P425/1
PURE MATHEMATICS
AUGUST - 2023
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS – AUGUST, 2023

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all the eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) will **not** be marked.*

*All working **must** be shown clearly.*

Begin each question on a fresh sheet of paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

 Turn Over

Answer all questions in this section

Answer all the questions in this section

1. A geometric progression has the sum of the first and second terms equal to -4. If the sum of the fourth and the fifth terms is 108. Calculate the
   (i) first term,
   (ii) Common ratio of the progression **(5 marks)**

2. Show that $3x^2 + 2y^2 + 6x - 8y = 7$ is an ellipse and hence determine its centre and eccentricity. **(5 marks)**

3. Find the coordinates on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is a horizontal line. **(5 marks)**

4. Solve the equation $3\cos^2\theta - 4\cos\theta\sin\theta + \sin^2\theta = 2 \text{ for } 0 \le \theta \le 180°$. **(5 marks)**

5. The line L_1 and L_2 are given by the equation $\frac{x-1}{k} = \frac{y}{-1} = \frac{z+3}{1}$ *and* $\frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ *respectively*.
   Find the,
   (i) value of K for which L_1 and L_2 intersect.
   (ii) point of intersection. **(5 marks)**

6. Prove by induction that for all positive integers $n, \sum_1^n \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$. **(5 marks)**

7. Evaluate $\int_2^6 \frac{\sqrt{x-2}}{x} dx$. **(5 marks)**

8. In order to post a parcel, the sum of the circumference of a cylindrical parcel and its height should add up to 6cm. Find the dimensions of the largest parcel that can be accepted. **(5 marks)**

SECTION B (60 MARKS)

Answer any five questions in this sections all questions carry equal marks

9. (a) Given that $z_1 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ and $z_2 = 8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Determine the following in the form $x+iy$:

(i) $\frac{z_1}{z_2}$ (ii) $\sqrt[3]{z_2}$. **(7 marks)**

(b) Find the locus of the complex number z represented by $\arg\left(\frac{z}{z-4+2i}\right) = \frac{\pi}{2}$. **(5 marks)**

10. (a) If y=mx is a tangent to a circle $x^2 + y^2 + 2fy + c = 0$.

$prove\ that\ c = \frac{f^2m^2}{1+m^2}$.

Hence find the equation of the tangents from origin to the circle

$x^2 + y^2 - 10y + 20 = 0$. **(7 marks)**

(b) Find the equation of a circle whose centre is at (5, 4) and touches the line joining (0, 5) and (4, 1). **(5 marks)**

Given the curve $y = \frac{12}{x^2-2x-3}$.

Determine the;

(a) range of values for y in which the curve does not lie and hence find the coordinates of the turning point. **(6 marks)**

(b) asymptotes and sketch the curve $y = \frac{12}{x^2-2x-3}$ **(6 marks)**

11.(a) Given the line $\mathbf{r} = (3+2\mu)i + (1-\mu)j + (-2+2\mu)k$. Find the :

(i) value of d if the line is in the plane $\mathbf{r}.(i - 2j - 2k) = d$

(ii) distance of the point (3, 1, 7) from the line. **(7 marks)**

(b) Given points A (2, −5, 3) and B (7, 0, −2) find the coordinates of point C which divides AB externally in the ratio 3:8. **(5 marks)**

12.(a) Prove that $log_{n^2}(ab) = \frac{1}{2}(log_{n^a} + log_{n^b})$.

Hence solve the simultaneous equations:

$2log_{9(xy)} = 5$

$log_{3^x}\, log_{3^y} + 6 = 0$ **(7 marks)**

(b) Find the first three terms of the Binomial expansion of $\sqrt{(1+x)(1+x^2)}$.

(5 marks)

(a) Show that $cos3\theta = 4cos^3\theta - 3cos\theta. Hence\ if\ cos\ \theta = \frac{1}{2}.\left(a + \frac{1}{a}\right)$,

Prove that $cos3\theta = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$. **(6 marks)**

(b) Given that A, B and C are angles of a triangle, prove that;

$sin^2\frac{1}{2}A + sin^2\frac{1}{2}B + sin^2\frac{1}{2}C = 1 - 2sin\frac{1}{2}A\, sin\frac{1}{2}B\, sin\frac{1}{2}C$ **(6 marks)**

13. (a) Differentiate the following with respect to x:

(i) $y = x^2 sin\left(\frac{1}{x}\right)$. (ii) $y = x(ln^3 x)$ **(6 marks)**

(c) Given $that\ y = log_e tan\left(\frac{\pi}{4} - \frac{x}{2}\right), Prove\ that\ \frac{dy}{dx} = -\sec x$. **(6 marks)**

16. (a) Solve the differential equation $x\frac{dy}{dx} = 2x - y$. **(3 marks)**

(b) In an agricultural plantation the proportion of the total area that has been destroyed by a bacterial disease is x. The rate of the destruction of the plantation is proportional to the product of the proportion already destroyed and that not yet. It was initially noticed that half of the plantation had been destroyed by the disease and that at this rate another quarter of the plantation would be destroyed in the next 6 hours.

(i) Form a differential equation relating x and time t

(ii) Calculate the percentage of the population destroyed 12 hours after the disease was noticed **(9 marks)**