Introduction to Mathematical Statistics I
Midterm

OSU-ID:

Name: Freya Zou.

Instructions:

- This exam consists of 6 problems. Each problem is worth 5 points.
- You must show your work and give concise yet complete answers to receive full credit.
- You are allowed to use a calculator. But you will not be allowed to share a calculator with other students and you will not be allowed to use your cell phone as a calculator.
- You are allowed to use one (two-sided) sheet of notes for this exam. The sheet cannot be larger than a standard printing size and cannot be shared with other students.
- Formulas will not be given during the exam.
- Answer the questions to the best of your knowledge or interpretation. No clarifications will be given during the exam.

Problem 1

Determine whether the following statements are true or false. Justify your answer.

(i) If A, B are mutually exclusive events, then A and B are independent.

(ii) If A, B are independent, then A and B^c (the complement of B) are also independent.

(iii) For a discrete random variable Y we have that $E(Y^2) \geq [E(Y)]^2$.

i) F, $P(A \cap B) = P(A)P(B) \neq$

ii) T $P(A)$ an B, are independent, then are indep

iii) T. $E(Y^2) - [E(Y)]^2 = Var(Y)$

Problem 2

Suppose that in a large shipment of automobile tires 5% are defective. An auto parts store purchases 50 tires from this shipment:

(i) What is the probability that none of the tires is defective?

(ii) What is the probability that at least one of the tires are defective?

(iii) What is the expected number of defective tires we can find at the store?

Given P(defective) = 0.05

i) P(none defective) = 1 - P(defective) = 1 - 0.05 = 0.95.

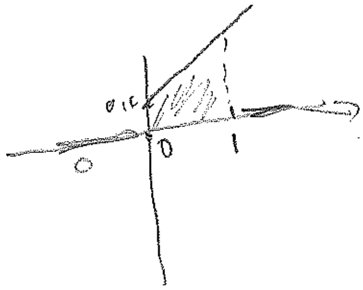
we have 50 tires : $(0.95)^{50}$ = 0.0769

P(none defective in 50 tires) = 0.769

ii) P(at least one of tires are defective) = $P(X \leq 1) = 1 - P(X=0)$

$\Rightarrow 1 - 0.769 = 0.231$

iii) $E(X) = n \cdot p = 50 * 0.05 = 2.5$



Problem 3
Let Y be a random variable with pdf

$$f(y) = \begin{cases} 0.4 + cy & , 0 \leq y \leq 1 \\ 0 & , \text{elsewhere} \end{cases}$$

(i) Find the value of c, that makes $f(y)$ a pdf.

(ii) Find the cdf $F(y)$.

(iii) Find $P(0 < Y < 1)$.

i) $1 = \int_{-\infty}^{\infty} f(x)dx = \int_0^1 (0.4 + cy)\,dy$

$1 = \int_0^1 0.4\,dy + c\int_0^1 y\,dy$

$1 = 0.4y\Big|_0^1 + 0.5c$

$1 = 0.9c$

$c = 1.11$

ii) $f(y) = \begin{cases} 0.4 + 1.11y & , 0 \leq y \leq 1 \longrightarrow \text{part (i)} \\ 0, & \text{elsewhere} \end{cases}$

For $0 \leq y \leq 1$, $\int_0^y (0.4 + 1.11t)\,dt \overset{(1)}{\Rightarrow} \int_0^y 0.4\,dt + \int_0^y 1.11t\,dt \overset{(2)}{\Rightarrow} 0.4t\Big|_0^y + \frac{1.11t^2}{2}\Big|_0^y$

$\overset{(3)}{\Rightarrow} 0.4y + \frac{1.11y^2}{2}$

$\therefore$ we have $F(y) = \begin{cases} 0.4y + \frac{1.11y^2}{2}, & 0 \leq y \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$

iii) $P(0<Y<1) = \int_0^1 (0.4 + 1.11y)\,dy = 0.955$

Problem 4

Suppose that A and B are two events such that $P(A) = 0.8$ and $P(B) = 0.7$. Suppose that A and B are two events such that $P(A) = 0.8$ and $P(B) = 0.7$.

(i) Is it possible that $P(A \cap B) = 0.1$. <u>Justify your answer.</u>

(ii) What is the smallest possible value for $P(A \cap B)$? <u>Justify your answer.</u>

(iii) What is the largest possible value for $P(A \cap B)$? <u>Justify your answer.</u>

i) Yes, P(A)

ii) $P(A) * P(B) = 0.8 * 0.7 = \boxed{0.56}$ → smallest

iii) $P(A \cap B) = P(B) - (P(A^c \cap B))$

$= 0.8 -$

Problem 5
A quality-control program at a plastic bottle production line involves inspecting finished bottles for flaws such as microscopic holes. The probability that a bottle has a flaw is 0.002. If a bottle has a flaw, the probability that it will fail the inspection is 0.995. If a bottle does not have a flaw, the probability that it will pass the inspection is 0.990.

(i) If a bottle does not have a flaw, what is the probability that it will fail the inspection?

(ii) What is the probability that a randomly selected bottle has a flaw and fails the inspection?

(iii) If a bottle fails the inspection, what is the probability that it has a flaw?

Given, $P(\text{flaw}) = 0.002$, $P(\text{fail} \mid \text{flaw}) = 0.995$

$P(\text{pass} \mid \text{no flaw}) = 0.990$ $\quad P(\text{no flaw}) = 1 - 0.002 = 0.998$

$= 1 - 0.990 = 0.11$

i) $P(\text{fail} \mid \text{no flaw}) = 1 - (\text{pass} \mid \text{no flaw}) = 1 - 0.990 = 0.11$

ii) $P(\text{flaw} \cap \text{fail}) = P(\text{fail} \mid \text{flaw}) \cdot P(\text{flaw}) = 0.995 * 0.002 = 1.99$

iii) $P(\text{flaw} \mid \text{fail}) = \dfrac{P(\text{fail} \mid \text{flaw})\, P(\text{flaw})}{P(\text{fail} \mid \text{flaw}) P(\text{flaw}) + P(\text{fail} \mid \text{no flaw})\, P(\text{no flaw})}$

$= \dfrac{0.995 * 0.002}{0.995 * 0.002 + \underset{\text{part i}}{0.11} * 0.998}$ (above in.)

$= \dfrac{1.99}{}$

Problem 6
The following table presents the probability function of the number of defects X in a randomly chosen printed-circuit board:

x	0	1	2	3	4	5
$p(x)$	0.10	??	0.20	0.25	0.20	0.10

(i) Find the probability that a randomly selected board has at least three defects.

(ii) Find $P(X=1)$ and compute $P(0.5 \le X \le 2.5)$.

(iii) Find the mean for the number of defects in a printed-circuit board.

i) $P(X=3) = 0.25$

ii) $P(X=1) = 1 - [P(X=0) + P(X=2) + P(X=3) + P(X=4) + P(X=5)]$

$= 1 - [0.1 + 0.2 + 0.25 + 0.2 + 0.1]$

$= 1 - 0.85$

$= 0.15$

$P(0.5 \le X \le 2.5) = P(X=1) + P(X=2) = 0.15 + 0.2 = 0.35$

iii) $V(X) = \sum_x x \cdot p(x) = 0(0.1) + 1(0.15) + 2(0.2) + 3(0.25) + 4(0.2) + 5(0.1)$

$= 2.6$

thus the mean is 2.6.