Numerical Linear Algebra - Sheet 5

to be handed in until November 22, 2023, 11am

Problem 1. Problem 2.4.16 in the Lecture Notes:

1. How many operations do the two versions of the Hessenberg QR step require?
2. Show that if $\mathbf{H}$ is Hermitian, the result of the Hessenberg QR step is Hermitian as well.

Problem 2. Problem 2.4.20 in the Lecture Notes:
Show that every (complex) Hermitian matrix is orthogonally similar to a symmetric tridiagonal matrix with real entries.

Problem 3.

(a) Implement the implicit Hessenberg QR step (Algorithm 2.4.7 in the lecture notes) in real arithmetic.

(b) Test your code with the tridiagonal matrix $\mathbf{A}_n = \mathrm{tridiag}(-1., 2., -1.)$ in dimension $n = 4$.

(c) Use your implementation to run several steps of the QR iteration (Algorithm 2.4.19 in the lecture notes) for the matrix $\mathbf{A}_{10}$. You should obtain the following eigenvalues:

$$\lambda_j = 2 - 2\cos(j\vartheta) = 4\sin^2\left(\frac{j\vartheta}{2}\right)$$

where $\vartheta = \frac{\pi}{n+1}$, $1 \leq j \leq n$.

(d) Discuss the observed convergence of the off-diagonal and diagonal entries, respectively.

Problem 4.

(a) Implement the QR iteration with shift (Algorithm 2.4.25 in the Lecture Notes) using the Wilkinson shift (Definition 2.4.30).

(b) Test your implementation with the matrix $\mathbf{A}_{10}$. Run several steps of the iteration and observe the behaviour of the subdiagonal elements, especially the last one.

(c) For the curious: Implement the QR iteration with deflation (cf. Algorithm 2.4.35).