UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g, to be 9.8 ms^{-2}.

SECTION A (40 MARKS)

*Answer **all** the questions in this section.*

1. A rough plane is inclined at an angle of $\sin^{-1}(^3/_5)$ to the horizontal. A car of mass 1500 kg moves up the plane with a steady speed. The coefficient of friction between the car and the plane is $^1/_4$. Calculate the driving force exerted by the engine of the car. (05 marks)

2. The table below shows the number of people in each of the 100 cars that passed through a check point on a certain day.

| Number of people	1	2	3	4	5
Number of cars	41	33	18	6	2

   Find:

   (a) the mean number of people in each car. (02 marks)

   (b) the variance of the data. (03 marks)

3. Use the trapezium rule with **seven** (7) ordinates to estimate

$$\int_0^2 \frac{1}{3+4x^2} \, dx$$

   correct to **three** decimal places. (05 marks)

4. The diagram below shows a mass of 5 kg lying on a rough plane inclined at 30° to the horizontal. From the 5 kg mass, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley carrying a mass of 3 kg hanging freely. The tension in the string is just sufficient to prevent the 5 kg mass from sliding down the slope.
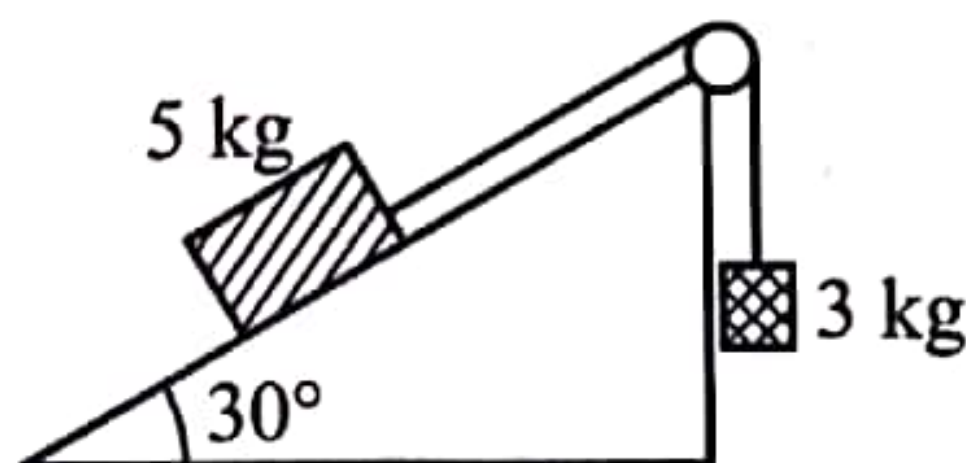
   Determine the coefficient of friction between the 5 kg mass and the rough plane. (05 marks)

5. Two events A and B are such that $P(A) = \dfrac{1}{2}$, $P(B) = \dfrac{7}{12}$ and $P(A' \cap B) = \dfrac{1}{2}$.

Find $P(B' \cap A)$. *(05 marks)*

6. The table below shows the amount of Euros equivalent to US dollars.

US dollars	79	97	105
Euros	64	78	85

Using linear interpolation/extrapolation, calculate the amount of:

(a) dollars equivalent to 92 Euros. *(03 marks)*

(b) Euros equivalent to 85 dollars. *(02 marks)*

7. A river which is 50 m wide flows with a speed of 3 ms^{-1}. A boat must be set to cross the river to a point directly opposite the starting point. The speed of the boat relative to the river is 4 ms^{-1}. Determine the;

(a) velocity with which the boat sets off to cross the river. *(03 marks)*

(b) time taken by the boat to cross the river. *(02 marks)*

8. Bag A contains 7 red balls and 4 blue balls. Bag B contains 5 red balls and 8 blue balls. A ball is randomly transferred from A to B. A ball is then randomly removed from bag B. Find the probability that:

(a) a red ball was removed from bag B. *(03 marks)*

(b) the transferred ball is blue given that a red ball is removed from B. *(02 marks)*

SECTION B (60 MARKS)

*Answer any **five** questions from this section.*
***All** questions carry equal marks.*

9. The table below shows the marks of eight students (A to H) in two tests.

Student	A	B	C	D	E	F	G	H
Test 1	67	73	50	75	42	44	95	40
Test 2	82	70	64	78	52	64	80	64

(a) (i) Plot a scatter diagram for the given data.

    (ii) Draw a line of best fit on the scatter diagram.

    (iii) Use your scatter diagram to obtain a mark in **Test 2** which corresponds to a mark of 63 in **Test 1**. *(06 marks)*

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(b) **(i)** Calculate the rank correlation coefficient for the data.

(ii) Comment on your result in b (i). *(06 marks)*

10. A particle of mass 4 kg is initially at rest at the point A $(2, -2, 3)$ metres. The particle is acted upon by a force of $F = \begin{pmatrix} 4t \\ t^2 \\ 5 \end{pmatrix}$ Newtons, where t is time in seconds. Find the;

(a) acceleration at time t. *(02 marks)*

(b) speed of the particle after 3 seconds. *(05 marks)*

(c) displacement of the particle after 3 seconds. *(05 marks)*

11. **(a)** Two numbers x and y are approximated by X and Y with errors Δx and Δy respectively. Show that the relative error in approximating $\dfrac{x}{y}$ by $\dfrac{X}{Y}$ is $\left|\dfrac{\Delta x}{X}\right| + \left|\dfrac{\Delta y}{Y}\right|$ *(06 marks)*

(b) The values in the expression $T = \dfrac{673.16}{40.345}$ are rounded off correct to the given decimal places. Determine the interval within which the exact value of T can be expected to lie. *(06 marks)*

12. A discrete random variable X has a probability distribution given by

$$f(x) = \begin{cases} kx^2, & x = 1, 2, 3 \\ k(7-x)^2, & x = 4, 5, 6 \\ 0, & elsewhere \end{cases}$$

Find the:

(a) **(i)** value of the constant k.

(ii) mean of X.

(iii) variance of X. *(09 marks)*

(b) **(i)** Determine the cumulative distribution function, $F(x)$. *(01 mark)*

(ii) Sketch the graph of $F(x)$. *(02 marks)*

13. Forces of magnitude 2N, 2N, 3N, 4N, $2\sqrt{2}$ N and $\sqrt{2}$ N act along sides $\overrightarrow{AB}$, $\overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{AD}, \overrightarrow{AC}$ and $\overrightarrow{DB}$ respectively. Where $ABCD$ is a square of side 2 m. Find the:

(a) resultant force. (09 marks)

(b) equation of the line of action of the resultant force. (03 marks)

14. The taxation system in a company is such that the tax T based on the amount A earned falls in three categories as shown in the table below:

AMOUNT EARNED A (DOLLARS)	TAX PAID T (DOLLARS)
$A < 2000$	$T = 0$
$2000 \leq A \leq 5000$	$T = \dfrac{2A}{100}$
$A > 5000$	$T = 60 + \dfrac{5}{100}(A\text{-}5000)$

Parts of a flow chart for computing the tax are given below.
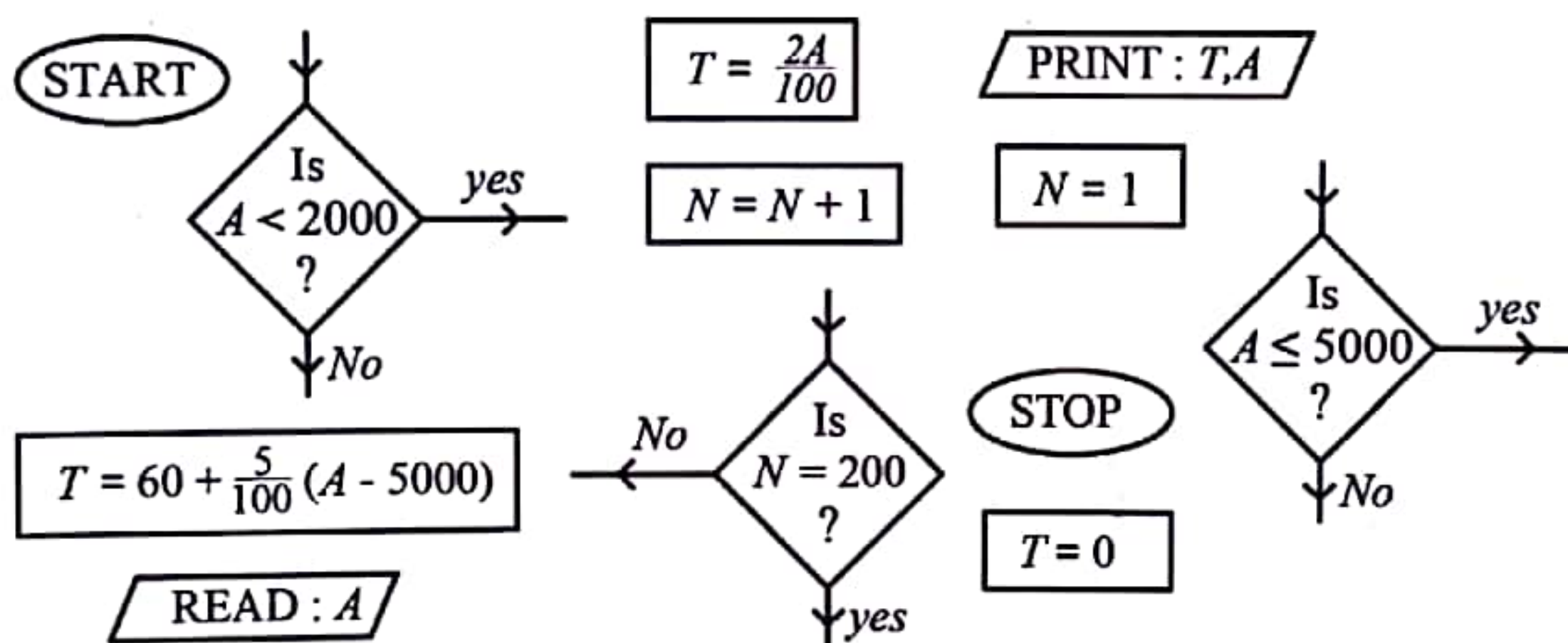
(a) (i) Arrange the given parts to form a complete logical flow chart. (08 marks)

(ii) State the purpose of the flow chart. (01 mark)

(b) Perform a dry run for the flow chart and complete the table below: (03 marks)

N	A	T
1	1500	–
2	3500	–
3	9000	–

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15. The masses of packets of tea leaves are normally distributed with mean mass 600 g and standard deviation 20 g.

(a) If a packet of tea leaves is chosen at random, find the mass exceeded by 7% of the packets. *(05 marks)*

(b) If 1,000 packets of tea leaves were sold, find the number of packets that weighed less than 545 g. *(07 marks)*

16. Two points A and B are on the same horizontal ground. A particle P is projected from point A towards point B with an initial speed of 49 ms^{-1} at an angle of $30°$ to the horizontal. Two seconds later, another particle Q is projected from point B towards A with the same speed and at the same angle of elevation as P.

If the particles collide at a height h m above the ground, determine the:

(a) value of h. *(06 marks)*

(b) distance between the points A and B. *(06 marks)*