in order to hit a mark h m above the ground at the same point, if the shot is projected at the same elevation, the velocity of projection must be increased to

$$\frac{v^2}{(v^2-gh)^{\frac{1}{2}}}.$$ (I.S.)

24. Prove that the time of flight T and the horizontal range X of a projectile are connected by the equation

$$gT^2 = 2X \tan \alpha,$$

where α is the angle of elevation.

Show that when the maximum horizontal range is 160 km the time of flight is about 3 minutes, and determine the muzzle velocity and the height of the trajectory. (I.E.)

25. A body is projected so that on its upward path it passes through a point x m horizontally and y m vertically from the point of projection. Show that, if R m is the range on a horizontal plane through the point of projection, the angle of elevation of projection is

$$\tan^{-1}\left(\frac{y}{x}\times\frac{R}{R-x}\right).$$ (I.S.)

26. A particle projected from a point meets the horizontal plane through the point of projection after describing a horizontal distance a, and in the course of its trajectory attains a greatest height b above the point of projection. Find the horizontal and vertical components of the velocity of projection in terms of a and b.

Show that when it has described a horizontal distance x it has attained a height of $4bx(a-x)/a^2$. (H.C.)

27. If the horizontal range of a particle projected with velocity V is a, show that the greatest height x attained is given by the equation

$$16gx^2-8V^2x+ga^2 = 0.$$

Explain why two values of x are to be expected. (I.S.)

28. Show that the relative velocity of two bodies moving in any direction under the acceleration of gravity remains constant. A stone is projected horizontally from the top of a tower 54 m high with a velocity of 15 $\mathrm{m\,s^{-1}}$, and at the same instant another stone is projected in the same vertical plane from the foot of the tower with a velocity of 30 $\mathrm{m\,s^{-1}}$ at an elevation of 60°. Show that the stones will meet, and find the height above the ground, and the distance from the tower at the instant of meeting. (I.E.)