

P425/1

PURE MATHEMATICS

PAPER 1

MID 2, 2024

3 hours

Uganda Advanced Certificate of Education

S.5 Pure Mathematics

Paper 1

3 hours

INSTRUCTIONS TO LEARNERS

*Answer **all** question in section **A** and section **B***

***All** working **must** be shown clearly.*

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

Neat work is a must

SECTION A

1. Solve for x in the equation $2^{2x+8} - 2^{x+5} + 1 = 0$ (5 marks)
2. If α and β are the roots of the equation $5x^2 - 3x - 1 = 0$, form an equation with integral coefficients whose roots are $\left(2 - \frac{\beta}{\alpha}\right)$ and $\left(2 - \frac{\alpha}{\beta}\right)$. (5 marks)
3. Solve the equation $\tan 2x = \cot 3x$ for $0^\circ \leq x \leq 180^\circ$ (5 marks)
4. Prove by induction that for all positive integral values of n , $7^n + 2^{2n+1}$ is divisible by 3. (5 marks)
5. Evaluate the third term for the series $\sqrt{2} - 1 + 3 - 2\sqrt{2} + \dots$ hence find the third term. (5 marks)
6. Without using tables or calculators, show that $\tan^2 22.5^\circ = 3 - 2\sqrt{2}$ (5 marks)
7. Solve the equation $\log_{25} 4x^2 = \log_5 3 - x^2$ (5 marks)
8. Find the square root of $18 - 12\sqrt{2}$ (5 marks)

SECTION B

9. a) Given that $(x - 2)$ and $\left(x - \frac{1}{2}\right)$ are factors of $ax^2 + 5x + b$. Find the values of a and b . (5 marks)
- b) The remainder when the expression $x^3 - 2x^2 + ax + b$ is divided by $x - 2$ is five times the remainder when the same expression is divided by $x - 1$ and 12 less than the remainder when the same expression is divided by $x - 3$, find the values of a and b . (7 marks)
10. a) The n th of a series is $3^n + 4n$. Calculate the sum of the first 20 terms of the series. (5 marks)
- b) Nankya opened a bank account with shs 50,000. She deposits the same amount every year and makes no withdrawals. After how many years will she accumulate more than one million on her account if the bank offers a 5% compound interest rate per annum. (7 marks)

11. Prove by induction that,

$$\frac{1}{1 \times 7} + \frac{1}{4 \times 7} + \dots + \frac{1}{(3n - 2)(3n + 1)} = \frac{n}{3n + 1}$$

- b) Given that $(n + 1)^2 + (n + 2)^2 + \dots + (2n + 2)^2 = \frac{(n+1)(n+2)(14n+15)}{6}$ (12 marks)

12. a) Solve the equation $9\cos x - 8\sin x = 12$ for $-360^\circ \leq x \leq 360^\circ$

b) Express $10\sin x \cos x + 12\cos 2x$ in the form $R\sin(2x + \alpha)$. hence solve

$10\sin x \cos x + 12\cos 2x + 7 = 0$ in the range $0^\circ \leq x \leq 360^\circ$ (12 marks)

END

