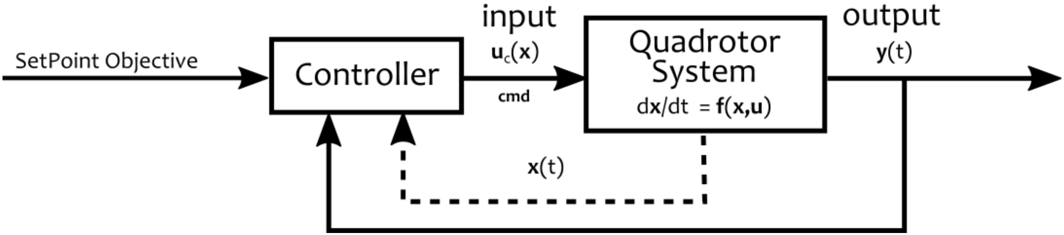
Controller Structure



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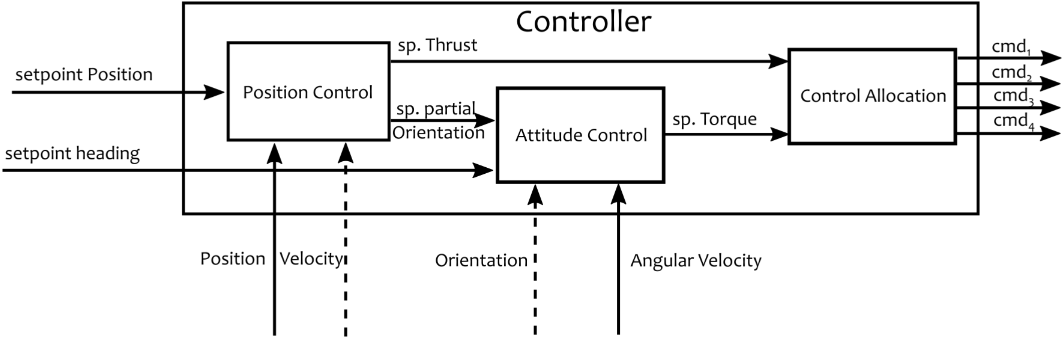

Small Angle Model Simplification

We look to simplify the kinematic and dynamic model, under a small angle assumption, meaning that the quadrotor state is close to the hoovering state, characterized by a horizontally level orientation (litlle tilt) and a thrust close to the $m \cdot g$ value. We do not restrict the heading angle. More specifically, this means:

- Roll angle assumption $\sin(\psi) = \psi$, $\cos(\psi) = 1$
- Pitch angle assumption $\sin(\theta) = \theta$, $\cos(\theta) = 1$
- $\psi^2 = 0$, $\theta^2 = 0$, $\psi\theta = 0$
- Thrust force $f_{\text{thrust}} \approx m \cdot g$

Small Angle Model Simplification

Under these assumptions, the rotation matrix looks like in the following:

$$\mathbf{R}_b^e = \begin{bmatrix} c\phi & -s\phi & \theta c\phi + \psi s\phi \\ s\phi & c\phi & \theta s\phi - \psi c\phi \\ -\theta & \psi & 1 \end{bmatrix} \tag{1}$$

and the relation between the derivatives of the Euler angles and the angular velocity,

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \theta \\ 0 & 1 & -\psi \\ 0 & \psi & 1 \end{bmatrix} \begin{bmatrix} \omega_x^b \\ \omega_y^b \\ \omega_z^b \end{bmatrix} \tag{2}$$

Model Simplification

Further simplifications:

$$\dot{\boldsymbol{p}}^e = \boldsymbol{v}^e \tag{3a}$$

$$\dot{\boldsymbol{v}}^e = \frac{1}{m}\mathbf{R}_b^e \left(\begin{bmatrix} 0 \\ 0 \\ f_{\text{thrust}} \end{bmatrix} + \overbrace{\boldsymbol{f}_{\text{aero}}^b}^{\approx 0} \right) - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \approx \frac{1}{m} \begin{bmatrix} f_{\text{thrust}}\left(\theta c\phi + \psi s\phi\right) \\ f_{\text{thrust}}\left(\theta s\phi - \psi c\phi\right) \\ f_{\text{thrust}} - mg \end{bmatrix} \tag{3b}$$

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \theta \\ 0 & 1 & -\psi \\ 0 & \psi & 1 \end{bmatrix} \begin{bmatrix} \omega_x^b \\ \omega_y^b \\ \omega_z^b \end{bmatrix} \approx \mathbf{I}_3 \boldsymbol{\omega}^{\boldsymbol{b}} \tag{3c}$$

$$\dot{\omega}^b = \left(\mathbf{J}^b\right)^{-1} \left(\tau_c^b - \left[\omega^b\right]_\times \mathbf{J}^b \omega^b \right) \tag{3d}$$

Model Decoupling

Horizontal movement model/channel:

$$\dot{\boldsymbol{p}}_h^e = \boldsymbol{v}_h^e \equiv \begin{bmatrix} \dot{p}_x^e \\ \dot{p}_y^e \end{bmatrix} = \begin{bmatrix} v_x^e \\ v_y^e \end{bmatrix} \tag{4a}$$

$$\dot{\boldsymbol{v}}_h^e = \begin{bmatrix} \dot{v}_x^e \\ \dot{v}_y^e \end{bmatrix} = \underbrace{\frac{f_{\text{thrust}}}{m}}_{\approx g} \begin{bmatrix} s\phi & c\phi \\ -c\phi & s\phi \end{bmatrix} \approx g \begin{bmatrix} s\phi & c\phi \\ -c\phi & s\phi \end{bmatrix} \begin{bmatrix} \psi \\ \theta \end{bmatrix} \tag{4b}$$

Vertical movement model/channel:

$$\dot{p}_z^e = v_z^e \tag{5a}$$

$$\dot{v}_z^e = \frac{f_{\text{thrust}}}{m} - g \tag{5b}$$

Model Decoupling

Attitude model:

$$\dot{\psi} = \omega_x^b,\ \dot{\omega}_x^b = \alpha_x^b \tag{6a}$$

$$\dot{\theta} = \omega_y^b,\ \dot{\omega}_y^b = \alpha_y^b \tag{6b}$$

$$\dot{\phi} = \omega_z^b,\ \dot{\omega}_z^b = \alpha_z^b \tag{6c}$$

Torque Model:

$$\boldsymbol{\alpha}^b = \left(\mathbf{J}^b\right)^{-1} \left(\boldsymbol{\tau}^b - \left[\boldsymbol{\omega}^b\right]_\times \mathbf{J}^b \boldsymbol{\omega}^b\right) \tag{7a}$$

Control Allocation

$$\begin{bmatrix} \omega_{r,1}^2 \\ \omega_{r,2}^2 \\ \omega_{r,3}^2 \\ \omega_{r,4}^2 \end{bmatrix} = \Gamma^{-1} \begin{bmatrix} f_z^b \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{8}$$

where the Γ matrix has different forms for the plus and cross configurations.