

- [4] 4. Given a fixed positive integer N , Claude and Rachel play a simple game. Claude secretly chooses an integer from the set $\{1, 2, \dots, N\}$, and Rachel guesses what he has chosen. If Rachel's guess is correct, Claude pays her 5 Galactic Currency Units. If her guess is not correct, no currency changes hands.

Find the optimal strategies for both players, and the expected payout to Rachel.

Suggestion: An efficient approach is to make smart conjectures about the desired quantities, and then to confirm the correctness of your proposals by applying well-known theorems.