2.) 1) $i = 1, 2, \dots, n$

$$Pr\{X > k\} = Pr\{A_1 \cap \dots \cap A_k\} = \frac{n}{m} \cdot \frac{n-1}{m-1} \cdots \frac{n-k+1}{m-k+1} \le \left(\frac{n}{m}\right)^k \le \left(\frac{m/2}{m}\right)^k$$

$$= 2^{-k} \Rightarrow Pr\{X > k\} \le 2^{-k}$$

2) $i = 1, \dots, n$

$$Pr\{X > 2\lg n\} = Pr\{A_1 \cap \dots \cap A_{2\log n}\} = \frac{n}{m} \cdot \frac{n-1}{m-1} \cdots \frac{n-2\lg n+1}{m-2\lg n+1} \le$$

$$\le \left(\frac{n}{m}\right)^{2\lg n} < \left(\frac{m/2}{m}\right)^{2\log_2 n} = \left(\frac{1}{2}\right)^{\log_2 n^2}$$

$$= \frac{1}{2^{\log_2 n^2}} = \frac{1}{n^2} \Rightarrow O\left(\frac{1}{n^2}\right)$$

$$Pr\{X > 2\lg n\} = O\left(\frac{1}{n^2}\right)$$

3) $Pr\{X > 2\lg n\} = O(1/n)$

$$Pr\{X > 2\lg n\} = \sum_{i=1}^{n} Pr\{X_i > 2\lg n\} \le \sum_{i=1}^{n} \frac{1}{n^2} = \frac{1}{n^2} \sum_{i=1}^{n} 1$$

$$= \frac{1}{n^2} \cdot n = \frac{1}{n}$$

$$\Rightarrow Pr\{X > 2\lg n\} \le \frac{1}{n}$$

$$\therefore Pr\{X > 2\lg n\} = O\left(\frac{1}{n}\right)$$

b) $h(k,i) = (k \bmod m + i \cdot (1 + (k \bmod (m-1)))) \bmod m, \; i = 0,1,2 \ldots$

$h(77,0) = 1$

$h(69,0) = 12$

$h(39,0) = 1$; $h(39,1) = 5$

$h(70,0) = 13$

$h(6,0) = 6$

$h(8,0) = 8$

$h(40,0) = 2$

$h(89,0) = 13$; $h(89,1) = 12$; $h(89,2) = 11$

$h(49,0) = 11$; $h(49,1) = 6$; $h(49,2) = 1$; $h(49,3) = 15$

$h(15,0) = 15$; $h(15,1) = 12$; $h(15,2) = 9$

0	
1	→ 77
2	→ 40
3	
4	
5	→ 39
6	→ 6
7	
8	→ 8
9	→ 15
10	
11	→ 89
12	→ 69
13	→ 70
14	
15	→ 49
16	
17	
18	

1. 2) $x_1 x_2 \ldots x_n$ $(x_i \in \{0, 1, \ldots, 9\})$

$f(x) = \sum_{i=1}^{n} a_i x_i \pmod 8$ nije univerzalna fja:

kontraprimjer:

$n = 4$, $a_1 = a_2 = a_3 = a_4 = 1$

$x = 6442$ $\Rightarrow$ $6a_1 + 4a_2 + 4a_3 + 2a_4 = 16 \bmod 8 = 0$

$y = 84164$ $\quad$ $8a_1 + 4a_2 + 16a_3 + 4a_4 = 32 \bmod 8 = 0$

vj. kolizije je 1 što je veće od $\frac{1}{8}$

2.) 1) $i = 1, 2, \dots, n$

$$Pr\{X > k\} = Pr\{A_1 \cap \dots \cap A_k\} = \frac{n}{n} \cdot \frac{n-1}{n-1} \cdots \frac{n-k+1}{n-k+1} \le \left(\frac{n}{n}\right)^k \le \left(\frac{n/2}{n}\right)^k$$

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2) $i = 1, \dots, n$

$$Pr\{X > 2\lg n\} = Pr\{A_1 \cap \dots \cap A_{2\log n}\} = \frac{n}{n} \cdot \frac{n-1}{n-1} \cdots \frac{n-2\lg n+1}{n-2\lg n+1} \le$$

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$$\Rightarrow Pr\{X > 2\lg n\} \le \frac{1}{n}$$

$$\therefore Pr\{X > 2\lg n\} = O\left(\frac{1}{n}\right)$$