

[16] **3.** Here is a pair of similar-looking linear programming problems:

$ \begin{aligned} (P) \text{ Maximize } f &= 2x_1 + 6x_2 + 4x_3 \\ \text{subject to } &2x_1 + x_2 + x_3 \leq 4 \\ &3x_1 - x_2 + x_3 \leq 8 \\ &2x_1 + 2x_2 \leq 6 \\ &x_1, x_2, x_3 \geq 0 \end{aligned} $	$ $	$ \begin{aligned} (\tilde{P}) \text{ Maximize } f &= 2x_1 + 6x_2 + 4x_3 + 6x_4 \\ \text{subject to } &2x_1 + x_2 + x_3 + x_4 \leq 4 \\ &3x_1 - x_2 + x_3 + 2x_4 \leq 8 \\ &2x_1 + 2x_2 + x_4 \leq 6 \\ &x_1, x_2, x_3, x_4 \geq 0 \end{aligned} $
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Let s_1, s_2, s_3 be the slack variables for the three constraints of (P) , in the given order.

- (a) Find a Basic Feasible Solution for (P) , using x_3, s_2 , and x_2 as the basic variables. [4 marks]
- (b) Prove that the BFS found in part (a) is optimal for (P) . [4 marks]
- (c) Find the maximum value in (\tilde{P}) , and all decision vectors that achieve it. [8 marks]