 3g – T = 3 a ………. (ii)

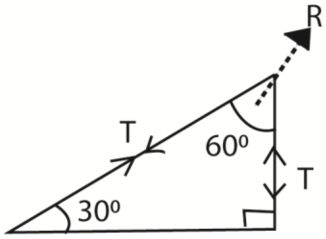
(ii) + (i): 3g – 2gsinθ – μ(2gcosθ) = 5a

$$\mu = \frac{(3 \times 9.8) - (2 \times 9.8 sin30 + 5 \times 3.375)}{2 \times 9.8 \times cos30} = 0.161$$

(iii)

Tension: 3g – T = 3a

T = 3 x 9.8 – 3 x 3.375 = 19.275N



Using parallelogram law of force

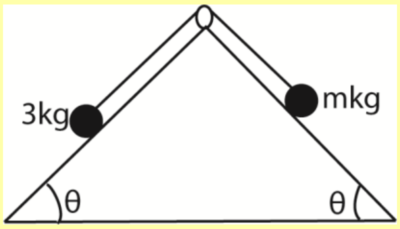
$R^2 = T^2 + T^2 + 2 \times T \cos 60 = 3T^2$

R = $19.275\sqrt{3}$ = 33.4N

Double inclined plane

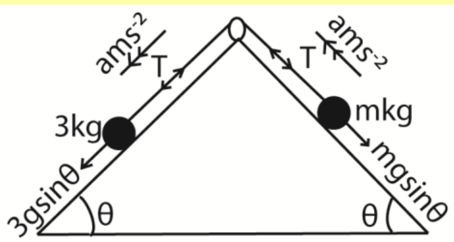
Example 11

The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where sinθ = 0.6. Two particles of mass 3kg and mkg, where m< 3kg are connected by a light inextensible string passing over a smooth fixed pulley.



The particles are released from rest with a string taut. After travelling a distance of 1.08m, the speed of the particle is 1.8ms-1. Calculate

(i) acceleration
(ii) tension in the string
(iii) value of m



(i) $v^2 = u^2 + 2as$

$1.8^2 = 0^2 + 2 \times a \times 1.08$

a = $1.5ms^{-2}$

(ii) F = ma

For 3kg mass: 3gsinθ – T = 3a

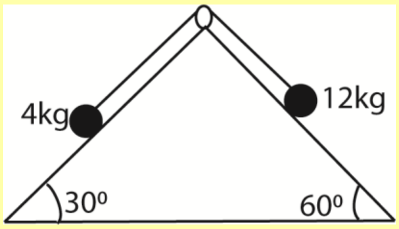
T = 3 x 9.8x 0.6 – 3 x 1.5 = 13.14N

(iii) For mkg mass; T – mgsinθ = ma

13.14 = m(9.8 + 1.5); m = 1.78kg

Example 12

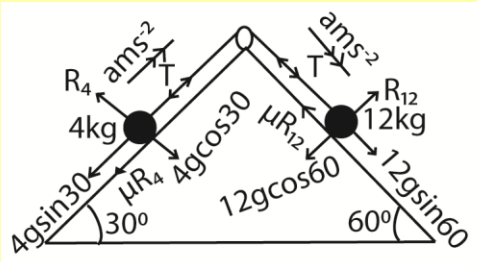
Two rough planes inclined at 300 and 600 to the horizontal and of the same height are placed back to back. Masses of 4kg and 12 kg are placed on the faces and connected by a light string passing over smooth pulley on the top of the planes.



If the coefficient of friction is 0.5 on both faces, find

(a) acceleration
(b) Tension in the strings

Solution



For 4kg mass: T -4gsin30 – 0.5 x 4gcos30 = 4a .... (i)

For 12kg mass: 12gsin60 –T – 0.5 x 12gcos 60 = 12a ..... (ii)

(i) + (ii)

12gsin60 – 4gsin30 - 0.5(4gcos30 + 12gcos 60) = 16a

a = 2.25ms^{-2}

(b) For 4kgmass

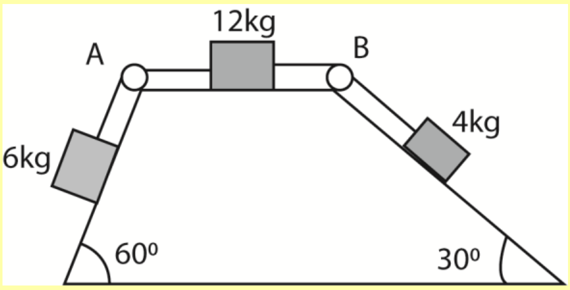
T -4gsin30 – 0.5 x 4gcos30 = 4a

T = 4gsin30 + 0.5 x 4gcos30 + 4x2.25

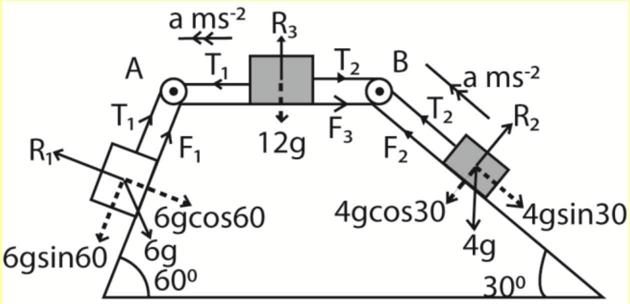
T = 45.54

Example 13

1. The diagram below shows a 12kg mass on a horizontal rough plane. The 6kg and 4kg masses are on rough planes inclined at angles 600 and 300 respectively. The masses are connected to each other by light inextensible strings over light smooth pulleys A and B.



The planes are equally rough with coefficient of friction$\frac{1}{12}$. If the system is released from rest find the;
(a) Acceleration of the system (08marks)



For 6kg mass

6gsin60-($T_1 + \frac{1}{12}\ x\ 6gcos60) = 6a$

6gsin60 – T_1 - $\frac{1}{2}cos60 = 6a$ ………………………… (i)

For 4kg mass

T_2 – ($\frac{1}{12}\ x\ 4gcos30 + 4gsin30) = 4a$

T_2 - $\frac{1}{3}gcos30 - 4gsin30$ = 4a …………………. (ii)

For 12kg mass

T_1 – (T_2 +$\frac{1}{12}\ R_3$) = 12a

T_1 – (T_2 +$\frac{1}{12}\ x\ 12g$) = 12a

T_1 –T_2 - g = 12a……………………………………… (iii)

Eqn. (i) + Eqn. (ii) + Eqn. (iii)

6gsin60 - $\frac{1}{2}gcos60 - \frac{1}{3}gcos30 - 4gsin\ 30 - g = 22a$

16.24327742 = 22a

a = $\frac{16.24327742}{22} = 0.73833ms^{-2}$

(b) Tensions in the strings. (04marks)

From equation (i)

$T_1 = 6gsin60 - \frac{1}{2}cos60 - 6a$

= $6gsin60 - \frac{1}{2}cos60 - 6\ x\ 0.73833$

= 44.0423N

From eqn. (ii)

T_2 = $\frac{1}{3}gcos30 + 4gsin30 + 4a$

= $\frac{1}{3}gcos30 + 4gsin30 + 4\ x\ 0.73833$

= 25.3823N

Revision exercise 2

1. A mass of 2kg lies on a smooth inclined plane 9m long and 3m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope

over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its other end. If the system is released from rest, find

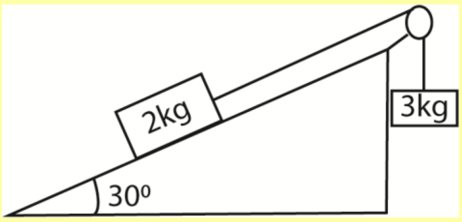
(i) acceleration of the system[$1.089ms^{-2}$]

(ii) tension in the string. [8.711N]

(iii) velocity with which the 1kg mass will hit the ground[$2.556ms^{-1}$]

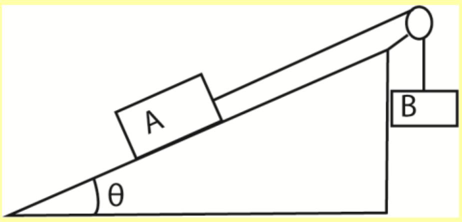
(iv) time the 1kg mass will hit the ground[2.347s]

2. A mass of 15kg lies on a smooth plane of inclination in 49. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 10kg at its other end. If the system is released from rest, find the acceleration of the masses and the distance each travel in the first 2s. [$3.8ms^{-2}$, 7.6m]
3. A mass of 2kg lies on a rough plane which is inclined at 300 to the horizontal. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 5kg at its other end. The system is released from rest as the 2kg mass accelerates up the slope, it experiences a constant resistance to motion of 14N down the slope due to friction. Find the tension of the string. [31N]
4. A mass of 10kg lies on a smooth plane which is inclined at θ to the horizontal. The mass is 5m from the top, measured along the plane. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 15kg at its other end. The 15kg mass is 4m above the floor. The system is released from rest and the string first goes slack $1\frac{3}{7}s$ latter. Find the value of θ. [30^0]
5. One of two identical masses lies on a smooth plane, which is inclines at $\sin^{-1}\left(\frac{1}{4}\right)$ to the horizontal and is 2m from the top. A light inextensible string attached to this mass passes along the line of greatest slope over a smooth pulley fixed at the top of the incline, the other end carries the other mass hanging freely 1m above the floor. If the system is released from rest, find the time taken for the hanging mass to reach the floor. [0.663s]
6. A particle of mass 2kg on a smooth plane inclined at 300 to the horizontal is attached by means of a light inextensible string passing over a smooth pulley at the edge of the plane to a particle of mass 4kg which hangs freely.



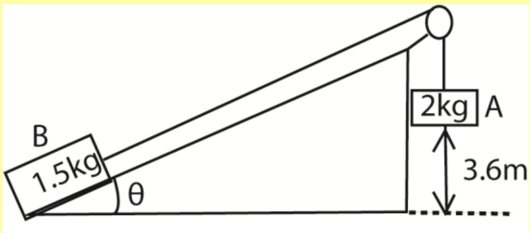
If the system is released from rest with above parts of the string taut, find the speed acquired by the particles when both have moved a distance of 1m [$2.8ms^{-1}$}

7. A body A of mass 13kg lying on a rough inclined plane, coefficient of friction, μ. From A, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley to a body B of mass mkg hanging freely, the plane makes an angle θ with the horizontal where sinθ = $\frac{5}{13}$.



When m = 1kgand the system is released from rest, B has upward acceleration of a ms^{-2}. When m = 11kg and the system released from rest, B has downward acceleration of ams^{-2}. Find a and μ. [1.96ms^{-2}, 0.1]

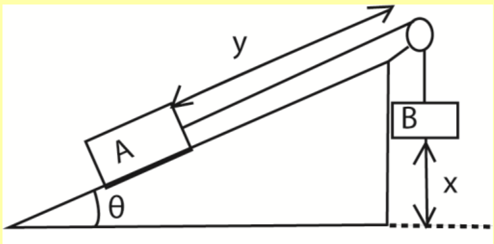
8. A particle A of mass 2kg and B of mass 1.5kg are connected by light inextensible string passing over a smooth pulley. The system is released from rest with A at height of 3.6m above the horizontal ground and B at the foot of a smooth slope inclined at an angle θ to the horizontal where sinθ = $\frac{1}{6}$. Take g = 10ms^{-2}.



Calculate

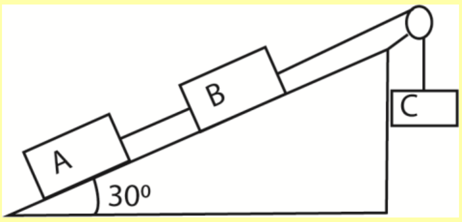
(i) the magnitude of the acceleration of particles [6ms^{-1}]
(ii) the speed with which A reaches the ground[5ms^{-2}]
(iii) the distance B moves up the slope before coming to instantaneous rest. [14.4m]

9. A mass A of 4kg and a mass B of 3kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest and mass accelerates up along a smooth slope inclined at an angle θ to horizontal where θ = 30^0.



If y = 3m and x = 2.8m, calculate the velocity with which A hits the pulley [2.42ms^{-1}]

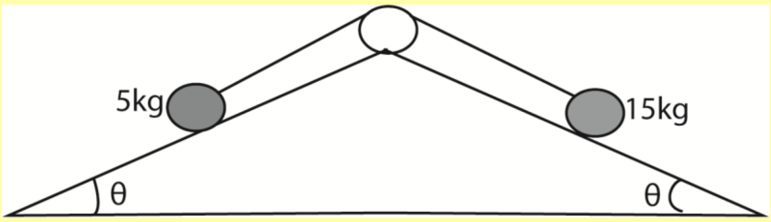
10. The diagram below shows particles A, B and C of masses 10kg, 8kg and 2kg respectively connected by a light inextensible strings. The string connecting B and C passes over a smooth light pulley fixed at the top of the plane.



If the coefficient of friction between the plane and particles A and B are 0.22 and 0.25 respectively, calculate

(i) acceleration of the system [1.6477ms^{-2}]
(ii) tension in the strings[22.89N, 13.851N]

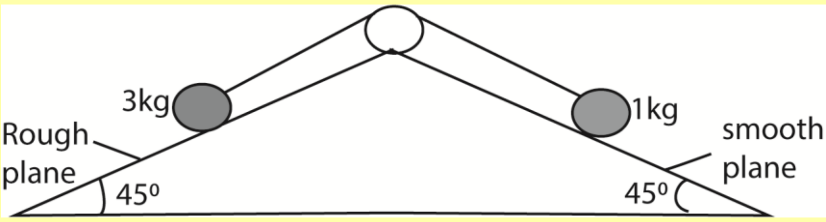
11. The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where sinθ =$\frac{3}{5}$. Two particles of mass 5kg and 15kg are connected by a light inextensible string over a smooth fixed pulley.



The particles are releases from rest with a string taut calculate

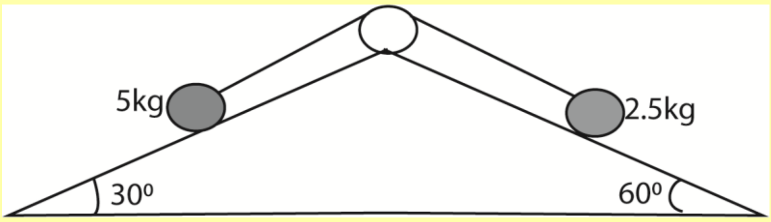
(i) Acceleration of the particles
(ii) Tension in the string

12. The diagram below shows two smooth plane and a rough plane both inclined at 450 to the horizontal. Two particles of mass of mass 1kg and 3kg are connected by light inextensible string passing over a smooth fixed pulley.



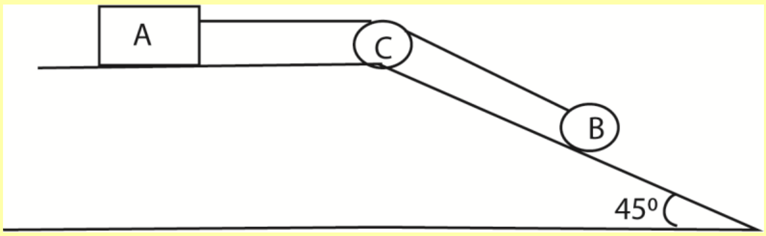
The particle are released from rest with a string taut. Calculate
(i) acceleration of the particle [1.4ms^{-2}]
(ii) tension in the string [.48N
(iii) coefficient of friction [0.4]

13. Two equally rough planes inclined at 30^0 and 60^0 to the horizontal and of the same height are placed back to back. Masses of 5kg and 2.5kg are placed on the faces and connected by a light string passing over a smooth pulley at the top of the planes.



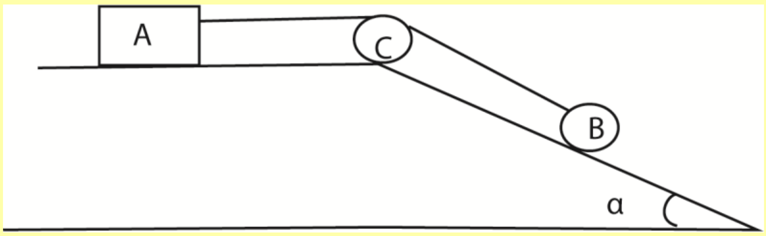
If the string is taut and 5kg is just about to slip downwards find the
(i) coefficient of friction[0.06]
(ii) tension in the string [21.9538N]

14. In the diagram, particle A and particle B are masses of 10kg and 8kg respectively and rest on planes as shown below. They are connected by a light inextensible string passing over a smooth pulley C.



Find the acceleration in the system and the tension in the string if
(i) the particles are in contact with smooth planes [3.08ms^{-2}, 30.N]
(ii) the particles are in contact with rough planes with coefficient of friction 0.25. [0.95ms^{-2}, 33.98N]

15. In the diagram particles A and B are of masses mkg and 5mkg respectively and rest on the planes as shown below. They are connected by a light inextensible string passing over a smooth fixed pulley at C

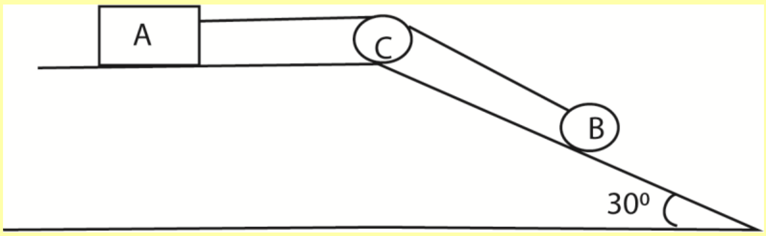


Find the acceleration of the system and the tension in the string if sinα = $\frac{4}{5}$ when;

(i) the particles are in contact with smooth plane[6.533ms-2, 6.533N]

(ii) the particles are in contact with rough plane of coefficient of friction $\frac{1}{3}$.
[4.356ms^{-2}, 7.622N]

16. In the diagram particles A and B of masses 2.4kg and 3.6kg respectively. A rests on a rough horizontal plane (coefficient of friction 0.5), it is connected by a light inextensible string passing over a smooth pulley C to particle B resting on smooth plane inclined at 30^0 to the horizontal.



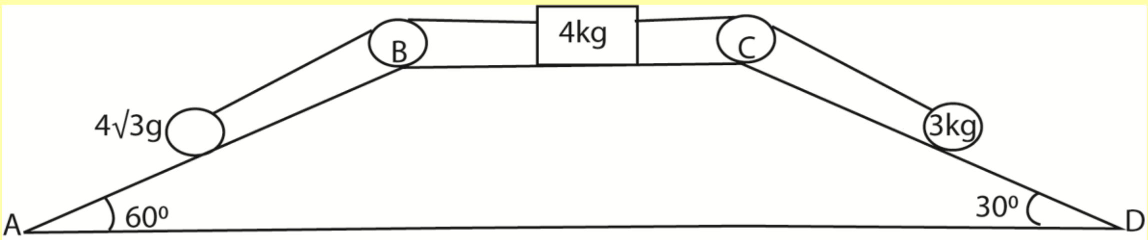
When the system is released from rest find

(i) acceleration of the system and tension in the string [0.98ms^{-2}, 14.112N]

(ii) the force on the pulley C [7.3049N]

(iii) the velocity of A mass after 2 seconds[1.96ms^{-2}]

17. The diagram below shows a 4kg mass on a horizontal rough plane with coefficient of friction 0.25. The 4$\sqrt{3}kg$ mass rests on a smooth plane inclined at angle 600 to the horizontal while the 3kg mass rests on a rough plane inclined at an angle 300 to the horizontal and coefficient of friction $\frac{1}{\sqrt{3}}$. the masses are connected to each other by a light inextensible strings over light smooth fixed pulleys B and C.



Find the

(i) acceleration of the system[1.407ms-2]

(ii) tension in the string [49.051N, 33.622N]

(iii) work done against frictional force when the particles each moved 0.5m [12.25J]

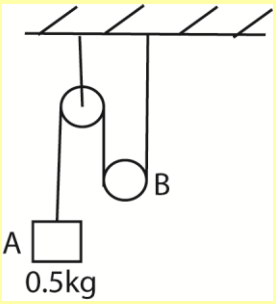
Multiple connections

- Acceleration of a particle moving between to portions of the string is equal to half the net acceleration of the particle (s) attached to the end of the string
- The tension in uninterrupted string is constant
- The tensions in interrupted strings are different.

Case I: A pulley moving between two portions of a string

Example 15

The diagram below shows particle A of mass 0.5kg attached to one end of alight inextensible string passing over a fixed pulley and under a movable light pulley B. The other end of the string is fixed as shown

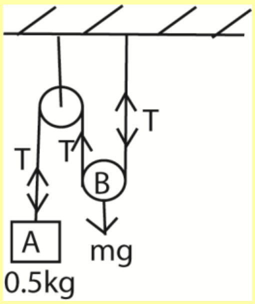


(i) What mass should be attached at B for the system to be in equilibrium
(ii) If B is 0.8kg what are the accelerations of particles A and pulley B?
(iii) find the tension in the string in (ii)

Solution

(i) Let T = tension in string

m = mass at B



For the system to be in equilibrium upward forces are equal to downward force. by resolving vertically

For mass A: T= 0.5g ……… (i)

For pulley B: 2T = mg

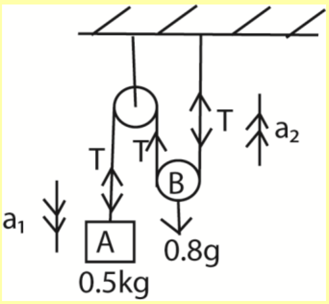
$T = \frac{mg}{2}$ ……… (i)

equating (i) to (ii)

$\frac{mg}{2} = 0.5g$; m = 1kg

(ii) Let a_1 = acceleration of A

a_2 = acceleration of B



For mass A: $0.5g – T = 0.5a_1$……… (i)

For pulley B: $2T – 0.8g = 0.8a_2$ but $a_2 = ½ a_1$

⇨ $2T – 0.8g = 0.4a_1$
$T – 0.4g = 0.2a_1$ …….. (i)

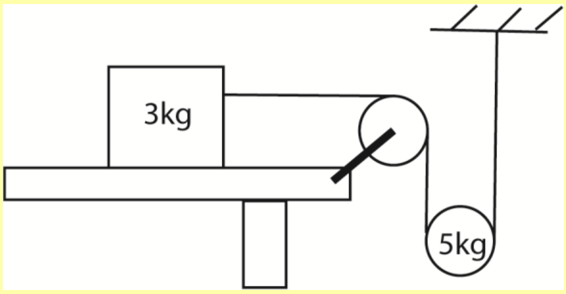
(i) + (ii) $a1 = \frac{9.9}{7} = 1.4ms^{-2}$ and $a2 = 0.7 ms^{-2}$

From eqn. (i)

T = 0.5 x 9.8 - 0.5 x 1.4 = 4.2N

Example 16

A particle of mass 3kg on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass 5kg, its other end being fixed so that the string beyond the table are vertical.

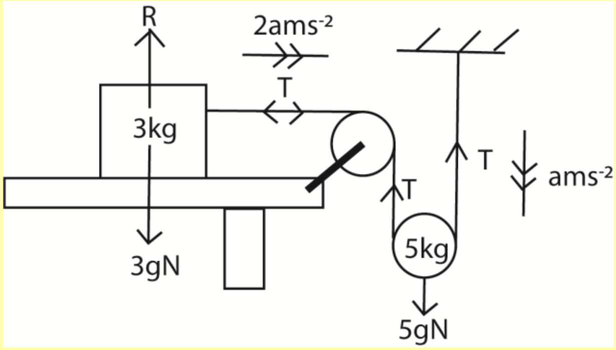


Find

(i) acceleration of 3kg and 5g

(ii) Tension in the string

Solution



F = ma

For 3kg: T = 3 x 2a ……… (i)

For 5kg: 5g – 2T = 5a …… (ii)

(ii) + 2 x (i)

5x 9.8 = 17a

a = $\frac{49}{17}$ = 2.8824ms^{-2}

Acceleration of 5kg: = 2.8824ms^{-2}

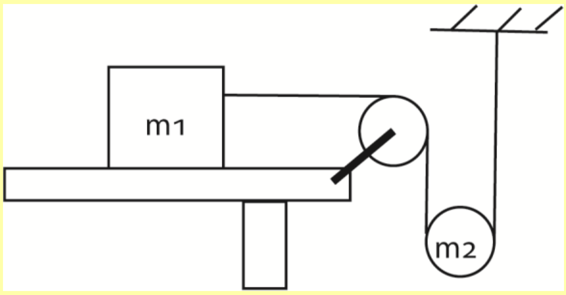
Acceleration of 3kg: = 2.8824 x 2ms^{-2}

= 5.7648ms^{-2}

T = 6a = 2.8824 x 6 = 17.2944N

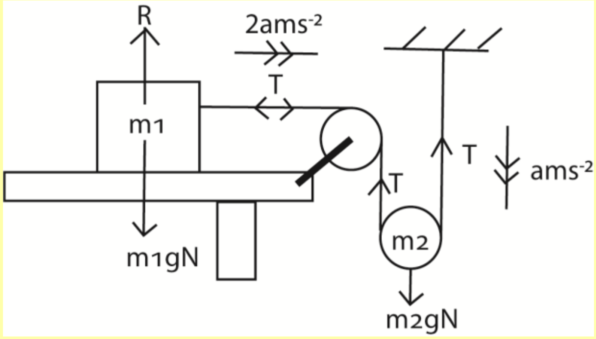
Example 17

A particle of mass m_1 on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass m_2, its other end being fixed so that the parts of the string beyond the table is vertical.



Show that m_2 descends with acceleration $\frac{m_2 g}{4m_1+m_2}$

Solution



F = ma

For m_1 kg mass: T = m_1 x 2a …… (i)

For m_2 kg mass: $m_2g - 2T = m_2a$ … (ii)

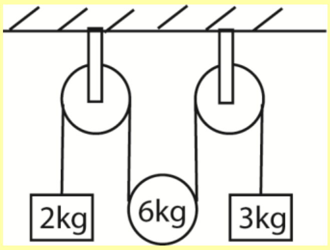
(ii) + 2 x (i)

$m_2g = 4m_1a + m_2a$

$a = \frac{m_2g}{4m_1+m_2}$

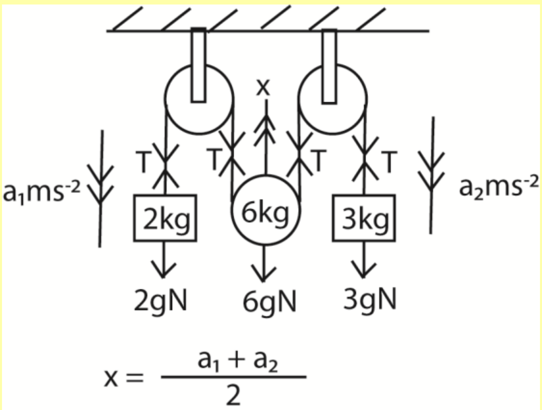
Example 18

A string has a load of mass 2kg attached at one end. The string passes over a smooth fixed pulley then under a movable pulley of mass 6kg and over another fixed pulley and has a load of mass 3kg attached to its end.



Find the acceleration of the movable pulley and the tension in the string

Solution



For 2kg mass: $2g - T = 2a_1$ …………………….. (i)

For 3kg mass: $3g - T = 3a_2$ …………………….. (ii)

For 6kg mass: $2T - 6g = 6\ x\ \frac{1}{2}(a_1 + a_2)$…. (iii)

(ii) – (i): $g = (3a_2 - 2a_1)$ …………………………… (iv)

2 x (ii) + (iii): $0 = 9a_2 + 3a_1$ …………………….. (v)

3 x (iv)- (v): $3g = -9a_1$

$a1 = \frac{-g}{3} = -3.267ms^{-2}$

From (v): $0 = 9a_2 + 3a_1$

$0 = 9a_2 + 3(-3.267)$

$a_2 = 1.089ms^{-2}$

Acceleration of pulley = $\frac{1}{2}(a_1 + a_2)$

$= \frac{1}{2}(-3.267 + 1.089)$

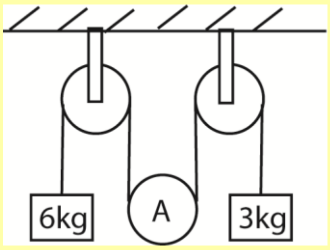
$= -1.089ms^{-2}$

Tension: $T = 2g - 2a_1$

T = 2 x 9.8 – 2 x -3.267 = 26.134N

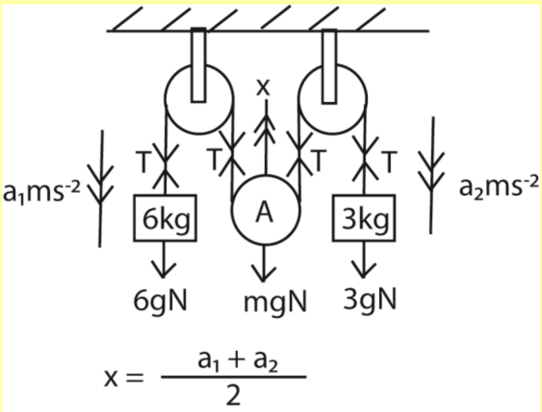
Example 19

In the pulley system below, A is a heavy pulley which is free to move



Find the mass of pulley A if it does not move upwards of downwards when the system is released from rest.

Solution



For 2kg mass: $6g - T = 6a_1$ …………………….. (i)

For 3kg mass: $3g - T = 3a_2$ …………………….. (ii)

For mkg mass: $2T - mg = 0$……………………. (iii)

$a_1 = -a_2$

$6g - T = -6a_2$ ……….. (iv)

$3g - T = 3a_2$ …………. (v)

(iv) – (v)

$3g = 9a_2$

$a_2 = \frac{-g}{3} = -3.267ms^{-2}$

$3g - T = 3a2$

$T = 3 \times 9.8 - 3(-3.267) = 39.201$

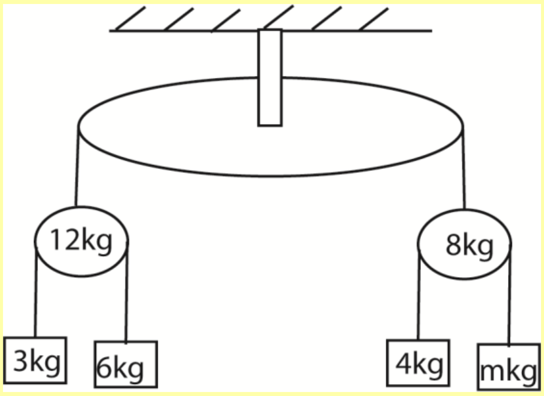
$2T - mg = 0$

$m = \frac{2 \times 39.201}{9.8} = 8kg$

Case 2: A pulley moving on one portion of a string

Example 20

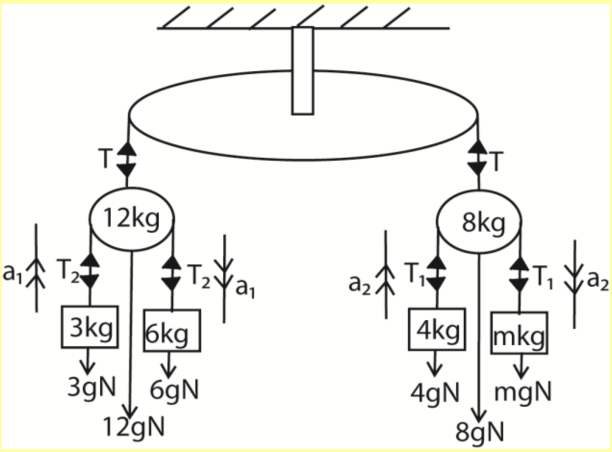
The diagram below shows two pulley to pulleys of masses 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.



The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the

(i) tensions in the string
(ii) value of m

Solution.



For 3kg mass: $T_2 - 3g = 3a_2$ .... (i)

For 6kg mass: $6g - T_2 = 6a_2$ .... (ii)

For 4kg mass: $T_1 - 4g = 4a_1$ .... (iii)

For mkg mass: $mg - T_1 = ma_1$ ....(iv)

For 8kg mass: $2T_1 + 8g = T$ .... (v)

For 12kg mass: $2T_2 + 12g = T$ .... (vi)

eqn. (i) + eqn. (ii): $3g = 3a_2$

$a_2 = \frac{3 \times 9.8}{9} = 3.2667\text{ms}^{-2}$

eqn. (i): $T_2 - 3g = 3a_2$

$T_2 = 3 \times 9.8 + 3 \times 3.2667 = 39.2001\text{N}$

eqn. (vi): $2T_2 + 12g = T$

$T = 2 \times 39.2001 + 12 \times 9.8 = 196.0002\text{N}$

eqn. (v): $2T_1 + 8g = T$

$T_1 = \frac{196.0002 - 8 \times 9.8}{2} = 58.8001\text{N}$

eqn. (iii): $T_1 - 4g = 4a_1$

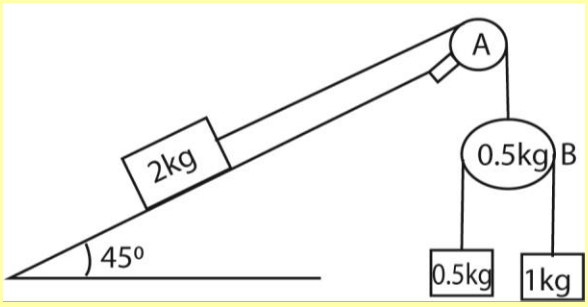
$a_1 = \frac{58.8001 - 4 \times 9.8}{4} = 4.9\text{ms}^{-2}$

eqn. $mg - T_1 = ma_1$

$m = \frac{58.8001}{9.8 - 4.9} = 12\text{kg}$

Example 21

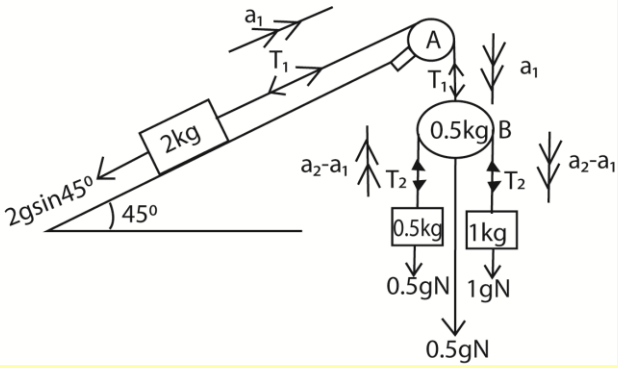
The diagram shows a particle of mass 2kg on a smooth plane inclined at 450 to the horizontal and attached by means of a light inextensible string over a smooth pulley, A at the top of the plane to pulley B of mass 0.5kg which hangs freely. Pulley B carries to particles of mass 0.5kg and 1kg on either side



Find

(a) acceleration of 2kg, 0.5kg and 1kg mass
(b) the tension in the strings

Solution



For 2kg mass: T_1 - 2gsin45 = $2a_1$ .... (i)

For 0.5kg mass: T_2 – 0.5g =(a_2 – a_1) ...... (ii)

For 1kg mass: gN – T_2 = 1(a_1 + a_2) .... (iii)

For pulley B: $2T_2$ + 0.5g – T_1 = $0.5a_1$ .... (iv)

eqn. (ii) + eqn (iii): 0.5g = 1.5a2 + 0.5a1

9.8 = $3a_2$ + a_1 .... (v)

eqn. (i) + eqn. (iv): $2T_2$ – 2gsin45 + 0.5g =$2.5a_1$

$2T_2$ -8.9593 = $2.5a_1$ .... (vi)

2 x eqn. (iii) + eqn. (vi): 10.6407 = $4.5a_1$ + $2a_2$

5.3204 = $2.5a_1$ + a2................ (vii)

eqn. (vii) – eqn. (v): $5.75a_1$ = 6.1612

$a_1 = \frac{6.1612}{5.75} = 1.0715\text{ms}^{-2}$

from eqn. (v): 9.8 = $3a_2$ + a_1

$a_2 = \frac{9.8-1.0715}{3} = 2.9095\text{ms}^{-2}$

Acceleration of 2kg mass = 1.0715ms^{-2}

Acceleration of 0.5kg mass = 2.9095ms^{-2}

Acceleration of 1kg = 2.9095 + 1.0715

= 3.981ms^{-2}

From eqn. (i): T_1 - 2gsin45 = $2a_1$

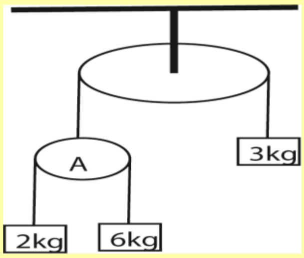
T1 = 2 x 1.0715 + 2 x 9.8sin45 =16.0023N

from eqn. (iv): $2T_2$ + 0.5g – T_1 = $0.5a_1$

$T_2 = \frac{0.5 \times 1.0715 + 16.0023 - 4.9}{2}$ =5.8190N

Example 22

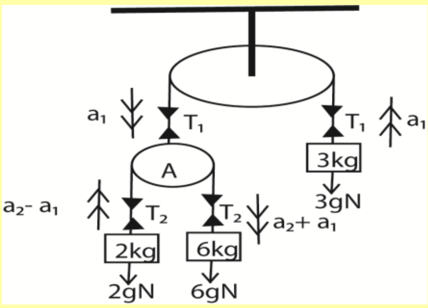
The diagram shows a fixed pulley carrying a string which has a mass of 3kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 6kg at one and a mass of 2kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 2kg, 6kg and 3kg masses
(c) tension in the string

solution



For 3kg mass: T_1 - 3g = $3a_1$ .... (i)

For 6kg mass: 6g – T_2 =6(a_2 + a_1) .... (ii)

For 2kg mass: T_2 – 2g = 2(a_2 – a_1) .... (iii)

For pulley A: $2T_2$ – T_1 = 0 x a_1 .... (iv)

eqn. (ii) and eqn. (iii): 4g = $8a_2$ + $4a_1$ ..... (v) eqn. (i) + eqn. (iv): $2T_2 - 3g = 3a_1$...... (vi)

2 x eqn. (iii) - eqn. (vi)

$-g = 4a_2 - 7a_1$ .... (vii)

2eqn. (vii) – eqn. (v)

$-18a_1 = -6g$

$a_1 = \frac{6 \times 9.8}{18} = 3.27ms^{-2}$

$4g = 8a_2 + 4a_1$

$a_2 = \frac{9.8-3.27}{2} = 3.27ms^{-2}$

Acceleration of pulley A $=3.27ms^{-2}$

Acceleration of 2kg $= 3.27ms^{-2} - 3.27ms^{-2} = 0$

Acceleration of 6kg $= 3.27ms^{-2} + 3.27ms^{-2}$

$= 6.54ms^{-2}$

Acceleration of 3kg $=3.27ms^{-2}$

$T_1 - 3g = 3a_1$

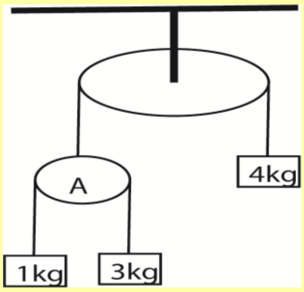
$T_1 = 3 \times 3.27 + 3 \times 9.8 = 39.21N$

$2T_2 - T_1 = 0 \times a_1$

$T_2 = \frac{39.21}{2} = 19.61N$

Example 23

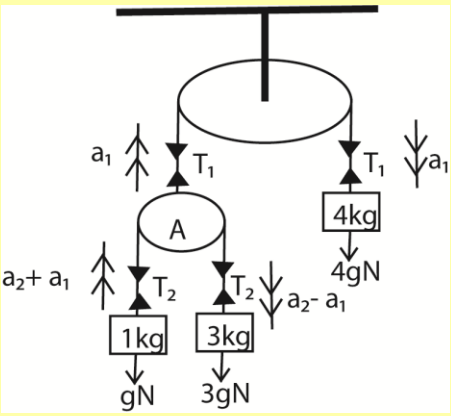
The diagram shows a fixed pulley carrying a string which has mass of 4kg attached at one end and a light pulley A at the other end. Another string passes over pulley A and carries a mass of 3kg at one end and a mass of 1kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 1kg, 3kg and 4kg masses
(c) tension in the string

Solution



For 4kg mass: $4g - T_1 = 4a_1$ .... (i)

For 3kg mass: $3g - T_2 = 3(a_2 - a_1)$ .... (ii)

For 1kg mass: $T_2 - g = (a_2 + a_1)$ .... (iii)

For pulley A: $T_1 - 2T_2 = 0 \times a_1$ .... (iv)

eqn. (ii) and eqn. (iii): $g = 2a_2 - a_1$ ..... (v)

eqn. (i) + eqn. (iv): $4g - 2T_2 = 4a1$ .... (vi)

2 x eqn.(iii) + eqn. (v): $2g = 2a_2 + 6a_1$ ..... (vii)

eqn. (vii) – eqn. (i): $7a_1 = g$

$a_1 = \frac{9.8}{7} = 1.4ms^{-2}$

$g = 2a_2 - a_1$

$a_2 = \frac{9.8+1.4}{2} = 5.6ms^{-2}$

Acceleration of pulley A $= 1.4ms^{-2}$

Acceleration of1kg mass $= 5.6 + 1.4 = 7ms^{-2}$

Acceleration of 3kg mass $= 5.6 - 1.4 = 4.2ms^{-2}$

Acceleration of 4kg mass $= 1.4ms^{-2}$

$4g - T_1 = 4a_1$

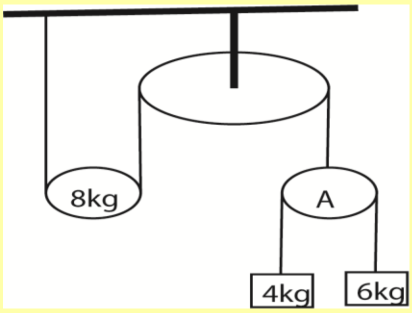
$T_1 = 4 \times 9.8 - 4 \times 1.4 = 33.6N$

$T_1 - 2T_2 = 0 \times a_1$

$T_2 = \frac{33.6}{2} = 16.8N$

Example 24

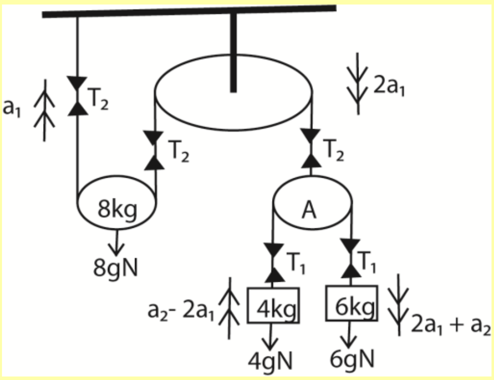
The diagram below shows a fixed pulley carrying a movable pulley of mass 8kg at one end and a light pulley A attached at the other end. A string passes over pulley A and carries a mass of 4kg at one end and a mass of 6kg at the other end.



Find

(a) acceleration of pulley A
(b) acceleration of 8kg, 6kg and 4kg masses
(c) tension in the string

Solution



For 8kg mass: $2T_2 - 8g = 8a_1$ .... (i)

For 4kg mass: $T_1 - 4g = 4(a_2 - 2a_1)$ .... (ii)

For 6kg mass: $6g - T_1 = 6(2a_1 + a_2)$ .... (iii)

For pulley A: $2T_1 - T_2 = 0 \times a_1$ .... (iv)

eqn. (ii) and eqn. (iii): $2g = 10a_2 + 4a_1$

$4.9 = 2.5a_2 + a_1$ ..... (v)

eqn. (i) + 2 x eqn. (iv): $4T_1 - 8g = 8a_1$.... (vi)

4 x eqn. (iii) + eqn. (vi): $16g = 56a_1 + 24a_2$

$2g = 7a_1 + 3a2$ .... (vii)

7 x eqn. (v) – eqn. (vii): $14.5a_2 = 14.7$

$a_2 = \frac{14.7}{14.5} = 1.0138ms^{-2}$

eqn. (v); $4.9 = 2.5a_2 + a_1$

$a_1 = 4.9 - 2.5 \times 1.0138 = 2.3655ms^{-2}$

Acceleration of pulley = $2a_1 = 2 \times 2.3655$

$= 4.731ms^{-2}$

Acceleration of 6kg = 4.731 + 1.0138

$= 5.7448ms^{-2}$

Acceleration of 3kg = $a_2 - 2a_1$

$1.0138 - 4.731 = -3.7172ms^{-2}$

From eqn. (i): $2T_2 - 8g = 8a_1$

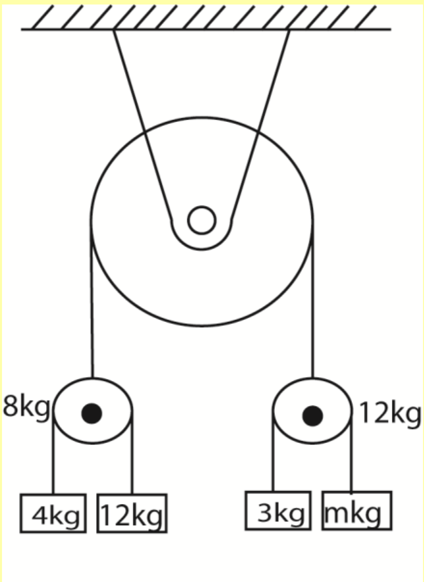
$T_2 = \frac{8 \times 4.731 + 8 \times 9.8}{2} = 58.124N$

From eqn. (iv)

$T_1 = \frac{T_2}{2} = \frac{58.124}{2} = 29.062N$

Example 25

The diagram below shows two pulleys of mass 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.



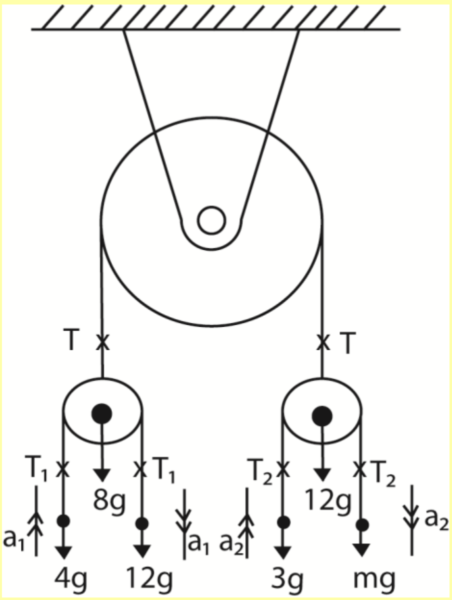
The acceleration of 4kg and 12kg masses are $\frac{g}{2}$ upward and $\frac{g}{2}$ downward respectively. The acceleration of the 3kg and m masses are $\frac{g}{3}$ upwards and $\frac{g}{3}$ downwards respectively. The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the

(a) tensions in the strings (09marks)

Let T = tension n the string joining masses 8kg and 12kg

T_1 = tension in the string joining masses 4kg and 12kg

T_2 = tension in string joining masses 3kg and mkg



Resolving vertically

2T = 8g + 12g

T = 10g = 10 x 9.8 = 98N

For 4kg mass; resultant force = T_1- 4g

$4a_1 = T_1 - 4g$

$T_1 = 4g + 4a_1 = 4g + 4 \times \frac{g}{2} = 6g$

= 6 x 9.8 = 58.8N

For 3kg mass; resultant force = T_2- 3g

⇨ $3a_2 = T_2 - 3g$

$T_2 = 3g + 3a_2 = 3g + 3 \times \frac{g}{3} = 4g$

= 4 x 9.8 = 39.2N

Hence the tensions in the strings are 98N, 58.8N and 39.2N

Since the string of the fixed pulley remains stationary, this means the pulleys of the 8kg and 12kg are stationary or fixed

(b) value of m. (03marks)

For the m kg mass

Resultant force = mg – T_2

$ma_2 = mg - T_2$

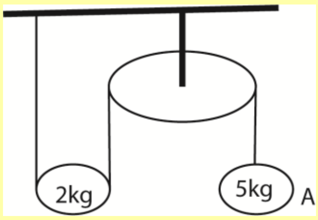
$m(\frac{g}{3}) = mg - 4g$

$\frac{2}{3}mg = 4g$

$m = \frac{12}{2} = 6kg$

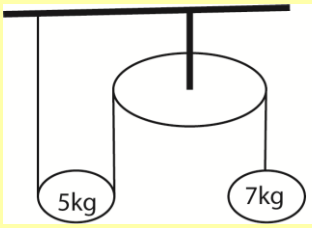
Revision exercise 3

1. A string with one end fixed passes under a movable pulley of mass 2kg, over a fixed pulley and carries a 5kg mass at its end



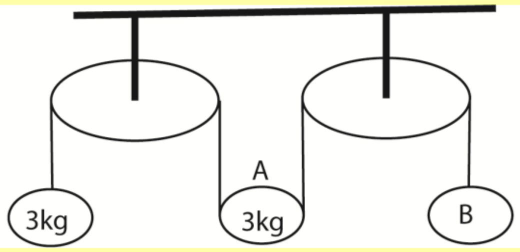
Find the acceleration of the movable pulley and the tension in the string. [$3.56ms^{-2}$, 13.36N]

2. a string with one end fixed passes under a movable pulley of mass 5kg, over a fixed pulley and carries a mass of7kg at its other end.



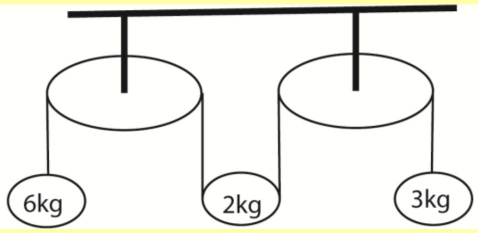
Find the acceleration of the movable pulley and the particle [$2.673ms^{-2}$, $5.146ms^{-2}$]

3. In the pulley system below, A is a heavy pulley which is free to move.



Find the mass B, if it does not move upwards or downwards when the system is released from rest. [1kg]

4. Two particles of mass 3kg and 6kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 2kg, the portions of the string not in contact are vertical

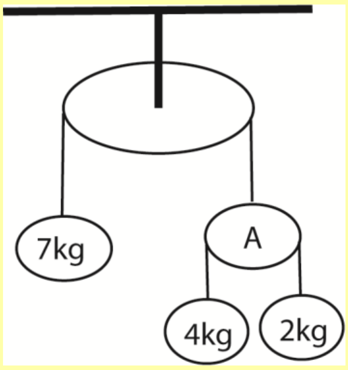


If the system is released from rest, find

(a) acceleration of movable pulley [$5.88ms^{-2}$]

(b) tension in the string [15.6N]

5. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over the pulley A and caries a mass of 4kg at one end and a mass of 2kg at the other end.

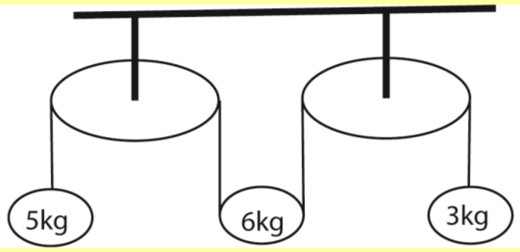


If the system is released from rest, find

(a) acceleration of 4kg mass [$2.38ms^{-2}$]

(b) tension in the strings [59.33N, 29.66N]

6. Two particles of mass 3kg and 5kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 6kg, the portions of the string not in contact are vertical

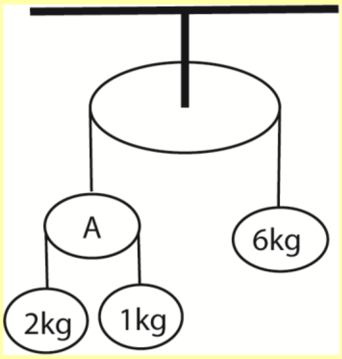


If the system is released from rest, find

(a) acceleration of movable pulley [1.089ms^{-2}]

(b) tension in the string [32.667N]

7. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 4kg at one end and a mass of 2kg at the other end.

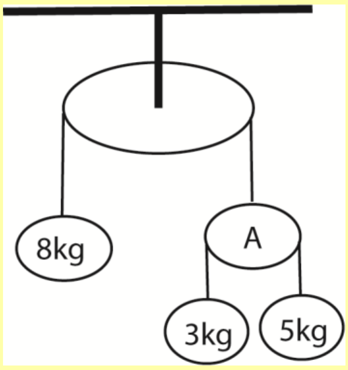


If the system is released from rest, find

(a) acceleration of 1kg mass [8.2923ms^{-2}]

(b) tension in the strings
[18.0923N, 36.1846N]

8. The diagram shows a system of masses and pulleys.

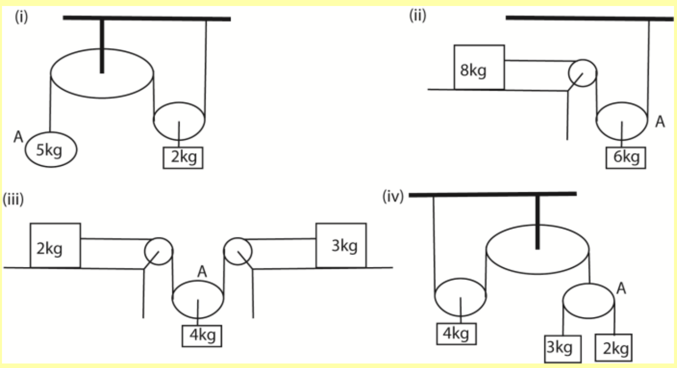


If the system is released from rest, find

(a) acceleration of 5kg mass [2.8451ms^{-2}]

(b) tension in the strings
[75.8712N, 37.9356N]

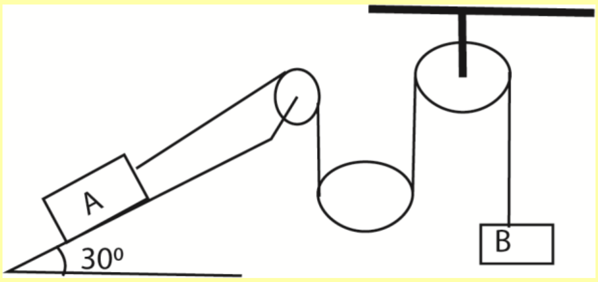
9. For each of the systems below: all the strings are light and inextensible, all pulleys are light and smooth and all surface are smooth. In each case find the acceleration of A and the tension in the string.



(i) [$7.127ms^{-2}$, 13.364N] (ii) [$1.547ms^{-2}$, 24.758N]

(iii) [$3.564ms^{-2}$, 10.691N] (iv) [$4.731ms^{-2}$, 12.166N, 24.331N]

10. Two particles A and B of mass 4kg and 2kg respectively and a movable pulley c of mass 6kg are connected by a light inextensible string as shown below



Given that the coefficient of friction between A and the plane is 0.2 and the system is released from rest, find the acceleration of A, B, C and the tension in the string.
[A = $-0.25ms^{-2}$, B = $2.9ms^{-2}$, C = $1.325ms^{-2}$]