On Applications of Filtering on Financial Markets

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***Abstract*— In this study, we implemented and applied multiple filtering techniques on physical market data and obtained the parameters for the underlying stochastic process through parameter estimation. Specifically, we implemented Extended/Unscented Kalman Filters (EKF, UKF) and Particle Filters (PF), also known as Sequential Monte Carlo, for Heston and Variance Gamma Stochastic Arrival (VGSA). Filtering is first applied on synthetic data of the underlying process and then applied for a case study. For the case study, we filtered 5-year Crude Oil Prices and the stock price of China Petroleum & Chemical Corporation (SNP). We also applied UKF to filter daily vol. for AAPL, AMZN, MSFT, FB based on their 5-year historical prices. The implementations can be found here: `https://github.com/ml4046/cfinance-filtering` and follows the derivations and algorithms in [1].**

I. Introduction

In order to understand the movements of the financial market, we first assume the market moves according to a stochastic process such as Heston (Geometric Brownian Motion with stochastic volatility) or Variance Gamma Stochastic Arrival (VGSA). One key difference between the two is that the former is a diffusion process whereas the latter has a 'jump' behavior, achieved through evaluating on random time. Under such processes, many applications require simulation of sample paths from a set of physical world statistical measures. Unlike pricing model calibration that is performed on the risk-neutral market (Q), filtering is applied on time-series data of the physical market (P) while we obtain the optimal set of parameters for the process on a given market in parameter estimation through maximizing likelihood of the observations given the (hidden) states of our process.

This study covers how we can implement Kalman Filters for non-linear transitions (EKF, UKF) and Particle Filter (PF) on Heston and VGSA, providing intuition on how to model such filtering problem on the market and technical details on the implementation of the filters.

The report is organized as follow: sec. II discusses the basics of the stochastic processes, filtering techniques, and what is uncovered in filtering, sec. III provides the algorithms and implementation specifics of the filtering techniques, sec. IV shows the results of filtering done of simulated data and our case study on oil prices, and sec. V concludes the report.

II. Background

Assuming the market moves under a stochastic process, the goal of filtering is obtaining the hidden states from a time series of observables (market prices). For example, observing the market prices we can uncover daily volatility, the hidden state, via. Heston process. Similarly, we can apply filtering for a VGSA process and model the hidden state as the arrival rate. Furthermore, we can find the set of parameters for a process that describes the market through parameter estimation, which can be achieved through maximizing the likelihood. We will give a brief introduction to the two process in sec. II-A and the filtering techniques in sec. II-B.

A. Stochastic Process

Heston Stochastic Volatility Process The Heston process extends a Geometric Brownian Motion by introducing stochastic volatility given as the following SDE:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^1$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2$$

with the parameter set $\{\mu, \kappa, \theta, \sigma, \rho, v_0\}$ where ρ is the correlation between W_t^1 and W_t^2 and v_0 is the initial volatility.

Variance Gamma Stochastic Arrival Unlike Heston which is a diffusion process, the VGSA is an extension of a stochastic time change model, adopted from Variance Gamma process, that introduces mean-reverting stochastic time changes by evaluating continuous time changes given by Cox-Ingersoll-Ross (CIR) process. VGSA and the CIR process y(t) (instantaneous time change) are defined as:

$$S(t) = \theta\gamma(Y(t);1,\nu) + \sigma W(\gamma(Y(t);1,\nu))$$

$$dy_t = \kappa(\eta - y_t)dt + \lambda\sqrt{y_t}dW_t$$

$$Y(t) = \int_0^t y(u)du$$

B. Filtering

We will briefly discuss the ideas behind the two main filtering techniques: Kalman Filters and Particle Filter. For technical details behind the filtering techniques, refer to Ch. 8 in [1].

Kalman Filters Kalman Filters is a filtering algorithm that assumes observables contain statistical (Gaussian) noise and estimates the unknown variables of the underlying process through a series of observations. The algorithm is split into two main steps: time update (prediction) and measurement update (filtering). During time update, the algorithm utilizes prior measures to predict the current state. In measurement update, the algorithm then utilizes observables to obtain a better estimate of the current state. One disadvantage of Kalman Filter is that it assumes both the observation and hidden state transitions to be linear. With Extended Kalman Filter (EKF), non-linear transitions are discretized by finding the Jacobian of the transition. However, EKF can suffer from poor estimations for highly non-linear processes as only the mean is perturbed in the process. Hence, unscented transform could be applied leading to Unscented Kalman Filter (UKF). In UKF, a set of points is sampled around the mean and propogated through the non-linear transition. A weighted mean and error covariance from the mapped set of points are then calculated, providing better estimates.

Particle Filter One challenge among Kalman Filters is that the algorithm assumes the probability density function $p(x_t|z_t)$ where x_t, z_t are the hidden state and observables, is Gaussian. Particle Filter (Sequential Monte Carlo) utilizes importances sampling to estimate $p(x_t|z_t)$ and hence provide a more generalized approach. The conditional expectation of state x_t given a set of particles $\{x_t^{(i)}\}$ is given as:

$$\mathbb{E}(x_t|z_{1:t}) = \sum_i \tilde{w}_t(x_t^{(i)})x_t^{(i)}$$

The three main components of Particle Filter are the likelihood $p(z_t|x_t^{(i)})$, the proposal probability $\pi(x_t^{(i)}|z_t, x_{t-1}^{(i)})$ and the transition probability $p(x_t|x_{t-1})$. The (unnormalized) weights are calculated as:

$$w_t = w_{t-1}\frac{p(z_t|x_t^{(i)})p(x_t|x_{t-1})}{\pi(x_t^{(i)}|z_t, x_{t-1}^{(i)})}$$

The parameter set is given by $\{\mu, \kappa, \theta, \sigma, \nu, \eta, \lambda\}$.

III. Methodology

In this section we will cover implementation insights from the algorithms and derivations described in [1]. We will focus on specifics such as how we initialized the process and approached negative values for volatility and arrival times while discuss what the filtering algorithm is estimating from the stochastic processes. In addition to filtering, we also showed how we can simulate a Heston, VG, and VGSA process. All implementations are done in Python and with scipy packages for optimization.

A. Filtering Heston via. EKF, UKF, and PF

The observable and state (non-linear) transitions are given by the SDE of Heston described in sec. II and we discretized it by finding the respective Jacobian matrices. By applying filtering, we can uncover the volatility (state) through the market prices (observables). For Kalman Filters, parameter estimation is done through maximizing the loglikelihood. To ensure parameters stay within a desired range, we applied periodic mapping on parameters outside of a desired bound.

For PF, the proposal, transition, and likelihood functions derivations are given and weights could be updated accordingly instead of assuming the proposal and transition functions are the same to simplify the process. To apply PF for an unknown set of parameters, we estimated the optimal parameters by utilizing an augmented states described in [2]. Specifically during initialization for each parameter in the set $\{\mu, \kappa, \theta, \sigma, \rho\}$, we sampled a set of points uniformly within a bound. Hence, each augmented state is $\boldsymbol{V_t} \in \mathbb{R}^{k\times N}$ where k is the number of parameters and N is the number of particles. Similarly, $\boldsymbol{v_0} \in \mathbb{R}^N$ is initialized by sampling from $\mathcal{N}(v_0, \sigma^2)$ for some $\sigma > 0$. Resampling is then performed on all the particles.

To ensure volatility is always positive, we set all negative values to be some small number; resampling could also be performed instead.

B. Filtering VGSA via. PF

We implemented filtering for VGSA by modeling the arrival rate as the state; similar to [1]. Hence, the initial state is $y_0 = \frac{1}{\nu}$. To find the optimal parameters, we followed the same augmented state for filtering Heston via. PF by sampling uniformly within a bound for the VGSA parameter set $\{\mu, \kappa, \theta, \sigma, \nu, \eta, \lambda\}$.

It is possible that arrival rate (state) to be negative. For all negative arrival rates, we resample until we have a non-negative value.

IV. Results

Filtering for Heston and VGSA is performed on both synthetic data and real market data. For synthetic data, we show that the filtering process indeed uncovers the state through simulated observations and compare the optimal parameters obtained to the true parameters. We simulated 1000 steps of data.

For real market data, filtering is performed for two oil related prices (Sinopec SNP, Crude Oil) and four major tech stocks: AAPL, AMZN, FB, MSFT. We collected 5-year daily data from 2013 to 2018.

We discretized the states by setting $dt = \frac{1}{250}$ to uncover the state daily. For all experiments, the number of particles is set as $N = 2000$ to ensure consistent comparisons. We applied Nelder-Mead simplex optimization to minimize the negative loglikelihood, running optimization for a maximum of 2500 iterations.

Although it is suggested that we first seek an optimal lower and upper bound for PF from UKF, results show that PF actually converges to a reasonable set of parameters, given a descent lower upper bound estimation. In addition, it is much faster than minimizing the negative loglikelihood as performed in parameter estimation for UKF. On the other hand, UKF converges to a better set of parameters since the set.

A. *Filtering for Synthetic Data*

We simulated 1000 steps of data based on the Heston and VGSA process based on a given set of parameters. The goal is to apply the filtering algorithms to uncover the state from observations. We will first present results on Heston and then VGSA.

The simulated path and volatility is shown in fig. 1 and fig. 2. For Heston, UKF provided the best predicted state result, giving the smallest error from the true simulated state. The true parameters, initial parameters and the bounds selected for PF for Heston is given in table I.

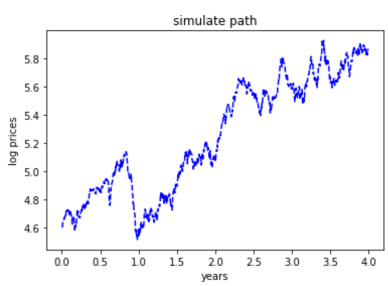

Fig. 1. Simulated Heston Path (observations) for 1000 steps

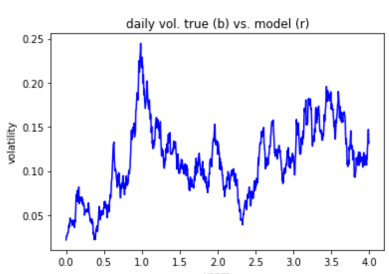

Fig. 2. Simulated Heston Volatility (state) for 1000 steps

TABLE I
TRUE AND INITIAL HESTON PARAMETERS

Param.	True	Initial	PF Bounds
μ	0.2	0.45	(0.05, 0.5)
κ	2.2	1.1	(1, 9)
θ	0.15	0.05	(0.05, 0.2)
σ	0.3	0.35	(0.01, 0.91)
ρ	-0.6	-0.4	(-0.5, 0)
v_0	0.02	0.05	0.05

For both EKF and UKF, optimization was completed within 2500 iterations. For PF, since parameters are found through resampling particles, we plot the parameters with respect to each filtering step by weighting the particles based on the likelihood weights. The parameters after optimization is shown in table II. The PF parameters are obtained from the final filtering step.

TABLE II
TRUE AND INITIAL HESTON PARAMETERS

Param.	True	EKF	UKF	PF
μ	0.2	0.3917	0.5634	0.2320
κ	2.2	1.0000	1.8627	1.3032
θ	0.15	0.1740	0.1056	0.1664
σ	0.3	0.6592	0.2259	0.2856
ρ	-0.6	0.6309	-0.9999	-0.1043
v_0	0.02	0.1166	0.0457	0.02

The predicted volatility based on the filtering results from EKF, UKF, and PF are shown in fig. 3, 4, and 5. The figures show that UKF obtained the best results whereas PF generally captured the pattern but has a larger variance.

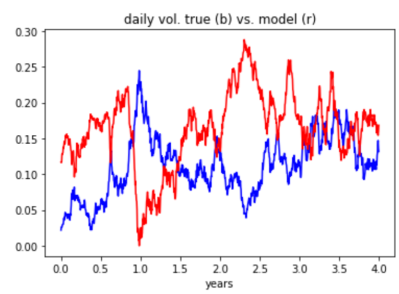

Fig. 3. EKF Predicted Volatility

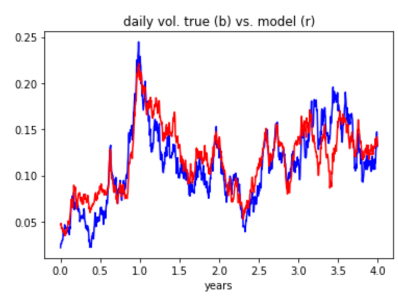

Fig. 4. UKF Predicted Volatility

In addition, we plot the convergence of σ and κ through resampling in fig. 6 and 7.

Similar analysis is done for VGSA through PF; the simulated path is shown in fig. 8 with true and parameter bounds shown in table. III.

VGSA is more sensitive to the initialization of the parameter bound. It is highly probable that most particles

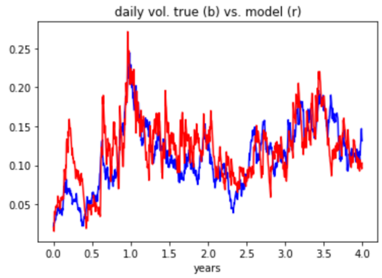

Fig. 5. PF Predicted Volatility

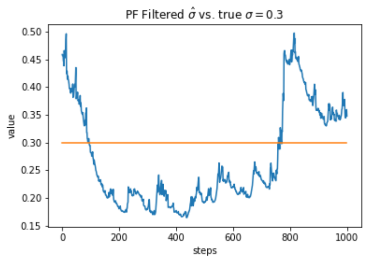

Fig. 6. Estimated σ vs. True

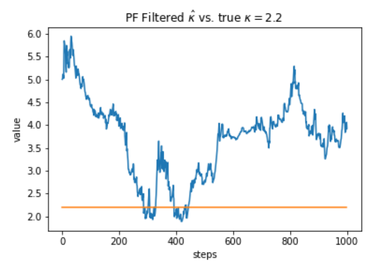

Fig. 7. Estimated κ vs. True

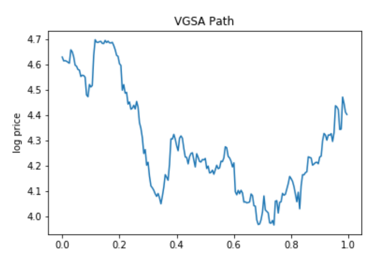

Fig. 8. Simulated VGSA Path

TABLE III
TRUE AND INITIAL VGSA PARAMETERS

Param.	True	PF Bounds
μ	0.02	(0.01, 0.05)
κ	3.5	(2, 4)
θ	0.05	(0.01, 0.1)
σ	0.13	(0.1, 0.2)
ν	0.07	(0.02, 0.1)
η	4	(3, 6)
λ	8	(6, 10)

sampled to have extremely small likelihood and hence driving weights to nan values; we simply replace them with 0 to avoid such problem. The estimated parameters are shown in table. IV. The filtered arrival rate is shown in fig. 9. Note we discarded the first few steps before the particles begin to converge.

TABLE IV
TRUE AND OPTIMAL VGSA PARAMETERS

Param.	True	PF Optimal
μ	0.02	0.0102
κ	3.5	2.3742
θ	0.05	0.0991
σ	0.13	0.1875
ν	0.07	0.0981
η	4	5.4726
λ	8	7.9876

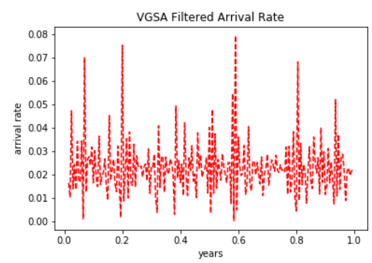

Fig. 9. Filtered VGSA Arrival Rate

B. Filtering Crude Oil and Sinopec (SNP) Prices

For the case study, we applied the filtering process on Crude Oil and SNP. As shown in fig. 10, the prices move simultaneously as SNP is an oil manufacturer; hence, it is affected by the oil price. With two correlated markets, we compared the filtered state on the selected markets.

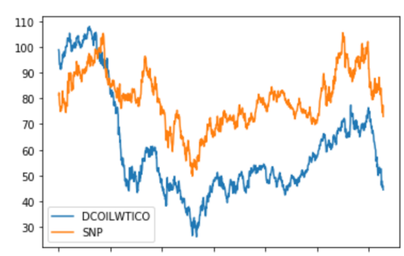

Fig. 10. 5-year Price of Crude Oil and SNP

The initial range chosen are: $\mu \in (-0.4, 0.4), \kappa \in (1e-3, 9), \theta \in (1e-3, 1), \sigma \in (1e-3, 1), \rho \in (-1, 1), v_0 \in (1e-3, 0.8)$. The initial parameters for EKF and UKF are the mean of the ranges. The parameters obtained for Crude Oil and SNP for the Heston process is shown in table. V and **??**.

Furthermore, we compared the daily volatility filtered by EKF, UKF, and PF in fig. 11 and show the VGSA filtered arrival rate in fig. 12. The volatility filtering showed that

TABLE V

FILTERED PARAMETERS FOR HESTON ON CRUDE OIL

Param.	EKF	UKF	PF
μ	1.0140	0.0087	-0.0491
κ	4.0551	0.6108	4.9624
θ	3.5633	0.0875	0.0698
σ	1.000	1.1033	0.7584
ρ	-3.0461	0.0057	-0.34307
v_0	1.3680	0.055	0.055

UKF and PF recovered similar patterns whereas EKF's predicted volatility is flatter compared to others.

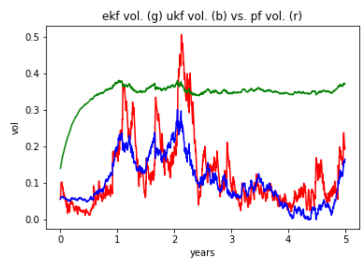

Fig. 11. Heston Daily Volatility for Crude Oil Prices

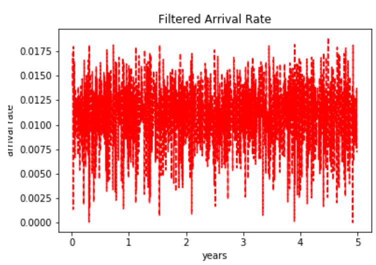

Fig. 12. VGSA Daily Arrival Rate for Crude Oil Prices

Similar procedures are applied for SNP; the optimal parameters as shown in table. VI and fig. 13.

TABLE VI

FILTERED PARAMETERS FOR HESTON ON SNP

Param.	EKF	UKF	PF
μ	-0.0322	-0.0038	0.0415
κ	8.9051	1.7503	6.3882
θ	0.0010	0.0811	0.1142
σ	0.7186	0.9999	0.9033
ρ	0.0179	-0.0001	-0.1313
v_0	0.0817	0.0922	0.0922

From the obtained parameters, we can see that EKF and UKF are similar except for κ. The vol. of vol σ is consistent through out the filtering methods. However, the predicted volatility from EKF is much smaller than the volatility recovered by the other two filtering methods whereas PF exhibits a more volatile behavior.

Finally, we compare the daily volatility of Crude Oil prices to SNP via. UKF and PF in fig. 14 and 15.

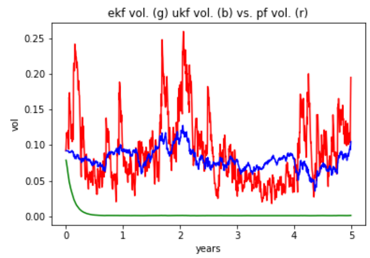

Fig. 13. Heston Daily Volatility for SNP

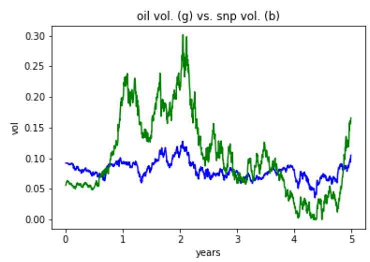

Fig. 14. Daily Vol. of Crude Oil vs. SNP Filtered from UKF

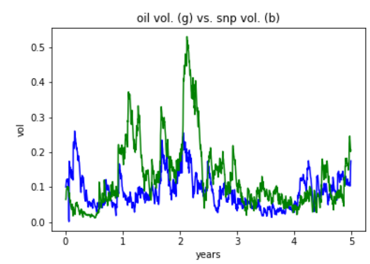

Fig. 15. Daily Vol. of Crude Oil vs. SNP Filtered from PF

Filtering results show consistency which Crude Oil prices are more volatile than SNP. Futhermore, it also shows that the daily vol. has positive correlation such that the they tend to move together.

One way to extend parameter estimation would be to minimize the negative loglikelihood in parameter estimation for PF; currently, parameters are updated through resampling a set of particles for the augmented state from importance weights.

C. Filtering for AAPL, AMZN, MSFT, FB

We applied UKF Hesotn to filter the daily vol. for the four major technology stock: AAPL, AMZN, MSFT, FB. The parameters obtained are shown in table. VII:

Note that although σ is the same across the four stocks, it is not caused by UKF drifting the the boundary since our bound is $\sigma \in (1e-3, 1)$. Fig. 16 of all the daily vol further shows AAPL and MSFT have similar daily vol. across the series.

TABLE VII
UKF Heston Estimated Params

Param.	AAPL	AMZN	MSFT	FB
μ	0.0196	0.0251	0.0201	0.0209
κ	1.3333	1.1444	1.3333	2.6504
θ	0.0673	0.1193	0.0670	0.0820
σ	0.6999	0.6999	0.6999	0.6999
ρ	-0.0006	0.0021	-0.0005	-0.0007
v_0	0.0537	0.1219	0.0357	0.1999

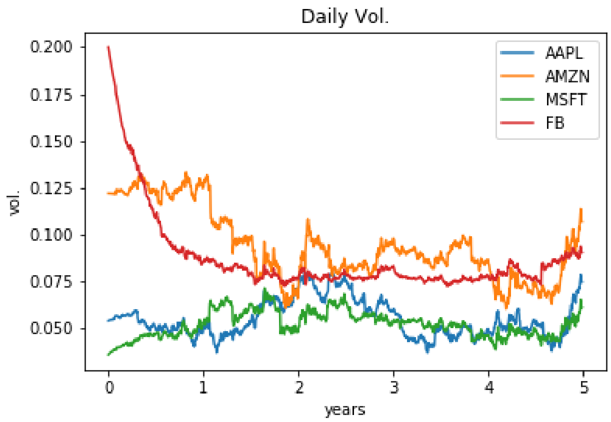

Fig. 16. Daily Vol. of AAPL, AMZN, MSFT, FB

V. Conclusions

In this study, we implemented multiple filtering and parameter estimation techniques for two stochastic processes, namely Heston and VGSA. Through these filtering techniques, we can uncover the underlying state (unobservables) through market prices (observables). Depending on the stochastic process, the state can be volatility or arrival rate. In addition, we can estimate the optimal parameter set from these physical market prices using a series of historical data. Filtering was applied on both simulated data and real market data such as Crude Oil prices and SNP.

VI. Acknowledgement

ML would like to thank Prof. Hirsa for his guidance and providing the derivations for the algorithms and insights through his sample codes.

References

[1] A. Hirsa. *Computational Methods in Finance*. Chapman and Hall/CRC Financial Mathematics Series. CRC Press, 2016.

[2] Shin Ichi Aihara, Arunabha Bagchi, and Saikat Saha. On parameter estimation of stochastic volatility models from stock data using particle filter-application to aex index.