[16] **7.** The linear program shown on the left below involves a scalar parameter k. When $k = 5$, the corresponding optimal dictionary is shown on the right:

$$\begin{aligned} \text{maximize } \zeta &= 4x_1 + 5x_2 \\ \text{subject to} \quad x_1 + x_2 &\le k \\ x_1 + 2x_2 &\le 8 \\ 2x_1 + x_2 &\le 8 \\ x_1, x_2 &\ge 0 \end{aligned}$$

$$\begin{aligned} \zeta &= 23 - 3x_3 - x_4 \\ \hline x_1 &= 2 - 2x_3 + x_4 \\ x_2 &= 3 + x_3 - x_4 \\ x_5 &= 1 + 3x_3 - x_4 \end{aligned}$$

(We have used $(x_3, x_4, x_5) = (w_1, w_2, w_3)$ as the slack variables for the constraints, in order.)

(a) Write a 3×5 matrix A and column vector $\mathbf{b}$ for which the constraints in the original problem can be expressed as $A\mathbf{x} = \mathbf{b}$, $\mathbf{x} \ge \mathbf{0}$. Then, in the notation of the Revised Simplex Method, find the matrices B and N associated with the optimal dictionary.

(b) Use the optimal dictionary above to find B^{-1}. Explain your method.
(*Most marks here are for a clear, RSM-based explanation/derivation. Simply finding B^{-1} by some unrelated method will not earn much.*)

(c) The given dictionary is correct only when $k = 5$. Find the general form of this dictionary in terms of k; of course, substituting $k = 5$ should restore the dictionary above.

(d) For each $k \ge 0$, find the set of maximizers for the stated problem. Also calculate and sketch the graph of the maximum value, $\zeta_{\max}(k)$, as a function of $k \ge 0$.

(e) Identify a specific value of $k > 0$ for which the dual problem has more than one minimizer. Find all dual minimizers for the k you choose.

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