ILLINOIS INSTITUTE OF TECHNOLOGY

Privacy Preserving Bag Preparation for Learning from Label Proportion

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CS-591 Thesis Defense

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M.S CDS & O.R

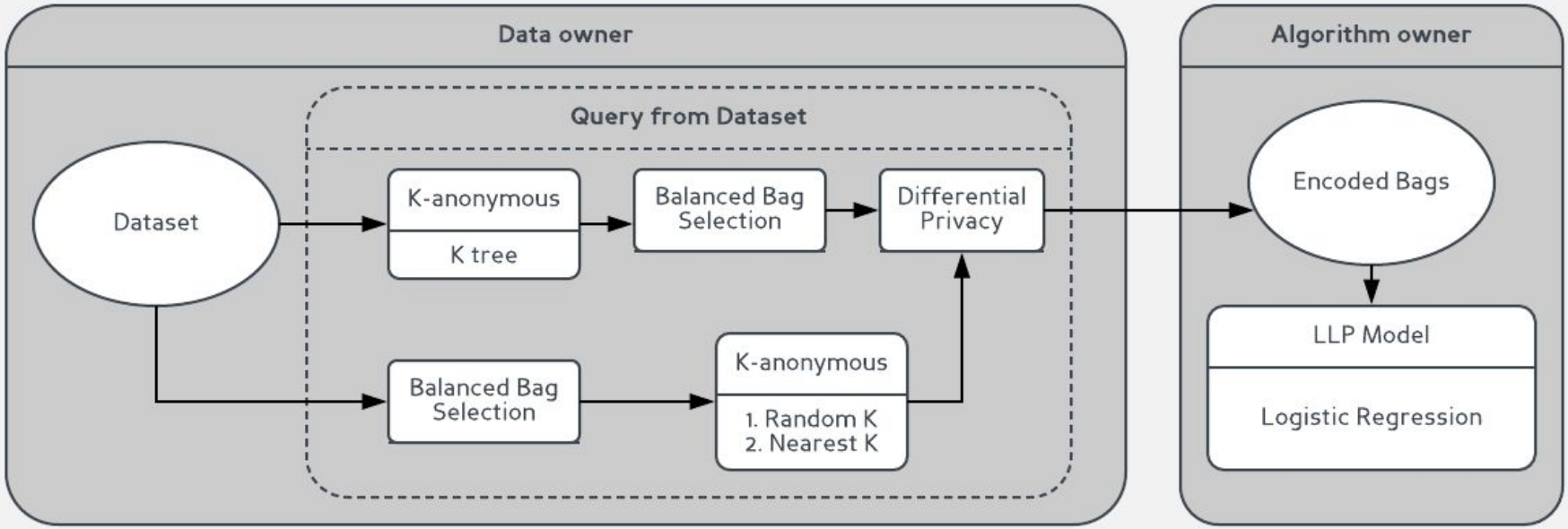
Main Target

- **Which Machine Learning model to choose?**
  (LLP: Learning from label proportion framework)
- **How to apply PPDM to LLP model?**
  (PPDM: Privacy preserving Data Mining standards)
- **How to ensure accuracy loss for PPDM LLP model?**

PPDM LLP Model

PPDM: Privacy preserving Data Mining standards
LLP: Learning from label proportion framework

- No sensitive information can be retrieved from the dataset
- No record can be traced traced in the dataset.



LLP Model

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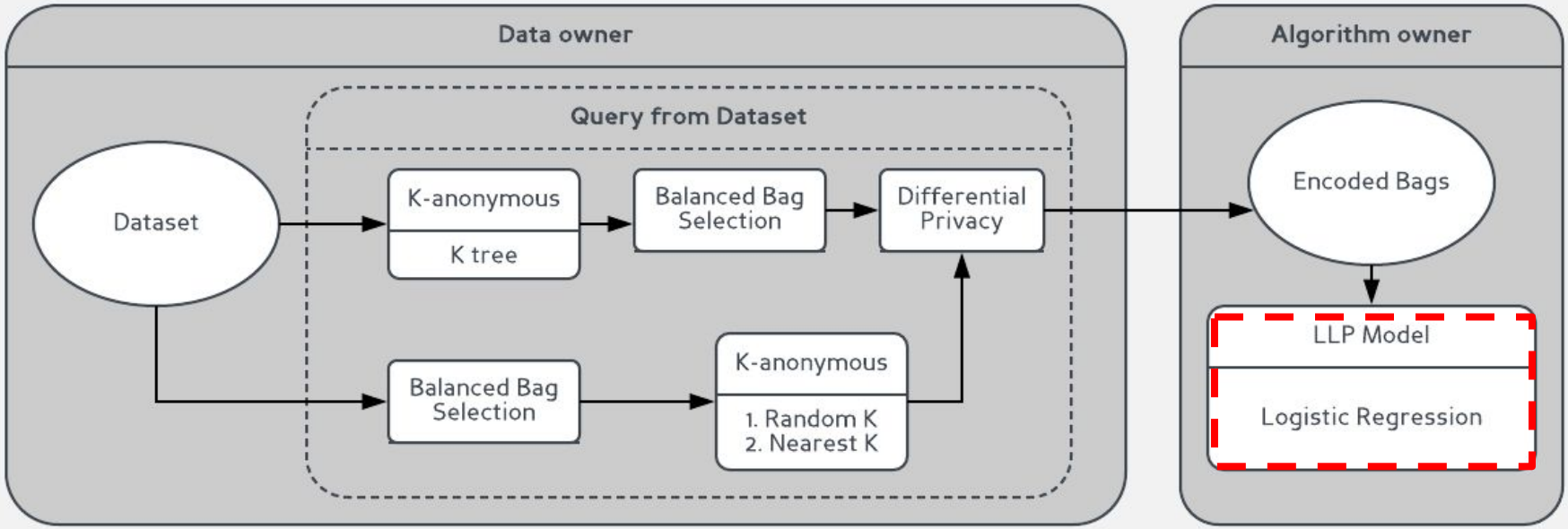
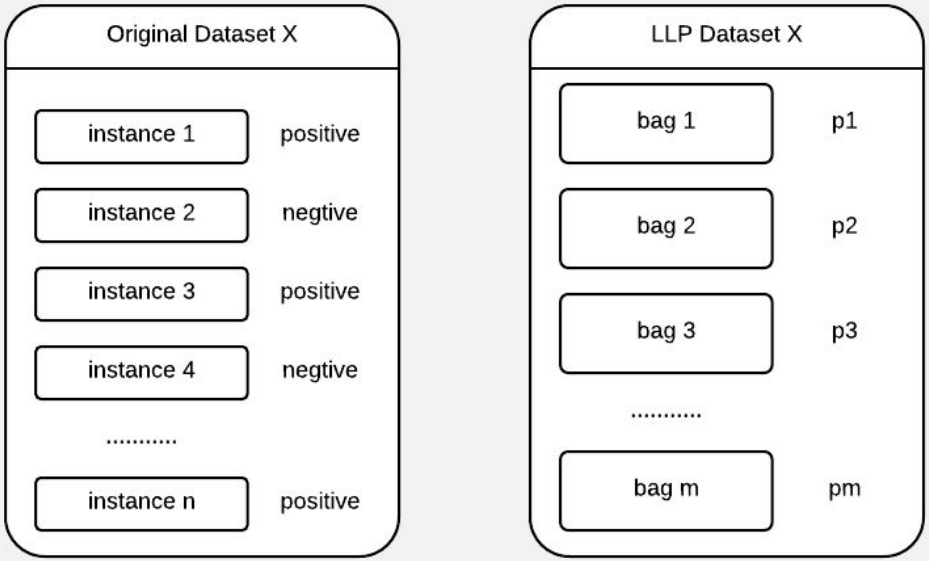


LLP model

LLP is a Learning from label proportion framework which enable the learner to train on unlabeled data

- In LR setting, each record has to be labeled.
- In LLp setting, Instances are put into bags with prior label distribution



LLP model

Use Logistic regression- LLP model to fit the dataset
Each bag's probability $p(X_i, p_i|\theta)$ **can be modeled as Beta distribution**

- X represents the feature matrix, Xi represents the feature matrix of ith bag
- pi represents the ith prior label distribution of ith bag
- p(Xij, θ) is the logistic function, which has domain [0, 1]
- p(Xi, θ) is the average label distribution which has domain [0, 1]

$$p(X_i, p_i|\theta) = p(X_i;\theta)^{p_i}(1-p(X_i;\theta))^{1-p_i}$$

$$p(X, p|\theta) = \prod_{i=1}^{n} p(X_i;\theta)^{p_i}(1-p(X_i;\theta))^{1-p_i}$$

$$p(\theta|X, p) \propto p(\theta)p(X, p|\theta) \propto p(X, p|\theta)$$

LLP model

Use Logarithmic Likelihood function to estimate the parameter
The optimal parameter is also the optimal parameter for the loss function

- X represents the feature matrix, Xi represents the feature matrix of ith bag
- Add up regularization term to ensure θ is close to zero vector.

$$l(\theta) = log(p(X, p|\theta))$$

$$= log \prod_{i=1}^{n} p(X_i; \theta)^{p_i} (1 - p(X_i; \theta))^{1-p_i}$$

$$= \sum_{i=1}^{n} (p_i log(p(X_i; \theta))) + \sum_{i=1}^{n} ((1 - p_i) log(1 - p(X_i; \theta)))$$

$$loss(P, p(X, \theta)) = -l(\theta) + \frac{\lambda \theta^2}{2}$$

LLP model

Prediction of a single instance in LLP model:

$$p(X_{i,j};\theta) = \frac{1}{1+e^{\theta^T X_{i,j}}}$$

Prediction of the average label distribution in the ith bag of the LLP model:

$$p(X_i;\theta) = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{1+e^{\theta^T X_{i,j}}}$$

Loss function:

$$loss(P, p(X,\theta)) = -l(\theta) + \frac{\lambda\theta^2}{2}$$

$$\frac{\partial loss(P, p(X,\theta))}{\partial\theta} = \frac{1}{m}\sum_{i=1}^{n}\left(\frac{p_i - p(X_i;\theta)}{p(X_i;\theta)(1-p(X_i;\theta))}\right)\sum_{j=1}^{m} p(X_{i,j};\theta)(1-p(X_{i,j};\theta))X_{i,j} + \lambda\theta$$

Balanced Bag Selection

PPDM: Privacy preserving Data Mining standards
LLP: Learning from label proportion framework

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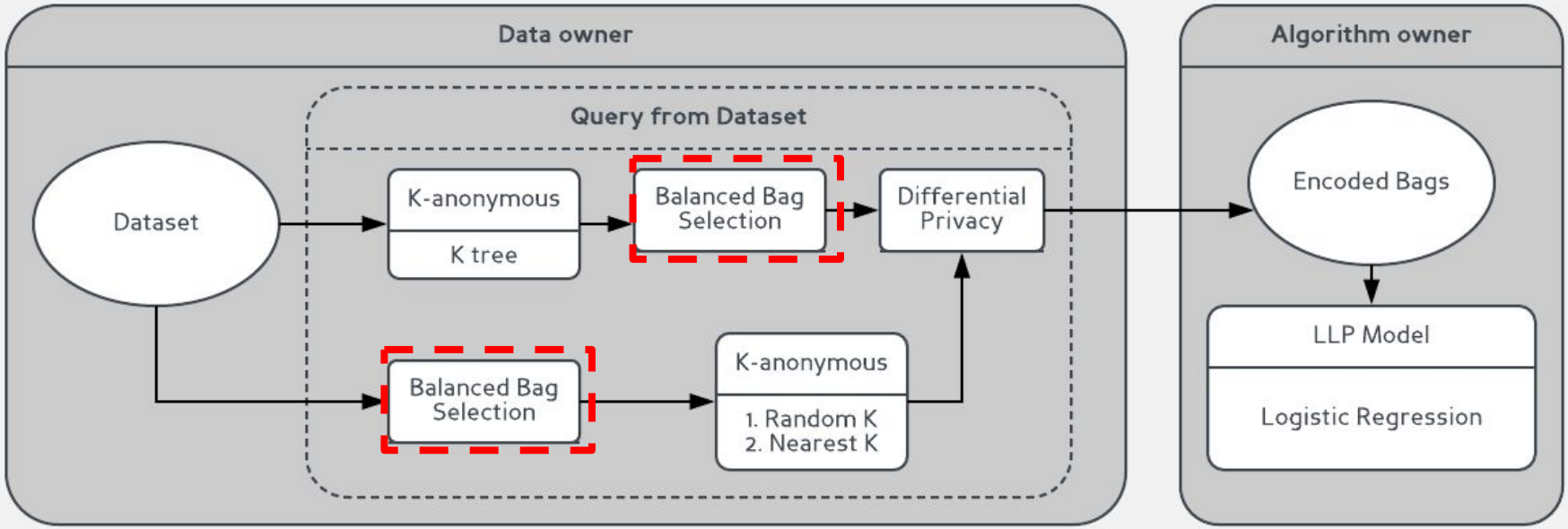
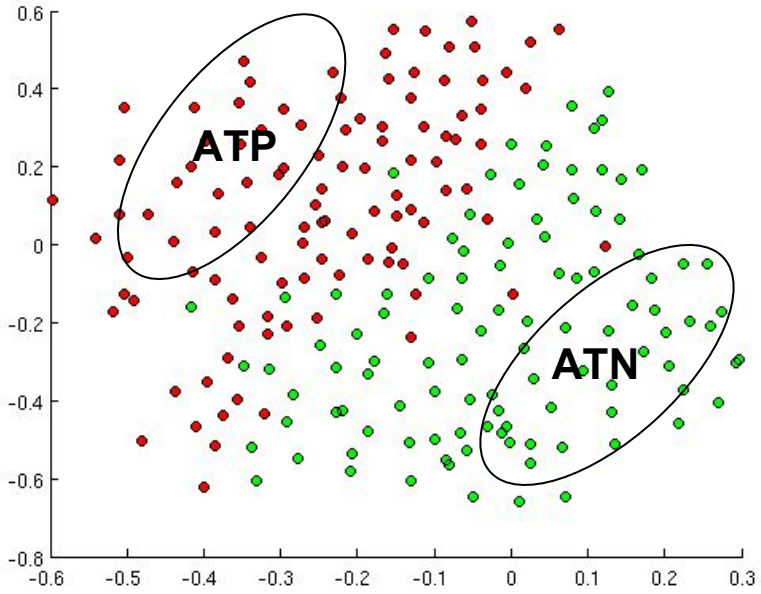
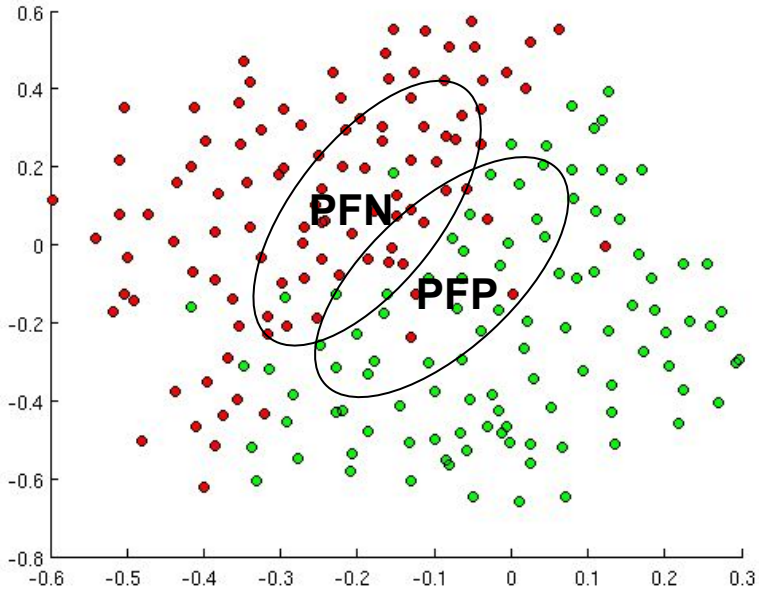


Balanced Bag Selection

We define highly confident instances as ”absolute true positive”(ATP) or ”absolute true negative”(ATN), some lowly confident instances as ”possible false positive”(PFP) or”possible false negative”(PFN).





[1] Image downloaded from Andrew Ng’s website

Experiment

Select bags i.i.d from confident , unconfident instances. The loss function punishes both the misclassification and low-precision model .

- Misclassification is punished by the wide margin, ATP&ATN
- The low precision is punished by the narrow margin: PFN&PFP

Table 4.3. Accuracy/F1 of different models

model	Train on	Positive F1	Negative F1	Average F1
logistic regression	Adult	72	91	86
LLP	Adult	45	85	75
Balanced-LLP	Adult	57	89	81
logistic regression	Instagram	64	96	92
LLP	Instagram	27	86	84
Balanced-LLP	Instagram	61	95	91

Experiment

Table 4.1. Average F1 of Bag selection Methods

Bag selection	Train on	Average F1
I.I.D Selection	Adult	0.75
ATP/ATN PFP/PFN	Adult	0.81
ATP/PFP PFN/ATN	Adult	0.81
ATP/ATN	Adult	77
PFP/PFN	Adult	72
I.I.D Selection	Instagram	0.86
ATP/ATN PFP/PFN	Instagram	0.91
ATP/PFP PFN/ATN	Instagram	0.87
ATP/ATN	Instagram	0.88
PFP/PFN	Instagram	0.30

PPDM Standards

PPDM: privacy preserving data mining.
PPDM ensures necessary security when querying the database.

- K-anonymous: any record in the database cannot be distinguished from other k-1 records.
- L-DIversity: at least L distinct labels for the records in each block
- Differential privacy: add Laplace noise to the dataset to ensure:

$$e^{-\varepsilon} \leq \frac{pr(M(X')=C)}{pr(M(X)=C)} \leq e^{\varepsilon}$$

X' is neighbour dataset , M is the query function, and C is the query result

PPDM Standards

Enough sensitivity, enough security for the differential privacy model

- Sensitivity is the maximal difference of the query result between two neighbour datasets.
- Add laplace noise to the query result to ensure differential privacy.

$$let\ \ \Delta f = max|M(X) - M(X')|,\ \ z \sim Lap(b),\ \ b = \Delta f/\varepsilon$$

$$\frac{pr(M(X')=C)}{pr(M(X)=C)} = \frac{\frac{1}{2b} e^{-|M(X')|/b}}{\frac{1}{2b} e^{-|M(X)|/b}} = e^{(|M(X)| - |M(X')|)/b} \le e^{(\Delta f)/b} \le e^{\varepsilon}$$

$$reversely,\ \ \frac{pr(M(X')=C)}{pr(M(X)=C)} \ge e^{(-\Delta f)/b} \ge e^{-\varepsilon}$$

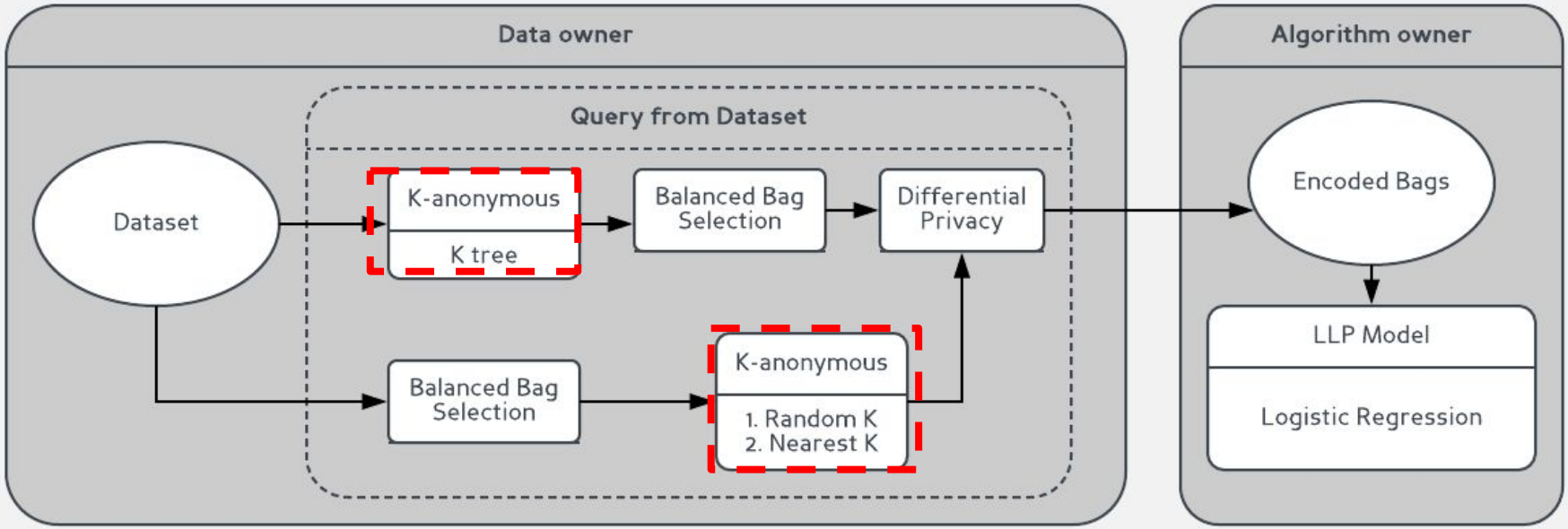
$$e^{-\varepsilon} \le \frac{pr(M(X')=C)}{pr(M(X)=C)} \le e^{\varepsilon}$$

[2] Deduction is quoted from Dwork's work.

K-Anonymous

Aggregate k instances in each bag and get mean feature vectors.

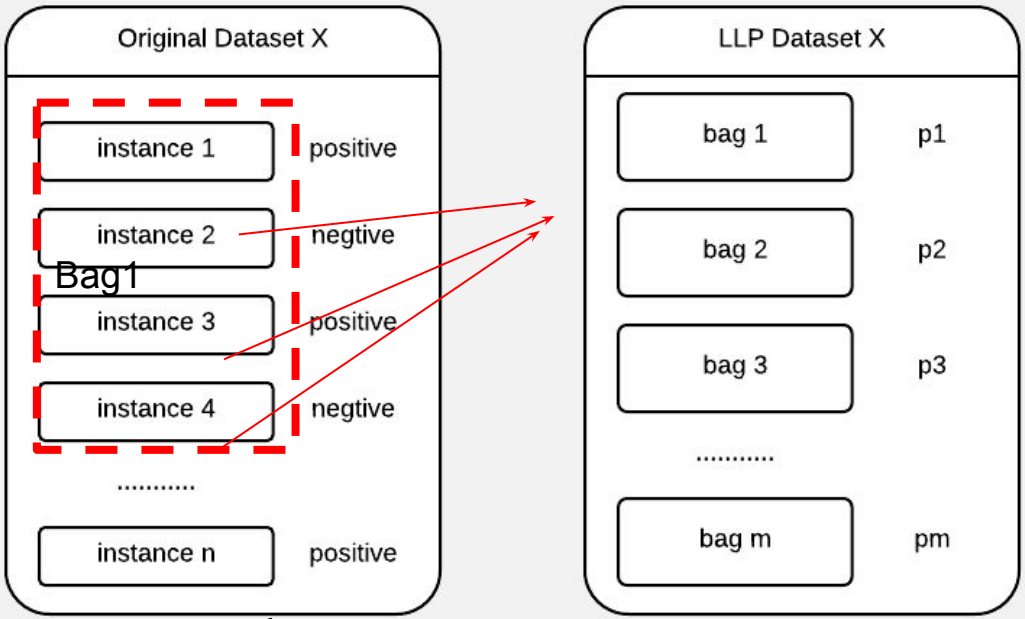
- Replace original instance with mean feature vector
- Label distribution unchanged



K-anonymous

Aggregation both positive and negative instances together

- Random K aggregation: randomly aggregate every k instances in each bag.
- Nearest K aggregation: find every k instances closest to one among k in each bag and aggregate them.



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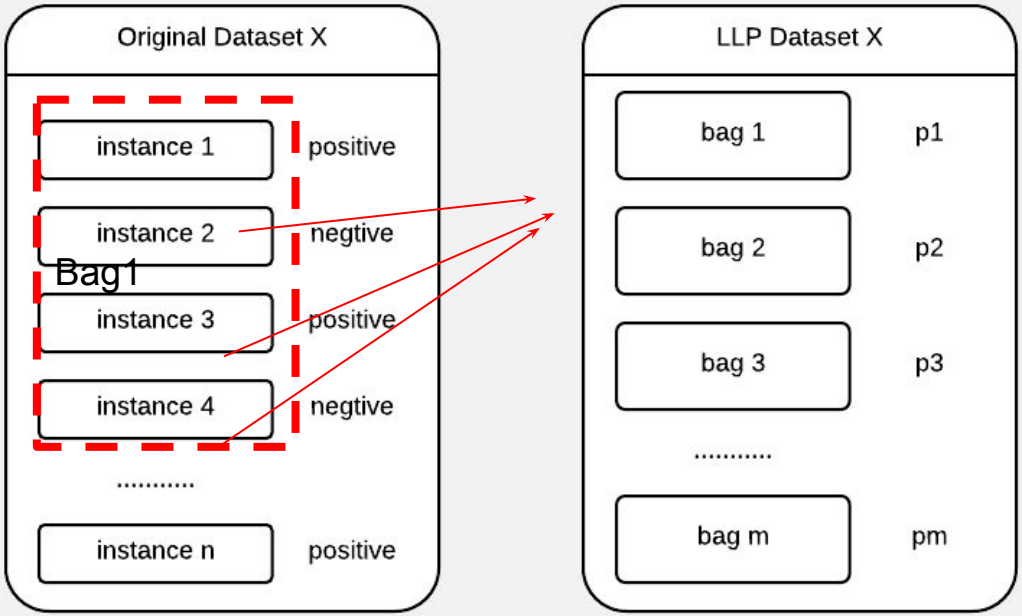

Accuracy Loss for LR

LR model guarantees accuracy loss when aggregation +/- instances separately.
LLP model guarantees accuracy loss and aggregate +/- instances together.

$$
\begin{aligned}
f(\theta) &= loss(\theta) \\
&= -\sum_{i=1}^{n} (y_i log P(x_i, \theta) + (1 - y_i) log(1 - P(x_i, \theta))) \\
&= -\sum_{i=1}^{n} (y_i log \frac{1}{1 + e^{\theta^T x_i}} + (1 - y_i) log(1 - \frac{1}{1 + e^{\theta^T x_i}})) + \frac{1}{2}\lambda\theta^2 \\
f'(\theta) &= -\sum_{i=1}^{n} (-y_i \frac{e^{\theta^T x_i}}{1 + e^{\theta^T x_i}} x_i + (1 - y_i) \frac{1}{1 + e^{\theta^T x_i}} x_i) + \lambda\theta \\
&= -\sum_{n=i}^{n} \frac{1}{1 + e^{\theta^T x_i}} x_i - y_i x_i + \lambda\theta \\
f(0) &= -\sum_{i=1}^{n} (y_i(-1) + (1 - y_i)(-1)) = n \\
f'(0) &= -\sum_{i=1}^{n} (\frac{1}{2} x_i - y_i x_i) = -\sum_{i=1}^{n} (\frac{1}{2} - y_i) x_i \\
f(\theta) &\approx f(0) + \theta^T f'(0) = n - \theta^T \sum_{i=1}^{n} (\frac{1}{2} - y_i) x_i \\
&= n + \frac{1}{2}\theta^T \sum_{i=1,pos}^{n} x_i = n + \frac{1}{2}\theta^T \sum_{i=1,pos}^{n} \bar{x}
\end{aligned}
$$

Accuracy Loss for LLP

LR model guarantees accuracy loss when aggregation +/- instances separately.

LLP model guarantees accuracy loss and aggregate +/- instances together.

$$
\begin{aligned}
f(\theta) &= loss(\theta) \\
&= -\sum_{i=1}^{n}(p_i log(p(X_i;\theta))) + \sum_{i=1}^{n}((1-p_i)log(1-p(X_i;\theta))) \\
&\quad + \frac{1}{2}\lambda\theta^2 \\
f'(\theta) &= \frac{1}{m}\sum_{i=1}^{n}(\frac{P_i - p(X_i;\theta)}{p(X_i;\theta)(1-p(X_i;\theta))})\sum_{j=1}^{m}p(X_{i,j};\theta)(1-p(X_{i,j};\theta))X_{i,j} \\
&\quad + \lambda\theta \\
f(0) &= -\sum_{i=1}^{n}(p_i(-1) + (1-P_i)(-1)) = n \\
f'(0) &= \frac{1}{m}\sum_{i=1}^{n}(\frac{P_i - \frac{1}{2}}{\frac{1}{2}*\frac{1}{2}}\sum_{j=1}^{m}\frac{1}{2}*\frac{1}{2}X_{i,j}) \\
&= \frac{1}{m}\sum_{i=1}^{n}((P_i - \frac{1}{2})\sum_{j=1}^{m}X_{i,j}) \\
&= \sum_{i=1}^{n}((P_i - \frac{1}{2})\bar{X}_i) \\
f(\theta) &\approx f(0) + \theta^T f'(0) = n + \theta^T\sum_{i=1}^{n}((P_i - \frac{1}{2})\bar{X}_i)
\end{aligned}
$$

Data Introduction

Adult Dataset: 48842 records, 25%+, 75% -. Categorical and numerical features

- Discretizing data, dimension 338
- Balanced bag selection
- Threshold is 0.9, naive Bayes model
- 100 bags, 100 instances in each bag.
- Prior label distribution vector as label for bags

Instagram Hostile Comment: 30987 records, 13%+, 87%. Text categorical features.

- Discretizing data, dimension 6046
- Balanced bag selection
- Threshold is 0.9
- 100 bags, 100 instances in each bag.
- Prior label distribution vector as label for bags

eg:

'This is me, though, emotionally ;D', 'Good', ...,
'Well Yeah,But they dont have an actor or a new suit yet,sadly',
'Maybe Another MCU film before civil war',
"Robert Downey Jr. said there's a big announcement in a few days. Maybe it's about Spidey.

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Instagram Hostile Comment: 30987 records, 13%+, 8

- Discretizing data, dimension 6046
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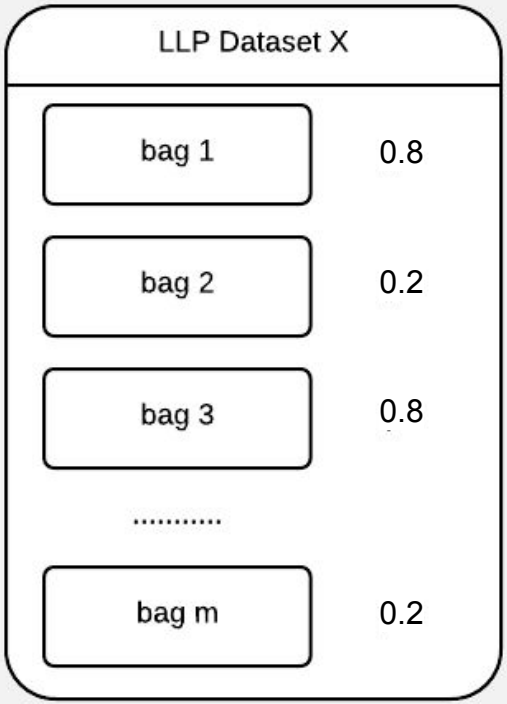

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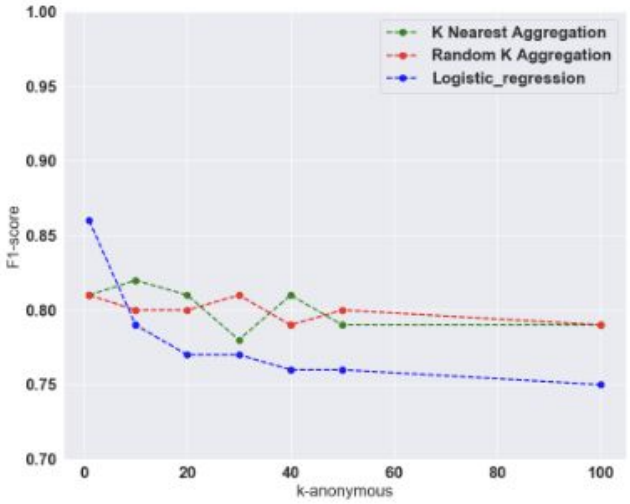
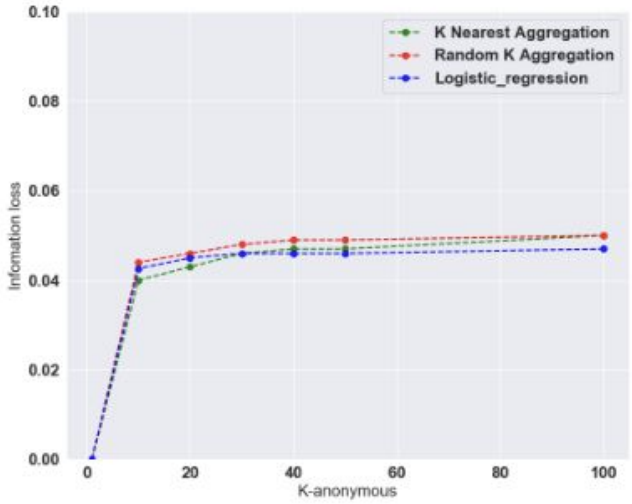
Performance

LLP model is much more robust than LR model when L-diversity is satisfied.



(a) K-value V.S. F1-score

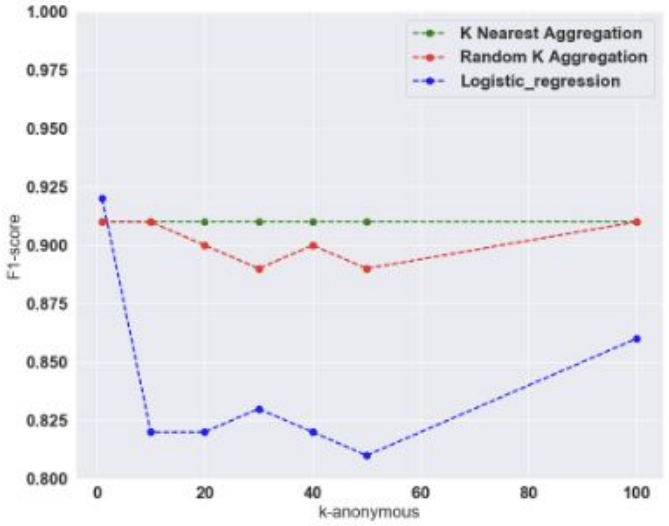


(b) Information loss V.S. K-value

Figure 4.2. Adult Dataset

Performance

LR model loses accuracy loss at all.



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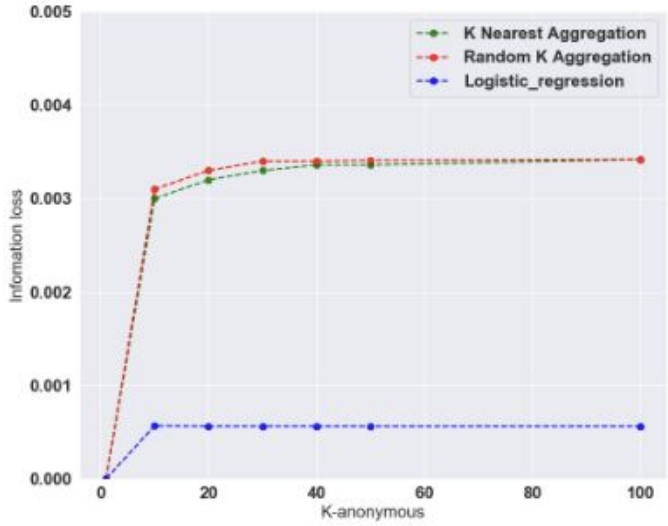


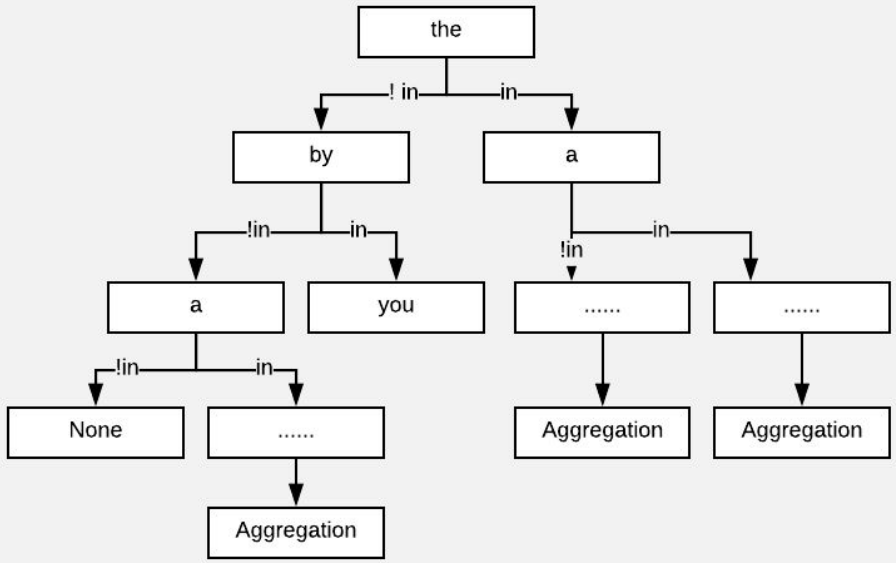
(b) Information loss V.S. K-value

Figure 4.3. Instagram Dataset

K-anonymous

K tree aggregation aggregates instances before putting into bags.
So the performance cannot be guaranteed by the formula above.

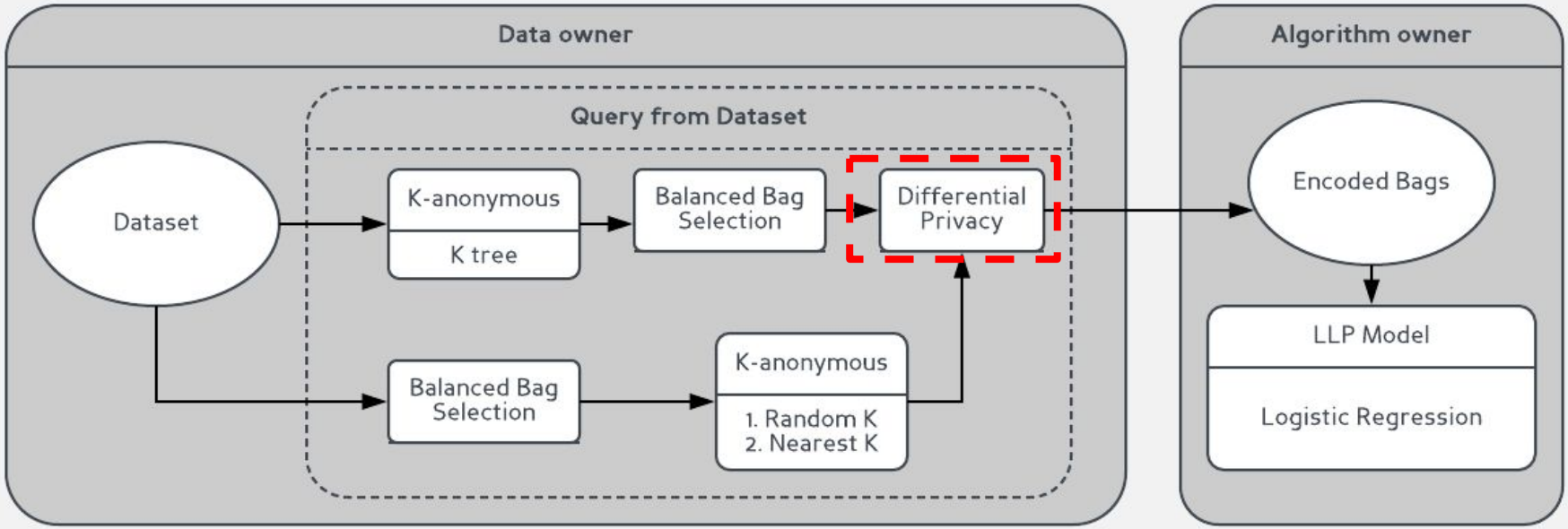
- K tree aggregates +/- instances separately, which cannot guarantee l-diversity.
- K tree allows big k value which equals to the maximal frequency of minority class' attributes.
- K tree is robust after applying differential privacy encoding.



Differential privacy

Ensure no record can be traced from the dataset

- Totally distorted feature matrix
- Totally distorted label distribution for each bag



Differential privacy

The sensitivity for the feature matrix:

- Query the dataset for the mean of k records' each attribute
- K is the anonymous value for the model
- The maximal difference of the query between 2 neighbour datasets is 1/k
- Prefer small ε and big k

The sensitivity for the prior label distribution:

- Query the dataset for each bag's label distribution
- m is the size of each bag
- The maximal difference of the query is 1/m
- Prefer small ε and big m

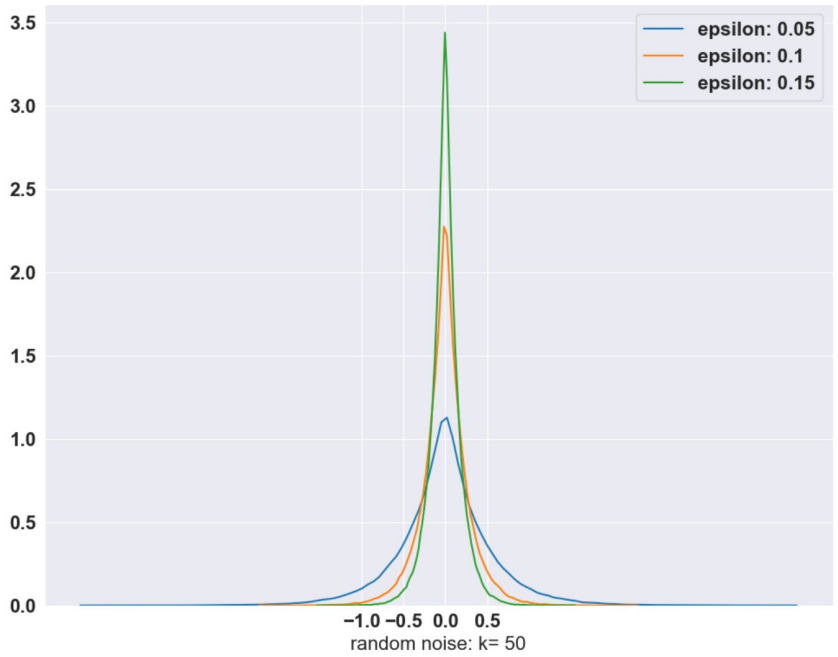
The sensitivity for the standard LR model:

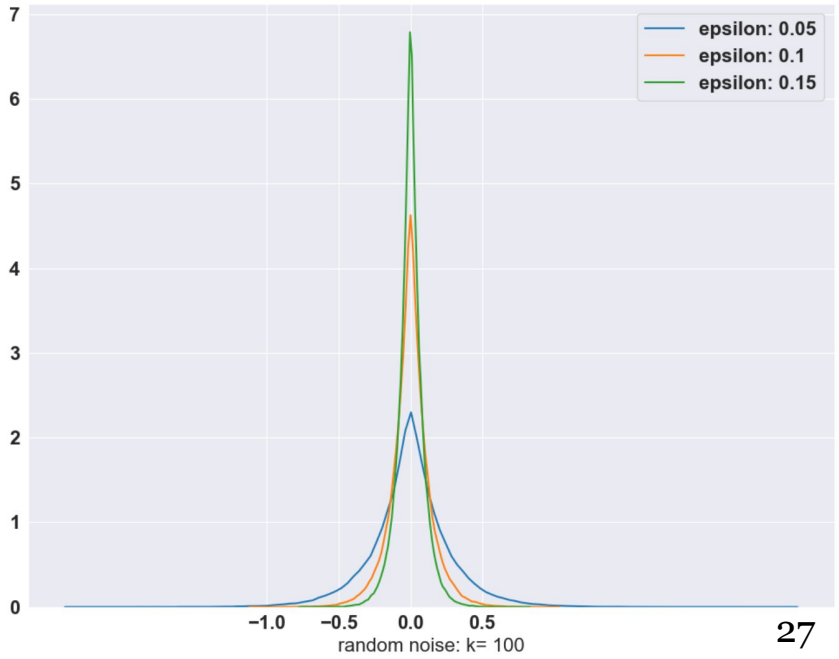
- Query the dataset for each instance's label
- The maximal difference of the query is 1 when querying the label
- Unbearable weight of noise

Differential privacy

Selection of epsilon:

- Domain of the feature matrix is [0,1]
- Domain of the label distribution is [0,1]
- Control noise domain above 90% in [-0.5, 0.5]





Prove

Laplace noise for feature matrix causes little accuracy loss.

- f_new(θ) is the loss function for distorted dataset
- Zij is random noise vector sampled from Laplace distribution
- When number of instance of each bag is big enough,
  the new loss function stays almost the same

$$f_{new}(\theta) \approx f_{new}(0) + \theta^T f'_{new}(0) = n + \theta^T \frac{1}{m} \sum_{i=1}^{n} ((P_i - \frac{1}{2}) \sum_{j=1}^{m} X_{i,j} + z_{i,j})$$

$$= n + \theta^T \frac{1}{m} \sum_{i=1}^{n} ((P_i - \frac{1}{2})(\sum_{j=1}^{m} X_{i,j} + \sum_{j=1}^{m} z_{i,j}))$$

$$As \quad m \quad \to \infty, \quad f_{new}(\theta) = n + \theta^T \frac{1}{m} \sum_{i=1}^{n} ((P_i - \frac{1}{2})(\sum_{j=1}^{m} X_{i,j} + \sum_{j=1}^{m} E(z_{i,j}))) = f(\theta)$$

Prove

Laplace noise for label distribution causes little accuracy loss.

- Odd bag from confident instances, even bag from unconfident instances i.i.d
- Mean vectors is almost the same, treat as constant
- Noise for label distribution and Noise for feature matrix is independent
- X_bar_even as the mean vector for even bag, so is the X_bar_odd

$$f_{new}(\theta) \approx f_{new}(0) + \theta^T f'_{new}(0) = n + \theta^T \sum_{i=1}^{n} ((P_i + z_i - \frac{1}{2})\bar{X}_i)$$

$$= n + \theta^T \sum_{i=1}^{n} ((P_i - \frac{1}{2}) + z_i\bar{X}_i) = f(\theta) + \theta^T \sum_{i=1}^{n} (z_i\bar{X}_i)$$

$$\sum_{i=1}^{n} (z_i\bar{X}_i) \approx \bar{X}_{even} \sum_{i=1,i\%2=0}^{n} z_i \quad + \bar{X}_{odd} \sum_{i=1,i\%2=0}^{n} z_i$$

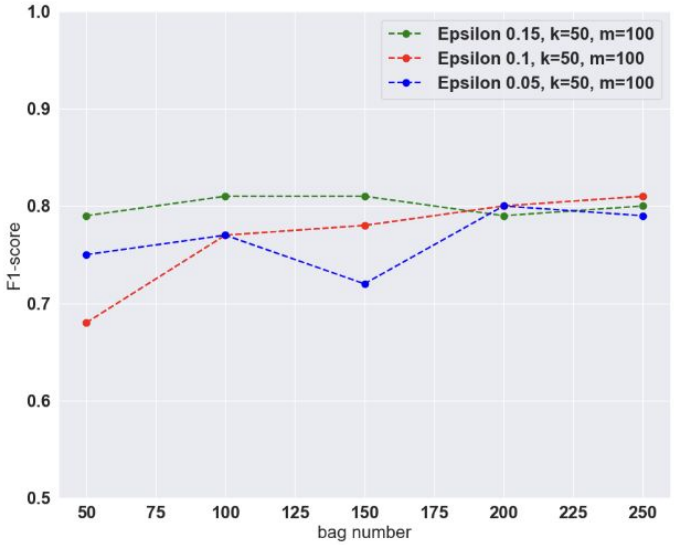
$$When \quad n \to \infty, \quad \bar{X}_{even} \sum_{i=1,i\%2=0}^{n} E(z_i) \quad + \bar{X}_{odd} \sum_{i=1,i\%2=1}^{n} E(z_i) = 0$$

$$f_{new}(\theta) = f(\theta)$$

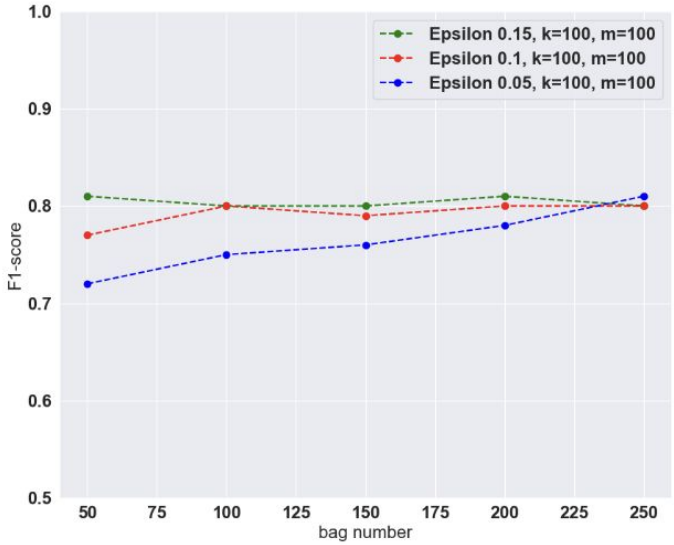
Performance

Performance increases with the number of bag increases.

- Based on Random K aggregation method
- For the dataset with less noise, the performance is more robust.
- Information loss does not change when number of bag increases
- The performance converges as the sum of the noise comes close to zero.



(a) K-value 50



(b) K-value 100

Performance

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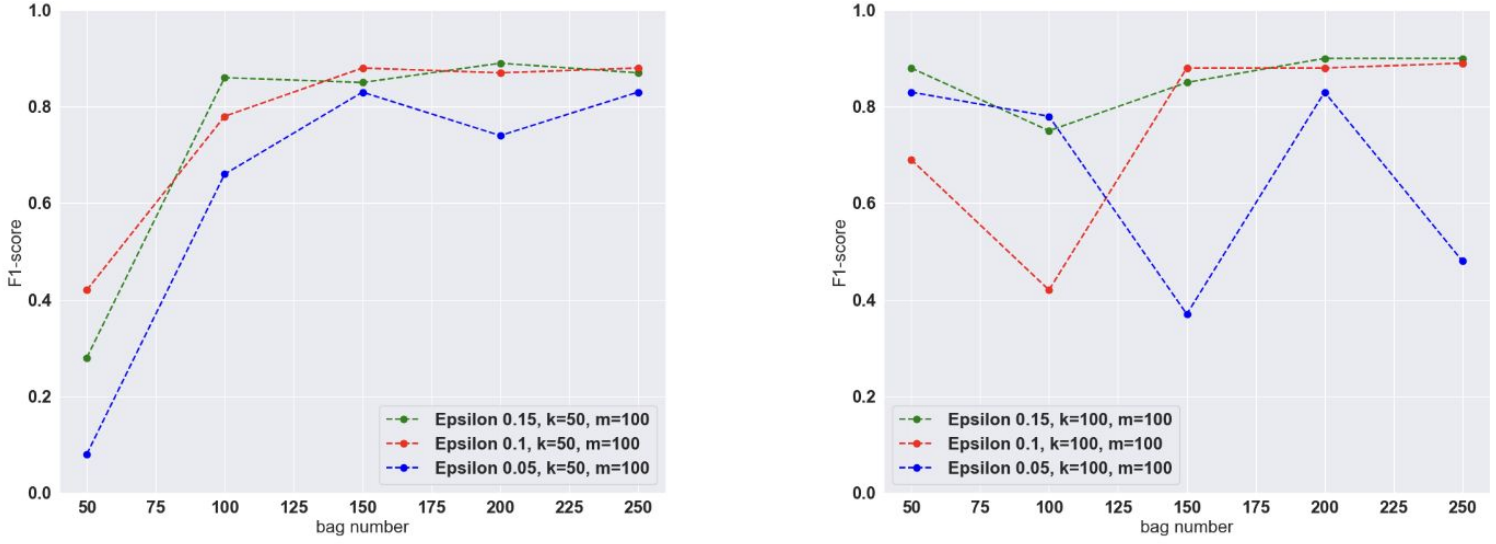
- Information loss does not change when number of bag increases
- The performance converges as the sum of the noise comes close to zero.



(a) K-value V.S. F1-score

(b) K-value V.S. Information loss

Figure 4.6. Instagram Comment Dataset