CS 331 **Introduction to Artificial Intelligence** Fall 2024

OREGON STATE UNIVERSITY

School of Electrical Engineering and Computer Science

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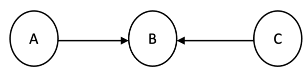
Final Exam

1. **[10 points]** Circle true or false for the following questions.

(a) Let h_1 and h_2 be two admissible heuristics for a graph. Heuristic $h_3 = \max\{h_1, h_2\}$ is a better estimate of the actual distance to the goal than $h_4 = \frac{h_1+h_2}{2}$. Answer: True

(b) Hill climbing algorithm with random restarts is always guaranteed to find the optimal solution. Answer: False

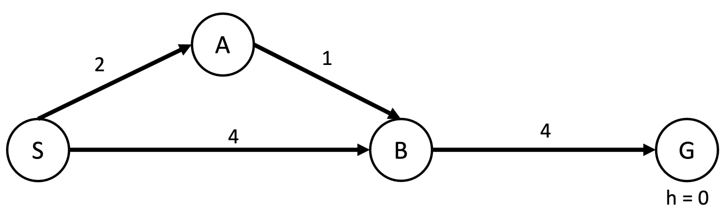
(c) In the following graph, A and C are d-separated when B is in the evidence set (I(A,C|B)). Answer: False



(d) Consider a knowledge base (KB) with a single sentence: $A \wedge B$. It can be concluded that KB $\models$B. Answer: True

(e) $(A \Rightarrow B) \equiv (\neg A \vee B) \wedge (\neg A \vee A)$. Answer: True

2. **(10 points)** Solve the following graph using A* search algorithm, using the heuristic values in the table. Report the best f value for each node, optimal path derived using these f values, and the solution cost of the optimal path.



Node	h
S	4
A	2
B	2
G	0

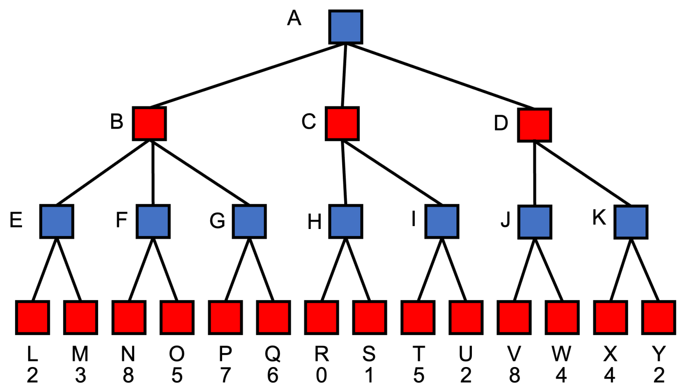
Answer:

Node	f value
S	4
A	4
B	5
G	7

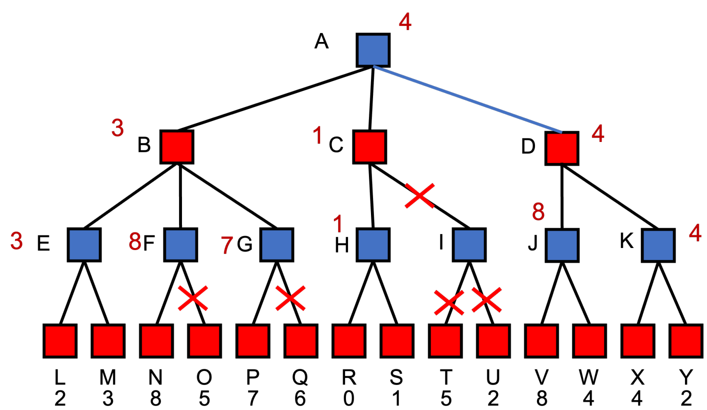
Solution path: S-A-B-G. Cost=7

3. **(20 points)** Adversarial Search.

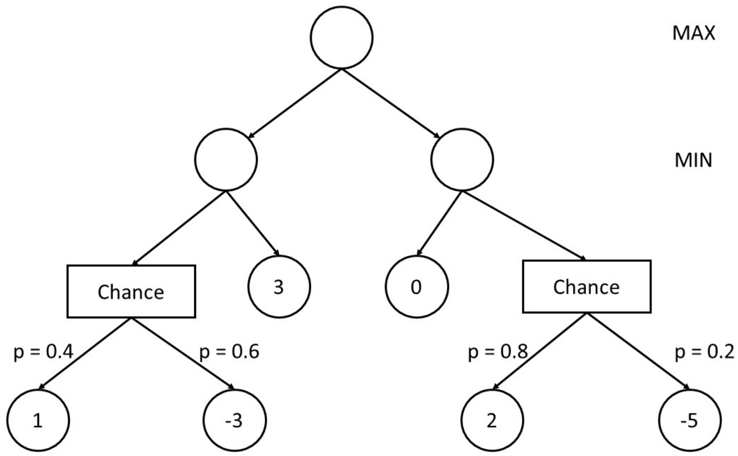
(a) [10 points] Consider the following graph, with blue nodes denoting MAX nodes and red nodes denoting MIN nodes. Solve the graph using alpha-beta pruning. What strategy should A choose and what is the corresponding payoff? List the nodes that are pruned.



Answer: A should choose to play D with a utility of 4. Nodes O,Q,I,T,U will be pruned.



(b) [10 points] Below is a portion of a game tree. What is the expectiminimax value at the root node? Show your work.



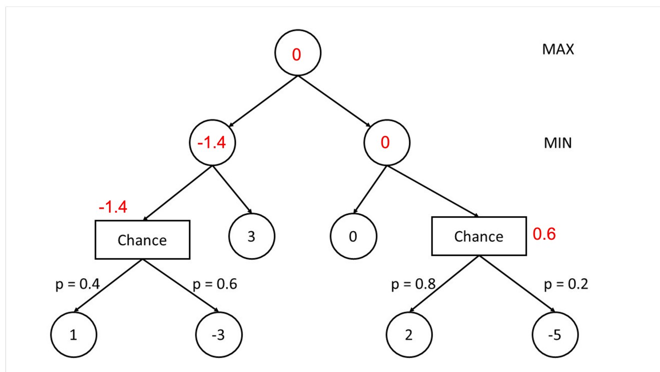
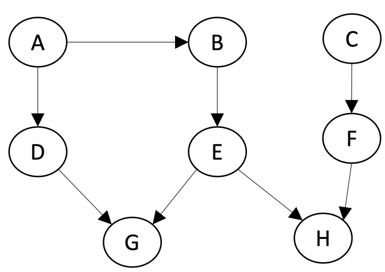


Figure 1: Solution for Q3b

4. **[25 points]** Bayesian Network.

(a) [11 points] Use the Bayesian network below to determine whether or not the following conditional independence relationships hold or not. Show the blocked/unblocked paths for each.



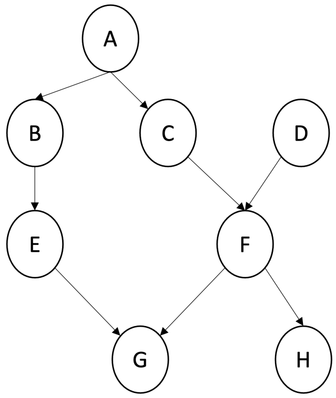
(i) [3 points] $I(A, C|H, E)$ Answer: Yes. There are two paths. P1: A-B-E-H-F-C. P2: A-D-G-E-H-F-C. P1 is blocked by E (case 1) even though case 3 is violated since H is observed. P2 is blocked by E (case 2) even though H is evidence set.

(ii) [3 points] $I(A, G|D)$ Answer: No. There are two paths. P1: A-D-G. P2: A-B-E-G. A and G are d-separated by D by case 3. P2 is not blocked by D since it is not in the chain (P2).

iii) [3 points] $I(D, E|A, G)$ Answer: No. There are two paths. P1: D-G-E which is not blocked since G is observed (case 3 violation). P2: D-A-B-E, which is blocked by A (case 2).

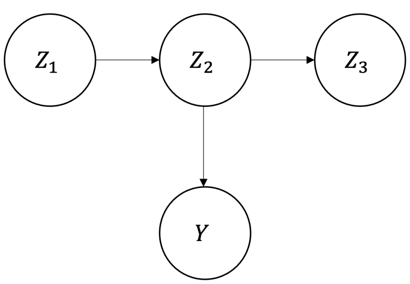
(iv) [2 points] $I(C, E|H)$ Answer: No. There is only path: E-H-F-C which is not blocked since H is observed (case 3 violation).

(b) [4 points] For the Bayesian network below, What nodes must be in the evidence set for E and F to be conditionally independent? That is, select a set of nodes β such $I(E, F|\beta)$ is true, from the options below. Show blocked/unblocked paths for partial credits. (Multiple choice question)



- $\{G\}$
- $\{A\} \rightarrow$ answer
- $\{A, G\}$
- $\{\}$

(c) [10 points] Consider the following graph with binary variables Y and Z. This section shows Z_t $t \in \{1, 2, 3\}$. Compute $P(Y = 0|Z_1 = 1)$. Show your work. Hint: the first step is to write it using joint probabilities (chain rule) and the second step is to apply marginalization over Z_t that are not involved in the query $P(Y = 0|Z_1 = 1)$. The third step is to decompose the joint probability based on the structure of the graph.



Z_1	$P(Z_1)$
1	0.5

Z_2	Y	$P(Y\|Z_2)$
0	1	0.8
1	1	0.3

Z_t	Z_{t+1}	$P(Z_{t+1}\|Z_t)$
0	1	0.1
1	1	0.9

Answer:

$$P(Y = 0|Z_1 = 1) = \frac{P(Y = 0, Z_1 = 1)}{P(Z_1 = 1)} \tag{1}$$

$$
\begin{aligned}
P(Y=0, Z_1=1) &= \sum_z \sum_x P(Z_1=1, Z_2=z, Y=0, Z_3=x) \\
&= \sum_z P(Z_1=1)P(Z_2=z|Z_1=1)P(Y=0|Z_2=z)\sum_x P(Z_3=x|Z_2=z) \\
&= P(Z_1=1)\sum_z P(Z_2=z|Z_1=1)P(Y=0|Z_2=z)\cdot 1 \\
&= 0.5 \times (P(Z_2=0|Z_1=1)P(Y=0|Z_2=0) + P(Z_2=1|Z_1=1)P(Y=0|Z_2=1)) \\
&= 0.5 \times (0.1 \times 0.2 + 0.9 \times 0.7) \\
&= 0.325 \\
P(Z_1=1) &= 0.5
\end{aligned}
$$

Plugging in the values in Eqn.1:

$$P(Y=0|Z_1=1) = \frac{0.325}{0.5} = 0.65$$

5. **[20 points]** Logic.

(a) [4 points] Convert the following to CNF: (a) [4 points] Convert the following to CNF:

- $\neg(P \wedge Q) \Rightarrow R$ Answer: $(P \vee R) \wedge (Q \vee R)$
- $\neg(Q \wedge S)$ Answer: $\neg Q \vee \neg S$

(b) [4 points] Is the following sentence valid, unsatisfiable, or neither? Justify your answer.

$$(\neg A \Rightarrow B) \wedge \neg A \wedge \neg B$$

Answer: $(\neg A \Rightarrow B) \wedge \neg A \wedge \neg B \equiv (A \vee B) \wedge \neg A \wedge \neg B$
We cannot have A or B true as well as both A and B false, so this is unsatisfiable.

(c) [5 points] Does $(A \Leftrightarrow B) \models (A \vee \neg B)$? Prove using truth table

A	B	$(A \Leftrightarrow B)$	$(A \vee \neg B)$
False	*False*	*True*	*True*
False	*True*	*False*	*False*
True	*False*	*False*	*True*
True	*True*	*True*	*True*

(d) [7 points] Use the resolution algorithm to determine whether the following KB entails $\neg P$.

KB:

1. $P \vee Q$
2. $\neg R \vee S$
3. $\neg S \vee Q$
4. $\neg P \vee Q \vee R$
5. $\neg Q$

Answer:

$$S6 : P(S5 + S1)$$
$$S7 : Q \vee R(S6 + S4)$$
$$S8 : R(S7 + S5)$$
$$S9 : S(S8 + S2)$$
$$S10 : Q(9 + 3)$$
$$S11 : \{\}(S10 + S5)$$

Therefore, KB entails $\neg P$.

6. **[15 points]** Consider the Normal form game representation below with two players A and B, each with two strategies S1 and S2.

		B	
		S1	S2
A	S1	2, -2	-4, 4
	S2	-3, 3	5, -5

(a) [4 points] Does this game have a Nash equilibrium? If yes, list them. If no, explain why. Answer: No Nash equilibrium because there is always a better strategy for the other player. For example, if A plays S1, B plays S2. If B plays S2, A plays S2. If A plays S2, B plays S1.

(b) [5 points] Is there a dominant pure strategy for both the players? If yes, list them. If no, explain why. Answer: No. Both players don't have a dominant strategy. For A, there is no strategy that is better irrespective of what B chooses to play. Similarly for B, their best response strategy varies based on A's strategy.

(c) [6 points] Suppose B chooses to play a mixed strategy such that B will select S1 with probability $p = 0.7$ and S2 with probability 0.3. Which pure strategy should A play (S1 or S2)? Briefly explain why. Answer: A's expected payoff if A plays S1 in response to B's mixed strategy: $0.7 * 2 + 0.3 * (-4) = 0.2$. A's expected payoff if A plays S2 in response to B's mixed strategy: $0.7 * -3 + 0.3 * 5 = -0.6$. So A should play S1 in response to B's mixed strategy.