WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
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MATHEMATICS P425/1

SB

No.1

$\frac{x}{y} + \frac{6y}{x} = 5$ …….. (1) and $2y = x - 2$ ………… *(2)*

Solution

Make y the subject from the equation

$y = \frac{x-2}{2}$ …………….*(3)*

B₁ subject

Substituting *equation (3)* into *equation (1)*

$\frac{\frac{x}{x+2}}{2} + 6\frac{\frac{(x-2)}{2}}{x} = 5$

M₁ substitu

$\frac{2x}{x-2} + 3\frac{(x-2)}{x} = 5$

OR $\frac{x^2 + 6y^2}{yx} = 5$

$x^2 + 6y^2 = 5yx$

$x^2 + 6\frac{(x-2)^2}{4} = 5x\left(\frac{x-2}{2}\right)$

$x^2 + 3\frac{(x-2)^2}{2} = 5x\frac{(x-2)}{2}$

$2x^2 + 3(x-2)^2 = 5x^2 - 10x$

$2x^2 + 3(x^2 - 4x + 4) = 5x^2 - 10x$

$2x^2 + 3x^2 - 12x + 12 = 5x^2 - 10x$

$5x^2 - 12x + 12 = 5x^2 - 10x$

$-12x + 10x = -12$

$\frac{-2x}{-2} = \frac{-12}{-12}$

$x = 6$

M₁ Rearra and scorin x

Substituting for $x = 6$ into *equation (3)*

$y = \frac{x-2}{2}$

$y = \frac{6-2}{2}$

$y = 2$

A₁ substitu x = 6

Therefore; the point of intersection of the curve is (6,2)

A₁ Cao

05

No.2

Let $u = \sqrt{1+x}$

$u^2 = 1 + x$

$x = u^2 - 1$

$dx = 2u\,du$

x	0	1
μ	1	$\sqrt{2}$

B₁ change of lim

$$\int_0^1 \frac{x}{\sqrt{1+x}}dx = \int_1^{\sqrt{2}} \frac{u^2-1}{u}\, 2u\,du$$

M₁ substituti the new varia

$$= 2\int_1^{\sqrt{2}} (u^2 - 1)\,du$$

$$= 2\left[{}^{u^3}/_{3} - u\right]_1^{\sqrt{2}}$$

M₁ integratio

$$=2\left[\left({}^{2}/_{3}{}^{\sqrt{2}} - \sqrt{2}\right) - \left({}^{1}/_{3} - 1\right)\right]$$

M₁ Substituti

$= 0.3905$ (4.dp)

Hence $\int_0^1 \frac{x}{\sqrt{1+x}}dx = 0.3905.$

A₁ Cao

05

No.3

(i) $\sin x + \sin y = \beta$……1 $\cos x + \cos y = \beta_2$ ……..2

Divide *equation 1* by *equation 2*

$$\frac{\sin x + \sin y = \beta_1}{\cos x + \cos y = \beta_2}$$

M₁ division se

From factor formulae.

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

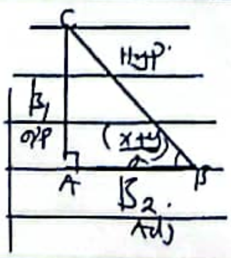
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\frac{2\sin(\frac{x+y}{2})\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{\beta_1}{\beta_2}$$

$\tan\left(\frac{x+y}{2}\right) = \frac{\beta_1}{\beta_2}$ hence proved

A₁ Cao

(ii)



$BC^2 = AC^2 + AB^2$

$BC^2 = (\beta_1)^2 + (\beta_2)^2$

$BC = \sqrt{B_1^2 + B_2^2}$ $BC = \sqrt{\beta_1^2 + \beta_2^2}$

$$\cos\left(\frac{x+y}{2}\right) = \frac{Adj}{Hyp} = \frac{\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}$$ M₁ application of double angle.

From double angle formulae.

$$\cos(x+y) = 2cos^2\left(\frac{x+y}{2}\right) - 1$$

$$=2\left[\frac{\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}\right]^2 - 1$$ M₁ substituting

$$=2\left[\frac{\beta_2^2}{\sqrt{\beta_1^2+\beta_2^2}}\right] - 1$$

$$= \frac{2\,\beta_2^2-\beta_1^2-\beta_2^2}{\beta_1^2+\beta_2^2}$$

$$\cos(x+y) = \frac{\beta_2^2-\beta_1^2}{\beta_1^2+\beta_2^2}$$ hence proved A₁ Cao

05

No. 4

$\sqrt[3]{27.15} = y$ M₁ introduce vanch

$y = x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3}x^{\frac{1}{3}-\frac{3}{3}} \to dy/dx = \frac{1}{3}\; x^{\frac{-2}{3}}$

$y + dy = x + dx$

$= \sqrt[3]{27+0.15}$; $dx = 0.15$ M₁ difference

$X = 27$

As $dx \to 0$

$$\frac{dy}{dx} = \frac{dy}{dx}$$

$$dy = \frac{dy}{dx}dx$$ M₁ make Dy subject

$$dy = \frac{1}{3}x^{-2/3}\,dx$$

$$dy = \frac{1}{3}\,(27)^{-2/3}\,X0.15$$ M₁ substituting

$$= \frac{1}{3}\left(\frac{1}{9}\right)x\ 0.15 = 0.0056$$ A₁ Cao

$$y + dy = \sqrt[3]{27} + 0.0056$$

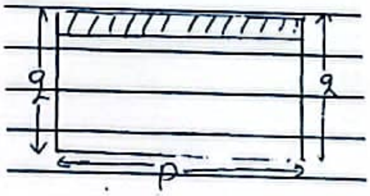
$$= 3 + 0.0056$$

$$= 3.0056$$

05

No. 5		
	Distance from P and A	
	d= $\overline{AP} = \sqrt{(x-3)^2+(y-0)^2} = \sqrt{(x-3)^2+y^2}$	M1 distance
	Distance of P from a line	
	D= $\left\lvert\frac{1(x)+0(y)+3}{\sqrt{1^2+0^2}}\right\rvert = \frac{x+3}{1} = x+3$	M1 distance
	since d = D	
	$\Rightarrow d^2 = D^2$	
	$(x-3)^2+(y-0)^2=(x+3)^2$	M1 equation
	$x^2-6x+9+y^2=x^2+6x+9$	
	$y^2=12x$	A1 equation
	Is a parabola with vertex at (0,0) focus at (3,0) and ~~direction~~ directrix $x=-3$	B1 description
		05
No. 6		
	Displacement of A from the plane	
	$S_1 = \frac{(2x^-2)-(1X0)+(3X6)+^-21}{\sqrt{(2)^2+(^-1)^2+(3)^2}}$	M1 substitution
	$=\frac{^-7}{\sqrt{14}}$	A1 displacement
	Displacement of B from the plane	
	$S_2 = \frac{2(3)-(1X^-4)+(3X5)+^-21}{\sqrt{(2)^2+(^-1)^2+(3)^2}}$	M1 substitution
	$\frac{4}{\sqrt{14}}$	A1 displacement
	Since S_1 and S_2 have different signs, hence A and B lie on opposite sides of the plane.	B1 conclusion
		05
No.7		
	G.P = $a + ar + ar^2 + ar^3 + \ldots\ldots\ldots$ 2^{nd} term = ar ar = 24. $ar^2 = 12(b+1)$ $a + ar + ar^2 = 76$ a+24+12(b+1) = 76 a+24+12b+12 = 76 a+ 12b =76-36	
	$a + ar + ar^2 = 76$ a+ ar(1 +r) = 76 a +24 (1 + r) = 76 24(1+r)=76-a $1+r = \frac{76-a}{24}$ $r= \frac{76-a-1}{24}$ -1	B1

CamScanner

	$a+12b = 40$ $ar^2 = 12(b+1)$ $r=\frac{76-a-24}{24}$ $r=\frac{52-a}{24}$ $a\left(\frac{52-a}{24}\right)=24$ $52a-a^2=24^2$ $a^2-52a+576=0$ $(a-16)(a-36)=0$ $a=16$ Or $a=36$ $\Rightarrow 16+12b=40$ $12b=40-16=24$ $b=2$ or $\Rightarrow 36+12b=40$ $12b=4$ $b=\frac{1}{3}$ Therefore $b=2$ or $b=\frac{1}{3}$.	M_1 solve for a B_1 correct values of a M_1 substitute and solve b A_1 Cao
		05
No. 8		
	Let p and q be the dimensions that will give the maximum possible area of the land. Perimeter $= p+q+q=200$ $=p+2q=200$ $\Rightarrow p=200-2q$  Area, A = $A=pq$ $A=q(200-2q)$ $A=200q-2q^2$ $\frac{dA}{dq}=200-4q$ At, maximum, $\frac{dA}{dq}=0 \quad 200-4q=0$ $\frac{200}{4}=\frac{4q}{4}$ $q=50m$ $p=200-2q$ $=200-2X50$ $=100m$ Maximum area = $pq=100X50 = 5000m^2$	B_1 expression for p B_1 differentiation M_1 finding value of q M_1 finding the value Ap A_1
		05

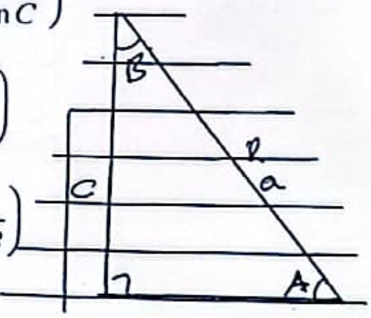
No.9

(a)

$$\frac{\operatorname{cosec}(B+C)}{\operatorname{cosec}B\operatorname{cosec}C} = \frac{bc}{ab+ac}$$

From the ~~hits~~ 6.1(d=5 $\dfrac{\operatorname{cosec}(B+c)}{\operatorname{cosec}B+\operatorname{cosec}c} = \dfrac{\dfrac{1}{\sin(B+C)}}{\dfrac{1}{\sin B}+\dfrac{1}{\sin C}}$ — M₁ simplifying

$$=\frac{1}{\sin(B+C)}\left(\frac{\sin c+\sin B}{\sin B\sin C}\right)$$



$$=\frac{1}{\sin(B+C)}\left(\frac{\sin B\sin C}{\sin C\sin B}\right)$$

$$=\frac{1}{\sin(B+C)}\left(\frac{\sin B\sin C}{\sin C+\sin B}\right)$$ — M₁ simplify

$$=\frac{\sin B\sin C}{\sin A(\sin C+\sin B)}$$

$$=\frac{R^2\sin B\sin C}{R^2\sin A(\sin C+\sin B)}$$ — M₁ introducing R^2

$$=\frac{R\sin BR\sin C}{R\sin A(R\sin C+R\sin B)}$$ but $R\sin A=a$, $R\sin B=b$, $R\sin C=c$ — B₁

$$\frac{\operatorname{cosec}(B+C)}{\operatorname{cosec}B\operatorname{cosec}C} = \frac{bc}{ab+ac}$$ hence proved — A₁ Cao

05

(b)

$$3\cot\theta+\operatorname{cosec}\theta=2$$

$$\frac{3}{\tan\theta}+\frac{1}{\sin\theta}$$

Let t = $\tan\theta/2$

$$\tan\theta=\frac{2t}{1-t^2} \qquad \sin\theta=\frac{2t}{1+t^2} \longrightarrow m$$

Substituting

$$\frac{3}{\frac{2t}{1-t^2}}+\frac{1}{\frac{2t}{1+t^2}}=2$$ — m — M₁ substitution

$$\frac{3(1-t^2)}{2t}+\frac{(1+t^2)}{2t}=2$$ — M₁ substitute

$3-3t^2+1+t^2=4t$

$2t^2+4t-4=0$

$t^2+2t-2=0$ m1

M1 Re-arranging to have quadratic equation of t

$t=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $t=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$t=\dfrac{-2\pm\sqrt{2^2-4\times1\times{}^-2}}{2\times1}$ $t=\dfrac{-2\pm\sqrt{(2)^2-4\times1\times-2}}{2\times1}$

M1 solve for θ

$t=\dfrac{-2\pm\sqrt{4+8}}{2}$ $t=\dfrac{-2\pm\sqrt{12}}{2}$

$t=\dfrac{-2\pm\sqrt{12}}{2}$ $=\dfrac{-2\pm2\sqrt{3}}{2}$

$t=\dfrac{-2\pm2\sqrt{3}}{2}$ $t=-1\pm\sqrt{3}$

$t={}^-1\pm\sqrt{3}$ m1

Alt:

$3\cot\theta+\operatorname{cosec}\theta=2$

$3\dfrac{\cos\theta}{\sin\theta}+\dfrac{1}{\sin\theta}=2$

$3\cos\theta-2\sin\theta=-1\rightarrow$ m1

Let $3\cos\theta-2\sin\theta=R\cos(\theta+\alpha)$

$=R\cos\theta\cos\alpha-R\sin\theta\sin\alpha.$

$R\cos\alpha=3$ → m1 | $R=\sqrt{3^2+2^2}=\sqrt{13}$ m1

$R\sin\alpha=2$

$\tan\alpha=\frac{2}{3}$, $\alpha=\tan^{-1}\left(\frac{2}{3}\right)=33.69^0$ m1

$\sqrt{13}\cos(\theta+33.69)=-1$

$\theta+33.69=\cos^{-1}\left(\frac{-1}{\sqrt{13}}\right)$ m1

$=106.1,\ 253.9$

$\theta=72.4^0,\ 220.2^0$ A1

Taking t= ${}^-1-\sqrt{3}$

$\tan\theta/2={}^-1-\sqrt{3}$

$\theta/2=\tan\left({}^-1-\sqrt{3}\right)$

$\theta/2=110.1^0,290.1^0$

$\theta=220.2^0$ m1

A1 θ = 220.2°

Taking t= ${}^-1+\sqrt{3}$

$\tan\theta/2={}^-1+\sqrt{3}$

$\theta/2=36.2^0,216.2^0$

A1

$\theta=74.4^0$ *Therefore;* $\theta=74.4^0$ *and* 220.2^0 *for* $0^0\le\theta\le360^0$

A1 - θ = 74.4°

(c) $2\sin(x)=\sin(x-60)$

$2\sin x=\sin X\cos60-\sin60\cos x$

$2\sin x=\frac{1}{2}\sin x-\dfrac{\sqrt{3}}{2}\cos x$

M1 method

$4\sin x=\sin x-\sqrt{3}\cos x$

$3\sin x=\sqrt{3}\cos x$

$\tan x=\dfrac{\sqrt{3}}{3}$

$x=\tan^{-1}\left(\dfrac{\sqrt{3}}{3}\right)$

$x={}^-30^0, 150^0$

Therefore ; x = ⁻30° and 150° for ⁻180° ≤ x ≤180°

A₁ – correct values of x

07

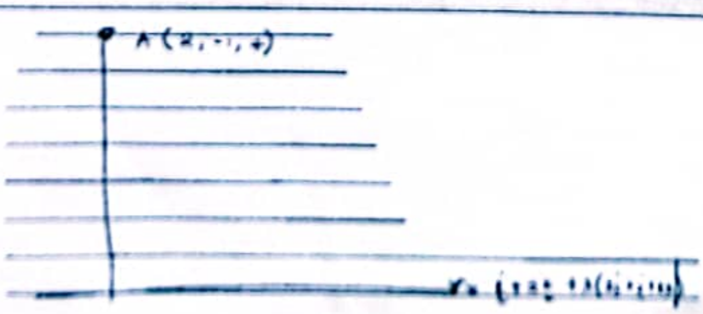
No. 10		
(a)	$\frac{(\sqrt{3})^8(\cos 8\theta + i\sin 8\theta)}{3^3(\cos 6\theta + i\sin 6\theta)}$	M_1 removing power from outside brackets
	$\frac{3^4}{3^3}(\cos(8\theta-6\theta)+i\sin(8\theta-6\theta))$	M_1 division of module M_1 subtract
	$3(\cos 2\theta + i\sin 2\theta)$	A_1 simplified solution
		04
(b)	$(1+3i)Z_1 = 5(1+i)$	
	$Z_1 = \frac{5+5i}{1+3i}$	B_1 (making Z_1 subject)
	$\frac{(5+5i)(1-3i)}{(1+3i)(1-3i)}$	M_1 realization
	$= \frac{5-15i+5i+15}{1+9}$	
	$= \frac{20-10i}{10}$	M_1
	$= 2-i$	A_1 values of Z_1
	$\lvert x+iy-(2-i)\rvert = \lvert 2-i\rvert$	
	$\lvert (x-2)+i(y+1)\rvert = \lvert 2-i\rvert$	
	$\sqrt{(x-2)^2+(y+1)^2} = \sqrt{2^2+(^-1)^2}$	
	Squaring both sides.	
	$(x-2)^2+(y+1) = 2^2+(^-1)^2$	M_1 squaring
	$(x-2)^2+(y+1)^2 = 5$	
	Or $x^2+y^2-4x+2y=0$	A_1
	Is a circle	
	Centre	
	$x-2=0$ and $y+1=0$	
	$x=2$ and $y={}^-1$	
	Centre is $(2,{}^-1)$ and $R^2 = 5$	M_1 method
	Radius,	
	$R = \sqrt{5}$	A_1 Centre and radius.
		08
No.11		
(a)	AP : a + (a+2) +(a+4)+…………..	
	G.P : $a+\frac{1}{3}a + \frac{1}{9}a + \ldots\ldots\ldots$	

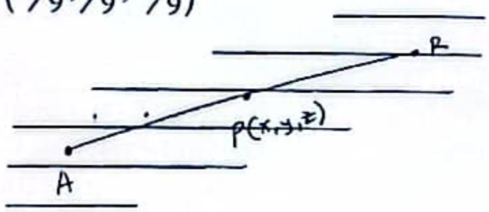
	$S_\infty = 9$ $S_\infty = \dfrac{a}{1-r}$ $9 = \dfrac{a}{1-\frac{1}{3}}$	M₁ state and substitute the value of r
	$9 = \dfrac{a}{2/3}$	M₁ Equating to 9
	$a = 9 \times 2/3 = 6$	A₁
	$S_{10} = \frac{n}{2}(2a+(n-1)d)$ $S_{10} = \frac{10}{2}(2\times6+(10-1)2)$	M₁ substituting for a and n
	$=5(12+18)$ $=5\times30$	M₁ evaluation
	$=150$	A₁ Cao
		06
(b)	Effective letters $\uparrow D \uparrow F \uparrow A \uparrow T \uparrow D$ Are 5 letters which when arranged without repetition 5!	B₁ 127: B₁ Total arrangement
	6 spaces are available for 3Es to enter possible arrangement $^6P_3$.	B₁ $\frac{8!}{3!2!} = 3360$ Arrangement with Es together DFATD EEE
	Arrange without repetition $(5!)\times(^6P_3)$	B₁ $\frac{6!}{2!} = 360\times6 = 3160$
	Arrangement with repetition of 2Ds and 3Es $\dfrac{(5!)(^6p_3)}{3!2!}$	M₁ Arrangement when Es are separated $= 3360 - 3160$
	= 1200 ways	A₁ $= 1200$ ways.

Arrangement of E DFATD E: $\dfrac{5!}{2!\times1!\times1!\times1!} = 60$
The 3 separated Es cons... in 6 spaces
$6C_3 \times 60 = 20\times60$
$= 1200$ ways

No.12

(a)	$AC = \begin{pmatrix}3\\2\\-1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$ $AB = \begin{pmatrix}1\\0\\1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\-1\\0\end{pmatrix}$ Let $n = \begin{pmatrix}d\\e\\f\end{pmatrix}$	B₁ finding both AC and AB

	$n.\overline{AC} = 0$	M_1 dotting the two vectors with normal.
	$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 0$	
	$2d + e - 2f = 0$	
	$n.\overline{AB} = 0$	
	$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = 0$	
	$0 - e + 0 = 0$ $e = 0$ $2d - 2f = 0$ $2d = 2f$ $d = f$	
	$n = \begin{pmatrix} d \\ 0 \\ d \end{pmatrix}$	
	$n = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow n = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$	A_1
	let $r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$	
	$r.n = n.a$	
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	M_1 finding equation of the…
	$x + z = 1 + 0 + 1$ $x + z = 2$	A_1 Cao
		05
(b)	 The vector parallel to the line is $\underline{n} = 2i + i + 2k$	

	Now AR n =0 (OR –OA) = 0	
	Given r = OR = $\begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\2\end{pmatrix} = \begin{pmatrix}2\lambda+1\\\lambda\\2\lambda+1\end{pmatrix}$	B_1 finding OR
	AR= OR-OA = $\begin{pmatrix}2\lambda+1\\\lambda\\2\lambda+1\end{pmatrix} - \begin{pmatrix}2\\-1\\2\end{pmatrix} = \begin{pmatrix}2\lambda-1\\\lambda+1\\2\lambda-2\end{pmatrix}$	B_1 finding AR
	Now AR·n =0 $= \begin{pmatrix}2\lambda-1\\\lambda+1\\2\lambda-2\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix} = 0$ $4\lambda-2+\lambda+1+4\lambda-4=0$ $9\lambda-5=0$	M_1 solve for scalar λ
	$\lambda = 5/9$	A_1
	OR = $\begin{pmatrix}1\\0\\2\end{pmatrix} + 5/9\begin{pmatrix}2\\1\\2\end{pmatrix}$ $R\left(19/9, 5/9, 28/9\right)$	
		
	AP = MAR OR= OA + M(OR –OA) OP= $\begin{pmatrix}2\\-1\\4\end{pmatrix} + M\left[\begin{pmatrix}19/9\\5/9\\28/9\end{pmatrix} - \begin{pmatrix}2\\-1\\4\end{pmatrix}\right]$	M_1 finding OP
	OP= $\begin{pmatrix}2\\-1\\4\end{pmatrix} + M\left[\begin{pmatrix}1/9\\14/9\\-8/9\end{pmatrix}\right]$ Which is the vector equation of the perpendicular line	

	$AR=\begin{pmatrix}2\left(\frac{5}{9}-1\right)\\ \frac{5}{9}+1\\ 2\left(\frac{5}{9}\right)-2\end{pmatrix}=\frac{1}{9}\begin{pmatrix}1\\14\\8\end{pmatrix}$ $AR==\frac{1}{9}\sqrt{1^2+14^2+8^2}$ $=1.795units$	M_1 find the ………of AR A_1
		07
No.13		
(a)	$\frac{2x}{25}+\frac{2y}{10}\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{-16x}{25y}$ At (cosθ, 4 sinθ); $\frac{dy}{dx}=\frac{-16(5\cos\theta)}{25(4\sin\theta)}$ $=\frac{-4\cos\theta}{5\sin\theta}$ $\quad\frac{5\sin\theta}{4\cos\theta}$ Gradient of normal at (5 cosθ, 4 sinθ) is $\frac{(5\cos\theta)}{(4\sin\theta)}$ $\Rightarrow\frac{y-4\sin\theta}{x-5\cos\theta}=\frac{(5\cos\theta)}{(4\sin\theta)}$ $4y$ cosθ- 16 sinθ cosθ = $5x$ sinθ - 25 sinθ cosθ $4y$ cosθ = $5x$ sinθ- 9 sinθ cosθ At A, y= 0 0 = $5x$ sinθ - 9 sinθ cosθ $x=\frac{9}{5}\cos\theta$ $A\left(\frac{9}{5}\cos\theta,0\right)$ At B, x = 0 $4y$ cosθ = 9 sinθ cosθ $y=\frac{^-9}{4}\sin\theta$ $B\left(0,\frac{^-9}{4}\sin\theta\right)$ Mid-point line AB is $B\left(\frac{9}{10}\cos\theta,\frac{^-9\sin\theta}{8}\right)$	M_1 identification A_1 gradient M_1 equating gradients A_1 method equation of normal M_1 method finding coordinates of A and B. A_1 mid-point
		06

(b)	Two circles are said to be orthogonal when the tangents at their points of intersections are at nighty angles. $x^2+y^2-2ax+c^2=0$ $x^2-2ax+y^2=-c^2$ $(x-a)^2+(y-0)^2=-c^2+a^2$ Centre (a,0) $r^2=-c^2+a^2$, or $r^2=a^2-c^2$ For, $x^2+y^2-2by-c^2=0$ $x^2+y^2-2by=c^2$ $(x+0)^2+(y-b)^2=c^2+b^2$ Centre (0,b) $r^2=c^2+b^2$ $d^2=a^2+b^2$ For orthogonal circles $d^2=r^2+R^2$ $(a^2+b^2)=(a^2-c^2)+(c^2+b^2)$ $a^2+b^2=a^2-\cancel{c^2}+\cancel{c^2}+b^2$ $(a^2+b^2)=a^2+b^2$ Therefore; the circles $x^2+y^2-2ax+c^2$ and $x^2+y^2-2by-c^2=0$ are orthogonal	 M_1 complete sarc M_1 complete square A_1 correct centre and radius for both equations M_1 applications of condition A_1 B_1 drawing conclusion
		06
No. 14		
(a)	$\frac{x^2+1}{x^3+4x^2+3x}=\frac{x^2+1}{x(x^2+4x^2+3)}$ $=\frac{x^2+1}{x(x+1)(x+3)}$ Let $\frac{x^2+1}{x^3+4x^2+3x}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x+3}$ $x^2+1=A(x+1)(x+3)+Bx(x+3)+Cx(x+1)$ Putting $x=-1$ $B=-1$ Putting $x=-3$ $C=\frac{5}{3}$ Putting $x=0$ $A=\frac{1}{3}$ $\int_1^3\frac{1}{3x}dx-\int_1^3\frac{1}{x+1}dx+\int_1^3\frac{5}{3(x+3)}dx$ $\int_1^3\frac{x^2+1}{x^3+4x^2+3x}dx=\frac{1}{3}[\ln x]_1^3-[\ln(x+1)]_1^3+\frac{5}{3}[\ln(x+3)]_1^3$ $\frac{1}{3}[\ln 3-\ln 1]-[\ln 4-\ln 1]+\frac{5}{3}[\ln 6-\ln 4]$ $=\frac{1}{3}\ln(3\div 1)-\ln(4\div 2)+\frac{5}{3}\ln(6\div 4)$ $=0.3662-0.6931+0.6758$ $=0.34887518$ $\int_1^3\frac{x^2+1}{x^3+4x^2+3x}dx\simeq 0.3489$	 M_1 spliting A_1 value of B A_1 value of C A_1 value of A. A_1 for integral M_1 substituting limits B_1 Cao
		07

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(b)	$\int \frac{1}{3x^2+5x+4}dx = \frac{1}{3}\int \frac{1}{\left(x+\frac{5}{6}\right)^2+\frac{23}{36}}dx$	M_1 complete the square
	$let\ \frac{23}{36}+\left(x+\frac{5}{6}\right)^2 = \frac{23}{36}\tan^2\theta$ $\frac{23}{36}+\frac{23}{36}\tan^2\theta$.	M_1 introducing
	$x = \frac{\sqrt{23}}{6}\tan\theta - \frac{5}{6}$ $\theta = \tan^{-1}\left(\frac{6x+5}{\sqrt{23}}\right)$	M_1 integration
	$dx = \frac{\sqrt{23}}{6}\sec^2\theta d\theta$ $= \frac{2\sqrt{23}}{23}\int d\theta$	M_1
	$\int \frac{1}{3x^2+5x+4}dx = \frac{2\sqrt{23}}{23}\tan^{-1}\left(\frac{6x+5}{\sqrt{23}}\right)+c$	A_1 cao
		05
No. 15		
	Let the number of people be x. $\frac{dx}{dt}\alpha(x-5)$	B_1 expression for differentia sign
	$\frac{dx}{dt} = k(x-5)$	M_1 introducing k constant o proportional
	$\int \frac{1}{x-5}dx = k\int dt$	M_1 separate variable
	$\ln(x-5) = Kt + c$ At $t=0$ $x=120$	M_1 solving for c
	$\ln(120-5) = 0 + c$ $C = \ln 115$	B_1 correct value of c
	$\ln(x-5) - \ln 115 = Kt$ $\ln\left(\frac{x-5}{115}\right) = Kt$ At $t = 1$ $x = 210$ ~~In~~ $\ln\left(\frac{210-5}{115}\right) = K\times 1$ $K = \ln\frac{205}{115}$	M_1 solving k
	$K = 0.5781$	B_1 correct value of K
a)	At $t = 5$ years $\ln\frac{x-5}{115} = 0.5781\ t$	M_1 substitute for t
	$\ln\frac{x-5}{115} = 0.5781 \times 5$	A_1 evaluation

	$x - 5 = 115\, e^{0.5781 \times 5}$ $x = 5 + 115\, e^{0.5781 \times 5}$ $x = 2075.265$ $x \simeq 2075$ people	A_1 cao
		10
(b)	$\ln \frac{x-5}{115} = 0.5781\, t$ $x = 37275$ $\ln\left(\frac{37275-5}{115}\right) = 0.5781\, t$ $t = \frac{1}{0.5781} \ln\left(\frac{3727-5}{115}\right)$ $t = 10.000205$ $t = 10$ years	M_1 solve for t A_1 Cao
		02
No.16		
	Given $y = x - \frac{8}{x^2}$ $y = \frac{x^3-8}{x^2}$	
(a)	(i) intercepts When x= 0, y= $= \frac{o^3-8}{0^2}$ $y = \alpha\,(doesnotexist)$ B_1 $(0, \alpha)$ $When y = 0, \frac{x^3-8}{x^2} = 0$ $x^3 - 8 = 0$ $x^3 = 8$ $x = 2$ $\text{int}\,ercept\,(2,0)$	B_1 drawing B_1 x intercept ✓
		02
	(ii) turning point $y = \frac{x^3-8}{x^2}$ $\frac{dy}{dx} = \frac{x^2(3x^2) - (x^3-8)2x}{x^4}$	M_1 differentiation

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	$= \frac{3x^4 - 2x^4 + 16x}{x^4}$ $= \frac{x^4 + 16x}{x^4}$ But at turning point, $\frac{dy}{dx} = 0$ $\frac{x^4 + 16x}{x^4} = \frac{0}{T}$ $x^4 + 16x = 0$ $x(x^3 + 16) = 0$ Either $x = 0$ Or $x^3 = {}^-16$ $x \quad = {}^-2.5$ $y = \frac{({}^-2.5)^3 - 8}{({}^-2.5)^2}$ $= {}^-3.8$ $\therefore$ Turning points are $({}^-2.5, {}^-3.8)$	B1 ✓ correct conclude
	(iii) Equation of asymptote vertical asymptote	03
	$y = x - \frac{8}{x^2}$ $asy \to \pm\alpha$ $x^2 = 0$ $x = 0$ Slanting asymptote $y = \frac{x^3 - 8}{x^2}$ $y = \sqrt[2]{x^3}$ $\frac{x^3}{-8}$ $y = x + \frac{{}^-8}{x^2}$ Slanting asymptote, $y=x$ When $x= 0$, $y = 0$ When $x= 1$, $y= 1$ Equation of asymptote, we have $x = 0$ and $y = x$	B1 ✓ finding vertical asymptote B1 ✓ Finding the slanting asymptote
		02
(b)	Nature of the curve Critical points are 0, 2 x | $x < 0$ | $0 < x < 2$ | $x > 2$ y | – | – | +	B1 Determining nature of turning point.

Nature of the curve

x	$L.H.S$	2.5	$R.H.S$
$\frac{dy}{dx}$	+	0	-

M1 $\frac{1}{2}$ - correct coordinate
Defining nature
Turning point

WAKISSHA JOINT MOCK EXAMINATIONS

JULY/AUGUST:__________

O&A

CANDIDATE'S NAME:... INDEX No.........................

SUBJECT:.. PAPER:.................. SIGNATURE..........................

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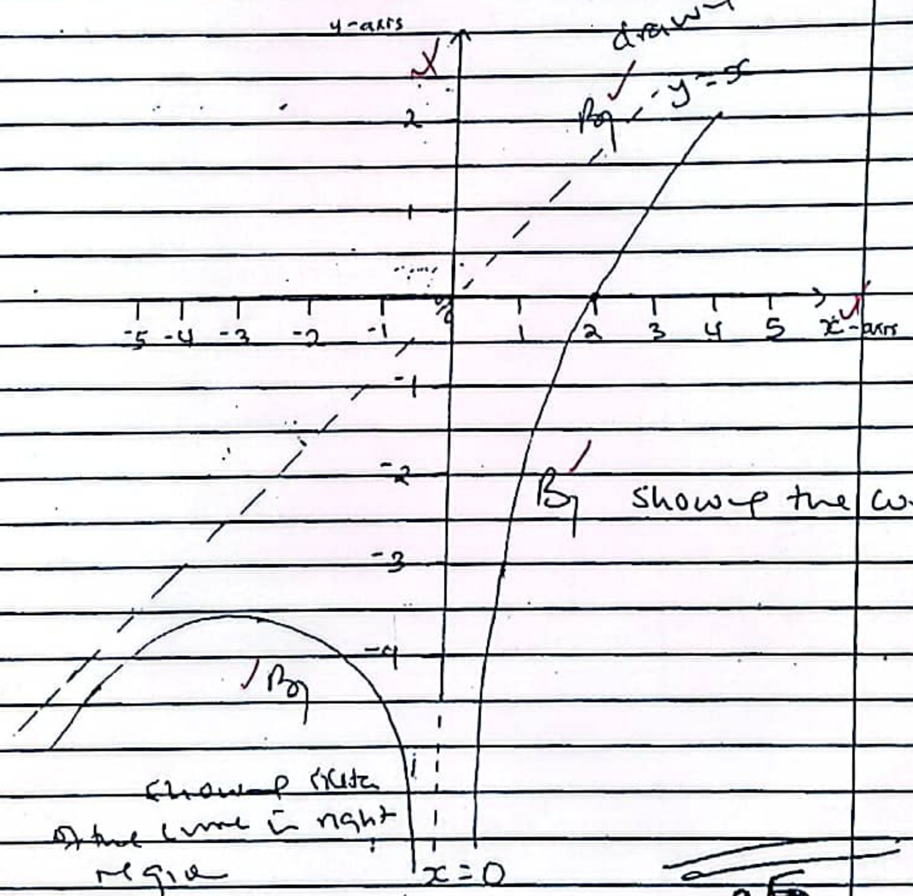

05

END