CS 331 **Introduction to Artificial Intelligence** Fall 2024

OREGON STATE UNIVERSITY

School of Electrical Engineering and Computer Science

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Total points: 100 HW 3: **Logic** Due date: Nov 16 2024

Instructions: This homework assignment consists of a written portion only. Collaboration is not allowed on any part of this assignment. Solutions must be typed (hand written and scanned submissions will not be accepted) and saved as a .pdf file.

1. (**25 points**) Consider three atomic sentences A, B and C. Construct a truth table for the following in propositional logic:

(i) $(A \wedge B \wedge C)$

A	B	C	$A \wedge B \wedge C$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

(ii) $(A \vee B \vee C)$

A	B	C	$A \vee B \vee C$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

(iii) $A \Rightarrow B$

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

(iv) $(A \wedge B) \vee C$

A	B	C	$(A \wedge B) \vee C$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

(v) $(\neg A) \vee (B \wedge C)$

A	B	C	$(\neg A) \vee (B \wedge C)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

2. (**14 points**) Inference.

(i) [7 points] Does $(A \wedge B) \models (A \Leftrightarrow B)$? Prove using a truth table. Answer:
True because the left-hand side has exactly one model (only one combination of values that results in true) that is one of the two models of the right-hand side.

A	B	$A \wedge B$	$A \Rightarrow B$	$B \Rightarrow A$	$A \Leftrightarrow B$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	F	T	T	T

(ii) [7 points] Does $A \Leftrightarrow B \models A \vee B$? Prove using a truth table.

Answer:
False. One of the models of $A \Leftrightarrow B$ has both A and B false, which does not satisfy $A \vee B$.

A	B	$A \vee B$	$A \Rightarrow B$	$B \Rightarrow A$	$A \Leftrightarrow B$
T	T	T	T	T	T
T	F	T	F	T	F
F	T	T	T	F	F
F	F	F	T	T	T

3. (**10 points**) For each of the following statements, prove if it is true or false using logical equivalence rules.

   (iii) $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$

   Answer:
   True. $(C \lor (\neg A \land \neg B)) \equiv (C \lor \neg A) \land (C \lor \neg B)$
   $\equiv (A \Rightarrow C) \land (B \Rightarrow C)$

   (iv) $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable

   Answer:
   True. $= (A \lor B) \land \neg(\neg A \lor B)$
   $= (A \lor B) \land (A \land \neg B)$
   $= (A \land A \land \neg B) \lor (B \land A \land \neg B)$
   $= (A \land \neg B) \lor false$
   Satisfiable with $(A \land \neg B)$

4. (**5 points**) Decide if the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or the equivalence rules.

   $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$

   Answer:
   Valid. $(Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire)$
   $(\neg Smoke \lor Fire) \lor (\neg Heat \lor Fire)$
   $\neg Smoke \lor Fire \lor \neg Heat \lor Fire$
   $\neg Smoke \lor Fire \lor \neg Heat$
   $\neg(Smoke \land Heat) \lor Fire$
   $(Smoke \land Heat) \Rightarrow Fire$

5. **(16 points)** Convert the following to CNF:

   (i) S1: $A \Leftrightarrow (B \lor E)$

   Answer:
   $(\neg A \lor B \lor E) \land (\neg B \lor A) \land (\neg E \lor A)$

   (ii) S2: $E \Rightarrow D$

   Answer:
   $\neg E \lor D$

   (iii) S3: $(C \land F) \Rightarrow \neg B$

   Answer:
   $\neg C \lor \neg F \lor \neg B$

   (iv) S4: $E \Rightarrow B$

   Answer:
   $(\neg E \lor B)$

   (v) S5: $B \Rightarrow F$

   Answer:
   $(\neg B \lor F)$

   (vi) S6: $B \Rightarrow C$

   Answer:
   $(\neg B \lor C)$

(vii) S7: $P \Rightarrow (Q \vee R)$

Answer:
$\neg P \vee Q \vee R$

(viii) S8: $\neg(P \wedge Q) \Leftrightarrow S$

Answer:
$(\neg(P \wedge Q) \Rightarrow S) \wedge (S \Rightarrow \neg(P \wedge Q))$
$((P \wedge Q) \vee S) \wedge (\neg S \vee \neg(P \wedge Q))$
$(P \vee S) \wedge (Q \vee S) \wedge (\neg S \vee (\neg P \vee \neg Q))$

6. **(10 points)** Using the sentences S1-S6 in Q5 in the CNF form as the knowledge base KB, prove $\neg A \wedge \neg B$ using resolution. Hint: To prove the conjunction, we can prove each literal separately.

To prove $\neg B$, add the negated goal

S7: B

Resolve S7 with S5 to form

S8: F

Resolve S7 with S6 to form

S9: C.

Resolve S8 with S3 to form

S10: $\neg C \vee \neg B$

Resolve S9 with S10 to form

S11: $\neg B$.

Resolve S7 with S11 to form the empty clause.

To prove $\neg A$ add the negated goal

S7: A.

Resolve S7 with the first clause of S1 to form

S8: $B \vee E$

Resolve S8 with S4 to form

S9: B.

Resolve S9 with S3 to form

S10: $\neg C \vee \neg F$

Resolve S9 with S5 to form

S11: F

Resolve S10 with S11 to form

S12: $\neg C$

Resolve S9 with S6 to form

S13: C

Resolve S12 with S13 to form the empty clause

7. **(20 points)** Convert the following sentences to first order logic sentences. We use the following predicates:
$E(x)$ means "x is an easy course"
$H(y)$ means "y is a happy student"
$T(y, x)$ means "student y takes the course x"
$F(x)$ means "x is a course with a final"

(i) "If a course is easy, some students are happy"

Answer:
$\forall x(E(x) \Rightarrow \exists y(T(y, x) \wedge H(y)))$

(ii) "If a course has a final, no students are happy"

Answer:
$\forall x(F(x) \Rightarrow \neg\exists y(T(y, x) \wedge H(y)))$ or $\forall x(F(x) \Rightarrow \forall y(T(y, x) \wedge \neg H(y)))$

(iii) "if a course has a final, the course is not easy"

Answer:
$\forall x(F(x) \Rightarrow \neg E(x))$

(iv) "If a student is happy, then they are taking an easy course with a final"

Answer:
$\forall y(H(y) \Rightarrow \exists x(T(y, x) \wedge E(x) \wedge F(x)))$