ДЗ №2

№2

а) $A_\mu = g_{\mu\nu} A^\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix} =$

$= \begin{pmatrix} -1 \\ -1 \\ -1 \\ -3 \end{pmatrix}$

б) $B^\mu = T^{\mu\nu} A_\nu = T^{\mu\nu} g_{\mu\nu} A^\mu =$

$\underset{\smile}{A_\nu} = g_{\mu\nu} A^\mu$

$A_\mu = g_{\nu\mu} A^\nu$

$= \begin{pmatrix} 2 & 1 & 1 & 4 \\ -2 & -1 & -2 & -2 \\ -2 & -2 & -1 & -2 \\ 1 & 1 & 1 & -1 \end{pmatrix}_{\mu\nu} \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix} =$

$= \begin{pmatrix} -16 \\ 11 \\ 11 \\ 0 \end{pmatrix}$

$$T^{\mu}{}_{\nu} = g_{\nu\lambda} T^{\mu\lambda} = T^{\mu\lambda} g_{\lambda\nu} =$$

$$= \begin{pmatrix} 2 & 1 & 1 & 4 \\ -2 & -1 & -2 & -2 \\ -2 & -2 & -1 & -2 \\ 1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & -1 & -4 \\ -2 & 1 & 2 & 2 \\ -2 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$T_{\mu}{}^{\nu} = g_{\mu\lambda} T^{\lambda\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 & 4 \\ -2 & -1 & -2 & -2 \\ -2 & -2 & -1 & -2 \\ 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & 1 & 1 & 4 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 \\ -1 & -1 & -1 & 1 \end{pmatrix}$$

$$T_{\mu\nu} = g_{\lambda\mu} T^{\lambda}{}_{\nu} = g_{\lambda\mu} g_{\sigma\nu} T^{\lambda\sigma} =$$

$$= g_{\lambda\mu} T^{\lambda}{}_{\nu} = -T^{\lambda}{}_{\nu} g_{\lambda\mu} = T_{\nu\mu} =$$

$$= \begin{pmatrix} 2 & -1 & -1 & 4 \\ 2 & -1 & -2 & -2 \\ 2 & -2 & -1 & -2 \\ -1 & 1 & 1 & -1 \end{pmatrix}$$

д) $A^\mu A_\mu$; $\begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix} =$

$$= \begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ -9 \end{pmatrix}$$

$$A^\mu A_\mu = 1-1-1-9 = -10$$

$$T^\mu_{\ \lambda} = g_{\lambda\nu} T^{\mu\nu}$$

$$T^\mu_{\ \mu} = tr\left(T^\mu_{\ \lambda}\right) = -5$$

$$T^{\mu\nu} T_{\mu\nu} = tr\left(\begin{pmatrix} 2 & 1 & 1 & 4 \\ -2 & -1 & -2 & -2 \\ -2 & -2 & -1 & -4 \\ 1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 & 4 \\ 2 & -1 & -2 & -2 \\ 2 & -2 & -1 & -2 \\ -1 & 1 & 1 & -1 \end{pmatrix} \right) =$$

$$= tr\begin{pmatrix} 4 & -1 & -1 & 0 \\ -8 & 5 & 4 & 0 \\ -8 & 4 & 5 & 0 \\ 7 & -5 & -5 & 1 \end{pmatrix} = 15$$

№ 3

$$\vec{x}' = \hat{\Lambda}\,\vec{x} \qquad x'^{\mu} = \Lambda^{\mu}_{\nu}\, x^{\nu}$$

Запишем по определению

дельта Крон по опред $\begin{pmatrix} 1 & 0 \\ 0 & \ddots \end{pmatrix}$

$$\delta_{\nu}^{\mu} = \Lambda^{\mu}_{\lambda}\, \delta^{\lambda}_{a} \left(\Lambda^{-1}\right)^{a}_{\nu} =$$

$$= \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \end{pmatrix} =$$

$$= \begin{pmatrix} \gamma^2(1-\beta^2) & 0 & 0 & 0 \\ 0 & \gamma^2(1-\beta^2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \delta_{\nu}^{\mu}$$

$$\gamma^2(1-\beta^2), \text{ но } \gamma^2 = \frac{1}{(1-\beta)^2}$$

$$\Downarrow$$

$$1$$

б) $\delta^\alpha_\beta \delta^\beta_\gamma \delta^\gamma_\alpha = \delta^\alpha_\beta \delta^\beta_\alpha = \delta^\beta_\beta =$

$$= tr(\hat{1}) = tr\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 4$$

№4

б) $(\vec{a} \cdot \vec{b}) = a^\mu b_\mu = a^\mu g_{\mu\nu} b^\nu$

рассмотрим обратнее

$a'^\mu = \Lambda^\mu_\nu a^\nu$ $\qquad$ $b'^\mu = \Lambda^\mu_\nu b^\nu$

$$a'^{\mu} g_{\mu\nu} b'^{\nu} = \Lambda^{\mu}_{\ \nu} a^{\nu} g_{\mu\nu} \Lambda_{\nu}^{\ \mu} b^{\mu} =$$

$$= a^{\mu} g_{\mu\nu} b^{\nu} = a^{\mu} b_{\mu}$$

а) По образцу и подобию предыдущей части

$$g_{\mu\nu} = (\Lambda^{-1})^{\lambda}_{\ \mu}\, g_{\lambda\sigma} (\Lambda^{-1})^{\sigma}_{\ \nu} =$$

$$= \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} =$$

$$\frac{1}{1-\beta^2}(1-\beta^2) \qquad \gamma^2 = \frac{1}{1-\beta^2}$$

$$= \begin{pmatrix} \gamma^2-\gamma^2\beta^2 & 0 & 0 & 0 \\ 0 & \gamma^2\beta^2-\gamma^2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$