

Numerical Linear Algebra - Sheet 10

discussion in the first tutorials of January, 2025

This exercise sheet reviews some topics of this lecture. Your answers do not have to be handed in, but the exercises will be discussed in the tutorials of the first week of January.

Problem 1. Recapitulate the concept of an orthogonal projection and an oblique projection. What are use cases of both and why are they important for numerical linear algebra?

Problem 2. What methods presented in the lecture can be used for computing or estimating the eigenvalues of a matrix \mathbf{A} ? Sort by properties of the methods, as well as by conditions on \mathbf{A} .

Problem 3. How can the QR factorization of a given matrix be computed? Discuss downsides and benefits of the different methods.

Problem 4. Householder reflections vs. Givens rotations: which one is more cost-efficient in the general case? When using the other one is advantageous?

Problem 5. Which of the discussed methods for solving eigenvalue problems can be implemented without explicitly forming a matrix?

Problem 6. Consider an $n \times n$ matrix that has n distinct eigenvalues such that $|\lambda_i| \neq |\lambda_j|$ for $i \neq j$. How can the eigenvalue with the second largest absolute value be computed?

Problem 7. When does one step of the Gauss-Seidel iteration provide the direct solution of a linear system? Consider an upper triangular matrix to visualize this matter.