MEBU EXAMINATIONS CONSULT

UGANDA ADVANCED CERTIFICATE OF EDUCATION

MATHEMATICS SEMINAR HELD ON 21ST-SEPTEMBER, 2024 AT KABEI SENIOR SECONDARY SCHOOL-BUKWO DISTRICT, SEBEI SUB REGION

PAPER 1

PURE MATHEMATICS

SECTON A (40 MARKS)

1. By row reduction to the echelon form, solve the simultaneous equations:

X-2y-2z =0
z + 3y = 1 -2x
3x = 3 + y + 3z

2. Show that $\frac{2\tan\Theta}{1+\tan^2\Theta} = \text{Sin } 2\Theta$

3. Given $x + 2x^3 + 4x^5 + 8x^7 + \dots = \frac{3}{7}$

i) Find the value of x. (03 marks)

ii) Find the 20th term. (02 marks)

4. If a line y= mx +c is a tangent to the curve $4x^2 + 3y^2 = 12$, show that $C^2 = 4 + 3m^2$

5. Solve the inequality $\frac{7-2x}{(x+1)(x-2)} > 0$

6. Solve $\tan^{-1}(x) + \tan^{-1}(x-1) = \tan^{-1}(3)$

7. Show that the area bounded by the curve $y = 10x - x^2$ and the line y = 4x is 36 square units.

8. The directional vectors $\boldsymbol{a} = 3\boldsymbol{i} + 2\boldsymbol{j} +$, $\boldsymbol{b} = -2\boldsymbol{i} + \boldsymbol{j} + p\boldsymbol{k}$ and $\boldsymbol{c} = 9\boldsymbol{i} + 9\boldsymbol{j}$ are such that $\boldsymbol{a}$ is perpendicular to $\boldsymbol{b}$.

Find the

i) Value of the scalar $\boldsymbol{p}$.

ii) Angle between $\boldsymbol{b}$ and $\boldsymbol{c}$

SECTION B (60 MARKS)

9. Given the curve

$$y = \frac{3x+3}{x(3-x)}$$

(a) Find the region where the curve does not lie, hence determine the turning points and their nature.

(b) State the asymptotes and find the intercepts.

(c) Sketch the curve.

10.(a) Prove that $\frac{\sin 5x - \sin 7x + \sin 8x - \sin 4x}{\cos 4x - \cos 5x - \cos 8x + \cos 7x} = \cot 6x$

(b) (i) Express $7\cos x - 24\sin x$ in the form $R\cos(x+a)$.

(ii) Write down the maximum and minimum value of the function f(x) = 12 + 7 cosx – 24 sin x

11.(a) Prove that $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$ for all positive values of n

(b) Given that $Z_1 = r_1(\cos A + i\sin A)$ and $Z_2 = r_2(\cos B + i\sin B)$,

Show that $Z_1Z_2 = r_1r_2(\cos(A+B) + i\sin(A+B))$

12. Express $\frac{6x+4}{(x^2-4)(x+2)}$ *into partial fractions hence evaluate* $\int_0^1 \frac{6x+4}{(x^2-4)(x+2)}dx$

13.(a) Find the coefficient of x^2 in $(1+x)^4(2+3x+x^2)$

(b) Expand $(1-8x)^{-1/2}$ up to the term in x^3 hence use $x = \frac{1}{100}$ to evaluate $\sqrt{23}$ to 3dp

14. (a) Show that the lines $r_1 = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ *and* $r_2 = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ intersect

(b) Find the;

(i) Point of intersection of the lines above

(ii) Cartesian equation of a plane which contains the lines above

15. $A(-3,\ 0)$ And $B(3,\ 0)$ are fixed points. Show that the locus of a point $P(x,\ y)$ which moves such that $PB = 2PA$ is a circle and find its Centre and radius.

b) Find the equation of a circle which passes through the points $A(1,\ 2)$, $B(2,\ 5)$ and $C(-3,\ 4)$.

16. Solve the differential equation: $\cos x\frac{dy}{dx} - 2y\sin x = 1$

b) According to Newton's law of cooling, the rate of cooling of a body in air is proportional to the difference between the temperature of the body and that of air. If the air temperature is kept at 25^oC and the body cools from 95^oC to 60^oC in 25 minutes, in what further time will the body cool to 32^oC?

PAPER 1

PURE MATHEMATICS

SECTION A: (40 MARKS)

Answer **ALL** questions in this section

1. Evaluate $\int_2^4 \frac{x^3-1}{x^2}dx$ (5 marks)
2. Solve the equation $5\cos\theta + 2\sin\theta = 3$ for $0^o \le \theta \le 360^o$ (5 marks)
3. The roots of a quadratic equation $x^2+8x+4=0$ are $\left(\alpha + \frac{1}{\beta}\right)$ and $\left(\beta + \frac{1}{\alpha}\right)$. Find the equation whose roots are α and β. (5 marks)
4. Find the acute angle between the lines $2x - y = 6$ and $x = 0$. (5 marks)
5. The sum of the first n terms of the progression is $S_n = n(n-2)$. Find the
   (i) Sum of the first twenty terms (2 marks)
   (ii) Twentieth term of the Progression. (3 marks)
6. Differentiate $\sqrt{\left(1+x^2\right)^3}$ with respect to x. (5 marks)
7. Prove that the points A, B and C whose position vectors are $2a$, $4b$ and $3a-2b$ respectively are collinear. (5marks)
8. The height of the cylinder increases by 2%, find the percentage change in the radius if the volume of the cylinder is to remain constant. (5 marks)

SECTION B (60 MARKS)

Answer only ***five*** questions from this section

9. (a) Use the substitution $y = x - \frac{1}{x}$ to express $x^2 + \frac{1}{x^2}$ in terms of y.

Hence solve the equation $2x^4 - 3x^3 - 4x^2 + 3x + 2 = 0$ (6 marks)

(b)Expand $(1-x)^{\frac{1}{3}}$ using binomial theorem in ascending powers of x up to and involving x^3, hence deduce the value of $\sqrt[3]{7}$ correct to three decimal places. (6 marks)

10 (a) Given that $z_1 = (4+5i)(7+2i)$ and that $z_2 = (4-5i)(7-2i)$. Write down the complex numbers z_1 and z_2 in the form $x+yi$. Hence express $18^2 + 43^2$ as a product of two prime factors. (5 marks)

(b) The point P representing a complex number z is such that

$$\arg(z+5-12i) = \frac{\pi}{4}$$

(i) Find the Cartesian equation of the locus of the point P. (4 marks)

(ii). Describe briefly the geometrical interpretation of the locus. (1 marks)

(iii) Compute the minimum value of $|z|$ (2 marks

11. For the curve $y = \frac{(x-2)^2}{x+2}$

i. Determine the nature turning points of the curve (7 marks)

ii. State the asymptotes of the curve. (3 marks)

iii. Sketch the curve. (2 marks)

12. The line l is drawn from A to B. B is the foot of the perpendicular drawn from the point A $(7,2,1)$ to the line $r = (1+\lambda)i + 2\lambda j + 3k$. Find the;

(a) Coordinates of the point B (8 marks)

(b) Cartesian equation of the line l (4 marks)

13 (a) Prove that $\frac{\cos 3\theta + \cos\theta}{\cos\theta + \sin\theta} = 1 + \cos 2\theta - \sin 2\theta$

Hence deduce $\frac{\cos 67\frac{1}{2}^o + \cos 22\frac{1}{2}^o}{\cos 22\frac{1}{2}^o + \sin 22\frac{1}{2}^o}$ (7 marks)

(b) Solve the equation: $2\cos^2\theta + \sin\theta = 1$ for $-180^o \le \theta \le 180^o$ (5 marks)

14 (a) Evaluate $\int_0^{\frac{\pi}{4}} x\tan^2 x\,dx$ (6 marks)

(b) Differentiate $x^2 + e^{2x} + x^x$ with respect to x. (6 marks)

15(a). The points $P\left(\frac{p}{3}, \frac{9}{p}\right)$ and $Q\left(\frac{q}{3}, \frac{9}{q}\right)$ lie on a rectangular hyperbola. Determine the equation of the chord PQ and deduce the equation of the tangent at a point whose parameter is 't' . (6 marks)

(b). Given that the chord PQ is parallel to the line $6x+2y=3$, Show that the locus of M the midpoint of PQ is a straight line. (6 marks)

16. An Antelope runs from point A towards B, 100m apart at a rate proportional to the square root of the distance yet to be covered. If the speed of the Antelope at a point A is $20ms^{-1}$. Find the;

(i) Time it takes for the Antelope to reach point B. (7 marks)

(ii) Distance covered from point A by the Antelope in 8 seconds and the speed of the Antelope at this instant. (5 marks)

END

PAPER 2

APPLIED MATHEMATICS

SECTION A (40 MARKS)

1. 40 percent of the students in a certain school own personal computers. If 24 students are selected at random, find the probability that between 8 and 15 students own personal computers. (05 marks)
2. Forces of (a*i* + bj) N and (6i- 4j) N act at points having position vectors (-2i-2j) m and (3i-j) m respectively. If the forces reduce to a couple, find;

   a).a and b

   b).the moments of the couple (05 marks)

3. Two examiners Y and Z each marked the scripts of ten candidates who sat a mathematics examination. The table below shows the examiners' ranking of the candidates.

Examiners	1	2	3	4	5	6	7	8	9	10
Y	**D**	**G**	**B**	**E**	**A**	**C**	**F**	**I**	**J**	**H**
Z	**G**	**D**	**B**	**F**	**E**	**A**	**C**	**J**	**I**	**H**

Calculate the spearman's rank correlation coefficient for the two examiners and show whether the examiners were in agreement at 1% level of significance. (05marks)

4. Two particles are travelling along a straight line AB of length 20m. At the same instance, one particle starts from rest at A and travels towards B with a constant acceleration of $2ms^{-2}$ and the other particle starts from rest at B and travels towards A with a constant acceleration of $5ms^{-2}$. Find how far from A the particles will collide. (5 marks)

5. The table below shows the velocity of a particle during the course of its motion.

Time (s)	5	9	12
Velocity (ms^{-1})	10	13	17

Use linear interpolation or extrapolation to estimate the:

i).Velocity when T=7s.

ii).Time when the velocity $=19ms^{-1}$ (05 marks)

6. Use the trapezium rule with 5 stripes to find an approximate value for $\int_1^2 xe^{-2x}dx$ correct to 3 significant figures. (05 marks)

7. An object performs Simple Harmonic Motion (SHM) at a rate of 20 oscillations per second between two points A and B which are 12cm.If C is the midpoint of AB,calculate the time taken to travel directly from C to the midpoint of CB. (05 marks)

8. The price index of an article in 2020 based on 2018 was 130. The price index for the article in 2022 based on 2020 was 80. Calculate the;

i).Price index of the article in 2022 based on 2018. (03 marks)

ii).Price of the article in 2018 if the price of the article was 45,000 in 2022. (02 marks)

SECTION B: (60 MARKS)

Answer only **FIVE** questions from this section. All questions carry equal marks.

9. A school took a survey of the ages of its employees. The results are shown in the frequency table below:

Ages (yrs)	18-	20-	30-	40-	50-
Frequency	8	22	15	7	2

(a) .Calculate the:

(i) .Mean age (03 marks)

(ii).Modal age (03 marks)

(iii).Standard deviation (02 marks)

(b).Draw a cummulative frequency curve and use it to estimate the middle 60% age range. (04 marks)

10. At 10:00am, ship A and ship B are 16km apart. Ship A is on a bearing $N35^0E$ from ship B. Ship A is travelling at $14kmh^{-1}$ on a bearing $S29^0E$.Ship B is travelling at 17 kmh^{-1} on a bearing $N50^0E$. Determine the;

(a).Velocity of ship B relative to ship A. (05 marks)

(b). Closest distance between the two ships and the time when it occurs. (07 marks)

11. (a). Two sides of a triangle PQR are p and q such that <PRQ=α

(i) Find the maximum possible error in the area of this triangle. (02 marks)

(ii) Hence, find the percentage error made in the area if p=4.5cm, q=8.4cm and $\alpha=30^0$. (06 marks) (b).Find the range with in which $\frac{3.679}{2}$ - $\frac{7.0}{5.48}$ lies. (04 marks)

12. The continuous random variable x is distributed between the values x=0 and x=2 and has a probability density function $mx^2 + nx$ with the mean of 1.25. Find;

(i).The value of m and n and hence f(x). (08 marks)

(ii).The mode of x. (04 marks)

13(a). At time, t, the position vector of a particle of mass 2kg is (Cost $\underline{i}$ + $t^2\underline{j}$) m.Show that the force acting on the particle when t=π seconds is of magnitude $2\sqrt{5}$N. (07 marks)

(b) A particle is moving such that at any instant, its velocity vector $\underline{v}$, is given by $\underline{v}$= (3t$\underline{i}$ +4$\underline{j}$ + $t^2\underline{k}$) ms^{-1}.When t=0, it is at the point (1, 0, 1). Show that the magnitude of its acceleration at t=2 seconds is $5ms^{-2}$. (05 marks)

14(a).Show that the Newton Raphson formula for approximating the root of the equation

e^x-2x-1=0 is given by $X_{n+1} = \frac{e^{xn}(xn-1)+1}{e^{xn}-2}$ (03 marks)

(b)(i) .Show that root of the equation e^x-2x-1=0 lies between 1 and 1.5. (04 marks)

(ii).Use linear interpolation to find the first approximation and hence find the root of the equation, correct to 3 s.f. (05 marks)

15. The weights of goats sold at a certain market are normally distributed with a mean of 26kg.Given that 8 of every 12 goats picked at random weighed more than 20kg.

(a) .Calculate the standard deviation of the masses of the goats, correct to the nearest whole number. (06 marks)

(b) .A random sample of 25 goats is picked, calculate the probability that their mean weight exceeds 15kg. (06 marks)

16(a).A practicle is projected from a point on a horizontal plane and has an initial speed of $42ms^{-1}$. If the particle passes through a point above the plane, 70m vertically and 60m horizontally from the point of projection.

(a).Find the possible angles of projection. (06 marks)

(b).A particle is projected at an angle of 30^0 to the horizontal with a speed of $21ms^{-1}$. If the point of projection is 5m above the horizontal ground, find the horizontal distance that the particle travels before hitting the ground. (06 marks)

END