[32] **5.** (4 points for each part) Briefly justify your answers:

(a) In the matrix game

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 5 & 4 & 1 \end{bmatrix}$$

someone claims that Alice's optimal mixed strategy is [1/2 1/2 0] and that Betty's is [1/2 1/2 0]. Make a quick calculation (without using the simplex method) to determine whether or not this is true; explain why your quick calculation works.

(b) Consider a $n \times n$ weighted bipartite matching problem (Assignment Problem), where $n \geq 6$ and where x_{ij} is 1 if person i does task j (entirely). Argue that $x_{16}, x_{36}, x_{34}, x_{14}$ cannot all be basic in any dictionary of the simplex method.

(c) Give an example of a linear program whose first three pivots must be degenerate. How do you know that these first three pivots must be degenerate?

Continued on page 7

(d) Give an example of a linear program such that the perturbation applied to it requires no more than two pivots, but if we change the order of the rows (or equivalently change the order of $\epsilon_1, \epsilon_2, \ldots$) the simplex method can take three iterations. Justify your claim, either in words or pictures.

(e) Consider a linear program that has various constraints including $x_1 - 2x_2 \geq 4$ and $x_1 - 2x_2 \geq 7$. Can both of the slack variables corresponding to these two constraints be nonbasic in some dictionary of the simplex method? Explain.

(f) In this class we viewed an $m \times n$ matrix, A, as a matrix game giving the payout to Alice with Alice playing m pure strategies represented by A's rows, and Betty playing n pure strategies columns represented by A's columns. What matrix would you get if Alice and Betty exchange roles? Explain.

Continued on page 8

(g) Find the value of the mixed strategy games for the matrix game

$$A = \begin{bmatrix} -1 & -4 & -9 & -16 & -25 & -36 & -49 \\ -49 & -36 & -25 & -16 & -9 & -4 & -1 \end{bmatrix}.$$

Justify your answer.

(h) Consider the line $y = a + bx$ which is the best "max approximation" regression line to the data points $(0,4)$, $(1,6)$, $(2,7)$, $(3,10)$, $(5,11)$, i.e., such that

$$d = \max(|4-a|, |6-a-b|, |7-a-2b|, |10-a-3b|, |11-a-5b|)$$

is minimized. Assume that you know that at optimality (i.e., when d is minimized), a, b, d are all positive. At the optimal a, b, d, at least how many of

$$|4-a|, |6-a-b|, |7-a-2b|, |10-a-3b|, |11-a-5b|$$

must equal d? What could you say if you had ten data points instead of five?

Continued on page 9