Discrete Mathematics

Solo Project nr.3

Matrices and Euclidian algorithm

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03/10/2023

Problem 1.

$$A = \begin{bmatrix} 1 & 0 \\ -5 & 4 \\ -7 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 3 & -3 & 5 \\ 0 & 2 & -2 & 4 & -4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+0 & -1+0 & 3+0 & -3+0 & 5+0 \\ -5+0 & 5+8 & -15-8 & 15+16 & -25-16 \\ -7+0 & 7-6 & -21+6 & 21-12 & -35+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & -1 & 3 & -3 & 5 \\ -5 & 13 & -23 & 31 & -41 \\ -7 & 1 & -15 & 9 & -23 \end{bmatrix}$$

Problem 2.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 14-12-1 & -16+15+1 & 10-9-1 \\ 7-8+1 & -8+10-1 & 5-6+1 \\ -7+4+3 & 8-5-3 & -5+3+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underbrace{I}_{3}$$

$$BA = \begin{bmatrix} 14-8-5 & 21-16-5 & -7-8+15 \\ -8+5+3 & -12+10+3 & 4+5-9 \\ 2-1-1 & 3-2-1 & -1-1+3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underbrace{I}_{3}$$

$$AB = BA = \underbrace{I}_{3}, \; A = B^{-1}$$

Problem 3.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

a)

$$A \wedge B = \begin{bmatrix} 1 \wedge 0 & 1 \wedge 1 \\ 0 \wedge 1 & 0 \wedge 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

b)

$$A \vee B = \begin{bmatrix} 1 \vee 0 & 1 \vee 1 \\ 0 \vee 1 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

c)

$$A \odot B = \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix} = \begin{bmatrix} 0 \vee 1 & 1 \vee 0 \\ 0 \vee 0 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Problem 4.

$r = a \bmod d$ $\quad$ $a = d * q + r$

$q = a \operatorname{div} d$

a)	b)	c)
-19 mod 7	352 mod 19	-115 mod 3
-3 = -19 div 7	18 = 352 div 19	-39 = -115 div 3
-19 = 7 * (-3) + r	352 = 19 * 18 + r	-115 = 3 * (-39) + r
-19 = -21 + r	352 = 342 + r	-115 = -117 + r
-19 + 21 = r	352-342 = r	-115+117 = r
r = 2	r = 10	r = 2
2 = -19 mod 7	10 = 352 mod 19	2 = -115 mod 3

d)

98 mod 10	98 = 90 + r
9 = 98 div 10	98 – 90 = r
98 = 10 * 9 + r	r = 8

Problem 5.

$\{a \mid a = 5 \bmod 15\}$

$5 = a \bmod 15$

$q = a \operatorname{div} 15$

$a = 15 * q + 5$

$a = 15 * 0 + 5$

$a = 5$

$a = 15 * 1 + 5$

$a = 20$

$a = 15 * 2 + 5$

$a = 35$

$a = 15 * 3 + 5$

$a = 50$

$\{\ldots -45, -25, -10, 5, 20, 35, 50 \ldots\}$

Problem 6.

a)

gcd(235,477)

$477 = 235 * q + r$

$477 = 235 * 2 + r$

$477 = 470 + r$

$r = 7$

$235 = 7 * 33 + r$

$235 = 231 + r$

$r = 235\text{-}231 = 4$

$7 = 4 * 1 +$

$r = 7 - 4 = 3$

$4 = 3 * 1 + r$

$r = 4 - 3 = 1$

$3 = 1 * 3 + r$

$r = 3 - 3 = 0$

$gcd(235,477) = 1$

b)

gcd(1529,14039)

$14039 = 1529 * 9 + r$

$14039 = 13761 + r$

$r = 14039\text{-}13761 = 278$

$1529 = 278 * 5 + r$

$1529 = 1390 + r$

$r = 1529 - 1390 = 139$

$278 = 139 * 2 + r$

$278 = 278 + r$

$$r = 0$$

$$\gcd(1529,14039) = 139$$