

P425/2

Applied Mathematics

Paper 2

March 2024

3 hours

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Applied Mathematics

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3 hours

INSTRUCTIONS:

- Answer ***all*** the ***eight*** questions in ***Section A*** and any ***five*** from ***Section B***.
- Any additional question(s) will ***not*** be marked.
- All necessary working must be shown clearly.
- Mathematical tables with a list of formulae and squared papers are provided.
- In numerical work, take g to be 9.8 ms^{-2} .

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SECTION: A (40 MARKS)

1. 40 percent of the students in certain school own personal computers. If 24 students are selected at random, find the probability that between 8 and 15 students own personal computers. (05 marks)
2. Forces of $(a\mathbf{i} + b\mathbf{j})\text{N}$ and $(6\mathbf{i} - 4\mathbf{j})\text{N}$ act at the points having position vectors $(-2\mathbf{i} - 2\mathbf{j})\text{ m}$ and $(3\mathbf{i} - \mathbf{j})\text{ m}$ respectively. If the forces reduce to a couple, find;
 - (a) a and b
 - (b) the moment of the couple (05 marks)
3. Use the trapezium rule with 5 strips to find an approximate value for

$$\int_1^2 x e^{-2x} dx \quad \text{Correct to 3 significant figures.} \quad (05 \text{ marks})$$

4. A random sample of 100 Adults is taken from a population. The sample is found to have a mean weight of 76.0 and variance of 144.00. Find the 97% confidence limits for the mean of the population (05 marks)
5. Two particles are travelling along a straight line AB of length 20m. At the same instant one particle starts from rest at A and travels towards B with a constant acceleration of 2ms^{-2} and the other particle starts from rest at B and travels towards A with a constant acceleration of 5m^{-2} . Find how far from A the particles collide. (05 marks)
6. An examination question has two parts A and B. The probability of a student getting part A correct is $\frac{2}{3}$. If she gets part A correct, the probability that she gets part B correct is $\frac{3}{4}$, otherwise its $\frac{1}{6}$. There are three marks for a correct solution of part A, two marks for part B and a bonus mark if both parts are correct. Calculate the expected student's total mark. (05 marks)
7. A light inextensible string of length 40 cm has its upper end fixed at a point A, and carries a mass of 2 kg at its lower end. A horizontal force applied to the mass keeps it in equilibrium, 20 cm from the vertical through A. Find the magnitude of this horizontal force and the tension in the string. (05 marks)
8. The table below shows the values of a function $f(x)$

X	1.0	1.1	1.2
f(x)	0.000	0.095	0.182

Use linear interpolation/extrapolation to find;

- (i) $f(1.16)$
- (ii) $f(x) = 0.24$ (05 marks)

SECTION B (60 MARKS)

9. (a) Show that there is real root of the equation $e^x - 2x - 5 = 0$ between -3 and -2.
(03 marks)
- (b) Derive the Newton-Raphson formula, to find the root of the equation in (a) above,
(03 marks)

Hence construct a flow chart that

- (i) reads the first approximation X_0 ,
 - (ii) computes the root to 3 decimal places,
 - (iii) prints the root.
(04 marks)
- (c) Perform a dry run for your flow chart using $x_0 = -2.5$ (02 marks)
10. (a) The height of pupils in a school is normally distributed with mean 130 cm and standard deviation of 7 cm. If a pupil is selected at random. Find the probability that his/her height is
- (i) above 144 cm (02 marks)
 - (ii) less than 123 cm (02 marks)
- (b) determine the lower and upper quartile heights (04 marks)
- (c) If 4000 pupils are selected at random, determine the number whose height will lie between 126.5 cm and 137 cm. (04 marks)
11. A car of mass 750 kg has a maximum power of 30 kW and moves against a constant resistance to motion of 800 N. Find;
- (a) the maximum speed of the car;
- (i) on the level road. (03 marks)
 - (ii) Up an incline of slope 1 in 10 (04 marks)
- (b) the acceleration down the same incline if it moves at 40 ms^{-1} (05 marks)

12. The table below shows the time taken to load vehicles in a day

Time (minutes)	Number of vehicles
40 - 45	4
45 - 50	13
50 - 55	17
55 - 60	44
60 - 70	59
70 - 80	7

- (a) Draw an histogram and use it to estimate the modal time. (05 marks)
- (b) Find the: (i) Mean time, (ii) Median time (iii) standard deviation of the time (07 marks)
13. (a) A particle is projected from a point which is 2 m above the ground level with a velocity of 40 ms^{-1} at an angle of 45° to the horizontal. Find its horizontal distance from the point of projection where it hits the ground. (06 marks)
- (b) A particle is projected inside a tunnel which is 2 m high. If the initial speed is u show that the maximum range inside the tunnel is $\sqrt[4]{\frac{u^2 - 4g}{g}}$ (06 marks)
14. The numbers y and z are approximated with possible errors of Δy and Δz respectively.
- (a) Show that the maximum relative error in the quotient $\frac{y}{z}$ is given by $\left| \frac{\Delta y}{y} \right| + \left| \frac{\Delta z}{z} \right|$ (05 marks)
- (b) Given that $x = 20.136$, $y = 15.3$ and $z = 9.5342$ are rounded to the given number of decimal places, find the limits within which the exact value of $\frac{y-x}{y+z}$ is expected to lie (07 marks)

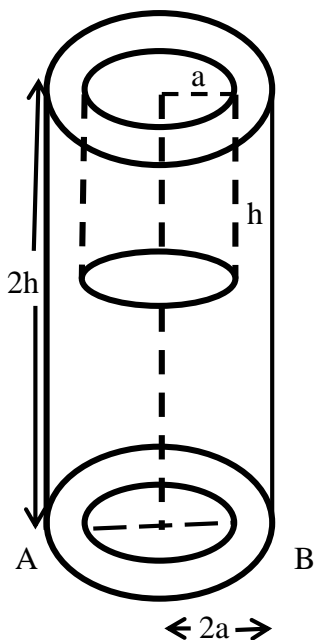
15. A continuous random variable X has a probability density function given by:

$$f(x) = \begin{cases} 0.5k x^2 & 0 \leq x < 2 \\ k(4 - x) & 2 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of $f(x)$ (03 marks)
 (b) Determine the value of k (03 marks)
 (c) Find the (i) Mean (ii) Median (06 marks)

16. (a) The rectangle ABCD has $AB = 4$ cm and $AD = 2$ cm. Particles of mass 3 kg, 5 kg, 1 kg and 7 kg are placed at the points A, B, C and D respectively. Find the distance of the centre of gravity of the system from each of lines AB and AD. (06 marks)

- (b) The diagram below shows a uniform cylinder of radius $2a$ and height $2h$ with a cylindrical hole of radius a and height h drilled centrally at one plane end.



Show that the centre of gravity of the remaining solid is $\frac{13}{14}h$ from the base AB. (06 marks)

END