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Homework 4

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Solution 1: Given X= GTATA and Y= GAGT

Generate step-by-step solution for Sequence Alignment problem for following scenarios.

1) penalty of gap mismatch is 4 and letter mismatch is 3.

The formula is **A[i][j] = min(A[i-1][j-1]+ letter mismatch, A[i-1][j] +gap, A[i][j-1]+ gap**)

Initial values of X and Y with a gap are

		j_0 0	j_1 G	j_2 T	j_3 A	j_4 T	j_5 A
i_0	0	0	4	8	12	16	20
i_1	G	4					
i_2	A	8					
i_3	G	12					
i_4	T	16					

Let's compute for each value of i,j

i=1, j=1

A[1,1] = min(A[0][0]+ mismatch, A[0][1]+ gap, A[1][0]+ gap)

= min(0+0, 4+4, 4+4)

= min(0,8,8)

= 0

		j_0 0	j_1 G	j_2 T	j_3 A	j_4 T	j_5 A
i_0	0	0	4	8	12	16	20
i_1	G	4	0				
i_2	A	8					
i_3	G	12					
i_4	T	16					

i=1, j=2

A[1,2] = min(A[0][1]+ mismatch, A[0][2]+ gap, A[1][1]+ gap)

= min(4+3, 8+4, 0+4)

= min(7,12,4)

= 4

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	4	8	12	16	20
i_1	G	4	0	4			
i_2	A	8					
i_3	G	12					
i_4	T	16					

i=1, j=3

A[1,3] = min(A[0][2]+ mismatch, A[0][3]+ gap, A[1][2]+ gap)

= min(8+3, 12+4, 4+4)

= min(11,16,8)

= 8

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	4	8	12	16	20
i_1	G	4	0	4	8		
i_2	A	8					
i_3	G	12					
i_4	T	16					

i=1, j=4

A[1,4] = min(A[0][3]+ mismatch, A[0][4]+ gap, A[1][3]+ gap)

= min(12+3, 16+4, 8+4)

= min(15,20,12)

= 12

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	4	8	12	16	20
i_1	G	4	0	4	8	12	
i_2	A	8					
i_3	G	12					
i_4	T	16					

i=1, j=5

A[1,5] = min(A[0][4]+ mismatch, A[0][5]+ gap, A[1][4]+ gap)

= min(16+3, 20+4, 12+4)

= min(19,24,16)

= 16

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	4	8	12	16	20
i_1	G	4	0	4	8	12	16
i_2	A	8					
i_3	G	12					
i_4	T	16					

i=2, j=1

A[2,1] = min(A[1][0]+ mismatch, A[1][1]+ gap, A[2][0]+ gap)

= min(4+3, 0+4, 8+4)

= min(7,4,12)

= 4

i=2, j=2

A[2,2] = min(A[1][1]+ mismatch, A[1][2]+ gap, A[2][1]+ gap)

= min(0+3, 4+4, 4+4)

= min(3,8,8)

= 3

i=2, j=3

A[2,3] = min(A[1][2]+ mismatch, A[1][3]+ gap, A[2][2]+ gap)

= min(4+0, 8+4, 3+4)

= min(4,12,7)

= 4

i=2, j=4

A[2,4] = min(A[1][3]+ mismatch, A[1][4]+ gap, A[2][3]+ gap)

= min(8+3, 12+4, 4+4)

= min(11,16,8)

= 8

i=2, j=5

A[2,5] = min(A[1][4]+ mismatch, A[1][5]+ gap, A[2][4]+ gap)

= min(12+0, 16+4, 8+4)

= min(12,20,12)

= 12

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	4	8	12	16	20
i_1	G	4	0	4	8	12	16
i_2	A	8	4	3	4	8	12
i_3	G	12					
i_4	T	16					

i=3, j=1

A[3,1] = min(A[2][0]+ mismatch, A[2][1]+ gap, A[3][0]+ gap)

= min(8+0, 4+4, 12+4)

= min(8,8,16)

= 8

i=3, j=2

A[3,2] = min(A[2][1]+ mismatch, A[2][2]+ gap, A[3][1]+ gap)

= min(4+3, 3+4, 8+4)

= min(7,7,12)

= 7

i=3, j=3

A[3,3] = min(A[2][2]+ mismatch, A[2][3]+ gap, A[3][2]+ gap)

= min(3+3, 4+4, 7+4)

= min(6,8,11)

= 6

i=3, j=4

A[3,4] = min(A[2][3]+ mismatch, A[2][4]+ gap, A[3][3]+ gap)

= min(4+3, 8+4, 6+4)

= min(7,12,10)

= 7

i=3, j=5

A[3,5] = min(A[2][4]+ mismatch, A[2][5]+ gap, A[3][4]+ gap)

= min(8+3, 12+4, 7+4)

= min(11,16,11)

= 11

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	4	8	12	16	20
i_1	G	4	0	4	8	12	16
i_2	A	8	4	3	4	8	12
i_3	G	12	8	7	6	7	11
i_4	T	16					

i=4, j=1

A[4,1] = min(A[3][0]+ mismatch, A[3][1]+ gap, A[4][0]+ gap)

= min(12+3, 8+4, 16+4)

= min(15,12,20)

= 12

i=4, j=2

A[4,2] = min(A[3][1]+ mismatch, A[3][2]+ gap, A[4][1]+ gap)

= min(8+0, 7+4, 12+4)

= min(8,11,16)

= 8

i=4, j=3

A[4,3] = min(A[3][2]+ mismatch, A[3][3]+ gap, A[4][2]+ gap)

= min(7+3, 6+4, 8+4)

= min(10,10,12)

= 10

i=4, j=4

A[4,4] = min(A[3][3]+ mismatch, A[3][4]+ gap, A[4][3]+ gap)

= min(6+0, 7+4, 10+4)

= min(6,11,14)

= 6

i=4, j=5

A[4,5] = min(A[3][4]+ mismatch, A[3][5]+ gap, A[4][4]+ gap)

= min(7+3, 11+4, 7+4)

= min(10,15,11)

= 10

		j_0 0	j_1 G	j_2 T	j_3 A	j_4 T	j_5 A
i_0	0	0	4	8	12	16	20
i_1	G	4	0	4	8	12	16
i_2	A	8	4	3	4	8	12
i_3	G	12	8	7	6	7	11
i_4	T	16	12	8	10	6	10

Case Analysis:

Case 1:

X and Y matched in last column

G T A T A

\- G A G T

Here we have one gap and three mismatches. Therefore according to the given penalty, 4+3+3+3 = 13

Case 2:

X and Y not matched in last column

G T A T A

G A G T -

Here we have one gap and 2 mismatches. Therefore according to the given penalty, $4 + 3 + 3 = 10$ So, we considered case 2.

2) penalty of gap mismatch is 5 and letter mismatch is 3.

The formula is **A[i][j] = min(A[i-1][j-1]+ letter mismatch, A[i-1][j] +gap, A[i][j-1]+ gap**)

Initial values of X and Y with a gap are

		j_0 0	j_1 G	j_2 T	j_3 A	j_4 T	j_5 A
i_0	0	0	5	10	15	20	25
i_1	G	5					
i_2	A	10					
i_3	G	15					
i_4	T	20					

Let's compute for each value of i,j

i=1, j=1

A[1,1] = min(A[0][0]+ mismatch, A[0][1]+ gap, A[1][0]+ gap)

= min(0+0, 5+5, 5+5)

= min(0,10,10)

= 0

i=1, j=2

A[1,2] = min(A[0][1]+ mismatch, A[0][2]+ gap, A[1][1]+ gap)

= min(5+3, 10+5, 0+5)

= min(8,15,5)

= 5

i=1, j=3

A[1,3] = min(A[0][2]+ mismatch, A[0][3]+ gap, A[1][2]+ gap)

= min(10+3, 15+5, 5+5)

= min(13,20,10)

= 10

i=1, j=4

A[1,4] = min(A[0][3]+ mismatch, A[0][4]+ gap, A[1][3]+ gap)

= min(15+3, 20+5, 10+5)

= min(18,25,15)

= 15

i=1, j=5

A[1,5] = min(A[0][4]+ mismatch, A[0][5]+ gap, A[1][4]+ gap)

= min(20+3, 25+5, 15+5)

= min(23,30,20)

= 20

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	5	10	15	20	25
i_1	G	5	0	5	10	15	20
i_2	A	10					
i_3	G	15					
i_4	T	20					

i=2, j=1

A[2,1] = min(A[1][0]+ mismatch, A[1][1]+ gap, A[2][0]+ gap)

= min(5+3, 0+5, 10+5)

= min(8,5,15)

= 5

i=2, j=2

A[2,2] = min(A[1][1]+ mismatch, A[1][2]+ gap, A[2][1]+ gap)

= min(0+3, 15+5, 5+5)

= min(3,20,10)

= 3

i=2, j=3

A[2,3] = min(A[1][2]+ mismatch, A[1][3]+ gap, A[2][2]+ gap)

= min(5+0, 10+5, 3+5)

= min(5,15,8)

= 5

i=2, j=4

A[2,4] = min(A[1][3]+ mismatch, A[1][4]+ gap, A[2][3]+ gap)

= min(10+3, 15+5, 5+5)

= min(13,20,10)

= 10

i=2, j=5

A[2,5] = min(A[1][4]+ mismatch, A[1][5]+ gap, A[2][4]+ gap)

= min(15+0, 20+5, 10+5)

= min(15,25,15)

= 15

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	5	10	15	20	25
i_1	G	5	0	5	10	15	20
i_2	A	10	5	3	5	10	15
i_3	G	15					
i_4	T	20					

i=3, j=1

A[3,1] = min(A[2][0]+ mismatch, A[2][1]+ gap, A[3][0]+ gap)

= min(10+0, 5+5, 15+5)

= min(10,10,20)

= 10

i=3, j=2

A[3,2] = min(A[2][1]+ mismatch, A[2][2]+ gap, A[3][1]+ gap)

= min(5+3, 3+5, 10+5)

= min(8,8,15)

= 8

i=3, j=3

A[3,3] = min(A[2][2]+ mismatch, A[2][3]+ gap, A[3][2]+ gap)

= min(3+3, 5+5, 8+5)

= min(6,10,13)

= 6

i=3, j=4

A[3,4] = min(A[2][3]+ mismatch, A[2][4]+ gap, A[3][3]+ gap)

= min(5+3, 10+5, 6+5)

= min(8,15,11)

= 8

i=3, j=5

A[3,5] = min(A[2][4]+ mismatch, A[2][5]+ gap, A[3][4]+ gap)

= min(10+3, 15+5, 8+5)

= min(13,20,13)

= 13

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	5	10	15	20	25
i_1	G	5	0	5	10	15	20
i_2	A	10	5	3	5	10	15
i_3	G	15	10	8	6	8	13
i_4	T	20					

i=4, j=1

A[4,1] = min(A[3][0]+ mismatch, A[3][1]+ gap, A[4][0]+ gap)
= min(15+3, 10+5, 20+5)
= min(18,15,25)
= 15

i=4, j=2

A[4,2] = min(A[3][1]+ mismatch, A[3][2]+ gap, A[4][1]+ gap)
= min(10+0, 8+5, 15+5)
= min(10,13,20)
= 10

i=4, j=3

A[4,3] = min(A[3][2]+ mismatch, A[3][3]+ gap, A[4][2]+ gap)
= min(8+3, 6+5, 10+5)
= min(11,11,15)
= 11

i=4, j=4

A[4,4] = min(A[3][3]+ mismatch, A[3][4]+ gap, A[4][3]+ gap)
= min(6+0, 8+5, 11+5)
= min(6,13,16)
= 6

i=4, j=5

A[4,5] = min(A[3][4]+ mismatch, A[3][5]+ gap, A[4][4]+ gap)
= min(8+3, 13+5, 6+5)
= min(11,18,11)
= 11

		j_0	j_1	j_2	j_3	j_4	j_5
		0	G	T	A	T	A
i_0	0	0	5	10	15	20	25
i_1	G	5	0	5	10	15	20
i_2	A	10	5	3	5	10	15
i_3	G	15	10	8	6	8	13
i_4	T	20	15	10	11	6	11

Case Analysis:

Case 1:

X and Y matched in last column

G T A T A

- G A G T

Here we have one gap and three mismatches. Therefore according to the given penalty, 5+3+3+3 = 14

Case 2:

X and Y not matched in last column

G T A T A

G A G T -

Here we have one gap and 2 mismatches. Therefore according to the given penalty, $5 + 3 + 3 = 11$ So, we considered case 2.

Yes, the optimal alignment is same for the two parameterizations.

Both the parameters have the same alignment that is

G T A T A

G A G T -

But here in the given parameterizations gap penalty is different for both.

In the first one gap penalty is 4 which is little variant than mismatch penalty. In the second one the gap penalty is 5 which is higher than mismatch penalty.

Solution 2: Given a value n with unlimited supply of denominations $d_1, .., d_k$

Find the minimum no.of coins required to get n.

Before implementing the algorithm, let's look at an example.

Consider n= 18 and you have 1,3,5,10 denomination coins.

Now, to get n=20 we can have many ways like: 1 - 20 coins, 3 - 7 coins, 5 - 4 coins, 10 - 2 coins

Now the least no.of coins required to get the value can be 2.

Algorithm :

1. Define a class with parameters coin-denominations and the target-val.
2. Store the length of coin denominations.
3. Initialize all the values to infinity except the 0th which is set to 0.
4. For i $->$ 1 to target-val do:
5. compute the minimum no.of coins required.
6. For j $->$ in target-val do:

7. Check if jth values is less than the ith values

8. Compute minimum coins as min(ith value, (ith - denominant of jth) +1)

9. Return the target-val

Proof Let C[d][t] be the minimum no.of coins required to make change for n using denominants d.

We need to show that

C[d][t]= min(C[d-1],[t], C[d][t-d]+1

We can prove this using proof of contradiction

Assume C[d][t] ¿ min(C[d-1],[t], C[d][t-d]+1

C[d-1],[t] ¡= min(C[d-1],[t], C[d][t-d]+1

C[d][t-d]+1 ¡= min(C[d-1],[t], C[d][t-d]+1

If we add these two conditions we get

C[d-1][t] + C[d][t-d]+1 ¡2 * min(C[d-1],[t], C[d][t-d]+1

This is a contradiction

Therefore our assumption that there exits C[d][t]!= min(C[d-1],[t], C[d][t-d]+1 is false, so a lemma holds