№1 $E = cp$ а) $p_0 - ?$ б) $\langle p \rangle - ?$ в) $\langle p^2 \rangle - ?$

$$P(s) = \frac{\int_{S_p} e^{-\beta p c}\, d^3p}{\int_S e^{-\beta p c}\, d^3p} = A\int_{S_p} e^{-\beta p c}\, d^3p = A\int_{S_p} e^{-\beta\sqrt{p_x^2+p_y^2+p_z^2}\,c}\, dp_x\, dp_y\, dp_z =$$

$$= A\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int p^2 \sin\theta\, e^{-\beta p c}\, dp = A\, 4\pi \int p^2 e^{-\beta p c}\, dp$$

$$A = \left(\int_S e^{-\beta p c}\, d^3p\right)^{-1} = \cancel{2\pi a}\left(2\pi \cdot 2 \cdot \int_S p^2 e^{-\beta p c}\, dp\right)^{-1}$$

$$\omega(p) = \pi p^2 e^{-\beta p c}$$

$$\frac{d\omega(p_0)}{dp} = 4\pi e^{-\beta p_0 c}(2p_0 - p_0^2 \cdot \beta c) = 0 \Rightarrow p_0 = \frac{2}{\beta c} = \frac{kT}{c}$$

а) $p_0 = \frac{kT}{c}$

$$\langle p \rangle = \frac{4\pi \cancel{A} \int_S p^3 e^{-\beta p c}\, dp}{4\pi \int_S p^2 e^{-\beta p c}\, dp}; \quad \int_S p^3 e^{-\beta p c}\, dp = \underbrace{-\frac{1}{\beta c} e^{-\beta p c} p^3 \Big|_S}_{\approx 0 \text{ — т.к.}} + \int_S \frac{e^{-\beta p c}}{\beta c} 3p^2\, dp$$

$$\cancel{p \approx mc \Rightarrow \frac{1}{\beta} e^{-\beta m c^2} m^3 c^2 \ll \frac{1}{\beta c}}$$

$$\Rightarrow \langle p \rangle \approx \frac{3}{\beta c} = \frac{3kT}{c}$$

б) $\langle p \rangle \approx \frac{3kT}{c}$

$$\langle p^2 \rangle = \frac{\int_S p^4 e^{-\beta p c}\, dp}{\int_S p^2 e^{-\beta p c}\, dp}; \quad \int p^4 e^{-\beta p c}\, dp = -\frac{1}{\beta c} e^{-\beta p c} p^4 \Big|_S +$$

$$+ \int 4p^3 \frac{e^{-\beta p c}}{\beta c}\, dp = \frac{-4}{\beta^2 c^2} e^{-\beta p c} p^3 \Big|_S +$$

$$+ \int \frac{12}{\beta^2 c^2} p^2 e^{-\beta p c}\, dp$$

$$\Rightarrow \langle p^2 \rangle \approx \frac{12}{\beta^2 c^2}$$

№2 $P - ?$

$$E = \frac{p^2}{2m} \Rightarrow \omega(p) = \frac{4\pi p^2 e^{-\frac{p^2}{2mkT}}}{(2\pi mkT)^{3/2}}$$

$$\langle v\rangle = \frac{1}{m}\int_0^\infty \frac{p^3\, 4\pi\, e^{-p^2/2mkT}}{(2\pi mkT)^{3/2}}\,dp = \frac{4\pi}{m(2\pi mkT)^{3/2}}\cdot\int_0^\infty t\, e^{-\frac{t}{2mkT}}\,\frac{dt}{2} =$$

$$= \frac{A}{2}\int_0^\infty t\, e^{-\beta t}\,dt = \frac{A}{2}\cdot\left(\underbrace{-\frac{1}{\beta} t e^{-\beta t}\Big|_0^\infty}_{0} + \int_0^\infty \frac{1}{\beta} e^{-\beta t}\,dt\right) =$$

$$= \frac{A}{2}\,\frac{1}{\beta}\cdot\left(+\frac{1}{\beta}\right) = \frac{2\pi}{m(2\pi mkT)^{3/2}}\cdot(2mkT)^2 = \sqrt{\frac{8kT}{\pi m}}$$

$$P = \int_0^{0{,}1\langle v\rangle m} \frac{4\pi p^2 e^{-\frac{p^2}{2mkT}}}{(2\pi mkT)^{3/2}}\,dp = \left\{0{,}1\langle v\rangle m = p_0\right\} =$$

$$= \int_0^{p_0} \frac{4\pi p^2}{(2\pi mkT)^{3/2}} e^{-\frac{p^2}{2mkT}}\,dp = C\cdot\int_0^{p_0} p^2 e^{-\beta p^2}\,dp =$$

$$= C\int_0^{p_0} \frac{p}{2} e^{-\beta p^2}\,dp^2 = \frac{C}{2}\int_0^{p_0^2} \sqrt{z}\, e^{-\beta z}\,dz = \frac{C}{2}\,\frac{1}{-\beta}\, e^{-\beta z}\sqrt{z}\Big|_0^{p_0^2} +$$

$$+ \int_0^{p_0^2} \frac{C}{2\beta} e^{-\beta z}\,\frac{1}{2}\,\frac{1}{\sqrt{z}}\,dz \Rightarrow$$

$$\Rightarrow \int_0^{p_0^2} e^{-\beta z}\left(\sqrt{z} - \frac{1}{\beta}\frac{1}{\sqrt{z}}\cdot\frac{1}{2}\right)dz = \frac{1}{-\beta} e^{-\beta z}\sqrt{z}\Big|_0^{p_0^2}$$

$$\Rightarrow \int_{p_0^2}^\infty e^{-\beta z}\left(\sqrt{z} - \frac{1}{\beta}\frac{1}{\sqrt{z}}\frac{1}{2}\right)dz \approx \int_{p_0^2}^\infty \sqrt{z}\, e^{-\beta z}\,dz \approx -\frac{1}{\beta} e^{-\beta z}\sqrt{z}\Big|_{p_0^2}^\infty =$$

$$= \frac{1}{\beta} p_0 e^{-\beta p_0^2} \Rightarrow 2\int_{p_0}^\infty p^2 e^{-\beta p^2}\,dp = +\frac{1}{\beta} p_0 e^{-\beta p_0^2} \Rightarrow$$

$$\Rightarrow P = \int_0^\infty C p^2 e^{-\beta p^2}\,dp - \int_{p_0}^\infty C p^2 e^{-\beta p^2}\,dp = 1 - \frac{C}{2}\,\frac{1}{\beta}\, p_0 e^{-\beta p_0^2} =$$

$$= 1 - \frac{4\pi}{(2\pi mkT)^{3/2}}\cdot\frac{1}{2}\cdot 2mkT\cdot 0{,}1\cdot m\cdot\sqrt{\frac{8kT}{\pi m}}\cdot e^{-0{,}1^2 m\,\frac{8kT}{\pi}\cdot\frac{1}{2mkT}} =$$

$\Rightarrow 1 - \frac{1}{\pi} \cdot 4 \cdot 0{,}1 \cdot e^{-0{,}01 \cdot \frac{4}{\pi}} \approx 1 - \frac{0{,}4}{\pi} \cdot 0{,}987 \approx 0{,}87$

Ответ: $P \approx 0{,}87$

№3

$dp = 2mv\, dN = \int 2mv\, n \cdot dS_0\ \omega(p_x)\ dp_x =$

$= 2m \cdot n \int dt \frac{p_x}{m} \cdot l_y v\ \omega(p_x)\ dp_x$

$F = \frac{dp}{dt} = 2m \cdot n \int \frac{p_x}{m} l_y v\, \omega(p_x)\ dp_x$

Давление: а) $P = \frac{dF}{dl_y} = 2mn \int v^2\ \omega(p_x)\ dp$

б) из (№1): $A = 4\pi \int_0^\infty p^2 e^{-\beta p c}\ dp = \frac{4\pi}{-\beta c} e^{-\beta p c} p^2 \Big|_0^\infty + \int \frac{4\pi}{\beta c} 2p\, e^{-\beta p c}\, dp =$

$= \frac{4\pi}{\beta^2}$

$P = \frac{2n}{m} \int p^2 \frac{e^{-\frac{p^2}{2mkT}}}{\sqrt{2\pi mkT}}\, dp$

№4 Дано: m, v_0, n_0, α $\quad \vec{v} - ?, n - ?, T - ?$

$dE = \frac{mv^2}{2} dN = \frac{mv^2}{2} j\, dt\, dS = \frac{mv_0^2}{2} n\, dS \cdot v_0\, dt$

$P_{изл} = \frac{dF}{dS} = \frac{d^2E}{dS\, dt} = \frac{mnv_0^3}{2}$

$\langle p \rangle = A \int_0^\infty p \cdot e^{-\beta\left(\frac{p^2}{2m} + \frac{I\omega^2}{2}\right)} d^3p\, d^2\omega$

$A = \left(2\pi mkT\right)^{3/2} \cdot \left(\int_{-\infty}^{\infty} e^{-\frac{\beta I \omega^2}{2}} d^2\omega\right)^{-1} = \left((2\pi mkT)^{3/2} \cdot \sqrt{\frac{\pi}{\beta I}} \int_{-\infty}^{\infty} 2\pi\omega\, e^{-\frac{\beta I \omega^2}{2}} d\omega\right)^{-1}$

$= A_1 \left(\int_{-\infty}^{\infty} \pi\, e^{-\frac{\beta I \omega^2}{2}} d\omega^2\right)^{-1} = A_1 \left(\pi \sqrt{\frac{\pi 2 kT}{I}}\right)^{-1} =$

$= \left(4\pi^3 (kT)^2\, m^{3/2} \sqrt{\frac{1}{I}}\right)^{-1}$

№ $\cancel{3}$ 4

$$\langle p\rangle = A_1 \cdot \int_0^\infty p\, e^{-\beta\frac{p^2}{2m}}\, d^3p \quad \left(= A\int p\, e^{-\beta\frac{p^2}{2m} - \beta\frac{I\omega^2}{2}}\, d^3p\, d^2\omega\right)$$

$$\langle p\rangle = \sqrt{\frac{8mkT}{\pi}}; \quad \cancel{\langle\omega\rangle = A_2 \int_{-\infty}^{\infty} \omega\, e^{-\beta\frac{I\omega^2}{2}}\, d^2\omega = A_2 \frac{\pi}{2}\frac{2kT}{I} =}$$

$$\cancel{= \frac{kT}{I}\cdot\sqrt{\frac{2I}{\pi^3 kT}} = \sqrt{\frac{2kT}{I\pi^3}}}$$

$$E = \left(\frac{p^2}{2m} + \frac{I\omega^2}{2m}\right)\cdot N = \left(\frac{\langle p^2\rangle}{2m} + \frac{I\langle\omega^2\rangle}{2m}\right)\cdot N;$$

$$\langle p^2\rangle = 3mkT; \quad \langle\omega^2\rangle = A_2 \int_0^\infty \omega^2 e^{-\frac{\beta\omega^2 I}{2}}\, d^2\omega =$$

$$= A_2 \int_0^\infty 2\pi\omega^3 e^{-\frac{\beta\omega^2 I}{2}}\, d^2\omega = A_2\pi\cdot\left(\frac{2}{-\beta I} e^{-\frac{\beta\omega^2 I}{2}}\omega^2 \Big|_0^\infty +\right.$$

$$\left. + \int_0^\infty \frac{2}{\beta I} e^{-\frac{\beta\omega^2 I}{2}}\, d\omega^2\right) = A_2\pi\cdot\frac{2}{\beta I}\cdot\frac{2}{\beta I} = \frac{4k^2T^2}{I^2}\frac{\pi}{\pi}\frac{\sqrt{2I}}{\sqrt{\pi kT}} =$$

$$= \cancel{\frac{8}{I}(kT)^{3/2}\frac{1}{\sqrt{\pi}}} \left(\frac{2}{I}kT\right)^{3/2}$$

$$E = \left(\frac{9m^2k^2T^2}{2m} + \frac{I\left(\frac{2}{I}kT\right)^3}{2m}\right)\cancel{N} = \frac{5}{2}\cancel{N}kT$$

№ 4

$$P_{in} = P_{out} = nKT = \frac{m n_0 v_0^3}{2} \Rightarrow n = \frac{m n_0 v_0^3}{2KT}$$

$$\frac{9}{2} mKT + \frac{4}{I^2 m} K^2 T^2 = \frac{5}{2}$$

$$\alpha T^2 + \beta T - \frac{5}{2} = 0$$

$$T = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha c}}{2\alpha} = \frac{-\frac{4K^2}{mI^2} \pm \sqrt{\frac{16K^4}{m^2 I^4} + 4 \frac{9}{8} mK \cdot \frac{5}{8}}}{9mK} =$$

$$= \sqrt{\frac{16}{81} \frac{K^2}{m^4 I^4} + \frac{5}{9mK}} - \frac{4K}{9m^2 I^2} > 0$$

$$n = \frac{m n_0 v_0^3}{2KT}$$

$$\langle v \rangle = \sqrt{\frac{8KT}{\pi m}}$$