[28] **6.** Consider the matrix game associated to the matrix

$$M = \begin{bmatrix} 1 & 2 \\ 3 & c \end{bmatrix},$$

where c is a given real number.

(a) Assuming that all pure strategies are involved in a unique equilibrium, what is player A's equilibrium strategy, $\vec{\alpha} = (\alpha_1, \alpha_2)$?

(b) For what values of c is it not the case that all pure strategies are involved in an equilibrium? Use domination to explain what are the equilibria for those values of c.

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(c) Consider a general LP, maximize $\vec{c}^{\mathrm{T}}\vec{x}$ subject to $A\vec{x} \leq \vec{b}$ and $\vec{x} \geq \vec{0}$. If the third inequality reads $x_1 + x_2 \leq 1$, and the fourth reads $x_1 + x_2 \leq 2$, can you say that one inequality "dominates" another in some reasonable sense? Explain. Same question with $3x_1 + 3x_2 \leq 1$ and $x_1 + 2x_2 \leq 2$ (recall that x_1, x_2 are both non-negative!).

(d) Write an LP for $\mathrm{MAXScream}_A$ in the matrix game above. Explain how one inequality "dominates" another (in your sense) for $c \geq 2$.

(e) Consider a general LP with an optimal solution in which a slack variable is positive. Can we discard the corresponding inequality and obtain the same optimal objective? Explain. Similarly, can we discard a decision variable that is zero? Explain.

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(f) Consider an LP with an optimal solution in which every slack variable is zero and every decision variable is positive. Can there be more decision variables than slack variables? [Hint: in a dictionary, how many variables are basic?]

(g) Explain how part (e) and (f) can reduce any to LP to linear algebra provided that you can guess which primal and dual decision variables can be 0 in certain optimal (primal and dual) solutions. Explain what this has to do with a similar observation made in class (and the notes) about matrix games that motives part (a) of this problem.

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