Student: Ya Zou

2.14 A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses to correct their reading vision and whether they use eyeglasses when reading. The proportions falling into the four resulting categories are given in the following table:

	Uses Eyeglasses for Reading	
Needs glasses	Yes	No
Yes	.44	.14
No	.02	.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult

a needs glasses.

$P(a) = 0.44 + 0.14 = 0.58$

b needs glasses but does not use them.

$P(b) = 0.14$

c uses glasses whether the glasses are needed or not.

$P(c) = 0.44 + 0.2 = 0.46$

2.27 In Exercise 2.12 we considered a situation where cars entering an intersection each could turn right, turn left, or go straight. An experiment consists of observing two vehicles moving through the intersection.

a How many sample points are there in the sample space? List them.

S = {(right, right), (right, left), (right, straight)
(left, left), (left, right), (left, straight)
(straight, straight), (straight, left) (straight, right)}

= 9 sample points

b Assuming that all sample points are equally likely, what is the probability that at least one car turns left?

P(at least one car turn left) = 5/9

(right, left), (left, left), (left, right)
(straight, left), (left, straight)

c Again assuming equally likely sample points, what is the probability that at most one vehicle turns?

P(at most one vechicle turns) = 5/9

(straight, straight) (Left, straight) (straight left)
(right, straight) (straight, right)

2.51 A local fraternity is conducting a raffle where 50 tickets are to be sold—one per customer. There are three prizes to be awarded. If the four organizers of the raffle each buy one ticket, what is the probability that the four organizers win

a all of the prizes?

The number of ways to choose all 3 winners from the 4 orgs: $\binom{4}{3}=4$

total number of ways to choose 3 from 50 tickets: $\binom{50}{3}=19600$

$$P(a)=\frac{\binom{4}{3}\binom{46}{0}}{\binom{50}{3}}=\frac{4}{19600}=\frac{1}{4900}\approx 0.00020$$

b exactly two of the prizes?

choose 2 winners from 4 orgs: $\binom{4}{2}=6$

choose 1 winner from the other orgs: $\binom{46}{1}=46$

The number of ways to choose 3 winners from 50 tickese $\binom{50}{3}=19600$

Thus $P(\text{exactly two prizes})=\frac{6\times 46}{19600}=\frac{69}{4900}\approx 0.014$

c exactly one of the prizes?

The number of ways choose 1 winner from 4 organizers is: $\binom{4}{1}=4$

The number of ways to choose 2 winners from the 46: $\binom{46}{2}=1035$

The total number of ways to choose 3 winners from 50 tickets: $\binom{50}{3}=19600$

$\therefore P(\text{exacely one prize})=\frac{4\times 1035}{19600}=\frac{4140}{19600}=\frac{207}{980}\approx 0.211$

d none of the prizes?

$$P(\text{no prize})=\frac{\#\text{ ways choose 3 from 46}}{\#\text{ ways none of org win a prize}}=\frac{\binom{46}{3}}{\binom{50}{3}}=\frac{15180}{19600}\approx 0.774$$

2.75 Cards are dealt, one at a time, from a standard 52-card deck.

a If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades?

spades remining $=13-2=11$

total cards remining $=52-2=50$

$$P(a)=\frac{11}{50}\times\frac{10}{49}\times\frac{9}{48}=\frac{990}{117600}=0.008418$$

b If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades?

spades remining $=13-3=10$

total cards remining $=52-3=49$

$$P(b)=\frac{10}{49}\times\frac{9}{48}=\frac{90}{2352}=\frac{15}{392}\approx 0.038$$

c If the first 4 cards are all spades, what is the probability that the next card is also a spade?

spades remining $=13-4=9$

total cards reminiy: $52-4=48$

$$P(c)=9/48=\frac{3}{16}=0.1875$$

2.86 Suppose that A and B are two events such that $P(A) = .8$ and $P(B) = .7$.

a Is it possible that $P(A \cap B) = .1$? Why or why not?

First, we need to ensure the intersection cannot exceed the individual probabilities of A or B, and it can not be less than difference between their sum and 1. thus we can get constrain: $\max(0, P(A)+P(B)-1) \le P(A \cap B) \le \min(P(A), P(B))$

Subsituting the value for P(A) and P(B):

$$\max(0, 0.8+0.7-1) \le P(A \cap B) \le \min(0.8, 0.7)$$

$$\max(0, 0.5) \le P(A \cap B) \le 0.7$$

Thus, $0.5 \le P(A \cap B) \le 0.7$, Since 0.1 is not in the range, It is Not Possible for $P(A \cap B) = 0.1$

b What is the smallest possible value for $P(A \cap B)$?

$\max(0, P(A) + P(B) - 1)$

$\Rightarrow \max(0, 0.8 + 0.7 - 1)$

$\Rightarrow \max(0, 0.5) = 0.5$

thus, the smallest possible value for $P(A \cap B)$ is 0.5.

c Is it possible that $P(A \cap B) = .77$? Why or why not?

From Part (A), we know that the maximum possible value for $P(A \cap B)$ is

$\min(P(A), P(B)) = \min(0.8, 0.7) = 0.7$

Since 0.77 is exceeds this maximum, thus 0.7 is Not Possible.

d What is the largest possible value for $P(A \cap B)$?

Form Part (a) the largest possible value for $P(A \cap B)$:

$$\min(P(A), P(B)) = \min(0.8, 0.7) = 0.7$$

thus, the largest possible value for $P(A \cap B)$ is 0.7

2.90 Suppose that there is a 1 in 50 chance of injury on a single skydiving attempt.

a If we assume that the outcomes of different jumps are independent, what is the probability that a skydiver is injured if she jumps twice?

If there is a 1 in 50 chance of injury and outcomes of different jumps are independent, the probability that a skydiver is injured on at least one of two jumps,

$P(\text{no injury in a single jump}): 1-(1/50) = 49/50$

$P(\text{no injury in two jumps}): 49/50 * 49/50 = 0.9604$

$\therefore P(\text{at least one injury over two jumps}) = 1-0.9604 = 0.0396$

b A friend claims if there is a 1 in 50 chance of injury on a single jump then there is a 100% chance of injury if a skydiver jumps 50 times. Is your friend correct? Why?

my friend's claims are incorrect, Because the outcomes are independent, The probability of at least one injury over 50 jumps should be less than 100%. $P(\text{at least one injury in 50 jumps}) = 1-\left(\frac{49}{50}\right)^{50} = 63.58\%$

2.95 Two events A and B are such that $P(A) = .2$, $P(B) = .3$, and $P(A \cup B) = .4$. Find the following:

a $P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cap B)$

$0.4 = 0.2 + 0.3 - P(A \cap B)$

$P(A \cap B) = 0.5 - 0.4 = 0.1$, thus $P(A \cap B) = 0.1$

b $P(\overline{A} \cup \overline{B})$

According to De Morgan's law:

$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B) = 1 - 0.1 = 0.9$

c $P(\overline{A} \cap \overline{B})$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - 0.4 = 0.6$

d $P(\overline{A}|B)$

$P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)}$, thus we need to find

$P(\bar{A} \cap B) = P(B) - P(A \cap B) = 0.3 - 0.1 = 0.2$, so we can subsitude to the formula

$$P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{0.2}{0.3} = \frac{2}{3} \approx 0.6667$$

thus $P(\bar{A}|B) = 0.6667$

2.102 Diseases I and II are prevalent among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract disease II eventually, and 3% will contract both diseases.

a Find the probability that a randomly chosen person from this population will contract at least one disease.

Let $P(A) = P(\text{disease I}) = 0.1$, $P(B) = P(\text{disease II}) = 0.15$

$P(A \cap B) = 0.03$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.1 + 0.15 - 0.03$$
$$= 0.22$$

Thus, a randomly chosen person from this populaion will contract at least one disease 0.22.

b Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.

$P(\text{both disease} \mid \text{at least one disease})$

$$= \frac{P(A \cap B)}{P(\text{at least one disease})} = \frac{0.03}{0.22} = \frac{0.03}{0.22} = \frac{3}{22} \approx 0.136$$

2.125 A diagnostic test for a disease is such that it (correctly) detects the disease in 90% of the individuals who actually have the disease. Also, if a person does not have the disease, the test will report that he or she does not have it with probability .9. Only 1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that she has the disease, what is the conditional probability that she does, in fact, have the disease? Are you surprised by the answer? Would you call this diagnostic test reliable?

Given: $P(\text{Positive Test} \mid \text{Disease}) = 0.9$, $P(\text{Disease}) = 0.01$

$P(\text{Positive Test} \mid \text{No Disease}) = 1 - 0.9 = 0.1$

$P(\text{No Disease}) = 1 - P(\text{Disease}) = 0.99$

we want to find: $P(\text{Disease} \mid \text{Positive Test})$, according to Bays theoerem: $P(\text{Disease} \mid \text{Positive Test}) = \dfrac{P(\text{Posive Test} \mid \text{Disease})\, P(\text{Disease})}{P(\text{Positive test})}$

$$= \frac{P(\text{Positive Test} \mid \text{Disease})\, P(\text{Disease})}{P(\text{Positive Test} \mid \text{Disease}) * P(\text{Disease}) + P(\text{Positive Test} \mid \text{No Disease}) * P(\text{No Disease})}$$

$$= \frac{0.9 * 0.01}{(0.9 * 0.01) + (0.1 * 0.99)} = \frac{0.009}{0.108} \approx 0.0833$$

Thus, the conditional probability that the person has the disease given that the test indicates she has the disease is appximatey 0.0833 or 8.33%.

Yes, I'm very superised by my answer, I think test can be considered realiable in terms of sensitivity which means it catches 90% of actual cases, but the probability of a false positive is relative high, which make the test less useful in a low-prevalence setting.

2.128 Use Theorem 2.8, the law of total probability, to prove the following:

a If $P(A|B) = P(A|\overline{B})$, then A and B are independent.

To prove A and B are independent, which means we have to prove $P(A \cap B) = P(A) \cdot P(B)$, Alternatively, we can use the conditional probability of independence, If A and B are independent, then: $P(A|B) = P(A)$

we are given that: $P(A|B) = P(A|\bar{B})$, from law of total probability of $P(A)$, we have $P(A) = P(A|B) \cdot P(B) + P(A|\bar{B}) * P(\bar{B})$

since $P(A|B) = P(A|\bar{B})$, Lets call this common value p. Thus, the above equation becomes: $P(A) = p \cdot P(B) + p \cdot P(\bar{B})$

Factor out p: $P(A) = p \cdot (P(B) + P(\bar{B})) = p \cdot 1 = p$

then we find: $P(A|B) = p = P(A)$, Since $P(A|B) = P(A)$, we conclude that A and B are independent.

b If $P(A|C) > P(B|C)$ and $P(A|\overline{C}) > P(B|\overline{C})$, then $P(A) > P(B)$.

according to the law of total probability for both $P(A)$ and $P(B)$, First for $P(A)$: $P(A) = P(A|C) \cdot P(C) + P(A|\bar{C}) \cdot P(\bar{C})$

for $P(B)$: $P(B) = P(B|C) \cdot P(C) + P(B|\bar{C}) \cdot P(\bar{C})$

we are given that: $P(A|C) > P(B|C)$ and $P(A|\bar{C}) > P(B|\bar{C})$

since both of these inequalities hold, it implies that each term in equation for $P(A)$ is greater than the corresponding term in the equation for $P(B)$, that is: $P(A|C) \cdot P(C) > P(B|C) \cdot P(C)$ and $P(A|\bar{C}) \cdot P(\bar{C}) > P(B|\bar{C}) \cdot P(\bar{C})$, thus we can add these two inequalities together, we get:

$P(A) = P(A|C) \cdot P(C) + P(A|\bar{C}) \cdot P(\bar{C}) > P(B|C) \cdot P(C) + P(B|\bar{C}) \cdot P(\bar{C})$

$= P(B)$, Therefore, $P(A) > P(B)$.