①

	Men	Women
Total	4	6
Atleast 2 men in 7.	2	5
	3	4
	4	3

$^4C_2 \cdot {}^6C_5 + {}^4C_3 \cdot {}^6C_4 + {}^4C_4 \cdot {}^6C_3$

$= 116$ ways.

UNEB PURE MTC MARKING GUIDE

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② $V = \pi r^2 h = 1000$

$\pi x^2 h = 1000.$

$h = \frac{1000}{\pi x^2}.$

$A = \pi r^2 + 2\pi rl$

$A = \pi x^2 + 2\pi x \left(\frac{1000}{\pi x^2}\right)$

$A = \pi x^2 + \frac{2000}{x}.$

$\frac{dA}{dx} = 2\pi x - \frac{2000}{x^2} = 0.$

$\pi x^3 = 1000$

$x = 10\pi^{-3}$

③ $4x^2 + 8x + 25y^2 - 100y = -4$

$4(x^2 + 2x) + 25(y^2 - 4y) = -4$

$4[(x+1)^2 - 1] + 25[(y-2)^2 - 4] = -4$

$4(x+1)^2 + 25(y-2)^2 = -4 + 100 + 4$

$\frac{(x+1)^2}{5^2} + \frac{(y-2)^2}{2^2} = 1$

$c(-1, 2)$

$4 = 25(1 - e^2)$

$4 = 25 - 25e^2.$

$e^2 = \frac{21}{25}$

$e = \pm\frac{\sqrt{21}}{5}$

(4) $\frac{1}{9-x2} = \frac{1}{(3-x)(3+x)} \equiv \frac{A}{3+x} + \frac{B}{3-x}$.

$1 = A(3-x) + B(3+x)$

Put, $x = 3$, $B = 1/6$ Put $x = -3$, $B = 2$.

$$1/6 \int \left[\frac{1}{3-x} + \frac{1}{3+x}\right] dx$$

$$1/6 \ln\left(\frac{3+x}{3-x}\right)\Big|_0^1$$

$$1/6 \left[\ln 2 - \ln 0\right] = 1/6 \ln 2$$

(5) $P + (P + 0.0275P) + \text{----}$

$a = P, \quad r = 1.0275$

$S_n = 2P.$

$$2P = \frac{P(1.0275^n - 1)}{1.0275 - 1}$$

$$n = \frac{\ln 1.055}{\ln 1.0275}$$

$n = 1.973586$

$n = 2$ years.

It will take 2 years.

$$\frac{1-\cos 2x + 2\sin x \cos 2x}{1+\cos 2x}$$

$$\frac{2\sin^2 x + 2\sin x \cos 2x}{2\cos 2x}$$

Split the numerator; $\tan^2 x + \sin x$.

⑦ $AC : CB = \lambda : 3$.

$$3\begin{bmatrix} a-1 \\ +2 \\ +2 \end{bmatrix} = \lambda \begin{bmatrix} 6-a \\ 3 \\ 3 \end{bmatrix}$$

$+6 = 3\lambda$

$\lambda = +2$.

$3a - 3 = 12 - 2a$.

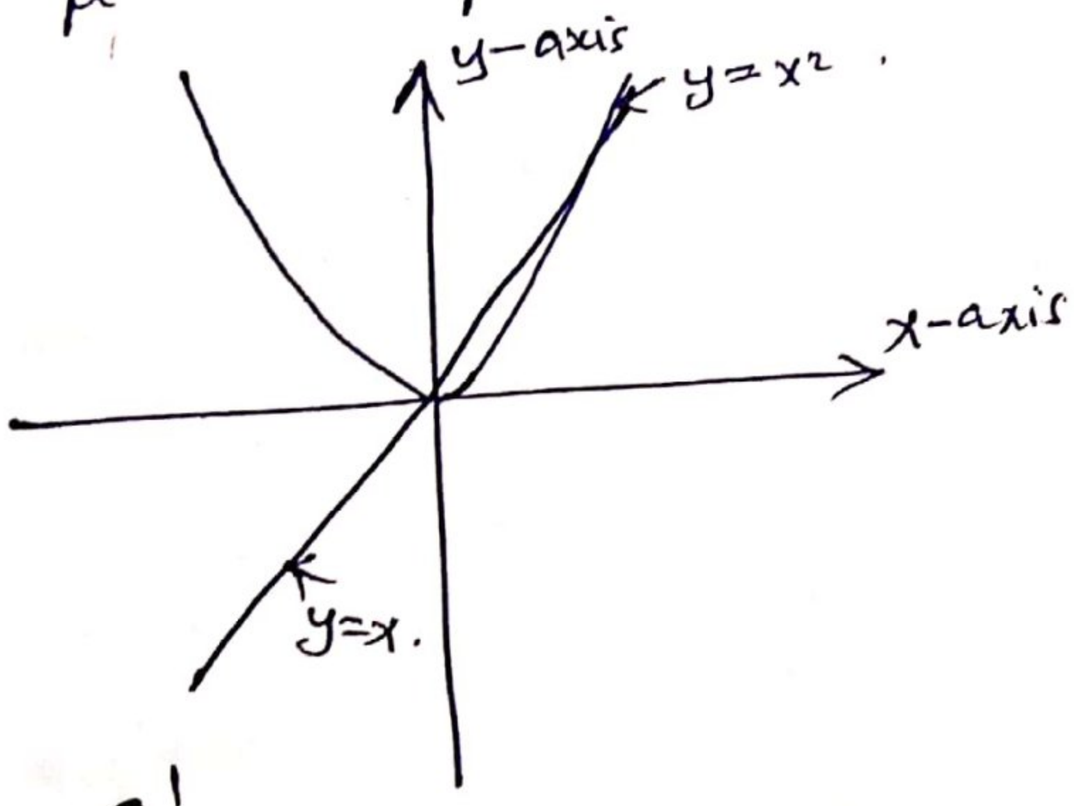
$5a = 15$

$a = 3$.

$3a - 3 = -12 + 2a$

$a = -9$

⑧



$$A = \int_0^1 (x - x^2)\,dx.$$

$$A = \left. \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1$$

$A = 1/6$ units.

(9) $12\cos\theta + 16\sin\theta \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$.

$R\cos\alpha = 12 \qquad R\sin\alpha = 16$.

$R = 20 \qquad \tan\alpha = 16/12$

$\alpha = 53.13°$

$12\cos\theta + 16\sin\theta = R\cos(\theta - \alpha)$

$= 20\cos(\theta - 53.13°)$

for min, $\cos(\theta - 53.13°) = -1$

$= -20$

for max, $\cos(\theta - 53.13°) = 1$

$= 20$

$20\cos(\theta - 53.13°) = 15$.

$\cos \quad \theta - 53.13° = 41.41°$

$\theta = 94.54°$

10 a, $f(x) = x^3 - 13x + p$.

$f(4) = 64 - 52 + p = 0$.

$p = -12$.

$f(x) = x3 - 13x - 12$.

$(x-4)(Ax^2 + Bx + C) \equiv x^3 - 13x - 12$.

$A = 1$, $C = 3$ and $B - 4A = 0$.

$B = 4$.

$x^2 + 4x + 3 = 0$.

$(x+1)(x+3) = 0$

$x = -1$ and $x = -3$.

$3/2 \quad -1/2 = 1/2$.

$1/2 \quad -3/2$.

$= -1/2$.

Alt.

$$x-4 \;\overline{)\; x^3 + 0x^2 - 13x + p \;\;\big|\;\; x^2 + 4x + 3}$$

$x^3 - 4x^2$

$4x^2 - 13x + p$

$4x^2 - 16x$

$3x + p$.

$3x - 12$

$p + 12$.

$p + 12 = 0$

$p = -12$

$x^2 + 4x + 3 = 0$.

$(x+2)^2 = \pm 1$

$x = -2 \pm 1$

$x = -1$ and $x = -3$.

(b), $\frac{x^2-x-18}{x+3} - x/_2 \geq 0$.

$\frac{2x^2-2x-36-x^2-3x}{2(x+3)} \geq 0$.

$\frac{x^2-5x-36}{2(x+3)} \geq 0$.

$\frac{(x+4)(x-9)}{2(x+3)} \geq 0$.

Critical values, $x=-4$, $x=9$ and $x=-3$

$x<-4$, $-4<x<-3$, $-3<x<9$, $x>9$.

$-$ $+$ $-$ $+$

$-4 \leq x < -3$ and $x \geq 9$

11 a), $\sin\theta = x$.

$\cos\theta d\theta = dx$.

x	0	1
θ	0	$\pi/2$.

$$\int_0^{\pi/2} \frac{(1+\sin\theta)\cos\theta\, d\theta}{\cos\theta}$$

$$\int_0^{\pi/2} (\theta + \sin\theta)\, d\theta.$$

$$\theta - \cos\theta \Big|_0^{\pi/2}.$$

$$\left(\pi/2 - \cos\pi/2\right) - \left(0 - \cos 0\right).$$

$$\pi/2 + 1$$

b), $\sin y = \frac{x}{\sqrt{1+x^2}}$

$\cos y \frac{dy}{dx} = \frac{(1+x^2)^{1/2} - x(1/2)(1+x^2)^{-1/2}(2x)}{1+x^2}$.

$\cos y \frac{dy}{dx} = \frac{1+x^2-x^2}{(1+x^2)^{3/2}}$.

$\cos y\, dy = \frac{1}{\sqrt{1+x^2}}$.

$\frac{dy}{dx} = \frac{1}{(1+x^2)^{3/2}} \cdot (1+x^2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

(12) $x = -t^3 + t^2 + 1 \quad y = t^2$. parallel to $3y - 2x - 1 = 0$.

a) $\frac{dx}{dt} = 2t - 3t^2 \quad \frac{dy}{dt} = 2t$.

$3y - 2x - 1 = 0$ in $y = mx + c$.

$\frac{dy}{dx} = \frac{2}{2 - 3t}$

$y = 2/3 x + 1/3$.

$m = 2/3$.

$2 = 2/3 (2 - 3t)$.

$6 = 4 - 6t$.

$6t = -2$.

$t = -1/3$

B $\left(\frac{21}{27}, 1/9\right)$

$x = \frac{31}{27}$ and $y = t^2$.

$y = 1/9$

$\frac{y - 31/27}{x - 1/9} = +2/3$.

$3y - \frac{21}{9} = +2x + 2/9$

$y = 2/3 x + 33/9$

$y = 2/3 x + 11/3$.

(13) a) $f(x) = \ln(1 - 2x)$.

$f(0) = \ln 1 = 0$

$f'(x) = \frac{-2}{1 - 2x}$.

$f'(0) = -2$.

$f''(x) = \frac{(-2)(-1)(-2)}{(1 - 2x)^2}$.

$f''(0) = -4$

$f'''(x) = \frac{-4(-2)(-2)}{(1 - 2x)^3}$

$f'''(0) = -16$

$\ln(1 - 2x) = -2x - \frac{4x^2}{2!} - \frac{16x^3}{3!}$

$= -2x - 2x^2 - 8/3 x^3$.

b) $y = \tan x$

$y + \delta y = \tan(x + \delta x)$

$x + \delta x = 45° + 1°$.

$x = 45°, \quad \delta x = 1°$.

$y = \tan 45, \quad y = 1$

$\frac{dy}{dx} = \sec^2 x$

$\delta y \simeq (\sec^2 x)\left(\frac{1 \cdot \pi}{180}\right)$

$\delta y = \frac{2\pi}{180} \simeq 0.034907$.

$\tan 46° = 1 + 0.034907$

$\simeq 1.034907$

14 3|z−2| = |z−6i|

Let z = x+yi

3 |(x−2)+yi| = |x+(y−6)i|

9 [x²−4x+4+y²] = x² + y² − 12y + 36.

$\frac{8}{8}x^2 + \frac{8}{8}y^2 - 4x + 12y = 0.$

$x^2 + y^2 - \frac{1}{2}x + \frac{3}{2}y = 0.$

b, Let the square root be a+bi

$a^2 - b^2 + 2abi = -5 + 12i$

$a^2 - b^2 = -5.$

$2ab = 12.$

$b = \frac{6}{a}.$

$a^2 - \frac{36}{a^2} = -5.$

Let $a^2 \equiv y.$

$y^2 + 5y - 36 = 0.$

$(y+9)(y-4) = 0.$

$y = -9$ and $y = 4.$

$a = \pm 2$ when $a = 2.$

$b = 3$

when $a = -2$

$b = -3$

Square root, $2+3i$.

and $-2+3i$

... dal ... in-M.

15) P (0, 2, 5) and Q (-1, 3, 1)

$$\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$$

$$d = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

a) n.r = n.a.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

$$2x - 2y - z = -9.$$

b) Q (-1, 3, 1)

$2(-1) - 2(3) - 1 = -9$; it lies on the plane.

$x = 3 + 2\lambda$ $y = 2 - 2\lambda$ and $z = 2 - \lambda$

$$2(3+2\lambda) - 2(2-2\lambda) - (2-\lambda) = -9.$$

$$6 + 4\lambda - 4 + 4\lambda - 2 + \lambda = -9$$

$$9\lambda = -9$$

$$\lambda = -1$$

$x = 1$ and $y = 4$ and $z = 3$.

R (1, 4, 3)

PR . QR = 0.

$$PR = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$QR = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} = \begin{matrix} 2 \\ 1 \\ 2. \end{matrix}$$

$$PR \cdot QP = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 2 + 2 - 4 = 0.$$

6

Let $\frac{dM}{dt} \propto 10 - M$.

$$\int \frac{dM}{10-M} = \int k\,dt$$

$$-\ln(10-M) = kt + c$$

at $t = 0$, M_0.

$c = -\ln 10$

$-\ln(10-M) = kt - \ln 10$

$\ln \frac{10}{10-M} = kt$

$\frac{10}{10-M} = e^{kt}$

$\frac{10}{10-M} = e^{t \ln 1.25}$

$10 = 10e^{t \ln 5/4} - Me^{t \ln 5/4}$

$Me^{t \ln 5/4} = 10(e^{t \ln 5/4} - 1)$

$M = 10\left(1 - e^{-t \ln 5/4}\right)$

$M = 10\left(1 - e^{t \ln 4/5}\right)$

when $t = 5$.

$-\ln(10-M) = 5 \ln 5/4 + \ln k$

$M = 10\left(1 - e^{5 \ln 4/5}\right)$.

$M = 6.7232$ tomes.

when $t = 1$, $M = 2$.

$-\ln 8 = k - \ln 10$.

$k = \ln 5/4$