

**P425/1**

**PURE MATHEMATICS**

Paper 1

**June 2023**

3 hours

**Uganda Advanced Certificate of Education**

**Pure Mathematics**

Paper 1

3 hours

**INSTRUCTIONS TO CANDIDATES:**

*Answer **all** the **eight** questions in Section **A** and **five** questions from Section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** working **must** be shown clearly.*

*Begin each answer on a fresh sheet of paper.*

*Graph paper is provided.*

*Silent, non – programmable scientific calculators and Mathematical tables with a list of formulae may be used.*

## SECTION A: (40 MARKS)

Attempt **ALL** questions in this section

1. Solve the inequality  $\left| \frac{x^2-4}{x} \right| \leq 3$ . (05 marks)
2. Find the cartesian equation of the locus of  $z = |z - 2 + i| = 1$  if and describe it fully. (05 marks)
3. Prove that  $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$ . (05 marks)
4. Use small changes to show that  $244^{\frac{1}{3}} = 3 \frac{1}{405}$ . (05 marks)
5.  $ABCD$  is a square inscribed in a circle  $x^2 + y^2 - 4x - 3y = 36$ . Find the length of the diagonals and the area of the square. (05 marks)
6.  $PQRS$  is a quadrilateral with vertices  $P(1, -2)$ ,  $Q(4, -1)$ ,  $R(5, 2)$  and  $S(2, 1)$ . Show that the quadrilateral is a Rhombus. (05 marks)
7. Evaluate  $\int_0^\pi x \cos^2 x \, dx$  and leave  $\pi$  in your answer. (05 marks)
8. Solve the equation  $e^y \frac{dy}{dx} = e^{-x}$  given that  $y(0) = 0$ . (05 marks)

## SECTION B: (60 MARKS)

Attempt **FIVE** questions in this section

9. (a) Given that  $y = Ae^{3t} + Be^{-2t}$ , show that  $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 0$ . (05 marks)
- (b) The period,  $T$  of a swing of a simple pendulum of length  $l$  is given by the equation  $T^2 = \frac{4\pi^2 l}{g}$  where  $g$  is the acceleration due to gravity. An error of 2% is made in measuring the length  $l$ , determine the resulting percentage error in the period. (07 marks)

10. (a) Determine the acute angle between the line  $\frac{x+4}{8} = \frac{y-2}{2} = \frac{1-z}{4}$  and the plane  $4x + 3y - 3z + 1 = 0$ . (05 marks)
- (b) Show that lines  $\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} - 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$  and  $\frac{x-7}{3} = \frac{y+1}{-2} = \frac{z+4}{-2}$  intersect, hence find the coordinates of the point of intersection. (07 marks)
11. (a) Prove by induction that  $4^{n+3} - 3n - 10$  is divisible by 3 for all positive integers of  $n$ . (05 marks)
- (b) Nansubuga opened up a bank account with shs 50,000. She deposits the same amount every year and makes no withdraws. After how many years will she accumulate more than one million on her account if the bank offers a 5% compound interest rate per annum? (07 marks)
12. (a) Use the substitution  $y = x + \frac{1}{x}$  to solve the equation  $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$ . (05 marks)
- (b) The expression  $py^2 + qy + r$  is divisible by  $y - 1$ , has a remainder of 2 when divided by  $y + 1$  and has a remainder of 8 when divided by  $y - 2$ . Find the values of  $p, q$  and  $r$ . (07 marks)
13. (a) By expression  $5\cos x + 12\sin x$  in the form  $R\cos(x - \beta)$  where  $R$  is a constant and  $\beta$  is an acute angle, solve  $5\cos x + 12\sin x = 7$  for  $0^\circ \leq \theta \leq 360^\circ$ . (06 marks)
- (b) If  $\sin 3\theta = p$  and  $\sin^2 \theta = \frac{3}{4} - q$ , prove that  $p^2 + 16q^3 = 12q^2$ . (06 marks)

14. (a) Find the equation of the normal to the curve  $y^2(y - 3x) = 3 - x^2$  at the point  $(-1, 2)$ . (06 marks)
- (b) Evaluate  $\int_9^{25} \frac{dx}{\sqrt{x} - \sqrt{x-9}}$ . (06 marks)
15. (a) Show that the parametric equations  $x = 5 + \frac{\sqrt{3}}{2} \cos \theta$  and  $y = -3 + \frac{\sqrt{3}}{2} \sin \theta$  represent a circle. Find the radius and the centre of the circle. (06 marks)
- (b) If the line  $3x - 4y - 12 = 0$  is the tangent to a circle with a centre  $(1, 1)$ . Find the equation of the circle. (06 marks)
16. Evaluate  $\int_3^4 \frac{2x+1}{(x-2)(x+1)^2} dx$  giving your answer correct to 3 decimal places.

**END**