P425/1
PURE MATHEMATICS
Paper 1
July/August 2023
3 hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.
- *Any additional question(s) answered will* ***not*** *be marked.*
- *Show* **all** *necessary working clearly.*
- *Begin each answer on a fresh page of paper.*
- *Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Answer **all** questions in this section.

1. If $a^3 + b^3 = 6ab(a + b)$, show that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}[\log a + \log b]$. (05 marks)

2. Given that $y = \text{cosec}^{-1}(x)$. Hence prove that $\frac{dy}{dx} = \frac{-1}{x\left[\sqrt{(x^2-1)}\right]}$ (05 marks)

3. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan x}\,dx$ (05 marks)

4. Solve equation $\cos x + \sin 2x = 0$ for $0 \leq x \leq 2\pi$. (05 marks)

5. Show that the circles whose equations are $x^2+y^2-4x-5=0$ and $x^2+y^2-8x+2y+1=0$ cut orthogonally. (05 marks)

6. Expand $\left(\frac{1+3x}{1-3x}\right)^{\frac{1}{2}}$ as far as the term in x^3.
   By putting $x=\frac{1}{7}$ in your expansion, estimate $\sqrt{10}$, correct to two decimal places. (05 marks)

7. Find the perpendicular distance of a point P(3,1,7) from the line $r = 3i + j - 2k + \lambda(2i - j + 2k)$. (05 marks)

8. An inverted cone with vertical angle 60^0 has water in it dripping out through a hole at the vertex at the rate of $9\ \text{cm}^3$ per minute. Find the rate at which it's level will be decreasing at an instant when the volume of water left in the cone is $9\pi\ \text{cm}^3$. (05 marks)

SECTION B (60 marks)

Answer any **five** questions from his section.

9. (a) Given that; $Z = \frac{(2-i)^2(3i-1)}{(i+3)^3}$. Find;

(i) modulus of Z. (02 marks)

(ii) argument of Z. (02 marks)

Hence express Z in polar form. (02 marks)

(b) Show the region represented by $|Z+i-2| \leq 1$ on an argand diagram and state the complex number of the centre of the wanted region. (06marks)

10. (a) A and B are points whose position vectors are $\mathbf{a} = \mathbf{i} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ respectively. Determine the position vectors of a point P that divides line $\overline{AB}$ internally in the ratio 5:1. (04 marks)

(b) If vectors $\mathbf{a} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{k} - \mathbf{i} - 3\mathbf{j}$ are parallel to a plane containing point (1, –2, 3). Determine;

(i) the equation of the plane. (04 marks)

(ii) the angle the line $\frac{x-4}{4} = \frac{y}{3} = \frac{Z-1}{2}$ makes with the plane in (i) above. (04 marks)

11. The curve is given parametrically by the equations $x = \frac{t}{1+t}$ and $y = \frac{t^2}{1+t}$

(a) Find the Cartesian equation of the curve. (02 marks)

(b) Determine the turning points of the curve. (05 marks)

(c) Sketch the curve. (05 marks)

12. (a) If $y = e^{2x}\sin 3x$, Prove that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$ (06 marks)

(b) By using a suitable substitution $x = \sin\theta$,

evaluate $\int_0^{\frac{\sqrt{3}}{2}} \left(\frac{x^3}{\sqrt{(1-x^2)}} \right) dx$ (06 marks)

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13. (a) Prove that $\sin\left[2\sin^{-1}(x)+\cos^{-1}(x)\right] = \sqrt{(1-x^2)}$. (05 marks)

(b) Show that $\sin 3x = 3\sin x - 4\sin^3 x$. Hence solve the equation $8t^3 - 6t + 1 = 0$ correct to 4 significant figures. (07 marks)

14. If the line $y = mx + c$ is a tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $c^2 = b^2 + a^2m^2$. (04 marks)

Hence determine;

(i) equations of four common tangent to the ellipses $\frac{x^2}{23} + \frac{y^2}{3} = 1$ and $\frac{x^2}{14} + \frac{y^2}{4} = 1$ (04 marks)

(ii) the equations of the tangents at the point (-3, 3) to ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ (04 marks)

15. (a) The eighth term of an arithmetic progression is twice the third term and the sum of the first eight terms is 39. Find the first three terms of the progression and show that its sum to n term is $\frac{3n}{8}(n+5)$.

(06marks)

(b) Find how many terms of the series $1+\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\ldots\ldots$ must be taken so that the sum will differ from the sum to infinity by less than 10^{-6}.

(06 marks)

16. On 1st march 2020. There were 60 female antelopes kept a side to feed lions. It was discovered that the rate at which the antelopes were eaten was proportional to sum of 5 and the number of antelopes present at any given time per month. On 31st August, 40 antelopes were present.

(a) Form a differential equation and solve it. (09 marks)

(b) How many antelopes were left by end of 15th November, 2020? (Assume each month is 30 days and none of antelope dies on itself on.) (03 marks)

END