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HW 4: Probability and Bayes Nets

1.

(i) P(Toothache= true) = 0.064+0.016+0.012+0.108 = **0.2**

(ii) P(Cavity= true) = 0.008+0.072 +0.012+0.108 = **0.2**

(iii) P(Cavity= false)= 0.576+0.144+0.064+0.016 = **0.8**

(iv) P(Toothache= false|Cavity= false) = P(Toothache= false, Cavity= false)/ P(Cavity = false) = (0.576+0.144)/0.8 = 0.72/0.8 = **0.9**

(v) P(Toothache= true|Cavity= false) = 1- P(Toothache= false|Cavity= false) = 1-0.9 = **0.1**

(vi) P(Toothache= false) = 0.576+0.144+0.008+0.072 = **0.8**

(vii) P(Toothache= true|Cavity= true) = P(Toothache= true,Cavity= true) / P(Cavity = true) = (0.012+0.108)/0.2 =0.12/0.2= **0.6**

2.

(i) Why is it good news that the disease is rare?

It is good news because the rarity of the disease significantly reduces the likelihood that a positive test result means Alice has the disease. The test's high false positive rate relative to the low disease prevalence leads to most positive results being false positives.

Thus, despite the test's high accuracy, Alice's chances of having the disease are still very low.

(ii) What is the chance that Alice actually has the disease?

Given: P(Disease) = 0.0001

P(No Disease) = 1- P(Disease) = 0.9999

P(Positive | Disease) = 0.99

P(Negative | No Disease) = 0.99 (True negative)

P(Positive | No Disease) = 1- P(Negative | No Disease) = 0.01

We need to find **P(Disease | Positive)** for Alice by using Bayes's theorem:

P(Positive)= P(Positive | Disease)*P(Disease)

+P(Positive | No Disease)*P(No Disease) =

$(0.99 \cdot 0.0001)+(0.01 \cdot 0.9999)$ = 0.000099 + 0.009999 = 0.010098

P(Disease | Positive) = (Positive|Disease)*P(Disease)/P(Positive)= $0.99 \cdot 0.0001/0.010098 \approx$ **0.0098**

Thus, there is a **0.98%** chance that Alice has the disease.

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3. Prove the conditionalized version of the general product rule : P(A,B |E) = P(A| B,E)*P(B|E)

We can start with the definition of conditional probability as shown in hints:

(a) P(A,B |E) = P(A,B,E)/ P(E)

W can rewrite P(A,B,E) using the Chian rule of probability as follows:

(b) P(A,B,E) = P(A| B,E)*P(B,E)

Substitute this back into a (The equation for P(A, B| E)):

(c) P(A,B |E) = P(A| B,E)*P(B,E)/ P(E)

We can simplify the P(B,E) using the definition of conditional probability:

(d) P(B,E) = P(B|E)P(E)

Now we can substitute (d) back into the equation (c):

(e) P(A,B |E) = P(A| B,E)*P(B|E)P(E)/ P(E)

Now we can cancel P(E) we get as follows:

P(A,B |E) = P(A| B,E)*P(B|E)

Therefore we proved the conditionalized version of the general product rule : P(A,B |E) = P(A| B,E)*P(B|E)

4. (20 points) Consider the Bayesian network below. Answer true or false for the following questions on d-separation. Show the blocked paths and explain your answer.

• I(B, C|{A, G}) **False**

Path 1: B←A →C. The path between B and C is <u>blocked by A</u> because A is observed (case2: Forks)

Path 2: B→E→G←F←C. The path between B and C is <u>Not blocked</u> by G because G is in the evidence set (**Case 3: colliders**).

Therefore, **I(B, C|{A, G}) is False**

• I(C, D|H) **False**

Path 1: D→F→G ←E←B←A →C. The path between C and D is <u>blocked by G</u> because G is <u>not observed</u>(Case 3: colliders)

Path 2: C→F←D(F→H). C and *D* are <u>not independent</u> because F has a child node H, which is in the evidence set (Case 3: colliders).

Therefore, **I(C, D|H) is False**

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• I(G, H|F) **True**

Path 1: G ← F→H. The path from G and H is blocked by F, because F is observed (Case 2: Forks).
Path 2: G ←E ←B←A→C→F→H. The path is blocked by F because F is observed(case 1: Chains). Thus observing F cuts the flow of information from G to H. Whether A is observed or not, The path from A to H is blocked.
Therefore, **I(G, H|F) is True**

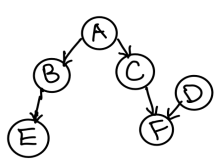
• I(A, H|F) **True**

Path 1: A→C→ F→ H. The path between A and H is blicked by F because F is observed(case 1: Chains)
Path 2:A →B→ E→ G← F→ H. The path between A and H is blocked by G, because G is not in the evidence set(Case 3: Colliders) Also is blocked by F, F is observed(Case 2: Forks)
Therefore, **I(A, H|F) is True**

• I(E, D|C) **True**
Path 1: E ←B ←A→ C→ F ←D. The path between E and D is blocked by F because F is not observed(Case 3: Colliders)
Path 2: E→ G ←F←D. The path between E and D is blocked by G because G is not observed(Case 3: colliders)
Therefore, **I(E, D|C) is True**.

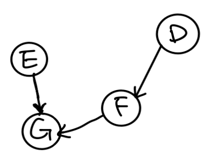
(v) I(E, D|C)? Yes

Path 1:



The path between E and D is blocked by F, Because F is not observed (case 3: Colliders)

Path 2:



path between E and D is blocked by G, Because G is not observed (case 3: colliders)

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5. Calculate the probability that the battery is low, given the observer's report that Orville dropped the ball.

Given:
P(B = Low) = 0.1
P(B= High) = 1- P(B = Low) = 0.9

P(Drops|B = Low) = 0.9
P(Drops|B = High)) = 0.01

P(O = Drops|Drops) = 0.9
P((O = Drops |¬Drops) = 0.2

We need to find that **P(Battery = Low | O = Drops)** according to Baye's rule:
P(Battery = Low | O = Drops) = P(O = Drops | B= Low) P(B= Low)/ P(O = drops):

P(O= Drops | B=Low) =
P(O = Drops|Drops)*P(Drops|B=Low)
+P(O = Drops|¬ Drops)* P(¬Drops|B=Low)
= (0.9 *0.9) +(0.2*0.1) = 0.83

P(O= Drops | B =High) =
P(O = Drops|Drops)P(Drops|B=High)
+P(O=Drops|¬Drops)P(¬Drops|B=High)
= (0.9 *0.01)+(0.2* 0.99) = 0.207

P(O= Drops) = P(O=Drops | B = Low) P(B = Low) + P(O = Drops | Battery = High) P(B = High)
= (0.83 *0.1)+(0.207 *0.9) = 0.2693

Now we can substitute all values we found in the equation as follows:
P(B = Low | O= Drops) = P(O= Drops| B = Low) P(B = Low)/ P(O = drops) = (0.83*0.1)/0.2693
= **0.3082**

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6.

(i)P(A=true, B=true, C=false, D=true, E=true)

P(A = true) = 0.6
P(B= true|A = ture)= 0.75
P(C = false |A = ture) = 0.8
P(D = true|C = false) = 0.25
P(E = ture|B= true,D= true) = 0.6
P(A=true, B=true, C=false, D=true, E=true)= P(A = true)*P(B= true|A = ture) *P(C = false |A = ture)*P(D = true|C = false)*P(E = ture|B= true,D= true) = 0.6*0.75*0.8*0.25*0.6 = **0.054**

(ii)P(A=true, B=true, D=false)

P(A = true) = 0.6
P(B= true|A = ture) = 0.75
P(C = true| A = true)= 0.2
P(C = false| A = true)= 0.8
P(D = false| C = true)= 0.9
P(D = false| C = false) = 0.75

$\sum$[P(C|A = true) * P(D= false|C)]= P(C= true| A = true)*P(D = false| C= true)
+P(C = false| A = true)* P(D = false|C= false)
= (0.2 *0.9)+(0.8*0.75)
= 0.18+0.6
= 0.78

P(A=true, B=true, D=false) = P(A = true)* P(B= true|A = ture) * $\sum$[P(C|A = true) * P(D= false|C)] = 0.6*0.75*0.78 = **0.351**

(iii) P(D=true | A=false)

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We can use the given probabilities as follows:

$P(C=true|A=false) = 0.75$

$P(D=true|C=true) = 0.1$

$P(C=false|A=false) = 0.25$

$P(D=true|C=false) = 0.25$

To calculate $P(D=true | A=false)$, we use marginalization over C, because D depends on C and C depends A. The equation is:

$$P(D=true|A=false) = \sum\left[P(C| A = false)*P(D= true|C)\right]$$

$$=[P(C=true|A=false)*P(D=true|C=true)]$$

$$+[P(C=false|A=false)*P(D=true|C=false)]$$

$$= (0.75 * 0.1)+(0.25 *0.25) = 0.075+0.0625 \mathbf{= 0.1375}$$