	Undergraduate Course
ELEMENTS OF COMPUTATION THEORY	College of Computer Science
Chapter 2	**Zhejiang University**
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2.1.1 Let M be a deterministic finite automaton. Under exactly what circumstances is $e \in L(M)$? Prove your answer.

Solution:

$e \in L(M)$ if and only if $s \in F$.

☐ Suppose $e \in L(M)$. Then, by definition of $L(M)$, $(s, e) \vdash_M^* (q, e)$, where $q \in F$. Because it is not the case that $(s, e) \vdash_M (q, w)$ for any configuration (q, w)($w \neq e$). $(s, e) \vdash_M^* (q, e)$ must be in the reflexive transitive closure of $\vdash_M$ by virtue of reflexivity – that is, $(s, e) = (q, e)$.
Therefore, $s = q$ and thus $s \in F$.
☐ Suppose $s \in F$. Because $\vdash_M^*$ is reflexive, $(s, e) \vdash_M^* (s, e)$. Because $s \in F$, we have $e \in L(M)$ by definition of $L(M)$.

2.1.2 Describe informally the languages accepted by the following DFA.

Solution:
(c) All strings with the same number of as and bs and in which no prefix has more than two bs than as, or as than bs.

(d)All strings with the same number of as and bs and in which no prefix has more than one more a than b, or vice-versa.

2.1.3 Construct DFA accepting each of the following languages.
(c) $\{w \in \{a, b\}^* : w$ has neither aa nor bb as a substring$\}$.
(e) $\{w \in \{a, b\}^* : w$ has both ab and ba as a substring$\}$.

Solution: (c) $M = (K, \Sigma, \delta, sF)$, where
$K = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $s = q_0$, $F = \{q_0, q_1, q_2\}$

q	a	$\delta(q, a)$
q_0	a	q_1
q_0	b	q_2
q_1	a	q_3
q_1	b	q_2
q_2	a	q_1
q_2	b	q_3
q_3	a	q_3
q_3	b	q_3

(e) $M = (K, \Sigma, \delta, sF)$, where
$K = \{q_0, q_1, q_2, q_3, q_4, q_5\}$, $\Sigma = \{a, b\}$, $s = q_0$, $F = \{q_5\}$

q	a	$\delta(q, a)$
q_0	a	q_1
q_0	b	q_2
q_1	a	q_1
q_1	b	q_3
q_2	a	q_4
q_2	b	q_2
q_3	a	q_5
q_3	b	q_3
q_4	a	q_4
q_4	b	q_5
q_5	a	q_5
q_5	b	q_5

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2.2.2 Which regular expression for the languages accepted by the NFA of Problem 2.2.1 .

Solution:

a) a^*

b) $a(ba \cup baa)^*(b \cup ba)$

2.2.6 (a) Find a simple NFA accepting $(ab \cup aab \cup aba)^*$.

(b) Convert the NFA of part (a) to a DFA by the method in section 2.2.

Solution:

(a) $M = (K, \Sigma, \Delta, sF)$, where $K = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $s = q_0$, $F = \{q_0\}$

$(q$	σ	$q_i)$
$(q_0$	a	$q_1)$
$(q_1$	a	$q_2)$
$(q_1$	b	$q_0)$
$(q_1$	b	$q_3)$
$(q_2$	a	$q_0)$
$(q_3$	b	$q_0)$

(b) Determinizing the above machine results in the following DFA:

$K = \{\{q_0\}, \{q_1\}, \{q_3\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_3\}, \emptyset\}$, $\Sigma = \{a, b\}$, $s = \{q_0\}$,
$F = \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}\}$

$\{q\}$	σ	$\{\delta(q, \sigma)\}$
$\{q_0\}$	a	$\{q_1\}$
$\{q_0\}$	b	$\emptyset$
$\{q_1\}$	a	$\{q_3\}$
$\{q_1\}$	b	$\{q_0, q_2\}$
$\{q_0, q_2\}$	a	$\{q_0, q_1\}$
$\{q_0, q_2\}$	b	$\emptyset$
$\{q_0, q_1\}$	a	$\{q_1, q_3\}$
$\{q_0, q_1\}$	b	$\{q_0, q_2\}$
$\{q_3\}$	a	$\emptyset$
$\{q_3\}$	b	$\{q_0\}$
$\{q_1, q_3\}$	a	$\{q_3\}$
$\{q_1, q_3\}$	b	$\{q_0, q_2\}$
$\emptyset$	a	$\emptyset$
$\emptyset$	b	$\emptyset$

2.2.10 Describe exactly what happens when the construction of this section

applied to a FA that is already deterministic.

Solution:
Only $|K|$ of the $2^{|K|}$ states of the new automaton will be reachable.
Each of these states will have $\{q\}$ for some $q \in K$. If we identify $\{q\}$ with q, we have a bijection between the states of the old automata and the reachable states of the new one. With respect to this bijection, δ, s, and F will be identical between the old machine and the new. Since $\sum$ is the same, there is a natural isomorphism between the old and the automaton formed from the new one by discarding unreachable states.

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2.3.4 Using the construction in the proof of theorem 2.3.1, construct FA accepting these languages.
(b) $((a \cup b)^*(e \cup c)^*)^*$.

Solution: (b) Ommited.

~~2.3.7~~ **Apply the construction in Example 2.3.2 to obtain regular expressions responding to each of the FA above. Simplify the resulting regular expressions as much as you can.**

Solution:
(a) $a^*b(ba^*b \cup a)^*$
(b) $((a \cup b)(a \cup b))^*$
(c) $(a \cup b)^*abaa(a \cup b)^*$
(d) $(a \cup \emptyset^*)(ba^*a)^*b(b \cup a)^*$

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2.4.5 Using the pumping theorem and closure under intersection, show that the following are not regular.
(a) $\{ww^R : w \in \{a, b\}^*\}$

Solution:

Assume L is regular, by the closure property under intersection so is $L_1 = L \cap a^*bba^*$.

Consider language L_1.
Let k be the constant whose existence the pumping theorem guarantees.
Choose string $w = a^k bba^k \in L_1$.
Clearly $|w| \geq k$. So the pumping theorem must hold.
Let $w = xyz$ such that $|xy| \leq k$ and $y \neq e$, then $y = a^i$ where $i > 0$.
But then $xy^n z = a^{k+(n-1)i}bba^k$, which is clearly asymmetric for any $n \neq 1$.
The theorem fails, and thus that L_1 is not regular, therefore L is not regular.

~~2.4.8~~ Are the following statements true od false? Explain you answer in each case.

(a) Every subset of a regular language is regular.
(b) Every regular language has a regular proper subset.
(c) If L is regular, then so is $\{xy \mid x \in L \text{ and } y \notin L\}$.
(d) $\{w \mid w = w^R\}$ is regular.
(e) If L is regular, then so is $\{w \mid w \in L \text{ and } w^R \in L\}$.
(f) If C is any set of regular languages, then $\cup C$ is a regular language.
(g) $\{xyx^R \mid x, y \in \Sigma^*\}$ is regular.

Solution:
(a) **False.** Every language, including those we know not to be regular, is a subset of the regular language $\sum^*$.
(b) **False.** The empty set, which is a regular language, has no proper subsets at all, so it certainly cannot have a proper subset which is also a regular.
(c) **True.** $\{xy \mid x \in L \text{ and } y \notin L\} = L \circ \bar{L}$. Since L is regular, so is its complement, and thus their concatenation is regular.
(d) **False.** This can be shown by trying to pump the string $a^k ba^k$. y will have to consist only of as and the resulting xy^2z will unbalanced. Note, however, that this language is regular over an alphabet of one symbol.(It is true when $C$is required to be finite).
(e) **True.** This language is $L \cap L^R$. If L is regular, then so is L^R. Since both L and L^R are regular, so is their intersection.

Solution:

(f) **False.** Any language can be written as the (possibly infinite) union of the singleton sets containing its individual elements. Since not every language is regular, this claim is false.

(g) **True.** $\{xyx^R \mid x, y \in \Sigma^*\} = \Sigma^*$. By letting $x = e$, y can vary over all the strings of Σ^*.)