Townside High School, Mbale

Department of Mathematics, 2019

Principle Mathematics [P425/1]

Discussion Questions

Quadratic and Miscellaneous Equations

1. a) Solve: $2x^2 + 5x - 12 = 0$

   b) Find the value of k so that the equation $4x^2 - 8x + k = 0$ shall have equation roots.

   c) Solve the following equations

   I. $\sqrt{(3-x)} - \sqrt{(7+x)} = \sqrt{(16+2x)}$

   II. $\sqrt{(x-1)} + 4\sqrt{(x-4)} = 4$

2. a) If $\alpha\ and\ \beta$ are roots of the equation $x^2 - px + q = 0$, form an equation whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$

   b) Show that if the equations $x^2 + bx + c = 0$ and $x^2 + px + q = 0$ have a common root, then $(c-q)^2 = (b-q)(cp-bq)$

   c) Show that if the equations $x^2 + ax + 1 = 0$ and $x^2 + x + b = 0$ have a common root, then$(b-1)^2 = (a-1)(1-ab)$.

3. Given that $\alpha\ and\ \beta$ are roots of the equation $x^2 + bx + c = 0$

   i) show that $(\alpha^2+1)(\beta^2+1) = (c-1)^2 + b^2 = 0$

   ii) Find in terms of b and c the equation whose roots are $\frac{\alpha}{\alpha^2+1}$ and$\frac{\beta}{\beta^2+1}$.

4. a) Use the substitution $y = x + \frac{1}{x}$ to solve the equation

$$2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$$

   b) Solve the simultaneous equations below

$$2x + 3y + 4z = 8$$
$$5x + 4y + 3z = 3$$
$$3x - 2y - 3z = -2$$

Indices, surds and logarithms

5. simplify

a) $\frac{x^{3/2}+xy}{xy-y^3}-\frac{\sqrt{x}}{\sqrt{x}-y}$

b) $\frac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form $p+q\sqrt{3}$ where p and q are rational numbers.

6. solve the following equations

i) $\log_6 36x+\log_{36} 6x=6$

ii) $2^{2x+8}-32(2^x)+1=0$

iii) $5^{\log_{25} x}=3^{\log_{27} 2x}$

7. a) Given that $\log_3 x=p$ and $\log_{18} x=q$, show that $\log_6 3=\frac{p}{p-q}$

c) If$a=\log_b c$, $b=\log_c a$ and$c=\log_a b$, prove that$abc=1$.

Polynomials and remainder theorem

8. a) Find the value of k if the remainder when the polynomial $2x^4+kx^3-11x^2+4x+12$ Is divided by $x-3$ is 60.

b) Find the constants $a, b\ and\ c$ such that $2x^2-9x+14\equiv a(x-1)+b(x-1)+c$

9. a) The expression $6x^2+x+7$ leaves the same remainder of 72 when divided by $(x+a)$ and$(x+2a)$, where $a\neq 0$, determine the value $of\ a$.

b) The function $f(x)=x^3+px^2-5x+q$ has a factor $(x-2)$ and has a value of 5 when $x=-3$. Find the value p and q.

Partial fractions

10. Resolve the following into partial fractions

a) $\frac{5}{x^2+x-6}$

b) $\frac{9}{(x-1)(x+2)^2}$

c) $\frac{16x}{x^4-16}$

d) $\frac{x^3}{x^2-3x+2}$

11. a) Express the function $f(x)=\frac{x+2}{(x+1)(2x-1)}$ as a sum of partial fractions

b) Given that $(x) = \frac{x^3+2x^2+6}{(x+3)^2(4+x^2)}$. Express $f(x)$ into partial fractions.

Proof by induction

12. Prove that $5^n + 4n - 1$ is divisible by 8 for all positive integers
13. Proof by induction
    a) $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$
    b) $\sum_{r=1}^{n} \frac{1}{(3r-2)(3r+1)} = \frac{n}{3n+1}$
    c) $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$
    d) $1^3 + 2^3 + \cdots + n^2 = \frac{1}{4}n^2(n+1)^2$

Series (Arithmetic and Geometric Progression)

14. a) The first term of an A.P is 73 and the ninth term is 25. Determine the
    i) Common difference
    ii) Number of terms that must be added to give a sum of 96

    b) A G.P and an A.P have the same first term. The sums of their first, second and third terms are 6, 10.5 and 18 respectively. Calculate the sum of their fifth terms

15. a) An A.P contains n terms. The first term is 2 and the common difference is $\frac{2}{3}$. If the sum of the last four terms is 72 more than the sum of the first four terms, find n.

    b) Show that the sum of the first n terms of an A.P with a as the first term and a common difference of d is $\frac{n}{2}[2a + (n-1)d]$.
16. A) In an A.P, the thirteenth term is 27, and the seventh term is three times the second term. Find the first term, common difference and the sum of the first ten terms
    e) The second, fourth and eighth terms of an A.P are in a G.P and the sum of the third and fifth terms is 20. Find the terms of the progression.
17. a) The ninth term of an A.P is -1 and the sum of the nine terms is 45. Find the common difference and the sum of the first twenty terms.

    b) In a G.P, the first term is 7 and the n^{th} term is 448. The sum of the first n^{th} is 889, find the common ratio.

18. In a Geometric Progression (G.P), the difference between the fifth and the second term is 156. The difference between the seventh and the fourth term is 1404. Find the possible values of the common ratio.

19. a) The tenth term of an arithmetic progression (A.P) is 29 and the fifteenth term is 44.

    i)Find the value of the common difference and the first term. Hence find the sum of the first 60 terms.

    b) A cable 10m long is divided into ten pieces whose lengths are in a geometrical progression. The length of the longest pieces is 8 times the length of the shortest piece. Calculate to the nearest centimeter the length of the third piece.

Binomial Theorem and Expansion

20. Express $\ln\sqrt{\frac{(1+x)}{(1-x)}}$ as a series of terms in ascending powers of x up to and including the terms in x^3. Hence or otherwise find the values of $\ln\sqrt{\frac{11}{3}}$ to six decimal places

21. Expand $\frac{(1+x)}{(1-2x)^3}$ in ascending powers of x up to and including the term in x^5

22. Prove that if x is too small that its cube and higher powers are neglected, $\sqrt{\frac{(1+x)}{(1-)}} = 1 + x + \frac{x^2}{2}$. By taking $= \frac{1}{9}$, prove that $\sqrt{5} = \frac{181}{81}$.

Curve sketching

23. Sketch the following curves

    a) $f(x) = 2x^3 + 5x - 3$

    b) $y = \frac{(x-1)(x+2)}{(x+1)(x-3)}$

24. Prove that $\frac{3x-9}{x^2-x-2}$ cannot lie between two certain values. Hence illustrate the curve graphically

25. Find the Cartesian equation of the curve $x = \frac{1+t}{1-t}$, $y = \frac{2t^2}{1-t}$. Hence sketch the curve $y = f(x)$ where $f(x)$ is the Cartesian equation of the curve.

26. Given that x is a real number, prove that the function

$f(x) = \frac{x^2+2x-11}{2(x-3)}$ does not lie between 2 and 6.

i) Determine the turning points and distinguish between them

ii) State the equations of the asymptotes

iii) Sketch the graph of $y = f(x)$.

Trigonometry

27. Prove the following identities

a) $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{2}{\sin\theta}$

b) $\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$

c) $\frac{\cos\theta - 1}{\sec\theta+\tan\theta} + \frac{\cos\theta+1}{\sec\theta-\tan\theta} = 2(1+\tan\theta)$

d) $\frac{\cos 3\theta}{\cos\theta} - \frac{\cos 6\theta}{\cos 2\theta} = 2(\cos 2\theta - \cos 4\theta)$

e) $\frac{\sin 3A\sin 6A + \sin A\sin 2A}{\sin 3A\cos 6A + \sin A\cos 2A} = \tan 5A$

f) $\tan^{-1} 1/3 + \sin^{-1} {}^{1}/_{\sqrt{5}} = \frac{\pi}{4}$

28. Prove that in any triangle ABC,

a) $\frac{b-c}{b+c} = \tan\frac{B-C}{2}\cot\frac{B-C}{2}$

b) $\frac{a^2-b^2}{c2} = \frac{\sin(A-B)}{\sin(A+B)}$

c) $\sin\frac{B-C}{2} = \frac{b-c}{a}\cos\frac{A}{2}$

d) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

e) $\frac{a+b-c}{a+b+c} = \tan\frac{A}{2}\tan\frac{B}{2}$

29. Solve the following trigonometric equations

a) $\cos\theta + \sin 2\theta = 0\ for\ 0 < \theta < 2\pi$

b) $\cos(\theta + 60°) = 1 + \cos\theta\ \ for - 180 < \theta < 180.$

c) $5\sin 2x - 10sin^2x + 4 = 0\ for - 180 < x < 180.$

27. Solve

(a) $4\sin^2\theta - 12\sin 2\theta + 35\cos^2\theta = 0$, for $0° \le \theta \le 90°$.

(b) $3\cos\theta - 2\sin\theta = 2$, for $0° \le \theta \le 360°$.

30. Given that $\cos 2A - \cos 2B = -p$ and $\sin 2A - \sin 2B = q$, prove that

sec $(A + 5) = \frac{1}{q}\sqrt{p^2 + q^2}$

31. a) Prove that tan (A-B) = $\frac{\tan A - \tan B}{1+\tan A \tan B}$. Hence show that $\frac{1-\tan 15^0}{1+\tan 15^0} = \frac{1}{\sqrt{3}}$.

b) given that cos A = $^3/_5$ and cos B = $^{12}/_{13}$ where A and B are acute, find the value of;

i) tan(A + B).

ii) Cosec(A + B).

32. a) show that $\frac{\sin 3\theta \sin 6\theta + \sin \theta \sin 2\theta}{\sin 3\theta \cos 6\theta + \sin \theta \cos 2\theta}$ = tan 5θ.

b) Express 4cosθ - 5 sin θ in the for R cos $(\theta + \beta)$, where R is a constant and β an acute angle.

Determine the maximum value of the expression and the value of O constant and β an acute angle.

c) Solve the equation 4cos θ - 5 sin θ = 2.2 for $0^0 < \theta < 360^0$

33. a) Show that cot A + tan 2A = cot A sec 2A.

b) Show that tan 3θ = $3t - t^3/1 - 3t^2$, where t = tanθ. Hence or otherwise show that tan $\frac{\pi}{12}$ = 2 - $\sqrt{3}$.

34. a) Solve the equation cos x + cos 2x = 1 for values of x from 0^0 to 360^0 inclusive

b) i) prove that $\frac{\cos A + \cos B}{\sin A + \sin B}$ = cot $\frac{A+B}{2}$.

ii)Deduce that $\frac{\cos A + \cos B}{\sin A + \sin B}$ = tan$\frac{C}{2}$ where A, B and C are angles of triangle.

Complex numbers

35. a) Given that $= \frac{(1+2i)}{(3-i)^2}$, find the modulus and argument of $\bar{Z} + \frac{5}{Z}$ where $\bar{z}$ is the conjugate of Z.

b) If $Z = x + iy$, determine the Cartesian equation of the locus given by $\left|\frac{Z-1}{Z+1-i}\right| = \frac{2}{5}$

36. a) Solve the equation $2Z - i\bar{Z} = 5 - i$ where $Z = x + iy$

b) Find in Cartesian form, the cube of $-\sqrt{2} + i\sqrt{6}$

c) Find the cube root of $8i$.

37. a) Find and sketch the locus of $Arg(iZ + 1) = \frac{\pi}{4}$ where $Z = x + iy$.

b) Given that $Z = \frac{7-i}{-4-3i}$, find the modulus and argument of Z. hence express Z in polar form.

38. a) One of the roots of the equation$x^2 + px + q = 0$ is $(2 - 3i)$, find the values of p and q

b) Find the modulus and argument of $\frac{7-i}{3-4i}$

39. Express the following in the form $r(cos\theta + isin\theta)$

a) $\sqrt{3} - i$

b) $-5i$

c) $-2 + 2i$

40. (a) Without using tables, simplify

$$\frac{\left(cos\,{}^{\pi}/_{9} + isin\,{}^{\pi}/_{9}\right)^4}{\left(cos\frac{\pi}{9} - isin\frac{\pi}{9}\right)^5}$$

(b) Express $z = \frac{7+4i}{3-2i}$ in the form $p + qi$ where p and q are real. Hence sketch Z on the Argand diagram.

(c) Describe the locus in Argand diagram representing Z such that

$arg\left(\frac{z-1}{z+1}\right) = \frac{1}{2}\pi$

41. Show that 2 + i is a root of the equation $2z^3 - 9z^2 + 14z - 5 = 0$. Hence find the other roots.

Differential equations and applications

42. The D.E $\frac{dy}{dx} = -\frac{1}{20}x$ represents the rate at which a radioactive substance disintegrates.

a) Given that x_0 takes 20 minutes to reduce to $\frac{1}{3}x_0$, calculate how long it takes an amount $2x_0$ reduce to $\frac{1}{3}x_0$.

b) Find the mass of the substance that reduces to 15gm in 15 minutes.

43. a) Solve the differential equation $x\frac{dy}{dx} = y + kx^2 \cos x$ given that $y = 2\pi$ when $x = \pi$.

b) A certain chemical reaction is such that the rate of transformation of the reacting substance is proportional to the concentration. If initially the concentration of the reagent was 9.5gm per litre and after five minutes the concentration was 3.5gm per litre, find what the concentration was after 2 minutes.

44. (a) Solve the differential equation,

$\frac{dy}{dx} + y = e^{-x}Cos\frac{1}{2}x$ given that $y = -1$ where $x = 0$.

(b) The rate of growth of a disease causing virus increases at a rate proportional to the number of virus present in the body. If the number increases from 1000 to 2000 in 1 hour.

I. How many viruses will be present after 1½ hours?

II. How long will it take the number of virus in the body to be 40.

45. A substance loses mass at a rate which is proportional to the amount *M* present at time *t,*

a) Form a differential equation connecting M, t and the constant of proportionality k.

b) If initially the mass of the substance is M_o, show that $M = M_0 e^{-kt}$.

c) Given that half of the substance is lost in 1600 years, determine the number of years 15 g of the substance would take to reduce to 13.6 g.

46. The differential equation $\frac{dp}{dt}$ = kp(c – p) shows the rate at which information flows in a student population c. P represents the number who have heard the information in t days k is a constant.

a) Solve the differential equation.

b) A school has a population of 1000 students. Initially, 20 students had heard the information. A day later, 50 students had heard the information. How many students heard the information by the tenth day?

47. A 3: 00 pm, the temperature of a hot metal was 80^0C and that of the surroundings 20^0C. At 3:03pm the temperature of the metal had dropped to 42^0C. The rate of cooling of the metal was directly proportional to the difference between its temperature θ and that of the surroundings.

a) i) Write a differential equation to represent the rate of cooling of the metal.

ii) Solve the differential equation using the given conditions.

b) Find the temperature of the metal at 3:05pm.

48. a) Solve the differential equation

$\frac{dy}{dx} = \frac{sin^2 x}{y^2}$, given that y =1 when x = 0,

b) It is observed that the rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. A body at 78%C is placed in a room at

20^0C and after 5 minutes the body has cooled to 65^0C. What will be its temperature after a further 5 minutes?

Differentiation and applications

49. If $y = \frac{1}{x^2}$, find $\frac{dy}{dx}$ from first principles

50. Find the equation of the normal to the curve $y = (x^2 + x + 1)(x - 3)$ at the point where it cuts the x-axis.

51. Find the Cartesian equation of a curve given parametrically by $x = acos2t$ and $y = asint$

52. Given that $ye^x = \sin x + \cos x$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

53. Find the equation of the normal to the curve $x^2y + 3y^2 - 4x - 12 = 0$

54. a) Given that y = $\frac{1+sin^2 x}{cos^2 x+1}$, show that $\frac{dy}{dx} = \frac{3\sin 2x}{(cos^2 x+1)^2}$.

Hence find $\frac{dy}{dx}$ where x $\frac{2\pi}{3}$.

b) A curve is represented by the parametric equations x = 3t and y = $\frac{4}{t^2+1}$. Find the general equation of the tangent to the curve in terms of x, y and t. Hence determine the equation of the tangent at the point (3,2).

55. a) Differentiate from first principles y = $\frac{x}{x^2+1}$ with respect to x.

b) i) Determine the turning points of the curve y = $x^2(x - 4)$.

ii)Sketch the curve in (i) above for $-2 \leq x \leq 5$.

iii)Find the area enclosed by the curve above and the x-axis

56. a) Differentiate the following with respect to x:

i) $(x + 1)^{1/2} (x + 2)^2$.

ii) $\frac{2x^2+3x}{(x-4)^2}$.

b) The base radius of a right circular cone increases and the volume changes by 2%. If the height of the cone remains constant, find the percentage in the circumference of the base.

57. If

a) $x^2 + y^2 = 2y$, show that $(1 - y)^3\frac{d^2y}{dx^2} = 1$

b) $y = \frac{x\sqrt{(x^2-1)}}{x+2}$, prove that $\frac{dy}{dx} = \frac{x^3+4x^2-2}{(x+2)^2\sqrt{(x^2-1)}}$

c) $y = \sec^{-1} x$, show that $\frac{dy}{dx} = \frac{1}{x\sqrt{(x^2-1)}}$

d) $y = \cos^{-1}(\frac{1-x^2}{1+x^2})$, show that $\frac{dy}{dx} = \frac{2}{1+x^2}$

Integration and its applications

58. Find the following integrals

a) $\int \frac{dx}{\sqrt{(1-\cos 2x)}}$

b) $\int \frac{1}{x^2(x^2-1)} dx$

c) $\int \csc\theta sec\theta\, d\theta$

d) $\int \frac{cosx}{4+sin^2 x} dx$

59. Prove that if $x = cos2\theta$, then $\int_0^1 \sqrt{\frac{1-x}{1+x}}\ dx = \frac{\pi}{2} - 1$

60. Resolve y =$\frac{x^3+5x^2-6x+6}{(x-1)^2(x^2+2)}$ into partial fractions. Hence find $\int ydx$ and $\frac{dy}{dx}$.

61. If y = (x – 0.5)e^{2x}, find $\frac{dy}{dx}$. hence determine $\int_0^1 xe^{2x}$ dx.

62. The region bounded by the curve y = cos x, the y-axis and the x-axis from x=0 to x = $\frac{\pi}{2}$ is rotated about the x-axis. Find the volume of the solid formed.

63. Using a suitable substitution, find $\int \frac{\sin^{-1} 2x}{\sqrt{(1-4x^2)}} dx$

64. a) On the same axes, sketch the curves y = x(x + 2) and y = x(4 – x)

b) Find the area enclosed by the two curves in (a)

c) Determine the volume of the solid generated when the area enclosed by the two curves in (a) is rotated about the x- axis.

65. Evaluate $\int_0^{\pi/2} \frac{\cos x}{1+ sin^2 x}$ dx.

66. a) Find $\int x^3 e^{x^4}$dx.

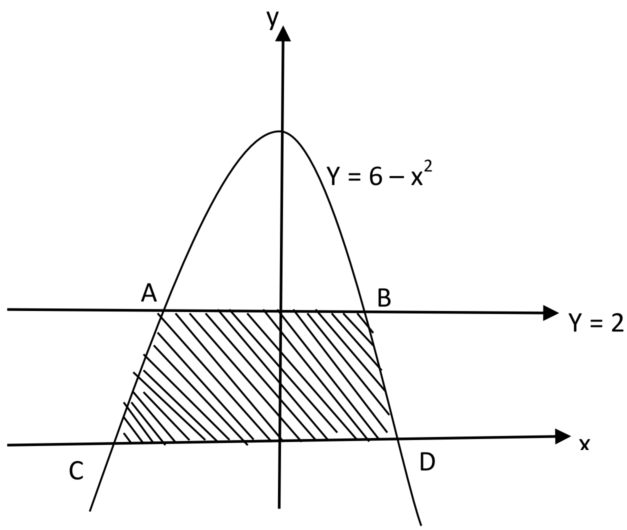
b)Use the substitution t = tan x to find $\int \frac{1}{1+ sin^2 x}$ dx.

67. a) Integrate $\frac{2x}{\sqrt{(x^2+4)}}$ with respect to x.

b) Evaluate$\int_0^{\pi/6} \sin x \sin 3x\, dx$.

c) Using the substitution x = 3 sin θ, evaluate $\int_0^3 \sqrt{\left[\frac{3+x}{3-x}\right]}\,dx$.

68. In the diagram below, the curve $y = 6 - x^2$ meets the line y = 2 at A and B, and the x – axis at C and D.



Find the

a) Coordinates of A, B, C and D.

b) Area of the shaded region, correct to **one** decimal place.

Vectors and planes

69. a) Show that the equation of the line passing through the points (1,2,1) and (4,-2,2) is given by $\frac{x-1}{3} = \frac{y-2}{-4} = \frac{z-1}{1}$

b) If the line in (a) above meets the line $\frac{x-1}{1} = \frac{y-2}{0} = \frac{z-1}{3}$ at P, find the

i) Coordinates of P

ii) Angle between the two lines.

70. a) Given the two vectors $\boldsymbol{a}=3\boldsymbol{i}-2\boldsymbol{j}+\boldsymbol{k}$ and $\boldsymbol{b}=\boldsymbol{i}-2\boldsymbol{j}+2\boldsymbol{k}$, find

i) The acute angle between the two vectors.

ii) Vector ***c*** such that it is perpendicular to both vectors ***a*** and ***b***.

b) Given that $\boldsymbol{OA} = \boldsymbol{a}$ and $\boldsymbol{OB} = \boldsymbol{b}$, point $\boldsymbol{R}$ is on $\boldsymbol{OB}$ such that $\boldsymbol{OR}:\boldsymbol{RB} = 4:1$. Point $\boldsymbol{P}$ is on $\boldsymbol{BA}$ such that $\boldsymbol{BP}:\boldsymbol{PA} = 2:3$ and when $\boldsymbol{RP}$ and $\boldsymbol{OA}$ are produced, they meet at point $\boldsymbol{Q}$. find:

i) $\boldsymbol{OR}$ and $\boldsymbol{OP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

ii) $\boldsymbol{OQ}$ in terms of $\boldsymbol{a}$.

71. a) If the point A, position vector $a\boldsymbol{i} + b\boldsymbol{j} + 3\mathbf{k}$ lie on the line L, vector equation $r = 2i + 4j - k + \mu(i + j + k)$, find the values of a and b.

    b) Find the equation of the perpendicular line from $A = (2, -1, 4)$ onto the line $r = i + 2k) + \mu(2i + j + 2k)$. Find also the perpendicular distance of the point A from the line r.

72. a) Given the points A(-3,3,4), B(5,7,2) and C(1,1,4), find the vector equation of a line which joins the mid-points of AB and BC.

    b) Two lines are given by the parametric equations L_1: $r = (-i + 2j + k) + t(i - 2j + 3k)$ and L_2: $r = (-3i + pj + 7k) + s(i - j + 2k)$. If the lines intersect, find

    i) The values of t, s and p.

    ii) The coordinates of the point of intersection.

73. a) Given that the vector $ai - 2j + k$ and $2ai + aj - 4k$ are perpendicular, find the values of a.

    b) Show that the vector $2i - 5j + 3.5k$ is perpendicular to the line $r = 2i - j + \mu(4i + 3j + 2k)$.

    c) Calculate the angle between vector $3i - 2j + k$ and the line in b above.

74. (a) Determine the Cartesian equation of a plane passing through a point $\underset{\sim}{A}(-8, 3, 2)$ which is parallel to the vectors $2\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$.

    (b) Find the angle between the line $r = i - j + k + \beta(2i + 2j - k)$ and the plane obtained in (a) above.

75. Points A and B are (-1, -2, 3) and (2, 1, -3) respectively. If point P divides the line AB externally in the ratio 1: 4. Find the Cartesian equation of the plane containing P and perpendicular to the line AB.

76. (a) A line $\boldsymbol{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix}$ is parallel to the plane $3x + 6y - 2z = 15$.

    Find;

i) the value of a .

ii) the shortest distance between the line and the plane.

(b) Find the acute angle between the line $\boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ and the plane $4x - 7y - 4z = 20$.

77. a) Find the Cartesian equation of a plane passing through the midpoint of $P(-1, 0, 5)$ and $Q(7, -4, 1)$ which is perpendicular to the line $\frac{x-6}{7} = -2 - y = \frac{z-5}{2}$

78. Two lines L_1 and L_2 are given by $\frac{x+1}{5} = \frac{y+2}{-1} = Z - 6$ and $\frac{x-6}{2} = y - 3 = \frac{Z}{5}$ respectively. L_1 meets $y = 0$ at A, while L_2 meets $Z = -5$ at B.

a) Find the;

(i) Coordinates of A and$B$,

(ii) Cartesian equation of line AB

b) Determine the angle between the plane $5x + 7y - Z = 11$ and line in (a) (ii) above.

79. a) Given two vectors $\underset{\sim}{\boldsymbol{a}} = 3\underset{\sim}{\boldsymbol{i}} - 12\underset{\sim}{\boldsymbol{j}} + 4\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{b}} = \underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{k}}$; find:

(i). the angle between $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$,

(ii). a vector that makes a right angle with $\underset{\sim}{\boldsymbol{a}}$ and with $\underset{\sim}{\boldsymbol{b}}$.

b) Find the equation of the plane passing through the points $\boldsymbol{A}(1, 1, 0)$, $\boldsymbol{B}(3, -1, 1)$, $\boldsymbol{C}(-1, 0, 3)$ and find the shrotest distance of the point $(3, 2, 1)$ to the plane.

Circles

80. a) Write down the equation of the circle with Centre $(1,2)$ and radius 3.

b) Find the centre and radius of the circle whose equation is $x^2 + y^2 + 8x - 2y + 13 = 0$

c)Find the equation of the circle whose diameter is the line joining the points $A(1,5)$ and $B(-2,3)$

d) Obtain the equation of the circle passing through the points A(0,1), B(4,7) and C(4,-1).

81. a) Determine the equation of the tangent at the point (3,1) on the circle $x^2 + y^2 - 4x + 10y - 8 = 0$

b) What is the angle between this tangent and the positive direction of the x-axis

c) Determine whether the lines $5y = 12x - 13$ and $3x + 4y = 9$ are tangents to the circle $x^2 + y^2 + 2x - 8y = 8$

d) show that the circles $(x - 8)^2 + (y - 6)^2 = 25$ and $(5x - 16)^2 + (5y - 12)^2 = 25$ are orthogonal.

82. (a) Prove that the circles $x^2 + y^2 + 4x - 2y - 11 = 0$ and $x^2 + y^2 - 6y - 8x = 0$ are orthogonal

(b) Find the equations of the normal and the tangent to the circle $x^2 + y^2 - 6y - 8x = 0$ at the point (0, 1)

Coordinate Geometry

83. a) Find the equation of the locus of a point which moves such that distance from D(4,5) is thrice its distance from the origin.

b) The line y = mx intersects the curve y = $2x^2$ – x at the points and B. Find the equation of the locus of the point P which divides AB in the ratio 3:5.

84. The line $y = mx + c$is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when$c = \pm\sqrt{(a^2m^2 + b^2)}$.

Find the equations of the tangents to the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ from the point (0, $\sqrt{5}$).

85. a) A point P is twice a s far from the line x + y = 5 as from the point (3,0). Find the locus of P.

b)A point Q is given parametrically by x = 2t, y = $\frac{2}{t}$ + 1. Determine the equation of Q and sketch it.

86. Show that the line x – 2y + 10 = 0 is a tangent to the ellipse $\frac{x^2}{64} + \frac{y^2}{9} = 1$.

87. a) i) Find the equation of the chord through the points (αt_1^2, $2\alpha t_2$) and (αt_2^2,$2\alpha t_2$) of the parabola $y^2 = 4\alpha x$.

ii)Show that the chord cuts the directrix when y = $\frac{2a(t_2t_1 - 1)}{t_1 + t_2}$.

b) Find the equation of the normal to the parabola $y^2 = 4\alpha$ x at (αt^2, $2\alpha t$) and determine its point of intersection with the directrix.

88. P is a variable point given by the parametric equations.

$$X = \frac{a}{2}\left(t + {}^{1}/_{t}\right) ; y = \frac{b}{2}\left(t - {}^{1}/_{t}\right).$$ Show that the locus of is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

State the asymptotes. Determine the coordinates of line points where the tangent from P meets the asymptotes.

89. a) A conic section is given by x = 4 cosθ ; y = 3 sinθ. Show that the conic section is an ellipse and determine its eccentricity.

b)Given that the line y = mx + c is a tangent to the ellipse $\frac{x^2}{\alpha^2}$ + $\frac{y^2}{b^2}$ = 1, show that $c^2 = \alpha^2 m^2 + b^2$. Hence determine the equations of the tangents at the point (-3,3) to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

90. a) show that the equation of the tangent to the hyperbola $(\alpha\ sec\theta\ , b\tan\theta)$ is

$$bx - \alpha y\sin\theta - \alpha b\cos\theta = 0$$

b) Find the equations of the tangent to $\frac{x^2}{4} - \frac{y^2}{9} = 1$, at the points where $\theta = 45^0$ and where $\theta = -135^0$.

c) Find the asymptotes.

91. a) Find the equation of the tangent and normal to the ellipse

$\frac{x^2}{4} + \frac{y^2}{1} = 1$ at the point P(2cosθ , sinθ).

b) If the tangent in (a) cuts the y-axis at point A and the x-axis at point B, and the normal cuts the x-axis at point C, find the co-ordinates of the points A,B and C.

92. a) Find the equation of the tangent to the parabola $y^2 = \frac{x}{16}$ at the point $\left(t^2, \frac{t}{4}\right)$.

b)If the tangent to the parabola in (a) above at the points p $\left(p^2, \frac{p}{4}\right)$ and Q $\left(p^2, \frac{q}{4}\right)$ meet on the line y = 2,

i) show that p + q = 16

ii) deduce that the mid-point of PQ lies on the line y = 2

- *Success doesn't come to those who cry but to those who work hard.*
- *Work hard in silence and let success be your noise.*