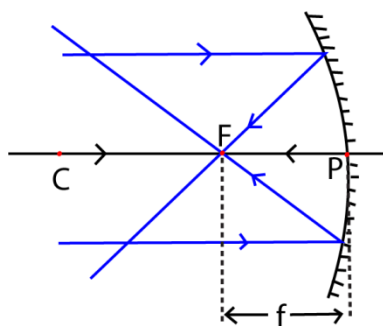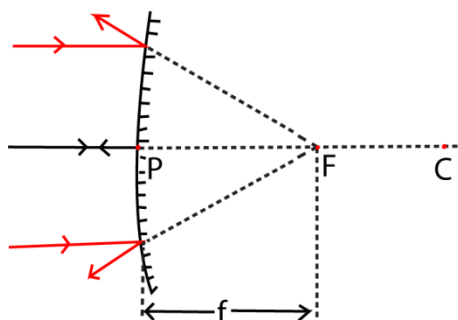## Reflection at curved mirrors

Curved mirrors are of two types namely;

(i) Concave (Converging) mirror: it is part of the sphere whose centre **C** is in front of its reflecting surface.

(ii) Convex (Diverging) mirror: it is part of the sphere whose centre **C** is behind its reflecting surface.

Consider the reflection of a parallel narrow beam of light at curved mirrors as shown.

Concave mirror   Convex

Definitions
1. **Centre of curvature C:** it is the centre of the sphere of which the mirror forms part.
2. **Radius of curvature r:** it is the radius of the sphere of which the mirror forms part.
3. **Pole of the mirror:** it is the mid-point (centre) of the mirror surface.
4. **Principal axis CP**: it is the line that passes through the centre of curvature and the pole of the mirror.
5. **Secondary axes:** These are lines parallel to the principal axis of the mirror.
6. **Paraxial rays:** These are rays drawn close and parallel to the principal axis
7. **Marginal rays:** These are parallel rays furthest from the principal axis of the mirror.
8. **(i) Principal focus "F" of a concave mirror:** it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis converge after reflection by the mirror.

(ii). Principal focus "F" of a convex mirror: it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis appear to diverge from after reflection by the mirror

9. **(i) Focal length "f" of a concave mirror:** it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis converge after reflection by the mirror.

   (ii) Focal length "f" of a convex mirror: it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after reflection by the mirror.

10. **Aperture of the mirror:** it is the length of the mirror surface.

Reflection of a parallel wide beam of light at curved mirrors

Consider the reflection of a wide parallel beam of light incident on a concave mirror as shown.
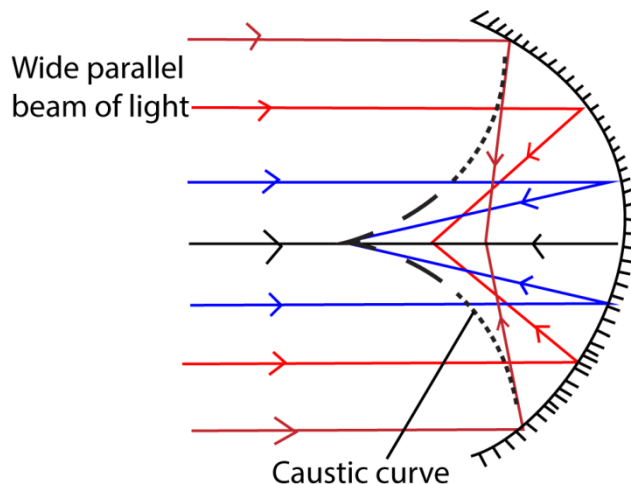
When a wide parallel beam of light is incident on a concave mirror, the different reflected rays are converged to different points. However these reflected rays appear to touch a surface known as a **caustic surface** having a cusp (an apex) at the principal focus **F**.
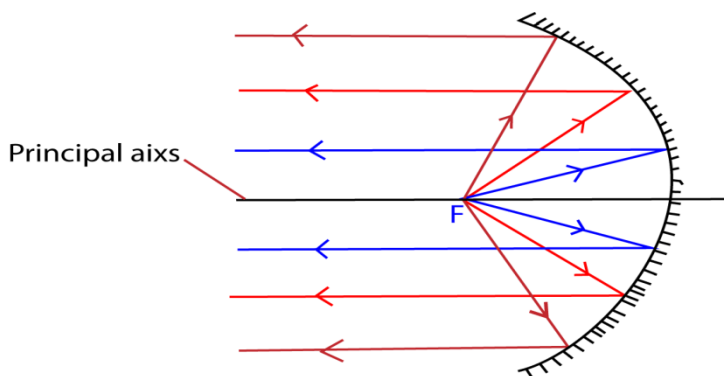
NOTE
(i) The marginal rays furthest from the principal axis are converged nearer to the pole of the mirror than the paraxial rays.

(ii) Similarly, if a wide parallel beam of light is incident on a convex mirror, the different reflected rays appear to have diverged from different points.

Hyperbolic mirror

These is a type of concave mirror that reflects all the parallel incident beam on to a unique point F, the principal focus of the mirror.

Note that in car headlight and torches, the reflecting surfaces are hyperbolic. When a bulb is placed at the principal focus of reflecting surface, a strong parallel beam of light is obtained. Hyperbolic mirrors are also used in reflecting telescopes.

Example
Explain why parabolic surface is preferred to concave surface in car headlights?

Solution
Parabolic mirror reflect a wide beam of light of high intensity from a lamp at its principal focus.

Concave mirror reflect light of low intensity which diminish with distance since they do not reflect parallel beam when a lamp a lamp is placed at their principal focus.

Geometrical rules for the construction of ray diagrams
The following is a set of rules for easy location of the images formed by spherical Mirrors.

1. Rays parallel to the principal axis are reflected through the principal focus.
2. Rays through the principal focus are reflected parallel to the principal axis.
3. Rays passing through the centre of curvature are reflected back along their own paths.
4. Rays incident to the pole are reflected back, making the same angle with the principal axis.

    NOTE:
    (i) The normal due to reflection at the mirror surface at any point must pass through the centre of curvature.

    (ii) The image position can be located by the intersection of two reflected rays initially coming from the object.

Real and virtual images

A REAL IMAGE: This is the image formed by the actual intersection of light rays from an object and can be received on the screen.

A VIRTUAL IMAGE: This is the image formed by the apparent intersection of light rays and cannot be received on the screen

IMAGES FORMED BY A CONCAVE MIRROR
The nature of the image formed by a concave mirror is either real or virtual depending on the object distance from the mirror as shown below;
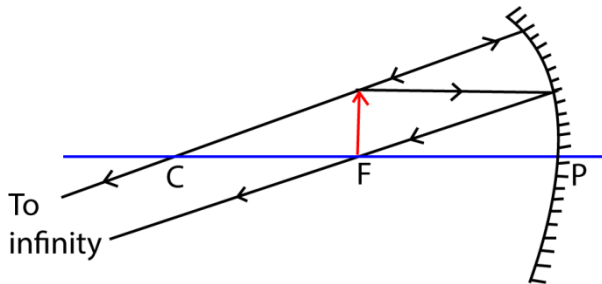
Object between F and P the image is
(a)   Behind the mirror
(b) Virtual
(c) Erect
(d) Magnified
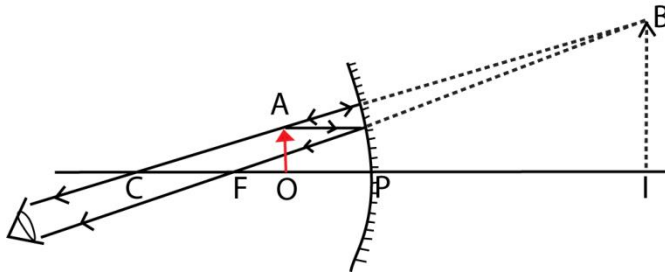
The property of a concave mirror to form erect, virtual and a magnified image when the object is nearer to the mirror than its focus makes it useful as a shaving mirror and also used by dentists for teeth examination.
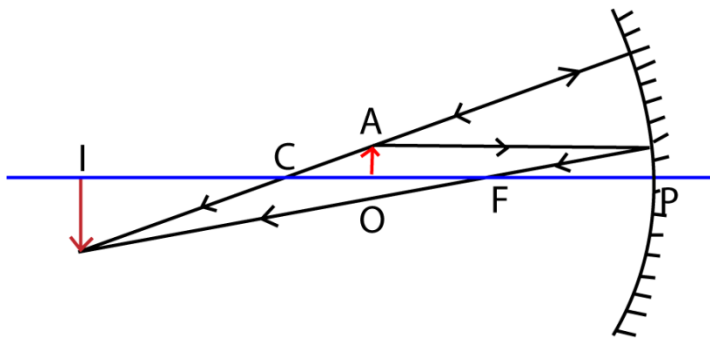
Object at F the image is
   (a) at infinity
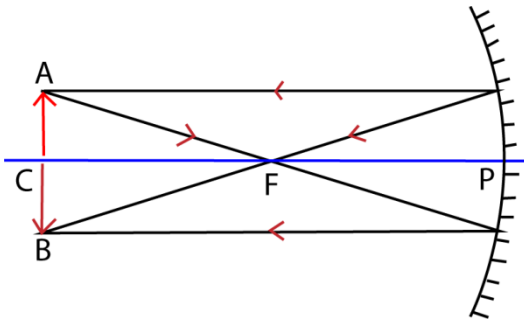

Object between F and C the image is
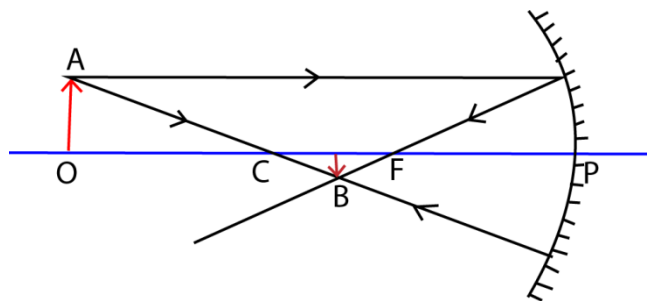(a) Beyond C
(b) Real
(c) Inverted
(d) Magnified

Object at C the image is
 (a) At C
 (b) Real
 (c) Inverted
 (d) Same size as the object
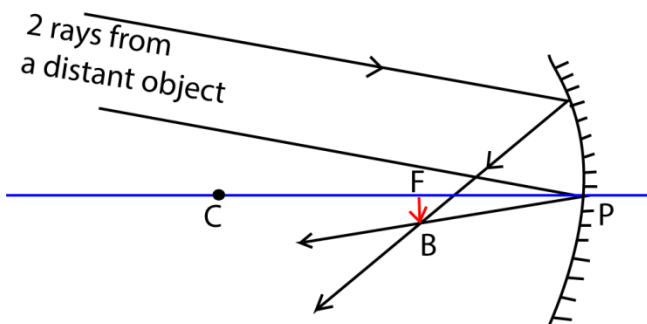

Object beyond C the image is
 (a) Between C and F
 (b) Real
 (c) Inverted
 (d) Diminished

Object at infinity the image is
 1) At F
 2) Real
 3) Inverted
 4) Diminished

2 rays from a distant object

C   F   B   P

NOTE;
Generally the image of an object in a concave mirror is virtual only when the object is nearer to the mirror than its focus.

Uses of concave mirrors
(i)   They are used as shaving mirrors.
(ii) They are used by dentists for teeth examination.
(iii)They are used as solar concentrators in solar panels.
(iv)They are used in reflecting telescopes, a device for viewing distant objects
(v) They are used in projectors, a device for showing slides on a screen.

IMAGES FORMED BY A CONVEX MIRROR
The image of an object in a convex mirror is erect, virtual, and diminished in size no matter where the object is situated as shown below

A   B

O   P   I   F   C
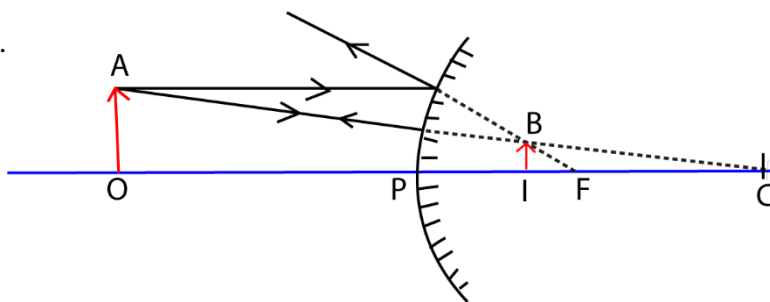
In addition to providing an erect image, convex mirrors have got a wide field of view as illustrated below.
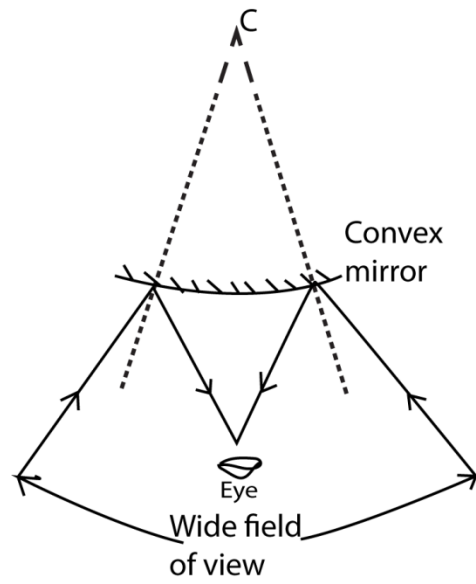
Uses of convex mirrors
(i)They are used as car driving mirrors
(ii)They are used in reflecting telescopes, a device for viewing distant objects

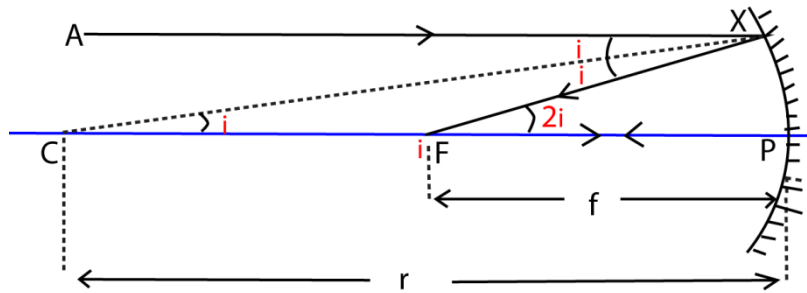**Relationship between the focal length f and the radius of curvature r of curved mirrors**

Consider the reflection of a paraxial ray, AX, parallel to the principal axis of a concave mirror as shown.

Taking **FP** = focal length and **C** = centre of curvature, then , **CX**, is the normal to the mirror surface and, **CP**, is the radius of curvature.

$\tan 2i \approx 2i$

If, X, is close to P, then $\tan 2i \approx 2i = \dfrac{XP}{FP}$ --------------- (for small angles)

$$\Rightarrow \quad 2i = \frac{XP}{FP} \quad \text{--(i)}$$

Similarly $\tan i \approx i = \dfrac{XP}{CP}$

$$\Rightarrow \quad i = \frac{XP}{CP} \quad \text{-- (ii)}$$

Combining equations (i) and (ii) gives,
$$2\frac{XP}{CP} = \frac{XP}{FP}$$

Canceling XP throughout and simplifying for CP gives

$$CP = 2FP \quad \text{where } CP = r, \ FP = f$$

$$\therefore \ \mathbf{r} = \mathbf{2f}.$$

Thus, the radius of curvature of a concave mirror is twice its focal length.

NOTE:
It can be shown that the relation between f and r holds for both concave and convex mirrors**.**

Mirror formula and sign convention
In order to obtain a formula which holds for both concave and convex mirrors, a Sign rule or convention must be obeyed and the following shall be adopted.

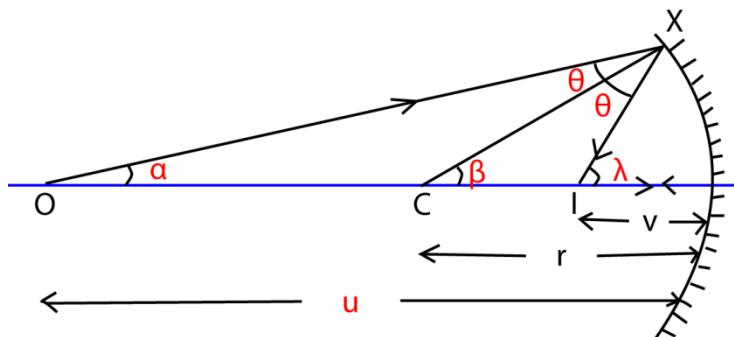**(i)** Distances of real objects and images are positive.

(ii) Distances of virtual objects and images are negative.

NOTE: A concave mirror has a positive focal length while a convex mirror has negative focal length.

Concave (converging) mirror formula

Consider the incidence of ray **OX** on to a concave mirror from a point object **O** placed along the principal axis and then suddenly reflected in the direction XI making an angle θ with the normal CX.

 Ray OP strikes the mirror incident normally at P and thus reflected back along its own path. The point of intersection **I** of the two reflected rays is the image position.

From Δ OXC, α + θ = β ------------------------------(i)
From Δ CXI, β + θ = λ ------------------------------(ii)
Equation (i) − Equation (ii) eliminates θ to give.
α − β = β − λ
⇒ α + λ = 2β --(a)

If X is very close to P, then

$\alpha \approx \tan\alpha = \frac{XP}{u}, \ \beta \approx \tan\beta = \frac{XP}{r}, \lambda \approx \tan\lambda = \frac{XP}{v}$

Equation (a) becomes $= \frac{XP}{u} + \frac{XP}{v} = \frac{2XP}{r}$

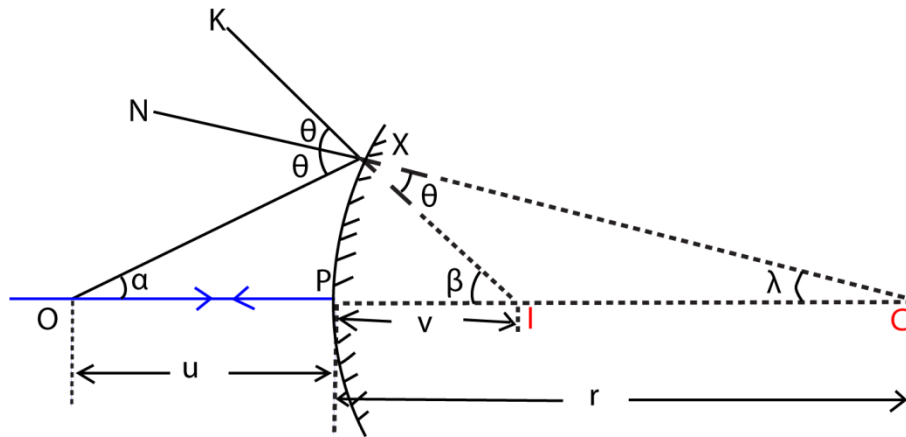$$= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

But 2 = 2f

$$= \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

NOTE:
The focal length of a concave mirror is a positive distance.

Convex (diverging) mirror formula

Consider the incidence of ray **OX** on to a convex mirror from a point object **O** placed along the principle axis and then suddenly reflected in the direction **XK** making an angle θ with the normal **XN**.

From Δ XIC, θ + λ = β

$\Rightarrow$ θ = β − λ ------------------------------(i)

From Δ OXI, α + β = 2θ ------------------------------(ii)

Substituting equation (i) into (ii) gives.

α + β = 2 (β − α)

$\Rightarrow$ α − β = −2λ --------------------------------------(a)

If X is very close to P, then

α ≈ tanα = $\frac{XP}{u}$, β ≈ $\frac{XP}{-v}$ (I is virtual) and λ ≈ tan λ = $\frac{XP}{-r}$ (C is virtual)

Equation (a) becomes

$$\frac{XP}{u} - \frac{XP}{-v} = \frac{-2XP}{-r}$$

$$= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

But 2 = 2f

$$= \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Note that the focal length of a convex mirror is a negative distance.

Formula for magnification

Linear magnification, m, $= \frac{\text{Image height}}{\text{Object height}} = \frac{\text{Image distance}}{\text{Object distance}}$.

Proof

Consider the incidence of ray **AP** on to the pole of a concave mirror from an object of height **h** placed a distance, **u,** from the mirror and then reflected back making the same angle with the principal axis to form an image of height **h₁**, located at distance, **v**, from the mirror as shown
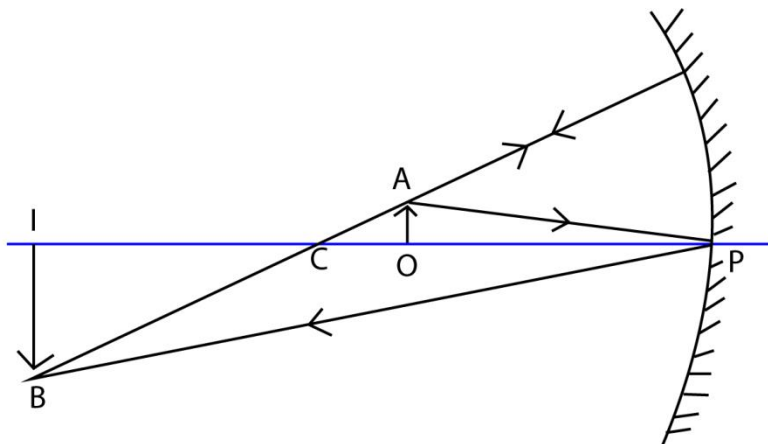
ray **AP** makes an angle θ with the normal **OP**, then,

From Δ OAP, $\tan \theta = \frac{h}{u}$ -------------------(i)

From Δ IPB, $\tan \theta = \frac{h_1}{v}$ -------------------------(ii)

Equating equation (i) and (ii) gives.

$$\frac{v}{u} = \frac{h_1}{h}$$

Thus magnification, m= $\frac{v}{u} = \frac{h_1}{h}$

NOTE:

(i) No signs need be inserted in the magnification formula.

(ii) Using the mirror formula, a connection relating magnification to the focal length of the mirror with either the object distance or the image distance can be established.

Relationship connecting **m, v and f**

Using the mirror formula; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Multiplying, v, throughout we get

$$\frac{v}{f} = \frac{v}{u} + 1$$

Since $\frac{v}{u} = m$

Then, $\frac{v}{f} = m + 1$ or $m = \frac{v}{f} - 1$

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Examples 1

An object is placed 10cm in front of a concave mirror of focal length 15cm. Find the image position and magnification.

Solution:

For a concave mirror, focal length f = +15cm, u = 10cm.

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{f \times u}{u-f} = \frac{15 \times 10}{10-15} = \text{-30cm}$$

The negative sign implies that the image formed is **virtual** and it is formed 30cm from the mirror.

∴ **Magnification, m** $= \frac{v}{u} = \frac{30}{10} = 3$

Examples 2

(2) The image of an object in a convex mirror is 6cm from the mirror. If the radius of curvature of the mirror is 20cm, find the object position and the magnification.

Solution:

For a convex mirror, $f = -\frac{r}{2} = -\frac{20}{2} = -10\text{cm}$

$$= \frac{-r}{2} = \frac{-20}{1} = 10\text{cm}$$

v = − 6cm   (The image in a convex mirror is always virtual)

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, gives

$$u = \frac{f \times v}{v-f} = \frac{-10 \times -6}{-6-(-10)} = 15\text{cm}$$

Magnification, m $\frac{v}{u} = \frac{6}{15} = 0.4$

Examples 3

Show that an object and its image coincide in position at the centre of curvature of a concave mirror. Hence find the magnification produced in this case.

Solution

At the centre of curvature of a concave mirror, object distance **u = r** where r is the radius of curvature of the mirror.

Using the mirror formula $= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$ and substituting r for u gives

$$\frac{1}{r} + \frac{1}{v} = \frac{2}{r}$$

V= r

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Thus the image is also formed at the centre of curvature and therefore it coincides in position with its object.

Hence magnification, $m = \dfrac{v}{u} = \dfrac{r}{r} = 1$

Therefore, he object and its image are of the same size in this case.

Examples 4

A concave mirror forms on a screen a real image of three times the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is **30cm,** determine the
 (i) focal length of the mirror
(ii) shift of the object

Solution

(i)   Using the relationship $m = \dfrac{u}{f} - 1$ for values $m_1 = 3$ and $m_2 = $ gives

$v_1 = (m_1 +1)f$ …………………………….a
$v_2 = (m_2 + 1)f$ ……………………… b
For $m_2 > m_1$, it follows that $v_2 > v_1$

thus, the given shift in image $= v_2 - v_1 = 30$cm

Equation a – equation b gives
$v_2 - v_1 = (m_2 - m_1)f$

$f = \dfrac{v_2 - v_1}{m_2 - m_1} = \dfrac{30}{5-3} = 15$cm

(ii) Using the relationship $\dfrac{1}{m} = \dfrac{u}{f} - 1$ for values $m_1 = 3$ and $m_2 = $ gives

$u_1 = \dfrac{f}{m_1} +1$ …………………….(a)

$u_2 = \dfrac{f}{m_2} +1$ …………………….(b)

For $m_2 > m_1$, follows that $u_1 > u_2$

The required shift in object distance is equal to $u_1 - u_2 = (\dfrac{f}{m_1} +1) - (\dfrac{f}{m_1} +1)$

$= (\dfrac{f}{m_1} - \dfrac{f}{m_1})$

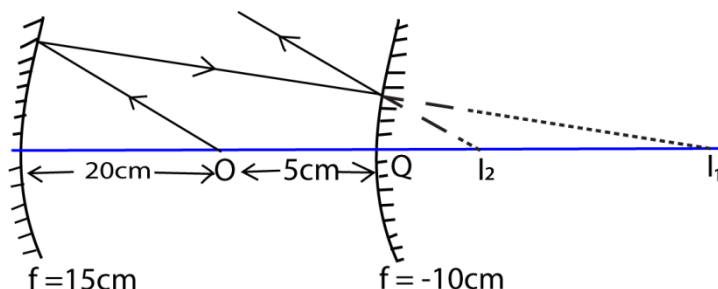$= \dfrac{15}{3} - \dfrac{15}{5} = 2$cm

Examples 5

A concave mirror **P** of focal length 15cm faces a convex mirror **Q** of focal length

10cm placed 25cm from it. An object is placed between **P** and **Q** at a point 20cm from **P**.
(i) Determine the distance from **Q** of the image formed by reflection, first in **P** and then in
   Q
(ii) Find the magnification of the image formed in **(i)** above

Solution

f =15cm   f = -10cm

Consider the action of a concave mirror
$$u = 20cm, \quad and \quad f = 15cm$$

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, gives, $v = \frac{fu}{u-f} = \frac{20 \times 15}{20-15} = 60cm$

$\therefore$ The image distance from a concave mirror = **60cm**

Thus, the image distance behind a convex mirror = **60 − (5 + 20)cm = 35cm.**
Consider the action of a convex mirror

The image formed by a concave mirror acts as a virtual object for the convex mirror.
Thus $u = -35cm$ and $f = -10cm$

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, gives, $v = \frac{fu}{u-f} = \frac{-10 \times -35}{-35--15} = -14cm$

$\therefore$ A final virtual image is **14cm** behind the convex mirror

Magnification, $m = \frac{h_2}{h_o} = \frac{h_2}{h_1} \; x \; \frac{h_1}{h_0} = m2 \; x \; m1$

$m = \frac{v_1}{u_1} \; x \; \frac{v_2}{v_2}$ where u_1 = 20cm, v_1 = 60cm, u_2 = 35cm, v_2 = 14cm

thus $m = \frac{60}{20} \; x \; \frac{14}{35} = 1.2$

Examples 6

A small convex mirror is placed **60cm** from the pole and on the axis of a large concave
mirror of radius of curvature **200cm.** The position of the convex mirror is such that a real
image of a distant object is formed in the plane of a hole drilled through the concave

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mirror at its pole.

 (a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non- axial
        point of a distant object
    (ii) Suggest a practical application for the arrangement of the mirrors in **a (i)** above**.**

(b) Calculate the
    (i) radius of curvature of the convex mirror.
    (ii) height of the real image if the distant object subtends an angle of **0·5°** at the pole
        of the convex mirror.

Solution
(a)(i)
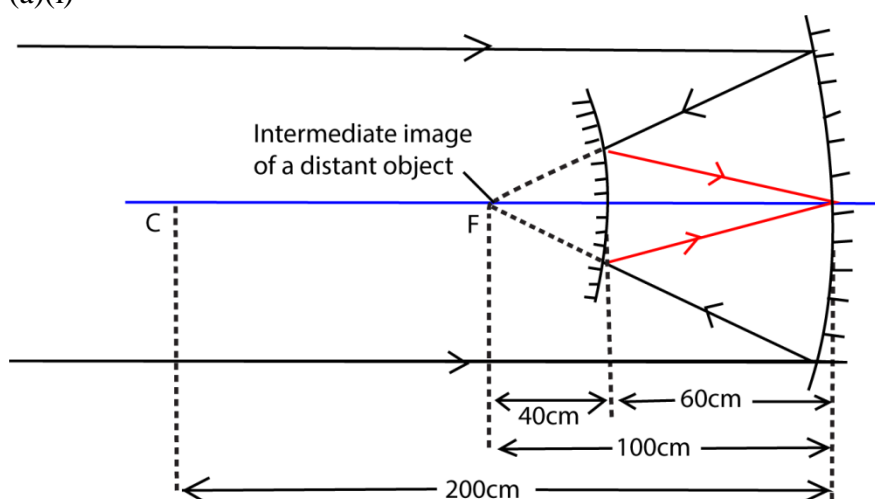
(ii) The mirror arrangement finds application in a reflecting telescope, a device for viewing distant objects

(b) (i) Consider the action of a concave mirror
    The image of a distant object is formed at the principal focus of the concave mirror. This image acts as a virtual object for a convex mirror.
    Consider the action of a convex mirror
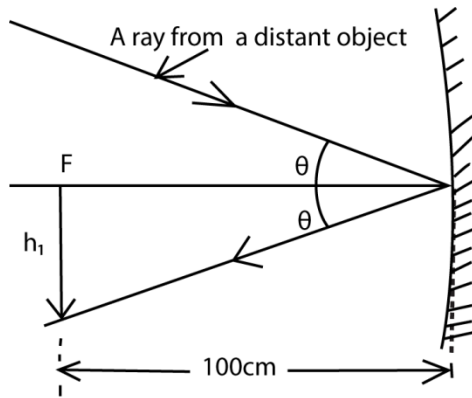   u = -40cm and v = 60cm

Applying the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$, gives

$$r = \frac{2uv}{(u+v)} = \frac{2\,x-40\,x\,60}{-40+60} = \text{-240 cm}$$

The required radius of curvature r = 240cm

(ii)   Consider the magnification produced by a convex mirror
    Let **h₁** = height of the intermediate image formed by a concave mirror as shown.

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From above, $\tan 0.50 = \dfrac{h_1}{100}$

$h_1 = 0.873\text{cm}$

Let h2 = height of the image formed by a convex mirror

Magnification, $m = \dfrac{h}{h_1} = \dfrac{v}{u}$

$h_2 = \dfrac{v}{u} \times h_1 = \dfrac{60}{40} \times 0.873 = 1.3 \text{ cm}$

Required image height = 1·3cm

EXERCISE

1.Define the terms centre of curvature, radius of curvature, principal focus and focal length of a converging mirror.

2.Distinguish between real and virtual images.

3. Explain with the aid of a concave mirror the term **a caustic surface.**

Explan the following as applied to

.

4.Explain why a parabolic mirror is used in searchlights instead of a concave mirror

5 An object is placed a distance **u** from a concave mirror. The mirror forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:

      (i) real

      (ii) virtual

6. Give two instances in each case where concave mirrors and convex mirrors are useful.

7. **(i)** Explain the suitability of a concave mirror as a shaving mirror.
  (ii) Explain with the aid of a ray diagram why a convex mirror is used as a car driving mirror.

8.Show with the aid of a ray diagram, that the radius of curvature of a concave mirror is twice the focal length of the mirror

9. Use a geometrical ray diagram to derive the relation $\frac{1}{f} = \frac{1}{u} + \frac{1}{V}$ for a concave mirror.

10. Derive the relation connecting the radius of curvature **r** object distance **u** and image distance **v** of a diverging mirror.

11. An object is placed perpendicular to the principal axis of a concave mirror of focal length f at a distance **(f+ x)** and a real image of the object is formed at a distance **(f + y).** Show that the radius of curvature **r** of the mirror is given by

  $r = 2\sqrt{xy}$

11. (i) Define the term **linear magnification.**

   (ii) Show that in a concave mirror,
     Linear magnification$= \dfrac{Image\ distance}{objective\ distance}$

(iii) A concave mirror of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.

(iv) A concave mirror of focal length 10cm forms an image five times the height of its object. Find the possible object and corresponding image positions.

[**Ans: (iii) u = 10cm, v = −30cm (iv) u = 12cm, v = 60cm OR u = 8cm, v =−40cm**]

13. A concave mirror forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 30cm, determine the

(**i**) focal length of the mirror
(**ii**) shift of the object

[**Answers: (i) f = 10cm   (ii) 3cm**]

14. A concave mirror of radius of curvature **20cm** faces a convex mirror of radius of curvature **10cm** and is **28cm** from it. If an object is placed midway between the mirrors, find the nature and position of the image formed by reflection first at the concave mirror and then at the convex mirror.

[**Answer:** A final virtual image is **17·5cm** behind the convex mirror]

15. A small convex mirror is placed **100cm** from the pole and on the axis of a large concave mirror of radius of curvature **320cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(**a**) (**i**) Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

(**ii**) Suggest a practical application for the arrangement of mirrors in **a (i)**

above**. (iii)** Calculate the radius of curvature of the convex mirror

(**b**) If the distant object subtends an angle of **3 × 10⁻³ radians** at the pole of the concave mirror, calculate the
(**i**) size of the real image that would have been formed at the focus of the concave mirror.
(**ii**) size of the image formed by the convex mirror

[**Ans: (a) (iii) 150cm (b) (i) 0·48cm (ii) 0·8cm**]

Determination of the focal length of a concave mirror.
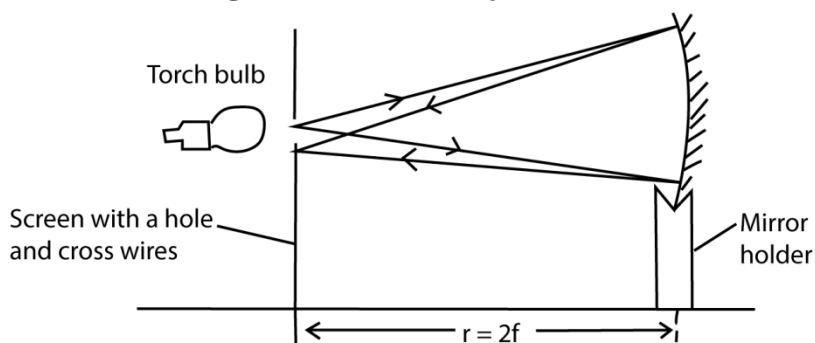
Method (1) using a pin at C
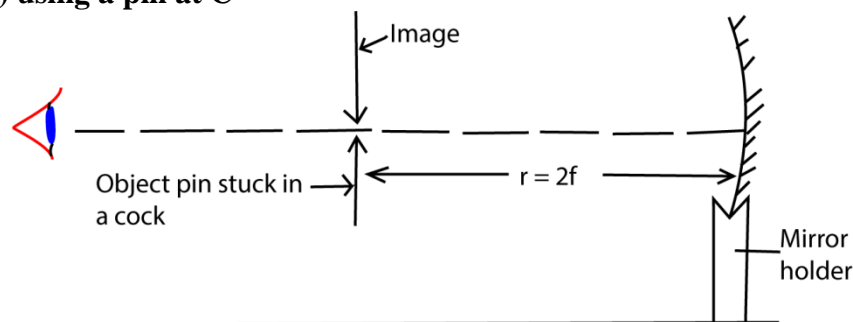
An object pin is placed in front of a mounted concave mirror so that its tip lies along the axis of the mirror. The position of the pin is adjusted until it coincides with its image such that there is no parallax between the pin and its image. The distance **r** of the pin from the mirror is measured.

The required focal length $\mathbf{f} = \dfrac{r}{2}$

NOTE :

(i) In the position where there is no parallax between the object pin and its image, there is no relative motion between the object and its image when the observer moves the head from side to side.

(ii) When the pin coincides with its image, the rays are incident normal to the mirror and are thus reflected along their own path. Therefore the pin coincides with its image at the centre of curvature of the mirror.

Method (2) using an illuminated object at C

Align an illuminated bulb, a screen and a concave mirror mounted in a holder as shown above. The mirror position is adjusted to or from the screen until a sharp image of the cross-wire is formed on the screen besides the object. The distance **r** of the mirror from the screen is measured.

The required focal length $\mathbf{f} = \dfrac{r}{2}$

Method (3) Using no parallax method in locating V
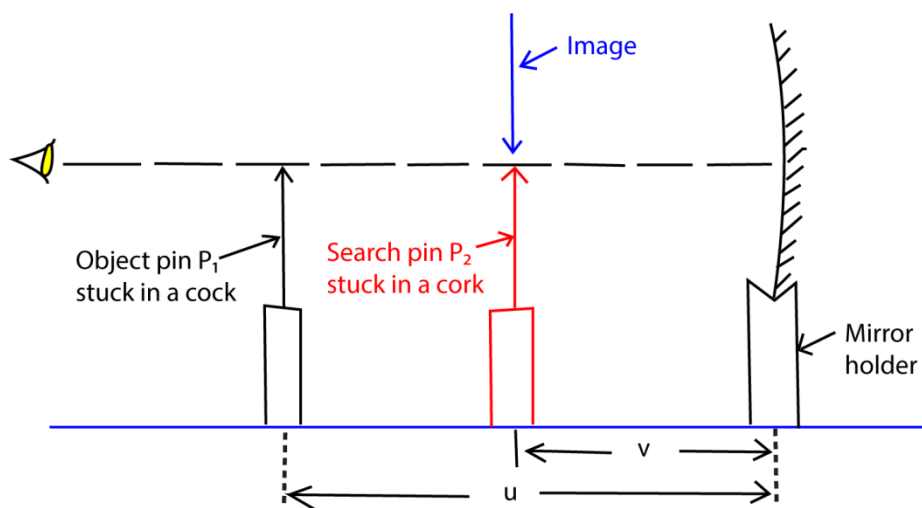
An object pin P_1 is placed at a distance **u** in front of a mounted concave mirror so that its tip lies along the axis of the mirror. A search pin P_2 placed between the mirror and pin p_1 is adjusted until it coincides with the image of pin p_1 by no-parallax method. The distance **v** of pin p_2 from the mirror is measured**.** The procedure is repeated for several values of **u** and the results are tabulated including values of **uv,** and **u+v.**
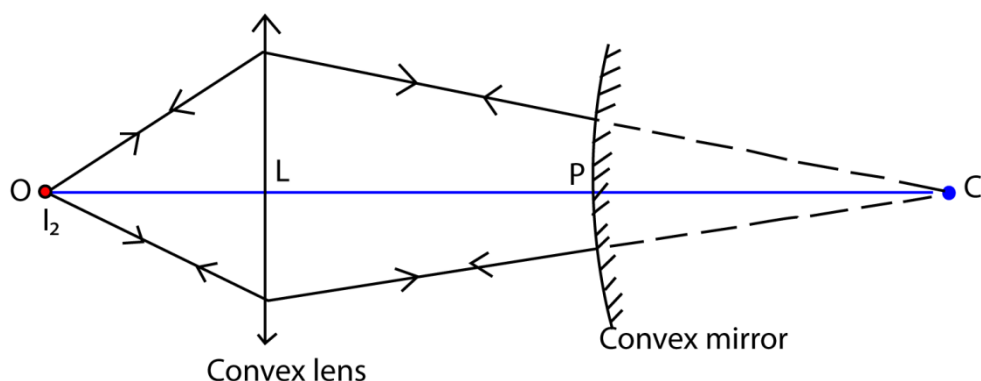
A graph of **uv** against **u+v** is plotted and the slope **s** of such a graph is equal to the focal length **f** of the mirror.

NOTE:

If a graph of $\frac{1}{u}$ against $\frac{1}{v}$ is plotted, then each intercept $C = \frac{1}{f}$

Determination of focal length of a convex mirror.
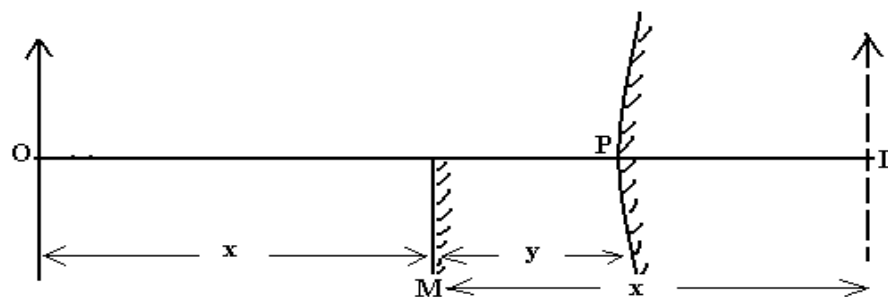
Method (1) Using a convex lens.

Convex lens

Convex mirror

(i)   The apparatus is arranged as shown above
(ii) An object, O is placed in front of a convex lens L and its image formed at C
(iii) The distance LC is measured and recorded.
(iv) The convex mirror whose focal length, f, is required is placed is placed
      between L and C with its reflecting surface facing the lens.
(v)   The lens is then moved along the axis, OC until a converging beam incident
      normally on the mirror forms its image at O
(vi) Distance LP is measured

PC = LC − LP thus, f can be determined from $f = \dfrac{PC}{2}$

Method (2) using No parallax.

(i)   An object pin **O** is placed in front of a convex mirror as shown in the
       diagram above
(ii)   A virtual diminished image is formed at **I**.
(iii)   A plane mirror **M** is placed between **O** and **P** so as to intercept half the field
       of view of the convex mirror.
(iv)   Mirror **M** is adjusted until its own image of **O** coincides with **I** by no
       parallax method.
(v)   Measure the distances **x** and **y.**
(vi)   The focal length of the mirror is calculated from $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$

Where u = (x + y) and v = (y-x)

Note:
(i) The two images coincides when they are as far behind the plane mirror as

the object is in front.

(ii) Substituting for u = x + y and v = y − x in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$
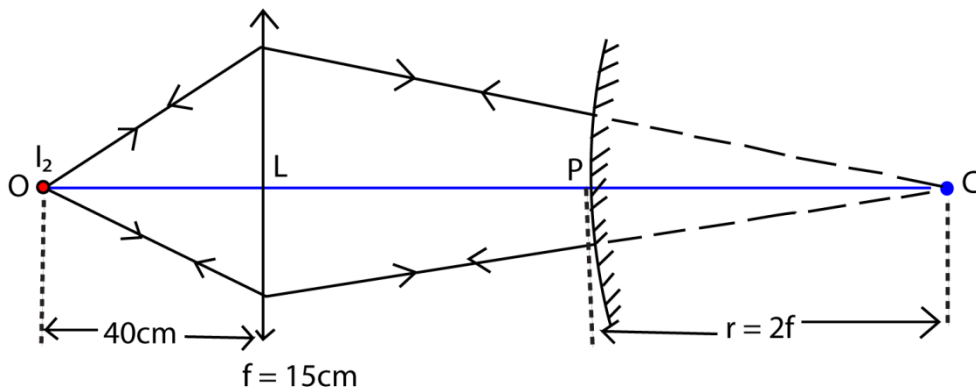
Examples 7

 An object **O** is placed **40cm** in front of a convex lens of focal length **15cm** forming an image on the screen. A convex mirror situated **4cm** from the lens in the region between the lens and the screen forms the final image besides object **O**.

      (i) Draw a ray diagram to show how the final image is formed.

      (ii) Determine the focal length of the convex mirror.

Solution

Consider the action of a convex
  lens u = 40cm, and f = 15cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ give

Image distance v of the lens $= \frac{fu}{u-f} = \frac{15 \times 40}{40-15} = 24cm$

The radius of curvature **r** = **(24 − 4)cm** = **20cm**

Using the relation r = 2f
⟹   2f = 20cm
  ∴ f = 10cm
Thus f = -10cm "The centre of curvature of a convex mirror is virtual"

Examples 8

A plane mirror is placed **10cm** in front of a convex mirror so that it covers about half of the mirror surface. A pin **20cm** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.
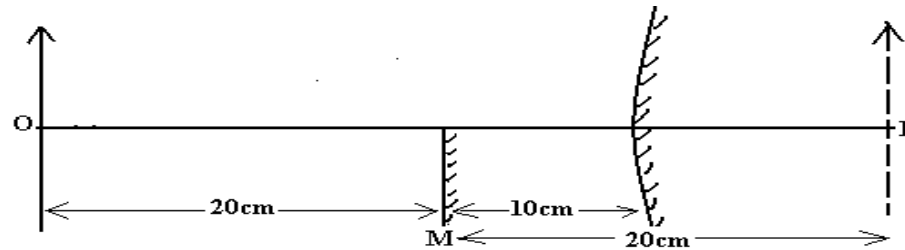
Consider the action of a convex mirror

u = 30cm and v = − (20 − 10) = −10cm   "The image formed is virtual "

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ give

Image distance v of the lens $= \frac{fu}{u-f} = \frac{-10 \; x \; 30}{30-10} = -15cm$

Exercise.

1. (a) Distinguish between a real image and virtual image

A real image is an image formed by intersection of rays from an object in a concave mirror and can be formed on a screen while a virtual image is an image formed by apparent intersection of rays in concave and convex mirrors and cannot be formed on a screen.

(b) Derive an expression relating the focal length f, of a convex mirror to the object distance, u, and image distance, v.

Solution

Consider the incidence of ray **OX** on to a convex mirror from a point object **O** placed along the principle axis and then suddenly reflected in the direction **XK** making an angle θ with the normal **XN**.

From Δ XIC, $\theta + \lambda = \beta$

$\Rightarrow \theta = \beta - \lambda$ ------------------------------(i)
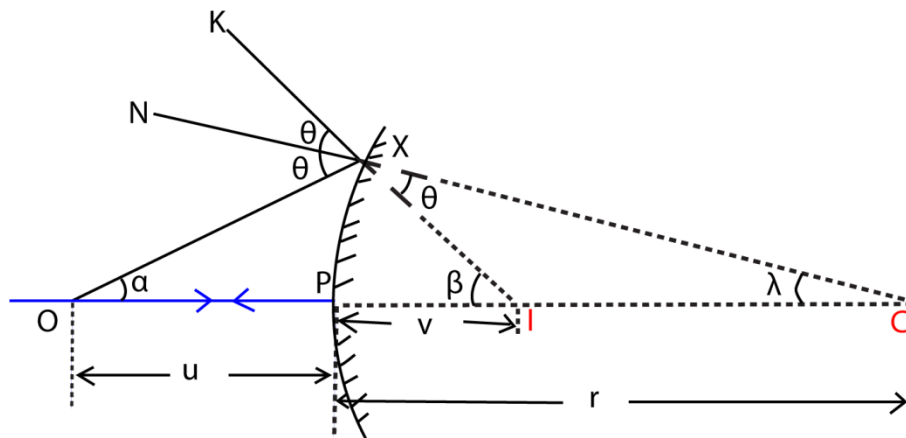
From Δ OXI, $\alpha + \beta = 2\theta$ --------------------------------(ii)

Substituting equation (i) into (ii) gives.

$\alpha + \beta = 2 (\beta - \alpha)$

$\Rightarrow \alpha - \beta = -2\lambda$ ----------------------------------(a)

If X is very close to P, then

$\alpha \approx \tan\alpha = \frac{XP}{u}$, $\beta \approx \frac{XP}{-v}$ (I is virtual) and $\lambda \approx \tan\lambda = \frac{XP}{-r}$ (C is $virtual$)

Equation (a) becomes

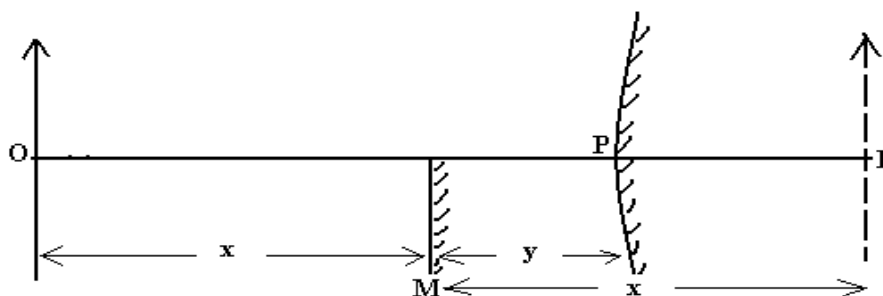$$\frac{XP}{u} - \frac{XP}{-v} = \frac{-2XP}{-r}$$

$$= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

But 2 = 2f

$$= \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Note that the focal length of a convex mirror is a negative distance.

2. (a) (i) Describe how the focal length of a convex mirror can be measured using a convex less of known focal length.

(i) An object pin **O** is placed in front of a convex mirror as shown in the diagram above
(ii) A virtual diminished image is formed at **I**.
(iii) A plane mirror **M** is placed between **O** and **P** so as to intercept half the field of view of the convex mirror.
(iv) Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method.
(v) Measure the distances **x** and **y.**
(vi) The focal length of the mirror is calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where u = (x + y) and v = (y-x)

Note:
(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

(ii) Substituting for u = x + y and v = y − x in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$

(ii)   The plane mirror, P, in figure 1 is adjusted to a position 8.0cm from the optical pin, the image of the pin in P coincides with image in M.
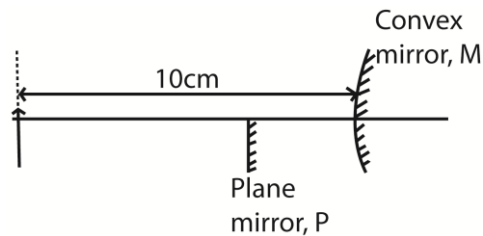
Calculate the focal length of the convex mirror

Solution
Distance from the plane mirror to the image = 8cm
Distance from the plane mirror to convex mirror = 10 -8 = 2cm

Object distance, u = 10cm
Image distance, v = 8 -2cm = 6cm
From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{10} + \frac{1}{-6}$ (image is virtual)
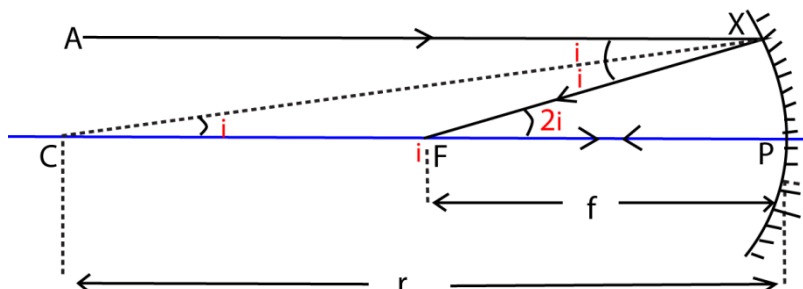f = -15cm (principle focus is virtual)

3.   (a) state the laws of reflection
Law 1: the incident ray, reflected ray and the normal at the point of incidence all lie in the same plane
Law 2: the angle of incidence is equal to the angle of reflection

(b) Show with the aid of a ray diagram that the radius of curvature of a concave mirror is twice the focal length of the mirror. (5marks)

Consider the reflection of a paraxial ray, AX, parallel to the principal axis of a concave mirror as shown.

Taking **FP** = focal length and **C** = centre of curvature, then , **CX**, is the normal to the mirror surface and, **CP**, is the radius of curvature.

$\tan 2i \approx 2i$

If, X, is close to P, then $\tan 2i \approx 2i = \dfrac{XP}{FP}$--------------- (for small angles)

$\Rightarrow \quad 2i = \dfrac{XP}{FP}$ --(i)

Similarly $\tan i \approx i = \dfrac{XP}{CP}$

$\Rightarrow \quad i = \dfrac{XP}{CP}$ -- (ii)

Combining equations (i) and (ii) gives,

$2\dfrac{XP}{CP} = \dfrac{XP}{FP}$

Canceling XP throughout and simplifying for CP gives

$CP = 2FP$ where $CP = r, \ FP = f$

$\therefore r = 2f$.

Thus, the radius of curvature of a concave mirror is twice its focal length.

4. (a) Define the focal length of concave mirror.

It is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis converge after reflection by the mirror.

(b) Use a geometrical to derive the relation $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ for concave mirror.

Consider the incidence of ray **OX** on to a concave mirror from a point object **O** placed along the principal axis and then suddenly reflected in the direction XI making an angle θ with the normal CX.
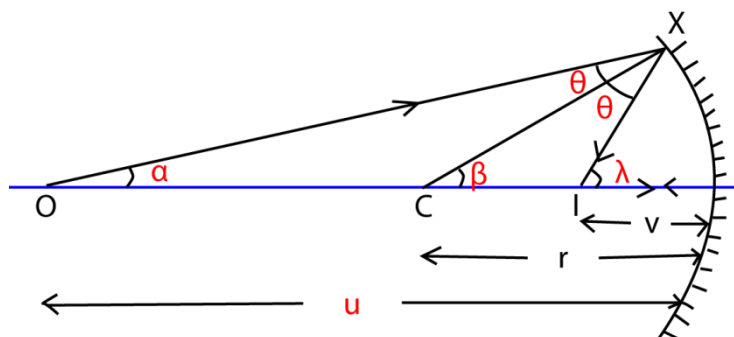
Ray OP strikes the mirror incident normally at P and thus reflected back along its own

path. The point of intersection **I** of the two reflected rays is the image position.

$$\text{From } \Delta \text{ OXC, } \alpha + \theta = \beta \text{ -----------------------------(i) From}$$
$$\Delta \text{ CXI, } \beta + \theta = \lambda \text{ ----------------------------(ii) Equation}$$
$$\text{(i) } - \text{Equation (ii) eliminates } \theta \text{ to give.}$$
$$\alpha - \beta = \beta - \lambda$$
$$\Rightarrow \quad \alpha + \lambda = 2\beta \text{ --(a)}$$

If X is very close to P, then

$$\alpha \approx \tan \alpha = \frac{XP}{u}, \ \beta \approx \tan \beta = \frac{XP}{r}, \lambda \approx \tan \lambda = \frac{XP}{v}$$

Equation (a) becomes $= \dfrac{XP}{u} + \dfrac{XP}{v} = \dfrac{2XP}{r}$

$$= \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

But 2 = 2f

$$= \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

5. (a)(i) state the laws of reflection
    (ii) Draw a ray diagram to show a concave mirror forms a real image of real object placed
       perpendicular to its principal axis.

    Concave mirror forms real images when an object is placed beyond F.

    (iii) Describe an experiment, including a graphical analysis of the results to determine the focal
       length of a concave mirror using the No-Parallax method. (06 marks)

    Or

    You are provided with the following pieces of apparatus.
       A screen with cross-wire, lamp, a concave mirror, and meter rule.
       Describe an experiment to determine the focal length of concave using the above apparatus.
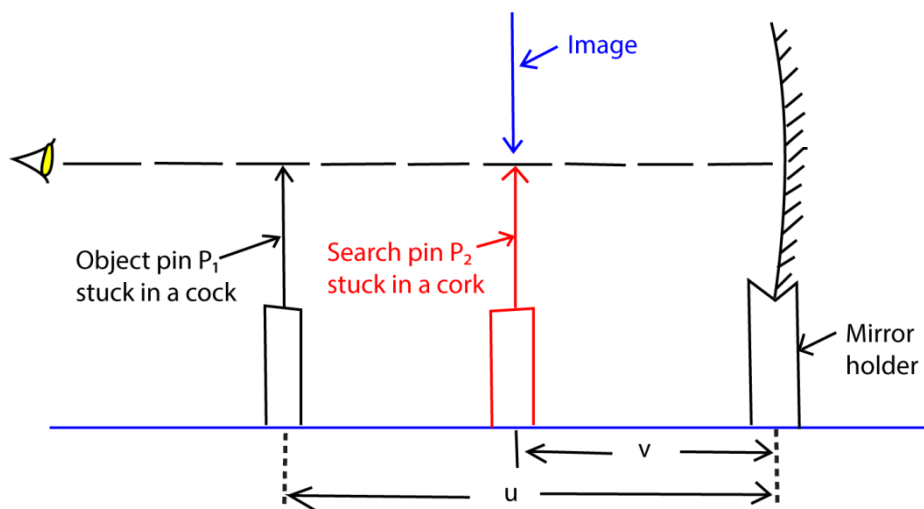
An object pin P_1 is placed at a distance **u** in front of a mounted concave mirror so that its tip lies along the axis of the mirror.

A search pin P_2 placed between the mirror and pin p_1 is adjusted until it coincides with the image of pin p_1 by no-parallax method.

The distance **v** of pin p_2 from the mirror is measured.

The procedure is repeated for several values of **u** and the results are tabulated including values of **uv**, and **u+v.**

A graph of **uv** against **u+v** is plotted and the slope **s** of such a graph is equal to the focal length **f** of the mirror.
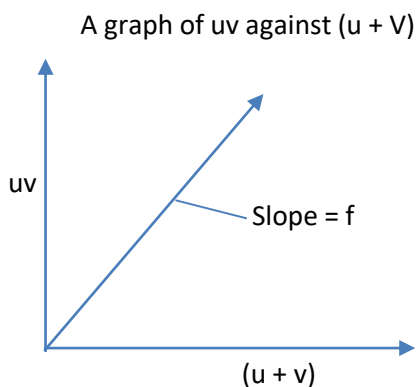
6. A concave mirror forms an image half the size of the object. The object is then moved towards the mirror until the image size is three quarter that of the object. Is the image is moved by a distance of 0.6cm, calculate the

(i) Focal length of the mirror (3marks)

Case I

Let image distance be u

From magnification, M = $\frac{v}{u} = \frac{1}{2}$

$$v = \frac{1}{2}u \quad ....(i)$$

Case II

$$\frac{v + 0.6}{u} = \frac{3}{4}$$

$$v + 0.6 = \frac{3}{4}u \quad ....\ (ii)$$

combining equations (i) and (ii)

$$\frac{1}{2}u + 0.6 = \frac{3}{4}u$$

$u = 2.4$cm

$v = 1.2 cm$

From $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{2.4} + \frac{1}{1.2} = \frac{1}{f}$$

f = 0.8cm

(ii) New position of object (3marks)

v = 1.8cm, f = 0.8cm, let object distance be u
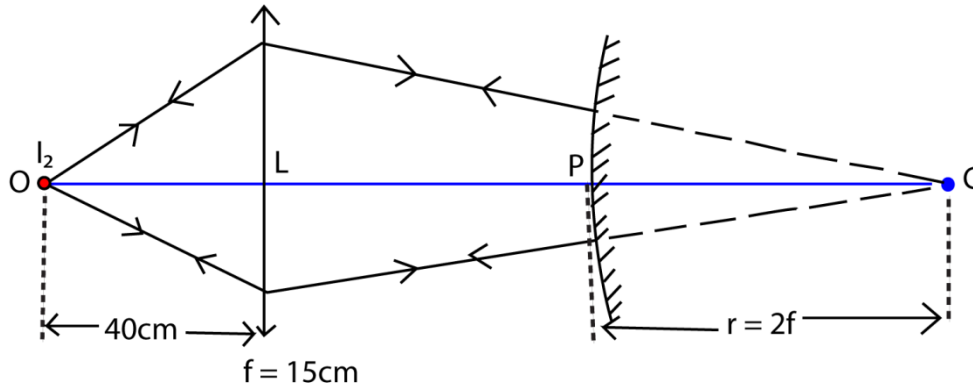
$$\frac{1}{u} + \frac{1}{1.8} = \frac{1}{0.8}$$

u = 1.44cm

7. An object **O** is placed **40cm** in front of a convex lens of focal length **15cm** forming an image on the screen. A convex mirror situated **4cm** from the lens in the region between the lens and the screen forms the final image besides object **O**.

(i) Draw a ray diagram to show how the final image is formed.

(ii) Determine the focal length of the convex mirror.

Solution

Consider the action of a convex lens u $=$ 40cm, and f $=$ 15cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ give

Image distance v of the lens $= \frac{fu}{u-f} = \frac{15 \; x \; 40}{40-15} = 24cm$

The radius of curvature **r** $=$ **(24 − 4)cm** $=$ **20cm**

Using the relation r = 2f
$\Rightarrow$ 2f $=$ 20cm
     $\therefore$ f $=$ 10cm
Thus f $=$ **-10cm**   "The centre of curvature of a convex mirror is virtual"

8. A plane mirror is placed **10cm** in front of a convex mirror so that it covers about half of the mirror surface. A pin **20cm** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.


Consider the action of a convex mirror

u = 30cm and v = – (20 – 10) = –10cm   "The image formed is virtual "

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ give

Image distance v of the lens = $\frac{fu}{u-f}$ = $\frac{-10 \; x \; 30}{30-10}$ = $-15cm$

9. A plane mirror is placed at a distance **d** in front of a convex mirror of focal length **f** such that it covers about half of the mirror surface. A pin placed at a distance **L** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, Show that **$2df = d^2 - L^2$**

Solution
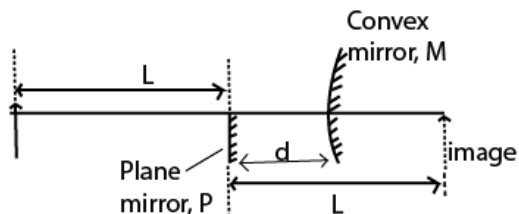
For convex object distance, u = (L+d) and image distance, v = (L – d)

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{(L+d)} + \frac{1}{-(L-d)} = \frac{-(L-d)+(L+d)}{(L+d)(L-d)} = \frac{2d}{L^2 - d^2}$

$2df = L^2 - d^2$