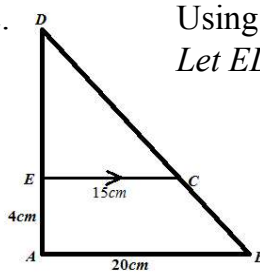
MATEMATICS 456/1

1. $x-2(x-5)\le x+2$
$x-2x+10\le x+2$
$-x-x\le -8$
$-2x\le -8$
$x\ge 4$
$x=\{4,5,6,7\ldots\ldots\}$

M1 for opening brackets
B1 simplification

A1 c.a.o
B1 for correct values

2. Using similarity of sides,



Let ED $= y$

$\frac{AB}{EC}=\frac{AD}{ED}$

$\frac{20}{15}=\frac{4+y}{y}$

M1 for substn

$\frac{4}{3}=\frac{4+y}{y}$

$4y=12+3y$

$y=12.$

B1 for multiplication
A1 c.a.o

$ED=12$ *and* $AD=12+4$

$=16cm$

B1 for 16cm seen

3. $2\begin{pmatrix}4&3\\-2&1\end{pmatrix}-3\begin{pmatrix}5&0\\2&3\end{pmatrix}$

M1 for substn.

$\begin{pmatrix}8&6\\-4&2\end{pmatrix}-\begin{pmatrix}15&0\\6&9\end{pmatrix}$

B1 for all values correct

$2M-2N=\begin{pmatrix}-7&6\\-10&-7\end{pmatrix}$

A1 c.a.o

$\det 2M-2N=(-7\times -7)-(-10\times 6)$

$=49--60$

$=109$

B1 for det. correct

$10y^2-3y-1$

M1 correctly subtute factor

$10y^2-5y+2y-1$

4. $5y(2y-1)+1(2y-1)$

M1

$(5y+1)(2y-1)$ *hence factorised*

Solving

Either $5y+1=0$ *or* $2y-1=0$

B1

$y=-\frac{1}{5}$ $\quad y=\frac{1}{2}.$

A1 c.a.o

B1 for -1/5 seen
-1/2 seen

5.

	Die					
	1	2	3	4	5	6
Coin H	1H	2H	3H	4H	5H	6H
Coin T	1T	2T	3T	4T	5T	6T

	Die					
	1	2	3	4	5	6
Coin 4	5	6	7	8	9	10
Coin 9	10	11	12	13	14	15

B1 correct values

probabil $y=\frac{n(E)}{n(S)}$

$n(E)=$ *is triangle number*

$=\{6,10,10,15\}$

A1 c.a.o

$n(S)$ *is sample space*

$n(E)=4$

probabil $y=\frac{4}{12}=\frac{1}{3}$

B1

6. $\begin{pmatrix}x&0\\0&2x-5\end{pmatrix}$ *sin ce* $\begin{pmatrix}k&0\\0&k\end{pmatrix}$

then $k=k$, *meaning*

$x=2x-5$

M1 for equati

$-x=-5$

$x=5$

.o

$\begin{pmatrix}5&0\\0&5\end{pmatrix}\begin{pmatrix}3\\-7\end{pmatrix}=A^1$

M1 for multip

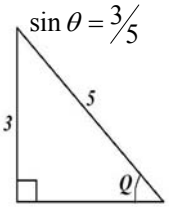
$\begin{pmatrix}15+0\\0+-35\end{pmatrix}=A^1$

$A^1(15,-35)$

A1 c.a.o

7. $5\sin\theta-3=0$

$\sin\theta=\frac{3}{5}$



$3^2+Adj^2=5^2$

M1 pythog the

$Adj=\sqrt{5^2-3^2}$

$=4$

A1 c.a.o

$\cos\theta=\frac{Adj}{hyp}=\frac{4}{5}$

B1 for $\frac{4}{5}$ *seen*

$\tan\theta=\frac{opp}{adj}=\frac{3}{4}$

B1 for $\frac{3}{4}$ *seen*

8. $n=a\sqrt{b-c}$

$\frac{n}{a}=\sqrt{b-c}$

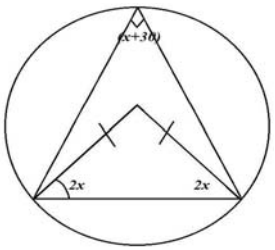
$\left(\frac{n}{a}\right)^2=b-c$

M1 squaring (

$c=b-\left(\frac{n}{a}\right)^2$

B1A1 for c see

9.



$180-(2x+2x)=2(x+30)$

M1 for $180-4x$ *s*

$180-4x=2x+60$

M1 for $2(x+30)$

$X=20^o$

A1 c.a.oA1 c.a.o

10 $P \downarrow Q = \dfrac{2PQ}{P-Q}$

$x \downarrow 4 = \dfrac{2 \cdot x \cdot 4}{x-4}$ M_1 *for substn*

$40 = \dfrac{8x}{x-4}$ A_1 *for equating to* 40

$40(x-4) = 8x$ M_1 *for multiptn*

$5(x-4) = x$

$5x - 4x = 20$

$4x = 20$

$x = 5$ A_1 *c.a.o*

SECTION B

11(*a*)(*i*).

$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$R(90) = \begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix}$

$R(90) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ B_1 *for rotation matrix*

(*ii*). $x + y = 0$

$y = -x$

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ B_1 *for reflection matrix*

(*b*) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 4 & 4 \\ 0 & 0 & 3 \end{pmatrix} = A'B'C'$

$\begin{pmatrix} 0+0 & 0+0 & 0+-3 \\ 2+0 & 4+0 & 4+0 \end{pmatrix} = A'B'C'$

$A'(0,2)\ B'(0,4)\ C'(-3,4)$ $B_1B_1B_1$ *for all values correct*

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & -3 \\ 2 & 4 & 4 \end{pmatrix} = A''B''C''$

$\begin{pmatrix} 0+-2 & 0+-4 & 0+-4 \\ 0+0 & 0+0 & 3+0 \end{pmatrix} = A''B''C''$

$A''(-2,0)\ B''(-4,0)\ C''(-4,3)$ $B_1B_1B_1$ *for all values correct*

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ M_1

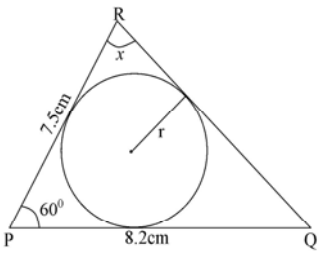
$\begin{pmatrix} 0+-1 & 0+0 \\ 0+0 & 1+0 \end{pmatrix}$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ B_1

$\dfrac{1}{-1}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ B_1 *for* det *of* -1 *seen*

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ *maps* $A''B''C''$ *back onto* ABC A_1 *c.a.o*

12.



$QR = 7.9cm. \pm 0.2$ B_1B_1

$Radius = 2.2cm. \pm 0.2$ B_1B_1

$Angle\ PRQ = (64-67)^o$ B_1

13. *let Jovan's speed be xkm*

Jovia's speed will be $(x+2)km/hr$

Now using time of travell

$Jovan's\ time = \dfrac{20}{x}$ B_1

$Jovia's\ time = \dfrac{20}{x+2}$ B_1

Difference in their arrival $= 1/2.$

$\dfrac{20}{x} - \dfrac{20}{x+2} = 1/2$ M_1

$\dfrac{20(x+2)-20x}{x(x+2)} = 1/2$

$\dfrac{40}{x(x+2)} = 1/2$ A_1

$x^2 + 2x = 80$ M_1

$x^2 + 2x - 80 = 0$

$x^2 + 10x - 8x - 80 = 0$ B_1

$x(x+10) - 8(x+10) =)$

$(x-8)(x+10) = 0$ B_1

$x = 8.$ *Ignore* -10 A_1

So Jovan travels at $8km/hr.$ B_1

While Jovia travels at $(x+2) = 10km/hr$ B_1

(*b*) *Jovan cycles a distances of* $5 \times 10 = 40km$ B_1

Jovia cycles adistance of $5 \times 10 = 50km$ B_1

14. $AB = A \times B$

$$\begin{pmatrix} -13 & 1 \\ 11 & 11 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

M_1 *for perfect substn*

$$\begin{pmatrix} -13 & 1 \\ 11 & 11 \end{pmatrix} = \begin{matrix} 4a-c & 4b-d \\ 2a+3c & 2b+3d \end{matrix}$$

$4a - c = -13 \quad 4b - d = 1$

$2a + 3c = 11 \quad 2b + 3d = 11$

B_1 *for all equations correctly*

$4a - c = -13 \quad 4b - d = 1$

$-4a + 6c = 22 \quad -4b + 6d = 22$

$-7c = -35 \quad -7d = -21$

$c = 5 \quad d = 3$

B_1 *for every correct value got*

U sing *from*

$4a - c = -13 \quad 4b - 3 = 1$

$4a = -13 + 5 \quad 4b = 4$

$4a = -8 \quad b = 1$

$a = -2$

B_1

$\therefore Matrix\ B = \begin{pmatrix} -2 & 1 \\ 5 & 3 \end{pmatrix}$

A_1 *for stating matrix correctly*

14.(*b*) $2x - y = 8$

$4x - 3y = 14$

$$\begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 8 \\ 14 \end{pmatrix}$$

M_1 *correct ma*

$$\begin{pmatrix} -6+4 & 3-3 \\ -8+8 & 4+-6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -24+14 \\ -32+28 \end{pmatrix}$$

B_1 *for correct*

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ -4 \end{pmatrix}$$

B_1 *for simplif*

$-2x = -10,\ x = 5$

A_1 *c.a.o*

$-2y = -4,\ y = 2$

A_1 *c.a.o*

15. $200x + 300y \geq 12000$

B_1 *for inequa*

$x + y \leq 60$

B_1 *formed co*

$y \leq 2x$

B_1

$x \geq 0,\ y \geq 0$

B_1 *for both x*

$y = 0,\ x = 0.$

for $y = 2x$

(0,0) (5,10)

B_1 *for c*

for $x + y = 60$

(10,50) (40,20)

B_1 *for any co*

for $2x + 3y = 120$. *reduced*

(0,40) (60,0)

B_1 *for correct*

$P = 300{,}000x + 60{,}000y$

B_1 *for objecti*

u sing (20,40) (15,45) (10,50)

(*c*) $profit(P) = 300{,}000x + 60{,}000y$ B_1

using points (20,40) (15,45) (10,50)

Points	*calculation*	*profits*
(20,40)	$300{,}000 \times 20 + 60{,}000 \times 40$	8,400,000
(15,45)	$300{,}000 \times 15 + 60{,}000 \times 45$	7,200,000
(10,50)	$300{,}000 \times 10 + 60{,}000 \times 50$	6,000,00

Truck A should make 20 *trips while truck B* 40 *trips to have* max *imum profit.*

B_1 *for number of trips got*

16 .(*a*) Grouped frequency table

Marks	*tally*	*f*	*x*	*fx*	*cf*	*C/B*
12 - 19	𝍸 𝍸 ||	12	15.5	186	12	11.5 - 19.5
20 -27	𝍸 𝍸 ||||	15	23.5	352.5	27	19.5 - 27.5
28 - 35	𝍸 |||	9	31.5	283.5	36	27.5 - 35.5
36 - 43	𝍸 |||	9	39.5	355.5	45	35.5 - 43.5
44 - 51	𝍸	5	47.5	237.5	50	43.5 - 51.5
		$\Sigma f = 50$		$\Sigma fx = 1415$		

(*i*) mod *al frequency* = 15

(*ii*) *class size* $(19 - 12) + 1 = 8$ B_1

(*b*) *mean mark* $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{1415}{50}$ M_1

$= 28.5$ A_1

(*c*) *Medium* $= \frac{1}{2}(N)^{th}$

$= \frac{1}{2} \times 50^{th}$

$= 25^{th}$

$\Rightarrow 19.5 + (0.8 \times 9\ or\ 8)$ M_1

$\Rightarrow 26.7\ or\ 25.9$ A_1

17.

x	-4.0	-3.5	-3.0	-2.5	-2.0	-1.5	-1	-0.5	0	0.5	1
$2x$	32	24.5	18	12.5	8	4.5	2	0.5	0	0.5	2
$5x - 3$	-23	-20.5	-18	-15.5	-13	-10.5	-8	-5.5	-3	-0.5	2
$y=2x+5x-3$	9	4	0	-3	-5	-6	-6	-5	-3	0	4
1	1	1	1	1	1	1	1	1	1	1	1
$y=x+1$	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2

coordinates for $y = 2x^2 + 5x - 3$

(–4,9)(–3.5,4)(–3.0,0)(–2.5,–3)(–2.0,–5)(–1.5,–6)

(–1,–6)(–0.5,–5)(0,3)(0.5,0)(1,4)

some coordinates for $y = x + 1$

(–4,–3)(–3,–2)(0,1)(1,2)

(*b*) $2x^2 + 5x - 3 = x + 1$. Read $x-$*values where line meets the curve on graph*

$x_1 = -2.7 \pm 0.1,\ x_2 = 0.7 \pm 0.1$