P425/1

PURE MATHEMATICS

PAPER 1

EOT 1 2024

3 hours

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Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

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INSTRUCTIONS TO CANDIDATES

*Answer **all** the **eight** questions in section **A** and **five** questions from section **B.***

*Any additional question (s) answered will **not** be marked.*

***All** working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper.

Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.

Neat work** is a **must.

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Turn over

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Prepared by Mukidi Andrew 0708934062

SECTION A: (40 MARKS)

*Answer **all** the questions in this section*

1. Evaluate $\int_{-1}^{\frac{-1}{2}} \frac{4x+2}{(x^2+x-2)^4} dx$ (05 marks)

2. Solve for x in $\log_a x + 3 + \frac{1}{\log_x a} = 2 \log_a 2$ (05 marks)

3. The roots of the quadratic equation $x^2 - bx + 2 = 0$ are α and β. Given that α=$\sqrt{5} + \sqrt{3}$. Show that $b = 2\sqrt{5}$. Hence find the value of $\alpha^2 - \beta^2$.   (05 marks)

4. Given that $y = 3^x$, find $\frac{d^2y}{dx^2}$ when $x = -1$ (05 marks)

5. If $\log 2$, $\log 2^x - 1$ and $\log 2^x + 3$ are in an Arithmetic progression (A.P), Find the value of x.   (05 marks)

6. Solve for y in $4sin^2y - 12 \sin 2y + 35cos^2y = 0$. For $0^o \le y \le \frac{\pi}{2}$ (05 marks)

7. Find the area bounded by the curve$x = y^2 - 4$ and the y- axis.   (05 marks)

8. Given that the plane $4x + 3y - 3z - 4 = 0$

   (a) Show that the point A (1, 1, 1) lies on the plane.   (02 marks)

   (b) Find the perpendicular distance from the plane to the point B (1, 5, 1).   (03 marks)

SECTION B: (60 MARKS)

*Answer any **five** questions from this section*

9. **(a)** Prove by induction that $3^{2n+2} - 8n - 9$ **is** divisible by 64 for $n \geq 1$. (05marks)

    (b) The expression $6x^3 + 7x^2 + ax + b$ has a remainder of 72 when divided by $x - 2$ and is exactly divisible by $x + 1$. Find the values of a and b. Show that $2x - 1$ is also a factor of the polynomial and obtain the third factor. (07 marks)

10. (a) Find the position vector of the point of intersection of the line $x - 2 = 2y + 1 = 3 - z$ and the plane $x + 2y + z = 3$. (04 marks)

    (b) Show that the position vectors $\boldsymbol{OA} = 4\boldsymbol{i} - 8\boldsymbol{j} - 13\boldsymbol{k}$, $\boldsymbol{OB} = 3\boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{OC} = 3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ are vertices of a triangle. (04 marks)

    (c) Determine the Cartesian equation of the plane defined by the vector equation

$$ \boldsymbol{r} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}. $$

(04 marks)

11. Given that $y = \dfrac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x}$

    (i) Prove that $y + tan^2 15^o$ in the form $p + q\sqrt{r}$ where p, q and r. are integers

    (ii) Hence find the value of x between 0° and 360° for which $2y + sec^2 \left(\frac{x}{2}\right) = 0$

    (12 marks)

12. Differentiate the following functions with respect to x

    (a) $3x \ln x^2$

    (b) $sin^3 2x$

    (c) $e^{\tan x}$ (12 marks)

13. (a) Given that $y = \theta - \cos\theta$ and $x = \sin\theta$, find $\dfrac{dy}{dx}$ when $\theta = 0^o$

    (b) The curve $y = ax^2 + bx + c$ cuts the x-axis at -1 and 3 and also passes through the point (1, 12). Sketch the curve hence find the area enclosed by the curve and the x-axis

    (12marks)

14. Given that $\boldsymbol{y} = \dfrac{64x^2 - 148x + 78}{(4x-5)^3}$, Express y into partial fractions hence find $\int_4^6 y\, dy$

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15. (a) The complex number has modulus 1 and argument120^o. Find the fourth root of z.

(b) If z is a complex number, describe and illustrate on the argand diagram the locus given by$\left|\frac{z+i}{z-2}\right| = 3$. (12 marks)

16. Evaluate (a) $\int_0^{\frac{\pi}{2}} \sin 2x \cos x \, dx$

(b) $\int \sqrt{(1 - x^2)} \, dx$ (12 marks)

END

The only way to learn mathematics is to do mathematics