MARKING GUIDE
UTEC P425/2
APPLIED MATHEMATICS 2023

NO	SOLUTION	Mks	Comments
1	(a) From $\mathrm{P}(A' \cup B) = 1 - \mathrm{P}(A \cap B')$ $\frac{1}{2} = 1 - \frac{5}{8}\mathrm{P}(B')$ $\frac{5}{8}\mathrm{P}(B') = \frac{1}{2} \qquad \therefore \mathrm{P}(B') = \frac{4}{5}$ ⇨ $\mathrm{P}(A \cup B') = 1 - \mathrm{P}(A' \cap B)$ $= 1 - \left(\frac{3}{8} \times \frac{1}{5}\right)$ $= \frac{37}{40}$ (b) $\mathrm{P}(A' \cup B') = \mathrm{P}(A \cap B)^1$ $= 1 - \mathrm{P}(A \cap B)$ $= 1 - \left(\frac{5}{8} \times \frac{1}{5}\right)$ $= \frac{7}{8}$		
		05	
2	i) $\begin{array}{\|c\|c\|c\|} 0.5 & 0.8 & 1.2 \\ \hline A & -0.24 & 0.18 \end{array}$ $\frac{A+0.24}{0.5-0.8} = \frac{-0.24-0.18}{0.8-1.2}$ $A = -0.555$ ii)		

	0.8	B	1.2
	−0.24	−0.12	0.18

$$\frac{B-0.8}{-0.12+0.24} = \frac{1.2-0.8}{0.18+0.24}$$

$$B = 0.9143$$

05

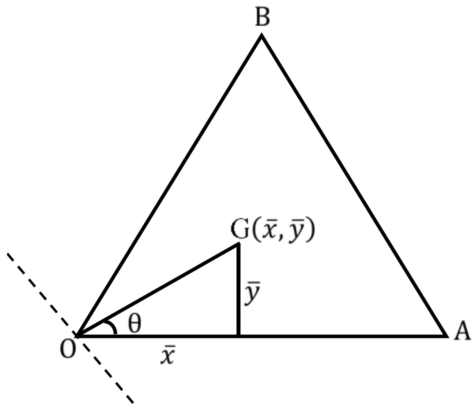
3

(a) $G(\bar{x}, \bar{y}) = G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= G\left(\frac{0+9+6}{3}, \frac{0+0+6}{3}\right)$

$= G(5, 2)$

(b)



$\tan\theta = \frac{2}{5}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta = 21.80^0$

05

4

R_H	R_M	d	d^2
1	2	−1	1
2	1	1	1
3	4	−1	1
4	3	1	1
5	7	−2	4
6	5	1	1
7	6	1	1
			$\sum d^2 = 10$

	$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$ $\rho = 1 - \frac{6\times 10}{7(7^2-1)}$ $\rho = 0.8214$ ***Very high positive correlation*** Or ***Significant at 5%*** Or ***Not significant at 1%***		
		05	
5	$\boldsymbol{v} = \begin{pmatrix} 12t^2 \\ 8t+23 \end{pmatrix} ms^{-1}$ Position vector, $\boldsymbol{r} = \int \boldsymbol{v}\, dt$ $= 4t^3\boldsymbol{i} + (4t^2 + 23t)\boldsymbol{J} + \boldsymbol{c}$ Average velocity $= \frac{\Delta \boldsymbol{r}}{\Delta t}$ $= \frac{r_{(t=3)} - r_{(t=1)}}{3-1}$ $= \frac{1}{2}[(108\boldsymbol{i} + 105\boldsymbol{j} + \boldsymbol{c}) - (4\boldsymbol{i} + 27\boldsymbol{j} + \boldsymbol{c})]$ $= 52\boldsymbol{i} + 39\boldsymbol{j}$ Average speed $= \sqrt{52^2 + 39^2}$ $= 65\, ms^{-1}$		
		05	
6	$e_x = 0.005, e_y = 0.0005$ Max value $= (xy)_{max}$ $= (1.25 + 0.005) \times (1.600 + 0.0005)$ $= 2.0086$ Min value $= (xy)_{min}$ $= (1.25 - 0.005) \times (1.600 - 0.0005)$ $= 1.9914$ Interval $= 1.9914 \leq xy \leq 2.0086$ Or $= [1.9914, 2.0086]$		

	Maximum error $= \frac{1}{2}(2.0086 - 1.9914)$ $= 0.0086$		
		05	
7	$\mathrm{P}(H) = 3\,\mathrm{P}(T)$ $\mathrm{P}(H) + \mathrm{P}(T) = 1$ $3P(T) + P(T) = 1$ $4\mathrm{P}(T) = 1 \qquad \therefore \mathrm{P}(T) = \frac{1}{4} = 0.25, \mathrm{P}(H) = 0.75$ Let X = Number of heads that occurs $X \sim B(15, 0.75)$ $\mathrm{P}(X \geq 7) = \mathrm{P}(X' \leq 8)$ $= 1 - P(X' \geq 9)$ $= 1 - 0.0042$ $= 0.9958$		
		05	
8	From $v = u + at$ $0 = 12 + 5a$ $a = -2.4\,ms^{-2}$ $s = s_{(t=5)} - s_{(t=4)}$ $s = \left(12 \times 5 - \frac{1}{2} \times 2.4 \times 5^2\right) - \left(12 \times 4 - \frac{1}{2} \times 2.4 \times 4^2\right)$ $s = 30 - 28.8$ $s = 1.2\,m$		
		05	
9	(table below)		

$c.b$	f	x	fx	c	$f.d$	$c.f$
0 – 10	8	5	40	10	0.8	8
10 – 15	10	12.5	125	5	2	18
15 – 25	25	20	500	10	2.5	43
25 – 40	15	32.5	487.5	15	1	58
40 – 50	4	45	180	10	0.4	62
50 – 60	2	55	110	10	0.2	64
$\sum$	64		1442.5			

(a) (i) mean $= \frac{\sum fx}{\sum f}$

$$= \frac{1442.5}{64}$$

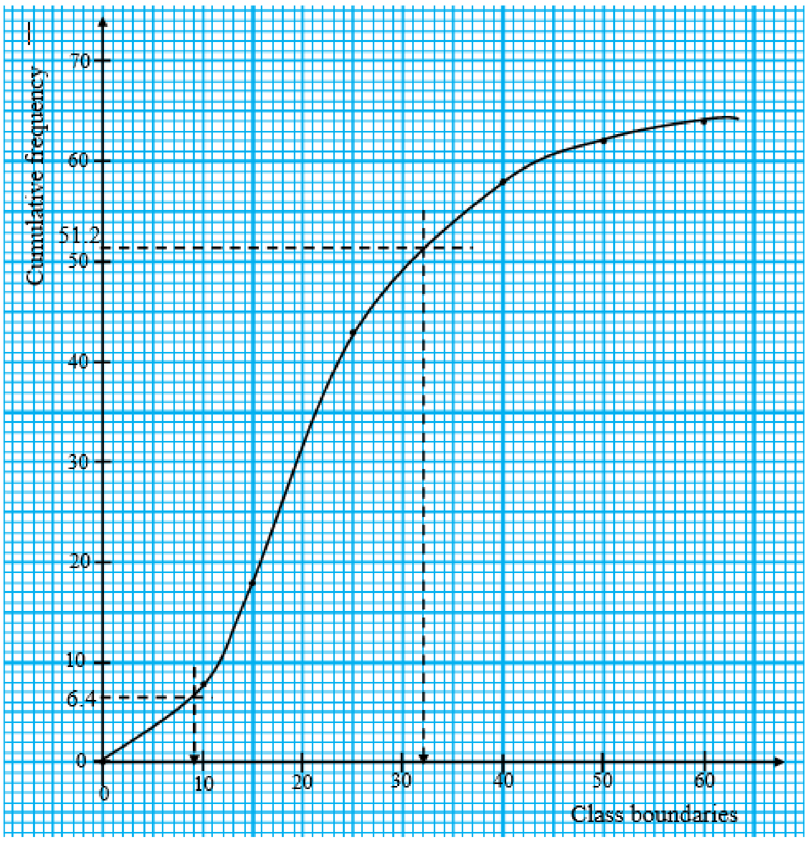
$$= 22.5391$$

(ii) mode $= l_1 + \left(\frac{d_1}{d_1+d_2}\right) \times c$

$$= 15 + \left(\frac{0.5}{0.5+1.5}\right) \times 10$$

$$= 17.5$$

(b)



Percentile deviation $= P_{80} - P_{10}$

$$= \left(\frac{80}{100} \times 64\right)^{th} - \left(\frac{10}{100} \times 64\right)^{th}$$

$$= 51.2^{th} - 6.4^{th}$$

$$= 32 - 9$$

$$= 23$$

		12	

10

(a) Let $y = 2x + \cos x$, $h = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$

x	y	
0	1.00000	
$\pi/12$		1.48952
$\pi/6$		1.91322
$\pi/4$		2.27790
$\pi/3$		2.59440
$5\pi/12$		2.87681
$\pi/2$	3.14159	
Total	**4.14159**	**11.15185**

$$\int_0^{\pi/2} (2x + \cos x)dx \approx \frac{1}{2} \times \frac{\pi}{12} [4.14159 + 2(11.15185)]$$

$$\approx 3.461680366$$

$$\approx 3.4617 \text{ (4dps)}$$

(b) Exact $= [x^2 + \sin x]_0^{\frac{\pi}{2}}$

$$= \left(\frac{\pi^2}{4} + \sin\left(\frac{\pi}{2}\right)\right) - 0$$

$$= 3.4674011$$

$$\approx 3.4674 \text{ (4dps)}$$

%age error $= \frac{|3.4674 - 3.4617|}{3.4674} \times 100$

$= 0.1644$ % or 0.16 %

It can be minimized by increasing the number of ordinates

		12	

11

(a) From $v^2 = \omega^2(a^2 - x^2)$

When $x = 3m, v = 8\ ms^{-1}$

$64 = \omega^2(a^2 - 9)$ …………………(i)

When $x = 4\ m, v = 6\ ms^{-1}$

$36 = \omega^2(a^2 - 16)$ ………………..(ii)

$(i) \div (ii)$;

$\frac{64}{36} = \frac{a^2-9}{a^2-16}$

$16(a^2 - 16) = 9(a^2 - 9)$

$16a^2 - 256 = 9a^2 - 81$

$7a^2 = 175$

$a^2 = 25$ $\quad\quad \therefore a = 5\ m$

From (i); $64 = \omega^2(25 - 9)$

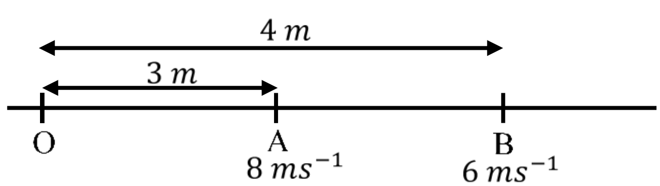
$\omega^2 = 4$ $\quad\quad \therefore \omega = 2$ rads^{-1}

From T $= \frac{2\pi}{\omega}$

$T = \frac{2\pi}{2}$

$T = \pi = 3.1416$ s

(b)



From $x = a\sin\omega t$

Time at point A from centre, O

$3 = 5\sin(2t_1)$

$t_1 = \frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

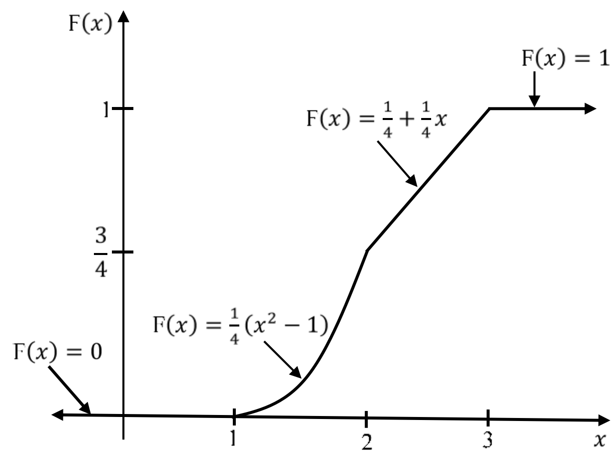
Time at point B from centre, O

$4 = 5\sin(2t_2)$

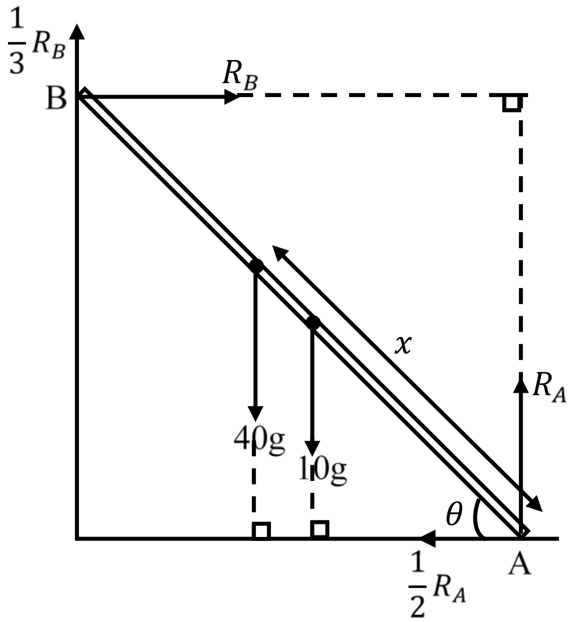
$t_2 = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$

Time from A to B,

$t = t_2 - t_1$

	$t = \frac{1}{2}\left[\sin^{-1}\left(\frac{4}{5}\right) - \sin^{-1}\left(\frac{3}{5}\right)\right]$ $t = 0.14187\text{s}$ $\approx 0.1419\text{s}$		
		12	
12	(a) $\text{F}(2);\ 3a = a + 2b$ $2a = 2b$ $a = b$ $\text{F}(3) = 1;$ $a + 3b = 1$ $b + 3b = 1$ $4b = 1 \qquad \therefore b = \frac{1}{4}, a = \frac{1}{4}$  (b) $\text{P}(X < 2.5/X < 1.5) = \frac{P(X<2.5 \cap X>1.5)}{P(X>1.5)}$ $= \frac{P(1.5<X<2.5)}{P(X>1.5)}$ $= \frac{F(2.5)-F(1.5)}{1-F(1.5)}$ $= \frac{\left(\frac{1}{4}+\frac{1}{4}(2.5)\right)-\left(\frac{1}{4}(1.5^2-1)\right)}{1-\frac{1}{4}(1.5^2-1)}$ $= \left(\frac{7}{8} - \frac{5}{16}\right) \div \left(1 - \frac{5}{16}\right)$ $= \frac{9}{16} \times \frac{16}{11}$		

	$= \frac{9}{11}$ (c) For $1 \le x \le 2, f(x) = \frac{d}{dx}\left[\frac{1}{4}(x^2 - 2)\right] = \frac{x}{2}$ For $2 \le x \le 3, f(x) = \frac{d}{dx}\left[\frac{1}{4} + \frac{1}{4}x\right] = \frac{1}{4}$ For $x \ge 1, f(x) = \frac{d}{dx}(1) = 0$ $f(x) = \begin{cases} \frac{x}{2}, 1 \le x \le 2 \\ \frac{1}{4}, 2 \le x \le 3 \\ 0, elsewhere \end{cases}$ $E(X) = \int_1^2 \frac{1}{2}x^2\,dx + \int_2^3 \frac{1}{4}x\,dx$ $\mathrm{E}(X) = \left[\frac{x^3}{6}\right]_1^2 + \left[\frac{x^2}{8}\right]_2^3$ $\mathrm{E}(X) = \frac{1}{6}(8-1) + \frac{1}{8}(9-4)$ $\mathrm{E}(X) = \frac{43}{24}$ or 1.7917		
		12	
13	(a) Let $2l$ =length of the ladder, $x =$ distance the man climbs before the ladder slides. Let $\theta = \tan^{-1}\frac{3}{4};\ \tan\theta = \frac{3}{4}$		



$(\rightarrow)$; $R_B = \frac{1}{2}R_A$…………(i)

$(\uparrow)$; $R_A + \frac{1}{3}R_B = 40\text{g} + 10\text{g}$

$$R_A + \frac{1}{3}R_B = 50\text{g}$$

$$2R_B + \frac{1}{3}R_B = 50g$$

$$\frac{7}{3}R_B = 50g$$

$$R_B = \frac{3\times50\times9.8}{7} = 210\text{N}$$

From (i);

$R_A = 2 \times 210 = 420\text{N}$

Taking moments about point A;

$40g \times x\cos\theta + 10g \times l\cos\theta = R_B \times 2l\sin\theta + \frac{1}{3}R_B \times 2l\cos\theta$

Dividing through by $\cos\theta$;

$40gx + 10gl = 2lR_B\tan\theta + \frac{2}{3}lR_B$

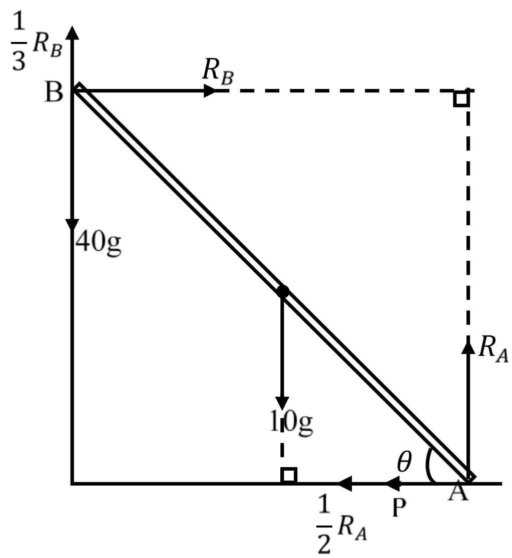
$40gx = \frac{2}{3} \times l \times 210 + 2 \times l \times 210 \times \frac{3}{4} - 10 \times 9.8 \times l$

$40gx = 140l + 315l - 98l$

$40gx = 357l$

$x = \frac{357}{392} l = \frac{51}{56} l$ m or $= 0.9107l$ m from A

(b) Let P be the minimum horizontal force



$(\uparrow)$; $\frac{1}{3} R_B + R_A = 50g$

$R_B = 150g - 3R_A$

$R_B = 150 \times 9.8 - 3R_A$

$R_B = 1470 - 3R_A$……………….(i)

$(\rightarrow)$; $R_B = P + \frac{1}{2} R_A$……………….(ii)

$(i) = (ii)$;

$1470 - 3R_A = P + \frac{1}{2} R_A$

$\frac{7}{2} R_A = 1470 - P$

$R_A = 420 - \frac{2}{7} P$

Taking moments about point B;

$10g \times l \cos\theta + \frac{1}{2} R_A \times 2l \sin\theta + P \times 2l \sin\theta = R_A \times 2l \cos\theta$

$10 \times 9.8 + R_A \tan\theta + 2P \tan\theta = 2R_A$

$98 + \frac{3}{4}R_A + 2P \times \frac{3}{4} = 2R_A$

$98 + \frac{3}{2}P = \frac{5}{4}R_A$

$392 + 6P = 5R_A$

But $R_A = 420 - \frac{2}{7}P$

$\Rightarrow\ 392 + 6P = 5\left(420 - \frac{2}{7}P\right)$

$392 + 6P = 2100 - \frac{10}{7}P$

$\frac{52}{7}P = 1708$ $\quad\quad\therefore P = \frac{2989}{13}N$ or 229.9231N

12

14 (a) Let X = weights of the goats sold

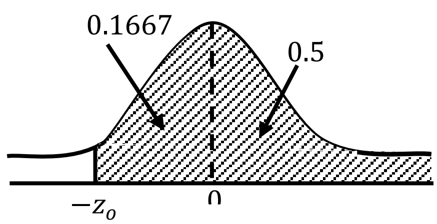
$\mu = 26$ kg, $\delta = ?$

$\mathrm{P}(X > 20) = \frac{8}{12} = 0.6667$

$\mathrm{P}\left(z > \frac{20-26}{\delta}\right) = 0.6667$

Let $\frac{20-26}{\delta} = z_0$

$\mathrm{P}(z > z_0) = 0.6667$



$P(0 < z < z_0) = 0.1667$

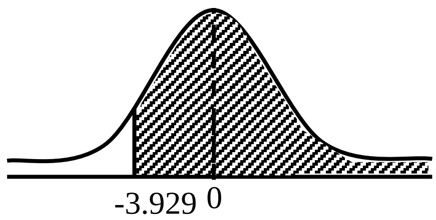
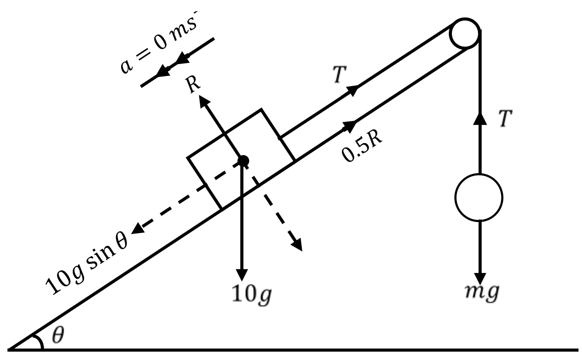
$z_0 = -0.431$

$\frac{20-26}{\delta} = -0.431$

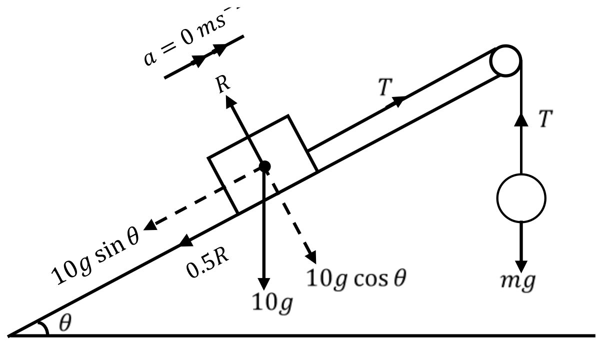
$-0.431\delta = -6$ $\quad\quad\therefore \delta = 13.92111369 \approx 14$ g

(b) Let $\bar{X}$ =sample mean, $n = 25$

$\mathrm{P}(\bar{X} > 15) = \mathrm{P}\left(z > \frac{15-26}{14/\sqrt{25}}\right)$

	$=\text{P}(z > -3.929)$  $\text{P}(z > -3.929) = 0.5 + \text{P}(0 < z < 3.929)$ $= 0.5 + 0.49996$ $= 0.99996$ (*Cal*)			
		12		
15	(a) From $\theta = \tan^{-1}\frac{4}{3}$, $\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}$ ***For minimum***: This is when the particle is just at the point of moving down the plane  At equilibrium; $T = mg$ ……….(i) Along the plane; $T + o.5R = 10g\sin\theta$ But R $= 10g\cos\theta$ $mg + 0.5 \times 10g\cos\theta = 10g\sin\theta$ $m = 10 \times \frac{4}{5} - 5 \times \frac{3}{5}$ $m = 8 - 3 = 5$ kg ***For maximum***;			

This is when the particle is just at the point of moving up the plane



At equilibrium;

$\mathrm{T} = mg$

Along the plane;

$\mathrm{T} = 0.5R + 10g\sin\theta$

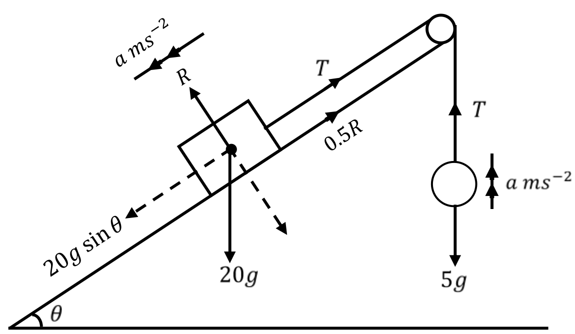
But $\mathrm{R} = 10g\cos\theta$

$mg = 0.5 \times 10g \times \cos\theta + 10g \times \sin\theta$

$m = 5 \times \frac{3}{5} + 10 \times \frac{4}{5}$

$m = 3 + 8 = 11$ kg

(b) When the mass of B is 5kg



For 5 kg mass;

$T - 5g = 5a$

$T = 5a + 5g$ ……………………(i)

For 20 kg mass;

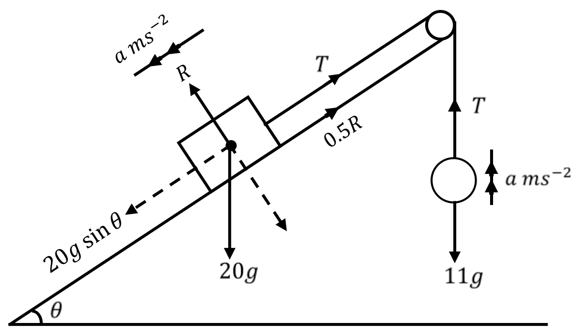
$20g\sin\theta - 0.5R - T = 20a$

But R= $20g\cos\theta$

$20 \times 9.8 \times \frac{4}{5} - 0.5 \times 20 \times 9.8 \times \frac{3}{5} - 5a - 5 \times 9.8 = 20a$

$25a = 49$ $\quad\quad\quad\quad$ $\therefore a = 1.96\ ms^{-2}$

When he mass of B is 11 kg;



For 11 kg mass;

$T - 11g = 11a$

$T = 11a + 11g$

For 20 kg mass;

$20g\sin\theta - 0.5R - T = 20a$

But R= $20g\cos\theta$

$20 \times 9.8 \times \frac{4}{5} - 0.5 \times 20 \times 9.8 \times \frac{3}{5} - 11a - 11 \times 9.8 = 20a$

$31a = -9.8;$ $\quad\quad\quad$ $a = -\frac{49}{155}\ ms^{-2}$

$\therefore a = \frac{49}{155}\ ms^{-2}$ or $0.3161\ ms^{-2}$

12

16 (a) Let $f(x) = x\sin x - 1$

$f(1) = 1\sin(1) - 1 = -0.15853$

$f(1.5) = 1.5\sin(1.5) - 1 = 0.49624$

$\therefore$ Since $f(1)\cdot f(1.5) < 0$,then $1 <$ root < 1.5

1	x_0	1.5
−0.15853	0	0.49624

	$\frac{x_0-1}{0+0.15853}=\frac{1.5-1}{0.49624+0.15853}$ $x_0=1.121057776$ $x_0\approx 1.12106$ (b) $f'(x)=x\cos x+\sin x$ $x_{n+1}=x_n-\left(\frac{x_n\sin x_n-1}{x_n\cos x_n+\sin x_n}\right)$ Taking $x_0=1.12106$ $x_1=1.12106-\left(\frac{1.12106\sin(1.12106)-1}{1.12106\cos(1.12106)+\sin(1.12106)}\right)$ $=1.11415$ $x_2=1.11415-\left[\frac{1.11415\sin(1.11415)-1}{1.11415\cos(1.11415)+\sin(1.11415)}\right]$ $=1.11416$ Since $\lvert x_2-x_1\rvert=0.00001<0.00005$, then the root is 1.1142		
		12	