"PROPOSED" MARKING GUIDE

WAKISSHA MOCK EXAMINATIONS

P425/2 APPLIED MATHEMATICS 2024

SECTION A

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No 1

$P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{1}{3}$, $P(B) = \frac{1}{4}$

a)

$P(A \text{ or } B \text{ but not both } A \text{ and } B) = P(A \cup B) - P(A \cap B)$

$$\frac{1}{3} = \frac{1}{2} + \frac{1}{4} - 2P(A \cap B)$$

$$P(A \cap B) = \frac{5}{24}$$

You can use B1, M1

$$P(B'/A) = \frac{P(B' \cap A)}{P(A)}$$

05

Be mindful, fair and demanding while marking

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$\frac{1}{2} - \frac{5}{24} = \frac{12-5}{24} = \frac{7}{24}$$

$$P(B'/A) = \frac{7}{24} \times \frac{2}{1} = \frac{7}{12}$$

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No 2

(a)

A	B	C
0.35	0.4	m
0.50	0.65	0.8

$$\frac{0.4-0.35}{0.65-0.50} = \frac{m-0.35}{0.8-0.50}$$

$m = 0.45 \times 100\%$
$= 45\%$

b)

0.35	0.37	0.4
0.50	H	0.65

$$\frac{H-0.50}{0.37-0.35} = \frac{0.65-0.50}{0.4-0.35}$$

$H = 0.56$

$0.56 \times 100\%$
$= 56\%$

05

No 3

90m 240m
2s 4s

$S = ut + \frac{1}{2}at^2$

$S_1 = 90m, t_1 = 2s$
$S_2 = 330m, t_2 = 6s$
$90 = 2u + \frac{1}{2}a(4)$
$90 = 2u + 2a$
$45 = u + a$ — (I)

$330 = 6u + 18a$
$55 = u + 3a$ — (II)

(II) − (I) 05

$10 = 2a$
$a = 5ms^{-2}$
$u = 40ms^{-1}$

4

$$P(Y=y)=\begin{cases} ky^2 , & y=1,2,3 \\ k(y+1), & y=4,5 \\ 0 & , \text{otherwise} \end{cases}$$

Y	P(Y=y)	Y P(Y=y)
1	K	K
2	4K	8K
3	9K	27K
4	5K	20K
5	6K	30K
	25K	86K

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4(a)

$\sum P(Y=y) = 1$

$k + 4k + 9k + 5k + 6k = 1$

$25k = 1$

$k = \frac{1}{25}$

4b)

mean of y

$E(y) = \sum y P(y=y)$

$= 86k = 86 \times \frac{1}{25}$

$E(y) = \underline{\underline{3.44}}$

05

4C)

$P(2 < y \leq 4) = P(y=3) + P(y=4)$

$= 9 \times \frac{1}{25} + 5 \times \frac{1}{25}$

$0.36 + 0.2$

$= 0.56$

No 5

$P = \frac{kT}{V}$

$P + e_p = k\frac{T + e_1}{V + e_2}$

$P + e_p = \frac{k(T + e_1)}{(V + e_2)} \frac{(V - e_2)}{(V - e_2)}$

$P + e_p = \frac{k(TV - Te_2 + Ve_1 - e_1 e_2)}{V^2 - e_2^2}$

$P + e_p = \frac{kTV - kTe_2 + kVe_1}{V^2}$

$e_p = \frac{kTV}{V^2} - \frac{kTe_2}{V^2} + \frac{kVe_1}{V^2} - \frac{kT}{V}$

$e_p = \frac{kVe_1}{V^2} - \frac{kTe_2}{V^2}$

But $k = \frac{PV}{T}$

05

$e_p = \frac{Pe_1}{T} - \frac{Pe_2}{V}$

$\frac{e_p}{P} = \frac{e_1}{T} - \frac{e_2}{V}$

$\frac{e_p}{P} = \left|\frac{e_1}{T} - \frac{e_2}{V}\right| \leq \left|\frac{e_1}{T}\right| + \left|\frac{-e_2}{V}\right|$

$\therefore$ maximum possible relative error $= \left|\frac{e_1}{T}\right| + \left|\frac{e_2}{V}\right|$.

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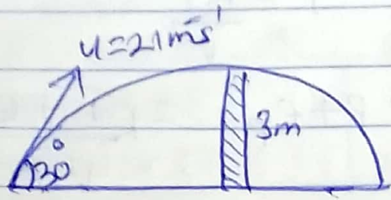
No. 6



A ball reached a maximum height.

Time of flight $= \frac{2u\sin\theta}{g}$

$= \frac{2 \times 21 \sin 30}{9.8}$

$= 2.14s$

Time taken to reach maximum height.

$t = \frac{u\sin\theta}{g} = \underline{\underline{1.07s}}$

05

$\rightarrow \quad x = u\cos\theta t$

$x = 21\cos 30 \times 1.07$

$x = 19.46m$

$V = \sqrt{V_x^2 + V_y^2}$

$V = \sqrt{(21\cos 30)^2 + (21\sin 30 - 9.8 \times 1.07)^2}$

$V = \sqrt{330.75 + 1.96 \times 10^{-4}}$

$\underline{\underline{V = 18.49ms^{-1}}}$

No. 7

a)

Item	2000	2004	weight	WP_0	WP_1
Food	100	125	35	3500	4375
Water	100	121	11	1100	1331
Rent	100	112	8	800	896
Electricity	100	108	6	600	648
Transport	100	118	22	2200	2596
			82	8200	9846

Cost of living index $= \frac{9846}{82} = \underline{\underline{120.07}}$

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b)

Cost of cloth in 2004

$120.07 = \frac{2004}{11200} \times 100$

$2004 = 13447.84 \approx$

49x 13448

25

40/40

No 8.

$V^2 = \omega^2(A^2 - x^2)$

4 free

25

$V = 3\sqrt{3}ms^{-1}$, $x = 1m$

$V = 3ms^{-1}$, $x = 0.268m$

$(3\sqrt{3})^2 = \omega^2(A^2 - 1^2)$

$27 = \omega^2(A^2 - 1)$ —— (i)

$3^2 = \omega^2(A^2 - 0.268^2)$

$9 = \omega^2(A^2 - (0.268)^2)$ —— (ii)

(i) ÷ (ii)

$\frac{27}{9} = \frac{A^2 - 1}{A^2 - (0.268)^2}$

Gives a complex ??
How ??

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SECTION B

9)

Class limits	f	x	fx	c.f	c.b

a)

(i)

mean time $= \frac{\Sigma fx}{\Sigma f} = 96.6$ minutes

(ii)

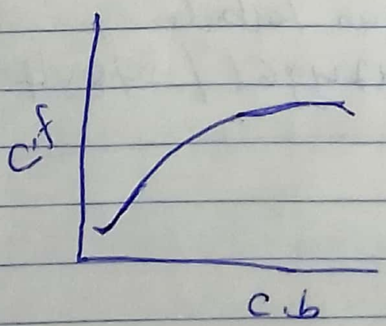
median $= (96-97)$ minutes

$$\left[L_1 + \left(\frac{N/2 - cfb}{fm}\right) i\right]$$

Graph is

B)

i)



(ii)

(118 - 122) times

12

HO 10

9)

$$\frac{20m}{3(8-\pi)}$$

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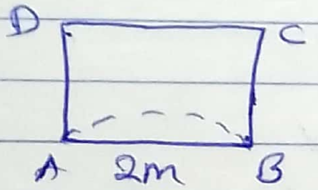


Lamina	Area	Weight	Distance of C.O.G from AB
ABCD	$4m^2$	$4m^2w$	m
Semi Circle	$\frac{1}{2}\pi m^2$	$\frac{1}{2}\pi m^2 w$	$\frac{4m}{3\pi}$
Remainder	$4-\frac{1}{2}\pi m^2$	$(4-\frac{1}{2}\pi)m^2w$	$\bar{y}$

C.O.G from AB: $\left(4-\frac{\pi}{2}\right)m^2\bar{y} = 4m^2 \times m - \frac{\pi}{2}m^2 \times \frac{4m}{3\pi}$

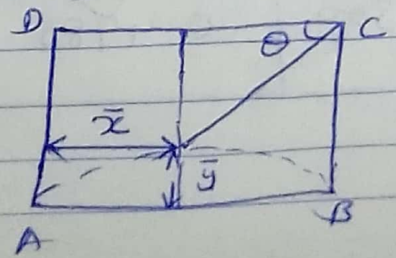
$$\left(\frac{8-\pi}{2}\right)\bar{y} = 4m - \frac{2m}{3}$$

$$\left(\frac{8-\pi}{2}\right)\bar{y} = \frac{10m}{3}$$

$$y = \frac{20m}{3(8-\pi)} \quad \text{---}\#$$

10b)

C.O.G from AB



C.O.G from CD $= 2m - \frac{20m}{3(8-\pi)}$

$$\frac{48m - 6m\pi - 20m}{3(8-\pi)}$$

$$= \frac{28m - 6m\pi}{3(8-\pi)}$$

$$\theta = \tan^{-1}\left(\frac{28m - 6m\pi}{3(8-\pi)}\right) / m$$

$$= \frac{28 - 6\pi}{3(8-\pi)} = \frac{2(14-3\pi)}{3(8-\pi)} \text{ —\#}$$

12

No 11

$$\int_1^2 \left(\frac{x}{7x^2-3}\right) dx \quad , 6 \text{ ordinates}$$

$n-1$

$6-1 = 5$

x	y	
1.0	0.2500	
1.2		0.1695
1.4		0.1306
1.6		0.1072
1.8		0.0946
2.0	0.08000	
	0.3300	0.4988

$$d = \frac{2-1}{5}$$

$$d = 0.2$$

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$$\int_1^2 \left(\frac{x}{7x^2-3}\right) dx = \frac{1}{2}d\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$\frac{1}{2}\times 0.2(0.3300+2(0.4986))$$

$$\approx 0.1328$$

b i)

$$\int_1^2 \left(\frac{x}{7x^2-3}\right) dx \quad , \text{let } u = 7x^2-3$$

$$du = 14x\,dx$$

$$dx = \frac{du}{14x}$$

$$\int_1^2 \left(\frac{x}{7x^2-3}\right) dx = \int_1^2 \frac{x}{u} \cdot \frac{du}{14x} = \frac{1}{14}\int_1^2 \frac{1}{u} du$$

$$= \frac{1}{14}\left[\ln u\right]_1^2 = \frac{1}{14}\left[\ln(7x^2-3)\right]$$

$$= \frac{1}{14}\left[\ln 25 - \ln 4\right]$$

$$= \frac{1}{14}\left[\ln\left(\frac{25}{4}\right)\right]$$

$$= 0.130899 \approx \underline{\underline{0.1309}}$$

12

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b ii)

$$\text{Percentage error} = \frac{\text{Inexact} - \text{exact}}{\text{exact}} \times 100\%$$

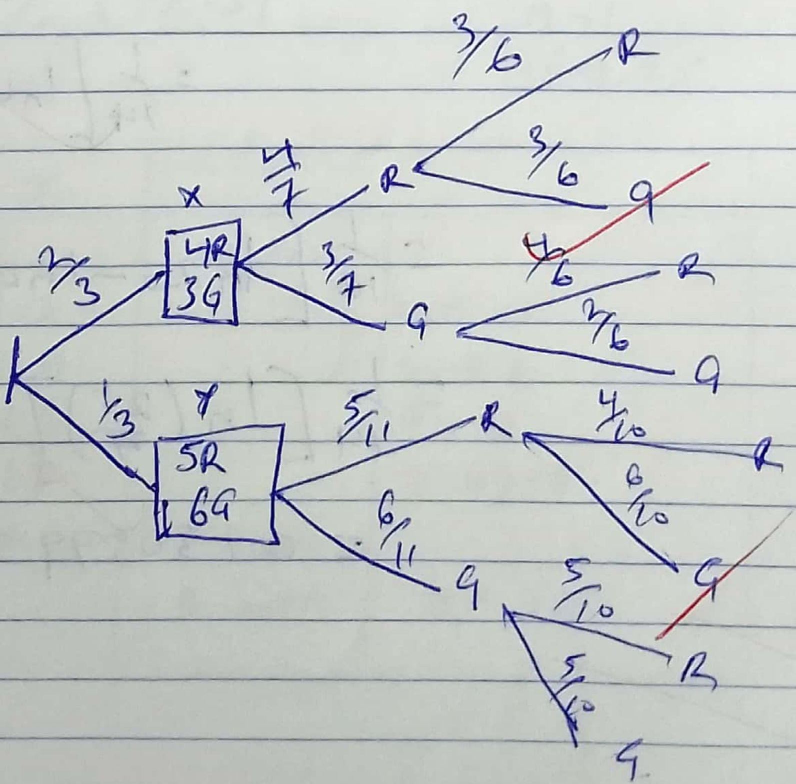
$$\frac{0.1328 - 0.1309}{0.1309} \times 100\%$$

$$= 1.45\%$$

b iii)

- Increasing the number of sub intervals can reduce the error
- Increasing number of dps.

No 12 a)



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P (same colour) = P(X) × P(same colour from X) + P(Y) P(same colour from Y)

$$= \frac{2}{3} \times \left(\frac{4}{7} \times \frac{3}{6} + \frac{3}{7} \times \frac{2}{6} \right) + \frac{1}{3} \left(\frac{5}{11} \times \frac{4}{10} + \frac{6}{11} \times \frac{5}{10} \right)$$

$$= \frac{3}{10}$$

12b)

Number of red buttons	probability
0	$\frac{2}{3} \times \frac{3}{7} \times \frac{2}{6} + \frac{1}{3} \times \frac{6}{11} \times \frac{5}{10} = \frac{22}{165}$
1	$2\left(\frac{4}{7} \times \frac{3}{6}\right) \times \frac{2}{3} + 2\left(\frac{5}{11} \times \frac{6}{10}\right) \times \frac{1}{3} = \frac{44}{165}$
2	$\frac{2}{3} \times \frac{4}{7} \times \frac{3}{6} + \frac{1}{3} \times \frac{5}{11} \times \frac{4}{10} = \frac{23}{165}$

12c)

$$E(x) = 0P(0) + 1P(1) + 2P(2)$$

$$= \frac{0 \times 22}{165} + \frac{1 \times 44}{165} + \frac{2 \times 23}{165}$$

$$E(x) = \frac{6}{11}$$

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Pb. 13

$a = (4e^{-3t}i + 12\sin t j - 7\cos t k)\, ms^{-2}$

at point $(5, -6, 2)$ and $v = (11i - 8j + 3k)\, ms^{-1}$

13a)

$|a| = \sqrt{(4e^{-3t})^2 + (12\sin t)^2 + (-7\cos t)^2}$

at $t = 0$

$|a| = \sqrt{4^2 + 0 + 7^2} = \sqrt{65}\, ms^{-2}$

13b)

V at time t

$a = \frac{dv}{dt}$, $\int dv = \int a\, dt$

$V = \int (4e^{-3t}i + 12\sin t j - 7\cos t k)\, dt$

$V = -\frac{4}{3}e^{-3t}i - 12\cos t j - 7\sin t k + C$

at $t = 0$, $v = 11i - 8j + 3k$

$$\begin{pmatrix} 11 \\ -8 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3} \\ -12 \\ 0 \end{pmatrix} + C$$

$$C = \begin{pmatrix} \frac{37}{3} \\ 4 \\ 3 \end{pmatrix}$$

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$$V(t=t) = \begin{pmatrix} -\frac{4}{3}e^{-3t} - \frac{37}{3} \\ -12\cos t - 4 \\ -7\sin t - 3 \end{pmatrix} ms^{-1}$$

c)

r at $t=1$

$$V = \frac{dr}{dt}, \quad r = \int v\,dt$$

$$r = \int \begin{pmatrix} -\frac{4}{3}e^{-3t} - \frac{37}{3} \\ -12\cos t - 4 \\ -7\sin t - 3 \end{pmatrix} dt$$

$$r = \begin{pmatrix} \frac{4}{9}e^{-3t} - \frac{37}{3}t \\ -12\sin t - 4t \\ 7\cos t - 3t \end{pmatrix} + C$$

at $t=0$, $r = (5, -6, 2)$

$$\begin{pmatrix} 5 \\ -6 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{4}{9} \\ 0 \\ 7 \end{pmatrix} + C$$

$$C = \begin{pmatrix} \frac{41}{9} \\ -6 \\ -5 \end{pmatrix}$$

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$$r(t=t) = \begin{pmatrix} \frac{4}{9}e^{-3t} - \frac{37t}{3} + \frac{49}{9} \\ -12\sin t - 4t - 6 \\ 7\cos t - 3t - 8 \end{pmatrix} m$$

12

No 14)

$e^x + x - 4 = 0$

$f(x) = e^x + x - 4$

x	-2	-1	0	1	2
$f(x)$	-5.9	-4.6	-3.0	-0.28	5.4

bi)

$f(x) = e^x + x - 4$

$f'(x) = e^x + 1$

$f(x_n) = e^{x_n} + x_n - 4$

$f'(x_n) = e^{x_n} + 1$

$$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}$$

$$= \frac{X_n}{1} - \frac{(e^{x_n} + x_n - 4)}{(e^{x_n} + 1)}$$

$$= \frac{x_n(e^{x_n}+1) - e^{x_n} - x_n + 4}{e^{x_n}+1}$$

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 4}{e^{x_n}+1}, \quad n = 0, 1, 2, 3$$

b(i)

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + 4}{e^{x_n}+1}$$

$x_0 =$ ______ from the graph

$x_1 =$

$x_2 =$

$x_3 =$

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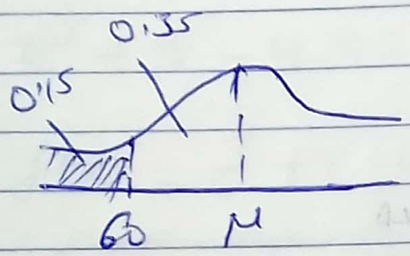
No 15

let X be a random variable of number of goats owned

$X \sim N(\mu, \sigma)$

$P(X < 60) = 0.15$

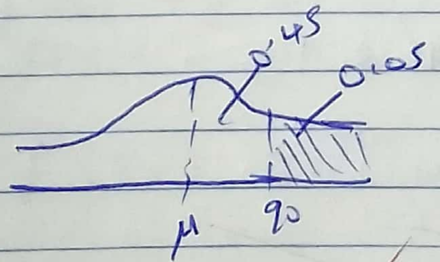


$p = 0.35$, $q = 0.15$

From tables critical points

$Z_1 = -1.036$

$P(X > 90) = 0.05$



$p = 0.45$, $q = 0.05$

Critical points (table)

$Z_2 = 1.645$

From $Z_1 = \dfrac{x - \mu}{\sigma}$

$-1.036\sigma = 60 - \mu$

$-1.036\sigma + \mu = 60$ — (i)

From $Z_2 = \dfrac{X - \mu}{\sigma}$

$1.645\sigma + \mu = 90$ — (ii)

(ii) − (i)

$2.681\sigma = 30$

$\sigma = 11.1899 \approx 11$

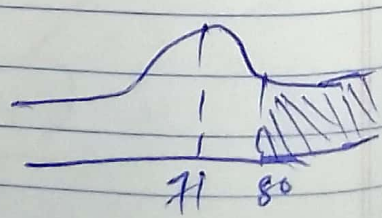
Substituting into (i)

$\mu = 71.398 \approx 71$

$\mu = 71$

b)

$P(X > 80)$



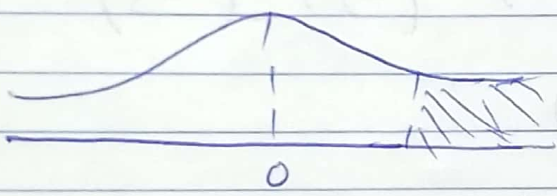
Standardise.

$$Z = \frac{x-\mu}{\sigma}$$

$$Z = \frac{80-71}{11} = 0.818$$

$P(Z > 0.818)$

Standardised curve.



$P(Z > 0.818) = 0.5 - \phi\ 0.818$

$0.5 - (0.29333)$ cal

$= 0.2067$ (4dps).

$E(x) = np$

300×0.2067

$= 62$ residents had more than 80 goats.

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No 16

$(-2\underset{\sim}{i}+3\underset{\sim}{j})N$, $(-\underset{\sim}{i}+2\underset{\sim}{j})N$, $(4\underset{\sim}{i}-2\underset{\sim}{j})N$ and $(-\underset{\sim}{i}-3\underset{\sim}{j})N$. points $A(-2,3)$, $B(3,1)$, $C(-1,-3)$ and $D(3,1)$

(i)
couple.

$$R=\begin{pmatrix}-2\\3\end{pmatrix}+\begin{pmatrix}-1\\2\end{pmatrix}+\begin{pmatrix}4\\-2\end{pmatrix}+\begin{pmatrix}-1\\-3\end{pmatrix}$$

$$R=\begin{pmatrix}0\\0\end{pmatrix}$$

$$G=\begin{vmatrix}-2 & -2\\3 & 3\end{vmatrix}+\begin{vmatrix}3 & -1\\1 & 2\end{vmatrix}+\begin{vmatrix}-1 & 4\\-3 & -2\end{vmatrix}+\begin{vmatrix}3 & -1\\1 & -3\end{vmatrix}$$

$$G=0+7+14-8$$

$$G=13Nm$$

Since the resultant force is zero and $G\neq0$, then the forces reduce to a couple.

b)

$$G=\begin{vmatrix}x & F_x\\y & F_y\end{vmatrix}$$

$G=xF_y+yF_x=0$

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Resultant force $= \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}4\\-2\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \begin{pmatrix}3\\4\end{pmatrix}$

Since forces are concurrent (act at the same point) point of application is $(3,1)$

$$m = \frac{F_y}{f_x} = \frac{4}{3}$$

$$m = \frac{y-y_1}{x-x_1}$$

$$\frac{4}{3} = \frac{y-1}{x-3}$$

$$y-1 = \left(\frac{4}{3}\right)(x-3)$$

∴ Equation of line of action

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For x-intercept $y=0$

$$y-1 = \left(\frac{4}{3}\right)(x-3)$$

$$0-1 = \left(\frac{4}{3}\right)(x-3)$$

$$-\frac{3}{4} = x-3$$

$$x = \frac{9}{4}$$

∴ line crosses the x-axis at $x = \frac{9}{4}$

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