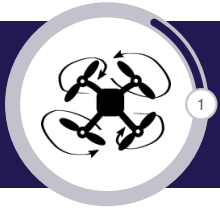
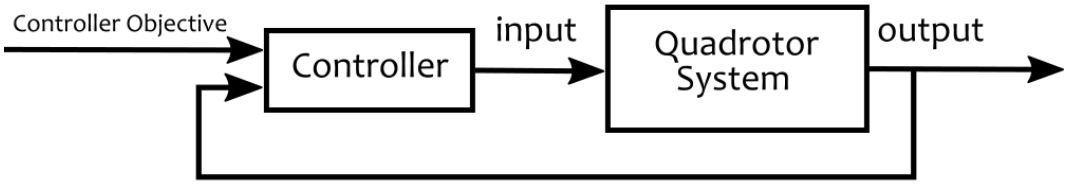
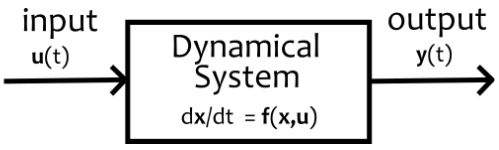
Feedback & Controllers





Dynamical System



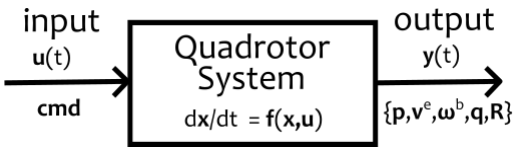
A general time invariant dynamical system is defined as follows:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \quad (1a)$$

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t)) \quad (1b)$$

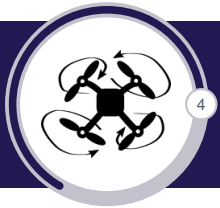
where $\boldsymbol{x} \in \mathbb{R}^n$ are states, $\boldsymbol{u} \in \mathbb{R}^m$ are inputs, $\boldsymbol{y} \in \mathbb{R}^p$ are the outputs, function $\boldsymbol{f} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, and function $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^p$.

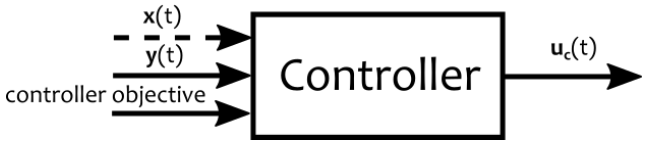
Quadrotor Dynamical System



- $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}^e \\ \boldsymbol{v}^e \\ \boldsymbol{q}^T \\ \omega^b \end{bmatrix} \in \mathbb{R}^{13}$ or $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}^e \\ \boldsymbol{v}^e \\ \boldsymbol{R}_b^e(:) \\ \omega^b \end{bmatrix} \in \mathbb{R}^{18}$.
- The system inputs are $\boldsymbol{u} = \begin{bmatrix} \text{cmd}_1 & \text{cmd}_2 & \text{cmd}_3 & \text{cmd}_4 \end{bmatrix}^T \in \mathbb{R}^4$.
- The system output is $\boldsymbol{y}(t)$ is in the case of the quadrotor a sub-set of the states

Controller





- The inputs of the controller block : the control objective, the system output $\boldsymbol{y}(t)$, and optionally the dynamical system states $\boldsymbol{x}(t)$ (state-controller)
- The output of the controller is $\boldsymbol{u}_c(t)$, the input to the dynamical system

Closed-Loop System

- The “Open-Loop” dynamical system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t))$$
$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t))$$

- The Feedback or “Closed-Loop” system:

$$\boldsymbol{u}(t) = \boldsymbol{u}_c(\boldsymbol{y})$$
$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}_c(\boldsymbol{y}))$$

Closed-Loop System