A LEVEL

P425/1 , PAPER 1
PURE MATHEMATICS
Nov.Dec.2013. 3 hours.
SECTION A

1. Solve $\log_x 5 + 4 \log_x 5 = 4$. ***(05 marks)***

2. In a Geometric progression (G.P), the difference between the fifth and the second term is 156. The difference between the seventh and the fourth term is 404. Find the possible values of the common ratio.
(05 marks)

3. Given that $r = 3\cos\theta$ is an equation of a circle , find it's Cartesian form. ***(05 marks)***

4. The position vector of point *A* IS $2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$, of *B* is

5***j*** +4***k*** and of *C* is i +2j +12***k***. Show that *ABC* is a triangle. ***(05 marks)***

5.Solve $5\cos^2 3\theta = 3(1 + \sin 3\theta)$ for 90^0. ***(05 marks)***

6. If $y = x(x-0.5)e^{2x}$, find $\frac{dy}{dx}$.

Hence determine $\int_0^1 xe^{2x}\,dx$. ***(05 marks)***

7. The region bounded by the curve y = cos x, the y-axis and the x-axis from x=0 to $x = \frac{\pi}{2}$ is rotated about the x-axis . Find the volume of the solid formed.

(05 marks)

8. Solve $(1-x^2)\frac{dy}{dx} - xy^2 = 0$, given that y=1 when x=0. ***(05 marks)***

SECTION B: (60 MARKS)

9. (a) The complex number $Z = \sqrt{3} + i$. Z is the conjugate of Z.

(i) Express Z in the modulus argument form.

(ii) On the same Argand diagram plot Z and 2Z+3i.

(08 marks)

(b)What are the greatest and least values of lZl

if I$|z - 4i| \leq 3$?

10. Given the equation $x^3 + x - 10 = 0$,

Show that x=2 is a root of the equation.

(05 marks)

(b) deduce the values of $\alpha + \beta$ and $\alpha\beta$ where α and β are the other roots of the equation.

Hence form a quadratic equation whose roots are α^2 and β^2. ***(09 marks)***

11. (a)Find the point of intersection of the lines

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$$

(06 marks)

(b) The equations of a line and a plane are

$\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{2}$ and 2x +y +4z = 9 respectively.

P is a point on the line where x =3. *N* is the foot of the perpendicular from point P to the plane.

Find the coordinates of *N*. ***(08 marks)***

12. (a) Find the equation of the tangents to the hyperbola whose points are of the parametric form $(2t, 2/t)$in (a), ***(05 marks)***

(b) (i) Find the equations of the tangents in (a), which are parallel to $y + 4x = 0$. ***(04 marks)***

(ii) Determine the distance between the tangents in (i) ***(03 marks)***

13. A curve has the equation $y = \frac{2}{1+x^2}$.

(a) Determine the nature of the turning point on the curve. ***(07 marks)***

(b) Find the equation of the asymptote. Hence sketch the curve. ***(05 marks)***

14. (a) Prove that tan (A-B) = $\frac{\tan A - \tan B}{1+\tan A \tan B}$.

Hence show that $\frac{1-\tan 15^0}{1+\tan 15^0} = \frac{1}{\sqrt{3}}$. ***(06 marks)***

(b)Given that cos A =3/5 and cos B=12/13 where A and B are acute, find the value of

(i) tan $(A+B)$

(ii) cosec $(A+B)$ ***(06 marks)***

15. Resolve y= $\frac{x^3+5x^2-6x6}{(x-1)^2(x^2+2)}$ into partial fractions.

Hence find $\int y dx$ and $\frac{dy}{dx}$. ***(12 marks)***

16. The differential equation $\frac{dp}{dt}$ =kp(c-p). shows the rate at which information flows in a student population c. p represents the number who have heard the information in t days and k is a constant.

(a) Solve the differential equation ***(06 marks)***

(b) A school has a population of 1000 students. Initially, 20 students had heard the information. A day later, 50 students had heard the information. How many students heard the information by the tenth day? ***(06 marks)***

P425/2 ,

PAPER 2

PURE MATHEMATICS

Nov.Dec.2013.

3 hours.

1. A class performed an experiment to estimate the diameter of a circular object. A sample of five students had the following results in centimeters; 3,12,3.16,2.94,3.33, and 3.0.

Determine the sample;

(a) mean

(b) standard deviation. ***(05 marks)***

2. The table below shows the values of a function f(x)

x	1.8	2.0	2.2	2.4
f(x)	0.52	0.484	0.436	0.384

Use linear interpolation to find the value of

(a) f(2.08).

(b) x corresponding to f(x) = 0.5 ***(05 marks)***

3. The speed of a taxi decreased from 90 kmh^{-1} to 18kmh^{-1} in a distance of 120 metres. Find the speed of the taxi when it had covered a distance of 50 metres.

4. Events A and B are such that $P(A \cap B) = \frac{1}{12}$

And $P(A/B) = \frac{1}{3}$

Find $P(A \cap B')$. ***(05 marks)***

5. Find the approximate value of $\int_0^2 \frac{1}{1+x^2}\,dx$ using the trapezium rule with 6 ordinates. Give your answer to 3 decimal places.

(05 marks)

6. Forces of 7N and 4 N act away from a common point and make and angle of θ^0 with each other.Given that magnitude of their resultant is 10.75N, Find the;

(a) value of θ .

(b) direction of the resultant. ***(05 marks)***

7. An industry manufactures iron sheets of mean length 3.0 m and standard deviation of 0.05 m. Given that the lengths are normally distributed, find the probability that the length of any iron sheet picked at random will be between 2.95 m and 3.15 m. ***(05 marks)***

8. A particle of mass m kg is released at rest from the highest point of asolid spherical object of radius a metres. Find the angle to the vertical at which the particle leaves the sphere. ***(05 marks)***

SECTION B: (60 MARKS)

9. The heights (cm) and ages (years) of a random a random sample of ten farmers are given in the table below.

Height(cm)	156	151	152	160	146	157	149	142	158	140
Age(years)	47	38	44	55	46	49	45	30	45	30

(a) (i) Calculate the rank correlation coeffient.

(ii) Comment on your result . ***(05 marks)***

(b) Plot a scatter diagram for the data.

Hence draw a line of best fit. ***(05 marks)***

(c) Use your diagram in (b) to find

(i) y when x= 147.

(ii) x when y =43. ***(05 marks)***

10. A mass of 12 kg rests on a smooth inclined plane which is 6 m long and 1 m high. The mass is connected by a light inextensible string, which passes over a smooth pulley fixed at the top of the plane, to a mass of 4 kg which is hanging freely. With the string taut, the system is released from rest.

(a) Find the

(i) acceleration of the system.

(ii) tension in the string. ***(08 marks)***

(b) Determine the;

(i) velocity with which the 4kg mass hits the ground.

(ii) time the 4kg mass takes to hit the ground. .

(04 marks)

11. The probability density function (p.d.f) of a continous random variable X is given by

$$f(x)=\begin{cases} kx(16-x^2), & 0 \le x \le 4 \\ 0 \quad , & \text{elsewhere;} \end{cases}$$

Where k is a constant.

Find the;

(a) value of k. ***(04 marks)***

(b) mode of X. ***(04 marks)***

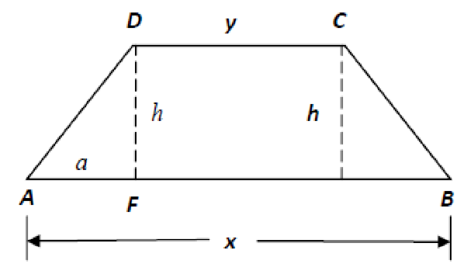
(c) mean of X. ***(04 marks)***

12. (a) Particles of masses, 5kg, 2kg, 3kg and 2kg act at points with position vectors 3**i**-**j**,2**i** +3**j**, -2**i** +5**j** and –**i** -2**j** respectively.

Find the position vector of their centre of gravity.

(06 marks)

(b) The figure ABCD below shows a metal sheet of uniform material cut in the shape of trapezium .AB=x, CD=y, AF =A, EB = b and h is the vertical distance between AB and CD.



Show that the centre of gravity of the sheet is at a distance.

$\frac{h}{3}\left[\frac{3y+a+b}{x+y}\right]$ from side AB. ***(06 marks)***

13. The numbers x and yare measured with possible errors of Δx and Δy respectively.

(a) Show that the maximum absolute error in the quotient $\frac{x}{y}$ is given by $\frac{|y||\Delta x|+|x||\Delta y|}{y^2}$ ***(06 marks)***

(b) Find the interval within which the exact value of $\frac{2.58}{3.4}$ is expected to lie. ***(06 marks)***

14. A particle is projected with speed of 36 ms^{-1} at an angle of 40^0 to the horizontal from the point 0.5

m above the level ground. It just clears a wall which is 70 metres on the horizontal plane from the point of projection.

Find the

(a) (i) time taken for the particle to reach the wall.

(ii) height of the wall. ***(08 marks)***

(b) maximum height reached by the particle from the point of projection. ***(04 marks)***

15. (a) Show that the iterative formula based on Newton Raphson's method for solving equation λn x + x-2 = 0 is given by

$x_{n+1} = \dfrac{x_n(3 - \lambda n x_n)}{1 + x_n}$; n=0,1,2….. ***(04 marks)***

(b) (i) Construct a flow chart that:

- reads the initial approximation as r
- computers, using the iterative formula in (a)

, and prints the root of the equation λnx+x-2=0, when the error is less that 1.0 $x10^{-4}$.

(ii) Perform a dry run of the flow chart when r=1.6. ***(08 marks)***

16. A research station supplies three varieties of seeds S_1, S_2 *and* S_3 in the ratio 4:2:1. The probabilities of germination of S_1, S_2 *and* S_3 are 50% ,60% and 80% respectively.

(a) Find the probability that a seed selected at random will germinate. ***(05 marks)***

(b) Given that 150 seeds are selectedat random, find the probability that less than 90 of the seeds will germinate. Give your answer to two decimal places. ***(07 marks)***