Weak Convergence of the Conditional U-process

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Abstract

Stute [8] introduced a class of estimators called conditional U-statistics. They could be seen as a generalization of the Nadaraya-Watson estimator for the regression function

$$U(\varphi,t)=\mathbb{E}(\varphi(Y_{i_1},\cdots,Y_{i_m})|(X_{i_1},\cdots,X_{i_m})=t),\quad t\in\mathbb{R}^{dm}.$$

In this work we are intersted in establishing the weak convergence for conditional U- process under $\beta-$ mixing observations.

Let consider the strictly stationary $\beta-$mixing sequence of random elements $\{(X_i,Y_i),i\in\mathbb{Z}\}$ with $X_i\in\mathbb{R}^d$ and $Y_i\in\mathcal{X}$ some polish space.The conditional U-statistic based on $\{(X_i,Y_i),i\in\mathbb{Z}\}$ and the kernel $\varphi\tilde{K}$ is defined by

$$U_n^m(\varphi,t)=\frac{\sum_{(i_1,\cdots,i_m)\in I_n^m}\varphi(Y_{i_1},\cdots,Y_{i_m})K\left(\frac{t_{i_1}-X_{i_1}}{h}\right)\cdots K\left(\frac{t_{i_m}-X_{i_m}}{h}\right)}{\sum_{(i_1,\cdots,i_m)\in I_n^m}K\left(\frac{t_{i_1}-X_{i_1}}{h}\right)\cdots K\left(\frac{t_{i_m}-X_{i_m}}{h}\right)}\tag{0.1}$$

where

$$I_n^m=\{(i_1,\cdots,i_m):i_j\in\mathbb{Z}\;and\;i_r\neq i_j\;if\;r\neq j\},\;\tilde{K}(\mathbf{t})=\prod_{i=1}^m K(t_i),\;\mathbf{t}=(t_1,\cdots,t_m)\in\mathbb{R}^m$$

and $\varphi\in\mathcal{F}$ which is a calss of measurable functions from $\mathcal{X}^m$ to $\mathbb{R}$ and $\mathcal{K}=\left\{K\left(\frac{\cdot-x}{h}\right),x\in\mathbb{R}\right\}$.

The conditional U-process on which the study is, is indexed by $\mathcal{FK}^m$ and given by

$$\{\eta_n^m(\varphi,\mathbf{t})\}_{\mathcal{FK}^m}:=\{\sqrt{nh^m}(U_n^m(\varphi,\mathbf{t})-U(\varphi.\mathbf{t}))\}_{\mathcal{FK}^m},\tag{0.2}$$

Keywords: absolute regularity, β-mixing, conditional U-process, conditional U-statistic, Nadaraya-Watson estim tor, central limit theorem, weak convergence.

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