SMATA

4TH ANNUAL POST MOCK SEMINAR 2023

S.6 SUBSIDIARY MATHEMATICS

AT

ST. JOSEPH OF NAZARETH HIGH SCHOOL

24th September 2023

SEMINAR QUESTIONS

PART I

PURE MATH QUESTIONS

1. Solve the differential equation $\frac{dy}{dx} = \frac{7x^2+1}{8y}$; given that y =2 when x =0
2. Evaluate i). $\int_1^2 (3x^2 - 4x + 2)\,dx$

   ii). $\int_1^4 \left(\sqrt{x} - \frac{4}{\sqrt{x}} + 2\right) dx$

3. Given that $\frac{1}{\sqrt{2}} - \frac{\sqrt{2}+1}{1+3\sqrt{2}} = a\sqrt{2+b}$ where a and b are constants, find the values of a and b.

4. a). Solve the equation $2\,sin\,2\,\theta = 3\,cos\,\theta$, for $0^0 \leq \theta \leq 360^0$

   b). Without using tables or a calculator, find in surd form the value of tan 105^0.

   c). If $Sin\,x = a$, find in terms of a, i). $cosx$ ii).$cosecx$.

5. Solve the equation $3(1-x)^2 + 7(1-x) + 4 = 0$

6. If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$are the roots of the equation$4x^2 - 8x + 1 = 0$, find the equation whose roots are $\alpha \ \ and\ \beta$.

7. In a geometric progression, the second term exceeds the first term by 20 and the fourth term exceeds the second by 15. Find the possible values of the first term.

8. An Arithmetic Progression (A.P) has a common difference of -3. The sum of the first twenty terms is ten times the second term. Find the sum of the first 15 terms of the A.P.

9. Given that x+1 is a factor of the polynomial $2x^3 + 5x^2 + x - 2$, find the other two factors.

10. The points P,Q,and R have position vectors $2i + 2j$, $i + 6j$ and $-7i + 4j$respectively.
    a).i). Find the vector QR and PQ
    ii). Show that triangle PQR is a right angled at Q.

    b). Find the angle between PR and PQ

11. a). Given that $A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$, evaluate $det(A^2 - 2A)$

    b). Given the Matrix $N = \begin{pmatrix} a & a+1 \\ a-1 & a+2 \end{pmatrix}$. Find the value of a for which N is singular.

12. Solve the simultaneous equations below using matrix method

$$5x - 3y = 16$$

$$4y + x = -6$$

13. Two types of bread A and B are manufactured by a certain bakery. Type A requires 2kg of wheat flour, 1.5 litres of oil,3 eggs and 0.2kg of sugar. And type B requires 4kg of wheat flour, 2 litres of oil, 4 eggs and 0.4kgs of sugar. The cost of wheat flour is shs2500 per kg, oil is shs3000 per kg shs 350 per egg and shs 2800 per kg of sugar. If shop X ordered for 5 breads of

type A and 8 of type B while shop Y ordered for 7 of type A and 10 of type B.

i. Form three matrices from the information above

ii. Use matrix multiplication to find the cost of each type of bread

iii. Find the total cost required to work on the two orders

14. Polynomial $P(x) = x^4 + \boldsymbol{m}x^3 + \boldsymbol{n}x^2 + 5x + 3$ has a remainder of $2x + 1$ when divided by $x^2 + 3x + 2$. Find the values of **m** and **n.**

15. The population of a certain organism grows at a rate proportional to the number (N) of organisms present at time t . initially the number was 500 and it doubles every after 36 hours. Find the time for the number of the organisms to triple.

16. solve the differential equation $\frac{dy}{dx} = \frac{x}{1-y}$ when $y = 5$ and $x = 0$

PART II

APPLIED MATH TOPICAL AREAS

STATISTICS AND PROBALITY

DISCRETE RANDOM VARIABLE

17. A random variable x has a probability distribution given in the table below.

x	-1	0	1	2
$p(x = x)$	a	0.3	0.4	b

a) If the mean E(x) = 0.7. Find the values of a and b hence mode and median.

b) Determine;

i). variance and standard deviation

ii). $p(x \leq 0)$

iii). $E(3x - 1)$

BINORMIAL DISTRIBUTION

18.a). Given that $X \approx B(10, p)$ and variance $\frac{15}{8}$. Determine the possible value of p hence find $p(x \geq 7)$if P is less than 0.5.

b). In a certain family, the chance of having a baby boy is double that of a girl. A family plans to have five children. Determine the probability that;

i). Atleast two will be girls.

ii). None will be a boy.

CONTINOUS RANDOM VARIABLE

19. A random variable has a pdf given by

$$f(x) = \begin{cases} cx\,, x = 1,2,3,4 \\ c(8-x), \quad x = 5,6,7 \end{cases}$$

Find i). the value of c

ii). $P(3 \leq x \leq 5)$

iii). $P(x \geq 2 | x \leq 6)$

20. a). The pdf of a continuous random variable is given by

$$f(x) = \begin{cases} k(x^2 - 1)\,, & 0 \leq x \leq 2 \\ 0 \; otherwise \end{cases}$$

Find i). value of k

ii). $p(x \leq 1.5)$

iii). Mean

iv). Variance

v). Median

21. 21. A continuous random variable is given by the pdf

$$f(x) = \begin{cases} ax, & 0 \leq x \leq 3 \\ 3a(4-x)\,, & 3 \leq x \leq 4 \\ 0, & \text{Otherwise} \end{cases}$$

Where a is a constant

i) Sketch the pdf, hence value the value of **a**

ii) Determine $P(1 \leq x \leq 3)$

NORMAL DISTRIBUTION

22.a). A random variable x is given as X ~N (500,900), find $P(x \geq 500)$

b). A certain type of sweet potato as a mass which is normally distributed with mean 1 Kg and standard deviation 0.15Kg sack loaded with potatoes. Find the probability that potato picked at random from the sack weighs;

i) More than 0.79Kg
ii) at most 1.13 Kg
iii) Between 0.85Kg and 1.15Kg
iv) Number of potatoes between 0.75Kg and 1.29 Kg if 10,000 potatoes were in a sack.

STATISTICS

23.a). The height in centimeters of 6 tree seedlings are 60, 55, 46, 43, 56 and 58. Using assumed mean of 50. Find the mean height of the seedlings hence variance.

24. Below are marks obtained by 50 candidates in a Sub-math test.

Marks	-<30	-<40	-<50	-<60	-<70	-<80
No. of Candidates	3	11	29	40	47	50

Draw a frequency distribution table and use it to determine:

i) Mean mark
ii) Standard deviation

c) Represent the data on an-ogive and use it to

i. Estimate the median mark
ii. Interquartile range
iii. Range of middle 50 percent mark

25. The following grades were obtained by 8 candidates in Mathematics and General paper.

Math (x)	A	O	B	F	E	C	D	B
G.P (y)	C3	D2	D1	P8	P8	D2	C3	D2

(a) Calculate the rank correlation coefficient for the grades.
(b) Comment on your results at 1% level of significance.

26. The table below shows the performance of Candidates in two mock examinations.

Mock 1	35	65	55	25	45	75	20	51	60	90
Mock 2	86	70	84	92	79	68	96	86	77	58

(a) Calculate the rank correlation coefficient between Mock 1 and Mock 2 performance and comment on your result at 5% significance level.

(b) Plot the data on the same graph and use it to comment on the performance in two mocks.

(c) Draw the best line of fit.

27. (a) The events A and B are such that P(A') = 0.3, P(B) = 0.1, P(A/B) =0.2
Find (i) P(AUB)
(ii) P(A/B')

(b). If P(A/B) = 0.2, P(A' ∩ B) = 0.3 and P(B/A′) = 0.4, find
(i) P($A \cap B$)
(ii) P($A' \cup B$)

(c). If A and B are mutually exclusive events such that P(A) = 0.5 and $P(A \cup B) = 0.9$, find (i) $P(A' \cap B')$ (ii) $P(A' \cup B)$

28.a). A committee of 8 people in a village council is to be selected from 7 men and 6 women in how many ways can this be done if not more than 4 women are to be selected.

b). (i) How many arrangements can be made from the word ***MATHEMATICS***

ii). What is the probability of those arrangements when the A s are not together.

PRICE INDICES

29. The table below shows the prices in Ugshs of some food items in 2020, 2021 and 2022 and their corresponding weights.

Items	**Price (shs)**			**Weight**
	2020	**2021**	**2022**	
Matooke	15000	13000	18000	4
Meat	6500	6000	7150	1
Posho	2000	1800	1600	3
Beans	2200	2000	2860	2

Taking 2020 as the base year, calculate;

a) Simple aggregate price index for 2021 and comment on your results.
b) Price index for 2022 for each item.
c) Weighted aggregate price index for 2022
d) Simple price index for 2021.

30. Three items required by a certain family were Food(F), Water(W) and Shelter(S). The monthly expenditure on F, W and S were shillings 150,000, 108,000 and 120,000. If F and W were each twice as important as S. Using Shelter as the base price, determine the cost of living index hence comment.

MOVING AVERAGES

31. The table below shows the number of boxes of pens sold by a certain wholesale shop from the year 2019 to 2022.

Year	**Quarter**			
	1st	**2nd**	**3rd**	**4th**
2019	192	280	320	260
2020	300	360	380	270
2021	342	420	430	320
2022	424	480	510	412

a) Calculate the for-point moving averages for the data.
b) i). On the same axes plot the graphs of original data and the four-point moving averages.

ii). Comment on the number of boxes of pens sold over the four year period.

c) Use your graph to estimate the number of boxes to be sold in the first-quarter of 2023.

CURVE SKETCHING

32. Given the curve $y = 5 + 8x - 4x^2$; Determine,
    i). Turning points and the nature
    ii). Sketch the curve
    iii). Calculate the area bounded by the curve $y = 5 + 8x - 4x^2$ and the line $y = 5$

INEQUALITIES

33. Birungi has a maximum of shs 8000 to spend on bracelets. She will make two types of Bracelets. Type A which costs shs 1000 and type B which costs shs 800 each. Birungi plans to make more Bracelets of type B than type A. Also she wants atleast 2 bracelets of type A and over 6 bracelets of both types. Assuming she makes x bracelets of type A and y bracelets of type B.

    a) Write down four inequalities from the above information.
    b) Represent the inequalities on a graph taking 2cm to represent 1 unit on both axes.
    c) If Birungi gets a profit of shs 300 from each bracelet of type A and shs 200 from each bracelet of type B, find how many bracelets of each type she should make in order to realize maximum profits and state the maximum profit realized by Birungi.

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