A COMPARISON OF DENOISING METHODS FOR ONE DIMENSIONAL TIME SERIES

FINAL PROJECT REPORT FOR IEOR4732 COMPUTATIONAL METHODS IN FINANCE

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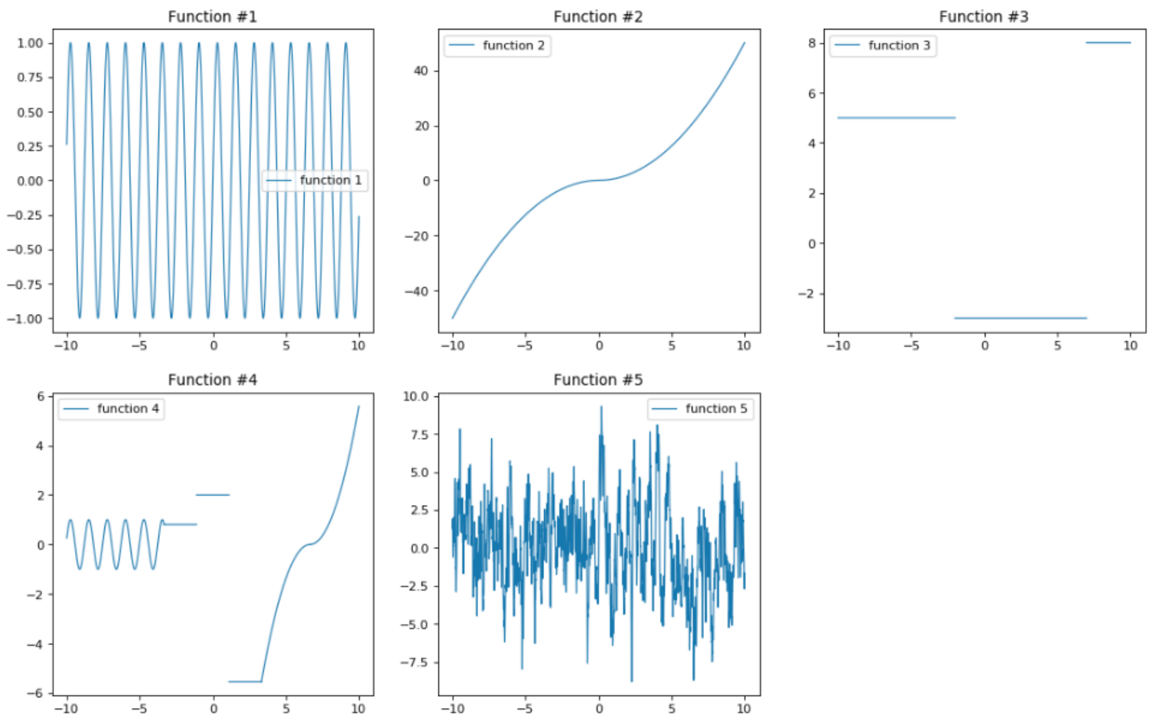
INTRODUCTION

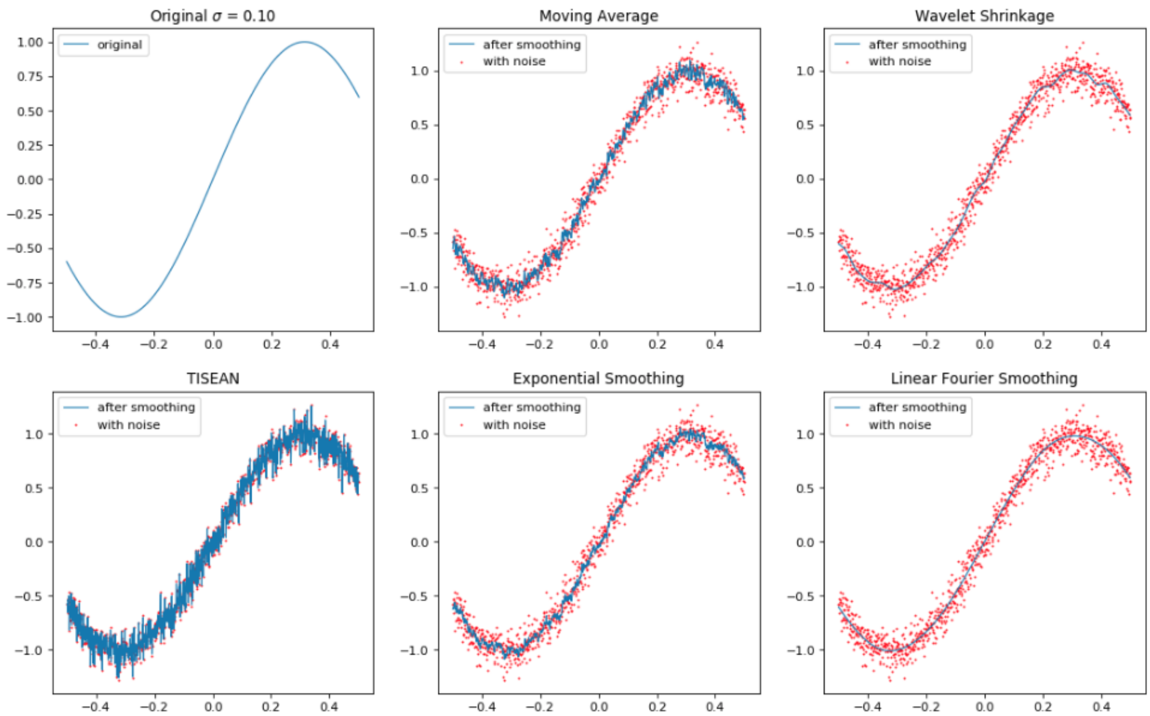
This report replicates the paper written by Kohler and Dirk in 2005 regarding to the analysis on various denoising methods on one dimensional time series. From the paper, we present 5 different functions and utilize them to demonstrate our smoothing techniques. Here is a sample output of the 5 different functions:



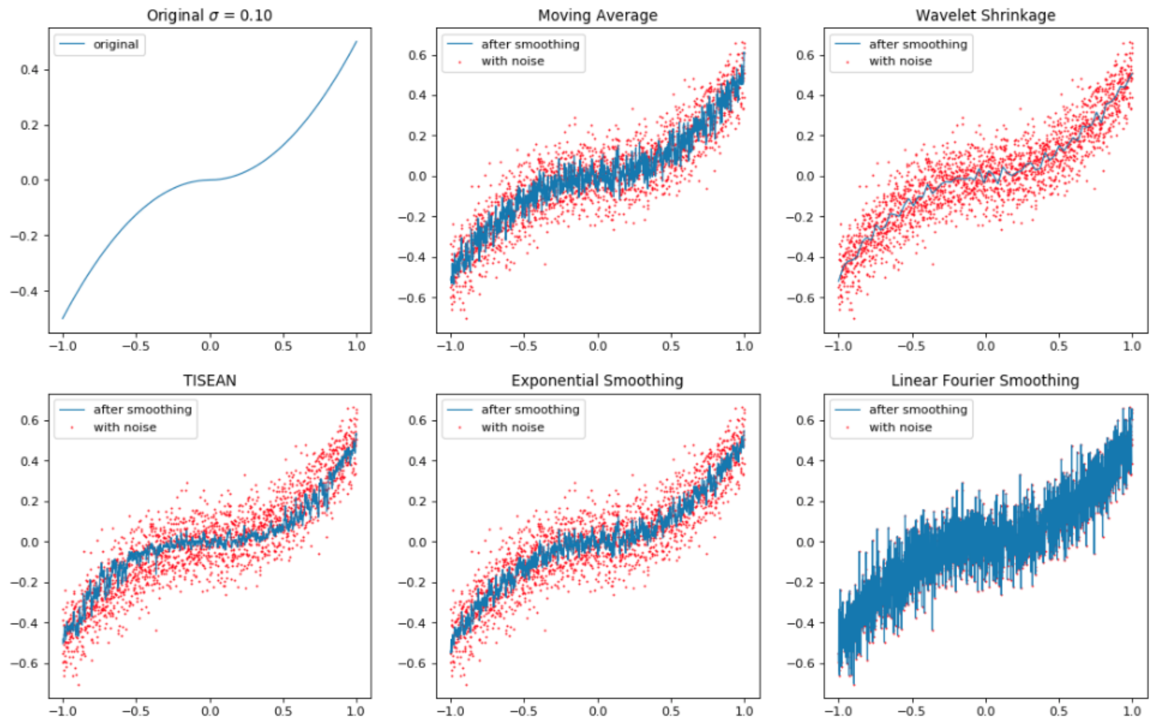
SMOOTHING TECHNIQUES

Similar to the procedure explained in the paper, we've generated 20,001 noisy data points for 5 different functions by adding the noise of standard normal with $\sigma = 0.01$, 0.05 and 0.10. We've implemented 5 different smoothing methods for each function, which includes moving average filter, exponential smoothing filter, linear Fourier smoothing, nonlinear wavelet shrinkage and simple nonlinear noise reduction. Here is a sample plot of the smoothing results for those 5 functions when $\sigma = 0.10$:

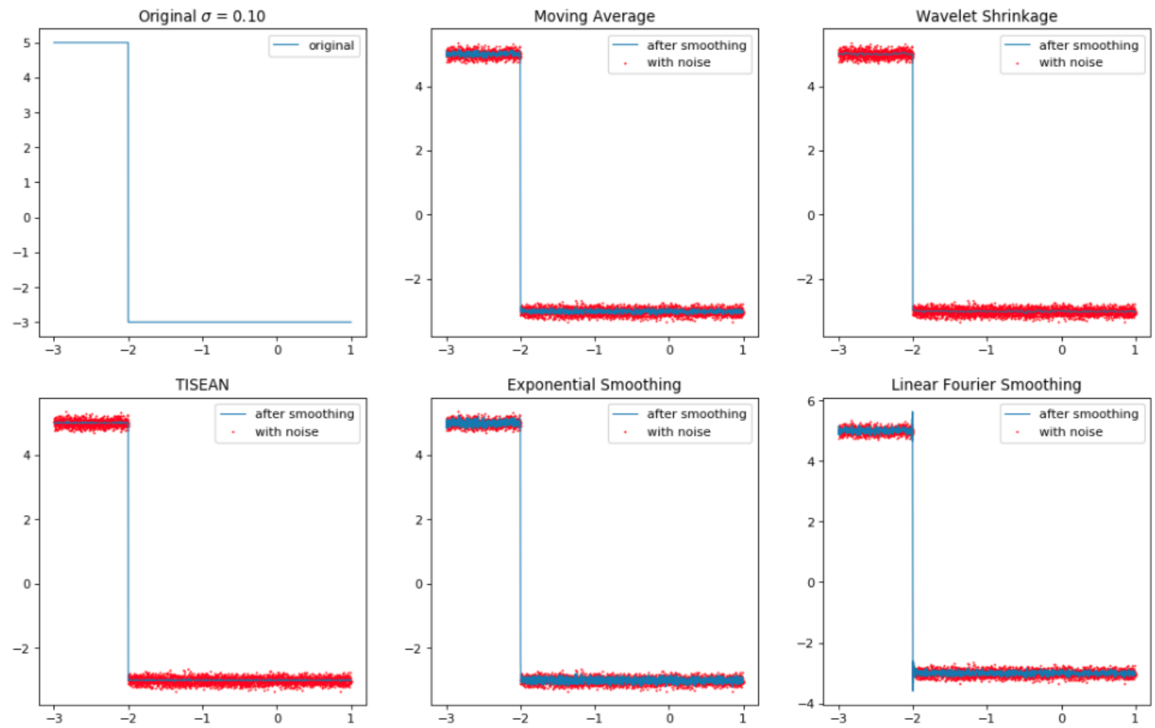
1. The simple sinusoid:



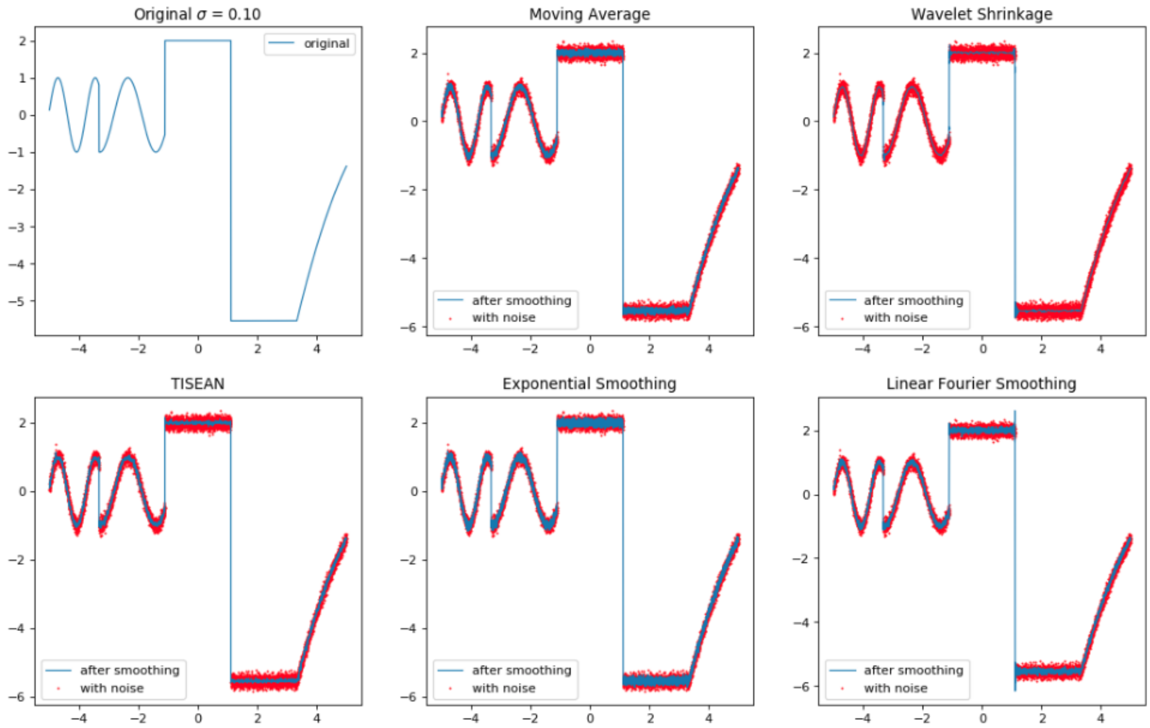
2. The piecewise quadratic function:



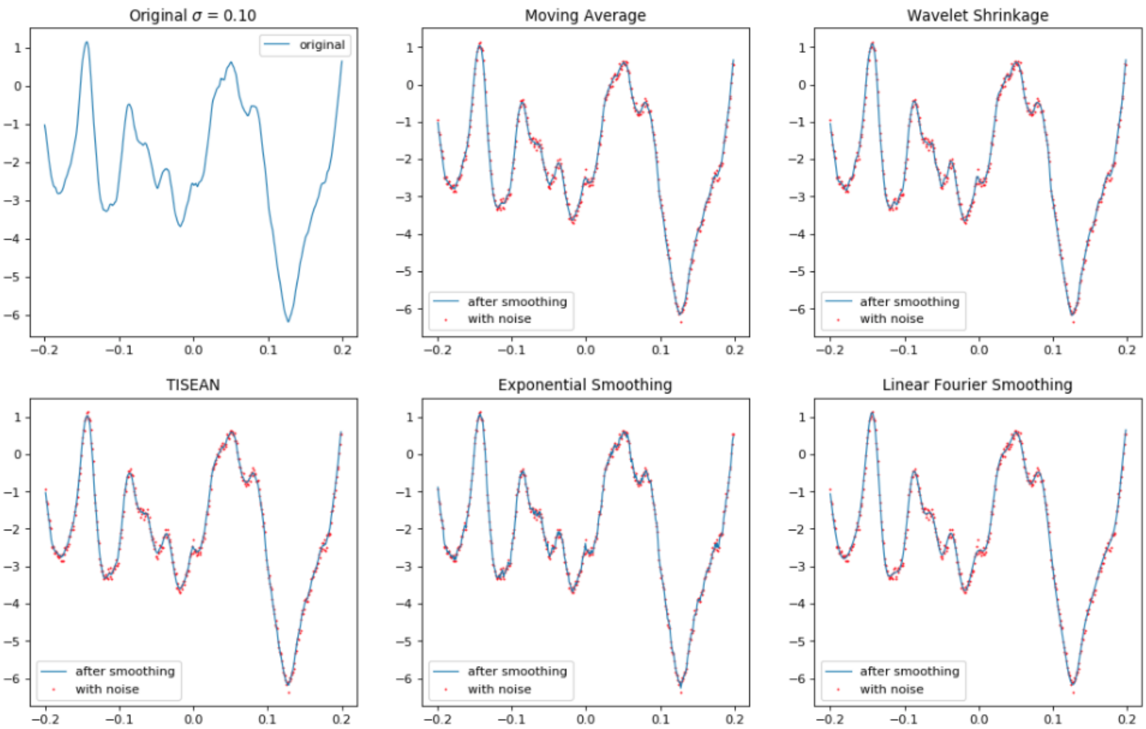
3. The piecewise constant function:



4. The mixture of the first three functions:



5. The time series:



Error Analysis

Then, we could analyze the smoothing results from various error metrics, which includes the L^1 norm, the L^2 norm, the L^∞ norm and the symmetrical visual error measure. We could also generate the similar error metrics as stated in the paper. However, since we don't have enough details from the implementation of the wavelet shrinkage method in the paper, we could only generate close results in wavelet shrinkage analysis, which is not exactly the same as the paper. Generally, we could achieve very similar evaluation outputs:

1. The simple sinusoid:

	L^1	L^2	L^∞	SE_2
Noisy data	0.16 ± 0.002	0.045 ± 0.001	0.042 ± 0.016	0.024 ± 0.0
MA filter	0.071 ± 0.002	0.02 ± 0.001	**0.018 ± 0.003**	0.014 ± 0.0
Wavelet filter	**0.029 ± 0.003**	**0.008 ± 0.001**	0.023 ± 0.007	**0.008 ± 0.0**
TISEAN	0.06 ± 0.003	0.017 ± 0.001	0.019 ± 0.005	0.012 ± 0.0
Exp. smoothing	0.108 ± 0.002	0.03 ± 0.001	0.027 ± 0.005	0.017 ± 0.0
FFT filter	0.058 ± 0.003	0.022 ± 0.001	0.231 ± 0.005	0.018 ± 0.0

$\sigma = 0.01$

	L^1	L^2	L^∞	SE_2
Noisy data	0.798 ± 0.01	0.223 ± 0.003	0.21 ± 0.044	0.102 ± 0.002
MA filter	0.357 ± 0.01	0.1 ± 0.003	0.093 ± 0.017	0.049 ± 0.002
Wavelet filter	0.14 ± 0.01	**0.04 ± 0.003**	**0.039 ± 0.02**	**0.025 ± 0.003**
TISEAN	0.248 ± 0.01	0.075 ± 0.003	0.094 ± 0.02	0.027 ± 0.001
Exp. smoothing	0.361 ± 0.012	0.101 ± 0.004	0.093 ± 0.02	0.048 ± 0.002
FFT filter	**0.125 ± 0.012**	0.049 ± 0.002	0.26 ± 0.013	0.036 ± 0.001

$\sigma = 0.05$

	L^1	L^2	L^∞	SE_2
Noisy data	1.596 ± 0.023	0.447 ± 0.005	0.42 ± 0.114	0.202 ± 0.005
MA filter	0.714 ± 0.021	0.2 ± 0.006	0.186 ± 0.034	0.093 ± 0.004
Wavelet filter	0.28 ± 0.024	0.079 ± 0.007	**0.067 ± 0.035**	**0.046 ± 0.007**
TISEAN	1.443 ± 0.027	0.412 ± 0.007	0.419 ± 0.115	0.184 ± 0.006
Exp. smoothing	0.578 ± 0.021	0.162 ± 0.005	0.147 ± 0.037	0.073 ± 0.003
FFT filter	**0.183 ± 0.029**	**0.069 ± 0.006**	0.265 ± 0.02	0.047 ± 0.005

$\sigma = 0.05$

2. The piecewise quadratic function:

	L^1	L^2	L^∞	SE_2
Noisy data	0.16 ± 0.003	0.045 ± 0.001	0.042 ± 0.008	0.025 ± 0.0
MA filter	0.071 ± 0.001	0.02 ± 0.0	**0.019 ± 0.003**	0.015 ± 0.0
Wavelet filter	**0.028 ± 0.002**	**0.008 ± 0.0**	0.034 ± 0.008	**0.009 ± 0.001**
TISEAN	0.095 ± 0.004	0.029 ± 0.001	0.042 ± 0.009	0.026 ± 0.002
Exp. smoothing	0.122 ± 0.002	0.034 ± 0.0	0.031 ± 0.007	0.02 ± 0.0
FFT filter	0.16 ± 0.003	0.045 ± 0.001	0.042 ± 0.008	0.025 ± 0.0

$\sigma = 0.01$

	L^1	L^2	L^∞	SE_2
Noisy data	0.798 ± 0.009	0.223 ± 0.003	0.209 ± 0.074	0.091 ± 0.002
MA filter	0.357 ± 0.008	0.1 ± 0.002	0.093 ± 0.017	0.046 ± 0.002
Wavelet filter	**0.14 ± 0.012**	**0.04 ± 0.003**	**0.051 ± 0.031**	**0.024 ± 0.002**
TISEAN	0.346 ± 0.016	0.098 ± 0.004	0.103 ± 0.076	0.045 ± 0.003
Exp. smoothing	0.4 ± 0.011	0.111 ± 0.003	0.1 ± 0.024	0.045 ± 0.002
FFT filter	0.798 ± 0.009	0.223 ± 0.003	0.209 ± 0.074	0.091 ± 0.002

$\sigma = 0.05$

	L^1	L^2	L^∞	SE_2
Noisy data	1.595 ± 0.02	0.447 ± 0.006	0.418 ± 0.062	0.176 ± 0.005
MA filter	0.714 ± 0.016	0.2 ± 0.004	0.185 ± 0.051	0.083 ± 0.003
Wavelet filter	**0.281 ± 0.021**	**0.08 ± 0.006**	**0.078 ± 0.034**	**0.041 ± 0.004**
TISEAN	0.637 ± 0.029	0.18 ± 0.009	0.182 ± 0.053	0.082 ± 0.01
Exp. smoothing	0.653 ± 0.023	0.182 ± 0.007	0.166 ± 0.18	0.068 ± 0.003
FFT filter	1.595 ± 0.02	0.447 ± 0.006	0.418 ± 0.062	0.176 ± 0.005

$\sigma = 0.05$

3. The piecewise constant function:

	L^1	L^2	L^∞	SE_2
Noisy data	0.16 ± 0.002	0.045 ± 0.001	0.041 ± 0.011	0.046 ± 0.001
MA filter	0.094 ± 0.002	0.273 ± 0.0	4.403 ± 0.006	0.273 ± 0.0
Wavelet filter	0.028 ± 0.003	0.008 ± 0.001	0.063 ± 0.021	0.011 ± 0.001
TISEAN	**0.002 ± 0.003**	**0.001 ± 0.001**	**0.0 ± 0.0**	**0.001 ± 0.001**
Exp. smoothing	0.16 ± 0.002	0.045 ± 0.001	0.041 ± 0.011	0.046 ± 0.001
FFT filter	0.16 ± 0.002	0.045 ± 0.001	0.041 ± 0.011	0.046 ± 0.001

$\sigma = 0.01$

	L^1	L^2	L^∞	SE_2
Noisy data	0.798 ± 0.012	0.224 ± 0.003	0.21 ± 0.031	0.225 ± 0.003
MA filter	0.38 ± 0.011	0.29 ± 0.002	4.417 ± 0.035	0.291 ± 0.002
Wavelet filter	0.143 ± 0.014	0.042 ± 0.004	0.321 ± 0.148	0.053 ± 0.005
TISEAN	**0.01 ± 0.011**	**0.004 ± 0.002**	**0.065 ± 0.053**	**0.005 ± 0.003**
Exp. smoothing	0.657 ± 0.009	0.203 ± 0.002	2.188 ± 0.084	0.204 ± 0.002
FFT filter	0.797 ± 0.012	0.224 ± 0.003	0.22 ± 0.051	0.225 ± 0.003

$\sigma = 0.05$

	L^1	L^2	L^∞	SE_2
Noisy data	1.595 ± 0.02	0.447 ± 0.005	0.419 ± 0.072	0.449 ± 0.005
MA filter	0.735 ± 0.021	0.337 ± 0.003	4.433 ± 0.06	0.339 ± 0.003
Wavelet filter	0.284 ± 0.022	0.084 ± 0.007	0.611 ± 0.243	0.099 ± 0.008
TISEAN	**0.034 ± 0.018**	**0.017 ± 0.003**	**0.103 ± 0.093**	**0.018 ± 0.004**
Exp. smoothing	1.055 ± 0.018	0.347 ± 0.005	4.379 ± 0.159	0.349 ± 0.005
FFT filter	0.966 ± 0.022	0.346 ± 0.004	3.677 ± 0.1	0.348 ± 0.004

$\sigma = 0.05$

4. The mixture of the first three functions:

	L^1	L^2	L^∞	SE_2
Noisy data	0.159 ± 0.003	0.045 ± 0.001	**0.042 ± 0.009**	0.033 ± 0.001
MA filter	0.085 ± 0.002	0.164 ± 0.0	3.021 ± 0.008	0.146 ± 0.0
Wavelet filter	**0.032 ± 0.002**	**0.013 ± 0.001**	0.092 ± 0.022	**0.013 ± 0.001**
TISEAN	0.059 ± 0.002	0.021 ± 0.001	0.049 ± 0.008	0.014 ± 0.001
Exp. smoothing	0.159 ± 0.003	0.045 ± 0.001	**0.042 ± 0.009**	0.033 ± 0.001
FFT filter	0.159 ± 0.003	0.045 ± 0.001	**0.042 ± 0.009**	0.033 ± 0.001

$\sigma = 0.01$

	L^1	L^2	L^∞	SE_2
Noisy data	0.799 ± 0.013	0.224 ± 0.004	0.21 ± 0.067	0.153 ± 0.003
MA filter	0.371 ± 0.01	0.191 ± 0.002	3.037 ± 0.062	0.161 ± 0.001
Wavelet filter	**0.157 ± 0.011**	**0.061 ± 0.003**	0.419 ± 0.089	**0.057 ± 0.003**
TISEAN	0.273 ± 0.013	0.088 ± 0.005	**0.128 ± 0.031**	**0.057 ± 0.006**
Exp. smoothing	0.591 ± 0.009	0.183 ± 0.003	2.266 ± 0.107	0.138 ± 0.003
FFT filter	0.587 ± 0.012	0.2 ± 0.002	1.913 ± 0.107	0.153 ± 0.001

$\sigma = 0.05$

	L^1	L^2	L^∞	SE_2
Noisy data	1.594 ± 0.027	0.447 ± 0.007	0.419 ± 0.077	0.302 ± 0.005
MA filter	0.727 ± 0.015	0.258 ± 0.004	3.055 ± 0.057	0.199 ± 0.003
Wavelet filter	**0.308 ± 0.023**	**0.116 ± 0.007**	0.791 ± 0.178	0.104 ± 0.007
TISEAN	0.519 ± 0.019	0.16 ± 0.006	**0.216 ± 0.08**	**0.092 ± 0.004**
Exp. smoothing	0.932 ± 0.012	0.298 ± 0.005	3.772 ± 0.129	0.209 ± 0.004
FFT filter	0.79 ± 0.021	0.293 ± 0.003	2.98 ± 0.069	0.22 ± 0.004

$\sigma = 0.05$

5. The time series:

	L^1	L^2	L^∞	SE_2
Noisy data	0.159 ± 0.002	0.045 ± 0.001	**0.041 ± 0.007**	0.054 ± 0.001
MA filter	0.354 ± 0.002	0.099 ± 0.0	0.088 ± 0.008	0.1 ± 0.0
Wavelet filter	0.332 ± 0.001	0.094 ± 0.0	0.096 ± 0.009	0.095 ± 0.0
TISEAN	0.217 ± 0.003	0.061 ± 0.001	0.058 ± 0.01	0.07 ± 0.001
Exp. smoothing	0.159 ± 0.002	0.045 ± 0.001	**0.041 ± 0.007**	0.054 ± 0.001
FFT filter	**0.149 ± 0.001**	**0.042 ± 0.0**	0.043 ± 0.012	**0.051 ± 0.001**

$\sigma = 0.01$

	L^1	L^2	L^∞	SE_2
Noisy data	0.798 ± 0.011	0.224 ± 0.004	0.207 ± 0.059	0.149 ± 0.002
MA filter	0.498 ± 0.007	0.14 ± 0.002	**0.131 ± 0.022**	**0.12 ± 0.002**
Wavelet filter	0.515 ± 0.007	0.144 ± 0.002	0.137 ± 0.033	0.121 ± 0.002
TISEAN	**0.494 ± 0.009**	**0.139 ± 0.003**	0.154 ± 0.061	0.121 ± 0.002
Exp. smoothing	0.756 ± 0.011	0.212 ± 0.003	0.195 ± 0.04	0.155 ± 0.003
FFT filter	0.57 ± 0.006	0.16 ± 0.001	0.144 ± 0.022	0.128 ± 0.001

$\sigma = 0.05$

	L^1	L^2	L^∞	SE_2
Noisy data	1.595 ± 0.022	0.447 ± 0.007	0.417 ± 0.08	0.182 ± 0.003
MA filter	0.795 ± 0.017	0.223 ± 0.005	0.209 ± 0.043	0.145 ± 0.003
Wavelet filter	0.861 ± 0.015	0.242 ± 0.004	0.23 ± 0.042	0.147 ± 0.002
TISEAN	0.806 ± 0.023	0.227 ± 0.006	0.239 ± 0.128	0.148 ± 0.003
Exp. smoothing	1.451 ± 0.022	0.407 ± 0.006	0.373 ± 0.07	0.168 ± 0.002
FFT filter	**0.781 ± 0.016**	**0.219 ± 0.004**	**0.198 ± 0.047**	**0.143 ± 0.002**

$\sigma = 0.05$

SUMMARY

There are some thoughts after we have generated all the error metrics from the smoothing methods. For the sinusoid function, wavelet and FFT filter tend to perform better. For the piecewise quadratic function, wavelet still performs much more stable than the other four methods. TISEAN filter works the best with the piecewise constant function while wavelet filter performs similarly stable with it in the mixture function. Last but not least, FFT filter seems to work the best in time series smoothing. There is one more thing to notice that moving average and exponential smoothing filter do not often perform the best but they are rather stable compared to the other techniques.

REFERENCES

Kohler, T., Lorenz, D., 2005. *A comparison of denoising methods for one dimensional time series*, Available at:<http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.565.1807&rep=rep1&type=pdf>.

Stephanie M., 1999., Academic Press, San Diego, CA, USA, *A Wavelet Tour of Signal Processing*, Available at: <https://www.di.ens.fr/~mallat/papiers/WaveletTourChap1-2-3.pdf>.

Michel, M., Yves, M., Georges O., Jean-Michel, P., 2004, The MathWorks, Inc., third edition, *Wavelet Toolbox User's Guide.*

H. Kantz, R. A. Schreiber, 1997, Cambridge University Press, *Nonlinear Time Series Analysis.*

Plotting and manipulating FFTs for filtering, Available at:<https://www.scipy-lectures.org/intro/scipy/auto_examples/plot_fftpack.html>.

Holger, N., 2010-2015, *pyyawt, Yet another python wavelet toolbox*, Available at: <https://github.com/holgern/pyyawt>.

Thomas, S., 1999, Simple nonlinear noise reduction, *Description of the Program: nrlazy*, Available at: <https://www.pks.mpg.de/~tisean/TISEAN_2.1/docs/docs_c/nrlazy.html>.