[12] **8.** Consider the following problem, in which $\mathbf{b} = (b_1, b_2)$ is not given explicitly:

$$\begin{array}{lll} (P) & \text{Maximize} & f = 2x_1 + 3x_2 + x_3 \\ & \text{subject to} & x_1 - x_2 + 2x_3 + x_4 = b_1 \\ & & 4x_1 + 2x_2 - x_3 + x_5 = b_2 \\ & & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

(a) Find the set of all pairs (b_1, b_2) for which the given problem has an optimal basic solution with x_2 and x_3 as basic variables. Sketch this set on a Cartesian plane with axes labelled "b_1" and "b_2". [8 marks]

(b) Let $V = V(b_1, b_2)$ denote the maximum value in problem (P) as a function of the parameters. Give a simple formula for $V(b_1, b_2)$ that is valid on the set of $\mathbf{b}$-values found in part (a). [3 marks]

(c) Show that the formula from part (c) is *not* valid for all $\mathbf{b} \in \mathbb{R}^2$. (Suggestion: Show that the true value $V(-1, -1)$ is obviously larger than the formula's preduction.) [1 mark]

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