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Discrete mathematics

Solo assignment 4.

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Problem 1.

Let's prove the rule that if n is an even integer, then n**2 is also even.

To start let n be an even integer,

Now let k be some integer so that $n = 2k$.

Now we can write the original equation as:

$$(2k)^{**}2 = 4k^{**}2 = 2(2k^{**}2)$$

2k**2 is equal to some integer so we can rewrite it as x and then we get 2x.

2x is the pattern that all even numbers follow and therefore we have proven what we wanted to prove.

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Problem 2.

Let's prove the rule that if n is an integer and 5n-1 is even then n is odd using contraposition.

So if n is even then 5n-1 should be odd.

Let's make a new integer k such that n=2k.

Now we get:

$$5(2k)\text{-}1 = 10k\text{-}1 = 2(5k)\text{-}1$$

5k is some integer that can be rewritten to x.

Then we get 2x-1, 2x is the pattern of even numbers so 2x-1 must be odd and therefore we've proven what we wanted to prove.

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Problem 3.

Let us prove that the absorption law e.g. $A \cup (A \cap B) = A$ holds true.

A union is defined in set builder notation as being:

$$A \cup B = \{x|x \in A \vee x \in B\}$$

meaning a set where x is an element in set A or x is an element in set B, and an intersection is defined as such:

$$A \cap B = \{x|x \in A \wedge x \in B\}$$

meaning a set where x is an element in both set A and set B.

Now we can rewrite our problem as $\{x|x \in A \vee x \in \{x|x \in A \wedge x \in B\}\}$ $= \{x|x \in A\}$.

Meaning create a set where element x is in set A OR in the set where element x is in set A AND in set B.

The inner set of $\{x|\ x \in A \wedge x \in B\}$ contains no elements that are exclusive to B, cause x must also be included in set A in this new set.

In conclusion, because no elements are exclusive to the set B, if you take all elements that are in $\{x|x \in A \wedge x \in B\}$ OR in A OR both you will get set A back as a result.

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Problem 4.

a)

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$1100 = 370 + 410 + 500 - 200 - 150 - 100 + |A \cap B \cap C|$$

$$1100 = 830 + |A \cap B \cap C|$$

$$|A \cap B \cap C| = 270 - 200$$

$$|A \cap B \cap C| = 70$$

b)

$$|A| - |A \cap B| - |A \cap C| + |A \cap B \cap C| = |A|$$

$$370 - 200 - 150 + 70 = 90$$

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Problem 5.

Let's solve $B \cup !(!A \cap B) = U$ using set identities.

According to De Morgan's laws our equation can be written as such:

$B \cup (!!A \cup !B) = U$

According to Complementation law:

$B \cup (A \cup !B) = U$

According to Commutative laws:

$(A \cup !B) \cup B = U$

According to Associative laws:

$A \cup (!B \cup B) = U$

According to Complement laws:

$A \cup U = U$

According to Domination laws:

$U = U$

And now that proves the proposition true that $B \cup !(!A \cap B) = U$.