P425/1
Pure Mathematics
Paper 1
July/August 2022
3 hours

BUGANDA EXAMINATIONS COUNCIL MOCKS

Uganda advanced Certificate of Education

Pure Mathematics

Paper I

3hours

INSTRUCTIONS TO CANDIDATES

- *Answer eight questions in Section **A** and ONLY 5 from Section **B**.*
- *All working **MUST** be clearly shown.*
- *Mathematical tables with a list of formulars and graph paper will be provided.*
- *Use a silent non programmable calculator.*
- *State the level of accuracy for answers got and indicate (tab) for Mathematics tables and (cal) for calculator used.*
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SECTION A (40MARKS)

1. If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx} - y\right)$. *(05marks)*

2. Given that $\log_{10} 2 = a$, prove that $\log_8 5 = \frac{1-a}{3a}$. *(05marks)*

3. Solve by echelon method the following set of simultaneous equation:

$$x + 3y + z = 6$$
$$2x + y - 4z = 7$$
$$5x - 6y + z = 9$$

(05marks)

4. Partialise $f(x) = \frac{1}{(x+1)(x-3)}$ hence evaluate $\int f(x) \quad dx$. *(05marks)*

5. The letters of the word ENGLISH are to be arranged in a row. In how many ways can this be done? In how many of these arrangements are the vowels separate? *(05 marks)*

6. Solve in the range $0 \leq x \leq 360$ for tan x + cot x = 2. *(05marks)*

7. Given that the vectors ai – 2j + k and 2ai + aj – 4k are perpendicular, find the possible values of the constant a. *(05marks)*

8. Form the equation of the circle through points A(1, 1) B(3, 0) and C(0, -2). Hence state its radius and the length of its tangent from P (0, 10). *(05marks)*

SECTION B (60MARKS)

9. If x is small enough so that terms in x^3 and higher powers may be ignored, use binomial expansion to show that $\sqrt{\left(\frac{1-x}{1+2x}\right)} = 1 - \frac{3x}{3} + \frac{15x^2}{8}$. (05*marks)*

b) Expand by use of Maclaurin's series up to the term in x^2 the function $f(x) = \sin x$. Hence evaluate $\sin 30^0$ to 4 decimal places. *(04marks)*

c) The tenth term of an arithmetic progression (A.p) is 69 and the sum of the first 30 terms is four times the sum of the first ten terms. Find the first term and the common difference of the A.p. *(04 marks)*

10. a) Solve the differential equation using the substitution $y = vx$ for $(x^2 - y^2)\frac{dy}{dx} = xy$ *when* $x = 3$ *and* $y = -2$. *(06marks)*

b) A small metal piece initially at 20^0C is dropped into a large container of water kept at 100^0C. It was observed that the temperature of the metal increased by 2^0C in one minute.

(i) How long will it take for the temperature of the metal to increase to 90^0C?

(ii) Find the temperature of metal after 20minutes. *(06marks)*

11a)(i) Form the equation of the plane perpendicular to line $\frac{x-3}{2} = \frac{y+1}{-5} = \frac{z-4}{2}$ passing through a point A(5, -6, 6). *(04marks)*

(ii) Determine the point B where the formed plane meets the line in (i) above. *(04 marks)*

b) Determine the shortest distance from the point P(2, -5, 3) to the line $\frac{x-1}{4} = \frac{y+3}{1} = \frac{z-2}{-2}$.

(04 marks)

13a) Prove that $\frac{\sin 3A \sin 6A + \sin A \sin 2A}{\sin 3A \cos 6A + \sin A \cos 2A} = \tan 5A$. *(04 marks)*

b) Show that $\tan^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi}{4}$. *(04 marks)*

c) Solve for θ in the range $0 \le \theta \le 2\pi$ if $4\cos\theta + 3\sin\theta = 5$ *04 marks)*

14. Sketch the following curve systematically $y = \frac{3(x-3)}{(x+1)(x-2)}$. *(12marks)*

15a) Determine the square root of the complex number 15 + 8i. *(04 marks)*

b) Given a complex number $Z = \frac{(1+3i)(i-2)^2}{i-3}$, determine;

(i) Z in the form a + b*i* where a and b are constants.

(ii) arg(Z) *(04marks)*

c) Evaluate $\dfrac{\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^4}{\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)3}$ and give the solution in modulus-argument form. *(04marks)*

16a) Show from first principles that $if\ y = sin2x\ then \frac{dy}{dx} = 2cos2x$ *(06 marks)*

(b) A right circular cone and cylinder have equal height and equal radius. The area of the cone is joined to one circular end of the cylinder. The slant length of the cone is $6\sqrt{3}cm$ and constant. Prove that as the height x of the cone varies the volume V of the combined body is given as $V=\frac{4}{3}\pi x\left(108-x^2\right)cm^3$. Hence find the value of x at which V is maximum and state the maximum values of V. *(06 marks)*

17a) A conic section is given by $y^2 = 16x$. Determine the focus, length of latus rectum and equation of the tangent at P(1,4). *(08 marks)*

b) The cord PQ to the parabola $y^2 = 4ax$ for point P(a$p^2, 2ap)\ and\ Q(aq^2, 2aq)$ subtend a right angle at the origin O(0,0). Show that $pq = -4$. *(04 marks)*

END