P425/1
PURE MATHEMATICS
July/ August 2023
3 Hours

ASSHU ANKOLE JOINT MOCK EXAMINATIONS 2023

Uganda Advanced Certificate of Education

PURE MATHEMATICS

3 Hours

INSTRUCTIONS TO CANDIDATES

- *Answer **all eight** questions in section A and any **five** questions from section B*
- *Any additional question(s) answered will **not** be marked.*
- *All necessary working **must** be shown clearly*
- *Begin each answer on a fresh sheet of paper*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulas may be used.*

SECTION A (40 MARKS)

Answer all the questions

1. Solve the equation $\sin x + \sin 3x + \sin 5x + \sin 7x = 0 \ for\ 0^o \leq x \leq 90^o$ (5 marks)

2. The polynomial $x^3 + ax^2 + bx - 7$ when divided by $x^2 - x - 2$ leaves a remainder of $8x - 1$. Find the values of a and b (5 marks)

3. Use the substitution $x = 3\sin\theta$ to evaluate $\int_o^3 x\sqrt{9 - x^2}\ dx$ (5 marks)

4. Find (in vector form) equation of a line of intersection of the two planes $3x + 2y - 3z = -18$ and $x - 2y + z = 12$ (5 marks)

5. Find the equation of a circle whose centre lies on the line $x + 3y = 8$ and which touches the positive axes. (5 marks)

6. Solve the simultaneous equations

$$2(5^x) + 3^{y+2} = 53 \text{ and}$$

$$5^{x+2} - 3^{y+5} = 544$$ (5 marks)

7. Differentiate $\sqrt{\left(\frac{(2x+3)^3}{1+x^2}\right)}$ with respect to x (5 marks)

8. Given that $\frac{d}{dx}\left(\frac{x}{1-x}\right) = \frac{1}{(1-x)^2}$. Find the equation of the tangent to the curve $\frac{y}{1-y} + \frac{x}{1-x} + 5x - 3y = 0$ at the point (2, 2) (5 marks)

SECTION B (60 MARKS)

Do any five questions from this section

9. (a) Given that $z^3 = \frac{^-(5+i)}{2+3i}$. Find the three possible values of z.

(b) Given that x and its conjugate $\bar{z}$ satisfy the equation

$z\bar{z} + 2(z - i\bar{z})$=3+2i. Find the possible values of z. (6 marks)

10. The plane P has equation $x + 2y + 3z = 13$ and plane Q which is perpendicular to P has equation $ax + y + z = 4$ where a is constant. The line r_1 passes through the point (2, 9, 13) and is perpendicular to plane P and meets the line $r_2 = \begin{pmatrix} -3 \\ 7 \\ t \end{pmatrix} + \mu\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$ at point A. Find

i. Value of constant a (3 marks)

ii. Vector equation of line , r_1 perpendicular to plane P. (3 marks)

iii. Coordinates of A(intersection of r_1 and r_2) (4 marks)

iv. Value of t (2 marks)

11. (a) If $T = 2\pi\sqrt{\frac{l}{10}}$. Find the approximate increase in T if l increases from 10.0 m to 10.1 m

(5 marks)

(b) A circular cylinder open at the top is made so as to have a volume of 1cm3. If r is the radius of the base, prove that the total outside surface is $\left(\pi r^2 + \frac{2}{r}\right)$. Hence prove that this surface area is minimum when

$h = r = \frac{1}{\sqrt[3]{\pi}}$, where h is the height of the cylinder. (7 marks)

12. (a) Prove that $tan^2\left(\frac{\theta}{2} + \frac{\pi}{4}\right) = \frac{1+\sin\theta}{1-\sin\theta}$ (5 marks)

(b) Express $\sqrt{6}\ \sin 2x - \sqrt{3}\cos 2x$ in the form $R\sin(2x - \alpha)$ where $R > 0$ and $0^o < \alpha < 90^o$. Hence find the minimum and maximum values of $\frac{1}{\sqrt{6}\sin 2x - \sqrt{3}\cos 2x + 5}$ and smallest positive values of x for which they occur.

(5 marks)

13. (a) Show that $\int_1^e \frac{Inx}{x^2}\ dx = 1 - \frac{2}{e}$ (6 marks)

(b) Evaluate $\int_0^4 x\sin(5x^2)\cos(7x^2)dx$ (6 marks)

14. (a) Expand $(1 + 2x - 3x^2)^6$ in ascending powers of x up to the term in x^3

(5 marks)

(b) The first terms of the Arithmetic Progression (A.P) and Geometric Progression (G.P) are equal. The common ratio of a G.P is equal to the common difference of the A.P while the third term of the AP is also equal to the second term of the G.P. if the fourth term of an A.P is 10 find the two possible values of their first terms.

(7 marks)

15. (a) Prove that the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point P$(a\cos\theta, b\sin\theta)$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$. The tangent at P meets the x- and y-axes at A and B respectively. Show that the area of triangle AOB where O is the origin is $\frac{ab}{\sin 2\theta}$. If M is the midpoint of AB, find the locus of m. (12 marks)

(a) Form a differential equation by eliminating the constant A from

$y = Ae^{x^2}$ (3 marks)

(b) A chapati had reached at a temperature of 160degrees in an oven. It was pulled out and allowed to cool in a room temperature of 70 degrees. After 20 minutes the chapati had a temperature of 140 degrees. Given that the rate of cooling of the chapati was directly proportional to the difference between its temperature T and that of its surrounding, how much longer would it take for chapati to cool to 120 degrees.

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