

N1 $E = cp$ a) $P_0 = ?$ b) $\langle p \rangle = ?$ c) $\langle p^2 \rangle = ?$

$$P(s) = \frac{\int_{S_0} e^{-\beta p c} d^3 p}{\int_S e^{-\beta p c} d^3 p} = A \int_{S_0} e^{-\beta p c} d^3 p = A \int_{S_0} e^{-\beta \sqrt{p_x^2 + p_y^2 + p_z^2} c} dp_x dp_y dp_z$$

$$= A \int_0^{2\pi} d\varphi \int_0^\pi d\theta \int p^2 \sin\theta e^{-\beta p c} dp = 4\pi A \int p^2 e^{-\beta p c} dp$$

$$A = \left(\int_S e^{-\beta p c} d^3 p \right)^{-1} = 2\pi \cdot (2\pi \cdot 2 \cdot \int_S p^2 e^{-\beta p c} dp)^{-1}$$

$$\omega(p) = 4\pi p^2 e^{-\beta p c}$$

$$\frac{d\omega(p_0)}{dp} = 4\pi e^{-\beta p_0 c} (2p_0 - p_0^2 \beta c) = 0 \Rightarrow p_0 = \frac{2}{\beta c} = \frac{kT}{c}$$

a) $P_0 = \frac{kT}{c}$

$$\langle p \rangle = \frac{4\pi \int_S p^3 e^{-\beta p c} dp}{4\pi \int_S p^2 e^{-\beta p c} dp} ; \int_S p^3 e^{-\beta p c} dp = -\frac{1}{\beta c} e^{-\beta p c} p^3 \Big|_S + \int_S \frac{e^{-\beta p c}}{\beta c} 3p^2 dp$$

$$\Rightarrow \langle p \rangle \approx \frac{3}{\beta c} = \frac{3kT}{c}$$

d) $\langle p \rangle \approx \frac{3kT}{c}$

$$\langle p^2 \rangle = \frac{\int_S p^4 e^{-\beta p c} dp}{\int_S p^2 e^{-\beta p c} dp} ; \int_S p^4 e^{-\beta p c} dp = -\frac{1}{\beta c} e^{-\beta p c} p^4 \Big|_S + \int_S 4p^3 \frac{e^{-\beta p c}}{\beta c} dp = -\frac{4}{\beta^2 c^2} e^{-\beta p c} p^3 \Big|_S + \int_S \frac{12}{\beta^2 c^2} p^2 e^{-\beta p c} dp$$

$$\Rightarrow \langle p^2 \rangle \approx \frac{12}{\beta^2 c^2}$$

$p \approx mc \Rightarrow \frac{1}{\beta} e^{-\beta mc^2} m^3 c^2 \ll 1$
 $\frac{1}{\beta c}$

1/2 p-1

$$E = \frac{p^2}{2m} \Rightarrow \omega(p) = \frac{4\pi p^2 e^{-\frac{p^2}{2mKT}}}{(2\pi mKT)^{3/2}}$$

$$\langle v \rangle = \frac{1}{m} \int_0^\infty \frac{p^3 4\pi e^{-\frac{p^2}{2mKT}}}{(2\pi mKT)^{3/2}} dp = \frac{4\pi}{m (2\pi mKT)^{3/2}} \cdot \int_0^\infty t e^{-\frac{t}{2mKT}} \frac{dt}{2} =$$

$$= \frac{A}{2} \int_0^\infty t e^{-\beta t} dt = \frac{A}{2} \cdot \left(-\frac{1}{\beta} t e^{-\beta t} \Big|_0^\infty + \int_0^\infty \frac{1}{\beta} e^{-\beta t} dt \right) =$$

$$= \frac{A}{2} \frac{1}{\beta} \cdot \left(+\frac{1}{\beta} \right) = \frac{2\pi}{m (2\pi mKT)^{3/2}} \cdot (2mKT)^2 = \sqrt{\frac{8KT}{\pi m}}$$

$$P = \int_0^{p_0} \frac{4\pi p^2 e^{-\frac{p^2}{2mKT}}}{(2\pi mKT)^{3/2}} dp = \left\{ 0, 1 < v > m = p_0 \right\} =$$

$$= \int_0^{p_0} \frac{4\pi p^2}{(2\pi mKT)^{3/2}} e^{-\frac{p^2}{2mKT}} dp = C \cdot \int_0^{p_0} p^2 e^{-\beta p^2} dp =$$

$$= C \int_0^{p_0} \frac{p}{2} e^{-\beta p^2} dp^2 = \frac{C}{2} \int_0^{p_0^2} \sqrt{z} e^{-\beta z} dz = \frac{C}{2} \frac{1}{-\beta} e^{-\beta z} \sqrt{z} \Big|_0^{p_0^2} +$$

$$+ \int_0^{p_0^2} \frac{C}{2\beta} e^{-\beta z} \frac{1}{2} \frac{1}{\sqrt{z}} dz \Rightarrow$$

$$\Rightarrow \int_0^{p_0^2} e^{-\beta z} \left(\sqrt{z} - \frac{1}{\beta} \frac{1}{\sqrt{z}} \cdot \frac{1}{2} \right) dz = \frac{1}{-\beta} e^{-\beta z} \sqrt{z} \Big|_0^{p_0^2} -$$

$$\Rightarrow \int_{p_0^2}^\infty e^{-\beta z} \left(\sqrt{z} - \frac{1}{\beta} \frac{1}{\sqrt{z}} \cdot \frac{1}{2} \right) dz \approx \int_{p_0^2}^\infty \sqrt{z} e^{-\beta z} dz - \frac{1}{\beta} e^{-\beta z} \sqrt{z} \Big|_{p_0^2}^\infty =$$

$$= \frac{1}{\beta} p_0 e^{-\beta p_0^2} \Rightarrow 2 \int_{p_0}^\infty p^2 e^{-\beta p^2} dp = + \frac{1}{\beta} p_0 e^{-\beta p_0^2} \Rightarrow$$

$$\Rightarrow P = \int_0^\infty C p^2 e^{-\beta p^2} dp - \int_{p_0}^\infty C p^2 e^{-\beta p^2} dp = 1 - \frac{C}{2} \frac{1}{\beta} p_0 e^{-\beta p_0^2} =$$

$$= 1 - \frac{4\pi}{(2\pi mKT)^{3/2}} \cdot \frac{1}{2} \cdot 2mKT \cdot 0,1 \cdot m \cdot \sqrt{\frac{8KT}{\pi m}} \cdot e^{-0,1^2 \cdot \frac{8KT}{\pi} \cdot \frac{1}{2mKT}} \quad \text{---}$$

$$\textcircled{+} 1 - \frac{1}{\pi} \cdot 4 \cdot 0,1 \cdot e^{-0,01 \cdot \frac{4}{\pi}} \approx 1 - \frac{0,4}{\pi} \cdot 0,984 \approx 0,87$$

Problem: $P \approx 0,87$

$\sqrt{3}$

$$dP = 2m v dN = \int 2m v n \cdot dS_0 \quad \omega(p_x) \quad dp_x =$$

$$= 2m \cdot n \int \frac{p_x}{m} \cdot \lg v \cdot \omega(p_x) \quad dp_x$$

$$F = \frac{dP}{dt} = 2m \cdot n \int \frac{p_x}{m} \lg v \cdot \omega(p_x) \quad dp_x$$

Давление: а) $P = \frac{dF}{dS} = 2mn \int v^2 \omega(p_x) \quad dp$

б) $\omega(N)$: $A = 4\pi \int_0^\infty p^2 e^{-\beta p^2} dp = \frac{4\pi}{-\beta p} e^{-\beta p^2} p^2 \Big|_0^\infty + \int \frac{4\pi}{\beta} p e^{-\beta p^2} dp =$

$$= \frac{4\pi}{\beta^2} \quad P = \frac{2n}{m} \int p^2 \frac{e^{-\frac{p^2}{2mKT}}}{\sqrt{2\pi mKT}} dp$$

$\sqrt{4}$ Вопрос: $m, v_0, n_0, \bar{v} - ?, n - ?, F - ?$

$$dE = \frac{mv^2}{2} dN = \frac{mv^2}{2} n dt dS = \frac{mv_0^2}{2} n dS \cdot v \cdot dt$$

$$P_{out} = \frac{dF}{dS} = \frac{d^2 E}{dS dt} = \frac{mn v_0^3}{2}$$

$$\langle p \rangle = A \int_0^\infty p \cdot e^{-\beta \left(\frac{p^2}{2m} + \frac{I \omega^2}{2} \right)} d^3 p d^2 \omega$$

$$A = (2\pi mKT)^{3/2} \cdot \int_{-\infty}^\infty e^{-\frac{\beta I \omega^2}{2}} d^2 \omega = (2\pi mKT)^{3/2} \cdot \sqrt{\frac{\pi}{\beta I}} \int_{-\infty}^\infty 2\pi \omega e^{-\frac{\beta I \omega^2}{2}} d\omega =$$

$$= A_1 \left(\int_{-\infty}^\infty \pi e^{-\frac{\beta I \omega^2}{2}} d\omega^2 \right)^{-1} = A_1 \left(\pi \sqrt{\frac{\pi 2KI}{I}} \right)^{-1} =$$

$$= (4\pi^3 (KT)^2 m^{3/2} \sqrt{\frac{1}{I}})^{-1}$$

$\langle p \rangle = A_1 \cdot \int_0^{\infty} p e^{-\beta \frac{p^2}{2m}} d^3 p \quad (= A \int p e^{-\beta \frac{p^2}{2m} - \beta \frac{I \omega^2}{2}} d^3 p d^2 \omega)$
 $\langle p \rangle = \sqrt{\frac{8 m K T}{\pi}} ; \quad \langle \omega \rangle = A_2 \int_0^{\infty} \omega e^{-\beta \frac{I \omega^2}{2}} d^2 \omega = A_2 \frac{1}{2} \frac{2 K T}{\frac{I}{2}} =$
 $= \frac{K T}{I} \cdot \frac{2 I}{\sqrt{\pi^3 K T}} = \sqrt{\frac{2 K T}{I \pi^3}}$

$E = \left(\frac{p^2}{2m} + \frac{I \omega^2}{2m} \right) \cdot N = \left(\frac{\langle p^2 \rangle}{2m} + \frac{I \langle \omega^2 \rangle}{2m} \right) \cdot N$

$\langle p^2 \rangle = 3 m K T ; \quad \langle \omega^2 \rangle = A_2 \int_0^{\infty} \omega^2 e^{-\beta \frac{\omega^2 I}{2}} d^2 \omega =$
 $= A_2 \int_0^{\infty} 2 \pi \omega^3 e^{-\frac{\beta \omega^2 I}{2}} d^2 \omega = A_2 \pi \cdot \left(-\frac{2}{\beta I} e^{-\frac{\beta \omega^2 I}{2}} \omega^2 \right) \Big|_0^{\infty} +$
 $+ \int_0^{\infty} \frac{2}{\beta I} e^{-\frac{\beta \omega^2 I}{2}} d(\omega^2) = A_2 \pi \cdot \frac{2}{\beta I} \cdot \frac{2}{\beta I} = \frac{4 K^2 T^2}{I^2} \frac{\pi}{\pi} \frac{2 I}{\sqrt{\pi K T}} =$
 $= \frac{8}{I} (K T)^{3/2} \frac{1}{\sqrt{\pi}} \left(\frac{2}{I} K T \right)^{3/2}$

$E = \left(\frac{9 m^2 K^2 T^2}{2 m} + \frac{I \left(\frac{2}{I} K T \right)^3}{2 m} \right) N = \frac{5}{2} N K T$

$$N_4 \quad P_{in} = P_{out} = nKT = \frac{m n_0 v_0^3}{2} \Rightarrow n = \frac{m n_0 v_0^3}{2KT}$$

$$\frac{9}{2} mKT + \frac{4 K^2 T^2}{I^2 m} = \frac{5}{2}$$

$$\alpha T^2 + \beta T - \frac{5}{2} = 0,$$

$$T = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha c}}{2\alpha} = \frac{-\frac{4K^2}{mI^2} \pm \sqrt{\frac{16K^4}{m^2 I^4} + 4 \frac{9}{8} mK \cdot \frac{5}{2}}}{2 \cdot \frac{9}{8} mK} =$$

$$= \sqrt{\frac{16}{81} \frac{K^2}{m^4 I^4} + \frac{5}{9 mK}} - \frac{4K}{9 m^2 I^2} > 0$$

$$n = \frac{m n_0 v_0^3}{2KT}$$

$$\langle v \rangle = \sqrt{\frac{8KT}{\pi m}}$$