Pure Mathematics.
Revision Series.
EXERCISE 1.

SOME USEFUL TOPICS IN ALGEBRA.

1. Given that α and β are the roots of the equation $x^2 + px + q = 0$ express

   $(\alpha^2 - \beta^2)$ and $(\alpha^3 + \beta^3)$ in terms of p and q. {Ans:$-p^3 + 3pq$}

2. (a) The function f(x) = x^3+ px^2-5x + q has a factor (x-2) and has a value 5 when x = -3 find p and q {Ans: p = 3, q -10}

   (b) The roots of the equation ax^2 + bx + c= 0 are α and β. Form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

   {Ans: $acx^2 - (b^2 - 2ac)x + ac = 0$}

3. Given the polynomial f (x) = Q(x)g(x)+ R(x) where Q(x) is the quotient, g(x)=(x-α)(x-β) and R(x) the remainder show that :
   $R(x) = \frac{(x-\beta)f(\alpha)+(\alpha-x)f(\beta)}{\alpha-\beta}$ hence find the remainder when f(x) is divided by x-2 is 2 and when divided by x+3 is -3

   {Ans: $\frac{5x-3}{6}$ }

4. Solve for x in the equation $\log_4(6 - x) = \log_2 x$ {Ans: x =2}
5. Solve the equation $\log_2 x - \log_x 8 = 2.$ {Ans: x = 8,$\frac{1}{2}$ }
6. Given that $\log_3 x = p$ and $\log_{18} x = q$ show that $\log_6 3 = \frac{q}{p-q}$
7. (a) Given that one of the roots of the equation x^2 +px + q = 0 is twice the other root. Show that 2p^2 = 9q. hence or other wise, find the value of k, if the equation x^2 - 2(k +2) x + (k ^{2} + 3k +2) = 0 has one root twice the other. {Ans, K = -7 or 2}

   (b) If a^2 + b^2 = 23ab, show that log a + log b = 2 log($\frac{a+b}{5}$).
8. Prove that $\log_b N = \frac{\log_a N}{\log_a b}$ hence solve the equation

   $\log_{10} x + \log_x 100 = 3$ {Ans: x = 10,100}

9. If α and β are the roots of the equation ax^2 + bx + c = 0, find the equation whose roots are (α^2 +β^2) and (α^{-2} +ß$^{-2}$)

   {Ans: $(ac)^2x^2$- (b ^{2} - 2ac) (c^2 + a^2) x + (b^2 – 2ac) = 0}

10. Solve the equation. $\log_x 64 + \log_4 x^2 = 7$ {Ans. $x = 2, 64$}

11. (a) When the polynomial P(x) is divided by x - 3 the remainder is 2 and when divided by x+3 the remainder is -3, find the remainder when P (x) is divided by $x^2 - 9$ {Ans: R=3}

(b) If α and ß are the roots of the equation $ax^2 + bx + c = 0$ find the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$

{Ans: $c^2x^2 - (b^2 - 2ac)x + a^2 = 0$}

12. The polyomial P(x) is dived by (x-2) the remainder is 7 and when divided by (x-2) the remainder is 9 when divided by x the remainder is 5. Find the remainder when the polynomial P(x) is divided x(x-2) (x+2). {Ans: $\frac{3}{4}x^2 - \frac{1}{2}x + 5$}

13. The roots of the equation $px^2 + qx + r = 0$ are α and β. find the equation whose roots are (α-2) and (ß-2).

{Ans. $px^2 + (q+4p)x + (r+2q+4p) = 0$}

14. If α and ß are roots of the equation $ax^2 + bx + c = 0$. find the value of $\alpha^2 - \beta^2$ {Ans: $\frac{\pm b\sqrt{b^2-4ac}}{a^2}$ }

15. If the equation $x^2 + px + q = 0$ has roots α and β. Find the value of

(i) $\alpha^3\beta + \alpha\beta^3$. {Ans:$q(p^2 - 2q)$}

(ii) $\alpha^4 + \alpha^2\beta^2 + \beta^4$. {Ans: $((p - 2q)^2 - 2q^2$}

16. When the polynomial f (x) = $x^7 - ax^3 + 4b$ is divided by x-2 the remainder is 8 and x-1 is a factor find the value of a and b. {Ans: a = 17, b = 16}

17. If α and β are roots of the equation $ax^2 + px + q = 0$ find the equation P whose roots are α^2 and β^2. {Ans: $a^2x^2 - (p^2 - 2aq)x + q^2 = 0$}

18. Show that $\log_b a = \frac{\log_c a}{\log_c b}$ hence solve the simultaneous equations

$x + y = 20$, $\log_3 x = \log_9 y$. {Ans. (4,16)(-5,25)}

19. If $\log_2 x = p$ and $\log_6 x = q$,Show that $2^{(p-q)} = 3^q$.

20. The function, f (x) = $x^3 + px^2 - 5x + q$ has a factor (x - 2) and has a remainder 5 when divided by (x + 3). Find p and q.

{Ans: p = 3, q = -10}

21. Given that the roots of the equation $ax^2 + bx + c = 0$ are β and nβ show that $(n+1)^2 ac = nb^2$

22. Solve the equation $\log_3 x - 4\log_x 3 + 3 = 0$. {Ans:$x = \frac{1}{81}, 3$}

23. Given that $\log_3 6 = m$ and $\log_6 5 = n$ express $\log_3 10$ in terms of m and n {Ans, nm + m -1}

24. Without using tables or a calculator, solve $\log_8\{\frac{x}{2}\} = \frac{\log_8 x}{\log_8 2}$. {Ans. $\frac{1}{\sqrt{2}}$ }

25. Simplify. $\frac{\sqrt{2}}{\sqrt{2}-\sqrt{3}-\sqrt{5}} + \frac{\sqrt{3}}{\sqrt{2}-\sqrt{3}+\sqrt{5}}$. {Ans: $\frac{1}{12}(\sqrt{6} + 3\sqrt{10} - 2\sqrt{15})$}