Домашнее задание

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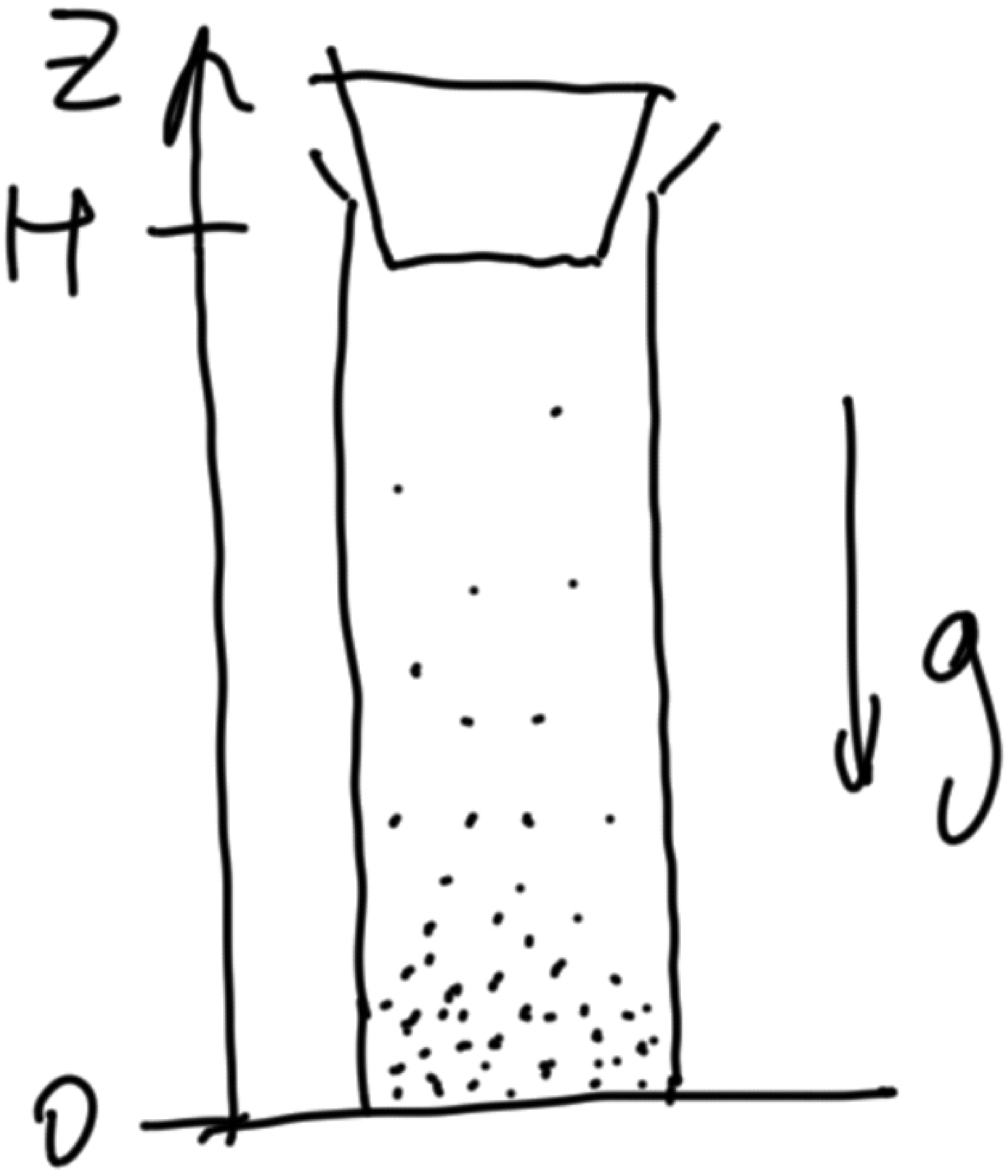
№1

молярная масса

$$\nu = 1\text{ моль},\ M,\ g$$
$$T,\ H$$

$$E = \frac{p^2}{2m} + mgz$$



$$W(p,z) = A\cdot e^{-\frac{E}{k_B T}}$$ (плотность распределения)

$$W(p,z) = A\cdot e^{-\frac{p^2}{2mk_BT}}\cdot e^{-\frac{mgz}{k_BT}}$$

$$W_z(z) = \int d^3p\,dx\,dy\cdot W(p,z) = \tilde{A}\cdot e^{-\frac{mgz}{k_BT}}$$

$$\int_0^H W_z(z)\,dz = \tilde{A}\cdot\int_0^H e^{-\frac{mg}{k_BT}z}\,dz =$$

$$= \tilde{A}\cdot\left(-\frac{k_BT}{mg}\right)\int_0^{-\frac{mgH}{k_BT}} e^{\xi}\,d\xi = \tilde{A}\left(-\frac{k_BT}{mg}\right)\cdot\left(e^{-\frac{mgH}{k_BT}} - 1\right) =$$

$$= \tilde{A}\,\frac{k_BT}{mg}\left(1 - e^{-\frac{mgH}{k_BT}}\right) = 1$$

(для норм. коэфф.)

$$\Rightarrow \tilde{A} = \frac{mg}{k_B T\left(1 - e^{-\frac{mgH}{k_B T}}\right)}$$

$$\Downarrow$$

$$W_z = \frac{mg}{k_B T} \cdot \frac{e^{-\frac{mgz}{k_B T}}}{1 - e^{-\frac{mgH}{k_B T}}}$$

$$\overline{z} = \int_0^H z \cdot W_z \, dz = \frac{mg}{k_B T \cdot \left(1 - e^{-\frac{mgH}{k_B T}}\right)} \times$$

$$\times \int_0^H z \cdot e^{-\frac{mgz}{k_B T}} dz =$$

$$\int_0^H z e^{-\beta z} dz = \int_0^H z \cdot \left(-\frac{1}{\beta}\right) d\left(e^{-\beta z}\right) =$$

$$= -\frac{1}{\beta} z e^{-\beta z}\Big|_0^H - \left(-\frac{1}{\beta}\right) \int_0^H e^{-\beta z} dz =$$

$$= -\frac{1}{\beta} z e^{-\beta z}\Big|_0^H + \frac{1}{\beta} \cdot \left(-\frac{1}{\beta}\right) \cdot e^{-\beta z}\Big|_0^H =$$

$$= -\frac{H e^{-\beta H}}{\beta} - \frac{1}{\beta^2}\left(e^{-\beta H} - 1\right) =$$

$$= \frac{1}{\beta^2} - \frac{1}{\beta^2} e^{-\beta H} - \frac{1}{\beta} H e^{-\beta H}$$

$$= \frac{mg}{k_B T} \cdot \frac{1}{\underbrace{1-e^{-\frac{mgH}{k_B T}}}} \cdot \left(\frac{k_B^2 T^2}{m^2 g^2} \underbrace{\left(1-e^{-\frac{mgH}{k_B T}}\right)} - \right.$$

$$\left. - \frac{k_B T}{mg} H e^{-\frac{mgH}{k_B T}} \right) = \frac{k_B T}{mg} - H e^{-\frac{mgH}{k_B T}}$$

$$\Delta E = \nu \cdot N_a \cdot mg\bar{z} = \qquad f(g) = f'(g) g'$$

$$= \nu N_a \left(k_B T - mgH e^{-\frac{mgH}{k_B T}} \right)$$

$$\frac{\partial}{\partial T}(\Delta E) = \nu N_a \left(k_B - mgH \cdot e^{-\frac{mgH}{k_B T}} \cdot \left(-\frac{mgH}{k_B T^2} \right) \right)$$

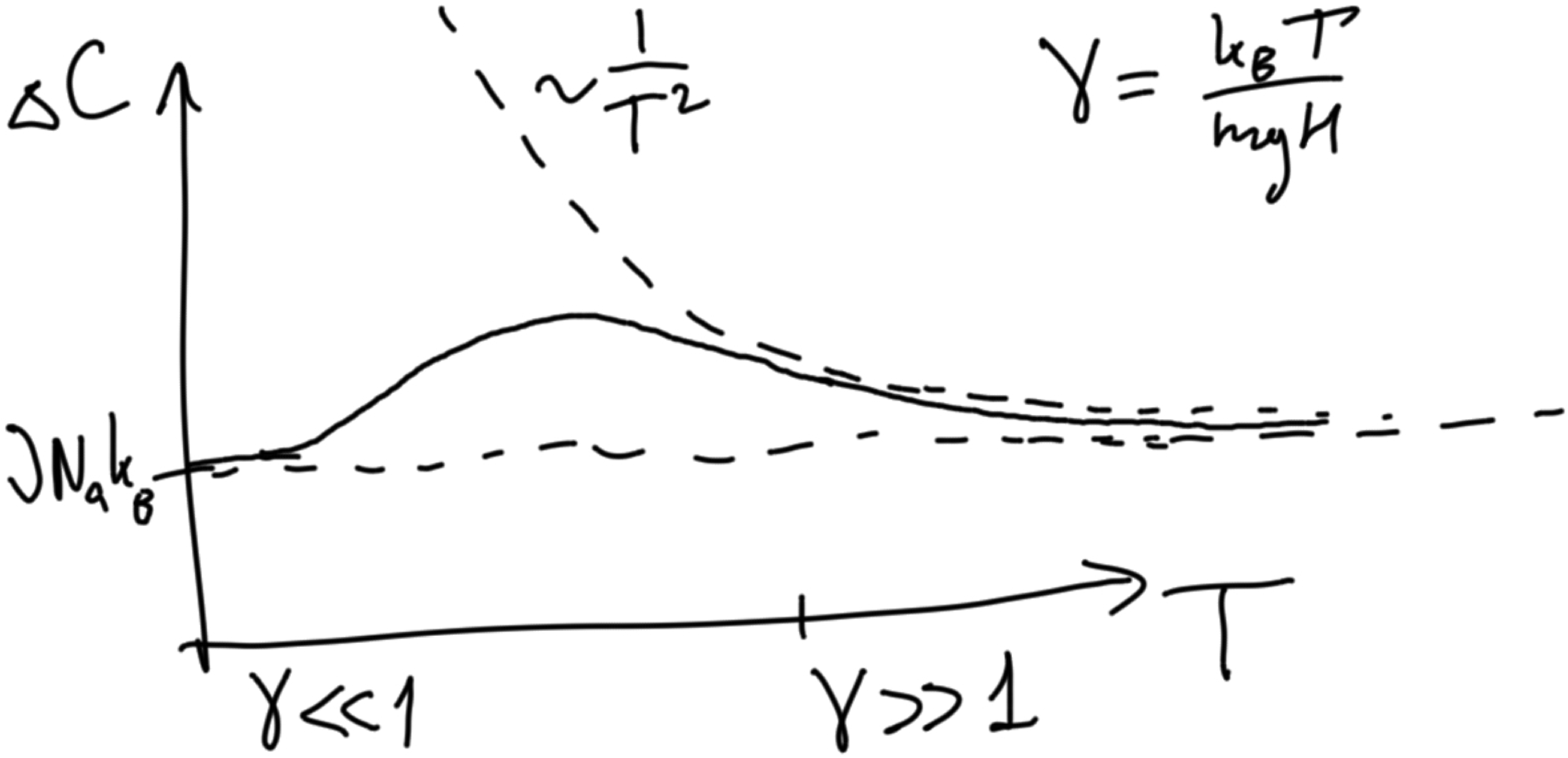
$$= \nu N_a k_B \left(1 + \frac{(mgH)^2}{k_B^2 T^2} e^{-\frac{mgH}{k_B T}} \right) = \Delta C$$

при $mgH \ll k_B T$:

$$\left(\frac{mgH}{k_B T} \right)^2 e^{-\frac{mgH}{k_B T}} \approx \left(\frac{mgH}{k_B T} \right)^2$$

при $mgH \gg k_B T$:

$$\left(\frac{mgH}{k_B T} \right)^2 e^{-\frac{mgH}{k_B T}} \approx 0$$



N2

$$E(\vec{p}) = c|p_x| \quad , T$$

$$W(p_x) = A \cdot e^{-\frac{E}{k_B T}}$$

$$\int_{-\infty}^{+\infty} W(p_x)\,dp_x = A \int_{-\infty}^{+\infty} e^{-\frac{c|p_x|}{k_B T}}\,dp_x =$$

$$= 2A \int_0^{+\infty} e^{-\frac{cp_x}{k_B T}}\,dp_x = 2A\left(-\frac{k_B T}{c}\right) \cdot \int_0^{+\infty} e^{-\xi}\,d\xi =$$

$$= 2A\left(-\frac{k_B T}{c}\right) \cdot \left(e^{-\infty} - e^{-0}\right) = \frac{2A k_B T}{c} = 1$$

$$\Rightarrow A = \frac{c}{2k_B T}$$

$$\Rightarrow W = \frac{c}{2k_B T} e^{-\frac{c|p_x|}{k_B T}}$$

$$\overline{E} = \int_{-\infty}^{\infty} E(p_x) \cdot W(p_x)\, dp_x =$$

$$= \int_{-\infty}^{\infty} c \cdot |p_x| \cdot \frac{c}{2k_B T} e^{-\frac{c|p_x|}{k_B T}} dp_x =$$

$$= \frac{c^2}{k_B T} \int_0^{\infty} p_x e^{-\frac{c p_x}{k_B T}} dp_x =$$

$$= \frac{c^2}{k_B T} \cdot \left(\frac{k_B T}{c}\right)^2 \cdot \int_0^{\infty} \xi e^{-\xi} d\xi =$$

$$= k_B T \cdot \left(-\int_0^{\infty} \xi\, d(e^{-\xi})\right) = k_B T \left(\cancel{-\xi e^{-\xi}\Big|_0^{\infty}} + \right.$$

$$\left. + \int_0^{\infty} e^{-\xi} d\xi \right) = k_B T \left(-e^{-\infty} + e^{0}\right) = k_B T$$

N3

3 состояния: $E_1 = -MB$;

$E_2 = 0$; $E_2 = MB$.

$$W_i = A \cdot e^{-\frac{E_i}{k_B T}}$$

$$\sum_{i \in \{1,2,3\}} W_i = A\left(e^{\frac{MB}{k_B T}} + 1 + e^{-\frac{MB}{k_B T}}\right) = 1$$

$$\Rightarrow A = \frac{1}{1 + e^{\frac{MB}{k_B T}} + e^{-\frac{MB}{k_B T}}}$$

$$W_1 = \frac{\overbrace{e^{\frac{MB}{kT}}}^{\gamma}}{1 + e^{\frac{MB}{kT}} + e^{-\frac{MB}{k_B T}}} = 0,99$$

$$\frac{\gamma}{1 + \gamma + \gamma^{-1}} = 0,99$$

$$\gamma^2 = 0,99\gamma + 0,99\gamma^2 + 0,99 \quad \Big| \cdot 100$$

$$\gamma^2 - 99\gamma - 99 = 0$$

$$\Rightarrow \gamma = \frac{99 \pm \sqrt{99^2 + 4 \cdot 99}}{2} \approx 99,99$$

Выбираем „+", т.к. $\gamma > 0$.

$$e^{\frac{MB}{kT}} = \gamma$$

$$\frac{MB}{kT} \approx 4{,}6$$

$$\Rightarrow T = \frac{MB}{4{,}6k}$$

<u>№4</u>

$$L \in \mathbb{Z},\ L \geq 0$$

$$M \in \mathbb{Z}:\ M \in [-L; L]$$

$$E_L = \hbar^2 L(L+1)/(2I)$$

$$W(L, M) = A \cdot e^{-\frac{E_L}{k_B T}}$$

$$\sum_{L=0}^{\infty} \sum_{M=-L}^{L} W(L, M) = A \cdot \sum_{L=0}^{\infty} \sum_{M=-L}^{L} e^{-\frac{\hbar^2 L(L+1)/(2I)}{k_B T}} =$$

$$= A \cdot \sum_{L=0}^{\infty} (2L+1) \cdot e^{-\frac{\hbar^2 L(L+1)}{2I k_B T}} = 1$$

$$\Rightarrow A = \frac{1}{\sum\limits_{L=0}^{\infty} (2L+1) e^{-\frac{\hbar^2 L(L+1)}{2I k_B T}}}$$

$$\overline{E} = \sum_{L=0}^{\infty} \sum_{M=-L}^{L} W(L, M) \cdot E_L =$$

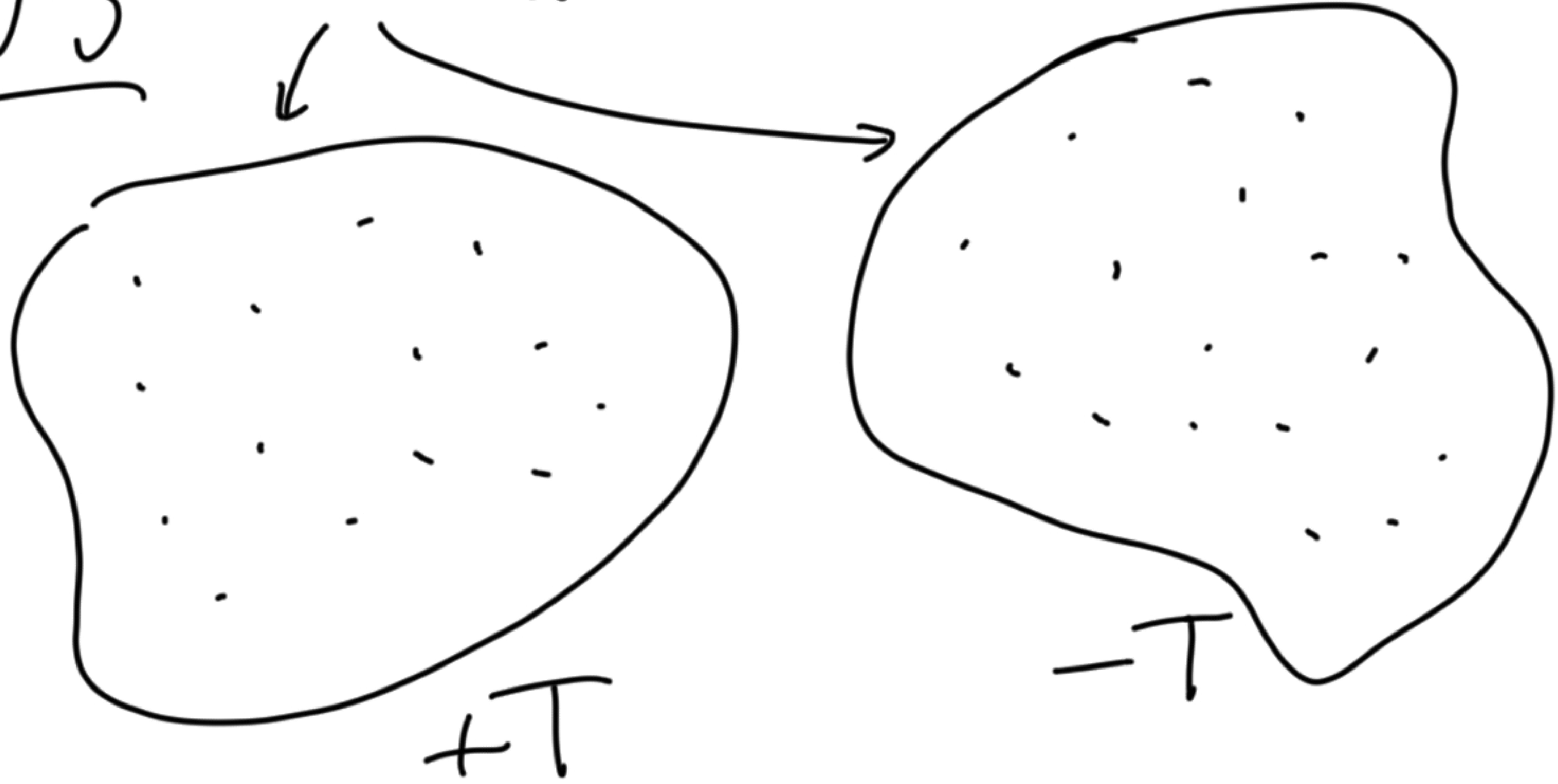
$$= \frac{\sum\limits_{L=0}^{\infty} (2L+1) \frac{\hbar^2 L(L+1)}{2I} \cdot e^{-\frac{\hbar^2 L(L+1)}{2I k_B T}}}{\sum\limits_{L=0}^{\infty} (2L+1) e^{-\frac{\hbar^2 L(L+1)}{2I k_B T}}} =$$

$$= \frac{\hbar^2}{2I} \cdot \frac{\sum\limits_{L=0}^{\infty} (2L+1)(L+1)L \cdot e^{-\frac{\hbar^2 L(L+1)}{2I k_B T}}}{\sum\limits_{L=0}^{\infty} (2L+1) e^{-\frac{\hbar^2 L(L+1)}{2I k_B T}}}$$

Дорешивайте сами!

N5

N систем



Средняя энергия двухуровневой системы в зависимости от температуры:

$$\overline{E} = -E_0 \tanh\left(\frac{E_0}{kT}\right)$$

$$\Rightarrow E_\Sigma = \left[-E_0 \tanh\left(\frac{E_0}{kT}\right) - E_0 \tanh\left(\frac{E_0}{k(-T)}\right)\right]\cdot N = 0$$

(tanh — нечетная ф-я)

Пусть T^* — конечная температура

$$E_\Sigma = \left[-E_0 \tanh\left(\frac{E_0}{kT^*}\right)\right]\cdot 2N$$

$$\Rightarrow \tanh\left(\frac{E_0}{kT^*}\right) = 0$$

$$\Rightarrow T^* = \frac{E_0}{k \operatorname{arctanh}(0)}$$

$$\Rightarrow T^* = \pm \infty$$