Control Theory Homework №5

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Consider classical benchmark system in control theory - inverted pendulum on a cart (Figure 1). It is nonlinear under-actuated system that has the following dynamics

$$(M+m)\ddot{x} - ml\cos(\theta)\ddot{\theta} + ml\sin(\theta)\dot{\theta}^2 = F \tag{1}$$

$$-\cos(\theta)\ddot{x} + l\ddot{\theta} - g\sin(\theta) = 0 \tag{2}$$

where $g = 9.81$ is gravitational acceleration.

The system dynamics can be written in state space form:

$$\dot{z} = f(z) + g(z)u$$
$$y = h(z) = \begin{bmatrix} x & \theta \end{bmatrix}^T$$

where $z = \begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} \end{bmatrix}^T$ is the state vector of the system, y is the output vector. The dynamics of the system around unstable equilibrium of the pendulum ($\bar{z} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$) can be described by a linear system that is obtained from linearization of the nonlinear dynamics around $\bar{z}$.

$$\delta\dot{z} = A\delta z + B\delta u$$
$$\delta y = C\delta z$$

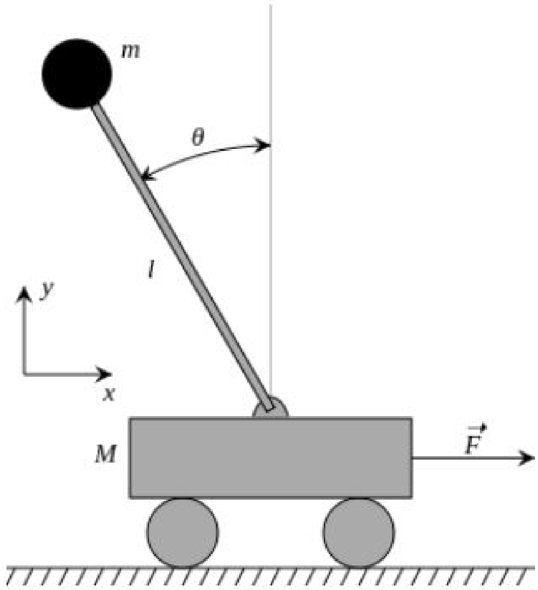

Figure 1: A schematic drawing of the inverted pendulum on a cart. The rod is considered massless. The mass of the cart and the point mass at the end of the rod are denoted by M and m. The rod has a length l.

(d) $M = 5.3, m = 3.2, l = 1.15$

A)

(A) prove that it is possible to design state observer of the linearized system

Firstly, I should find linearized system matrices. Doing the same steps, like in my previous homework, I got this matrices A, B, C.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 5.92 & 0 & 0 \\ 0 & 22.65 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{5.3} \\ \frac{1}{9.775} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

Next, I need to create a observable matrix:

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 5.92 & 0 & 0 \\ 0 & 0 & 0 & 5.92 \end{bmatrix}$$

System is observable, if rank of this matrix equal to rank(A). Rank(A) = 4, rank of this matrix is 4, so system is observable.

B)

(B) for open loop state observer, is the error dynamics stable?

To know is error dynamics stable or not, I should find eigenvalues of matrix A.

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eigenvalues of A [ 0.          0.          4.75920161 -4.75920161]
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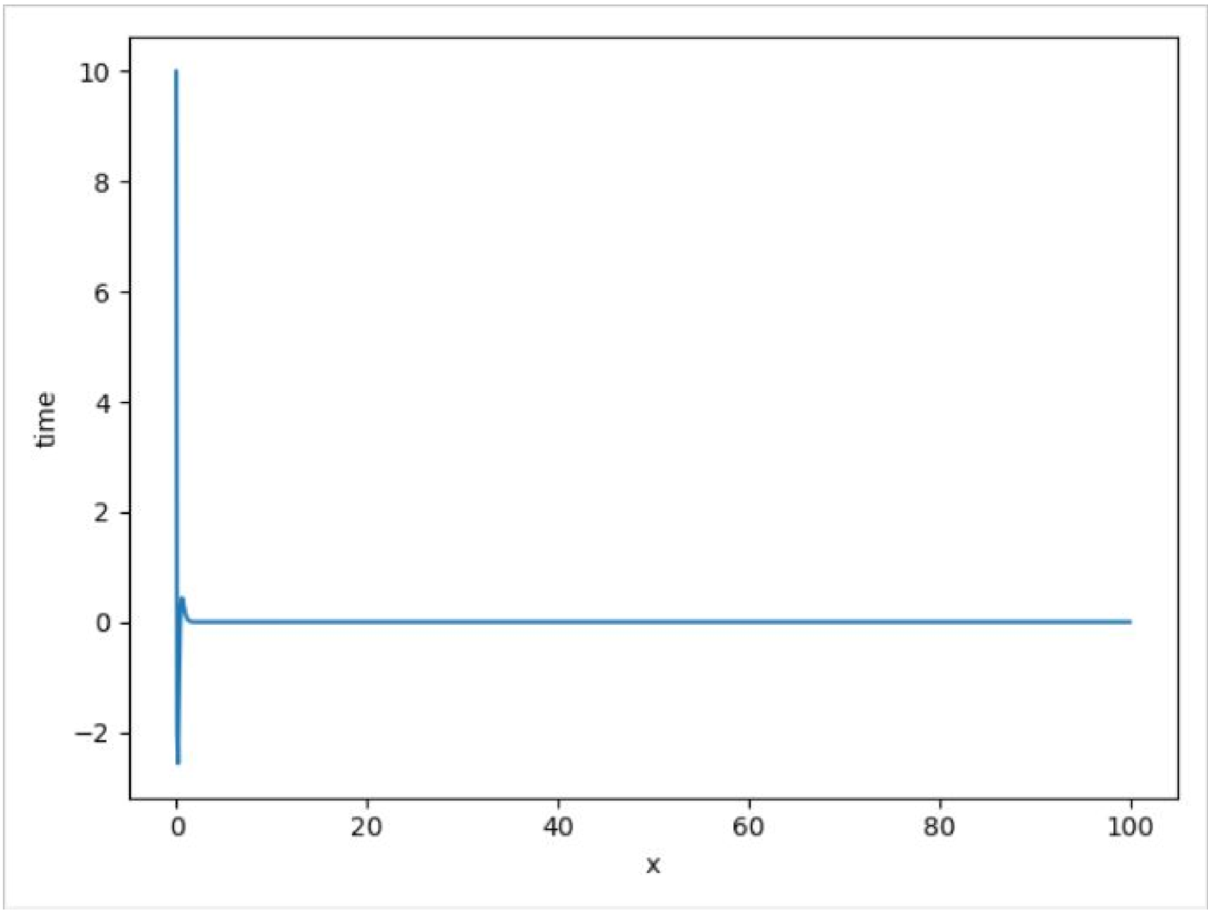
For stable system all eigenvalues should be less than zero, so error dynamics is not stable.

C)

(C) design Luenberger observer for linearized system using both pole placement and LQR methods

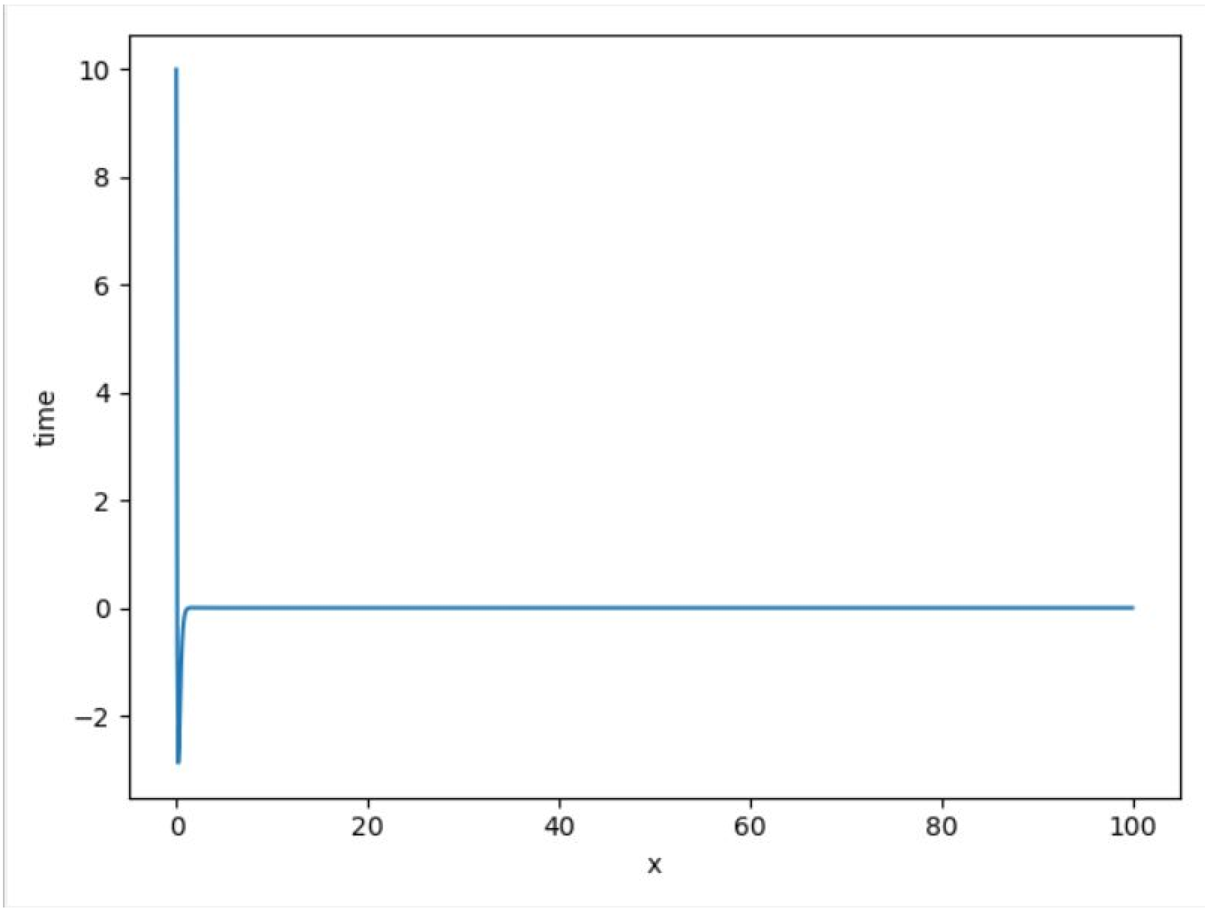
1) Poles are [-5, -6, -7 , -8], initial conditions are [10, 10, 10, 10]



2) Initial conditions are the same.

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



D)

(D) design state feedback controller for linearized system

I am using same controller from previous assignment.

E)

(E) simulate nonlinear system with Luenberger observer and state feedback controller that uses estimated states ($u = K\hat{x}$). Make sure that the system is stabilized for various initial conditions around $\bar{z}$.

F)

(F) add white gaussian noise to the output ($\delta y = C\delta z + v$). What happens to the state estimation?

G)

(G) add white gaussian noise to the dynamics ($\delta\dot{z} = A\delta z + B\delta u + w$). What happens to the state estimation and control system?

H)

(H) implement Kalman Filter (you can use libraries with KF if this task is not for you to get some points in next ones)

I)

(I) generate some data and show that your implementation of KF is correct

J)

(J) using KF function implement LQG controller