CS 331 **Introduction to Artificial Intelligence** Fall 2024

OREGON STATE UNIVERSITY

School of Electrical Engineering and Computer Science

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Total points: 100 HW 4 Solutions: **Probability and Bayes Nets** Due date: Dec 2 2024

Instructions: This homework assignment consists of a written portion only. Collaboration is not allowed on any part of this assignment. Solutions must be typed (hand written and scanned submissions will not be accepted) and saved as a .pdf file.

1. (i) P(Toothache= true) = 0.064 + 0.016 + 0.012 + 0.108 = 0.2

   (ii) P(Cavity= true) = 0.008 + 0.072 + 0.012 + 0.108 = 0.2

   (iii) P(Cavity= false)) = 0.576 + 0.144 + 0.064 + 0.016 = 0.8

   (iv) $P(Toothache = false|Cavity = false)$ = P(Toothache = false, Cavity = false) / P(Cavity = false)

   = (0.576+0.144)/(0.8)

   = 0.9

   (v) $P(Toothache = true|Cavity = false)$ = P(Toothache = true, Cavity = false) / P(Cavity = false)

   = (0.064+0.016)/(0.8)

   = 0.1

   (vi) P(Toothache= false) = 0.576 + 0.144 + 0.008 + 0.072 = 0.8

   (vii) $P(Toothache = true|Cavity = true)$ = P(Toothache = true, Cavity = true) / P(Cavity = true)

   = (0.012 + 0.108) / (0.2)

   = 0.6

2. Let us define the following random variables: D = Disease; T= Test

   $P(T = true|D = true) = 0.99$

   $P(T = false|D = false) = 0.99$

   $P(T = true|D = false) = 0.01$

   $P(D = true) = 0.0001$

   $P(D = false) = 0.9999$

   $P(D = true|T = true) = \frac{P(T=true|D=true)P(D=true)}{P(T=true|D=true)P(D=true)+P(T=true|D=false)P(D=false)}$

   $= ((0.99)(0.0001))/((0.99)(0.0001) + (0.01)(0.9999))$

   $= 0.000099/(0.000099 + 0.009999) = 0.000099/0.010098 = 0.009804$

   It is good news that the disease is rare because $P(Disease = true|Test = true)$ is proportional to P(Disease=true). The lower P(Disease=true) is, the lower $P(Disease = true|Test = true)$ will be.

3.

$$\begin{aligned} P(A,B|E) &= \frac{P(A,B,E)}{P(E)} \\ &= \frac{P(A|B,E)P(B,E)}{P(E)} \\ &= \frac{P(A|B,E)P(B|E)P(E)}{P(E)} \\ &= P(A|B,E)P(B|E) \end{aligned}$$

4.
- $I(B,C|\{A,G\})$: False. There are two paths. Path 1: B-A-C. Path 2: B-E-G-F-C. Path 2 is not blocked since G is a child node and is in the evidence set (case 3 violation).
- $I(C,D|H)$: False. There are two paths. Path 1: C-F-D. Path 2: C-A-B-E-G-F-D. Path 1 is not blocked since H is a child of F and is in the evidence set (case 3 violation).
- $I(G,H|F)$: True. There are two paths. Path 1: G-F-H. Path 2: G-E-B-A-C-F-H. All paths are blocked.
- $I(A,H|F)$: True. There are two paths. Path 1: A-C-F-H. Path 2: A-B-E-G-F-H. All paths are blocked.
- $I(E,D|C)$: True. Path 1: E-G-F-D. Path 2: E-B-A-C-F-D. All paths are blocked.

5. Let o represent the report that Orville dropped the ball, and $\neg o$ represents the report that Orville did not drop the ball. Let b represent the battery level is low. Let d denote the robot actually dropped the ball and $\neg d$ denote that the robot did not drop the ball.

   From the table, $P(d|B = low) = 0.9$. Therefore, $P(\neg d|B = low) = 1 - P(d|B = low) = 0.1$. Similarly, $P(d|B = high) = 0.01$. Therefore, $P(\neg d|B = high) = 1 - P(d|B = high) = 0.99$.

   $P(o|d) = 0.9$. Therefore, $P(\neg o|d) = 1 - P(o|d) = 0.1$. Similarly, $P(o|\neg d) = 0.2$. Therefore, $P(\neg o|\neg d) = 1 - P(o|d) = 0.8$.

   Using the chain rule (or product rule), $P(b|o) = \frac{P(b,o)}{P(o)}$.

   $P(o) = \sum_B \sum_D P(o|D)P(d|B)P(B)$. To calculate this value, we will consider all four combinations of $D = \{d, \neg d\}$ and $B = \{b, \neg b\}$. Substituting the values, we get $P(o) = 0.2693$.

$$\begin{aligned} P(b,o) &= \sum_d P(b,o,d) \\ &= \sum_d P(b) \cdot P(d|b) \cdot P(o|d) \\ &= P(b) \cdot \sum_d P(d|b) \cdot P(o|d) \end{aligned}$$

   $P(b,o) = P(b) * [P(D = drop|b) * P(o = drop|d = drop) + P(D = \neg drop|b) * P(o = drop|d = \neg drop)]$

   $P(b,o) = 0.1(0.9 \cdot 0.9 + 0.1 \cdot 0.2) = 0.083$

   Substituting the values in $P(b|o) = \frac{P(b,o)}{P(o)}$, $P(b|o) = 0.083/0.2693 = 0.308$.

6.
- P(A=true, B=true, C=false, D=true, E=true)
  = P(A=true)P(B=true|A=true)P(C=false|A=true) P(D=true|C=false) P(E=true|B=true,D=true)
  = 0.6× 0.75× 0.8× 0.25× 0.6 = 0.054

- P(A=true, B=true, D=false)
  = P(A=true) P(B=true|A=true)$\sum_{c\in\{true,false\}}$ [P(C=c|A=true)P(D=false|C=c)$\times$ $\sum_{e\in\{true,false\}}$ [P(E=e|B=true,D=false)]]
  = P(A=true) P(B=true|A=true) $\sum_{c\in\{true,false\}}$ P(C=c|A=true)P(D=false|C=c) $\times 1$
  =0.6×0.75× (P(C=true|A=true)P(D=false|C=true) +P(C=false|A=true)P(D=false|C=false))
  =0.6×0.75×(0.2× 0.9+0.8×0.75) = 0.351

- P(D=true |A=false)
  $= \frac{P(A=false,D=true)}{P(A=false)}$
  $= \frac{1}{P(A=false)} P(A = false) \times \sum_b P(B = b|A = false) \sum_c P(C = c|A = false)P(D = true|C = c) \times \sum_e P(E = e|B = b, D = true)$
  $= \sum_b P(B = b|A = false) \sum_c P(C = c|A = false)P(D = true|C = c) \times 1$
  $= (P(B = true|A = false) + P(B = false|A = false)) \times (P(C = true|A = false)P(D = true|C = true) + (P(C = false|A = false)P(D = true|C = false))$
  = (0.1+0.9)×(0.75×0.1+0.25×0.25) = 0.1375