P425/1
PURE
MATHEMATICS
Paper 1
April/ May 2023
3 hours

QUESTLYFT EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in* ***section A*** *and any* ***five*** *from* ***section B.***

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section ***B*** *will* **not** *be marked.*

SECTION A (40 MARKS)

Answer all questions in this section

1. Solve the inequality $(\log_4 x)\ (\log_2 x) > 2.$ *(05 marks)*

2. Prove the identity, $4\, Cos\, 3\theta Cos\,\theta + 1 = \frac{Sin 5\theta}{Sin\theta}.$ *(05 marks)*

3. Given that $\sqrt{x^2 + 4} = x + ax^{-1} + bx^{-3} + 4x^{-5} + ...,$ find the values of the constants a and b. *(05 marks)*

4. Find $\int 10^{\ln x}\ dx.$ *(05 marks)*

5. Find the acute angle between the lines;

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 5 \\ -8 \end{pmatrix} \ and \ \frac{x-1}{-8} = \frac{y-2}{3} = \frac{Z-3}{5}.$$ *(05 marks)*

6. Given that ; $x = \theta - \sin\theta \ and \ y = \cos\theta$, show that $\frac{dy}{dx} = -\cot\frac{1}{2}\theta.$ *(05 marks)*

7. Given that P($2t^2$, 4t) is a variable point on a curve. Find the equation of the tangent to the curve at the point (8, -8). *(08 marks)*

8. Show that the area bounded by the curve y = 1 + sin x, the coordinate axes and the line $\pi = \pi/2$ is $1/2(\pi + 2)\ sq.units.$ *(05 marks)*

SECTION B (60 MARKS)

Attempt FIVE questions only in this section

9. (a) Given that $\tan\beta = \frac{3 - 4\tan\alpha}{4 + 3\tan\alpha}$, find the values of sin $(\alpha + \beta)$. *(06 marks)*

   (b) Given that $Sin\theta + \cos(\theta - 60^0) = Cos\,\theta$, show that $\tan\theta = 2 - \sqrt{3}$, hence solve for θ in the interval 0^0 to 360^0. *(06 marks)*

10. (a) A, B, C and D are points in a straight line in that order such that;
    AB : BC : CD = 4:1:3.
    State the ratio in which;
    (i) C divides AD,
    (ii) B divides CD,
    (iii) D divides BA respectively. *(04 marks)*

(b) The line $\frac{x}{2} = \frac{y-b}{1} = \frac{z-4}{-2}$ passes through the point P (a, a, a) and is perpendicular to the plane containing the point P(a, a, a). Find the;

(i) values of a and b

(ii) scalar product equation of the plane. *(08 marks)*

11. (a) Find the smallest positive integral value of n such that;

$$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^n = -1.$$ *(06 marks)*

(b) Given that Z = x + iy. Find the equation of the locus;

$|Z - 2 + 1| = |Z + 1 - 2i|$; hence sketch the locus $|Z - 2 + i| > |Z + 1 - 2i|$

(06 marks)

12. (a) Given that $x^2 + y^2 = \sqrt{10x - 5y}$. Find the value of $\frac{dy}{dx}$ when x = 4 and y = 3. *(05 marks)*

(b) A builder is to construct a rectangular fence of height 2m using iron sheets. One side is an existing wall. If the volume of space fenced off is $100m^3$ calculate the minimum area of the iron sheets required. *(07 marks)*

13. Given the curve; $y = 1 + \frac{20x}{x^2 - 10x + 9}$,

(a) Find the; (i) equations of the asymptotes.

(ii) coordinates of the stationary points

(b) Sketch the curve. *(12 marks)*

14. Find; (a) $\int x \ln\left(x^2 - 1\right) dx$ *(05 marks)*

(b) $\int \frac{4}{4x^3 + 4x^2 + x} dx$ *(07 marks)*

15. (a) Prove by induction: $\sum_{r=1}^{n}(5r - 3) = \frac{1}{2}n(5n - 3)$. *(06 marks)*

(b) Find the Maclaurin's series for ln (cosx) up to the terms in x^4. *(06 marks)*

16. (a) Solve the exact differential equation : $e^{x+y}\dfrac{dy}{dx} + e^{x+y} = 2x$.*(04 marks)*

(b) Whenever two rival football teams play against each other the game takes 1½ hours. The sum of the scores in any such game after **t** hours of play is known to vary at a rate which is directly proportional to the time **t** and inversely as the sum of the scores.

In a particular game the first goal was scored after 15 minutes of play and the game ended in a 3 all draw. If by the hour mark one team had scored 3 goals, find how many goals the other team had scored. *(08 marks)*

END