|  |  |  |
| --- | --- | --- |
| *1.* | Solve for  given that .                *Comment:* |  |
| *2.* | Show that .  Let        Comment: |  |
| *3.* | A line is drawn through the point  making an angle of  with  the positive axis, and it meets the line  at . Find the  distance of  from the origin and the equation of the line through  perpendicular to .  , equation of the line is , ,  So when ,  thus .  Distance  Gradient of  So the gradient of the perpendicular line is  Equation is ,  Comment: |  |
| *4.* | Given the points ,  and  such that  divides externally in the ratio , find the coordinates of point .  for ratio is  or  Let , ,  Thus  so  and  so,  The coordinates are .  Comment: |  |
| *5* | Prove that  From the L.H.S      , divide through by  as the R.H.S  Comment: |  |
| *6* | Solve the differential equation: , given that  when .  Separating the variables,    when    thus,  To get  Comment: |  |
| *7* | If , prove that            *Comment:* |  |
| *8* | Solve the equations: and .  Let  ,  and  so  thus  , ,  to get    Comment: |  |

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| --- | --- | --- |
| *9a)* | Simplify: i)  =  ii)  = |  |
| *b)* | The complex number with modulus and argument  is denoted . Find  the four roots of :  Using the polar form .  for  For ,  For ,  For ,  For , |  |
| *10* | Prove that the curve  cannot lie in the region  and , determine the turning points and sketch the curve.  ,    For no real roots, , so    ,   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | |  |  |  |  | |  |  |  |  |   Thus the curve does not lie in the region .  Turning points,  For , we have,    To get,  so  Thus, ,  Intercepts:  so  has no real roots since  Vertical asymptotes:  Horizontal asymptote, , ,  , thus curve crosses horizontal asymptote at  Comment: |  |
| *11* | a) Prove that  Using the sine rule:  From the L.H.S  divide through by  but  as R.H.S  b) Express  in the form and hence, find the maximum and minimum values of the function  stating clearly the values of .  Let  , thus ,  ,    The maximum occurs when (minimum)  i.e when , thus when  the maximum is  The minimum occurs when (maximum)  i.e when , thus when  the minimum is |  |
| *12a)* | and  are fixed points. Show that the locus of a point  which moves such that  is a circle and find its centre  and radius.        ,  Which is an equation of a circle with centre  and  b) Find the equation of a circle which passes through the points  ,  and .  The general equation can be  ; , ……..(i)  ;  …..(ii)  ;  …..(iii)  Eqn(ii) – eqn(i):  Eqn(iii) –eqn(ii):  Solving: , ,  Equation is; ,  or  ALT: Perpendicular bisectors of two chords intersect at the centre of the circle.  Midpoint of , mid point    Gradient of , Gradient of  Gradient of normal to, that to  Equation through  is , to get …(i)  Equation through  is to get ..(ii)  Solve eqn(i) and eqn(ii) to get  and  Thus centre is  using , find the radius    Equation of circle is:    Comment: |  |
| *13* | Find the angle between the line  and the plane .  , is the direction vector of the line.  , vector normal to the plane.  Using        b) The vector equation of two lines are  and  where  is a constant. If the two lines intersect find:  (i)  and the position vector of the point of intersection.  and **,** if they do intersect then  Thus implies that **, so**  Also **,** for **,** then so,  Using **,** then **,** gives  Position vector for point of intersection is given by or **.**  (ii) the angle between the two lines giving your answer to the nearest degree.  Let and be the directional vectors.  Thus the angle is given by  to the nearest degree**.**  Comment: |  |
| *14a)* | 14a) By row reduction to the echelon form, solve the simultaneous equations: |  |
| *b)* | Solve the equation:  *Comment:* |  |
| *15a)* | Solve the differential equation:      So,  thus  ,  thus |  |
| *b)* | According to Newton’s law of cooling, the rate of cooling of a body in air is  proportional to the difference between the temperature of the body and that of air.  If the air temperature is kept at  and the body cools from  to  in  25 minutes, in what further time will the body cool to ?  Let be the temperature of the body at any time      , when  Thus,  When , so        *Comment:* |  |
| *16a)* | Find  Let      Comment: | *M1double angle*  *M1for parts.*  *M1for integration*  *A1for answer.* |
| *16b)* | Express in partial fractions and hence find  Let    , ,  Solve to get;  Thus    Or  Comment: | *M1factorising*  *M1for partial fr*  *M1solving*  *A1for all values*  *M1for substg.*  *M1 M1for integration*  *A1for answer.* |