

$$S(t) = s_0 + s_1 t + \dots$$

$$\text{RMST}(h) = \int_0^h S(t) dt = s_0 \cdot h + \frac{1}{2} s_1 \cdot h^2 + \dots$$

$$[\text{RMSDT}(h)]^2 = 2 \int_0^h t \cdot S(t) dt - \cancel{2S} [\text{RMST}(h)]^2$$

$$= 2 \cdot \left[\frac{1}{2} s_0 h^2 + \frac{1}{3} s_1 h^3 \right] - s_0^2 h^2 - 2s_0 s_1 h^3 + \dots$$

$$= s_0(1-s_0)h^2 + 2s_1\left(\frac{1}{3} - s_0\right)h^3 + \dots$$

$$Z(h) = \frac{\text{RMST}^{(1)}(h) - \text{RMST}^{(0)}(h)}{[(\text{RMSDT}^{(1)})^2 + (\text{RMSDT}^{(0)})^2]^{1/2}}$$

$$= \frac{(s_0^{(1)} - s_0^{(0)}) \cdot h + \frac{1}{2}(s_1^{(1)} - s_1^{(0)}) \cdot h^2}{\left[(s_0^{(1)}(1-s_0^{(0)}))h^2 + s_0^{(0)}(1-s_0^{(0)})h^2 + \left(2s_1^{(1)}\left(\frac{1}{3} - s_0^{(1)}\right) + 2s_1^{(0)}\left(\frac{1}{3} - s_0^{(0)}\right) \right)h^3 \right]^{1/2}}$$

But $s_0 = 1$. Therefore

$$Z(h) = \frac{\frac{1}{2}(s_1^{(1)} - s_1^{(0)})h^2}{\left[-\frac{2}{3}(s_1^{(1)} + s_1^{(0)})h^3 \right]^{1/2}} + \dots$$

$$\lim_{h \rightarrow 0} Z(h) = \mathcal{O}(h^{1/2}) = 0 \quad \left(\text{note } s_1 \leq 0, \right. \\ \left. \text{so denominator never zero unless numerator zero} \right)$$