

- [12] 5. Consider the LP maximize  $x_1 + ax_2$  subject to  $x_1 + x_2 \leq 1$  and  $x_1 + 2x_2 \leq b$  and  $x_1, x_2 \geq 0$ , where  $a$  and  $b$  are real parameters. Introducing slack variables,  $x_3, x_4$  ( $x_3$  for the first inequality), here is one possible dictionary:

$$\begin{array}{rcll} x_1 & = & 2 - b & -2x_3 & +x_4 \\ x_2 & = & b - 1 & +x_3 & -x_4 \\ z & = & 2 - a - b + ab & +(a - 2)x_3 & +(1 - a)x_4 \end{array}$$

- (a) For what range of  $a, b$  is the above dictionary optimal? Explain.
- (b) Perform one regular simplex pivot to obtain a dictionary optimal when  $b = 3/2$  and  $a = 3$ . Explain how you are choosing entering and leaving variables. For what range of  $a, b$  is this dictionary optimal?
- (c) Perform one dual pivot to obtain a dictionary optimal when  $b = 3$  and  $a = 3/2$ . Explain how you are choosing entering and leaving variables. For what range of  $a, b$  is this dictionary optimal?