Formeln

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1 Exponential

1. $S(t) = \exp(-\lambda t)$
2. $h(t) = \lambda$
3. $RMST = \int_0^\tau S(t)\,dt$
4. $RMSTR = \frac{RMST_1}{RMST_0} = \frac{\int_0^\tau S_1(t)\,dt}{\int_0^\tau S_0(t)\,dt} = \frac{\lambda_0[1-\exp(-\lambda_1\tau)]}{\lambda_1[1-\exp(-\lambda_0\tau)]}$

1.1 RMSTR against HR, and RMSTR against τ

let hazard ratio $HR = \frac{\lambda_1}{\lambda_0}$, then
$RMSTR = \frac{1-\exp(-HR\lambda_0\tau)}{HR[1-\exp(-\lambda_0\tau)]}$

1.2 RMSTR against HR within mixed cure fraction model

$S_i(t) = p + (1-p)\exp(-\lambda_i\tau)$ for $i = 0, 1$ with cure fraction p
$RMSTR = \frac{\int_0^\tau S_1(t)\,dt}{\int_0^\tau S_0(t)\,dt} = \frac{\exp(-\lambda_1\tau)\{\exp(\lambda_1\tau)[p(\lambda_1\tau-1)+1]+p-1\}}{\exp(-\lambda_0\tau)\{\exp(\lambda_0\tau)[p(\lambda_0\tau-1)+1]+p-1\}}$
$= \frac{\exp(-HR\lambda_0\tau)\{\exp(HR\lambda_0\tau)[p(HR\lambda_0\tau-1)+1]+p-1\}}{\exp(-\lambda_0\tau)\{\exp(\lambda_0\tau)[p(\lambda_0\tau-1)+1]+p-1\}}$

1.3 RMSTR against acceleration factor ϕ and against τ

let $S_1(t) = S_0(\phi t) = \exp(-\lambda\phi t)$, then
$RMSTR = \frac{\int_0^\tau S_1(t)\,dt}{\int_0^\tau S_0(t)\,dt} = \frac{1-\exp(-\lambda\tau\phi)}{\phi-\phi\exp(-\lambda\tau)}$

2 Weibull

$S(t) = \exp(-\lambda^p t^p)$

2.1 Weibull RMSTR against τ

$RMST = \int_0^\tau S(t)\,dt = -\frac{\Gamma(1/p,(\tau\lambda)^p)}{\lambda p} - [-\frac{\Gamma(1/p,(0\lambda)^p)}{\lambda p}] = \frac{\Gamma(1/p)}{\lambda p} - \frac{\Gamma(1/p,(\tau\lambda)^p)}{\lambda p} = \frac{\Gamma(1/p)-\Gamma(1/p,(\tau\lambda)^p)}{\lambda p}$
$RMSTR = \frac{RMST_1}{RMST_0} = \frac{\Gamma(1/p_1)-\Gamma(1/p_1,(\tau\lambda_1)^{p_1})}{\lambda_1 p_1} \times \frac{\lambda_0 p_0}{\Gamma(1/p_0)-\Gamma(1/p_0,(\tau\lambda_0)^{p_0})}$

2.2 Weibull RMSTR against acceleration factor ϕ

let $S_1(t) = S_0(\phi t) = \exp(-\lambda^p \phi^p t^p)$, then
$RMSTR = \frac{\int_0^\tau S_1(t)\,dt}{\int_0^\tau S_0(t)\,dt} = \frac{\Gamma(1/p_1)-\Gamma(1/p_1,(\phi\tau\lambda_1)^{p_1})}{\lambda_1 p_1} \times \frac{\lambda_0 p_0}{\Gamma(1/p_0)-\Gamma(1/p_0,(\phi\tau\lambda_0)^{p_0})}$