Total points: 100   HW 4: **Probability and Bayes Nets**   Due date: Dec 2 2024

Instructions: This homework assignment consists of a written portion only. Collaboration is not allowed on any part of this assignment. Solutions must be typed (hand written and scanned submissions will not be accepted) and saved as a .pdf file.

1. **(15 points)** Use the full joint probability distribution in the table below to calculate the following.

Toothache	Cavity	Catch	P(Toothache, Cavity, Catch)
false	false	false	0.576
false	false	true	0.144
false	true	false	0.008
false	true	true	0.072
true	false	false	0.064
true	false	true	0.016
true	true	false	0.012
true	true	true	0.108

    (i) P(Toothache= true)

    (ii) P(Cavity= true)

    (iii) P(Cavity= false)

    (iv) $P(Toothache = false|Cavity = false)$

    (v) $P(Toothache = true|Cavity = false)$

    (vi) P(Toothache= false)

    (vii) $P(Toothache = true|Cavity = true)$

2. (**10 points**) Alice goes to her yearly checkup. After examination, the doctor says Alice tested positive for a serious disease and that the test is 99% accurate (i.e. the test is correct with probability 0.99 in determining if someone has the disease and is correct with probability 0.99 in determining if someone does not have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that Alice actually has the disease?

3. (**10 points**) Prove the conditionalized version of general product rule: $P(A, B|E) = P(A|B, E)P(B|E)$. Hint: The first step of this derivation is $P(A, B|E) = \frac{P(A,B,E)}{P(E)}$ and then simplify the joint distribution using conditional independence rules. For example, rewrite $P(A, B, E)$ as $P(A|B, E)P(B, E)$.
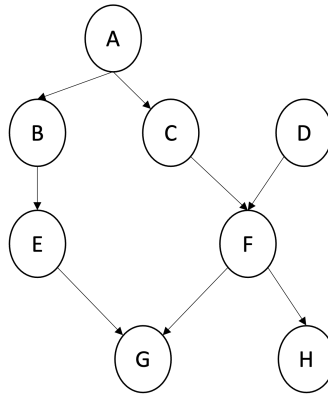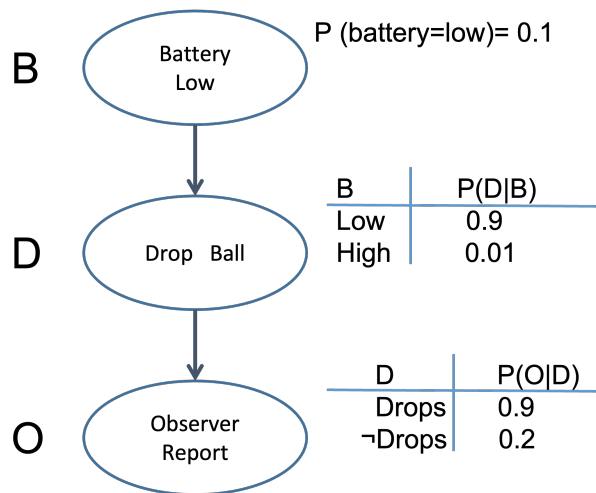
4. **(20 points)** Consider the Bayesian network below. Answer true or false for the following questions on d-separation. Show the blocked paths and explain your answer.

- $I(B, C|\{A, G\})$
- $I(C, D|H)$
- $I(G, H|F)$
- $I(A, H|F)$
- $I(E, D|C)$

5. **(20 points)** Orville, the robot juggler, drops balls quite often when its battery is low. A robot observer, with a somewhat unreliable vision system, reports that Orville dropped a ball. Variables and domain: the battery can either be low or high, drop ball can be true or false, and the observer's report may indicate whether Orville dropped the ball or not.

For the graph below, calculate the probability that the battery is low, given the observer's report that Orville dropped the ball.

P (battery=low)= 0.1

B	P(D|B)
Low	0.9
High	0.01

D	P(O|D)
Drops	0.9
¬Drops	0.2

6. **(25 points)** Calculate the following probabilities using the Bayesian network below. You may use various probability formulas such as marginalization, the chain rule, conditional independence, Bayes rule, etc.

- (5 points) P(A=true, B=true, C=false, D=true, E=true)

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- (10 points) P(A=true, B=true, D=false)
- (10 points) P(D=true |A=false)
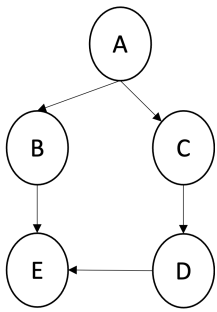
A	P(A)
false	0.4
true	0.6

A	B	P(B|A)
false	false	0.9
false	true	0.1
true	false	0.25
true	true	0.75

C	D	P(D|C)
false	false	0.75
false	true	0.25
true	false	0.9
true	true	0.1

B	D	E	P(E|B,D)
false	false	false	0.1
false	false	true	0.9
false	true	false	0.2
false	true	true	0.8
true	false	false	0.3
true	false	true	0.7
true	true	false	0.4
true	true	true	0.6

A	C	P(C|A)
false	false	0.25
false	true	0.75
true	false	0.8
true	true	0.2