APPLIED MATHEMATICS

STATISTICS

1. For a particular set of observations, $\sum f = 20, \sum fx^2 = 16143$ and $\sum fx = 56.3$. Find the value of the;

(i) Mean

(ii) Standard deviation ***(05 marks)***

2. A set of digits consists of m zeros and n ones.

(i) Find the mean of this set of data

(ii) Hence show that the standard deviation of the set of digits is; $\frac{\sqrt{mn}}{m+n}$

(05 marks)

3. A bag contained five balls each bearing one of the numbers 1, 2, 3, 4, 5. A ball was drawn from the bag its number noted and then replaced. This was done 50 times in all and the table below shows the resulting frequency distribution. If the mean is 2.7, Find the value of x and y.

Number	1	2	3	4	5
Frequency	x	11	y	8	9

(05 marks)

4. The data below shows the weights of some students in senior five class.

Mass	-<45	-<50	-<55	-<60	-<65	-<70	-<75
Frequency	3	30	39	33	13	1	1

Calculate the mean mass of the students. ***(05 marks)***

5. The table below is the distribution of weights of a group of animals.

Mass(Kg)	21 – 25	26 – 30	31 – 35	36 - 40	41 – 50	51 – 65	66 – 75
Frequency	10	20	15	10	30	45	5

(a) Construct a histogram for the above data and use it to estimate the mode.

(b) Calculate the medium for the above data.

(c) Calculate mode. ***(05 marks)***

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6. The numbers $a, b, 8, 5, 7$ have a mean of 6 and variance 2. Find the values of a and b if $a > b$. ***(05 marks)***

7. The table shows the speed of 200 vehicles passing a particular point.

Speed (Km/h^{-1})	30-	40-	50-	60-	70-
Frequency	14	30	52	71	77

Find the mean speed ***(05 marks)***

8. The table gives the frequency distribution of heights (in cm) of 400 children in a certain school.

Height	<110	<120	<130	<140	<150	<160	<170
Frequency	27	58	130	105	50	25	5

(a) Draw a cumulative frequency curve for the above data. Hence estimate

(i) The median

(ii) The interquatile range

(iii) The 10th to 90th percentile range

(b) Calculate the modal height ***(12 marks)***

9. The table below shows the height distribution of seedlings.

Height(cm)	5 – 9	10 – 14	15 – 19	20 – 24	24 - 29
Frequency	18	24	46	32	30

Calculate;

(i) The modal height

(ii) The number of seedlings of height less than 17 cm. ***(05 marks)***

10. A class performed an experiment to estimate the diameter of a circular object. A sample of five students had the following results in centimeters.
3.12, 3.16, 2.94, 3.33 and 3.00

Determine the sample

(a) Mean

(b) Standard deviation ***(05 marks)***

11. The table below shows marks obtained by 120 students in a test.

Marks	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
No of students	5	15	35	10	25	8	7	5

(a). Calculate;

(i) The mean

(ii) Standard deviation

(b). Plot an Ogive for this data and use it to estimate;

(i) Mean

(ii) The number of students with marks that are within one standard deviation of the mean. **(12 marks)**

12. The table below shows the cumulative marks scored by 100 students in a math test.

Marks (%)	<10	<20	<30	<40	<50	<60	<70	<80	<90	<100
Cumulative frequency	5	9	16	26	41	60	77	90	97	100

(a). Calculate the;

(i) mean mark

(ii) standard mark

(iii) standard deviation

(b). Draw a cumulative frequency and use it to estimate the;

(i) Median mark

(ii) Pass mark if 70 students passed **(12 marks)**

13. The distribution below shows the weighs of babies in Gombe hospital.

3, 5, 3, 9, 6, 8, 20, 19, 24, 14, 12

(a). Find the;

(i) Upper quartile

(ii) Standard deviation **(05 marks)**

14. The table below shoes the frequency distribution of marks obtained in paper one of the mathematics contest by GLISS students.

Marks (%)	10-	20-	40-	50-	60-	70-	80 - 90
Frequency	18	34	42	24	10	6	8

(a). Calculate the;

(i) Mean mark

(ii) Standard deviation

(iii) Number of student who scored above 54%

(b). Draw a cumulative frequency curve and use it to estimate the;

(i) 5^{th} decile

(ii) Number of students that would not qualify for paper two if the pass mark is fixed at 40%. **(12 marks)**

15. A discrete random variable X has the following probability distribution.

X	0	1	2	3	4	5
$P(X = x)$	0.11	0.17	0.2	0.13	P	0.09

(a). Find the;

(i) Value of P **(02 marks)**

(ii) Expected value of X **(03 marks)**

16. The table below shows the marks obtained in a mathematics test by a group of students.

Marks	5-<15	15-<25	25-<35	35-<45	45-<55	55-<65	65-<75	75-<85
Number of students	5	7	19	17	7	4	2	3

(a). Construct a cumulative frequency curve (Ogive) for the data. **(05 marks)**

(b). Use your Ogive to find the;

(i) Range between the 10^{th} and 70^{th} percentiles

(ii) Probability that a student selected at random scored below 50 marks.

(07 marks)

17. The times taken for 55 students to have their lunch to the nearest minute are given in the table below.

Time(minutes)	3 – 4	5 – 9	10 – 19	20 – 29	30 – 44
Number of students	2	7	16	21	9

(a). Calculate the mean time for students to have lunch **(04 marks)**

(b). Draw a histogram for the given data

(c). Use your histogram to estimate the modal time for students to have lunch. **(08 marks)**

18. The data below shows the length in centimeters of different calendars produced by a printing press. A cumulative frequency distribution was formed.

Length (cm)	<20	<30	<35	<40	<50	<60
Cumulative frequency	4	20	32	42	48	50

(a). Construct a frequency distribution table. **(02 marks)**

(b). Draw a histogram and use it to estimate the modal length. **(06 marks)**

(c). Find the mean length of calendars **(04 marks)**

19. The cumulative frequency table below shows ages in years of employees of a certain company.

Age (years)	<15	<20	<30	<40	<50	<60	<65
Cumulative frequency	0	17	39	69	87	92	98

(a). Use the data in the table to draw a cumulative frequency curve(Ogive)

(b). Use the curve to estimate the semi-interqurtile range **(06 marks)**

(c). Calculate the mean age of the employees. **(06 marks)**

20. The heights (cm) and ages (years) of a random sample of ten farmers are given in the table below.

Heights(cm)	156	151	152	160	146	149	142	158	140
Ages(years)	47	38	44	55	46	49	30	45	30

(a). Calculate the;

(i) Rank coefficient

(ii) Comment on your results **(06 marks)**

(b). Plot a scatter diagram for the data. Hence draw lines of best fit **(02 marks)**

(c). Use your diagram in (b) to find;

(i) y when $x = 147$

(ii) x when $y = 43$ **(04 marks)**

21. The table below shows the marks obtained in examination by 200 candidates.

Marks (%)	Number of candidates
10 – 19	18
20 – 29	34
30 – 39	58
40 – 49	42
50 – 59	24
60 – 69	10
70 – 79	6
80 – 89	8

(a). Calculate the;

(i) Mean mark

(ii) Modal mark **(05 marks)**

(b). Draw a cumulative frequency curve for the data
Hence estimate the lowest mark for a distinction one if top 50% of the candidates qualify for the distinction. **(07 marks)**

22. The heights and masses of ten students are given in the table below.

Heights(cm)	156	151	152	146	160	157	149	142	158	141
Mass (kg)	62	58	63	58	70	60	55	57	68	56

(a). (i) Plot the data on a scatter diagram

(ii) Draw the line of best fit hence estimate the mass corresponding to height of 55cm. **(06 marks)**

(b). (i) Calculate the rank correlation coefficient for the data

(ii) Comment on the significance of the heights on masses of the students (spearman's ϑ= 0.79 and Kendall's = τ = 0.64 at 1% level of significance based on 10 observations) **(06 marks)**

23. A teacher gave tow tests in chemistry, five students were graded as follows:

	GRADE				
Test 1	A	B	C	D	E
Test 2	B	A	C	D	E

Determine the rank correlation coefficient between the two tests. Comment on your results. **(05 marks)**

24. The table shows scores of students in mathematics and English tests.

Mathematics	72	65	82	54	32	74	40	53
English	58	50	86	35	76	43	40	60

Calculate the rank correlation coefficient for the students' performance in the two subjects. **(05 marks)**

25. The table below shows the values of two variables P and Q

P	14	15	25	20	15	7
Q	30	25	20	18	5	22

Calculate the rank correlation coefficient between the two variables. **(05 marks)**

26. The table below shows heights in centimeters of 25 students in GLISS.

Heights(cm)	<10	<20	<25	<30	<50	<55	<65
No. of students	0	3	7	15	17	23	25

(a). Calculate the;

(i) Mean height

(ii) Variance

(iii) Mode

(iv) Middle 80% range of the heights **(12 marks)**

REGRESSIONS AND CORRELATION

27. The following table gives the test results for 10 children.

Child	A	B	C	D	E	F	G	H	I	J
Math(x)	1	8	15	18	23	28	33	39	45	45
English(y)	3	14	8	20	19	17	36	26	14	29

(a) Draw a scatter diagram for the above data. Hence comment on the relationship between the maths and English marks.

(b) Draw line of best fit on the scatter diagram and use it to estimate the mark of a student who scored 30 marks I English.

(c) Calculate the rank correlation coefficient for the above data. Hence comment your result at 5% level of significance. **(12 marks)**

28. Five students obtained the following A – level grades in mid-term and end of term II examinations in a certain subject.

Mid-term(M)	A	B	C	D	E
End of term (E)	B	A	C	D	E

(a) Determine the rank correlation coefficient between mid-term and end of term examinations. **(04 marks)**

(b) Comment on your results in (a) above. **(01 mark)**

29. The height (cm) and ages (years) of a random sample of 10 farmers are given in the table below.

Height x (cm)	156	151	152	160	146	157	149	142	158	140
Age, Y (years)	47	38	44	55	46	49	45	30	45	30

(a) Calculate the rank correlation coefficient

(b) Comment on your results

(c) Plot a scatter diagram for the data and comment on the relationship of the data. Draw line of best fit.

(d) Use your graph to find Y when X = 147.

30. The table shows the percentage of sand, y into soil at different depths, X (cm).

Soil depth(x)	35	65	55	25	45	75	20	90	51	60
Percentage of sand (y)	86	70	84	92	79	68	97	58	86	77

(a) Calculate the rank correlation between the two variables.

(b) Comment on the significance at 5% level.

(c) Draw a scatter diagram for the data and comment on our results.

(d) Draw the line of best fit, hence estimate the;

(i). Percentage of sand in the soil at a depth of 31cm

(ii). Depth of the soil with 54 sand

31. The table below shows the grades scored by 8 candidates in applied maths and overall score in mathematics on mocks of a certain school.

Applied maths	D1	D2	C4	C6	C6	F9	C5	C6
Overall grade	A	C	B	D	E	F	C	O

(i). Calculate the rank correlation coefficient for the data.

(ii). Test whether it is significant at 5% level.

INDEX NUMBERS

32. In 1991, the index number of the value of commodity was 135 when 1989 was taken as base year. The value of the commodity in 1991 was Sh.5400 and in 1990 was Sh,4600. Find;

(i). The value of the commodity in 1989. **(02 marks)**

(ii). The index number of the value of the commodity in 1990 when 1989 was taken as the base year. **(03 marks)**

33. Calculate a weighted price index for the following figures for 1994 based on 1990. (Give your answer to the nearest integer) Hence comment on your results.

Item	1990 price (£)	1994 price (£)	Weight, W
Food	55	60	4
Housing	48	52	2
Transport	16	20	1

34. The table below shows the prices in shillings of flour and eggs in 1990 and 2000.

Item	Price (Shs)	
	1990	2000
Flour (1kg)	3,000	5,400
Eggs (1 dozen)	5,000	7,800

Calculate the simple aggregate price index for the above data, taking 1990 as the base year. **(05 marks)**

35. The table below shows the expenditure of restaurant for the years 2014 and 2016.

Item	Price (Shs)		Weight
	2014	2016	
Milk (per litre)	1,000	1,300	0.5
Eggs (per tray)	6,500	8,300	1
Sugar (per kg)	3,000	3,800	2
Blue band	7,000	9,000	1

(a) Taking 2014 as the base year, calculate for 2016 for;

(i). Price relative for each item

(ii). Simple aggregate price index

(iii). Weighted aggregate price index and comment on your result.

(b) In 2016, the restaurant spent Shs.45,000 on buying these items. Using the index obtained in (c), find how much money the restaurant could have spent in 2014. **(12 marks)**

36. The table below shows the wages of workers in thousands of shillings.

Category	Monthly wage		Number of workers
	2016	2017	
1	1,200	1,920	180
2	1,500	2,850	165
3	1,650	3,300	100
4	1,700	4,250	55

(a) Calculate the weighted aggregate index number for the monthly wage in 2017.

(b) Comment on your results in (A) above,

37. The table below shows the average monthly wage in thousands of shillings of workers in category in 2014 and 2016 for a certain soft drinks factory.

Category	Monthly wage		Index number	Number of workers
	2014	2016		
1	120	192	160	180
2	150	285	X	165
3	Y	330	200	100
4	170	Z	250	55

Taking 2014 = 100%, find the;

(i). Values of X, Y and Z

(ii). Weighted index number for the monthly wage of the whole factory in 2016.

(05 marks)

38. The table below shows the retail prices (shs) and amount of each item bought weekly by a restaurant in 2002 and 2003.

Item	**Price (shs)**		**Amount bought**
	2002	**2003**	
Milk (per litre)	400	500	200
Eggs (per tray)	2,500	3,000	18
Cooking oil (per litre)	2,400	2,100	2
Baking flour (per packet)	2,000	2,200	15

(a) Taking 2002 as the base year, calculate the weighted aggregate price index.

(b) In 2003, the restaurant spent shs450,000 on buying these items. Using the weighted aggregate price index obtains in (a), calculate what the restaurant could have spent in 2002.

39. The following information relates to three products sold by a company in the year 2001 and 2004.

Product	2001		2004	
	Quantity in thousands	Selling price	Quantity in thousands	Selling price per unit
A	76	0.60	72	0.18
B	52	0.75	60	1.00
C	28	1.10	40	1.32

Calculate the;

(i). Percentage increase in the sales over the period.

(ii). Corresponding percentage increase in income over the period.

PROBABILITY THEORY

1. Two events and N are such that $P(M) = 0.7, P(M \cap N) = 0.45$ and $P(M' \cap N') = 0.18$. Find;

(i). $P(N')$ **(03 marks)**

(ii). $P(M \text{ or } N \text{ but not } M \text{ and } N)$ **(02 marks)**

2. X and Y are two independent events such that P(X') = 0.6 and P(X∩Y) = 0.2.

(a) Show that X' and Y' are also independent

(b) Find;

(i). P(Y)

(ii). P(X'∩Y')

3. A and B are two events such that P(A) = $\frac{2}{3}$, P($^{A}/_{B}$) = $\frac{1}{2}$ and P($^{B}/_{A}$) = $\frac{2}{3}$. Find;

(i). P(A∩B)

(ii). P(AUB) **(05 marks)**

4. Two ordinary dice are thrown. Find the possibility space for the sum of the scores obtained. Hence find the probability that the sum of the scores obtained.

(i). Is a multiple of 5 **(01 mark)**

(ii). Is greater than 9 **(01 mark)**

(iii). Is a multiple of 5 or greater than 9 **(02 marks)**

(iv). Is a multiple of 5 and is greater than 9 (02 marks)

5. Given that A and B are mutually exclusive events and that P(A) = $\frac{2}{5}$ and P(B) = $\frac{1}{2}$.

Find;

(i). P(AUB) **(02 marks)**

(ii). P(A'∩B') **(02 marks)**

6. In how many ways can the letters of the "FACETIOUS" be arranged in a line. What is the probability that an arrangement begins with "F" and ends with "S"?

7. (a) In a game, a player tosses three fair coins. He wins £10 if 3 heads occur, £x if 2 heads occur, £3 if 1 head occurs and £2 if no head occur. Express in terms of x, his expected gain from each game. Given that he pays £4.50 to play each game, Calculate the value of x for which the game is fair.

(b) A committee of three (3) is to be chosen from 4 girls and 7 boys.

(i). Form a probability distribution for the number of girls on the committee and show that the experiment is random.

(ii). Find the expected number of girls on the committee. **(12 marks)**

8. The events A and B are such that $P(A/B) = 0.4$, $P(B/A) = 0.25$, $P(A \cap B) = 0.12$. Find $P(A \cup B')$ **(05 marks)**

9. The probability that a fisherman catches fish on a clear day is $\frac{2}{5}$ and on cloudy day is $\frac{7}{10}$. If the probability that the day is cloudy is $\frac{3}{5}$, find the probability that the day is cloudy given that the fisherman does not catch fish.

10. A discrete random variable X has a probability distribution given below.

x	-1	0	1	2	3
$P(X = x)$	$1/8$	a	$1/8$	$3/8$	$3/2$a

Find the;

(i). Value of a

(ii). $E(X^2)$

11. A discrete random variable X has its probability distribution given below.

X	1	2	5	7	10
$P(X = x)$	$2c + k$	$2(c + k)$	$5c + k$	$5c + 0.1$	$6c + 0.1$

Given that P(X = 1) = P(X = 5),

Find;

(i). The values of C and K

(ii). The mean and mode of X

(iii). The standard deviation of X **(12 marks)**

12. Events X and Y are such that $3P(X \cap Y) = 2P(X' \cap Y) = q$ and $P(X) = \frac{3}{5}$ Use Venn diagram to find;

(i). The value of a

(ii). $P(X' \cup Y)$ **(05 marks)**

13. A continuous random variable has a p.d.f given as;

$$f(x) = \begin{cases} K(x+3) & : -1 < x \le 1 \\ K(5-x) & : \quad 1 < x \le 3 \\ 0 \; ; & \text{elsewhere} \end{cases}$$

Sketch f(y) and hence find the value of K.

14. The probability density functions of a continuous random variable X is given as;

$$f(x) = \begin{cases} \frac{2}{13}(x+1) \; ; & 0 \le x \le 2 \\ \frac{2}{13}(5-x) \; ; & 2 \le x \le 3 \\ 0 \; ; & elsewhere \end{cases}$$

(a) Calculate the;

(i). P(X < 2.5)

(ii). mean of X

(b) Determine the cumulative distribution function, $F(x)$ **(12 marks)**

15. (a) A discrete random variable X has a function given by P(X ≤ x) = $\frac{x^2}{9}$, for $x = 1, 2, 3$.

(i). Write out the probability distribution of X

(ii). find the mode

(iii). find E(3X + 2) **(06 marks)**

(b) A and B are two independent events with P(A) > P(B) such that P(A∩B) = $\frac{1}{3}$ and P(AUB) = 0.9. Find;

(i). P(A)

(ii). P(B) **(06 marks)**

16. Two events A and B are such that P(A'/B') $\frac{2}{7}$ and P(B) = $\frac{2}{7}$ and P(B) = $\frac{2}{3}$. Find the;

(i). P(AUB)

(ii). P(A∩B')

17. A random variable X is uniformly distributed over the interval (-3 , 9). Find the;

(i). $Var(x)$

(ii). $(P|x| > 1.5)$

18. A random variable R has the probability distribution function given below.

$$P(R = r) = \begin{cases} \frac{K}{17}\left(2^{(9-r)}\right) & ; \quad r = 0, 1, 2, \ldots . n \\ 0 & ; \quad elsewhere \end{cases}$$

Given that $P(R = n) = \frac{1}{255}$

(i). Find the value of n and show that $K = \frac{1}{60}$

(ii). Find $P(R > 2/R \neq 2)$

19. A discrete random variable X has the following probability distribution.

X	1	2	3
P(X = x)	0.1	0.6	0.3

Find;

(i). $E(5x + 3)$

(ii). $Var(5x + 3)$

20. X is a random variable such that; $f(x) = \begin{cases} \beta(1 - 2x) & ; \ -1 \leq x \leq 0 \\ \beta(1 + 2x) & ; \ 0 \leq x \leq 2 \\ 0 & ; \ elsewhere \end{cases}$

(a) (i) Sketch the $P.d.f, f(x)$ **(03 marks)**

(ii) Determine the value of the constant, β **(03 marks)**

(b) Find the;

(i). mean of X **(03 marks)**

(ii). 60^{th} percentile **(03 marks)**

21. A random variable Y has a probability distribution function given by;

$$P(Y = y) = \begin{cases} Cy & ; \quad y = 1, 2, 3, \ldots \ldots . N \\ 0 & ; \quad elsewhere \end{cases}$$

If $P(Y = 2) = \frac{2}{15}$, find the values of C and N **(05 marks)**

22. A discrete random variable X has a probability distribution given by;

$P(X = x) = \begin{cases} Kx & ; \quad x = 1, 2, 3, 4, \\ 0 & ; \quad otherwise \end{cases}$ where K is a constant.

Determine;

(a) Value of K

(b) $P(2 < X < 5)$

(c) Expectation, E(X)

(d) Variance, Var(X)

23. A continuous random variable has a cumulative probability function given by;

$$F(x) = \begin{cases} \log_2(x^k) & ; \quad 0 \le x \le e \\ 1 & ; \quad x \ge e \end{cases}$$

(a) Show that $K = 1n2$

(b) Obtain the P.d.f of x

24. A random variable of X has its P.d.f given by $P(X = x) = \begin{cases} \frac{K}{x} & ; \quad x = 1, 2, 3 \\ 0 & ; \quad elsewhere \end{cases}$

Find;

(i). The value of constant K

(ii). $E(X + 1)^2$

(iii). Median

(iv). 3rd decile

25. Evatory stays in Uganda and Brian stays in Tanzania. The probability that Evatory will go to Sweden in December this year is $\frac{3}{4}$ and that of Brian is $\frac{3}{5}$.

(a) Find the probability that they are likely to be in different countries next year. **(06 marks)**

(b) The probability that Mugerwa passes Physics, Economics and Mathematics is 0.65, 0.8 and 0.7 respectively.

(i). Find the probability that he passes at most one subject.

26. Two events A and B are such that $P(B) = \frac{1}{8}, P(A \cap B) = \frac{1}{10}$ and $P(A/_B) = \frac{1}{3}$.

Determine the;

(a) P(A) **(03 marks)**

(b) P(AUB) **(03 marks)**

(c) P($A/_{\bar{B}}$) **(06 marks)**

27. (a) Two events A and B are such that $P(A/_B) = \frac{5}{11}$, P(AUB) $\frac{9}{10}$, and $P(B) = x$

(i). Show that $P(A) = \frac{9}{10} - \frac{6x}{11}$. **(03 marks)**

(ii). If $P(A \cap B) = 2\,P(A \cap B')$, find the values of x.

(b) In a survey conducted in S.6 Mathematics class, 35% of the students watched football and not cricket. 10% watched cricket but not football and 40% did not watch either game. If a student is chosen at random from those in the survey, find the probability that he watches;

(i). Football given that he watches cricket

(ii). Football given that he does not watch cricket. **(06 marks)**

28. A box A contains 1 white ball and 1 blue ball. Box B contains only 2 white balls. If a ball is picked at random, find the probability that it is;

(a) White

(b) From box A given that is white

29. A bag contains 5 Pepsi Cola and 4 Miranda bottle tops. Three bottle tops are picked at random from the bag one after the other without replacement. Find the probability that the bottle tops picked are of the same type.

30. (a) A bag contains 30 white (W), 20 blue (B) and 20 red (R) balls. Three balls are drawn at random one after the other without replacement. Determine the probability that the first ball is white and the third ball is also white.

(b) Events A and B are such that $P(A) = \frac{4}{7}, P(A \cap B') = \frac{1}{3}$ and $P\left({}^{A}/_{B}\right) = \frac{5}{14}$. Find;

(i). $P(B)$

(ii). $P(A' \cap B')$

31. Events A and B are such that $P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{2}$. Find $P(AUB)$ when A and B are;

(a) Independent events

(b) Mutually exclusive events

32. Two events A and B are such that $P(A' \cap B) = 3x, P(A \cap B') = 2x$, $P(A' \cap B) = x$ and $P(B) = \frac{4}{7}$. Using a Venn diagram, find he values of;

(a) x

(b) $P(A \cap B)$

33. Two events M and N are such that $P(M) = 0.7, P(M \cap N = 0.45$ and $P(M' \cap N') = 0.18.\ Find$;

(a) P(N')

(b) P(M or N but not both M and N)

34. If A and B are independent events;

(i). Show that the events A and B' are also independent events

(ii). Find P(B) given that $P(AP\ =\ 0.4$ and $P(AUB) =\ 0.8$

PROBABILITY

NORMAL DISTRIBUTION

1. The heights (cm) of Senior six candidates in a certain school were recorded as in the table below.

Height (cm)	148-<152	152-<156	156-<160	160-<164	164-<168	168 - < 172
Frequency	5	8	12	15	6	4

(a) Calculate the;

(i). Mean

(ii). Standard deviation **(07 marks)**

(b) Calculate the unbiased estimate of the variance and hence construct a 95% confidence interval for the mean height of all the Senior six candidates.

(05 marks)

2. The mean life of a certain make of dry cells is 150 days and standard deviation 32 days. Their duration is normally distributed.

(a) Find the probability that the cells will last between 125 and 210 days.

(b) If there are 300 dry cells, calculate how many will need replacement after 225 days.

(c) After how many days will a quarter of the cells have expired? **(12 marks)**

3. A biased die is tossed such that the probability of obtaining a six is $\frac{1}{10}$. If it is tossed 120 times, find the probability that there are less than 15 sixes. **(05 marks)**

4. A biased die with faces labelled 1, 2, 2, 3, 5 and 6 is tossed 45 times. Calculate the probability that 2 will appear;

(a) More than 18 times **(07 marks)**

(b) Exactly 11 times **(05 marks)**

5. The amount of meat sold by a butcher is normally distributed with mean 43 Kg and standard deviation 4Kg. Determine the probability that the amount of meat sold is between 40Kg and 50Kg.

6. A random variable X is such that $X \sim N\ (102, 16)$. Find $P(|X - 100| < 7.2)$.

(05 marks)

7. The time taken to complete an application form is normally distributed with mean 17.2 minutes and standard deviation 3.6 minutes. If five applicants are chosen at random, find the probability that exactly three of them take over 20 minutes to complete the form. **(07 marks)**

8. The weights of apples from an orchard are approximately normally distributed with the mean 82.36g and standard deviation 15g. If a random sample of 400 apples is chosen, find the 97.5% confidence limits for the mean weight of all the apples in the orchard. **(05 marks)**

9. The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows. 5% of the residents have over 90 cows.
   (a) Determine the values of the mean and standard deviation of the cows
   (b) If there are 200 residents. Find how many have more than 80 cows.

10. The probability that a patient suffering from a certain disease recovers is 0.4. If 15 people contracted the disease, find the probability that;
   (a) More than 9 will recover
   (b) Between 5 and 8 will recover

11. The drying time of a newly manufactured paint is normally distributed with mean 110.5 minutes and standard deviation 12 minutes.
   (a) Find the probability that the paint dries between 104 and 109 minutes
   (b) If a random sample of 20 tins of the paint was taken, find the probability that the mean drying time of the sample is between 108 and 112 minutes.

12. The daily number of patients visiting a hospital is uniformly distributed between 150 and 210.
   (a) Write down the probability distribution function (P.d.f) of the number of patients.
   (b) Find the probability that between 170 and 194 patients visit the hospital on a particular day.

13. Tom's chance of passing an examination is $\frac{2}{3}$. If he sits for four examinations, find the probability that he passes;
   (a) Only two examinations
   (b) More than half of the examinations

14. The marks in an examination were normally distributed with mean $\mu$and standard deviation δ. 20% of the candidates scored less than 40 marks and 10% scored more than 75 marks. Find the;
   (a) Values of $\mu$and δ
   (b) Percentage of candidates who scored more than 50 marks **(08 marks)**

15. An industry manufactures iron sheets of mean length 3.0m and standard deviation of 0.05m. Given that the lengths are normally distributed, find the probability that the length of any iron sheet picked at random will be between 2.95m and 3.15m. **(05 marks)**

16. A research station supplies three varieties of seeds S_1, S_2 and S_3 are 50%, 60% and 80% respectively.

(a) Find the probability that a seed selected at random will germinate.

(05 marks)

(b) Given that 150 seeds are selected at random. Find the probability that less than 90 of the seeds will germinate. Give your answer to two decimal places.

(07 marks)

17. (a) A box of oranges contains 20 good and 4 bad oranges. If 5 oranges are picked at random, determine the probability that 4 are good and the other is bad.

(b) An examination has 100 questions. A student has 60% chance of getting each question correct. A student fails the examination for a mark less than 55. A student gets a distribution for a mark of 68 or more. Calculate the probability that a student;

(a) Fails the examination

(b) Gets a distinction

18. Box A contains 4 red sweets and 3 green sweets. Box B contains 5 red sweets and 6 green sweets. Box A is twice as likely to be picked as box B. If a box is chosen at random and two sweets are removed from it, one at a time without replacement;

(a) Find the probability that the two sweets removed are of the same colour

(b) (i) Construct a probability distribution table for the number of red sweets removed. **(06 marks)**

(ii) Find the mean number of red sweets removed. **(06 marks)**

19. (a) The chance that a cow recovers from a certain mouth disease when treated is 0.72. If 100 cows are treated by the same vaccine, find the 95% confidence limits for the mean number of cows that recover. **(05 marks)**

(b) The ages of the taxis in a route are normally distributed with standard deviation of 1.5 years. A sample of 100 taxis inspected on a particular day gave a mean age of 5.6 years.

Determine;

(i). A 99% confidence interval for the mean age of all taxis that operate on the route. **(03 marks)**

(ii). The probability that the taxi were of ages between 5.4 and 5.8 years.

(04 marks)

20. (a) Among the spectators watching football match, 80% were the home team's supporters while the rest were the visitors' teams' supporters. If 2500 of the spectators are selected at random, what is the probability that there were more than 540 visitors in this sample? **(06 marks)**

(b) The times a factory takes to make units of a product are approximately normally distributed. A sample of 49 units of the product was taken and found to take an average of 50 minutes with a standard deviation if 2 minutes. Calculate the 99% confidence limits of the mean of making all the units of the product. **(06 marks)**

NUMERICAL METHODS

1. The information in the table below gives a system of tax calculation for the amount of money, A (in pounds) earned annually.

Annual earnings (A)	**Tax (T)**
< £2000	0
≥ £ 2000 but< ≠ £5000	2% 0f A
£5000 ≤ A	£60 plus 5% 0f the amount over £5000

(i). Draw a flow chart using the above data given that the process stops when 200 counts (N) are made. **(09 marks)**

(ii). Calculate the tax a man who earns £6000 annually. **(03 marks)**

(iii).

2. (a) Use a graphical method to show that the equation $e^x - x^2 - 2 = 0$ has only one real root. **(05 marks)**

(b) Using the Newton-Raphson method, find the root of the equation in (a) above correct to three significant figures. **(07 marks)**

3. The table below gives values of x and the corresponding values of f(x).

X	0.1	0.2	.03	0.4	0.5	0.7
F(x)	4.21	3.83	3.25	2.85	2.25	1.43

Use linear interpolation /extrapolation to find;

(a) $f(x)$ when $x = 0.6$

(b) the value of x when $f(x) = 0.75$

4. The table below is an extract from the table of $SinX$

x	0.1	0.2	0.3	0.4	0.5
$Sin\ X$	0.0998	0.1987	0.2955	0.3894	

Use linear interpolation to find;

(a) Sin 0.29
(b) Sin 0.52
(c) Sin^{-1} 0.2598
(d) Sin^{-1} 0.4900 **(12 marks)**

5. (a) Use the trapezium rule to estimate $\int_0^1 (3x+5)dx$ using 5 sub-intervals. Give your answer correct to 2 decimal places.

(b) Find the exact value of $\int_0^1 3x + 5)\ dx$

(c)(i) Determine the percentage error in the two calculations in (a0 and (b) above.

(ii) State how the percentage error in (c) (i) can be reduced. **(12 marks)**

6. Given that $x = 3.7$ and $y = 70$ are each rounded of with percentage error of 0.2 and 0.05 respectively while number $Z\ =\ 26.23$ is calculated with relative error of 0.04. Find the interval in which the exact value of $\frac{x}{y-z}$ lies. Correct to 4 significant figures.

7. The numbers $D = 5.241$ and $F = 3.6$ have been rounded to give number of decimal places.

(a) Find the maximum possible error in DF. **(06 marks)**

(b) Determine the interval within which $\frac{D^2}{F}$ can be expected to lie. **(06 marks)**

8. The height and radius of a cylindrical water tank are given as $H = 3.5 \pm 0.2m$ and $R = 1.4 \pm 0.1m$ respectively. Determine in m^3, the least and greatest amount of water the tank can contain hence calculate the maximum possible error in your calculation.

9. (a) Use the trapezium rule with six ordinates to find the approximate value of $\int_2^5 xe^{-x}\ dx$ correct to 3 significant figures.

(b) Find the area bounded by the curve $y = xe^{-x}$ between $x = 2$ and $x = 5$.

(c) Find the percentage error in (a) and (b) above. **(12 marks)**

10. (a) The numbers .6754, 4.802, 15.18 and 0.925 are rounded off to the given number of decimal places. Find the range within which the exact value of $2.6754\left[4.802 - \frac{15.18}{0.925}\right]$ can be expected to lie. **(06 marks)**

(b) The numbers a, b, c and d are rounded off with errors e_1, e_2, e_3 and e_4 respectively. Show that the expression for the maximum absolute error e_Z is;

$$Z = \frac{ab}{c+d} \text{ is } \frac{ab}{c+d}\left|\frac{e_1}{a}\right| + \left|\frac{e_2}{b}\right| + \left|\frac{e_{3}+e_4}{c+d}\right| .$$ **(06 marks)**

11. Given that $y = \sin\theta$ and θ is measured with maximum possible error of 2%. If $\theta = 30^0$, determine the;

(i). Absolute error in y

(ii). Interval within which the value of y lies. Give your answer correct to 4 significant figures. **(05marks)**

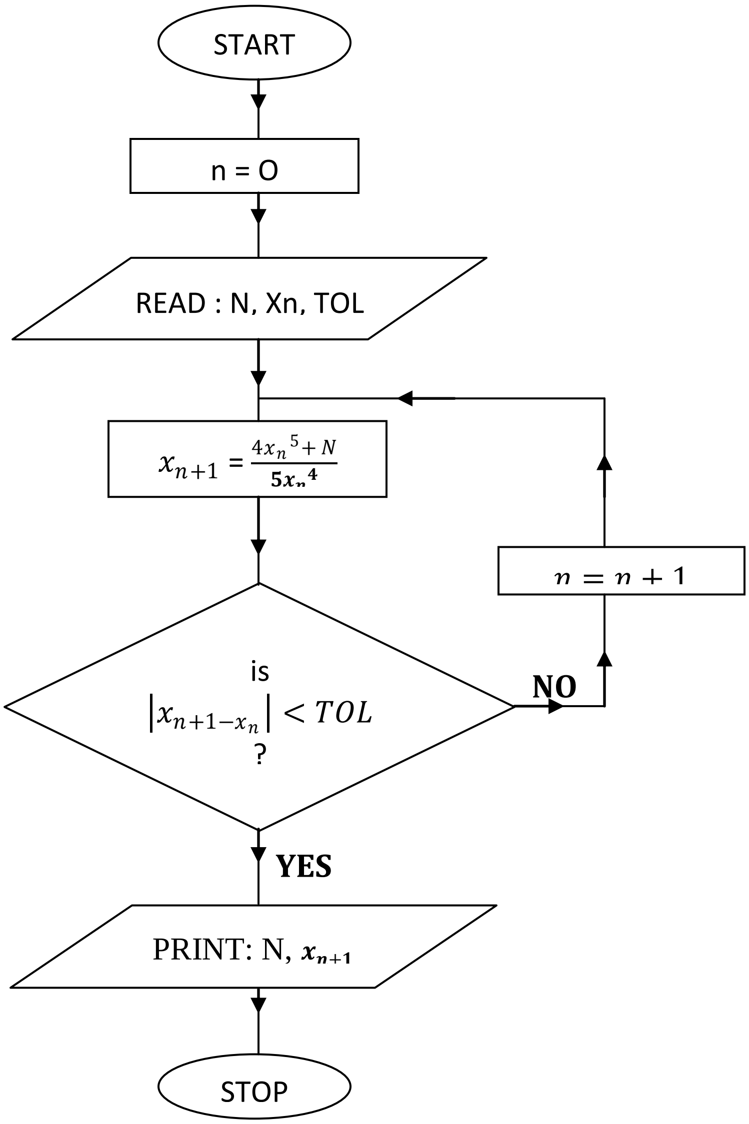
12. The temperatures (^{0}C) of a cooling body measured every 10 minutes were recorded as 82, 70, 56 and 42. If the body's initial temperature is 93 ^{0}C. Find using linear interpolation /extrapolation, the;

(i). Time taken for the body to cool to 63 ^{0}C

(ii). Temperature of the body after 45 minutes

13. Use the trapezium rule with equal strips of width $\frac{\pi}{6}$ to find an approximation for $\int_0^{\frac{\pi}{6}} x\sin x dx$, Give your answer to 4 significant figures

14. Study the flow chart below.



(i). Using the flow chart, perform a dry run for $x_0 = 2.1$ and $N = 50$, $TOL = 0.0005$. **(04 marks)**

(ii). What is the purpose of the flow chart? **(01 mark)**

15. The numbers A and B are rounded off to a and b with errors e1 and e2 respectively.

(a) Show that the absolute relative error in the product AB is given by; $\frac{|a|\ |e_2|+|b|\ |e_1|}{ab}$ **(05 marks)**

(b) Given that $A = 6.43$ and $B = 37.2$ are rounded off to the given number of decimal places indicated.

(i). State the maximum possible errors in A and B. **(02 marks)**

(ii). Determine the absolute error in AB. **(02 marks)**

(iii). Find the limits within which the product AB lies. Give your answer to 4 decimal places. **(03 marks)**

16. Given that $x = 4.3$ and $Z = 84.001$ are rounded off with corresponding errors of $0.5\ and\ 0.05$. Find the relative error of $(x - z)$, correct your answer to 3 significant figures. **(05 marks)**

17. Use the trapezium rule with six ordinates to evaluate giving your answer truncated to three significant figures; $\int_0^{\frac{\pi}{2}} e^{sinX}\,dx$ **(05 marks)**

18. (a) Use a graphical method to show that he equation $e^x - x^2 - 2 = 0$ has only one real root. **(05 marks)**

(b) Using the Newton-Raphson method, fond the root of the equation in (a) above correct to three significant figures. **(07 marks)**

19. Use the trapezium rule with seven ordinates to estimate $\int_0^3 [(1.2)^x - 1]^{\frac{1}{2}}\,dx$. Correct to 2 decimal places. **(05 marks)**

20. (a) Show that the equation $x - 3sinx = 0$ has a root between 2 and 3.(03 marks)

(b) Show that the Newton-Raphson iterative formula for estimating the root of the equation in (a) is given by; $x_{n+1} = \frac{3(Sinx_n - x_n\ Cosx_n)}{1-3Cosx_n}$, $n = 0, 1, 2, \dots\dots$ hence find the root of the equation. Correct to 2 decimal places. **(09 marks)**

21. (a) Given that $y = e^x$ and $x = 0.62$ correct to two decimal places. Find the interval within which the exact value of y lies. **(05 marks)**

(b) Show that the maximum possible error in $y\ sin^2X$ is $\left|\frac{\Delta y}{y}\right| + 2\ cotx|\Delta x|$, where Δx and Δy are errors in x and y respectively hence find the percentage error in calculating $ysin^2X$ if $y = 5.2\ \pm 0.05$ and $x = \frac{\pi}{6} \pm \frac{\pi}{360}$. **(07 marks)**

22. Given that $y = \sec(45^0 \pm 10\%)$. Find the limit within which exact value of y lies. **(05 marks)**

23. (a) Use the trapezium rule with five ordinates to evaluate $\int_0^{\frac{\pi}{4}} \frac{2}{1-x^2}\,dx$ to 3 decimal places.

(b) Find the exact value of $\int_0^{\frac{\pi}{4}} \frac{2}{\sqrt{1-x^2}}\,dx$ to 3 decimal places.

(c) Find the absolute error in the function and state one way how this error can be reduced. **(12 marks)**

24. The table below gives of x and the corresponding values of $f(x)$.

x	0.1	0.2	0.3	0.4	0.5	0.7
$f(x)$	4.21	3.83	3.25	2.85	2.25	1.43

Use linear interpolation/extrapolation to find;

(i). $f(x)$ when $x = 0.6$

(ii). The value of x when $f(x) = 0.75$

25. Given that $y = \frac{1}{x} + x$ and $x = 2.4$ correct to one decimal place, find;

26. By plotting graphs of $y = x$ and $y = 4sinx$ on the same axes, show that the root at the equation $x - 4sinx = 0$ lies between 2 and 3.

Hence use Newton-Raphson's method to find the root of the equation correct to 3 decimal places.

27. Use the trapezium rule with 4 sub-intervals to estimate; $\int_0^{\frac{\pi}{2}} cosxdx$ correct to three decimal places.

28. The table below shows the values of f(x) for the given values of x

x	0.4	0.6	0.8
$f(x)$	-0.9613	-0.5108	-0.2231

Use linear interpolation to determine $f^{-1}(-0.4305)$ correct to 2 decimal places.

29. Given the equation $x^3 - 6x^2 + 9x + 2 = 0$

(a) Show that the equation has a root between -1 and 0.

(b) (i) Show that the Newton-Raphon formula for approximating the root of the equation is given by; $x_{n+1} = \frac{2}{3}\left[\frac{x_n^3 - 3x_n^2 - 1}{x_n^2 - 4x_n + 3}\right]$

(ii) Use the formula in b (i) above with an initial approximation $x_0 = -0.5$ to find the root of the equation correct to two decimal places.

30. The numbers x and y are approximated by X and Y with error Δx and Δy respectively.

(a) Show that the maximum relative error in xy is given by; $\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$.

(b) If $x = 4.95$ and $y = 2.013$ are each rounded off to a given number of decimal places.

(i). Calculate the percentage error in xy

(ii). Limits with which xy are expected to lie

31. Use the trapezium rule with 4 sub-intervals to estimate; $\int_0^1 \frac{2x+1}{x^2+x} dx$ correct to 2 dps.

32. Given the table below:

x	0	10	20	30
y	6.6	2.9	-0.1	-2.9

Use linear interpolation/extrapolation to find;

$y \text{ when } x = 16$

$x \text{ when } y = -1$

33. (a) The number x and y are approximated with errors of Δx *and* Δy respectively. Show that the maximum absolute error in $\frac{x}{y}$ is given; $\frac{|y||\Delta x| + |x||\Delta y|}{y^2}$

(b) Given that x = 2.68 and y = 0.9 are rounded off to a given number of decimal places. Find the interval within which the expected value of $\frac{x}{y}$ is expected to lie.

34. Show that the Newton-Raphson formula for finding the root of the equation $x = N^{\frac{1}{5}}$ is given by; $x_{n+1} = \frac{4x_n^5 + N}{5x_n^4}$ $\quad$ $n = 0, 1, 2 \ldots$

Construct a flow chart that;

(i). Reads n and the first approximation x_0

(ii). Computes 16 roots to 3 dps

(iii). Prints the root x_n and the number of iterations (n)

Taking N = S0, $x_0 = 2.2$ perform a dry run for the flow chart. Give your root to 3 dps.

35. The table below shows the values of a function f(x)

x	1.8	2.0	2.2	2.4
$f(x)$	0.532	0.484	0.436	0.384

Use linear interpolation to find the value of;

(i). $f\,(2.08)$

(ii). x corresponding to $f(x) = 0.5$

36. Find the approximate value of $\int_0^2 \frac{1}{1+x^2}$dx rule of 6 ordinates. Give your answer to 3 decimal places.

37. The number x and y are measured with possible errors of $\Delta x \; and \; \Delta y$ respectively.

(a) Show that the maximum absolute error in the equation $\frac{x}{y}$ is given by

$\frac{|y||\Delta x| + |x||\Delta y|}{y^2}$

(b) Find the interval within which the exact value of $\frac{2.58}{3.4}$ is expected to lie.

38. (a) Show that the iterative formula based on Newton-Raphson method for solving the equation $1nx + x - 2 = 0$ is given by; $x_{n+1} = \frac{x_n(3 - 1n\, x_n)}{1 + x_n}$

(b)(i) Construct a flow chart that;

Reads the initial approximation as r

Computes using the iterative formula in (a) and prints the root of the equation $1nx + x - 2 = 0$, when the errors is less than 1.0 x 10^{-4}

(ii) Perform a dry run of the flow chart when r=1.6

39. Use a trapezium rule with 4 subintervals to estimate; $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx} dx$. Give your answer correct to three decimal places.

40. The table below show the cost Y shillings for hiring a motorcycle for a distance x kilometers.

Distance x km	10	20	30	40
Cost (shs)	2800	3600	4400	5200

Use linear interpolation/extrapolation to calculate;

(a) Cost of hiring the motorcycle for a distance of 45km

(b) Distance Mukasa travelled if he paid Shs.4000

41. (a)(i)Show that the equation $e^x - 2x - 1 = 0$ has a root between $x = 1$ and $x = -1.5$.

(ii)Use linear interpolation to obtain an approximation of the root.

(b)(i) Solve the equation in a(i) using each formula below twice. Take the approximation in a(ii) as the initial value.

Formula 1: $x_{n+1} = \frac{1}{3}(e^{x_n} + 1)$

Formula 2:$x_{n+1} = \frac{e^{x_n}(x_{n+1}) + 1}{e^{x_n - 2}}$

(ii) Deduce with a reason which of the two formulae is appropriate for solving the given equation in a(i). Hence write down a better approximate root, correct to two decimal places.

42. The table below shows delivery charges by a courier company.

Mass(gm)	200	400	600
Charges(shs)	700	1200	3000

Use linear interpolation or extrapolation, find the;

(a) Delivery of a parcel weighing 352gm

(b) Mass of a parcel whose delivery charge is shs.3,300

43. Use the trapezium rule with five sub-intervals to estimate; $\int_0^{\frac{\pi}{3}} tanxdx$, correct to three decimal places.

(i). Find the value of $\int_0^{\frac{\pi}{3}} tanxdx$, to three decimal places.

(ii). Calculate the percentage error in your estimation in (a) above.

(iii). Suggest how the percentage error may be reduced.

44. Find the approximate value, to one decimal place of $\int_0^1 \frac{dx}{1+x}$, using the trapezium rule with 5 trips.

45. The table below shows the values of a continuous function f with respect to t.

t	0	0.3	0.6	1.2	1.8
f(t)	2.72	3.00	3.22	4.06	4.95

Use linear interpolation, find;

(i). f(t) when t = 0.9

(ii). t when f(t) = 4.48

46. (a) If a is the first approximation to the root of the equation $x^5 - 6 = 0$, show that the second approximation is given by; $\frac{4a+ {}^{b}/_{a^4}}{5}$

(b) Show that the positive real root of the equation $x^5 - 17 = 0$ lies between 1.5 and 1.8. Hence use the formula in (a) above to determine the root to three decimal places.

47. (a) Two positive real numbers N_1 and N_2 are rounded off to give n_1 and n_2 respectively. Determine the maximum relative error I using n_1n_2 for N_1N_2. State any assumptions made.

(b) If $N_1 = 2.765, N_2 = 0.72$. Determine the range within which the exact values.

(i). $N_1 N_2(N_1 - N_2)$

(ii). $\frac{N_2-N_1}{N_1N_2}$ are expected to lie. Give your answer to three decimal places.

48. The table below shows the distance in kilometers (km) a truck man can move with a given amount of fuel in litres (l)

Distance (km)	20	28	33	42
Fuel (l)	10	13	21	2.4

Estimate;

(a) How far the truck can move on 27.5l of fuel

(b) The amount of fuel required to cover 29.8km

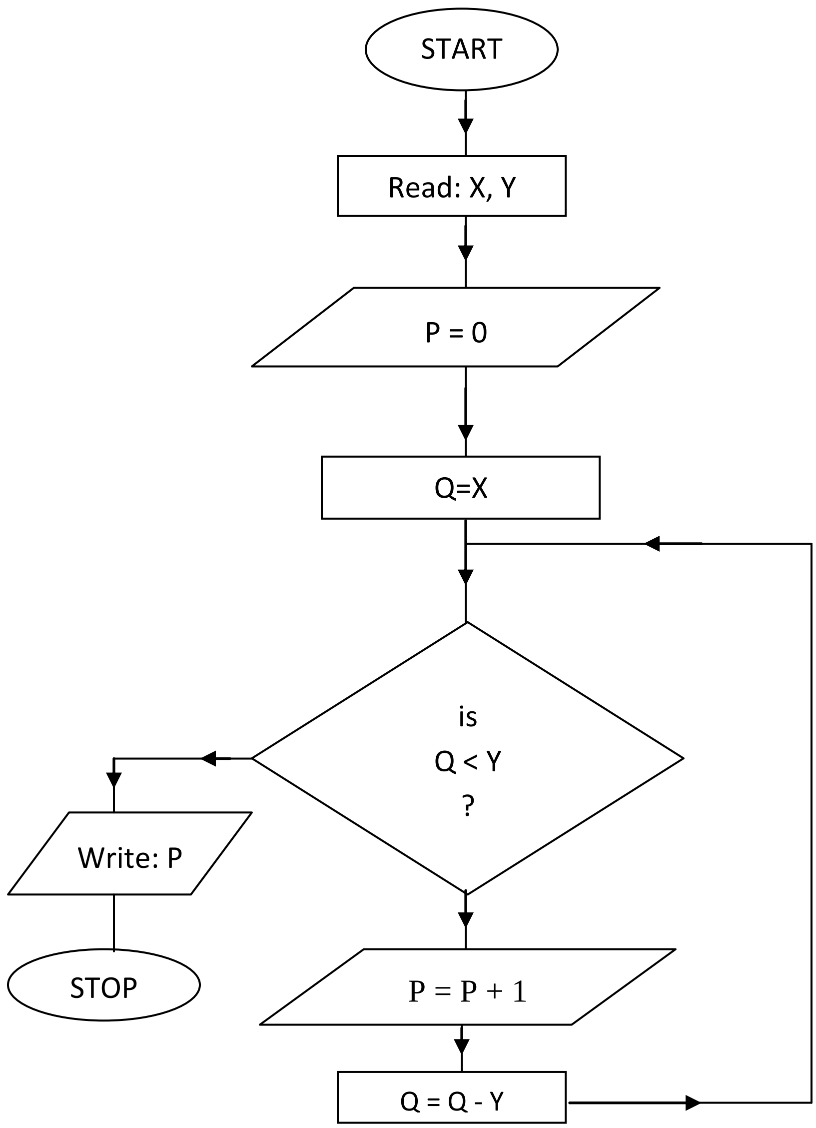
49. Use the trapezium rule with six ordinates to estimate; $\int_1^2 \frac{lnx}{x}\, dx$. Give your answer correct to three decimal places.

50. (a)Show that the root of the equation $2x - 3\cos\left(\frac{x}{2}\right) = 0$ lies between 1 and 2.

(b) Use Newton's Raphson's method to find the root of the equation in (a) above. Give your answer correct to two decimal places.

51. Use the trapezium rule with 6 ordinates to evaluate $\int_0^1 e^{-x^2}\, dx$ correct to 2 decimal places.

52. Study the flow chart below.



Use the flow chart, perform a dry run and complete the table below for $X = 22$ and $Y = 7$

P	Q
0	22

Thus record P =

Q =

What is the purpose of the flow chart?

53. The number X and Y were estimated with maximum possible errors of ΔX *and* ΔY respectively. Show that the percentage relative error in XY is $\left(\frac{\Delta X}{\Delta Y} + \frac{\Delta \mathrm{Y}}{\mathrm{Y}}\right)$ x 100.
Obtain the range of values within which the exact value of 3.551 x 2.71635 lies.
Locate each of the three roots of the equation $x^3 - 5x^2 + 5 = 0$

54. Show graphically that there is only one positive real roots of the equation
$e^x - 2x - 1 = 0$
Use the Newton Raphson method to calculate the root of equation in (a) correct to 2 decimal places.

END