

[20] **3.** Consider the problem maximize $x_1 + 2x_2$ subject to $x_1 + x_2 \leq 1$ and $x_1, x_2 \geq 0$.

(a) Write the dual LP.

(b) Consider a proposed optimal solution $x_1 = 1$ and $x_2 = 0$. Use complementary slackness to find the dual solution to this proposed solution. Does the dual solution verify that the proposed solution is optimal?

(c) Same questions as in part (b) for a proposed optimal solution $x_1 = 0$ and $x_2 = 1$.

- (d) Consider the problem maximize $x_1 + 2x_2$ subject to $x_1 + x_2 \leq 1$ and $x_1 + x_2 \leq 1$ and $x_1, x_2 \geq 0$ (yes, this is essentially the same LP, except that the constraint $x_1 + x_2 \leq 1$ appears twice). Write the dual LP. Will complementary slackness always yield a unique dual solution? Explain.

- (e) Consider the LP maximize $x_1 + x_2$ subject to $2x_1 \leq 1$ and $x_1 + x_2 \leq 1$ and $2x_2 \leq 1$ and $x_1, x_2 \geq 0$. Will complementary slackness always yield a unique dual solution? Explain. (You are not required to write down the dual LP if you can argue convincingly without it.)