1 Problem Formulation

acados can handle the following optimization problem

/* Cost function, see section 3 */

$$\min_{x(\cdot), u(\cdot), z(\cdot), s(\cdot), s^{e}} \int_{0}^{T} l(x(\tau), u(\tau), z(\tau), p) + \frac{1}{2} \begin{bmatrix} s_{l}(\tau) \\ s_{u}(\tau) \\ 1 \end{bmatrix}^{\top} \begin{bmatrix} Z_{l} & 0 & z_{l} \\ 0 & Z_{u} & z_{u} \\ z_{l}^{\top} & z_{u}^{\top} & 0 \end{bmatrix} \begin{bmatrix} s_{l}(\tau) \\ s_{u}(\tau) \\ 1 \end{bmatrix} d\tau + m(x(T), z(T), p) + \frac{1}{2} \begin{bmatrix} s_{l}^{e} \\ s_{u}^{e} \\ 1 \end{bmatrix}^{\top} \begin{bmatrix} Z_{l}^{e} & 0 & z_{l}^{e} \\ 0 & Z_{u}^{e} & z_{u}^{e} \\ z_{l}^{e}^{\top} & z_{u}^{e}^{\top} & 0 \end{bmatrix} \begin{bmatrix} s_{l}^{e} \\ s_{u}^{e} \\ 1 \end{bmatrix} \tag{1}$$

/* Initial values, see section 4.1 */

s.t.
$$\underline{x}_0 \le J_{\text{bx},0} x(0) \le \bar{x}_0, \tag{2}$$

/* Dynamics, see section 2 */

$$f_{\text{impl}}(x(t), \dot{x}(t), u(t), z(t), p) = 0,$$
 $t \in [0, T),$ (3)

/* Path constraints with lower bounds, see section 4.2 */

$$\underline{h} \le h(x(t), u(t), p) + J_{\rm sh} s_{\rm l,h}(t), \qquad t \in [0, T), \tag{4}$$

$$\underline{x} \le J_{\text{bx}} x(t) + J_{\text{sbx}} s_{\text{l,bx}}(t), \qquad t \in (0, T), \tag{5}$$

$$\underline{u} \le J_{\text{bu}} u(t) + J_{\text{sbu}} s_{\text{l,bu}}(t), \qquad t \in [0, T), \tag{6}$$

$$g \le C x(t) + D u(t) + J_{sg} s_{l,g}(t),$$
 $t \in [0, T),$ (7)

$$s_{l,h}(t), s_{l,bx}(t), s_{l,bu}(t), s_{l,g}(t) \ge 0,$$
 $t \in [0, T),$ (8)

/* Path constraints with upper bounds, see section 4.2 */

$$h(x(t), u(t), p) - J_{\rm sh} s_{\rm u,h}(t) \le \bar{h},$$
 $t \in [0, T), (9)$

$$J_{\text{bx}}x(t) - J_{\text{sbx}} s_{\text{u,bx}}(t) \le \bar{x}, \tag{10}$$

$$J_{\text{bu}}u(t) - J_{\text{sbu}}s_{\text{u,bu}}(t) \le \bar{u}, \tag{11}$$

$$Cx(t) + Du(t) - J_{sg} s_{u,g} \le \bar{g}, \qquad t \in [0, T), \quad (12)$$

$$s_{u,h}(t), s_{u,bx}(t), s_{u,bu}(t), s_{u,g}(t) \ge 0,$$
 $t \in [0, T), (13)$

/* Terminal constraints with lower bounds, see section 4.3 */

$$\underline{h}^{\mathrm{e}} \leq h^{\mathrm{e}}(x(T), p) + J_{\mathrm{sh}}^{\mathrm{e}} \, s_{\mathrm{lh}}^{\mathrm{e}}, \tag{14}$$

$$\underline{x}^{e} \le J_{\text{bx}}^{e} x(T) + J_{\text{sbx}} s_{\text{l.bx}}^{e},\tag{15}$$

$$g^{e} \le C^{e} x(T) + J_{sg}^{e} s_{l,g}^{e} \le \bar{g}^{e}, \tag{16}$$

$$s_{l,h}^{e}, s_{l,bu}^{e}, s_{l,bu}^{e}, s_{l,e}^{e} \ge 0,$$
 (17)

/* Terminal constraints with upper bound, see section 4.3 */

$$h^{e}(x(T), p) - J^{e}_{sh} s^{e}_{uh} \leq \bar{h}^{e},$$
 (18)

$$J_{\text{bx}}^{\text{e}} x(T) - J_{\text{sbx}} s_{\text{u,bx}}^{\text{e}} \le \bar{x}^{e}, \tag{19}$$

$$C^{e} x(T) - J_{sg}^{e} s_{u,g}^{e} \le \bar{g}^{e} \tag{20}$$

$$s_{\mathbf{u},h}^{\mathbf{e}}, s_{\mathbf{u},b\mathbf{u}}^{\mathbf{e}}, s_{\mathbf{u},b\mathbf{u}}^{\mathbf{e}}, s_{\mathbf{u},g}^{\mathbf{e}} \ge 0,$$
 (21)

with

- state vector $x: \mathbb{R} \to \mathbb{R}^{n_x}$
- control vector $u: \mathbb{R} \to \mathbb{R}^{n_{\mathrm{u}}}$
- algebraic state vector $z: \mathbb{R} \to \mathbb{R}^{n_z}$
- model parameters $p \in \mathbb{R}^{n_{\mathrm{p}}}$
- slacks for path constraints $s_l(t) = (s_{l,bu}, s_{l,bx}, s_{l,g}, s_{l,h}) \in \mathbb{R}^{n_s}$ and $s_u(t) = (s_{u,bu}, s_{u,bx}, s_{u,g}, s_{u,h}) \in \mathbb{R}^{n_s}$

• slacks for terminal constraints $s^{\text{e}}_{\text{l}}(t) = (s^{\text{e}}_{\text{l,bx}}, s^{\text{e}}_{\text{l,g}}, s^{\text{e}}_{\text{l,h}}) \in \mathbb{R}^{n^{\text{e}}_{\text{s}}}$ and $s^{\text{e}}_{\text{u}}(t) = (s^{\text{e}}_{\text{u,bx}}, s^{\text{e}}_{\text{u,g}}, s^{\text{e}}_{\text{u,h}}) \in \mathbb{R}^{n^{\text{e}}_{\text{s}}}$

Some of the following restrictions may apply to matrices in the formulation:

DIAG diagonal

SPUM horizontal slice of a permuted unit matrix SPUME like SPUM, but with empty rows intertwined

Document Purpose This document is only associated to the MATLAB interface of acados. Here, the focus is to give a mathematical overview of the problem formulation and possible options to model it within acados. The problem formulation and the possibilities of acados are also found in the Python interface (the string identifiers are different). The documentation is not exhaustive and does not contain a full description for the MATLAB interface.

You can find examples under the <ACADOS>/examples/acados_matlab_octave directory. The MATLAB source code of CasADi is found here: <ACADOS>/interfaces/acados_matlab_octave and should serve as a more extensive documentation about the possibilities.

2 Dynamics

The system dynamics term is used to connect state trajectories from adjacent shooting nodes by means of equality constraints. The system dynamics equation (3) is a placeholder for equations (22), (23) or (24). Therefore, the dynamics can be formulated in different ways in acados. This section and table 1 summarizes the options.

2.1 Implicit Dynamics

The most general way to provide a continuous time ODE in acados is to define the function $f_{\text{impl}}: \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_x+n_z}$ which is fully implicit DAE formulation describing the system as:

$$f_{\text{impl}}(x, \dot{x}, u, z, p) = 0. \tag{22}$$

acados can discretize $f_{\rm impl}$ with a classical implicit Runge-Kutta (irk) or a structure exploiting implicit Runge-Kutta method (irk_gnsf). Both discretization methods are set using the 'sim_method' identifier in a acados_ocp_opts class instance.

2.2 Explicit Dynamics

Alternatively, acados offers an explicit Runge-Kutta integrator (erk), which can be used with explicit ODE models, i.e., models of the form

$$f_{\text{expl}}(x, u, p) = \dot{x}. \tag{23}$$

2.3 Discrete Dynamics

Another option is to provide a discrete function that maps state x_i , control u_i and parameters p_i from shooting node i to the state x_{i+1} of the next shooting node i + 1, i.e., a function

$$x_{i+1} = f_{\operatorname{disc}}(x_i, u_i, p_i). \tag{24}$$

Table 1: Dynamics definitions.

Term	String identifier	Data type	Required
f_{impl} respectively f_{expl}	dyn_expr_f	CasADi expression	yes
$f_{ m disc}$	dyn_exp_phi	CasADi expression	yes
-	<pre>dyn_type</pre>	<pre>string ('explicit', 'implicit' or 'discrete')</pre>	yes

3 Cost

There are different acados modules to model the cost functions in equation (1).

- $l: \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \to \mathbb{R}$ is the Lagrange objective term.
- $m: \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}$ is the Mayer objective term.

to define which one is used set $cost_type$ for l, $cost_type_e$ for m.

Setting the slack penalties in equation (1) is done in the same way for all cost modules, see table 2 for an overview.

Table 2: Cost module slack variable options.

Term	String id	Data type	Required
Z_{l}	cost_Zl	double, \mathbf{DIAG}	no
$Z_{ m u}$	${\tt cost_Zu}$	double, \mathbf{DIAG}	no
$z_{ m l}$	cost_zl	double	no
$z_{ m u}$	cost_zu	double	no
$Z_{ m l}^{ m e}$	cost_Zl_e	double, \mathbf{DIAG}	no
$Z_{ m u}^{ m e}$	cost_Zu_e	double, \mathbf{DIAG}	no
$z_{ m l}^{ m e}$	cost_zl_e	double	no
$z_{ m u}^{ m e}$	cost_zu_e	double	no

Moreover, you can specify $cost_Z$, to set Z_l , Z_u to the same values, i.e., use a symmetric L2 slack penalty. Similarly, $cost_Z$, $cost_Z_e$, $cost_Z_e$ can be used to set symmetric slack L1 penalties, respectively penalties for the terminal slack variables.

Note, that you don't have to set slack variables $s_l(t)$, $s_l^e(t)$, $s_u(t)$ and $s_u^e(t)$ manually. They are part of the solution and its dimensions are determined by acados from the associated matrices $(Z_l, Z_u, J_{sh}, J_{sbx}, J_{sbu}, \text{ etc.})$.

3.1 Cost module: auto

Set cost_type to auto (default). In this case acados detects if the cost function specified is a linear least squares term and transcribe it in the corresponding form. Otherwise, it is formulated using the external cost module. Note: slack penalties are optional and we plan to detected them form the expressions in future versions. Table 3 shows the available options.

Table 3: Cost module auto options.

Term	String identifier	Data type	Required
l	cost_expr_ext_cost	CasADi expression	yes

3.2 Cost module: external

Set cost_type to ext_cost. See table 4 for the available options.

Table 4: Cost module external options.

Term	String identifier Data type		Required
l	cost_expr_ext_cost	CasADi expression	yes
m	cost_expr_ext_cost_e	CasADi expression	yes

3.3 Cost module: linear least squares

In order to activate the linear least squares cost module, set cost_type to linear_ls. The Lagrange cost term has the form

$$l(x, u, z) = \frac{1}{2} \left\| \underbrace{V_x \, x + V_u \, u + V_z \, z}_{y} - y_{\text{ref}} \right\|_{W}^{2}$$
 (25)

where matrices $V_x \in \mathbb{R}^{n_y \times n_x}$, $V_u \in \mathbb{R}^{n_y \times n_u}$ are $V_z \in \mathbb{R}^{n_y \times n_z}$ map x, u and z onto y, respectively and $W \in \mathbb{R}^{n_y \times n_y}$ is the weighing matrix. The vector $y_{\text{ref}} \in \mathbb{R}^{n_y}$ is the reference.

Similarly, the Mayer cost term has the form

$$m(x, u, z) = \frac{1}{2} \left\| \underbrace{V_{x}^{\text{e}} x}_{y^{\text{e}}} - y_{\text{ref}}^{\text{e}} \right\|_{W^{\text{e}}}^{2}$$
 (26)

where matrix $V_x^{\mathrm{e}} \in \mathbb{R}^{n_{y^{\mathrm{e}}} \times n_x}$ maps x onto y^{e} and $W^{\mathrm{e}} \in \mathbb{R}^{n_{y^{\mathrm{e}}} \times n_{y^{\mathrm{e}}}}$ is the weighing matrix. The vector $y_{\mathrm{ref}}^{\mathrm{e}} \in \mathbb{R}^{n_{y^{\mathrm{e}}}}$ is the reference

See table 5 for the available options of this cost module.

Table 5: Cost module linear 1s options.

Table 6. Cost modale Timodi_iD options.			
Term	String identifier	Data type	Required
$\overline{V_x}$	cost_Vx	double	yes
V_u	${\tt cost_Vu}$	double	yes
V_z	cost_Vz	double	yes
W	cost_W	double	yes
y_{ref}	cost_y_ref	double	yes
V_x^{e}	cost_Vx_e	double	yes
W^{e}	cost_W_e	double	yes
$y_{ m ref}^{ m e}$	cost_y_ref_e	double	yes

3.4 Cost module: non-linear least squares

In order to activate the non-linear least squares cost module, set cost_type to nonlinear_ls.

The non-linear least squares cost function has the same basic form as eqns. (25 - 26) of the linear least squares cost module. The only difference is that y and y^e are defined by means of CasADi expressions, instead of via matrices V_x , V_u , V_z and V_x^e . See table 6 for the available options of this cost module.

Table 6: Cost module nonlinear_ls options.

Term	String identifier	Data type	Required
$y \ W \ y_{ m ref}$	<pre>cost_expr_y cost_W cost_y_ref</pre>	CasADi expression double double	yes yes
$y^{ m e} \ W^{ m e} \ y^{ m e}_{ m ref}$	<pre>cost_expr_y_e cost_W_e cost_y_ref_e</pre>	CasADi expression double double	yes yes

4 Constraints

This section is about how to define the constraints equations (2) and (4 - 21). The constraint type can be set using the identifier 'constr_type' and 'constr_type_e' for the path constraints and terminal constraints, respectively. The string identifier options are found in table 7. The default setting is 'bgh'.

Table 7: Constraint type string identifier.

	Supported constraints			
String identifier	simple bounds	polytopic constr.	general non- linear constr.	positive definite constr.
bgh bgp	yes yes	yes yes	yes yes	no yes

4.1 Initial State

Note: An initial state is not required. For example for moving horizon estimation (MHE) problems it should not be set.

Two possibilities exist to define the initial states equation (2): a simple syntax and an extended syntax.

Simple syntax defines the full initial state $x(0) = \bar{x}_0$. The options are found in table 8.

Table 8: Simple syntax for setting the initial state.

Term	String identifier	Data type	Required
\bar{x}_0	constr_x0	double	no

Extended syntax allows to define upper and lower bounds on a subset of states. The options for the extended syntax are found in table 9.

Table 9: Extended syntax for setting the initial state.

Term	String identifier	Data type	Required
\underline{x}_0	constr_lbx_0	double	no
\bar{x}_0	$constr_ubx_0$	double	no
$J_{ m bx,0}$	constr_Jbx_0	double	no

4.2 Path Constraints

Table 10 shows the options for defining the path constraints equations (4 - 13). Here, matrices

- $J_{\rm sh}$, maps lower slack vectors $s_{\rm l,h}(t)$ and upper slack vectors $s_{\rm u,h}(t)$ onto non-linear constraint expressions h(x(t), u(t), p).
- J_{bx} , J_{bu} map x(t) and u(t) onto its bounds vectors \underline{x} , \bar{x} and \underline{u} , \bar{u} , respectively.
- J_{sx} , J_{su} map lower slack vectors $s_{\text{l,bx}}(t)$, $s_{\text{l,bu}}(t)$ and upper slack vectors $s_{\text{u,bx}}(t)$, $s_{\text{u,bu}}(t)$ onto x(t) and u(t), respectively.
- J_{sg} map lower slack vectors $s_{\text{l,g}}(t)$ and upper slack vectors $s_{\text{u,g}}(t)$ onto lower and upper equality bounds \underline{g} , \bar{g} , respectively.
- C, D map x(t) and u(t) onto lower and upper inequality bounds g, \bar{g} (polytopic constraints)

Table 10: Path constraints options.

Term	String identifier	Data type	Required
		double, SPUM	
$J_{ m bx}$	constr_Jbx	,	no
$\frac{x}{\bar{x}}$	constr_lbx	double	no
\bar{x}	constr_ubx	double	no
$J_{ m bu}$	constr_Jbu	double, SPUM	no
\underline{u}	constr_lbu	double	no
$rac{u}{ar{u}}$	constr_ubu	double	no
	_		
C	constr_C	double	no
D	constr D	double	no
q	constr_lg	double	no
$rac{g}{ar{g}}$	constr_ug	double	no
9	0011201_46	404010	110
h	constr_expr_h	CasADi expression	no
h	constr_lh	double	no
$rac{\underline{h}}{ar{h}}$	constr_uh	double	no
	0011001_411	404010	110
$J_{ m sbx}$	constr_Jsbx	double, \mathbf{SPUME}	no
$J_{ m sbu}$	constr_Jsbu	double, SPUME	no
$J_{ m sg}$	constr_Jsg	double, SPUME	no
$J_{ m sbx}$	constr_Jsh	double, SPUME	no
- 552	_		

4.3 Terminal Constraints

Table 11 shows the options for defining the terminal constraints equations (14 - 21). Here, matrices

- $J_{\rm sh}^{\rm e}$, maps lower slack vectors $s_{\rm l,h}^{\rm e}(t)$ and upper slack vectors $s_{\rm u,h}^{\rm e}(t)$ onto non-linear terminal constraint expressions $h^{\rm e}(x(T),p)$.
- $J_{\mathrm{bx}}^{\mathrm{e}}$ maps x(T) onto its bounds vectors $\underline{x}^{\mathrm{e}}$ and \bar{x}^{e} .
- J_{sbx} maps lower slack vectors $s_{l,bx}^e$ and upper slack vectors $s_{u,bx}^e$ onto x(T).
- J_{sg}^{e} map lower slack vectors $s_{\text{l,g}}^{\text{e}}(t)$ and upper slack vectors $s_{\text{u,g}}^{\text{e}}(t)$ onto lower and upper equality bounds \underline{g}^{e} , \overline{g}^{e} , respectively.
- C^{e} maps x(T) onto lower and upper inequality bounds g^{e} , \bar{g}^{e} (polytopic constraints)

Table 11: Terminal constraints options.

Term	String identifier	Data type	Required
$J_{ m bx}^{ m e}$	constr_Jbx_e	double, \mathbf{SPUM}	no
$\frac{\underline{x}^{\mathrm{e}}}{\bar{x}^{\mathrm{e}}}$	constr_lbx_e	double	no
\bar{x}^{e}	constr_ubx_e	double	no
C^{e}	constr_C_e	double	no
$rac{g^{ m e}}{ar{g}^{ m e}}$	constr_lg	double	no
$\overline{ar{g}}^{\mathrm{e}}$	constr_ug	double	no
h^{e}	constr_expr_h_e	CasADi expression	no
$rac{\underline{h}}{ar{h}}^{\mathrm{e}}$	constr_lh_e	double	no
$ar{h}^{\mathrm{e}}$	constr_uh_e	double	no
$J_{ m sbx}^{ m e}$	constr Jsbx	double, SPUME	no
$J_{ m sg}^{ m e}$	constr Jsg e	double, SPUME	no
$J_{ m sbx}^{ m e}$	constr_Jsh_e	double, SPUME	no

Table 12: Model set(id, data) options

String id	Data type	Description	Required	
name	string	model name, used for code generation, default: 'ocp_model'	no	
T	double	end time	yes	
sym_x	CasADi expr.	state vector x in problem formulation in sec. 1	?	
sym_u	CasADi expr.	control vector u in problem formulation in sec. 1	?	
sym_xdot	CasADi expr.	\dot{x} in implicit dynamics eq. (3)	?	
	Additionally, options from tables $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ and 11 , apply here.			

5 External links

A table sheet with additional info is found here:

 $\verb|https://docs.google.com/spreadsheets/d/1rVRycLnCyaWJLwnV47u30Vokp7vRu68og30hlDbSjDU/edit?usp=sharing| | the continuous continuou$

6 Model

A model instance is created using ocp_model = acados_ocp_model(). It contains all model definitions for either simulating the system or using it in the solver.

7 Solver

The solver options are created as an instance from the acados_ocp_opts() class.

Table 13: Solver set(id, option) options

		able 13:	Solver set(id, option) options
String identifier	Type	Default	Description
Code generation			
compile_interface	string	'auto'	in ('auto', 'true', 'false')
codgen_model	string	'true'	in ('true', 'false')
compile_model	string	'true'	in ('true', 'false')
output_dir	string	'build'	codegen output directory
Shooting nodes			
param_scheme_N	int > 1	10	uniform grid: number of shooting nodes; acts together with time T from model.
<pre>shooting_nodes or param scheme_shooting_nodes</pre>	doubles	[]	nonuniform grid 1: direct definition of the shooting node times
time_steps	${\rm doubles}$	[]	nonuniform grid 2: definition of deltas between shooting nodes
NLP solver			
nlp_solver	string	'sqp'	in ('sqp', 'sqp_rti')
nlp_solver_exact_hessian	string	'false'	use exact hessian calculation: (")in ('true', 'false'), use exact
nlp_solver_max_iter	int > 1	100	maximum number of NLP iterations
nlp_solver_tol_stat	double	10^{-6}	
nlp_solver_tol_eq	double	10^{-6}	
nlp_solver_tol_ineq	double	10^{-6}	
nlp_solver_tol_comp	double	10^{-6}	
nlp_solver_ext_qp_res	int	0	compute QP residuals at each NLP iteration
nlp_solver_step_length	double	1.0	fixed step length in SQP algorithm
rti_phase	int	0	RTI phase: (1) preparation, (2) feedback, (0) both
QP solver			
qp_solver	string	\longrightarrow	Defines the quadratic programming solver and condensing strategy. See table 14
globalization	string,	fixed_step	globalization
alpha_min	double	0.05	
alpha_reduction	double	0.7	
qp_solver_iter_max	int	50	
qp_solver_cond_ric_alg	int	0	factorize hessian in the condensing: (0) no, (1) yes
qp_solver_ric_alg	int	0	HPIPM specific
qp_solver_warm_start	$_{ m int}$	0	$\left(0\right)$ cold start, $\left(1\right)$ warm start primal variables, $\left(2\right)$ warm start and dual variables
warm_start_first_qp	int	0	warm start even in first SQP iteration: (0) no, (1) yes
sim_method	string	'irk'	'erk', 'irk', 'irk_gnsf'
${\tt sim_method_num_stages}$	int	4	Runge-Kutta int. stages: (1) RK1, (2) RK2, (4) RK4
${\tt sim_method_num_steps}$	int	1	
sim_method_newton_iter	int	3	
<pre>gnsf_detect_struct</pre>	string	'true'	
${\tt regularize_method}$	string	\longrightarrow	Defines the hessian regularization method. See table 15
print_level	$int \ge 0$	0	verbosity of the solver: (0) silent, (>0) print solver output during solution finding
${\tt levenberg_marquardt}$	double	0.0	
exact_hess_dyn	int	1	in $(0, 1)$, compute and use hessian in dynamics, only if 'nlp_solver
			exact_hessian' = 'true'
exact_hess_cost	int	1	in (0, 1), only if 'nlp_solver_exact_hessian' = 'true'
exact_hess_constr	$_{ m int}$	1	in $(0, 1)$, only if 'nlp_solver_exact_hessian' = 'true'

Table 14: Solver set(id, option) options, 'qp_solver' options. The availability depends on for which solver interfaces acados was linked to.

Solver lib	Condensing	String identifier
HPIPM	partial full	<pre>partial_condensing_hpipm (default) full_condensing_hpipm</pre>
HPMPC	partial	partial_condensing_hpmpc
OOQP	partial full	${ t partial_condensing_ooqp}$ ${ t full_condensing_ooqp}$
OSQP QORE qpDUNES qpOASES	partial full partial full	<pre>partial_condensing_osqp full_condensing_qore partial_condensing_qpdunes full_condensing_qpoases</pre>

Table 15: Solver set(id, option) options, 'regularize_method' options.

String identifier	Description
no_regularize mirror project project_reduc_hess convexify	don't regularize (default)