Numerical Linear Algebra - Sheet 4

to be handed in until November 15, 2023,11am.

Problem 1. Write a program `COMPUTE_EIGENVALUES` that

1. computes the dominant eigenvalue of a matrix using the power method.
2. computes all further eigenvalues using suitable matrix polynomials.

Test your program with the following matrix

$$\mathbf{A} = \begin{pmatrix} 1+i & 1 & 0 & 0 & 0 \\ 0 & 1+i & 1 & 0 & 0 \\ 0 & 0 & 1+i & 1 & 0 \\ 0 & 0 & 0 & 1+i & 1 \\ 1 & 0 & 0 & 0 & 1+i \end{pmatrix}.$$

Problem 2. Let $\mathbf{A}$ be a symmetric tridiagonal matrix. Show that the QR-iteration (see Algorithm 2.4.2 in the lecture notes) preserves the tridiagonal structure of the matrix, i.e., all iterates $\mathbf{A}^{(n)}$ generated by the QR-iteration are tridiagonal.

Problem 3. Rewrite the QR factorization of a tridiagonal (complex) symmetric matrix such that its complexity is of order $O(n)$ (this proves the second part of Corollary 2.4.17 in the lecture notes).