Stopping power of hot QCD plasma

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The partonic energy loss has been calculated taking both the hard and soft contributions for all the $2 \to 2$ processes, revealing the importance of the individual channels. Cancellation of the intermediate separation scale has been exhibited. Subtleties related to the identical final state partons have properly been taken into account. The estimated collisional loss is compared with its radiative counter part. We show that there exists a critical energy (E_c) below which the collisional loss is more than its radiative counterpart. In addition, we present closed form formulas for both the collision probabilities and the stopping power (dE/dx).

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I. INTRODUCTION

The partonic energy loss in a QCD plasma has received significant attention in recent years. Experimentally, the partonic energy loss can be probed by measuring the high p_T hadrons emanating from ultrarelativistic heavy ion collisions. This idea was first proposed by Bjorken [1] where "ionization loss" of the quarks and gluons in a QCD plasma was estimated. In fact, the "stopping power" (dE/dx) of the plasma is proportional to $\sqrt{\epsilon}$, where, ϵ is the energy density of the partonic medium. Therefore, by measuring various high p_T observables one can probe the initial parton density [1].

Hard partons, injected into hot QCD medium, can dissipate energy in two ways, vis-à-vis, by two body collisions or via the bremsstrahlung emission of gluons, commonly referred to as collisional and radiative loss, respectively. For electromagnetic processes, it is well known that at large energies, radiative losses are much higher than the collisional loss. In fact, there is a critical energy E_c, at which, both the processes contribute equally [2–4]. In QCD plasma, however, the situation is more involved because of the non-Abelian nature of the interaction. We shall show that for low energy partons $2 \to 2$ processes are equally important. This is particularly so for heavy quarks [5]. In QCD plasma, to our knowledge, such estimation of E_c is not known yet. We address this issue in the present work. Obviously this requires complete treatment of both $2 \to 2$ and $2 \to 3$ (or higher order) processes. While significant progress has been made over the past decade to estimate bremsstrahlung induced partonic energy loss [6–14], collisional loss, as we uncover, begs further attention. The energy loss of partons in a QCD plasma due to the dissociation of the possible binary bound states in a strongly coupled quark-gluon plasma (QGP) has recently been considered by Shuryak *et al.* [15]. They have shown that the partonic energy loss due to this process is important in a narrow interval of plasma temperatures. It may be noted that unlike radiative loss in which mostly relativistic gluons are produced, collisional/ionization loss, on the contrary, injects energy-momentum and entropy into the plasma. However, in this work we discuss only parton parton scattering, not the dissociation of the binary bound state.

The partonic energy loss in a QCD plasma was first estimated by Bjorken [1]. Considering two body scattering of the parton off thermal quarks and gluons, the following expression for the energy loss is obtained.

$$\frac{dE}{dx} = \frac{8\pi}{3}\left(1 + \frac{n_f}{6}\right)\alpha_s^2 T^2 \ln\left(\frac{q_{\max}}{q_{\min}}\right). \tag{1}$$

In the above equation, $n_f/6$ term is for quark sector while the other one is responsible for the gluonic loss. Bjorken retains only the infrared divergent part of the integral. Here $q_{\min}$ and $q_{\max}$ denote maximum and minimum momentum transfer. Evidently, as indicated in Ref. [1] itself, that this expression breaks down in the infrared region due to plasma effects. The presence of $q_{\max}$ and $q_{\min}$ should be noted here, for which one takes reasonable values from physical argument. In principle, however, should emerge from the theory itself in a natural way.

Physically energy losses for partons traversing plasma can be divided into two parts. One due to close collisions involving hard momentum transfer, to be treated microscopically in terms of individual scatterings, and the other for the distant collisions involving soft momentum transfer. Evidently, for the latter, the de Broglie wave length of the exchanged particle becomes comparable with the inter particle distance ($\sim T^{-1}$), which renders the concept of individual scattering meaningless and necessitates the inclusion of the plasma effects.

In the long wave length regime the problem of energy loss can be treated macroscopically in terms of classical chromoelectric field. This coherent regime of partonic energy loss was first calculated by Thoma and Gyulassy [16]. They actually express relative energy loss in terms of the chromodielectric field tensor by combining hard thermal loop corrected gluon propagator with the techniques of classical plasma physics as

$$\frac{dE}{dx} = \frac{C_F \alpha_s}{2\pi^2 v} \int \frac{\omega d^3k}{k^2} [\mathrm{Im}\epsilon_l^{-1} + (v^2k^2 - \omega^2) \times \mathrm{Im}(\omega^2 \epsilon_t - k^2)^{-1}]. \quad (2)$$

In this formalism, the ambiguity related with the lower cutoff is naturally removed by the inclusion of the polarization effects. However, this description also breaks down at some ultraviolet scale [17,18] and should therefore be cut off at some hard momentum scale.

Evidently, Eqs. (1) and (2) capture physics relevant for two different kinematic regime and therefore complimentary to each other. Any complete calculation should include both the infrared and ultraviolet part as discussed at length by Mrowczynski [17] and Braaten *et al.* [18] to evaluate the total energy loss. In Ref. [17] for the first time a complete calculation for the partonic energy loss was presented. Here the hard part was treated along the line of Ref. [1] and the soft part was calculated classically. However, the ambiguity related to the separation of scale remained till the full field theoretic techniques was developed by Braaten and Pisarski [19] which was subsequently used to calculate the energy loss of heavy fermions by Braaten and Yuan [20]. In their formalism it was shown how the infrared cutoff naturally arise from the hard thermal loop resummation scheme and at the same time the intermediate scale dependence is removed from the theory where the separation of scale appears in the argument of the logarithms and cancels automatically when contributions of these two regimes are added together [18,21].

In retrospect of these developments we revisit the problem of light quark and gluon energy loss in a QCD plasma. The treatment of light quarks (or gluons) is different from that of heavy quarks in many different ways. Primarily this is related to the presence of the light quarks or gluons in the thermal bath. This would modify the thermal phase space. In addition there will be back reactions which should be taken into account. Moreover, unlike heavy quark, their light partners can even be annihilated with the thermal constituents. Most importantly, light quarks or gluons involve subtleties related to the processes having identical final state species which was not properly taken into account in Ref. [1].

The motivation of the present work is to unravel the relative contribution of individual processes. Therefore, we take all possible channels including elastic and inelastic scatterings. Consequently, estimated $(dE/dx)_{\mathrm{coll}}$, is higher than reported before. In fact, the two body compton like scattering, proves to be quite efficient in transferring energy into the plasma. In addition, we also evaluate explicitly the collisional loss of gluon energy. For this, $gg \rightarrow gg$ and $qg \rightarrow qg$ are found to be most important.

It might be mentioned that calculations reported in [1] (see also [22]) were restricted only to the t channel processes, thereby, excluding the interference and exchange terms, which contribute significantly. They are particularly important for processes having u channel divergences[1].

The plan of the paper is as follows. In Sec. II, we develop the formalism, which will be used afterwards. In Sec. III contributions from various channels on the collisional energy loss of partons have been presented. These results are then compared with radiative loss. In Sec. IV, we explicitly treat both the soft and hard momentum transfer regime and show how infrared divergence, originally present in the previous sections are automatically removed by the plasma effects. In addition we demonstrate that the final expression is free from the momentum cutoff introduced to regulate the divergences in two (soft and hard) kinematic regimes. Sec. V is devoted to summary and discussion. Various sum rules used in Sec. IV are collected in the appendix.

II. FORMALISM

While the heavy quark energy loss is very similar to the muon energy loss in a plasma of electrons and positrons, light quark energy loss is analogous to the electron energy loss in a QED plasma. Therefore to calculate the relevant dE/dx arising out of two body scatterings, we introduce a formalism along the line similar to what is employed to study cosmic ray showers [2,3]. Accordingly, we define a differential collision probability, $\Theta(E, E')dE'dx$ which represents the probability of a parton with energy E to transfer an amount of energy between E' and $E' + dE'$ to a plasma constituent in traversing a thickness dx. Energy loss can be obtained by convoluting $\Theta(E, E')dE'$ with the energy transfer (E') for each processes which generically is given by

$$\frac{dE}{dx} = \int_{E_{\min}}^{E_{\max}} E'\Theta(E, E')dE'. \quad (3)$$

In the above equation, $E_{\max}$ is the maximum energy transfer, while $E_{\min}$ is a cutoff used to regulate infrared divergence related to the usual small angle limit. However, in case of energy loss calculation $\int \frac{d\theta}{\theta^3}$ divergence that appears in the cross section becomes softer as schematically $dE/dx \sim n\sigma E'$, where, $E'/E = \frac{1}{2}(1 - \cos\theta) \sim \theta^2/4$, tames the divergence. This cutoff procedure can be avoided by incorporating appropriate screening effects [16,18,23]. The differential collision probability is defined as

$$\Theta(E, E')dE' = \frac{\pi\alpha_s^2}{2E^2} \int \frac{d^3k}{k} \mid \mathcal{M} \mid^2 f(k), \quad (4)$$

where

[1]For compton like scattering involving heavy quarks both u and s channel have been considered in Ref. [18]. However, in case of light quarks, there are additional sources of such diagrams e.g. QCD Moller and Bhaba like scattering.

$$f = \frac{\nu}{(2\pi)^3} \frac{1}{\exp(k/T) \pm 1}, \tag{5}$$

where ν is the statistical degeneracy of the particles in the thermal bath. It might be mentioned that for coulomb like scattering, $\Theta(E, E') \propto 1/E'^2$ and therefore $dE/dx \propto \int \frac{dE'}{E'}$. This, evidently, is divergent and gives the logarithmic dependence of dE/dx [1]. We also assume that the energy of the incoming parton $E \gg T$, where T is the temperature of the system.

Now we consider a specific process $qq' \to qq'$, the matrix element for which is given by the following expression,

$$|\mathcal{M}|^2 = \frac{4}{9} g^4 \frac{s^2 + u^2}{t^2}, \tag{6}$$

where $g^2 = 4\pi\alpha_s$ is the color charge and α_s is the strong coupling constant.

For $qq' \to qq'$, the differential collision probability can be expressed as

$$\Theta(E, E') = \frac{\pi\alpha_s^2}{18} \frac{T^2}{E^2}\left(1 - 2\frac{E}{E'} + 2\frac{E^2}{E'^2}\right), \tag{7}$$

where T is the temperature of the medium.

The last equation together with Eq. (3) gives the following expression for the energy loss,

$$\frac{dE}{dx} \approx \frac{\pi\alpha_s^2 T^2}{9} \ln(E/\omega_0), \tag{8}$$

where ω_0 is the lower cutoff. In Ref. [1] this is taken as $\mu^2/2k$ while in Ref. [22] $\omega_0 \sim \alpha_s T$. We in Sec. IV, show that how this cutoff follows from the full calculation. A complete treatment of this singularity in the context of heavy quark energy loss both for soft and hard momentum transfer within hard thermal loop resummation scheme has been discussed in [18]. We present corresponding calculations for the light quarks in a slightly different approach.

In the subsequent sections, we present explicit expressions of $\Theta(E, E')dE'$ for various processes. Corresponding QED results are also derived for comparison [2,3]. The quantities E and E' are defined in the laboratory frame.

III. CONTRIBUTION OF VARIOUS CHANNELS

A. Quarks

Let us first consider propagation of a hard quark through a QCD plasma. The collisions which would contribute to its energy loss are $qq \to qq$, $qq' \to qq'$, $q\bar{q}' \to q\bar{q}'$, $qg \to qg$, and $q\bar{q} \to q\bar{q}, gg, q'\bar{q}'$. In the above processes primes indicate different flavors. It might be mentioned that in a baryon free region, i.e., in absence of a net baryonic chemical potential, quark and antiquark energy loss will be the same. Therefore, we do not treat them separately.

The most dominant process for quark energy loss, as mentioned before, is compton like scattering, i.e. $qg \to qg$. Beside the t channel, contributions from additional diagrams are found to be non-negligible.

The differential collision probability for the compton like scattering is as follows:

$$\Theta_{qg\to qg}(E, E')dE' = \frac{2\pi\alpha_s^2}{3E^2} T^2\left[1 - 2\frac{E}{E'} + 2\left(\frac{E}{E'}\right)^2 + \frac{4}{9}\left\{1 - \frac{E'}{E} + \frac{E}{E - E'}\right\}\right]dE'. \tag{9}$$

In Eq. (9), the first three terms come from the t channel and others originate from the exchange diagram and s channel. Evidently, $E/(E - E')$ gives rise to logarithmic enhancement. Thus overall contribution becomes significant if one retains all the possible diagrams. To get the energy loss, one integrates the differential collision probability weighted with the energy transfer as shown in Eq. (3) to yield

$$\left(\frac{dE}{dx}\right)_{qg\to qg} = \frac{2\pi\alpha_s^2}{3} T^2\left[\frac{22}{9} \ln(E/\omega_0) - 0.176\right]. \tag{10}$$

In writing the above equation, we have used $E_{\min} = \omega_0/2$, and $E_{\max} = E$, the energy of the hard parton, it is also assumed here that $E \gg \omega_0$. This is justified, as for the present problem, partonic jets have very high energy compared to the energy of the plasma constituents which could be $\sim$3 T. It should be noted that the coefficient of the logarithmic term is different from that one obtains by restricting to the t channel alone.

Next we consider Möller type $qq \to qq$ scattering for which the differential collision probability [24] reads

$$\Theta_{qq\to qq}(E, E')dE' = \frac{\pi\alpha_s^2}{9} T^2\left[\frac{E^2}{E'^2(E - E')^2} + \frac{\Delta}{E'(E - E')} + \frac{1}{E^2}\right]dE', \tag{11}$$

where $\Delta = -10/3$. In the last expression if we replace $\frac{2}{9}\alpha_s^2$ by α_{em}^2 and Δ by -2, the electron energy loss due to Möller scattering ensues [2,3]. This reaction deserves special attention as it involves two identical particles in the final state. Therefore, $\Theta(E, E')dE'$, in this case, should be interpreted as the probability of a collision which leaves one parton in the energy state E' and the other in the energy state $E - E'$. To take into account all the possibilities, E' is varied from $\omega_0/2$ to $E/2$ [2,3]. Similar subtlety is involved for processes like $gg \to gg$ or $q\bar{q} \to gg$ etc. The final expression for $qq \to qq$ is given by

$$\left(\frac{dE}{dx}\right)_{qq\to qq} = \frac{\pi\alpha_s^2}{9} T^2[\ln(E/\omega_0) + \Delta \ln 2 + 1.125]. \tag{12}$$

The other important reaction is $q\bar{q} \to q\bar{q}$, which also has t^{-2} divergence [25] and therefore, found to contribute significantly to the total energy loss. It should be noted that there is no u^{-2} divergence involved in this process hence the collision is dominated by soft scattering and result do not differ much if the relevant s channel diagram is excluded. We, nevertheless, retain all the diagrams. The differential probability for "bhabha" like scattering, therefore, takes the form

$$\Theta_{q\bar{q}\to q\bar{q}}(E,E')dE' = \frac{\pi\alpha_s^2}{9E'^2}T^2\bigg[1 - \Delta'\frac{E'}{E} + (2\Delta' - 1)\frac{E'^2}{E^2} - \Delta'\frac{E'^3}{E^3} + \frac{E'^4}{E^4}\bigg]dE'. \tag{13}$$

Corresponding energy loss turns out to be

$$\left(\frac{dE}{dx}\right)_{q\bar{q}\to q\bar{q}} = \frac{\pi\alpha_s^2}{9}T^2\bigg[\ln\frac{E}{\omega_0} - \frac{\Delta'}{3} + 0.443\bigg], \tag{14}$$

where $\Delta' = 2/3$. The QED limit for the last two equations can be taken by replacing $\frac{2}{9}\alpha_s^2$ with α_{em}^2 and $\Delta' = 2$ [2,3].

Finally we present results for the process, $q\bar{q} \to gg$. This again involved identical particles in the final channel for which appropriate limit is taken. This process is also suppressed because of less sensitive infrared divergences as evident from the expression:

$$\Theta_{q\bar{q}\to gg}(E,E')dE' = \frac{\pi\alpha_s^2}{3E^2}T^2\bigg[\frac{4}{9}\bigg\{\frac{E}{E'} + \frac{E'}{E-E'}\bigg\} + 2\frac{E'}{E} - 2\frac{E'^2}{E^2} - \frac{13}{9}\bigg]dE'. \tag{15}$$

Other important reactions for which we do not present explicit results include $qq' \to qq'$ and $q\bar{q}' \to q\bar{q}'$. They contribute equally to the energy loss (for baryon free matter). It should be mentioned that $q\bar{q} \to q'\bar{q}'$ induced energy loss is small because of the absence of infrared enhancement. This is less divergent (no t^{-2} or u^{-2}), and, therefore, found to be less effective means of energy dissipation.

B. Gluons

Similar to quarks, hard gluons can also dissipate energy while colliding with the plasma constituents. Most important process by which gluons can transfer energy to the plasma is the $gg \to gg$.

$$\Theta_{gg\to gg}(E,E')dE' = \frac{3\pi\alpha_s^2}{E^2}T^2\bigg[3 - \frac{E'(E-E')}{E^2} + \frac{E^2}{E'^2} - \frac{E}{E'} + \frac{EE'}{(E-E')^2}\bigg]dE'. \tag{16}$$

Corresponding expression for the energy loss can be written as

$$\left(\frac{dE}{dx}\right)_{gg\to gg} = 3\pi\alpha_s^2T^2\bigg[\ln\left(\frac{E}{\omega_0}\right) - 0.038\bigg]. \tag{17}$$

Like quark, the QCD compton scattering also proves to be quite efficient in transferring gluon energy into the plasma. Relevant expressions for the differential collisional probability $\Theta(E,E')dE'$ and dE/dx induced by $gq \to gq$ scattering can be obtained from Eqs. (9) and (10) respectively by appropriately replacing the phase space factor (factor $2/3$ in Eqs. (9) and (10) should be replaced by $3/4$ for three flavour QGP). It should be mentioned that $gg \to q\bar{q}$ is also suppressed as there is no t^{-2} or u^{-2} singularity involved in this process. Gluonic energy loss induced by this process can be obtained from Eq. (15).

In Fig. 1 we present stopping power as function of energy of the incoming parton at a temperature $T = 250$ MeV. The result is to be compared with previous estimates [1,22]. Evidently, bulk contribution to the total collisional energy loss of quark comes from the $qg \to qg$ channel. Net energy loss of a light quark is given by the sum of all these diagrams including scattering and annihilation processes. Contribution of inelastic channels are found to be small and, therefore, have not been shown explicitly. However, the total loss, as demonstrated in Fig. 1 include effect of all the channels. It should be mentioned that present treatment can be extended for heavy quarks for which collision probabilities will be modified [24]. Quantitatively, we find $dE/dx \sim 0.8$ GeV/fm for a 20 GeV parton, *vis-à-vis* 0.2 GeV/fm of Refs. [1,18,22]. This may be attributed to the diagrams other than t channel and $\alpha_s = 0.3$ instead of $\alpha_s = 0.2$ in [18].

The results for gluon energy loss is presented in Fig. 2 below. Evidently gluon energy loss is mostly driven by $gg \to gg$ scattering. Also comparable is the contribution of $qg \to qg$ channel.

C. Comparison with radiative loss

To bring the importance of collisional loss into bold relief, we estimate the possible parton density relevant

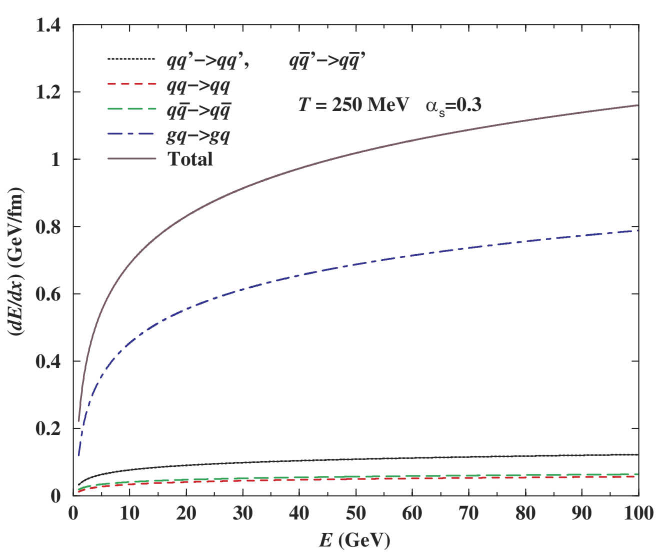

FIG. 1 (color online). Individual contributions of various processes responsible for quark energy loss are shown. The aggregated collisional loss is also presented.

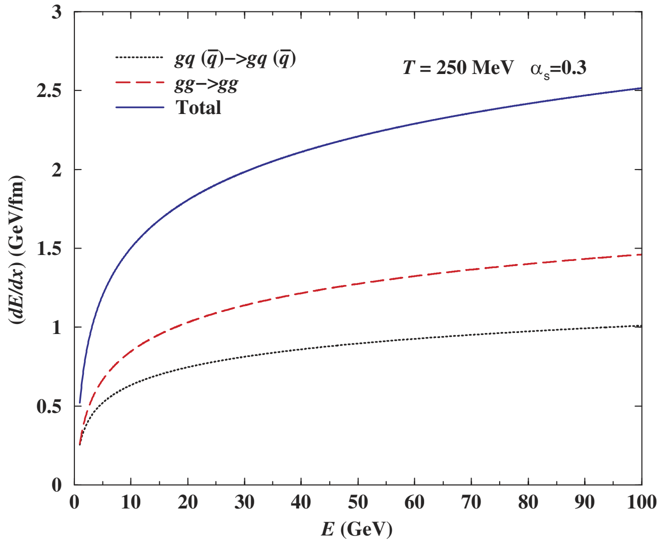

FIG. 2 (color online). Individual contributions of various processes responsible for gluon energy loss are shown. Dashed, dot-dashed and solid line represent $gq(\bar{q}) \rightarrow gq(\bar{q})$, $gg \rightarrow gg$ and total, respectively.

for the RHIC energies. The gluon rapidity density in this case can be taken as $dN_g/dy \sim 1000$, which, when plugged into the Bjorken formula [26] $\rho_g = \frac{dN_g}{dy}/\tau_0 \pi R_{Au}^2$ with formation time $\tau_0 = 0.5$ fm$/c$, we get a value of $T \sim 400$ MeV. It might be mentioned that this density is consistent with the one used in Ref. [7]. Corresponding values of the total (integrated over plasma length) energy loss for quark and gluon is significantly large as depicted in Fig. 3. We also compare total collisional energy loss with its radiative counterpart. For the latter, we take [27]

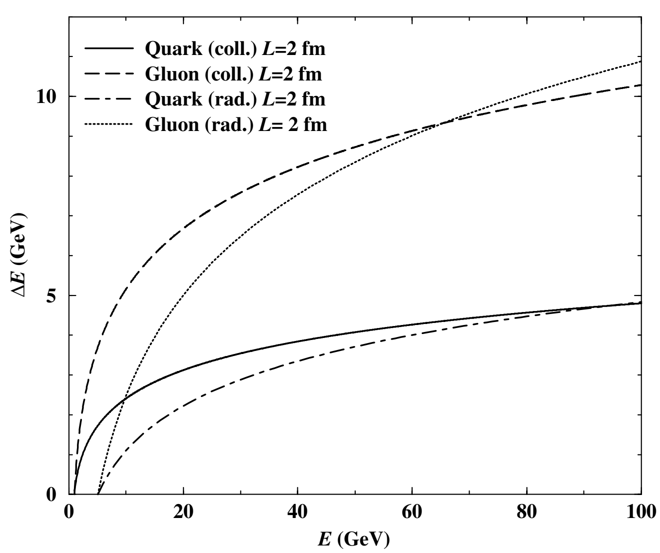

FIG. 3. The collisional and radiative energy loss of quarks and gluons passing through quark gluon plasma.

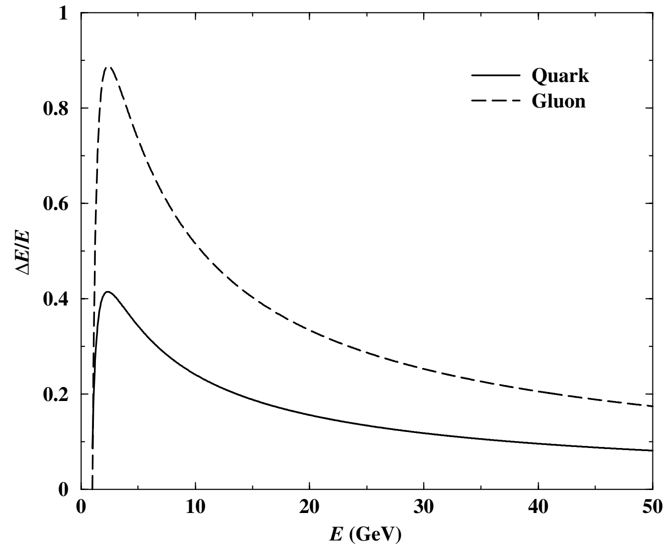

FIG. 4. The fractional energy loss of quarks and gluons due to collisions. The length of the medium $L = 2$ fm, $T = 400$ MeV.

$$\Delta E_{\rm rad} = C_2 \frac{\alpha_s \mu^2 L^2}{N(E)\lambda} \ln\left(\frac{2E}{\mu^2 L}\right), \tag{18}$$

where L is the length of the plasma traversed by the partons and λ is the mean free path. Collisional energy loss is more than its radiative counterpart for parton energy up to $E = E_c \sim 85$ GeV for quarks and 60 GeV for gluons, respectively. The results shown in Fig. 3 correspond to $N(E) = 10$, $\mu = 1$ GeV, $\alpha_s = 0.3$, and $L/\lambda = 4$. It is important to point out here that $N(E) = 7.3, 10.1, 24.4$ for $E = 500, 50, 5$ GeV respectively and $N(E \rightarrow \infty) = 4$ [7].

In Fig. 4 we depict the variation of fractional energy loss due to the collision of quarks and gluons passing through a QGP medium of length $L = 2$ fm at $T = 400$ MeV. This

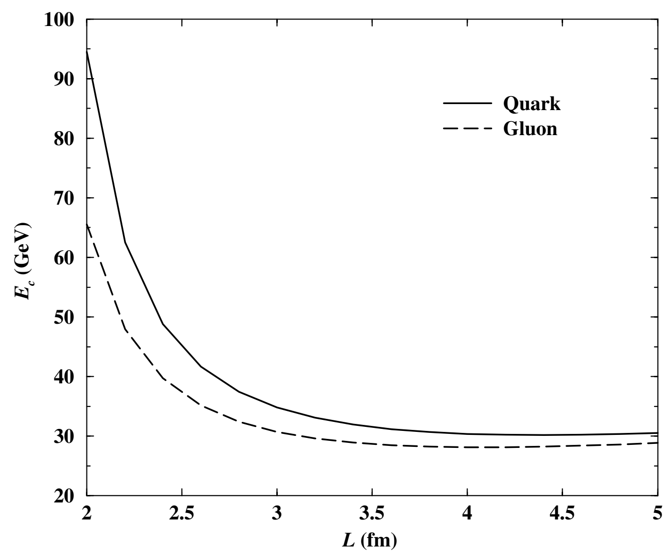

FIG. 5. The variation of E_c as function of path length of the high energy partons moving through the QGP at $T = 400$ MeV.

result may be contrasted with its radiative counter part as given in [28]. It is important to point out here that a value of $\Delta E/E \sim 1/5$ [15] can reproduce the high p_T suppression of pion spectra observed at RHIC energy.

In Fig. 5 we show the variation of the critical energy of the high energy parton as a function of its path length in the medium where the radiative and the collisional losses contribute equally. Here the results are obtained for $N(E) = 10$, $\mu = 1$ GeV, $\alpha_s = 0.3$, and $\lambda = 0.5$ fm.

IV. CANCELLATION OF THE INTERMEDIATE SCALE

In the previous section the results for the quark and gluon energy loss are presented with a lower cutoff ω_0. This was introduced to regulate the infrared divergence. In this section, we explicitly show that infrared divergence is automatically screened by the plasma effects while there exists a separation scale coming from the two kinematic regimes. Ultimately it is removed from the final expression once both the hard scattering and the collective plasma effects are added. We also assume that the energy of the incoming parton is much larger than the temperature, i.e., $E \gg T$. The soft and hard momentum transfer (q) is determined by the fact whether $q \sim gT$ or $q \sim T$ and while for the former the interaction is screened by the Debye mass (for the electric mode or dynamically for the magnetic part). The intermediate scale q^* is chosen such that $gT < q^* < T$. We, then show that the final result is independent of this scale q^*.

Following Braaten *et al.* [18], we define the energy loss dE/dx as a product of collision rate and energy transfer per scattering divided by the velocity [18] :

$$\frac{dE}{dx} = \frac{1}{v}\int dE_3(E - E_3)\frac{d\Gamma}{dE_3}, \tag{19}$$

where Γ is the interaction rate. The equivalence with the previously defined $\Theta(E, E')$ can be established easily by identifying $\Theta(E, E') = v^{-1} d\Gamma/dE'$, where $E' = E - E_3$ is the energy transfer.

$$\frac{dE}{dx} = \frac{\nu}{2E}\int \frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3p_3}{2E_3(2\pi)^3}\frac{(E-E_3)}{v} \times [f_1(1-f_2)(1-f_3) \pm (1-f_1)f_2f_3] \times (2\pi)^4\delta^4(P + P_1 - P_2 - P_3)|\mathcal{M}|^2. \tag{20}$$

Note the difference of the thermal phase space here with that of the heavy quarks where only the factor $f_1(1 - f_2)$ appears [18] due to the absence of heavy quarks in the thermal bath, deleting the possibility of reverse reactions. In the above equation, ν stands for the statistical degeneracy factor. For the processes under consideration $qq' \to qq'$, we have the following matrix element

$$|\mathcal{M}|^2 = 4g^4C_{qq}D_{\mu\nu}(q)D^*_{\alpha\beta}(q)[(P^\mu P_3^\alpha + P_3^\mu P^\alpha - g^{\mu\alpha}(P\cdot P_3))(P_1^\nu P_2^\beta + P_2^\nu P_1^\beta - g^{\nu\beta}(P_1\cdot P_2)] \tag{21}$$

where C_{qq} is the color factor. The matrix element Eq. (21), in general is very complicated, which takes a simple form in the limit of soft momentum transfer or small angle scatterings. This is justified because of the infrared sensitivity, energy loss is dominated by the soft collisions.

In the coulomb gauge, we can define $D_{00} = \Delta_l$ and $D_{ij} = (\delta_{ij} - q^iq^j/q^2)\Delta_t$. Δ_l and Δ_t denote the longitudinal and transverse gluon propagators given by

$$\Delta_l(q_0, q)^{-1} = q^2 - \frac{3}{2}\omega_p^2\left[\frac{q_0}{q}\ln\frac{q_0+q}{q_0-q} - 2\right], \tag{22}$$

$$\Delta_t(q_0, q)^{-1} = q_0^2 - q^2 + \frac{3}{2}\omega_p^2\left[\frac{q_0(q_0^2-q^2)}{2q^3}\ln\frac{q_0+q}{q_0-q} - \frac{q_0^2}{q^2}\right]. \tag{23}$$

With this, matrix element in the limit of small angle scattering, for which $P\cdot P_1 = P_2\cdot P_3 \gg P\cdot P_3$ or $P_1\cdot P_4$, we get the following expression for the squared matrix element.

$$|\mathcal{M}|^2 = g^4C_{qq}16(EE_1)^2|\Delta_l(q_0, q) + (v\times\hat{q})\cdot(v_1\times\hat{q})\Delta_t(q_0, q)|^2 \tag{24}$$

with $v = \hat{p}$ and $v_1 = \hat{p}_1$. We also use energy conservation

$$q_0 = E - E_3 = E_2 - E_1$$

this in the soft limit, i.e. $q \ll T$, becomes

$$q_0 \simeq v\cdot q \simeq v_1\cdot q. \tag{25}$$

Another useful identity that helps to cast Eq. (20) in a simplified form is the following

$$\int\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}(2\pi)^4\delta^4(P + P_1 - P_2 - P_3) \simeq \int\frac{dq_0d^3q}{2\pi(2\pi)^3}2\pi\delta(q_0 - v\cdot q)2\pi\delta(q_0 - v_1\cdot q). \tag{26}$$

These delta functions can be used to perform the angular integrations, while the integration over p_1 can be obtained by means of partial integration,

$$\int dp_1p_1^2\left(-\frac{df_1}{dp_1}\right) = 2\int dp_1p_1f_1 = \frac{\pi^2T^2}{6}. \tag{27}$$

In case of fermionic initial and final states we have

$$f_1(1-f_2)(1-f_3) + (1-f_1)f_2f_3 = (f_1 - f_2)[1 + N(q_0) - f_3] \tag{28}$$

$$\simeq -\frac{df_1}{dp_1}q_0\left[\frac{T}{q_0} - \frac{1}{2}\right]. \tag{29}$$

Here $N(q_0) = (\exp(q_0/T) - 1)^{-1}$.

$$\frac{dE}{dx} = \frac{\nu g^4}{2E} \int \frac{d^3 p_1}{2E_1(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \frac{dq_0}{2\pi} \frac{df_1}{dp_1} q_0^2 \left(-\frac{T}{q_0} + \frac{1}{2}\right) \times \frac{1}{4E_2E_3} |\mathcal{M}|^2 2\pi\delta(q_0 - \mathbf{v}\cdot\mathbf{q}) 2\pi\delta(q_0 - \mathbf{v}_1\cdot\mathbf{q}). \tag{30}$$

It might be noted that for the calculation of collisional rate, Γ, the term T/q_0 contributes at the leading order, while for the dE/dx the with $1/2$ inside the bracket gives nonzero contribution. This is related to the parity of the spectral function of the gluons.

Final expression for the energy loss is given by

$$\frac{dE}{dx} = \frac{g^4 C_{qq} T^2}{96\pi} \nu \int dq \int_{-q}^{q} q_0^2 dq_0 \times \left[|\Delta_l|^2 + \frac{1}{2}\left(1 - \frac{q_0^2}{q^2}\right)^2 |\Delta_t|^2 \right]. \tag{31}$$

Before proceeding further, let us see what happens if we use the bare propagator in the last equation which is given by $|\Delta_l(q_0, q)|^2 = 1/q^4$ and $|\Delta_t(q_0, q)|^2 = 1/(q_0^2 - q^2)^2$. With this the expression for dE/dx turns out to be

$$-\frac{dE}{dx} = \frac{g^4 T^2}{96\pi} \nu C_{qq} \int_{q^*}^{T} \frac{dq}{q} = \frac{g^4 T^2}{96\pi} \nu C_{qq} \ln\frac{T}{q^*}, \tag{32}$$

which clearly is logarithmically divergent. The upper limit indicates break down of the approximation beyond T and the lower limit is to regulate the infrared divergence.

It is instructive to compare this with the corresponding limit of the collision rate which diverges quadratically [29]

$$\Gamma = \frac{g^4 T^2}{96\pi} \nu C_{qq} \int \frac{dq}{q^3}. \tag{33}$$

Equation (31), can be expressed in terms of the spectral functions and directly be compared with Ref. [18]. For this we recall that the transverse and longitudinal propagators have the following spectral representations:

$$\Delta_l(q_0, q) = -\frac{1}{q^2} + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho_l(\omega, q)}{\omega - q_0}, \tag{34}$$

$$\Delta_t(q_0, q) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho_t(\omega, q)}{\omega - q_0}, \tag{35}$$

where,

$$\rho_{l,t} = 2\,\mathrm{Im}\Delta_{l,t}(q_0 + i\epsilon, q). \tag{36}$$

The spectral function contains contributions both from the residue at the pole and the discontinuity due to the branch cuts,

$$\rho_{l,t}(q_0, q) = 2\pi\epsilon(q_0) z_{l,t}(q) \delta[q_0^2 - \omega_{l,t}^2(q)] + \beta_{l,t}(q_0, q)\theta(q^2 - q_0^2). \tag{37}$$

Here $z_{l,t}(q)$ is the residue of the time like pole at $\omega_{l,t}$ and $\beta_{l,t}$ is the contribution from the branch cuts.

$$\beta_l(q_0, q) = 3\pi\omega_p^2 \frac{q_0}{q} |\Delta_l(q_0, q)|^2, \tag{38}$$

$$\beta_t(q_0, q) = 3\pi\omega_p^2 \frac{q_0(q^2 - q_0^2)}{2q^3} |\Delta_t(q_0, q)|^2. \tag{39}$$

With these equations, the energy loss can be expressed as

$$-\frac{dE}{dx} = \frac{g^2 C_{qq} \nu}{16\pi} \int q dq \int_{-q}^{+q} \frac{q_0 dq_0}{2\pi} \times \left[\rho_l(q_0, q) + \left(1 - \frac{q_0^2}{q^2}\right) \rho_t \right]. \tag{40}$$

To calculate the soft part, we make use of the identities listed in the appendix for $\omega_p < q^* < T$.

$$-\frac{dE}{dx} = \frac{g^2 C_{qq} \nu}{16\pi} \int_{\omega_p}^{q^*} q dq \left[I_{(1)}^l + I_{(1)}^t - \frac{1}{q^2} I_{(3)}^t \right] = \frac{3\nu g^2 C_{qq}}{32\pi} \omega_p^2 \ln\frac{q^*}{\omega_p}. \tag{41}$$

It might be mentioned that the hard part can also be calculated from Eq. (40) by taking

$$\rho_l(q_0, q) \simeq \frac{3\omega_p^2 q_0}{2q^5}, \tag{42}$$

$$\rho_t(q_0, q) \simeq \frac{3\omega_p^2 q_0}{4q^5(1 - \frac{q_0^2}{q^2})}, \tag{43}$$

yielding,

$$-\frac{dE}{dx} = \frac{3\nu g^2 C_{qq}}{32\pi} \omega_p^2 \ln\frac{T}{q^*}. \tag{44}$$

Equation (40) and the last equation clearly shows that the intermediate scale gets canceled when both the contributions are added together.

In the present treatment, to show the cancellation, we focussed only on the leading log part. However, full calculation can be done along the line of [18] with appropriate modification of the kinematics, i.e. $q_0 \le p - p_1$ and $q \le q_0 + 2p_1$. Using the fact that $p = E \gg T$, one finds $q_{\max} \simeq E$. With these, by adding the soft and hard contributions, one obtains

$$-\frac{dE}{dx} = \frac{3\nu g^2 C_{qq}}{64\pi} \omega_p^2 \ln\frac{E}{g^2 T}. \tag{45}$$

This expression coincides with Eq. (8) with appropriate degeneracy and color factors.

V. SUMMARY AND DISCUSSIONS

To summarize, in the present work, we have studied collisional loss of light partons in hot QCD plasma. We have identified some of the important diagrams previously ignored which include u channel divergences contributing to the leading log results. Subtleties related to the identical final state particles which were overlooked earlier (leads to overestimation in dE/dx) in dealing with light quarks and gluons. This has properly been taken into account in the present work. Our results are free from any arbitrary cut-off that was present in [1,17,30]. Moreover, the conditions where the collisional and the radiative losses are comparable is clearly revealed. Furthermore, note that Relativistic Heavy Ion Collider (RHIC) data suggests only a tiny amount of "quenching," $Q_t(p_T) = 0.2$ [15,31]. This corresponds to a small amount of energy loss which might be accommodated with the collisional loss. In addition, collisional energy loss has a different qualitative importance as it injects energy into the plasma. Implication of this has recently been discussed in Ref. [15].

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APPENDIX

In this appendix we present various sum rules used in this work (see [32] for more details). Expressions for the spectral sum rules:

$$\int_{-q}^{q} \frac{dq_0}{2\pi} \frac{\beta_l(q_0, q)}{q_0} = \frac{1}{q^2} - \frac{1}{q^2 + 3\omega_p^2} - \frac{z_l(q)}{\omega_l^2(q)}, \tag{A1}$$

$$\int_{-q}^{q} \frac{dq_0}{2\pi} q_0 \beta_l(q_0, q) = \frac{\omega_p^2}{q^2} - z_l(q), \tag{A2}$$

$$\int_{-q}^{q} \frac{dq_0}{2\pi} q_0^3 \beta_l(q_0, q) = \frac{3}{5}\omega_p^2 + \frac{\omega_p^4}{q^2} - z_l(q)\omega_l^2(q), \tag{A3}$$

where

$$z_l(q) = \frac{2\omega_l^2(\omega_l^2 - q^2)}{q^2(3\omega_p^2 + q^2 - \omega_l^2)}, \tag{A4}$$

$$\int_{-q}^{q} \frac{dq_0}{2\pi} \frac{\beta_t(q_0, q)}{q_0} = \frac{1}{q^2} - \frac{z_t(q)}{\omega_t^2(q)}, \tag{A5}$$

$$\int_{-q}^{q} \frac{dq_0}{2\pi} q_0 \beta_t(q_0, q) = 1 - z_t(q), \tag{A6}$$

$$\int_{-q}^{q} \frac{dq_0}{2\pi} q_0^3 \beta_t(q_0, q) = q^2 + \omega_p^2 - z_t(q)\omega_t^2(q), \tag{A7}$$

where,

$$z_t(q) = \frac{2\omega_t^2(\omega_t^2 - q^2)}{3\omega_p^2\omega_t^2 - (\omega_t^2 - q^2)^2}, \tag{A8}$$

$$z_l(q)_{q\to 0} = \frac{\omega_p^2}{q^2}, \tag{A9}$$

$$z_l(q)_{q\to\infty} = 0, \tag{A10}$$

$$z_t(q)_{q\to 0} = 1 - \frac{q^2}{5\omega_p^2}, \tag{A11}$$

$$z_t(q)_{q\to\infty} = 1 + \frac{3\omega_p^2}{4q^2}. \tag{A12}$$

Similarly,

$$\omega_l^2(q)_{q\to 0} = \omega_p^2 + \frac{3}{5}q^2, \tag{A13}$$

$$\omega_l^2(q)_{q\to\infty} = q^2[1 + 4e^{-2/3(q^2)/(3\omega_p^2)-2}], \tag{A14}$$

$$\omega_t^2(q)_{q\to 0} = \omega_p^2 + \frac{6}{5}q^2, \tag{A15}$$

$$\omega_t^2(q)_{q\to\infty} = q^2 + \frac{3}{2}\omega_p^2. \tag{A16}$$

With the help of these limiting values one can readily evaluate following integrals which can be used to evaluate the integrals involving $\rho_{l,t}$ as appear in the expression for the energy loss.

Defining

$$I_{(n)}^{l,t} = \int_{-q}^{+q} \frac{dq_0}{2\pi} \beta_{l,t}(q_0, q) q_0^n. \tag{A17}$$

For $q \gg \omega_p$:
Longitudinal:

$$I_{(-1)}^{l} = \frac{3\omega_p^2}{q^4}, \tag{A18}$$

$$I_{(1)}^{l} = \frac{\omega_p^2}{q^2}, \tag{A19}$$

$$I_{(3)}^{l} = \frac{3}{5}\omega_p^2 + \frac{\omega_p^4}{q^2}. \tag{A20}$$

Transverse:

$$I_{(-1)}^{t} = \frac{3\omega_p^2}{4q^4}, \tag{A21}$$

$$I^t_{(1)} = -\frac{3\omega_p^2}{4q^2}, \tag{A22}$$

$$I^t_{(3)} = -\frac{5}{4}\omega_p^2. \tag{A23}$$

For $q \ll \omega_p$:
Longitudinal:

$$I^l_{(-1)} = \frac{4}{15}\frac{1}{\omega_p^2}, \tag{A24}$$

$$I^l_{(1)} = 0, \tag{A25}$$

$$I^l_{(3)} = 0. \tag{A26}$$

Transverse:

$$I^t_{(-1)} = \frac{1}{q^2} - \frac{1}{\omega_p^2}, \tag{A27}$$

$$I^t_{(1)} = \frac{q^2}{5\omega_p^2}, \tag{A28}$$

$$I^t_{(3)} = 0. \tag{A29}$$

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