N.S.S.S
END OF TERM ONE EXAMS 2013
S6 MATHEMATICS PAPER 1(P425/1)
PURE MATHEMATICS
TIME: 3 hours

Instructions:
*Attempt **all** questions in Section **A** and any **five** from Section **B**.*

SECTION A

1. Solve for the real value of x in the equation $16^x - 4^x = 12(4^x + 2^x)$. (*05 Marks*)

2. Calculate the angle between the vectors $3\hat{i} + 2\hat{j} + 5k$ and $2\hat{i} + 5\hat{j} - 3k$. (*05 Marks*)

3. Given that $x^2 + y^2 + 4x - 6y = -6$, show that $\frac{d^2y}{dx^2} = \frac{-7}{(y-3)^2}$. (*05 Marks*)

4. In a triangle ABC, $\overline{AB}$= 8cm, $\overline{BC}$ = 5cm and $\overline{AC}$ = 7cm. Prove that the height of the triangle through vertex C has length $4\sqrt{3}$. (*05 Marks*)

5. Solve the equation; $Cosec\theta + Cot\theta = 1$ for $0 < \theta < 2\pi$. (*05 Marks*)

6. Find $\int \frac{dx}{Secx - 1}$. (*05 Marks*)

7. Express $\frac{8-6i}{3+4i}$ in modulus-argument form. (*05 Marks*)

8. Use small changes to estimate the cube root of 7.97. (*05 Marks*)

SECTION B (*60 Marks*)

9. (a) Expand $\sqrt{\frac{1+4x}{1-4x}}$ up to the term in x^3.

   (b) (i) Use the expansion in (a) above to evaluate $\left(\frac{1.4}{0.6}\right)^{\frac{1}{2}}$ to 4dps hence evaluate $\sqrt{21}$.

   (ii) Taking x = $\frac{1}{16}$ in the expansion in (a), estimate $\sqrt{15}$ to 4dp/s. (*12 Marks*)

10. (a) Calculate the area bounded by the curve y = x(2 – x) and the x-axis. (*05 Marks*)

(b) The area in (a) above is rotated about the y-axis through 360°. Find the volume of the solid generated. (*07 Marks*)

11. (a) Prove the identity; $\tan(\alpha + \beta) - \tan(\alpha + \beta) = \frac{2Sins\beta}{Cos2\alpha + Cos2\beta}$. (*05 Marks*)

(b) Express R = $\sqrt{3}Sin\theta - Cos\theta$ in the form λSin(θ – x); hence solve the equation 3(Sinθ – 1) = Cosθ; for $0^o \leq \theta \leq 360^o$. (*07 Marks*)

12. The line passing through the points A(1, 1, 3) and B(2, -1, 5) meets the plane $r.\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} =$ 19 in point C. Find;

(i) the equation of the line AB (*02 Marks*)
(ii) the coordinates of point C (*04 Marks*)
(iii) the angle between the line AB and the plane and also determine the equation of the projection of the line AB on the plane. (*06 Marks*)

13. (a) Solve the equation $z^3 - 2z^2 + 4z = 0$. (*05 Marks*)

(b) Given z = x + îy, find and sketch the locus |z – 3 - 4î| = 3 hence state the coordinates of the point representing z when |z| is maximum. (*07 Marks*)

14. (a) Differentiate with respect to x;

(i) $5x^3$ (*03 Marks*)
(ii) $xSin(In\sqrt{x})$ (*03 Marks*)

(b) A curve whose second derivative $\frac{d^2y}{dx^2} = 6x + c$ has a point of inflection at (1, 2). Find the value of the constant c and the equation of the curve. (*06 Marks*)

15. (a) Solve the differential equation $x\frac{dy}{dx} = y + \tan\left(\frac{y}{x}\right)$, using the substitution y = ux. (*05 Marks*)

(b) At a black spot, the number of accidents decrease with the number of policemen deployed, at a rate proportional to the number of policemen deployed along the stretch containing the spot. The weekly number of accidents decreases from 7 to 1 when the number of policemen increases from 2 to 4. Find the expected number of;

(i) accidents in a week when there is no police deployment.
(ii) policemen men required to stump out accidents from this spot completely. (*07 Marks*)

16. Given the curve $y = \frac{(x+3)(x-4)}{(x-1)(x-2)}$;

(i) Find the equations of the three asymptotes, stating the coordinates of the point where the curve crosses one of the asymptotes.
(ii) State the coordinates of the x and y intercept.
(iii) Sketch the curve. (*12 Marks*)

END