

Numerical Linear Algebra - Sheet 10

discussion in the first tutorials of January, 2025

This exercise sheet reviews some topics of this lecture. Try first to answer without looking at the notes. Your answers do not have to be handed in, but the exercises will be discussed in the tutorials of the first week of January.

Problem 1. Recapitulate the concept of an orthogonal projection and an oblique projection. What are use cases of both and why are they important for numerical linear algebra?

Problem 2. How can the QR factorization of a given matrix be computed? Discuss downsides and benefits of the different methods.

Problem 3. Householder reflections vs. Givens rotations: Which one is more cost-efficient in the general case? When is using the other one advantageous?

Problem 4. Which of the discussed methods for solving eigenvalue problems can be implemented without explicitly forming a matrix?

Problem 5. Consider an $n \times n$ matrix that has n distinct eigenvalues such that $|\lambda_i| \neq |\lambda_j|$ for $i \neq j$. How can the eigenvalue with the second largest absolute value be computed?

Problem 6. When does one step of the Gauss-Seidel iteration provide the direct solution of a linear system? Consider an upper triangular matrix to visualize this matter.

Problem 7. What are drawbacks of the orthogonal subspace iteration? When does it converge?

Problem 8. Why is the implicit double shift better than the explicit?

Problem 9. What is deflation? Why do we use it?

Problem 10. What is polynomial filtering?

Problem 11. Why do we use iterative solvers?

Problem 12. How does the Gauss-Seidel method converge for an upper triangular matrix?

Problem 13. What is the difference between the steepest descent and the minimal residual method?

Problem 14. What is the difference between the steepest descent and the conjugate gradient method?

Problem 15. Why do we use conjugate gradient and not a straight forward implementation of a Lanczos solver?

Problem 16. What is a Krylov space?

Problem 17. What is the difference between the Arnoldi and the Lanczos processes?

Problem 18. Is there a short recurrence for the GMRES method?

Problem 19. What is preconditioning?

Problem 20. What complicates preconditioning for the conjugate gradient method?