Zadaća 1.

1. a) $k \in \{77, 69, 33, 70, 6, 8, 40, 89, 49, 15\}$

$m = 19$

$h(k) = k \bmod m$

$h(77) = 1$

$h(69) = 12$

$h(33) = 1$

$h(70) = 13$

$h(6) = 6$

$h(8) = 8$

$h(40) = 2$

$h(89) = 13$

$h(49) = 11$

$h(15) = 15$

0	
1	→ 77 → 33
2	→ 40
3	
4	
5	
6	→ 6
7	
8	→ 8
9	
10	
11	→ 49
12	→ 69
13	→ 70 → 89
14	
15	→ 15
16	
17	
18	

b) $h(k,i) = (k \bmod m + i \cdot (1+(k \bmod (m-1)))) \bmod m, \; i=0,1,2...$

$h(77,0) = 1$

$h(69,0) = 12$

$h(33,0) = 1; \; h(33,1) = 5$

$h(70,0) = 13$

$h(6,0) = 6$

$h(8,0) = 8$

$h(40,0) = 2$

$h(89,0) = 13; \; h(89,1) = 12; \; h(89,2) = 11$

$h(49,0) = 11; \; h(49,1) = 6; \; h(49,2) = 1; \; h(49,3) = 15$

$h(15,0) = 15; \; h(15,1) = 12; \; h(15,2) = 9$

0	
1 →	77
2 →	40
3	
4	
5 →	33
6 →	6
7	
8 →	8
9 →	15
10	
11 →	89
12 →	69
13 →	70
14	
15 →	49
16	
17	
18	

1. 2) $\quad x_1 x_2 ... x_4 \quad (x_i \in \{0,1,...,9\})$

$l(x) = \sum\limits_{i=1}^{n} a_i x_i \pmod 8$ nije univerzalna jer:

kontraprimjer:

$n=4, \; a_1 = a_2 = a_3 = a_4 = 1$

$x = 6+42$

$y = 8+164$

$\Rightarrow$

$6 a_1 + 4 a_2 + 4 a_3 + 2 a_4 = 16 \bmod 8 = 0$

$8 a_1 + 4 a_2 + 16 a_3 + 4 a_4 = 32 \bmod 8 = 0$

vj kolizije je 1 što je veće od $\frac{1}{8}$

2) a)

$i = 1, 2, \dots, m$

$P_n\{x > k\} = P_n\{A_1 \cap A_2 \cap \dots \cap A_k\} = \frac{m}{m} \cdot \frac{m-1}{m-1} \cdots \frac{m-k+1}{m-k+1} \leq \left(\frac{m}{m}\right)^{\frac{k}{?}}$

$= 2^{-k} \implies P_n\{x > k\} \leq 2^{-k}$

2) i = 1, \dots, m

$P_n\{x > 2\log_2 3\} = P_n\{A_1 \cap A_2 \cap \dots \cap A_{2\log_2 n}\} = \frac{m}{m} \cdot \frac{m-1}{m-1} \cdots \frac{m-2\log_2 n + 1}{m-2\log_2 n + 1} \leq$

$\leq \left(\frac{m}{m}\right)^{2\log_2 n} < \left(\frac{m-1}{m}\right)^{2\log_2 n} = \left(\frac{1}{2}\right)^{2\log_2 n}$

$= \frac{1}{2^{2\log_2 n^2}} = \frac{1}{n^2} \implies O\left(\frac{1}{n^2}\right)$

$P_n\{x > 2\log_2 n\} = O\left(\frac{1}{n^2}\right)$

3) $P_n\{x > 2\log_2 3\} = O\left(\frac{1}{m}\right)$

$P_n\{x > 2\log_2 3\} = \sum_{i=1}^{m} P_n\{x_i > 2\log_2 3\} \leq \sum_{i=1}^{m} \frac{1}{n^2} = \frac{1}{n^2}\sum_{i=1}^{m} 1$

$= \frac{1}{n^2} \cdot m = \frac{1}{m}$

$\implies P_n\{x > 2\log_2 n3\} \leq \frac{1}{n}$

$\implies P_n\{x > 2\log_2 n3\} = O\left(\frac{1}{n}\right)$