SOLUTIONS TO CONCEPTS
CHAPTER – 1

1. a) Linear momentum : mv $= [MLT^{-1}]$

   b) Frequency : $\frac{1}{T} = [M^0L^0T^{-1}]$

   c) Pressure : $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

2. a) Angular speed $\omega = \theta/t = [M^0L^0T^{-1}]$

   b) Angular acceleration $\alpha = \frac{\omega}{t} = \frac{M^0L^0T^{-2}}{T} = [M^0L^0T^{-2}]$

   c) Torque $\tau = F\,r = [MLT^{-2}]\,[L] = [ML^2T^{-2}]$

   d) Moment of inertia $= Mr^2 = [M]\,[L^2] = [ML^2T^0]$

3. a) Electric field $E = F/q = \frac{MLT^{-2}}{[IT]} = [MLT^{-3}I^{-1}]$

   b) Magnetic field $B = \frac{F}{qv} = \frac{MLT^{-2}}{[IT][LT^{-1}]} = [MT^{-2}I^{-1}]$

   c) Magnetic permeability $\mu_0 = \frac{B \times 2\pi a}{I} = \frac{MT^{-2}I^{-1}] \times [L]}{[I]} = [MLT^{-2}I^{-2}]$

4. a) Electric dipole moment $P = qI = [IT] \times [L] = [LTI]$

   b) Magnetic dipole moment $M = IA = [I]\,[L^2]\,[L^2I]$

5. $E = h\nu$ where E = energy and ν = frequency.

   $h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]}[ML^2T^{-1}]$

6. a) Specific heat capacity = $C = \frac{Q}{m\Delta T} = \frac{[ML^2T^{-2}]}{[M][K]} = [L^2T^{-2}K^{-1}]$

   b) Coefficient of linear expansion = $\alpha = \frac{L_1 - L_2}{L_0 \Delta T} = \frac{[L]}{[L][R]} = [K^{-1}]$

   c) Gas constant = $R = \frac{PV}{nT} = \frac{[ML^{-1}T^{-2}][L^3]}{[(mol)][K]} = [ML^2T^{-2}K^{-1}(mol)^{-1}]$

7. Taking force, length and time as fundamental quantity

   a) Density $= \frac{m}{V} = \frac{(\text{force/acceleration})}{\text{Volume}} = \frac{[F/LT^{-2}]}{[L^2]} = \frac{F}{L^4T^{-2}} = [FL^{-4}T^2]$

   b) Pressure $= F/A = F/L^2 = [FL^{-2}]$

   c) Momentum = mv (Force / acceleration) × Velocity $= [F / LT^{-2}] \times [LT^{-1}] = [FT]$

   d) Energy $= \frac{1}{2}mv^2 = \frac{\text{Force}}{\text{acceleration}} \times (\text{velocity})^2$

   $= \left[\frac{F}{LT^{-2}}\right] \times [LT^{-1}]^2 = \left[\frac{F}{LT^{-2}}\right] \times [L^2T^{-2}] = [FL]$

8. $g = 10\,\frac{\text{metre}}{\text{sec}^2} = 36 \times 10^5\ \text{cm/min}^2$

9. The average speed of a snail is 0.02 mile/hr

   Converting to S.I. units, $\frac{0.02 \times 1.6 \times 1000}{3600}$ m/sec [1 mile = 1.6 km = 1600 m] $= 0.0089\ ms^{-1}$

   The average speed of leopard = 70 miles/hr

   In SI units = 70 miles/hour $= \frac{70 \times 1.6 \times 1000}{3600}$ = 31 m/s

10. Height h = 75 cm, Density of mercury = 13600 kg/m^3, g = 9.8 ms^{-2} then
    Pressure = hfg = 10×10^4 N/m^2 (approximately)
    In C.G.S. Units, P = 10×10^5 dyne/cm^2
11. In S.I. unit 100 watt = 100 Joule/sec
    In C.G.S. Unit = 10^9 erg/sec
12. 1 micro century = $10^4 \times 100$ years = $10^{-4} \times 365 \times 24 \times 60$ min
    So, 100 min = 10^5 / 52560 = 1.9 microcentury
13. Surface tension of water = 72 dyne/cm
    In S.I. Unit, 72 dyne/cm = 0.072 N/m
14. $K = kI^a \omega^b$ where k = Kinetic energy of rotating body and k = dimensionless constant
    Dimensions of left side are,
    $K = [ML^2T^{-2}]$
    Dimensions of right side are,
    $I^a = [ML^2]^a$, $\omega^b = [T^{-1}]^b$
    According to principle of homogeneity of dimension,
    $[ML^2T^{-2}] = [ML^2T^{-2}]\ [T^{-1}]^b$
    Equating the dimension of both sides,
    $2 = 2a$ and $-2 = -b \Rightarrow a = 1$ and $b = 2$
15. Let energy $E \propto M^aC^b$ where M = Mass, C = speed of light
    $\Rightarrow E = KM^aC^b$ (K = proportionality constant)
    Dimension of left side
    $E = [ML^2T^{-2}]$
    Dimension of right side
    $M^a = [M]^a$, $[C]^b = [LT^{-1}]^b$
    $\therefore [ML^2T^{-2}] = [M]^a[LT^{-1}]^b$
    $\Rightarrow a = 1;\ b = 2$
    So, the relation is $E = KMC^2$
16. Dimensional formulae of R = $[ML^2T^{-3}I^{-2}]$
    Dimensional formulae of V = $[ML^2T^3I^{-1}]$
    Dimensional formulae of I = [I]
    $\therefore [ML^2T^3I^{-1}] = [ML^2T^{-3}I^{-2}]\ [I]$
    $\Rightarrow V = IR$
17. Frequency $f = KL^aF^bM^c$ M = Mass/unit length, L = length, F = tension (force)
    Dimension of f = $[T^{-1}]$
    Dimension of right side,
    $L^a = [L^a]$, $F^b = [MLT^{-2}]^b$, $M^c = [ML^{-1}]^c$
    $\therefore [T^{-1}] = K[L]^a\ [MLT^{-2}]^b\ [ML^{-1}]^c$
    $M^0L^0T^{-1} = KM^{b+c}\ L^{a+b-c}\ T^{-2b}$
    Equating the dimensions of both sides,
    $\therefore b + c = 0$ …(1)
    $-c + a + b = 0$ …(2)
    $-2b = -1$ …(3)
    Solving the equations we get,
    $a = -1$, $b = 1/2$ and $c = -1/2$
    $\therefore$ So, frequency $f = KL^{-1}F^{1/2}M^{-1/2} = \frac{K}{L}F^{1/2}M^{-1/2} = \frac{K}{L} = \sqrt{\frac{F}{M}}$

18. a) h = $\frac{2S\text{Cos}\theta}{\rho rg}$

LHS = [L]

Surface tension = S = F/l = $\frac{MLT^{-2}}{L} = [MT^{-2}]$

Density = ρ = M/V = $[ML^{-3}T^{0}]$

Radius = r = [L], g = $[LT^{-2}]$

RHS = $\frac{2S\cos\theta}{\rho rg} = \frac{[MT^{-2}]}{[ML^{-3}T^{0}][L][LT^{-2}]} = [M^{0}L^{1}T^{0}] = [L]$

LHS = RHS

So, the relation is correct

b) v = $\sqrt{\frac{p}{\rho}}$ where v = velocity

LHS = Dimension of v = $[LT^{-1}]$

Dimension of p = F/A = $[ML^{-1}T^{-2}]$

Dimension of ρ = m/V = $[ML^{-3}]$

RHS = $\sqrt{\frac{p}{\rho}} = \sqrt{\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}} = [L^{2}T^{-2}]^{1/2}$ = $[LT^{-1}]$

So, the relation is correct.

c) V = $(\pi pr^{4}t) / (8\eta l)$

LHS = Dimension of V = $[L^{3}]$

Dimension of p = $[ML^{-1}T^{-2}]$, $r^{4} = [L^{4}]$, t = [T]

Coefficient of viscosity = $[ML^{-1}T^{-1}]$

RHS = $\frac{\pi pr^{4}t}{8\eta l} = \frac{[ML^{-1}T^{-2}][L^{4}][T]}{[ML^{-1}T^{-1}][L]}$

So, the relation is correct.

d) v = $\frac{1}{2\pi}\sqrt{(mgl/I)}$

LHS = dimension of v = $[T^{-1}]$

RHS = $\sqrt{(mgl/I)} = \sqrt{\frac{[M][LT^{-2}][L]}{[ML^{2}]}} = [T^{-1}]$

LHS = RHS

So, the relation is correct.

19. Dimension of the left side = $\int\frac{dx}{\sqrt{(a^{2}-x^{2})}} = \int\frac{L}{\sqrt{(L^{2}-L^{2})}} = [L^{0}]$

Dimension of the right side = $\frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right) = [L^{-1}]$

So, the dimension of $\int\frac{dx}{\sqrt{(a^{2}-x^{2})}} \neq \frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right)$

So, the equation is dimensionally incorrect.

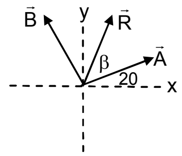
20. Important Dimensions and Units :

Physical quantity	Dimension	SI unit
Force (F)	$[M^1L^1T^{-2}]$	newton
Work (W)	$[M^1L^2T^{-2}]$	joule
Power (P)	$[M^1L^2T^{-3}]$	watt
Gravitational constant (G)	$[M^{-1}L^3T^{-2}]$	$N\text{-}m^2/kg^2$
Angular velocity (ω)	$[T^{-1}]$	radian/s
Angular momentum (L)	$[M^1L^2T^{-1}]$	$kg\text{-}m^2/s$
Moment of inertia (I)	$[M^1L^2]$	$kg\text{-}m^2$
Torque (τ)	$[M^1L^2T^{-2}]$	N-m
Young's modulus (Y)	$[M^1L^{-1}T^{-2}]$	N/m^2
Surface Tension (S)	$[M^1T^{-2}]$	N/m
Coefficient of viscosity (η)	$[M^1L^{-1}T^{-1}]$	$N\text{-}s/m^2$
Pressure (p)	$[M^1L^{-1}T^{-2}]$	N/m^2 (Pascal)
Intensity of wave (I)	$[M^1T^{-3}]$	$watt/m^2$
Specific heat capacity (c)	$[L^2T^{-2}K^{-1}]$	J/kg-K
Stefan's constant (σ)	$[M^1T^{-3}K^{-4}]$	$watt/m^2\text{-}k^4$
Thermal conductivity (k)	$[M^1L^1T^{-3}K^{-1}]$	watt/m-K
Current density (j)	$[I^1L^{-2}]$	$ampere/m^2$
Electrical conductivity (σ)	$[I^2T^3M^{-1}L^{-3}]$	$\Omega^{-1}\ m^{-1}$
Electric dipole moment (p)	$[L^1I^1T^1]$	C-m
Electric field (E)	$[M^1L^1I^{-1}T^{-3}]$	V/m
Electrical potential (V)	$[M^1L^2I^{-1}T^{-3}]$	volt
Electric flux (Ψ)	$[M^1T^3I^{-1}L^{-3}]$	volt/m
Capacitance (C)	$[I^2T^4M^{-1}L^{-2}]$	farad (F)
Permittivity (ε)	$[I^2T^4M^{-1}L^{-3}]$	$C^2/N\text{-}m^2$
Permeability (μ)	$[M^1L^1I^{-2}T^{-3}]$	$Newton/A^2$
Magnetic dipole moment (M)	$[I^1L^2]$	N-m/T
Magnetic flux (ϕ)	$[M^1L^2I^{-1}T^{-2}]$	Weber (Wb)
Magnetic field (B)	$[M^1I^{-1}T^{-2}]$	tesla
Inductance (L)	$[M^1L^2I^{-2}T^{-2}]$	henry
Resistance (R)	$[M^1L^2I^{-2}T^{-3}]$	ohm (Ω)

* * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 2

1. As shown in the figure,



The angle between $\vec{A}$ and $\vec{B}$ = 110° – 20° = 90°

$|\vec{A}|$ = 3 and $|\vec{B}|$ = 4m

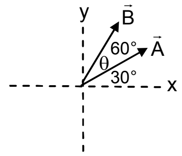
Resultant R = $\sqrt{A^2 + B^2 + 2AB\cos\theta}$ = 5 m

Let β be the angle between $\vec{R}$ and $\vec{A}$

$\beta = \tan^{-1}\left(\dfrac{4\sin 90°}{3 + 4\cos 90°}\right) = \tan^{-1}(4/3) = 53°$

∴ Resultant vector makes angle (53° + 20°) = 73° with x-axis.

2. Angle between $\vec{A}$ and $\vec{B}$ is θ = 60° – 30° =30°



$|\vec{A}|$ and $|\vec{B}|$ = 10 unit

R = $\sqrt{10^2 + 10^2 + 2.10.10.\cos 30°}$ = 19.3

β be the angle between $\vec{R}$ and $\vec{A}$

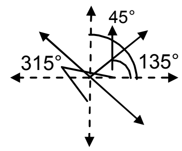
$\beta = \tan^{-1}\left(\dfrac{10\sin 30°}{10 + 10\cos 30°}\right) = \tan^{-1}\left(\dfrac{1}{2+\sqrt{3}}\right) = \tan^{-1}(0.26795) = 15°$

∴ Resultant makes 15° + 30° = 45° angle with x-axis.

3. x component of $\vec{A}$ = 100 cos 45° = $100/\sqrt{2}$ unit

x component of $\vec{B}$ = 100 cos 135° = $100/\sqrt{2}$

x component of $\vec{C}$ = 100 cos 315° = $100/\sqrt{2}$



Resultant x component = $100/\sqrt{2} - 100/\sqrt{2} + 100/\sqrt{2} = 100/\sqrt{2}$

y component of $\vec{A}$ = 100 sin 45° = $100/\sqrt{2}$ unit

y component of $\vec{B}$ = 100 sin 135° = $100/\sqrt{2}$

y component of $\vec{C}$ = 100 sin 315° = $-100/\sqrt{2}$

Resultant y component = $100/\sqrt{2} + 100/\sqrt{2} - 100/\sqrt{2} = 100/\sqrt{2}$

Resultant = 100

Tan α = $\dfrac{\text{y component}}{\text{x component}}$ = 1

⇒ α = $\tan^{-1}(1)$ = 45°

The resultant is 100 unit at 45° with x-axis.

4. $\vec{a} = 4\vec{i} + 3\vec{j}$, $\vec{b} = 3\vec{i} + 4\vec{j}$

a) $|\vec{a}| = \sqrt{4^2 + 3^2}$ = 5

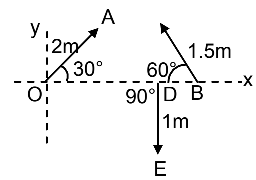
b) $|\vec{b}| = \sqrt{9+16}$ = 5

c) $|\vec{a} + \vec{b}| = |7\vec{i} + 7\vec{j}| = 7\sqrt{2}$

d) $\vec{a} - \vec{b} = (-3+4)\hat{i} + (-4+3)\hat{j} = \hat{i} - \hat{j}$

$|\vec{a} - \vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$.

5. x component of $\overrightarrow{OA}$ = 2cos30° = $\sqrt{3}$



x component of $\overrightarrow{BC}$ = 1.5 cos 120° = –0.75

x component of $\overrightarrow{DE}$ = 1 cos 270° = 0

y component of $\overrightarrow{OA}$ = 2 sin 30° = 1

y component of $\overrightarrow{BC}$ = 1.5 sin 120° = 1.3

y component of $\overrightarrow{DE}$ = 1 sin 270° = –1

R_x = x component of resultant = $\sqrt{3} - 0.75 + 0 = 0.98$ m

R_y = resultant y component = 1 + 1.3 – 1 = 1.3 m

So, R = Resultant = 1.6 m

If it makes and angle α with positive x-axis

Tan α = $\dfrac{\text{y component}}{\text{x component}}$ = 1.32

$\Rightarrow \alpha = \tan^{-1} 1.32$

6. $|\vec{a}|$ = 3m $|\vec{b}|$ = 4

a) If R = 1 unit $\Rightarrow \sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 1

θ = 180°

b) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 5

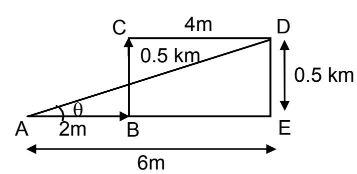
θ = 90°

c) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 7

θ = 0°

Angle between them is 0°.

7. $\overrightarrow{AD} = 2\hat{i} + 0.5\hat{J} + 4\hat{K}$ = $6\hat{i} + 0.5\hat{j}$



AD = $\sqrt{AE^2 + DE^2}$ = 6.02 KM

Tan θ = DE / AE = 1/12

$\theta = \tan^{-1}$ (1/12)

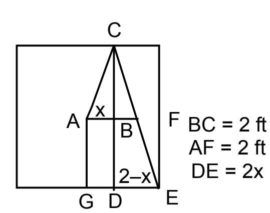
The displacement of the car is 6.02 km along the distance $\tan^{-1}$ (1/12) with positive x-axis.

8. In ΔABC, tanθ = x/2 and in ΔDCE, tanθ = (2 – x)/4 tan θ = (x/2) = (2 – x)/4 = 4x

$\Rightarrow$ 4 – 2x = 4x

$\Rightarrow$ 6x = 4 $\Rightarrow$ x = 2/3 ft

a) In ΔABC, AC = $\sqrt{AB^2 + BC^2}$ = $\dfrac{2}{3}\sqrt{10}$ ft

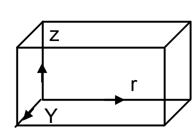


b) In ΔCDE, DE = 1 – (2/3) = 4/3 ft

CD = 4 ft. So, CE = $\sqrt{CD^2 + DE^2}$ = $\dfrac{4}{3}\sqrt{10}$ ft

c) In ΔAGE, AE = $\sqrt{AG^2 + GE^2}$ = $2\sqrt{2}$ ft.

9. Here the displacement vector $\vec{r} = 7\hat{i} + 4\hat{j} + 3\hat{k}$



a) magnitude of displacement = $\sqrt{74}$ ft

b) the components of the displacement vector are 7 ft, 4 ft and 3 ft.

10. $\vec{a}$ is a vector of magnitude 4.5 unit due north.

a) $3|\vec{a}| = 3 \times 4.5 = 13.5$

$3\vec{a}$ is along north having magnitude 13.5 units.

b) $-4|\vec{a}| = -4 \times 1.5 = -6$ unit

$-4\vec{a}$ is a vector of magnitude 6 unit due south.

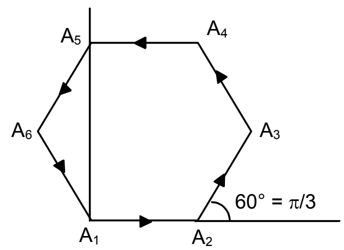
11. $|\vec{a}| = 2$ m, $|\vec{b}| = 3$ m

angle between them $\theta = 60°$

a) $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos 60° = 2 \times 3 \times 1/2 = 3$ m^2

b) $|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\sin 60° = 2 \times 3 \times \sqrt{3/2} = 3\sqrt{3}$ m^2.

12. We know that according to polygon law of vector addition, the resultant of these six vectors is zero.



Here A = B = C = D = E = F (magnitude)

So, Rx = A cosθ + A cos $\pi/3$ + A cos $2\pi/3$ + A cos $3\pi/3$ + A cos $4\pi/4$ + A cos $5\pi/5$ = 0

[As resultant is zero. X component of resultant R_x = 0]

= cos θ + cos $\pi/3$ + cos $2\pi/3$ + cos $3\pi/3$ + cos $4\pi/3$ + cos $5\pi/3$ = 0

Note : Similarly it can be proved that,

sin θ + sin $\pi/3$ + sin $2\pi/3$ + sin $3\pi/3$ + sin $4\pi/3$ + sin $5\pi/3$ = 0

13. $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$; $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$

$$\vec{a}\cdot\vec{b} = ab\cos\theta \Rightarrow \theta = \cos^{-1}\frac{\vec{a}\cdot\vec{b}}{ab}$$

$$\Rightarrow \cos^{-1}\frac{2\times3+3\times4+4\times5}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+4^2+5^2}} = \cos^{-1}\left(\frac{38}{\sqrt{1450}}\right)$$

14. $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$ (claim)

As, $\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$

AB sin θ $\hat{n}$ is a vector which is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, this implies that it is also perpendicular to $\vec{A}$. As dot product of two perpendicular vector is zero.

Thus $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$.

15. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 4 & 3 & 2\end{vmatrix} \Rightarrow \hat{i}(6-12) - \hat{j}(4-16) + \hat{k}(6-12) = -6\hat{i} + 12\hat{j} - 6\hat{k}.$$

16. Given that $\vec{A}$, $\vec{B}$ and $\vec{C}$ are mutually perpendicular

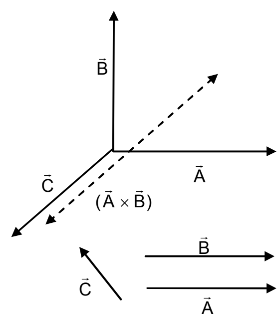
$\vec{A} \times \vec{B}$ is a vector which direction is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

Also $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$

$\therefore$ Angle between $\vec{C}$ and $\vec{A} \times \vec{B}$ is 0° or 180° (fig.1)

So, $\vec{C} \times (\vec{A} \times \vec{B}) = 0$

The converse is not true.



For example, if two of the vector are parallel, (fig.2), then also

$\vec{C} \times (\vec{A} \times \vec{B}) = 0$

So, they need not be mutually perpendicular.

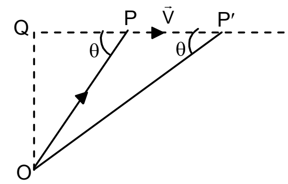
17. The particle moves on the straight line PP' at speed v.

From the figure,

$\overrightarrow{OP} \times v = (OP)v \sin\theta \hat{n}$ = v(OP) sin θ $\hat{n}$ = v(OQ) $\hat{n}$

It can be seen from the figure, OQ = OP sin θ = OP' sin θ'

So, whatever may be the position of the particle, the magnitude and direction of $\overrightarrow{OP} \times \vec{v}$ remain constant.



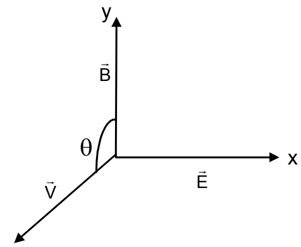
$\therefore$ $\overrightarrow{OP} \times \vec{v}$ is independent of the position P.

18. Give $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0$

$\Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$

So, the direction of $\vec{v} \times \vec{B}$ should be opposite to the direction of $\vec{E}$. Hence, $\vec{v}$ should be in the positive yz-plane.

Again, E = vB sin θ ⇒ v = $\dfrac{E}{B\sin\theta}$



For v to be minimum, θ = 90° and so v_{min} = F/B

So, the particle must be projected at a minimum speed of E/B along +ve z-axis (θ = 90°) as shown in the figure, so that the force is zero.

19. For example, as shown in the figure,

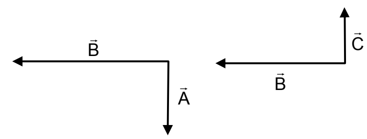
$\vec{A} \perp \vec{B}$ $\quad$ $\vec{B}$ along west

$\vec{B} \perp \vec{C}$ $\quad$ $\vec{A}$ along south

$\vec{C}$ along north

$\vec{A} \cdot \vec{B} = 0$ $\quad$ $\therefore\ \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C}$

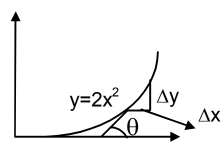
$\vec{B} \cdot \vec{C} = 0$ $\quad$ But $\vec{B} \neq \vec{C}$



20. The graph y = $2x^2$ should be drawn by the student on a graph paper for exact results.

To find slope at any point, draw a tangent at the point and extend the line to meet x-axis. Then find tan θ as shown in the figure.

It can be checked that,



Slope = tan θ = $\dfrac{dy}{dx} = \dfrac{d}{dx}(2x^2)$ = 4x

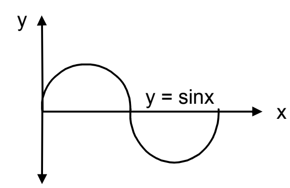
Where x = the x-coordinate of the point where the slope is to be measured.

21. y = sinx

So, y + Δy = sin (x + Δx)

Δy = sin (x + Δx) – sin x

= $\left(\dfrac{\pi}{3} + \dfrac{\pi}{100}\right) - \sin\dfrac{\pi}{3}$ = 0.0157.



22. Given that, i = $i_0 e^{-t/RC}$

$\therefore$ Rate of change of current = $\dfrac{di}{dt} = \dfrac{d}{dt} i_0 e^{-i/RC} = i_0 \dfrac{d}{dt} e^{-t/RC}$ = $\dfrac{-i_0}{RC} \times e^{-t/RC}$

When $\quad$ a) t = 0, $\dfrac{di}{dt} = \dfrac{-i}{RC}$

b) when t = RC, $\dfrac{di}{dt} = \dfrac{-i}{RCe}$

c) when t = 10 RC, $\dfrac{di}{dt} = \dfrac{-i_0}{RCe^{10}}$

23. Equation i = $i_0 e^{-t/RC}$

i_0 = 2A, R = $6 \times 10^{-5}\ \Omega$, C = 0.0500×10^{-6} F = 5×10^{-7} F

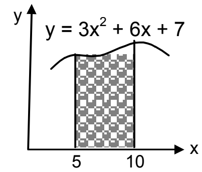
a) $i = 2 \times e^{\left(\frac{-0.3}{6\times 0^3 \times 5\times 10^{-7}}\right)} = 2 \times e^{\left(\frac{-0.3}{0.3}\right)} = \frac{2}{e}\text{amp}$.

b) $\frac{di}{dt} = \frac{-i_0}{RC}e^{-t/RC}$ when t = 0.3 sec ⇒ $\frac{di}{dt} = -\frac{2}{0.30}e^{(-0.3/0.3)} = \frac{-20}{3e}\text{Amp/sec}$

c) At t = 0.31 sec, i = $2e^{(-0.3/0.3)} = \frac{5.8}{3e}\text{Amp}$.

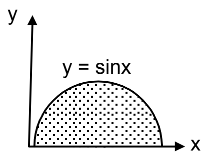
24. y = $3x^2 + 6x + 7$

∴ Area bounded by the curve, x axis with coordinates with x = 5 and x = 10 is given by,



Area = $\int_0^y dy = \int_5^{10}(3x^2+6x+7)dx = 3\frac{x^3}{3}\Big]_5^{10} + 5\frac{x^2}{3}\Big]_5^{10} + 7x]_5^{10}$ = 1135 sq.units.

25. Area = $\int_0^y dy = \int_0^{\pi}\sin x dx = -[\cos x]_0^{\pi}$ = 2

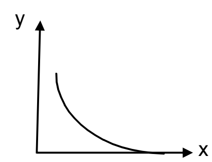


26. The given function is y = e^{-x}

When x = 0, y = e^{-0} = 1

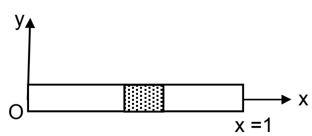
x increases, y value deceases and only at x = ∞, y = 0.

So, the required area can be found out by integrating the function from 0 to ∞.



So, Area = $\int_0^{\infty} e^{-x}dx = -[e^{-x}]_0^{\infty} = 1$.

27. $\rho = \frac{\text{mass}}{\text{length}} = a + bx$



a) S.I. unit of ‘a’ = kg/m and SI unit of ‘b’ = kg/m^2 (from principle of homogeneity of dimensions)

b) Let us consider a small element of length ‘dx’ at a distance x from the origin as shown in the figure.

∴ dm = mass of the element = ρ dx = (a + bx) dx

So, mass of the rod = m = $\int dm = \int_0^L (a+bx)dx = \left[ax + \frac{bx^2}{2}\right]_0^L = aL + \frac{bL^2}{2}$

28. $\frac{dp}{dt}$ = (10 N) + (2 N/S)t

momentum is zero at t = 0

∴ momentum at t = 10 sec will be

dp = [(10 N) + 2Ns t]dt

$\int_0^p dp = \int_0^{10} 10dt + \int_0^{10}(2tdt) = 10t]_0^{10} + 2\frac{t^2}{2}\Big]_0^{10}$ = 200 kg m/s.

29. The change in a function of y and the independent variable x are related as $\frac{dy}{dx} = x^2$.

$\Rightarrow dy = x^2\,dx$

Taking integration of both sides,

$\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + c$

$\therefore$ y as a function of x is represented by $y = \frac{x^3}{3} + c$.

30. The number significant digits

a) 1001 No.of significant digits = 4

b) 100.1 No.of significant digits = 4

c) 100.10 No.of significant digits = 5

d) 0.001001 No.of significant digits = 4

31. The metre scale is graduated at every millimeter.

1 m = 100 mm

The minimum no.of significant digit may be 1 (e.g. for measurements like 5 mm, 7 mm etc) and the maximum no.of significant digits may be 4 (e.g.1000 mm)

So, the no.of significant digits may be 1, 2, 3 or 4.

32. a) In the value 3472, after the digit 4, 7 is present. Its value is greater than 5.

So, the next two digits are neglected and the value of 4 is increased by 1.

$\therefore$ value becomes 3500

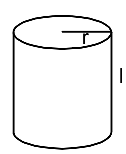
b) value = 84

c) 2.6

d) value is 28.

33. Given that, for the cylinder

Length = l = 4.54 cm, radius = r = 1.75 cm



Volume = $\pi r^2 l = \pi \times (4.54) \times (1.75)^2$

Since, the minimum no.of significant digits on a particular term is 3, the result should have 3 significant digits and others rounded off.

So, volume $V = \pi r^2 l = (3.14) \times (1.75) \times (1.75) \times (4.54) = 43.6577\ cm^3$

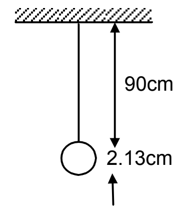
Since, it is to be rounded off to 3 significant digits, $V = 43.7\ cm^3$.

34. We know that,

Average thickness = $\frac{2.17 + 2.17 + 2.18}{3}$ = 2.1733 mm

Rounding off to 3 significant digits, average thickness = 2.17 mm.

35. As shown in the figure,



Actual effective length = (90.0 + 2.13) cm

But, in the measurement 90.0 cm, the no. of significant digits is only 2.

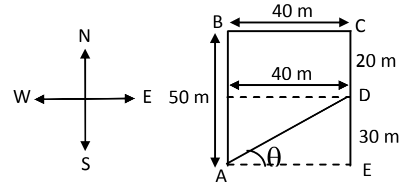
So, the addition must be done by considering only 2 significant digits of each measurement.

So, effective length = 90.0 + 2.1 = 92.1 cm.

* * * *

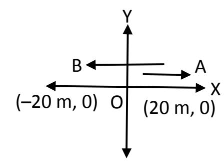
SOLUTIONS TO CONCEPTS
CHAPTER – 3

1. a) Distance travelled = 50 + 40 + 20 = 110 m
   b) AF = AB – BF = AB – DC = 50 – 20 = 30 M
   His displacement is AD
   AD = $\sqrt{AF^2 - DF^2} = \sqrt{30^2 + 40^2} = 50m$
   In ΔAED tan θ = DE/AE = 30/40 = 3/4
   $\Rightarrow \theta = \tan^{-1}(3/4)$
   His displacement from his house to the field is 50 m,
   $\tan^{-1}(3/4)$ north to east.

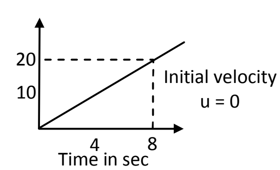


A → Initial point
(starting point)

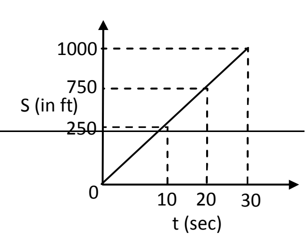
2. O → Starting point origin.
   i) Distance travelled = 20 + 20 + 20 = 60 m
   ii) Displacement is only OB = 20 m in the negative direction.
   Displacement → Distance between final and initial position.



3. a) V_{ave} of plane (Distance/Time) = 260/0.5 = 520 km/hr.
   b) V_{ave} of bus = 320/8 = 40 km/hr.
   c) plane goes in straight path
   velocity = $\vec{V}_{ave}$ = 260/0.5 = 520 km/hr.
   d) Straight path distance between plane to Ranchi is equal to the displacement of bus.
   $\therefore$ Velocity = $\vec{V}_{ave}$ = 260/8 = 32.5 km/hr.
4. a) Total distance covered 12416 – 12352 = 64 km in 2 hours.
   Speed = 64/2 = 32 km/h
   b) As he returns to his house, the displacement is zero.
   Velocity = (displacement/time) = 0 (zero).
5. Initial velocity u = 0 ($\therefore$ starts from rest)
   Final velocity v = 18 km/hr = 5 sec
   (i.e. max velocity)
   Time interval t = 2 sec.
   $\therefore$ Acceleration = $a_{ave} = \frac{v-u}{t} = \frac{5}{2} = 2.5\ m/s^2$.
6. In the interval 8 sec the velocity changes from 0 to 20 m/s.
   Average acceleration = 20/8 = 2.5 m/s^2 $\left(\frac{\text{change in velocity}}{\text{time}}\right)$
   Distance travelled S = ut + 1/2 at^2
   $\Rightarrow 0 + 1/2(2.5)8^2 = 80$ m.



7. In 1st 10 sec $S_1 = ut + 1/2\ at^2 \Rightarrow 0 + (1/2 \times 5 \times 10^2) = 250$ ft.
   At 10 sec v = u + at = 0 + 5 × 10 = 50 ft/sec.
   $\therefore$ From 10 to 20 sec (Δt = 20 – 10 = 10 sec) it moves with uniform velocity 50 ft/sec,



Distance $S_2 = 50 \times 10 = 500$ ft

Between 20 sec to 30 sec acceleration is constant i.e. -5 ft/s^2. At 20 sec velocity is 50 ft/sec.

$t = 30 - 20 = 10$ s

$S_3 = ut + 1/2\ at^2$

$= 50 \times 10 + (1/2)(-5)(10)^2 = 250$ m

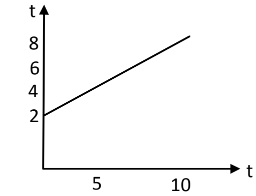
Total distance travelled is 30 sec $= S_1 + S_2 + S_3 = 250 + 500 + 250 = 1000$ ft.

8. a) Initial velocity $u = 2$ m/s.

final velocity $v = 8$ m/s

time = 10 sec,

acceleration $= \frac{v-u}{ta} = \frac{8-2}{10} = 0.6$ m/s^2



b) $v^2 - u^2 = 2aS$

$\Rightarrow$ Distance $S = \frac{v^2 - u^2}{2a} = \frac{8^2 - 2^2}{2 \times 0.6} = 50$ m.

c) Displacement is same as distance travelled.

Displacement = 50 m.

9. a) Displacement in 0 to 10 sec is 1000 m.

time = 10 sec.

$V_{ave} = s/t = 100/10 = 10$ m/s.

b) At 2 sec it is moving with uniform velocity $50/2.5 = 20$ m/s.

at 2 sec. $V_{inst} = 20$ m/s.

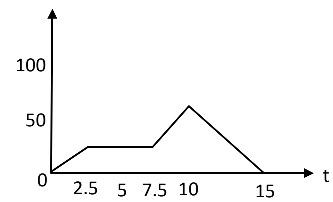
At 5 sec it is at rest.

V_{inst} = zero.

At 8 sec it is moving with uniform velocity 20 m/s

$V_{inst} = 20$ m/s

At 12 sec velocity is negative as it move towards initial position. $V_{inst} = -20$ m/s.

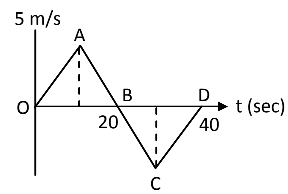


(slope of the graph at t = 2 sec)

10. Distance in first 40 sec is, Δ OAB + ΔBCD

$= \frac{1}{2} \times 5 \times 20 + \frac{1}{2} \times 5 \times 20 = 100$ m.

Average velocity is 0 as the displacement is zero.



11. Consider the point B, at t = 12 sec

At t = 0 ; s = 20 m

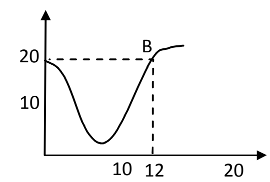
and t = 12 sec s = 20 m

So for time interval 0 to 12 sec

Change in displacement is zero.

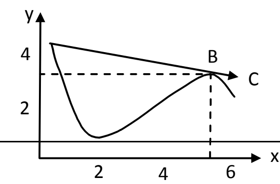
So, average velocity = displacement/ time = 0

$\therefore$ The time is 12 sec.



12. At position B instantaneous velocity has direction along $\overrightarrow{BC}$. For average velocity between A and B.

V_{ave} = displacement / time = $(\overrightarrow{AB}/t)$ t = time



We can see that $\overrightarrow{AB}$ is along $\overrightarrow{BC}$ i.e. they are in same direction.

The point is B (5m, 3m).

13. u = 4 m/s, a = 1.2 m/s^2, t = 5 sec

Distance = s = $ut + \frac{1}{2}at^2$

= 4(5) + 1/2 (1.2)5^2 = 35 m.

14. Initial velocity u = 43.2 km/hr = 12 m/s

u = 12 m/s, v = 0

a = –6 m/s^2 (deceleration)

Distance S = $\frac{v^2 - u^2}{2(-6)}$ = 12 m

15. Initial velocity u = 0
Acceleration a = 2 m/s^2. Let final velocity be v (before applying breaks)
t = 30 sec
$v = u + at \Rightarrow 0 + 2 \times 30 = 60$ m/s
a) $S_1 = ut + \frac{1}{2}at^2 = 900$ m
when breaks are applied u′ = 60 m/s
v′ = 0, t = 60 sec (1 min)
Declaration a′ = (v – u)/t = = (0 – 60)/60 = –1 m/s^2.
$S_2 = \frac{v'^2 - u'^2}{2a'} = 1800$ m
Total $S = S_1 + S_2 = 1800 + 900 = 2700$ m = 2.7 km.
b) The maximum speed attained by train v = 60 m/s
c) Half the maximum speed = 60/2= 30 m/s
Distance $S = \frac{v^2 - u^2}{2a} = \frac{30^2 - 0^2}{2 \times 2} = 225$ m from starting point
When it accelerates the distance travelled is 900 m. Then again declarates and attain 30 m/s.
$\therefore$ u = 60 m/s, v = 30 m/s, a = –1 m/s^2
Distance $= \frac{v^2 - u^2}{2a} = \frac{30^2 - 60^2}{2(-1)} = 1350$ m
Position is 900 + 1350 = 2250 = 2.25 km from starting point.

16. u = 16 m/s (initial), v = 0, s = 0.4 m.
Deceleration $a = \frac{v^2 - u^2}{2s} = -320$ m/s^2.
Time = $t = \frac{v - u}{a} = \frac{0 - 16}{-320} = 0.05$ sec.

17. u = 350 m/s, s = 5 cm = 0.05 m, v = 0
Deceleration = $a = \frac{v^2 - u^2}{2s} = \frac{0 - (350)^2}{2 \times 0.05} = -12.2 \times 10^5$ m/s^2.
Deceleration is 12.2×10^5 m/s^2.

18. u = 0, v = 18 km/hr = 5 m/s, t = 5 sec
$a = \frac{v - u}{t} = \frac{5 - 0}{5} = 1$ m/s^2.
$s = ut + \frac{1}{2}at^2 = 12.5$ m
a) Average velocity V_{ave} = (12.5)/5 = 2.5 m/s.
b) Distance travelled is 12.5 m.

19. In reaction time the body moves with the speed 54 km/hr = 15 m/sec (constant speed)
Distance travelled in this time is $S_1 = 15 \times 0.2 = 3$ m.
When brakes are applied,
u = 15 m/s, v = 0, a = –6 m/s^2 (deceleration)

$S_2 = \frac{v^2 - u^2}{2a} = \frac{0 - 15^2}{2(-6)} = 18.75$ m

Total distance $s = s_1 + s_2 = 3 + 18.75 = 21.75 = 22$ m.

20.

	Driver X Reaction time 0.25	Driver Y Reaction time 0.35
A (deceleration on hard braking = 6 m/s^2)	Speed = 54 km/h Braking distance a= 19 m Total stopping distance b = 22 m	Speed = 72 km/h Braking distance c = 33 m Total stopping distance d = 39 m.
B (deceleration on hard braking = 7.5 m/s^2)	Speed = 54 km/h Braking distance e = 15 m Total stopping distance f = 18 m	Speed = 72 km/h Braking distance g = 27 m Total stopping distance h = 33 m.

$a = \dfrac{0^2 - 15^2}{2(-6)} = 19$ m

So, b = 0.2 × 15 + 19 = 33 m

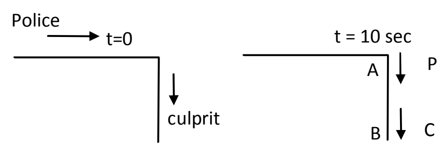
Similarly other can be calculated.

Braking distance : Distance travelled when brakes are applied.

Total stopping distance = Braking distance + distance travelled in reaction time.

21. V_P = 90 km/h = 25 m/s.

V_C = 72 km/h = 20 m/s.



In 10 sec culprit reaches at point B from A.

Distance converted by culprit S = vt = 20 × 10 = 200 m.

At time t = 10 sec the police jeep is 200 m behind the culprit.

Time = s/v = 200 / 5 = 40 s. (Relative velocity is considered).

In 40 s the police jeep will move from A to a distance S, where

S = vt = 25 × 40 = 1000 m = 1.0 km away.

∴ The jeep will catch up with the bike, 1 km far from the turning.

22. v_1 = 60 km/hr = 16.6 m/s.

v_2 = 42 km/h = 11.6 m/s.

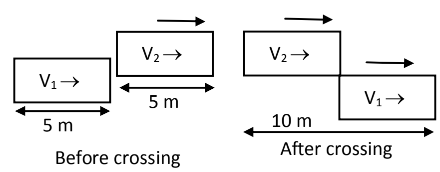
Relative velocity between the cars = (16.6 – 11.6) = 5 m/s.

Distance to be travelled by first car is 5 + t = 10 m.

Time = t = s/v = 0/5 = 2 sec to cross the 2nd car.

In 2 sec the 1st car moved = 16.6 × 2 = 33.2 m

H also covered its own length 5 m.



∴ Total road distance used for the overtake = 33.2 + 5 = 38 m.

23. u = 50 m/s, g = –10 m/s^2 when moving upward, v = 0 (at highest point).

a) $S = \dfrac{v^2 - u^2}{2a} = \dfrac{0 - 50^2}{2(-10)} = 125$ m

maximum height reached = 125 m

b) t = (v – u)/a = (0 – 50)/–10 = 5 sec

c) s′ = 125/2 = 62.5 m, u = 50 m/s, a = –10 m/s^2,

$v^2 - u^2 = 2as$

$\Rightarrow v = \sqrt{(u^2 + 2as)} = \sqrt{50^2 + 2(-10)(62.5)}$ = 35 m/s.

24. Initially the ball is going upward

$u = -7$ m/s, s = 60 m, a = g = 10 m/s^2

$s = ut + \frac{1}{2}at^2 \Rightarrow 60 = -7t + 1/2\ 10t^2$

$\Rightarrow 5t^2 - 7t - 60 = 0$

$t = \frac{7 \pm \sqrt{49 - 4.5(-60)}}{2 \times 5} = \frac{7 \pm 35.34}{10}$

taking positive sign $t = \frac{7 + 35.34}{10}$ = 4.2 sec ($\therefore\ t \neq -ve$)

Therefore, the ball will take 4.2 sec to reach the ground.

25. u = 28 m/s, v = 0, a = –g = –9.8 m/s^2

a) $S = \frac{v^2 - u^2}{2a} = \frac{0^2 - 28^2}{2(9.8)}$ = 40 m

b) time $t = \frac{v - u}{a} = \frac{0 - 28}{-9.8}$ = 2.85

$t' = 2.85 - 1 = 1.85$

$v' = u + at' = 28 - (9.8)(1.85) = 9.87$ m/s.

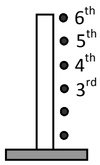
$\therefore$ The velocity is 9.87 m/s.

c) No it will not change. As after one second velocity becomes zero for any initial velocity and deceleration is g = 9.8 m/s^2 remains same. Fro initial velocity more than 28 m/s max height increases.

26. For every ball, u = 0, a = g = 9.8 m/s^2

$\therefore$ 4th ball move for 2 sec, 5th ball 1 sec and 3rd ball 3 sec when 6th ball is being dropped.

For 3rd ball t = 3 sec



$S_3 = ut + \frac{1}{2}at^2 = 0 + 1/2\ (9.8)3^2 = 4.9$ m below the top.

For 4th ball, t = 2 sec

$S_2 = 0 + 1/2\ gt^2 = 1/2\ (9.8)2^2 = 19.6$ m below the top (u = 0)

For 5th ball, t = 1 sec

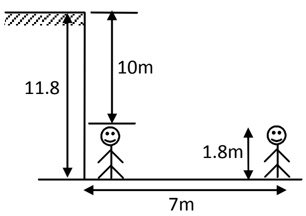
$S_3 = ut + 1/2\ at^2 = 0 + 1/2\ (9.8)t^2 = 4.98$ m below the top.

27. At point B (i.e. over 1.8 m from ground) the kid should be catched.

For kid initial velocity u = 0

Acceleration = 9.8 m/s^2

Distance S = 11.8 – 1.8 = 10 m



$S = ut + \frac{1}{2}at^2 \Rightarrow 10 = 0 + 1/2\ (9.8)t^2$

$\Rightarrow t^2 = 2.04 \Rightarrow t = 1.42.$

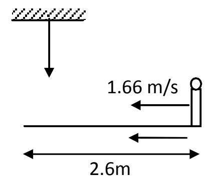
In this time the man has to reach at the bottom of the building.

Velocity s/t = 7/1.42 = 4.9 m/s.

28. Let the true of fall be ‘t’ initial velocity u = 0

Acceleration a = 9.8 m/s^2

Distance S = 12/1 m



$\therefore$ $S = ut + \frac{1}{2}at^2$

$\Rightarrow$ 12.1 = 0 + 1/2 (9.8) × t^2

$\Rightarrow t^2 = \frac{12.1}{4.9} = 2.46 \Rightarrow t = 1.57$ sec

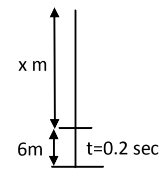
For cadet velocity = 6 km/hr = 1.66 m/sec

Distance = vt = 1.57 × 1.66 = 2.6 m.

The cadet, 2.6 m away from tree will receive the berry on his uniform.

29. For last 6 m distance travelled s = 6 m, u = ?

t = 0.2 sec, a = g = 9.8 m/s^2



$S = ut + \frac{1}{2}at^2 \Rightarrow 6 = u(0.2) + 4.9 \times 0.04$

$\Rightarrow$ u = 5.8/0.2 = 29 m/s.

For distance x, u = 0, v = 29 m/s, a = g = 9.8 m/s^2

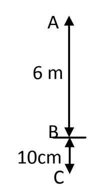
$$S = \frac{v^2 - u^2}{2a} = \frac{29^2 - 0^2}{2 \times 9.8} = 42.05 \text{ m}$$

Total distance = 42.05 + 6 = 48.05 = 48 m.

30. Consider the motion of ball form A to B.

B $\rightarrow$ just above the sand (just to penetrate)

u = 0, a = 9.8 m/s^2, s = 5 m



$S = ut + \frac{1}{2}at^2$

$\Rightarrow$ 5 = 0 + 1/2 (9.8)t^2

$\Rightarrow t^2$ = 5/4.9 = 1.02 $\Rightarrow$ t = 1.01.

$\therefore$ velocity at B, v = u + at = 9.8 × 1.01 (u = 0) =9.89 m/s.

From motion of ball in sand

u_1 = 9.89 m/s, v_1 = 0, a = ?, s = 10 cm = 0.1 m.

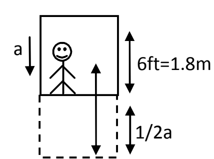
$$a = \frac{v_1^2 - u_1^2}{2s} = \frac{0 - (9.89)^2}{2 \times 0.1} = -490 \text{ m/s}^2$$

The retardation in sand is 490 m/s^2.

31. For elevator and coin u = 0

As the elevator descends downward with acceleration a′ (say)

The coin has to move more distance than 1.8 m to strike the floor. Time taken t = 1 sec.



$S_c = ut + \frac{1}{2}a't^2 = 0 + 1/2\ g(1)^2 = 1/2\ g$

$S_e = ut + \frac{1}{2}at^2 = u + 1/2\ a(1)^2 = 1/2\ a$

Total distance covered by coin is given by = 1.8 + 1/2 a = 1/2 g

$\Rightarrow$ 1.8 +a/2 = 9.8/2 = 4.9

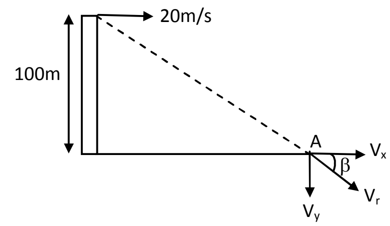
$\Rightarrow$ a = 6.2 m/s^2 = 6.2 × 3.28 = 20.34 ft/s^2.

32. It is a case of projectile fired horizontally from a height.

h = 100 m, g = 9.8 m/s^2

a) Time taken to reach the ground $t = \sqrt{(2h/g)}$

$= \sqrt{\frac{2 \times 100}{9.8}}$ = 4.51 sec.



b) Horizontal range x = ut = 20 × 4.5 = 90 m.

c) Horizontal velocity remains constant through out the motion.

At A, V = 20 m/s

A V_y = u + at = 0 + 9.8 × 4.5 = 44.1 m/s.

Resultant velocity $V_r = \sqrt{(44.1)^2 + 20^2}$ = 48.42 m/s.

Tan $\beta = \frac{V_y}{V_x} = \frac{44.1}{20}$ = 2.205

$\Rightarrow \beta = \tan^{-1}$ (2.205) = 60°.

The ball strikes the ground with a velocity 48.42 m/s at an angle 66° with horizontal.

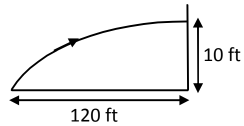
33. u = 40 m/s, a = g= 9.8 m/s^2, θ = 60° Angle of projection.

a) Maximum height $h = \frac{u^2 \sin^2 \theta}{2g} = \frac{40^2 (\sin 60°)^2}{2 \times 10}$ = 60 m

b) Horizontal range X = (u^2 sin 2θ) / g = (40^2 sin 2(60°)) / 10 = $80\sqrt{3}$ m.

34. $g = 9.8\ m/s^2$, $32.2\ ft/s^2$; 40 yd = 120 ft

horizontal range x = 120 ft, u = 64 ft/s, $\theta = 45°$



We know that horizontal range $X = u \cos \theta t$

$$\Rightarrow t = \frac{x}{u\cos\theta} = \frac{120}{64\cos 45°} = 2.65 \text{ sec.}$$

$$y = u \sin \theta(t) - 1/2\, gt^2 = 64\frac{1}{\sqrt{2}(2.65)} - \frac{1}{2}(32.2)(2.65)^2$$

= 7.08 ft which is less than the height of goal post.

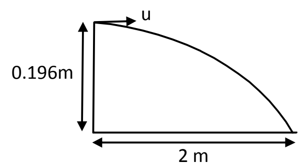
In time 2.65, the ball travels horizontal distance 120 ft (40 yd) and vertical height 7.08 ft which is less than 10 ft. The ball will reach the goal post.

35. The goli move like a projectile.

Here h = 0.196 m

Horizontal distance X = 2 m

Acceleration $g = 9.8\ m/s^2$.



Time to reach the ground i.e.

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 0.196}{9.8}} = 0.2 \text{ sec}$$

Horizontal velocity with which it is projected be u.

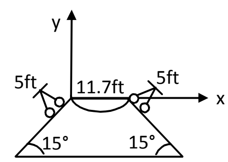
$\therefore$ x = ut

$$\Rightarrow u = \frac{x}{t} = \frac{2}{0.2} = 10 \text{ m/s.}$$

36. Horizontal range X = 11.7 + 5 = 16.7 ft covered by te bike.

$g = 9.8\ m/s^2 = 32.2\ ft/s^2$.

$$y = x \tan \theta - \frac{gx^2 \sec^2\theta}{2u^2}$$



To find, minimum speed for just crossing, the ditch

y = 0 ($\because$ A is on the x axis)

$$\Rightarrow x \tan \theta = \frac{gx^2 \sec^2\theta}{2u^2} \Rightarrow u^2 = \frac{gx^2\sec^2\theta}{2x\tan\theta} = \frac{gx}{2\sin\theta\cos\theta} = \frac{gx}{\sin 2\theta}$$

$$\Rightarrow u = \sqrt{\frac{(32.2)(16.7)}{1/2}} \text{ (because } \sin 30° = 1/2)$$

$\Rightarrow$ u = 32.79 ft/s = 32 ft/s.

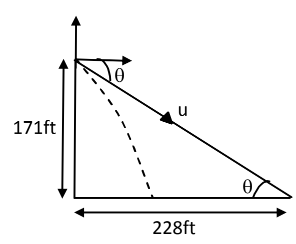
37. $\tan \theta = 171/228 \Rightarrow \theta = \tan^{-1}(171/228)$

The motion of projectile (i.e. the packed) is from A. Taken reference axis at A.

$\therefore$ $\theta = -37°$ as u is below x-axis.

u = 15 ft/s, $g = 32.2\ ft/s^2$, y = –171 ft

$$y = x \tan \theta - \frac{x^2 g \sec^2\theta}{2u^2}$$



$$\therefore -171 = -x\,(0.7536) - \frac{x^2 g(1.568)}{2(225)}$$

$$\Rightarrow 0.1125x^2 + 0.7536\,x - 171 = 0$$

x = 35.78 ft (can be calculated)

Horizontal range covered by the packet is 35.78 ft.

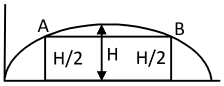
So, the packet will fall 228 – 35.78 = 192 ft short of his friend.

38. Here u = 15 m/s, θ = 60°, g = 9.8 m/s^2

Horizontal range X = $\frac{u^2 \sin 2\theta}{g} = \frac{(15)^2 \sin(2 \times 60°)}{9.8}$ = 19.88 m

In first case the wall is 5 m away from projection point, so it is in the horizontal range of projectile. So the ball will hit the wall. In second case (22 m away) wall is not within the horizontal range. So the ball would not hit the wall.

39. Total of flight T = $\frac{2u \sin\theta}{g}$



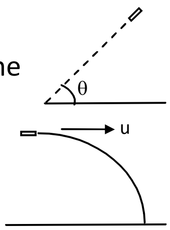
Average velocity = $\frac{\text{change in displacement}}{\text{time}}$

From the figure, it can be said AB is horizontal. So there is no effect of vertical component of the velocity during this displacement.

So because the body moves at a constant speed of 'u cos θ' in horizontal direction.

The average velocity during this displacement will be u cos θ in the horizontal direction.

40. During the motion of bomb its horizontal velocity u remains constant and is same as that of aeroplane at every point of its path. Suppose the bomb explode i.e. reach the ground in time t. Distance travelled in horizontal direction by bomb = ut = the distance travelled by aeroplane. So bomb explode vertically below the aeroplane.

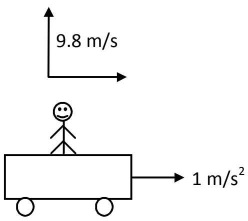


Suppose the aeroplane move making angle θ with horizontal. For both bomb and aeroplane, horizontal distance is u cos θ t. t is time for bomb to reach the ground.

So in this case also, the bomb will explode vertically below aeroplane.

41. Let the velocity of car be u when the ball is thrown. Initial velocity of car is = Horizontal velocity of ball.

Distance travelled by ball B S_b = ut (in horizontal direction)



And by car S_c = ut + 1/2 at^2 where t $\rightarrow$ time of flight of ball in air.

$\therefore$ Car has travelled extra distance $S_c - S_b$ = 1/2 at^2.

Ball can be considered as a projectile having θ = 90°.

$\therefore$ t = $\frac{2u \sin\theta}{g} = \frac{2 \times 9.8}{9.8}$ = 2 sec.

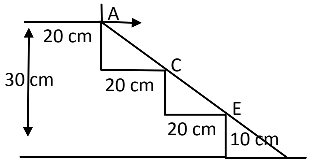
$\therefore$ $S_c - S_b$ = 1/2 at^2 = 2 m

$\therefore$ The ball will drop 2m behind the boy.

42. At minimum velocity it will move just touching point E reaching the ground.

A is origin of reference coordinate.

If u is the minimum speed.



X = 40, Y = –20, θ = 0°

$\therefore$ Y = x tan θ – g $\frac{x^2 \sec^2\theta}{2u^2}$ (because g = 10 m/s^2 = 1000 cm/s^2)

$\Rightarrow$ –20 = x tan θ – $\frac{1000 \times 40^2 \times 1}{2u^2}$

$\Rightarrow$ u = 200 cm/s = 2 m/s.

$\therefore$ The minimum horizontal velocity is 2 m/s.

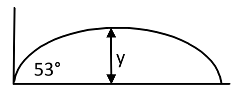
43. a) As seen from the truck the ball moves vertically upward comes back. Time taken = time taken by truck to cover 58.8 m.

$\therefore$ time = $\frac{s}{v} = \frac{58.8}{14.7}$ = 4 sec. (V = 14.7 m/s of truck)

u = ?, v = 0, g = –9.8 m/s^2 (going upward), t = 4/2 = 2 sec.

v = u + at $\Rightarrow$ 0 = u – 9.8 × 2 $\Rightarrow$ u = 19.6 m/s. (vertical upward velocity).

b) From road it seems to be projectile motion.



Total time of flight = 4 sec

In this time horizontal range covered 58.8 m = x

$\therefore$ X = u cos θ t

$\Rightarrow$ u cos θ = 14.7 ...(1)

Taking vertical component of velocity into consideration.

$$y = \frac{0^2 - (19.6)^2}{2 \times (-9.8)} = 19.6 \text{ m [from (a)]}$$

$\therefore$ y = u sin θ t – 1/2 gt^2

$\Rightarrow$ 19.6 = u sin θ (2) – 1/2 $(9.8)2^2$ $\Rightarrow$ 2u sin θ = 19.6 × 2

$\Rightarrow$ u sin θ = 19.6 ...(ii)

$$\frac{u\sin\theta}{u\cos\theta} = \tan\theta \Rightarrow \frac{19.6}{14.7} = 1.333$$

$\Rightarrow \theta = \tan^{-1}(1.333) = 53°$

Again u cos θ = 14.7

$$\Rightarrow u = \frac{14.7}{u\cos 53°} = 24.42 \text{ m/s.}$$

The speed of ball is 42.42 m/s at an angle 53° with horizontal as seen from the road.

44. θ = 53°, so cos 53° = 3/5

$Sec^2\,\theta$ = 25/9 and tan θ = 4/3

Suppose the ball lands on nth bench

So, y = (n – 1)1 ...(1) [ball starting point 1 m above ground]

Again $y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$ [x = 110 + n – 1 = 110 + y]

$$\Rightarrow y = (110 + y)(4/3) - \frac{10(110+y)^2(25/9)}{2\times 35^2}$$

$$\Rightarrow \frac{440}{3} + \frac{4}{3}y - \frac{250(110+y)^2}{18\times 35^2}$$

From the equation, y can be calculated.

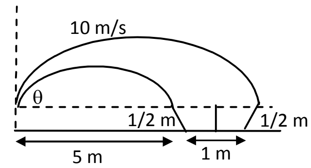
$\therefore$ y = 5

$\Rightarrow$ n – 1 = 5 $\Rightarrow$ n = 6.

The ball will drop in sixth bench.

45. When the apple just touches the end B of the boat.

x = 5 m, u = 10 m/s, g = 10 m/s^2, θ = ?



$x = \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow 5 = \frac{10^2 \sin 2\theta}{10} \Rightarrow 5 = 10 \sin 2\theta$

$\Rightarrow \sin 2\theta = 1/2 \Rightarrow \sin 30° \text{ or } \sin 150°$

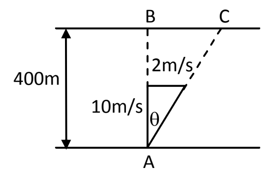
$\Rightarrow \theta = 15° \text{ or } 75°$

Similarly for end C, x = 6 m

Then $2\theta_1 = \sin^{-1}(gx/u^2) = \sin^{-1}(0.6) = 182°$ or 71°.

So, for a successful shot, θ may very from 15° to 18° or 71° to 75°.

46. a) Here the boat moves with the resultant velocity R. But the vertical component 10 m/s takes him to the opposite shore.



Tan θ = 2/10 = 1/5

Velocity = 10 m/s

distance = 400 m

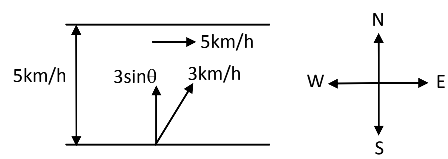
Time = 400/10 = 40 sec.

b) The boat will reach at point C.

In ΔABC, $\tan\theta = \frac{BC}{AB} = \frac{BC}{400} = \frac{1}{5}$

$\Rightarrow$ BC = 400/5 = 80 m.

47. a) The vertical component 3 sin θ takes him to opposite side.



Distance = 0.5 km, velocity = 3 sin θ km/h

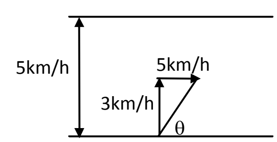
$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{0.5}{3\sin\theta}\text{hr}$

= 10/sinθ min.

b) Here vertical component of velocity i.e. 3 km/hr takes him to opposite side.

$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{0.5}{3} = 0.16 \text{ hr}$

$\therefore$ 0.16 hr = 60 × 0.16 = 9.6 = 10 minute.



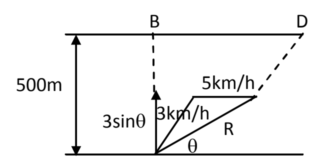
48. Velocity of man $\vec{V}_m$ = 3 km/hr

BD horizontal distance for resultant velocity R.

X-component of resultant $R_x = 5 + 3\cos\theta$

t = 0.5 / 3sinθ

which is same for horizontal component of velocity.



$H = BD = (5 + 3\cos\theta)(0.5 / 3\sin\theta) = \frac{5+3\cos\theta}{6\sin\theta}$

For H to be min $(dH/d\theta) = 0$

$\Rightarrow \frac{d}{d\theta}\left(\frac{5+3\cos\theta}{6\sin\theta}\right) = 0$

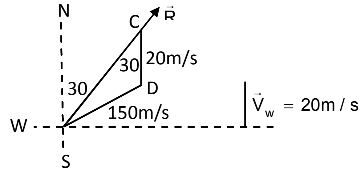
$\Rightarrow -18(\sin^2\theta + \cos^2\theta) - 30\cos\theta = 0$

$\Rightarrow -30\cos\theta = 18 \Rightarrow \cos\theta = -18/30 = -3/5$

Sin θ = $\sqrt{1-\cos^2\theta}$ = 4/5

$\therefore$ H = $\frac{5+3\cos\theta}{6\sin\theta} = \frac{5+3(-3/5)}{6\times(4/5)} = \frac{2}{3}$ km.

49. In resultant direction $\vec{R}$ the plane reach the point B.



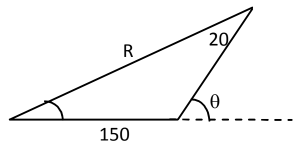
Velocity of wind $\vec{V}_w$ = 20 m/s

Velocity of aeroplane $\vec{V}_a$ = 150 m/s

In ΔACD according to sine formula

$\therefore \frac{20}{\sin A} = \frac{150}{\sin 30°} \Rightarrow \sin A = \frac{20}{150}\sin 30° = \frac{20}{150}\times\frac{1}{2} = \frac{1}{15}$

$\Rightarrow$ A = $\sin^{-1}$ (1/15)



a) The direction is $\sin^{-1}$ (1/15) east of the line AB.

b) $\sin^{-1}$ (1/15) = 3°48′

$\Rightarrow$ 30° + 3°48′ = 33°48′

R = $\sqrt{150^2+20^2+2(150)20\cos 33°48'}$ = 167 m/s.

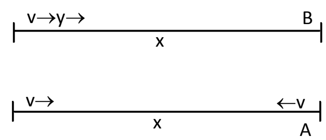
Time = $\frac{s}{v} = \frac{500000}{167}$ = 2994 sec = 49 = 50 min.

50. Velocity of sound v, Velocity of air u, Distance between A and B be x.

In the first case, resultant velocity of sound = v + u

$\Rightarrow$ (v + u) t_1 = x

$\Rightarrow$ v + u = x/t_1 ...(1)



In the second case, resultant velocity of sound = v – u

$\therefore$(v – u) t_2 = x

$\Rightarrow$ v – u = x/t_2 ...(2)

From (1) and (2) 2v = $\frac{x}{t_1}+\frac{x}{t_2} = x\left(\frac{1}{t_1}+\frac{1}{t_2}\right)$

$\Rightarrow$ v = $\frac{x}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right)$

From (i) u = $\frac{x}{t_1} - v = \frac{x}{t_1} - \left(\frac{x}{2t_1}+\frac{x}{2t_2}\right) = \frac{x}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)$

$\therefore$ Velocity of air V = $\frac{x}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right)$

And velocity of wind u = $\frac{x}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)$

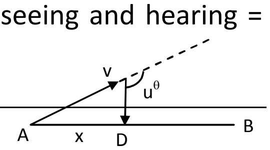
51. Velocity of sound v, velocity of air u

Velocity of sound be in direction AC so it can reach B with resultant velocity AD.

Angle between v and u is θ > π/2.

Resultant $\overrightarrow{AD} = \sqrt{(v^2-u^2)}$

Here time taken by light to reach B is neglected. So time lag between seeing and hearing = time to here the drum sound.



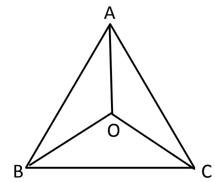
$$t = \frac{\text{Displacement}}{\text{velocity}} = \frac{x}{\sqrt{v^2 - u^2}}$$

$$\Rightarrow \frac{x}{\sqrt{(v+u)(v-u)}} = \frac{x}{\sqrt{(x/t_1)(x/t_2)}}$$ [from question no. 50]

$$= \sqrt{t_1 t_2}\ .$$

52. The particles meet at the centroid O of the triangle. At any instant the particles will form an equilateral ΔABC with the same centroid.

Consider the motion of particle A. At any instant its velocity makes angle 30°. This component is the rate of decrease of the distance AO.



Initially AO = $\frac{2}{3}\sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{3}}$

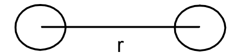
Therefore, the time taken for AO to become zero.

$$= \frac{a/\sqrt{3}}{v\cos 30^\circ} = \frac{2a}{\sqrt{3}v \times \sqrt{3}} = \frac{2a}{3v}\ .$$

* * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 4

1. m = 1 gm = 1/1000 kg



$F = 6.67 \times 10^{-17}$ N $\Rightarrow F = \dfrac{Gm_1m_2}{r^2}$

$\therefore 6.67 \times 20^{-17} = \dfrac{6.67 \times 10^{-11} \times (1/1000) \times (1/1000)}{r^2}$

$\Rightarrow r^2 = \dfrac{6.67 \times 10^{-11} \times 10^{-6}}{6.64 \times 10^{-17}} = \dfrac{10^{-17}}{10^{-17}} = 1$

$\Rightarrow \quad r = \sqrt{1} = 1$ metre.

So, the separation between the particles is 1 m.

2. A man is standing on the surface of earth

The force acting on the man = mg ………(i)

Assuming that, m = mass of the man = 50 kg

And g = acceleration due to gravity on the surface of earth = 10 m/s^2

W = mg = 50× 10= 500 N = force acting on the man

So, the man is also attracting the earth with a force of 500 N

3. The force of attraction between the two charges

$= \dfrac{1}{4\pi\varepsilon_o} \dfrac{q_1 q_2}{r^2} = 9 \times 10^9 \dfrac{1}{r^2}$

The force of attraction is equal to the weight

$Mg = \dfrac{9 \times 10^9}{r^2}$

$\Rightarrow r^2 = \dfrac{9 \times 10^9}{m \times 10} = \dfrac{9 \times 10^8}{m}$ [Taking g=10 m/s^2]

$\Rightarrow r = \sqrt{\dfrac{9 \times 10^8}{m}} = \dfrac{3 \times 10^4}{\sqrt{m}}$ mt

For example, Assuming m= 64 kg,

$r = \dfrac{3 \times 10^4}{\sqrt{64}} = \dfrac{3}{8} 10^4 = 3750$ m

4. mass = 50 kg

r = 20 cm = 0.2 m

$F_G = G\dfrac{m_1m_2}{r^2} = \dfrac{6.67 \times 10^{-11} \times 2500}{0.04}$

Coulomb's force $F_C = \dfrac{1}{4\pi\varepsilon_o} \dfrac{q_1 q_2}{r^2} = 9 \times 10^9 \dfrac{q^2}{0.04}$

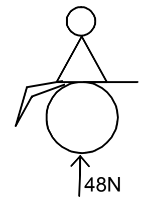
Since, $F_G = F_c = \dfrac{6.7 \times 10^{-11} \times 2500}{0.04} = \dfrac{9 \times 10^9 \times q^2}{0.04}$

$\Rightarrow q^2 = \dfrac{6.7 \times 10^{-11} \times 2500}{0.04} = \dfrac{6.7 \times 10^{-9}}{9 \times 10^9} \times 25$

$= 18.07 \times 10^{-18}$

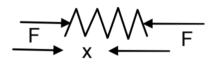
$q = \sqrt{18.07 \times 10^{-18}} = 4.3 \times 10^{-9}$ C.

5. The limb exerts a normal force 48 N and frictional force of 20 N. Resultant magnitude of the force,



$$R = \sqrt{(48)^2 + (20)^2}$$
$$= \sqrt{2304 + 400}$$
$$= \sqrt{2704}$$
$$= 52\text{ N}$$

6. The body builder exerts a force = 150 N.



Compression x = 20 cm = 0.2 m

$\therefore$ Total force exerted by the man = f = kx

$\Rightarrow$ kx = 150

$$\Rightarrow k = \frac{150}{0.2} = \frac{1500}{2} = 750\text{ N/m}$$

7. Suppose the height is h.

At earth station $F = GMm/R^2$

M = mass of earth

m = mass of satellite

R = Radius of earth

$$F = \frac{GMm}{(R+h)^2} = \frac{GMm}{2R^2}$$

$\Rightarrow 2R^2 = (R + h)^2 \Rightarrow R^2 - h^2 - 2Rh = 0$

$\Rightarrow h^2 + 2Rh - R^2 = 0$

$$H = \frac{\left(-2R \pm \sqrt{4R^2 + 4R^2}\right)}{2} = \frac{-2R \pm 2\sqrt{2}R}{2}$$

$= -R \pm \sqrt{2}R = R\left(\sqrt{2} - 1\right)$

$= 6400 \times (0.414)$

$= 2649.6 = 2650$ km

8. Two charged particle placed at a sehortion 2m. exert a force of 20m.

$F_1 = 20$ N. $\quad r_1 = 20$ cm

$F_2 = ?$ $\quad r_2 = 25$ cm

$$\text{Since, } F = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}, \qquad F \propto \frac{1}{r^2}$$

$$\frac{F_1}{F_2} = \frac{{r_2}^2}{{r_1}^2} \Rightarrow F_2 = F_1 \times \left(\frac{r_1}{r_2}\right)^2 = 20 \times \left(\frac{20}{25}\right)^2 = 20 \times \frac{16}{25} = \frac{64}{5} = 12.8\text{ N} = 13\text{ N.}$$

9. The force between the earth and the moon, $F = G\dfrac{m_m m_c}{r^2}$

$$F = \frac{6.67 \times 10^{-11} \times 7.36 \times 10^{22} \times 6 \times 10^{24}}{\left(3.8 \times 10^8\right)^2} = \frac{6.67 \times 7.36 \times 10^{35}}{(3.8)^2 \times 10^{16}}$$

$= 20.3 \times 10^{19} = 2.03 \times 10^{20}$ N $= 2 \times 10^{20}$ N

10. Charge on proton $= 1.6 \times 10^{-19}$

$$\therefore F_{electrical} = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1q_2}{r^2} = \frac{9 \times 10^9 \times (1.6)^2 \times 10^{-38}}{r^2}$$

mass of proton $= 1.732 \times 10^{-27}$ kg

$$F_{gravity} = G\frac{m_1m_2}{r^2} = \frac{6.67\times10^{-11}\times(1.732)\times10^{-54}}{r^2}$$

$$\frac{F_e}{F_g}=\frac{\frac{9\times10^9\times(1.6)^2\times10^{-38}}{r^2}}{\frac{6.67\times10^{-11}\times(1.732)\times10^{-54}}{r^2}} = \frac{9\times(1.6)^2\times10^{-29}}{6.67\ (1.732)^2\ 10^{-65}} = 1.24\times10^{36}$$

11. The average separation between proton and electron of Hydrogen atom is r= 5.3 10^{-11}m.

a) Coulomb's force = $F = 9\times10^9\times\frac{q_1q_2}{r^2} = \frac{9\times10^9\times(1.0\times10^{-19})^2}{(5.3\times10^{-11})^2} = 8.2\times10^{-8}$ N.

b) When the average distance between proton and electron becomes 4 times that of its ground state

Coulomb's force $F = \frac{1}{4\pi\varepsilon_o}\times\frac{q_1q_2}{(4r)^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{16\times(5.3)^2\times10^{-22}} = \frac{9\times(1.6)^2}{16\times(5.3)^2}\times10^{-7}$

$= 0.0512\times10^{-7} = 5.1\times10^{-9}$ N.

12. The geostationary orbit of earth is at a distance of about 36000km.

We know that, $g' = GM / (R+h)^2$

At h = 36000 km. $g' = GM / (36000+6400)^2$

$\therefore \frac{g`}{g}=\frac{6400\times6400}{42400\times42400}=\frac{256}{106\times106}=0.0227$

$\Rightarrow g' = 0.0227\times9.8 = 0.223$

[taking g = 9.8 m/s^2 at the surface of the earth]

A 120 kg equipment placed in a geostationary satellite will have weight

Mg` = 0.233 × 120 = 26.79 = 27 N

* * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 5

1. m = 2kg
S = 10m
Let, acceleration = a, Initial velocity u = 0.
S= ut + 1/2 at^2
$\Rightarrow 10 = ½ a (2^2) \Rightarrow 10 = 2a \Rightarrow a = 5\ m/s^2$
Force: F = ma = 2 × 5 = 10N (Ans)

2. u = 40 km/hr = $\frac{40000}{3600}$ = 11.11 m/s.
m = 2000 kg ; v = 0 ; s = 4m
acceleration 'a' = $\frac{v^2-u^2}{2s} = \frac{0^2-(11.11)^2}{2\times4} = -\frac{123.43}{8} = -15.42\ m/s^2$ (deceleration)
So, braking force = F = ma = 2000 × 15.42 = 30840 = 3.08 10^4 N (Ans)

3. Initial velocity u = 0 (negligible)
v = 5 × 10^6 m/s.
s = 1cm = 1 × 10^{-2}m.
acceleration a = $\frac{v^2-u^2}{2s} = \frac{(5\times10^6)^2-0}{2\times1\times10^{-2}} = \frac{25\times10^{12}}{2\times10^{-2}} = 12.5\times10^{14}ms^{-2}$
F = ma = 9.1 × 10^{-31} × 12.5 × 10^{14} = 113.75 × 10^{-17} = 1.1 × 10^{-15} N.

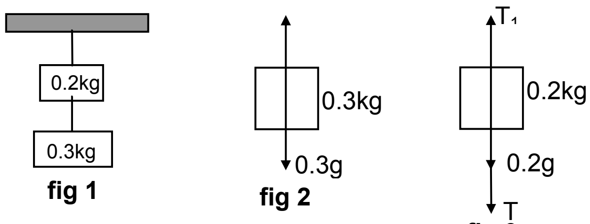
4.


fig 1 fig 2 fig 3

g = 10m/s^2 T – 0.3g = 0 $\Rightarrow$ T = 0.3g = 0.3 × 10 = 3 N
T_1 – (0.2g + T) =0 $\Rightarrow$ T_1 = 0.2g + T = 0.2 × 10 + 3 = 5N
$\therefore$ Tension in the two strings are 5N & 3N respectively.

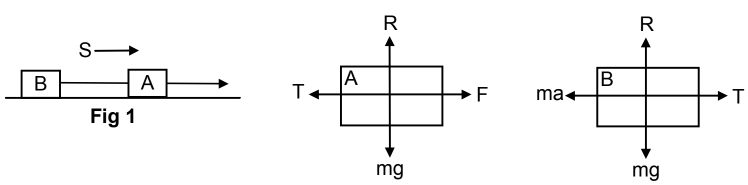
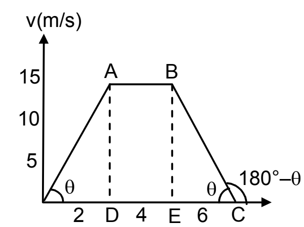
5.


Fig 1 Fig 2 Fig 3

T + ma – F = 0 T – ma = 0 $\Rightarrow$ T = ma ....(i)
$\Rightarrow$ F= T + ma $\Rightarrow$ F= T + T from (i)
$\Rightarrow$ 2T = F $\Rightarrow$ T = F / 2

6. m = 50g = 5 × 10^{-2} kg
As shown in the figure,
Slope of OA = Tanθ $\frac{AD}{OD} = \frac{15}{3}$ = 5 m/s^2
So, at t = 2sec acceleration is 5m/s^2
Force = ma = 5 × 10^{-2} × 5 = 0.25N along the motion



At t = 4 sec slope of AB = 0, acceleration = 0 [tan 0° = 0]

$\therefore$ Force = 0

At t = 6 sec, acceleration = slope of BC.

In $\Delta BEC = \tan\theta = \frac{BE}{EC} = \frac{15}{3} = 5.$

Slope of BC = tan (180° – θ) = – tan θ = –5 m/s^2 (deceleration)

Force = ma = 5 × 10^{-2} 5 = 0.25 N. Opposite to the motion.

7. Let, F $\rightarrow$ contact force between m_A & m_B.
And, f $\rightarrow$ force exerted by experimenter.

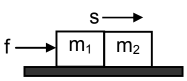


Fig 1

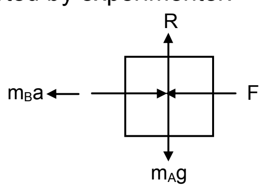


Fig 2

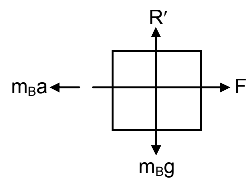


Fig 3

F + m_A a –f = 0 $\qquad$ m_B a –f =0

$\Rightarrow$ F = f – m_A a ……….(i) $\qquad$ $\Rightarrow$ F= m_B a ……...(ii)

From eqn (i) and eqn (ii)

$\Rightarrow$ f – m_A a = m_B a $\Rightarrow$ f = m_B a + m_A a $\Rightarrow$ f = a (m_A + m_B).

$\Rightarrow f = \frac{F}{m_B}(m_B + m_A) = F\left(1+\frac{m_A}{m_B}\right)$ [because a = F/m_B]

$\therefore$ The force exerted by the experimenter is $F\left(1+\frac{m_A}{m_B}\right)$

8. r = 1mm = 10^{-3}
'm' = 4mg = 4 × 10^{-6}kg
s = 10^{-3}m.
v = 0
u = 30 m/s.

So, $a = \frac{v^2-u^2}{2s} = \frac{-30\times30}{2\times10^{-3}}$ = –4.5 × 10^5 m/s^2 (decelerating)

Taking magnitude only deceleration is 4.5 × 10^5 m/s^2

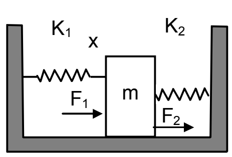
So, force F = 4 × 10^{-6} × 4.5 × 10^5 = 1.8 N

9. x = 20 cm = 0.2m, k = 15 N/m, m = 0.3kg.

Acceleration $a = \frac{F}{m} = \frac{-kx}{x} = \frac{-15(0.2)}{0.3} = -\frac{3}{0.3}$ = –10m/s^2 (deceleration)

So, the acceleration is 10 m/s^2 opposite to the direction of motion

10. Let, the block m towards left through displacement x.



$F_1 = k_1 x$ (compressed)

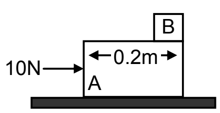
$F_2 = k_2 x$ (expanded)

They are in same direction.

Resultant F = $F_1 + F_2 \Rightarrow F = k_1 x + k_2 x \Rightarrow F = x(k_1 + k_2)$

So, a = acceleration = $\frac{F}{m} = \frac{x(k_1+k_2)}{m}$ opposite to the displacement.

11. m = 5 kg of block A.



ma = 10 N

$\Rightarrow$ a 10/5 = 2 m/s^2.

As there is no friction between A & B, when the block A moves, Block B remains at rest in its position.

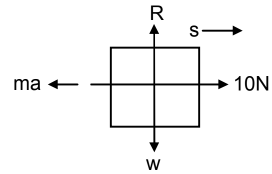
Initial velocity of A = u = 0.

Distance to cover so that B separate out s = 0.2 m.

Acceleration a = 2 m/s^2

$\therefore$ s= ut + ½ at^2

$\Rightarrow 0.2 = 0 + (½) \times 2 \times t^2 \Rightarrow t^2 = 0.2 \Rightarrow t = 0.44$ sec $\Rightarrow$ t= 0.45 sec.



12. a) at any depth let the ropes make angle θ with the vertical

From the free body diagram

$F \cos\theta + F\cos\theta - mg = 0$

$\Rightarrow 2F\cos\theta = mg \Rightarrow F = \dfrac{mg}{2\cos\theta}$

As the man moves up. θ increases i.e. cos θ decreases. Thus F increases.

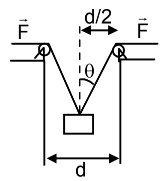


Fig-1

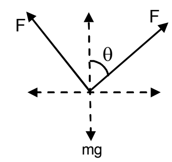
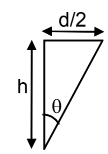


Fig-2

b) When the man is at depth h

$$\cos\theta = \frac{h}{\sqrt{(d/2)^2 + h^2}}$$

$$\text{Force} = \frac{mg}{\dfrac{h}{\sqrt{\dfrac{d^2}{4} + h^2}}} = \frac{mg}{4h}\sqrt{d^2 + 4h^2}$$



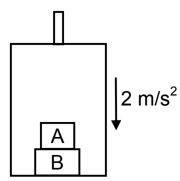
13. From the free body diagram

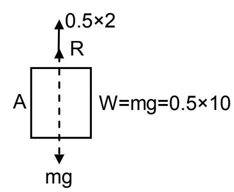
$\therefore R + 0.5 \times 2 - w = 0$

$\Rightarrow R = w - 0.5 \times 2$

$= 0.5\,(10 - 2) = 4N$.

So, the force exerted by the block A on the block B, is 4N.





14. a) The tension in the string is found out for the different conditions from the free body diagram as shown below.

$T - (W + 0.06 \times 1.2) = 0$

$\Rightarrow T = \;\; 0.05 \times 9.8 + 0.05 \times 1.2$

$= 0.55$ N.

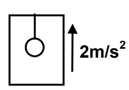


Fig-1

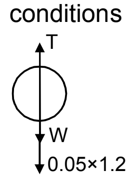


Fig-2

b) $\therefore T + 0.05 \times 1.2 - 0.05 \times 9.8 = 0$

$\Rightarrow T = \;\; 0.05 \times 9.8 - 0.05 \times 1.2$

$= 0.43$ N.

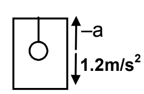


Fig-3

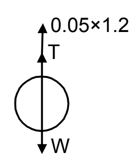


Fig-4

c) When the elevator makes uniform motion

$T - W = 0$

$\Rightarrow T = \;\; W = 0.05 \times 9.8$

$= 0.49$ N

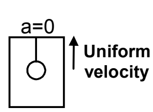


Fig-5

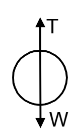


Fig-6

d) $T + 0.05 \times 1.2 - W = 0$

$\Rightarrow T = \;\; W - 0.05 \times 1.2$

$= 0.43$ N.

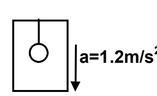


Fig-7

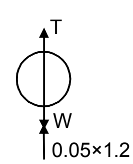


Fig-8

e) $T - (W + 0.05 \times 1.2) = 0$

$\Rightarrow T = \;\; W + 0.05 \times 1.2$

$= 0.55$ N

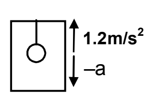


Fig-9

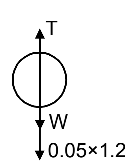


Fig-10

f) When the elevator goes down with uniform velocity acceleration = 0

$T - W = 0$

$\Rightarrow T = W = 0.05 \times 9.8$

$= 0.49$ N.

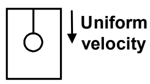


Fig-11

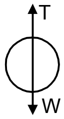


Fig-12

15. When the elevator is accelerating upwards, maximum weight will be recorded.

$R - (W + ma) = 0$

$\Rightarrow R = W + ma = m(g + a)$ max.wt.

When decelerating upwards, maximum weight will be recorded.

$R + ma - W = 0$

$\Rightarrow R = W - ma = m(g - a)$

So, $m(g + a) = 72 \times 9.9$ …(1)

$m(g - a) = 60 \times 9.9$ …(2)

Now, $mg + ma = 72 \times 9.9 \Rightarrow mg - ma = 60 \times 9.9$

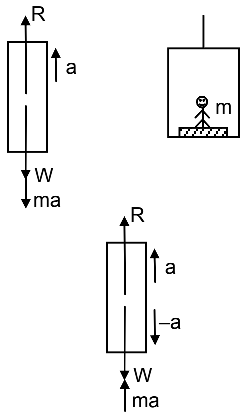
$\Rightarrow 2mg = 1306.8$

$$\Rightarrow m = \frac{1306.8}{2 \times 9.9} = 66 \text{ Kg}$$

So, the true weight of the man is 66 kg.

Again, to find the acceleration, $mg + ma = 72 \times 9.9$

$$\Rightarrow a = \frac{72 \times 9.9 - 66 \times 9.9}{66} = \frac{9.9}{11} = 0.9 \text{ m/s}^2.$$



16. Let the acceleration of the 3 kg mass relative to the elevator is 'a' in the downward direction.

As, shown in the free body diagram

$T - 1.5\,g - 1.5(g/10) - 1.5\,a = 0$ from figure (1)

and, $T - 3g - 3(g/10) + 3a = 0$ from figure (2)

$\Rightarrow T = 1.5\,g + 1.5(g/10) + 1.5a$ … (i)

And $T = 3g + 3(g/10) - 3a$ … (ii)

Equation (i) × 2 $\Rightarrow 3g + 3(g/10) + 3a = 2T$

Equation (ii) × 1 $\Rightarrow 3g + 3(g/10) - 3a = T$

Subtracting the above two equations we get, $T = 6a$

Subtracting $T = 6a$ in equation (ii)

$6a = 3g + 3(g/10) - 3a.$

$$\Rightarrow 9a = \frac{33g}{10} \Rightarrow a = \frac{(9.8)33}{10} = 32.34$$

$\Rightarrow a = 3.59 \;\therefore\; T = 6a = 6 \times 3.59 = 21.55$

$T^1 = 2T = 2 \times 21.55 = 43.1$ N cut is T_1 shown in spring.

$$\text{Mass} = \frac{wt}{g} = \frac{43.1}{9.8} = 4.39 = 4.4 \text{ kg}$$

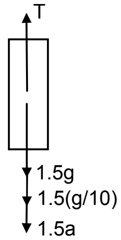


Fig-1

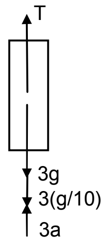


Fig-2

17. Given, m = 2 kg, k = 100 N/m

From the free body diagram, $kl - 2g = 0 \Rightarrow kl = 2g$

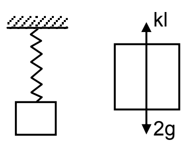
$$\Rightarrow l = \frac{2g}{k} = \frac{2 \times 9.8}{100} = \frac{19.6}{100} = 0.196 = 0.2 \text{ m}$$

Suppose further elongation when 1 kg block is added be x,

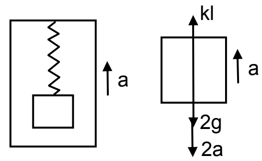
Then $k(1 + x) = 3g$

$\Rightarrow kx = 3g - 2g = g = 9.8$ N

$$\Rightarrow x = \frac{9.8}{100} = 0.098 = 0.1 \text{ m}$$



18. $a = 2\ m/s^2$



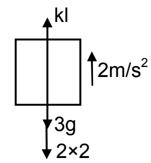
$kl - (2g + 2a) = 0$

$\Rightarrow kl = 2g + 2a$

$= 2 \times 9.8 + 2 \times 2 = 19.6 + 4$

$\Rightarrow l = \dfrac{23.6}{100} = 0.236\ m = 0.24\ m$

When 1 kg body is added total mass (2 + 1)kg = 3kg.



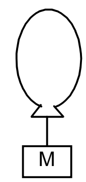
elongation be l_1

$kl_1 = 3g + 3a = 3 \times 9.8 + 6$

$\Rightarrow l_1 = \dfrac{33.4}{100} = 0.0334 = 0.36$

Further elongation $= l_1 - l = 0.36 - 0.12$ m.

19. Let, the air resistance force is F and Buoyant force is B.



Given that

$F_a \propto v$, where $v \rightarrow$ velocity

$\Rightarrow F_a = kv$, where $k \rightarrow$ proportionality constant.

When the balloon is moving downward,

$B + kv = mg$ ...(i)

$\Rightarrow M = \dfrac{B + kv}{g}$

For the balloon to rise with a constant velocity v, (upward)

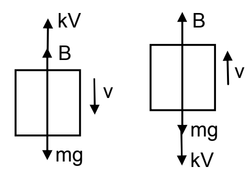


Fig-1 Fig-2

let the mass be m

Here, $B - (mg + kv) = 0$ ...(ii)

$\Rightarrow B = mg + kv$

$\Rightarrow m = \dfrac{B - kw}{g}$

So, amount of mass that should be removed = M – m.

$= \dfrac{B + kv}{g} - \dfrac{B - kv}{g} = \dfrac{B + kv - B + kv}{g} = \dfrac{2kv}{g} = \dfrac{2(Mg - B)}{G} = 2\{M - (B/g)\}$

20. When the box is accelerating upward,

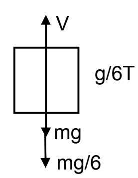


Fig-1

$U - mg - m(g/6) = 0$

$\Rightarrow U = mg + mg/6 = m\{g + (g/6)\} = 7\ mg/7$ ...(i)

$\Rightarrow m = 6U/7g.$

When it is accelerating downward, let the required mass be M.

$U - Mg + Mg/6 = 0$

$\Rightarrow U = \dfrac{6Mg - Mg}{6} = \dfrac{5Mg}{6} \Rightarrow M = \dfrac{6U}{5g}$

Mass to be added $= M - m = \dfrac{6U}{5g} - \dfrac{6U}{7g} = \dfrac{6U}{g}\left(\dfrac{1}{5} - \dfrac{1}{7}\right)$

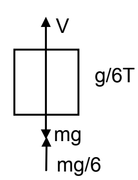


Fig-2

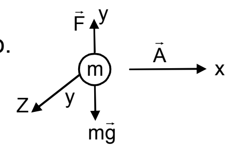
$= \dfrac{6U}{g}\left(\dfrac{2}{35}\right) = \dfrac{12}{35}\left(\dfrac{U}{g}\right)$

$= \dfrac{12}{35}\left(\dfrac{7mg}{6} \times \dfrac{1}{g}\right)$ from (i)

= 2/5 m.

$\therefore$ The mass to be added is 2m/5.

21. Given that, $\vec{F} = \vec{u} \times \vec{A}$ and $\overrightarrow{mg}$ act on the particle.



For the particle to move undeflected with constant velocity, net force should be zero.

$\therefore\ (\vec{u} \times \vec{A}) + \overrightarrow{mg} = 0$

$\therefore\ (\vec{u} \times \vec{A}) = -\overrightarrow{mg}$

Because, $(\vec{u} \times \vec{A})$ is perpendicular to the plane containing $\vec{u}$ and $\vec{A}$, $\vec{u}$ should be in the xz-plane.

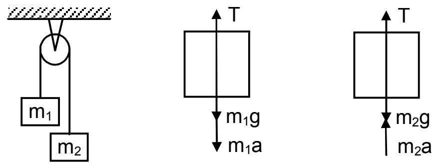
Again, u A sin θ = mg

$\therefore u = \dfrac{mg}{A \sin\theta}$

u will be minimum, when sin $\theta = 1 \Rightarrow \theta = 90°$

$\therefore u_{min} = \dfrac{mg}{A}$ along Z-axis.

22.



$m_1 = 0.3$ kg, $m_2 = 0.6$ kg

$T - (m_1g + m_1a) = 0$ …(i) $\Rightarrow T = m_1g + m_1a$

$T + m_2a - m_2g = 0$ …(ii) $\Rightarrow T = m_2g - m_2a$

From equation (i) and equation (ii)

$m_1g + m_1a + m_2a - m_2g = 0$, from (i)

$\Rightarrow a(m_1 + m_2) = g(m_2 - m_1)$

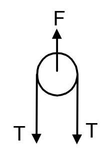
$\Rightarrow a = f\left(\dfrac{m_2 - m_1}{m_1 + m_2}\right) = 9.8\left(\dfrac{0.6 - 0.3}{0.6 + 0.3}\right) = 3.266\ ms^{-2}$.

a) t = 2 sec acceleration = 3.266 ms^{-2}

Initial velocity u = 0

So, distance travelled by the body is,

$S = ut + 1/2\ at^2 \Rightarrow 0 + ½(3.266)\ 2^2 = 6.5$ m



b) From (i) $T = m_1(g + a) = 0.3\ (9.8 + 3.26) = 3.9$ N

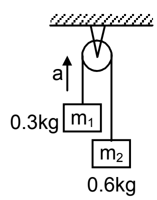
c) The force exerted by the clamp on the pully is given by

F – 2T = 0

F = 2T = 2 × 3.9 = 7.8 N.

23. a = 3.26 m/s^2

T = 3.9 N



After 2 sec mass m_1 the velocity

V = u + at = 0 + 3.26 × 2 = 6.52 m/s upward.

At this time m_2 is moving 6.52 m/s downward.

At time 2 sec, m_2 stops for a moment. But m_1 is moving upward with velocity 6.52 m/s.

It will continue to move till final velocity (at highest point) because zero.

Here, v = 0 ; u = 6.52

A = –g = –9.8 m/s^2 [moving up ward m_1]

$V = u + at \Rightarrow 0 = 6.52 + (-9.8)t$

$\Rightarrow t = 6.52/9.8 = 0.66 = 2/3$ sec.

During this period 2/3 sec, m_2 mass also starts moving downward. So the string becomes tight again after a time of 2/3 sec.

24. Mass per unit length 3/30 kg/cm = 0.10 kg/cm.

Mass of 10 cm part = m_1 = 1 kg

Mass of 20 cm part = m_2 = 2 kg.

Let, F = contact force between them.

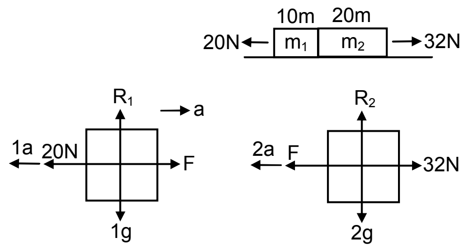
From the free body diagram

$F - 20 - 10 = 0$ …(i)

And, $32 - F - 2a = 0$ …(ii)

From eqa (i) and (ii) $3a - 12 = 0 \Rightarrow a = 12/3 = 4\ m/s^2$

Contact force F = 20 + 1a = 20 + 1 × 4 = 24 N.



25.

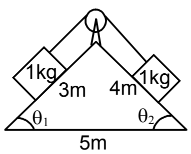


Fig-1

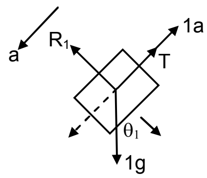


Fig-2

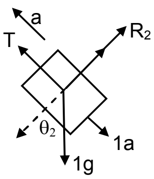


Fig-3

$\sin\theta_1 = 4/5$

$\sin\theta_2 = 3/5$

$g\sin\theta_1 - (a + T) = 0$

$\Rightarrow g\ \text{sing}\ \theta_1 = a + T$ …(i)

$\Rightarrow T + a - g\sin\theta_1 = 0$

$T - g\sin\theta_2 - a = 0$

$\Rightarrow T = g\sin\theta_2 + a$ …(ii)

From eqn (i) and (ii), $g\sin\theta_2 + a + a - g\sin\theta_1 = 0$

$$\Rightarrow 2a = g\sin\theta_1 - g\sin\theta_2 = g\left(\frac{4}{5} - \frac{3}{5}\right) = g/5$$

$$\Rightarrow a = \frac{g}{5} \times \frac{1}{2} = \frac{g}{10}$$

26.

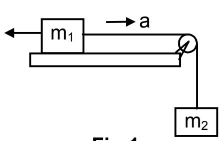


Fig-1

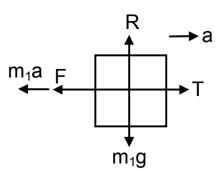


Fig-2

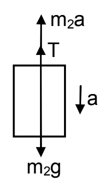


Fig-3

From the above Free body diagram

$M_1a + F - T = 0 \Rightarrow T = m_1a + F$ …(i)

From the above Free body diagram

$m_2a + T - m_2g = 0$ ….(ii)

$\Rightarrow m_2a + m_1a + F - m_2g = 0$ (from (i))

$\Rightarrow a(m_1 + m_2) + m_2g/2 - m_2g = 0$ {because $f = m^2g/2$}

$\Rightarrow a(m_1 + m_2) - m_2g = 0$

$$\Rightarrow a(m_1 + m_2) = m_2g/2 \Rightarrow a = \frac{m_2g}{2(m_1 = m_2)}$$

Acceleration of mass m_1 is $\dfrac{m_2g}{2(m_1 = m_2)}$ towards right.

27.

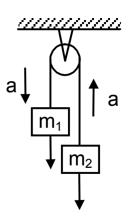


Fig-1

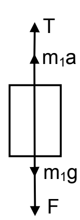


Fig-2

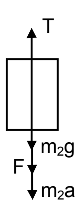


Fig-3

From the above free body diagram

$T + m_1a - m(m_1g + F) = 0$

From the free body diagram

$T - (m_2g + F + m_2a) = 0$

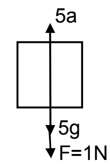
$\Rightarrow T = m_1g + F - m_1a \Rightarrow T = 5g + 1 - 5a$ …(i)

$\Rightarrow T = m_2g + F + m_2a \Rightarrow T = 2g + 1 + 2a$ …(ii)

From the eqn (i) and eqn (ii)

$5g + 1 - 5a = 2g + 1 + 2a \Rightarrow 3g - 7a = 0 \Rightarrow 7a = 3g$

$\Rightarrow a = \dfrac{3g}{7} = \dfrac{29.4}{7} = 4.2\ m/s^2$ [$g = 9.8 m/s^2$]



a) acceleration of block is $4.2\ m/s^2$

b) After the string breaks m_1 move downward with force F acting down ward.

$m_1a = F + m_1g = (1 + 5g) = 5(g + 0.2)$ Force = 1N, acceleration = 1/5= 0.2m/s.

So, acceleration $= \dfrac{\text{Force}}{\text{mass}} = \dfrac{5(g+0.2)}{5} = (g + 0.2)\ m/s^2$

28.

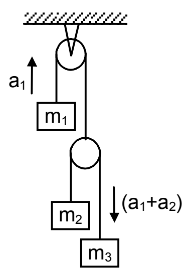


Fig-1

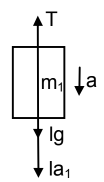


Fig-2

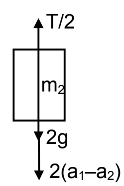


Fig-3

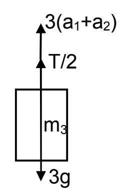


Fig-4

Let the block m+1+ moves upward with acceleration a, and the two blocks m_2 an m_3 have relative acceleration a_2 due to the difference of weight between them. So, the actual acceleration at the blocks m_1, m_2 and m_3 will be a_1.

$(a_1 - a_2)$ and $(a_1 + a_2)$ as shown

$T = 1g - 1a_2 = 0$ ...(i) from fig (2)

$T/2 - 2g - 2(a_1 - a_2) = 0$ ...(ii) from fig (3)

$T/2 - 3g - 3(a_1 + a_2) = 0$ ...(iii) from fig (4)

From eqn (i) and eqn (ii), eliminating T we get, $1g + 1a_2 = 4g + 4(a_1 + a_2) \Rightarrow 5a_2 - 4a_1 = 3g$ (iv)

From eqn (ii) and eqn (iii), we get $2g + 2(a_1 - a_2) = 3g - 3(a_1 - a_2) \Rightarrow 5a_1 + a_2 =$ (v)

Solving (iv) and (v) $a_1 = \dfrac{2g}{29}$ and $a_2 = g - 5a_1 = g - \dfrac{10g}{29} = \dfrac{19g}{29}$

So, $a_1 - a_2 = \dfrac{2g}{29} - \dfrac{19g}{29} = -\dfrac{17g}{29}$

$a_1 + a_2 = \dfrac{2g}{29} + \dfrac{19g}{29} = \dfrac{21g}{29}$ So, acceleration of m_1, m_2, m_3 ae $\dfrac{19g}{29}$ (up) $\dfrac{17g}{29}$ (doan) $\dfrac{21g}{29}$ (down) respectively.

Again, for m_1, u = 0, s= 20cm=0.2m and $a_2 = \dfrac{19}{29}g$ [$g = 10m/s^2$]

$\therefore S = ut + ½ at^2 = 0.2 = \dfrac{1}{2} \times \dfrac{19}{29} gt^2 \Rightarrow t = 0.25sec.$

29.

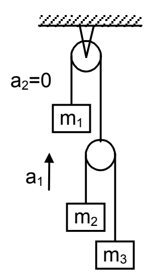


Fig-1

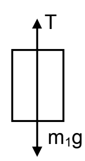


Fig-2

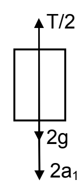


Fig-3

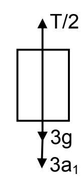


Fig-4

m_1 should be at rest.

$T - m_1g = 0$ $T/2 - 2g - 2a_1 = 0$ $T/2 - 3g - 3a_1 = 0$

$\Rightarrow T = m_1g$ …(i) $\Rightarrow T - 4g - 4a_1 = 0$ …(ii) $\Rightarrow T = 6g - 6a_1$ …(iii)

From eqn (ii) & (iii) we get

$3T - 12g = 12g - 2T \Rightarrow T = 24g/5 = 408g.$

Putting yhe value of T eqn (i) we get, $m_1 = 4.8$kg.

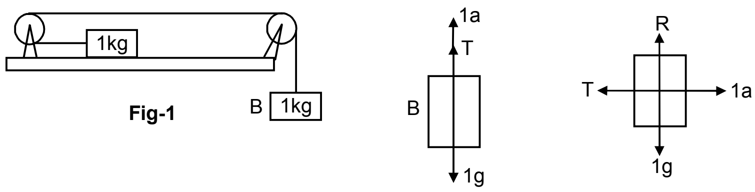
30. 

Fig-1 Fig-2 Fig-3

$T + 1a = 1g$ …(i) $T - 1a = 0 \Rightarrow T = 1a$ (ii)

From eqn (i) and (ii), we get

$1a + 1a = 1g \Rightarrow 2a = g \Rightarrow a = \frac{g}{2} = \frac{10}{2} = 5\text{m/s}^2$

From (ii) $T = 1a = 5$N.

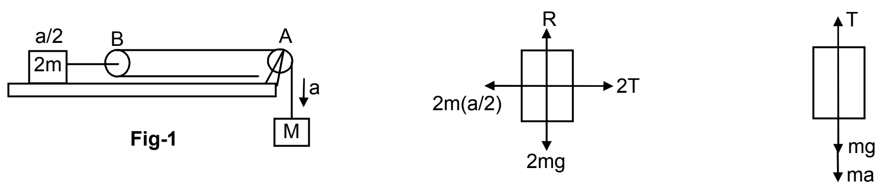
31. 

Fig-1 Fig-2 Fig-3

$Ma - 2T = 0$ $T + Ma - Mg = 0$

$\Rightarrow Ma = 2T \Rightarrow T = Ma/2.$ $\Rightarrow Ma/2 + ma = Mg.$ (because $T = Ma/2$)

$\Rightarrow 3\,Ma = 2\,Mg \Rightarrow a = 2g/3$

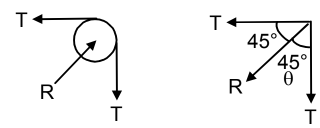
a) acceleration of mass M is 2g/3.

b) Tension $T = \frac{Ma}{2} = \frac{M}{2} = \frac{2g}{3} = \frac{Mg}{3}$

c) Let, R^1 = resultant of tensions = force exerted by the clamp on the pulley

$R^1 = \sqrt{T^2 + T^2} = \sqrt{2}T$

$\therefore R = \sqrt{2}T = \sqrt{2}\frac{Mg}{3} = \frac{\sqrt{2}Mg}{3}$



Again, $\text{Tan}\theta = \frac{T}{T} = 1 \Rightarrow \theta = 45°.$

So, it is $\frac{\sqrt{2}Mg}{3}$ at an angle of 45° with horizontal.

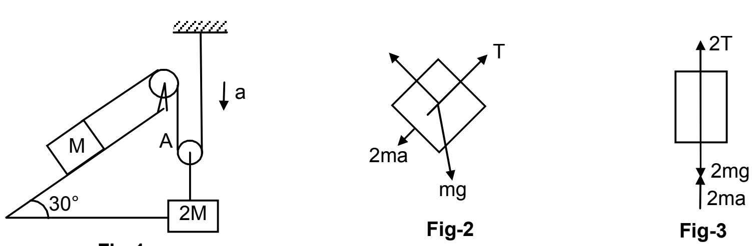
32. 

Fig-1 Fig-2 Fig-3

$2Ma + Mg\sin\theta - T = 0$

$\Rightarrow T = 2Ma + Mg\sin\theta$ …(i)

$2T + 2Ma - 2Mg = 0$

$\Rightarrow 2(2Ma + Mg\sin\theta) + 2Ma - 2Mg = 0$ [From (i)]

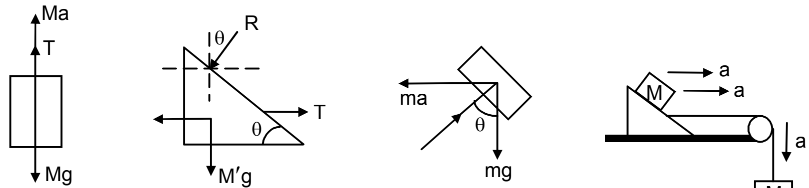
$\Rightarrow 4Ma + 2Mg\sin\theta + 2Ma - 2Mg = 0$

$\Rightarrow 6Ma + 2Mg\sin 30° - 2Mg = 0$

$\Rightarrow 6Ma = Mg \Rightarrow a = g/6$.

Acceleration of mass M is 2a = s × g/6 = g/3 up the plane.

33.



FBD-1 **FBD-2** **FBD-3** **FBD-4**

As the block 'm' does not slinover M′, ct will have same acceleration as that of M′

From the freebody diagrams.

$T + Ma - Mg = 0$ ...(i) (From FBD – 1)

$T - M'a - R\sin\theta = 0$ ...(ii) (From FBD -2)

$R\sin\theta - ma = 0$ ...(iii) (From FBD -3)

$R\cos\theta - mg = 0$ ...(iv) (From FBD -4)

Eliminating T, R and a from the above equation, we get $M = \dfrac{M' + m}{\cot\theta - 1}$

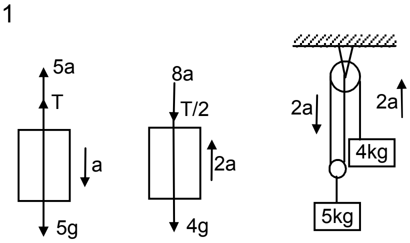
34. a) $5a + T - 5g = 0 \Rightarrow T = 5g - 5a$ ...(i) (From FBD-1)

Again $(1/2) - 4g - 8a = 0 \Rightarrow T = 8g - 16a$ ...(ii) (from FBD-2)

From equn (i) and (ii), we get

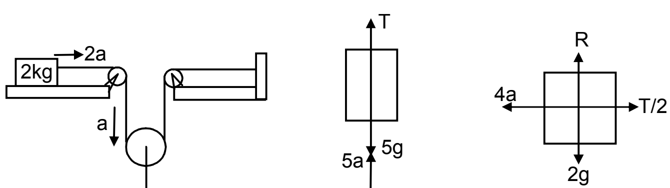
$5g - 5a = 8g + 16a \Rightarrow 21a = -3g \Rightarrow a = -1/7g$

So, acceleration of 5 kg mass is g/7 upward and that of 4 kg mass is 2a = 2g/7 (downward).



FBD-1 **FBD-2** **FBD-3**

b)



FBD-3 **FBD-4**

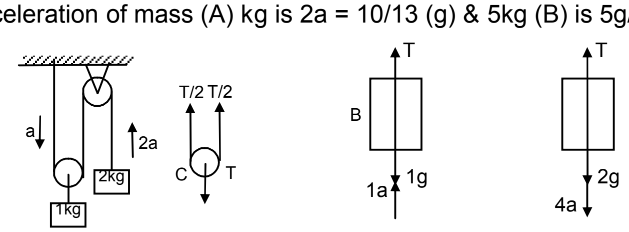
$4a - t/2 = 0 \Rightarrow 8a - T = 0 \Rightarrow T = 8a$ … (ii) [From FBD -4]

Again, $T + 5a - 5g = 0 \Rightarrow 8a + 5a - 5g = 0$

$\Rightarrow 13a - 5g = 0 \Rightarrow a = 5g/13$ downward. (from FBD -3)

Acceleration of mass (A) kg is 2a = 10/13 (g) & 5kg (B) is 5g/13.

c)



FBD-5 **FBD-6**

$T + 1a - 1g = 0 \Rightarrow T = 1g - 1a$ …(i) [From FBD – 5]

Again, $\dfrac{T}{2} - 2g - 4a = 0 \Rightarrow T - 4g - 8a = 0$ …(ii) [From FBD -6]

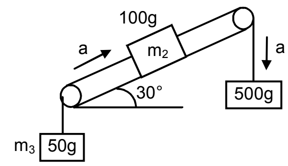
$\Rightarrow 1g - 1a - 4g - 8a = 0$ [From (i)]

$\Rightarrow a = -(g/3)$ downward.

Acceleration of mass 1kg(b) is g/3 (up)

Acceleration of mass 2kg(A) is 2g/3 (downward).

35. $m_1 = 100g = 0.1kg$



$m_2 = 500g = 0.5kg$

$m_3 = 50g = 0.05kg$.

$T + 0.5a - 0.5g = 0$ ...(i)

$T_1 - 0.5a - 0.05g = a$ ...(ii)

$T_1 + 0.1a - T + 0.05g = 0$ ...(iii)

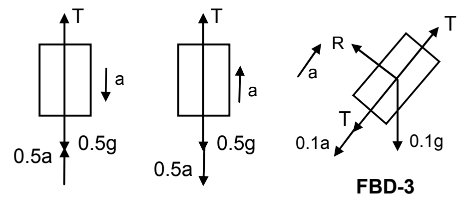
From equn (ii) $T_1 = 0.05g + 0.05a$ ...(iv)

From equn (i) $T_1 = 0.5g - 0.5a$ ...(v)

Equn (iii) becomes $T_1 + 0.1a - T + 0.05g = 0$

$\Rightarrow 0.05g + 0.05a + 0.1a - 0.5g + 0.5a + 0.05g = 0$ [From (iv) and (v)]

$\Rightarrow 0.65a = 0.4g \Rightarrow a = \frac{0.4}{0.65} = \frac{40}{65}g = \frac{8}{13}g$ downward

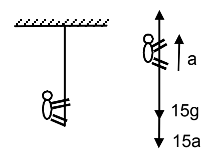


FBD-1 **FBD-2** **FBD-3**

Acceleration of 500gm block is 8g/13g downward.

36. m = 15 kg of monkey. $a = 1\ m/s^2$.

From the free body diagram



$\therefore T - [15g + 15(1)] = 0 \Rightarrow T = 15\ (10 + 1) \Rightarrow T = 15 \times 11 \Rightarrow T = 165$ N.

The monkey should apply 165N force to the rope.

Initial velocity u = 0 ; acceleration $a = 1m/s^2$; s = 5m.

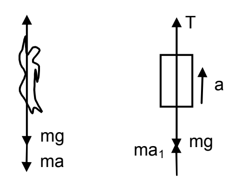
$\therefore s = ut + ½ at^2$

$5 = 0 + (1/2)1\ t^2 \quad \Rightarrow t^2 = 5 \times 2 \quad \Rightarrow t = \sqrt{10}$ sec.

Time required is $\sqrt{10}$ sec.

37. Suppose the monkey accelerates upward with acceleration 'a' & the block, accelerate downward with acceleration a_1. Let Force exerted by monkey is equal to 'T'

From the free body diagram of monkey



$\therefore T - mg - ma = 0$ ...(i)

$\Rightarrow T = mg + ma$.

Again, from the FBD of the block,

$T = ma_1 - mg = 0$.

$\Rightarrow mg + ma + ma_1 - mg = 0$ [From (i)] $\Rightarrow ma = -ma_1 \Rightarrow a = a_1$.

Acceleration '–a' downward i.e. 'a' upward.

$\therefore$The block & the monkey move in the same direction with equal acceleration.

If initially they are rest (no force is exertied by monkey) no motion of monkey of block occurs as they have same weight (same mass). Their separation will not change as time passes.

38. Suppose A move upward with acceleration a, such that in the tail of A maximum tension 30N produced.

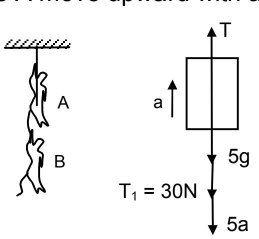


Fig-2

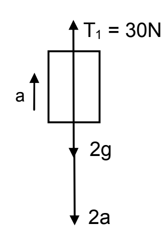


Fig-3

$T - 5g - 30 - 5a = 0$ ...(i)

$\Rightarrow T = 50 + 30 + (5 \times 5) = 105$ N (max)

$30 - 2g - 2a = 0$ ...(ii)

$\Rightarrow 30 - 20 - 2a = 0 \Rightarrow a = 5\ m/s^2$

So, A can apply a maximum force of 105 N in the rope to carry the monkey B with it.

For minimum force there is no acceleration of monkey 'A' and B. $\Rightarrow$ a = 0

Now equation (ii) is $T'_1 - 2g = 0 \Rightarrow T'_1 = 20$ N (wt. of monkey B)

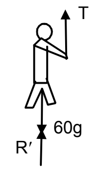
Equation (i) is $T - 5g - 20 = 0$ [As $T'_1 = 20$ N]

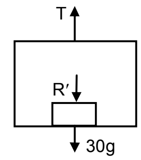
$\Rightarrow T = 5g + 20 = 50 + 20 = 70$ N.

$\therefore$ The monkey A should apply force between 70 N and 105 N to carry the monkey B with it.

39. (i) Given, Mass of man = 60 kg.

Let R′ = apparent weight of man in this case.





Now, $R' + T - 60g = 0$ [From FBD of man]

$\Rightarrow T = 60g - R'$ ...(i)

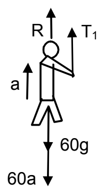
$T - R' - 30g = 0$ ...(ii) [From FBD of box]

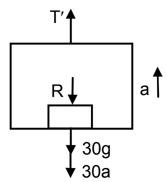
$\Rightarrow 60g - R' - R' - 30g = 0$ [From (i)]

$\Rightarrow R' = 15g$ The weight shown by the machine is 15kg.

(ii) To get his correct weight suppose the applied force is 'T' and so, acclerates upward with 'a'.

In this case, given that correct weight = R = 60g, where g = acc^n due to gravity





From the FBD of the man

$T^1 + R - 60g - 60a = 0$

$\Rightarrow T^1 - 60a = 0$ [$\therefore R = 60g$]

$\Rightarrow T^1 = 60a$ ...(i)

From the FBD of the box

$T^1 - R - 30g - 30a = 0$

$\Rightarrow T^1 - 60g - 30g - 30a = 0$

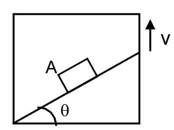
$\Rightarrow T^1 - 30a = 90g = 900$

$\Rightarrow T^1 = 30a - 900$ ...(ii)

From eqn (i) and eqn (ii) we get $T^1 = 2T^1 - 1800 \Rightarrow T^1 = 1800$N.

$\therefore$ So, he should exert 1800 N force on the rope to get correct reading.

40. The driving force on the block which n the body to move sown the plane is F = mg sin θ,



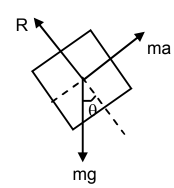
So, acceleration = g sin θ

Initial velocity of block u = 0.

$s = \ell$, $a = g \sin\theta$

Now, $S = ut + ½ at^2$

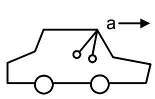
$\Rightarrow \ell = 0 + ½ (g \sin\theta) t^2 \Rightarrow g^2 = \dfrac{2\ell}{g\sin\theta} \Rightarrow t = \sqrt{\dfrac{2\ell}{g\sin\theta}}$

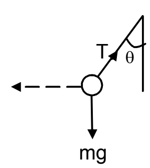


Time taken is $\sqrt{\dfrac{2\ell}{g\sin\theta}}$

41. Suppose pendulum makes θ angle with the vertical. Let, m = mass of the pendulum.

From the free body diagram





$T\cos\theta - mg = 0$

$\Rightarrow T\cos\theta = mg$

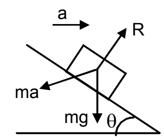
$\Rightarrow T = \dfrac{mg}{\cos\theta}$ ...(i)

$ma - T\sin\theta = 0$

$\Rightarrow ma = T\sin\theta$

$\Rightarrow t = \dfrac{ma}{\sin\theta}$ ...(ii)

From (i) & (ii) $\frac{mg}{\cos\theta} = \frac{ma}{\sin\theta} \Rightarrow \tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\frac{a}{g}$

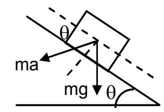


The angle is Tan^{-1}(a/g) with vertical.

(ii) m → mass of block.

Suppose the angle of incline is ‘θ’

From the diagram

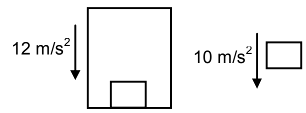


ma cos θ – mg sin θ = 0 ⇒ ma cos θ = mg sin θ ⇒ $\frac{\sin\theta}{\cos\theta} = \frac{a}{g}$

⇒ tan θ = a/g ⇒ θ = $\tan^{-1}$(a/g).

42. Because, the elevator is moving downward with an acceleration 12 m/s^2 (>g), the bodygets separated. So, body moves with acceleration g = 10 m/s^2 [freely falling body] and the elevator move with acceleration 12 m/s^2

Now, the block has acceleration = g = 10 m/s^2



u = 0

t = 0.2 sec

So, the distance travelled by the block is given by.

∴ s = ut + ½ at^2

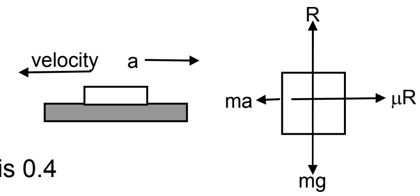
= 0 + (½) 10 $(0.2)^2$ = 5 × 0.04 = 0.2 m = 20 cm.

The displacement of body is 20 cm during first 0.2 sec.

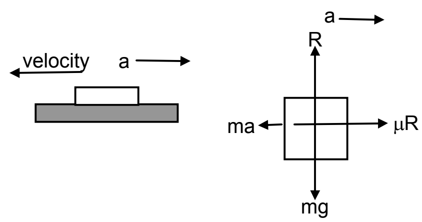
* * * *

SOLUTIONS TO CONCEPTS
CHAPTER 6

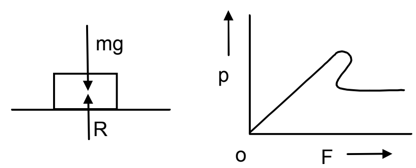
1. Let m = mass of the block
From the freebody diagram,
R – mg = 0 $\Rightarrow$ R = mg ...(1)
Again ma – μ R = 0 $\Rightarrow$ ma = μ R = μ mg (from (1))
$\Rightarrow$ a = μg $\Rightarrow$ 4 = μg $\Rightarrow$ μ = 4/g = 4/10 = 0.4
The co-efficient of kinetic friction between the block and the plane is 0.4



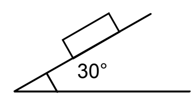
2. Due to friction the body will decelerate
Let the deceleration be ‘a’
R – mg = 0 $\Rightarrow$ R = mg ...(1)
ma – μ R = 0 $\Rightarrow$ ma = μ R = μ mg (from (1))
$\Rightarrow$ a = μg = 0.1 × 10 = 1m/s^2.
Initial velocity u = 10 m/s
Final velocity v = 0 m/s
a = –1m/s^2 (deceleration)
$$S = \frac{v^2 - u^2}{2a} = \frac{0 - 10^2}{2(-1)} = \frac{100}{2} = 50\text{m}$$
It will travel 50m before coming to rest.

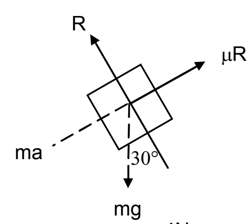


3. Body is kept on the horizontal table.
If no force is applied, no frictional force will be there
f $\rightarrow$ frictional force
F $\rightarrow$ Applied force
From grap it can be seen that when applied force is zero, frictional force is zero.

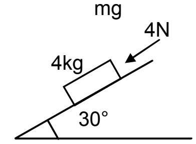


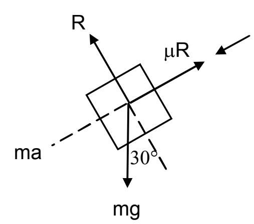
4. From the free body diagram,
R – mg cos θ = 0 $\Rightarrow$ R = mg cos θ ..(1)
For the block
U = 0, s = 8m, t = 2sec.
$\therefore$s = ut + ½ at^2 $\Rightarrow$ 8 = 0 + ½ a 2^2 $\Rightarrow$ a = 4m/s^2
Again, μR + ma – mg sin θ = 0
$\Rightarrow$ μ mg cos θ + ma – mg sin θ = 0 [from (1)]
$\Rightarrow$ m(μg cos θ + a – g sin θ) = 0
$\Rightarrow$ μ × 10 × cos 30° = g sin 30° – a
$\Rightarrow$ μ × 10 × $\sqrt{(3/3)}$ = 10 × (1/2) – 4
$\Rightarrow$ $(5/\sqrt{3})\,\mu = 1 \Rightarrow \mu = 1/(5/\sqrt{3}) = 0.11$
$\therefore$ Co-efficient of kinetic friction between the two is 0.11.



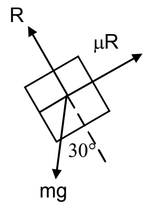


5. From the free body diagram
4 – 4a – μR + 4g sin 30° = 0 …(1)
R – 4g cos 30° = 0 ...(2)
$\Rightarrow$ R = 4g cos 30°
Putting the values of R is & in equn. (1)
4 – 4a – 0.11 × 4g cos 30° + 4g sin 30° = 0
$\Rightarrow$ 4 – 4a – 0.11 × 4 × 10 × ($\sqrt{3}/2$) + 4 × 10 × (1/2) = 0
$\Rightarrow$ 4 – 4a – 3.81 + 20 = 0 $\Rightarrow$ a $\approx$ 5 m/s^2
For the block u =0, t = 2sec, a = 5m/s^2
Distance s = ut + ½ at^2 $\Rightarrow$ s = 0 + (1/2) 5 × 2^2 = 10m
The block will move 10m.





6. To make the block move up the incline, the force should be equal and opposite to the net force acting down the incline = μ R + 2 g sin 30°



(body moving down)

$= 0.2 \times (9.8)\sqrt{3} + 2$ I $9.8 \times (1/2)$ [from (1)]

= 3.39 + 9.8 = 13N

With this minimum force the body move up the incline with a constant velocity as net force on it is zero.

b) Net force acting down the incline is given by,

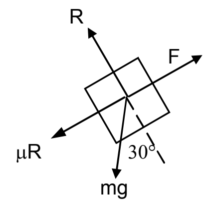
F = 2 g sin 30° – μR

= 2 × 9.8 × (1/2) – 3.39 = 6.41N

Due to F = 6.41N the body will move down the incline with acceleration.

No external force is required.

$\therefore$ Force required is zero.



(body moving us)

7. From the free body diagram

$g = 10 m/s^2$, m = 2kg, $\theta = 30°$, $\mu = 0.2$

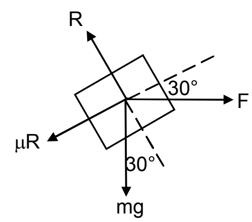
R – mg cos θ - F sin θ = 0

$\Rightarrow$ R = mg cos θ + F sin θ ...(1)

And mg sin θ + μR – F cos θ = 0

$\Rightarrow$ mg sin θ + μ(mg cos θ + F sin θ) – F cos θ = 0

$\Rightarrow$ mg sin θ + μ mg cos θ + μ F sin θ – F cos θ = 0



$$\Rightarrow F = \frac{(mg\sin\theta - \mu mg\cos\theta)}{(\mu\sin\theta - \cos\theta)}$$

$$\Rightarrow F = \frac{2\times10\times(1/2)+0.2\times2\times10\times(\sqrt{3}/2)}{0.2\times(1/2)-(\sqrt{3}/2)} = \frac{13.464}{0.76} = 17.7N \approx 17.5N$$

8. m $\to$ mass of child

R – mg cos 45° = 0

$\Rightarrow$ R = mg cos 45° = mg /v^2 ...(1)

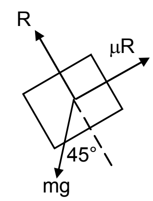
Net force acting on the boy due to which it slides down is mg sin 45° - μR

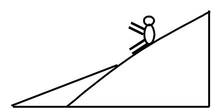
= mg sin 45° - μ mg cos 45°

$= m \times 10\ (1/\sqrt{2}) - 0.6 \times m \times 10 \times (1/\sqrt{2})$

$= m\ [(5/\sqrt{2}) - 0.6 \times (5/\sqrt{2})]$

$= m(2\sqrt{2})$





$$\text{acceleration} = \frac{\text{Force}}{\text{mass}} = \frac{m(2\sqrt{2})}{m} = 2\sqrt{2}\ m/s^2$$

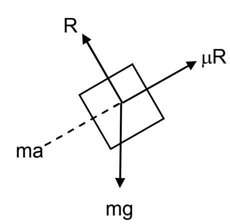
9. Suppose, the body is accelerating down with acceleration 'a'.

From the free body diagram

R – mg cos θ = 0

$\Rightarrow$ R = mg cos θ ...(1)

ma + mg sin θ – μ R = 0



$$\Rightarrow a = \frac{mg(\sin\theta - \mu\cos\theta)}{m} = g\ (\sin\theta - \mu\cos\theta)$$

For the first half mt. u = 0, s = 0.5m, t = 0.5 sec.

So, v = u + at = 0 + (0.5)4 = 2 m/s

$S = ut + ½ at^2 \Rightarrow 0.5 = 0 + ½ a (0/5)^2 \Rightarrow a = 4m/s^2$ ...(2)

For the next half metre

u\` = 2m/s, $a = 4m/s^2$, s= 0.5.

$\Rightarrow 0.5 = 2t + (1/2)\ 4\ t^2 \Rightarrow 2\ t^2 + 2\ t - 0.5 = 0$

$\Rightarrow 4t^2 + 4t - 1 = 0$

$\therefore = \dfrac{-4 \pm \sqrt{16+16}}{2 \times 4} = \dfrac{1.656}{8} = 0.207\text{sec}$

Time taken to cover next half meter is 0.21sec.

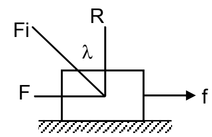
10. $f \rightarrow$ applied force

$F_i \rightarrow$ contact force

$F \rightarrow$ frictional force

$R \rightarrow$ normal reaction

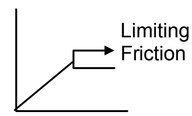
$\mu = \tan\lambda = F/R$



When $F = \mu R$, F is the limiting friction (max friction). When applied force increase, force of friction increase upto limiting friction (μR)

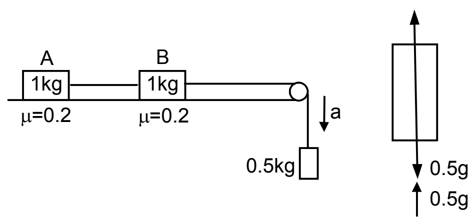
Before reaching limiting friction

$F < \mu R$



$\therefore \tan\lambda = \dfrac{F}{R} \le \dfrac{\mu R}{R} \Rightarrow \tan\lambda \le \mu \Rightarrow \lambda \le \tan^{-1}\mu$

11. From the free body diagram



$T + 0.5a - 0.5g = 0$ ...(1)

$\mu R + 1a + T_1 - T = 0$ ...(2)

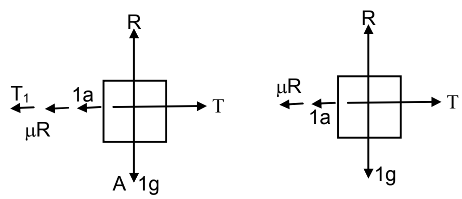
$\mu R + 1a - T_1 = 0$

$\mu R + 1a = T_1$ ...(3)

From (2) & (3) $\Rightarrow \mu R + a = T - T_1$

$\therefore T - T_1 = T_1$

$\Rightarrow T = 2T_1$



Equation (2) becomes $\mu R + a + T_1 - 2T_1 = 0$

$\Rightarrow \mu R + a - T_1 = 0$

$\Rightarrow T_1 = \mu R + a = 0.2g + a$ ...(4)

Equation (1) becomes $2T_1 + 0/5a - 0.5g = 0$

$\Rightarrow T_1 = \dfrac{0.5g - 0.5a}{2} = 0.25g - 0.25a$ ...(5)

From (4) & (5) $0.2g + a = 0.25g - 0.25a$

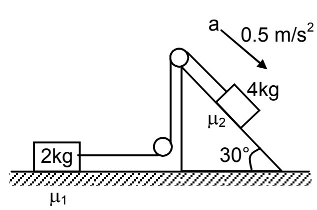
$\Rightarrow a = \dfrac{0.05}{1.25} \times 10 = 0.04 \text{ I } 10 = 0.4\text{m/s}^2$

a) Accln of 1kg blocks each is 0.4m/s^2

b) Tension $T_1 = 0.2g + a + 0.4 = 2.4\text{N}$

c) $T = 0.5g - 0.5a = 0.5 \times 10 - 0.5 \times 0.4 = 4.8\text{N}$

12. From the free body diagram



$\mu_1 R + 1 - 16 = 0$

$\Rightarrow \mu_1 (2g) + (-15) = 0$

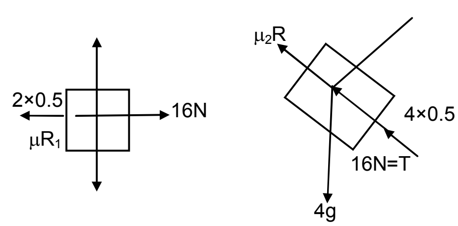
$\Rightarrow \mu_1 = 15/20 = 0.75$

$\mu_2 R_1 + 4 \times 0.5 + 16 - 4g \sin 30° = 0$

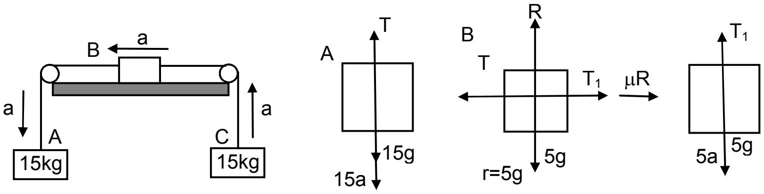
$\Rightarrow \mu_2 (20\sqrt{3}) + 2 + 16 - 20 = 0$

$\Rightarrow \mu_2 = \dfrac{2}{20\sqrt{3}} = \dfrac{1}{17.32} = 0.057 \approx 0.06$

$\therefore$Co-efficient of friction $\mu_1 = 0.75$ & $\mu_2 = 0.06$



13.



From the free body diagram

$T + 15a - 15g = 0$ $\Rightarrow T = 15g - 15a$ ...(i)

$T - (T_1 + 5a + \mu R) = 0$
$\Rightarrow T - (5g + 5a + 5a + \mu R) = 0$
$\Rightarrow T = 5g + 10a + \mu R$ ...(ii)

$T_1 - 5g - 5a = 0$ $\Rightarrow T_1 = 5g + 5a$ ...(iii)

From (i) & (ii) $15g - 15a = 5g + 10a + 0.2\,(5g)$

$\Rightarrow 25a = 90 \Rightarrow a = 3.6\text{m/s}^2$

Equation (ii) $\Rightarrow T = 5 \times 10 + 10 \times 3.6 + 0.2 \times 5 \times 10$

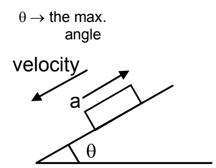
$\Rightarrow$ 96N in the left string

Equation (iii) $T_1 = 5g + 5a = 5 \times 10 + 5 \times 3.6 = 68\text{N}$ in the right string.

14. $s = 5\text{m}$, $\mu = 4/3$, $g = 10\text{m/s}^2$

$u = 36\text{km/h} = 10\text{m/s}$, $v = 0$,

$$a = \frac{v^2 - u^2}{2s} = \frac{0 - 10^2}{2 \times 5} = -10\text{m/s}^2$$



From the freebody diagrams,

$R - mg\cos\theta = 0$; $g = 10\text{m/s}^2$

$\Rightarrow R = mg\cos\theta$ ....(i) ; $\mu = 4/3$.

Again, $ma + mg\sin\theta - \mu R = 0$

$\Rightarrow ma + mg\sin\theta - \mu\, mg\cos\theta = 0$

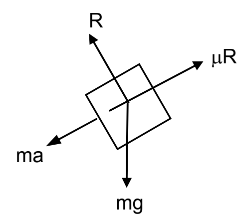
$\Rightarrow a + g\sin\theta - mg\cos\theta = 0$

$\Rightarrow 10 + 10\sin\theta - (4/3) \times 10\cos\theta = 0$

$\Rightarrow 30 + 30\sin\theta - 40\cos\theta = 0$

$\Rightarrow 3 + 3\sin\theta - 4\cos\theta = 0$

$\Rightarrow 4\cos\theta - 3\sin\theta = 3$



$\Rightarrow 4\sqrt{1 - \sin^2\theta} = 3 + 3\sin\theta$

$\Rightarrow 16\,(1 - \sin^2\theta) = 9 + 9\sin^2\theta + 18\sin\theta$

$$\sin\theta = \frac{-18 \pm \sqrt{18^2 - 4(25)(-7)}}{2 \times 25} = \frac{-18 \pm 32}{50} = \frac{14}{50} = 0.28 \text{ [Taking +ve sign only]}$$

$\Rightarrow \theta = \sin^{-1}(0.28) = 16°$

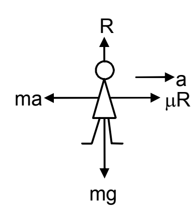
Maximum incline is $\theta = 16°$

15. to reach in minimum time, he has to move with maximum possible acceleration.

Let, the maximum acceleration is 'a'

$\therefore ma - \mu R = 0 \Rightarrow ma = \mu\, mg$

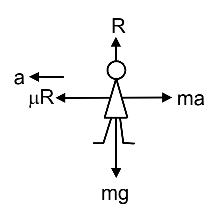
$\Rightarrow a = \mu\, g = 0.9 \times 10 = 9\text{m/s}^2$



a) Initial velocity $u = 0$, $t = ?$

$a = 9\text{m/s}^2$, $s = 50\text{m}$

$$s = ut + ½\,at^2 \Rightarrow 50 = 0 + (1/2)\,9\,t^2 \Rightarrow t = \sqrt{\frac{100}{9}} = \frac{10}{3}\text{ sec.}$$



b) After overing 50m, velocity of the athelete is

$V = u + at = 0 + 9 \times (10/3) = 30\text{m/s}$

He has to stop in minimum time. So deceleration ia $-a = -9\text{m/s}^2$ (max)

$$\begin{bmatrix} R = ma \\ ma = \mu R(\text{max frictional force}) \\ \Rightarrow a = \mu g = 9m/s^2(\text{Deceleration}) \end{bmatrix}$$

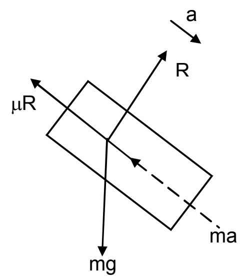
u^1 = 30m/s, v^1 = 0

$t = \frac{v^1 - u^1}{a} = \frac{0-30}{-a} = \frac{-30}{-a} = \frac{10}{3}$ sec.

16. Hardest brake means maximum force of friction is developed between car's type & road.

Max frictional force = μR

From the free body diagram



$R - mg\cos\theta = 0$

$\Rightarrow R = mg\cos\theta$ ...(i)

and $\mu R + ma - mg \sin\ ) = 0$ ...(ii)

$\Rightarrow \mu mg\cos\theta + ma - mg\sin\theta = 0$

$\Rightarrow \mu g\cos\theta + a - 10 \times (1/2) = 0$

$\Rightarrow a = 5 - \{1 - (2\sqrt{3})\} \times 10\ (\sqrt{3}/2) = 2.5\ m/s^2$

When, hardest brake is applied the car move with acceleration 2.5m/s^2

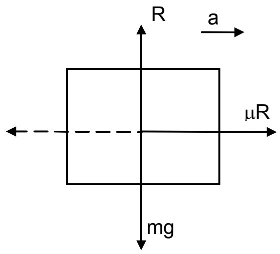
S = 12.8m, u = 6m/s

S0, velocity at the end of incline

$V = \sqrt{u^2 + 2as} = \sqrt{6^2 + 2(2.5)(12.8)} = \sqrt{36+64}$ = 10m/s = 36km/h

Hence how hard the driver applies the brakes, that car reaches the bottom with least velocity 36km/h.

17. Let, , a maximum acceleration produced in car.



$\therefore$ ma = μR [For more acceleration, the tyres will slip]

$\Rightarrow ma = \mu\ mg \Rightarrow a = \mu g = 1 \times 10 = 10m/s^2$

For crossing the bridge in minimum time, it has to travel with maximum acceleration

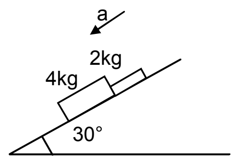
u = 0, s = 500m, a = 10m/s^2

s = ut + ½ at^2

$\Rightarrow 500 = 0 + (1/2)\ 10\ t^2 \Rightarrow t = 10$ sec.

If acceleration is less than 10m/s^2, time will be more than 10sec. So one can't drive through the bridge in less than 10sec.

18. From the free body diagram

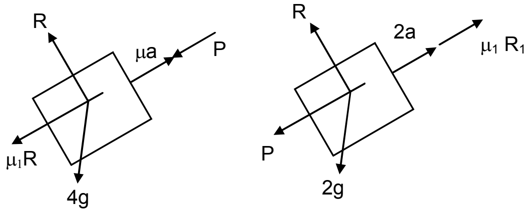


$R = 4g\cos 30° = 4 \times 10\ \times \sqrt{3}/2 = 20\sqrt{3}$ ...(i)

$\mu_2 R + 4a - P - 4g\sin 30° = 0 \Rightarrow 0.3\ (40)\cos 30°\ + 4a - P - 40\sin 20° = 0$ ...(ii)

$P + 2a + \mu_1 R_1 - 2g\sin 30° = 0$ ...(iii)

$R_1 = 2g\cos 30° = 2 \times 10 \times \sqrt{3}/2 = 10\sqrt{3}$ ...(iv)



Equn. (ii) $6\sqrt{3}\ + 4a - P - 20 = 0$

Equn (iv) $P + 2a + 2\sqrt{3}\ - 10 = 0$

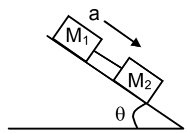
From Equn (ii) & (iv) $6\sqrt{3}\ + 6a - 30 + 2\sqrt{3}\ = 0$

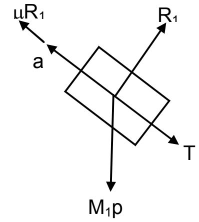
$\Rightarrow 6a = 30 - 8\sqrt{3}\ = 30 - 13.85 = 16.15$

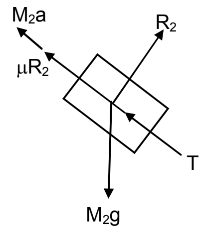
$\Rightarrow a = \frac{16.15}{6} = 2.69 = 2.7m/s^2$

b) can be solved. In this case, the 4 kg block will travel with more acceleration because, coefficient of friction is less than that of 2kg. So, they will move separately. Drawing the free body diagram of 2kg mass only, it can be found that, a = 2.4m/s^2.

19. From the free body diagram







$R_1 = M_1\, g \cos\theta$ ...(i)

$R_2 = M_2\, g \cos\theta$ ...(ii)

$T + M_1 g \sin\theta - m_1\, a - \mu\, R_1 = 0$ ...(iii)

$T - M_2 - M_2\, a + \mu\, R_2 = 0$ ...(iv)

Equn (iii) $\Rightarrow$ $T + M_1 g \sin\theta - M_1\, a - \mu\, M_1 g \cos\theta = 0$

Equn (iv) $\Rightarrow$ $T - M_2\, g \sin\theta + M_2\, a + \mu\, M_2 g \cos\theta = 0$ ...(v)

Equn (iv) & (v) $\Rightarrow$ $g \sin\theta\, (M_1 + M_2) - a(M_1 + M_2) - \mu g \cos\theta\, (M_1 + M_2) = 0$

$\Rightarrow a\, (M_1 + M_2) = g \sin\theta\, (M_1 + M_2) - \mu\, g \cos\theta\, (M_1 + M_2)$

$\Rightarrow a = g(\sin\theta - \mu\cos\theta)$

$\therefore$ The blocks (system has acceleration $g(\sin\theta - \mu\cos\theta)$

The force exerted by the rod on one of the blocks is tension.

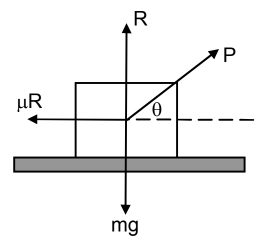
Tension $T = - M_1 g \sin\theta + M_1 a + \mu\, M_1 g \sin\theta$

$\Rightarrow T = - M_1 g \sin\theta + M_1(g \sin\theta - \mu\, g \cos\theta) + \mu\, M_1 g \cos\theta$

$\Rightarrow T = 0$

20. Let 'p' be the force applied to at an angle θ

From the free body diagram



$R + P \sin\theta - mg = 0$

$\Rightarrow R = - P \sin\theta + mg$ ...(i)

$\mu R - p \cos\theta$ ...(ii)

Equn. (i) is $\mu(mg - P\sin\theta) - P\cos\theta = 0$

$$\Rightarrow \mu\, mg = \mu\, \rho \sin\theta - P\cos\theta \Rightarrow \rho = \frac{\mu mg}{\mu\sin\theta + \cos\theta}$$

Applied force P should be minimum, when $\mu \sin\theta + \cos\theta$ is maximum.

Again, $\mu \sin\theta + \cos\theta$ is maximum when its derivative is zero.

$\therefore d/d\theta\, (\mu\sin\theta + \cos\theta) = 0$

$\Rightarrow \mu\cos\theta - \sin\theta = 0 \Rightarrow \theta = \tan^{-1}\mu$

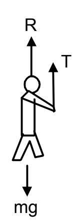
$$\text{So, } P = \frac{\mu mg}{\mu\sin\theta + \cos\theta} = \frac{\mu mg/\cos\theta}{\frac{\mu\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}} = \frac{\mu mg\sec\theta}{1+\mu\tan\theta} = \frac{\mu mg\sec\theta}{1+\tan^2\theta}$$

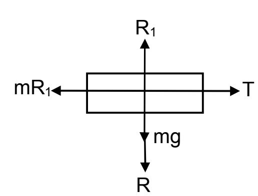
$$= \frac{\mu mg}{\sec\theta} = \frac{\mu mg}{\sqrt{(1+\tan^2\theta}} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Minimum force is $\dfrac{\mu mg}{\sqrt{1+\mu^2}}$ at an angle $\theta = \tan^{-1}\mu$.

21. Let, the max force exerted by the man is T.

From the free body diagram





$R + T - Mg = 0$

$\Rightarrow R = Mg - T$ ...(i)

$R_1 - R - mg = 0$

$\Rightarrow R_1 = R + mg$ ...(ii)

And $T - \mu\, R_1 = 0$

$\Rightarrow T - \mu (R + mg) = 0$ [From equn. (ii)]

$\Rightarrow T - \mu R - \mu mg = 0$

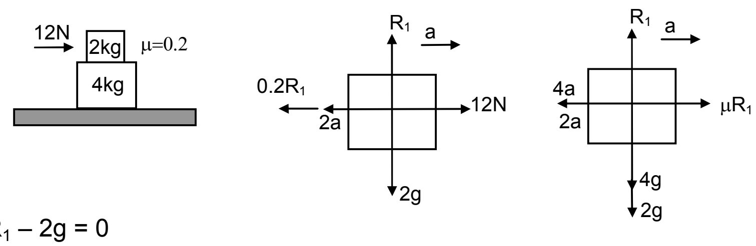
$\Rightarrow T - \mu (Mg + T) - \mu mg = 0$ [from (i)]

$\Rightarrow T (1 + \mu) = \mu Mg + \mu mg$

$\Rightarrow T = \dfrac{\mu(M+m)g}{1+\mu}$

Maximum force exerted by man is $\dfrac{\mu(M+m)g}{1+\mu}$

22.



$R_1 - 2g = 0$

$\Rightarrow R_1 = 2 \times 10 = 20$

$2a + 0.2 R_1 - 12 = 0$

$\Rightarrow 2a + 0.2(20) = 12$

$\Rightarrow 2a = 12 - 4 = 8$

$\Rightarrow a = 4m/s^2$

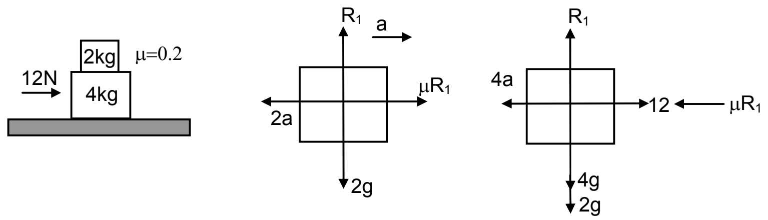
$4a_1 - \mu R_1 = 0$

$\Rightarrow 4a_1 = \mu R_1 = 0.2 (20)$

$\Rightarrow 4a_1 = 4$

$\Rightarrow a_1 = 1m/s^2$

2kg block has acceleration $4m/s^2$ & that of 4 kg is $1m/s^2$



(ii) $R_1 = 2g = 20$

$Ma - \mu R_1 = 0$

$\Rightarrow 2a = 0.2 (20) = 4$

$\Rightarrow a = 2m/s^2$

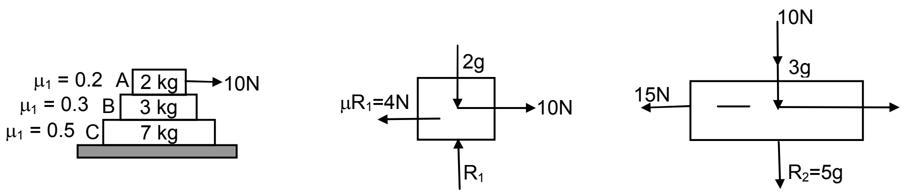
$4a + 0.2 \times 2 \times 10 - 12 = 0$

$\Rightarrow 4a + 4 = 12$

$\Rightarrow 4a = 8$

$\Rightarrow a = 2 m/s^2$

23.



a) When the 10N force applied on 2kg block, it experiences maximum frictional force $\mu R_1 = \mu \times 2kg = (0.2) \times 20 = 4N$ from the 3kg block.

So, the 2kg block experiences a net force of $10 - 4 = 6N$

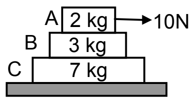
So, $a_1 = 6/2 = 3 m/s^2$

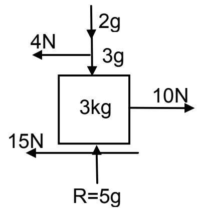
But for the 3kg block, (fig-3) the frictional force from 2kg block (4N) becomes the driving force and the maximum frictional force between 3kg and 7 kg block is

$\mu_2 R_2 = (0.3) \times 5kg = 15N$

So, the 3kg block cannot move relative to the 7kg block. The 3kg block and 7kg block both will have same acceleration ($a_2 = a_3$) which will be due to the 4N force because there is no friction from the floor.

$\therefore a_2 = a_3 = 4/10 = 0.4m/s^2$





b) When the 10N force is applied to the 3kg block, it can experience maximum frictional force of 15 + 4 = 19N from the 2kg block & 7kg block.

So, it can not move with respect to them.

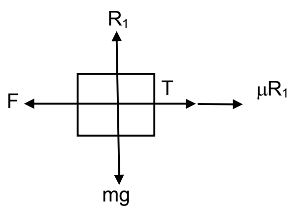
As the floor is frictionless, all the three bodies will move together

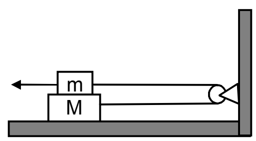
$\therefore a_1 = a_2 = a_3 = 10/12 = (5/6)\text{m/s}^2$

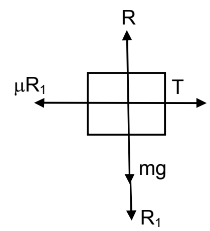
c) Similarly, it can be proved that when the 10N force is applied to the 7kg block, all the three blocks will move together.

Again $a_1 = a_2 = a_3 = (5/6)\text{m/s}^2$

24. Both upper block & lower block will have acceleration 2m/s^2







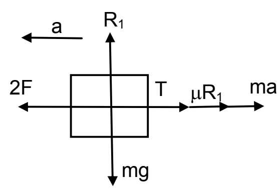
$R_1 = mg$ ...(i)

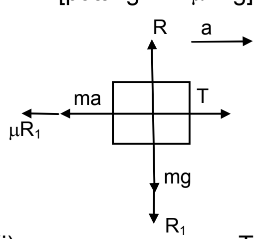
$F - \mu R_1 - T = 0 \Rightarrow F - \mu mg - T = 0$ ...(ii)

$\therefore F = \mu\, mg + \mu\, mg = 2\,\mu\, mg$ [putting $T = \mu$ mg]

$T - \mu R_1 = 0$

$\Rightarrow T = \mu mg$





b) $2F - T - \mu\, mg - ma = 0$ …(i)

$T - Ma - \mu\, mg = 0$ [$\therefore R_1 = mg$]

$\Rightarrow T = Ma + \mu\, mg$

Putting value of T in (i)

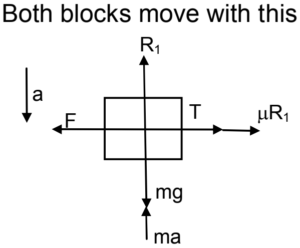
$2f - Ma - \mu mg - \mu\, mg - ma = 0$

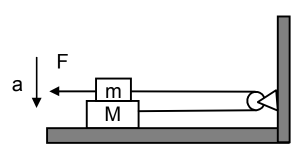
$\Rightarrow 2(2\mu mg) - 2\,\mu\, mg = a(M + m)$ [Putting $F = 2\,\mu mg$]

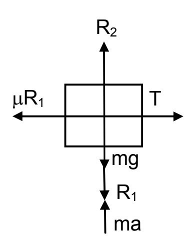
$\Rightarrow 4\mu\, mg - 2\,\mu\, mg = a\,(M + m)$ $\Rightarrow a = \dfrac{2\mu mg}{M+m}$

Both blocks move with this acceleration 'a' in opposite direction.

25.







$T = mR_1$
$= m\,(mg - ma)$

$R_1 + ma - mg = 0$

$\Rightarrow R_1 = m(g - a) = mg - ma$ ...(i)

$T - \mu\, R_1 = 0 \Rightarrow T = m\,(mg - ma)$ ...(ii)

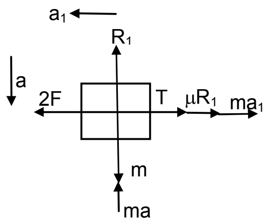
Again, $F - T - \mu\, R_1 = 0$

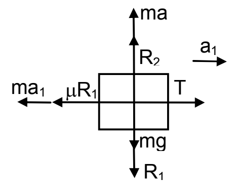
$\Rightarrow F - \{\mu(mg - ma)\} - u(mg - ma) = 0$

$\Rightarrow F - \mu\, mg + \mu\, ma - \mu\, mg + \mu\, ma = 0$

$\Rightarrow F = 2\,\mu\, mg - 2\mu\, ma \Rightarrow F = 2\mu\, m(g-a)$

b) Acceleration of the block be a_1





$R_1 = mg - ma$ ...(i)

$2F - T - \mu R_1 - ma_1 = 0$

$\Rightarrow 2F - t - \mu mg + \mu a - ma_1 = 0$ ...(ii)

$T - \mu R_1 - M\, a_1 = 0$

$\Rightarrow T = \mu R_1 + M\, a_1$

$\Rightarrow T = \mu\,(mg - ma) + Ma_1$

$\Rightarrow T = \mu\, mg - \mu\, ma + M\, a_1$

Subtracting values of F & T, we get

$2(2\mu m(g-a)) - 2(\mu mg - \mu ma + Ma_1) - \mu mg + \mu\, ma - \mu\, a_1 = 0$

$\Rightarrow 4\mu\, mg - 4\,\mu\, ma - 2\,\mu\, mg + 2\mu\, ma = ma_1 + M\, a_1$ $\qquad \Rightarrow a_1 = \dfrac{2\mu m(g-a)}{M+m}$

Both blocks move with this acceleration but in opposite directions.

26. $R_1 + QE - mg = 0$

$R_1 = mg - QE$ ...(i)

$F - T - \mu R_1 = 0$

$\Rightarrow F - T\ \ \mu(mg - QE) = 0$

$\Rightarrow F - T - \mu\, mg + \mu QE = 0$ ...(2)

$T - \mu\, R_1 = 0$

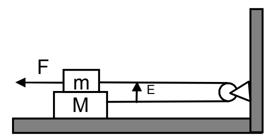
$\Rightarrow T = \mu\, R_1 = \mu\,(mg - QE) = \mu\, mg - \mu QE$

Now equation (ii) is $F - mg + \mu\, QE - \mu\, mg + \mu\, QE = 0$

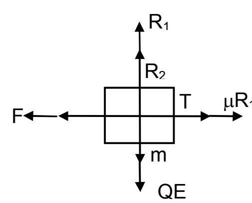
$\Rightarrow F - 2\,\mu\, mg + 2\mu\, QE = 0$

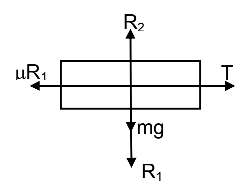
$\Rightarrow F = 2\mu mg - 2\mu\, QE$

$\Rightarrow F = 2\mu(mg - QE)$



F=QE





Maximum horizontal force that can be applied is $2\mu(mg - QE)$.

27. Because the block slips on the table, maximum frictional force acts on it.

From the free body diagram

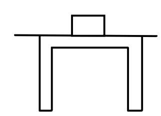
$R = mg$

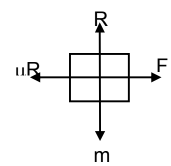
$\therefore F - \mu\, R = 0 \Rightarrow F = \mu R = \mu\, mg$

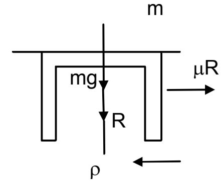
But the table is at rest. So, frictional force at the legs of the table is not $\mu\, R_1$. Let be f, so form the free body diagram.

$f_o - \mu\, R = 0 \Rightarrow f_o = \mu R = \mu\, mg$.

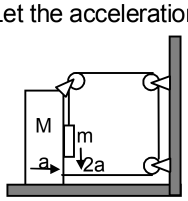
Total frictional force on table by floor is μ mg.

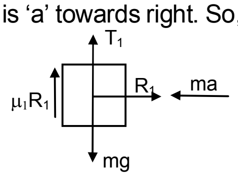




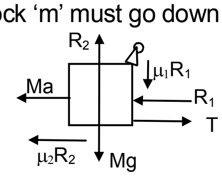


28. Let the acceleration of block M is ‘a’ towards right. So, the block ‘m’ must go down with an acceleration ‘2a’.





(FBD-1)



(FBD-2)

As the block ‘m’ is in contact with the block ‘M’, it will also have acceleration ‘a’ towards right. So, it will experience two inertia forces as shown in the free body diagram-1.

From free body diagram -1

$R_1 - ma = 0 \Rightarrow R_1 = ma$ ...(i)

Again, $2ma + T - mg + \mu_1 R_1 = 0$

$\Rightarrow T = mg - (2 - \mu_1)ma$ …(ii)

From free body diagram-2

$T + \mu_1 R_1 + mg - R_2 = 0$

$\Rightarrow R_2 = T + \mu_1\, ma + Mg$ [Putting the value of R_1 from (i)]

$= (mg - 2ma - \mu_1\, ma) + \mu_1\, ma + Mg$ [Putting the value of T from (ii)]

$\therefore R_2 = Mg + mg - 2ma$ …(iii)

Again, form the free body diagram -2

$T + T - R - Ma - \mu_2 R_2 = 0$

$\Rightarrow 2T - MA - mA - \mu_2\,(Mg + mg - 2ma) = 0$ [Putting the values of R_1 and R_2 from (i) and (iii)]

$\Rightarrow 2T = (M + m) + \mu_2(Mg + mg - 2ma)$ ...(iv)

From equation (ii) and (iv)

$2T = 2\,mg - 2(2 + \mu_1)mg = (M + m)a + \mu_2(Mg + mg - 2ma)$

$\Rightarrow 2mg - \mu_2(M + m)g = a\,(M + m - 2\mu_2 m + 4m + 2\mu_1 m)$

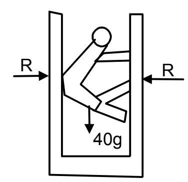
$\Rightarrow a = \dfrac{[2m - \mu_2(M+m)]g}{M + m[5 + 2(\mu_1 - \mu_2)]}$

29. Net force = *(202 + (15)2 – (0.5) × 40 = 25 – 20 = 5N

$\therefore \tan\theta = 20/15 = 4/3 \Rightarrow \mu = \tan^{-1}(4/3) = 53°$

So, the block will move at an angle 53 ° with an 15N force

30. a) Mass of man = 50kg. $g = 10\ m/s^2$



Frictional force developed between hands, legs & back side with the wall the wt of man. So he remains in equilibrium.

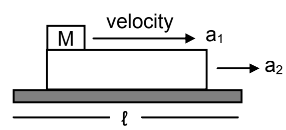
He gives equal force on both the walls so gets equal reaction R from both the walls. If he applies unequal forces R should be different he can't rest between the walls. Frictional force $2\mu R$ balance his wt.

From the free body diagram

$\mu R + \mu R = 40g \Rightarrow 2\,\mu R = 40 \times 10 \Rightarrow R = \dfrac{40 \times 10}{2 \times 0.8} = 250N$

b) The normal force is 250 N.

31. Let a_1 and a_2 be the accelerations of ma and M respectively.

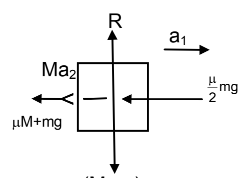


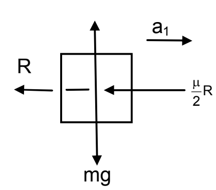
Here, $a_1 > a_2$ so that m moves on M

Suppose, after time 't' m separate from M.

In this time, m covers $vt + ½\, a_1 t^2$ and $S_M = vt + ½\, a_2 t^2$

For 'm' to m to 'm' separate from M. $vt + ½\, a_1 t^2 = vt + ½\, a_2 t^2 + \ell$ ...(1)





Again from free body diagram

$Ma_1 + \mu/2\ R = 0$

$\Rightarrow ma_1 = -(\mu/2)\,mg = -(\mu/2)m \times 10 \Rightarrow a_1 = -5\mu$

Again,

$Ma_2 + \mu\,(M + m)g - (\mu/2)mg = 0$

$\Rightarrow 2Ma_2 + 2\mu\,(M + m)g - \mu\,mg = 0$

$\Rightarrow 2\,M\,a_2 = \mu\,mg - 2\mu Mg - 2\,\mu mg$

$\Rightarrow a_2\ \dfrac{-\mu mg - 2\mu Mg}{2M}$

Putting values of a_1 & a_2 in equation (1) we can find that

$T = \sqrt{\left(\dfrac{4ml}{(M+m)\mu g}\right)}$

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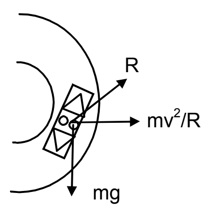
SOLUTIONS TO CONCEPTS circular motion;;
CHAPTER 7

1. Distance between Earth & Moon
   $r = 3.85 \times 10^5$ km $= 3.85 \times 10^8$m
   T = 27.3 days = $24 \times 3600 \times (27.3)$ sec $= 2.36 \times 10^6$ sec
   $v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 3.85 \times 10^8}{2.36 \times 10^6} = 1025.42$m/sec
   $a = \frac{v^2}{r} = \frac{(1025.42)^2}{3.85 \times 10^8} = 0.00273\text{m/sec}^2 = 2.73 \times 10^{-3}\text{m/sec}^2$
2. Diameter of earth = 12800km
   Radius R = 6400km = 64×10^5 m
   $V = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times 64 \times 10^5}{24 \times 3600}$ m/sec = 465.185
   $a = \frac{V^2}{R} = \frac{(46.5185)^2}{64 \times 10^5} = 0.0338\text{m/sec}^2$
3. V = 2t, r = 1cm
   a) Radial acceleration at t = 1 sec.
   $a = \frac{v^2}{r} = \frac{2^2}{1} = 4\text{cm/sec}^2$
   b) Tangential acceleration at t = 1sec.
   $a = \frac{dv}{dt} = \frac{d}{dt}(2t) = 2\text{cm/sec}^2$
   c) Magnitude of acceleration at t = 1sec
   $a = \sqrt{4^2 + 2^2} = \sqrt{20}\ \text{cm/sec}^2$
4. Given that m = 150kg,
   v= 36km/hr = 10m/sec, r = 30m
   Horizontal force needed is $\frac{mv^2}{r} = \frac{150 \times (10)^2}{30} = \frac{150 \times 100}{30} = 500$N
5. in the diagram
   $R\cos\theta = mg$ ..(i)
   $R\sin\theta = \frac{mv^2}{r}$ ..(ii)
   Dividing equation (i) with equation (ii)



   $\text{Tan}\,\theta = \frac{mv^2}{rmg} = \frac{v^2}{rg}$
   v = 36km/hr = 10m/sec, r = 30m
   $\text{Tan}\,\theta = \frac{v^2}{rg} = \frac{100}{30 \times 10} = (1/3)$
   $\Rightarrow \theta = \tan^{-1}(1/3)$
6. Radius of Park = r = 10m
   speed of vehicle = 18km/hr = 5 m/sec
   Angle of banking $\tan\theta = \frac{v^2}{rg}$
   $\Rightarrow \theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{25}{100} = \tan^{-1}(1/4)$

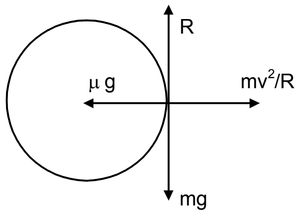
7. The road is horizontal (no banking)

$\frac{mv^2}{R} = \mu N$

and N = mg

So $\frac{mv^2}{R} = \mu\, mg$ v = 5m/sec, R = 10m

$\Rightarrow \frac{25}{10} = \mu g \Rightarrow \mu = \frac{25}{100} = 0.25$



8. Angle of banking = θ = 30°

Radius = r = 50m

$\tan\theta = \frac{v^2}{rg} \Rightarrow \tan 30° = \frac{v^2}{rg}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{v^2}{rg} \Rightarrow v^2 = \frac{rg}{\sqrt{3}} = \frac{50\times10}{\sqrt{3}}$

$\Rightarrow v = \sqrt{\frac{500}{\sqrt{3}}} = 17$m/sec.

9. Electron revolves around the proton in a circle having proton at the centre.

Centripetal force is provided by coulomb attraction.

r = 5.3 →t 10^{-11}m m = mass of electron = 9.1 × 10^{-3}kg.

charge of electron = 1.6 × 10^{-19}c.

$\frac{mv^2}{r} = k\frac{q^2}{r^2} \Rightarrow v^2 = \frac{kq^2}{rm} = \frac{9\times10^9\times1.6\times1.6\times10^{-38}}{5.3\times10^{-11}\times9.1\times10^{-31}} = \frac{23.04}{48.23}\times10^{13}$

$\Rightarrow v^2 = 0.477 \times 10^{13} = 4.7 \times 10^{12}$

$\Rightarrow v = \sqrt{4.7\times10^{12}} = 2.2 \times 10^6$ m/sec

10. At the highest point of a vertical circle

$\frac{mv^2}{R} = mg$

$\Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$

11. A celling fan has a diameter = 120cm.

∴Radius = r = 60cm = 0/6m

Mass of particle on the outer end of a blade is 1g.

n = 1500 rev/min = 25 rev/sec

$\omega = 2\,\pi\, n = 2\,\pi \times 25 = 157.14$

Force of the particle on the blade = $Mr\omega^2$ = (0.001) × 0.6 × (157.14) = 14.8N

The fan runs at a full speed in circular path. This exerts the force on the particle (inertia). The particle also exerts a force of 14.8N on the blade along its surface.

12. A mosquito is sitting on an L.P. record disc & rotating on a turn table at $33\frac{1}{3}$ rpm.

$n = 33\frac{1}{3}$ rpm $= \frac{100}{3\times60}$ rps

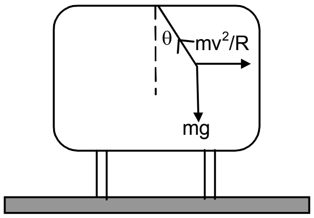
$\therefore \omega = 2\,\pi\, n = 2\,\pi \times \frac{100}{180} = \frac{10\pi}{9}$ rad/sec

r = 10cm =0.1m, g = 10m/sec^2

$\mu mg \ge mr\omega^2 \Rightarrow \mu = \frac{r\omega^2}{g} \ge \frac{0.1\times\left(\frac{10\pi}{9}\right)^2}{10}$

$\Rightarrow \mu \ge \frac{\pi^2}{81}$

13. A pendulum is suspended from the ceiling of a car taking a turn



$r = 10\text{m}$, $v = 36\text{km/hr} = 10\text{ m/sec}$, $g = 10\text{m/sec}^2$

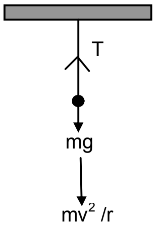
From the figure $T\sin\theta = \dfrac{mv^2}{r}$ ..(i)

$T\cos\theta = mg$ ..(ii)

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{mv^2}{rmg} \Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\frac{100}{10\times 10} = \tan^{-1}(1) \Rightarrow \theta = 45°$$

14. At the lowest pt.

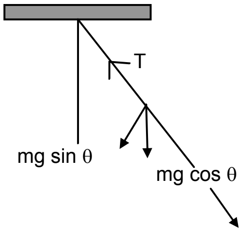


$$T = mg + \frac{mv^2}{r}$$

Here $m = 100\text{g} = 1/10\text{ kg}$, $r = 1\text{m}$, $v = 1.4\text{ m/sec}$

$$T = mg + \frac{mv^2}{r} = \frac{1}{10}\times 9.8\times\frac{(1.4)^2}{10} = 0.98 + 0.196 = 1.176 = 1.2\text{ N}$$

15. Bob has a velocity 1.4m/sec, when the string makes an angle of 0.2 radian.



$m = 100\text{g} = 0.1\text{kg}$, $r = 1\text{m}$, $v = 1.4\text{m/sec.}$

From the diagram,

$$T - mg\cos\theta = \frac{mv^2}{R}$$

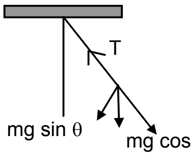
$$\Rightarrow T = \frac{mv^2}{R} + mg\cos\theta$$

$$\Rightarrow T = \frac{0.1\times(1.4)^2}{1} + (0.1)\times 9.8\times\left(1 - \frac{\theta^2}{2}\right)$$

$$\Rightarrow T = 0.196 + 9.8\times\left(1 - \frac{(.2)^2}{2}\right) \qquad (\therefore\ \cos\theta = 1 - \frac{\theta^2}{2}\text{ for small }\theta)$$

$$\Rightarrow T = 0.196 + (0.98)\times(0.98) = 0.196 + 0.964 = 1.156\text{N} \approx 1.16\text{ N}$$

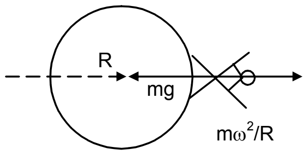
16. At the extreme position, velocity of the pendulum is zero.



So there is no centrifugal force.

So $T = mg\cos\theta_o$

17. a) Net force on the spring balance.



$R = mg - m\omega^2 r$

So, fraction less than the true weight (3mg) is

$$= \frac{mg - (mg - m\omega^2 r)}{mg} = \frac{\omega^2}{g} = \left(\frac{2\pi}{24\times 3600}\right)^2 \times \frac{6400\times 10^3}{10} = 3.5\times 10^{-3}$$

b) When the balance reading is half the true weight,

$$\frac{mg - (mg - m\omega^2 r)}{mg} = 1/2$$

$$\omega^2 r = g/2 \Rightarrow \omega = \sqrt{\frac{g}{2r}} = \sqrt{\frac{10}{2\times 6400\times 10^3}}\text{ rad/sec}$$

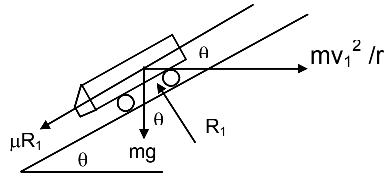
$\therefore$ Duration of the day is

$$T = \frac{2\pi}{\omega} = 2\pi\times\sqrt{\frac{2\times 6400\times 10^3}{9.8}}\text{ sec} = 2\pi\times\sqrt{\frac{64\times 10^6}{49}}\text{ sec} = \frac{2\pi\times 8000}{7\times 3600}\text{ hr} = 2\text{hr}$$

18. Given, v = 36km/hr = 10m/s, r = 20m, μ = 0.4

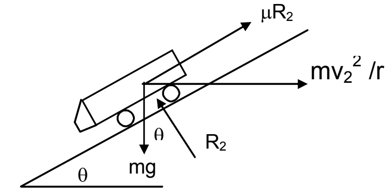
The road is banked with an angle,

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{100}{20\times10}\right) = \tan^{-1}\left(\frac{1}{2}\right)$ or $\tan\theta = 0.5$



When the car travels at max. speed so that it slips upward, μR_1 acts downward as shown in Fig.1

So, $R_1 - mg\cos\theta - \frac{mv_1^2}{r}\sin\theta = 0$ ..(i)



And $\mu R_1 + mg\sin\theta - \frac{mv_1^2}{r}\cos\theta = 0$ ..(ii)

Solving the equation we get,

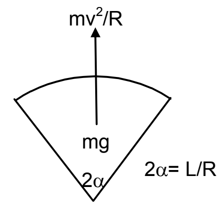
$V_1 = \sqrt{rg\frac{\tan\theta-\mu}{1+\mu\tan\theta}} = \sqrt{20\times10\times\frac{0.1}{1.2}}$ = 4.082 m/s = 14.7 km/hr

So, the possible speeds are between 14.7 km/hr and 54km/hr.

19. R = radius of the bridge

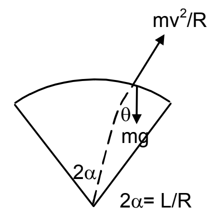
L = total length of the over bridge

a) At the highest pt.



$mg = \frac{mv^2}{R} \Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$

b) Given, $v = \frac{1}{\sqrt{2}}\sqrt{Rg}$



suppose it loses contact at B. So, at B, $mg\cos\theta = \frac{mv^2}{R}$

$\Rightarrow v^2 = Rg\cos\theta$

$\Rightarrow \left(\sqrt{\frac{Rv}{2}}\right)^2 = Rg\cos\theta \Rightarrow \frac{Rg}{2} = Rg\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = 60° = \pi/3$

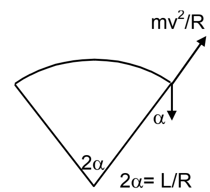
$\theta = \frac{\ell}{r} \to \ell = r\theta = \frac{\pi R}{3}$

So, it will lose contact at distance $\frac{\pi R}{3}$ from highest point

c) Let the uniform speed on the bridge be v.

The chances of losing contact is maximum at the end of the bridge for which $\alpha = \frac{L}{2R}$.

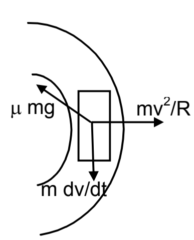
So, $\frac{mv^2}{R} = mg\cos\alpha \Rightarrow v = \sqrt{gR\cos\left(\frac{L}{2R}\right)}$

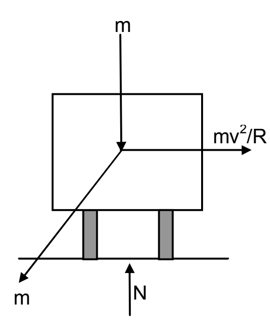


20. Since the motion is nonuniform, the acceleration has both radial & tangential component

$a_r = \frac{v^2}{r}$

$a_t = \frac{dv}{dt} = a$





Resultant magnitude = $\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$

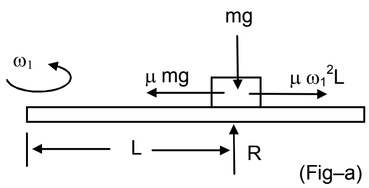
Now $\mu N = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu\, mg = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu^2 g^2 = \left(\frac{v^4}{r2}\right) + a^2$

$\Rightarrow v^4 = (\mu^2 g^2 - a^2)\, r^2 \Rightarrow v = [(\mu^2 g^2 - a^2)\, r^2]^{1/4}$

21. a) When the ruler makes uniform circular motion in the horizontal plane, (fig–a)

μ mg = $m\omega_1^2 L$

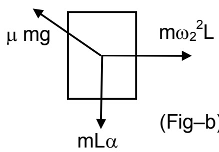
$\omega_1 = \sqrt{\dfrac{\mu g}{L}}$



(Fig–a)

b) When the ruler makes uniformly accelerated circular motion,(fig–b)

$\mu mg = \sqrt{(m\omega_2^2 L)^2 + (mL\alpha)^2} \Rightarrow \omega_2^4 + \alpha^2 = \dfrac{\mu^2 g^2}{L^2} \Rightarrow \omega_2 = \left[\left(\dfrac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$



(Fig–b)

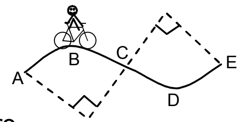
(When viewed from top)

22. Radius of the curves = 100m

Weight = 100kg

Velocity = 18km/hr = 5m/sec

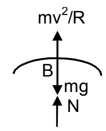
a) at B mg – $\dfrac{mv^2}{R}$ = N ⇒ N = (100 × 10) – $\dfrac{100 \times 25}{100}$ = 1000 – 25 = 975N



At d, N = mg + $\dfrac{mv^2}{R}$ = 1000 + 25 = 1025 N

b) At B & D the cycle has no tendency to slide. So at B & D, frictional force is zero.

At 'C', mg sin θ = F ⇒ F = 1000 × $\dfrac{1}{\sqrt{2}}$ = 707N



c) (i) Before 'C' mg cos θ – N = $\dfrac{mv^2}{R}$ ⇒ N = mg cos θ – $\dfrac{mv^2}{R}$ = 707 – 25 = 683N

(ii) N – mg cos θ = $\dfrac{mv^2}{R}$ ⇒ N = $\dfrac{mv^2}{R}$ + mg cos θ = 25 + 707 = 732N

d) To find out the minimum desired coeff. of friction, we have to consider a point just before C. (where N is minimum)

Now, μ N = mg sin θ ⇒ μ × 682 = 707

So, μ = 1.037

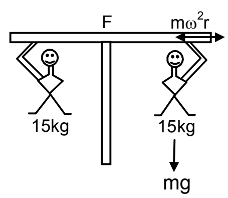
23. d = 3m ⇒ R = 1.5m

R = distance from the centre to one of the kids

N = 20 rev per min = 20/60 = 1/3 rev per sec

ω = 2πr = 2π/3

m = 15kg



∴ Frictional force F = $mr\omega^2$ = 15 × (1.5) × $\dfrac{(2\pi)^2}{9}$ = 5 × (0.5) × $4\pi^2$ = $10\pi^2$

∴ Frictional force on one of the kids is $10\pi^2$

24. If the bowl rotates at maximum angular speed, the block tends to slip upwards. So, the frictional force acts downward.

Here, r = R sin θ

From FBD –1

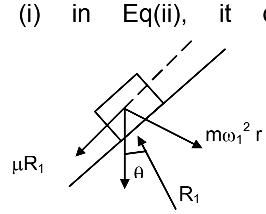
R_1 – mg cos θ – $m\omega_1^2$ (R sin θ) sin θ = 0 ..(i) [because r = R sin θ]

and μR_1 mg sin θ – $m\omega_1^2$ (R sin θ) cos θ = 0 ..(ii)

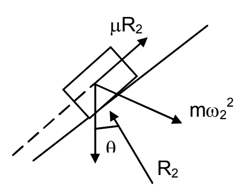
Substituting the value of R_1 from Eq (i) in Eq(ii), it can be found out that

$\omega_1 = \left[\dfrac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{1/2}$

Again, for minimum speed, the frictional force μR_2 acts upward. From FBD–2, it can be proved that,



(FBD – 1)



(FBD – 2)

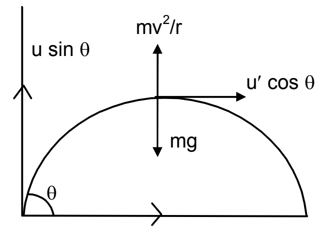
$\omega_2 = \left[\dfrac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$

$\therefore$ the range of speed is between ω_1 and ω_2

25. Particle is projected with speed 'u' at an angle θ. At the highest pt. the vertical component of velocity is '0'

So, at that point, velocity = u cos θ

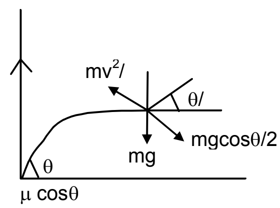
centripetal force = $m\,u^2\cos^2\left(\dfrac{\theta}{r}\right)$



At highest pt.

mg = $\dfrac{mv^2}{r} \Rightarrow r = \dfrac{u^2\cos^2\theta}{g}$

26. Let 'u' the velocity at the pt where it makes an angle $\theta/2$ with horizontal. The horizontal component remains unchanged



So, v cos $\theta/2$ = ω cos $\theta \Rightarrow v = \dfrac{u\cos\theta}{\cos\left(\dfrac{\theta}{2}\right)}$ ...(i)

From figure

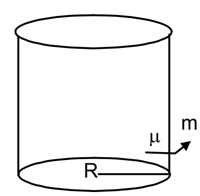
mg cos $(\theta/2)$ = $\dfrac{mv^2}{r} \Rightarrow r = \dfrac{v^2}{g\cos(\theta/2)}$

putting the value of 'v' from equn(i)

$r = \dfrac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

27. A block of mass 'm' moves on a horizontal circle against the wall of a cylindrical room of radius 'R' Friction coefficient between wall & the block is μ.

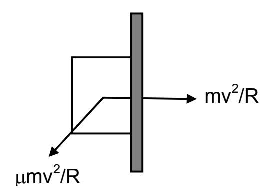
a) Normal reaction by the wall on the block is = $\dfrac{mv^2}{R}$



b) $\therefore$ Frictional force by wall = $\dfrac{\mu mv^2}{R}$

c) $\dfrac{\mu mv^2}{R}$ = ma $\Rightarrow a = -\dfrac{\mu v^2}{R}$ (Deceleration)

d) Now, $\dfrac{dv}{dt} = v\dfrac{dv}{ds} = -\dfrac{\mu v^2}{R} \Rightarrow ds = -\dfrac{R}{\mu}\dfrac{dv}{v}$



$\Rightarrow s = -\dfrac{R\mu}{}\ \text{In}\ V + c$

At s = 0, v = v_0

Therefore, c = $\dfrac{R}{\mu}$ In V_0

so, $s = -\dfrac{R}{\mu}\text{In}\dfrac{v}{v_0} \Rightarrow \dfrac{v}{v_0} = e^{-\mu s/R}$

For, one rotation s = $2\pi R$, so $v = v_0 e^{-2\pi\mu}$

28. The cabin rotates with angular velocity ω & radius R

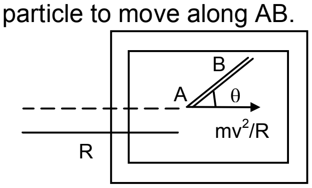
$\therefore$ The particle experiences a force $mR\omega^2$.

The component of $mR\omega^2$ along the groove provides the required force to the particle to move along AB.

$\therefore$ $mR\omega^2\cos\theta = ma \Rightarrow a = R\omega^2\cos\theta$

length of groove = L

$L = ut + ½ at^2 \Rightarrow L = ½ R\omega^2\cos\theta\, t^2$



$\Rightarrow t^2 = \dfrac{2L}{R\omega^2\cos\theta} = \Rightarrow t = 1\sqrt{\dfrac{2L}{R\omega^2\cos\theta}}$

29. v = Velocity of car = 36km/hr = 10 m/s
r = Radius of circular path = 50m
m = mass of small body = 100g = 0.1kg.
μ = Friction coefficient between plate & body = 0.58
a) The normal contact force exerted by the plate on the block

$$N = \frac{mv^2}{r} = \frac{0.1 \times 100}{50} = 0.2N$$

b) The plate is turned so the angle between the normal to the plate & the radius of the road slowly increases

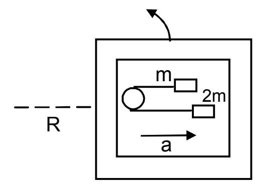
$$N = \frac{mv^2}{r} \cos\theta \quad ..(i)$$

$$\mu N = \frac{mv^2}{r} \sin\theta \quad ..(ii)$$

Putting value of N from (i)

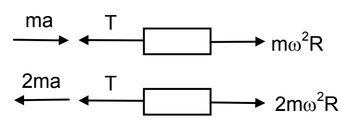
$$\mu \frac{mv^2}{r} \cos\theta = \frac{mv^2}{r} \sin\theta \Rightarrow \mu = \tan\theta \Rightarrow \theta = \tan^{-1}\mu = \tan^{-1}(0.58) = 30°$$

30. Let the bigger mass accelerates towards right with 'a'.



From the free body diagrams,
$T - ma - m\omega^2 R = 0$ …(i)
$T + 2ma - 2m\omega^2 R = 0$ …(ii)
Eq (i) – Eq (ii) $\Rightarrow$ $3ma = m\omega^2 R$

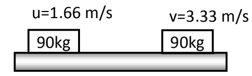
$$\Rightarrow a = \frac{m\omega^2 R}{3}$$

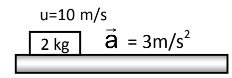


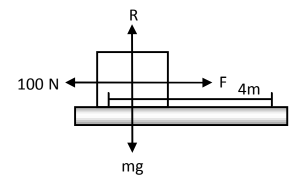
Substituting the value of a in Equation (i), we get T = 4/3 $m\omega^2 R$.

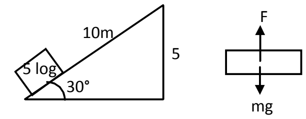
* * * *

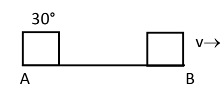
SOLUTIONS TO CONCEPTS
CHAPTER – 8

1. $M = m_c + m_b = 90kg$
   

   u = 6 km/h = 1.666 m/sec
   v = 12 km/h = 3.333 m/sec
   Increase in K.E. = $½ Mv^2 – ½ Mu^2$
   $= ½\ 90 \times (3.333)^2 – ½ \times 90 \times (1.66)^2 = 494.5 – 124.6 = 374.8 \approx 375$ J
2. $m_b = 2$ kg.
   u = 10 m/sec
   

   a = 3 m/aec^2
   t = 5 sec
   v = u + at = 10 + 3 I 5 = 25 m/sec.
   $\therefore$ F.K.E = $½\ mv^2 = ½ \times 2 \times 625 = 625$ J.
3. F = 100 N
   

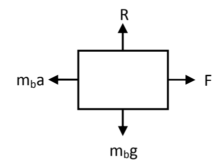
   S = 4m, $\theta = 0°$
   $\omega = \vec{F}.\vec{S} = 100 \times 4 = 400$ J
4. m = 5 kg
   $\theta = 30°$
   

   S = 10 m
   F = mg
   So, work done by the force of gravity
   $\omega = mgh = 5 \times 9.8 \times 5 = 245$ J
5. F= 2.50N, S = 2.5m, m =15g = 0.015kg.
   

   So, $w = F \times S \Rightarrow a = \frac{F}{m} = \frac{2.5}{0.015} = \frac{500}{3}\ m/s^2$
   =F × S cos 0° (acting along the same line)
   = 2.5 × 2.5 = 6.25J
   Let the velocity of the body at b = U. Applying work-energy principle $½\ mv^2 – 0 = 6.25$
   $\Rightarrow V = \sqrt{\frac{6.25 \times 2}{0.015}} = 28.86$ m/sec.
   So, time taken to travel from A to B.
   $\Rightarrow t = \frac{v-u}{a} = \frac{28.86 \times 3}{500}$
   $\therefore$ Average power $= \frac{W}{t} = \frac{6.25 \times 500}{(28.86) \times 3} = 36.1$
6. Given
   $\vec{r}_1 = 2\hat{i} + 3\hat{j}$
   $r_2 = 3\hat{i} + 2\hat{j}$
   So, displacement vector is given by,
   $\vec{r} = \vec{r}_1 - \vec{r}_2 \Rightarrow \vec{r} = (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j}) = \hat{i} - \hat{j}$

So, work done = $\vec{F} \times \vec{s}$ = 5 × 1 + 5(-1) = 0

7. m_b = 2kg, s = 40m, a = 0.5m/sec^2



So, force applied by the man on the box

F = $m_b a$ = 2 × (0.5) = 1 N

ω = FS = 1 × 40 = 40 J

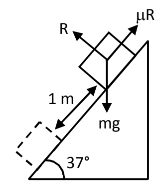
8. Given that F= a + bx

Where a and b are constants.

So, work done by this force during this force during the displacement x = 0 and x = d is given by

W = $\int_0^d F\,dx = \int_0^d (a+bx)dx$ = ax + (bx^2/2) = [a + ½ bd] d

9. m_b = 250g = .250 kg



θ = 37°, S = 1m.

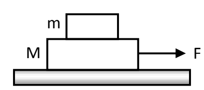
Frictional force f = μR

mg sin θ = μ R ..(1)

mg cos θ ..(2)

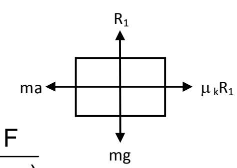
so, work done against μR = μRS cos 0° = mg sin θ S = 0.250 × 9.8 × 0.60 × 1 = 1.5 J

10. a = $\dfrac{F}{2(M+m)}$ (given)



a) from fig (1)

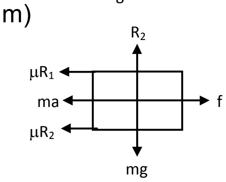
ma = μ_k R_1 and R_1 = mg



$\Rightarrow \mu = \dfrac{ma}{R_1} = \dfrac{F}{2(M+m)g}$

b) Frictional force acting on the smaller block f = μR = $\dfrac{F}{2(M+m)g} \times mg = \dfrac{m \times F}{2(M+m)}$

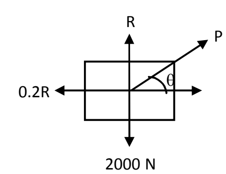
c) Work done w = fs s = d



$$w = \frac{mF}{2(M+m)} \times d = \frac{mFd}{2(M+m)}$$

11. Weight = 2000 N, S = 20m, μ = 0.2

a) R + Psinθ - 2000 = 0 ..(1)



P cosθ - 0.2 R =0 ..(2)

From (1) and (2) P cosθ – 0.2 (2000 – P sinθ)=0

P = $\dfrac{400}{\cos\theta + 0.2\sin\theta}$ ..(3)

So, work done by the person, W = PS cosθ = $\dfrac{8000\cos\theta}{\cos\theta + 0.2\sin\theta} = \dfrac{8000}{1+0.2\sin\theta} = \dfrac{40000}{5+\tan\theta}$

b) For minimum magnitude of force from equn(1)

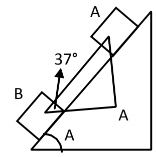
d/dθ (cos θ + 0.2 sinθ) = 0 $\Rightarrow$ tan θ = 0.2

putting the value in equn (3)

W = $\dfrac{40000}{5+\tan\theta} = \dfrac{40000}{(5.2)}$ = 7690 J

12. w = 100 N, θ = 37°, s = 2m

Force F= mg sin 37° = 100 × 0.60 = 60 N



So, work done, when the force is parallel to incline.

w = Fs cos θ = 60 × 2 × cos θ = 120 J

In ΔABC AB= 2m

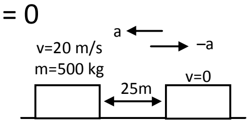
CB = 37°

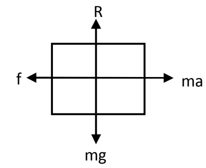
so, h = C = 1m

$\therefore$ work done when the force in horizontal direction

W = mgh = 100 × 1.2 = 120 J

13. m = 500 kg, s = 25m, u = 72km/h= 20 m/s, v = 0

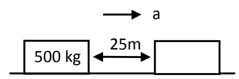


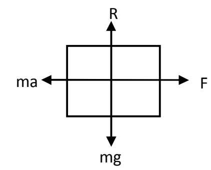


(-a) = $\dfrac{v^2 - u^2}{2S}$ $\Rightarrow$ a = $\dfrac{400}{50}$ = 8m/sec^2

Frictional force f = ma = 500 × 8 = 4000 N

14. m = 500 kg, u = 0, v = 72 km/h = 20m/s





a = $\dfrac{v^2 - u^2}{2s} = \dfrac{400}{50}$ = 8m/sec^2

force needed to accelerate the car F = ma = 500 × 8 = 4000 N

15. Given, v = a$\sqrt{x}$ (uniformly accelerated motion)

displacement s = d – 0 = d

putting x = 0, v_1= 0

putting x = d, v_2= a$\sqrt{d}$

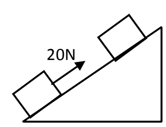
a = $\dfrac{{v_2}^2 - {u_2}^2}{2s} = \dfrac{a^2 d}{2d} = \dfrac{a^2}{2}$

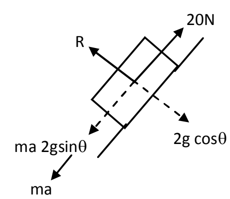
force f = ma = $\dfrac{ma^2}{2}$

work done w = FS cos θ = $\dfrac{ma^2}{2} \times d = \dfrac{ma^2 d}{2}$

16. a) m = 2kg, θ = 37°, F = 20 N

From the free body diagram





F = (2g sin θ) + ma $\Rightarrow$ a = (20 – 20 sinθ)/s = 4m/sec^2

S = ut + ½ at^2 (u = 0, t = 1s, a = 1.66)

= 2m

So, work, done w = Fs = 20 × 2 = 40 J

b) If W = 40 J

S = $\dfrac{W}{F} = \dfrac{40}{20}$

h = 2 sin 37° = 1.2 m

So, work done W = –mgh = – 20 × 1.2 = –24 J

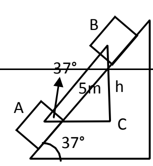
c) v = u + at = 4 × 10 = 40 m/sec

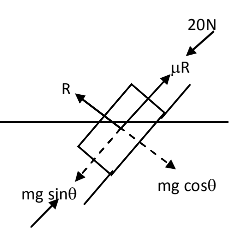
So, K.E. = ½ mv^2 = ½ × 2 × 16 = 16 J

17. m = 2kg, θ = 37°, F = 20 N, a = 10 m/sec^2

a) t = 1sec

So, s= ut + ½ at^2 = 5m





Work done by the applied force w = FS cos 0° = 20 × 5 = 100 J

b) BC (h) = 5 sin 37° = 3m

So, work done by the weight W = mgh = 2 × 10 × 3 = 60 J

c) So, frictional force f = mg sinθ

work done by the frictional forces w = fs cos0° = (mg sinθ) s = 20 × 0.60 × 5 = 60 J

18. Given, m = 25o g = 0.250kg,

u = 40 cm/sec = 0.4m/sec

μ = 0.1, v=0

Here, μ R = ma {where, a = deceleration}

$a = \frac{\mu R}{m} = \frac{\mu mg}{m} = \mu g = 0.1 \times 9.8 = 0.98 \text{ m/sec}^2$

$S = \frac{v^2 - u^2}{2a} = 0.082\text{m} = 8.2 \text{ cm}$

Again, work done against friction is given by

– w = μ RS cos θ

= 0.1 × 2.5 × 0.082 × 1 (θ = 0°) = 0.02 J

$\Rightarrow$ W = – 0.02 J

19. h = 50m, m = 1.8×10^5 kg/hr, P = 100 watt,

P.E. = mgh = $1.8 \times 10^5 \times 9.8 \times 50 = 882 \times 10^5$ J/hr

Because, half the potential energy is converted into electricity,

Electrical energy ½ P.E. = 441×10^5 J/hr

So, power in watt (J/sec) is given by = $\frac{441 \times 10^5}{3600}$

$\therefore$ number of 100 W lamps, that can be lit $\frac{441 \times 10^5}{3600 \times 100} = 122.5 \approx 122$

20. m = 6kg, h = 2m

P.E. at a height '2m' = mgh = 6 × (9.8) × 2 = 117.6 J

P.E. at floor = 0

Loss in P.E. = 117.6 – 0 = 117. 6 J $\approx$ 118 J

21. h = 40m, u = 50 m/sec

Let the speed be 'v' when it strikes the ground.

Applying law of conservation of energy

mgh + ½ mu^2 = ½ mv^2

$\Rightarrow 10 \times 40 + (1/2) \times 2500 = ½ v^2 \Rightarrow v^2 = 3300 \Rightarrow v = 57.4 \text{ m/sec} \approx 58 \text{ m/sec}$

22. t = 1 min 57.56 sec = 11.56 sec, p= 400 W, s =200 m

$p = \frac{w}{t}$, Work w = pt = 460 × 117.56 J

Again, W = FS = $\frac{460 \times 117.56}{200} = 270.3 \text{ N} \approx 270 \text{ N}$

23. S = 100 m, t = 10.54 sec, m = 50 kg

The motion can be assumed to be uniform because the time taken for acceleration is minimum.

a) Speed v = S/t = 9.487 e/s

So, K.E. = ½ mv^2 = 2250 J

b) Weight = mg = 490 J

given R = mg /10 = 49 J

so, work done against resistance W_F = – RS = – 49 × 100 = – 4900 J

c) To maintain her uniform speed, she has to exert 4900 j of energy to over come friction

$P = \frac{W}{t}$ = 4900 / 10.54 = 465 W

24. h = 10 m

flow rate = (m/t) = 30 kg/min = 0.5 kg/sec

power $P = \frac{mgh}{t}$ = (0.5) × 9.8 × 10 = 49 W

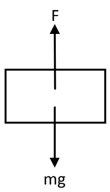
So, horse power (h.p) P/746 = 49/746 = 6.6×10^{-2}hp

25. m = 200g = 0.2kg, h = 150cm = 1.5m, v = 3m/sec, t = 1 sec

Total work done = ½ mv^2 + mgh = (1/2) × (0.2) ×9 + (0.2) × (9.8) × (1.5) = 3.84 J

h.p. used = $\frac{3.84}{746} = 5.14 \times 10^{-3}$

26. m = 200 kg, s = 12m, t = 1 min = 60 sec



So, work W = F cos θ = mgs cos0° [θ = 0°, for minimum work]

= 2000 × 10 × 12 = 240000 J

So, power $p = \frac{W}{t} = \frac{240000}{60}$ = 4000 watt

h.p = $\frac{4000}{746}$ = 5.3 hp.

27. The specification given by the company are

U = 0, m = 95 kg, P_m = 3.5 hp

V_m = 60 km/h = 50/3 m/sec t_m = 5 sec

So, the maximum acceleration that can be produced is given by,

$a = \frac{(50/3)-0}{5} = \frac{10}{3}$

So, the driving force is given by

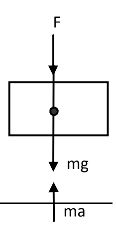
$F = ma = 95 \times \frac{10}{3} = \frac{950}{3}$ N

So, the velocity that can be attained by maximum h.p. white supplying $\frac{950}{3}$ will be

$v = \frac{p}{F} \Rightarrow v = \frac{3.5 \times 746 \times 5}{950}$ = 8.2 m/sec.

Because, the scooter can reach a maximum of 8.s m/sec while producing a force of 950/3 N, the specifications given are some what over claimed.

28. Given m = 30kg, v = 40 cm/sec = 0.4 m/sec s = 2m



From the free body diagram, the force given by the chain is,

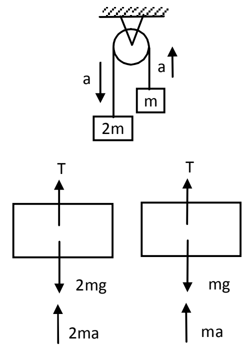
F = (ma – mg) = m(a – g) [where a = acceleration of the block]

$a = \frac{(v2\ \ u2)}{2s} = \frac{0.16}{0.4} = 0.04\ m/sec^2$

So, work done $W = Fs\cos\theta = m(a-g)\,s\cos\theta$

$\Rightarrow W = 30\,(0.04 - 9.8) \times 2 \Rightarrow W = -585.5 \Rightarrow W = -586$ J.

So, $W = -586$ J



29. Given, T = 19 N

From the freebody diagrams,

$T - 2\,mg + 2\,ma = 0$ ...(i)

$T - mg - ma = 0$ ...(ii)

From, Equation (i) & (ii) $T = 4ma \Rightarrow a = \dfrac{T}{4m} \Rightarrow A = \dfrac{16}{4m} = \dfrac{4}{m}$ m/s^2.

Now, $S = ut + \frac{1}{2}at^2$

$\Rightarrow S = \dfrac{1}{2} \times \dfrac{4}{m} \times 1 \Rightarrow S = \dfrac{2}{m}$ m [because u=0]

Net mass = 2m – m = m

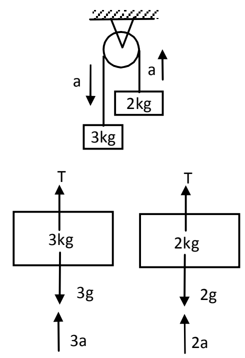
Decrease in P.E. = mgh $\Rightarrow$ P.E. $= m \times g \times \dfrac{2}{m} \Rightarrow$ P.E. $= 9.8 \times 2 \Rightarrow$ P.E. = 19.6 J

30. Given, $m_1 = 3$ kg, $m_2 = 2$kg, t = during 4th second

From the freebody diagram

$T - 3g + 3a = 0$ ..(i)

$T - 2g - 2a = 0$ ..(ii)



Equation (i) & (ii), we get $3g - 3a = 2g + 2a \Rightarrow a = \dfrac{g}{5}$ m/sec^2

Distance travelled in 4th sec is given by

$$S_{4th} = \frac{a}{2}(2n-1) = \frac{\left(\frac{g}{5}\right)}{s}(2\times4-1) = \frac{7g}{10} = \frac{7\times9.8}{10}\ \text{m}$$

Net mass ‘m’ $= m_1 - m_2 = 3 - 2 = 1$kg

So, decrease in P.E. = mgh $= 1 \times 9.8 \times \dfrac{7}{10} \times 9.8 = 67.2 = 67$ J

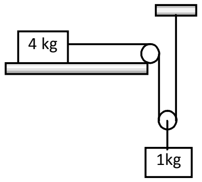
31. $m_1 = 4$kg, $m_2 = 1$kg, $V_2 = 0.3$m/sec $V_1 = 2 \times (0.3) = 0.6$ m/sec

($v_1 = 2x_2$ m this system)

h = 1m = height descent by 1kg block

s = 2 × 1 = 2m distance travelled by 4kg block

u = 0



Applying change in K.E. = work done (for the system)

$[(1/2)m_1v_1^2 + (1/2)\,m_2v_m^2] - 0 = (-\mu R)S + m_2g$ [R = 4g = 40 N]

$\Rightarrow \frac{1}{2} \times 4 \times (0.36) \times \frac{1}{2} \times 1 \times (0.09) = -\mu \times 40 \times 2 + 1 \times 40 \times 1$

$\Rightarrow 0.72 + 0.045 = -80\mu + 10$

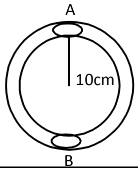
$\Rightarrow \mu = \dfrac{9.235}{80} = 0.12$

32. Given, m = 100g = 0.1kg, v = 5m/sec, r = 10cm

Work done by the block = total energy at A – total energy at B

$(1/2\ mv^2 + mgh) - 0$

$\Rightarrow W = \frac{1}{2}mv^2 + mgh - 0 = \frac{1}{2} \times (0.1) \times 25 + (0.1) \times 10 \times (0.2)$ [h = 2r = 0.2m]

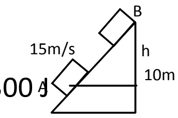


$\Rightarrow W = 1.25 - 0.2 \Rightarrow W = 1.45$ J

So, the work done by the tube on the body is

$W_t = -1.45$ J

33. m = 1400kg, v = 54km/h = 15m/sec, h = 10m



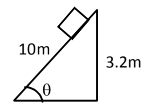
Work done = (total K.E.) – total P.E.

$= 0 + \frac{1}{2} mv^2 - mgh = \frac{1}{2} \times 1400 \times (15)^2 - 1400 \times 9.8 \times 10 = 157500 - 137200 = 20300$ J

So, work done against friction, $W_t = 20300$ J

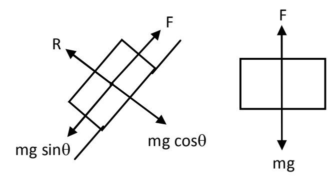
34. m = 200g = 0.2kg, s = 10m, h = 3.2m, g = 10 m/sec^2

a) Work done $W = mgh = 0.2 \times 10 \times 3.2 = 6.4$ J



b) Work done to slide the block up the incline

$w = (mg \sin\theta) = (0.2) \times 10 \times \frac{3.2}{10} \times 10 = 6.4$ J



c) Let, the velocity be v when falls on the ground vertically,

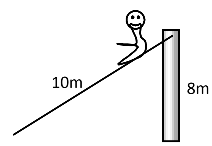
$\frac{1}{2} mv^2 - 0 = 6.4J \Rightarrow v = 8$ m/s

d) Let V be the velocity when reaches the ground by liding

$\frac{1}{2} mV^2 - 0 = 6.4 J \Rightarrow V = 8$m/sec

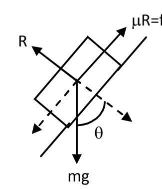
35. $\ell = 10$m, h = 8m, mg = 200N

$f = 200 \times \frac{3}{10} = 60$N



a) Work done by the ladder on the boy is zero when the boy is going up because the work is done by the boy himself.

b) Work done against frictional force, $W = \mu RS = f\ell = (-60) \times 10 = -600$ J



c) Work done by the forces inside the boy is

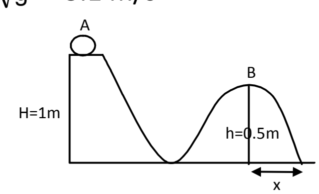
$W_b = (mg \sin\theta) \times 10 = 200 \times \frac{8}{10} \times 10 = 1600$ J

36. H = 1m, h = 0.5m

Applying law of conservation of Energy for point A & B

$mgH = \frac{1}{2} mv^2 + mgh \Rightarrow g = (1/2) v^2 + 0.5g \Rightarrow v^2\ 2(g - 0.59) = g \Rightarrow v = \sqrt{g} = 3.1$ m/s

After point B the body exhibits projectile motion for which



$\theta = 0°$, $v = -0.5$

So, $-0.5 = (u \sin\theta)\, t - (1/2) gt^2 \Rightarrow 0.5 = 4.9\, t^2 \Rightarrow t = 0.31$ sec.

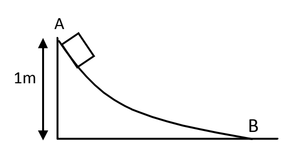
So, $x = (4 \cos\theta)\, t = 3.1 \times 3.1 = 1$m.

So, the particle will hit the ground at a horizontal distance in from B.

37. mg = 10N, $\mu = 0.2$, H = 1m, u = v = 0

change in P.E. = work done.

Increase in K.E.



$\Rightarrow w = mgh = 10 \times 1 = 10$ J

Again, on the horizontal surface the fictional force

$F = \mu R = \mu mg = 0.2 \times 10 = 2$ N

So, the K.E. is used to overcome friction

$\Rightarrow S = \frac{W}{F} = \frac{10J}{2N} = 5$m

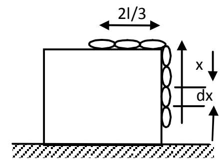
38. Let 'dx' be the length of an element at a distance × from the table

mass of 'dx' length = (m/ℓ) dx

Work done to put dx part back on the table

W = (m/ℓ) dx g(x)

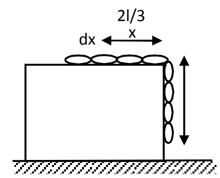
So, total work done to put $\ell/3$ part back on the table



$$W = \int_0^{1/3} (m/\ell)gx\,dx \Rightarrow w = (m/\ell)\,g\left[\frac{x^2}{2}\right]_0^{\ell/3} = \frac{mg\ell^2}{18\ell} = \frac{mg\ell}{18}$$

39. Let, x length of chain is on the table at a particular instant.

So, work done by frictional force on a small element 'dx'



$dW_f = \mu Rx = \mu\left(\frac{M}{L}dx\right)gx$ [where dx = $\frac{M}{L}dx$]

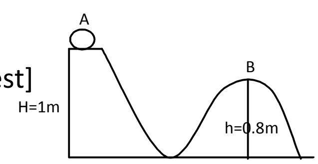
Total work don by friction,

$$W_f = \int_{2L/3}^{0} \mu\frac{M}{L}gx\ dx$$

$$\therefore W_f = \mu\frac{m}{L}g\left[\frac{x^2}{2}\right]_{2L/3}^{0} = \mu\frac{M}{L}\left[\frac{4L^2}{18}\right] = 2\mu Mg\ L/9$$

40. Given, m = 1kg, H = 1m, h = 0.8m

Here, work done by friction = change in P.E. [as the body comes to rest]



$\Rightarrow W_f = mgh - mgH$

= mg (h – H)

= 1 × 10 (0.8 – 1) = – 2J

41. m = 5kg, x = 10cm = 0.1m, v = 2m/sec,

h =? G = $10 m/sec^2$

S0, $k = \frac{mg}{x} = \frac{50}{0.1} = 500$ N/m

Total energy just after the blow E = $\frac{1}{2} mv^2 + \frac{1}{2} kx^2$ ...(i)

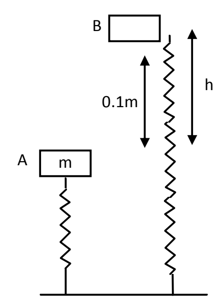
Total energy a a height h = $\frac{1}{2} k(h-x)^2 + mgh$ ...(ii)

$\frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} k(h-x)^2 + mgh$

On, solving we can get,

H = 0.2 m = 20 cm

42. m = 250 g = 0.250 kg,



k = 100 N/m, m = 10 cm = 0.1m

g = $10 m/sec^2$

Applying law of conservation of energy

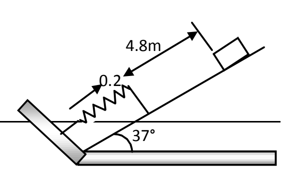
$$\frac{1}{2}kx^2 = mgh \Rightarrow h = \frac{1}{2}\left(\frac{kx^2}{mg}\right) = \frac{100\times(0.1)^2}{2\times0.25\times10} = 0.2\text{ m} = 20\text{ cm}$$

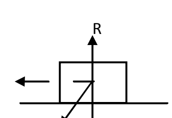
43. m = 2kg, s_1 = 4.8m, x = 20cm = 0.2m, s_2 = 1m,

sin 37° = 0.60 = 3/5, θ = 37°, cos 37° = .79 = 0.8 = 4/5

g = $10 m/sec^2$

Applying work – Energy principle for downward motion of the body





$0 - 0 = mg \sin 37° \times 5 - \mu R \times 5 - ½\, kx^2$

$\Rightarrow 20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½\, k\,(0.2)^2 = 0$

$\Rightarrow 60 - 80\mu - 0.02k = 0 \Rightarrow 80\mu + 0.02k = 60$ ...(i)

Similarly, for the upward motion of the body the equation is

$0 - 0 = (-mg \sin 37°) \times 1 - \mu\, R \times 1 + ½\, k\,(0.2)^2$

$\Rightarrow -20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½\, k\,(0.2)^2 = 0$

$\Rightarrow -12 - 16\mu + 0.02\,K = 0$ ..(ii)

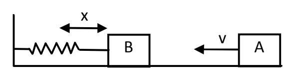
Adding equation (i) & equation (ii), we get $96\,\mu = 48$

$\Rightarrow \mu = 0.5$

Now putting the value of μ in equation (i) K = 1000N/m

44. Let the velocity of the body at A be v

So, the velocity of the body at B is v/2

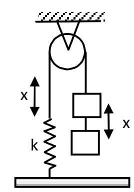


Energy at point A = Energy at point B

So, $½\, mv_A^2 = ½\, mv_B^2 + ½\, kx^{2+}$

$\Rightarrow ½\, kx^2 = ½\, mv_A^2 - ½\, mv_B^2 \Rightarrow kx^2 = m\,(v_A^{2+} - v_B^2) \Rightarrow kx^2 = m\left(v^2 - \frac{v^2}{4}\right) \Rightarrow k = \frac{3mv^2}{3x^2}$

45. Mass of the body = m



Let the elongation be x

So, $½\, kx^2 = mgx$

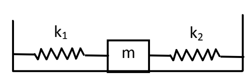
$\Rightarrow x = 2mg / k$

46. The body is displaced x towards right

Let the velocity of the body be v at its mean position

Applying law of conservation of energy

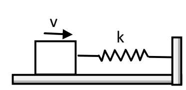
$½\, mv^2 = ½\, k_1x^2 + ½\, k_2x^2 \Rightarrow mv^2 = x^2\,(k_1 + k_2) \Rightarrow v^2 = \frac{x^2(k_1+k_2)}{m}$



$\Rightarrow v = x\sqrt{\frac{k_1+k_2}{m}}$

47. Let the compression be x

According to law of conservation of energy

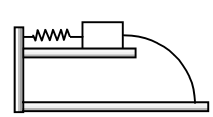


$½\, mv^2 = ½\, kx^2 \Rightarrow x^2 = mv^2 / k \Rightarrow x = v\sqrt{(m/k)}$

b) No. It will be in the opposite direction and magnitude will be less due to loss in spring.

48. m = 100g = 0.1kg, x = 5cm = 0.05m, k = 100N/m

when the body leaves the spring, let the velocity be v



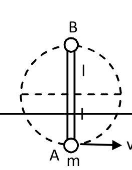
$½\, mv^2 = ½\, kx^2 \Rightarrow v = x\sqrt{k/m} = 0.05 \times \sqrt{\frac{100}{0.1}} = 1.58\text{m/sec}$

For the projectile motion, $\theta = 0°$, Y = –2

Now, $y = (u \sin \theta)t - ½\, gt^2$

$\Rightarrow -2 = (-1/2) \times 9.8 \times t^2 \Rightarrow t = 0.63$ sec.

So, $x = (u \cos \theta)\, t \Rightarrow 1.58 \times 0.63 = 1\text{m}$

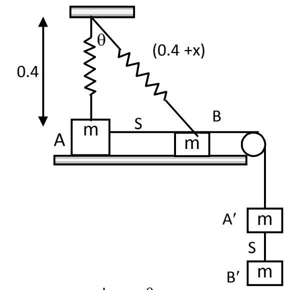


49. Let the velocity of the body at A is 'V' for minimum velocity given at A velocity of the body at point B is zero.

Applying law of conservation of energy at A & B

$\frac{1}{2} mv^2 = mg (2\ell) \Rightarrow v = \sqrt{(4g\ell)} = 2\sqrt{g\ell}$

50. m = 320g = 0.32kg



k = 40N/m

h = 40cm = 0.4m

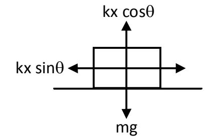
$g = 10\ m/s^2$

From the free body diagram,

$kx \cos\theta = mg$

(when the block breaks off R = 0)

$\Rightarrow \cos\theta = mg/kx$



So, $\frac{0.4}{0.4+x} = \frac{3.2}{40 \times x} \Rightarrow 16x = 3.2x + 1.28 \Rightarrow x = 0.1$ m

S0, s = AB = $\sqrt{(h+x)^2 - h^2} = \sqrt{(0.5)^2 - (0.4)^2} = 0.3$ m

Let the velocity of the body at B be v

Charge in K.E. = work done (for the system)

$(1/2\ mv^2 + \frac{1}{2} mv^2) = -1/2\ kx^2 + mgs$

$\Rightarrow (0.32) \times v^2 = -(1/2) \times 40 \times (0.1)^2 + 0.32 \times 10 \times (0.3) \Rightarrow v = 1.5$ m/s.

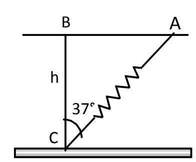
51. $\theta = 37°$; l = h = natural length

Let the velocity when the spring is vertical be 'v'.

Cos 37° = BC/AC = 0.8 = 4/5

Ac = (h + x) = 5h/4 (because BC = h)

So, x = (5h/4) – h = h/4



Applying work energy principle $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$\Rightarrow v = x\sqrt{(k/m)} = \frac{h}{4}\sqrt{\frac{k}{m}}$

52. The minimum velocity required to cross the height point c = $\sqrt{2gl}$

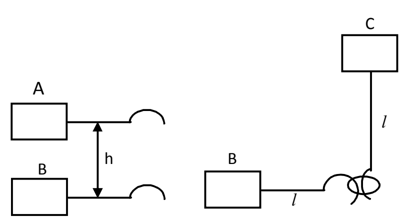
Let the rod released from a height h.

Total energy at A = total energy at B

$mgh = 1/2\ mv^2$; $mgh = 1/2\ m\ (2gl)$

[Because v = required velocity at B such that the block makes a complete circle. [Refer Q – 49]

So, h = l.



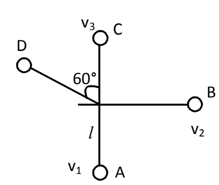
53. a) Let the velocity at B be v_2

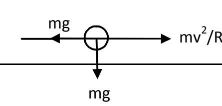
$1/2\ mv_1^2 = 1/2\ mv_2^2 + mgl$

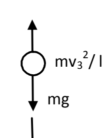
$\Rightarrow 1/2\ m\ (10\ gl) = 1/2\ mv_2^2 + mgl$

$v_2^2 = 8\ gl$

So, the tension in the string at horizontal position







$T = \frac{mv^2}{R} = \frac{m8gl}{l} = 8\ mg$

b) Let the velocity at C be V_3

$1/2\ mv_1^2 = 1/2\ mv_3^2 + mg\ (2l)$

$\Rightarrow 1/2\ m\ (\log l) = 1/2\ mv_3^2 + 2mgl$

$\Rightarrow v_3^2 = 6\ mgl$

So, the tension in the string is given by

$T_c = \frac{mv^2}{l} - mg = \frac{6\ glm}{l}\ mg = 5\ mg$

c) Let the velocity at point D be v_4

Again, $1/2\ mv_1^2 = 1/2\ mv_4^2 + mgh$

$1/2 \times m \times (10\ gl) = 1.2\ mv_4^2 + mgl\ (1 + \cos 60°)$

$\Rightarrow v_4^2 = 7\ gl$

So, the tension in the string is

$T_D = (mv^2/l) - mg \cos 60°$

$= m(7\ gl)/l - l - 0.5\ mg \Rightarrow 7\ mg - 0.5\ mg = 6.5\ mg.$

54. From the figure, $\cos\theta = AC/AB$

$\Rightarrow AC = AB \cos\theta \Rightarrow (0.5) \times (0.8) = 0.4.$

So, $CD = (0.5) - (0.4) = (0.1)\ m$

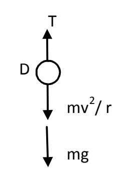
Energy at D = energy at B

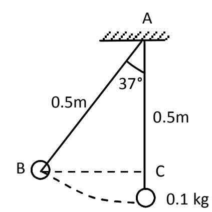
$1/2\ mv^2 = mg\ (CD)$

$v^2 = 2 \times 10 \times (0.1) = 2$

So, the tension is given by,

$T = \frac{mv^2}{r} + mg = (0.1)\left(\frac{2}{0.5} + 10\right) = 1.4\ N.$





55. Given, N = mg

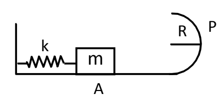
As shown in the figure, $mv^2 / R = mg$

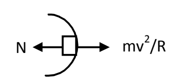
$\Rightarrow v^2 = gR$ ...(1)

Total energy at point A = energy at P

$1/2\ kx^2 = \frac{mgR + 2mgR}{2}$ [because $v^2 = gR$]

$\Rightarrow x^2 = 3mgR/k \Rightarrow x = \sqrt{(3mgR)/k}$.





56. $V = \sqrt{3gl}$

$1/2\ mv^2 - 1/2\ mu^2 = -mgh$

$v^2 = u^2 - 2g(l + l\cos\theta)$

$\Rightarrow v^2 = 3gl - 2gl\ (1 + \cos\theta)$ ...(1)

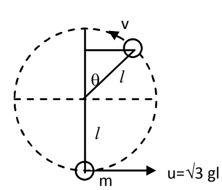
Again,

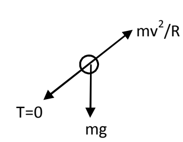
$mv^2/l = mg \cos\theta$

$v^2 = lg \cos\theta$

From equation (1) and (2), we get

$3gl - 2gl - 2gl \cos\theta = gl \cos\theta$





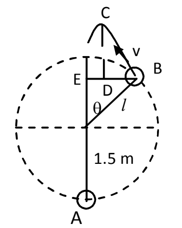
$3 \cos\theta = 1 \Rightarrow \cos\theta = 1/3$

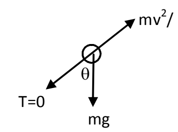
$\theta = \cos^{-1}(1/3)$

So, angle rotated before the string becomes slack

$= 180° - \cos^{-1}(1/3) = \cos^{-1}(-1/3)$

57. $l = 1.5$ m; $u = \sqrt{57}$ m/sec.





a) $mg \cos\theta = mv^2 / l$

$v^2 = lg \cos\theta$ ...(1)

change in K.E. = work done

$1/2\ mv^2 - 1/2\ mu^2 = mgh$

$\Rightarrow v^2 - 57 = -2 \times 1.5\ g\ (1 + \cos\theta)$...(2)

$\Rightarrow v^2 = 57 - 3g(1 + \cos\theta)$

Putting the value of v from equation (1)

$15 \cos\theta = 57 - 3g\ (1 + \cos\theta) \Rightarrow 15 \cos\theta = 57 - 30 - 30 \cos\theta$

$\Rightarrow 45 \cos\theta = 27 \Rightarrow \cos\theta = 3/5.$

$\Rightarrow \theta = \cos^{-1}(3/5) = 53°$

b) $v = \sqrt{57 - 3g(1+\cos\theta)}$ from equation (2)

$= \sqrt{9} = 3$ m/sec.

c) As the string becomes slack at point B, the particle will start making projectile motion.

$$H = OE + DC = 1.5 \cos\theta + \frac{u^2 \sin^2\theta}{2g}$$

$$= (1.5) \times (3/5) + \frac{9 \times (0.8)^2}{2 \times 10} = 1.2 \text{ m.}$$

58.

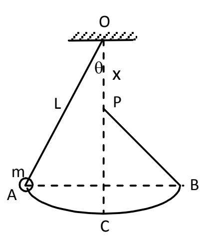


Fig-1

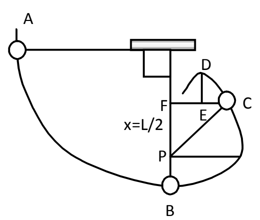


Fig-2

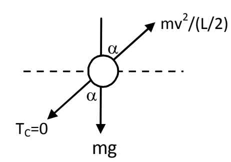


Fig-3

a) When the bob has an initial height less than the peg and then released from rest (figure 1), let body travels from A to B.

Since, Total energy at A = Total energy at B

$\therefore (K.E)_A = (PE)_A = (KE)_B + (PE)_B$

$\Rightarrow$ $(PE)_A = (PE)_B$ [because, $(KE)_A = (KE)_B = 0$]

So, the maximum height reached by the bob is equal to initial height.

b) When the pendulum is released with $\theta = 90°$ and $x = L/2$, (figure 2) the path of the particle is shown in the figure 2.

At point C, the string will become slack and so the particle will start making projectile motion. (Refer Q.No. 56)

$(1/2)mv_c^2 - 0 = mg\ (L/2)\ (1 - \cos\alpha)$

because, distance between A nd C in the vertical direction is L/2 (1 – cos α)

$\Rightarrow v_c^2 = gL(1 - \cos\theta)$ ..(1)

Again, form the freebody diagram (fig – 3)

$\frac{mv_c^2}{L/2} = mg\cos\alpha$ {because $T_c = 0$}

So, $V_C^2 = \frac{gL}{2}\cos\alpha$ ..(2)

From Eqn.(1) and equn (2),

$gL(1 - \cos\alpha) = \frac{gL}{2}\cos\alpha$

$\Rightarrow 1 - \cos\alpha = 1/2\cos\alpha$

$\Rightarrow 3/2\cos\alpha = 1 \Rightarrow \cos\alpha = 2/3$ ..(3)

To find highest position C, before the string becomes slack

$BF = \frac{L}{2} + \frac{L}{2}\cos\theta = \frac{L}{2} + \frac{L}{2} \times \frac{2}{3} = L\left(\frac{1}{2} + \frac{1}{3}\right)$

So, BF = (5L/6)

c) If the particle has to complete a vertical circle, at the point C.

$\frac{mv_c^2}{(L-x)} = mg$

$\Rightarrow v_c^2 = g(L - x)$ ..(1)

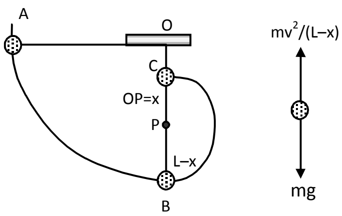
Again, applying energy principle between A and C,

$1/2\ mv_c^2 - 0 = mg\ (OC)$

$\Rightarrow 1/2\ v_c^2 = mg[L - 2(L - x)] = mg(2x - L)$

$\Rightarrow v_c^2 = 2g(2x - L)$ ..(2)

From equn. (1) and equn (2)



$g(L - x) = 2g(2x - L)$

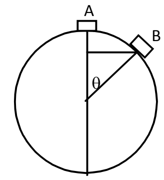
$\Rightarrow L - x = 4x - 2L$

$\Rightarrow 5x = 3L$

$\therefore \frac{x}{L} = \frac{3}{5} = 0.6$

So, the rates (x/L) should be 0.6

59. Let the velocity be v when the body leaves the surface.

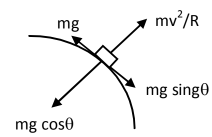


From the freebody diagram,

$\frac{mv^2}{R} = mg\cos\theta$ [Because normal reaction]

$v^2 = Rg\cos\theta$ ..(1)

Again, form work-energy principle,



Change in K.E. = work done

$\Rightarrow 1/2\ mv^2 - 0 = mg(R - R\cos\theta)$

$\Rightarrow v^2 = 2gR(1 - \cos\theta)$ ..(2)

From (1) and (2)

$Rg\cos\theta = 2gR(1 - \cos\theta)$

$3gR \cos\theta = 2\ gR$

$\text{Cos}\ \theta = 2/3$

$\theta = \cos^{-1}(2/3)$

60. a) When the particle is released from rest (fig–1), the centrifugal force is zero.

N force is zero = mg cos θ

$= mg \cos 30° = \frac{\sqrt{3}mg}{2}$

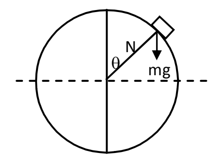


Fig-1

b) When the particle leaves contact with the surface (fig–2), N = 0.

So, $\frac{mv^2}{R}$ mg cos θ

$\Rightarrow v^2 = Rg \cos\theta \quad ..(1)$

Again, ½ $mv^2 = mgR\ (\cos 30° - \cos\theta)$

$\Rightarrow v^2 = 2Rg\left(\frac{\sqrt{3}}{2} - \cos\theta\right) \quad ..(2)$

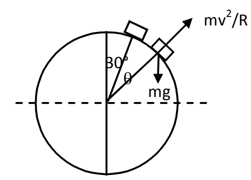


Fig-2

From equn. (1) and equn. (2)

$Rg \cos\theta = \sqrt{3}\ Rg - 2Rg \cos\theta$

$\Rightarrow 3 \cos\theta = \sqrt{3}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

So, the distance travelled by the particle before leaving contact,

$\ell = R(\theta - \pi/6)$ [because $30° = \pi/6$]

putting the value of θ, we get $\ell = 0.43R$

61. a) Radius =R

horizontal speed = v

From the free body diagram, (fig–1)

N = Normal force = mg - $\frac{mv^2}{R}$

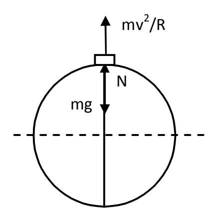


Fig-1

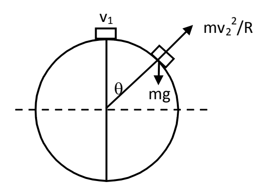


Fig-2

b) When the particle is given maximum velocity so that the centrifugal force balances the weight, the particle does not slip on the sphere.

$\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$

c) If the body is given velocity v_1

$v_1 = \sqrt{gR}/2$

$v_1^2 - gR/4$

Let the velocity be v_2 when it leaves contact with the surface, (fig–2)

So, $\frac{mv^2}{R} = mg \cos\theta$

$\Rightarrow v_2^2 = Rg \cos\theta \quad ..(1)$

Again, $1/2\ mv_2^2 - 1/2\ mv_1^2 = mgR\ (1 - \cos\theta)$

$\Rightarrow v_2^2 = v_1^2 + 2gR\ (1 - \cos\theta) \qquad ..(2)$

From equn. (1) and equn (2)

$Rg \cos\theta = (Rg/4) + 2gR(1 - \cos\theta)$

$\Rightarrow \cos\theta = (1/4) + 2 - 2\cos\theta$

$\Rightarrow 3\cos\theta = 9/4$

$\Rightarrow \cos\theta = 3/4$

$\Rightarrow \theta = \cos^{-1}(3/4)$

62. a) Net force on the particle between A & B, $F = mg\sin\theta$

work done to reach B, $W = FS = mg\sin\theta\,\ell$

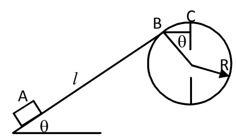
Again, work done to reach B to C = mgh = $mg R(1 - \cos\theta)$

So, Total workdone = $mg[\ell\sin\theta + R(1 - \cos\theta)]$

Now, change in K.E. = work done

$\Rightarrow 1/2\, mv_O^2 = mg[\ell\sin\theta + R(1 - \cos\theta)]$

$\Rightarrow v_o = \sqrt{2g(R(1-\cos\theta) + \ell\sin\theta)}$



b) When the block is projected at a speed $2v_o$.

Let the velocity at C will be V_c.

Applying energy principle,

$1/2\, mv_c^2 - 1/2\, m(2v_o)^2 = -mg[\ell\sin\theta + R(1 - \cos\theta)]$

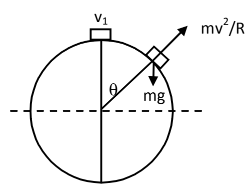
$\Rightarrow v_c^2 = 4v_o - 2g[\ell\sin\theta + R(1 - \cos\theta)]$

$4.2g[\ell\sin\theta + R(1 - \cos\theta)] - 2g[\ell\sin\theta + R(1 - \cos\theta)$

$= 6g[\ell\sin\theta + R(1 - \cos\theta)]$

So, force acting on the body,

$\Rightarrow N = \frac{mv_c^2}{R} = 6mg[(\ell/R)\sin\theta + 1 - \cos\theta]$



c) Let the block loose contact after making an angle θ

$\frac{mv^2}{R} = mg\cos\theta \Rightarrow v^2 = Rg\cos\theta$ ..(1)

Again, $1/2\, mv^2 = mg(R - R\cos\theta) \Rightarrow v^2 = 2gR(1 - \cos\theta)$ ..(2)........(?)

From (1) and (2) $\cos\theta = 2/3 \Rightarrow \theta = \cos^{-1}(2/3)$

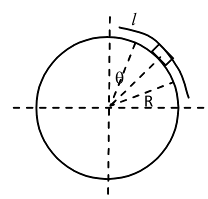
63. Let us consider a small element which makes angle ‘$d\theta$’ at the centre.

$\therefore dm = (m/\ell)R\,d\theta$

a) Gravitational potential energy of ‘dm’ with respect to centre of the sphere

$= (dm)g\,R\cos\theta$

$= (mg/\ell)\,R\cos\theta\; d\theta$



So, Total G.P.E. $= \int_0^{\ell/r} \frac{mgR^2}{\ell}\cos\theta\, d\theta$ [$\alpha = (\ell/R)$](angle subtended by the chain at the centre)..........

$= \frac{mR^2g}{\ell}[\sin\theta](\ell/R) = \frac{mRg}{\ell}\sin(\ell/R)$

b) When the chain is released from rest and slides down through an angle θ, the K.E. of the chain is given

K.E. = Change in potential energy.

$= \frac{mR^2g}{\ell}\sin(\ell/R) - m\int \frac{gR^2}{\ell}\cos\theta\, d\theta$..........?

$= \frac{mR^2g}{\ell}[\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

c) Since, K.E. = $1/2\, mv^2 = \frac{mR^2g}{\ell}[\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

Taking derivative of both sides with respect to 't'

$(1/2) \times 2v \times \frac{dv}{dt} = \frac{R^2g}{\ell}[\cos\theta \times \frac{d\theta}{dt} - \cos(\theta + \ell/R)\frac{d\theta}{dt}]$

$\therefore (R\frac{d\theta}{dt})\frac{dv}{dt} = \frac{R^2g}{\ell} \times \frac{d\theta}{dt}[\cos\theta - \cos(\theta + (\ell/R))]$

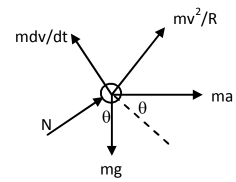
When the chain starts sliding down, $\theta = 0$.

So, $\frac{dv}{dt} = \frac{Rg}{\ell}[1 - \cos(\ell/R)]$

64. Let the sphere move towards left with an acceleration 'a

Let m = mass of the particle

The particle 'm' will also experience the inertia due to acceleration 'a' as it is on the sphere. It will also experience the tangential inertia force (m (dv/dt)) and centrifugal force (mv^2/R).



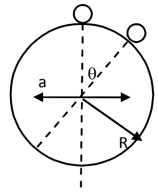
$m\frac{dv}{dt} = ma\cos\theta + mg\sin\theta \Rightarrow mv\frac{dv}{dt} = ma\cos\theta\left(R\frac{d\theta}{dt}\right) + mg\sin\theta\left(R\frac{d\theta}{dt}\right)$

Because, $v = R\frac{d\theta}{dt}$

$\Rightarrow v\,dv = aR\cos\theta\, d\theta + gR\sin\theta\, d\theta$

Integrating both sides we get,

$\frac{v^2}{2} = aR\sin\theta - gR\cos\theta + C$



Given that, at $\theta = 0$, $v = 0$, So, $C = gR$

So, $\frac{v^2}{2} = aR\sin\theta - gR\cos\theta + gR$

$\therefore v^2 = 2R(a\sin\theta + g - g\cos\theta) \Rightarrow v = [2R(a\sin\theta + g - g\cos\theta)]^{1/2}$

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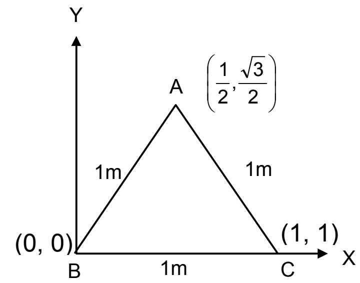
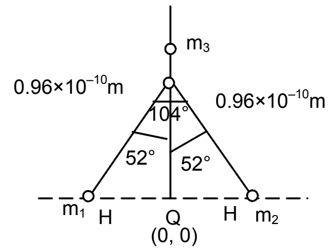
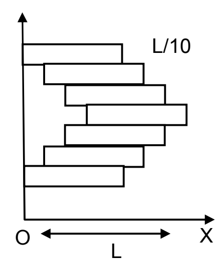
SOLUTIONS TO CONCEPTS
CHAPTER 9



1. $m_1 = 1kg$, $m_2 = 2kg$, $m_3 = 3kg$,
   $x_1 = 0$, $x_2 = 1$, $x_3=1/2$
   $y_1 = 0$, $y_2 = 0$, $y_3 = \sqrt{3}/2$
   The position of centre of mass is
   $$C.M = \left(\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}, \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}\right)$$
   $$= \left(\frac{(1\times0)+(2\times1)+(3\times1/2)}{1+2+3}, \frac{(1\times0)+(2\times0)+(3\times(\sqrt{3}/2))}{1+2+3}\right)$$
   $$= \left(\frac{7}{12}, \frac{3\sqrt{3}}{12}\right) \text{ from the point B.}$$

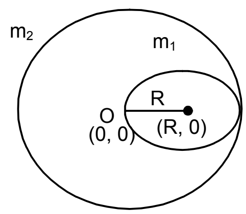


2. Let θ be the origin of the system
   In the above figure
   $m_1 = 1gm$, $x_1 = -(0.96\times10^{-10})\sin 52°$ $y_1 = 0$
   $m_2 = 1gm$, $x_2 = -(0.96\times10^{-10})\sin 52°$ $y_2 = 0$
   $x_3 = 0$ $y_3 = (0.96 \times 10^{-10}) \cos 52°$
   The position of centre of mass
   $$\left(\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}, \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}\right)$$
   $$= \left(\frac{-(0.96\times10^{-10})\times\sin52+(0.96\times10^{-10})\sin52+16\times0}{1+1+16}, \frac{0+0+16y_3}{18}\right)$$
   $$= \left(0,\ (8/9)0.96\times10^{-10}\cos52^\circ\right)$$



3. Let 'O' (0,0) be the origin of the system.
   Each brick is mass 'M' & length 'L'.
   Each brick is displaced w.r.t. one in contact by 'L/10'
   ∴The X coordinate of the centre of mass

$$\overline{X}_{cm} = \frac{m\left(\frac{L}{2}\right)+m\left(\frac{L}{2}+\frac{L}{10}\right)+m\left(\frac{L}{2}+\frac{2L}{10}\right)+m\left(\frac{L}{2}+\frac{3L}{10}\right)+m\left(\frac{L}{2}+\frac{3L}{10}-\frac{L}{10}\right)+m\left(\frac{L}{2}+\frac{L}{10}\right)+m\left(\frac{L}{2}\right)}{7m}$$
$$= \frac{\frac{L}{2}+\frac{L}{2}+\frac{L}{10}+\frac{L}{2}+\frac{L}{5}+\frac{L}{2}+\frac{3L}{10}+\frac{L}{2}+\frac{L}{5}+\frac{L}{2}+\frac{L}{10}+\frac{L}{2}}{7}$$
$$= \frac{\frac{7L}{2}+\frac{5L}{10}+\frac{2L}{5}}{7} = \frac{35L+5L+4L}{10\times7} = \frac{44L}{70} = \frac{11}{35}L$$



4. Let the centre of the bigger disc be the origin.
   2R = Radius of bigger disc
   R = Radius of smaller disc
   $m_1 = \pi R^2 \times T \times \rho$
   $m_2 = \pi(2R)^2\ I\ T \times \rho$
   where T = Thickness of the two discs
   ρ = Density of the two discs
   ∴ The position of the centre of mass

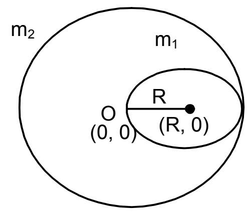
$$\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}, \frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$$

$x_1 = R \qquad y_1 = 0$
$x_2 = 0 \qquad y_2 = 0$

$$\left(\frac{\pi R^2T\rho R+0}{\pi R^2T\rho+\pi(2R)^2T\rho}, \frac{0}{m_1+m_2}\right) = \left(\frac{\pi R^2T\rho R}{5\pi R^2T\rho}, 0\right) = \left(\frac{R}{5}, 0\right)$$

At R/5 from the centre of bigger disc towards the centre of smaller disc.

5. Let '0' be the origin of the system.



R = radius of the smaller disc
2R = radius of the bigger disc
The smaller disc is cut out from the bigger disc
As from the figure

$m_1 = \pi R^2T\rho \qquad x_1 = R \qquad y_1 = 0$
$m_2 = \pi(2R)^2T\rho \qquad x_2 = 0 \qquad y_2 = 0$

The position of C.M. = $\left(\frac{-\pi R^2T\rho R+0}{-\pi R^2T\rho+\pi(2R)^2T\rho R}, \frac{0+0}{m_1+m_2}\right)$

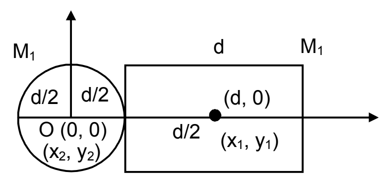
$$= \left(\frac{-\pi R^2T\rho R}{3\pi R^2T\rho}, 0\right) = \left(-\frac{R}{3}, 0\right)$$

C.M. is at R/3 from the centre of bigger disc away from centre of the hole.

6. Let m be the mass per unit area.

$\therefore$ Mass of the square plate = $M_1 = d^2m$

Mass of the circular disc = $M_2 = \frac{\pi d^2}{4}m$



Let the centre of the circular disc be the origin of the system.

$\therefore$ Position of centre of mass

$$= \left(\frac{d^2md+\pi(d^2/4)m\times 0}{d^2m+\pi(d^2/4)m}, \frac{0+0}{M_1+M_2}\right) = \left(\frac{d^3m}{d^2m\left(1+\frac{\pi}{4}\right)}, 0\right) = \left(\frac{4d}{\pi+4}, 0\right)$$

The new centre of mass is $\left(\frac{4d}{\pi+4}\right)$ right of the centre of circular disc.

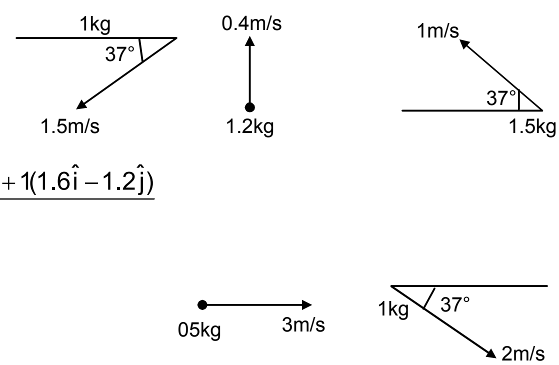
7. m_1 = 1kg. $\qquad \vec{v}_1 = -1.5\cos 37\,\hat{i} - 1.55\sin 37\,\hat{j} = -1.2\,\hat{i} - 0.9\,\hat{j}$

m_2 = 1.2kg. $\qquad \vec{v}_2 = 0.4\,\hat{j}$

m_3 = 1.5kg $\qquad \vec{v}_3 = -0.8\,\hat{i} + 0.6\,\hat{j}$

m_4 = 0.5kg $\qquad \vec{v}_4 = 3\,\hat{i}$

m_5 = 1kg $\qquad \vec{v}_5 = 1.6\,\hat{i} - 1.2\,\hat{j}$



So, $\vec{v}_c = \frac{m_1\vec{v}_1+m_2\vec{v}_2+m_3\vec{v}_3+m_4\vec{v}_4+m_5\vec{v}_5}{m_1+m_2+m_3+m_4+m_5}$

$$= \frac{1(-1.2\hat{i}-0.9\hat{j})+1.2(0.4\hat{j})+1.5(-0.8\hat{i}+0.6\hat{j})+0.5(3\hat{i})+1(1.6\hat{i}-1.2\hat{j})}{5.2}$$

$$= \frac{-1.2\hat{i}-0.9\hat{j}+4.8\hat{j}-1.2\hat{i}+.90\hat{j}+1.5\hat{i}+1.6\hat{i}-1.2\hat{j}}{5.2}$$

$$= \frac{0.7\hat{i}}{5.2} - \frac{0.72\hat{j}}{5.2}$$

8. Two masses m_1 & m_2 are placed on the X-axis
m_1 = 10 kg, m_2 = 20kg.
The first mass is displaced by a distance of 2 cm

$$\therefore \overline{X}_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{10 \times 2 + 20x_2}{30}$$

$$\Rightarrow 0 = \frac{20 + 20x_2}{30} \Rightarrow 20 + 20x_2 = 0$$

$\Rightarrow 20 = -20x_2 \Rightarrow x_2 = -1.$
$\therefore$ The 2^{nd} mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

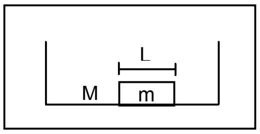
9. Two masses m_1 & m_2 are kept in a vertical line
m_1 = 10kg, m_2 = 30kg
The first block is raised through a height of 7 cm.
The centre of mass is raised by 1 cm.

$$\therefore 1 = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{10 \times 7 + 30y_2}{40}$$

$$\Rightarrow 1 = \frac{70 + 30y_2}{40} \Rightarrow 70 + 30y_2 = 40 \Rightarrow 30y_2 = -30 \Rightarrow y_2 = -1.$$

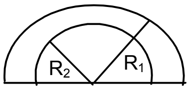
The 30 kg body should be displaced 1cm downward inorder to raise the centre of mass through 1 cm.

10. As the hall is gravity free, after the ice melts, it would tend to acquire a spherical shape. But, there is no external force acting on the system. So, the centre of mass of the system would not move.



11. The centre of mass of the blate will be on the symmetrical axis.

$$\Rightarrow \overline{y}_{cm} = \frac{\left(\frac{\pi R_2^2}{2}\right)\left(\frac{4R_2}{3\pi}\right) - \left(\frac{\pi R_1^2}{2}\right)\left(\frac{4R_1}{3\pi}\right)}{\frac{\pi R_2^2}{2} - \frac{\pi R_1^2}{2}}$$

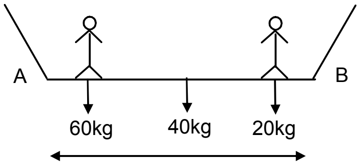


$$= \frac{(2/3)R_2^3 - (2/3)R_1^3}{\pi/2(R_2^2 - R_1^2)} = \frac{4}{3\pi}\frac{(R_2 - R_1)(R_2^2 + R_1^2 + R_1R_2)}{(R_2 - R_1)(R_2 + R_1)}$$

$$= \frac{4}{3\pi}\frac{(R_2^2 + R_1^2 + R_1R_2)}{R_1 + R_2}$$ above the centre.

12. m_1 = 60kg, m_2 = 40kg , m_3 = 50kg,
Let A be the origin of the system.
Initially Mr. Verma & Mr. Mathur are at extreme position of the boat.
$\therefore$ The centre of mass will be at a distance

$$= \frac{60 \times 0 + 40 \times 2 + 50 \times 4}{150} = \frac{280}{150} = 1.87\text{m from 'A'}$$

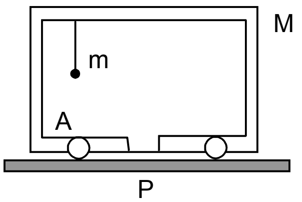


When they come to the mid point of the boat the CM lies at 2m from 'A'.
$\therefore$ The shift in CM = 2 – 1.87 = 0.13m towards right.
But as there is no external force in longitudinal direction their CM would not shift.
So, the boat moves 0.13m or 13 cm towards right.

13. Let the bob fall at A,. The mass of bob = m.
The mass of cart = M.
Initially their centre of mass will be at

$$\frac{m \times L + M \times 0}{M + m} = \left(\frac{m}{M + m}\right)L$$

Distance from P
When, the bob falls in the slot the CM is at a distance 'O' from P.



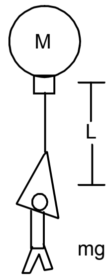
Shift in CM = $0 - \frac{mL}{M+m} = -\frac{mL}{M+m}$ towards left

$= \frac{mL}{M+m}$ towards right.

But there is no external force in horizontal direction.

So the cart displaces a distance $\frac{mL}{M+m}$ towards right.

14. Initially the monkey & balloon are at rest.
So the CM is at 'P'
When the monkey descends through a distance 'L'
The CM will shift



$t_o = \frac{m \times L + M \times 0}{M+m} = \frac{mL}{M+m}$ from P

So, the balloon descends through a distance $\frac{mL}{M+m}$

15. Let the mass of the to particles be m_1 & m_2 respectively
$m_1 = 1kg$, $m_2 = 4kg$
$\therefore$ According to question
$½\, m_1v_1^2 = ½\, m_2v_2^2$

$\Rightarrow \frac{m_1}{m_2} = \frac{v_2^2}{v_1^2} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$

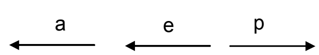
Now, $\frac{m_1v_1}{m_2v_2} = \frac{m_1}{m_2} \times \sqrt{\frac{m_2}{m_1}} = \frac{\sqrt{m_1}}{\sqrt{m_2}} = \frac{\sqrt{1}}{\sqrt{4}} = 1/2$

$\Rightarrow \frac{m_1v_1}{m_2v_2} = 1 : 2$

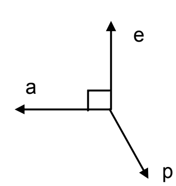
16. As uranium 238 nucleus emits a α-particle with a speed of 1.4×10^7m/sec. Let v_2 be the speed of the residual nucleus thorium 234.
$\therefore m_1v_1 = m_2v_2$
$\Rightarrow 4 \times 1.4 \times 10^7 = 234 \times v_2$
$\Rightarrow v_2 = \frac{4 \times 1.4 \times 10^7}{234} = 2.4 \times 10^5$ m/sec.

17. $m_1v_1 = m_2v_2$
$\Rightarrow 50 \times 1.8 = 6 \times 10^{24} \times v_2$
$\Rightarrow v_2 = \frac{50 \times 1.8}{6 \times 10^{24}} = 1.5 \times 10^{-23}$ m/sec
so, the earth will recoil at a speed of 1.5×10^{-23}m/sec.

18. Mass of proton = 1.67×10^{-27}
Let 'V_p' be the velocity of proton



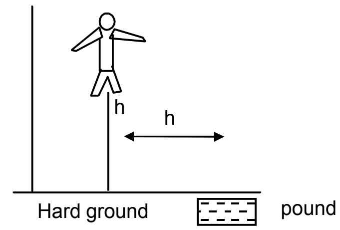
Given momentum of electron = 1.4×10^{-26}kg m/sec
Given momentum of antineutrino = 6.4×10^{-27} kg m/sec
a) The electron & the antineutrino are ejected in the same direction. As the total momentum is conserved the proton should be ejected in the opposite direction.
$1.67 \times 10^{-27} \times V_p = 1.4 \times 10^{-26} + 6.4 \times 10^{-27} = 20.4 \times 10^{-27}$
$\Rightarrow V_p = (20.4 / 1.67) = 12.2$ m/sec in the opposite direction.
b) The electron & antineutrino are ejected $\perp^r$ to each other.
Total momentum of electron and antineutrino,



$= \sqrt{(14)^2 + (6.4)^2} \times 10^{-27}$ kg m/s = 15.4×10^{-27} kg m/s
Since, $1.67 \times 10^{-27}\, V_p = 15.4 \times 10^{-27}$ kg m/s
So $V_p = 9.2$ m/s

19. Mass of man = M, Initial velocity = 0

Mass of bad = m



Let the throws the bag towards left with a velocity v towards left. So, there is no external force in the horizontal direction.

The momentum will be conserved. Let he goes right with a velocity

$mv = MV \Rightarrow V = \dfrac{mv}{M} \Rightarrow v = \dfrac{MV}{m}$ ..(i)

Let the total time he will take to reach ground = $\sqrt{2H/g} = t_1$

Let the total time he will take to reach the height h = $t_2 = \sqrt{2(H-h)/g}$

Then the time of his flying = $t_1 - t_2 = \sqrt{2H/g} - \sqrt{2(H-h)/g} = \sqrt{2/g}\left(\sqrt{H} - \sqrt{H-h}\right)$

Within this time he reaches the ground in the pond covering a horizontal distance x

$\Rightarrow x = V \times t \Rightarrow V = x / t$

$\therefore v = \dfrac{M}{m}\dfrac{x}{t} = \dfrac{M}{m} \times \dfrac{\sqrt{g}}{\sqrt{2}\left(\sqrt{H} - \sqrt{H-h}\right)}$

As there is no external force in horizontal direction, the x-coordinate of CM will remain at that position.

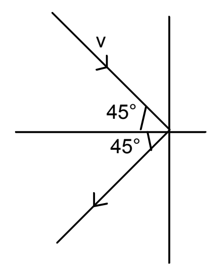
$\Rightarrow 0 = \dfrac{M \times (x) + m \times x_1}{M+m} \Rightarrow x_1 = -\dfrac{M}{m}x$

$\therefore$ The bag will reach the bottom at a distance (M/m) x towards left of the line it falls.

20. Mass = 50g = 0.05kg

$v = 2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$

$v_1 = -2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$



a) change in momentum = $m\vec{v} - m\vec{v}_1$

$= 0.05\,(2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}) - 0.05\,(-2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j})$

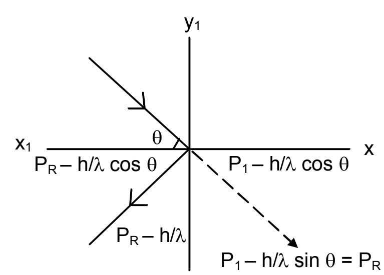
$= 0.1\cos 45°\,\hat{i} - 0.1\sin 45°\,\hat{j} + 0.1\cos 45°\,\hat{i} + 0.1\sin 45°\,\hat{j}$

$= 0.2\cos 45°\,\hat{i}$

$\therefore$ magnitude = $\sqrt{\left(\dfrac{0.2}{\sqrt{2}}\right)^2} = \dfrac{0.2}{\sqrt{2}} = 0.14$ kg m/s

c) The change in magnitude of the momentum of the ball

$-\left|\vec{P}_i\right| - \left|\vec{P}_f\right| = 2 \times 0.5 - 2 \times 0.5 = 0.$



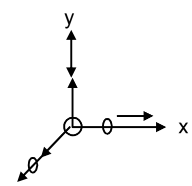
21. $\vec{P}_{incidence} = (h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$

$P_{Reflected} = -(h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$

The change in momentum will be only in the x-axis direction. i.e.

$\left|\Delta P\right| = (h/\lambda)\cos\theta - ((h/\lambda)\cos\theta) = (2h/\lambda)\cos\theta$

22. As the block is exploded only due to its internal energy. So net external force during this process is 0. So the centre mass will not change.



Let the body while exploded was at the origin of the co-ordinate system.

If the two bodies of equal mass is moving at a speed of 10m/s in + x & +y axis direction respectively,

$\sqrt{10^2 + 10^2 + 2 10.10\cos 90°} = 10\sqrt{2}$ m/s 45° w.r.t. + x axis

If the centre mass is at rest, then the third mass which have equal mass with other two, will move in the opposite direction (i.e. 135° w.r.t. + x- axis) of the resultant at the same velocity.

23. Since the spaceship is removed from any material object & totally isolated from surrounding, the missions by astronauts couldn't slip away from the spaceship. So the total mass of the spaceship remain unchanged and also its velocity.

24. d = 1cm, v = 20 m/s, u = 0, ρ = 900 kg/m^3 = 0.9gm/cm^3
volume = $(4/3)\pi r^3 = (4/3)\,\pi\,(0.5)^3$ = 0.5238cm^3
$\therefore$ mass = vρ = 0.5238 × 0.9 = 0.4714258gm
$\therefore$ mass of 2000 hailstone = 2000 × 0.4714 = 947.857
$\therefore$ Rate of change in momentum per unit area = 947.857 × 2000 = 19N/m^3
$\therefore$ Total force exerted = 19 × 100 = 1900 N.

25. A ball of mass m is dropped onto a floor from a certain height let ‘h’.
$\therefore v_1 = \sqrt{2gh}$, $v_1 = 0$, $v_2 = -\sqrt{2gh}$ & $v_2 = 0$
$\therefore$ Rate of change of velocity :-
$$F = \frac{m \times 2\sqrt{2gh}}{t}$$
$\therefore v = \sqrt{2gh}$, s = h, v = 0
$\Rightarrow$ v = u + at
$$\Rightarrow \sqrt{2gh} = g\,t \Rightarrow t = \sqrt{\frac{2h}{g}}$$
$$\therefore \text{Total time } 2\sqrt{\frac{2h}{t}}$$
$$\therefore F = \frac{m \times 2\sqrt{2gh}}{2\sqrt{\frac{2h}{g}}} = mg$$

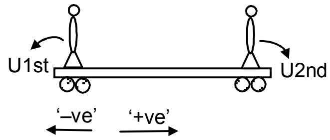
26. A railroad car of mass M is at rest on frictionless rails when a man of mass m starts moving on the car towards the engine. The car recoils with a speed v backward on the rails.
Let the mass is moving with a velocity x w.r.t. the engine.
$\therefore$The velocity of the mass w.r.t earth is (x – v) towards right
V_{cm} = 0 (Initially at rest)
$\therefore$ 0 = –Mv + m(x – v)
$$\Rightarrow Mv = m(x - v) \Rightarrow mx = Mv + mv \Rightarrow x = \left(\frac{M+m}{m}\right)v \Rightarrow x = \left(1+\frac{M}{m}\right)v$$

27. A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is 50m where m is the mass of one shell. The muzzle velocity of the shells is 200m/s.
Initial, V_{cm} = 0.
$$\therefore 0 = 49\,m \times V + m \times 200 \Rightarrow V = \frac{-200}{49}\ \text{m/s}$$
$$\therefore \frac{200}{49}\ \text{m/s towards left.}$$
When another shell is fired, then the velocity of the car, with respect to the platform is,
$$\Rightarrow V` = \frac{200}{49}\ \text{m/s towards left.}$$
When another shell is fired, then the velocity of the car, with respect to the platform is,
$$\Rightarrow v` = \frac{200}{48}\ \text{m/s towards left}$$
$$\therefore \text{Velocity of the car w.r.t the earth is } \left(\frac{200}{49}+\frac{200}{48}\right)\text{m/s towards left.}$$

28. Two persons each of mass m are standing at the two extremes of a railroad car of mass m resting on a smooth track.
Case – I
Let the velocity of the railroad car w.r.t the earth is V after the jump of the left man.
$\therefore$ 0 = – mu + (M + m) V

$\Rightarrow$ V = $\frac{mu}{M+m}$ towards right

Case – II



When the man on the right jumps, the velocity of it w.r.t the car is u.

$\therefore$ 0 = mu – Mv'

$\Rightarrow$ **v′ =** $\frac{mu}{M}$

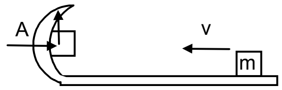
(V′ is the change is velocity of the platform when platform itself is taken as reference assuming the car to be at rest)

$\therefore$ So, net velocity towards left (i.e. the velocity of the car w.r.t. the earth)

$= \frac{mv}{M} - \frac{mv}{M+m} = \frac{mMu + m^2v - Mmu}{M(M+m)} = \frac{m^2v}{M(M+m)}$

29. A small block of mass m which is started with a velocity V on the horizontal part of the bigger block of mass M placed on a horizontal floor.

Since the small body of mass m is started with a velocity V in the horizontal direction, so the total initial momentum at the initial position in the horizontal direction will remain same as the total final momentum at the point A on the bigger block in the horizontal direction.



From L.C.K. m:

mv + M×O = (m + M) v $\Rightarrow$ v′ = $\frac{mv}{M+m}$

30. Mass of the boggli = 200kg, V_B = 10 km/hour.

$\therefore$ Mass of the boy = 2.5kg & V_{Boy} = 4km/hour.

If we take the boy & boggle as a system then total momentum before the process of sitting will remain constant after the process of sitting.

$\therefore$ $m_b V_b = m_{boy}V_{boy} = (m_b + m_{boy})$ v

$\Rightarrow$ 200 × 10 + 25 × 4 = (200 +25) × v

$\Rightarrow$ v = $\frac{2100}{225} = \frac{28}{3}$ = 9.3 m/sec

31. Mass of the ball = m_1 = 0.5kg, velocity of the ball = 5m/s

Mass of the another ball m_2 = 1kg

Let it's velocity = v′ m/s

Using law of conservation of momentum,

0.5 × 5 + 1 × v′ = 0 $\Rightarrow$ v′ = – 2.5

$\therefore$ Velocity of second ball is 2.5 m/s opposite to the direction of motion of 1^{st} ball.

32. Mass of the man = m_1 = 60kg

Speed of the man = v_1 = 10m/s

Mass of the skater = m_2 = 40kg

let its velocity = v′

$\therefore$ 60 × 10 + 0 = 100 × v′ $\Rightarrow$ v′ = 6m/s

loss in K.E. = $(1/2)60 \times (10)^2 - (1/2) \times 100 \times 36$ = 1200 J

33. Using law of conservation of momentum.

$m_1u_1 + m_2u_2 = m_1v(t) + m_2v'$

Where v′ = speed of 2^{nd} particle during collision.

$\Rightarrow m_1u_1 + m_2u_2 = m_1u_1 + m_1 + (t/\Delta t)(v_1 - u_1) + m_2v'$

$\Rightarrow \frac{m_2u_2}{m^2} - \frac{m_1}{m2}\frac{t}{\Delta t}(v_1 - u_1)v'$

$\therefore$ v′ = $u_2 - \frac{m_1}{m_2}\frac{t}{\Delta t}(v_1 - u)$

34. Mass of the bullet = m and speed = v

Mass of the ball = M

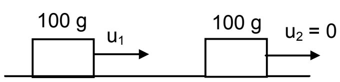
m′ = frictional mass from the ball.

Using law of conservation of momentum,
$mv + 0 = (m' + m)\, v' + (M - m')\, v_1$
where v' = final velocity of the bullet + frictional mass
$\Rightarrow v' = \dfrac{mv - (M + m')V_1}{m + m'}$

35. Mass of 1^{st} ball = m and speed = v
Mass of 2^{nd} ball = m
Let final velocities of 1^{st} and 2^{nd} ball are v_1 and v_2 respectively
Using law of conservation of momentum,
$m(v_1 + v_2) = mv.$
$\Rightarrow v_1 + v_2 = v \qquad \ldots(1)$
Also
$v_1 - v_2 = ev \qquad \ldots(2)$
Given that final K.E. = ¾ Initial K.E.
$\Rightarrow \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 = \frac{3}{4} \times \frac{1}{2} mv^2$
$\Rightarrow v_1^2 + v_2^2 = \frac{3}{4} v^2$
$\Rightarrow \dfrac{(v_1 + v_2)^2 + (v_1 - v_2)^2}{2} = \dfrac{3}{4} v^2$
$\Rightarrow \dfrac{(1 + e^2)v^2}{2} = \dfrac{3}{4} v^2 \Rightarrow 1 + e^2 = \dfrac{3}{2} \Rightarrow e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

36. Mass of block = 2kg and speed = 2m/s
Mass of 2^{nd} block = 2kg.
Let final velocity of 2^{nd} block = v
using law of conservation of momentum.
$2 \times 2 = (2 + 2)\, v \Rightarrow v = 1\text{m/s}$
$\therefore$ Loss in K.E. in inelastic collision
$= (1/2) \times 2 \times (2)^2\, v - (1/2)\,(2 + 2) \times (1)^2 = 4 - 2 = 2$ J
b) Actual loss $= \dfrac{\text{Maximum loss}}{2} = 1$J
$(1/2) \times 2 \times 2^2 - (1/2)\, 2 \times v_1^2 + (1/2) \times 2 \times v_2^2 = 1$
$\Rightarrow 4 - (v_1^2 + v_2^2) = 1$
$\Rightarrow 4 - \dfrac{(1 + e^2) \times 4}{2} = 1$
$\Rightarrow 2(1 + e^2) = 3 \Rightarrow 1 + e^2 = \dfrac{3}{2} \Rightarrow e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

37. Final K.E. = 0.2J
Initial K.E. $= \frac{1}{2} mV_1^2 + 0 = \frac{1}{2} \times 0.1\, u^2 = 0.05\, u^2$
$mv_1 = mv_2' = mu$
Where v_1 and v_2 are final velocities of 1^{st} and 2^{nd} block respectively.



$\Rightarrow v_1 + v_2 = u \qquad \ldots(1)$
$(v_1 - v_2) + \ell\,(a_1 - u_2) = 0 \Rightarrow \ell a = v_2 - v_1 \qquad ..(2)$
$u_2 = 0, \qquad u_1 = u.$
Adding Eq.(1) and Eq.(2)
$2v_2 = (1 + \ell)u \Rightarrow v_2 = (u/2)(1 + \ell)$
$\therefore v_1 = u - \dfrac{u}{2} - \dfrac{u}{2}\ell$
$v_1 = \dfrac{u}{2}(1 - \ell)$
Given $(1/2)mv_1^2 + (1/2)mv_2^2 = 0.2$
$\Rightarrow v_1^2 + v_2^2 = 4$

$\Rightarrow \frac{u^2}{4}(1-\ell)^2 + \frac{u^2}{4}(1+\ell)^2 = 4$ $\quad \Rightarrow \frac{u^2}{2}(1+\ell^2) = 4$ $\quad \Rightarrow u^2 = \frac{8}{1+\ell^2}$

For maximum value of u, denominator should be minimum,

$\Rightarrow \ell = 0.$

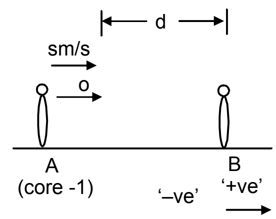
$\Rightarrow u^2 = 8 \Rightarrow u = 2\sqrt{2}$ m/s

For minimum value of u, denominator should be maximum,

$\Rightarrow \ell = 1$

$u^2 = 4 \Rightarrow u = 2$ m/s

38. Two friends A & B (each 40kg) are sitting on a frictionless platform some distance d apart A rolls a ball of mass 4kg on the platform towards B, which B catches. Then B rolls the ball towards A and A catches it. The ball keeps on moving back & forth between A and B. The ball has a fixed velocity 5m/s.



a) Case – I :– Total momentum of the man A & the ball will remain constant

$\therefore 0 = 4 \times 5 - 40 \times v$ $\quad \Rightarrow v = 0.5$ m/s towards left

b) Case – II :– When B catches the ball, the momentum between the B & the ball will remain constant.

$\Rightarrow 4 \times 5 = 44v \Rightarrow v = (20/44)$ m/s

Case – III :– When B throws the ball, then applying L.C.L.M

$\Rightarrow 44 \times (20/44) = -4 \times 5 + 40 \times v$ $\quad \Rightarrow v = 1$m/s (towards right)

Case – IV :– When a Catches the ball, the applying L.C.L.M.

$\Rightarrow -4 \times 5 + (-0.5)\times 40 = -44v$ $\quad \Rightarrow v = \frac{10}{11}$ m/s towards left.

c) Case – V :– When A throws the ball, then applying L.C.L.M.

$\Rightarrow 44 \times (10/11) = 4 \times 5 - 40 \times V$ $\quad \Rightarrow V = 60/40 = 3/2$ m/s towards left.

Case – VI :– When B receives the ball, then applying L.C.L.M

$\Rightarrow 40 \times 1 + 4 \times 5 = 44 \times v$ $\quad \Rightarrow v = 60/44$ m/s towards right.

Case – VII :– When B throws the ball, then applying L.C.L.M.

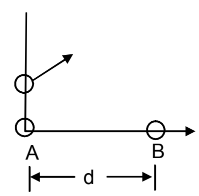
$\Rightarrow 44 \times (66/44) = -4 \times 5 + 40 \times V$ $\quad \Rightarrow V = 80/40 = 2$ m/s towards right.

Case – VIII :– When A catches the ball, then applying L.C.L.M

$\Rightarrow -4 \times 5 - 40 \times (3/2) = -44\,v$ $\quad \Rightarrow v = (80/44) = (20/11)$ m/s towards left.

Similarly after 5 round trips

The velocity of A will be (50/11) & velocity of B will be 5 m/s.



d) Since after 6 round trip, the velocity of A is 60/11 i.e.

> 5m/s. So, it can't catch the ball. So it can only roll the ball six.

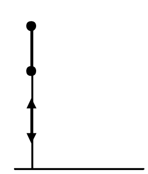
e) Let the ball & the body A at the initial position be at origin.

$\therefore X_C = \frac{40\times 0 + 4\times 0 + 40\times d}{40+40+4} = \frac{10}{11}d$

39. $u = \sqrt{2gh}$ = velocity on the ground when ball approaches the ground.

$\Rightarrow u = \sqrt{2\times 9.8\times 2}$

v = velocity of ball when it separates from the ground.

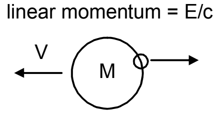


$\vec{v} + \ell\vec{u} = 0$

$\Rightarrow \ell\vec{u} = -\vec{v} \;\Rightarrow \ell = \frac{\sqrt{2\times 9.8\times 1.5}}{\sqrt{2\times 9.8\times 2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

40. K.E. of Nucleus $= (1/2)mv^2 = (1/2)\,m\left(\frac{E}{mc}\right)^2 = \frac{E^2}{2mc^2}$

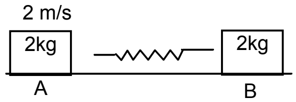
linear momentum = E/c



Energy limited by Gamma photon = E.

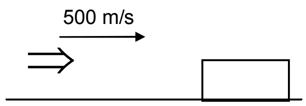
Decrease in internal energy $= E + \frac{E^2}{2mc^2}$

41. Mass of each block M_A and M_B = 2kg.
Initial velocity of the 1st block, (V) = 1m/s
V_A = 1 m/s, V_B = 0m/s
Spring constant of the spring = 100 N/m.
The block A strikes the spring with a velocity 1m/s/
After the collision, it's velocity decreases continuously and at a instant the whole system (Block A + the compound spring + Block B) move together with a common velocity.
Let that velocity be V.
Using conservation of energy, $(1/2)\,M_AV_A^2 + (1/2)M_BV_B^2 = (1/2)M_Av^2 + (1/2)M_Bv^2 + (1/2)kx^2$.
$(1/2) \times 2(1)^2 + 0 = (1/2) \times 2\times v^2 + (1/2) \times 2 \times v^2 + (1/2)\,x^2 \times 100$
(Where x = max. compression of spring)
$\Rightarrow 1 = 2v^2 + 50x^2$ …(1)
As there is no external force in the horizontal direction, the momentum should be conserved.
$\Rightarrow M_AV_A + M_BV_B = (M_A + M_B)V$.
$\Rightarrow 2 \times 1 = 4 \times v$
$\Rightarrow V = (1/2)$ m/s. …(2)



Putting in eq.(1)
$1 = 2 \times (1/4) + 50x+2+$
$\Rightarrow (1/2) = 50x^2$
$\Rightarrow x^2 = 1/100\text{m}^2$
$\Rightarrow x = (1/10)\text{m} = 0.1\text{m} = 10\text{cm}$.

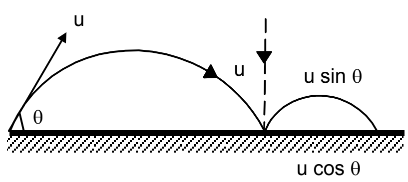
42. Mass of bullet m = 0.02kg.
Initial velocity of bullet V_1 = 500m/s
Mass of block, M = 10kg.
Initial velocity of block $u_2 = 0$.
Final velocity of bullet = 100 m/s = v.



Let the final velocity of block when the bullet emerges out, if block = v′.
$mv_1 + Mu_2 = mv + Mv'$
$\Rightarrow 0.02 \times 500 = 0.02 \times 100 + 10 \times v'$
$\Rightarrow v' = 0.8$m/s
After moving a distance 0.2 m it stops.
$\Rightarrow$ change in K.E. = Work done
$\Rightarrow 0 - (1/2) \times 10\times (0.8)^2 = -\mu \times 10 \times 10 \times 0.2 \Rightarrow \mu = 0.16$

43. The projected velocity = u.
The angle of projection = θ.
When the projectile hits the ground for the 1st time, the velocity would be the same i.e. u.
Here the component of velocity parallel to ground, u cos θ should remain constant. But the vertical component of the projectile undergoes a change after the collision.

$\Rightarrow e = \dfrac{u\sin\theta}{v} \Rightarrow v = eu\sin\theta$.



Now for the 2nd projectile motion,
U = velocity of projection = $\sqrt{(u\cos\theta)^2 + (eu\sin\theta)^2}$

and Angle of projection = $\alpha = \tan^{-1}\left(\dfrac{eu\sin\theta}{a\cos\theta}\right) = \tan^{-1}(e\tan\theta)$

or $\tan\alpha = e\tan\theta$ …(2)

Because, $y = x\tan\alpha - \dfrac{gx^2\sec^2\alpha}{2u^2}$ …(3)

Here, y = 0, $\tan\alpha = e\tan\theta$, $\sec^2\alpha = 1 + e^2\tan^2\theta$
And $u^2 = u^2\cos^2\theta + e^2\sin^2\theta$
Putting the above values in the equation (3),

x e tan θ = $\frac{gx^2(1+e^2\tan^2\theta)}{2u^2(\cos^2\theta+e^2\sin^2\theta)}$

$\Rightarrow$ x = $\frac{2eu^2\tan\theta(\cos^2\theta+e^2\sin^2\theta)}{g(1+e^2\tan^2\theta)}$

$\Rightarrow$ x = $\frac{2eu^2\tan\theta-\cos^2\theta}{g}$ = $\frac{eu^2\sin2\theta}{g}$

$\Rightarrow$ So, from the starting point O, it will fall at a distance

= $\frac{u^2\sin2\theta}{g}+\frac{eu^2\sin2\theta}{g}$ = $\frac{u^2\sin2\theta}{g}(1+e)$

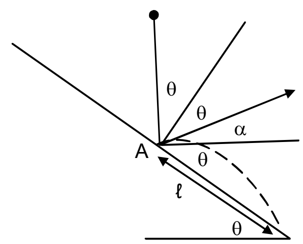
44. Angle inclination of the plane = θ

M the body falls through a height of h,

The striking velocity of the projectile with the indined plane v = $\sqrt{2gh}$

Now, the projectile makes on angle (90° – 2θ)

Velocity of projection = u = $\sqrt{2gh}$



Let AB = L.

So, x = ℓ cos θ, y = – ℓ sin θ

From equation of trajectory,

y = x tan α – $\frac{gx^2\sec^2\alpha}{2u^2}$

– ℓ sin θ = ℓ cos θ . tan (90° – 2θ) – $\frac{g\times\ell^2\cos^2\theta\sec^2(90^\circ-2\theta)}{2\times2gh}$

$\Rightarrow$ – ℓ sin θ = ℓ cos θ . cot 2θ – $\frac{g\ell^2\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4gh}$

So, $\frac{\ell\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4h}$ = sin θ + cos θ cot 2θ

$\Rightarrow$ ℓ = $\frac{4h}{\cos^2\theta\,\mathrm{cosec}^2 2\theta}$(sin θ + cos θ cot 2θ) = $\frac{4h\times\sin^2 2\theta}{\cos^2\theta}\left(\sin\theta+\cos\theta\times\frac{\cos2\theta}{\sin2\theta}\right)$

= $\frac{4h\times4\sin^2\theta\cos^2\theta}{\cos^2\theta}\left(\frac{\sin\theta\times\sin2\theta+\cos\theta\cos2\theta}{\sin2\theta}\right)$ = 16 h $\sin^2\theta$ × $\frac{\cos\theta}{2\sin\theta\cos\theta}$ = 8h sin θ

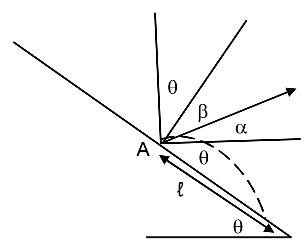
45. h = 5m, θ = 45°, e = (3/4)

Here the velocity with which it would strike = v = $\sqrt{2g\times5}$ = 10m/sec

After collision, let it make an angle β with horizontal. The horizontal component of velocity 10 cos 45° will remain unchanged and the velocity in the perpendicular direction to the plane after wllisine.

$\Rightarrow$ V_y = e × 10 sin 45°

= (3/4) × 10 × $\frac{1}{\sqrt{2}}$ = (3.75) $\sqrt{2}$ m/sec



V_x = 10 cos 45° = 5 $\sqrt{2}$ m/sec

So, u = $\sqrt{V_x^2+V_y^2}$ = $\sqrt{50+28.125}$ = $\sqrt{78.125}$ = 8.83 m/sec

Angle of reflection from the wall β = $\tan^{-1}\left(\frac{3.75\sqrt{2}}{5\sqrt{2}}\right)$ = $\tan^{-1}\left(\frac{3}{4}\right)$ = 37°

$\Rightarrow$ Angle of projection α = 90 – (θ + β) = 90 – (45° + 37°) = 8°

Let the distance where it falls = L

$\Rightarrow$ x = L cos θ, y = – L sin θ

Angle of projection (α) = –8°

Using equation of trajectory, $y = x \tan\alpha - \dfrac{gx^2 \sec^2\alpha}{2u^2}$

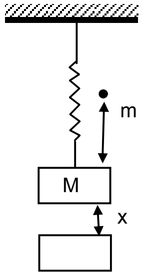
$\Rightarrow -\ell \sin\theta = \ell \cos\theta \times \tan 8° - \dfrac{g}{2} \times \dfrac{\ell \cos^2\theta \sec^2 8°}{u^2}$

$\Rightarrow -\sin 45° = \cos 45° - \tan 8° - \dfrac{10\cos^2 45° \sec 8°}{(8.83)^2}(\ell)$

Solving the above equation we get,

ℓ = 18.5 m.

46. Mass of block



Block of the particle = m = 120gm = 0.12kg.

In the equilibrium condition, the spring is stretched by a distance x = 1.00 cm = 0.01m.

$\Rightarrow 0.2 \times g = K.\ x.$

$\Rightarrow 2 = K \times 0.01 \Rightarrow K = 200$ N/m.

The velocity with which the particle m will strike M is given by u

$= \sqrt{2 \times 10 \times 0.45} = \sqrt{9} = 3$ m/sec.

So, after the collision, the velocity of the particle and the block is

$V = \dfrac{0.12 \times 3}{0.32} = \dfrac{9}{8}$ m/sec.

Let the spring be stretched through an extra deflection of δ.

$0 - (1/2) \times 0.32 \times (81/64) = 0.32 \times 10 \times \delta - (1/2 \times 200 \times (\delta + 0.1)^2 - (1/2) \times 200 \times (0.01)^2$

Solving the above equation we get

$\delta = 0.045 = 4.5$cm

47. Mass of bullet = 25g = 0.025kg.

Mass of pendulum = 5kg.

The vertical displacement h = 10cm = 0.1m

Let it strike the pendulum with a velocity u.

Let the final velocity be v.

$\Rightarrow mu = (M + m)v.$

$\Rightarrow v = \dfrac{m}{(M+m)}u = \dfrac{0.025}{5.025} \times u = \dfrac{u}{201}$

Using conservation of energy.

$0 - (1/2)(M + m).\ V^2 = -(M + m)\ g \times h \Rightarrow \dfrac{u^2}{(201)^2} = 2 \times 10 \times 0.1 = 2$

$\Rightarrow u = 201 \times \sqrt{2} = 280$ m/sec.

48. Mass of bullet = M = 20gm = 0.02kg.

Mass of wooden block M = 500gm = 0.5kg

Velocity of the bullet with which it strikes u = 300 m/sec.

Let the bullet emerges out with velocity V and the velocity of block = V′

As per law of conservation of momentum.

$mu = Mv' + mv$ ....(1)

Again applying work – energy principle for the block after the collision,

$0 - (1/2) M \times V'^2 = -Mgh$ (where h = 0.2m)

$\Rightarrow V'^2 = 2gh$

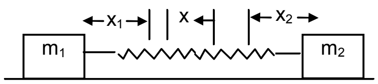
$V' = \sqrt{2gh} = \sqrt{20 \times 0.2} = 2$m/sec

Substituting the value of V′ in the equation (1), we get\

$0.02 \times 300 = 0.5 \times 2 + 0.2 \times v$

$\Rightarrow V = \dfrac{6.1}{0.02} = 250$m/sec.

49. Mass of the two blocks are m_1 , m_2.
Initially the spring is stretched by x_0
Spring constant K.
For the blocks to come to rest again,
Let the distance travelled by m_1 & m_2
Be x_1 and x_2 towards right and left respectively.
As o external forc acts in horizontal direction,

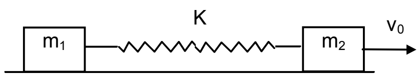


$m_1x_1 = m_2x_2$ ...(1)
Again, the energy would be conserved in the spring.
$\Rightarrow (1/2)\, k \times x^2 = (1/2)\, k\, (x_1 + x_2 - x_0)^2$
$\Rightarrow x_o = x_1 + x_2 - x_0$
$\Rightarrow x_1 + x_2 = 2x_0$ ...(2)
$\Rightarrow x_1 = 2x_0 - x_2$ similarly $x_1 = \left(\dfrac{2m_2}{m_1+m_2}\right)x_0$

$\Rightarrow m_1(2x_0 - x_2) = m_2x_2$ $\quad\Rightarrow 2m_1x_0 - m_1x_2 = m_2x_2$ $\quad\Rightarrow x_2 = \left(\dfrac{2m_1}{m_1+m_2}\right)x_0$

50. a) $\therefore$ Velocity of centre of mass = $\dfrac{m_2 \times v_0 + m_1 \times 0}{m_1+m_2} = \dfrac{m_2v_0}{m_1+m_2}$

b) The spring will attain maximum elongation when both velocity of two blocks will attain the velocity of centre of mass.
d) $x \to$ maximum elongation of spring.
Change of kinetic energy = Potential stored in spring.



$\Rightarrow (1/2)\, m_2\, v_0^2 - (1/2)\,(m_1 + m_2)\left(\left(\dfrac{m_2v_0}{m_1+m_2}\right)^2 = (1/2)\, kx^2\right.$

$\Rightarrow m_2\, v_0^2\left(1-\dfrac{m_2}{m_1+m_2}\right) = kx^2$ $\qquad\Rightarrow x = \left(\dfrac{m_1m_2}{m_1+m_2}\right)^{1/2} \times v_0$

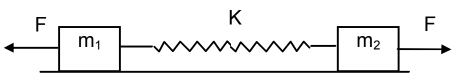
51. If both the blocks are pulled by some force, they suddenly move with some acceleration and instantaneously stop at same position where the elongation of spring is maximum.
$\therefore$ Let x_1, $x_2 \to$ extension by block m_1 and m_2
Total work done = $Fx_1 + Fx_2$ ...(1)
$\therefore$ Increase the potential energy of spring = $(1/2)\, K\, (x_1+ x_2)^2$ ...(2)
Equating (1) and (2)

$F(x_1 + x_2) = (1/2)\, K\, (x_1+ x_2)^2 \;\Rightarrow (x_1+ x_2) = \dfrac{2F}{K}$

Since the net external force on the two blocks is zero thus same force act on opposite direction.
$\therefore m_1x_1 = m_2x_2$ ...(3)

And $(x_1+ x_2) = \dfrac{2F}{K}$

$\therefore x_2 = \dfrac{m_1}{m_2} \times 1$



Substituting $\dfrac{m_1}{m_2} \times 1 + x_1 = \dfrac{2F}{K}$

$\Rightarrow x_1\left(1+\dfrac{m_1}{m_2}\right) = \dfrac{2F}{K}$ $\qquad\Rightarrow x_1 = \dfrac{2F}{K}\dfrac{m_2}{m_1+m_2}$

Similarly $x_2 = \dfrac{2F}{K}\dfrac{m_1}{m_1+m_2}$

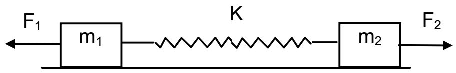
52. Acceleration of mass $m_1 = \dfrac{F_1 - F_2}{m_1 + m_2}$

Similarly Acceleration of mass $m_2 = \dfrac{F_2 - F_1}{m_1 + m_2}$

Due to F_1 and F_2 block of mass m_1 and m_2 will experience different acceleration and experience an inertia force.

$\therefore$ Net force on $m_1 = F_1 - m_1\,a$

$= F_1 - m_1 \times \dfrac{F_1 - F_2}{m_1 + m_2} = \dfrac{m_1F_1 + m_2F_1 - m_1F_1 + F_2m_1}{m_1 + m_2} = \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2}$



Similarly Net force on $m_2 = F_2 - m_2\,a$

$= F_2 - m_2 \times \dfrac{F_2 - F_1}{m_1 + m_2} = \dfrac{m_1F_2 + m_2F_2 - m_2F_2 + F_1m_2}{m_1 + m_2} = \dfrac{m_1F_2 + m_2F_2}{m_1 + m_2}$

$\therefore$ If m_1 displaces by a distance x_1 and x_2 by m_2 the maximum extension of the spring is $x_1 + m_2$.

$\therefore$ Work done by the blocks = energy stored in the spring.,

$\Rightarrow \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_1 + \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_2 = (1/2)\,K\,(x_1 + x_2)^2$

$\Rightarrow x_1 + x_2 = \dfrac{2}{K}\,\dfrac{m_2F_1 + m_1F_2}{m_1 + m_2}$

53. Mass of the man (M_m) is 50 kg.

Mass of the pillow (M_p) is 5 kg.

When the pillow is pushed by the man, the pillow will go down while the man goes up. It becomes the external force on the system which is zero.

$\Rightarrow$ acceleration of centre of mass is zero

$\Rightarrow$ velocity of centre of mass is constant

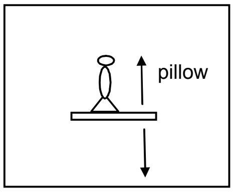
$\therefore$ As the initial velocity of the system is zero.

$\therefore M_m \times V_m = M_p \times V_p$ ...(1)

Given the velocity of pillow is 80 ft/s.

Which is relative velocity of pillow w.r.t. man.

$\vec{V}_{p/m} = \vec{V}_p - \vec{V}_m = V_p - (-V_m) = V_p + V_m \Rightarrow V_p = V_{p/m} - V_m$



Putting in equation (1)

$M_m \times V_m = M_p\,(V_{p/m} - V_m)$

$\Rightarrow 50 \times V_m = 5 \times (8 - V_m)$

$\Rightarrow 10 \times V_m = 8 - V_m \Rightarrow V_m = \dfrac{8}{11} = 0.727\text{m/s}$

$\therefore$ Absolute velocity of pillow = 8 – 0.727 = 7.2 ft/sec.

$\therefore$ Time taken to reach the floor $= \dfrac{S}{v} = \dfrac{8}{7.2} = 1.1$ sec.

As the mass of wall >>> then pillow

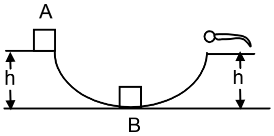
The velocity of block before the collision = velocity after the collision.

$\Rightarrow$ Times of ascent = 1.11 sec.

$\therefore$ Total time taken = 1.11 + 1.11 = 2.22 sec.

54. Let the velocity of A = u_1.

Let the final velocity when reaching at B becomes collision = v_1.



$\therefore (1/2)\,mv_1^2 - (1/2)mu_1^2 = mgh$

$\Rightarrow v_1^2 - u_1^2 = 2\,gh \qquad \Rightarrow v_1 = \sqrt{2gh - u_1^2}$ ...(1)

When the block B reached at the upper man's head, the velocity of B is just zero.

For B, block

$\therefore (1/2) \times 2m \times 0^2 - (1/2) \times 2m \times v^2 = mgh \qquad \Rightarrow v = \sqrt{2gh}$

$\therefore$ Before collision velocity of $u_A = v_1$, $u_B = 0$.

After collision velocity of $v_A = v$ (say) $v_B = \sqrt{2gh}$

Since it is an elastic collision the momentum and K.E. should be coserved.

$\therefore$ $m \times v_1 + 2m \times 0 = m \times v + 2m \times \sqrt{2gh}$

$\Rightarrow v_1 - v = 2\sqrt{2gh}$

Also, $(1/2) \times m \times v_1^2 + (1/2)$ I $2m \times 0^2 = (1/2) \times m \times v^2 + (1/2) \times 2m \times \left(\sqrt{2gh}\right)^2$

$\Rightarrow v_1^2 - v^2 = 2 \times \sqrt{2gh} \times \sqrt{2gh}$ …(2)

Dividing (1) by (2)

$$\frac{(v_1+v)(v_1-v)}{(v_1+v)} = \frac{2\times\sqrt{2gh}\times\sqrt{2gh}}{2\times\sqrt{2gh}} \Rightarrow v_1 + v = \sqrt{2gh} \quad \ldots(3)$$

Adding (1) and (3)

$$2v_1 = 3\sqrt{2gh} \Rightarrow v_1 = \left(\frac{3}{2}\right)\sqrt{2gh}$$

$$\text{But } v_1 = \sqrt{2gh+u^2} = \left(\frac{3}{2}\right)\sqrt{2gh}$$

$$\Rightarrow 2gh + u^2 = \frac{9}{4}\times 2gh$$

$\Rightarrow u = 2.5\sqrt{2gh}$

So the block will travel with a velocity greater than $2.5\sqrt{2gh}$ so awake the man by B.

55. Mass of block = 490 gm.

Mass of bullet = 10 gm.

Since the bullet embedded inside the block, it is an plastic collision.

Initial velocity of bullet $v_1 = 50\sqrt{7}$ m/s.

Velocity of the block is $v_2 = 0$.

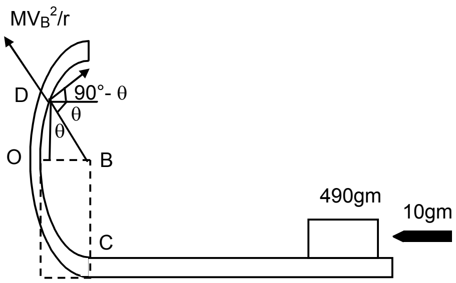
Let Final velocity of both = v.

$\therefore$ $10 \times 10^{-3} \times 50 \times \sqrt{7} + 10^{-3} \times 190$ I $0 = (490 + 10) \times 10^{-3} \times V_A$

$\Rightarrow V_A = \sqrt{7}$ m/s.

When the block losses the contact at 'D' the component mg will act on it.

$$\frac{m(V_B)^2}{r} = mg\sin\theta \Rightarrow (V_B)^2 = gr\sin\theta \quad \ldots(1)$$



Puttin work energy principle

$(1/2)\, m \times (V_B)^2 - (1/2) \times m \times (V_A)^2 = -mg\,(0.2 + 0.2\sin\theta)$

$\Rightarrow (1/2) \times gr\sin\theta - (1/2) \times \left(\sqrt{7}\right)^2 = -mg\,(0.2 + 0.2\sin\theta)$

$\Rightarrow 3.5 - (1/2) \times 9.8 \times 0.2 \times \sin\theta = 9.8 \times 0.2\,(1 + \sin\theta)$

$\Rightarrow 3.5 - 0.98\sin\theta = 1.96 + 1.96\sin\theta$

$\Rightarrow \sin\theta = (1/2) \Rightarrow \theta = 30°$

$\therefore$ Angle of projection = 90° - 30° = 60°.

$\therefore$ time of reaching the ground = $\sqrt{\dfrac{2h}{g}}$

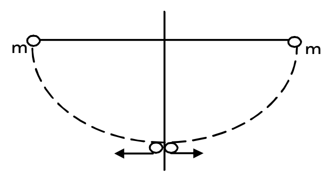
$$= \sqrt{\frac{2\times(0.2+0.2\times\sin 30°)}{9.8}} = 0.247 \text{ sec.}$$

$\therefore$ Distance travelled in horizontal direction.

$s = V\cos\theta \times t = \sqrt{gr\sin\theta}\times t = \sqrt{9.8\times 2\times(1/2)}\times 0.247 = 0.196$m

$\therefore$ Total distance = (0.2 – 0.2 cos 30°) + 0.196 = 0.22m.

56. Let the velocity of m reaching at lower end = V_1
From work energy principle.
$\therefore$ (1/2) × m × V_1^2 – (1/2) × m × 0^2 = mg ℓ
$\Rightarrow v_1 = \sqrt{2g\ell}$.
Similarly velocity of heavy block will be $v_2 = \sqrt{2gh}$.
$\therefore v_1 = V_2$ = u(say)
Let the final velocity of m and 2m v_1 and v_2 respectively.
According to law of conservation of momentum.
$m \times x_1 + 2m \times V_2 = mv_1 + 2mv_2$
$\Rightarrow m \times u - 2\,m\,u = mv_1 + 2mv_2$
$\Rightarrow v_1 + 2v_2 = -u$ …(1)
Again, $v_1 - v_2 = -(V_1 - V_2)$
$\Rightarrow v_1 - v_2 = -[u - (-v)] = -2V$ …(2)
Subtracting.



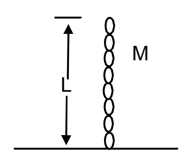
$3v_2 = u \Rightarrow v_2 = \frac{u}{3} = \frac{\sqrt{2g\ell}}{3}$

Substituting in (2)

$v_1 - v_2 = -2u \Rightarrow v_1 = -2u + v_2 = -2u + \frac{u}{3} = -\frac{5}{3}u = -\frac{5}{3} \times \sqrt{2g\ell} = -\frac{\sqrt{50g\ell}}{3}$

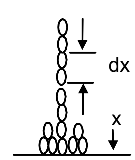
b) Putting the work energy principle
(1/2) × 2m × 0^2 – (1/2) × 2m × $(v_2)^2$ = – 2m × g × h
[h → height gone by heavy ball]
$\Rightarrow (1/2)\,\frac{2g}{9} = \ell \times h \qquad \Rightarrow h = \frac{\ell}{9}$
Similarly, (1/2) × m × 0^2 – (1/2) × m × v_1^2 = m × g × h_2
[height reached by small ball]
$\Rightarrow (1/2) \times \frac{50g\ell}{9} = g \times h_2 \qquad \Rightarrow h_2 = \frac{25\ell}{9}$
Someh_2 is more than 2ℓ, the velocity at height point will not be zero. And the 'm' will rise by a distance 2ℓ.

57. Let us consider a small element at a distance 'x' from the floor of length 'dy' .
So, $dm = \frac{M}{L}dx$



(Initial position)

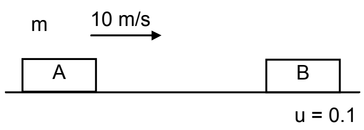
So, the velocity with which the element will strike the floor is, $v = \sqrt{2gx}$
$\therefore$ So, the momentum transferred to the floor is,
$M = (dm)v = \frac{M}{L} \times dx \times \sqrt{2gx}$ [because the element comes to rest]
So, the force exerted on the floor change in momentum is given by,
$F_1 = \frac{dM}{dt} = \frac{M}{L} \times \frac{dx}{dt} \times \sqrt{2gx}$
Because, $v = \frac{dx}{dt} = \sqrt{2gx}$ (for the chain element)
$F_1 = \frac{M}{L} \times \sqrt{2gx} \times \sqrt{2gx} = \frac{M}{L} \times 2gx = \frac{2Mgx}{L}$
Again, the force exerted due to 'x' length of the chain on the floor due to its own weight is given by,
$W = \frac{M}{L}(x) \times g = \frac{Mgx}{L}$
So, the total forced exerted is given by,
$F = F_1 + W = \frac{2Mgx}{L} + \frac{Mgx}{L} = \frac{3Mgx}{L}$



58. $V_1 = 10$ m/s $\quad V_2 = 0$

$V_1, v_2 \rightarrow$ velocity of ACB after collision.

a) If the edlision is perfectly elastic.



$mV_1 + mV_2 = mv_1 + mv_2$

$\Rightarrow 10 + 0 = v_1 + v_2$

$\Rightarrow v_1 + v_2 = 10 \quad \ldots(1)$

Again, $v_1 - v_2 = -(u_1 - v_2) = -(10 - 0) = -10 \quad \ldots(2)$

Subtracting (2) from (1)

$2v_2 = 20 \Rightarrow v_2 = 10$ m/s.

The deacceleration of B = μg

Putting work energy principle

$\therefore (1/2) \times m \times 0^2 - (1/2) \times m \times v_2^2 = - m \times a \times h$

$\Rightarrow -(1/2) \times 10^2 = -\mu g \times h \quad \Rightarrow h = \dfrac{100}{2 \times 0.1 \times 10} = 50$m

b) If the collision perfectly in elastic.

$m \times u_1 + m \times u_2 = (m + m) \times v$

$\Rightarrow m \times 10 + m \times 0 = 2m \times v \quad \Rightarrow v = \dfrac{10}{2} = 5$ m/s.

The two blocks will move together sticking to each other.

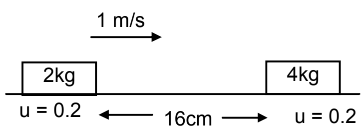
$\therefore$ Putting work energy principle.

$(1/2) \times 2m \times 0^2 - (1/2) \times 2m \times v^2 = 2m \times \mu g \times s$

$\Rightarrow \dfrac{5^2}{0.1 \times 10 \times 2} = s \quad \Rightarrow s = 12.5$ m.

59. Let velocity of 2kg block on reaching the 4kg block before collision $= u_1$.

Given, $V_2 = 0$ (velocity of 4kg block).



$\therefore$ From work energy principle,

$(1/2)\, m \times u_1^2 - (1/2)\, m \times 1^2 = - m \times ug \times s$

$\Rightarrow \dfrac{u_1^2 - 1}{2} = -2 \times 5 \quad \Rightarrow -16 = \dfrac{u_1^2 - 1}{4}$

$\Rightarrow 64 \times 10^{-2} = u_1^2 - 1 \quad \Rightarrow u_1 = 6$m/s

Since it is a perfectly elastic collision.

Let $V_1, V_2 \rightarrow$ velocity of 2kg & 4kg block after collision.

$m_1V_1 + m_2V_2 = m_1v_1 + m_2v_2$

$\Rightarrow 2 \times 0.6 + 4 \times 0 = 2v_1 + 4\, v_2 \quad \Rightarrow v_1 + 2v_2 = 0.6 \quad \ldots(1)$

Again, $V_1 - V_2 = -(u_1 - u_2) = -(0.6 - 0) = -0.6 \quad \ldots(2)$

Subtracting (2) from (1)

$3v_2 = 1.2 \quad \Rightarrow v_2 = 0.4$ m/s.

$\therefore v_1 = -0.6 + 0.4 = -0.2$ m/s

$\therefore$ Putting work energy principle for 1st 2kg block when come to rest.

$(1/2) \times 2 \times 0^2 - (1/2) \times 2 \times (0.2)^2 = -2 \times 0.2 \times 10 \times s$

$\Rightarrow (1/2) \times 2 \times 0.2 \times 0.2 = 2 \times 0.2 \times 10 \times s \quad \Rightarrow S_1 = 1$cm.

Putting work energy principle for 4kg block.

$(1/2) \times 4 \times 0^2 - (1/2) \times 4 \times (0.4)^2 = -4 \times 0.2 \times 10 \times s$

$\Rightarrow 2 \times 0.4 \times 0.4 = 4 \times 0.2 \times 10 \times s \quad \Rightarrow S_2 = 4$ cm.

Distance between 2kg & 4kg block = $S_1 + S_2 = 1 + 4 = 5$ cm.

60. The block 'm' will slide down the inclined plane of mass M with acceleration a_1 g sin α (relative) to the inclined plane.

The horizontal component of a_1 will be, $a_x = g \sin\alpha \cos\alpha$, for which the block M will accelerate towards left. Let, the acceleration be a_2.

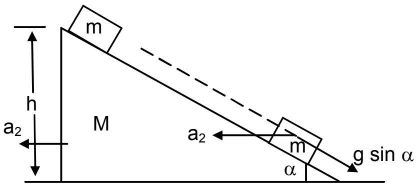
According to the concept of centre of mass, (in the horizontal direction external force is zero).

$ma_x = (M + m)\, a_2$

$\Rightarrow a_2 = \dfrac{ma_x}{M+m} = \dfrac{mg\sin\alpha\cos\alpha}{M+m}$ …(1)

So, the absolute (Resultant) acceleration of 'm' on the block 'M' along the direction of the incline will be,

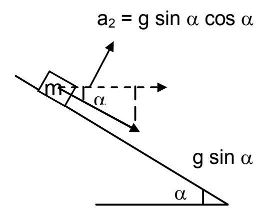
$a = g \sin \alpha - a_2 \cos \alpha$



$= g \sin \alpha - \dfrac{mg\sin\alpha\cos^2\alpha}{M+m} = g \sin \alpha \left[1 - \dfrac{m\cos^2\alpha}{M+m}\right]$

$= g \sin \alpha \left[\dfrac{M+m-m\cos^2\alpha}{M+m}\right]$

So, $a = g \sin \alpha \left[\dfrac{M+m\sin^2\alpha}{M+m}\right]$ …(2)



Let, the time taken by the block 'm' to reach the bottom end be 't'.

Now, $S = ut + (1/2)\, at^2$

$\Rightarrow \dfrac{h}{\sin\alpha} = (1/2)\, at^2 \qquad \Rightarrow t = \sqrt{\dfrac{2}{a\sin\alpha}}$

So, the velocity of the bigger block after time 't' will be.

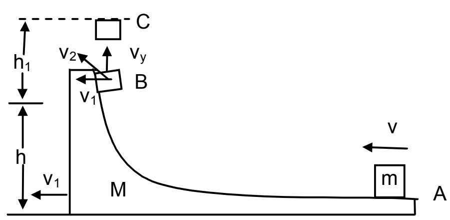
$V_m = u + a_2 t = \dfrac{mg\sin\alpha\cos\alpha}{M+m}\sqrt{\dfrac{2h}{a\sin\alpha}} = \sqrt{\dfrac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2 a\sin\alpha}}$

Now, subtracting the value of a from equation (2) we get,

$V_M = \left[\dfrac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2\sin\alpha} \times \dfrac{(M+m)}{g\sin\alpha(M+m\sin^2\alpha)}\right]^{1/2}$

or $V_M = \left[\dfrac{2m^2g^2h\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}\right]^{1/2}$

61.



The mass 'm' is given a velocity 'v' over the larger mass M.

a) When the smaller block is travelling on the vertical part, let the velocity of the bigger block be v_1 towards left.

From law of conservation of momentum, (in the horizontal direction)

$mv = (M + m)\, v_1$

$\Rightarrow v_1 = \dfrac{mv}{M+m}$

b) When the smaller block breaks off, let its resultant velocity is v_2.

From law of conservation of energy,

$(1/2)\, mv^2 = (1/2)\, Mv_1^2 + (1/2)\, mv_2^2 + mgh$

$\Rightarrow v_2^2 = v^2 - \dfrac{M}{m} v_1^2 - 2gh$ ..(1)

$\Rightarrow v_2^2 = v^2\left[1 - \dfrac{M}{m} \times \dfrac{m^2}{(M+m)^2}\right] - 2gh$

$\Rightarrow v_2 = \left[\dfrac{(m^2+Mm+m^2)}{(M+m)^2} v^2 - 2gh\right]^{1/2}$

e) Now, the vertical component of the velocity v_2 of mass 'm' is given by,

$v_y^2 = v_2^2 - v_1^2$

$= \frac{(M^2 + Mm + m^2)}{(M+m)^2} v^2 - 2gh - \frac{m^2v^2}{(M+m)^2}$

$[\therefore v_1 = \frac{mv}{M+v}]$

$\Rightarrow v_y^2 = \frac{M^2 + Mm + m^2 - m^2}{(M+m)^2} v^2 - 2gh$

$\Rightarrow v_y^2 = \frac{Mv^2}{(M+m)} - 2gh \quad \ldots(2)$

To find the maximum height (from the ground), let us assume the body rises to a height 'h', over and above 'h'.

Now, $(1/2)mv_y^2 = mgh_1 \Rightarrow h_1 = \frac{v_y^2}{2g} \quad \ldots(3)$

So, Total height $= h + h_1 = h + \frac{v_y^2}{2g} = h + \frac{mv^2}{(M+m)2g} - h$

[from equation (2) and (3)]

$\Rightarrow H = \frac{mv^2}{(M+m)2g}$

d) Because, the smaller mass has also got a horizontal component of velocity 'v_1' at the time it breaks off from 'M' (which has a velocity v_1), the block 'm' will again land on the block 'M' (bigger one).

Let us find out the time of flight of block 'm' after it breaks off.

During the upward motion (BC),

$0 = v_y - gt_1$

$\Rightarrow t_1 = \frac{v_y}{g} = \frac{1}{g}\left[\frac{Mv^2}{(M+m)} - 2gh\right]^{1/2} \quad \ldots(4)$ [from equation (2)]

So, the time for which the smaller block was in its flight is given by,

$T = 2t_1 = \frac{2}{g}\left[\frac{Mv^2 - 2(M+m)gh}{(M+m)}\right]^{1/2}$

So, the distance travelled by the bigger block during this time is,

$S = v_1T = \frac{mv}{M+m} \times \frac{2}{g} \frac{[Mv^2 - 2(M+m)gh]^{1/2}}{(M+m)^{1/2}}$

or $S = \frac{2mv[Mv^2 - 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$

62. Given h < < < R.

$G_{mass} = 6 \text{ I } 10^{24}$ kg.

$M_b = 3 \times 10^{24}$ kg.

Let $V_e \rightarrow$ Velocity of earth

$V_b \rightarrow$ velocity of the block.

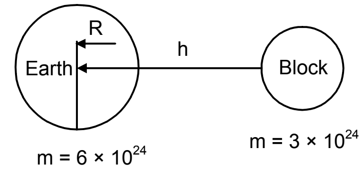
The two blocks are attracted by gravitational force of attraction. The gravitation potential energy stored will be the K.E. of two blocks.

$\overline{G}^{pim}\left[\frac{1}{R+(h/2)} - \frac{1}{R+h}\right] = (1/2)\, m_e \times v_e^2 + (1/2)\, m_b \times v_b^2$

Again as the an internal force acts.

$M_eV_e = m_bV_b \qquad \Rightarrow V_e = \frac{m_bV_b}{M_e} \quad \ldots(2)$

Putting in equation (1)



$G_{me} \times m_b \left[\frac{2}{2R+h} - \frac{1}{R+h}\right]$

$= (1/2) \times M_e \times \frac{m_b{}^2 v_b{}^2}{M_e{}^2} \times v_e{}^2 + (1/2)\, M_b \times V_b{}^2$

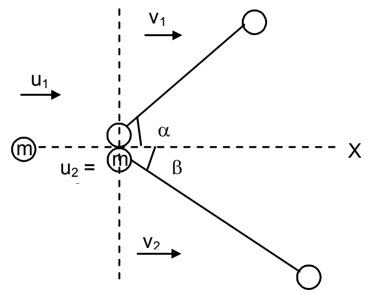
$= (1/2) \times m_b \times V_b{}^2 \left(\frac{M_b}{M_e} + 1\right)$

$\Rightarrow GM\left[\frac{2R+2h-2R-h}{(2R+h)(R+h)}\right] = (1/2) \times V_b{}^2 \times \left(\frac{3\times10^{24}}{6\times10^{24}}+1\right) \quad \Rightarrow \left[\frac{GM\times h}{2R^2+3Rh+h^2}\right] = (1/2) \times V_b{}^2 \times (3/2)$

As h < < < R, if can be neglected

$\Rightarrow \frac{GM\times h}{2R^2} = (1/2) \times V_b{}^2 \times (3/2) \qquad \Rightarrow V_b = \sqrt{\frac{2gh}{3}}$

63. Since it is not an head on collision, the two bodies move in different dimensions. Let $V_1, V_2 \rightarrow$ velocities of the bodies vector collision. Since, the collision is elastic. Applying law of conservation of momentum on X-direction.



$mu_1 + mxo = mv_1 \cos\alpha + mv_2 \cos\beta$

$\Rightarrow v_1 \cos a + v_2 \cos b = u_1 \quad \ldots(1)$

Putting law of conservation of momentum in y direction.

$0 = mv_1 \sin\alpha - mv_2 \sin\beta$

$\Rightarrow v_1 \sin\alpha = v_2 \sin\beta \qquad \ldots(2)$

Again $\frac{1}{2} m\, u_1{}^2 + 0 = \frac{1}{2} m\, v_1{}^2 + \frac{1}{2} m \,x\, v_2{}^2$

$\Rightarrow u_1{}^2 = v_1{}^2 + v_2{}^2 \qquad \ldots(3)$

Squaring equation(1)

$u_1{}^2 = v_1{}^2 \cos^2\alpha + v_2{}^2 \cos^2\beta + 2\, v_1 v_2 \cos\alpha \cos\beta$

Equating (1) & (3)

$v_1{}^2 + v_2{}^2 = v_1{}^2 \cos^2\alpha + v_2{}^2 \cos^2\beta + 2\, v_1 v_2 \cos\alpha \cos\beta$

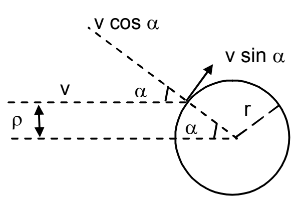
$\Rightarrow v_1{}^2 \sin^2\alpha + v_2{}^2 \sin^2\beta = 2\, v_1 v_2 \cos\alpha \cos\beta$

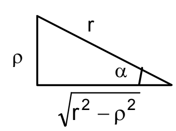
$\Rightarrow 2v_1{}^2 \sin^2\alpha = 2 \times v_1 \times \frac{v_1 \sin\alpha}{\sin\beta} \times \cos\alpha \cos\beta$

$\Rightarrow \sin\alpha \sin\beta = \cos\alpha \cos\beta \qquad \Rightarrow \cos\alpha \cos\beta - \sin\alpha \sin\beta = 0$

$\Rightarrow \cos(\alpha + \beta) = 0 = \cos 90° \qquad \Rightarrow (\alpha + \beta) = 90°$

64.





Let the mass of both the particle and the spherical body be 'm'. The particle velocity 'v' has two components, $v \cos\alpha$ normal to the sphere and $v \sin\alpha$ tangential to the sphere.

After the collision, they will exchange their velocities. So, the spherical body will have a velocity $v \cos\alpha$ and the particle will not have any component of velocity in this direction.

[The collision will due to the component $v \cos\alpha$ in the normal direction. But, the tangential velocity, of the particle $v \sin\alpha$ will be unaffected]

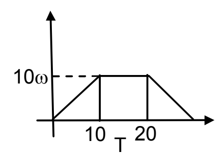
So, velocity of the sphere $= v \cos\alpha = \frac{v}{r}\sqrt{r^2 - \rho^2}$ [from (fig-2)]

And velocity of the particle $= v \sin\alpha = \frac{v\rho}{r}$

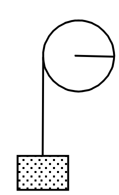
* * * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 10

1. $\omega_0 = 0$; $\rho = 100$ rev/s ; $\omega = 2\pi$; $\rho = 200\ \pi$ rad/s
$\Rightarrow \omega = \omega_0 = \alpha t$
$\Rightarrow \omega = \alpha t$
$\Rightarrow \alpha = (200\ \pi)/4 = 50\ \pi$ rad /s^2 or 25 rev/s^2
$\therefore \theta = \omega_0 t + 1/2\ \alpha t^2 = 8 \times 50\ \pi = 400\ \pi$ rad
$\therefore \alpha = 50\ \pi$ rad/s^2 or 25 rev/s^s
$\theta = 400\ \pi$ rad.
2. $\theta = 100\ \pi$; t = 5 sec
$\theta = 1/2\ \alpha t^2 \Rightarrow 100\pi = 1/2\ \alpha\ 25$
$\Rightarrow \alpha = 8\pi \times 5 = 40\ \pi$ rad/s = 20 rev/s
$\therefore \alpha = 8\pi$ rad/s^2 = 4 rev/s^2
$\omega = 40\pi$ rad/s^2 = 20 rev/s^2.
3. Area under the curve will decide the total angle rotated
$\therefore$ maximum angular velocity = 4 × 10 = 40 rad/s
Therefore, area under the curve = 1/2 × 10 × 40 + 40 × 10 + 1/2 × 40 × 10
= 800 rad
$\therefore$ Total angle rotated = 800 rad.



4. $\alpha = 1$ rad/s^2, $\omega_0 = 5$ rad/s; $\omega = 15$ rad/s
$\therefore w = w_0 + \alpha t$
$\Rightarrow t = (\omega - \omega_0)/\alpha = (15 - 5)/1 = 10$ sec
Also, $\theta = \omega_0 t + 1/2\ \alpha t^2$
= 5 ×10 + 1/2 × 1 × 100 = 100 rad.
5. $\theta = 5$ rev, $\alpha = 2$ rev/s^2, $\omega_0 = 0$; ω = ?
$\omega^2 = (2\ \alpha\ \theta)$
$\Rightarrow \omega = \sqrt{2 \times 2 \times 5} = 2\sqrt{5}$ rev/s.
or $\theta = 10\pi$ rad, $\alpha = 4\pi$ rad/s^2, $\omega_0 = 0$, ω = ?
$\omega = \sqrt{2\alpha\theta} = 2 \times 4\pi \times 10\pi$
$= 4\pi\sqrt{5}$ rad/s $= 2\sqrt{5}$ rev/s.
6. A disc of radius = 10 cm = 0.1 m
Angular velocity = 20 rad/s
$\therefore$ Linear velocity on the rim = ωr = 20 × 0.1 = 2 m/s
$\therefore$ Linear velocity at the middle of radius = $\omega r/2$ = 20 × (0.1)/2 = 1 m/s.
7. t = 1 sec, r = 1 cm = 0.01 m
$\alpha = 4$ rd/s^2
Therefore $\omega = \alpha t = 4$ rad/s
Therefore radial acceleration,
$A_n = \omega^2 r = 0.16$ m/s^2 = 16 cm/s^2
Therefore tangential acceleration, $a_r = \alpha r = 0.04$ m/s^2 = 4 cm/s^2.
8. The Block is moving the rim of the pulley
The pulley is moving at a ω = 10 rad/s
Therefore the radius of the pulley = 20 cm
Therefore linear velocity on the rim = tangential velocity = $r\omega$
= 20 × 20 = 200 cm/s = 2 m/s.

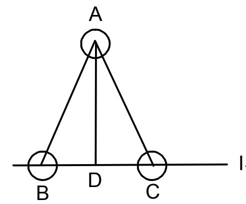


9. Therefore, the $\perp$ distance from the axis (AD) = $\sqrt{3}/2 \times 10 = 5\sqrt{3}$ cm.

Therefore moment of inertia about the axis BC will be

$I = mr^2 = 200\ K\ (5\sqrt{3})^2 = 200 \times 25 \times 3$

$= 15000$ gm – cm^2 = 1.5×10^{-3} kg – m^2.



b) The axis of rotation let pass through A and $\perp$ to the plane of triangle

Therefore the torque will be produced by mass B and C

Therefore net moment of inertia = $I = mr^2 + mr^2$

$= 2 \times 200 \times 10^2 = 40000$ gm-cm^2 = 4×10^{-3} kg-m^2.

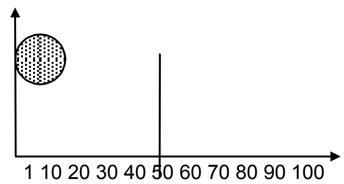
10. Masses of 1 gm, 2 gm ……100 gm are kept at the marks 1 cm, 2 cm, ……1000 cm on he x axis respectively. A perpendicular axis is passed at the 50th particle.

Therefore on the L.H.S. side of the axis there will be 49 particles and on the R.H.S. side there are 50 particles.

Consider the two particles at the position 49 cm and 51 cm.

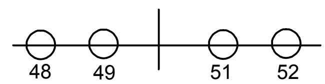
Moment inertial due to these two particle will be =

$$49 \times 1^2 + 51 + 1^2 = 100 \text{ gm-cm}^2$$



Similarly if we consider 48th and 52nd term we will get 100×2^2 gm-cm^2

Therefore we will get 49 such set and one lone particle at 100 cm.



Therefore total moment of inertia =

$100\ \{1^2 + 2^2 + 3^2 + \ldots + 49^2\} + 100(50)^2$.

$= 100 \times (50 \times 51 \times 101)/6 = 4292500$ gm-cm^2

$= 0.429$ kg-m^2 = 0.43 kg-m^2.

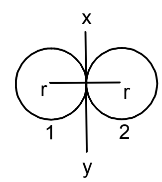
11. The two bodies of mass m and radius r are moving along the common tangent.

Therefore moment of inertia of the first body about XY tangent.

$= mr^2 + 2/5\ mr^2$

– Moment of inertia of the second body XY tangent = $mr^2 + 2/5\ mr^2 = 7/5\ mr^2$

Therefore, net moment of inertia = $7/5\ mr^2 + 7/5\ mr^2 = 14/5\ mr^2$ units.



12. Length of the rod = 1 m, mass of the rod = 0.5 kg

Let at a distance d from the center the rod is moving

Applying parallel axis theorem :

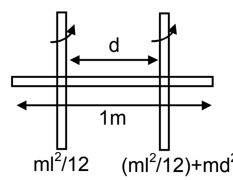
The moment of inertial about that point

$\Rightarrow (mL^2 / 12) + md^2 = 0.10$

$\Rightarrow (0.5 \times 1^2)/12 + 0.5 \times d^2 = 0.10$

$\Rightarrow d^2 = 0.2 - 0.082 = 0.118$

$\Rightarrow d = 0.342$ m from the centre.



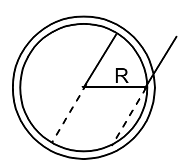
13. Moment of inertia at the centre and perpendicular to the plane of the ring.

So, about a point on the rim of the ring and the axis $\perp$ to the plane of the ring, the moment of inertia

$= mR^2 + mR^2 = 2mR^2$ (parallel axis theorem)

$\Rightarrow mK^2 = 2mR^2$ (K = radius of the gyration)

$\Rightarrow K = \sqrt{2R^2} = \sqrt{2}\ R$.



14. The moment of inertia about the center and $\perp$ to the plane of the disc of radius r and mass m is = mr^2.

According to the question the radius of gyration of the disc about a point = radius of the disc.

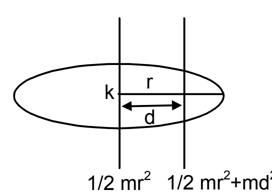
Therefore $mk^2 = ½\ mr^2 + md^2$

(K = radius of gyration about acceleration point, d = distance of that point from the centre)

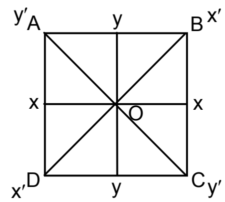
$\Rightarrow K^2 = r^2/2 + d^2$

$\Rightarrow r^2 = r^2/2 + d^2$ ($\because$ K = r)

$\Rightarrow r^2/2 = d^2 \Rightarrow d = r/\sqrt{2}$.



15. Let a small cross sectional area is at a distance x from xx axis.
Therefore mass of that small section = $m/a^2 \times ax\,dx$
Therefore moment of inertia about xx axis



$= I_{xx} = 2\int_0^{a/2}(m/a^2)\times(adx)\times x^2 = (2\times(m/a)(x^3/3)]_0^{a/2}$

$= ma^2/12$

Therefore $I_{xx} = I_{xx} + I_{yy}$

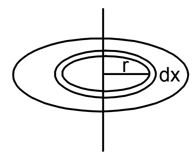
$= 2 \times {}^*ma^2/12) = ma^2/6$

Since the two diagonals are $\perp$ to each other

Therefore $I_{zz} = I_{x'x'} + I_{y'y'}$

$\Rightarrow ma^2/6 = 2 \times I_{x'x'}$ (because $I_{x'x'} = I_{y'y'}$) $\Rightarrow I_{x'x'} = ma^2/12$

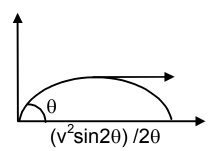
16. The surface density of a circular disc of radius a depends upon the distance from the centre as $P(r) = A + Br$
Therefore the mass of the ring of radius r will be
$\theta = (A + Br) \times 2\pi r\,dr \times r^2$
Therefore moment of inertia about the centre will be



$= \int_0^a (A+Br)2\pi r\times dr = \int_0^a 2\pi Ar^3 dr + \int_0^a 2\pi Br^4 dr$

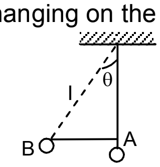
$= 2\pi A\,(r^4/4) + 2\pi\,B(r^5/5)]_0^a = 2\pi a^4\,[(A/4) + (Ba/5)].$

17. At the highest point total force acting on the particle id its weight acting downward.
Range of the particle = $u^2 \sin 2\pi / g$
Therefore force is at a $\perp$ distance, $\Rightarrow$ (total range)/2 = $(v^2 \sin 2\theta)/2g$
(From the initial point)



Therefore $\tau = F \times r$ (θ = angle of projection)
= $mg \times v^2 \sin 2\theta/2g$ (v = initial velocity)
= $mv^2 \sin 2\theta / 2 = mv^2 \sin\theta \cos\theta$.

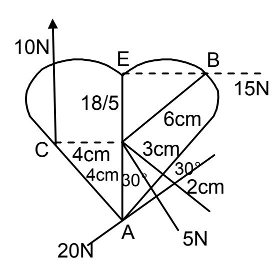
18. A simple of pendulum of length l is suspended from a rigid support. A bob of weight W is hanging on the other point.
When the bob is at an angle θ with the vertical, then total torque acting on the point of suspension = i = F × r



$\Rightarrow W\,r \sin\theta = W\,l \sin\theta$
At the lowest point of suspension the torque will be zero as the force acting on the body passes through the point of suspension.

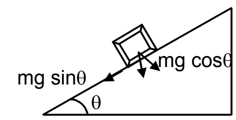
19. A force of 6 N acting at an angle of 30° is just able to loosen the wrench at a distance 8 cm from it.
Therefore total torque acting at A about the point 0
= 6 sin 30° × (8/100)
Therefore total torque required at B about the point 0
= F × 16/100 $\Rightarrow$ F × 16/100 = 6 sin 30° × 8/100
$\Rightarrow$ F = (8 × 3) / 16 = 1.5 N.

20. Torque about a point = Total force × perpendicular distance from the point to that force.
Let anticlockwise torque = + ve
And clockwise acting torque = –ve
Force acting at the point B is 15 N
Therefore torque at O due to this force
= $15 \times 6 \times 10^{-2} \times \sin 37°$
= $15 \times 6 \times 10^{-2} \times 3/5 = 0.54$ N-m (anticlock wise)



Force acting at the point C is 10 N
Therefore, torque at O due to this force
= $10 \times 4 \times 10^{-2} = 0.4$ N-m (clockwise)
Force acting at the point A is 20 N
Therefore, Torque at O due to this force = $20 \times 4 \times 10^{-2} \times \sin 30°$
= $20 \times 4 \times 10^{-2} \times 1/2 = 0.4$ N-m (anticlockwise)
Therefore resultant torque acting at 'O' = 0.54 – 0.4 + 0.4 = 0.54 N-m.

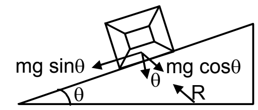
21. The force mg acting on the body has two components mg sin θ and mg cos θ and the body will exert a normal reaction. Let R =



Since R and mg cos θ pass through the centre of the cube, there will be no torque due to R and mg cos θ. The only torque will be produced by mg sin θ.

$\therefore$ i = F × r (r = a/2) (a = ages of the cube)

$\Rightarrow$ i = mg sin θ × a/2



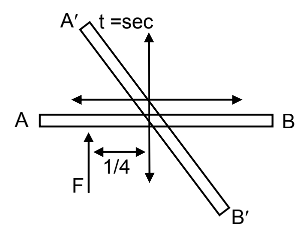
= 1/2 mg a sin θ.

22. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis passing through its centre.

A force F is acting perpendicular to the rod at a distance L/4 from the centre.

Therefore torque about the centre due to this force

i_i = F × r = FL/4.



This torque will produce a angular acceleration α.

Therefore $\tau_c = I_c \times \alpha$

$\Rightarrow i_c = (mL^2 / 12) \times \alpha$ (I_c of a rod = $mL^2 / 12$)

$\Rightarrow$ F i/4 = $(mL^2 / 12) \times \alpha \Rightarrow \alpha$ = 3F/ml

Therefore $\theta = 1/2\ \alpha t^2$ (initially at rest)

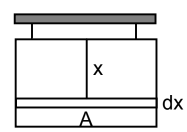
$\Rightarrow \theta = 1/2 \times (3F / ml)t^2 = (3F/2ml)t^2$.

23. A square plate of mass 120 gm and edge 5 cm rotates about one of the edge.

Let take a small area of the square of width dx and length a which is at a distance x from the axis of rotation.

Therefore mass of that small area

$m/a^2 \times a\,dx$ (m = mass of the square ; a = side of the plate)



$$I = \int_0^a (m/a^2) \times ax^2 dx = (m/a)(x^3/3)]_0^a$$

$= ma^2/3$

Therefore torque produced = I × α = $(ma^2/3) \times \alpha$

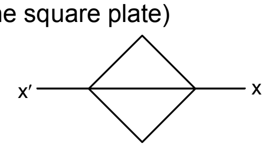
$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/3\}\ 0.2$

$= 0.2 \times 10^{-4} = 2 \times 10^{-5}$ N-m.

24. Moment of inertial of a square plate about its diagonal is $ma^2/12$ (m = mass of the square plate)

a = edges of the square

Therefore torque produced = $(ma^2/12) \times \alpha$



$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/12 \times 0.2$

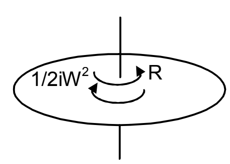
$= 0.5 \times 10^{-5}$ N-m.

25. A flywheel of moment of inertia 5 kg m is rotated at a speed of 60 rad/s. The flywheel comes to rest due to the friction at the axle after 5 minutes.

Therefore, the angular deceleration produced due to frictional force = $\omega = \omega_0 + \alpha t$

$\Rightarrow \omega_0 = -\alpha t$ (ω = 0+

$\Rightarrow \alpha = -(60/5 \times 60) = -1/5$ rad/s^2.



a) Therefore total workdone in stopping the wheel by frictional force

W = 1/2 $i\omega^2$ = 1/2 × 5 × (60 × 60) = 9000 Joule = 9 KJ.

b) Therefore torque produced by the frictional force (R) is

$I_R = I \times \alpha = 5 \times (-1/5)$ = IN – m opposite to the rotation of wheel.

c) Angular velocity after 4 minutes

$\Rightarrow \omega = \omega_0 + \alpha t = 60 - 240/5 = 12$ rad/s

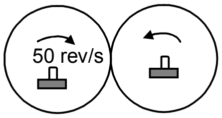
Therefore angular momentum about the centre = 1 × ω = 5 × 12 = 60 kg-m^2/s.

26. The earth's angular speed decreases by 0.0016 rad/day in 100 years.
Therefore the torque produced by the ocean water in decreasing earth's angular velocity
$\tau = I\alpha$
$= 2/5\ mr^2 \times (\omega - \omega_0)/t$
$= 2/6 \times 6 \times 10^{24} \times 64^2 \times 10^{10} \times [0.0016 /(26400^2 \times 100 \times 365)]$ (1 year = 365 days= 365 × 56400 sec)
$= 5.678 \times 10^{20}$ N-m.

27. A wheel rotating at a speed of 600 rpm.
ω_0 = 600 rpm = 10 revolutions per second.
T = 10 sec. (In 10 sec. it comes to rest)
$\omega = 0$
Therefore $\omega_0 = -\alpha t$
$\Rightarrow \alpha = -10/10 = -1$ rev/s^2
$\Rightarrow \omega = \omega_0 + \alpha t = 10 - 1 \times 5 = 5$ rev/s.
Therefore angular deacceleration = 1 rev/s^2 and angular velocity of after 5 sec is 5 rev/s.

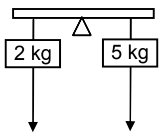
28. ω = 100 rev/min = 5/8 rev/s = $10\pi/3$ rad/s
θ = 10 rev = 20 π rad, r = 0.2 m
After 10 revolutions the wheel will come to rest by a tangential force
Therefore the angular deacceleration produced by the force = $\alpha = \omega^2/2\theta$
Therefore the torque by which the wheel will come to an rest = $I_{cm} \times \alpha$
$\Rightarrow F \times r = I_{cm} \times \alpha \rightarrow F \times 0.2 = 1/2\ mr^2 \times [(10\pi/3)^2 / (2 \times 20\pi)]$
$\Rightarrow F = 1/2 \times 10 \times 0.2 \times 100\ \pi^2 / (9 \times 2 \times 20\pi)$
$= 5\pi / 18 = 15.7/18 = 0.87$ N.

29. A cylinder is moving with an angular velocity 50 rev/s brought in contact with another identical cylinder in rest. The first and second cylinder has common acceleration and deacceleration as 1 rad/s^2 respectively.
Let after t sec their angular velocity will be same 'ω'.
For the first cylinder $\omega = 50 - \alpha t$
$\Rightarrow t = (\omega - 50)/-1$
And for the 2nd cylinder $\omega = \alpha_2 t$
$\Rightarrow t = \omega/I$
So, $\omega = (\omega - 50)/-1$
$\Rightarrow 2\omega = 50 \Rightarrow \omega = 25$ rev/s.
$\Rightarrow t = 25/1$ sec = 25 sec.

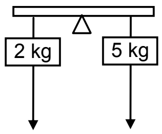


30. Initial angular velocity = 20 rad/s
Therefore $\alpha = 2$ rad/s^2
$\Rightarrow t_1 = \omega/\alpha_1 = 20/2 = 10$ sec
Therefore 10 sec it will come to rest.
Since the same torque is continues to act on the body it will produce same angular acceleration and since the initial kinetic energy = the kinetic energy at a instant.
So initial angular velocity = angular velocity at that instant
Therefore time require to come to that angular velocity,
$t_2 = \omega_2/\alpha_2 = 20/2 = 10$ sec
therefore time required = $t_1 + t_2$ = 20 sec.

31. $I_{net} = I_{net} \times \alpha$
$\Rightarrow F_1r_1 - F_2r_2 = (m_1r_1^2 + m_2r_2^2) \times \alpha - 2 \times 10 \times 0.5$
$\Rightarrow 5 \times 10 \times 0.5 = (5 \times (1/2)^2 + 2 \times (1/2)^2) \times \alpha$
$\Rightarrow 15 = 7/4\ \alpha$
$\Rightarrow \alpha = 60/7 = 8.57$ rad/s^2.



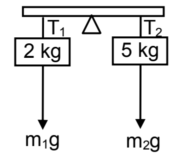
32. In this problem the rod has a mass 1 kg
a) $\tau_{net} = I_{net} \times \alpha$
$\Rightarrow 5 \times 10 \times 10.5 - 2 \times 10 \times 0.5$
$= (5 \times (1/2)^2 + 2 \times (1/2)^2 + 1/12) \times \alpha$



$\Rightarrow 15 = (1.75 + 0.084)\ \alpha$

$\Rightarrow \alpha = 1500/(175 + 8.4) = 1500/183.4 = 8.1\ rad/s^2$ (g = 10)

$= 8.01\ rad/s^2$ (if g = 9.8)



b) $T_1 - m_1g = m_1a$

$\Rightarrow T_1 = m_1a + m_1g = 2(a + g)$

$= 2(\alpha r + g) = 2(8 \times 0.5 + 9.8)$

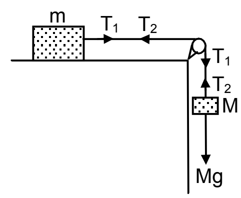
= 27.6 N on the first body.

In the second body

$\Rightarrow m_2g - T_2 = m_2a \Rightarrow T_2 = m_2g - m_2a$

$\Rightarrow T_2 = 5(g - a) = 5(9.8 - 8 \times 0.5) = 29$ N.

33. According to the question



$Mg - T_1 = Ma$ …(1)

$T_2 = ma$ …(2)

$(T_1 - T_2) = 1\ a/r^2$ …(3) [because $a = r\alpha$]…**[T.r =I(a/r)]**

If we add the equation 1 and 2 we will get

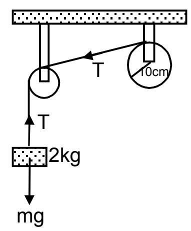
$Mg + (T_2 - T_1) = Ma + ma$ …(4)

$\Rightarrow Mg - Ia/r^2 = Ma + ma$

$\Rightarrow (M + m + I/r^2)a = Mg$

$\Rightarrow a = Mg/(M + m + I/r^2)$

34. $I = 0.20\ kg\text{-}m^2$ (Bigger pulley)



r = 10 cm = 0.1 m, smaller pulley is light

mass of the block, m = 2 kg

therefore $mg - T = ma$ …(1)

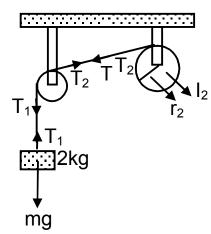
$\Rightarrow T = Ia/r^2$ …(2)

$\Rightarrow mg = (m + I/r^2)a \Rightarrow (2 \times 9.8) / [2 + (0.2/0.01)] = a$

$= 19.6 / 22 = 0.89\ m/s^2$

Therefore, acceleration of the block = $0.89\ m/s^2$.

35. $m = 2$ kg, $i_1 = 0.10\ kg\text{-}m^2$, $r_1 = 5$ cm = 0.05 m



$i_2 = 0.20\ kg\text{-}m^2$, $r_2 = 10$ cm = 0.1 m

Therefore $mg - T_1 = ma$ …(1)

$(T_1 - T_2)r_1 = I_1\alpha$ …(2)

$T_2r_2 = I_2\alpha$ …(3)

Substituting the value of T_2 in the equation (2), we get

$\Rightarrow (t_1 - I_2\ \alpha/r_1)r_2 = I_1\alpha$

$\Rightarrow (T_1 - I_2\ a/r_1^2) = I_1a/r_2^2$

$\Rightarrow T_1 = [(I_1/r_1^2) + I_2/r_2^2)]a$

Substituting the value of T_1 in the equation (1), we get

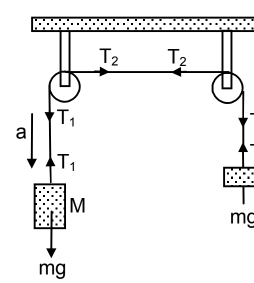
$\Rightarrow mg - [(I_1/r_1^2) + I_2/r_2^2)]a = ma$

$$\Rightarrow \frac{mg}{[(I_1/r_1^2)+(I_2/r_2^2)]+m} = a$$

$$\Rightarrow a = \frac{2\times 9.8}{(0.1/0.0025)+(0.2/0.01)+2} = 0.316\ m/s^2$$

$$\Rightarrow T_2 = I_2a/r_2^2 = \frac{0.20\times 0.316}{0.01} = 6.32\ N.$$

36. According to the question



$Mg - T_1 = Ma$ …(1)

$(T_2 - T_1)R = Ia/R \Rightarrow (T_2 - T_1) = Ia/R^2$ …(2)

$(T_2 - T_3)R = Ia/R^2$ …(3)

$\Rightarrow T_3 - mg = ma$ …(4)

By adding equation (2) and (3) we will get,

$\Rightarrow (T_1 - T_3) = 2\ Ia/R^2$ …(5)

By adding equation (1) and (4) we will get

$-\,mg + Mg + (T_3 - T_1) = Ma + ma$ …(6)

Substituting the value for $T_3 - T_1$ we will get

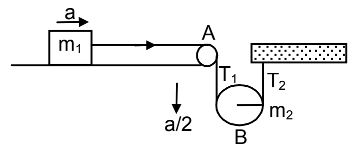
$\Rightarrow Mg - mg = Ma + ma + 2Ia/R^2$

$\Rightarrow a = \dfrac{(M - m)G}{(M + m + 2I/R^2)}$

37. A is light pulley and B is the descending pulley having $I = 0.20\ kg - m^2$ and $r = 0.2$ m

Mass of the block = 1 kg

According to the equation



$T_1 = m_1a$ …(1)

$(T_2 - T_1)r = I\alpha$ …(2)

$m_2g - m_2\ a/2 = T_1 + T_2$ …(3)

$T_2 - T_1 = Ia/2R^2 = 5a/2$ and $T_1 = a$ (because $\alpha = a/2R$)

$\Rightarrow T_2 = 7/2\ a$

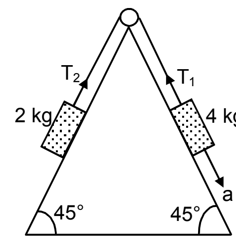
$\Rightarrow m_2g = m_2a/2 + 7/2\ a + a$

$\Rightarrow 2I / r^2g = 2I/r^2\ a/2 + 9/2\ a$ $(1/2\ mr^2 = I)$

$\Rightarrow 98 = 5a + 4.5\ a$

$\Rightarrow a = 98/9.5 = 10.3\ ms^2$

38. $m_1g \sin\theta - T_1 = m_1a$ …(1)



$(T_1 - T_2) = Ia/r^2$ …(2)

$T_2 - m_2g \sin\theta = m_2a$ …(3)

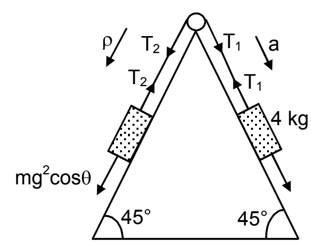
Adding the equations (1) and (3) we will get

$m_1g \sin\theta + (T_2 - T_1) - m_2g \sin\theta = (m_1 + m_2)a$

$\Rightarrow (m_1 - m_2)g \sin\theta = (m_1 + m_2 + 1/r^2)a$

$\Rightarrow a = \dfrac{(m_1 - m_2)g\sin\theta}{(m_1 + m_2 + 1/r^2)} = 0.248 = 0.25\ ms^{-2}$.

39. $m_1 = 4$ kg, $m_2 = 2$ kg



Frictional co-efficient between 2 kg block and surface = 0.5

R = 10 cm = 0.1 m

$I = 0.5\ kg - m^2$

$m_1g \sin\theta - T_1 = m_1a$ …(1)

$T_2 - (m_2g \sin\theta + \mu\ m_2g \cos\theta) = m_2a$ …(2)

$(T_1 - T_2) = Ia/r^2$

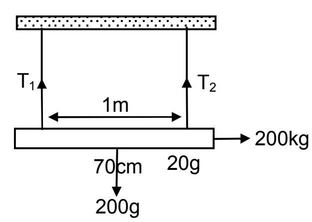
Adding equation (1) and (2) we will get

$m_1g \sin\theta - (m_2g \sin\theta + \mu m_2g \cos\theta) + (T_2 - T_1) = m_1a + m_2a$

$\Rightarrow 4 \times 9.8 \times (1/\sqrt{2}) - \{(2 \times 9.8 \times (1/\sqrt{2}) + 0.5 \times 2 \times 9.8 \times (1/\sqrt{2})\} = (4 + 2 + 0.5/0.01)a$

$\Rightarrow 27.80 - (13.90 + 6.95) = 65\ a \Rightarrow a = 0.125\ ms^{-2}$.

40. According to the question



$m_1 = 200$ g, $l = 1$ m, $m_2 = 20$ g

Therefore, $(T_1 \times r_1) - (T_2 \times r_2) - (m_1f \times r_3g) = 0$

$\Rightarrow T_1 \times 0.7 - T_2 \times 0.3 - 2 \times 0.2 \times g = 0$

$\Rightarrow 7T_1 - 3T_2 = 3.92$ …(1)

$T_1 + T_2 = 0.2 \times 9.8 + 0.02 \times 9.8 = 2.156$ …(2)

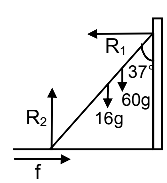
From the equation (1) and (2) we will get

$10\ T_1 = 10.3$

$\Rightarrow T_1 = 1.038\ N = 1.04$ N

Therefore $T_2 = 2.156 - 1.038 = 1.118 = 1.12$ N.

41. $R_1 = \mu R_2$, $R_2 = 16g + 60\ g = 745$ N

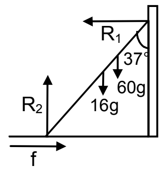


$R_1 \times 10 \cos 37° = 16g \times 5 \sin 37° + 60\ g \times 8 \times \sin 37°$

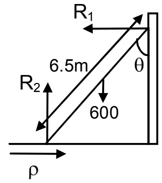
$\Rightarrow 8R_1 = 48g + 288$ g

$\Rightarrow R_1 = 336g/8 = 412\ N = f$

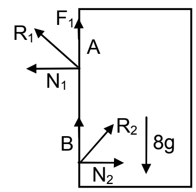
Therefore $\mu = R_1 / R_2 = 412/745 = 0.553$.



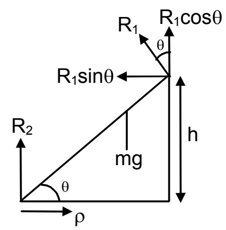
42. $\mu = 0.54$, $R_2 = 16g + mg$; $R_1 = \mu R_2$
$\Rightarrow R_1 \times 10 \cos 37° = 16g \times 5 \sin 37° + mg \times 8 \times \sin 37°$
$\Rightarrow 8R_1 = 48g + 24/5\ mg$
$$\Rightarrow R_2 = \frac{48g + 24/5\ mg}{8 \times 0.54}$$
$$\Rightarrow 16g + mg = \frac{24.0g + 24mg}{5 \times 8 \times 0.54} \Rightarrow 16 + m = \frac{240 + 24m}{40 \times 0.54}$$
$\Rightarrow m = 44$ kg.



43. m = 60 kg, ladder length = 6.5 m, height of the wall = 6 m
Therefore torque due to the weight of the body
a) $\tau = 600 \times 6.5 / 2 \sin \theta = i$
$\Rightarrow \tau = 600 \times 6.5 / 2 \times \sqrt{[1-(6/6.5)^2]}$
$\Rightarrow \tau = 735$ N-m.
b) $R_2 = mg = 60 \times 9.8$
$R_1 = \mu R_2 \Rightarrow 6.5\ R_1 \cos \theta = 60g \sin \theta \times 6.5/2$
$\Rightarrow R_1 = 60\ g \tan \theta = 60\ g \times (2.5/12)$ [because $\tan \theta = 2.5/6$]
$\Rightarrow R_1 = (25/2)\ g = 122.5$ N.



44. According to the question
$8g = F_1 + F_2$; $N_1 = N_2$
Since, $R_1 = R_2$
Therefore $F_1 = F_2$
$\Rightarrow 2F_1 = 8\ g \Rightarrow F_1 = 40$
Let us take torque about the point B, we will get $N_1 \times 4 = 8\ g \times 0.75$.
$\Rightarrow N_1 = (80 \times 3) / (4 \times 4) = 15$ N
Therefore $\sqrt{(F_1^2 + N_1^2} = R_1 = \sqrt{40^2 + 15^2} = 42.72 = 43$ N.



45. Rod has a length = L
It makes an angle θ with the floor
The vertical wall has a height = h
$R_2 = mg - R_1 \cos \theta$ …(1)
$R_1 \sin \theta = \mu R_2$ …(2)
$R_1 \cos \theta \times (h/\tan \theta) + R_1 \sin \theta \times h = mg \times 1/2 \cos \theta$
$\Rightarrow R_1 (\cos^2 \theta / \sin \theta)h + R_1 \sin \theta\ h = mg \times 1/2 \cos \theta$
$$\Rightarrow R_1 = \frac{mg \times L/2\cos\theta}{\{(\cos^2\theta/\sin\theta)h + \sin\theta h\}}$$
$$\Rightarrow R_1 \cos\theta = \frac{mgL/2\cos^2\theta\sin\theta}{\{(\cos^2\theta/\sin\theta)h + \sin\theta h\}}$$
$$\Rightarrow \mu = R_1\sin\theta / R_2 = \frac{mg\ L/2\ \cos\theta.\sin\theta}{\{(\cos^2\theta/\sin\theta)h + \sin\theta h)\}mg - mg\ 1/2\ \cos^2\theta}$$
$$\Rightarrow \mu = \frac{L/2\ \cos\theta.\sin\theta \times 2\sin\theta}{2(\cos^2\theta h + \sin^2\theta h) - L\cos^2\theta\sin\theta}$$
$$\Rightarrow \mu = \frac{L\cos\theta\sin^2\theta}{2h - L\cos^2\theta\sin\theta}.$$

46. A uniform rod of mass 300 grams and length 50 cm rotates with an uniform angular velocity = 2 rad/s about an axis perpendicular to the rod through an end.
a) $L = I\omega$
I at the end $= mL^2/3 = (0.3 \times 0.5^2\}/3 = 0.025$ kg-m^2
$= 0.025 \times 2 = 0.05$ kg – m^2/s
b) Speed of the centre of the rod
$V = \omega r = w \times (50/2) = 50$ cm/s = 0.5 m/s.
c) Its kinetic energy $= 1/2\ I\omega^2 = (1/2) \times 0.025 \times 2^2 = 0.05$ Joule.

47. I = 0.10 N-m; a = 10 cm = 0.1 m; m = 2 kg

Therefore $(ma^2/12) \times \alpha = 0.10$ N-m

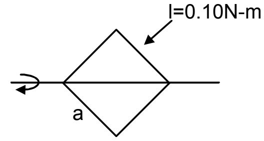
$\Rightarrow$ α = 60 rad/s

Therefore $\omega = \omega_0 + \alpha t$

$\Rightarrow$ ω = 60 × 5 = 300 rad/s

Therefore angular momentum = $I\omega$ = (0.10 / 60) × 300 = 0.50 kg-m^2/s

And 0 kinetic energy = 1/2 $I\omega^2$ = 1/2 × (0.10 / 60) × 300^2 = 75 Joules.



48. Angular momentum of the earth about its axis is

= 2/5 mr^2 × (2π / 85400)   (because, I = 2/5 mr^2)

Angular momentum of the earth about sun's axis

= mR^2 × (2π / 86400 × 365) (because, I = mR^2)

Therefore, ratio of the angular momentum = $\dfrac{2/5mr^2 \times (2\pi/86400)}{mR^2 \times 2\pi/(86400 \times 365)}$

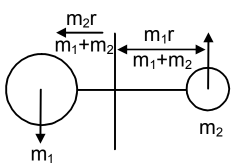
$\Rightarrow$ $(2r^2 \times 365) / 5R^2$

$\Rightarrow$ $(2.990 \times 10^{10}) / (1.125 \times 10^{17}) = 2.65 \times 10^{-7}$.

49. Angular momentum due to the mass m_1 at the centre of system is = $m_1\ r^{12}$.

$$= m1\left(\frac{m_2}{m_1+m_2}\right)^2 \omega = \frac{m_1 m_2^2 r^2}{(m_1+m_2)^2}\omega \quad \ldots(1)$$

Similarly the angular momentum due to the mass m_2 at the centre of system is m_2 $r^{112}\omega$



$$= m_2\left(\frac{m_1 r}{m_1 m_2}\right)^2 \omega = \frac{m_2 m_1^2}{(m_1+m_2)^2}\omega \quad \ldots(2)$$

Therefore net angular momentum = $\dfrac{m_1 m_2^2 r^2\omega}{(m_1+m_2)^2} + \dfrac{m_2 m_1^2 r^2\omega}{(m_1+m_2)^2}$

$\Rightarrow$ $\dfrac{m_1 m_2 (m_1+m_2) r^2\omega}{(m_1+m_2)^2}$ = $\dfrac{m_1 m_2}{(m_1+m_2)} r^2\omega = \mu r^2\omega$ (proved)

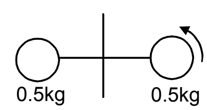
50. $\tau = I\alpha$

$\Rightarrow$ F × r = $(mr^2 + mr^2)\alpha$ $\Rightarrow$ 5 × 0.25 = $2mr^2 \times \alpha$

$\Rightarrow$ α = $\dfrac{1.25}{2 \times 0.5 \times 0.025 \times 0.25} = 20$

ω_0 = 10 rad/s, t = 0.10 sec, $\omega = \omega_0 + \alpha t$

$\Rightarrow$ ω = 10 + 010 × 230 = 10 + 2 = 12 rad/s.



51. A wheel has

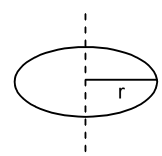
I = 0.500 Kg-m^2, r = 0.2 m, ω = 20 rad/s

Stationary particle = 0.2 kg

Therefore $I_1\omega_1 = I_2\omega_2$ (since external torque = 0)

$\Rightarrow$ 0.5 × 10 = (0.5 + 0.2 × 0.2^2)ω_2

$\Rightarrow$ 10/0.508 = ω_2 = 19.69 = 19.7 rad/s



52. I_1 = 6 kg-m^2, ω_1 = 2 rad/s , I_2 = 5 kg-m^2

Since external torque = 0

Therefore $I_1\omega_1 = I_2\omega_2$

$\Rightarrow$ ω_2 = (6 × 2) / 5 = 2.4 rad/s

53. ω_1 = 120 rpm = 120 × (2π / 60) = 4π rad /s.

I_1 = 6 kg – m^2, I_2 = 2 kgm^2

Since two balls are inside the system

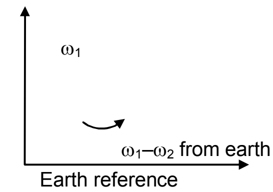
Therefore, total external torque = 0

Therefore, $I_1\omega_1 = I_2\omega_2$

$\Rightarrow$ 6 × 4π = 2ω_2

$\Rightarrow$ ω_2 = 12 π rad/s = 6 rev/s = 360 rev/minute.

54. $I_1 = 2 \times 10^{-3}$ kg-m^2 ; $I_2 = 3 \times 10^{-3}$ kg-m^2 ; $\omega_1 = 2$ rad/s
From the earth reference the umbrella has a angular velocity $(\omega_1 - \omega_2)$
And the angular velocity of the man will be ω_2
Therefore $I_1(\omega_1 - \omega_2) = I_2\omega_2$
$\Rightarrow\ 2 \times 10^{-3}\ (2 - \omega_2) = 3 \times 10^{-3} \times \omega_2$
$\Rightarrow\ 5\omega_2 = 4 \Rightarrow \omega_2 = 0.8$ rad/s.



55. Wheel (1) has
$I_1 = 0.10$ kg-m^2, $\omega_1 = 160$ rev/min
Wheel (2) has
$I_2 = ?$; $\omega_2 = 300$ rev/min
Given that after they are coupled, $\omega = 200$ rev/min
Therefore if we take the two wheels to bean isolated system
Total external torque = 0
Therefore, $I_1\omega_1 + I_1\omega_2 = (I_1 + I_1)\omega$
$\Rightarrow\ 0.10 \times 160 + I_2 \times 300 = (0.10 + I_2) \times 200$
$\Rightarrow\ 5I_2 = 1 - 0.8 \Rightarrow I_2 = 0.04$ kg-m^2.

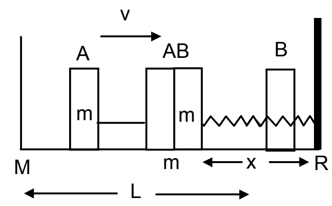
56. A kid of mass M stands at the edge of a platform of radius R which has a moment of inertia I. A ball of m thrown to him and horizontal velocity of the ball v when he catches it.
Therefore if we take the total bodies as a system
Therefore $mvR = \{I + (M + m)R^2\}\omega$
(The moment of inertia of the kid and ball about the axis = $(M + m)R^2$)
$\Rightarrow\ \omega = \dfrac{mvR}{1+(M+m)R^2}$.

57. Initial angular momentum = Final angular momentum
(the total external torque = 0)
Initial angular momentum = mvR (m = mass of the ball, v = velocity of the ball, R = radius of platform)
Therefore angular momentum = $I\omega + MR^2\omega$
Therefore $mVR = I\omega + MR^2\ \omega$
$\Rightarrow \omega = \dfrac{mVR}{(1+MR^2)}$.

58. From a inertial frame of reference when we see the (man wheel) system, we can find that the wheel moving at a speed of ω and the man with $(\omega + V/R)$ after the man has started walking.
(ω' = angular velocity after walking, ω = angular velocity of the wheel before walking.
Since $\Sigma I = 0$
Extended torque = 0
Therefore $(1 + MR^2)\omega = I\omega' + mR^2\ (\omega' + V/R)$
$\Rightarrow\ (I + mR^2)\omega + I\omega' + mR^2\omega' + mVR$
$\Rightarrow\ \omega' = \omega - \dfrac{mVR}{(1+mR^2)}$.

59. A uniform rod of mass m length ℓ is struck at an end by a force F. $\perp$ to the rod for a short time t
a) Speed of the centre of mass
$mv = Ft \Rightarrow v = \dfrac{Ft}{m}$
b) The angular speed of the rod about the centre of mass
$\ell\omega - r \times p$
$\Rightarrow (m\ell^2 / 12) \times \omega = (1/2) \times mv$
$\Rightarrow m\ell^2 / 12 \times \omega = (1/2)\ \ell\omega^2$
$\Rightarrow \omega = 6Ft / m\ell$
c) K.E. = $(1/2)\ mv^2 + (1/2)\ \ell\omega^2$
$= (1/2)\ \times m(Ft / m)^2\ (1/2)\ \ell\omega^2$
$= (1/2)\ \times m \times (\ F^2t^2/m^2) + (1/2)\ \times (m\ell^2/12)\ (36 \times(\ F^2t^2/m^2\ell^2))$

So, the time period of B is $\frac{2\pi\sqrt{\frac{m}{k}}}{2} = \pi\sqrt{\frac{m}{k}}$



The block B collides with the block A and comes to rest at that point. The block A again moves a further distance 'L' to return to its original position.

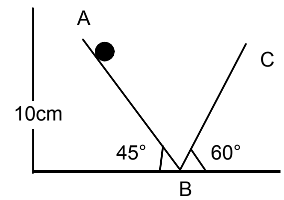
$\therefore$ Time taken by the block to move from M $\rightarrow$ N and N $\rightarrow$ M

is $\frac{L}{V} + \frac{L}{V} = 2\left(\frac{L}{V}\right)$

$\therefore$ So time period of the periodic motion is $2\left(\frac{L}{V}\right) + \pi\sqrt{\frac{m}{k}}$

29. Let the time taken to travel AB and BC be t_1 and t_2 respectively

Fro part AB, $a_1 = g \sin 45°$. $s_1 = \frac{0.1}{\sin 45°} = 2m$



Let, v = velocity at B

$\therefore v^2 - u^2 = 2a_1 s_1$

$\Rightarrow v^2 = 2 \times g \sin 45° \times \frac{0.1}{\sin 45°} = 2$

$\Rightarrow v = \sqrt{2}$ m/s

$\therefore t_1 = \frac{v-u}{a_1} = \frac{\sqrt{2}-0}{\frac{g}{\sqrt{2}}} = \frac{2}{g} = \frac{2}{10} = 0.2$ sec

Again for part BC, $a_2 = -g \sin 60°$, $u = \sqrt{2}$, $v = 0$

$\therefore t_2 = \frac{0-\sqrt{2}}{-g\left(\frac{\sqrt{3}}{2}\right)} = \frac{2\sqrt{2}}{\sqrt{3}g} = \frac{2\times(1.414)}{(1.732)\times 10} = 0.165$sec.

So, time period = 2 $(t_1 + t_2)$ = 2(0.2 + 0.155) = 0.71sec

30. Let the amplitude of oscillation of 'm' and 'M' be x_1 and x_2 respectively.

a) From law of conservation of momentum,

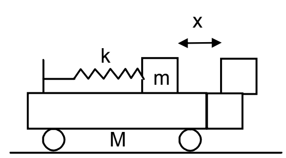
$mx_1 = Mx_2$ ...(1) [because only internal forces are present]

Again, $(1/2)\, kx_0^2 = (1/2)\, k\, (x_1 + x_2)^2$

$\therefore x_0 = x_1 + x_2$ ...(2)

[Block and mass oscillates in opposite direction. But x $\rightarrow$ stretched part]

From equation (1) and (2)



$\therefore x_0 = x_1 + \frac{m}{M} x_1 = \left(\frac{M+m}{M}\right) x_1$

$\therefore x_1 \frac{Mx_0}{M+m}$

So, $x_2 = x_0 - x_1 = x_0 \left[1 - \frac{M}{M+m}\right] = \frac{mx_0}{M+m}$ respectively.

b) At any position, let the velocities be v_1 and v_2 respectively.

Here, v_1 = velocity of 'm' with respect to M.

By energy method

Total Energy = Constant

$(1/2)\, Mv^2 + (1/2)\, m(v_1 - v_2)^2 + (1/2)\, k(x_1 + x_2)^2$ = Constant ...(i)

[$v_1 - v_2$ = Absolute velocity of mass 'm' as seen from the road.]

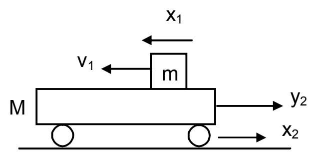
Again, from law of conservation of momentum,

$mx_2 = mx_1 \Rightarrow x_1 = \frac{M}{m} x_2$ ...(1)

$mv_2 = m(v_1 - v_2) \Rightarrow (v_1 - v_2) = \frac{M}{m} v_2$ ...(2)

Putting the above values in equation (1), we get

$\frac{1}{2} M v_2^2 + \frac{1}{2} m \frac{M^2}{m^2} v_2^2 + \frac{1}{2} k x_2^2 \left(1 + \frac{M}{m}\right)^2 = \text{constant}$



$\therefore M\left(1 + \frac{M}{m}\right) v_2 + k\left(1 + \frac{M}{m}\right)^2 x_2^2 = \text{Constant.}$

$\Rightarrow mv_2^2 + k\left(1 + \frac{M}{m}\right) x_2^2 = \text{constant}$

Taking derivative of both sides,

$M \times 2v_2 \frac{dv_2}{dt} + k\frac{(M+m)}{m} - ex_2^2 \frac{dx_2}{dt} = 0$

$\Rightarrow ma_2 + k\left(\frac{M+m}{m}\right) x_2 = 0$ [because, $v_2 = \frac{dx_2}{dt}$]

$\Rightarrow \frac{a_2}{x_2} = -\frac{k(M+m)}{Mm} = \omega^2$

$\therefore \omega = \sqrt{\frac{k(M+m)}{Mm}}$

So, Time period, $T = 2\pi\sqrt{\frac{Mm}{k(M+m)}}$

31. Let 'x' be the displacement of the plank towards left. Now the centre of gravity is also displaced through 'x'
In displaced position
$R_1 + R_2 = mg.$
Taking moment about G, we get
$R_1(\ell/2 - x) = R_2(\ell/2 + x) = (mg - R_1)(\ell/2 + x)$ ...(1)\
So, $R_1 (\ell/2 - x) = (mg - R_1)(\ell/2 + x)$

$\Rightarrow R_1 \frac{\ell}{2} - R_1 x = mg \frac{\ell}{2} - R_1 x + mgx - R_1 \frac{\ell}{2}$

$\Rightarrow R_1 \frac{\ell}{2} + R_1 \frac{\ell}{2} = mg\,(x + \frac{\ell}{2})$

$\Rightarrow R_1 \left(\frac{\ell}{2} + \frac{\ell}{2}\right) = mg \left(\frac{2x+\ell}{2}\right)$

$\Rightarrow R_1 \ell = \frac{mg(2x+\ell)}{2}$

$\Rightarrow R_1 = \frac{mg(2x+\ell)}{2\ell}$ ...(2)

Now $F_1 = \mu R_1 = \frac{\mu mg(\ell + 2x)}{2\ell}$

Similarly $F_2 = \mu R_2 = \frac{\mu mg(\ell - 2x)}{2\ell}$

Since, $F_1 > F_2. \Rightarrow F_1 - F_2 = ma = \frac{2\mu mg}{\ell} x$

$\Rightarrow \frac{a}{x} = \frac{2\mu g}{\ell} = \omega^2 \Rightarrow \omega = \sqrt{\frac{2\mu g}{\ell}}$

$\therefore$ Time period $= 2\pi\sqrt{\frac{\ell}{2rg}}$

32. T = 2sec.

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow 2 = 2\pi\sqrt{\frac{\ell}{10}} \Rightarrow \frac{\ell}{10} = \frac{1}{\pi^2} \Rightarrow \ell = 1\text{cm} \qquad (\therefore \pi^2 \approx 10)$

33. From the equation,

$\theta = \pi \sin[\pi \sec^{-1} t]$

$\therefore \omega = \pi \sec^{-1}$ (comparing with the equation of SHM)

$\Rightarrow \frac{2\pi}{T} = \pi \Rightarrow T = 2$ sec.

We know that $T = 2\pi\sqrt{\frac{\ell}{g}} \quad \Rightarrow 2 = 2\sqrt{\frac{\ell}{g}} \quad \Rightarrow 1 = \sqrt{\frac{\ell}{g}} \quad \Rightarrow \ell = 1\text{m}.$

$\therefore$ Length of the pendulum is 1m.

34. The pendulum of the clock has time period 2.04sec.

Now, No. or oscillation in 1 day = $\frac{24 \times 3600}{2} = 43200$

But, in each oscillation it is slower by (2.04 – 2.00) = 0.04sec.

So, in one day it is slower by,

= 43200 × (0.04) = 12 sec = 28.8 min

So, the clock runs 28.8 minutes slower in one day.

35. For the pendulum, $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

Given that, T_1 = 2sec, g_1 = 9.8m/s^2

$T_2 = \frac{24 \times 3600}{\left(\frac{24 \times 3600 - 24}{2}\right)} = 2 \times \frac{3600}{3599}$

Now, $\frac{g^2}{g_1} = \left(\frac{T_1}{T_2}\right)^2$

$\therefore g_2 = (9.8)\left(\frac{3599}{3600}\right)^2 = 9.795\text{m/s}^2$

36. L = 5m.

a) $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{0.5} = 2\pi(0.7)$

$\therefore$ In $2\pi(0.7)$sec, the body completes 1 oscillation,

In 1 second, the body will complete $\frac{1}{2\pi(0.7)}$ oscillation

$\therefore f = \frac{1}{2\pi(0.7)} = \frac{10}{14\pi} = \frac{0.70}{\pi}$ times

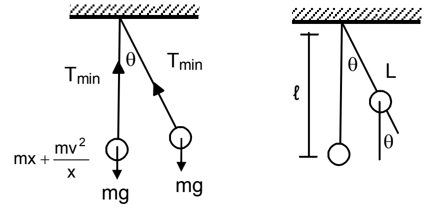
b) When it is taken to the moon

$T = 2\pi\sqrt{\frac{\ell}{g'}}$ where $g' \rightarrow$ Acceleration in the moon.

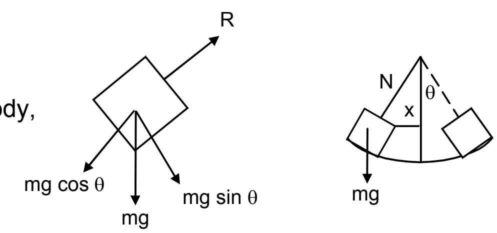
$= 2\pi\sqrt{\frac{5}{1.67}}$

$\therefore f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{1.67}{5}} = \frac{1}{2\pi}(0.577) = \frac{1}{2\pi\sqrt{3}}$ times.

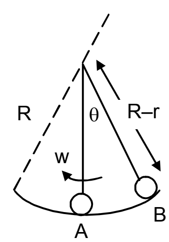
37. The tension in the pendulum is maximum at the mean position and minimum on the extreme position.
Here $(1/2)\ mv^2 - 0 = mg\ \ell(1 - \cos\theta)$
$v^2 = 2g\ell(1 - \cos\theta)$
Now, $T_{max} = mg + 2\ mg\ (1 - \cos\theta)$ $\quad$ $[\ T = mg + (mv^2/\ell)]$
Again, $T_{min} = mg \cos\theta$.
According to question, $T_{max} = 2T_{min}$
$\Rightarrow mg + 2mg - 2mg\cos\theta = 2mg\cos\theta$
$\Rightarrow 3mg = 4mg\cos\theta$
$\Rightarrow \cos\theta = 3/4$
$\Rightarrow \theta = \cos^{-1}(3/4)$



38. Given that, R = radius.
Let N = normal reaction.
Driving force F = mg sinθ.
Acceleration $= a = g\sin\theta$
As, $\sin\theta$ is very small, $\sin\theta \to \theta$
$\therefore$ Acceleration $a = g\theta$
Let 'x' be the displacement from the mean position of the body,
$\therefore\ \theta = x/R$
$\Rightarrow a = g\theta = g(x/R) \Rightarrow (a/x) = (g/R)$
So the body makes S.H.M.
$\therefore T = 2\pi\sqrt{\dfrac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\dfrac{x}{gx/R}} = 2\pi\sqrt{\dfrac{R}{g}}$



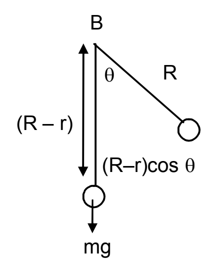
39. Let the angular velocity of the system about the point os suspension at any time be 'ω'
So, $v_c = (R - r)\omega$
Again $v_c = r\omega_1$ [where, ω_1 = rotational velocity of the sphere]
$\omega_1 = \dfrac{v_c}{r} = \left(\dfrac{R + -r}{r}\right)\omega$ $\qquad$ …(1)



By Energy method, Total energy in SHM is constant.
So, $mg(R - r)(1 - \cos\theta) + (1/2)\ mv_c^2 + (1/2)\ I\omega_1^2 = \text{constant}$
$\therefore\ mg(R - r)\ (1 - \cos\theta) + (1/2)\ m(R - r)^2\ \omega^2 + (1/2)\ mr^2\left(\dfrac{R - r}{r}\right)^2\ \omega^2 = \text{constant}$

$\Rightarrow g(R - r)\ 1 - \cos\theta) + (R - r)^2\ \omega^2\left[\dfrac{1}{2} + \dfrac{1}{5}\right] = \text{constant}$

Taking derivative, $g(R - r)\sin\theta\dfrac{d\theta}{dt} = \dfrac{7}{10}(R - r)^2 2\omega\dfrac{d\omega}{dt}$



$\Rightarrow g\sin\theta = 2 \times \dfrac{7}{10}(R - r)\alpha$

$\Rightarrow g\sin\theta = \dfrac{7}{5}\ (R - r)\alpha$

$\Rightarrow \alpha = \dfrac{5g\sin\theta}{7(R - r)} = \dfrac{5g\theta}{7(R - r)}$

$\therefore\ \dfrac{\alpha}{\theta} = \omega^2 = \dfrac{5g\theta}{7(R - r)} = \text{constant}$

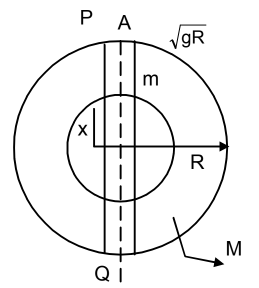
So the motion is S.H.M. Again $\omega = \omega\sqrt{\dfrac{5g}{7(R - r)}} \Rightarrow T = 2\pi\sqrt{\dfrac{7(R - r)}{5g}}$

40. Length of the pendulum = 40cm = 0.4m.
Let acceleration due to gravity be g at the depth of 1600km.
$\therefore gd = g(1\text{-}d/R) = 9.8\left(1 - \dfrac{1600}{6400}\right) = 9.8\left(1 - \dfrac{1}{4}\right) = 9.8 \times \dfrac{3}{4} = 7.35\text{m/s}^2$

$\therefore$ Time period $T' = 2\pi\sqrt{\dfrac{\ell}{g\delta}}$

$= 2\pi\sqrt{\dfrac{0.4}{7.35}} = 2\pi\sqrt{0.054} = 2\pi \times 0.23 = 2 \times 3.14 \times 0.23 = 1.465 \approx 1.47$sec.

41. Let M be the total mass of the earth.
At any position x,



$$\therefore \frac{M'}{M} = \frac{\rho \times \left(\frac{4}{3}\right)\pi \times x^3}{\rho \times \left(\frac{4}{3}\right)\pi \times R^3} = \frac{x^3}{R^3} \Rightarrow M' = \frac{Mx^3}{R^3}$$

So force on the particle is given by,

$$\therefore F_X = \frac{GM'm}{x^2} = \frac{GMm}{R^3}x \qquad \ldots(1)$$

So, acceleration of the mass 'M' at that position is given by,

$$a_x = \frac{GM}{R^2}x \Rightarrow \frac{a_x}{x} = w^2 = \frac{GM}{R^3} = \frac{g}{R} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

So, $T = 2\pi\sqrt{\dfrac{R}{g}}$ = Time period of oscillation.

a) Now, using velocity – displacement equation.

$V = \omega\sqrt{(A^2 - R^2)}$ [Where, A = amplitude]

Given when, $y = R$, $v = \sqrt{gR}$, $\omega = \sqrt{\dfrac{g}{R}}$

$\Rightarrow \sqrt{gR} = \sqrt{\dfrac{g}{R}}\sqrt{(A^2 - R^2)}$ [because $\omega = \sqrt{\dfrac{g}{R}}$]

$\Rightarrow R^2 = A^2 - R^2 \Rightarrow A = \sqrt{2}\,R$

[Now, the phase of the particle at the point P is greater than $\pi/2$ but less than π and at Q is greater than π but less than $3\pi/2$. Let the times taken by the particle to reach the positions P and Q be t_1 & t_2 respectively, then using displacement time equation]

$y = r \sin \omega t$

We have, $R = \sqrt{2}\,R \sin \omega t_1 \qquad \Rightarrow \omega t_1 = 3\pi/4$

& $-R = \sqrt{2}\,R \sin \omega t_2 \qquad \Rightarrow \omega t_2 = 5\pi/4$

So, $\omega(t_2 - t_1) = \pi/2 \Rightarrow t_2 - t_1 = \dfrac{\pi}{2\omega} = \dfrac{\pi}{2\sqrt{(R/g)}}$

Time taken by the particle to travel from P to Q is $t_2 - t_1 = \dfrac{\pi}{2\sqrt{(R/g)}}$ sec.

b) When the body is dropped from a height R, then applying conservation of energy, change in P.E. = gain in K.E.

$\Rightarrow \dfrac{GMm}{R} - \dfrac{GMm}{2R} = \dfrac{1}{2}mv^2 \qquad \Rightarrow v = \sqrt{gR}$

Since, the velocity is same at P, as in part (a) the body will take same time to travel PQ.

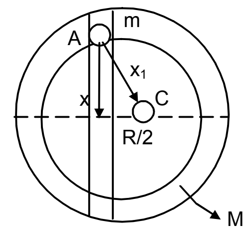
c) When the body is projected vertically upward from P with a velocity $\sqrt{gR}$, its velocity will be Zero at the highest point.

The velocity of the body, when reaches P, again will be $v = \sqrt{gR}$, hence, the body will take same time $\dfrac{\pi}{2\sqrt{(R/g)}}$ to travel PQ.

42. $M = 4/3\ \pi R^3 \rho$.

$M^1 = 4/3\ \pi x_1^3 \rho$

$M^1 = \left(\frac{M}{R^3}\right) x_1^3$

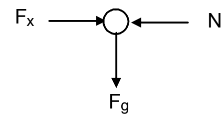


a) F = Gravitational force exerted by the earth on the particle of mass 'x' is,

$$F = \frac{GM^1 m}{x_1^2} = \frac{GMm}{R^3}\frac{x_1^3}{x_1^2} = \frac{GMm}{R^3} x_1 = \frac{GMm}{R^3}\sqrt{x^2 + \left(\frac{R^2}{4}\right)}$$

b) $F_y = F \cos\theta = \frac{GMmx_1}{R^3}\frac{x}{x_1} = \frac{GMmx}{R^3}$

$F_x = F \sin\theta = \frac{GMmx_1}{R^3}\frac{R}{2x_1} = \frac{GMm}{2R^2}$



c) $F_x = \frac{GMm}{2R^2}$ [since Normal force exerted by the wall $N = F_x$]

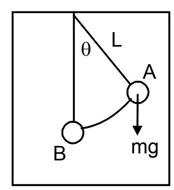
d) Resultant force $= \frac{GMmx}{R^3}$

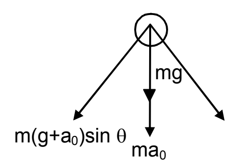
e) Acceleration $= \frac{\text{Driving force}}{\text{mass}} = \frac{GMmx}{R^3 m} = \frac{GMx}{R^3}$

So, a α x (The body makes SHM)

$$\therefore \frac{a}{x} = w^2 = \frac{GM}{R^3} \Rightarrow w = \sqrt{\frac{GM}{R^3}} \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}}$$

43. Here driving force $F = m(g + a_0)\sin\theta$ …(1)





Acceleration $a = \frac{F}{m} = (g + a_0)\sin\theta = \frac{(g+a_0)x}{\ell}$

(Because when θ is small $\sin\theta \to \theta = x/\ell$)

$\therefore a = \frac{(g+a_0)x}{\ell}$.

$\therefore$ acceleration is proportional to displacement.

So, the motion is SHM.

Now $\omega^2 = \frac{(g+a_0)}{\ell}$

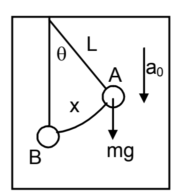
$\therefore T = 2\pi\sqrt{\frac{\ell}{g+a_0}}$

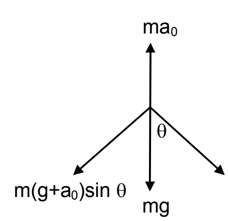
b) When the elevator is going downwards with acceleration a_0

Driving force $= F = m(g - a_0)\sin\theta$.

Acceleration $= (g - a_0)\sin\theta = \frac{(g-a_0)x}{\ell} = \omega^2 x$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g-a_0}}$





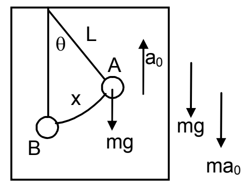
c) When moving with uniform velocity $a_0 = 0$.

For, the simple pendulum, driving force $= \frac{mgx}{\ell}$

$\Rightarrow a = \frac{gx}{\ell} \Rightarrow \frac{x}{a} = \frac{\ell}{g}$

$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{\ell}{g}}$

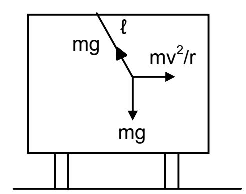
44. Let the elevator be moving upward accelerating 'a_0'
Here driving force $F = m(g + a_0) \sin\theta$
Acceleration $= (g + a_0)\sin\theta$
$= (g + a_0)\theta$ $\quad$ $(\sin\theta \to \theta)$
$= \dfrac{(g+a_0)x}{\ell} = \omega^2 x$
$T = 2\pi\sqrt{\dfrac{\ell}{g+a_0}}$



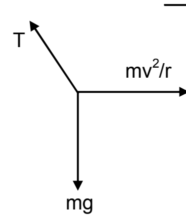
Given that, $T = \pi/3$ sec, ℓ = 1ft and g = 32 ft/sec^2
$\dfrac{\pi}{3} = 2\pi\sqrt{\dfrac{1}{32+a_0}}$
$\dfrac{1}{9} = 4\left(\dfrac{1}{32+a}\right)$
$\Rightarrow 32 + a = 36$ $\quad$ $\Rightarrow a = 36 - 32 = 4$ ft/sec^2

45. When the car moving with uniform velocity
$T = 2\pi\sqrt{\dfrac{\ell}{g}} \Rightarrow 4 = 2\pi\sqrt{\dfrac{\ell}{g}}$ $\quad$ …(1)
When the car makes accelerated motion, let the acceleration be a_0
$T = 2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2+a_0^2}}}$
$\Rightarrow 3.99 = 2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2+a_0^2}}}$
Now $\dfrac{T}{T'} = \dfrac{4}{3.99} = \dfrac{(g^2+a_0^2)^{1/4}}{\sqrt{g}}$
Solving for 'a_0' we can get $a_0 = g/10$ ms^{-2}

46. From the freebody diagram,

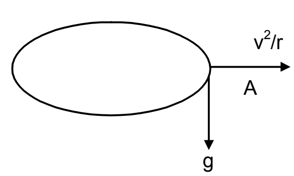


$T = \sqrt{(mg)^2 + \left(\dfrac{mv^2}{r^2}\right)}$
$= m\sqrt{g^2 + \dfrac{v^4}{r^2}} = ma$, where a = acceleration $= \left(g^2 + \dfrac{v^4}{r^2}\right)^{1/2}$



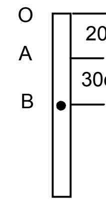
The time period of small accellations is given by,
$T = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{\ell}{\left(g^2 + \dfrac{v^4}{r^2}\right)^{1/2}}}$

47. a) ℓ = 3cm = 0.03m.
$T = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{0.03}{9.8}} = 0.34$ second.



b) When the lady sets on the Merry-go-round the ear rings also experience centrepetal acceleration
$a = \dfrac{v^2}{r} = \dfrac{4^2}{2} = 8$ m/s^2
Resultant Acceleration $A = \sqrt{g^2 + a^2} = \sqrt{100+64} = 12.8$ m/s^2
Time period $T = 2\pi\sqrt{\dfrac{\ell}{A}} = 2\pi\sqrt{\dfrac{0.03}{12.8}} = 0.30$ second.

48. a) M.I. about the pt A = I = $I_{C.G.}$ + Mh^2

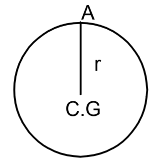


$$= \frac{m\ell^2}{12} + MH^2 = \frac{m\ell^2}{12} + m(0.3)^2 = M\left(\frac{1}{12}+0.09\right) = M\left(\frac{1+1.08}{12}\right) = M\left(\frac{2.08}{12}\right)$$

$$\therefore T = 2\pi\sqrt{\frac{I}{mg\ell'}} = 2\pi\sqrt{\frac{2.08m}{m\times 9.8\times 0.3}}$$ (ℓ' = dis. between C.G. and pt. of suspension)

$\approx$ 1.52 sec.

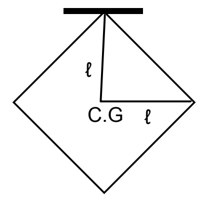
b) Moment of in isertia about A



$I = I_{C.G.} + mr^2 = mr^2 + mr^2 = 2\,mr^2$

$$\therefore \text{Time period} = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$$

c) I_{ZZ} (corner) = $m\left(\frac{a^2+a^2}{3}\right) = \frac{2ma^2}{3}$



In the ΔABC, $\ell^2 + \ell^2 = a^2$

$$\therefore \ell = \frac{a}{\sqrt{2}}$$

$$\therefore T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{2ma^2}{3mg\ell}} = 2\pi\sqrt{\frac{2a^2}{3ga\sqrt{2}}} = 2\pi\sqrt{\frac{\sqrt{8}a}{3g}}$$

d) h = r/2, ℓ = r/2 = Dist. Between C.G and suspension point.

$$\text{M.I. about A, } I = I_{C.G.} + Mh^2 = \frac{mc^2}{2} + n\left(\frac{r}{2}\right)^2 = mr^2\left(\frac{1}{2}+\frac{1}{4}\right) = \frac{3}{4}mr^2$$

$$\therefore T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{3mr^2}{4mg\ell}} = 2\pi\sqrt{\frac{3r^2}{4g\left(\frac{r}{2}\right)}} = 2\pi\sqrt{\frac{3r}{2g}}$$

49. Let A $\rightarrow$ suspension of point.

B $\rightarrow$ Centre of Gravity.

$\ell' = \ell/2$, $\quad h = \ell/2$

Moment of inertia about A is

$$I = I_{C.G.} + mh^2 = \frac{m\ell^2}{12} + \frac{m\ell^2}{4} = \frac{m\ell^2}{3}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{mg\left(\frac{\ell}{2}\right)}} = 2\pi\sqrt{\frac{2m\ell^2}{3mgl}} = 2\pi\sqrt{\frac{2\ell}{3g}}$$

Let, the time period 'T' is equal to the time period of simple pendulum of length 'x'.

$$\therefore T = 2\pi\sqrt{\frac{x}{g}}. \text{ So, } \frac{2\ell}{3g} = \frac{x}{g} \Rightarrow x = \frac{2\ell}{3}$$

$$\therefore \text{Length of the simple pendulum} = \frac{2\ell}{3}$$

50. Suppose that the point is 'x' distance from C.G.

Let m = mass of the disc., Radius = r

Here ℓ = x

M.I. about A = $I_{C.G.} + mx^2 = mr^2/2 + mx^2 = m(r^2/2 + x^2)$

$$T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{m\left(\frac{r^2}{2}+x^2\right)}{mgx}} = 2\pi\sqrt{\frac{m\left(r^2+2x^2\right)}{2mgx}} = 2\pi\sqrt{\frac{r^2+2x^2}{2gx}} \qquad \ldots(1)$$

For T is minimum $\frac{dt^2}{dx}=0$

$$\therefore \frac{d}{dx}T^2 = \frac{d}{dx}\left(\frac{4\pi^2 r^2}{2gx}+\frac{4\pi^2 2x^2}{2gx}\right)$$

$$\Rightarrow \frac{2\pi^2 r^2}{g}\left(-\frac{1}{x^2}\right)+\frac{4\pi^2}{g} = 0$$

$$\Rightarrow -\frac{\pi^2 r^2}{gx^2}+\frac{2\pi^2}{g} = 0$$

$$\Rightarrow \frac{\pi^2 r^2}{gx^2}=\frac{2\pi^2}{g} \Rightarrow 2x^2 = r^2 \Rightarrow x = \frac{r}{\sqrt{2}}$$

So putting the value of equation (1)

$$T = 2\pi\sqrt{\frac{r^2+2\left(\frac{r^2}{2}\right)}{2gx}} = 2\pi\sqrt{\frac{2r^2}{2gx}} = 2\pi\sqrt{\frac{r^2}{g\left(\frac{r}{\sqrt{2}}\right)}} = 2\pi\sqrt{\frac{\sqrt{2}\,r^2}{gr}} = 2\pi\sqrt{\frac{\sqrt{2}r}{g}}$$

51. According to Energy equation,

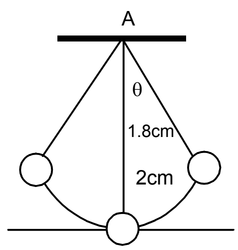
$mg\ell(1-\cos\theta) + (1/2)I\omega^2 = \text{const.}$

$mg(0.2)(1-\cos\theta) + (1/2)I\omega^2 = C.$ (I)

Again, $I = 2/3\, m(0.2)^2 + m(0.2)^2$

$$= m\left[\frac{0.008}{3}+0.04\right]$$

$= m\left(\frac{0.1208}{3}\right)$m. Where $I \rightarrow$ Moment of Inertia about the pt of suspension A



From equation

Differenting and putting the value of I and 1 is

$$\frac{d}{dt}\left[mg(0.2)(1-\cos\theta)+\frac{1}{2}\frac{0.1208}{3}m\omega^2\right] = \frac{d}{dt}(C)$$

$$\Rightarrow mg(0.2)\sin\theta\frac{d\theta}{dt}+\frac{1}{2}\left(\frac{0.1208}{3}\right)m2\omega\frac{d\omega}{dt} = 0$$

$$\Rightarrow 2\sin\theta = \frac{0.1208}{3}\alpha \text{ [because, } g = 10\text{m/s}^2\text{]}$$

$$\Rightarrow \frac{\alpha}{\theta} = \frac{6}{0.1208} = \omega^2 = 58.36$$

$$\Rightarrow \omega = 7.3. \text{ So } T = \frac{2\pi}{\omega} = 0.89\text{sec.}$$

For simple pendulum $T = 2\pi\sqrt{\frac{0.19}{10}} = 0.86$sec.

$$\% \text{ more} = \frac{0.89-0.86}{0.89} = 0.3.$$

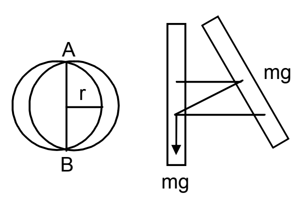
$\therefore$ It is about 0.3% larger than the calculated value.

52. (For a compound pendulum)

a) $T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{I}{mgr}}$

The MI of the circular wire about the point of suspension is given by

$\therefore I = mr^2 + mr^2 = 2\,mr^2$ is Moment of inertia about A.



$\therefore 2 = 2\pi\sqrt{\dfrac{2mr^2}{mgr}} = 2\pi\sqrt{\dfrac{2r}{g}}$

$\Rightarrow \dfrac{2r}{g} = \dfrac{1}{\pi^2} \Rightarrow r = \dfrac{g}{2\pi^2} = 0.5\pi = 50\text{cm. (Ans)}$

b) $(1/2)\,\omega^2 - 0 = mgr\,(1 - \cos\theta)$

$\Rightarrow (1/2)\, 2mr^2 - \omega^2 = mgr\,(1 - \cos 2°)$

$\Rightarrow \omega^2 = g/r\,(1 - \cos 2°)$

$\Rightarrow \omega = 0.11$ rad/sec [putting the values of g and r]

$\Rightarrow v = \omega \times 2r = 11$ cm/sec.

c) Acceleration at the end position will be centripetal.

$= a_n = \omega^2\,(2r) = (0.11)^2 \times 100 = 1.2\ \text{cm/s}^2$

The direction of 'a_n' is towards the point of suspension.

d) At the extreme position the centrepetal acceleration will be zero. But, the particle will still have acceleration due to the SHM.

Because, T = 2 sec.

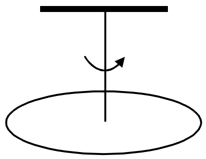
Angular frequency $\omega = \dfrac{2\pi}{T}$ $(\pi = 3.14)$

So, angular acceleration at the extreme position,

$\alpha = \omega^2\theta = \pi^2 \times \dfrac{2\pi}{180} = \dfrac{2\pi^3}{180}$ $[1° = \dfrac{\pi}{180}$ radious]

So, tangential acceleration $= \alpha\,(2r) = \dfrac{2\pi^3}{180} \times 100 = 34\ \text{cm/s}^2$.

53. M.I. of the centre of the disc. $= mr^2/2$



$T = 2\pi\sqrt{\dfrac{I}{k}} = 2\pi\sqrt{\dfrac{mr^2}{2K}}$ [where K = Torsional constant]

$T^2 = 4\pi^2\,\dfrac{mr^2}{2K} = 2\pi^2\,\dfrac{mr^2}{K}$

$\Rightarrow 2\pi^2\,mr^2 = KT^2 \quad \Rightarrow K = \dfrac{2mr^2\pi^2}{T^2}$

$\therefore$ Torsional constant $K = \dfrac{2mr^2\pi^2}{T^2}$

54. The M.I of the two ball system

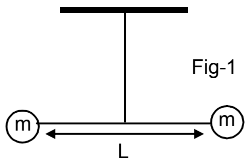


Fig-1

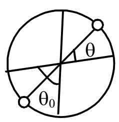
$I = 2m\,(L/2)^2 = m\,L^2/2$

At any position θ during the oscillation, [fig-2]

Torque $= k\theta$

So, work done during the displacement 0 to θ_0,

$W = \int_0^{\theta} k\theta\, d\theta = k\,\theta_0^2/2$



By work energy method,

$(1/2)\, I\omega^2 - 0 = \text{Work done} = k\,\theta_0^2/2$

$\therefore \omega^2 = \dfrac{k\theta_0^2}{2I} = \dfrac{k\theta_0^2}{mL^2}$

Now, from the freebody diagram of the rod,

$T_2 = \sqrt{(m\omega^2 L)^2 + (mg)^2}$

$= \sqrt{\left(m\dfrac{k\theta_0^2}{mL^2}\times L\right)^2 + m^2g^2} = \dfrac{k^2\theta_0^4}{L^2} + m^2g^2$

55. The particle is subjected to two SHMs of same time period in the same direction/
Given, r_1 = 3cm, r_2 = 4cm and ϕ = phase difference.

Resultant amplitude = R = $\sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos\phi}$

a) When ϕ = 0°,

R = $\sqrt{(3^2 + 4^2 + 2 \times 3 \times 4 \cos 0°}$ = 7 cm

b) When ϕ = 60°

R = $\sqrt{(3^2 + 4^2 + 2 \times 3 \times 4 \cos 60°}$ = 6.1 cm

c) When $\phi = 90°$

R = $\sqrt{(3^2 + 4^2 + 2 \times 3 \times 4 \cos 90°}$ = 5 cm

56. Three SHMs of equal amplitudes 'A' and equal time periods in the same dirction combine.
The vectors representing the three SHMs are shown it the figure.
Using vector method,
Resultant amplitude = Vector sum of the three vectors
= A + A cos 60° + A cso 60° = A + A/2 + A/2 = 2A
So the amplitude of the resultant motion is 2A.

57. x_1 = 2 sin 100 πt
x_2 = w sin (120πt + π/3)
So, resultant displacement is given by,
x = x_1 + x_2 = 2 [sin (100πt) + sin (120πt + π/3)]
a) At t = 0.0125s,
x = 2 [sin (100π × 0.0125) + sin (120π ×0.0125 + π/3)]
= 2 [sin 5π/4 + sin (3π/2 + π/3)]
= 2 [(–0.707) + (–0.5)] = – 2.41cm.
b) At t = 0.025s.
x = 2 [sin (100π × 0.025) + sin (120π ×0.025 + π/3)]
= 2 [sin 5π/2 + sin (3π + π/3)]
=2[1+(–0.8666)] = 0.27 cm.

58. The particle is subjected to two simple harmonic motions represented by,
x = x_0 sin wt
s = s_0 sin wt
and, angle between two motions = θ = 45°
$\therefore$Resultant motion will be given by,
R = $\sqrt{}$

$v = \omega/k = 6000$ cm/sec = 60 m/s

53. The equation of the standing wave is given by

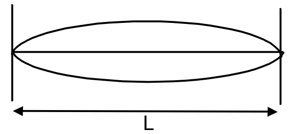
$y = (0.4\text{ cm}) \sin[(0.314\text{ cm}^{-1})x] \cos[(6.00\ \pi s^{-1})t]$

$\Rightarrow k = 0.314 = \pi/10$

$\Rightarrow 2\pi/\lambda = \pi/10 \Rightarrow \lambda = 20$ cm

for smallest length of the string, as wavelength remains constant, the string should vibrate in fundamental frequency

$\Rightarrow l = \lambda/2 = 20\text{ cm} / 2 = 10$ cm



54. L = 40 cm = 0.4 m, mass = 3.2 kg = 3.2×10^{-3} kg

$\therefore$ mass per unit length, m = (3.2)/(0.4) = 8×10^{-3} kg/m

change in length, $\Delta L = 40.05 - 40 = 0.05 \times 10^{-2}$ m

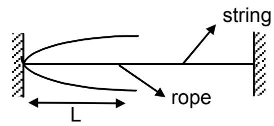
strain = $\Delta L/L = 0.125 \times 10^{-2}$ m

f = 220 Hz

$$f = \frac{1}{2l'}\sqrt{\frac{T}{m}} = \frac{1}{2\times(0.4005)}\sqrt{\frac{T}{8\times10^{-3}}} \Rightarrow T = 248.19\text{ N}$$

Strain = 248.19/1 mm^2 = 248.19×10^6

Y = stress / strain = 1.985×10^{11} N/m^2



55. Let, $\rho \to$ density of the block

Weight ρ Vg where V = volume of block

The same turning fork resonates with the string in the two cases

$$f_{10} = \frac{10}{2l}\sqrt{\frac{T-\rho_w Vg}{m}} = \frac{11}{2l}\sqrt{\frac{(\rho-\rho_w)Vg}{m}}$$

As the f of tuning fork is same,

$$f_{10} = f_{11} \Rightarrow \frac{10}{2l}\sqrt{\frac{\rho Vg}{m}} = \frac{11}{2l}\sqrt{\frac{(\rho-\rho_w)Vg}{m}}$$

$$\Rightarrow \frac{10}{11} = \sqrt{\frac{\rho-\rho_w}{m}} \Rightarrow \frac{\rho-1}{\rho} = \frac{100}{121} \qquad \text{(because, } \rho_w = 1\text{ gm/cc)}$$

$\Rightarrow 100\rho = 121\ \rho - 121 \Rightarrow 5.8 \times 10^3$ kg/m^3

56. l = length of rope = 2 m

M = mass = 80 gm = 0.8 kg

mass per unit length = m = 0.08/2 = 0.04 kg/m

Tension T = 256 N

Velocity, V = $\sqrt{T/m}$ = 80 m/s

For fundamental frequency,

$l = \lambda/4 \Rightarrow \lambda = 4l = 8$ m

$\Rightarrow f = 80/8 = 10$ Hz

a) Therefore, the frequency of 1st two overtones are

1st overtone = 3f = 30 Hz

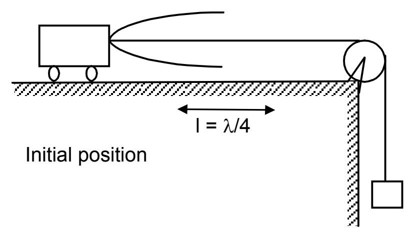
2nd overtone = 5f = 50 Hz

b) $\lambda_1 = 4l = 8$ m

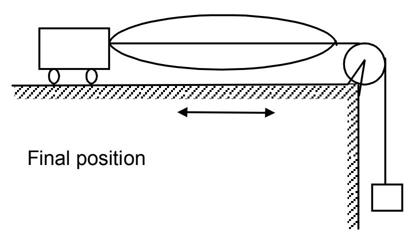
$\lambda_1 = V/f_1 = 2.67$ m

$\lambda_2 = V/f_2 = 1.6$ mt

so, the wavelengths are 8 m, 2.67 m and 1.6 m respectively.



Initial position



Final position

57. Initially because the end A is free, an antinode will be formed.

So, $l = Ql_1 / 4$

Again, if the movable support is pushed to right by 10 m, so that the joint is placed on the pulley, a node will be formed there.

So, $l = \lambda_2 / 2$

Since, the tension remains same in both the cases, velocity remains same.

As the wavelength is reduced by half, the frequency will become twice as that of 120 Hz i.e. 240 Hz.

SOLUTIONS TO CONCEPTS
CHAPTER – 16

1. V_{air}= 230 m/s. V_s = 5200 m/s. Here S = 7 m

   So, $t = t_1 - t_2 = \left(\frac{1}{330} - \frac{1}{5200}\right)$ = 2.75 × 10^{-3} sec = 2.75 ms.

2. Here given S = 80 m × 2 = 160 m.

   v = 320 m/s

   So the maximum time interval will be

   t = 5/v = 160/320 = 0.5 seconds.

3. He has to clap 10 times in 3 seconds.

   So time interval between two clap = (3/10 second).

   So the time taken go the wall = (3/2 × 10) = 3/20 seconds.

   = 333 m/s.

4. a) for maximum wavelength n = 20 Hz.

   as $\left(\eta \propto \frac{1}{\lambda}\right)$

   b) for minimum wavelength, n = 20 kHz

   $\therefore \lambda$ = 360/ (20 × 10^3) = 18 × 10^{-3} m = 18 mm

   $\Rightarrow$ x = (v/n) = 360/20 = 18 m.

5. a) for minimum wavelength n = 20 KHz

   $\Rightarrow v = n\lambda \Rightarrow \lambda = \left(\frac{1450}{20 \times 10^3}\right)$ = 7.25 cm.

   b) for maximum wavelength n should be minium

   $\Rightarrow v = n\lambda \Rightarrow \lambda = v/n \Rightarrow$ 1450 / 20 = 72.5 m.

6. According to the question,

   a) λ = 20 cm × 10 = 200 cm = 2 m

   v = 340 m/s

   so, n = v/λ = 340/2 = 170 Hz.

   N = v/$\lambda \Rightarrow \frac{340}{2 \times 10^{-2}}$ = 17.000 Hz = 17 KH_2 (because λ = 2 cm = 2 × 10^{-2} m)

7. a) Given V_{air} = 340 m/s , n = 4.5 ×10^6 Hz

   $\Rightarrow \lambda_{air}$ = (340 / 4.5) × 10^{-6} = 7.36 × 10^{-5} m.

   b) V_{tissue} = 1500 m/s $\Rightarrow \lambda_t$ = (1500 / 4.5) × 10^{-6} = 3.3 × 10^{-4} m.

8. Here given r_y = 6.0 × 10^{-5} m

   a) Given $2\pi/\lambda = 1.8 \Rightarrow \lambda = (2\pi/1.8)$

   So, $\frac{r_y}{\lambda} = \frac{6.0 \times (1.8) \times 10^{-5} m/s}{2\pi}$ = 1.7 × 10^{-5} m

   b) Let, velocity amplitude = V_y

   V = dy/dt = 3600 cos (600 t – 1.8) × 10^{-5} m/s

   Here V_y = 3600 × 10^{-5} m/s

   Again, $\lambda = 2\pi/1.8$ and $T = 2\pi/600 \Rightarrow$ wave speed = v = λ/T = 600/1.8 = 1000 / 3 m/s.

   So the ratio of $(V_y/v) = \frac{3600 \times 3 \times 10^{-5}}{1000}$.

9. a) Here given n = 100, v = 350 m/s

   $\Rightarrow \lambda = \frac{v}{n} = \frac{350}{100}$ = 3.5 m.

   In 2.5 ms, the distance travelled by the particle is given by

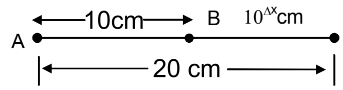
   Δx = 350 × 2.5 × 10^{-3}

So, phase difference $\phi = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \frac{2\pi}{(350/100)} \times 350 \times 2.5 \times 10^{-3} = (\pi/2)$.

b) In the second case, Given $\Delta\eta$ = 10 cm = 10^{-1} m

So, $\phi = \frac{2\pi}{x} \Delta x = \frac{2\pi \times 10^{-1}}{(350/100)} = 2\pi/35$.

10. a) Given Δx = 10 cm, λ = 5.0 cm



$\Rightarrow \phi = \frac{2\pi}{\lambda} \times \Delta\eta = \frac{2\pi}{5} \times 10 = 4\pi$.

So phase difference is zero.

b) Zero, as the particle is in same phase because of having same path.

11. Given that p = 1.0 × 10^5 N/m^2, T = 273 K, M = 32 g = 32 × 10^{-3} kg

V = 22.4 litre = 22.4 × 10^{-3} m^3

C/C_v = r = 3.5 R / 2.5 R = 1.4

$\Rightarrow V = \sqrt{\frac{rp}{f}} = \sqrt{\frac{1.4 \times 1.0 \times 10^{-5}}{32/22.4}}$ = 310 m/s (because ρ = m/v)

12. V_1 = 330 m/s, V_2 = ?

T_1 = 273 + 17 = 290 K, T_2 = 272 + 32 = 305 K

We know $v \propto \sqrt{T}$

$\frac{\sqrt{V_1}}{\sqrt{V_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \Rightarrow V_2 = \frac{V_1 \times \sqrt{T_2}}{\sqrt{T_1}}$

$= 340 \times \sqrt{\frac{305}{290}}$ = 349 m/s.

13. T_1 = 273 $V_2 = 2V_1$

V_1 = v T_2 = ?

We know that $V \propto \sqrt{T} \Rightarrow \frac{T_2}{T_1} = \frac{V_2^2}{V_1^2} \Rightarrow T_2 = 273 \times 2^2 = 4 \times 273$ K

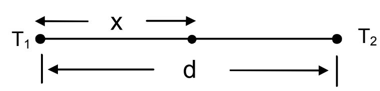
So temperature will be (4 × 273) – 273 = 819°c.

14. The variation of temperature is given by

$T = T_1 + \frac{(T_2 - T_2)}{d} x$ ...(1)

We know that $V \propto \sqrt{T} \Rightarrow \frac{V_T}{V} = \sqrt{\frac{T}{273}} \Rightarrow VT = v\sqrt{\frac{T}{273}}$

$\Rightarrow dt = \frac{dx}{V_T} = \frac{du}{V} \times \sqrt{\frac{273}{T}}$



$\Rightarrow t = \frac{273}{V} \int_0^d \frac{dx}{[T_1 + (T_2 - T_1)/d)x]^{1/2}}$

$= \frac{\sqrt{273}}{V} \times \frac{2d}{T_2 - T_1} [T_1 + \frac{T_2 - T_1}{d} x]_0^d = \left(\frac{2d}{V}\right)\left(\frac{\sqrt{273}}{T_2 - T_1}\right) \times \sqrt{T_2} - \sqrt{T_1}$

$= T = \frac{2d}{V} \frac{\sqrt{273}}{\sqrt{T_2} + \sqrt{T_1}}$

Putting the given value we get

$= \frac{2 \times 33}{330} = \frac{\sqrt{273}}{\sqrt{280} + \sqrt{310}}$ = 96 ms.

15. We know that v = $\sqrt{K/\rho}$

Where K = bulk modulus of elasticity

$\Rightarrow K = v^2 \rho = (1330)^2 \times 800$ N/m^2

We know K = $\left(\dfrac{F/A}{\Delta V/V}\right)$

$\Rightarrow \Delta V = \dfrac{\text{Pressures}}{K} = \dfrac{2\times10^5}{1330\times1330\times800}$

So, $\Delta V = 0.15$ cm^3

16. We know that,

Bulk modulus B = $\dfrac{\Delta p}{(\Delta V/V)} = \dfrac{P_0\lambda}{2\pi S_0}$

Where P_0 = pressure amplitude $\Rightarrow P_0 = 1.0 \times 10^5$

S_0 = displacement amplitude $\Rightarrow S_0 = 5.5 \times 10^{-6}$ m

$\Rightarrow B = \dfrac{14\times35\times10^{-2}\text{m}}{2\pi(5.5)\times10^{-6}\text{m}} = 1.4 \times 10^5$ N/m^2.

17. a) Here given V_{air} = 340 m/s., Power = E/t = 20 W

f = 2,000 Hz, ρ = 1.2 kg/m^3

So, intensity I = E/t.A

$= \dfrac{20}{4\pi r^2} = \dfrac{20}{4\times\pi\times6^2} = 44$ mw/m^2 (because r = 6m)

b) We know that $I = \dfrac{P_0^2}{2\rho V_{air}} \Rightarrow P_0 = \sqrt{1\times2\rho V_{air}}$

$= \sqrt{2\times1.2\times340\times44\times10^{-3}} = 6.0$ N/m^2.

c) We know that $I = 2\pi^2 S_0^2 v^2 \rho V$ where S_0 = displacement amplitude

$\Rightarrow S_0 = \sqrt{\dfrac{I}{\pi^2\rho^2\rho V_{air}}}$

Putting the value we get $S_g = 1.2 \times 10^{-6}$ m.

18. Here $I_1 = 1.0 \times 10^{-8}$ W_1/m^2 ; I_2 = ?

r_1 = 5.0 m, r_2 = 25 m.

We know that $I \propto \dfrac{1}{r^2}$

$\Rightarrow I_1 r_1^2 = I_2 r_2^2 \Rightarrow I_2 = \dfrac{I_1 r_1^2}{r_2^2}$

$= \dfrac{1.0\times10^{-8}\times25}{625} = 4.0 \times 10^{-10}$ W/m^2.

19. We know that $\beta = 10 \log_{10}\left(\dfrac{I}{I_0}\right)$

$\beta_A = 10\log\dfrac{I_A}{I_o}$, $\beta_B = 10\log\dfrac{I_B}{I_o}$

$\Rightarrow I_A / I_0 = 10^{(\beta_A/10)} \Rightarrow I_B/I_o = 10^{(\beta_B/10)}$

$\Rightarrow \dfrac{I_A}{I_B} = \dfrac{r_B^2}{r_A^2} = \left(\dfrac{50}{5}\right)^2 \Rightarrow 10^{(\beta_A \beta_B)} = 10^2$

$\Rightarrow \dfrac{\beta_A - \beta_B}{10} = 2 \Rightarrow \beta_A - \beta_B = 20$

$\Rightarrow \beta_B = 40 - 20 = 20$ dβ.

20. We know that, $\beta = 10 \log_{10} J/I_0$
According to the questions
$\beta_A = 10 \log_{10} (2I/I_0)$
$\Rightarrow \beta_B - \beta_A = 10 \log (2I/I) = 10 \times 0.3010 = 3$ dB.
21. If sound level = 120 dB, then I = intensity = 1 W/m^2
Given that, audio output = 2W
Let the closest distance be x.
So, intensity = $(2 / 4\pi x^2) = 1 \Rightarrow x^2 = (2/2\pi) \Rightarrow x = 0.4$ m = 40 cm.
22. $\beta_1 = 50$ dB, $\beta_2 = 60$ dB
$\therefore I_1 = 10^{-7}$ W/m^2, $I_2 = 10^{-6}$ W/m^2
(because $\beta = 10 \log_{10} (I/I_0)$, where $I_0 = 10^{-12}$ W/m^2)
Again, $I_2/I_1 = (p_2/p_1)^2 = (10^{-6}/10^{-7}) = 10$ (where p = pressure amplitude).
$\therefore (p_2 / p_1) = \sqrt{10}$.
23. Let the intensity of each student be I.
According to the question

$$\beta_A = 10\log_{10}\frac{50\ I}{I_0}\ ;\ \beta_B = 10\log_{10}\left(\frac{100\ I}{I_0}\right)$$

$$\Rightarrow \beta_B - \beta_A = 10\log_{10}\frac{50\ I}{I_0} - 10\log_{10}\left(\frac{100\ I}{I_0}\right)$$

$$= 10\log\left(\frac{100\ I}{50\ I}\right) = 10\log_{10} 2 = 3$$

So, $\beta_A = 50 + 3 = 53$ dB.
24. Distance between tow maximum to a minimum is given by, $\lambda/4 = 2.50$ cm
$\Rightarrow \lambda = 10$ cm $= 10^{-1}$ m
We know, V = nx

$$\Rightarrow n = \frac{V}{\lambda} = \frac{340}{10^{-1}} = 3400 \text{ Hz} = 3.4 \text{ kHz}.$$

25. a) According to the data
$\lambda/4 = 16.5$ mm $\Rightarrow \lambda = 66$ mm $= 66 \times 10^{-6=3}$ m

$$\Rightarrow n = \frac{V}{\lambda} = \frac{330}{66 \times 10^{-3}} = 5 \text{ kHz}.$$

b) $I_{minimum} = K(A_1 - A_2)^2 = I \Rightarrow A_1 - A_2 = 11$
$I_{maximum} = K(A_1 + A_2)^2 = 9 \Rightarrow A_1 + A_2 = 31$

$$\text{So, } \frac{A_1 + A_2}{A_1 + A_2} = \frac{3}{4} \Rightarrow A_1/A_2 = 2/1$$

So, the ratio amplitudes is 2.
26. The path difference of the two sound waves is given by
$\Delta L = 6.4 - 6.0 = 0.4$ m

$$\text{The wavelength of either wave} = \lambda = \frac{V}{\rho} = \frac{320}{\rho} \text{ (m/s)}$$

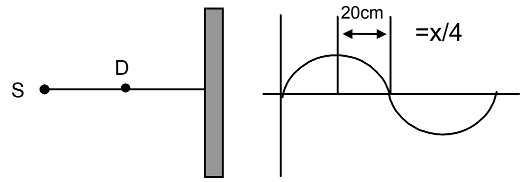
For destructive interference $\Delta L = \frac{(2n+1)\lambda}{2}$ where n is an integers.

$$\text{or } 0.4 \text{ m} = \frac{2n+1}{2} \times \frac{320}{\rho}$$

$$\Rightarrow \rho = n = \frac{320}{0.4} = 800\frac{2n+1}{2}\text{Hz} = (2n + 1)\ 400 \text{ Hz}$$

Thus the frequency within the specified range which cause destructive interference are 1200 Hz, 2000 Hz, 2800 Hz, 3600 Hz and 4400 Hz.

27. According to the given data
$V = 336$ m/s,
$\lambda/4$ = distance between maximum and minimum intensity
= (20 cm) $\Rightarrow \lambda = 80$ cm
$\Rightarrow$ n = frequency = $\frac{V}{\lambda} = \frac{336}{80 \times 10^{-2}}$ = 420 Hz.



28. Here given $\lambda = d/2$

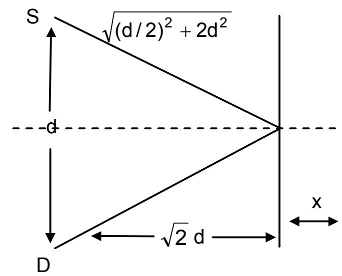
Initial path difference is given by = $2\sqrt{\left(\frac{d}{2}\right)^2 + 2d^2} - d$

If it is now shifted a distance x then path difference will be

= $2\sqrt{\left(\frac{d}{2}\right)^2 + (\sqrt{2}d + x)^2} - d = \frac{d}{4}\left(2d + \frac{d}{4}\right)$

$\Rightarrow \left(\frac{d}{2}\right)^2 + (\sqrt{2}d + x)^2 = \frac{169d^2}{64} \Rightarrow \frac{153}{64}d^2$

$\Rightarrow \sqrt{2}d + x = 1.54$ d $\Rightarrow$ x = 1.54 d – 1.414 d = 0.13 d.



29. As shown in the figure the path differences 2.4 = $\Delta x = \sqrt{(3.2)^2 + (2.4)^2} - 3.2$

Again, the wavelength of the either sound waves = $\frac{320}{\rho}$

We know, destructive interference will be occur

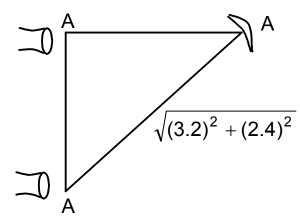
If $\Delta x = \frac{(2n+1)\lambda}{2}$

$\Rightarrow \sqrt{(3.2)^2 + (2.4)^2 - (3.2)} = \frac{(2n+1)}{2}\frac{320}{\rho}$

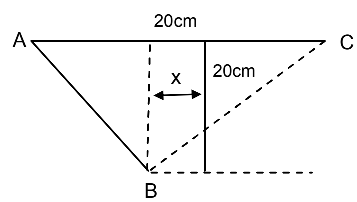
Solving we get

$\Rightarrow$ V = $\frac{(2n+1)400}{2} = 200(2n+1)$

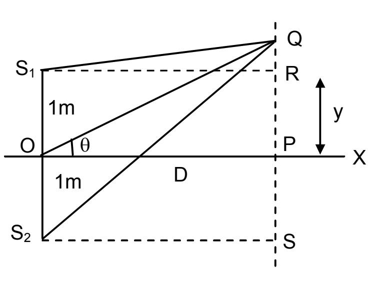
where n = 1, 2, 3, …… 49. (audible region)

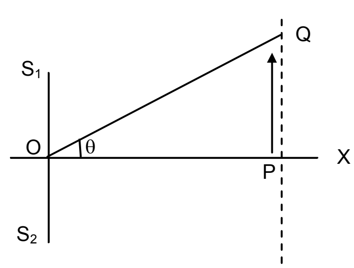


30. According to the data
λ = 20 cm, S_1S_2 = 20 cm, BD = 20 cm
Let the detector is shifted to left for a distance x for hearing the minimum sound.
So path difference AI = BC – AB
= $\sqrt{(20)^2 + (10+x)^2} - \sqrt{(20)^2 + (10-x)^2}$
So the minimum distances hearing for minimum
= $\frac{(2n+1)\lambda}{2} = \frac{\lambda}{2} = \frac{20}{2}$ = 10 cm
$\Rightarrow \sqrt{(20)^2 + (10+x)^2} - \sqrt{(20)^2 + (10-x)^2}$ = 10 solving we get x = 12.0 cm.



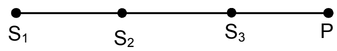
31.

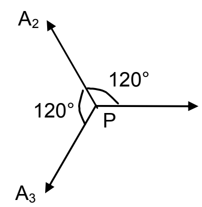




Given, F = 600 Hz, and v = 330 m/s $\Rightarrow \lambda$ = v/f = 330/600 = 0.55 mm

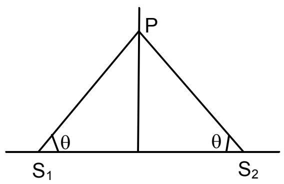
Let OP = D, PQ = y $\Rightarrow \theta = y/R$ …(1)
Now path difference is given by, $x = S_2Q - S_1Q = yd/D$
Where d = 2m
[The proof of x = yd/D is discussed in interference of light waves]
a) For minimum intensity, $x = (2n + 1)(\lambda/2)$
$\therefore$ $yd/D = \lambda/2$ [for minimum y, $x = \lambda/2$]
$\therefore$ $y/D = \theta = \lambda/2 = 0.55 / 4 = 0.1375$ rad $= 0.1375 \times (57.1)° = 7.9°$
b) For minimum intensity, $x = 2n(\lambda/2)$
$yd/D = \lambda \Rightarrow y/D = \theta = \lambda/D = 0.55/2 = 0.275$ rad
$\therefore$ $\theta = 16°$
c) For more maxima,
$yd/D = 2\lambda, 3\lambda, 4\lambda, \ldots$
$\Rightarrow y/D = \theta = 32°, 64°, 128°$
But since, the maximum value of θ can be 90°, he will hear two more maximum i.e. at 32° and 64°.

32. 



Because the 3 sources have equal intensity, amplitude are equal
So, $A_1 = A_2 = A_3$
As shown in the figure, amplitude of the resultant = 0 (vector method)
So, the resultant, intensity at B is zero.

33. The two sources of sound S_1 and S_2 vibrate at same phase and frequency.
Resultant intensity at P = I_0



a) Let the amplitude of the waves at S_1 and S_2 be ‘r’.
When $\theta = 45°$, path difference $= S_1P - S_2P = 0$ (because $S_1P = S_2P$)
So, when source is switched off, intensity of sound at P is $I_0/4$.
b) When $\theta = 60°$, path difference is also 0.
Similarly it can be proved that, the intensity at P is $I_0 / 4$ when one is switched off.

34. If V = 340 m/s, I = 20 cm = 20×10^{-2} m
Fundamental frequency = $\dfrac{V}{21} = \dfrac{340}{2\times 20\times 10^{-2}}$ = 850 Hz
We know first over tone = $\dfrac{2V}{21} = \dfrac{2\times 340}{2\times 20\times 10^{-2}}$ (for open pipe) = 1750 Hz
Second over tone = 3 (V/21) = 3 × 850 = 2500 Hz.

35. According to the questions V = 340 m/s, n = 500 Hz
We know that V/4l (for closed pipe)
$\Rightarrow$ l = $\dfrac{340}{4\times 500}$ m = 17 cm.

36. Here given distance between two nodes is = 4. 0 cm,
$\Rightarrow \lambda = 2 \times 4.0 = 8$ cm
We know that $v = n\lambda$
$\Rightarrow \eta = \dfrac{328}{8\times 10^{-2}}$ = 4.1 Hz.

37. V = 340 m/s
Distances between two nodes or antinodes
$\Rightarrow \lambda/4 = 25$ cm
$\Rightarrow \lambda = 100$ cm = 1 m
$\Rightarrow n = v/\lambda = 340$ Hz.

38. Here given that 1 = 50 cm, v = 340 m/s
As it is an open organ pipe, the fundamental frequency $f_1 = (v/21)$
$= \dfrac{340}{2\times 50\times 10^{-2}}$ = 340 Hz.

So, the harmonies are

$f_3 = 3 \times 340 = 1020$ Hz

$f_5 = 5 \times 340 = 1700$, $f_6 = 6 \times 340 = 2040$ Hz

so, the possible frequencies are between 1000 Hz and 2000 Hz are 1020, 1360, 1700.

39. Here given $l_2 = 0.67$ m, $l_1 = 0.2$ m, f = 400 Hz

We know that

$\lambda = 2(l_2 - l_1) \Rightarrow \lambda = 2(62 - 20) = 84$ cm = 0.84 m.

So, $v = n\lambda = 0.84 \times 400 = 336$ m/s

We know from above that,

$l_1 + d = \lambda/4 \Rightarrow d = \lambda/4 - l_1 = 21 - 20 = 1$ cm.

40. According to the questions

f_1 first overtone of a closed organ pipe $P_1 = 3v/4l = \frac{3 \times V}{4 \times 30}$

f_2 fundamental frequency of a open organ pipe $P_2 = \frac{V}{2l_2}$

Here given $\frac{3V}{4 \times 30} = \frac{V}{2l_2} \Rightarrow l_2 = 20$ cm

$\therefore$ length of the pipe P_2 will be 20 cm.

41. Length of the wire = 1.0 m

For fundamental frequency $\lambda/2 = l$

$\Rightarrow \lambda = 2l = 2 \times 1 = 2$ m

Here given n = 3.8 km/s = 3800 m/s

We know $\Rightarrow v = n\lambda \Rightarrow n = 3800 / 2 = 1.9$ kH.

So standing frequency between 20 Hz and 20 kHz which will be heard are

$= n \times 1.9$ kHz where n = 0, 1, 2, 3, … 10.

42. Let the length will be l.

Here given that V = 340 m/s and n = 20 Hz

Here $\lambda/2 = l \Rightarrow \lambda = 2l$

We know $V = n\lambda \Rightarrow l = \frac{V}{n} = \frac{340}{2 \times 20} = \frac{34}{4} = 8.5$ cm (for maximum wavelength, the frequency is minimum).

43. a) Here given l = 5 cm = 5×10^{-2} m, v = 340 m/s

$\Rightarrow n = \frac{V}{2l} = \frac{340}{2 \times 5 \times 10^{-2}} = 3.4$ KHz

b) If the fundamental frequency = 3.4 KHz

$\Rightarrow$ then the highest harmonic in the audible range (20 Hz – 20 KHz)

$= \frac{20000}{3400} = 5.8 = 5$ (integral multiple of 3.4 KHz).

44. The resonance column apparatus is equivalent to a closed organ pipe.

Here l = 80 cm = 10×10^{-2} m ; v = 320 m/s

$\Rightarrow n_0 = v/4l = \frac{320}{4 \times 50 \times 10^{-2}} = 100$ Hz

So the frequency of the other harmonics are odd multiple of $n_0 = (2n + 1)$ 100 Hz

According to the question, the harmonic should be between 20 Hz and 2 KHz.

45. Let the length of the resonating column will be = 1

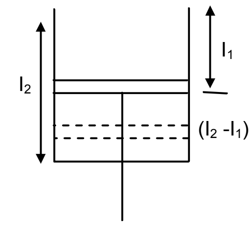
Here V = 320 m/s

Then the two successive resonance frequencies are $\frac{(n+1)v}{4l}$ and $\frac{nv}{4l}$

Here given $\frac{(n+1)v}{4l} = 2592$; $\lambda = \frac{nv}{4l} = 1944$

$\Rightarrow \frac{(n+1)v}{4l} - \frac{nv}{4l} = 2592 - 1944 = 548$ cm = 25 cm.

46. Let, the piston resonates at length l_1 and l_2
Here, l = 32 cm; v = ?, n = 512 Hz
Now $\Rightarrow 512 = v/\lambda$
$\Rightarrow v = 512 \times 0.64 = 328$ m/s.



47. Let the length of the longer tube be L_2 and smaller will be L_1.
According to the data $440 = \frac{3 \times 330}{4 \times L_2}$ ...(1) (first over tone)
and $440 = \frac{330}{4 \times L_1}$ ...(2) (fundamental)
solving equation we get $L_2 = 56.3$ cm and $L_1 = 18.8$ cm.

48. Let n_0 = frequency of the turning fork, T = tension of the string
L = 40 cm = 0.4 m, m = 4g = 4×10^{-3} kg
So, m = Mass/Unit length = 10^{-2} kg/m
$n_0 = \frac{1}{2l}\sqrt{\frac{T}{m}}$.
So, 2^{nd} harmonic $2n_0 = (2/2l)\sqrt{T/m}$
As it is unison with fundamental frequency of vibration in the air column
$\Rightarrow 2n_0 = \frac{340}{4 \times 1} = 85$ Hz
$\Rightarrow 85 = \frac{2}{2 \times 0.4}\sqrt{\frac{T}{14}} \Rightarrow T = 85^2 \times (0.4)^2 \times 10^{-2} = 11.6$ Newton.

49. Given, m = 10 g = 10×10^{-3} kg, l = 30 cm = 0.3 m
Let the tension in the string will be = T
μ = mass / unit length = 33×10^{-3} kg
The fundamental frequency $\Rightarrow n_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ ...(1)
The fundamental frequency of closed pipe
$\Rightarrow n_0 = (v/4l)\frac{340}{4 \times 50 \times 10^2} = 170$ Hz ...(2)
According equations (1) × (2) we get
$170 = \frac{1}{2 \times 30 \times 10^{-2}} \times \sqrt{\frac{T}{33 \times 10^{-3}}}$
$\Rightarrow$ T = 347 Newton.

50. We know that $f \propto \sqrt{T}$
According to the question $f + \Delta f \propto \sqrt{\Delta T} + T$
$\Rightarrow \frac{f + \Delta f}{f} = \sqrt{\frac{\Delta t + T}{T}} \Rightarrow 1 + \frac{\Delta f}{f} = \left(1 + \frac{\Delta T}{T}\right)^{1/2} = 1 + \frac{1}{2}\frac{\Delta T}{T} + \ldots$ (neglecting other terms)
$\Rightarrow \frac{\Delta f}{f} = (1/2)\frac{\Delta T}{T}$.

51. We know that the frequency = f, T = temperatures
$f \propto \sqrt{T}$
So $\frac{f_1}{f_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \Rightarrow \frac{293}{f_2} = \frac{\sqrt{293}}{\sqrt{295}}$
$\Rightarrow f_2 = \frac{293 \times \sqrt{295}}{\sqrt{293}} = 294$

52. V_{rod} = ?, V_{air} = 340 m/s, $L_r = 25 \times 10^{-2}$, $d_2 = 5 \times 10^{-2}$ metres

$$\frac{V_r}{V_a} = \frac{2L_r}{D_a} \Rightarrow V_r = \frac{340 \times 25 \times 10^{-2} \times 2}{5 \times 10^{-2}} = 3400 \text{ m/s.}$$

53. a) Here given, $L_r = 1.0/2 = 0.5$ m, $d_a = 6.5$ cm $= 6.5 \times 10^{-2}$ m

As Kundt's tube apparatus is a closed organ pipe, its fundamental frequency

$$\Rightarrow n = \frac{V_r}{4L_r} \Rightarrow V_r = 2600 \times 4 \times 0.5 = 5200 \text{ m/s.}$$

b) $$\frac{V_r}{V_a} = \frac{2L_r}{d_a} \Rightarrow v_a = \frac{5200 \times 6.5 \times 10^{-2}}{2 \times 0.5} = 338 \text{ m/s.}$$

54. As the tunning fork produces 2 beats with the adjustable frequency the frequency of the tunning fork will be $\Rightarrow n = (476 + 480) / 2 = 478$.

55. A tuning fork produces 4 beats with a known tuning fork whose frequency = 256 Hz

So the frequency of unknown tuning fork = either 256 – 4 = 252 or 256 + 4 = 260 Hz

Now as the first one is load its mass/unit length increases. So, its frequency decreases.

As it produces 6 beats now original frequency must be 252 Hz.

260 Hz is not possible as on decreasing the frequency the beats decrease which is not allowed here.

56.

Group – I	Group – II
Given V = 350	v = 350
λ_1 = 32 cm	λ_2 = 32.2 cm
$= 32 \times 10^{-2}$ m	$= 32.2 \times 10^{-2}$ m
So η_1 = frequency = 1093 Hz	$\eta_2 = 350 / 32.2 \times 10^{-2} = 1086$ Hz

So beat frequency = 1093 – 1086 = 7 Hz.

57. Given length of the closed organ pipe, $l = 40$ cm $= 40 \times 10^{-2}$ m

$V_{air} = 320$

So, its frequency $\rho = \frac{V}{4l} = \frac{320}{4 \times 40 \times 10^{-2}} = 200$ Hertz.

As the tuning fork produces 5 beats with the closed pipe, its frequency must be 195 Hz or 205 Hz.

Given that, as it is loaded its frequency decreases.

So, the frequency of tuning fork = 205 Hz.

58. Here given $n_B = 600 = \frac{1}{2l}\sqrt{\frac{TB}{14}}$

As the tension increases frequency increases

It is given that 6 beats are produces when tension in A is increases.

So, $n_A \Rightarrow 606 = \frac{1}{2l}\sqrt{\frac{TA}{M}}$

$$\Rightarrow \frac{n_A}{n_B} = \frac{600}{606} = \frac{(1/2l)\sqrt{(TB/M)}}{(1/2l)\sqrt{(TA/M)}} = \frac{\sqrt{TB}}{\sqrt{TA}}$$

$$\Rightarrow \frac{\sqrt{T_A}}{\sqrt{T_B}} = \frac{606}{600} = 1.01 \qquad \Rightarrow \frac{T_A}{T_B} = 1.02.$$

59. Given that, $l = 25$ cm $= 25 \times 10^{-2}$ m

By shortening the wire the frequency increases, $[f = (1/2l)\sqrt{(TB/M)}]$

As the vibrating wire produces 4 beats with 256 Hz, its frequency must be 252 Hz or 260 Hz.

Its frequency must be 252 Hz, because beat frequency decreases by shortening the wire.

$$\text{So, } 252 = \frac{1}{2 \times 25 \times 10^{-2}}\sqrt{\frac{T}{M}} \qquad \ldots(1)$$

Let length of the wire will be l, after it is slightly shortened,

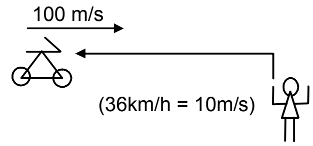
$\Rightarrow 256 = \frac{1}{2 \times l_1}\sqrt{\frac{T}{M}}$ ...(2)

Dividing (1) by (2) we get

$\frac{252}{256} = \frac{l_1}{2 \times 25 \times 10^{-2}} \Rightarrow l_1 = \frac{252 \times 2 \times 25 \times 10^{-2}}{260}$ = 0.2431 m

So, it should be shorten by (25 – 24.61) = 0.39 cm.

60. Let u = velocity of sound; V_m = velocity of the medium;
v_o = velocity of the observer; v_a = velocity of the sources.



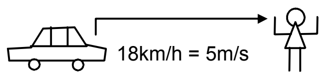
$f = \left(\frac{\vec{u} + \vec{V}_m - \vec{v}_o}{v + V_m - v_s}\right)F$

using sign conventions in Doppler's effect,

$V_m = 0$, u = 340 m/s, $v_s = 0$ and $\vec{v}_o = -10$ m (36 km/h = 10 m/s)

$= \left(\frac{340 + 0 - (-10)}{340 + 0 - 0}\right) \times 2\text{KHz}$ = 350/340 × 2 KHz = 2.06 KHz.

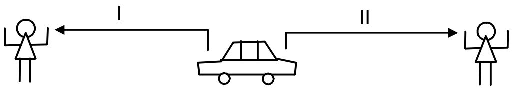
61. $f^1 = \left(\frac{\vec{u} + \vec{V}_m - \vec{v}_o}{\vec{u} + \vec{V}_m - \vec{v}_s}\right) f$ [18 km/h = 5 m/s]



using sign conventions,

app. Frequency = $\left(\frac{340 + 0 - 0}{340 + 0 - 5}\right) \times 2400$ = 2436 Hz.

62.



a) Given v_s = 72 km/hour = 20 m/s, ρ = 1250

apparent frequency = $\frac{340 + 0 + 0}{340 + 0 - 20} \times 1250$ = 1328 H_2

b) For second case apparent frequency will be = $\frac{340 + 0 + 0}{340 + 0 - (-20)} \times 1250$ = 1181 Hz.

63. Here given, apparent frequency = 1620 Hz
So original frequency of the train is given by

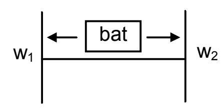
$1620 = \left(\frac{332 + 0 + 0}{332 - 15}\right) f \Rightarrow f = \left(\frac{1620 \times 317}{332}\right)$ Hz

So, apparent frequency of the train observed by the observer in

$f^1 = \left(\frac{332 + 0 + 0}{332 + 15}\right) f \times \left(\frac{1620 \times 317}{332}\right) = \frac{317}{347} \times 1620$ = 1480 Hz.

64. Let, the bat be flying between the walls W_1 and W_2.
So it will listen two frequency reflecting from walls W_2 and W_1.

So, apparent frequency, as received by wall W = $fw_2 = \frac{330 + 0 + 0}{330 - 6} \times f$ = 330/324



Therefore, apparent frequency received by the bat from wall W_2 is given by

F_{B_2} of wall W_1 = $\left(\frac{330 + 0 - (-6)}{330 + 0 + 0}\right) f_{w_2} = \left(\frac{336}{330}\right) \times \left(\frac{330}{324}\right) f$

Similarly the apparent frequency received by the bat from wall W_1 is

$f_{B_1} = (324/336)f$

So the beat frequency heard by the bat will be = 4.47 × 10^4 = 4.3430 × 10^4 = 3270 Hz.

65. Let the frequency of the bullet will be f
Given, u = 330 m/s, v_s = 220 m/s

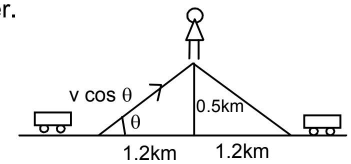
a) Apparent frequency before crossing = $f' = \left(\dfrac{330}{330-220}\right) f = 3f$

b) Apparent frequency after crossing = $f'' = \left(\dfrac{330}{530+220}\right) f = 0.6\ f$

So, $\left(\dfrac{f''}{f'}\right) = \dfrac{0.6f}{3f} = 0.2$

Therefore, fractional change = 1 – 0.2 = 0.8.

66. The person will receive, the sound in the directions BA and CA making an angle θ with the track.
Here, $\theta = \tan^{-1}(0.5/2.4) = 22°$
So the velocity of the sources will be '$v \cos \theta$' when heard by the observer.
So the apparent frequency received by the man from train B.

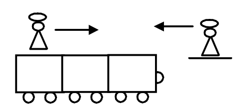


$$f' = \left(\frac{340+0+0}{340 - v\cos 22°}\right) 500 = 529\ \text{Hz}$$

And the apparent frequency heard but the man from train C,

$$f'' = \left(\frac{340+0+0}{340 - v\cos 22°}\right) \times 500 = 476\ \text{Hz}.$$

67. Let the velocity of the sources is = v_s



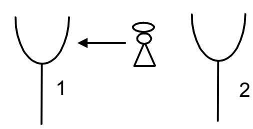
a) The beat heard by the standing man = 4
So, frequency = 440 + 4 = 444 Hz or 436 Hz

$$\Rightarrow 440 = \left(\frac{340+0+0}{340 - v_s}\right) \times 400$$

On solving we get V_s = 3.06 m/s = 11 km/hour.

b) The sitting man will listen less no.of beats than 4.

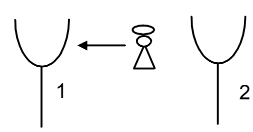
68. Here given velocity of the sources $v_s = 0$
Velocity of the observer v_0 = 3 m/s



So, the apparent frequency heard by the man = $\left(\dfrac{332+3}{332}\right) \times 256 = 258.3$ Hz.

from the approaching tuning form = f′
$f'' = [(332–3)/332] \times 256 = 253.7$ Hz.
So, beat produced by them = 258.3 – 253.7 = 4.6 Hz.

69. According to the data, V_s = 5.5 m/s for each turning fork.
So, the apparent frequency heard from the tuning fork on the left,



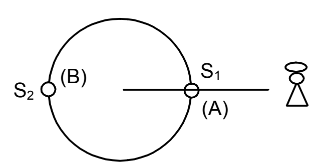
$$f' = \left(\frac{330}{330-5.5}\right) \times 512 = 527.36\ \text{Hz} = 527.5\ \text{Hz}$$

similarly, apparent frequency from the tunning fork on the right,

$$f'' = \left(\frac{330}{330+5.5}\right) \times 512 = 510\ \text{Hz}$$

So, beats produced 527.5 – 510 = 17.5 Hz.

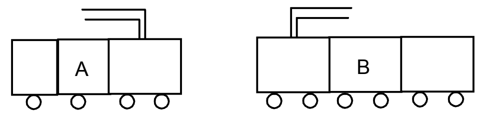
70. According to the given data
Radius of the circle = $100/\pi \times 10^{-2}$ m = $(1/\pi)$ metres; ω = 5 rev/sec.
So the linear speed $v = \omega r = 5/\pi = 1.59$
So, velocity of the source V_s = 1.59 m/s
As shown in the figure at the position A the observer will listen maximum and at the position B it will listen minimum frequency.



So, apparent frequency at A = $\dfrac{332}{332-1.59} \times 500 = 515$ Hz

Apparent frequency at B = $\dfrac{332}{332+1.59} \times 500 = 485$ Hz.

71. According to the given data V_s = 90 km/hour = 25 m/sec.
v_0 = 25 m/sec
So, apparent frequency heard by the observer in train B or observer in = $\left(\frac{350+25}{350-25}\right) \times 500$ = 577 Hz.



72. Here given $f_s = 16 \times 10^3$ Hz
Apparent frequency $f' = 20 \times 10^3$ Hz (greater than that value)
Let the velocity of the observer = v_o
Given $v_s = 0$

So $20 \times 10^3 = \left(\frac{330+v_o}{330+0}\right) \times 16 \times 10^3$

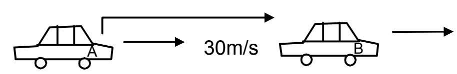
$\Rightarrow (330 + v_o) = \frac{20 \times 330}{16}$

$\Rightarrow v_o = \frac{20 \times 330 - 16 \times 330}{4} = \frac{330}{4}$ m/s = 297 km/h

b) This speed is not practically attainable ordinary cars.

73. According to the questions velocity of car A = V_A = 108 km/h = 30 m/s
V_B = 72 km/h = 20 m/s, f = 800 Hz
So, the apparent frequency heard by the car B is given by,

$f' = \left(\frac{330-20}{330-30}\right) \times 800 \Rightarrow 826.9 = 827$ Hz.



74. a) According to the questions, v = 1500 m/s, f = 2000 Hz, v_s = 10 m/s, v_o = 15 m/s
So, the apparent frequency heard by the submarine B,

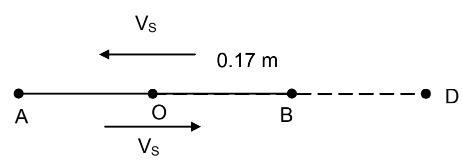
$= \left(\frac{1500+15}{1500-10}\right) \times 2000 = 2034$ Hz

b) Apparent frequency received by submarine A,

$= \left(\frac{1500+10}{1500-15}\right) \times 2034 = 2068$ Hz.

75. Given that, r = 0.17 m, F = 800 Hz, u = 340 m/s
Frequency band = $f_1 - f_2$ = 6 Hz
Where f_1 and f_2 correspond to the maximum and minimum apparent frequencies (both will occur at the mean position because the velocity is maximum).

Now, $f_1 = \left(\frac{340}{340-v_s}\right) f$ and $f_2 = \left(\frac{340}{340+v_s}\right) f$



$\therefore f_1 - f_2 = 8$

$\Rightarrow 340\, f \left(\frac{1}{340-v_s} - \frac{1}{340+v_s}\right) = 8$

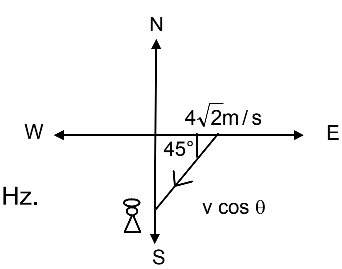
$\Rightarrow \frac{2v_s}{340^2 - {v_s}^2} = \frac{8}{340 \times 800}$

$\Rightarrow 340^2 - {v_s}^2 = 68000\, v_s$

Solving for v_s we get, v_s = 1.695 m/s
For SHM, $v_s = r\omega \Rightarrow \omega = (1.695/0.17) = 10$
So, $T = 2\pi / \omega = \pi/5 = 0.63$ sec.

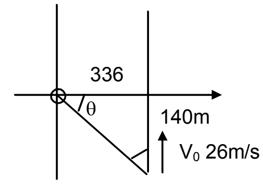
76. u = 334 m/s, $v_b = 4\sqrt{2}$ m/s, $v_o = 0$
so, $v_s = V_b \cos\theta = 4\sqrt{2} \times (1/\sqrt{2}) = 4$ m/s.

so, the apparent frequency $f' = \left(\frac{u+0}{u - v_b \cos\theta}\right) f = \left(\frac{334}{334-4}\right) \times 1650 = 1670$ Hz.



77. u = 330 m/s, v_0 = 26 m/s

a) Apparent frequency at, y = – 336

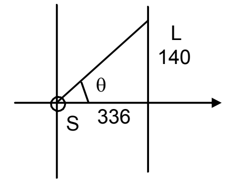


$$m = \left(\frac{v}{v - u\sin\theta}\right) \times f$$

$$= \left(\frac{330}{330 - 26\sin 23^\circ}\right) \times 660$$

[because, θ = $\tan^{-1}$ (140/336) = 23°] = 680 Hz.

b) At the point y = 0 the source and listener are on a x-axis so no apparent change in frequency is seen. So, f = 660 Hz.



c) As shown in the figure θ = $\tan^{-1}$ (140/336) = 23°

Here given, = 330 m/s ; v = V sin 23° = 10.6 m/s

So, $F'' = \frac{u}{u + v\sin 23^\circ} \times 660 = 640$ Hz.

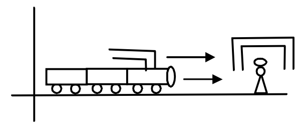
78. V_{train} or V_s = 108 km/h = 30 m/s; u = 340 m/s

a) The frequency by the passenger sitting near the open window is 500 Hz, he is inside the train and does not hair any relative motion.

b) After the train has passed the apparent frequency heard by a person standing near the track will be,

so $f'' = \left(\frac{340 + 0}{340 + 30}\right) \times 500 = 459$ Hz

c) The person inside the source will listen the original frequency of the train.



Here, given V_m = 10 m/s

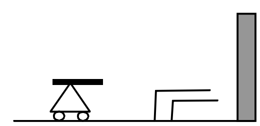
For the person standing near the track

Apparent frequency = $\frac{u + V_m + 0}{u + V_m - (-V_s)} \times 500$ = 458 Hz.

79. To find out the apparent frequency received by the wall,

a) V_s = 12 km/h = 10/3 = m/s

V_o = 0, u = 330 m/s



So, the apparent frequency is given by = $f' = \left(\frac{330}{330 - 10/3}\right) \times 1600 = 1616$ Hz

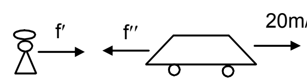
b) The reflected sound from the wall whistles now act as a sources whose frequency is 1616 Hz.

So, u = 330 m/s, V_s = 0, V_o = 10/3 m/s

So, the frequency by the man from the wall,

$\Rightarrow f'' = \left(\frac{330 + 10/3}{330}\right) \times 1616 = 1632$ m/s.

80. Here given, u = 330 m/s, f = 1600 Hz



So, apparent frequency received by the car

$f' = \left(\frac{u - V_o}{u - V_s}\right) f = \left(\frac{330 - 20}{330}\right) \times 1600$ Hz … [V_o = 20 m/s, V_s = 0]

The reflected sound from the car acts as the source for the person.

Here, V_s = –20 m/s, V_o = 0

So $f'' = \left(\frac{330 - 0}{330 + 20}\right) \times f' = \frac{330}{350} \times \frac{310}{330} \times 160 = 1417$ Hz.

∴ This is the frequency heard by the person from the car.

81. a) f = 400 Hz,, u = 335 m/s

⇒ λ (v/f) = (335/400) = 0.8 m = 80 cm

b) The frequency received and reflected by the wall,

$f' = \left(\frac{u - V_o}{u - V_s}\right) \times f = \frac{335}{320} \times 400$ …[V_s = 54 m/s and V_o = 0]

$\Rightarrow x' = (v/f) = \dfrac{320 \times 335}{335 \times 400} = 0.8$ m = 80 cm

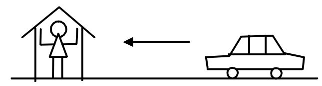
c) The frequency received by the person sitting inside the car from reflected wave,

$f' = \left(\dfrac{335-0}{335-15}\right)f = \dfrac{335}{320} \times 400 = 467$ $\qquad$ [$V_s = 0$ and $V_o = -15$ m/s]

d) Because, the difference between the original frequency and the apparent frequency from the wall is very high (437 – 440 = 37 Hz), he will not hear any beats.mm)

82. f = 400 Hz, u = 324 m/s, $f' = \dfrac{u-(-v)}{u-(0)}f = \dfrac{324+v}{324} \times 400$ $\qquad$ …(1)

for the reflected wave,



$f'' = 410 = \dfrac{u-0}{u-v}f'$

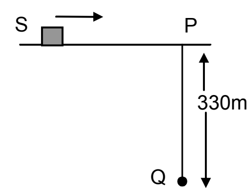
$\Rightarrow 410 = \dfrac{324}{324-v} \times \dfrac{324+v}{324} \times 400$

$\Rightarrow 810\,v = 324 \times 10$

$\Rightarrow v = \dfrac{324 \times 10}{810} = 4$ m/s.

83. f = 2 kHz, v = 330 m/s, u = 22 m/s

At t = 0, the source crosses P



a) Time taken to reach at Q is

$t = \dfrac{S}{v} = \dfrac{330}{330} = 1$ sec

b) The frequency heard by the listner is

$f' = f\left(\dfrac{v}{v - u\cos\theta}\right)$

since, $\theta = 90°$

f′ = 2 × (v/u) = 2 KHz.

c) After 1 sec, the source is at 22 m from P towards right.

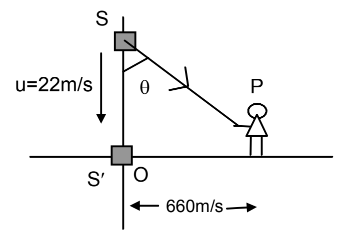
84. t = 4000 Hz, u = 22 m/s

Let ‘t’ be the time taken by the source to reach at ‘O’. Since observer hears the sound at the instant it crosses the ‘O’, ‘t’ is also time taken to the sound to reach at P.

$\therefore$ OQ = ut and QP = vt

$\cos\theta = u/v$

Velocity of the sound along QP is $(u\cos\theta)$.

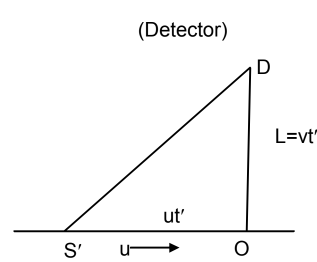


$$f' = f\left(\frac{v-0}{v-u\cos\theta}\right) = f\left(\frac{v}{v-\dfrac{u^2}{v}}\right) = f\left(\frac{v^2}{v^2-u^2}\right)$$

Putting the values in the above equation, $f' = 4000 \times \dfrac{330^2}{330^2 - 22^2} = 4017.8 = 4018$ Hz.

85. a) Given that, f = 1200 Hz, u = 170 m/s, L = 200 m, v = 340 m/s

From Doppler’s equation (as in problem no.84)



$f' = f\left(\dfrac{v^2}{v^2-u^2}\right) = 1200 \times \dfrac{340^2}{340^2 - 170^2} = 1600$ Hz.

b) v = velocity of sound, u = velocity of source

let, t be the time taken by the sound to reach at D

DO = vt′ = L, and S′O = ut′

t′ = L/V

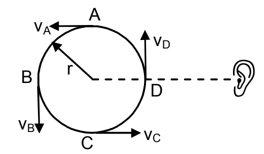
$S'D = \sqrt{S'O^2 + DO^2} = \sqrt{u^2\frac{L^2}{v^2} + L^2} = \frac{L}{v}\sqrt{u^2 + v^2}$

Putting the values in the above equation, we get

$S'D = \frac{220}{340}\sqrt{170^2 + 340^2}$ = 223.6 m.

86. Given that, r = 1.6 m, f = 500 Hz, u = 330 m/s

a) At A, velocity of the particle is given by



$v_A = \sqrt{rg} = \sqrt{1.6 \times 10} = 4\ \text{m/s}$

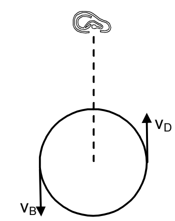
and at C, $v_c = \sqrt{5rg} = \sqrt{5 \times 1.6 \times 10} = 8.9\ \text{m/s}$

So, maximum frequency at C,

$f'_c = \frac{u}{u - v_s} f = \frac{330}{330 - 8.9} \times 500 = 513.85\ \text{Hz.}$

Similarly, maximum frequency at A is given by $f'_A = \frac{u}{u - (-v_s)} f = \frac{330}{330 + 4}(500) = 494\ \text{Hz.}$

b) Velocity at B = $\sqrt{3rg} = \sqrt{3 \times 1.6 \times 10} = 6.92\ \text{m/s}$



So, frequency at B is given by,

$f_B = \frac{u}{u + v_s} \times f = \frac{330}{330 + 6.92} \times 500$ = 490 Hz

and frequency at D is given by,

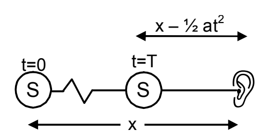
$f_D = \frac{u}{u - v_s} \times f = \frac{330}{330 - 6.92} \times 500$

87. Let the distance between the source and the observer is 'x' (initially)

So, time taken for the first pulse to reach the observer is $t_1 = x/v$

and the second pulse starts after T (where, T = 1/v)

and it should travel a distance $\left(x - \frac{1}{2}aT^2\right)$.



So, $t_2 = T + \frac{x - 1/2\, aT^2}{v}$

$t_2 - t_1 = T + \frac{x - 1/2\, aT^2}{v} = \frac{x}{v} = T - \frac{1}{2}\frac{aT^2}{v}$

Putting = T = 1/v, we get

$t_2 - t_1 = \frac{2uv - a}{2vv^2}$

so, frequency heard = $\frac{2vv^2}{2uv - a}$ (because, f = $\frac{1}{t_2 - t_1}$)

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SOLUTIONS TO CONCEPTS
CHAPTER 17

1. Given that, 400 m < λ < 700 nm.

$$\frac{1}{700\text{nm}} < \frac{1}{\lambda} < \frac{1}{400\text{nm}}$$

$$\Rightarrow \frac{1}{7\times10^{-7}} < \frac{1}{\lambda} < \frac{1}{4\times10^{-7}} \Rightarrow \frac{3\times10^{8}}{7\times10^{-7}} < \frac{c}{\lambda} < \frac{3\times10^{8}}{4\times10^{-7}}$$ (Where, c = speed of light = 3×10^8 m/s)

$\Rightarrow 4.3 \times 10^{14} < c/\lambda < 7.5 \times 10^{14}$

$\Rightarrow 4.3 \times 10^{14}$ Hz < f < 7.5×10^{14} Hz.

2. Given that, for sodium light, λ = 589 nm = 589×10^{-9} m

a) $f_a = \dfrac{3\times10^{8}}{589\times10^{-9}} = 5.09 \times 10^{14}\ \text{sec}^{-1}\left[\because f = \dfrac{c}{\lambda}\right]$

b) $\dfrac{\mu_a}{\mu_w} = \dfrac{\lambda_w}{\lambda_a} \Rightarrow \dfrac{1}{1.33} = \dfrac{\lambda_w}{589\times10^{-9}} \Rightarrow \lambda_w$ = 443 nm

c) $f_w = f_a = 5.09 \times 10^{14}\ \text{sec}^{-1}$ [Frequency does not change]

d) $\dfrac{\mu_a}{\mu_w} = \dfrac{v_w}{v_a} \Rightarrow v_w = \dfrac{\mu_a v_a}{\mu_w} = \dfrac{3\times10^{8}}{1.33} = 2.25 \times 10^{8}$ m/sec.

3. We know that, $\dfrac{\mu_2}{\mu_1} = \dfrac{v_1}{v_2}$

So, $\dfrac{1472}{1} = \dfrac{3\times10^{8}}{v_{400}} \Rightarrow v_{400} = 2.04\times10^{8}\,\text{m/sec}.$

[because, for air, μ = 1 and v = 3×10^8 m/s]

Again, $\dfrac{1452}{1} = \dfrac{3\times10^{8}}{v_{760}} \Rightarrow v_{760} = 2.07\times10^{8}\,\text{m/sec}.$

4. $\mu_t = \dfrac{1\times3\times10^{8}}{(2.4)\times10^{8}} = 1.25\ \left[\text{since, } \mu = \dfrac{\text{velocity of light in vaccum}}{\text{velocity of light in the given medium}}\right]$

5. Given that, d = 1 cm = 10^{-2} m, $\lambda = 5 \times 10^{-7}$ m and D = 1 m

a) Separation between two consecutive maxima is equal to fringe width.

So, $\beta = \dfrac{\lambda D}{d} = \dfrac{5\times10^{-7}\times1}{10^{-2}}$ m = 5×10^{-5} m = 0.05 mm.

b) When, β = 1 mm = 10^{-3} m

$10^{-3}\text{m} = \dfrac{5\times10^{-7}\times1}{D} \Rightarrow D = 5 \times 10^{-4}$ m = 0.50 mm.

6. Given that, β = 1 mm = 10^{-3} m, D = 2.t m and d = 1 mm = 10^{-3} m

So, $10^{-3}\text{m} = \dfrac{25\times\lambda}{10^{-3}} \Rightarrow \lambda = 4 \times 10^{-7}$ m = 400 nm.

7. Given that, d = 1 mm = 10^{-3} m, D = 1 m.

So, fringe with = $\dfrac{D\lambda}{d}$ = 0.5 mm.

a) So, distance of centre of first minimum from centre of central maximum = 0.5/2 mm = 0.25 mm

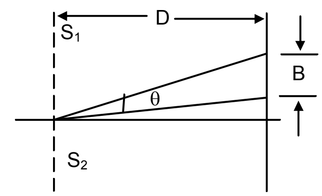
b) No. of fringes = 10 / 0.5 = 20.

8. Given that, d = 0.8 mm = 0.8×10^{-3} m, λ = 589 nm = 589×10^{-9} m and D = 2 m.

So, $\beta = \dfrac{D\lambda}{d} = \dfrac{589\times10^{-9}\times2}{0.8\times10^{-3}} = 1.47 \times 10^{-3}$ m = 147 mm.

9. Given that, λ = 500 nm = 500×10^{-9} m and d = 2×10^{-3} m

As shown in the figure, angular separation $\theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$



So, $\theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{2 \times 10^{-3}}$ = 250×10^{-6}

= 25×10^{-5} radian = 0.014 degree.

10. We know that, the first maximum (next to central maximum) occurs at $y = \frac{\lambda D}{d}$

Given that, λ_1 = 480 nm, λ_2 = 600 nm, D = 150 cm = 1.5 m and d = 0.25 mm = 0.25×10^{-3} m

So, $y_1 = \frac{D\lambda_1}{d} = \frac{1.5 \times 480 \times 10^{-9}}{0.25 \times 10^{-3}}$ = 2.88 mm

$y_2 = \frac{1.5 \times 600 \times 10^{-9}}{0.25 \times 10^{-3}}$ = 3.6 mm.

So, the separation between these two bright fringes is given by,

$\therefore$ separation = $y_2 - y_1$ = 3.60 – 2.88 = 0.72 mm.

11. Let m^{th} bright fringe of violet light overlaps with n^{th} bright fringe of red light.

$\therefore \frac{m \times 400\text{nm} \times D}{d} = \frac{n \times 700\text{nm} \times D}{d} \Rightarrow \frac{m}{n} = \frac{7}{4}$

$\Rightarrow$ 7^{th} bright fringe of violet light overlaps with 4^{th} bright fringe of red light (minimum). Also, it can be seen that 14^{th} violet fringe will overlap 8^{th} red fringe.

Because, m/n = 7/4 = 14/8.

12. Let, t = thickness of the plate

Given, optical path difference = $(\mu - 1)t = \lambda/2$

$\Rightarrow t = \frac{\lambda}{2(\mu - 1)}$

13. a) Change in the optical path = $\mu t - t = (\mu - 1)t$

b) To have a dark fringe at the centre the pattern should shift by one half of a fringe.

$\Rightarrow (\mu - 1)t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{2(\mu - 1)}$.

14. Given that, μ = 1.45, t = 0.02 mm = 0.02×10^{-3} m and λ = 620 nm = 620×10^{-9} m

We know, when the transparent paper is pasted in one of the slits, the optical path changes by $(\mu - 1)t$.

Again, for shift of one fringe, the optical path should be changed by λ.

So, no. of fringes crossing through the centre is given by,

$n = \frac{(\mu - 1)t}{\lambda} = \frac{0.45 \times 0.02 \times 10^{-3}}{620 \times 10^{-9}}$ = 14.5

15. In the given Young's double slit experiment,

μ = 1.6, t = 1.964 micron = 1.964×10^{-6} m

We know, number of fringes shifted = $\frac{(\mu - 1)t}{\lambda}$

So, the corresponding shift = No.of fringes shifted $\times$ fringe width

$= \frac{(\mu - 1)t}{\lambda} \times \frac{\lambda D}{d} = \frac{(\mu - 1)tD}{d}$ … (1)

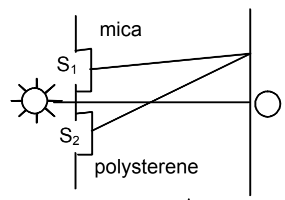
Again, when the distance between the screen and the slits is doubled,

Fringe width = $\frac{\lambda(2D)}{d}$ …(2)

From (1) and (2), $\frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$

$\Rightarrow \lambda = \frac{(\mu - 1)t}{\lambda} = \frac{(1.6 - 1) \times (1.964) \times 10^{-6}}{2}$ = 589.2×10^{-9} = 589.2 nm.

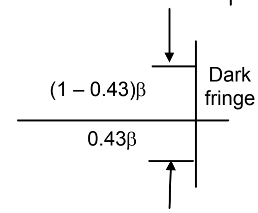
16. Given that, $t_1 = t_2 = 0.5$ mm $= 0.5 \times 10^{-3}$ m, $\mu_m = 1.58$ and $\mu_p = 1.55$,
$\lambda = 590$ nm $= 590 \times 10^{-9}$ m, d = 0.12 cm $= 12 \times 10^{-4}$ m, D = 1 m



a) Fringe width $= \dfrac{D\lambda}{d} = \dfrac{1 \times 590 \times 10^{-9}}{12 \times 10^{-4}} = 4.91 \times 10^{-4}$ m.

b) When both the strips are fitted, the optical path changes by
$\Delta x = (\mu_m - 1)t_1 - (\mu_p - 1)t_2 = (\mu_m - \mu_p)t$
$= (1.58 - 1.55) \times (0.5)(10^{-3}) = 0.015 \times 10^{-13}$ m.

So, No. of fringes shifted $= \dfrac{0.015 \times 10^{-3}}{590 \times 10^{-3}} = 25.43$.



$\Rightarrow$ There are 25 fringes and 0.43 th of a fringe.
$\Rightarrow$ There are 13 bright fringes and 12 dark fringes and 0.43 th of a dark fringe.
So, position of first maximum on both sides will be given by
$\therefore$ $x = 0.43 \times 4.91 \times 10^{-4} = 0.021$ cm
$x' = (1 - 0.43) \times 4.91 \times 10^{-4} = 0.028$ cm (since, fringe width $= 4.91 \times 10^{-4}$ m)

17. The change in path difference due to the two slabs is $(\mu_1 - \mu_2)t$ (as in problem no. 16).
For having a minimum at P_0, the path difference should change by $\lambda/2$.

So, $\Rightarrow \lambda/2 = (\mu_1 - \mu_2)t \Rightarrow t = \dfrac{\lambda}{2(\mu_1 - \mu_2)}$.

18. Given that, t = 0.02 mm $= 0.02 \times 10^{-3}$ m, $\mu_1 = 1.45$, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m
a) Let, I_1 = Intensity of source without paper = I
b) Then I_2 = Intensity of source with paper = (4/9)I

$\Rightarrow \dfrac{I_1}{I_2} = \dfrac{9}{4} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{3}{2}$ [because $I \propto r^2$]

where, r_1 and r_2 are corresponding amplitudes.

So, $\dfrac{I_{max}}{I_{min}} = \dfrac{(r_1 + r_2)^2}{(r_1 - r_2)^2} = 25 : 1$

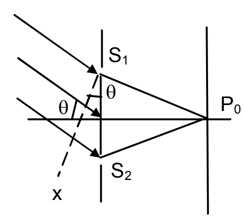
b) No. of fringes that will cross the origin is given by,

$n = \dfrac{(\mu - 1)t}{\lambda} = \dfrac{(1.45 - 1) \times 0.02 \times 10^{-3}}{600 \times 10^{-9}} = 15.$

19. Given that, d = 0.28 mm $= 0.28 \times 10^{-3}$ m, D = 48 cm = 0.48 m, $\lambda_a = 700$ nm in vacuum
Let, λ_w = wavelength of red light in water
Since, the fringe width of the pattern is given by,

$\beta = \dfrac{\lambda_w D}{d} = \dfrac{525 \times 10^{-9} \times 0.48}{0.28 \times 10^{-3}} = 9 \times 10^{-4}$ m = 0.90 mm.

20. It can be seen from the figure that the wavefronts reaching O from S_1 and S_2 will have a path difference of S_2X.

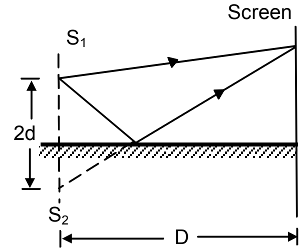


In the $\Delta\, S_1S_2X$,

$\sin\theta = \dfrac{S_2X}{S_1S_2}$

So, path difference $= S_2X = S_1S_2 \sin\theta = d\sin\theta = d \times \lambda/2d = \lambda/2$
As the path difference is an odd multiple of $\lambda/2$, there will be a dark fringe at point P_0.

21. a) Since, there is a phase difference of π between direct light and reflecting light, the intensity just above the mirror will be zero.



b) Here, 2d = equivalent slit separation
D = Distance between slit and screen.

We know for bright fringe, $\Delta x = \dfrac{y \times 2d}{D} = n\lambda$

But as there is a phase reversal of $\lambda/2$.

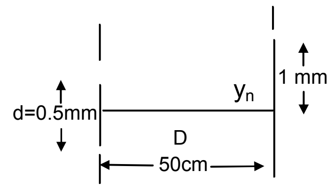
$\Rightarrow \dfrac{y \times 2d}{D} + \dfrac{\lambda}{2} = n\lambda$ $\qquad \Rightarrow \dfrac{y \times 2d}{D} = n\lambda - \dfrac{\lambda}{2} \Rightarrow y = \dfrac{\lambda D}{4d}$

22. Given that, D = 1 m, λ = 700 nm = 700×10^{-9} m
Since, a = 2 mm, d = 2a = 2mm = 2×10^{-3} m (L loyd's mirror experiment)
Fringe width = $\frac{\lambda D}{d} = \frac{700 \times 10^{-9} m \times 1m}{2 \times 10^{-3} m}$ = 0.35 mm.

23. Given that, the mirror reflects 64% of energy (intensity) of the light.
So, $\frac{I_1}{I_2} = 0.64 = \frac{16}{25} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5}$
So, $\frac{I_{max}}{I_{min}} = \frac{(r_1 + r_2)^2}{(r_1 - r_2)^2}$ = 81 : 1.

24. It can be seen from the figure that, the apparent distance of the screen from the slits is,
$D = 2D_1 + D_2$
So, Fringe width = $\frac{D\lambda}{d} = \frac{(2D_1 + D_2)\lambda}{d}$

25. Given that, λ = (400 nm to 700 nm), d = 0.5 mm = 0.5×10^{-3} m,
D = 50 cm = 0.5 m and on the screen y_n = 1 mm = 1×10^{-3} m



a) We know that for zero intensity (dark fringe)
$y_n = \left(\frac{2n+1}{2}\right)\frac{\lambda_n D}{d}$ where n = 0, 1, 2, ...

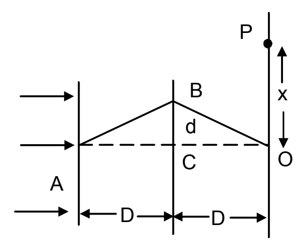
$\Rightarrow \lambda_n = \frac{2}{(2n+1)} \frac{\lambda_n d}{D} = \frac{2}{2n+1} \times \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} \Rightarrow \frac{2}{(2n+1)} \times 10^{-6} m = \frac{2}{(2n+1)} \times 10^{3} nm$

If n = 1, λ_1 = (2/3) × 1000 = 667 nm
If n = 1, λ_2 = (2/5) × 1000 = 400 nm

So, the light waves of wavelengths 400 nm and 667 nm will be absent from the out coming light.

b) For strong intensity (bright fringes) at the hole
$y_n = \frac{n\lambda_n D}{d} \Rightarrow \lambda_n = \frac{y_n d}{nD}$
When, n = 1, $\lambda_1 = \frac{y_n d}{D} = \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} = 10^{-6} m = 1000 nm$.
1000 nm is not present in the range 400 nm – 700 nm
Again, where n = 2, $\lambda_2 = \frac{y_n d}{2D}$ = 500 nm
So, the only wavelength which will have strong intensity is 500 nm.

26. From the diagram, it can be seen that at point O.
Path difference = (AB + BO) – (AC + CO)
= 2(AB – AC) [Since, AB = BO and AC = CO] = $2(\sqrt{d^2 + D^2} - D)$

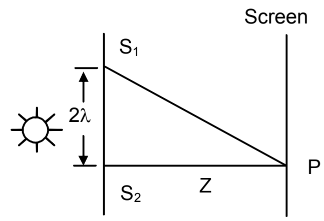


For dark fringe, path difference should be odd multiple of $\lambda/2$.
So, $2(\sqrt{d^2 + D^2} - D)$ = (2n + 1)(λ/2)
$\Rightarrow \sqrt{d^2 + D^2}$ = D + (2n + 1) λ/4
$\Rightarrow D^2 + d^2 = D^2 + (2n+1)^2 \lambda^2/16 + (2n + 1) \lambda D/2$
Neglecting, $(2n+1)^2 \lambda^2/16$, as it is very small
We get, d = $\sqrt{(2n+1)\frac{\lambda D}{2}}$
For minimum 'd', putting n = 0 $\Rightarrow d_{min} = \sqrt{\frac{\lambda D}{2}}$.

27. For minimum intensity

$\therefore$ $S_1P - S_2P = x = (2n + 1)\,\lambda/2$

From the figure, we get



$$\Rightarrow \sqrt{Z^2 + (2\lambda)^2} - Z = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow Z^2 + 4\lambda^2 = Z^2 + (2n+1)^2\frac{\lambda^2}{4} + Z(2n+1)\lambda$$

$$\Rightarrow Z = \frac{4\lambda^2 - (2n+1)^2(\lambda^2/4)}{(2n+1)\lambda} = \frac{16\lambda^2 - (2n+1)^2\lambda^2}{4(2n+1)\lambda} \quad \ldots(1)$$

Putting, $n = 0 \Rightarrow Z = 15\lambda/4$ $\quad$ $n = -1 \Rightarrow Z = -15\lambda/4$

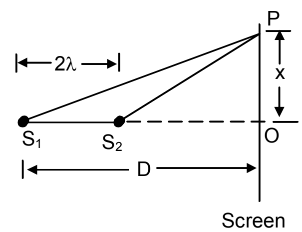
$n = 1 \Rightarrow Z = 7\lambda/12$ $\quad$ $n = 2 \Rightarrow Z = -9\lambda/20$

$\therefore$ $Z = 7\lambda/12$ is the smallest distance for which there will be minimum intensity.

28. Since S_1, S_2 are in same phase, at O there will be maximum intensity.

Given that, there will be a maximum intensity at P.

$\Rightarrow$ path difference $= \Delta x = n\lambda$



From the figure,

$(S_1P)^2 - (S_2P)^2 = (\sqrt{D^2 + X^2})^2 - (\sqrt{(D-2\lambda)^2 + X^2})^2$

$= 4\lambda D - 4\lambda^2 = 4\,\lambda D$ (λ^2 is so small and can be neglected)

$$\Rightarrow S_1P - S_2P = \frac{4\lambda D}{2\sqrt{x^2 + D^2}} = n\lambda$$

$$\Rightarrow \frac{2D}{\sqrt{x^2 + D^2}} = \nu$$

$$\Rightarrow n^2(X^2 + D^2) = 4D^2 = \Delta X = \frac{D}{n}\sqrt{4 - n^2}$$

when $n = 1$, $x = \sqrt{3}\,D$ $\quad$ (1^{st} order)

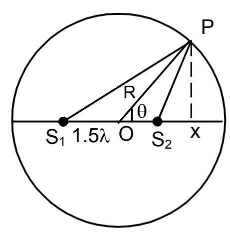
$n = 2$, $x = 0$ $\quad$ (2^{nd} order)

$\therefore$ When $X = \sqrt{3}\,D$, at P there will be maximum intensity.

29. As shown in the figure,

$(S_1P)^2 = (PX)^2 + (S_1X)^2$ $\quad \ldots(1)$

$(S_2P)^2 = (PX)^2 + (S_2X)^2$ $\quad \ldots(2)$



From (1) and (2),

$(S_1P)^2 - (S_2P)^2 = (S_1X)^2 - (S_2X)^2$

$= (1.5\,\lambda + R\cos\theta)^2 - (R\cos\theta - 15\,\lambda)^2$

$= 6\lambda\, R\cos\theta$

$$\Rightarrow (S_1P - S_2P) = \frac{6\lambda R\cos\theta}{2R} = 3\lambda\cos\theta.$$

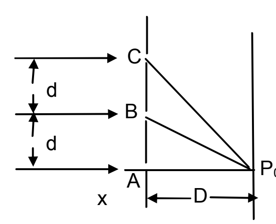
For constructive interference,

$(S_1P - S_2P)^2 = x = 3\lambda\cos\theta = n\lambda$

$\Rightarrow \cos\theta = n/3 \Rightarrow \theta = \cos^{-1}(n/3)$, where $n = 0, 1, 2, \ldots.$

$\Rightarrow \theta = 0°, 48.2°, 70.5°, 90°$ and similar points in other quadrants.

30. a) As shown in the figure, $BP_0 - AP_0 = \lambda/3$



$\Rightarrow \sqrt{(D^2 + d^2)} - D = \lambda/3$

$\Rightarrow D^2 + d^2 = D^2 + (\lambda^2/9) + (2\lambda D)/3$

$\Rightarrow d = \sqrt{(2\lambda D)/3}$ $\quad$ (neglecting the term $\lambda^2/9$ as it is very small)

b) To find the intensity at P_0, we have to consider the interference of light waves coming from all the three slits.

Here, $CP_0 - AP_0 = \sqrt{D^2 + 4d^2} - D$

$$= \sqrt{D^2 + \frac{8\lambda D}{3}} - D = D\left\{1 + \frac{8\lambda}{3D}\right\}^{1/2} - D$$

$$= D\left\{1 + \frac{8\lambda}{3D \times 2} + ...\right\} - D = \frac{4\lambda}{3} \quad \text{[using binomial expansion]}$$

So, the corresponding phase difference between waves from C and A is,

$$\phi_c = \frac{2\pi x}{\lambda} = \frac{2\pi \times 4\lambda}{3\lambda} = \frac{8\pi}{3} = \left(2\pi + \frac{2\pi}{3}\right) = \frac{2\pi}{3} \quad \text{...(1)}$$

Again, $\phi_B = \frac{2\pi x}{3\lambda} = \frac{2\pi}{3}$ ...(2)

So, it can be said that light from B and C are in same phase as they have some phase difference with respect to A.

So, R $= \sqrt{(2r)^2 + r^2 + 2 \times 2r \times r\cos(2\pi/3)}$ (using vector method)

$= \sqrt{4r^2 + r^2 - 2r^2} = \sqrt{3}r$

$\therefore\ I_{P_0} - K(\sqrt{3}r)^2 = 3Kr^2 = 3I$

As, the resulting amplitude is $\sqrt{3}$ times, the intensity will be three times the intensity due to individual slits.

31. Given that, d = 2 mm = 2×10^{-3} m, λ = 600 nm = 6×10^{-7} m, I_{max} = 0.20 W/m^2, D = 2m

For the point, y = 0.5 cm

We know, path difference = x = $\frac{yd}{D} = \frac{0.5 \times 10^{-2} \times 2 \times 10^{-3}}{2}$ = 5×10^{-6} m

So, the corresponding phase difference is,

$$\phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 5 \times 10^{-6}}{6 \times 10^{-7}} \Rightarrow \frac{50\pi}{3} = 16\pi + \frac{2\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$$

So, the amplitude of the resulting wave at the point y = 0.5 cm is,

$$A = \sqrt{r^2 + r^2 + 2r^2\cos(2\pi/3)} = \sqrt{r^2 + r^2 - r^2} = r$$

Since, $\frac{I}{I_{max}} = \frac{A^2}{(2r)^2}$ [since, maximum amplitude = 2r]

$\Rightarrow \frac{I}{0.2} = \frac{A^2}{4r^2} = \frac{r^2}{4r^2}$

$\Rightarrow I = \frac{0.2}{4} = 0.05\ \text{W/m}^2.$

32. i) When intensity is half the maximum $\frac{I}{I_{max}} = \frac{1}{2}$

$\Rightarrow \frac{4a^2\cos^2(\phi/2)}{4a^2} = \frac{1}{2}$

$\Rightarrow \cos^2(\phi/2) = 1/2 \Rightarrow \cos(\phi/2) = 1/\sqrt{2}$

$\Rightarrow$ $\phi/2 = \pi/4 \Rightarrow \phi = \pi/2$

$\Rightarrow$ Path difference, x = $\lambda/4$

$\Rightarrow$ y = xD/d = λD/4d

ii) When intensity is 1/4th of the maximum $\frac{I}{I_{max}} = \frac{1}{4}$

$\Rightarrow \frac{4a^2\cos^2(\phi/2)}{4a^2} = \frac{1}{4}$

$\Rightarrow \cos^2(\phi/2) = 1/4 \Rightarrow \cos(\phi/2) = 1/2$

$\Rightarrow$ $\phi/2 = \pi/3 \Rightarrow \phi = 2\pi/3$

$\Rightarrow$ Path difference, x = $\lambda/3$

$\Rightarrow$ y = xD/d = λD/3d

33. Given that, D = 1 m, d = 1 mm = 10^{-3} m, λ = 500 nm = 5×10^{-7} m

For intensity to be half the maximum intensity.

$$y = \frac{\lambda D}{4d} \qquad \text{(As in problem no. 32)}$$

$$\Rightarrow y = \frac{5\times10^{-7}\times1}{4\times10^{-3}} \Rightarrow y = 1.25 \times 10^{-4} \text{ m.}$$

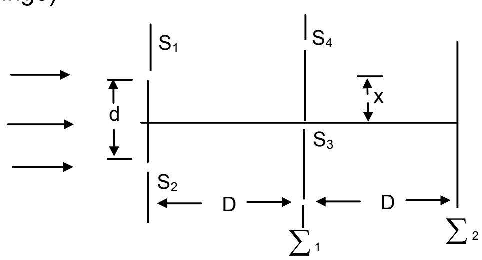
34. The line width of a bright fringe is sometimes defined as the separation between the points on the two sides of the central line where the intensity falls to half the maximum.

We know that, for intensity to be half the maximum

$$y = \pm\frac{\lambda D}{4d}$$

$$\therefore \text{ Line width } = \frac{\lambda D}{4d} + \frac{\lambda D}{4d} = \frac{\lambda D}{2d}.$$

35. i) When, z = $\lambda D/2d$, at S_4, minimum intensity occurs (dark fringe)



$\Rightarrow$ Amplitude = 0,

At S_3, path difference = 0

$\Rightarrow$ Maximum intensity occurs.

$\Rightarrow$ Amplitude = 2r.

So, on $\Sigma 2$ screen,

$$\frac{I_{max}}{I_{min}} = \frac{(2r+0)^2}{(2r-0)^2} = 1$$

ii) When, z = $\lambda D/2d$, At S_4, minimum intensity occurs. (dark fringe)

$\Rightarrow$ Amplitude = 0.

At S_3, path difference = 0

$\Rightarrow$ Maximum intensity occurs.

$\Rightarrow$ Amplitude = 2r.

So, on $\Sigma 2$ screen,

$$\frac{I_{max}}{I_{min}} = \frac{(2r+2r)^2}{(2r-0)^2} = \infty$$

iii) When, z = $\lambda D/4d$, At S_4, intensity = $I_{max}/2$

$\Rightarrow$ Amplitude = $\sqrt{2}r$.

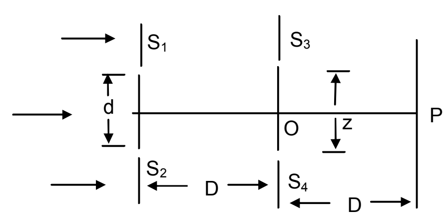
$\therefore$ At S_3, intensity is maximum.

$\Rightarrow$ Amplitude = 2r

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(2r+\sqrt{2}r)^2}{(2r-\sqrt{2}r)^2} = 34.$$

36. a) When, z = $D\lambda/d$

So, $OS_3 = OS_4 = D\lambda/2d \Rightarrow$ Dark fringe at S_3 and S_4.



$\Rightarrow$ At S_3, intensity at $S_3 = 0 \Rightarrow I_1 = 0$

At S_4, intensity at $S_4 = 0 \Rightarrow I_2 = 0$

At P, path difference = 0 $\Rightarrow$ Phase difference = 0.

$\Rightarrow I = I_1 + I_2 + \sqrt{I_1I_2}\cos 0° = 0 + 0 + 0 = 0 \Rightarrow$ Intensity at P = 0.

b) Given that, when z = $D\lambda/2d$, intensity at P = I

Here, $OS_3 = OS_4 = y = D\lambda/4d$

$$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda}\times\frac{yd}{D} = \frac{2\pi}{\lambda}\times\frac{D\lambda}{4d}\times\frac{d}{D} = \frac{\pi}{2}. \qquad \text{[Since, x = path difference = yd/D]}$$

Let, intensity at S_3 and S_4 = I′

$\therefore$ At P, phase difference = 0

So, I′ + I′ + 2I′ cos 0° = I.

$\Rightarrow$ 4I′ = I $\Rightarrow$ I′ = 1/4.

When, $z = \frac{3D\lambda}{2d}$, $\Rightarrow y = \frac{3D\lambda}{4d}$

$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{3D\lambda}{4d} \times \frac{d}{D} = \frac{3\pi}{2}$

Let, I″ be the intensity at S_3 and S_4 when, $\phi = 3\pi/2$

Now comparing,

$\frac{I''}{I} = \frac{a^2 + a^2 + 2a^2 \cos(3\pi/2)}{a^2 + a^2 + 2a^2 \cos \pi/2} = \frac{2a^2}{2a^2} = 1$ $\Rightarrow I'' = I' = I/4$.

$\therefore$ Intensity at P = I/4 + I/4 + 2 × (I/4) cos 0° = I/2 + I/2 = I.

c) When $z = 2D\lambda/d$

$\Rightarrow y = OS_3 = OS_4 = D\lambda/d$

$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{D\lambda}{d} \times \frac{d}{D} = 2\pi$.

Let, I‴ = intensity at S_3 and S_4 when, $\phi = 2\pi$.

$\frac{I'''}{I'} = \frac{a^2 + a^2 + 2a^2 \cos 2\pi}{a^2 + a^2 + 2a^2 \cos \pi/2} = \frac{4a^2}{2a^2} = 2$

$\Rightarrow$ I‴ = 2I′ = 2(I/4) = I/2

At P, $I_{resultant}$ = I/2 + I/2 + 2(I/2) cos 0° = I + I = 2I.

So, the resultant intensity at P will be 2I.

37. Given $d = 0.0011 \times 10^{-3}$ m

For minimum reflection of light, $2\mu d = n\lambda$

$\Rightarrow \mu = \frac{n\lambda}{2d} = \frac{2n\lambda}{4d} = \frac{580 \times 10^{-9} \times 2n}{4 \times 11 \times 10^{-7}} = \frac{5.8}{44}(2n) = 0.132\ (2n)$

Given that, μ has a value in between 1.2 and 1.5.

$\Rightarrow$ When, n = 5, $\mu = 0.132 \times 10 = 1.32$.

38. Given that, $\lambda = 560 \times 10^{-9}$ m, $\mu = 1.4$.

For strong reflection, $2\mu d = (2n + 1)\lambda/2 \Rightarrow d = \frac{(2n+1)\lambda}{4d}$

For minimum thickness, putting n = 0.

$\Rightarrow d = \frac{\lambda}{4d} \Rightarrow d = \frac{560 \times 10^{-9}}{14} = 10^{-7}$ m = 100 nm.

39. For strong transmission, $2\mu d = n\lambda \Rightarrow \lambda = \frac{2\mu d}{n}$

Given that, $\mu = 1.33$, $d = 1 \times 10^{-4}$ cm = 1×10^{-6} m.

$\Rightarrow \lambda = \frac{2 \times 1.33 \times 1 \times 10^{-6}}{n} = \frac{2660 \times 10^{-9}}{n}$ m

when, n = 4, λ_1 = 665 nm
n = 5, λ_2 = 532 nm
n = 6, λ_3 = 443 nm

40. For the thin oil film,

$d = 1 \times 10^{-4}$ cm = 10^{-6} m, $\mu_{oil} = 1.25$ and $\mu_x = 1.50$

$\lambda = \frac{2\mu d}{(n + 1/2)} \frac{2 \times 10^{-6} \times 1.25 \times 2}{2n+1} = \frac{5 \times 10^{-6} m}{2n+1}$

$\Rightarrow \lambda = \frac{5000 \text{ nm}}{2n+1}$

For the wavelengths in the region (400 nm – 750 nm)

When, n = 3, $\lambda = \frac{5000}{2 \times 3 + 1} = \frac{5000}{7}$ = 714.3 nm

When, n = 4, $\lambda = \frac{5000}{2 \times 4 + 1} = \frac{5000}{9}$ = 555.6 nm

When, n = 5, $\lambda = \frac{5000}{2 \times 5 + 1} = \frac{5000}{11}$ = 454.5 nm

41. For first minimum diffraction, b sin $\theta = \lambda$
Here, θ = 30°, b = 5 cm
$\therefore \lambda = 5 \times \sin 30° = 5/2 = 2.5$ cm.

42. λ = 560 nm = 560×10^{-9} m, b = 0.20 mm = 2×10^{-4} m, D = 2 m

Since, $R = 1.22\frac{\lambda D}{b} = 1.22 \times \frac{560 \times 10^{-9} \times 2}{2 \times 10^{-4}}$ = 6.832×10^{-3} M = 0.683 cm.

So, Diameter = 2R = 1.37 cm.

43. λ = 620 nm = 620×10^{-9} m,
D = 20 cm = 20×10^{-2} m, b = 8 cm = 8×10^{-2} m

$\therefore R = 1.22 \times \frac{620 \times 10^{-4} \times 20 \times 10^{-2}}{8 \times 10^{-2}}$ = $1891 \times 10^{-9} = 1.9 \times 10^{-6}$ m

So, diameter = 2R = 3.8×10^{-6} m

♠ ♠ ♠

SOLUTIONS TO CONCEPTS
CHAPTER – 18

SIGN CONVENTION :

1) The direction of incident ray (from object to the mirror or lens) is taken as positive direction.
2) All measurements are taken from pole (mirror) or optical centre (lens) as the case may be.

1. u = –30 cm, R = – 40 cm

From the mirror equation,

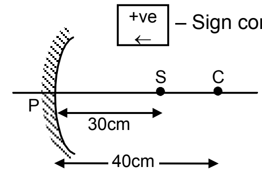
$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

$$\Rightarrow \frac{1}{v}=\frac{2}{R}-\frac{1}{u}=\frac{2}{-40}-\frac{1}{-30} = -\frac{1}{60}$$

or, v = –60 cm

So, the image will be formed at a distance of 60 cm in front of the mirror.

+ve ← – Sign convertion



2. Given that,

H_1 = 20 cm, v = –5 m = –500 cm, h_2 = 50 cm

Since, $\frac{-v}{u}=\frac{h_2}{h_1}$

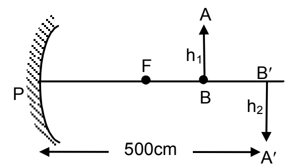
or $\frac{500}{u}=-\frac{50}{20}$ (because the image in inverted)

or u = $-\frac{500\times 2}{5}$ = –200 cm = – 2 m

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ or } \frac{1}{-5}+\frac{1}{-2}=\frac{1}{f}$$

or f = $\frac{-10}{7}$ = –1.44 m

So, the focal length is 1.44 m.



+ve ← – Sign convertion

3. For the concave mirror, f = –20 cm, M = –v/u = 2

$\Rightarrow$ v = –2u

<u>1st case</u>

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{2u}-\frac{1}{u}=-\frac{1}{f}$$

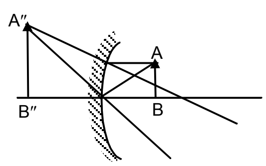
$\Rightarrow$ u = f/2 = 10 cm

<u>2nd case</u>

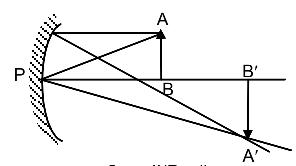
$$\frac{-1}{2u}-\frac{1}{u}=-\frac{1}{f}$$

$$\Rightarrow \frac{3}{2u}=\frac{1}{f}$$

$\Rightarrow$ u = 3f/2 = 30 cm



Case I (Virtual)



Case II(Real)

$\therefore$ The positions are 10 cm or 30 cm from the concave mirror.

4. m = –v/u = 0.6 and f = 7.5 cm = 15/2 cm

From mirror equation,

$$\Rightarrow \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{0.6u}-\frac{1}{u}=\frac{1}{f}$$

$\Rightarrow$ u = 5 cm

5. Height of the object AB = 1.6 cm

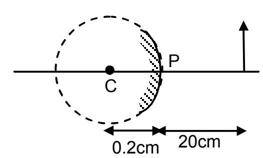
Diameter of the ball bearing = d = 0.4 cm

$\Rightarrow$ R = 0.2 cm

Given, u = 20 cm

We know, $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$

+ve ← – Sign convertion



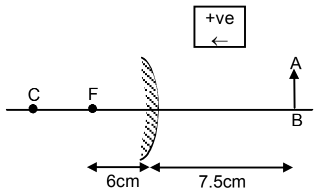
Putting the values according to sign conventions $\frac{1}{-20}+\frac{1}{v}=\frac{2}{0.2}$

$\Rightarrow \frac{1}{v}=\frac{1}{20}+10=\frac{201}{20}$ $\Rightarrow$ v = 0.1 cm = 1 mm inside the ball bearing.

Magnification = m = $\frac{A'B'}{AB}=-\frac{v}{u}=-\frac{0.1}{-20}=\frac{1}{200}$

$\Rightarrow$ A′B′ = $\frac{AB}{200}=\frac{16}{200}$ = +0.008 cm = +0.8 mm.

6. Given AB = 3 cm, u = –7.5 cm, f = 6 cm.



Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

Putting values according to sign conventions,

$\frac{1}{v}=\frac{1}{6}-\frac{1}{-7.5}=\frac{3}{10}$

$\Rightarrow$ v = 10/3 cm

$\therefore$ magnification = m = $-\frac{v}{u}=\frac{10}{7.5\times 3}$

$\Rightarrow \frac{A'B'}{AB}=\frac{10}{7.5\times 3}\Rightarrow A'B'=\frac{100}{72}=\frac{4}{3}=1.33$ cm.

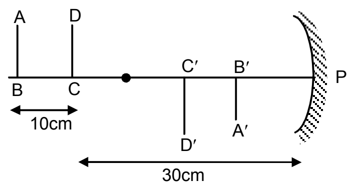
$\therefore$ Image will form at a distance of 10/3 cm. From the pole and image is 1.33 cm (virtual and erect).

7. R = 20 cm, f = R/2 = –10 cm

For part AB, PB = 30 + 10 = 40 cm

So, u = –40 cm $\Rightarrow \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(\frac{1}{-40}\right)=-\frac{3}{40}$

$\Rightarrow$ v = $-\frac{40}{3}$ = –13.3 cm.



So, PB′ = 13.3 cm

m = $\frac{A'B'}{AB}=-\left(\frac{v}{u}\right)=-\left(\frac{-13.3}{-40}\right)=-\frac{1}{3}$

$\Rightarrow$ A′B′ = –10/3 = –3.33 cm

For part CD, PC = 30, So, u = –30 cm

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(-\frac{1}{30}\right)=-\frac{1}{15}$ $\Rightarrow$ v = –15 cm = PC′

So, m = $\frac{C'D'}{CD}=-\frac{v}{u}=-\left(\frac{-15}{-30}\right)=-\frac{1}{2}$

$\Rightarrow$ C′D′ = 5 cm

B′C′ = PC′ – PB′ = 15 – 13.3 = 17 cm

So, total length A′B′ + B′C′ + C′D′ = 3.3 + 1.7 + 5 = 10 cm.

8. u = –25 cm

m = $\frac{A'B'}{AB}=-\frac{v}{u}\Rightarrow 1.4=-\left(\frac{v}{-25}\right)\Rightarrow\frac{14}{10}=\frac{v}{25}$

$\Rightarrow$ v = $\frac{25\times 14}{10}$ = 35 cm.

Now, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

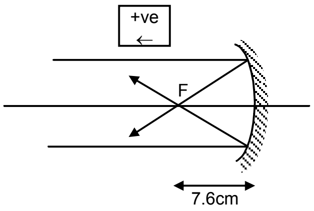
$\Rightarrow \frac{1}{f}=\frac{1}{35}-\left(\frac{1}{-25}\right)=\frac{5-7}{175}=-\frac{2}{175}$ $\Rightarrow$ f = –87.5 cm.

So, focal length of the concave mirror is 87.5 cm.

9. $u = -3.8 \times 10^5$ km

diameter of moon = 3450 km ; f = –7.6 m

$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \left(-\frac{1}{3.8 \times 10^5}\right) = \left(-\frac{1}{7.6}\right)$



Since, distance of moon from earth is very large as compared to focal length it can be taken as ∞.

⇒ Image will be formed at focus, which is inverted.

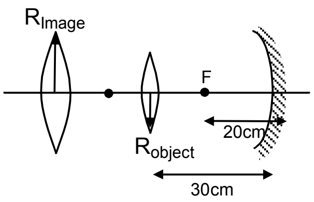
$\Rightarrow \frac{1}{v} = -\left(\frac{1}{7.6}\right) \Rightarrow v = -7.6$ m.

$m = -\frac{v}{u} = \frac{d_{image}}{d_{object}} \Rightarrow \frac{-(-7.6)}{(-3.8 \times 10^8)} = \frac{d_{image}}{3450 \times 10^3}$

$d_{image} = \frac{3450 \times 7.6 \times 10^3}{3.8 \times 10^8}$ = 0.069 m = 6.9 cm.

10. u = –30 cm, f = –20 cm

We know, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$



$\Rightarrow \frac{1}{v} + \left(-\frac{1}{30}\right) = \left(-\frac{1}{20}\right) \Rightarrow v = -60$ cm.

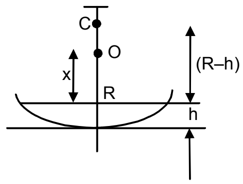
Image of the circle is formed at a distance 60 cm in front of the mirror.

$\therefore m = -\frac{v}{u} = \frac{R_{image}}{R_{object}} \Rightarrow -\frac{-60}{-30} = \frac{R_{image}}{2}$

$\Rightarrow R_{image} = 4$ cm

Radius of image of the circle is 4 cm.

11. Let the object be placed at a height x above the surface of the water.



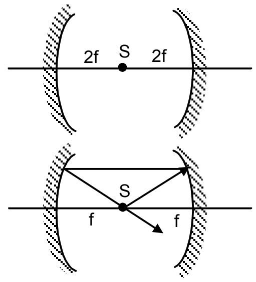
The apparent position of the object with respect to mirror should be at the centre of curvature so that the image is formed at the same position.

Since, $\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$ (with respect to mirror)

Now, $\frac{x}{R-h} = \frac{1}{\mu} \Rightarrow x = \frac{R-h}{\mu}$.

12. Both the mirrors have equal focal length f.

They will produce one image under two conditions.



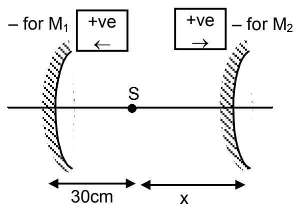
Case I : When the source is at distance '2f' from each mirror i.e. the source is at centre of curvature of the mirrors, the image will be produced at the same point S. So, d = 2f + 2f = 4f.

Case II : When the source S is at distance 'f' from each mirror, the rays from the source after reflecting from one mirror will become parallel and so these parallel rays after the reflection from the other mirror the object itself. So, only sine image is formed.

Here, d = f + f = 2f.

13. As shown in figure, for 1st reflection in M_1, u = –30 cm, f = –20 cm

$\Rightarrow \frac{1}{v} + \frac{1}{-30} = -\frac{1}{20} \Rightarrow v = -60$ cm.



So, for 2nd reflection in M_2

u = 60 – (30 + x) = 30 – x

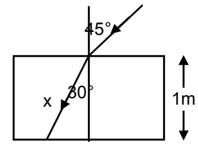
v = –x ; f = 20 cm

$\Rightarrow \frac{1}{30-x} - \frac{1}{x} = \frac{1}{20} \Rightarrow x^2 + 10x - 600 = 0$

$\Rightarrow$ x = $\dfrac{10 \pm 50}{2} = \dfrac{40}{2}$ = 20 cm or –30 cm

$\therefore$ Total distance between the two lines is 20 + 30 = 50 cm.

14. We know, $\dfrac{\sin i}{\sin r} = \dfrac{3 \times 10^8}{v} = \dfrac{\sin 45°}{\sin 30°} = \sqrt{2}$



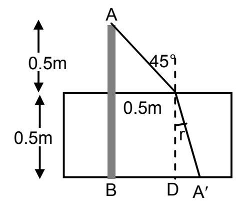
$\Rightarrow$ v = $\dfrac{3 \times 10^8}{\sqrt{2}}$ m/sec.

Distance travelled by light in the slab is,

x = $\dfrac{1\,m}{\cos 30°} = \dfrac{2}{\sqrt{3}}$ m

So, time taken = $\dfrac{2 \times \sqrt{2}}{\sqrt{3} \times 3 \times 10^8}$ = 0.54 × 10^{-8} = 5.4 × 10^{-9} sec.

15. Shadow length = BA′ = BD + A′D = 0.5 + 0.5 tan r



Now, 1.33 = $\dfrac{\sin 45°}{\sin r}$ $\Rightarrow$ sin r = 0.53.

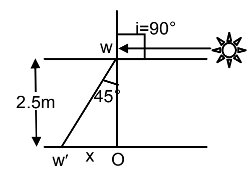
$\Rightarrow$ cos r = $\sqrt{1 - \sin^2 r} = \sqrt{1 - (0.53)^2}$ = 0.85

So, tan r = 0.6235

So, shadow length = (0.5) (1 + 0.6235) = 81.2 cm.

16. Height of the lake = 2.5 m

When the sun is just setting, θ is approximately = 90°



$\therefore \dfrac{\sin i}{\sin r} = \dfrac{\mu_2}{\mu_1} \Rightarrow \dfrac{1}{\sin r} = \dfrac{4/3}{1} \Rightarrow \sin r = \dfrac{3}{4} \Rightarrow r = 49°$

As shown in the figure, x/2.5 = tan r = 1.15

$\Rightarrow$ x = 2.5 × 1.15 = 2.8 m.

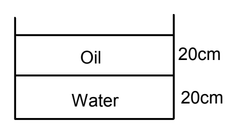
17. The thickness of the glass is d = 2.1 cm and μ =1.5

Shift due to the glass slab

$\Delta T = \left(1 - \dfrac{1}{\mu}\right)d = \left(1 - \dfrac{1}{1.5}\right)2.1$ = 0.7 CM

So, the microscope should be shifted 0.70 cm to focus the object again.

18. Shift due to water $\Delta t_w = \left(1 - \dfrac{1}{\mu}\right)d = \left(1 - \dfrac{1}{1.33}\right)20$ = 5 cm



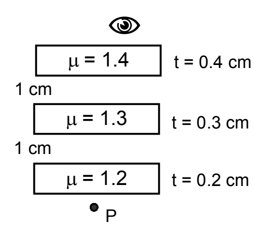
Shift due to oil, $\Delta t_o = \left(1 - \dfrac{1}{1.3}\right)20$ = 4.6 cm

Total shift Δt = 5 + 4.6 = 9.6 cm

Apparent depth = 40 – (9.6) = 30.4 cm below the surface.

19. The presence of air medium in between the sheets does not affect the shift.

The shift will be due to 3 sheets of different refractive index other than air.



$= \left(1 - \dfrac{1}{1.2}\right)(0.2) + \left(1 - \dfrac{1}{13}\right)(0.3) + \left(1 - \dfrac{1}{14}\right)(0.4)$

= 0.2 cm above point P.

20. Total no. of slabs = k, thickness = $t_1, t_2, t_3 \ldots t_k$

Refractive index = $\mu_1, \mu_2, \mu_3, \mu_4, \ldots \mu_k$

$\therefore$ The shift $\Delta t = \left(1 - \dfrac{1}{\mu_1}\right)t_1 + \left(1 - \dfrac{1}{\mu_2}\right)t_2 + \ldots\ldots + \left(1 - \dfrac{1}{\mu_k}\right)t_k$ …(1)

If, $\mu \rightarrow$ refractive index of combination of slabs and image is formed at same place,

$\Delta t = \left(1 - \dfrac{1}{\mu}\right)(t_1 + t_2 + \ldots + t_k)$ …(2)

Equation (1) and (2), we get,

$$\left(1-\frac{1}{\mu}\right)(t_1+t_2+\ldots+t_k) = \left(1-\frac{1}{\mu_1}\right)t_1+\left(1-\frac{1}{\mu_2}\right)t_2+\ldots\ldots+\left(1-\frac{1}{\mu_k}\right)t_k$$

$$= (t_1+t_2+\ldots+t_k)-\left(\frac{t_1}{\mu_1}+\frac{t_2}{\mu_2}+\ldots+\frac{t_k}{\mu_k}\right)$$

$$= -\frac{1}{\mu}\sum_{i=1}^{k} t_1 = -\sum_{i=1}^{k}\left(\frac{t_1}{\mu_1}\right) \Rightarrow \mu = \frac{\sum_{i=1}^{k} t_i}{\sum_{i=1}^{k}(t_1/\mu_1)}.$$

21. Given r = 6 cm, r_1 = 4 cm, h_1 = 8 cm

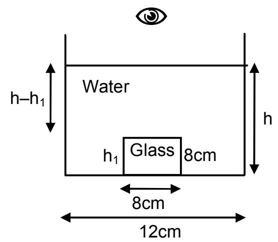
Let, h = final height of water column.

The volume of the cylindrical water column after the glass piece is put will be,

$\pi r^2 h = 800\pi + \pi r_1^2 h_1$

or $r^2 h = 800 + r_1^2 h_1$

or $6^2 h = 800 + 4^2 \times 8 = 25.7$ cm



There are two shifts due to glass block as well as water.

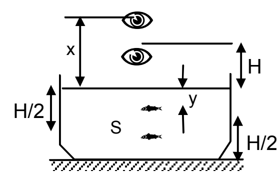
So, $\Delta t_1 = \left(1-\frac{1}{\mu_0}\right)t_0 = \left(1-\frac{1}{3/2}\right)8 = 2.26$ cm

And, $\Delta t_2 = \left(1-\frac{1}{\mu_w}\right)t_w = \left(1-\frac{1}{4/3}\right)(25.7-8) = 4.44$ cm.

Total shift = (2.66 + 4.44) cm = 7.1 cm above the bottom.

22. a) Let x = distance of the image of the eye formed above the surface as seen by the fish

So, $\frac{H}{x} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$ or x = μH



So, distance of the direct image = $\frac{H}{2}+\mu H = H(\mu+\frac{1}{2})$

Similarly, image through mirror = $\frac{H}{2}+(H+x) = \frac{3H}{2}+\mu H = H(\mu+\frac{3}{2})$

b) Here, $\frac{H/2}{y} = \mu$, so, $y = \frac{H}{2\mu}$

Where, y = distance of the image of fish below the surface as seen by eye.

So, Direct image = H + y = $H+\frac{H}{2\mu} = H\left(1+\frac{1}{2\mu}\right)$

Again another image of fish will be formed H/2 below the mirror.

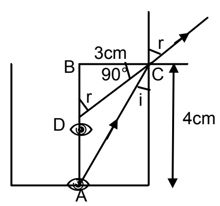
So, the real depth for that image of fish becomes H + H/2 = 3H/2

So, Apparent depth from the surface of water = $3H/2\mu$

So, distance of the image from the eye = $H+\frac{3H}{2\mu} = H(1+\frac{3}{2\mu})$.

23. According to the figure, x/3 = cot r …(1)

Again, $\frac{\sin i}{\sin r} = \frac{1}{1.33} = \frac{3}{4}$



$\Rightarrow \sin r = \frac{4}{3}\sin i = \frac{4}{3}\times\frac{3}{5} = \frac{4}{5}$ (because $\sin i = \frac{BC}{AC} = \frac{3}{5}$)

$\Rightarrow$ cot r = 3/4 …(2)

From (1) and (2) $\Rightarrow$ x/3 = ¾

$\Rightarrow$ x = 9/4 = 2.25 cm.

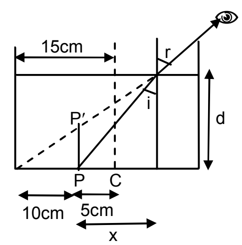
$\therefore$ Ratio of real and apparent depth = 4 : (2.25) = 1.78.

24. For the given cylindrical vessel, dimetre = 30 cm

$\Rightarrow$ r = 15 cm and h = 30 cm

Now, $\frac{\sin i}{\sin r} = \frac{3}{4}\left[\mu_w = 1.33 = \frac{4}{3}\right]$

$\Rightarrow$ sin i = $3/4\sqrt{2}$ [because r = 45°]



The point P will be visible when the refracted ray makes angle 45° at point of refraction.

Let x = distance of point P from X.

Now, tan 45° = $\frac{x+10}{d}$

$\Rightarrow$ d = x + 10 ...(1)

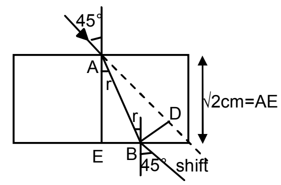
Again, tan i = x/d

$\Rightarrow \frac{3}{\sqrt{23}} = \frac{d-10}{d}$ $\left[\text{since, } \sin i = \frac{3}{4\sqrt{2}} \Rightarrow \tan i = \frac{3}{\sqrt{23}}\right]$

$\Rightarrow \frac{3}{\sqrt{23}} - 1 = -\frac{10}{d} \Rightarrow d = \frac{\sqrt{23} \times 10}{\sqrt{23} - 3}$ = 26.7 cm.

25. As shown in the figure,

$\frac{\sin 45°}{\sin r} = \frac{2}{1} \Rightarrow \sin r = \frac{\sin 45°}{2} = \frac{1}{2\sqrt{2}} \Rightarrow r = 21°$



Therefore, θ = (45° – 21°) = 24°

Here, BD = shift in path = AB sin 24°

$= 0.406 \times AB = \frac{AE}{\cos 21°} \times 0.406$ = 0.62 cm.

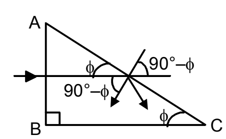
26. For calculation of critical angle,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\sin C}{\sin 90} = \frac{15}{1.72} = \frac{75}{86}$

$\Rightarrow$ C = $\sin^{-1}\left(\frac{75}{26}\right)$.

27. Let θ_c be the critical angle for the glass

$\frac{\sin\theta_c}{\sin 90°} = \frac{1}{x} \Rightarrow \sin\theta_c = \frac{1}{1.5} = \frac{2}{3} \Rightarrow \theta_c = \sin^{-1}\left(\frac{2}{3}\right)$



From figure, for total internal reflection, 90° – ϕ > θ_c

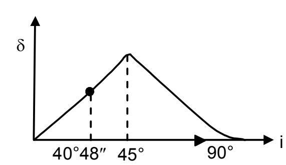
$\Rightarrow \phi < 90° - \theta_c \Rightarrow \phi < \cos^{-1}(2/3)$

So, the largest angle for which light is totally reflected at the surface is $\cos^{-1}(2/3)$.

28. From the definition of critical angle, if refracted angle is more than 90°, then reflection occurs, which is known as total internal reflection.

So, maximum angle of refraction is 90°.

29. Refractive index of glass μ_g = 1.5



Given, 0° < i < 90°

Let, C $\rightarrow$ Critical angle.

$\frac{\sin C}{\sin r} = \frac{\mu_a}{\mu_g} \Rightarrow \frac{\sin C}{\sin 90°} = \frac{1}{15}$ = 0.66

$\Rightarrow$ C = 40°48″

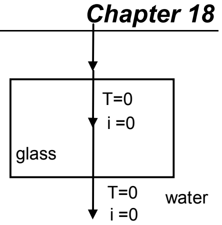
The angle of deviation due to refraction from glass to air increases as the angle of incidence increases from 0° to 40°48″. The angle of deviation due to total internal reflection further increases for 40°48″ to 45° and then it decreases.

30. μ_g = 1.5 = 3/2 ; μ_w = 1.33 = 4/3

For two angles of incidence,

1) When light passes straight through normal,

$\Rightarrow$ Angle of incidence = 0°, angle of refraction = 0°, angle of deviation = 0

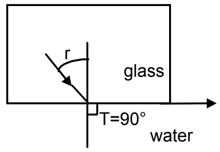


2) When light is incident at critical angle,

$$\frac{\sin C}{\sin r} = \frac{\mu_w}{\mu_g} \quad \text{(since light passing from glass to water)}$$

$\Rightarrow$ sin C = 8/9 $\Rightarrow$ C = $\sin^{-1}(8/9)$ = 62.73°.

$\therefore$ Angle of deviation = 90° – C = 90° – $\sin^{-1}(8/9)$ = $\cos^{-1}(8/9)$ = 37.27°



Here, if the angle of incidence is increased beyond critical angle, total internal reflection occurs and deviation decreases. So, the range of deviation is 0 to $\cos^{-1}(8/9)$.

31. Since, μ = 1.5, Critial angle = $\sin^{-1}(1/\mu)$ = $\sin^{-1}$ (1/1.5) = 41.8°

We know, the maximum attainable deviation in refraction is (90° – 41.8°) = 47.2°

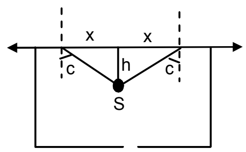
So, in this case, total internal reflection must have taken place.

In reflection,

Deviation = 180° – 2i = 90° $\Rightarrow$ 2i = 90° $\Rightarrow$ i = 45°.

32. a) Let, x = radius of the circular area

$$\frac{x}{h} = \tan C \quad \text{(where C is the critical angle)}$$



$$\Rightarrow \frac{x}{h} = \frac{\sin C}{\sqrt{1-\sin^2 C}} = \frac{1/\mu}{\sqrt{1-\frac{1}{\mu^2}}} \quad \text{(because sin C = } 1/\mu)$$

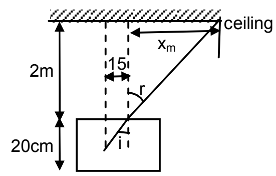
$$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{\mu^2 - 1}} \text{ or } x = \frac{h}{\sqrt{\mu^2-1}}$$

So, light escapes through a circular area on the water surface directly above the point source.

b) Angle subtained by a radius of the area on the source, C = $\sin^{-1}(1/\mu)$.

33. a) As shown in the figure, sin i = 15/25

So, $\frac{\sin i}{\sin r} = \frac{1}{\mu} = \frac{3}{4}$



$\Rightarrow$ sin r = 4/5

Again, x/2 = tan r (from figure)

So, sin r = $\frac{\tan r}{\sqrt{1+\tan^2 r}} = \frac{x/2}{\sqrt{1-x^2/4}}$

$$\Rightarrow \frac{x}{\sqrt{4+x^2}} = \frac{4}{5}$$

$\Rightarrow$ $25x^2 = 16(4 + x^2)$ $\Rightarrow$ $9x^2 = 64$ $\Rightarrow$ x = 8/3 m

$\therefore$ Total radius of shadow = 8/3 + 0.15 = 2.81 m

b) For maximum size of the ring, i = critical angle = C

Let, R = maximum radius

$\Rightarrow$ sin C = $\frac{\sin C}{\sin r} = \frac{R}{\sqrt{20^2+R^2}} = \frac{3}{4}$ (since, sin r = 1)

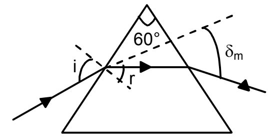
$\Rightarrow$ $16R^2 = 9R^2 + 9 \times 400$

$\Rightarrow$ $7R^2 = 9 \times 400$

$\Rightarrow$ R = 22.67 cm.

34. Given, A = 60°, μ = 1.732

Since, angle of minimum deviation is given by,



$$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin A/2} \Rightarrow 1.732 \times ½ = \sin(30 + \delta m/2)$$

$\Rightarrow \sin^{-1}(0.866) = 30 + \delta m/2 \Rightarrow 60° = 30\ \delta m/2 \Rightarrow \delta m = 60°$

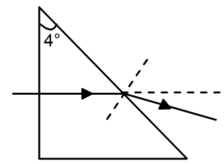
Now, $\delta m = i + i' - A$

$\Rightarrow$ 60° = i + i′ – 60° (δ = 60° minimum deviation)

$\Rightarrow$ i = 60°. So, the angle of incidence must be 60°.

35. Given μ = 1.5

And angle of prism = 4°



$$\therefore \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{(A+\delta_m)/2}{(A/2)} \quad \text{(for small angle } \sin\theta = \theta)$$

$$\Rightarrow \mu = \frac{A+\delta_m}{2} \Rightarrow 1.5 = \frac{4°+\delta_m}{4°} \Rightarrow \delta_m = 4° \times (1.5) - 4° = 2°.$$

36. Given A = 60° and δ = 30°

We know that,

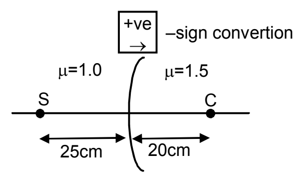
$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{\sin\frac{60°+\delta_m}{2}}{\sin 30°} = 2\sin\frac{60°+\delta_m}{2}$$

Since, one ray has been found out which has deviated by 30°, the angle of minimum deviation should be either equal or less than 30°. (It can not be more than 30°).

So, $\mu \le 2\sin\frac{60°+\delta_m}{2}$ (because μ will be more if δ_m will be more)

or, $\mu \le 2 \times 1/\sqrt{2}$ or, $\mu \le \sqrt{2}$.

37. $\mu_1 = 1$, $\mu_2 = 1.5$, R = 20 cm (Radius of curvature), u = –25 cm



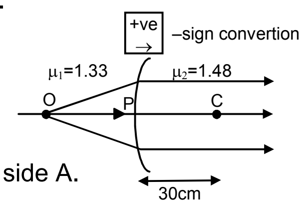
$$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{25} = \frac{1}{40} - \frac{1}{25} = \frac{-3}{200}$$

$\Rightarrow v = -200 \times 0.5 = -100$ cm.

So, the image is 100 cm from (P) the surface on the side of S.

38. Since, paraxial rays become parallel after refraction i.e. image is formed at ∞.

$v = \infty$, $\mu_1 = 1.33$, u = ?, $\mu_2 = 1.48$, R = 30 cm



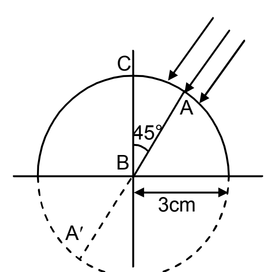
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1.48}{\infty} - \frac{1.33}{u} = \frac{1.48-1.33}{30} \Rightarrow -\frac{1.33}{u} = \frac{0.15}{30}$$

$\Rightarrow$ u = –266.0 cm

$\therefore$ Object should be placed at a distance of 266 cm from surface (convex) on side A.

39. Given, $\mu_2 = 2.0$

So, critical angle = $\sin^{-1}\left(\frac{1}{\mu_2}\right) = \sin^{-1}\left(\frac{1}{2}\right)$ = 30°



a) As angle of incidence is greater than the critical angle, the rays are totally reflected internally.

b) Here, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$

$$\Rightarrow \frac{2}{v} - \left(-\frac{1}{\infty}\right) = \frac{2-1}{3} \quad \text{[For parallel rays, } u = \infty]$$

$$\Rightarrow \frac{2}{v} = \frac{1}{3} \Rightarrow v = 6 \text{ cm}$$

$\Rightarrow$ If the sphere is completed, image is formed diametrically opposite of A.

c) Image is formed at the mirror in front of A by internal reflection.

40. a) Image seen from left :

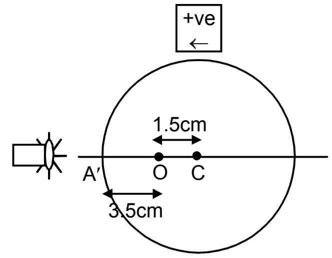
$u = (5 - 15) = -3.5$ cm

$R = -5$ cm

$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} + \frac{1.5}{3.5} = -\frac{1-1.5}{5}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{3}{7} \Rightarrow v = \frac{-70}{23} = -3$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2 cm left to centre.



b) Image seen from right :

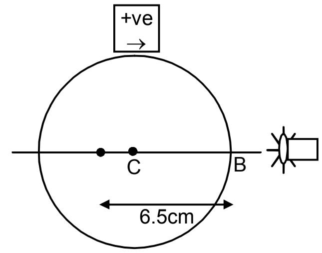
$u = -(5 + 1.5) = -6.5$ cm

$R = -5$ cm

$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} + \frac{1.5}{6.5} = \frac{1-1.5}{-5}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{3}{13} \Rightarrow v = -\frac{130}{17} = -7.65$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2.65 cm left to centre.



41. $R_1 = R_2 = 10$ cm, $t = 5$ cm, $u = -\infty$

For the first refraction, (at A)

$\frac{\mu_g}{v} - \frac{\mu_a}{u} = \frac{\mu_g - \mu_a}{R_1}$ or $\frac{1.5}{v} - 0 = \frac{1.5}{10}$

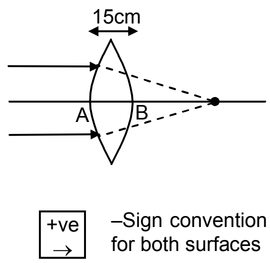
$\Rightarrow v = 30$ cm.

Again, for 2nd surface, $u = (30 - 5) = 25$ cm (virtual object)

$R_2 = -10$ cm

So, $\frac{1}{v} - \frac{15}{25} = \frac{-0.5}{-10} \Rightarrow v = 9.1$ cm.

So, the image is formed 9.1 cm further from the 2nd surface of the lens.



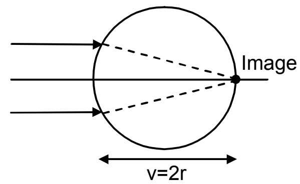
42. For the refraction at convex surface A.

$\mu = -\infty$, $\mu_1 = 1$, $\mu_2 = ?$

a) When focused on the surface, $v = 2r$, $R = r$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{\mu_2}{2r} = \frac{\mu_2 - 1}{r} \Rightarrow \mu_2 = 2\mu_2 - 2 \Rightarrow \mu_2 = 2$



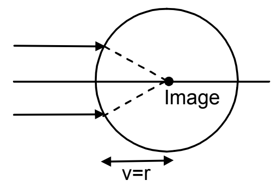
b) When focused at centre, $u = r_1$, $R = r$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{\mu_2}{R} = \frac{\mu_2 - 1}{r} \Rightarrow \mu_2 = \mu_2 - 1.$

This is not possible.

So, it cannot focus at the centre.



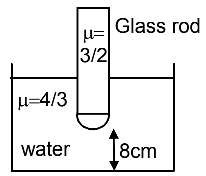
43. Radius of the cylindrical glass tube = 1 cm

We know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Here, $u = -8$ cm, $\mu_2 = 3/2$, $\mu_1 = 4/3$, $R = +1$ cm

So, $\frac{3}{2v} + \frac{4}{3 \times 8} \Rightarrow \frac{3}{2v} + \frac{1}{6} = \frac{1}{6}$ $\quad v = \infty$

$\therefore$ The image will be formed at infinity.



44. In the first refraction at A.

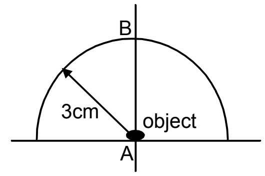
$\mu_2 = 3/2$, $\mu_1 = 1$, u = 0, R = ∞

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow$ v = 0 since (R $\Rightarrow \infty$ and u = 0)

$\therefore$ The image will be formed at the point, Now for the second refraction at B,

u = –3 cm, R = –3 cm, $\mu_1 = 3/2$, $\mu_2 = 1$



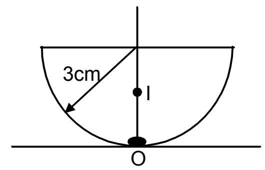
So, $\frac{1}{v} + \frac{3}{2 \times 3} = \frac{1 - 1.5}{-3} = \frac{1}{6}$

$\Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$

$\Rightarrow$ v = –3 cm, $\therefore$ There will be no shift in the final image.

45. Thickness of glass = 3 cm, $\mu_g = 1.5$

Image shift = $3\left(1 - \frac{1}{1.5}\right)$



[Treating it as a simple refraction problem because the upper surface is flat and the spherical surface is in contact with the object]

= $3 \times \frac{0.5}{1.5}$ = 1 cm.

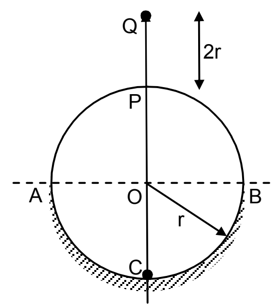
The image will appear 1 cm above the point P.

46. As shown in the figure, OQ = 3r, OP = r

So, PQ = 2r

For refraction at APB

We know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$\Rightarrow \frac{1.5}{v} - \frac{1}{-2r} = \frac{0.5}{r} = \frac{1}{2r}$ [because u = –2r]

$\Rightarrow$ v = ∞

For the reflection in concave mirror

u = ∞

So, v = focal length of mirror = r/2

For the refraction of APB of the reflected image.

Here, u = –3r/2

$\frac{1}{v} - \frac{1.5}{-3r/2} = \frac{-0.5}{-r}$ [Here, $\mu_1 = 1.5$ and $\mu_2 = 1$ and R = –r]

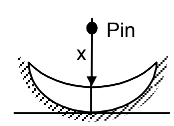
$\Rightarrow$ v = –2r

As, negative sign indicates images are formed inside APB. So, image should be at C.

So, the final image is formed on the reflecting surface of the sphere.

47. a) Let the pin is at a distance of x from the lens.

Then for 1st refraction, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



Here $\mu_2 = 1.5$, $\mu_1 = 1$, u = –x, R = –60 cm

$\therefore \frac{1.5}{v} - \frac{1}{-x} = \frac{0.5}{-60}$

$\Rightarrow$ 120(1.5x + v) = –vx …(1)

$\Rightarrow$ v(120 + x) = –180x

$\Rightarrow$ v = $\frac{-180x}{120 + x}$

This image distance is again object distance for the concave mirror.

$u = \frac{-180x}{120+x}$, f = –10 cm ($\therefore$ f = R/2)

$\therefore \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v_1}=\frac{1}{-10}-\frac{-(120+x)}{180x}$

$\Rightarrow \frac{1}{v_1}=\frac{120+x-18x}{180x} \Rightarrow v_1 = \frac{180x}{120-17x}$

Again the image formed is refracted through the lens so that the image is formed on the object taken in the 1st refraction. So, for 2nd refraction.

According to sign conversion v = –x, μ_2 = 1, μ_1 = 1.5, R = –60

Now, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ $\quad [u = \frac{180x}{120-17x}]$

$\Rightarrow \frac{1}{-x}-\frac{1.5}{180x}(120-17x)=\frac{-0.5}{-60}$

$\Rightarrow \frac{1}{x}+\frac{120-17x}{120x}=\frac{-1}{120}$

Multiplying both sides with 120 m, we get

120 + 120 – 17x = –x

$\Rightarrow$ 16x = 240 $\Rightarrow$ x = 15 cm

$\therefore$ Object should be placed at 15 cm from the lens on the axis.

48. For the double convex lens

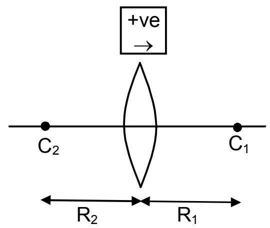
f = 25 cm, R_1 = R and R_2 = –2R (sign convention)

$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{25}=(15-1)\left(\frac{1}{R}-\frac{1}{-2R}\right) = 0.5\left(\frac{3R}{2}\right)$

$\Rightarrow \frac{1}{25}=\frac{3}{4}\frac{1}{R} \Rightarrow$ R = 18.75 cm

R_1 = 18.75 cm, R_2 = 2R = 37.5 cm.



49. R_1 = +20 cm ; R_2 = +30 cm ; μ = 1.6

a) If placed in air :

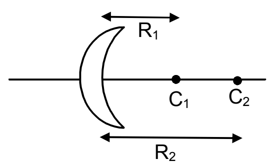
$\frac{1}{f}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1.6}{1}-1\right)\left(\frac{1}{20}-\frac{1}{30}\right)$

$\Rightarrow$ f = 60/6 = 100 cm

b) If placed in water :

$\frac{1}{f}=(\mu_w-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1.6}{1.33}-1\right)\left(\frac{1}{20}-\frac{1}{30}\right)$

$\Rightarrow$ f = 300 cm



50. Given μ = 1.5

Magnitude of radii of curvatures = 20 cm and 30 cm

The 4types of possible lens are as below.

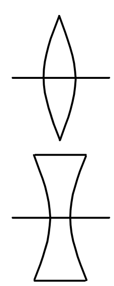
$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Case (1) : (Double convex) [R_1 = +ve, R_2 = –ve]

$\frac{1}{f}=(15-1)\left(\frac{1}{20}-\frac{1}{-30}\right) \Rightarrow$ f = 24 cm

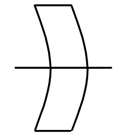
Case (2) : (Double concave) [R_1 = –ve, R_2 = +ve]

$\frac{1}{f}=(15-1)\left(\frac{-1}{20}-\frac{1}{30}\right) \Rightarrow$ f = –24 cm



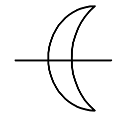
Case (3) : (Concave concave) [R_1 = –ve, R_2 = –ve]

$$\frac{1}{f} = (15-1)\left(\frac{1}{-20} - \frac{1}{-30}\right) \Rightarrow f = -120 \text{ cm}$$



Case (4) : (Concave convex) [R_1 = +ve, R_2 = +ve]

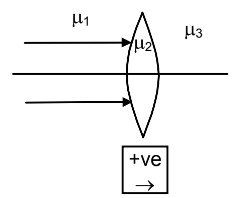
$$\frac{1}{f} = (15-1)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow f = +120 \text{ cm}$$



51. a) When the beam is incident on the lens from medium μ_1.

Then $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ or $\frac{\mu_2}{v} - \frac{\mu_1}{(-\infty)} = \frac{\mu_2 - \mu_1}{R}$

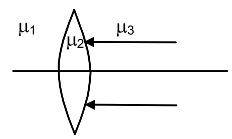
or $\frac{1}{v} = \frac{\mu_2 - \mu_1}{\mu_2 R}$ or v = $\frac{\mu_2 R}{\mu_2 - \mu_1}$



Again, for 2[nd] refraction, $\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R}$

or, $\frac{\mu_3}{v} = -\left[\frac{\mu_3 - \mu_2}{R} - \frac{\mu_2}{\mu_2 R}(\mu_2 - \mu_1)\right] \Rightarrow -\left[\frac{\mu_3 - \mu_2 - \mu_2 + \mu_1}{R}\right]$

or, v = $-\left[\frac{\mu_3 R}{\mu_3 - 2\mu_2 + \mu_1}\right]$



So, the image will be formed at = $\frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$

b) Similarly for the beam from μ_3 medium the image is formed at $\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$.

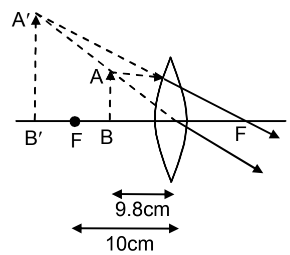
52. Given that, f = 10 cm

a) When u = –9.5 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{9.8} = \frac{-0.2}{98}$$

$\Rightarrow$ v = – 490 cm

So, $\Rightarrow$ m = $\frac{v}{u} = \frac{-490}{-9.8}$ = 50 cm



(Virtual image)

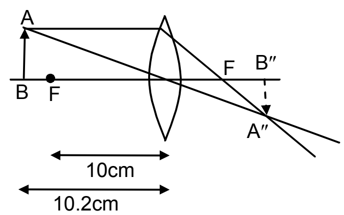
So, the image is erect and virtual.

b) When u = –10.2 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{-10.2} = \frac{102}{0.2}$$

$\Rightarrow$ v = 510 cm

So, m = $\frac{v}{u} = \frac{510}{-9.8}$



(Real image)

The image is real and inverted.

53. For the projector the magnification required is given by

m = $\frac{v}{u} = \frac{200}{3.5} \Rightarrow$ u = 17.5 cm

[35 mm > 23 mm, so the magnification is calculated taking object size 35 mm]

Now, from lens formula,

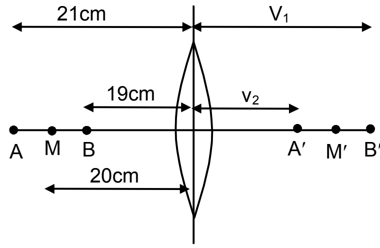
$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{1000} + \frac{1}{17.5} = \frac{1}{f}$$

$\Rightarrow$ f = 17.19 cm.

54. When the object is at 19 cm from the lens, let the image will be at, v_1.

$\Rightarrow \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f} \Rightarrow \frac{1}{v_1} - \frac{1}{-19} = \frac{1}{12}$



$\Rightarrow v_1 = 32.57$ cm

Again, when the object is at 21 cm from the lens, let the image will be at, v_2

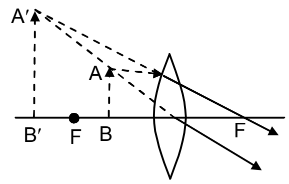
$\Rightarrow \frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{v_2} + \frac{1}{21} = \frac{1}{12}$

$\Rightarrow v_2 = 28$ cm

$\therefore$ Amplitude of vibration of the image is $A = \frac{A'B'}{2} = \frac{v_1 - v_2}{2}$

$\Rightarrow A = \frac{32.57 - 28}{2} = 2.285$ cm.

55. Given, u = –5 cm, f = 8 cm



So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{8} - \frac{1}{5} = \frac{-3}{40}$

$\Rightarrow v = -13.3$ cm (virtual image).

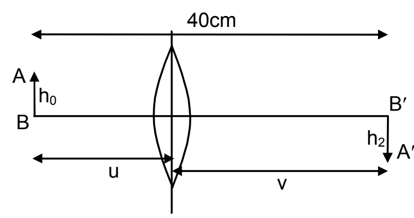
56. Given that,

(–u) + v = 40 cm = distance between object and image

$h_o = 2$ cm, $h_i = 1$ cm

Since $\frac{h_i}{h_o} = \frac{v}{-u}$ = magnification

$\Rightarrow \frac{1}{2} = \frac{v}{-u} \Rightarrow u = -2v$ …(1)



Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{2v} = \frac{1}{f}$

$\Rightarrow \frac{3}{2v} = \frac{1}{f} \Rightarrow f = \frac{2v}{3}$ …(2)

Again, (–u) + v = 40

$\Rightarrow 3v = 40 \Rightarrow v = 40/3$ cm

$\therefore f = \frac{2 \times 40}{3 \times 3}$ = 8.89 cm = focal length

From eqn. (1) and (2)

u = –2v = –3f = –3(8.89) = 26.7 cm = object distance.

57. A real image is formed. So, magnification m = –2 (inverted image)

$\therefore \frac{v}{u} = -2 \Rightarrow v = -2u = (-2)(-18) = 36$

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{36} - \frac{1}{-18} = \frac{1}{f}$

$\Rightarrow f = 12$ cm

Now, for triple sized image m = –3 = (v/u)

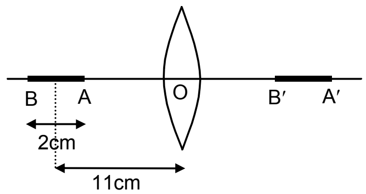
$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-3u} - \frac{1}{u} = \frac{1}{12}$

$\Rightarrow 3u = -48 \Rightarrow u = -16$ cm

So, object should be placed 16 cm from lens.

58. Now we have to calculate the image of A and B. Let the images be A′, B′. So, length of A′ B′ = size of image.

For A, u = –10 cm, f = 6 cm



Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-10} = \frac{1}{6}$

$\Rightarrow$ v = 15 cm = OA′

For B, u = –12 cm, f = 6 cm

Again, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{12}$

$\Rightarrow$ v = 12 cm = OB′

$\therefore$A′B′ = OA′ – OB′ = 15 – 12 = 3 cm.

So, size of image = 3 cm.

59. $u = -1.5 \times 10^{11}$ m ; $f = +20 \times 10^{-2}$ m

Since, f is very small compared to u, distance is taken as ∞. So, image will be formed at focus.

$\Rightarrow v = +20 \times 10^{-2}$ m

$\therefore$ We know, $m = \frac{v}{u} = \frac{h_{image}}{h_{object}}$

$\Rightarrow \frac{20 \times 10^{-2}}{1.5 \times 10^{11}} = \frac{D_{image}}{1.4 \times 10^{9}}$

$\Rightarrow D_{image} = 1.86$ mm

So, radius = $\frac{D_{image}}{2}$ = 0.93 mm.

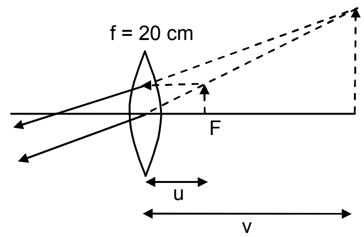
60. Given, P = 5 diopter (convex lens)

$\Rightarrow$ f = 1/5 m = 20 cm

Since, a virtual image is formed, u and v both are negative.

Given, v/u = 4

$\Rightarrow$ v = 4u ...(1)



From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{f} = \frac{1}{4u} - \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{1-4}{4u} = -\frac{3}{4u}$

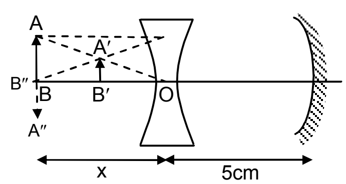
$\Rightarrow$ u = –15 cm

$\therefore$ Object is placed 15 cm away from the lens.

61. Let the object to placed at a distance x from the lens further away from the mirror.

For the concave lens (1st refraction)

u = –x, f = –20 cm



From lens formula,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-20} + \frac{1}{-x}$

$\Rightarrow v = -\left(\frac{20x}{x+20}\right)$

So, the virtual image due to fist refraction lies on the same side as that of object. (A′B′)

This image becomes the object for the concave mirror.

For the mirror,

$u = -\left(5 + \frac{20x}{x+20}\right) = -\left(\frac{25x+100}{x+20}\right)$

f = –10 cm

From mirror equation,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{x+20}{25x+100}$

$\Rightarrow v = \dfrac{50(x+4)}{3x-20}$

So, this image is formed towards left of the mirror.

Again for second refraction in concave lens,

$u = -\left[5 - \dfrac{50(x+4)}{3x-20}\right]$ (assuming that image of mirror is formed between the lens and mirro)

v = +x (Since, the final image is produced on the object)

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} + \frac{1}{5 - \dfrac{50(x+4}{3x-20}} = \frac{1}{-20}$$

$\Rightarrow$ x = 60 cm

The object should be placed at a distance 60 cm from the lens further away from the mirror.

So that the final image is formed on itself.

62. It can be solved in a similar manner like question no.61, by using the sign conversions properly. Left as an exercise for the student.

63. If the image in the mirror will form at the focus of the converging lens, then after transmission through the lens the rays of light will go parallel.

Let the object is at a distance x cm from the mirror

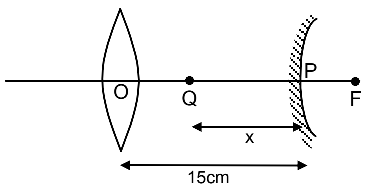
$\therefore$ u = –x cm ; v = 25 – 15 = 10 cm (because focal length of lens = 25 cm)

f = 40 cm

$$\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} = \frac{1}{10} - \frac{1}{40}$$

$\Rightarrow$ x = 400/30 = 40/3

$\therefore$ The object is at distance $\left(15 - \dfrac{40}{3}\right) = \dfrac{5}{3}$ = 1.67 cm from the lens.



64. The object is placed in the focus of the converging mirror.

There will be two images.

a) One due to direct transmission of light through lens.

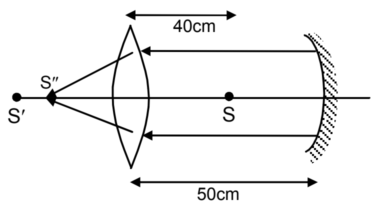
b) One due to reflection and then transmission of the rays through lens.

Case I : (S′) For the image by direct transmission,

u = –40 cm, f = 15 cm

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{-40}$$

$\Rightarrow$ v = 24 cm (left of lens)



Case II : (S″) Since, the object is placed on the focus of mirror, after reflection the rays become parallel for the lens.

So, u = ∞

$\Rightarrow$ f = 15 cm

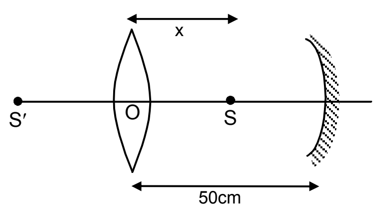
$\Rightarrow \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow$ v = 15 cm (left of lens)

65. Let the source be placed at a distance ‘x’ from the lens as shown, so that images formed by both coincide.

For the lens, $\dfrac{1}{v_\ell} - \dfrac{1}{-x} = \dfrac{1}{15} \Rightarrow v_\ell = \dfrac{15x}{x-15}$ …(1)

Fro the mirror, u = –(50 – x), f = –10 cm

So, $\dfrac{1}{v_m} + \dfrac{1}{-(50-x)} = -\dfrac{1}{10}$



$\Rightarrow \frac{1}{v_m} = \frac{1}{-(50-x)} - \frac{1}{10}$

So, $v_m = \frac{10(50-x)}{x-40}$ ...(2)

Since the lens and mirror are 50 cm apart,

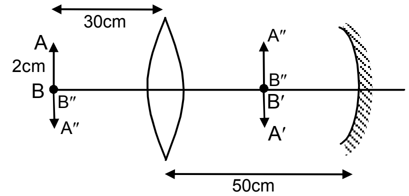
$v_\ell - v_m = 50 \Rightarrow \frac{15x}{x-15} - \frac{10(50-x)}{(x-40)} = 50$

$\Rightarrow$ x = 30 cm.

So, the source should be placed 30 cm from the lens.

66. Given that, f_1 = 15 cm, F_m = 10 cm, h_o = 2 cm

The object is placed 30 cm from lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.



$\Rightarrow v = \frac{uf}{u+f}$

Since, u = –30 cm and f = 15 cm

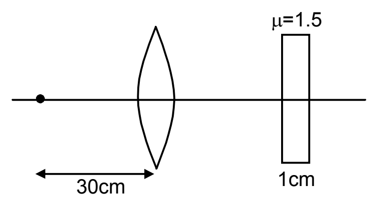
So, v = 30 cm

So, real and inverted image (A′B′) will be formed at 30 cm from the lens and it will be of same size as the object. Now, this real image is at a distance 20 cm from the concave mirror. Since, f_m = 10 cm, this real image is at the centre of curvature of the mirror. So, the mirror will form an inverted image A″B″ at the same place of same size.

Again, due to refraction in the lens the final image will be formed at AB and will be of same size as that of object. (A‴B‴)

67. For the lens, f = 15 cm, u = –30 cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{30} = \frac{1}{30} \Rightarrow$ v = 30 cm

The image is formed at 30 cm of right side due to lens only.

Again, shift due to glass slab is,

$= \Delta t = \left(1 - \frac{1}{15}\right)1$ [since, μ_g = 1.5 and t = 1 cm]

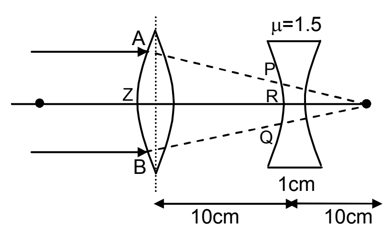
= 1 – (2/3) = 0.33 cm

$\therefore$ The image will be formed at 30 + 0.33 = 30.33 cm from the lens on right side.

68. Let, the parallel beam is first incident on convex lens.

d = diameter of the beam = 5 mm

Now, the image due to the convex lens should be formed on its focus (point B)



So, for the concave lens,

u = +10 cm (since, the virtual object is on the right of concave lens)

f = –10 cm

So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{1}{10} = 0 \Rightarrow v = \infty$

So, the emergent beam becomes parallel after refraction in concave lens.

As shown from the triangles XYB and PQB,

$\frac{PQ}{XY} = \frac{RB}{ZB} = \frac{10}{20} = \frac{1}{2}$

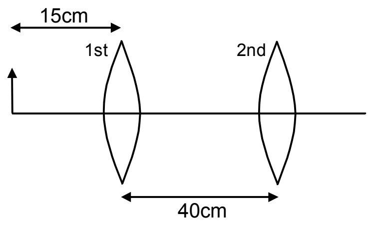
So, PQ = ½ $\times$ 5 = 25 mm

So, the beam diameter becomes 2.5 mm.

Similarly, it can be proved that if the light is incident of the concave side, the beam diameter will be 1cm.

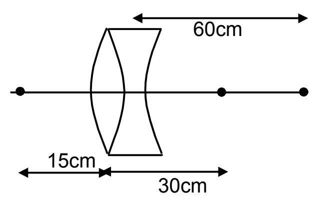
69. Given that, f_1 = focal length of converging lens = 30 cm
f_2 = focal length of diverging lens = –20 cm
and d = distance between them = 15 cm
Let, F = equivalent focal length
So, $\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{30} + \left(-\frac{1}{20}\right) - \left(\frac{15}{30(-200}\right) = \frac{1}{120}$
$\Rightarrow$ F = 120 cm
$\Rightarrow$ The equivalent lens is a converging one.
Distance from diverging lens so that emergent beam is parallel (image at infinity),
$d_1 = \frac{dF}{f_1} = \frac{15 \times 120}{30}$ = 60 cm
It should be placed 60 cm left to diverging lens
$\Rightarrow$ Object should be placed (120 – 60) = 60 cm from diverging lens.
Similarly, $d_2 = \frac{dF}{f_2} = \frac{15 \times 120}{20}$ = 90 cm
So, it should be placed 90 cm right to converging lens.
$\Rightarrow$ Object should be placed (120 + 90) = 210 cm right to converging lens.

70. **a) First lens** :
u = –15 cm, f = 10 cm
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{15}\right) = -\frac{1}{10}$
$\Rightarrow$ v = 30 cm
So, the final image is formed 10 cm right of second lens.



b) m for 1st lens :
$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{30}{-15}\right) = \frac{h_{image}}{5mm}$
$\Rightarrow h_{image}$ = –10 mm (inverted)
Second lens :
u = –(40 – 30) = –10 cm ; f = 5 cm
[since, the image of 1st lens becomes the object for the second lens].
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{10}\right) = \frac{1}{5}$
$\Rightarrow$ v = 10 cm
m for 2nd lens :
$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{10}{10}\right) = \frac{h_{image}}{-10}$
$\Rightarrow h_{image}$ = 10 mm (erect, real).
c) So, size of final image = 10 mm

71. Let u = object distance from convex lens = –15 cm
v_1 = image distance from convex lens when alone = 30 cm
f_1 = focal length of convex lens



Now, $\therefore \frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$
or, $\frac{1}{f_1} = \frac{1}{30} - \frac{1}{-15} = \frac{1}{30} + \frac{1}{15}$
or f_1 = 10 cm
Again, Let v = image (final) distance from concave lens = +(30 + 30) = 60 cm
v_1 = object distance from concave lens = +30 m

f_2 = focal length of concave lens

Now, $\therefore \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1}$

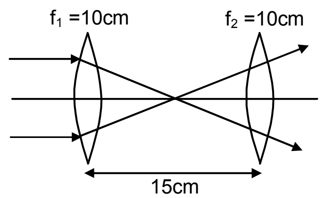
or, $\frac{1}{f_1} = \frac{1}{60} - \frac{1}{30} \Rightarrow f_2 = -60$ cm.

So, the focal length of convex lens is 10 cm and that of concave lens is 60 cm.

72. a) The beam will diverge after coming out of the two convex lens system because, the image formed by the first lens lies within the focal length of the second lens.

b) For 1st convex lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10}$ (since, $u = -\infty$)

or, v = 10 cm



for 2nd convex lens, $\frac{1}{v'} = \frac{1}{f} + \frac{1}{u}$

or, $\frac{1}{v'} = \frac{1}{10} + \frac{1}{-(15-10)} = \frac{-1}{10}$

or, v′ = –10 cm

So, the virtual image will be at 5 cm from 1st convex lens.

c) If, F be the focal length of equivalent lens,

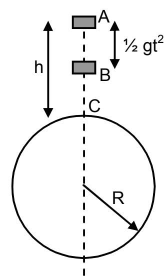
Then, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{10} + \frac{1}{10} - \frac{15}{100} = \frac{1}{20}$

$\Rightarrow$ F = 20 cm.

73. Let us assume that it has taken time ‘t’ from A to B.

$\therefore AB = \frac{1}{2}gt^2$

$\therefore BC = h - \frac{1}{2}gt^2$



This is the distance of the object from the lens at any time ‘t’.

Here, $u = -\left(h - \frac{1}{2}gt^2\right)$

$\mu_2 = \mu$(given) and μ_1 = i (air)

So, $\Rightarrow \frac{\mu}{v} - \frac{1}{-(h - \frac{1}{2}gf^2)} = \frac{\mu - 1}{R}$

$$\Rightarrow \frac{\mu}{v} = \frac{\mu-1}{R} - \frac{1}{(h - \frac{1}{2}gt^2)} = \frac{(\mu-1)(h - \frac{1}{2}gt^2) - R}{R(h - \frac{1}{2}gt^2)}$$

So, v = image distance at any time ‘t’ = $\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h - \frac{1}{2}gt^2) - R}$

So, velocity of the image = $V = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h - \frac{1}{2}gt^2) - R}\right] = \frac{\mu R^2 gt}{(\mu-1)(h - \frac{1}{2}gt^2) - R}$ (can be found out).

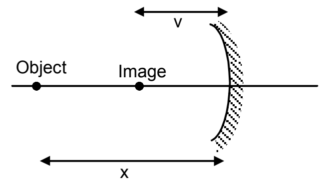
74. Given that, u = distance of the object = –x

f = focal length = –R/2

and, V = velocity of object = dx/dt

From mirror equation, $\frac{1}{-x} + \frac{1}{v} = -\frac{2}{R}$

$\frac{1}{v} = -\frac{2}{R} + \frac{1}{x} = \frac{R-2x}{Rx} \Rightarrow v = \frac{Rx}{R-2x}$ = Image distance



So, velocity of the image is given by,

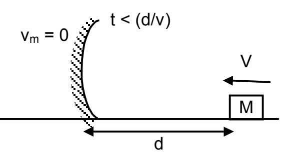
$$V_1 = \frac{dv}{dt} = \frac{[\frac{d}{dt}(xR)(R-2x)] - [\frac{d}{dt}(R-2x)][xR]}{(R-2x)^2}$$

$$= \frac{R[\frac{dx}{dt}(R-2x)] - [-2\frac{dx}{dt}x]}{(R-2x)^2} = \frac{R[v(R-2x)+2vx0}{(R-2x)^2}$$

$$= \frac{VR^2}{(2x-R)^2} = \frac{R[VR-2xV+2xV}{(R-2x)^2}.$$

75. a) When t < d/V, the object is approaching the mirror

As derived in the previous question,



$$V_{image} = \frac{\text{Velocity of object} \times R^2}{[2 \times \text{distance between them } - R]^2}$$

$$\Rightarrow V_{image} = \frac{VR^2}{[2(d-Vt)-R]^2} \quad \text{[At any time, x = d – Vt]}$$

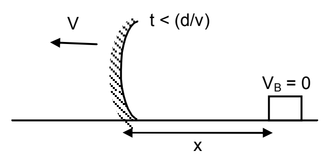
b) After a time t > d/V, there will be a collision between the mirror and the mass.

As the collision is perfectly elastic, the object (mass) will come to rest and the mirror starts to move away with same velocity V.

At any time t > d/V, the distance of the mirror from the mass will be

$$x = V\left(t - \frac{d}{V}\right) = Vt - d$$

Here, u = –(Vt – d) = d – Vt ; f = –R/2



So, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{d-Vt} + \frac{1}{(-R/2)} = -\left[\frac{R+2(d-Vt)}{R(d-Vt)}\right]$

$\Rightarrow v = -\left[\frac{R(d-Vt)}{R-2(d-Vt)}\right]$ = Image distance

So, Velocity of the image will be,

$$V_{image} = \frac{d}{dt}(\text{Image distance}) = \frac{d}{dt}\left[\frac{R(d-Vt)}{R+2(d-Vt)}\right]$$

Let, y = (d – Vt)

$$\Rightarrow \frac{dy}{dt} = -V$$

So, $V_{image} = \frac{d}{dt}\left[\frac{Ry}{R+2y}\right] = \frac{(R+2y)R(-V) - Ry(+2)(-V)}{(R+2y)^2}$

$$= -Vr\left[\frac{R+2y-2y}{(R+2y)^2}\right] = \frac{-VR^2}{(R+2y)^2}$$

Since, the mirror itself moving with velocity V,

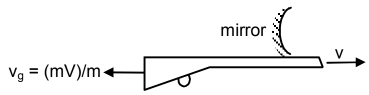
Absolute velocity of image = $V\left[1 - \frac{R^2}{(R+2y)^2}\right]$ (since, V = V_{mirror} + V_{image})

$$= V\left[1 - \frac{R^2}{[2(Vt-d)-R^2}\right].$$

76. Recoil velocity of gun = $V_g = \frac{mV}{M}$.

At any time 't', position of the bullet w.r.t. mirror = $Vt + \frac{mV}{M}t = \left(1+\frac{m}{M}\right)Vt$

For the mirror, u = $-\left(1+\frac{m}{M}\right)Vt = kVt$



v = position of the image

From lens formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-f} + \frac{1}{kVt} = \frac{1}{kVt} - \frac{1}{f} = \frac{f-kVt}{kVtf}$$

Let $\left(1+\frac{m}{M} = k\right)$,

So, v = $\frac{kVft}{-kVt+f} = \left(\frac{kVtf}{f-kVt}\right)$

So, velocity of the image with respect to mirror will be,

$$v_1 = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{kVtf}{f-kVt}\right] = \frac{(f-kVt)kVf - kVtf(-kV)}{(f-kVt)^2} = \frac{kVt^2}{(f-kVt)^2}$$

Since, the mirror itself is moving at a speed of mV/M and the object is moving at 'V', the velocity of separation between the image and object at any time 't' will be,

$$v_s = V + \frac{mV}{M} + \frac{kVf^2}{(f-kVt)^2}$$

When, t = 0 (just after the gun is fired),

$$v_s = V + \frac{mV}{M} + kV = V + \frac{m}{M}V + \left(1+\frac{m}{M}\right)V = 2\left(1+\frac{m}{M}\right)V$$

77. Due to weight of the body suppose the spring is compressed by which is the mean position of oscillation.

$m = 50 \times 10^{-3}$ kg, $g = 10\ ms^{-2}$, $k = 500\ Nm^{-2}$, h = 10 cm = 0.1 m

For equilibrium, $mg = kx \Rightarrow x = mg/k = 10^{-3}m = 0.1$ cm

So, the mean position is at 30 + 0.1 = 30.1 cm from P (mirror).

Suppose, maximum compression in spring is δ.

Since, E.K.E. – I.K.E. = Work done

$\Rightarrow 0 - 0 = mg(h+\delta) - \frac{1}{2}k\delta^2$ (work energy principle)

$\Rightarrow mg(h+\delta) = \frac{1}{2}k\delta^2 \Rightarrow 50 \times 10^{-3} \times 10(0.1+\delta) = \frac{1}{2}\,500\,\delta^2$

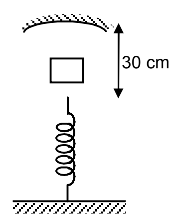


Fig-A

So, $\delta = \frac{0.5 \pm \sqrt{0.25+50}}{2\times 250}$ = 0.015 m = 1.5 cm.

From figure B,

Position of B is 30 + 1.5 = 31.5 cm from pole.

Amplitude of the vibration = 31.5 – 30.1 – 1.4.

Position A is 30.1 – 1.4 = 28.7 cm from pole.

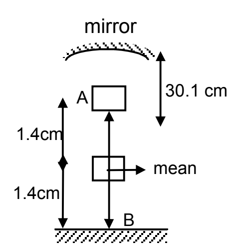


Fig-B

For A u = –31.5, f = –12 cm

$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{31.5}$

$\Rightarrow v_A = -19.38$ cm

For B f = –12 cm, u = –28.7 cm

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{28.7}$

$\Rightarrow v_B = -20.62$ cm

The image vibrates in length (20.62 – 19.38) = 1.24 cm.

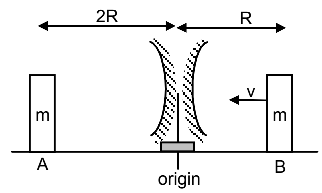
78. a) In time, t = R/V the mass B must have moved (v × R/v) = R closer to the mirror stand

So, For the block B :

u = –R, f = –R/2

$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{2}{R} + \frac{1}{R} = -\frac{1}{R}$

$\Rightarrow$ v = –R at the same place.



For the block A : u = –2R, f = –R/2

$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{-2}{R} + \frac{1}{2R} = \frac{-3}{2R}$

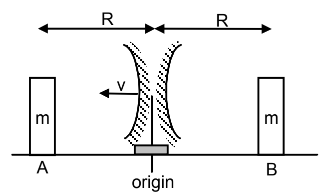
$\Rightarrow v = \frac{-2R}{3}$ image of A at $\frac{2R}{3}$ from PQ in the x-direction.

So, with respect to the given coordinate system,

$\therefore$ Position of A and B are $\frac{-2R}{3}$, R respectively from origin.

b) When t = 3R/v, the block B after colliding with mirror stand must have come to rest (elastic collision) and the mirror have travelled a distance R towards left form its initial position.

So, at this point of time,



For block A :

u = –R, f = –R/2

Using lens formula, v = –R (from the mirror),

So, position x_A = –2R (from origin of coordinate system)

For block B :

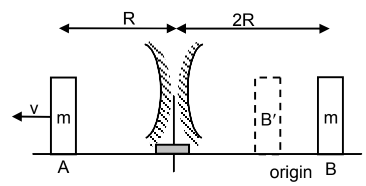
Image is at the same place as it is R distance from mirror. Hence, position of image is ‘0’.

Distance from PQ (coordinate system)

$\therefore$ positions of images of A and B are = –2R, 0 from origin.

c) Similarly, it can be proved that at time t = 5R/v,

the position of the blocks will be –3R and –4R/3 respectively.



79. Let a = acceleration of the masses A and B (w.r.t. elevator). From the freebody diagrams,

T – mg + ma – 2m = 0 …(1)

Similarly, T – ma = 0 …(2)

From (1) and (2), 2ma – mg – 2m = 0

$\Rightarrow$ 2ma = m(g + 2)

$\Rightarrow a = \frac{10+2}{2} = \frac{12}{2} = 6\ ms^{-2}$

so, distance travelled by B in t = 0.2 sec is,

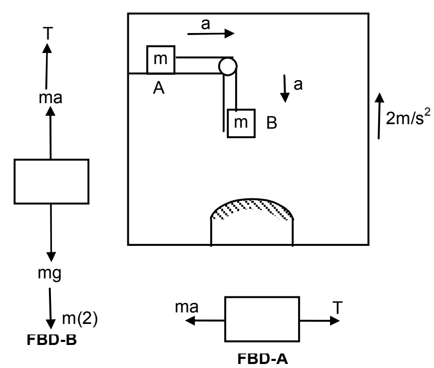
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 6 \times (0.2)^2 = 0.12\ m = 12\ cm.$

So, Distance from mirror, u = –(42 – 12) = –30 cm ; f = +12 cm

From mirror equation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \left(-\frac{1}{30}\right) = \frac{1}{12}$

$\Rightarrow$ v = 8.57 cm

Distance between image of block B and mirror = 8.57 cm.



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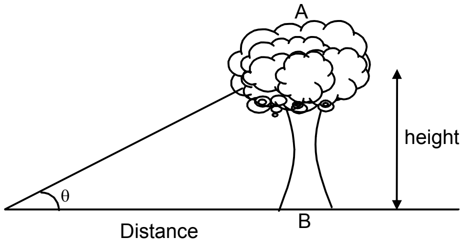
SOLUTIONS TO CONCEPTS
CHAPTER 19

1. The visual angles made by the tree with the eyes can be calculated be below.

$$\theta = \frac{\text{Height of the tree}}{\text{Distance from the eye}} = \frac{AB}{OB} \Rightarrow \theta_A = \frac{2}{50} = 0.04$$



similarly, $\theta_B = 2.5 / 80 = 0.03125$

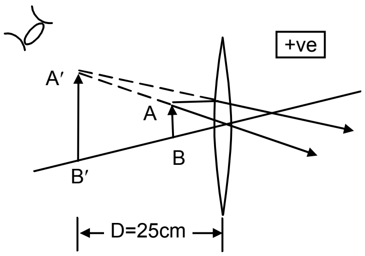
$\theta_C = 1.8 / 70 = 0.02571$

$\theta_D = 2.8 / 100 = 0.028$

Since, $\theta_A > \theta_B > \theta_D > \theta_C$, the arrangement in decreasing order is given by A, B, D and C.

2. For the given simple microscope,

f = 12 cm and D = 25 cm



(Simple Microscope)

For maximum angular magnification, the image should be produced at least distance of clear vision.

So, v = – D = –25 cm

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{12} = -\frac{37}{300}$

$\Rightarrow$ u = –8.1 cm

So, the object should be placed 8.1 cm away from the lens.

3. The simple microscope has, m = 3, when image is formed at D = 25 cm

a) $m = 1 + \frac{D}{f} \Rightarrow 3 = 1 + \frac{25}{f}$

$\Rightarrow$ f = 25/2 = 12.5 cm

b) When the image is formed at infinity (normal adjustment)

Magnifying power = $\frac{D}{f} = \frac{25}{12.5} = 2.0$

4. The child has D = 10 cm and f = 10 cm

The maximum angular magnification is obtained when the image is formed at near point.

$m = 1 + \frac{D}{f} = 1 + \frac{10}{10} = 1 + 1 = 2$

5. The simple microscope has magnification of 5 for normal relaxed eye (D = 25 cm).

Because, the eye is relaxed the image is formed at infinity (normal adjustment)

So, $m = 5 = \frac{D}{f} = \frac{25}{f} \Rightarrow f = 5$ cm

For the relaxed farsighted eye, D = 40 cm

So, $m = \frac{D}{f} = \frac{40}{5} = 8$

So, its magnifying power is 8X.

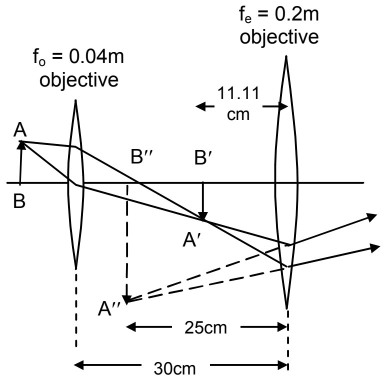
6. For the given compound microscope

$f_0 = \dfrac{1}{25 \text{ diopter}} = 0.04 \text{ m} = 4 \text{ cm}, f_e = \dfrac{1}{5 \text{ diopter}} = 0.2 \text{ m} = 20 \text{ cm}$

D = 25 cm, separation between objective and eyepiece = 30 cm

The magnifying power is maximum when the image is formed by the eye piece at least distance of clear vision i.e. D = 25 cm

for the eye piece, $v_e = -25$ cm, $f_e = 20$ cm



For lens formula, $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

$\Rightarrow \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} \Rightarrow \dfrac{1}{-25} - \dfrac{1}{20} \quad \Rightarrow u_e = 11.11 \text{ cm}$

So, for the objective lens, the image distance should be

$v_0 = 30 - (11.11) = 18.89$ cm

Now, for the objective lens,

$v_0 = +18.89$ cm (because real image is produced)

$f_0 = 4$ cm

So, $\dfrac{1}{u_o} = \dfrac{1}{v_o} - \dfrac{1}{f_o} \Rightarrow \dfrac{1}{18.89} - \dfrac{1}{4} = 0.053 - 0.25 = -0.197$

$\Rightarrow u_o = -5.07$ cm

So, the maximum magnificent power is given by

$m = -\dfrac{v_o}{u_o}\left[1 + \dfrac{D}{f_e}\right] = -\dfrac{18.89}{-5.07}\left[1 + \dfrac{25}{20}\right]$

$= 3.7225 \times 2.25 = 8.376$

7. For the given compound microscope

$f_o = 1$ cm, $f_e = 6$ cm, D = 24 cm

For the eye piece, $v_e = -24$ cm, $f_e = 6$ cm

Now, $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

$\Rightarrow \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} \Rightarrow -\left[\dfrac{1}{24} + \dfrac{1}{6}\right] = -\dfrac{5}{24}$

$\Rightarrow u_e = -4.8$ cm

a) When the separation between objective and eye piece is 9.8 cm, the image distance for the objective lens must be (9.8) – (4.8) = 5.0 cm

Now, $\dfrac{1}{v_0} - \dfrac{1}{u_0} = \dfrac{1}{f_0}$

$\Rightarrow \dfrac{1}{u_0} = \dfrac{1}{v_0} - \dfrac{1}{f_0} = \dfrac{1}{5} - \dfrac{1}{1} = -\dfrac{4}{5}$

$\Rightarrow u_0 = -\dfrac{5}{4} = -1.25$ cm

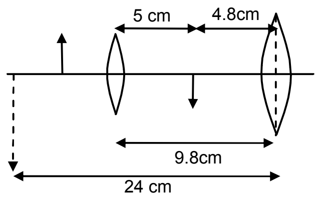


Fig-A

So, the magnifying power is given by,

$m = \dfrac{v_0}{u_o}\left[1 + \dfrac{D}{f}\right] = \dfrac{-5}{-1.25}\left[1 + \dfrac{24}{6}\right] = 4 \times 5 = 20$

(b) When the separation is 11.8 cm,

$v_0 = 11.8 - 4.8 = 7.0$ cm, $\quad f_0 = 1$ cm

$\Rightarrow \dfrac{1}{u_0} = \dfrac{1}{v_0} - \dfrac{1}{f_0} = \dfrac{1}{7} - \dfrac{1}{1} = -\dfrac{6}{7}$

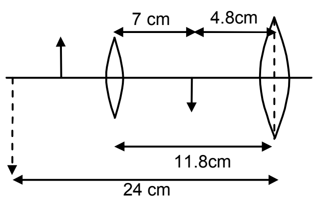


Fig-B

So, $m = -\frac{v_0}{u_o}\left[1+\frac{D}{f}\right] = \frac{-7}{-\left(\frac{7}{6}\right)}\left[1+\frac{24}{6}\right] = 6 \times 5 = 30$

So, the range of magnifying power will be 20 to 30.

8. For the given compound microscope.

$f_0 = \frac{1}{20D} = 0.05$ m = 5 cm, $f_e = \frac{1}{10D} = 0.1$ m = 10 cm.

D = 25 cm, separation between objective & eyepiece= 20 cm

For the minimum separation between two points which can be distinguished by eye using the microscope, the magnifying power should be maximum.

For the eyepiece, $v_0 = -25$ cm, $f_e = 10$ cm

So, $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{10} = -\left[\frac{2+5}{50}\right] \Rightarrow u_e = -\frac{50}{7}$ cm

So, the image distance for the objective lens should be,

$V_0 = 20 - \frac{50}{7} = \frac{90}{7}$ cm

Now, for the objective lens,

$\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{7}{90} - \frac{1}{5} = -\frac{11}{90}$

$\Rightarrow u_0 = -\frac{90}{11}$ cm

So, the maximum magnifying power is given by,

$m = \frac{-v_0}{u_0}\left[1+\frac{D}{f_e}\right]$

$= \frac{\left(\frac{90}{7}\right)}{\left(-\frac{90}{11}\right)}\left[1+\frac{25}{10}\right]$

$= \frac{11}{7} \times 3.5 = 5.5$

Thus, minimum separation eye can distinguish = $\frac{0.22}{5.5}$ mm = 0.04 mm

9. For the give compound microscope,

$f_0 = 0.5$cm, tube length = 6.5cm

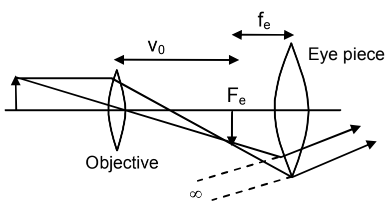
magnifying power = 100 (normal adjustment)

Since, the image is formed at infinity, the real image produced by the objective lens should lie on the focus of the eye piece.

So, $v_0 + f_e = 6.5$ cm ...(1)

Again, magnifying power= $\frac{v_0}{u_0} \times \frac{D}{f_e}$ [for normal adjustment]

$\Rightarrow m = -\left[1-\frac{v_0}{f_0}\right]\frac{D}{f_e}$ $\left[\because \frac{v_0}{u_0} = 1 - \frac{v_0}{f_0}\right]$



$\Rightarrow 100 = -\left[1-\frac{v_0}{0.5}\right] \times \frac{25}{f_e}$ [Taking D = 25 cm]

$\Rightarrow 100 f_e = -(1 - 2v_0) \times 25$

$\Rightarrow 2v_0 - 4f_e = 1$ ...(2)

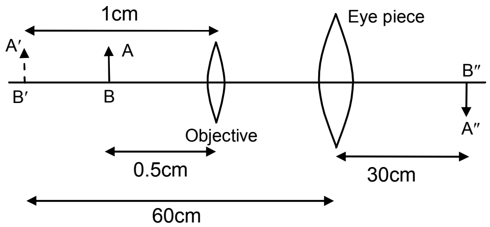
Solving equation (1) and (2) we can get,

V_0 = 4.5 cm and f_e = 2 cm

So, the focal length of the eye piece is 2cm.

10. Given that,

f_o = = 1 cm, f_e = 5 cm, u_0 = 0.5 cm, v_e = 30 cm



For the objective lens, u_0 = – 0.5 cm, f_0 = 1 cm.

From lens formula,

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{v_0} = \frac{1}{u_0} + \frac{1}{f_0} = \frac{1}{-0.5} + \frac{1}{1} = -1$$

$\Rightarrow v_0 = -1$ cm

So, a virtual image is formed by the objective on the same side as that of the object at a distance of 1 cm from the objective lens. This image acts as a virtual object for the eyepiece.

For the eyepiece,

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{30} - \frac{1}{5} = \frac{-5}{30} = \frac{-1}{6} \Rightarrow u_0 = -6 \text{ cm}$$

So, as shown in figure,

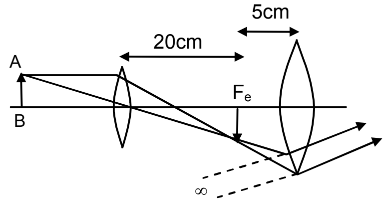
Separation between the lenses = $u_0 - v_0$ = 6 – 1 = 5 cm

11. The optical instrument has

$$f_0 = \frac{1}{25D} = 0.04 \text{ m} = 4 \text{ cm}$$

$$f_e = \frac{1}{20D} = 0.05 \text{ m} = 5 \text{ cm}$$

tube length = 25 cm (normal adjustment)



(a) The instrument must be a microscope as $f_0 < f_e$

(b) Since the final image is formed at infinity, the image produced by the objective should lie on the focal plane of the eye piece.

So, image distance for objective = v_0 = 25 – 5 = 20 cm

Now, using lens formula.

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{20} - \frac{1}{4} = \frac{-4}{20} = \frac{-1}{5} \Rightarrow u_0 = -5 \text{ cm}$$

So, angular magnification = $m = -\frac{v_0}{u_0} \times \frac{D}{f_e}$ [Taking D = 25 cm]

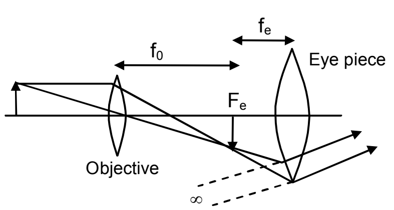
$$= -\frac{20}{-5} \times \frac{25}{5} = 20$$

12. For the astronomical telescope in normal adjustment.

Magnifying power = m = 50, length of the tube = L = 102 cm

Let f_0 and f_e be the focal length of objective and eye piece respectively.

$$m = \frac{f_0}{f_e} = 50 \Rightarrow f_0 = 50\, f_e \quad \ldots(1)$$



and, L = $f_0 + f_e$ = 102 cm ...(2)

Putting the value of f_0 from equation (1) in (2), we get,

$f_0 + f_e = 102 \Rightarrow 51f_e = 102 \Rightarrow f_e$ = 2 cm = 0.02 m

So, f_0 = 100 cm = 1 m

$\therefore$ Power of the objective lens = $\frac{1}{f_0}$ = 1D

And Power of the eye piece lens = $\frac{1}{f_e} = \frac{1}{0.02}$ = 50D

13. For the given astronomical telescope in normal adjustment,
F_e = 10 cm, L = 1 m = 100cm
S0, $f_0 = L - f_e = 100 - 10 = 90$ cm
and, magnifying power = $\frac{f_0}{f_e} = \frac{90}{10} = 9$

14. For the given Galilean telescope, (When the image is formed at infinity)
f_0 = 30 cm, L = 27 cm
Since $L = f_0 - |f_e|$
[Since, concave eyepiece lens is used in Galilean Telescope]
$\Rightarrow f_e = f_0 - L = 30 - 27 = 3$ cm

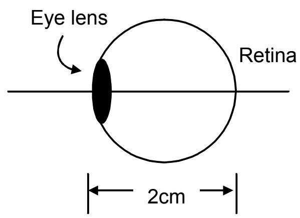
15. For the far sighted person,
u = – 20 cm, v = – 50 cm
from lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
$\frac{1}{f} = \frac{1}{-50} - \frac{1}{-20} = \frac{1}{20} - \frac{1}{50} = \frac{3}{100}$ $\Rightarrow f = \frac{100}{3}$ cm = $\frac{1}{3}$ m
So, power of the lens = $\frac{1}{f}$ = 3 Diopter

16. For the near sighted person,
$u = \infty$ and v = – 200 cm = – 2m
So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-2} - \frac{1}{\infty} = -\frac{1}{2} = -0.5$
So, power of the lens is –0.5D

17. The person wears glasses of power –2.5D
So, the person must be near sighted.
$u = \infty$, v = far point, f= $\frac{1}{-2.5}$ = – 0.4m = – 40 cm
Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
$\Rightarrow \frac{1}{v} = \frac{1}{u} + \frac{1}{f} = 0 + \frac{1}{-40}$ $\Rightarrow v = -40$ cm
So, the far point of the person is 40 cm

18. On the 50^{th} birthday, he reads the card at a distance 25cm using a glass of +2.5D.
Ten years later, his near point must have changed.
So after ten years,
u = – 50 cm, f = $\frac{1}{2.5D}$ = 0.4m = 40 cm v = near point
Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\Rightarrow \frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{-50} + \frac{1}{40} = \frac{1}{200}$
So, near point = v = 200cm
To read the farewell letter at a distance of 25 cm,
U = – 25 cm
For lens formula,
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{200} - \frac{-}{-25} = \frac{1}{200} + \frac{1}{25} = \frac{9}{200}$ $\Rightarrow f = \frac{200}{9}$ cm = $\frac{2}{9}$ m
$\Rightarrow$ Power of the lens = $\frac{1}{f} = \frac{9}{2} = 4.5$D
$\therefore$ He has to use a lens of power +4.5D.

19. Since, the retina is 2 cm behind the eye-lens
 v = 2cm
 (a) When the eye-lens is fully relaxed
 $u = \infty$, $v = 2\text{cm} = 0.02 \text{ m}$
 $\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{\infty} = 50D$
 So, in this condition power of the eye-lens is 50D
 (b) When the eye-lens is most strained,
 $u = -25 \text{ cm} = -0.25 \text{ m}$, $v = +2 \text{ cm} = +0.02 \text{ m}$
 $\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-0.25} = 50 + 4 = 54D$
 In this condition power of the eye lens is 54D.



20. The child has near point and far point 10 cm and 100 cm respectively.
 Since, the retina is 2 cm behind the eye-lens, v = 2cm
 For near point $u = -10 \text{ cm} = -0.1 \text{ m}$, $v = 2 \text{ cm} = 0.02 \text{ m}$
 So, $\frac{1}{f_{near}} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-0.1} = 50 + 10 = 60D$
 For far point, $u = -100 \text{ cm} = -1 \text{ m}$, $v = 2 \text{ cm} = 0.02 \text{ m}$
 So, $\frac{1}{f_{far}} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-1} = 50 + 1 = 51D$
 So, the rage of power of the eye-lens is +60D to +51D
21. For the near sighted person,
 v = distance of image from glass
 = distance of image from eye – separation between glass and eye
 = 25 cm – 1cm = 24 cm = 0.24m
 So, for the glass, $u = \infty$ and $v = -24 \text{ cm} = -0.24\text{m}$
 So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-0.24} - \frac{1}{\infty} = -4.2 \text{ D}$
22. The person has near point 100 cm. It is needed to read at a distance of 20cm.
 (a) When contact lens is used,
 $u = -20 \text{ cm} = -0.2\text{m}$, $v = -100 \text{ cm} = -1 \text{ m}$
 So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-0.2} = -1 + 5 = +4D$
 (b) When spectacles are used,
 $u = -(20 - 2) = -18 \text{ cm} = -0.18\text{m}$, $v = -100 \text{ cm} = -1 \text{ m}$
 So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-0.18} = -1 + 5.55 = +4.5D$
23. The lady uses +1.5D glasses to have normal vision at 25 cm.
 So, with the glasses, her least distance of clear vision = D = 25 cm
 Focal length of the glasses $= \frac{1}{1.5} \text{ m} = \frac{100}{1.5} \text{ cm}$
 So, without the glasses her least distance of distinct vision should be more
 If, $u = -25\text{cm}$, $f = \frac{100}{1.5} \text{ cm}$
 Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \frac{1.5}{100} - \frac{1}{25} = \frac{1.5 - 4}{100} = \frac{-2.5}{100}$ $\Rightarrow v = -40\text{cm}$ = near point without glasses.
 Focal length of magnifying glass $= \frac{1}{20} \text{ m} = 0.05\text{m} = 5 \text{ cm} = f$

(a) The maximum magnifying power with glasses

$$m = 1+\frac{D}{f} = 1+\frac{25}{5} = 6 \qquad [\because D = 25cm]$$

(b) Without the glasses, D = 40cm

$$\text{So, } m = 1+\frac{D}{f} = 1+\frac{40}{5} = 9$$

24. The lady can not see objects closer than 40 cm from the left eye and 100 cm from the right eye.

For the left glass lens,

v = – 40 cm, u = – 25 cm

$$\therefore \frac{1}{f}=\frac{1}{v}-\frac{1}{u} = \frac{1}{-40}-\frac{1}{-25} = \frac{1}{25}-\frac{1}{40} = \frac{3}{200} \qquad \Rightarrow f = \frac{200}{3}\text{ cm}$$

For the right glass lens,

v = – 100 cm, u = – 25 cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} = \frac{1}{-100}-\frac{1}{-25} = \frac{1}{25}-\frac{1}{100} = \frac{3}{100} \qquad \Rightarrow f = \frac{100}{3}\text{ cm}$$

(a) For an astronomical telescope, the eye piece lens should have smaller focal length. So, she should use the right lens ($f = \frac{100}{3}$ cm) as the eye piece lens.

(b) With relaxed eye, (normal adjustment)

$$f_0 = \frac{200}{3}\text{ cm}, \quad f_e = \frac{100}{3}\text{ cm}$$

$$\text{magnification} = m = \frac{f_0}{f_e} = \frac{(200/3)}{(100/3)} = 2$$

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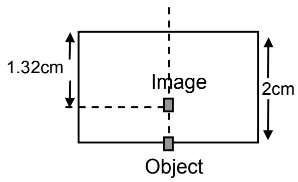
SOLUTIONS TO CONCEPTS
CHAPTER – 20

1. Given that,
Refractive index of flint glass = μ_f = 1.620
Refractive index of crown glass = μ_c = 1.518
Refracting angle of flint prism = A_f = 6.0°
For zero net deviation of mean ray
$(\mu_f - 1)A_f = (\mu_c - 1) A_c$
$\Rightarrow A_c = \frac{\mu_f - 1}{\mu_c - 1} A_f = \frac{1.620 - 1}{1.518 - 1}(6.0)^\circ = 7.2^\circ$

2. Given that
$\mu_r = 1.56$, $\mu_y = 1.60$, and $\mu_v = 1.68$
(a) Dispersive power = $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.68 - 1.56}{1.60 - 1} = 0.2$
(b) Angular dispersion = $(\mu_v - \mu_r)A = 0.12 \times 6^\circ = 7.2^\circ$

3. The focal length of a lens is given by
$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
$\Rightarrow (\mu - 1) = \frac{1}{f} \times \frac{1}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{K}{f}$ …(1)
So, $\mu_r - 1 = \frac{K}{100}$ …(2)
$\mu_y - 1 = \frac{K}{98}$ …(3)
And $\mu_v - 1 = \frac{K}{96}$ (4)
So, Dispersive power = $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \frac{\frac{K}{96} - \frac{K}{100}}{\frac{K}{98}} = \frac{98 \times 4}{9600} = 0.0408$

4. Given that, $\mu_v - \mu_r = 0.014$
Again, $\mu_y = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{2.00}{1.30} = 1.515$
So, dispersive power = $\frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{0.014}{1.515 - 1} = 0.027$



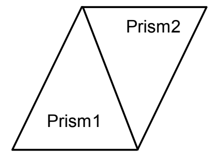
5. Given that, $\mu_r = 1.61$, $\mu_v = 1.65$, $\omega = 0.07$ and $\delta_y = 4^\circ$
Now, $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$
$\Rightarrow 0.07 = \frac{1.65 - 1.61}{\mu_y - 1}$
$\Rightarrow \mu_y - 1 = \frac{0.04}{0.07} = \frac{4}{7}$
Again, $\delta = (\mu - 1) A$
$\Rightarrow A = \frac{\delta_y}{\mu_y - 1} = \frac{4}{(4/7)} = 7^\circ$

6. Given that, $\delta_r = 38.4°$, $\delta_y = 38.7°$ and $\delta_v = 39.2°$

Dispersive power = $\dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \dfrac{\left(\dfrac{\delta_v}{A}\right) - \left(\dfrac{\delta_r}{A}\right)}{\left(\dfrac{\delta_v}{A}\right)}$ $\quad [\because\ \delta = (\mu - 1)\,A]$

$= \dfrac{\delta_v - \delta_r}{\delta_y} = \dfrac{39.2 - 38.4}{38.7} = 0.0204$

7. Two prisms of identical geometrical shape are combined.



Let A = Angle of the prisms

$\mu'_v = 1.52$ and $\mu_v = 1.62$, $\delta_v = 1°$

$\delta_v = (\mu_v - 1)A - (\mu'_v - 1)\,A$ [since A = A′]

$\Rightarrow \delta_v = (\mu_v - \mu'_v)A$

$\Rightarrow A = \dfrac{\delta_v}{\mu_v - \mu'_v} = \dfrac{1}{1.62 - 1.52} = 10°$

8. Total deviation for yellow ray produced by the prism combination is

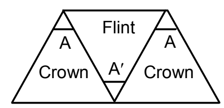
$\delta_y = \delta_{cy} - \delta_{fy} + \delta_{cy} = 2\,\delta_{cy} - \delta_{fy} = 2(\mu_{cy} - 1)A - (\mu_{cy} - 1)A'$

Similarly the angular dispersion produced by the combination is

$\delta_v - \delta_r = [(\mu_{vc} - 1)A - (\mu_{vf} - 1)A' + (\mu_{vc} - 1)A] - [(\mu_{rc} - 1)\,A - (\mu_{rf} - 1)A' + (\mu_r - 1)\,A)]$

$= 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A'$

(a) For net angular dispersion to be zero,



$\delta_v - \delta_r = 0$

$\Rightarrow\ 2(\mu_{vc} - 1)A = (\mu_{vf} - 1)A'$

$\Rightarrow\ \dfrac{A'}{A} = \dfrac{2(\mu_{cv} - \mu_{rc})}{(\mu_{vf} - \mu_{rf})} = \dfrac{2(\mu_v - \mu_r)}{(\mu'_v - \mu'_r)}$

(b) For net deviation in the yellow ray to be zero,

$\delta_y = 0$

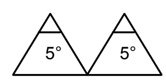
$\Rightarrow 2(\mu_{cy} - 1)A = (\mu_{fy} - 1)A'$

$\Rightarrow\ \dfrac{A'}{A} = \dfrac{2(\mu_{cy} - 1)}{(\mu_{fy} - 1)} = \dfrac{2(\mu_y - 1)}{(\mu'_y - 1)}$

9. Given that, $\mu_{cr} = 1.515$, $\mu_{cv} = 1.525$ and $\mu_{fr} = 1.612$, $\mu_{fv} = 1.632$ and $A = 5°$

Since, they are similarly directed, the total deviation produced is given by,

$\delta = \delta_c + \delta_r = (\mu_c - 1)A + (\mu_r - 1)\,A = (\mu_c + \mu_r - 2)A$



So, angular dispersion of the combination is given by,

$\delta_v - \delta_y = (\mu_{cv} + \mu_{fv} - 2)A - (\mu_{cr} + \mu_{fr} - 2)A$

$= (\mu_{cv} + \mu_{fv} - \mu_{cr} - \mu_{fr})A = (1.525 + 1.632 - 1.515 - 1.612)\ 5 = 0.15°$

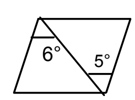
10. Given that, $A' = 6°$, $\omega' = 0.07$, $\mu'_y = 1.50$

$A = ?$, $\omega = 0.08$, $\mu_y = 1.60$

The combination produces no deviation in the mean ray.

(a) $\delta_y = (\mu_y - 1)A - (\mu'_y - 1)A' = 0$ [Prism must be oppositely directed]

$\Rightarrow (1.60 - 1)A = ((1.50 - 1)A'$



$\Rightarrow A = \dfrac{0.50 \times 6°}{0.60} = 5°$

(b) When a beam of white light passes through it,

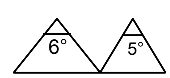
Net angular dispersion = $(\mu_y - 1)\omega A - (\mu'_y - 1)\omega' A'$

$\Rightarrow (1.60 - 1)(0.08)(5°) - (1.50 - 1)(0.07)(6°)$

$\Rightarrow 0.24° - 0.21° = 0.03°$

(c) If the prisms are similarly directed,

$\delta_y = (\mu_y - 1)A + (\mu'_y - 1)A$

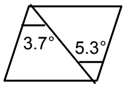


$= (1.60 - 1)5° + (1.50 - 1)6° = 3° + 3° = 6°$

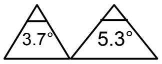
(d) Similarly, if the prisms are similarly directed, the net angular dispersion is given by,

$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\ \omega' A' = 0.24° + 0.21° = 0.45°$

11. Given that, $\mu'_v - \mu'_r = 0.014$ and $\mu_v - \mu_r = 0.024$
A′ = 5.3° and A = 3.7°



(a) When the prisms are oppositely directed,
angular dispersion = $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$
$= 0.024 \times 3.7° - 0.014 \times 5.3° = 0.0146°$



(b) When they are similarly directed,
angular dispersion = $(\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A'$
$= 0.024 \times 3.7° + 0.014 \times 5.3° = 0.163°$

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SOLUTIONS TO CONCEPTS
CHAPTER 21

1. In the given Fizeau'' apparatus,
   $D = 12$ km $= 12 \times 10^3$ m
   $n = 180$
   $c = 3 \times 10^8$ m/sec
   We know, $c = \frac{2Dn\omega}{\pi}$
   $\Rightarrow \omega = \frac{\pi c}{2Dn}$ rad/sec $= \frac{\pi c}{2Dn} \times \frac{180}{\pi}$ deg/sec
   $\Rightarrow \omega = \frac{180 \times 3 \times 10^8}{24 \times 10^3 \times 180} = 1.25 \times 10^4$ deg/sec
2. In the given Focault experiment,
   R = Distance between fixed and rotating mirror = 16m
   ω = Angular speed = 356 rev/′ = 356 × 2π rad/sec
   b = Distance between lens and rotating mirror = 6m
   a = Distance between source and lens = 2m
   s = shift in image = 0.7 cm = 0.7×10^{-3} m
   So, speed of light is given by,
   $C = \frac{4R^2\omega a}{s(R+b)} = \frac{4 \times 16^2 \times 356 \times 2\pi \times 2}{0.7 \times 10^{-3}(16+6)} = 2.975 \times 10^8$ m/s
3. In the given Michelson experiment,
   $D = 4.8$ km $= 4.8 \times 10^3$ m
   $N = 8$
   We know, $c = \frac{D\omega N}{2\pi}$
   $\Rightarrow \omega = \frac{2\pi c}{DN}$ rad/sec $= \frac{c}{DN}$ rev/sec $= \frac{3 \times 10^8}{4.8 \times 10^3 \times 8} = 7.8 \times 10^3$ rev/sec

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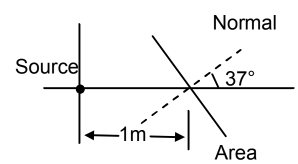
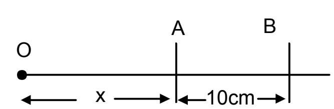
SOLUTIONS TO CONCEPTS
CHAPTER 22

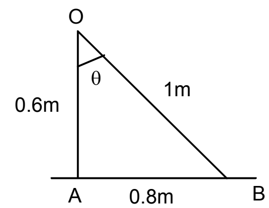
1. Radiant Flux = $\frac{\text{Total energy emitted}}{\text{Time}} = \frac{45}{15s}$ = 3W
2. To get equally intense lines on the photographic plate, the radiant flux (energy) should be same.
   S0, 10W × 12sec = 12W × t
   $\Rightarrow t = \frac{10W \times 12\,\text{sec}}{12W}$ = 10 sec.
3. it can be found out from the graph by the student.
4. Relative luminousity = $\frac{\text{Luminous flux of a source of given wavelength}}{\text{Luminous flux of a source of 555 nm of same power}}$
   Let the radiant flux needed be P watt.
   Ao, 0.6 = $\frac{\text{Luminous flux of source 'P' watt}}{685\text{ P}}$
   $\therefore$ Luminous flux of the source = (685 P)× 0.6 = 120 × 685
   $\Rightarrow P = \frac{120}{0.6}$ = 200W
5. The luminous flux of the given source of 1W is 450 lumen/watt
   $\therefore$ Relative luminosity = $\frac{\text{Luminous flux of the source of given wavelength}}{\text{Luminous flux of 555 nm source of same power}} = \frac{450}{685}$ = 66%
   [$\therefore$ Since, luminous flux of 555nm source of 1W = 685 lumen]
6. The radiant flux of 555nm part is 40W and of the 600nm part is 30W
   (a) Total radiant flux = 40W + 30W = 70W
   (b) Luminous flux = $(\text{L.Fllux})_{555nm} + (\text{L.Flux})_{600nm}$
   = 1 × 40× 685 + 0.6 × 30 × 685 = 39730 lumen
   (c) Luminous efficiency = $\frac{\text{Total luminous flux}}{\text{Total radiant flux}} = \frac{39730}{70}$ = 567.6 lumen/W
7. Overall luminous efficiency = $\frac{\text{Total luminous flux}}{\text{Power input}} = \frac{35 \times 685}{100}$ = 239.75 lumen/W
8. Radiant flux = 31.4W, Solid angle = 4π
   Luminous efficiency = 60 lumen/W
   So, Luminous flux = 60 × 31.4 lumen
   And luminous intensity = $\frac{\text{Luminous Flux}}{4\pi} = \frac{60 \times 31.4}{4\pi}$ = 150 candela
9. I = luminous intensity = $\frac{628}{4\pi}$ = 50 Candela
   r = 1m, $\theta = 37°$
   So, illuminance, E = $\frac{I\cos\theta}{r^2} = \frac{50 \times \cos 37°}{1^2}$ = 40 lux



10. Let, I = Luminous intensity of source
    E_A = 900 lumen/m^2
    E_B = 400 lumen/m^2
    Now, $E_a = \frac{I\cos\theta}{x^2}$ and $E_B = \frac{I\cos\theta}{(x+10)^2}$
    So, I = $\frac{E_A x^2}{\cos\theta} = \frac{E_B (x+10)^2}{\cos\theta}$
    $\Rightarrow 900x^2 = 400(x + 10)^2 \Rightarrow \frac{x}{x+10} = \frac{2}{3} \Rightarrow 3x = 2x + 20 \Rightarrow x = 20$ cm
    So, The distance between the source and the original position is 20cm.



11. Given that, E_a = 15 lux = $\frac{I_0}{60^2}$



$\Rightarrow I_0 = 15 \times (0.6)^2 = 5.4$ candela

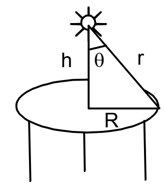
So, $E_B = \frac{I_0 \cos\theta}{(OB)^2} = \frac{5.4 \times \left(\frac{3}{5}\right)}{1^2} = 3.24$ lux

12. The illuminance will not change.

13. Let the height of the source is 'h' and the luminous intensity in the normal direction is I_0.

So, illuminance at the book is given by,

$$E = \frac{I_0 \cos\theta}{r^2} = \frac{I_0 h}{r^3} = \frac{I_0 h}{(r^2 + h^2)^{3/2}}$$



For maximum E, $\frac{dE}{dh} = 0 \Rightarrow \frac{I_0\left[(R^2 + h^2)^{3/2} - \frac{3}{2}h \times (R^2 + h^2)^{1/2} \times 2h\right]}{(R^2 + h^2)^3}$

$\Rightarrow (R^2 + h^2)^{1/2}[R^2 + h^2 - 3h^2] = 0$

$\Rightarrow R^2 - 2h^2 = 0 \Rightarrow h = \frac{R}{\sqrt{2}}$

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