Summer 2022 CS 4641\7641 A: Machine Learning Homework 4

Instructor: Dr. Mahdi Roozbahani

Deadline: Monday, July 25, 2022 11:59 pm AOE

- No unapproved extension of the deadline is allowed. Late submission will lead to 0 credit.
- Discussion is encouraged on Ed as part of the Q/A. However, all assignments should be done individually.
- Plagiarism is a **serious offense**. You are responsible for completing your own work. You are not allowed to copy and paste, or paraphrase, or submit materials created or published by others, as if you created the materials. All materials submitted must be your own.
- All incidents of suspected dishonesty, plagiarism, or violations of the Georgia Tech Honor Code will be subject to the institute's Academic Integrity procedures. If we observe any (even small) similarities/plagiarisms detected by Gradescope or our TAs, **WE WILL DIRECTLY REPORT ALL CASES TO OSI**, which may, unfortunately, lead to a very harsh outcome. **Consequences can be severe, e.g., academic probation or dismissal, grade penalties, a 0 grade for assignments concerned, and prohibition from withdrawing from the class.**

Instructions for the assignment

- This assignment consists of both programming and theory questions.
- Unless a theory question explicitly states that no work is required to be shown, you must provide an explanation, justification, or calculation for your answer.
- To switch between cell for code and for markdown, see the menu -> Cell -> Cell Type
- You can directly type Latex equations into markdown cells.
- If a question requires a picture, you could use this syntax `<img src="" style="width: 300px;"/>` to include them within your ipython notebook.
- Your write up must be submitted in PDF form. You may use either Latex, markdown, or any word processing software. We will **NOT** accept handwritten work. Make sure that your work is formatted correctly, for example submit $\sum_{i=0} x_i$ instead of \text{sum_{i=0} x_i}
- When submitting the non-programming part of your assignment, you must correctly map pages of your PDF to each question/subquestion to reflect where they appear. **Improperly mapped questions may not be graded correctly and/or will result in point deductions for the error.**
- All assignments should be done individually, and each student must write up and submit their own answers.
- **Graduate Students**: You are required to complete any sections marked as Bonus for Undergrads

Using the autograder

- You will find three assignments (for grads) on Gradescope that correspond to HW4: "Assignment 4 Programming", "Assignment 4 - Non-programming" and "Assignment 4 Programming - Bonus for all". Undergrads will have an additional assignment called "Assignment 4 Programming - Bonus for Undergrads".
- You will submit your code for the autograder in the Assignment 4 Programming sections. Please refer to the Deliverables and Point Distribution section for what parts are considered required, bonus for undergrads, and bonus for all".
- We provided you different .py files and we added libraries in those files please DO NOT remove those lines and add your code after those lines. Note that these are the only allowed libraries that you can use for the homework
- You are allowed to make as many submissions until the deadline as you like. Additionally, note that the autograder tests each function separately, therefore it can serve as a useful tool to help you debug your code if you are not sure of what part of your implementation might have an issue
- **For the "Assignment 4 - Non-programming" part, you will download your Jupyter Notebook as html and submit it as a PDF on Gradescope. To download the notebook as html, click on "File" on the top left corner of this page and select "Download as > html". Then, open the html file and print to PDF.** Please refer to the Deliverables and Point Distribution section for an outline of the non-programming questions.
- **When submitting to Gradescope, please make sure to mark the page(s) corresponding to each problem/sub-problem. The pages in the PDF should be of size 8.5" x 11", otherwise there may be a deduction in points for extra long sheets.**

Using the local tests

- For some of the programming questions we have included a local test using a small toy dataset to aid in debugging. The local test sample data and outputs are stored in .py files in the **local_tests_folder**
- There are no points associated with passing or failing the local tests, you must still pass the autograder to get points.
- **It is possible to fail the local test and pass the autograder** since the autograder has a certain allowed error tolerance while the local test allowed error may be smaller. Likewise, passing the local tests does not guarantee passing the autograder.
- **You do not need to pass both local and autograder tests to get points, passing the Gradescope autograder is sufficient for credit.**
- It might be helpful to comment out the tests for functions that have not been completed yet.
- It is recommended to test the functions as it gets completed instead of completing the whole class and then testing. This may help in isolating errors. Do not solely rely on the local tests, continue to test on the autograder regularly as well.

Deliverables and Points Distribution

Q1: Two Layer NN [65 pts; 50pts + 15pts Undergrad Bonus]

Deliverables: NN.py and Notebook Graphs

- **1.1 NN Implementation** [55pts; 45pts + 10pts **Bonus for Undergrad**] - *programming*
  - Leaky relu [5pts]
  - tanh [5pts]
  - loss [5pts]
  - forward [10pts]
  - backward [10pts]
  - Gradient Descent [10pts]
  - Batch Gradient Descent [10pts **Bonus for Undergrad**]
- **1.2 Loss plot and MSE for Gradient Descent** [5pts] - *non-programming*
- **1.3 Loss plot and MSE for Batch Gradient Descent** [5pts **Bonus for Undergrad**] - *non-programming*

Q2: CNN [15pts; 12pts Bonus for Undergrad + 3pts Bonus for All]

Deliverables: cnn.py and Written Report

- **2.1 Image Classification using Keras CNN** [12pts **Bonus for Undergrad**]
  - 2.1.3 Building the Model [2pts **Bonus for Undergrad**] - *non-programming*
  - 2.1.4 Training the Model [8pts **Bonus for Undergrad**] - *non-programming*
  - 2.1.5 Examining Accuracy and Loss [2pts **Bonus for Undergrad**] - *non-programming*
- **2.2 Exploring Deep CNN Architectures** [3pts **Bonus for All**] - *non-programming*

Q3: Random Forest [50pts; 40pts + 10pts Bonus for All]

Deliverables: random_forest.py and Written Report

- 3.1 Random Forest Implementation [35pts] - *programming*
- 3.2 Hyperparameter Tuning with a Random Forest [5pts] - *programming*
- 3.3 Plotting Feature Importance [5pts **Bonus for All**] - *non-programming*
- 3.4 Improvement [5pts **Bonus for All**] - *non-programming*

Q4: SVM [30pts Bonus for all]

Deliverables: feature.py and Written Report

- 4.1: Fitting an SVM Classifier by hand [20pts] - *non programming*
- 4.2: Feature Mapping [10pts] - *programming*

Environment Setup

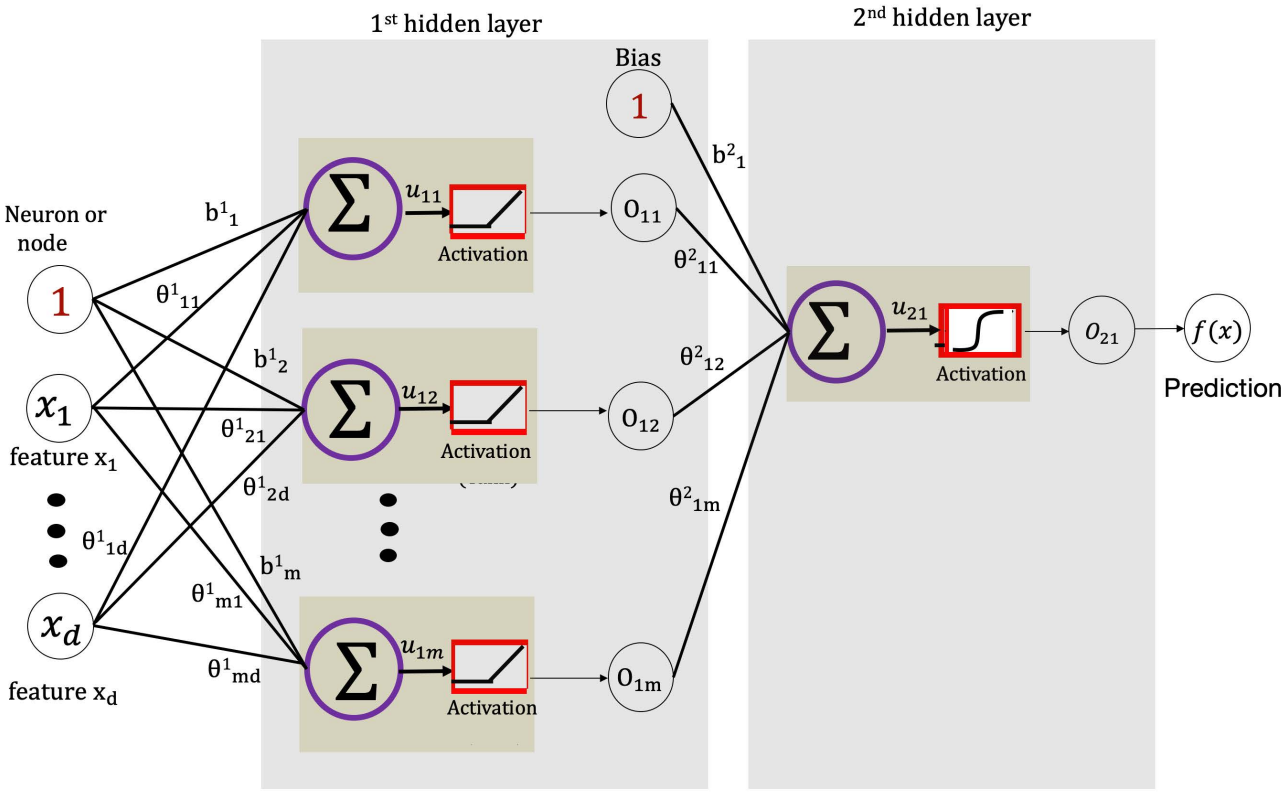
In [1]:

```
import sys
import matplotlib
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import load_diabetes
from sklearn.preprocessing import MinMaxScaler

from sklearn.model_selection import train_test_split
from sklearn.metrics import classification_report
from sklearn.metrics import plot_confusion_matrix
from sklearn.metrics import mean_squared_error

from collections import Counter
from scipy import stats
from math import log2, sqrt
import pandas as pd
import time
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import LabelEncoder
from sklearn.tree import DecisionTreeClassifier

from sklearn.datasets import make_moons
from sklearn.metrics import accuracy_score
from sklearn import svm

print('Version information')

print('python: {}'.format(sys.version))
print('matplotlib: {}'.format(matplotlib.__version__))
print('numpy: {}'.format(np.__version__))

%load_ext autoreload
%autoreload 2
%reload_ext autoreload
```

```
Version information
python: 3.7.4 (default, Aug 13 2019, 15:17:50)
[Clang 4.0.1 (tags/RELEASE_401/final)]
matplotlib: 3.1.0
numpy: 1.17.2
```

1: Two Layer Neural Network [65pts; 50pts + 15pts Undergrad Bonus] [P] [W]

Perceptron



Notation clarification – superscript represents the layer number, subscripts represent the specific units in two adjacent layers being connected by theta

θ^1_{21} - theta of the 1st layer connecting the 2nd hidden unit of the 1st layer and the 1st input unit

θ^2_{12} - theta of the 2nd layer connecting the 1st hidden unit of the 2nd layer and the 2nd hidden unit of the 1st layer

b^1_1 – bias of the 1st hidden unit of the 1st layer

A single layer perceptron can be thought of as a linear hyperplane as in logistic regression followed by a non-linear activation function.

$$u_i = \sum_{j=1}^{d} \theta_{ij} x_j + b_i$$

$$o_i = \phi\left(\sum_{j=1}^{d} \theta_{ij} x_j + b_i\right) = \phi(\theta_i^T x + b_i)$$

where $\boldsymbol{x}$ is a d-dimensional vector i.e. $\boldsymbol{x} \in \boldsymbol{R^d}$. It is one datapoint with $\boldsymbol{d}$ features. $\boldsymbol{\theta_i} \in \boldsymbol{R^d}$ is the weight vector for the $\boldsymbol{i^{th}}$ hidden unit, $\boldsymbol{b_i} \in \boldsymbol{R}$ is the bias element for the $\boldsymbol{i^{th}}$ hidden unit and $\boldsymbol{\phi(.)}$ is a non-linear activation function that has been described below. $\boldsymbol{u_i}$ is a linear combination of the features in $\boldsymbol{x_j}$ weighted by $\boldsymbol{\theta_i}$ whereas $\boldsymbol{o_i}$ is the $\boldsymbol{i^{th}}$ output unit from the activation layer.

Fully connected Layer

Typically, a modern neural network contains millions of perceptrons as the one shown in the previous image. Perceptrons interact in different configurations such as cascaded or parallel. In this part, we describe a fully connected layer configuration in a neural network which comprises multiple parallel perceptrons forming one layer.

We extend the previous notation to describe a fully connected layer. Each layer in a fully connected network has a number of input/hidden/output units cascaded in parallel. Let us a define a single layer of the neural net as follows:
$\boldsymbol{m}$ denotes the number of hidden units in a single layer $\boldsymbol{l}$ whereas $\boldsymbol{n}$ denotes the number of units in the previous layer $\boldsymbol{l-1}$.

$$\boldsymbol{u^{[l]} = \theta^{[l]} o^{[l-1]} + b^{[l]}}$$

where $\boldsymbol{u^{[l]} \in R^m}$ is a m-dimensional vector pertaining to the hidden units of the $\boldsymbol{l^{th}}$ layer of the neural network after applying linear operations. Similarly, $\boldsymbol{o^{[l-1]}}$ is the n-dimensional output vector corresponding to the hidden units of the $\boldsymbol{(l-1)^{th}}$ activation layer. $\boldsymbol{\theta^{[l]} \in R^{m \times n}}$ is the weight matrix of the $\boldsymbol{l^{th}}$ layer where each row of $\boldsymbol{\theta^{[l]}}$ is analogous to $\boldsymbol{\theta_i}$ described in the previous section i.e. each row corresponds to one hidden unit of the $\boldsymbol{l^{th}}$ layer. $\boldsymbol{b^{[l]} \in R^m}$ is the bias vector of the layer where each element of b pertains to one hidden unit of the $\boldsymbol{l^{th}}$ layer. This is followed by element wise non-linear activation function $\boldsymbol{o^{[l]} = \phi(u^{[l]})}$. The whole operation can be summarized as,

$$\boldsymbol{o^{[l]} = \phi(\theta^{[l]} o^{[l-1]} + b^{[l]})}$$

where $\boldsymbol{o^{[l-1]}}$ is the output of the previous layer.

Activation Function

There are many activation functions in the literature but for this question we are going to use Leaky Relu and Tanh only.

HINT 1: When calculating the tanh and leaky relu function, make sure you are not modifying the values in the original passed in matrix. You may find np.copy() helpful (`u` should not be modified in the method.) .

Relu and Leaky Relu

The rectified linear unit (Relu) is one of the most commonly used activation functions in deep learning models. The mathematical form is

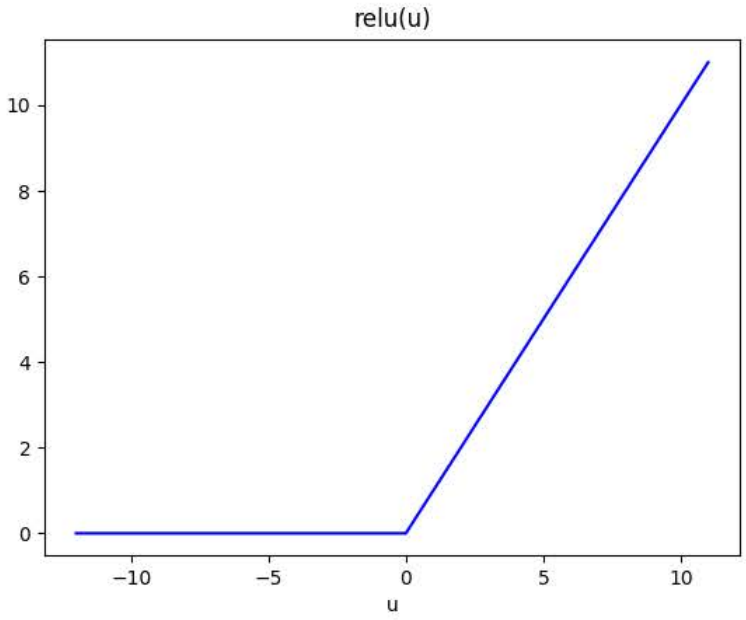
$$o = \phi(u) = max(0, u)$$

One of the advantages of Relu is that it is a fast nonlinearity.

The derivative of relu function is given as $o' = \phi'(u) = \begin{cases} 0 & u \leq 0 \\ 1 & u > 0 \end{cases}$

Leaky ReLU is a type of activation function based on a ReLU. The difference is that it has a small slope for negative values instead of a flat slope. The slope coefficient is determined before training. Leaky relu is a popular solution for sparse gradients, for example training generative adversarial networks.

Here in our homework, we are going to implement leaky relu.



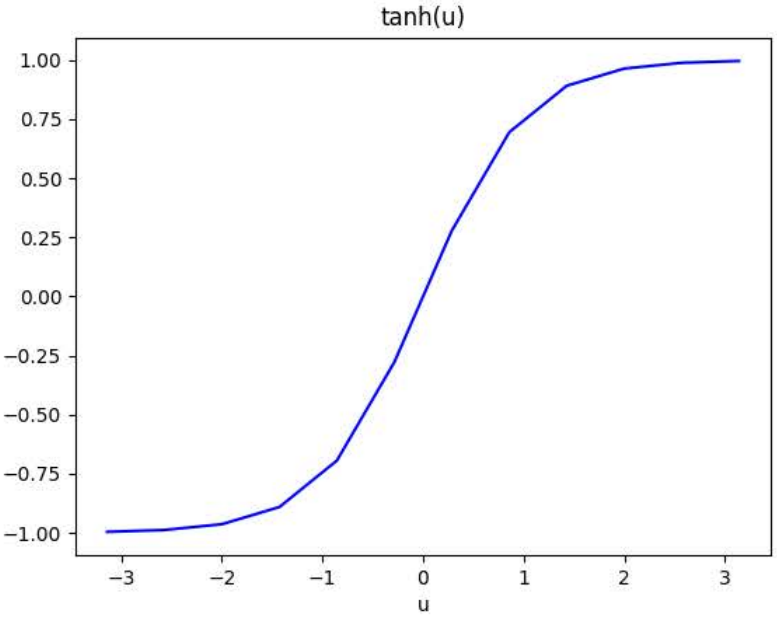
Tanh

Tanh also known as hyperbolic tangent is like a shifted version of sigmoid activation function with its range going from -1 to 1. Tanh almost always proves to be better than the sigmoid function since the mean of the activations are closer to zero. Tanh has an effect of centering data that makes learning for the next layer a bit easier. The mathematical form of tanh is given as

$$o = \phi(u) = tanh(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

The derivative of tanh is given as

$$o' = \phi'(u) = 1 - \left(\frac{e^u - e^{-u}}{e^u + e^{-u}}\right)^2 = 1 - o^2$$



Sigmoid

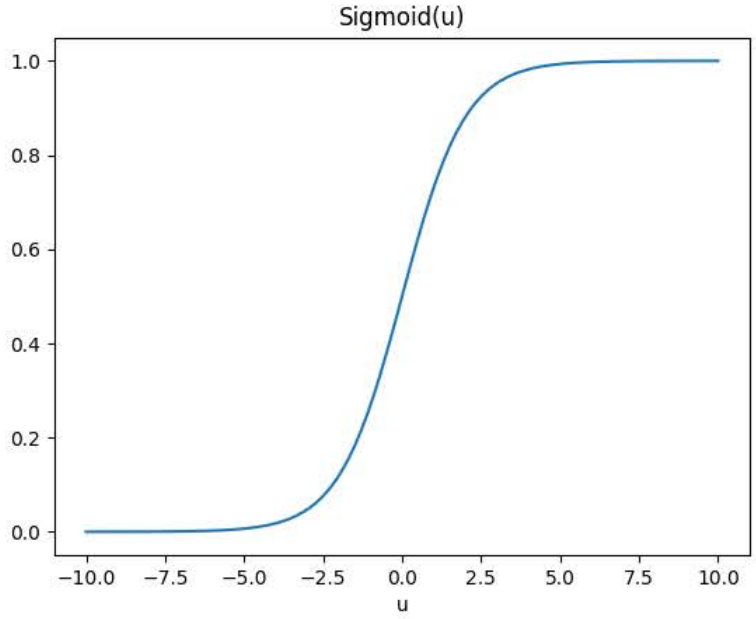
The sigmoid function is another non-linear function with S-shaped curve. This function is useful in the case of binary classification as its output is between 0 and 1. The mathematical form of the function is

$$o = \phi(u) = \frac{1}{1+e^{-u}}$$

The derivation of the sigmoid function has a nice form and is given as

$$o' = \phi'(u) = \frac{1}{1+e^{-u}}\left(1 - \frac{1}{1+e^{-u}}\right) = \phi(u)(1-\phi(u))$$

Note: We will not be using sigmoid activation function for this assignment. This is included only for the sake of completeness.



Mean Squared Error

It is an estimator that measures the average of the squares of the errors i.e. the average squared difference between the actual and the estimated values. It estimates the quality of the learnt hypothesis between the actual and the predicted values. It's non-negative and closer to zero, the better the learnt function is.

Implementation details

For regression problems as in this exercise, we compute the loss as follows:

$$MSE = \frac{1}{2N}\sum_{i=1}^{N}\left(y_i - \hat{y_i}\right)^2$$

where y_i is the true label and $\hat{y_i}$ is the estimated label. We use a factor of $\frac{1}{2N}$ instead of $\frac{1}{N}$ to simply the derivative of loss function.

Forward Propagation

We start by initializing the weights of the fully connected layer using Xavier initialization Xavier initialization (http://proceedings.mlr.press/v9/glorot10a/glorot10a.pdf) (At a high level, we are using a uniform distribution for weight initialization). During training, we pass all the data points through the network layer by layer using forward propagation. The main equations for forward prop have been described below.

$$\begin{aligned} u^{[0]} &= x \\ u^{[1]} &= \theta^{[1]}u^{[0]} + b^{[1]} \\ o^{[1]} &= Leaky_Relu(u^{[1]}) \\ u^{[2]} &= \theta^{[2]}o^{[1]} + b^{[2]} \\ \hat{y} = o^{[2]} &= Tanh(u^{[2]}) \end{aligned}$$

Then we get the output and compute the loss

$$l = \frac{1}{2N}\sum_{i=1}^{N}\left(y_i - \hat{y_i}\right)^2$$

Backward propagation

After the forward pass, we do back propagation to update the weights and biases in the direction of the negative gradient of the loss function. So, we update the weights and biases using the following formulas

$$\begin{aligned} \theta^{[2]} &:= \theta^{[2]} - lr \times \frac{\partial l}{\partial \theta^{[2]}} \\ b^{[2]} &:= b^{[2]} - lr \times \frac{\partial l}{\partial b^{[2]}} \\ \theta^{[1]} &:= \theta^{[1]} - lr \times \frac{\partial l}{\partial \theta^{[1]}} \\ b^{[1]} &:= b^{[1]} - lr \times \frac{\partial l}{\partial b^{[1]}} \end{aligned}$$

where lr is the learning rate. It decides the step size we want to take in the direction of the negative gradient.

To compute the terms $\frac{\partial l}{\partial \theta^{[i]}}$ and $\frac{\partial l}{\partial b^{[i]}}$ we use chain rule for differentiation as follows:

$$\begin{aligned} \frac{\partial l}{\partial \theta^{[2]}} &= \frac{\partial l}{\partial o^{[2]}}\frac{\partial o^{[2]}}{\partial u^{[2]}}\frac{\partial u^{[2]}}{\partial \theta^{[2]}} \\ \frac{\partial l}{\partial b^{[2]}} &= \frac{\partial l}{\partial o^{[2]}}\frac{\partial o^{[2]}}{\partial u^{[2]}}\frac{\partial u^{[2]}}{\partial b^{[2]}} \end{aligned}$$

So, $\frac{\partial l}{\partial o^{[2]}}$ is the differentiation of the loss function at point $o^{[2]}$

$\frac{\partial o^{[2]}}{\partial u^{[2]}}$ is the differentiation of the Tanh function at point $u^{[2]}$

$\frac{\partial u^{[2]}}{\partial \theta^{[2]}}$ is equal to $o^{[1]}$

$\frac{\partial u^{[2]}}{\partial b^{[2]}}$ is equal to 1.

To compute $\frac{\partial l}{\partial \theta^{[2]}}$, we need $o^{[2]}, u^{[2]}$ & $o^{[1]}$ which are calculated during forward propagation. So we need to store these values in cache variables during forward propagation to be able to access them during backward propagation. Similarly for calculating other partial derivatives, we store the values we'll be needing for chain rule in cache. These values are obtained from the forward propagation and used in backward propagation. The cache is implemented as a dictionary here where the keys are the variable names and the values are the variables values.

Also, the functional form of the MSE differentiation and Leaky Relu differentiation are given by

$$\frac{\partial l}{\partial o^{[2]}} = \left(o^{[2]} - y\right)$$
$$\frac{\partial l}{\partial u^{[2]}} = \frac{\partial l}{\partial o^{[2]}} * (1 - (tanh(u^{[2]}))^2)$$
$$\frac{\partial u^{[2]}}{\partial \theta^{[2]}} = o^{[1]}$$
$$\frac{\partial u^{[2]}}{\partial b^{[2]}} = 1$$

On vectorization, the above equations become:

$$\frac{\partial l}{\partial o^{[2]}} = \frac{1}{n}\left(o^{[2]} - y\right)$$
$$\frac{\partial l}{\partial \theta^{[2]}} = \frac{1}{n}\frac{\partial l}{\partial u^{[2]}} o^{[1]}$$
$$\frac{\partial l}{\partial b^{[2]}} = \frac{1}{n}\sum \frac{\partial l}{\partial u^{[2]}}$$

HINT 2: Division by N only needs to occur ONCE for any derivative that requires a division by N. Make sure you avoid cascading divisions by N where you might accidentally divide your derivative by N^2 or greater.

This completes the differentiation of loss function w.r.t to parameters in the second layer. We now move on to the first layer, the equations for which are given as follows:

$$\frac{\partial l}{\partial \theta^{[1]}} = \frac{\partial l}{\partial o^{[2]}} \frac{\partial o^{[2]}}{\partial u^{[2]}} \frac{\partial u^{[2]}}{\partial o^{[1]}} \frac{\partial o^{[1]}}{\partial u^{[1]}} \frac{\partial u^{[1]}}{\partial \theta^{[1]}}$$
$$\frac{\partial l}{\partial b^{[1]}} = \frac{\partial l}{\partial o^{[2]}} \frac{\partial o^{[2]}}{\partial u^{[2]}} \frac{\partial u^{[2]}}{\partial o^{[1]}} \frac{\partial o^{[1]}}{\partial u^{[1]}} \frac{\partial u^{[1]}}{\partial b^{[1]}}$$

Where

$$\frac{\partial u^{[2]}}{\partial o^{[1]}} = \theta^{[2]}$$
$$\frac{\partial o^{[1]}}{\partial u^{[1]}} = 1(o^{[1]} > 0) \text{ and } \alpha(o^{[1]} <= 0)$$
$$\frac{\partial u^{[1]}}{\partial \theta^{[1]}} = x$$
$$\frac{\partial u^{[1]}}{\partial b^{[1]}} = 1$$

Note that $\frac{\partial o^{[1]}}{\partial u^{[1]}}$ is the differentiation of the leaky ReLU function at $u^{[1]}$.

The above equations outline the forward and backward propagation process for a 2-layer fully connected neural net with leaky Relu as the first activation layer and Tanh has the second one. The same process can be extended to different neural networks with different activation layers.

Code Implementation:

$$dLoss\_o2 = \frac{\partial l}{\partial o^{[2]}} \implies dim = (1, 331)$$
$$dLoss\_u2 = dLoss\_o2 \frac{\partial o^{[2]}}{\partial u^{[2]}} \implies dim = (1, 331)$$
$$dLoss\_theta2 = dLoss\_u2 \frac{\partial u^{[2]}}{\partial \theta^{[2]}} \implies dim = (1, 15)$$
$$dLoss\_b2 = dLoss\_u2 \frac{\partial u^{[2]}}{\partial b^{[2]}} \implies dim = (1, 1)$$
$$dLoss\_o1 = dLoss\_u2 \frac{\partial u^{[2]}}{\partial o^{[1]}} \implies dim = (15, 331)$$
$$dLoss\_u1 = dLoss\_o1 \frac{\partial o^{[1]}}{\partial u^{[1]}} \implies dim = (15, 331)$$
$$dLoss\_theta1 = dLoss\_u1 \frac{\partial u^{[1]}}{\partial \theta^{[1]}} \implies dim = (15, 10)$$
$$dLoss\_b1 = dLoss\_u1 \frac{\partial u^{[1]}}{\partial b^{[1]}} \implies dim = (15, 1)$$

Note: Training set has 331 examples.

1.1 NN Implementation [55pts; 45pts + 10pts Bonus for Undergrad] [P]

In this section, you will implement a two layer fully connected neural network. You will also experiment with different activation functions and optimization techniques. We provide two activation functions here - Leaky Relu and Tanh. You will implement a neural network that would have Leaky Relu activation followed by a Tanh layer.

You'll also implement Gradient Descent (GD) and Batch Gradient Descent (BGD) algorithms for training these neural nets. **GD is mandatory for all. BGD is bonus for undergraduate students but mandatory for graduate students.**

In the **NN.py** file, complete the following functions:

- **Leaky Relu**: Recall Hint 1
- **Tanh**: Recall Hint 1
- **nloss**
- **forward**
- **backward**: Recall Hint 2, if you still have issues passing the autograder make sure to address Hint 1 as well.
- **gradient_descent**
- **batch_gradient_descent**: **Mandatory for graduate students, bonus for undergraduate students.** Please batch your data in a wraparound manner. For example, given a dataset of 9 numbers, [1, 2, 3, 4, 5, 6, 7, 8, 9], and a batch size of 6, the first iteration batch will be [1, 2, 3, 4, 5, 6], the second iteration batch will be [7, 8, 9, 1, 2, 3], the third iteration batch will be [4, 5, 6, 7, 8, 9], etc...

We'll train this neural net on sklearn's diabetes dataset.

1.1.1 Local Tests: Neural Network [No Points]

You may test your implementation of the functions contained in **NN.py** in the cell below. Feel free to comment out tests for functions that have not been completed yet. See Using the Local Tests for more details.

In [2]:

```
##############################
### DO NOT CHANGE THIS CELL ###
##############################

from NN import dlnet
from local_tests.nn_test import NN_Test

test_nn = NN_Test()
nn = dlnet(test_nn.x_train, test_nn.y_train, lr=0.01, batch_size=6)

print('Local Tests Neural Network \n')
# Local test for LeakyRelu
output = nn.Leaky_Relu(0.05, test_nn.x_train)
leaky_relu_test = np.allclose(output, test_nn.leaky_relu)
print('Your Leaky_Relu works within the expected range:', leaky_relu_test)

# Local test for Tanh
output = nn.Tanh(test_nn.x_train)
tanh_test = np.allclose(output, test_nn.tanh)
print('Your Tanh works within the expected range:', tanh_test)

# Local test for nloss
output = nn.nloss(test_nn.y_train, test_nn.yh)
nloss_test = np.allclose(output, test_nn.nloss)
print('Your nloss works within the expected range:', nloss_test)

# Local test for forward
# Values for u1, o1, u2 are also saved in nn_test.py if further debugging is needed
nn.nInit()
output = nn.forward(test_nn.x_train)
forward_test = np.allclose(output, test_nn.forward_o2)
print('Your forward works within the expected range:', forward_test)

# Local test for backward
print('\nTest Losses in backward function:')
output = nn.backward(test_nn.y_train, test_nn.forward_o2)
dLoss_theta2_test = np.allclose(output[0], test_nn.dLoss_theta2)
print('Your dLoss_theta2 works within the expected range:', dLoss_theta2_test)
dLoss_b2_test = np.allclose(output[1], test_nn.dLoss_b2)
print('Your dLoss_b2 works within the expected range:', dLoss_b2_test)
dLoss_theta1_test = np.allclose(output[2], test_nn.dLoss_theta1)
print('Your dLoss_theta1 works within the expected range:', dLoss_theta1_test)
dLoss_b1_test = np.allclose(output[3], test_nn.dLoss_b1)
print('Your dLoss_b1 works within the expected range:', dLoss_b1_test)
```

```
Local Tests Neural Network

Your Leaky_Relu works within the expected range: True
Your Tanh works within the expected range: True
Your nloss works within the expected range: True
Your forward works within the expected range: True

Test Losses in backward function:
Your dLoss_theta2 works within the expected range: True
Your dLoss_b2 works within the expected range: True
Your dLoss_theta1 works within the expected range: True
Your dLoss_b1 works within the expected range: True
```

1.1.2 Local Test: Gradient Descent [No Points]

There is no local test for the Gradient Descent function, however section 1.2 may help with local debugging.

1.1.3 Local Test: Batch Gradient Descent [No Points]

You may test your implementation of the BGD function contained in **NN.py** in the cell below. See Using the Local Tests for more details.

In [3]:

```
##############################
### DO NOT CHANGE THIS CELL ###
##############################

from NN import dlnet
from local_tests.nn_test import NN_Test

test_nn = NN_Test()
nn = dlnet(test_nn.x_train, test_nn.y_train, lr=0.01, batch_size=6)

# Local test batch gradient descent
nn.batch_gradient_descent(test_nn.x_train, test_nn.y_train, iter=3, local_test=True)

batch_str = 'batch_y at iteration %i: '
print('\ny_train input:', test_nn.y_train)
[print(batch_str %(i), batch_y) for i, batch_y in enumerate(nn.batch_y)]

bgd_loss_test = np.allclose(np.array(nn.loss), test_nn.bgd_loss)
print('\nYour BGD losses works within the expected range:', bgd_loss_test)
batch_y_test = np.allclose(np.array(nn.batch_y), test_nn.batch_y)
print('Your batch_y works within the expected range:', batch_y_test)
```

```
Loss after iteration 0: 7.573928
Loss after iteration 1: 17.098338
Loss after iteration 2: 21.757258

y_train input: [[1. 2. 3. 4. 5. 6. 7. 8. 9.]]
batch_y at iteration 0:  [[1. 2. 3. 4. 5. 6.]]
batch_y at iteration 1:  [[7. 8. 9. 1. 2. 3.]]
batch_y at iteration 2:  [[4. 5. 6. 7. 8. 9.]]

Your BGD losses works within the expected range: True
Your batch_y works within the expected range: True
```

1.2 Loss plot and MSE value for NN with Gradient Descent [5pts] [W]

Train your neural net implementation with gradient descent and print out the loss at every 1000th iteration (starting at iteration 0). The following cells will plot the loss vs epoch graph and calculate the final test MSE.

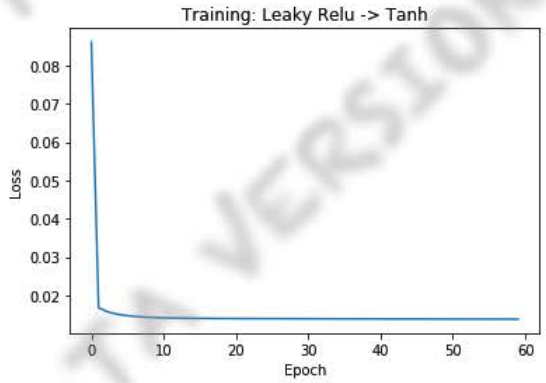
In [4]:

```
################################
### DO NOT CHANGE THIS CELL ###
################################

from NN import dlnet

dataset = load_diabetes() # load the dataset
x, y = dataset.data, dataset.target
y = y.reshape(-1,1)
perm = np.random.RandomState(seed=3).permutation(x.shape[0])[:500]
x = x[perm]
y = y[perm]



x_train, x_test, y_train, y_test = train_test_split(x, y, random_state=1) #split data


x_scale = MinMaxScaler()
x_train = x_scale.fit_transform(x_train) #normalize data
x_test = x_scale.transform(x_test)

y_scale = MinMaxScaler()
y_train = y_scale.fit_transform(y_train)
y_test = y_scale.transform(y_test)

x_train, x_test, y_train, y_test = x_train.T, x_test.T, y_train.reshape(1,-1), y_test #condition data

nn = dlnet(x_train,y_train,lr=0.01) # initalize neural net class
nn.gradient_descent(x_train, y_train, iter = 60000) #train

# create figure
fig = plt.plot(np.array(nn.loss).squeeze())
plt.title(f'Training: {nn.neural_net_type}')
plt.xlabel("Epoch")
plt.ylabel("Loss")

```

```
Loss after iteration 0: 0.086064
Loss after iteration 1000: 0.016759
Loss after iteration 2000: 0.015858
Loss after iteration 3000: 0.015304
Loss after iteration 4000: 0.014940
Loss after iteration 5000: 0.014680
Loss after iteration 6000: 0.014493
Loss after iteration 7000: 0.014354
Loss after iteration 8000: 0.014250
Loss after iteration 9000: 0.014172
Loss after iteration 10000: 0.014113
Loss after iteration 11000: 0.014070
Loss after iteration 12000: 0.014038
Loss after iteration 13000: 0.014013
Loss after iteration 14000: 0.013994
Loss after iteration 15000: 0.013979
Loss after iteration 16000: 0.013966
Loss after iteration 17000: 0.013956
Loss after iteration 18000: 0.013947
Loss after iteration 19000: 0.013940
Loss after iteration 20000: 0.013933
Loss after iteration 21000: 0.013925
Loss after iteration 22000: 0.013919
Loss after iteration 23000: 0.013912
Loss after iteration 24000: 0.013907
Loss after iteration 25000: 0.013902
Loss after iteration 26000: 0.013897
Loss after iteration 27000: 0.013891
Loss after iteration 28000: 0.013886
Loss after iteration 29000: 0.013880
Loss after iteration 30000: 0.013875
Loss after iteration 31000: 0.013870
Loss after iteration 32000: 0.013865
Loss after iteration 33000: 0.013861
Loss after iteration 34000: 0.013856
Loss after iteration 35000: 0.013853
Loss after iteration 36000: 0.013849
Loss after iteration 37000: 0.013845
Loss after iteration 38000: 0.013841
Loss after iteration 39000: 0.013837
Loss after iteration 40000: 0.013833
Loss after iteration 41000: 0.013828
Loss after iteration 42000: 0.013825
Loss after iteration 43000: 0.013821
```

```
Loss after iteration 44000: 0.013817
Loss after iteration 45000: 0.013813
Loss after iteration 46000: 0.013810
Loss after iteration 47000: 0.013807
Loss after iteration 48000: 0.013804
Loss after iteration 49000: 0.013801
Loss after iteration 50000: 0.013799
Loss after iteration 51000: 0.013796
Loss after iteration 52000: 0.013794
Loss after iteration 53000: 0.013791
Loss after iteration 54000: 0.013788
Loss after iteration 55000: 0.013785
Loss after iteration 56000: 0.013782
Loss after iteration 57000: 0.013779
Loss after iteration 58000: 0.013776
Loss after iteration 59000: 0.013773
```

Out[4]: `Text(0, 0.5, 'Loss')`



In [5]:

```
################################
### DO NOT CHANGE THIS CELL ###
################################

y_predicted = nn.predict(x_test) # predict
y_test = y_test.reshape(1,-1)
print("Mean Squared Error (MSE)", mean_squared_error(y_test, y_predicted))

```

```
Mean Squared Error (MSE) 0.036208836333320266
```

1.3 Loss plot and MSE value for NN with BGD [5pts Bonus for Undergrad] [W]

Train your neural net implementation with batch gradient descent and print out the loss at every 1000th iteration (starting at iteration 0). The following cells will plot the loss vs epoch graph and calculate the final test MSE.

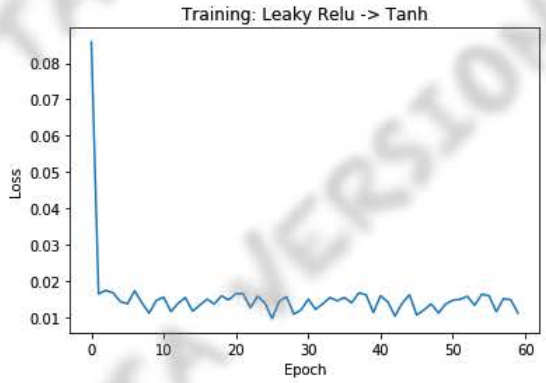
In [6]:

```
################################
### DO NOT CHANGE THIS CELL ###
################################

from NN import dlnet

dataset = load_diabetes() # load the dataset
x, y = dataset.data, dataset.target
y = y.reshape(-1,1)
perm = np.random.RandomState(seed=3).permutation(x.shape[0])[:500]
x = x[perm]
y = y[perm]

x_train, x_test, y_train, y_test = train_test_split(x, y, random_state=1) #split data

x_scale = MinMaxScaler()
x_train = x_scale.fit_transform(x_train) #normalize data
x_test = x_scale.transform(x_test)

y_scale = MinMaxScaler()
y_train = y_scale.fit_transform(y_train)
y_test = y_scale.transform(y_test)

x_train, x_test, y_train, y_test = x_train.T, x_test.T, y_train.reshape(1,-1), y_test #condition data

nn = dlnet(x_train,y_train,lr=0.01) # initalize neural net class
nn.batch_gradient_descent(x_train, y_train, iter = 60000) #train


# create figure
fig = plt.plot(np.array(nn.loss).squeeze())
plt.title(f'Training: {nn.neural_net_type}')
plt.xlabel("Epoch")
plt.ylabel("Loss")
```

```
Loss after iteration 0: 0.085896
Loss after iteration 1000: 0.016408
Loss after iteration 2000: 0.017377
Loss after iteration 3000: 0.016717
Loss after iteration 4000: 0.014299
Loss after iteration 5000: 0.013671
Loss after iteration 6000: 0.017274
Loss after iteration 7000: 0.013904
Loss after iteration 8000: 0.011122
Loss after iteration 9000: 0.014634
Loss after iteration 10000: 0.015535
Loss after iteration 11000: 0.011515
Loss after iteration 12000: 0.013827
Loss after iteration 13000: 0.015403
Loss after iteration 14000: 0.011659
Loss after iteration 15000: 0.013299
Loss after iteration 16000: 0.014965
Loss after iteration 17000: 0.013652
Loss after iteration 18000: 0.015844
Loss after iteration 19000: 0.014786
Loss after iteration 20000: 0.016463
Loss after iteration 21000: 0.016482
Loss after iteration 22000: 0.012612
Loss after iteration 23000: 0.015773
Loss after iteration 24000: 0.014004
Loss after iteration 25000: 0.009581
Loss after iteration 26000: 0.014435
Loss after iteration 27000: 0.015644
Loss after iteration 28000: 0.010839
Loss after iteration 29000: 0.011997
Loss after iteration 30000: 0.014938
Loss after iteration 31000: 0.012163
Loss after iteration 32000: 0.013640
Loss after iteration 33000: 0.015405
Loss after iteration 34000: 0.014512
Loss after iteration 35000: 0.015420
Loss after iteration 36000: 0.013922
Loss after iteration 37000: 0.016699
Loss after iteration 38000: 0.016166
Loss after iteration 39000: 0.011267
Loss after iteration 40000: 0.015912
Loss after iteration 41000: 0.014213
Loss after iteration 42000: 0.010281
Loss after iteration 43000: 0.013881
Loss after iteration 44000: 0.016178
Loss after iteration 45000: 0.010587
```

```
Loss after iteration 46000: 0.011988
Loss after iteration 47000: 0.013655
Loss after iteration 48000: 0.011156
Loss after iteration 49000: 0.013621
Loss after iteration 50000: 0.014682
Loss after iteration 51000: 0.014920
Loss after iteration 52000: 0.015768
Loss after iteration 53000: 0.013299
Loss after iteration 54000: 0.016318
Loss after iteration 55000: 0.015912
Loss after iteration 56000: 0.011519
Loss after iteration 57000: 0.015081
Loss after iteration 58000: 0.014805
Loss after iteration 59000: 0.011134
```

Out[6]: `Text(0, 0.5, 'Loss')`



In [7]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################

y_predicted = nn.predict(x_test) # predict
y_test = y_test.reshape(1,-1)
print("Mean Squared Error (MSE)", mean_squared_error(y_test, y_predicted))

```

```
Mean Squared Error (MSE) 0.036203929663209 03
```

2: Image Classification based on Convolutional Neural Networks [15pts; 12pts Bonus for Undergrad + 3pts Bonus for all] [W]

2.1 Image Classification using Keras and CNN

Keras is a deep learning API written in Python, running on top of the machine learning platform TensorFlow. It was developed with a focus on enabling fast experimentation. Being able to go from idea to result as fast as possible is key to doing good research. In this part, you will build a convolutional neural network based on TF/Keras to solve the image classification task for the CIFAR-10 dataset. If you haven't installed TensorFlow, you can install the package by **pip** command or train your model by uploading HW4 notebook to Colab (https://colab.research.google.com/) directly. Colab contains all packages you need for this section.

Hint1: First contact with Keras (https://keras.io/about/)

Hint2: How to Install Keras (https://www.pyimagesearch.com/2016/07/18/installing-keras-for-deep-learning/)

Hint3: CS231n Tutorial (Layers used to build ConvNets) (https://cs231n.github.io/convolutional-networks/)

Environment Setup

In [8]:
```
from __future__ import print_function
import tensorflow as tf
from keras.datasets import cifar10
from sklearn.model_selection import train_test_split
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Conv2D, MaxPooling2D, Flatten, Dense, Activation, Dropout
from tensorflow.keras.layers import LeakyReLU
from tensorflow.keras.layers import BatchNormalization
from sklearn.utils import shuffle
import numpy as np
import matplotlib.pyplot as plt
```

2.1.1 Load CIFAR-10 dataset [Setup - No points]

We use CIFAR-10 (https://www.cs.toronto.edu/~kriz/cifar.html) dataset to train our model. This is a dataset of 60,000 32x32 colour images in 10 classes, with 6,000 images per class. There are 50,000 training images and 10,000 test images. We provide code for you to download CIFAR-10 dataset below.

In [9]:
```
###############################
### DO NOT CHANGE THIS CELL ###
###############################

# split data between train and test sets

(x_train, y_train),(x_test, y_test)=cifar10.load_data()

# input image dimensions
img_rows, img_cols = 32, 32
number_channels = 3
#set num of classes
num_classes = 10

if tf.keras.backend.image_data_format() == 'channels_first':
    x_train = x_train.reshape(x_train.shape[0], number_channels, img_rows, img_cols)
    x_test = x_test.reshape(x_test.shape[0], number_channels, img_rows, img_cols)
    input_shape = (number_channels, img_rows, img_cols)
else:
    x_train = x_train.reshape(x_train.shape[0], img_rows, img_cols, number_channels)
    x_test = x_test.reshape(x_test.shape[0], img_rows, img_cols, number_channels)
    input_shape = (img_rows, img_cols, number_channels)

x_train = x_train.astype('float32')
x_test = x_test.astype('float32')
x_train /= 255
x_test /= 255
print('x_train shape:', x_train.shape)
print('x_test shape:', x_test.shape)
print(x_train.shape[0], 'train samples')
print(x_test.shape[0], 'test samples')

class_names=['airplane', 'automobile', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
# convert class vectors to binary class matrices
y_train = tf.keras.utils.to_categorical(y_train, num_classes)
y_test = tf.keras.utils.to_categorical(y_test, num_classes)
```

```
x_train shape: (50000, 32, 32, 3)
x_test shape: (10000, 32, 32, 3)
50000 train samples
10000 test samples
```

2.1.2 Load some sample images from CIFAR-10 [Setup - No points]

In [10]:

```
################################
### DO NOT CHANGE THIS CELL ###
################################

# Show some images from CIFAR-10

fig = plt.figure(figsize=(10, 10))
for i in range(9):
    random_index = np.random.randint(0, len(y_train))
    ax = fig.add_subplot(3, 3, i+1)
    ax.imshow(x_train[random_index, :])
plt.show()
```

As you can see from above, the CIFAR-10 dataset contains different types of objects. The images have been size-normalized and objects remain centered in fixed-size images.

2.1.3 Build convolutional neural network model [2pts] [W]

In this part, you need to build a convolutional neural network as described below. The architecture of the model is outlined below.

In the **cnn.py** file, complete the following functions:

- **__init__**: See Defining Variables section
- **create_net**: See Defining Model section
- **compile_net**: See Compiling Model section

[INPUT - CONV - CONV - MAXPOOL - DROPOUT - CONV - CONV - MAXPOOL - DROPOUT - FC1 - DROPOUT - FC2 - DROPOUT - FC3]

> INPUT: [$32 \times 32 \times 3$] will hold the raw pixel values of the image, in this case, an image of width 32, height 32. This layer should give 8 filters and have appropriate padding to maintain shape.
>
> CONV: Conv. layer will compute the output of neurons that are connected to local regions in the input, each computing a dot product between their weights and a small region they are connected to the input volume. We decide to set the kernel_size 3×3 for both Conv. layers. For example, the output of the Conv. layer may look like $[32 \times 32 \times 32]$ if we use 32 filters. Again, we use padding to maintain shape.
>
> MAXPOOL: MAXPOOL layer will perform a downsampling operation along the spatial dimensions (width, height). With pool size of 2×2, resulting shape takes form 16×16.
>
> DROPOUT: DROPOUT layer with the dropout rate of 0.30 to prevent overfitting.
>
> CONV: Additonal Conv. layer takes outputs from above layers and applies more filters. The Conv. layer may look like $[16 \times 16 \times 32]$. We set the kernel_size 3×3 and use padding to maintain shape for both Conv. layers.
>
> CONV: Additonal Conv. layer takes outputs from above layers and applies more filters. The Conv. layer may look like $[16 \times 16 \times 64]$.
>
> MAXPOOL: MAXPOOL layer will perform a downsampling operation along the spatial dimensions (width, height).
>
> DROPOUT: Dropout layer with the dropout rate of 0.30 to prevent overfitting.
>
> FC1: Dense layer which takes output from above layers, and has 256 neurons. Flatten() operations may be useful.
>
> DROPOUT: Dropout layer with the dropout rate of 0.5 to prevent overfitting.
>
> FC2： Dense layer which takes output from above layers, and has 128 neurons.
>
> DROPOUT: Dropout layer with the dropout rate of 0.5 to prevent overfitting.
>
> FC3: Dense layer with 10 neurons, and Softmax activation, is the final layer. The dimension of the output space is the number of classes.

Activation function: Use LeakyReLU with negative_slope 0.1 as the activation function for Conv. layers and Dense layers unless otherwise indicated to build you model architecture

Note that while this is a suggested model design, you may use other architectures and experiment with different layers for better results.

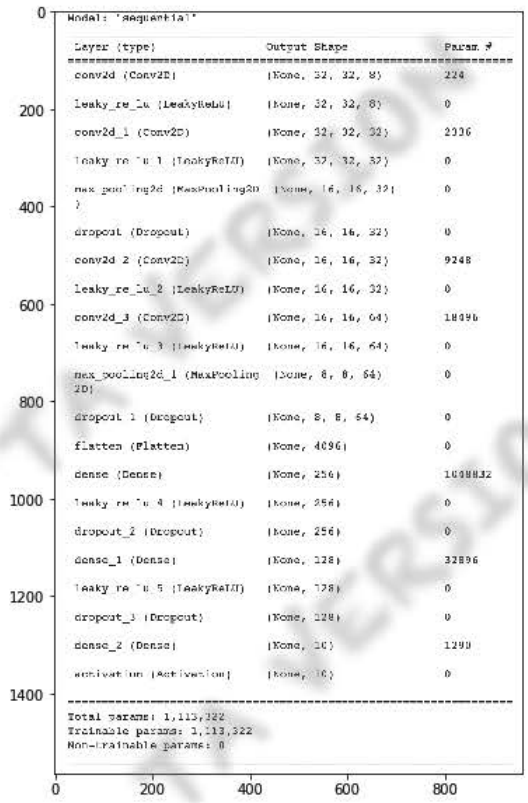
Use the following keras links to reference crucial layers of the model in keras API:

- Conv2d: https://keras.io/api/layers/convolution_layers/convolution2d/ (https://keras.io/api/layers/convolution_layers/convolution2d/)
- Dense: https://keras.io/api/layers/core_layers/dense/ (https://keras.io/api/layers/core_layers/dense/)
- Flatten: https://keras.io/api/layers/reshaping_layers/flatten/ (https://keras.io/api/layers/reshaping_layers/flatten/)
- MaxPool: https://keras.io/api/layers/pooling_layers/max_pooling2d/ (https://keras.io/api/layers/pooling_layers/max_pooling2d/)
- Dropout: https://keras.io/api/layers/regularization_layers/dropout/ (https://keras.io/api/layers/regularization_layers/dropout/)
- LeakyReLU: https://keras.io/api/layers/activation_layers/leaky_relu/ (https://keras.io/api/layers/activation_layers/leaky_relu/)

And explore the keras layers API if you would like to experiment with additional layers: https://keras.io/api/layers/ (https://keras.io/api/layers/)

In [11]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################

# Show the architecture of the model
achi=plt.imread('./data/images/Architecture.png')
fig = plt.figure(figsize=(10,10))
plt.imshow(achi)
```

Out[11]: <matplotlib.figure.AxesImage at 0x7ff17b1651d0>



Defining Variables [No Points]

You now need to set training variables in the `__init__()` function in **cnn.py**. Once you have defined variables you may use the cell below to see them.

- Recommended Batch Sizes fall in the range 32-512 (use powers of 2)
- Recommended Epoch Counts fall in the range 5-20
- Recommended Learning Rates fall in the range .0001-.01

In [12]:

```
##############################
### DO NOT CHANGE THIS CELL ###
##############################

# You can adjust parameters to train your model in __init__() in cnn.py

from cnn import CNN

net = CNN()
batch_size, epochs, init_lr = net.get_vars()
print(f'Batch Size\t: {batch_size} \nEpochs\t\t: {epochs} \nLearning Rate\t: {init_lr} \n')

```

```
Batch Size      : 64
Epochs          : 5
Learning Rate   : 0.001
```


Defining model [2pts Bonus for Undergrad]

You now need to complete the `create_net()` function in **cnn.py** to define your model structure. Once you have defined a model structure you may use the cell below to examine your architecture.

Your model is required to have at least 2 convolutional layers and at least 2 dense layers. Ensuring that these requirements are met will earn you 2pts.

In [13]:

```
##############################
### DO NOT CHANGE THIS CELL ###
##############################

# model.summary() gives you details of your architecture.
#You can compare your architecture with the 'Architecture.png'

from cnn import CNN
net = CNN()

s = tf.keras.backend.clear_session()
model=net.create_net()
model.summary()
```

```
Model: "sequential"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d (Conv2D)              (None, 32, 32, 8)         224
_________________________________________________________________
leaky_re_lu (LeakyReLU)      (None, 32, 32, 8)         0
_________________________________________________________________
conv2d_1 (Conv2D)            (None, 32, 32, 32)        2336
_________________________________________________________________
leaky_re_lu_1 (LeakyReLU)    (None, 32, 32, 32)        0
_________________________________________________________________
max_pooling2d (MaxPooling2D) (None, 16, 16, 32)        0
_________________________________________________________________
dropout (Dropout)            (None, 16, 16, 32)        0
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 16, 16, 32)        9248
_________________________________________________________________
leaky_re_lu_2 (LeakyReLU)    (None, 16, 16, 32)        0
_________________________________________________________________
conv2d_3 (Conv2D)            (None, 16, 16, 64)        18496
_________________________________________________________________
leaky_re_lu_3 (LeakyReLU)    (None, 16, 16, 64)        0
_________________________________________________________________
max_pooling2d_1 (MaxPooling2 (None, 8, 8, 64)          0
_________________________________________________________________
dropout_1 (Dropout)          (None, 8, 8, 64)          0
_________________________________________________________________
flatten (Flatten)            (None, 4096)              0
_________________________________________________________________
dense (Dense)                (None, 256)               1048832
_________________________________________________________________
leaky_re_lu_4 (LeakyReLU)    (None, 256)               0
_________________________________________________________________
dropout_2 (Dropout)          (None, 256)               0
_________________________________________________________________
dense_1 (Dense)              (None, 128)               32896
_________________________________________________________________
leaky_re_lu_5 (LeakyReLU)    (None, 128)               0
_________________________________________________________________
dropout_3 (Dropout)          (None, 128)               0
_________________________________________________________________
dense_2 (Dense)              (None, 10)                1290
_________________________________________________________________
activation (Activation)      (None, 10)                0
=================================================================
Total params: 1,113,322
Trainable params: 1,113,322
Non-trainable params: 0
_________________________________________________________________
```

Compiling model [No Points]

Next prepare the model for training by completing `compile_model()` in **cnn.py** Remember we are performing 10-way clasification when selecting a loss function. Loss function can be categorical crossentropy

In [14]:
```
##############################
### DO NOT CHANGE THIS CELL ###
##############################

# Complete compile_model() in cnn.py.
from cnn import CNN
net = CNN()
model = net.compile_net(model)
print(model)
```

```
<tensorflow.python.keras.engine.sequential.Sequential object at 0x7ff17b21d890>
```

2.1.4 Train the network [8pts total (3pts, 3pts, 2pts) Bonus for Undergrad] [W]

Tuning: Training the network is the next thing to try. You can set your parameter at the **Defining Variable** section. If your parameters are set properly, you should see the loss of the validation set decreased and the value of accuracy increased. **It may take more than 15 minutes to train your model.**

Expected Result: You should be able to achieve more than **80%** accuracy on the test set to get full points. If you achieve accuracy between **60%** to **70%**, you will only get 3 points. An accuracy between **70%** to **80%** will earn an additional 3pts.

- **60%** to **70%** earns 3pts
- **70%** to **80%** earns 3pts more (6pts total)
- **80%**+ earns 2pts more (8pts total)

Train your own CNN model

In [15]:

```
##############################
### DO NOT CHANGE THIS CELL ###
##############################


from cnn import CNN

net = CNN()
batch_size, epochs, init_lr = net.get_vars()

def lr_scheduler(epoch):
    new_lr = init_lr * 0.9 ** epoch
    print("Learning rate:", new_lr)
    return new_lr

history = model.fit(
    x_train, y_train,
    batch_size=batch_size,
    epochs=epochs,
    callbacks=[tf.keras.callbacks.LearningRateScheduler(lr_scheduler)],
    shuffle=True,
    verbose=1,
    initial_epoch=0,
    validation_data=(x_test, y_test)
)
score = model.evaluate(x_test, y_test, verbose=0)
print('Test loss:', score[0])
print('Test accuracy:', score[1])
```

```
Learning rate: 0.001
Epoch 1/5
782/782 [==============================] - 73s 94ms/step - loss: 1.8301 - accuracy: 0.3239 - val_loss: 1.4812 -
val_accuracy: 0.4562
Learning rate: 0.0009000000000000001
Epoch 2/5
782/782 [==============================] - 73s 93ms/step - loss: 1.4708 - accuracy: 0.4670 - val_loss: 1.2705 -
val_accuracy: 0.5454
Learning rate: 0.0008100000000000001
Epoch 3/5
782/782 [==============================] - 70s 90ms/step - loss: 1.3316 - accuracy: 0.5242 - val_loss: 1.1630 -
val_accuracy: 0.5916
Learning rate: 0.0007290000000000002
Epoch 4/5
782/782 [==============================] - 70s 90ms/step - loss: 1.2372 - accuracy: 0.5647 - val_loss: 1.1071 -
val_accuracy: 0.6126
Learning rate: 0.0006561000000000001
Epoch 5/5
782/782 [==============================] - 71s 91ms/step - loss: 1.1604 - accuracy: 0.5913 - val_loss: 1.0349 -
val_accuracy: 0.6391
Test loss: 1.034911870956421
Test accuracy: 0.6391000151634216
```

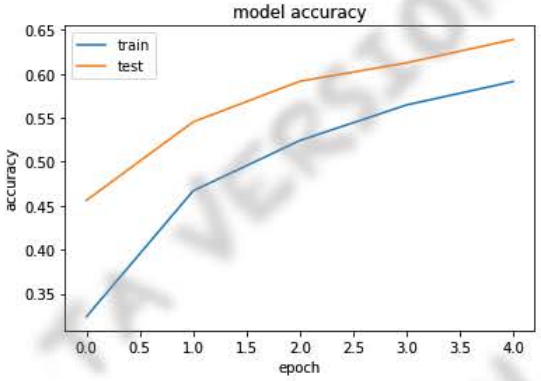
2.1.5 Examine accuracy and loss [2pts Bonus for Undergrad] [W]

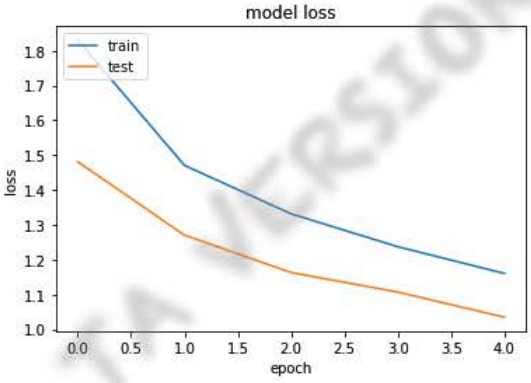
You should expect to see gradually decreasing loss and gradually increasing accuracy. Examine loss and accuracy by running the cell below, no editing is necessary. Having appropriate looking loss and accuracy plots will earn you the last 2pts for your convolutional neural net.

In [16]:

```
################################
### DO NOT CHANGE THIS CELL ###
################################

# list all data in history
print(history.history.keys())

# summarize history for accuracy and loss
plt.plot(history.history['accuracy'])
plt.plot(history.history['val_accuracy'])
plt.title('model accuracy')
plt.ylabel('accuracy')
plt.xlabel('epoch')
plt.legend(['train', 'test'], loc='upper left')
plt.show()

plt.plot(history.history['loss'])
plt.plot(history.history['val_loss'])
plt.title('model loss')
plt.ylabel('loss')
plt.xlabel('epoch')
plt.legend(['train', 'test'], loc='upper left')
plt.show()

```

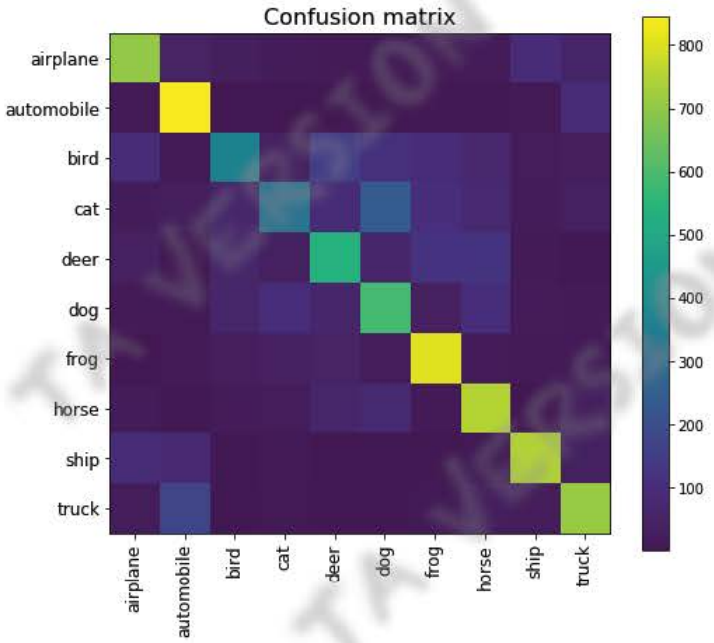
```
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy', 'lr'])
```





In [17]:

```
################################
### DO NOT CHANGE THIS CELL ###
################################

# make predictions
y_pred = model.predict(x_test)
y_pred_classes = np.argmax(y_pred, axis=1)
y_pred_prob = np.max(y_pred, axis=1)
y_gt_classes = np.argmax(y_test, axis=1)

from sklearn.metrics import confusion_matrix, accuracy_score
plt.figure(figsize=(8, 7))
plt.imshow(confusion_matrix(y_gt_classes, y_pred_classes))
plt.title('Confusion matrix', fontsize=16)
plt.xticks(np.arange(10), class_names, rotation=90, fontsize=12)
plt.yticks(np.arange(10), class_names, fontsize=12)
plt.colorbar()
plt.show()
```



2.2 Exploring Deep CNN Architectures [3pts Bonus for All] [W]

The network you have produced is rather simple relative to many of those used in industry and research. Researchers have worked to make CNN models deeper and deeper over the past years in an effort to gain higher accuracy in predictions. While your model is only a handful of layers deep, some state of the art deep architectures may include up to 150 layers. However, this process has not been without challenges.

One such problem is the problem of the vanishing gradient. The weights of a neural network are updated using the backpropagation algorithm. The backpropagation algorithm makes a small change to each weight in such a way that the loss of the model decreases. Using the chain rule, we can find this gradient for each weight. But, as this gradient keeps flowing backwards to the initial layers, this value keeps getting multiplied by each local gradient. Hence, the gradient becomes smaller and smaller, making the updates to the initial layers very small, increasing the training time considerably.

Many tactics have been used in an effort to solve this problem. One architecture, named ResNet, solves the vanishing gradient problem in a unique way. ResNet was developed at Microsoft Research to find better ways to train deep networks. Take a moment to explore how ResNet tackles the vanishing gradient problem by reading the original research paper here: https://arxiv.org/pdf/1512.03385.pdf (https://arxiv.org/pdf/1512.03385.pdf) (also included as PDF in papers directory).

Question: In your own words, explain how ResNet addresses the vanishing gradient problem in 1-2 sentences below: (Please type answers directly in the cell below.)

Answer:

3: Random Forests [50pts; 40pts + 10pts Bonus for All] [P] [W]

NOTE: Please use sklearn's DecisionTreeClassifier in your Random Forest implementation. You can find more details about this classifier here. (https://scikit-learn.org/stable/modules/generated/sklearn.tree.DecisionTreeClassifier.html#sklearn.tree.DecisionTreeClassifier)

3.1 Random Forest Implementation [35pts] [P]

The decision boundaries drawn by decision trees are very sharp, and fitting a decision tree of unbounded depth to a list of examples almost inevitably leads to **overfitting**. In an attempt to decrease the variance of a decision tree, we're going to use a technique called 'Bootstrap Aggregating' (often abbreviated 'bagging'). This stems from the idea that a collection of weak learners can learn decision boundaries as well as a strong learner. This is commonly called a Random Forest.

We can build a Random Forest as a collection of decision trees, as follows:

1. For every tree in the random forest, we're going to

   a) Subsample the examples with replacement. Note that in this question, the size of the subsample data is equal to the original dataset.

   b) From the subsamples in part a, choose attributes at random without replacement to learn on in accordance with a provided attribute subsampling rate. Based on what it was mentioned in the class, we randomly pick features in each split. We use a more general approach here to make the programming part easier. Let's randomly pick some features (70% percent of features) and grow the tree based on the pre-determined randomly selected features. Therefore, there is no need to find random features in each split.

   c) Fit a decision tree to the subsample of data we've chosen to a certain depth.

Classification for a random forest is then done by taking a majority vote of the classifications yielded by each tree in the forest after it classifies an example.

In the **random_forest.py** file, complete the following functions:

- **_bootstrapping**: this function will be used in `bootstrapping()`
- **fit**: Fit the decision trees initialized in `__init__` with the datasets created in `bootstrapping()`. You will need to call `bootstrapping()`.

NOTES:

1. In the Random Forest Class, X is assumed to be a matrix with num_training rows and num_features columns where num_training is the number of total records and num_features is the number of features of each record. y is assumed to be a vector of labels of length num_training.
2. Look out for TODO's for the parts that need to be implemented
3. If you receive any `SettingWithCopyWarning` warnings from the Pandas library, you can safely ignore them.

3.2 Hyperparameter Tuning with a Random Forest [5pts] [P]

In machine learning, hyperparameters are parameters that are set before the learning process begins. The max_depth, num_estimators, or max_features variables from 3.1 are examples of different hyperparameters for a random forest model. In this section, you will tune your random forest model on a heart disease to achieve a high accuracy on determining the likelihood of patient having narrowing arteries.

Let's first review the dataset in a bit more detail.

Dataset Objective

Imagine that we are doctors working on a cure for heart disease by using machine learning to categorize patients. We know that narrowing arteries are an early indicator of disease. We are tasked with the responsibility of coming up with a method for determining the likelihood of patient having narrowing arteries. We will then use this information to decide which patients to run further tests on for treatment.

After much deliberation amongst the team, you come to a conclusion that we can use past patient data to predict the future occurence of disease.

We will use our random forest algorithm from Q3.1 to predict if a patient may have indicators of heart disease.

You can find more information on the dataset here (https://archive.ics.uci.edu/ml/datasets/heart+disease).

Loading the dataset

The dataset that the company has collected has the following features:

Only 13 features used out of a potential 76 to train. We also use the "num" feature as our label we are trying to predict.

Inputs:

1. (age)
2. (type)
3. (cp) chest pain type
4. (trestbps) resting blood pressure (in mm Hg on admission to the hospital)
5. (chol) serum cholestoral in mg/dl
6. (fbs) (fasting blood sugar > 120 mg/dl) (1 = true; 0 = false)
7. (restecg) resting electrocardiographic results:
   - Value 0: normal
   - Value 1: having ST-T wave abnormality (T wave inversions and/or ST elevation or depression of > 0.05 mV)
   - Value 2: showing probable or definite left ventricular hypertrophy by Estes' criteria
8. (thalach) maximum heart rate achieved
9. (exang) exercise induced angina (1 = yes; 0 = no)
10. (oldpeak) ST depression induced by exercise relative to rest
11. (slope) the slope of the peak exercise ST segment
    - Value 1: upsloping
    - Value 2: flat
    - Value 3: downsloping
12. (ca) number of major vessels (0-3) colored by flourosopy
13. (thal) 3 = normal; 6 = fixed defect; 7 = reversable defect

Output:

1. (num) target value:
   - 0 means <50% chance of narrowing arteries
   - 1 means greater than 50% chance of narrowing arteries

Your random forest model will try to predict this variable.

In [18]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################
from sklearn import preprocessing
import pandas as pd
preprocessor = preprocessing.LabelEncoder()

data_train = pd.read_csv("./data/heart_disease_cleaveland_train.csv")
data_test = pd.read_csv("./data/heart_disease_cleaveland_test.csv")

X_train = data_train.drop(columns = 'num')
y_train = data_train['num']
y_train = y_train.to_numpy()
y_train[y_train > 1] = 1
X_test = data_test.drop(columns = 'num')
X_test = np.array(X_test)
y_test = data_test['num']
y_test = y_test.to_numpy()
y_test[y_test > 1] = 1
X_train, y_train, X_test, y_test = np.array(X_train), np.array(y_train), np.array(X_test), np.array(y_test)
```

In [19]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################
assert X_train.shape == (200, 13)
assert y_train.shape == (200,)
assert X_test.shape == (98, 13)
assert y_test.shape == (98,)
X_train.shape, y_train.shape, X_test.shape, y_test.shape
```

Out[19]: `((200, 13), (200,), (98, 13), (98,))`

In the following codeblock, train your random forest model with different values for max_depth, n_estimators, or max_features and evaluate each model on the held-out test set. Try to choose a combination of hyperparameters that maximizes your prediction accuracy on the test set (aim for 75%+).

In **random_forest.py**, once you are satisfied with your chosen parameters, update the following function:

- **select_hyperparameters**: change the values for `max_depth`, `n_estimators`, and `max_features` to your chosen values

Submit this file to Gradescope. You must achieve at least a **75% accuracy** against the test set in Gradescope to receive full credit for this section.

In [20]:

```
"""
TODO:
n_estimators defines how many decision trees are fitted for the random forest.
max_depth defines a stop condition when the tree reaches to a certain depth.
max_features controls the percentage of features that are used to fit each decision tree.

Tune these three parameters to achieve a better accuracy. While you can use the provided test set to
evaluate your implementation, you will need to obtain 75% on the test set to receive full credit
for this section.
"""
from random_forest import RandomForest
import sklearn.ensemble

################# DO NOT CHANGE THIS RANDOM SEED ###################
student_random_seed = 4641 + 7641
####################################################################

################# CHANGE THESE VALUES ##############################
n_estimators = 1 #Hint: Consider values between 5-12.
max_depth = 1 # Hint: Consider values betweeen 3-12.
max_features = 0.1 # Hint: Consider values betweeen 0.6-1.0.
####################################################################

random_forest = RandomForest(n_estimators, max_depth, max_features, random_seed=student_random_seed)
random_forest.fit(X_train, y_train)

accuracy=random_forest.OOB_score(X_test, y_test)

print("accuracy: %.4f" % accuracy)
```

```
accuracy: 0.7523
```

DON'T FORGET: Once you are satisfied with your chosen parameters, change the values for `max_depth`, `n_estimators`, and `max_features` in the `select_hyperparameters()` function of your RandomForest class in `random_forest.py` to your chosen values, and then submit this file to Gradescope. You must achieve at least a **75% accuracy** against the test set in Gradescope to receive full credit for this section.

3.3 Plotting Feature Importance [5pts Bonus for All] [W]

While building tree-based models, it's common to quantify how well splitting on a particular feature in a decision tree helps with predicting the target label in a dataset. Machine learning practicioners typically use "Gini importance", or the (normalized) total reduction in entropy brought by that feature to evaluate how important that feature is for predicting the target variable.

Gini importance is typically calculated as the reduction in entropy from reaching a split in a decision tree weighted by the probability of reaching that split in the decision tree. Sklearn internally computes the probability for reaching a split by finding the total number of samples that reaches it during the training phase divided by the total number of samples in the dataset. This weighted value is our feature importance.

Let's think about what this metric means with an example. A high probabiity of reaching a split on "Age" in a decision tree trained on our patient dataset (many samples will reach this split for a decision) and a large reduction in entropy from splitting on "Age" will result in a high feature importance value for "Age". This could mean "Age" is a very important feature for predicting a patients probability of disease. On the other hand, a low probability of reaching a split on "Cholesterol (chol)" in a decision tree (few samples will reach this split for a decision) and a low reduction in entropy from splitting on "Cholesterol (chol)" will result in a low feature importance value. This could mean "Cholesterol (chol)" is not a very informative feature for predicting a patients probability of disease in our decision tree. **Thus, the higher the feature importance value, the more important the feature is to predicting the target label.**

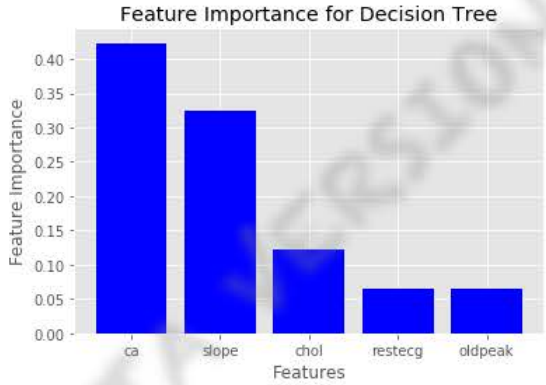
Fortunately for us, fitting a sklearn.DecisionTreeClassifier to a dataset auomatically computes the Gini importance for every feature in the decision tree and stores these values in a **feature_importances_** variable. Review the docs for more details on how to access this variable (https://scikit-learn.org/stable/modules/generated/sklearn.tree.DecisionTreeClassifier.html#sklearn.tree.DecisionTreeClassifier.feature_importances_)

In the **random_forest.py** file, complete the following function:

- **plot_feature_importance**: Make sure to sort the bars in descending order and remove any features with feature importance of 0

In the cell below, call your implementation of `plot_feature_importance()` and display a bar plot that shows the feature importance values for at least one decision tree in your tuned random forest from Q3.2.

In [21]:
```
# TODO: Complete plot_feature_importance() in random_forest.py

random_forest.plot_feature_importance(data_train)
```



Note that there isn't a 'correct' answer here. We simply want you to investigate how different features in your random forest contribute to predicting the target variable.

3.4 Improvement [5 pts Bonus for All] [W]

For this question, we only ask that the Random Forest model have an accuracy of at least 75%. In the real world, we would not be satisified with this accuracy and would most likely not immediately deploy this model to identify narrowing arteries.

Please answer the following two questions:

1. What are some potential causes for why a Random Forest decision tree model may not produce high accuracies?
2. What are some ways to improve on these potential causes?

Answer:

4: (Bonus for All) SVM [30 pts] [W] [P]

4.1 Fitting an SVM classifier by hand [20 pts] [W]

Consider a dataset with the following points in 2-dimensional space:

x_1	x_2	y
0	0	-1
0	2	-1
2	0	-1
2	2	1
4	0	1
4	4	1

Here, x_1 and x_2 are features and y is the label.

The max margin classifier has the formulation,

$$\min \|\theta\|^2$$

$$s.t.\ y_i(x_i\theta + b) \geq 1 \quad \forall\, i$$

Hint: $\mathbf{x}_4$ are the suppport vectors. Margin is equal to $\frac{1}{\|\theta\|}$ and full margin is equal to $\frac{2}{\|\theta\|}$. You might find it useful to plot the points in a 2D plane. When calculating the θ you don't need to consider the bias term.

(1) Are the points linearly separable? Does adding the point $\mathbf{x} = (4, 2), \mathbf{y} = -1$ change the separability? (2 pts)

(2) According to the max-margin formulation, find the separating hyperplane. Do not consider the new point from part 1 in your calculations for this current question or subsequent parts. (You should give some kind of explanation or calculation on how you found the hyperplane.) (4 pts)

(3) Find a vector parallel to the optimal vector θ. (4 pts)

(4) Calculate the value of the margin (single-sided) achieved by this θ? (4 pts)

(5) Solve for θ, given that the margin is equal to $1/\|\theta\|$. (4 pts)

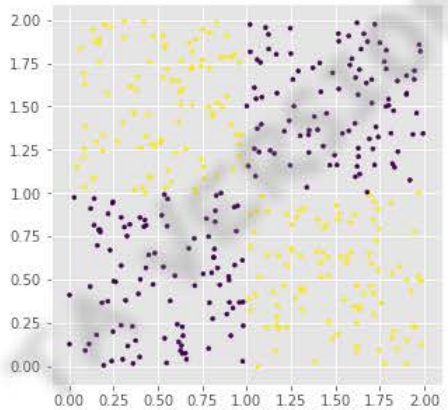
(6) If we remove one of the points from the original data the SVM solution might change. Find all such points which change the solution. (2 pts)

4.2 Feature Mapping [10 pts] [P]

Let's look at a dataset where the datapoint can't be classified with a good accuracy using a linear classifier. Run the below cell to generate the dataset.

We will also see what happens when we try to fit a linear classifier to the dataset.

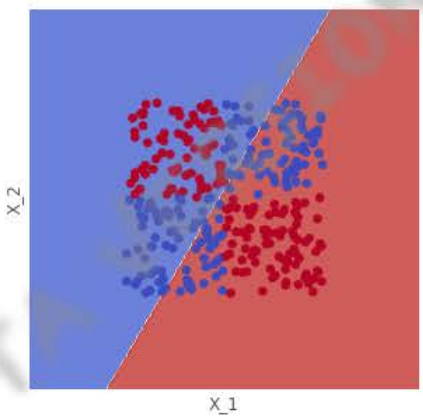
In [29]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################
# Generate dataset

random_state = 1

X_1 = np.random.uniform(size=(100, 2))
y_1 = np.zeros((100,)) - 1

X_2 = np.random.uniform(size=(100, 2))
X_2[:, 0] = X_2[:, 0] + 1.0
y_2 = np.ones((100,))

X_3 = np.random.uniform(size=(100, 2))
X_3[:, 1] = X_3[:, 1] + 1.0
y_3 = np.ones((100,))

X_4 = np.random.uniform(size=(100, 2))
X_4[:, 0] = X_4[:, 0] + 1.0
X_4[:, 1] = X_4[:, 1] + 1.0
y_4 = np.zeros((100,)) - 1

X = np.concatenate([X_1, X_2, X_3, X_4], axis=0)
y = np.concatenate([y_1, y_2, y_3, y_4], axis=0)
X_train, X_test, y_train, y_test = train_test_split(X, y,
                                                    test_size=0.20,
                                                    random_state=random_state)

f, ax = plt.subplots(nrows=1, ncols=1,figsize=(5,5))
plt.scatter(X[:, 0], X[:, 1], c = y, marker = '.')
plt.show()
```



In [30]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################

def visualize_decision_boundary(X, y, feature_new=None, h=0.02):
    '''
    You don't have to modify this function

    Function to vizualize decision boundary

    feature_new is a function to get X with additional features
    '''
    x1_min, x1_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    x2_min, x2_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx_1, xx_2 = np.meshgrid(np.arange(x1_min, x1_max, h),
                         np.arange(x2_min, x2_max, h))

    if X.shape[1] == 2:
        Z = svm_cls.predict(np.c_[xx_1.ravel(), xx_2.ravel()])
    else:
        X_conc = np.c_[xx_1.ravel(), xx_2.ravel()]
        X_new = feature_new(X_conc)
        Z = svm_cls.predict(X_new)
    Z = Z.reshape(xx_1.shape)

    f, ax = plt.subplots(nrows=1, ncols=1,figsize=(5,5))
    plt.contourf(xx_1, xx_2, Z, cmap=plt.cm.coolwarm, alpha=0.8)
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.coolwarm)
    plt.xlabel('X_1')
    plt.ylabel('X_2')
    plt.xlim(xx_1.min(), xx_1.max())
    plt.ylim(xx_2.min(), xx_2.max())
    plt.xticks(())
    plt.yticks(())

    plt.show()
```

In [31]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################
# Try to fit a linear classifier to the dataset

svm_cls = svm.LinearSVC()
svm_cls.fit(X_train, y_train)
y_test_predicted = svm_cls.predict(X_test)

print("Accuracy on test dataset: {}".format(accuracy_score(y_test,
                                                           y_test_predicted)))

visualize_decision_boundary(X_train, y_train)
```

```
Accuracy on test dataset: 0.375
```



We can see that we need a non-linear boundary to be able to successfully classify data in this dataset. By mapping the current feature x to a higher space with more features, linear SVM could be performed on the features in the higher space to learn a non-linear decision boundary. In the function below add additional features which can help classify in the above dataset. After creating the additional features use code in the further cells to see how well the features perform on the test set.

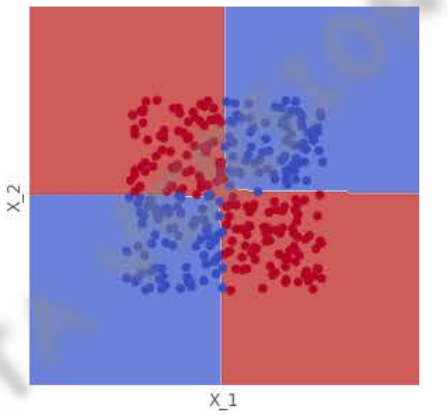
Note: You should get a test accuracy above 90%

Hint: Think of the shape of the decision boundary that would best separate the above points. What additional features could help map the linear boundary to the non-linear one? Look at this (https://xavierbourretsicotte.github.io/Kernel_feature_map.html) for a detailed analysis of doing the same for points separable with a circular boundary

In [32]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################
from feature import create_nl_feature

X_new = create_nl_feature(X)
X_train, X_test, y_train, y_test = train_test_split(X_new, y,
                                                    test_size=0.20,
                                                    random_state=random_state)
```

In [33]:
```
################################
### DO NOT CHANGE THIS CELL ###
################################
# Fit to the new features and vizualize the decision boundary
# You should get more than 90% accuracy on test set

svm_cls = svm.LinearSVC()
svm_cls.fit(X_train, y_train)
y_test_predicted = svm_cls.predict(X_test)

print("Accuracy on test dataset: {}".format(accuracy_score(y_test, y_test_predicted)))

visualize_decision_boundary(X_train, y_train, create_nl_feature)
```

```
Accuracy on test dataset: 0.9375
```



In [ ]:
```

```