

## Solving Abrupt Silicon PN Junction Question

1. Calculate  $V_{bi}$ .
2. Look up  $D_p$  on the diffusion coefficient chart.
3. Calculate the diffusion length:  $L_p = \sqrt{D_p \tau_p}$  (for  $p^+n$ ) - or -  $L_n = \sqrt{D_n \tau_n}$  (for  $pn^+$ )
4. If:  $L_p > x_p$  (for  $p^+n$ ) - or -  $L_n > x_n$  (for  $pn^+$ ), the diode is short-base.

## Bipolar Junction Transistor

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}; \beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$$

$$\alpha_F = \gamma_F \alpha_T; \alpha_{dc} = \gamma_{dc} \alpha_T$$

$$\alpha_R = \gamma_R \alpha_T$$

$$\alpha_{T(npn)} = \frac{I_{Cn}}{I_{E2n}}; \alpha_{T(pnp)} = \frac{I_{Cp}}{I_{Ep}}$$

$$\alpha_T = 1 - \frac{x_B^2}{2D_n \tau_n} = 1 - \frac{x_p^2}{2L_n^2}$$

$$I_{pE} = \frac{-qA_E n_i^2 D_p}{N_{dE} x_E} \exp\left(\frac{qV_{BE}}{k_B T} - 1\right)$$

$$I_{pE} = \frac{-qA_E n_i^2 D_p}{N_{dE} L_p} \exp\left(\frac{qV_{BE}}{k_B T} - 1\right)$$

$$\gamma_F = \left[1 + \frac{x_B N_{aB} D_{pE}}{x_E N_{dE} D_{nB}}\right]^{-1}$$

$$\gamma_R = \left[1 + \frac{x_B N_{aB} D_{pC}}{x_E N_{dC} D_{nB}}\right]^{-1}$$

$$\gamma_F = \left[1 + \frac{x_B N_{aB} D_{pE}}{L_{pE} N_{dE} D_{nB}}\right]^{-1}$$

$$\gamma_R = \left[1 + \frac{x_B N_{aB} D_{pC}}{L_{pC} N_{dC} D_{nB}}\right]^{-1}$$

$$\gamma_{(nnp)} = \frac{I_{En}}{I_E} = \frac{|I_{En}|}{|I_{En}| + |I_{Ep}|}$$

$$\gamma_{(pnp)} = \frac{I_{Ep}}{I_E} = \frac{|I_{Ep}|}{|I_{En}| + |I_{Ep}|}$$

$$I_E = I_{Ep} + I_{En} \quad I_C = I_{Cp} + I_{Cn} \quad I_B = \frac{I_C - I_{CE0}}{\beta_{dc}}$$

$$I_C = \alpha_{dc} I_E + I_{CB0} \quad I_C = \beta I_B + I_{CE0} \quad I_{Cn} \approx I_{BC0}$$

$$I_{Cn} \approx I_{BC0}$$

$$I_{Cn} \approx I_{BC0}$$

Electron Current Density (ECD) - constant base doping

$$J_n = \frac{qD_n n_i^2}{x_B N_{aB}} \left[ \exp\left(\frac{qV_{BC}}{k_B T}\right) - \exp\left(\frac{qV_{BE}}{k_B T}\right) \right] \quad (\text{A/cm}^2)$$

ECD - non-constant base doping (A/cm<sup>2</sup>)

$$J_n = J_0 \left[ \exp\left(\frac{qV_{BC}}{k_B T}\right) - \exp\left(\frac{qV_{BE}}{k_B T}\right) \right]$$

$$J_0 = \frac{q^2 n_i^2 \tilde{D}_n}{Q_B}, \quad \tilde{D}_n = \text{avg}(D_n)$$

Collector Current Density (under active bias)

$$J_C \approx J_0 \exp\left(\frac{qV_{BE}}{k_B T}\right)$$

Recombination of excess minority carriers in the base

$$I_{rB} = \frac{qA_g n_i^2 x_B}{2N_{aB} \tau_n} \left[ \exp\left(\frac{qV_{BE}}{k_B T}\right) - 1 \right]$$

Collector-Emitter Breakdown Voltage in terms of the Collector-Base Breakdown. Note that  $m \approx 4$

$$BV_{CE0} = \frac{BV_{CB0}}{\beta^{1/m}}$$

## Finding $\beta$ Of a BJT

1. Look-up  $D_{pE}$  and  $D_{pC}$  on chart
2. Find  $L_{pE} = \sqrt{D_{pE} \tau_{pE}}$  and  $L_{pC} = \sqrt{D_{pC} \tau_{pC}}$
3. Check if emitter is long or short  $L_{pE} > x_B \rightarrow$  long emitter or  $L_{pE} < x_B \rightarrow$  short emitter.
4. Find  $\gamma_F$ , (Short Emitter Forward Efficiency)
5. Find  $\alpha_T$  (Base Transport Factor)
6. Find  $\alpha_F$  (Base Transport Factor)
7. Find  $\beta_F$  (Current Gain)

## Designing an Prototype NPN Structure for an Amplifier

1. Assume these doping levels:  $N_{dC} = 10^{16} \text{cm}^{-3}$  and  $N_{aB} = 5 \times 10^{16} \text{cm}^{-3}$
2. Calculate  $V_{bi} = \frac{k_B T}{q} \ln \left[ \frac{N_d N_a}{n_i^2} \right]$
3. Use  $V_a$ , desired punch through voltage.
4. Calculate (shown as a design parameter)  $x_B = \left( \frac{N_{aB}}{N_{dC}} \right)^{-1} \left[ \frac{2\epsilon_s}{q} \left( \frac{1}{N_{aB}} + \frac{1}{N_{dC}} \right) (V_A - V_{bi}) \right]^{1/2}$
5. Calculate  $\alpha_T = 1 - \frac{x_B^2}{2D_n \tau_n}$  use hole curve ( $D_n = 23 \text{cm}^2 \text{s}^{-1}$ ) for doping levels above.
6. With these doping levels  $\alpha_t \cong 1$ . Since  $\alpha_T \cong 1$ , assume  $\alpha_F = \gamma_F$ .
7. Find the ratio =  $\frac{N_{dE}}{D_{pE}} = \underbrace{\left( \frac{x_B N_{aB}}{x_E D_{nB}} \right)}_{\text{ratio}} \left( \frac{1}{\gamma_F} - 1 \right)$
8. Using  $D_{pE} = \frac{N_{dE}}{\text{ratio}}$ , find a good value for  $N_{dE}$  that allows you to look up  $D_{pE}$  on the diffusion chart. Use the curve for holes.

## Ebers-Moll Equations

$$I_E = I_F - \alpha_R I_R \quad \text{Emitter Current}$$

$$I_C = \alpha_F I_F - I_R \quad \text{Collector Current}$$

$$I_B = I_E - I_C$$

$$I_B = (1 - \alpha_F) I_F + (1 - \alpha_R) I_R \quad \text{Base Current}$$

$$I_{F0} = qA \left[ \frac{D_E n_{E0}}{L_e} + \frac{D_B p_{B0}}{W} \right]$$

$$I_F = I_{F0} \left[ e^{(qV_{cB}/k_B T)} - 1 \right] \quad \text{Forward Coefficient}$$

$$I_{R0} = qA \left[ \frac{D_E n_{C0}}{L_C} + \frac{D_B p_{B0}}{W} \right]$$

$$I_R = I_{R0} \left[ e^{(qV_{cB}/k_B T)} - 1 \right] \quad \text{Reverse Coefficient}$$

$$\alpha_F I_{F0} = \alpha_R I_{R0} = I_S \quad \text{Rev Cur Component}$$

$$\frac{I_{F0}}{I_{R0}} = \frac{\alpha_R}{\alpha_F} \quad \text{reciprocity Relation.}$$

$$\beta_f = \frac{\alpha_F}{1-\alpha_F} \quad \text{Normal Forward } \beta$$

$$\alpha_R I_R = \frac{qA D_B p_{B0}}{W} \left[ e^{(qV_{cB}/k_B T)} - 1 \right] \quad \text{Ebers-Moll Eqns}$$

$$\alpha_R I_R = \frac{qA D_B p_{B0}}{W} \left[ e^{(qV_{cB}/k_B T)} - 1 \right] \quad \text{Vol III - 47.}$$

## Transit Time and Frequency Response

$$\tau_1 = r_e C_{jE} \quad \text{Emitter-Base Capacitance Charging Time}$$

$$\tau_2 = \quad \text{Collector Capacitance Charging Time}$$

$$r_C C_{jC}$$

$$\tau_B = \frac{x_B^2}{2D_{nB}} \quad \text{Base Transit Time}$$

$$\tau_C = \frac{x_{dc}}{v_{sat}} \quad \text{Collector Depletion Region Transit Time}$$

$$(x_{dc}: \text{Depletion region width of collector})$$

$$(v_{sat}: \text{Saturation velocity, } \approx 10^7 \text{ m/s})$$

$$\tau_{EC} = \tau_1 + \tau_2 + \tau_B + \tau_C \quad \tau_{EC} = \tau_C + \tau_B + \tau_E$$

$$\text{Emitter to Collector Transit Time. } f_T = \frac{1}{2\pi \tau_{EC}}$$

$$\text{Cut-off Frequency.}$$

**Amplification:** For amplifying BJTs, the thickness and resistivity of the collector are both large. Th is results in an increased breakdown voltage and reduces the early effect.

**Switching:** For switching BJT's, saturation (On-State) resistance must be minimized, which requires a very thin collector layer with a resistivity of a few tenths of an  $\Omega$ -cm.

The Early Effect results in an increase in  $I_C$  due to base-width modulation when  $V_{CB}$  is increased.

**Finding  $V_T$ :** Using substrate resistivity  $\rho \rightarrow N_A \& N_D$  (Vol I - pg 71) 2. Calculate  $\phi_p$  3. Calculate  $Q_f$  from given data  $Q_f/q$  4. Calculate  $C_{ox}$  5. Find  $\phi_{MS}$  (depends on gate material, use Vol I - pg 96. 6. Calculate  $V_{FB}$  7. Calculate  $V_T$ .

## MOSFET's

$K_s = 11.8$  Dielectric Constant of Si (at 300 K)  
 $K_o = 3.9$  Dielectric Constant of SiO<sub>2</sub> (at 300K)  
 $\epsilon_s = K_s \epsilon_0 = 1.1045 \times 10^{-12} \frac{F}{cm}$  Permittivity of Si (at 300K)  
 $\epsilon_{ox} = K_o \epsilon_0 = 345.15 \times 10^{-15} \frac{F}{cm}$  Permittivity SiO<sub>2</sub> (300K)

P-type Si MOS Structure → N-channel Device

N-type Si MOS Structure → P-channel Device

Ref Voltage rel to the semicond doping concent.

$$\phi_{F(p-type)} = \phi_p = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right)$$

$$\phi_{F(n-type)} = \phi_p = -\frac{kT}{q} \ln \left( \frac{N_D}{n_i} \right) \text{ Semicond Surf Pot}$$

at Depletion-Inversion Transition Point  $\phi_S = 2\phi_F$

Flat-band voltage (voltage that produces flat energy bands in the oxide and silicon)

$V_{FB} = \phi_{MS} - \frac{Q_f}{C_{ox}}$  With a charge on the oxide layer.

$V_{FB} = \phi_M - \phi_S = \phi_{MS}$  oxide layer free of charge.

$$W_T = \left[ \frac{2K_s \epsilon_0}{q N_A} (2\phi_p) \right]^{1/2} \text{ Depletion Width Iv p 43}$$

$$\phi(x) = \frac{1}{q} [E_f - E_i(x)] \text{ Potential in Silicon}$$

$$\phi_S = \phi(0) = \frac{1}{q} (E_f - E_i(0)) \text{ Surface Potential}$$

$$C_{ox} = \frac{\epsilon_{ox}}{x_{ox}} \dot{A} = 10^{-10} m \text{ Oxide Layer Capacitance}$$

$$Q_{d(max)} = \sqrt{4k_s \epsilon_0 q N_A |\phi_P|} \text{ Space Charge Density (max)}$$

$$V_T = V_{FB} + 2|\phi_p| + \frac{|Q_{d(max)}|}{C_{ox}}, \text{ Threshold Voltage (T.V.)}$$

$$V_T = 2\phi_F - \frac{K_s x_{ox}}{K_o \epsilon_0} \left[ \frac{4q N_A}{K_s \epsilon_0} \phi_F \right]$$

$$\Delta V_G = V'_T - V_T \text{ (Threshold Adjustment)} \Delta V_G = \frac{-Q_l}{C_{ox}}$$

$Q_l = \pm q N_l \rightarrow N_l = \pm \frac{Q_l}{q} V'_T$  Un-adjusted T.V.  $V_T$ :  
 Adjusted T.V.  $N_l$ : # of implanted ions  $Q_l$ : Implant-related charge/cm<sup>2</sup> Donor{+} or Acceptor {-}

$$I_D = \frac{Z \bar{\mu}_n C_{ox}}{L} \left[ (V_G - V_T) V_D - \frac{V_D^2}{2} \right] \text{ Square-law theory}$$

Z: Width of MOSFET  $\bar{\mu}_n$  Eff hole mobil Vol IV. Pg 73

**Long Channel MOSFET Equation** Bulk Charge Factor ( $\alpha$ )  $I_D = \mu C_{ox} \frac{W}{L} \left[ (V_G - V_T - \frac{1}{2} V_D) V_D \right]$

$$I_D = \mu C_{ox} \frac{W}{2\alpha L} \left[ (V_G - V_T - \frac{1}{2} V_D) V_D \right]$$

**Channel Carrier Velocity** (Using Long-Channel Theory)  $V = \frac{\mu_n C_{ox} \left[ (V_G - V_T - \frac{1}{2} V_D) V_D \right]}{Q_N L}$

Note: The saturation velocity of carriers in Silicon is:  $v_{sat} \approx 10^7$  cm/s. If this equation yields a velocity  $v > v_{sat}$ , long channel theory does not apply in this situation.  $Q_{n(source)} = C_{ox}(V_G - V_T)$ ,  $Q_{n(drain)} = C_{ox}(V_G - V_D - V_T)$ .

**Drain Saturation Voltage**  $V_{D(sat)} = V_G - V_T$

$$V_{D(sat)} = \frac{V_G - V_T}{\alpha}$$

$g_d, g_m$  Small Signal Parameters and Conductance

$$f_{max} = \frac{g_m}{2\pi C_{ox}} = \frac{\bar{\mu}_n V_D}{2\pi L^2} \text{ Cutoff f } V_D \leq V_{D(sat)}$$

$$\text{Using } I_{D(sat)} = \mu_n C_{ox} \frac{W}{2L} (V_G - V_T)^2$$

$$\text{Channel Dimensions } \frac{W}{L} = \frac{2I_{D(sat)}}{\mu_n C_{ox} (V_G - V_T)^2}$$

## MOSFET Integrated Circuit Applications

$$V_T = V_{FB} + 2|\phi_p| + \frac{Q_{d(max)}}{\epsilon_{ox}} (d_1 + d_2) + \frac{Q_{fg}}{\epsilon_{ox}} d_1 \text{ Charge}$$

stored on a floating gate memory cell

$$|Q_{fg}| = \frac{\epsilon_{ox}}{d_1} \left[ V_T - V_{FB} - 2\phi_p - \frac{Q_{d(max)}}{\epsilon_{ox}} (d_1 + d_2) \right],$$

$Q_{fg}$  = Floating Gate Charge Density  $V_{FB} = \phi_{MS}$

Step 1: Find  $V_{bi}$  (n-well to source/drain junction) using  $N_d$  from the n-well and  $N_a$  from the p-channel source/drain.

Step 2: Find  $x_d$  using the doping levels and  $V_{bi}$ .

Step 3: Find  $V_{bi}$  (n-well to p-substrate junction) using  $N_d$  from the n-well and  $N_a$  from the p-substrate.

Step 4: Find  $x_n$  using  $V_{bi}$  and  $V_a = V_{DD}$ , and the same doping level as step 3.

Step 5: The minimum required n-well depth to prevent punchthrough at this voltage is:

$$d_{n-well} = d_{p-channelsrc/drn} + x_{d(step2)} + x_{n(step4)}$$

$$V_{bi} = \frac{k_B T}{q} \ln \left( \frac{N_d N_a}{n_i^2} \right)$$

$$x_d = x_n + x_p = \left[ \frac{2\epsilon_s}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) (V_a - V_{bi}) \right]^{1/2},$$

$x_n = \left\{ \frac{2K_s \epsilon_0}{q} (V_{bi} + V_a) \left[ \frac{N_a}{N_d(N_a + N_d)} \right] \right\}^{1/2}$  Note: The  
 p-channel source/drain depth  $d_{p-channelsrc/drn}$  should be given in the question.

CMOS Well-Depth Design: Finding minimum well depth to prevent vertical punch through.

**CMOS Structures** P-Well: The substrate is N-Type. The N-Channel device is built into a P-Type well within the parent N-Type substrate. The P-channel device is built directly on the substrate.

N-Well: The substrate is P-Type. The N-channel device is built directly on the substrate, while the P-channel device is built into a N-type well within the parent P-Type substrate.

$$g_d = \frac{Z \bar{\mu}_n C_o}{L} (V_G - V_T) \quad (V_D = 0)$$

**Practise Test 2** 1) Which of the following can reduce the base transit time? c) Short base width.

2) Design the doping levels and dimensions of a silicon npn bipolar transistor such that the dc current gain is 320 and the Gummel Number is  $10^{12} \text{cm}^{-2}$ . Assume that  $\tau_n = 10^{-7} \text{s}$  in the base,  $\tau_p = 10^{-8} \text{s}$  in the collector.

$$GN = Q_B = \int_0^{x_B} N_{aB}(x) dx.$$

**Table 3.1** MOSFET Small Signal Parameters.\*

	Below pinch-off ( $V_D \leq V_{Dsat}$ )	Post pinch-off ( $V_D > V_{Dsat}$ )
Square law	$g_d = \frac{Z \bar{\mu}_n C_o}{L} (V_G - V_T - V_D)$	$g_d = 0$
Bulk charge	$g_d = \frac{Z \bar{\mu}_n C_o}{L} [V_G - V_T - V_D - V_w(\sqrt{1 + V_D/2\phi_F} - 1)]$	$g_d = 0$
Square law	$g_m = \frac{Z \bar{\mu}_n C_o}{L} V_D$	$g_m = \frac{Z \bar{\mu}_n C_o}{L} (V_G - V_T)$
Bulk charge	$g_m = \frac{Z \bar{\mu}_n C_o}{L} V_D$	$g_m = \frac{Z \bar{\mu}_n C_o}{L} V_{Dsat}$ with $V_{Dsat}$ per Eq. (3.27)

\*Entries in the table were obtained by direct differentiation of Eqs. (3.15), (3.20), and (3.26). The variation of  $\bar{\mu}_n$  with  $V_G$  was neglected in establishing the  $g_m$  expressions.

Figure 1: Vol IV Mosfet table

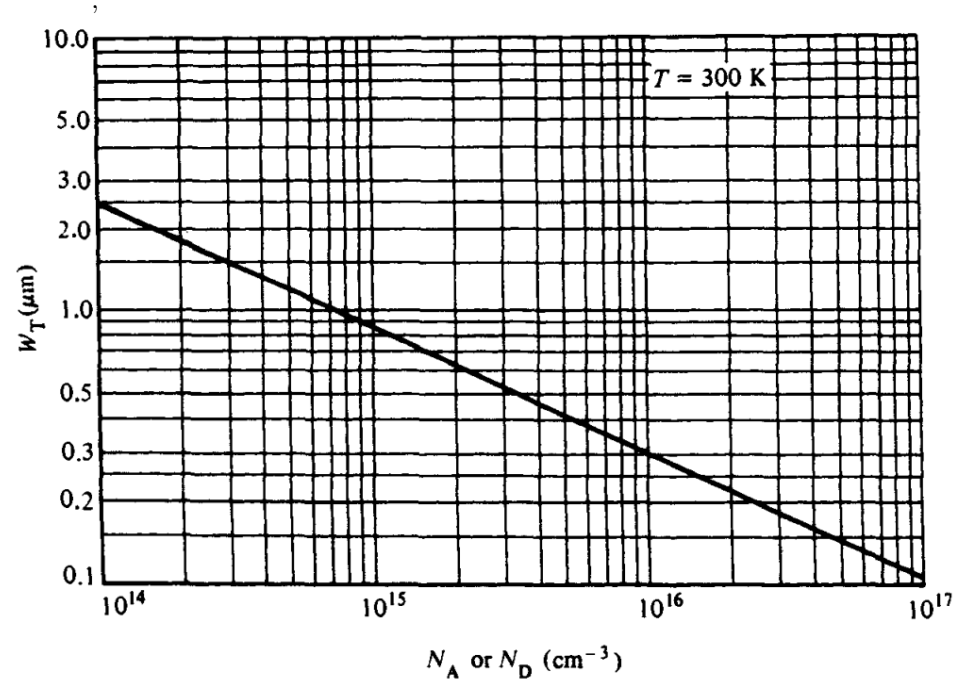


Figure 2: Doping dependence of the maximum equilibrium depletion width inside silicon devices maintained at 300 K.

**Table 12.1** | Notation used in the analysis of the bipolar transistor

Notation	Definition
<b>For both the npn and pnp transistors</b>	
$N_E, N_B, N_C$	Doping concentrations in the emitter, base, and collector
$x_E, x_B, x_C$	Widths of neutral emitter, base, and collector regions
$D_E, D_B, D_C$	Minority carrier diffusion coefficients in emitter, base, and collector regions
$L_E, L_B, L_C$	Minority carrier diffusion lengths in emitter, base, and collector regions
$\tau_{E0}, \tau_{B0}, \tau_{C0}$	Minority carrier lifetimes in emitter, base, and collector regions
<b>For the npn</b>	
$p_{E0}, n_{B0}, p_{C0}$	Thermal-equilibrium minority carrier hole, electron, and hole concentrations in the emitter, base, and collector
$p_E(x'), n_B(x), p_C(x'')$	Total minority carrier hole, electron, and hole concentrations in the emitter, base, and collector
$\delta p_E(x'), \delta n_B(x), \delta p_C(x'')$	Excess minority carrier hole, electron, and hole concentrations in the emitter, base, and collector
<b>For the pnp</b>	
$n_{E0}, p_{B0}, n_{C0}$	Thermal-equilibrium minority carrier electron, hole, and electron concentrations in the emitter, base, and collector
$n_E(x'), p_B(x), n_C(x'')$	Total minority carrier electron, hole, and electron concentrations in the emitter, base, and collector
$\delta n_E(x'), \delta p_B(x), \delta n_C(x'')$	Excess minority carrier electron, hole, and electron concentrations in the emitter, base, and collector

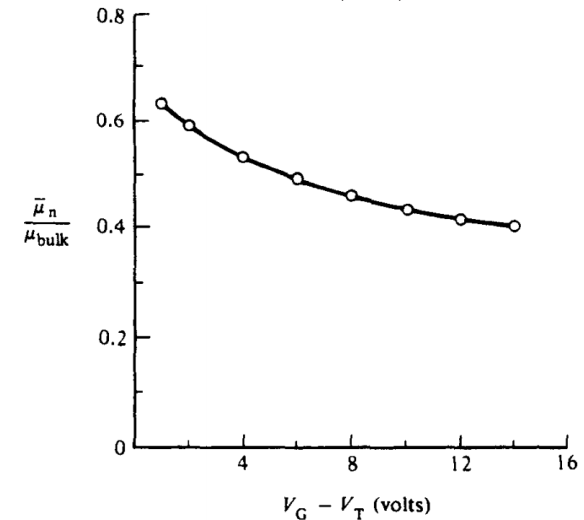


Figure 3: Vol IV Mosfet Gate voltages

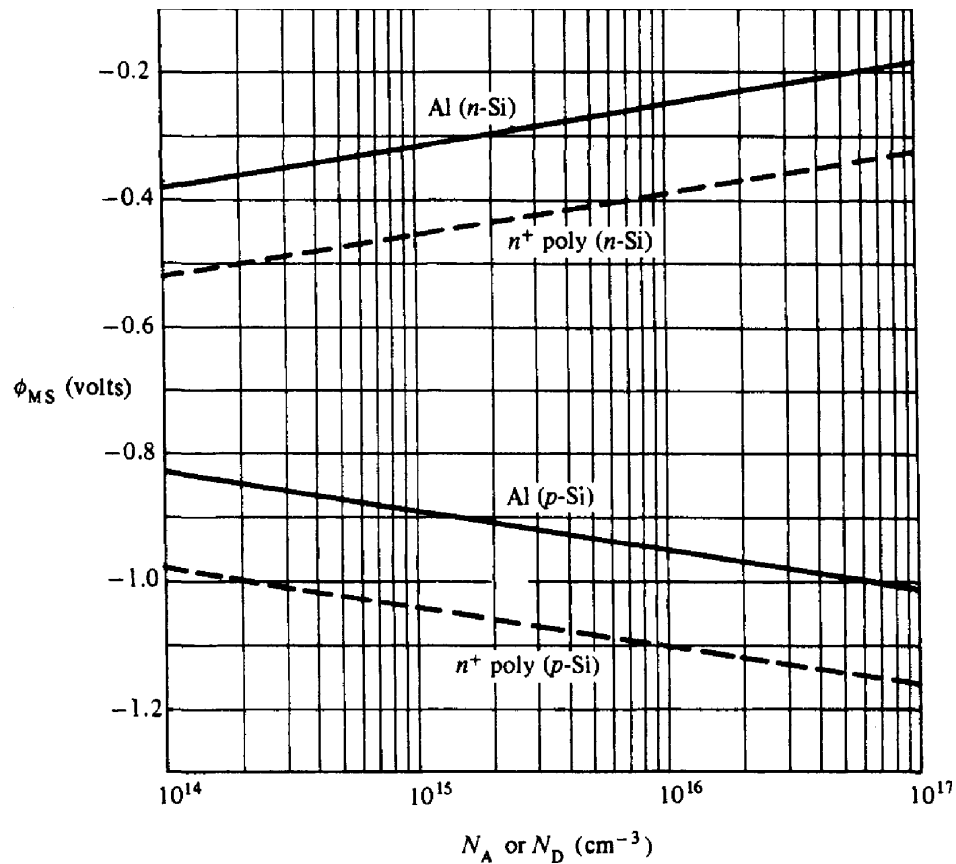


Figure 4: Workfunction difference as a function of a n- and p-type dopant concentration in  $n^+$  poly-Si-gate and Al-gate  $SiO_2 - Si$  structures. ( $T = 300K$ .  $\phi'_M - \chi' = -0.18 eV$  for the  $n'$  poly-Si-gate structure;  $\phi'_M - \chi' = 0.03eV$  for the Al-gate structure.)

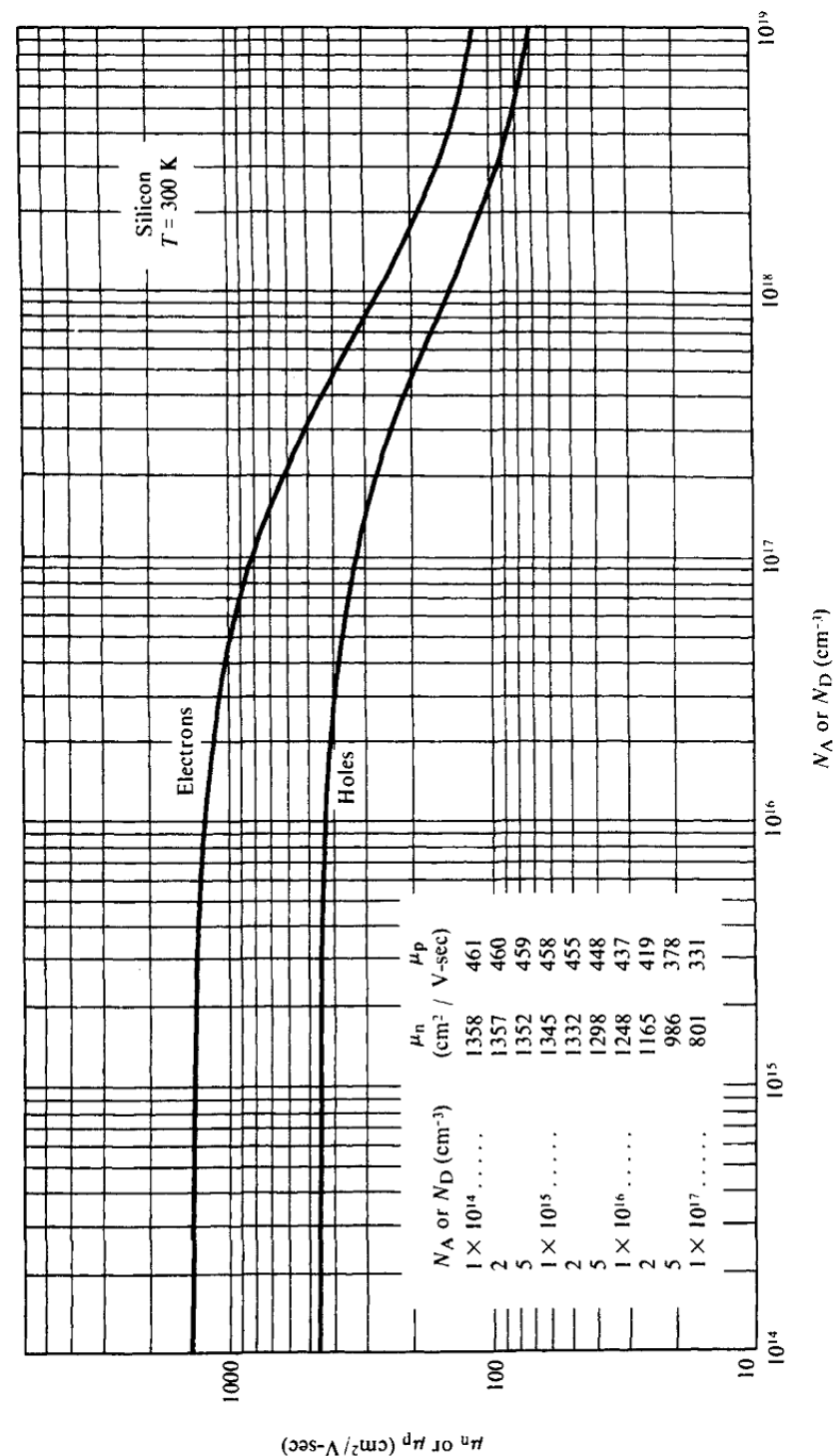
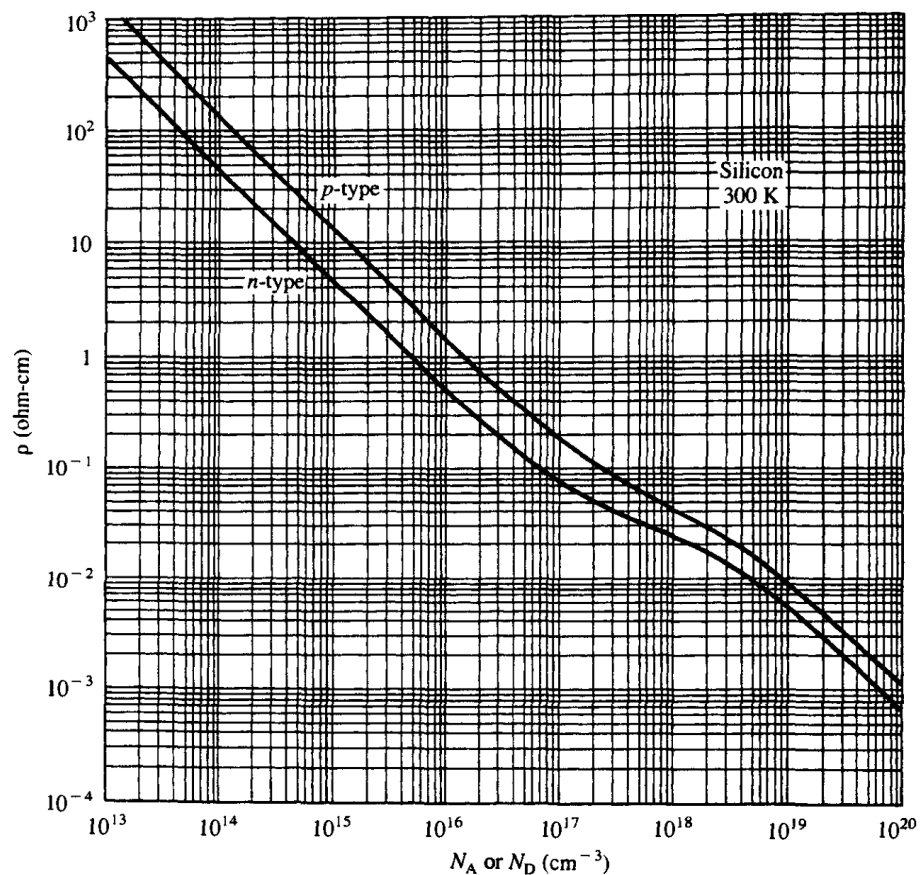
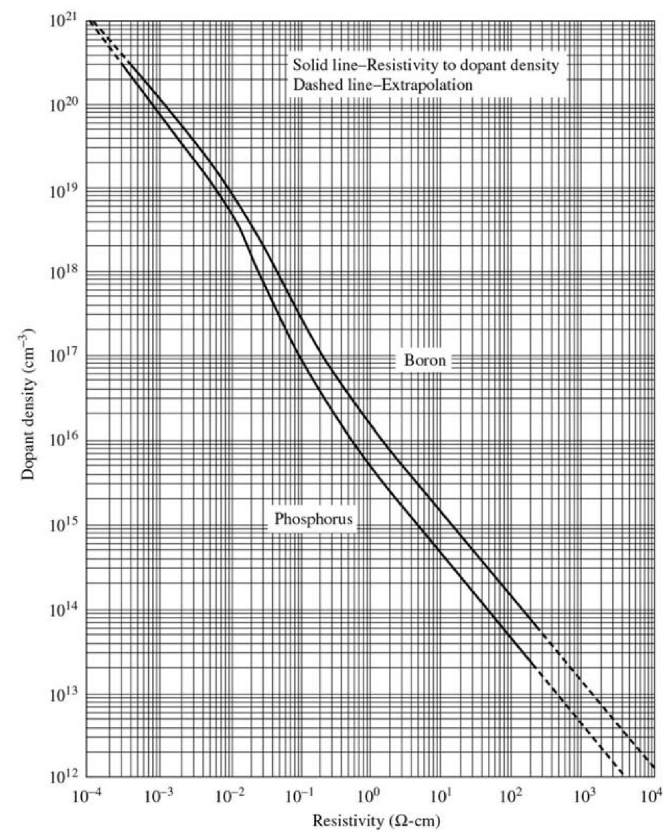
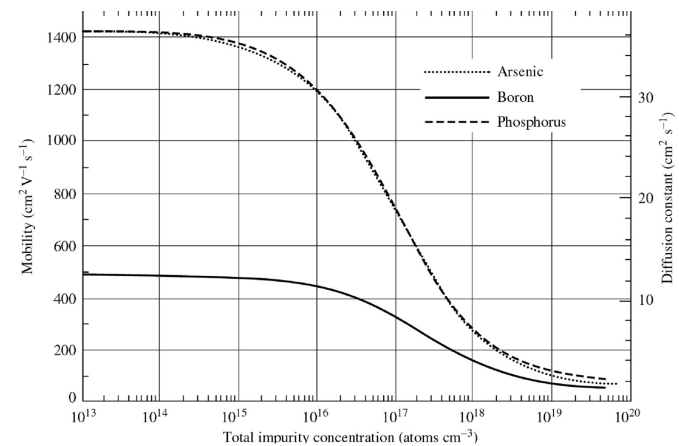


Fig. 3.5 Room-temperature carrier mobilities in silicon as a function of the dopant concentration.  $\mu_n$  is the electron mobility;  $\mu_p$  is the hole mobility.

Figure 5: Diffusion Constant



**Fig. 3.7** Si resistivity versus impurity concentration at 300 K.



**Figure 6: Boron Chart**

