INTERSECONDARY SCHOOLS EXAMINATION SERIES
ISESE
FORM SIX SERIES 1
ADVANCED MATHEMATICS 1

142/1

Time: 3 Hours **Saturday 6th July 2024**

INSTRUCTIONS

1. This paper consists of **ten (10)** questions

2. Answer **ALL** questions.

3. Each question carries **ten (10)** marks

4. All necessary working and answers of each question must be shown clearly

5. Mathematical tables and non-programmable calculators may be used

6. Cellular phones and any other unauthorized materials are not allowed in the examination room.

1.(a)By using a non-programmable calculator, evaluate the following and give your answer to 4 decimal places.

(i) $\sqrt[3]{\frac{(41.67)^8 \text{ x } (34.35)^3}{(2.351)^4}}$

(ii) $\sqrt[5]{\frac{(46.92)^6 \text{ x } (\sin 47.72)^3}{(\cosh 2.82)^2 \text{ x } (164.8)^{1/2}}}$

(b) By using a non-programmable calculator, determine the determinant and inverse of the following matrix $\begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{bmatrix}$

2.(a) Draw graph of $\cot \text{hx}$ and $\sec \text{hx}$ on separate axes

(b) Solve for x given that $\text{Tanh}^{-1}\left({}^{X}\!/_{3}\right) + \text{Tanh}^{-1}(x) = \text{Tanh}^{-1}\left({}^{1}\!/_{4}\right)$

(c) Show that $\frac{\text{d}}{\text{dx}}\{\coth^{-1}(\sin \text{x})\} = \sec \text{x}$

3. A Company produces two types of Leather belts, A and B. A is of supentr quality and B of lower quality. Profits on the two types of belts are 40 and 30 percent per belt respectively. Each belt of type A requires twice as much time as required by belt of type B. If all belts were of type B, the Company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day are available. Belt A requires a fancy buckle and only400 fancy buckles are available for this per day. For belt of type B, only 700 buckles are available per day. How should the Company manufacture the two types of belts in order to have a maximum Profit?

4. Consider the following distribution of marks students in a certain mathematics test.

Class Interval	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	6	12	14	16	8	6	6

(i) Find the mean and mode of the distribution
(ii) Draw a Cumulative frequency Curve and use it to estimate median
(iii) Calculate the semi inter-quartile range

5.(a) Using laws of algebra of sets, Simplify each of the following expressions.

(i) $[A \cap (A' \cup B)] \cup [B \cap (A' \cap B')]$

(ii) $(\text{A} \cap \text{B}') \cup (\text{A}' \cup \text{B}')$

(b) Sets A and B are defined as

$A = \{x: x \in R; 3 < x \leq 6\}$and $B = \{x: x \in R; -4 < X \leq 4\}$. Find the following using a number line.

(i) $A \cap B$

(ii) $A' \cap (A \cup B)$

(iii) $A - B$

6. (a) (i) Prove that $f(x) = \sqrt{2x-4}, x \geq 22$ and $g(x) = \frac{x^2+4}{2}, x \geq 0$ are inverse functions of one another.

(b) Sketch the graph of F(x) and g(x) on the same coordinate plane.

(b) A function f(x) is defined as follows

$$f(x) = \begin{cases} x^2 \text{ if } x < 0 \\ 3x \text{ if } 0 \leq x < 2 \\ 6 \text{ if } x \geq 2 \end{cases}$$

(i) Sketch the graph of F(x)

(ii) State domain and range of F(x)

(iii) Find the value of $fofof(-1)$

7.(a)(i) show that the iteration formula.$X_{n+1} = {}^1/_2 \left[X_n + \frac{A}{Xn}\right]$ where A is a positive number can be used to determine an approximation for the square root of A

(ii) The equation X^3 -3x-20 = 0 has a single real root in the interval [3, 4]. Approximate the root in four iterations using the secant method.

(b)Apply Simpson's rule with n=4 need to obtain an approximation for the integral $I = \int_0^1 \frac{1}{1+X} dx$. Give your answer to three decimal places.

8. (a) Find the locus of a point P(x,y) which moves such that its distance from (-1/2) is twice its distance from the origin.

(b) Derive the condition for the points A (X_1, Y_1), B(X_2, Y_2) and (X_3, Y_3) to be Collinear.

(c)(i) Translate the line Y=3X+1 by a factor of (2,3)

(ii) Dilate y= 3x-1 by a factor of (2,5)

9.(a) Evaluate the following integrals.

(i) $\int \frac{\ln x}{x\sqrt{1-4\ln x - \ln^2 x}}$

(ii) $\int \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}$

(b)Given two Curves represented by y = 2 - X^2 and $y^3 = x^2$.

(i) Sketch to show the region S enclosed by the Curves

(ii) Evaluate the area S enclosed between the Curves.

10. (a) Find the derivative of the following

(i) $y = \ln(\sec x + \tan x)$
(ii) $y = \sinh^{-1}(\tan x)$

(b) By using first principle of differentiation, differentiate $y = \frac{1}{x+1}$

(c) By using Maclaurin's theorem expand $\tan^{-1} x$ in ascending powers of x as far as the term in x^7. Use the resulting expression to evaluate $\tan^{-1} 0.1$. Correct to four decimal places.