Equilibrium of forces

several forces acting on a particle are said to be in equilibrium when the resultant force is equal to zero

i.e. $F_R = \begin{pmatrix}0\\0\end{pmatrix}$

Example 1

For the following set of forces in equilibrium find the values of a and b in each case

(i) (6i + 4j)N, (-2i -5j)N, (ai + bj)N

$\begin{pmatrix}6\\4\end{pmatrix} + \begin{pmatrix}-2\\-5\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

6 – 2 + a = 0 => a = -4

4 – 5 + b = 0 => b = 1

(ii) (5i + aj + ck)N, (bi – 6j – k)N, and (-3i + 2j + ck)

$\begin{pmatrix}5\\a\\c\end{pmatrix} + \begin{pmatrix}b\\-6\\-1\end{pmatrix} + \begin{pmatrix}-3\\2\\c\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

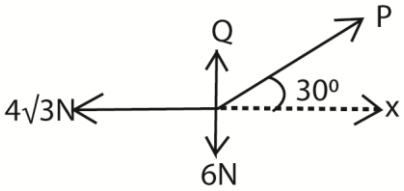
5 + b – 3 = 0 => b = -2

a – 6 +2 = 0 => a = 4

2c – 1 = 0 => c = 0.5

Example 2

In the diagram below, the particle is in equilibrium, find the values of P and Q.



Solution

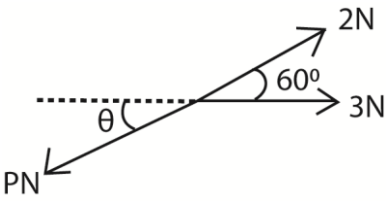
$\begin{pmatrix}0\\Q\end{pmatrix} + \begin{pmatrix}Pcos30\\Qcos30\end{pmatrix} + \begin{pmatrix}0\\-6\end{pmatrix} + \begin{pmatrix}-4\sqrt{3}\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

0 + Pcos30 + 0 -$4\sqrt{3}$ = 0; => P = 8N

Q + Qcos 30 – 6 + 0 = 0; => Q = 2N

Example 3

Diagram below shows three coplanar forces of magnitude 2N, 3N and PN all acting at point O in the direction shown. Given that the forces are in equilibrium, Find the value of P



Solution

-Pcos θ + 2cos 60 + 3 = 0 …. (i)

cos θ = $\frac{2cos60+3}{P}$

-Psinθ + 2sin60 = 0

sin θ = $\frac{2sin60}{P}$ …….. (ii)

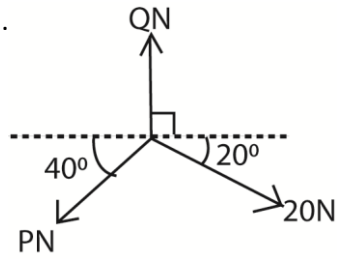
Eqn. (i) and (ii)

θ = $\tan^{-1}\left(\frac{2sin60}{2cos60+3}\right)$= 23.413

from (ii) P = $\frac{2\sin 60}{\sin 23.413}$ =4.3589N

Revision Exercise

1 .



(i) The diagram above shows three coplanar forces in equilibrium. Find the value of P and Q.
(ii) If the direction of Q s now reversed, find the magnitude and direction of the resultant
[(i) 24.5N, 22.6N; (ii) 45.2N]

2. Forces F_1 = (-3i + 7j)N, F_2= (i – j)N and F_3 =(pi +qj) act on a particle.

(i) If the particle is in equilibrium, find the vlues of p and q. [p = 2, q= -6]

(ii) Find the magnitude and direction of the resultant of F1 and F2 . [6.3246N, 71.57^0]

3. Forces of 6N, 5N, 8N, 5N and 9N act on a particles in the direction $N30^0E$, $N30^0W$, $S50^0E$, $N60^0W$, $N80^0E$ and $S40^0W$ respectively. Find the additional force that will keep the system of force in equilibrium. [5.358N at 68.920 above the positive axis]

4. Forces of 7N, 2N, 4N and 5N act on a particle in directions of 0600, 1600 2000 and 3150 respectively. Find the additional force that will keep the system of forces in equilibrium. [2.3125N at 37.18^0 below the negative axis]

5. Forces of 2N, 1N, 3N and 4N act on a particle in the direction 0^0, 90^0, 270^0 and 330^0 respectively. Find the additional force that will keep the system of forces in equilibrium. [6.8N at 36^0 above the negative axis]

6. Forces of 6N, 5N, 7N, 4N, $3\sqrt{2}\,N$ and $7\sqrt{2}$N act in direction AB, CB, CD, DA, CA and DB respectively on a square ABCD. Find the additional force that will keep the system of forces in equilibrium. [19.2N at 81^0 above the negative axis]

7. Forces 8N, 7N, 6N, 4N, 7N and 6N act along the sides of a regular hexagon ABCDEF in direction AB, CB, CD, DE, EF and FA respectively. Find the additional force that will keep the system of forces in equilibrium. [12.49N at 76^0 above AB]

Thank you
Dr. Bbosa Science