[16] **4.** Let B_i denote the basis variables in the i-th dictionary of the revised simplex method. Say that $A_{B_7} = 2I$ with I the identity matrix, and that

$$A_{B_8} = A_{B_7}E, \qquad A_{B_9} = A_{B_8}F,$$

where

$$E = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}, \qquad F = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

(a) Compute $c_B^{\mathrm{T}} A_{B_9}^{-1}$ as you would in the revised simplex method, where

$$c_B^{\mathrm{T}} = [\,7 \quad 8 \quad 9\,].$$

(b) Let A, B, C be matrices that are $1 \times n$, $n \times n$, and $n \times n$ respectively. How many multiplications are required to multiply ABC, if we first multiply AB? How many if we first multiply BC? Which method requires fewer multiplications when n is large? Assume that you multiply matrices naively (taking dot products of each row of the first matrix with the columuns of the second matrix).

(c) In the revised simplex method, explain why the constants in the $\vec{x}_B$ row, namely $A_B^{-1}\vec{b}$, should be easily available from the previous iteration. [Hint: this also holds for the plain old simplex method.]

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