HeLP
One World One Education

SENIOR FIVE HeLP MATHEMATICS SEMINAR TO BE HELD ON SATURDAY 11TH SEPTEMBER 2021 AT 10:00AM

INSTRUCTIONS TO STUDENTS AND TEACHERS:

Dear students and teachers we would like to welcome you to participate in the forthcoming mathematics seminar for senior five at 10:00a.m as they revise their mathematics concepts. The seminar is organized by teachers under the Holistic eLearning programme .This is a free seminar and no one should charge you any fees. The process to be followed by both the teachers and students is suggested below:

1. Teachers share the Seminar questions with their students and ask for volunteers to discuss any of the questions.

2. Teachers talk to the parents to allow the children participate as presenters in the seminar on Saturday **11th September from 10:00am-1:00pm.** Other students will just be participants.

3. If your student is going to present then as the teacher(s) prepare her/him by looking through the calculations made by the student. Then encourage the student to write out the solution neatly in black pen including any graph. Then they scan or take a picture and send to the teacher. They can also type out the solution in a word or PowerPoint document and share with the teacher. .

4. The teacher could now train the student on how to present in zoom as far as sharing a screen and using the whiteboard. Alternatively the students' presentation will be loaded on the computer screen and they explain to us their solution.

5. When we receive all the solutions before the seminar day we shall build an online course for the students' work that all other students will access and download the solutions at www.help.sc.ug. This is our home as an initiative by teachers.

6. The teacher or student will hand in the solutions to Ronald Ddungu (0701433878(W)) or Kaziba Stephen (0787698238(W)) by Thursday **9th September 2021**and attend the presenters practice session at 4:00p.m using the same seminar link below.The process here is very important

7. The seminar will also be live on the Big Blue Button(BBB)at zero data cost for MTN users and for airtel users some little data will be required.The link below will be used https://meet.renu.ac.ug/b/kaz-blr-znx-a1r

> Holistic eLearning Platform is inviting you to a scheduled A level mathematics seminar.
> **Topic**: SENIOR FIVE MATHEMATICS SEMINAR
> **Time**: SEPTEMBER 11TH, 2021 10:00 AM
> Join Zoom Meeting: https://bit.ly/3jHO8D0
> **Meeting ID**:840 200 3818
> **Passcode**: HeLP

PURE MATHEMATICS (P425/1)

ALGEBRA

1. Solve the equation: $\sqrt{4x+13} - \sqrt{x+1} = \sqrt{12+x}$

2. Solve the simultaneous equations below using the echelon form:

$$2x + 3y - 4z = 9$$
$$3x + 2y + z = 4$$
$$5x + y + 3z = 3$$

3. Rationalise: $\frac{1}{\sqrt{5}+\sqrt{2}+\sqrt{3}}$

4. Solve the equations:

$$x + 2y = 2$$
$$x^3 + 8y^3 = 56$$

5. Express $\frac{1+\sqrt{2}}{(\sqrt{2}-1)^3}$ in the form $a + b\sqrt{c}$

6. Solve for x : $9^{3x} - 6(9^{2x}) + 11(9^x) - 6 = 0$

7. Solve for x given: $0.7 = (1.2)^{2x-1}$

8. Solve the equations:

$$\log_2 x^2 + \log_2 y^3 = 1$$
$$\log_2 x - \log_2 y^2 = 4$$

9. The roots of the quadratic equation $x^2 + (7+p)x + p = 0$ are α and β .Given that α and β differ by 5, find the possible values of p

10. Prove by induction that : $\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \frac{n+2}{2^n}$

11. Use the binomial theorem to expand $\sqrt{1 - \frac{x}{2}}$ as far as the term in x^3 and by substituting $x = 0.02$,find the value of $\sqrt{11}$ correct to 3 s.f

12. When the quadratic expression $px^2 + qx + r$ is divided by $x - 1$,$x - 2$ and $x + 1$,has the remainders 1,1 and 25 respectively.Determine the value of p, q and r then factorise the expression completely

13. When a polynomial $P(x)$ is divided by $x - 2$,the remainder is 4 and when $P(x)$ is divided by $x - 3$,the remainder is 7.Find the remainder when $P(x)$ is divided by $(x - 2)(x - 3)$

14. Solve the equation:

$$x^{\frac{1}{3}} + 3 = 4x^{-\frac{1}{3}}$$

15. Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$; give your answer correct to 3S.Fs

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16. Find the value of x in the following equations;

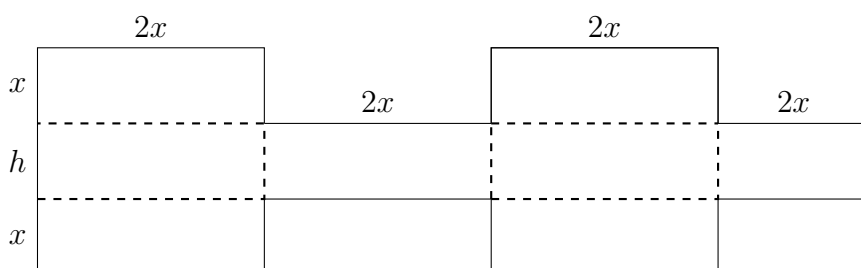   (a) $log_{10}(x+9) = 2 + log_{10}x$

    (b) $e^x = 3^{x-2}$

DIFFERENTIATION

17. Find the area enclosed by the curve $y = x(8-x)$ and the line $y = 12$

18. The curve is given parametrically by the equations $x = \frac{t}{1+t^3}$,$y = \frac{t}{1+t^3}$. Show that $\frac{dy}{dx} = \frac{3t}{(2-t^3)}$ and that $\frac{d^2y}{dx^2} = 48$ at a point $t = 1$

19. Differentiate from first principles. $y = \frac{1}{x^2}$

20. A carton of volume V m^3 is made from a piece of cardboard as shown below.

    The area of the cardboard used is $6m^3$, find expressions for h and V in terms of x and find the value of x which will give a maximum volume.

21. A 4% error is made in measuring the radius of a sphere, find the percentage error in measuring the volume.

22. A container in the shape of a hollow cone of semi-vertical angle 30^0 is held with its vertex pointing downwards. Water is poured into the cone at the rate of $5cm^3s^{-1}$ .Find the rate at which the depth of water in the cone is increasing when this depth is 10 cm .

COMPLEX NUMBERS

23. Given that $z_1 = 3 + 2i$,$z_2 = 3 - 2i$ find the modulus and argument of

    (a) $\frac{z_1^2}{z_2}$

    (b) $z_1^2 z_2^2$

24. Solve the equation giving the three roots: $z^3 + 8 = 0$

25. Show that $1 + 2i$ is a root of the equation $2z^3 - z^2 + 4z + 15 = 0$, hence find the other roots.

26. Express z in the form $a + bi$, given that $z = \frac{1}{3+i} - \frac{1}{1+7i}$ and hence find the modulus and argument of z

TRIGONOMETRY

27. Solve: $3 + 2\cos^2\theta\tan\theta = 4\cos^2\theta$, for $0 \le \theta \le 360^0$.

28. Solve:$3\cot 2\theta + \cot\theta = 1$, for $0 \le \theta \le 360^0$.

29. Prove that : $\tan(45^0 + \theta) - \tan(45^0 - \theta) = 2\tan 2\theta$

30. Solve for x given $7\cos x + 24\sin x = 10$ and $0 \leq x \leq 360^0$.

31. (a) Solve the simultaneous equations :

$$\cos x + 4\sin y = 1$$
$$4\sec x - 3\text{cosec } y = 5$$

(b) Determine the solution of the equation $\tan 2x + 2\sin x = 0$ for $0 \leq x \leq 90^0$.

32. Prove the following identities

(a) $2\text{cosec}2\theta = \text{cosec}\theta \sec \theta$

(b) $\frac{1+\tan^2 \beta}{2-\tan^2 \beta} = \sec 2\beta$

33. Prove that :

$$\tan 5\theta = \frac{5\tan \theta - \tan^3 \theta + \tan^5 \theta}{1 - 10\tan^2 \theta + \tan^4 \theta}$$

CIRCLES

34. Fin the radii and coordinates of the centres of the following circles

(a) $x^2 + y^2 + 4x - 6y + 12 = 0$

(b) $x^2 + y^2 - 2x - 4y + 1 - 0$

(c) $x^2 + y^2 - 3x = 0$

35. Find the equation of a circle which passes through the points $(-2, 1)$,$(6, 1)$ and $(-2, 7)$

36. A $(-3, 0)$ and B$(3,0)$ are fixed points. Show that the locus of a point$P(x, y)$ which moves such that PB $=2$ PA is a circle and find its centre and radius.

INTERGRATION

37. Find the following intergrals

(a) $\int x(x + 4)^2 dx$

(b) $\int x\sqrt{x - 1} dx$

(c) $\int \frac{x}{\sqrt{x-3}} dx$

(d) $\int \frac{1}{x(x-2)} dx$

(e) $\int \frac{1}{9+36x^2} dx$

(f) $\int \frac{x-3}{\sqrt{x+1}}$

38. Evaluate $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$

39. Evaluate $\int_0^{\frac{1}{2}} \frac{4dx}{(1-x)(1-x^2)^{\frac{1}{2}}}$

40. Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx$$

END

STATISTICS

1. (a) Find the lower and upper quartiles and the 9th decile for the following data :

20	30	35	25	20	30	35	25
20	30	35	40	30	35	35	25
20	40	20	25	25	30	20	20

(b) The table below shows the time taken to complete a puzzle by 95 students in a certain school

Time(seconds)	Number of students
10−	5
20−	11
30−	16
40−	19
45−	14
50−	12
60−	9
70−	6
80 − 100	3

(i) Draw a Histogram using this data and use it to estimate the mode

(ii) Draw a cummulative frequency curve and From the curve estimate the

(a) median

(b) Interquartile range

2. The marks of 6 students in Mathematics and Physics were as follows:

Mathematics(x)	90	60	80	54	86	70
Physics(y)	48	72	60	78	50	65

(a) Plot a scatter diagram for the data .Comment on the relationship between the scores in the two tests

(b) Draw a line of best fit for the scatter diagram ,hence find y when $x = 68$

(c) Calculate the rank correlation coefficient for the data .Comment on your result at 5% level of significance

3. The table below shows the prices and price indices for three items in the years 2019 and 2020.

Items	Price in 2019	Price indices in 2020 using 2019 as base
A	3200	125
B	4000	105
C	4500	120

(a) Calculate the price of each item in 2020

(b) Using weights $3, 5,$ and 2 for item A,B and C respectively,calculate the weighted average price index of the items in 2020

PROBABLITY

4. (a) A and B are mutually exclusive events such that P(A)=0.5 and P(B)=0.2 .Find :

   (i) $P(A \cup B)$

   (ii) $P(A' \cap B')$

   (b) The probability that 3 people A,B and C will survive the Covid 19 pandemic in the next 8 months are $0.4, 0.3$ and 0.2 respectively.Find the probability that in the next eight months

   (i) All will be alive

   (ii) None will be alive

   (iii) Only one will be alive

   (iv) Atleast one will be alive

5. (a) The probability that a student passes mechanics is 0.75 and the probability that she passes statistics is 0.65 ,the probability that she passes neither of the units is 0.20.Find the probability that she passes mechanics but not statistics

   (b) A bag contains 4 white balls ,3 black balls and 1 red ball.Two balls are picked in succession at random without replacement .Find the probability that

   (i) Both are of the same color

   (ii) At least one black ball is picked

6. Three events A,B and C are such that A and B are independent ,A and C are mutually exclusive.Given that $P(A) = 0.4, P(B) = 0.2, P(C) = 0.3$ and $P(CnB) = 0.1$.find $P(A/AUC)$

7. Events M and N are such that $3P(M \cap N) = 2P(\overline{M} \cap N) = P(\overline{M} \cap \overline{N}), P(M) = 0.6$.Find the probability that:

   (a) Neither events occur

   (b) Only one event occurs

8. Independent events A and B are such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{3}{5}$.Find

   (a) $P(A \cup B)$

   (b) $P(A \cap B)$

   (c) $P(\overline{A} \cup \overline{B})$

9. The probability density function of the discrete r.v is given by $P(X = x) = k(\frac{1}{4})$ for $x = 0, 1, 2, 3, \therefore$ Find the value of the constant k.

NUMERICAL METHODS

10.   (a) Find the exact area under the curve $f(x) = 2x^2 + 4$ between $x = 2$ to $x = 5$

   (b) Use the trapezium rule with :

     (i) 6 ordinates

     (ii) 8 sub intervals ,to estimate the area in (a) above

   (c) Calculate the percentage error in your answers in part(b)above .Comment on your answers

11. Use the trapezium rule with 7 ordinates to find the approximate value of $\int_{1}^{2.2}(1 + x \sin 2x)dx$, correct to four significant figures

12. By constructing a table of values for $f(x) = 3xe^x - 1$ in the range $0.1 \leq x \leq 1.1$, using intervals of 0.2, obtain;

   (i) the value of f (1.13) using Linear extrapolation.

   (ii) Using linear interpolation find the root of f(x) correct to 3d.p .

13. Given that f(0.9) = 0.2661, f(1.0) = 0.2420 and f(1.1.) = 0.2179, Estimate:

   (i) f(0.98) and f(1.2)

   (ii) $f^{-1}(0.2372)$ using linear interpolation or extrapolation

14. The table values of $\tan \Theta$ have been extracted from four figure tables

| Θ	35	36	37	38	39
$\tan \Theta$	0.7002	0.7265	0.7536	0.7813	0.8098

   Estimate

   (i) $\tan^{-1}(0.7673)$

   (ii) $\tan 39^0 36'$

15. Show that the iterative formula based on NRM for solving the equation $e^{2x} + 4x - 5 = 0$ is given by :

$$x_{n+1} = \frac{e^{2x_n}(2x_n - 1) + 5}{2e^{2x_n} + 4} \qquad\qquad n = 0, 2, 3, \cdots$$

MECHANICS

16. Three forces $F_1 = (2i + bj + 4k)N$, $F_2 = (-ai - 2j + ck)N$ and $F_3 = (4i + 3k)N$ act on a particle,if the particle doesnot move .Find the values of $a, b,$ and, c

17. A particle with position vector $40i + 10j + 20k$ moves with a constant speed of $5ms^{-1}$in the direction of the vector $4i + 7j + 4k$.Find its distance from the origin after $5 seconds$.

18. A cyclist passes point Q and accelerates uniformly at a rate of ams^{-2}.If the cyclist covers distances of $3.5m$ and $4.5m$ in the **fourth** and in **sixth** seconds respectively.Determine the value of a

19. A Force of magnitude $6\sqrt{2}$N acts in direction making an angle of 60^0 with the horizontal.Find the horizontal and vertical components of the force

20. The figure below shows a system of forces acting on a particle placed at O.Find the magnitude and direction of their resultant.
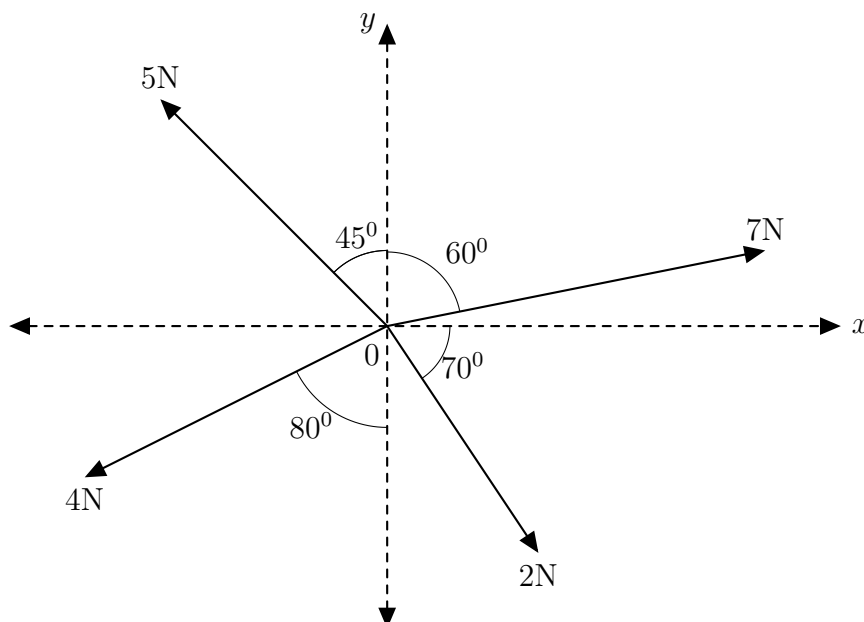
21. Express the force shown below in the form $ai + bj$
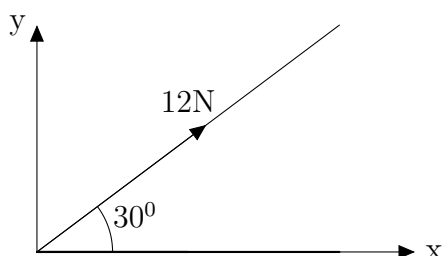
22. The magnitude of the resultant of the two forces P and q is $q\sqrt{3}$ and it makes an angle of 30^0 with direction P.prove that the magnitude of P is either equal to ,or double the magnitude of q

23. A particle **P** is projected vertically upwards with a speed of ums^{-1} from a point **O** on the ground,while at its highest point a second particle is projected vertically upwards with a speed $2ums^{-1}$ from point **O** .The particles collide at apoint which is at a distance xm from the highest point of **P**.Prove that :$u^2 = 32gx$

24. An object of mass $4kg$ is initially at rest at a point whose position vector is $(-4i+2j)m$.If its acted upon by a force $F = (14i + 21j + 24k)N$. Find

   (i) acceleration of the object

   (ii) its velocity and speed after $2seconds$

   (iii) its distance from the origin after 4seconds

25. A particle is projected from a point on level ground such that its initial velocity is $60ms^{-1}$ at an angle of elevation 30^0 and taking $g = 10ms^{-2}$,find

   (a) the time taken for the particle to reach its maximum height

   (b) the maximum height

   (c) the time of flight

   (d) the horizontal range of the particle

26. A bullet leaves the barrel of a gun 1.5m above the ground level ,travelling horizontally at $70ms^{-1}$.If it experiences a downward acceleration of $10ms^{-2}$,find the :

   (a) time taken for the bullet to hit the ground

   (b) distance it had travelled horizontally

27. A particle of mass 2.5kg is acted upon by a resultant force of magnitude 15N acting in the direction $2i - j - 2k$.Determine the:

   (a) Force in vector form

   (b) acceleration and its magnitude

28. A block of mass 10kg is placed on an inclined plane at angle of 30^0 to the horizontal where the coefficient of friction between the plane and the surface is 0.5.Find the

   (a) Force required to make the block move up the plane

   (b) Force required to keep the block at rest

   (c) acceleration of the block down the plane

29. In a square ABCD ,three forces of magnitude $5N, 6N$ and, $3N$ act along AB,AD and CA respectively.Their directions are in the order of the letters.Find the magnitude and direction of the resultant force.

30. The initial velocity of a particle moving with constant acceleration is (4i+3j)ms^{-1}.After 3 seconds the velocity of the particle is of magnitude 8ms^{-1} and parallel to 2i+j.Find the magnitude of acceleration of the particle

END