P425/1
PURE MATHEMATICS
Paper 1
July/ August, 2023
3 HOURS

MATIGO MOCK EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES:

- *Answer **all the eight** questions in section **A** and any **five** from section **B**.*

- *Any additional question(s) answered will **not** be marked.*

- *All working **must** be shown clearly.*

- *Begin each question on a fresh sheet of paper.*

- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

Turn Over

SECTION A: (40MARKS)
*Answer **all** questions in this section*

1. Given that $nC_3 = 35$, find the value of n. (5 marks)

2. Prove that $\sqrt{3}\,cosec\,20° - sec\,20° = 4$ (5 marks)

3. Evaluate $\int_2^4 \frac{x+2}{x^2+4x-7}\,dx$, give your answer to 3 significant figures. (5 marks)

4. Given that $3ln2 + 2ln5 - \frac{1}{2}\,ln10000 = lnP$, find the value of P (5 marks)

5. Find the point of intersection of the line;

   $\frac{x-1}{3} = \frac{y+2}{2} = \frac{z-1}{5}$ and the plane $2x - y + 4z = 6$ (5 marks)

6. $x^2 + bx + c$ and $x^2 + dx + e$ have a common factor $(x + 2)$;
   Show that $2(d - b) = e - c$ (5 marks)

7. Solve the inequality; $\frac{x^2+4x+5}{x+3} \leq 1$ (5 marks)

8. Find the particular solution to the differential equation $3e^{4x} - \frac{dV}{dx} = 2$
   where the graph of V against x passes through the point $(0, -4)$ (5 marks)

SECTION B: (60 MARKS)
*Answer any **five** questions from this section*

9. **(a)** Express $p(x) = \frac{x^2+7x+2}{(1+x^2)(2-x)}$ into partial fractions (6 marks)
   (b) Hence prove that $\int_0^1 P(x)\,dx = \frac{11}{2}\,ln2 - \frac{\pi}{4}$ (6 marks)

10. **(a)** Differentiate with respect to x;
    (i) $sin^{-1}\left(\tan\frac{x}{2}\right)$ (4 marks)
    (ii) x^{lnx} (4 marks)

    (b) If $y = e^x \sin x$ show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ (4 marks)

11. **(a)** The coordinates of two points A and B are $(3, -4, 2)$ and $(-5, 2, -8)$
    respectively, find the distance from A to B giving your answer in the form
    $a\sqrt{b}$ where a and b are integers to be found. (5 marks)
    (b) Find the perpendicular distance of a point; $(1, 1, 4)$ from the line
    $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{3}$ (7 marks)

Turn Over

12. Given the curve $y = \frac{4x - 10}{x^2 - 4}$;

   (i) Find the range of values of y within which the curve doesn't lie and hence, determine the stationary points of the curve.   (6 marks)

   (ii) State the equations of the asymptotes to the curve.   (3 marks)

   (iii) Sketch the curve   (3 marks)

13.(a) Find the equation of the locus of point $P(x, y)$ which moves so that its distance from point $A(1, 3)$ is twice its distance from point $B(4,6)$. Show that the locus is a circle giving its centre and radius   (5 marks)

(b) The normal at a point $P(3cos\beta, 4sin\beta)$ on the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ meets the x and y axes at point A and B respectively. Show that the locus of M, the midpoint of AB., is an ellipse with the same eccentricity as the given ellipse.   (7 marks)

14.(a) Given that $-3 + 2i$ is one of the roots of the quadratic equation, $z^2 + bz + c = 0$ Where b and c are real constants find values of b and c.   (5 marks)

(b) If point P in the complex plane correspond to the complex number Z; find the locus of P when $|Z + 2i| = |Z - 1|$ (3 marks)

(c) The complex number z satisfies the equation, $(2 + 5i)(z + 2i) = -7 - 32i$, Find z, giving your answer in the form $a + bi$, where a and b are real numbers   (4 marks)

15.(a) Find the A. P whose 10th term is 5 and 18th term is 77.   (6 marks)

   (b) Solve the simultaneous equations

$x + y - 4 = 0$

$x^2 - 4x - 3y = 0$ (6 marks)

16. A large weather balloon is being inflated at a rate that is inversely proportional to the square of its volume

   (a) Defining variables for the volume of the balloon (m^3) and time $(seconds)$ write down a differential equation to describe the relationship between volume and time as the weather balloon is inflated.   (2 marks)

   (b) Given that initially the balloon may be considered to have a volume of zero and that after $400\ seconds$ of inflating its volume is $600m^3$, find the particular solution to your differential equation.   (5 marks)

   (c) Although it can be inflated further, the balloon is considered ready for release when its volume reaches $1250m^3$, if the balloon needs to be ready for a midday release, what is the latest time it can start being inflated.   (5 marks)

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