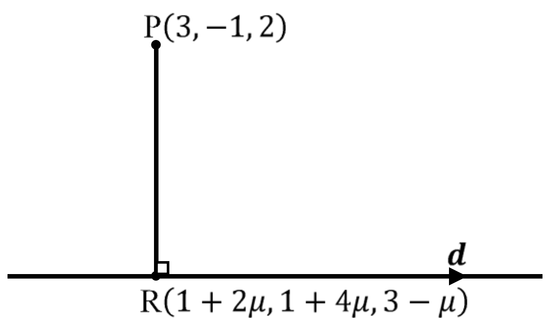
PROPOSED
MARKING GUIDE
KAMSSA
2022

NO	SOLUTION	MKS	COMMENT
1	$4\cos y = 3\tan y + 3\sec y$ $4\cos y = \frac{3\sin y}{\cos y} + \frac{3}{\cos y}$ $4cos^2 y = 3\sin y + 3$ $4(1 - sin^2 y) = 3\sin y + 3$ $4 - 4sin^2 y = 3\sin y + 3$ $4sin^2 y + 3\sin y - 1 = 0$ $(4\sin y - 1)(\sin y + 1) = 0$ Either; $4\sin y - 1 = 0$ $\sin y = 0.25$ $y = \sin^{-1}(0.25)$ $y = 14.5^0, 165.5^0$ Or; $\sin y + 1 = 0$ $y = \sin^{-1}(1)$ $y = 270^0$ $\therefore y = \{14.5^0, 165.5^0, 270^0\}$		
		05	
2	$\int_0^{\frac{\pi}{2}} x \sin 2x\, dx$ Let $u = x, \frac{dv}{dx} = \sin 2x$ $\frac{du}{dx} = 1, v = -\frac{1}{2}\cos 2x$ $\int_0^{\frac{\pi}{2}} x\sin 2x\, dx = \left[-\frac{1}{2}x\cos 2x\right]_0^{\frac{\pi}{2}} + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 2x\, dx$		

	$= \left[-\frac{1}{2}x\cos 2x\right]_0^{\frac{\pi}{2}} + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$ $= \left[\left(-\frac{\pi}{4} \times -1\right) - (0)\right] + [(0) - (0)]$ $= \frac{\pi}{4}$		
		05	
3	$5^{2t} = 5^{t+1} - 6$ $(5^t)^2 = 5^t \cdot 5^1 - 6$ Let $m = 5^t$ $m^2 = 5m - 6$ $m^2 - 5m + 6 = 0$ $(m-3)(m-2) = 0$ $m = 3, m = 2$ But $m = 5^t$ For $m = 3, 5^t = 3$ $t\log_{10} 5 = \log_{10} 3\,, t = \frac{\log_{10} 3}{\log_{10} 5} = 0.6826$ For $m = 2, 5^t = 2$ $t\log_{10} 5 = \log_{10} 2\,, t = \frac{\log_{10} 2}{\log_{10} 5} = 0.4307$		
		05	
4	$\overline{AP}:\overline{PB} = 1:2$ $\overline{PB} = 2\overline{AP}$ $\sqrt{(x-3)^2 + (y-4)^2} = 2\sqrt{(x-2)^2 + (y+3)^2}$ $(x-3)^2 + (y-4)^2 = 4[(x-2)^2 + (y+3)^2]$ $x^2 - 6x + 9 + y^2 - 8y + 16 = 4(x^2 - 4x + 4 + y^2 + 6y + 9)$ $x^2 + y^2 - 6x - 8y + 25 = 4x^2 + 4y^2 - 16x + 24y + 52$ $\therefore 3x^2 + 3y^2 - 10x + 32y + 27 = 0$ Hence, $x^2 + y^2 - \frac{10}{3}x + \frac{32}{3}y + 9 = 0$ $\left(x - \frac{5}{3}\right)^2 + \left(y + \frac{16}{3}\right)^2 = -9 + \left(\frac{5}{3}\right)^2 + \left(\frac{16}{3}\right)^2$		

	$\left(x-\frac{5}{3}\right)^2+\left(y+\frac{16}{3}\right)^2=\frac{200}{9}$ Centre, C$\left(\frac{5}{3},-\frac{16}{3}\right)$ and radius, $r=\frac{10}{3}\sqrt{2}$ units		
		05	
5	$y=\sqrt{(4+3\sin x)}$ $\frac{dy}{dx}=\frac{1}{2}(4+3\sin x)^{-1/2}\cdot 3\cos x$ $\frac{dy}{dx}=\frac{3\cos x}{2\sqrt{(4+3\sin x)}}$ $\frac{dy}{dx}=\frac{3\cos x}{2y}$ $2y\frac{dy}{dx}=3\cos x$ $2\left(y\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{dy}{dx}\right)=-3\sin x$ $2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=-3\sin x$ But $-3\sin x=4-y^2$ $2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=4-y^2$ $\therefore 2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2+y^2=4$		
		05	
6	 Distance $=\lvert \boldsymbol{PR}\rvert$ $\boldsymbol{PR}=\begin{pmatrix}1+2\mu\\1+4\mu\\3-\mu\end{pmatrix}-\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}-2+2\mu\\2+4\mu\\1-\mu\end{pmatrix}$ $\boldsymbol{PR}\cdot\boldsymbol{d}=0$		

	$\begin{pmatrix}-2+2\mu\\2+4\mu\\1-\mu\end{pmatrix}\cdot\begin{pmatrix}2\\4\\-1\end{pmatrix}=0$ $-4+4\mu+8+16\mu-1+\mu=0$ $21\mu=-3 \qquad \therefore \mu=-\frac{1}{7}$ $\boldsymbol{PR}=\begin{pmatrix}-2+2\left(-\frac{1}{7}\right)\\2+4\left(-\frac{1}{7}\right)\\1+\frac{1}{7}\end{pmatrix}=\begin{pmatrix}-\frac{16}{7}\\\frac{10}{7}\\\frac{8}{7}\end{pmatrix}$ Distance $=\sqrt{\left(-\frac{16}{7}\right)^2+\left(\frac{10}{7}\right)^2+\left(\frac{8}{7}\right)^2}=2.9277$ units		
		05	
7	Let the common root be t $t^2+kt-6k=0$ ……………(i) $t^2-2t-k=0$ ……………(ii) $(i)-(ii); t(k+2)-5k=0$ $t=\frac{5k}{k+2}$ Substituting for t into (i) $\left(\frac{5k}{k+2}\right)^2+k\left(\frac{5k}{k+2}\right)-6k=0$ $25k^2+5k^2(k+2)-6k(k+2)^2=0$ Dividing through by k $25k+5k(k+2)-6(k+2)^2=0$ $25k+5k^2+10k-6(k^2+4k+4)=0$ $25k+5k^2+10k-6k^2-24k-24=0$ $k^2-11k+24=0$ $(k-3)(k-8)=0$ $\therefore k=3, k=8$		
		05	
8	$\frac{dv}{dt}=200\ cm^3s^{-1}$ $v=\frac{4}{3}\pi r^3$		

	$\frac{dv}{dr} = 4\pi r^2$ $\frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt}$ $200 = 4\pi r^2 \cdot \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{50}{\pi r^2}\ cms^{-1}$ $A = 4\pi r^2$ $\frac{dA}{dr} = 8\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$ $= 8\pi r \times \frac{50}{\pi r^2} = \frac{400}{r}$ When $r = 80\ mm = 8\ cm$ $\therefore \frac{dA}{dt} = \frac{400}{8} = 50\ cm^2 s^{-1}$		
		05	
9	(a) $\int \frac{1}{e^{2x}-1} dx$ Let $e^{2x} - 1 = u$ $2e^{2x} dx = du$ $dx = \frac{du}{2e^{2x}} = \frac{du}{2(1+u)}$ $\int \frac{1}{e^{2x}-1} dx = \int \frac{1}{u} \cdot \frac{du}{2(1+u)}$ Let $\frac{1}{u(1+u)} \equiv \frac{A}{u} + \frac{B}{1+u}$ $1 \equiv A(1+u) + Bu$ When $u = 0; 1 = A$ $\therefore A = 1$ When $u = -1; 1 = -B$ $\therefore B = -1$ $\int \frac{1}{e^{2x}-1} dx = \frac{1}{2} \int \frac{1}{u} du - \frac{1}{2} \int \frac{1}{1+u} du$ $= \frac{1}{2} \ln u - \frac{1}{2} \ln(1+u) + c$ $= \frac{1}{2} \ln\left(\frac{u}{1+u}\right) + c$ $= \frac{1}{2} \ln\left(\frac{e^{2x}-1}{e^{2x}}\right) + c$		

	(b) Let $u = \tan {}^{t}/_{2}$ $du = \frac{1}{2} sec^2 \, {}^{t}/_{2} \, dt$ $2du = (1+u^2)dt$ $dt = \frac{2du}{1+u^2}$ $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos t} dt = \int_0^1 \frac{1}{1+\frac{1-u^2}{1+u^2}} \cdot \frac{2du}{1+u^2}$ $= \int_0^1 \frac{2}{2} du$ $= u]_0^1$ $= 1 - 0 = 1$		
		12	
10	(a) $u_{r+1} = {}^{n}C_r \cdot a^{n-r} \cdot x^r$ $= {}^{18}C_r \cdot \left(\frac{1}{x^2}\right)^{18-r} \cdot (-x)^r$ $= {}^{18}C_r \cdot (x^{-2})^{18-r} \cdot (-1)^r \cdot x^r$ ⇨ $-36 + 2r + r = 3$ $3r = 39 \qquad \therefore r = 13$ $u_{14} = {}^{18}C_{13} \cdot x^{-10} \cdot (-1)^{13} \cdot x^3$ $= -8568\, x^3$ $\therefore$ the coefficient of x^3 in the expansion is -8568 (b) $\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}}$ $= \sqrt{\frac{(1+x)^2}{1-x^2}}$ $= \frac{1+x}{\sqrt{1-x^2}}$ $= (1+x)(1-x^2)^{-1/2}$ $= (1+x)\left(1 - \frac{1}{2}(-x^2) + \cdots\right)$ $= (1+x)\left(1 + \frac{1}{2}x^2 + \cdots\right)$		

	$= 1 + \frac{1}{2}x^2 + x + \cdots$ $\therefore \sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2 + \cdots$ Hence putting $x = \frac{1}{7}$ $\sqrt{\left(\frac{1+\frac{1}{7}}{1-\frac{1}{7}}\right)} \approx 1 + \frac{1}{7} + \frac{1}{2}\left(\frac{1}{7}\right)^2$ $\sqrt{\frac{4}{3}} \approx \frac{113}{98}$ $\frac{2}{\sqrt{3}} \approx \frac{113}{98}$ $\therefore \sqrt{3} \approx \frac{196}{113}$		
		12	
11	(a) $2A + B = 45^0$ $B = 45^0 - 2A$ Taking tan on both sides, $\tan B = \tan(45^0 - 2A)$ $= \frac{\tan 45^0 - \tan 2A}{1 + \tan 45^0 \tan 2A}$ $= \frac{1 - \frac{2\tan A}{1 - tan^2 A}}{1 + \frac{2\tan A}{1 - tan^2 A}}$ $= \frac{1 - 2\tan A - tan^2 A}{1 + 2\tan A - tan^2 A}$ (b) Let $\tan^{-1}(2x) = A, \tan^{-1}(3x) = B$ $\tan A = 2x, \tan B = 3x$ $A + B = \frac{\pi}{4}$ $\tan(A + B) = \tan\frac{\pi}{4}$ $\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$ $2x + 3x = 1 - (2x)(3x)$		

	$5x = 1 - 6x^2$ $6x^2 + 5x - 1 = 0$ $(6x - 1)(x + 1) = 0$ $x = \frac{1}{6}$ or $x = -1$ $\therefore x = \frac{1}{6}$		
		12	
12	(a) Let $y = \tan x$ $y + \delta y = \tan(x + \delta x)$ $x = 60^0, \delta x = 1^0 = \frac{\pi}{180}$ $\frac{dy}{dx} = sec^2 x$ $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ $\delta y \approx \frac{dy}{dx} \cdot \delta x$ $\delta y \approx sec^2(60^0) \cdot \frac{\pi}{180} \approx \frac{\pi}{45}$ $\tan 61^0 \approx y + \delta y$ $\approx \tan 60^0 + \frac{\pi}{45}$ $\approx \sqrt{3} + \frac{\pi}{45}$ $\approx \frac{45\sqrt{3}+\pi}{45}$ (b) Let $y = cosecx = \frac{1}{\sin x}$ $y + \delta y = \frac{1}{\sin(x+\delta x)}$ $\delta y = \frac{1}{\sin(x+\delta x)} - \frac{1}{\sin x}$ $\delta y = \frac{\sin x - \sin(x+\delta x)}{\sin x \sin(x+\delta x)}$ $\delta y = \frac{2\cos\left(x+\frac{\delta x}{2}\right)\sin\left(-\frac{\delta x}{2}\right)}{\sin x \sin(x+\delta x)}$		

	$\frac{\delta y}{\delta x} = \frac{2\cos\left(x+\frac{\delta x}{2}\right)\frac{\sin\left(-\frac{\delta x}{2}\right)}{\delta x}}{\sin x \sin(x+\delta x)}$ As $\delta x \to 0, \frac{\sin\left(-\frac{\delta x}{2}\right)}{\delta x} \to -\frac{1}{2}, \frac{\delta y}{\delta x} \to \frac{dy}{dx}$ $\frac{dy}{dx} = -\frac{\cos x}{\sin x \cdot \sin x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \cdot cosecx$ $\therefore \frac{d}{dx}(cosecx) = -cosecx \cot x$		
		12	
13	(a) $x^2 + 4x - 8y - 4 = 0$ $x^2 + 4x = 8y + 4$ $x^2 + 4x + (2)^2 = 8y + 4 + (2)^2$ $(x+2)^2 = 8y + 8$ $(x+2)^2 = 8(y+1)$ Let $x + 2 = X, y + 1 = Y$ $X^2 = 8Y$ $\therefore$ since $X^2 = 8Y$ is in the form $x^2 = 4ay$, it's a parabola. ii) compare with $(x-h)^2 = 4a(y-k)$ $h = -2, k = -1$ $\therefore$ coordinates for the vertex $= (-2, -1)$ (b) From $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $b^2x^2 + a^2y^2 = a^2b^2$ $b^2x^2 + a^2(mx+c)^2 = a^2b^2$ $b^2x^2 + a^2(m^2x^2 + 2mcx + c^2) = a^2b^2$ $b^2x^2 + a^2m^2x^2 + 2a^2mcx + a^2c^2 = a^2b^2$ $(b^2 + a^2m^2)x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$ For tangency; $b^2 = 4ac$ $(2a^2mc)^2 = 4(b^2 + a^2m^2)(a^2c^2 - a^2b^2)$		

	$2a^4m^2c^2 = 4[a^2b^2c^2 - a^2b^4 + a^4m^2c^2 - a^4b^2m^2]$ $0 = a^2b^2c^2 - a^2b^4 - a^4b^2m^2$ $a^2b^2c^2 = a^2b^4 + a^4b^2m^2$ Dividing through by a^2b^2 $c^2 = b^2 + a^2m^2$ Comparing, $a^2 = 9, b^2 = 4$ If they are parallel, then $m = 1$ $c^2 = 4 + 9 \times 1$ $c^2 = 13 \qquad \therefore c = \pm\sqrt{13}$ $\therefore y = x \pm \sqrt{13}$ are the tangents to the ellipse		
		12	
14	$\arg\left(\frac{z-3}{z-2i}\right) = \frac{\pi}{4}$ $\arg(z-3) - arg(z-2i) = \frac{\pi}{4}$ $\arg(x-3+iy) - \arg\big(x + i(y-2)\big) = \frac{\pi}{4}$ $\tan^{-1}\left(\frac{y}{x-3}\right) - \tan^{-1}\left(\frac{y-2}{x}\right) = \frac{\pi}{4}$ Let $\tan^{-1}\left(\frac{y}{x-3}\right) = A, \tan^{-1}\left(\frac{y-2}{x}\right) = B$ $\tan A = \frac{y}{x-3}, \tan B = \frac{y-2}{x}$ $A - B = \frac{\pi}{4}$ $\tan(A-B) = \tan\frac{\pi}{4}$ $\frac{\tan A - \tan B}{1 + \tan A \tan B} = 1$ $\tan A - \tan B = 1 + \tan A \tan B$ $\frac{y}{x-3} - \frac{y-2}{x} = 1 + \frac{y}{x-3} \times \frac{y-2}{x}$ $\frac{xy - xy + 2x + 3y - 6}{x^2 - 3x} = 1 + \frac{y^2 - 2y}{x^2 - 3x}$ $2x + 3y - 6 = x^2 - 3x + y^2 - 2y$		

	$x^2 + y^2 - 5x - 5y + 6 = 0$ $\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2$ $\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{13}{2}$ Description: The locus is half a circle with centre, C$\left(\frac{5}{2}, \frac{5}{2}\right)$ and radius, $r = \sqrt{\frac{13}{2}}$ units		
		12	
15	(a) $\boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & -\boldsymbol{j} & \boldsymbol{k} \\ 6 & -2 & 1 \\ -1 & 3 & -7 \end{vmatrix}$ $= \boldsymbol{i}\begin{vmatrix} -2 & 1 \\ 3 & -7 \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} 6 & 1 \\ -1 & -7 \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} 6 & -2 \\ -1 & 3 \end{vmatrix}$ $= 11\boldsymbol{i} + 41\boldsymbol{j} + 16\boldsymbol{k}$ $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 11 \\ 41 \\ 16 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ 41 \\ 16 \end{pmatrix}$ $11x + 41y + 16z = 11 + 0 - 16$ $\therefore 11x + 41y + 16z = -5$ (b) If the line is perpendicular to the plane, then the normal to the plane it's the direction of the line $\boldsymbol{r} = \boldsymbol{a} + \mu\boldsymbol{d}$ $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} 11 \\ 41 \\ 16 \end{pmatrix}$ Let $\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} 11 \\ 41 \\ 16 \end{pmatrix}$		

	$\therefore \frac{x-4}{11} = \frac{y-4}{41} = \frac{z+1}{16}$		
		12	
16	Let T = temperature of the liquid Difference = $(T-15)$ $\frac{dT}{dt} \propto (T-15)$ $\frac{dT}{dt} = -k(T-15)$ $\frac{dT}{T-15} = -kdt$ $\int \frac{dT}{T-15} = \int -kdt$ $\ln(T-15) = -kt + c$ $T - 15 = e^{-kt+c}$ $T - 15 = e^{-kt} \cdot e^{c}$ $T - 15 = Ae^{-kt}$ When $t = 0, T = 50^0$C $50 - 15 = Ae^0 \qquad \therefore A = 35$ $T = 15 + 35e^{-kt}$ When $t = 20\ mins, T = 35^0$C $35 - 15 = 35e^{-20k}$ $e^{-20k} = \frac{20}{35} = \frac{4}{7}$ $20k = \ln\left(\frac{7}{4}\right) \qquad \therefore k = \frac{1}{20}\ln\left(\frac{7}{4}\right)$ When $t = 26\ mins, T = ?$ $T = 15 + 35e^{-26\times\frac{1}{20}\ln\left(\frac{7}{4}\right)}$ $T = 31.90902558$ $\therefore T = 31.9^0$C		
		12	