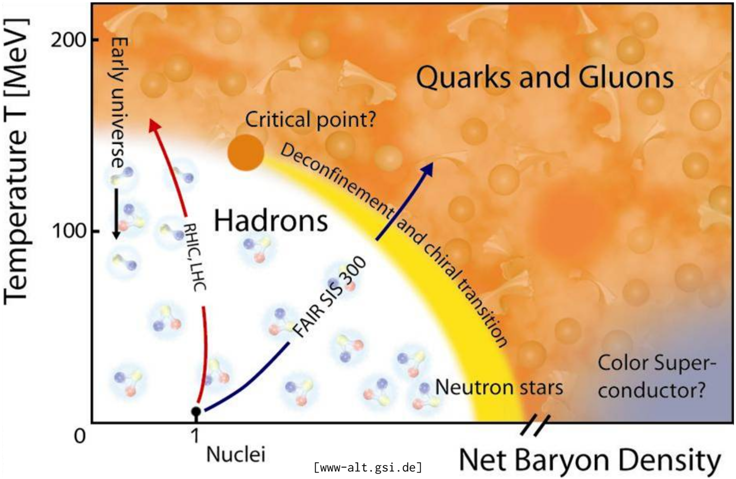
The QCD Phase Diagram

Ralf-Arno Tripolt



[www-alt.gsi.de]

Outline

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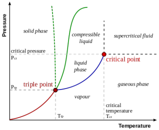
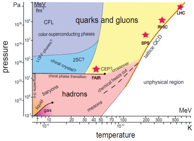
introduction

- phases, phase transitions and thermodynamics



introduction to QCD

- QCD basics and symmetries

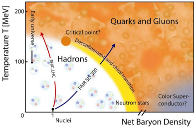


QCD phase diagram from theory

- different approaches and results

QCD phase diagram from experiment

- early universe, neutron stars and heavy ion collisions



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What is a Thermodynamic Phase?

Matter may exist in different phases, throughout which all physical properties, like density, index of refraction, magnetisation and chemical composition, are essentially uniform and homogeneous.

homogeneous or inhomogeneous?





[wikipedia.org]

...it depends on the scale

More on Inhomogeneous (Chiral Symmetry Breaking) Phases

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Michael Buballa

Stefano Carignano

Jamil Arif

Marco Schramm

Daniel Nowakowski

[S. Carignano, D. Nickel, M. Buballa, PRD (2010)]

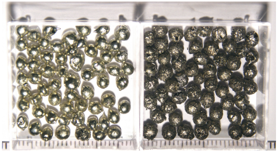
[S. Carignano, M. Buballa, arXiv:1203.5343]

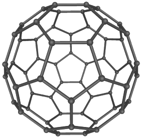
[M. Buballa, S. Carignano, D. Nickel, Phys.Atom.Nucl. 75 (2012)]

[M. Buballa, S. Carignano, arXiv:1210.7155]

Thermodynamic Equilibrium

In thermodynamic equilibrium there are no net flows of matter or of energy, no phase changes, and no unbalanced potentials (or driving forces).
However, a system may not reach this stable equilibrium due to energy barriers - such a quasi-equilibrium is metastable.





[wikipedia.org]

The Phase Diagram of Water

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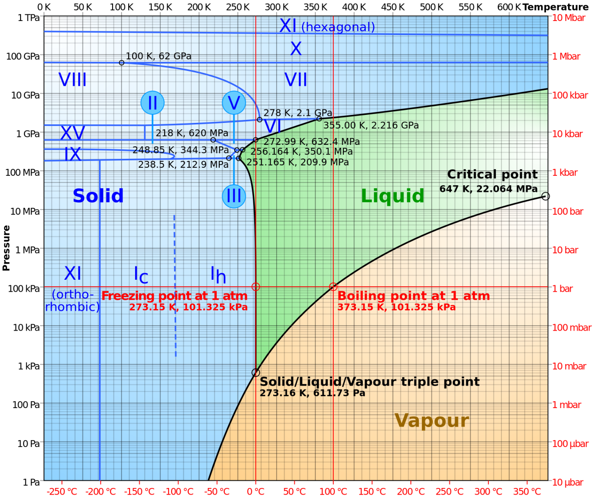

[wikipedia.org/wiki/Phase_diagram]

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Is There a Critical Point?

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THE

ANNALS

OF

PHILOSOPHY.

NEW SERIES.

JANUARY TO JUNE, 1823.

290 *M. de la Tour on the* [APRIL,

ARTICLE IX.

An Account of some Results obtained by the combined Action of Heat and Compression upon certain Fluids, such as Water, Alcohol, Sulphuric Ether, and the rectified Oil of Petroleum. By M. le Baron Cagniard de la Tour.*

The last experiment was made with a glass tube about one-third full of water; this tube lost its transparency, and broke a few seconds afterwards. It appears that at a high temperature water is capable of decomposing glass by combining with its alkali; this suggests the idea that some other result interesting to chemistry may, perhaps, be obtained by increasing the applications of this process of decomposition.

Reminder on Thermodynamics

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thermodynamic potentials:

energy:	$E(S,V,N)$	$dE = T\,dS - P\,dV + \mu\,dN$
free energy:	$F(T,V,N) = E - TS$	$dF = -S\,dT - P\,dV + \mu\,dN$
enthalpy:	$H(S,P,N) = E + PV$	$dH = T\,dS + V\,dP + \mu\,dN$
free enthalpy:	$G(T,P,N) = E - TS + PV$	$dG = -S\,dT + V\,dP + \mu\,dN$

Legendre transformations w.r.t. $-\mu N$ give further potentials, e.g.:

grand potential: $\Omega(T,V,\mu) = E - TS - \mu N$ $\qquad d\Omega = -S\,dT - P\,dV - N\,d\mu$

some important relations following from this:

- $\mu = \left(\partial G/\partial N\right)_{T,P} = G/N$
- $\Omega = -PV$

Equilibrium Conditions (I)

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second law of thermodynamics (Clausius, 1865):

- a closed system with fixed energy E and fixed external parameters (V, N) tends to maximise its entropy S

it follows that G is minimal for fixed T and P in equilibrium:

$$
\begin{aligned}
S(E_A, V_A) &= S_A(E_A, V_A) + S_B(E - E_A, V - V_A) \\
&= S_A + S_B(E, V) - \frac{\partial S_B(E_B, V_B)}{\partial E_B} E_A - \frac{\partial S_B(E_B, V_B)}{\partial V_B} V_A \\
&= \text{const.} + S_A - \frac{E_A}{T} - \frac{PV_A}{T} \\
&= \text{const.} - G_A / T \to \text{maximal}
\end{aligned}
$$

Equilibrium Conditions (II)

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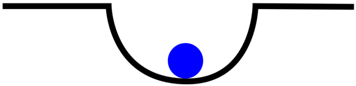
summary of extremal conditions for equilibrium:

	$V = \text{const.}$	$p = \text{const.}$
$S = \text{const.}$	$E \to \text{min.}$	$H \to \text{min.}$
$T = \text{const.}$	$F, \Omega \to \text{min.}$	$G \to \text{min.}$



[wikipedia.org]

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Equilibrium Conditions at Phase Transitions

consider two phases, vapour (A) and water (B):

$$E = E_A + E_B = \text{const.},$$

$$V = V_A + V_B = \text{const.},$$

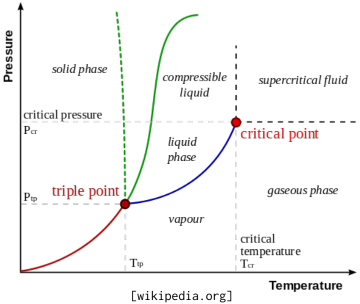
$$N = N_A + N_B = \text{const.},$$

the entropy is maximal in equilibrium,

$$S(E_A, V_A, N_A) = S_A + S_B \to \text{max}.$$

from $\partial S/\partial E_A = 0$, $\partial S/\partial V_A = 0$ and $\partial S/\partial N_A = 0$ it follows that, in equilibrium,

$$T_A = T_B, \quad P_A = P_B, \quad \mu_A = \mu_B$$



[wikipedia.org]

The Clausius-Clapeyron Equation

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consider vapour (A) and water (B) along phase-transition line:

$$\mu_A(T, P(T)) = \mu_B(T, P(T))$$

the total T-derivative gives, with $d\mu = -s\,dT + v\,dP$ (Duhem-Gibbs),

$$\begin{aligned}
\left(\frac{\partial \mu_A}{\partial T}\right)_P + \left(\frac{\partial \mu_A}{\partial P}\right)_T \frac{dP(T)}{dT} &= \left(\frac{\partial \mu_B}{\partial T}\right)_P + \left(\frac{\partial \mu_B}{\partial P}\right)_T \frac{dP(T)}{dT} \\
(v_A - v_B)\frac{dP(T)}{dT} &= s_A - s_B,
\end{aligned}$$

with $s = S/N$ and $v = V/N$; for a fixed pressure P we have $\Delta s = \Delta h/T$, so

$$\frac{dP(T)}{dT} = \frac{\Delta h}{T\Delta v}$$

The Clausius-Clapeyron Equation for QCD

to express the Clausius-Clapeyron equation in terms of μ and T, we use the grand potential $\Omega(\mu, T(\mu))$, where Ω, S and N are now taken to be divided by V:

$$\Omega_A(\mu, T(\mu)) = \Omega_B(\mu, T(\mu)),$$

taking the total μ-derivative and using $d\Omega = -SdT - Nd\mu$ gives

$$\begin{aligned}
\left(\frac{\partial \Omega_A}{\partial \mu}\right)_T + \left(\frac{\partial \Omega_A}{\partial T}\right)_\mu \frac{dT(\mu)}{d\mu} &= \left(\frac{\partial \Omega_B}{\partial \mu}\right)_T + \left(\frac{\partial \Omega_B}{\partial T}\right)_\mu \frac{dT(\mu)}{d\mu} \\
\frac{dT(\mu)}{d\mu}(S_A - S_B) &= -(N_A - N_B) \\
\frac{dT(\mu)}{d\mu} &= -\frac{\Delta N}{\Delta S}
\end{aligned}$$

Classification of Phase Transitions (I)

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order paramter ψ:

is a (macroscopic) quantity that changes characteristically at a phase transition, e.g. M/M_0 for paramagnetic $\leftrightarrow$ ferromagnetic or $(n - n_c)/n_c$ for liquid $\leftrightarrow$ gas; it is zero in one phase (usually above the critical point), and non-zero in the other.

order of phase transitions - modern classification:

- ψ changes discontinuously: first-order
- ψ changes continuously: second-order

order of phase transitions - Ehrenfest classification:

- $\partial F/\partial y$ changes discontinuously: first-order
- $\partial F/\partial y$ is continuous but $\partial^2 F/\partial y^2$ discontinuous: second-order

Classification of Phase Transitions (II)

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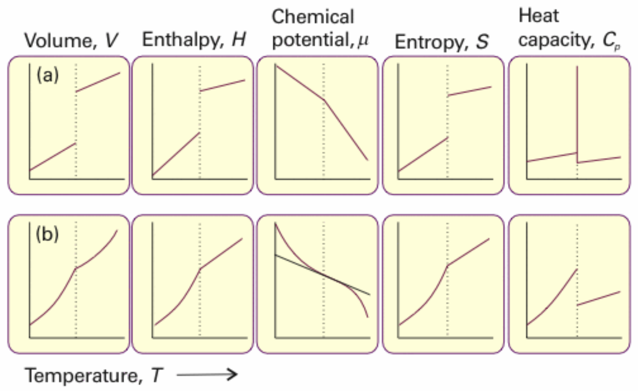
first-order (a) and second-order (b) phase transitions:



[www.chem.queensu.ca]

Classification of Phase Transitions (III) - Critical Exponents and Universality

critical exponents describe the behaviour of physical quantities near continuous (second order) phase transitions and are universal, i.e. they do not depend on the details of the physical system

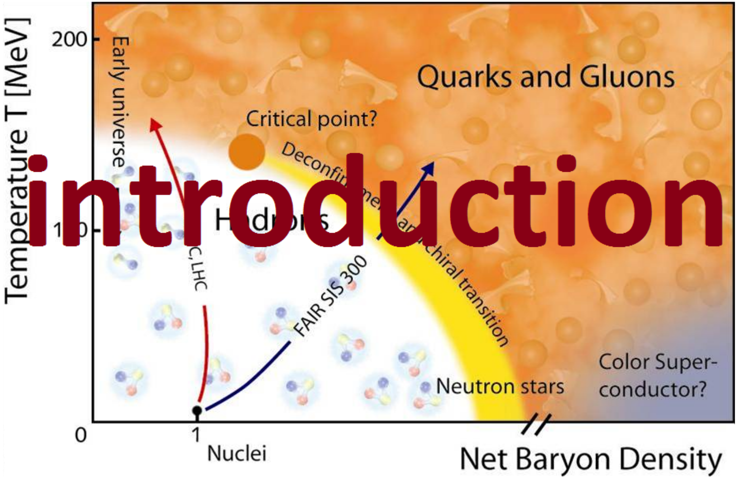
most important critical exponents, $t = (T - T_c)/T_c > 0$:

- specific heat: $C = -T\partial^2 F/\partial T^2 \propto t^{-\alpha}$
- susceptibility: $\chi = \partial^2 F/\partial J^2 \propto t^{-\gamma} \propto t^{-\nu(2+\eta)}$
- correlation length: $\langle \psi(x)\psi(y) \rangle \sim e^{-(x-y)/\xi}$, $\xi \propto t^{-\nu}$

universality classes:

in general, microscopic theories which have the same global symmetries, have equal critical exponents and form broad universality classes, e.g. 3D Ising model: $\eta \approx 0.04$ and $\nu \approx 0.63$

Introduction to QCD



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QCD Lagrangian:

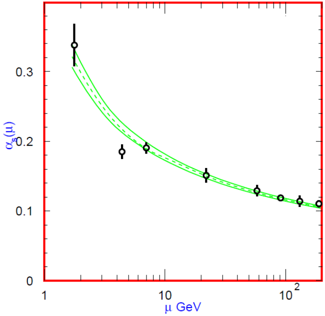
$$\mathcal{L}_{QCD} = -\frac{1}{2} G_{\mu\nu} G^{\mu\nu} + \sum_q \bar{q} \left(i\gamma^\mu D_\mu - m_q \right) q$$

with the covariant derivative $D_\mu = \partial_\mu - igA_\mu$

and the gluon field strength

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu$$

running coupling:



[W.M. Yao, et al., J. Phys. G33, 1 (2006)]

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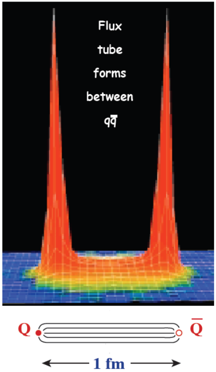
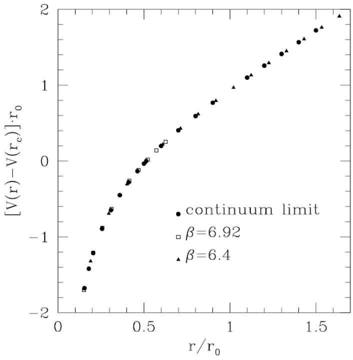
Introduction to QCD - Quark Confinement

lattice QCD shows flux tube and linear potential for static quarks:



[G. Bali et al., PRD 51, 5165 (1995), arXiv:9409005]



[S. Necco and R. Sommer, NPB 622 (2002), arXiv:0306005]

Introduction to QCD - Center Symmetry and Confinement (I)

boundary conditions at finite temperature:

$$q(t + 1/T, \vec{x}) = -q(t, \vec{x}), \qquad A_\mu(t + 1/T, \vec{x}) = A_\mu(t, \vec{x})$$

this constrains the possible gauge transformations $g(t + 1/T, \vec{x}) = h\, g(t, \vec{x})$

gluons respect the Z(SU(3)) center symmetry:

$$A'_\mu(t + 1/T, \vec{x}) = h\, A'_\mu(t, \vec{x})\, h^\dagger \Rightarrow h = z\mathbb{1}, \quad z = \exp(i2\pi n/3), \quad n = 1, 2, 3$$

dynamical quarks break center symmetry:

$$q'(t + 1/T, \vec{x}) = g(t + 1/T, \vec{x})\, q(t + 1/T, \vec{x}) = -z\, g(t, \vec{x})\, q(t, \vec{x}) = -z\, q'(t, \vec{x})$$

Introduction to QCD - Center Symmetry and Confinement (II)

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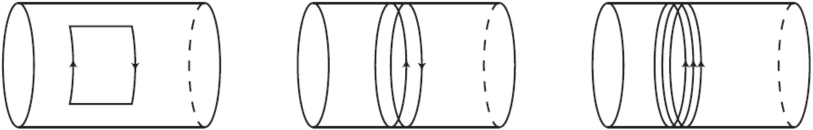
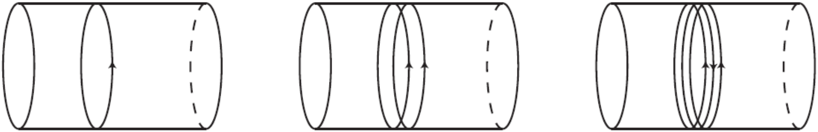
thermal quark excitations respecting center symmetry:



thermal quark excitations breaking center symmetry:



[K. Fukushima, Ann. Phys. 304, 72 (2003), hep-ph/0204302]

Introduction to QCD - Chiral Symmetry

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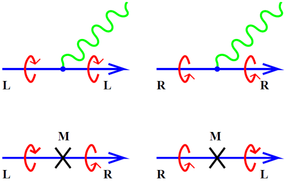
chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ is conserved for massless quarks:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \bar{q}_L (i\not{D}) q_L + \bar{q}_R (i\not{D}) q_R \\
& + \bar{q}_L M q_R + \bar{q}_R M q_L
\end{aligned}
$$

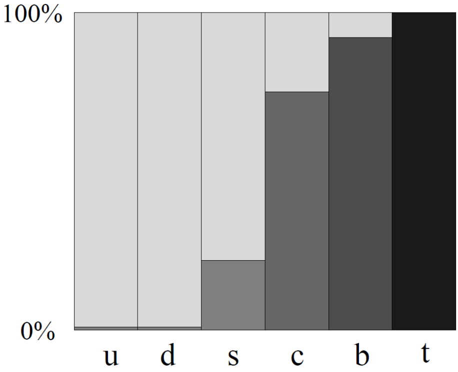


[T. Schaefer, Lecture on The Phases of QCD, NC State U.]

symmetry is spontaneously broken, $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$:

$$\langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \approx -(230\ \text{MeV})^3$$

dynamical mass generation vs. Higgs mass:



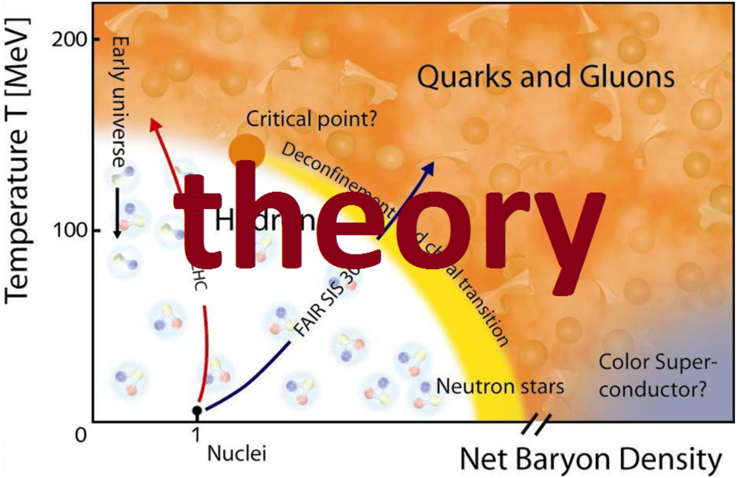
[P. Braun-Munzinger, J. Wambach, arXiv:0801.4256]

Introduction to QCD - Summary of QCD symmetries

symmetry	vacuum	high T	low T, high μ	order parameter	consequences
(local) color $SU(3)$	unbroken	unbroken	broken	diquark condensate	color super-conductivity
$Z(3)$ center symmetry	unbroken	broken	broken	Polyakov loop	confinement/ deconfinement
scale invariance	anomaly			gluon condensate	scale (Λ_{QCD}), running coupling
chiral symmetry $U_L(N_f) \times U_R(N_f) = U_V(1) \times SU_V(N_f) \times SU_A(N_f) \times U_A(1)$					
$U_V(1)$	unbroken	unbroken	unbroken	—	baryon number conservation
flavor $SU_V(N_f)$	unbroken	unbroken	unbroken	—	multiplets
chiral $SU_A(N_f)$	broken	unbroken	broken	quark condensate	Goldstone bosons, no degenerate states with opposite parity
$U_A(1)$	anomaly			topological susceptibility	violation of intrinsic parity

[CBM Physics Book, 2011]

The QCD Phase Diagram from Theory



The QCD Phase Diagram from Theory - Standard-Lore Phase Diagram

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hadron gas:

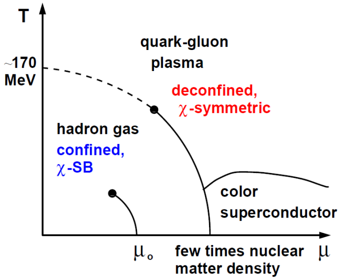
- moderate temperatures and densities
- quarks and gluons are confined
- chiral symmetry is spontaneously broken

quark-gluon plasma:

- very high temperatures and densities
- deconfined quarks and gluons
- chiral symmetry is restored

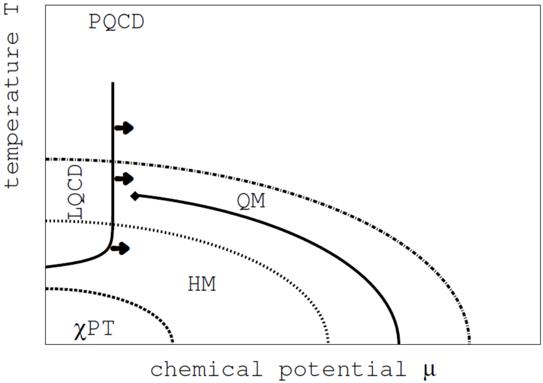
color superconductor:

- $T < 100$ MeV and very high densities
- quarks form bosonic pairs in analogy to BCS theory



[CBM physics book (2011)]

The QCD Phase Diagram from Theory - Overview of Theoretical Approaches



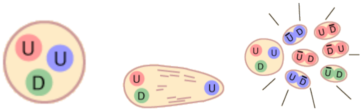
[CBM Physics Book, 2011]

The QCD Phase Diagram from Theory - Models (I) - MIT Bag Model

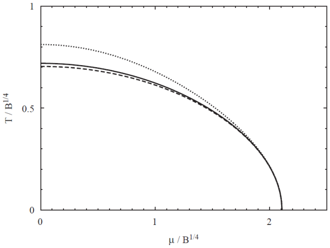
Bag Model (Chodos et al., 1974):

- simple implementation of confinement
- weakly interacting quarks are confined to a speherical cavity with radius $R \sim \Lambda_{\mathrm{QCD}}^{-1} \approx 1$ fm
- cost in energy density for creating the cavity is the bag constant B
- for a free gas of massless quarks and gluons,

$$p_{\mathrm{QGP}} = 37\frac{\pi^2}{90}T^4 + \mu_q^2 T^2 + \frac{\mu_q^4}{2\pi^2} - B$$



[hyperphysics.phy-astr.gsu.edu]



[M. Buballa, Phys. Rept. 407, 205 (2005)]

The QCD Phase Diagram from Theory - Models (II) - NJL and QM Models

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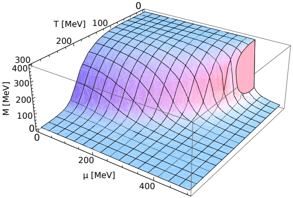
NJL model (Nambu and Jona-Lasinio, 1961):

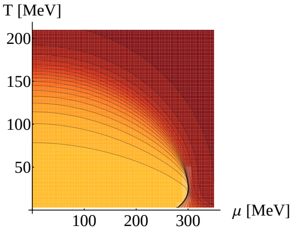
$$\mathcal{L}_{\mathrm{NJL}} = \bar{q}(i\gamma^\mu\partial_\mu - m_q)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$



[courtesy S. Möller]

quark-meson model:

$$\mathcal{L}_{\mathrm{QM}} = \bar{q}(i\gamma^\mu\partial_\mu - h(\sigma + i\gamma_5\vec{\tau}\vec{\pi}))q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi)^2 - U(\sigma,\vec{\pi})$$

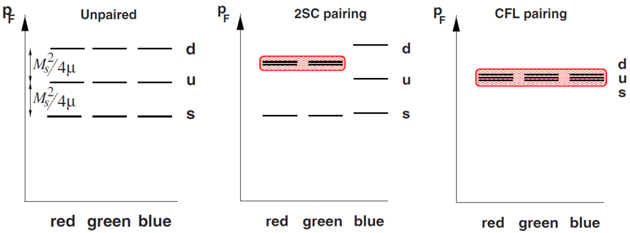


[RAT]

The QCD Phase Diagram from Theory - Models (III) - Color Superconducting Phases

consider quark matter at $T = 0$ and large density n_q:

- quarks near the Fermi surface interact only weakly
- attractive single-gluon exchange channel leads to formation of Cooper pairs
- 2SC: only quarks of two colors (e.g. red and green) participate in pairing
- CFL: all colors and flavors pair with each other



[M.Alford, A.Schmitt, Rev.Mod.Phys.80 (2008)]

The QCD Phase Diagram from Theory - Perturbative QCD and Chiral Perurbation Theory

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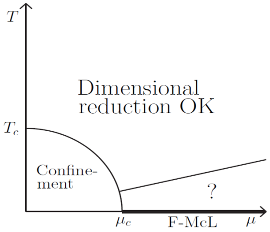
Perturbative QCD:

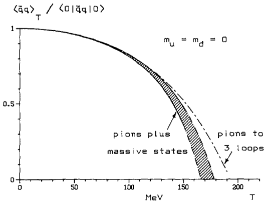
- expansion in α_s for high energies an
- quarks and gluons are relevant degrees of freedom

Chiral Perurbation Theory:

- expansion in terms of energies and (Goldstone boson) masses for low momenta
- stable (light) hadrons are relevant degrees of freedom



[A. Vuorinen, Phys.Rev. D68, 054017 (2003)]



[P. Gerber, H. Leutwyler, Nucl. Phys. B321, 387 (1989)]

The QCD Phase Diagram from Theory - Lattice QCD

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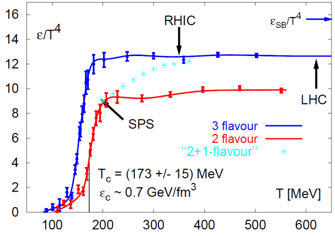
→ **talk by D. Scheffler on *Lattice QCD*, 15.11.2012**

- full QCD is solved from first principles on a discrete Euclidean space-time grid
- predicts a cross-over from hadronic matter to QGP for $\mu_q = 0$ at $T_c \approx 170$ MeV
- confinement and chiral transition seem to appear at exactly the same temperature for $\mu_q = 0$



[F.Karsch, E.Laermann, arXiv:0305025]

problems:

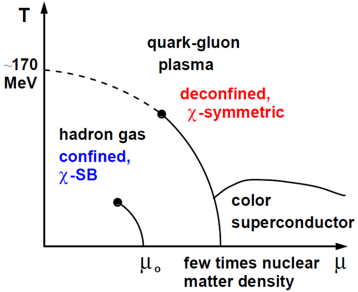
- most lattice calculations still use unrealistically large light-quark masses
- fermion-sign problem at finite μ_q

The QCD Phase Diagram from Theory - Summary

→ talk by S. Finkbeiner on *Fluctuations and the QCD phase boundary - theory*, 6.12.2012

what to expect:

- bag model predicts that T_c decreases with increasing μ_q
- models (e.g. NJL, QM) predict first-order transition for low T and large μ_q
- lattice results indicate cross over at small μ_q
- together, this implies the existence of a critical endpoint



[CBM physics book (2011)]

The QCD Phase Diagram from Theory - Summary

what to expect:

- bag model predicts that T_c decreases with increasing μ_q
- models (e.g. NJL, QM) predict first-order transition for low T and large μ_q
- lattice results indicate cross over at small μ_q
- together, this implies the existence of a critical endpoint

what we really know:

[M.Nahrgang, M.Bleicher, arXiv:1111.3771]

The QCD Phase Diagram from Experiment

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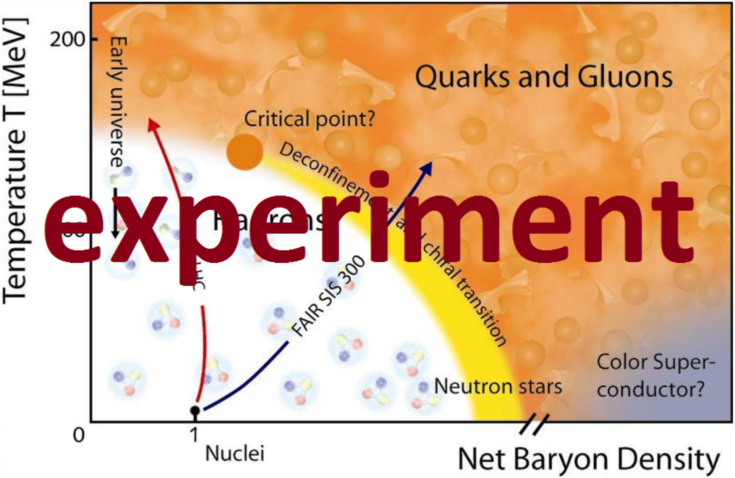
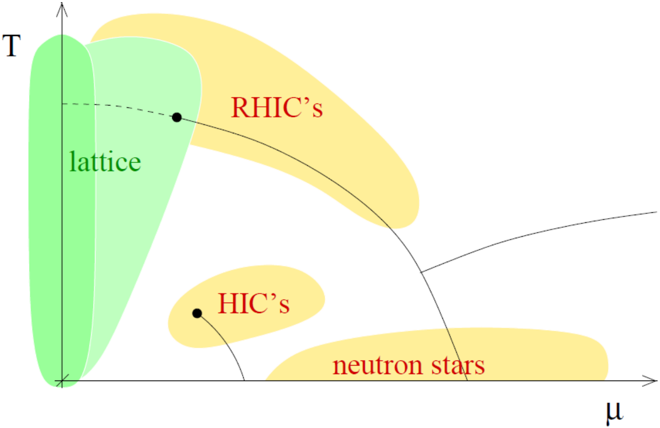


The QCD Phase Diagram from Experiment - Overview



[T. Schaefer, Lecture on The Phases of QCD, NC State U.]

The QCD Phase Diagram from Experiment - Early Universe

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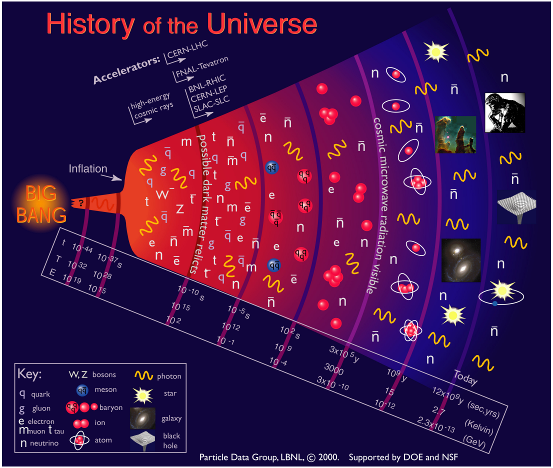

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The QCD Phase Diagram from Experiment - Neutron Stars

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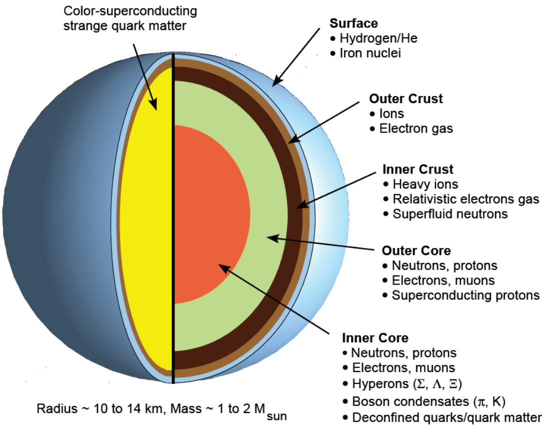
- density around five times nuclear saturation density
- temperature reaches a few *keV*
- possibility to study strongly interacting matter under extreme conditions not available in the laboratory



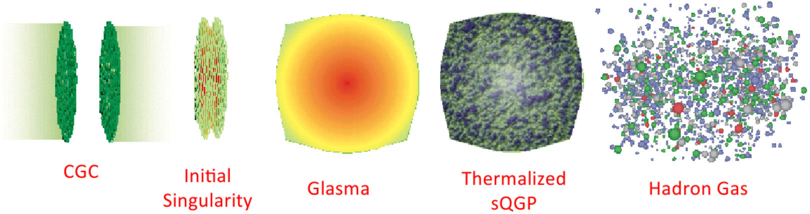
[CBM physics book (2011)]

The QCD Phase Diagram from Experiment - Heavy Ion Collisions (I)

$\rightarrow$ **talk by D. Nowakowski on *Hydrodynamics - theory*, 22.11.2012**

stages of a heavy-ion collision:

- color glass condensate: dense, weakly-interacting, contracted gluon-walls
- glasma: immediately after the collision, chromo E and B fields have only longitudinal components forming flux tubes along the collision axis
- sQGP: strongly coupled QGP, not a perturbatively describable plasma
- chemical freeze-out: inelastic collisions stop, composition of hadrons is fixed
- thermal freeze-out: interaction rates insufficient for thermal equilibrium



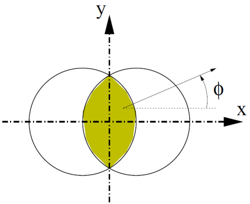
[L. McLerran, HPT Workshop (2012)]

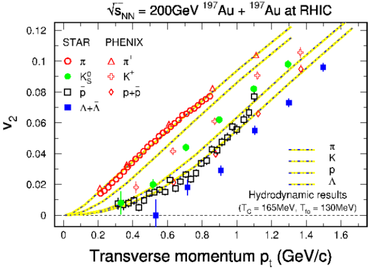
The QCD Phase Diagram from Experiment - Heavy Ion Collisions (II)

→ **talk by NN on *Collective flow at RHIC and LHC - experimental data*, 29.11.2012**

evidence for quark gluon plasma:

- the Fourier coefficient $v_2 = \langle\cos(2\Phi)\rangle$ of the distribution of the momenta of produced particles can be compared to predictions from hydrodynamical calculations
- close agreement implies that the quark-gluon matter produced at RHIC flows collectively like a liquid with negligible η/s, i.e. is a strongly coupled QGP





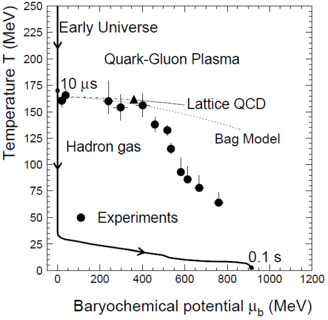
[P. Braun-Munzinger, J. Wambach, arXiv:0801.4256]
[J. Adams, et al. (STAR), Phys. Rev. C72, 014904 (2005)]
[P. Huovinen et al., Phys. Lett. B503, 58 (2001)]

The QCD Phase Diagram from Experiment - Heavy Ion Collisions (III)

→ **talk by G. Dentinger on *Thermal fits of hadron yields and the QCD phase diagram*, 8.11.2012**

evidence for a phase transition:

- information about the location of the freeze-out point is obtained by measuring the ratios of particle yields, and fitting to a statistical model with T and μ_B as parameters
- at least for small μ_B the extracted temperature is closely linked to the (calculated) phase boundary



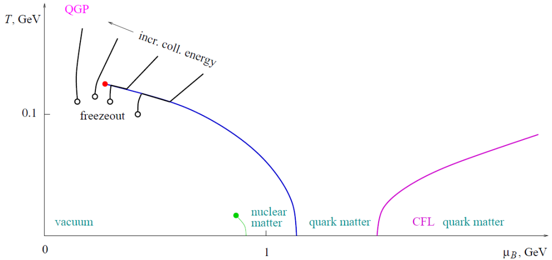
[P. Braun-Munzinger, J. Wambach, arXiv:0801.4256]

The QCD Phase Diagram from Experiment - Heavy Ion Collisions (IV)

→ **talk by P. Piasecki on *Search for the QCD phase boundary with experiments*, 13.12.2012**

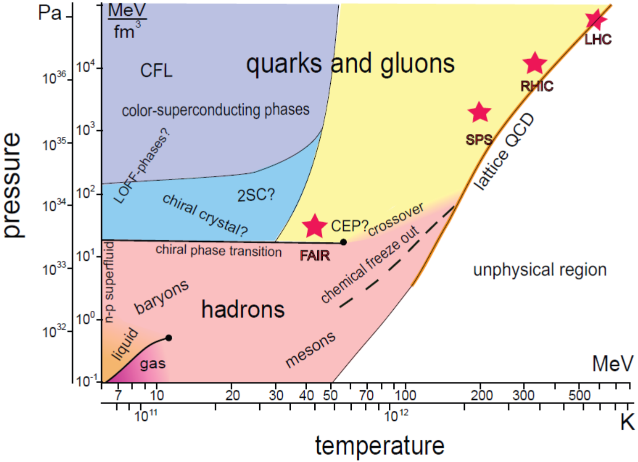
approaching the critical endpoint:

- in the approximation of ideal hydrodynamics, entropy is conserved and the expanding fireball follows a trajectory of constant baryon per entropy ratio n/s
- T increases at phase transition since latent heat is released and the lower number of dof's in the hadronic phase would result in a smaller entropy



[M. Stephanov, arXiv:0402115 (2006)], [M. Stephanov, K. Rajagopal and E. Shuryak, arXiv:9806219]

Summary -
Contemporary View of the QCD Phase Diagram



[J. Wambach, K. Heckmann and M. Buballa, arXiv: 1111.5475]

Appendix

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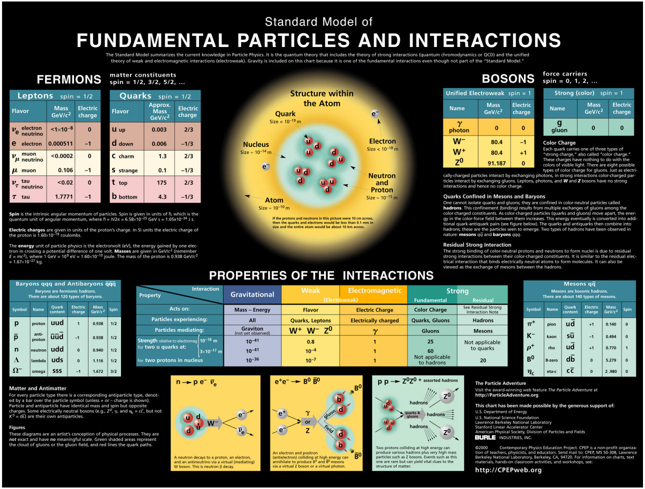

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