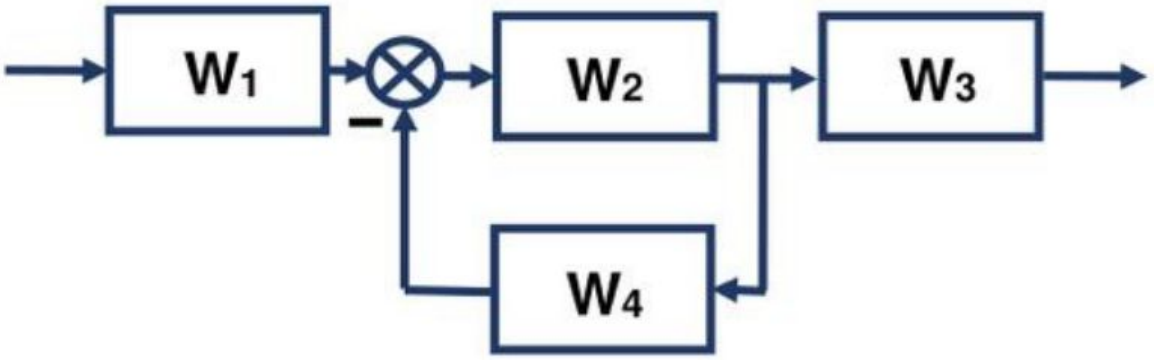
Control Theory
Homework № 2

Daniil Fronts, Group № 3, Variant f

№2.



A) Calculate the total Transfer Function of the system.

$$W_1 = \frac{s+3}{s+1}, \quad W_2 = \frac{1}{s+2}, \quad W_3 = \frac{1}{s+0.1}, \quad W_4 = \frac{1}{s+2.5}$$

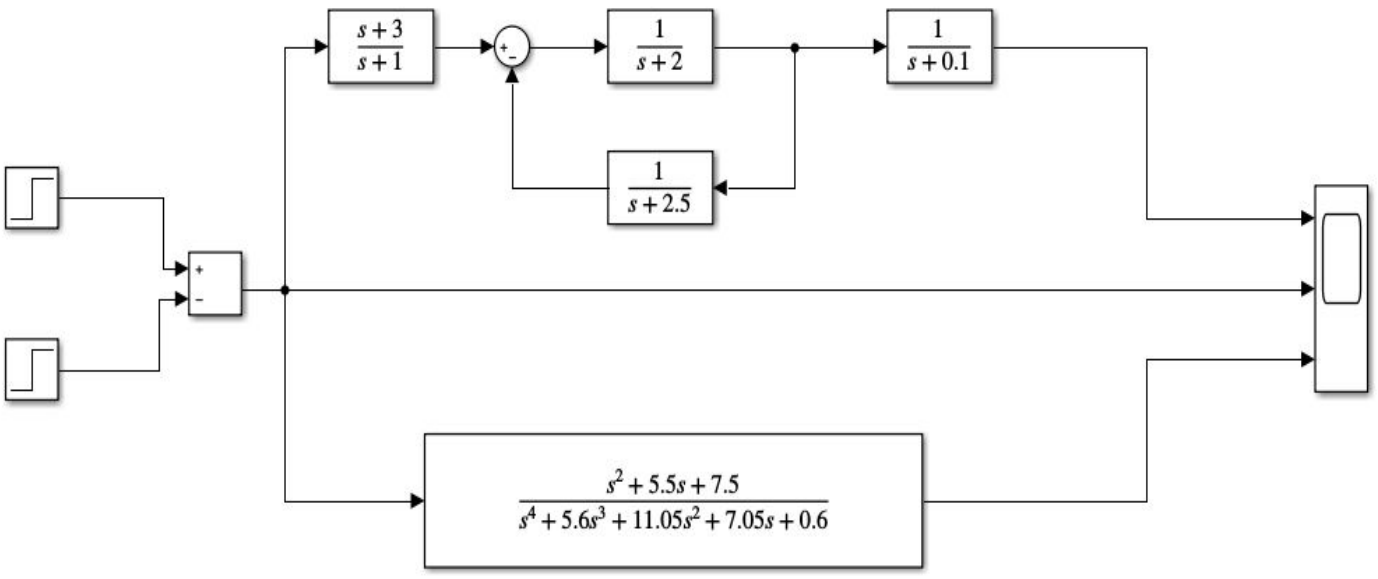
$$W(s) = W_1 * \frac{W_2}{1+W_4*W_2} * W_3$$

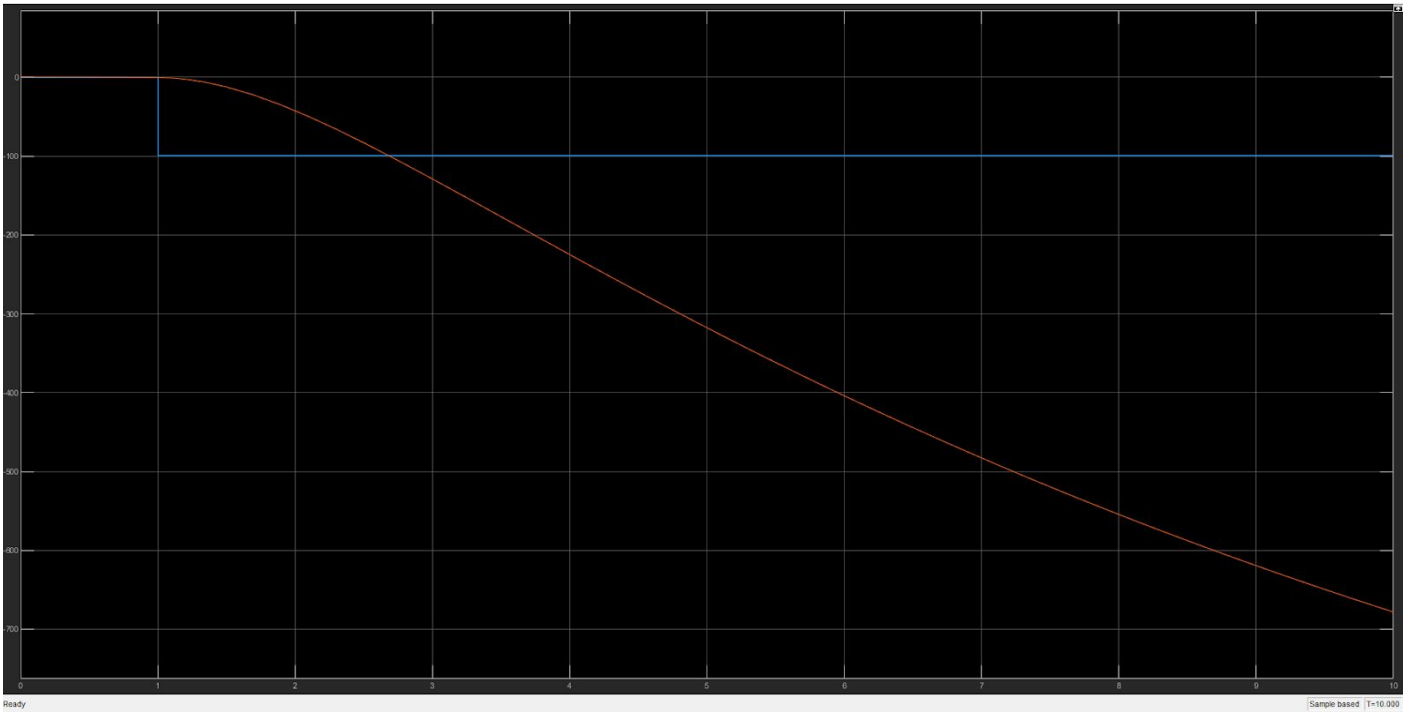
$$W(s) = \frac{s+3}{s+1} * \frac{\frac{1}{s+2}}{1+\frac{1}{s+2.5}*\frac{1}{s+2}} * \frac{1}{s+0.1} = \frac{10(s+3)(2s^2+9s+10)}{(10s+1)(s+1)(s+2)(2s^2+13s+19)}$$

$$W(s) = \frac{s^2+5.5s+7.5}{s^4+5.6s^3+11.05s^2+7.05s+0.6}$$

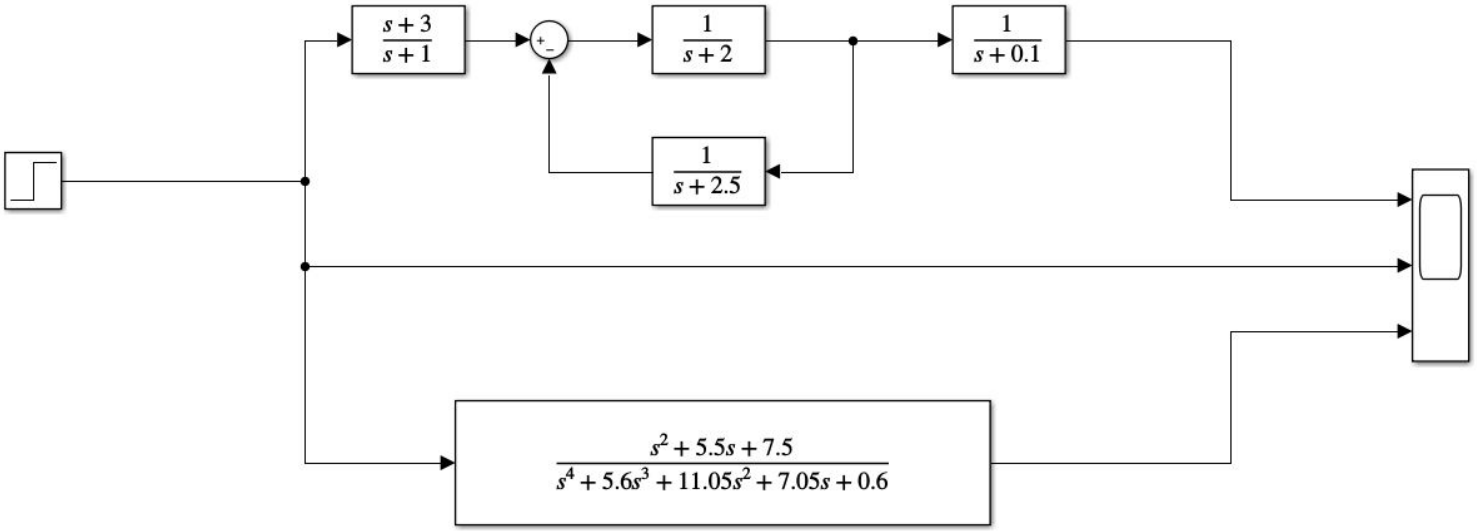
B) Build initial system shown in the block diagram and simplified in one Simulink schema and analyze its Step, Impulse and Frequency responses. Results should have a schema with both systems and 3 Scope plots(for each input). Each plot should have 3 lines - input signal, and two outputs from each system.

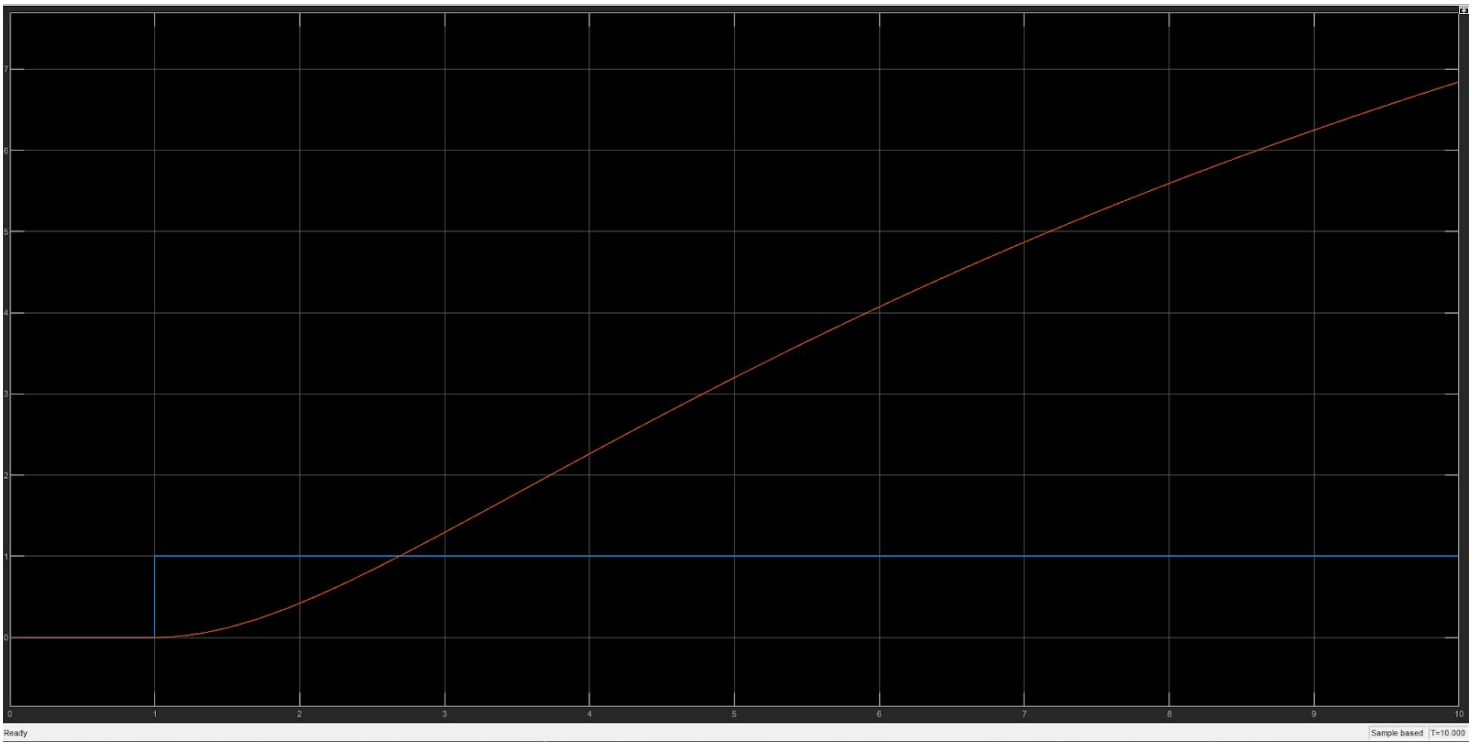
Impulse:



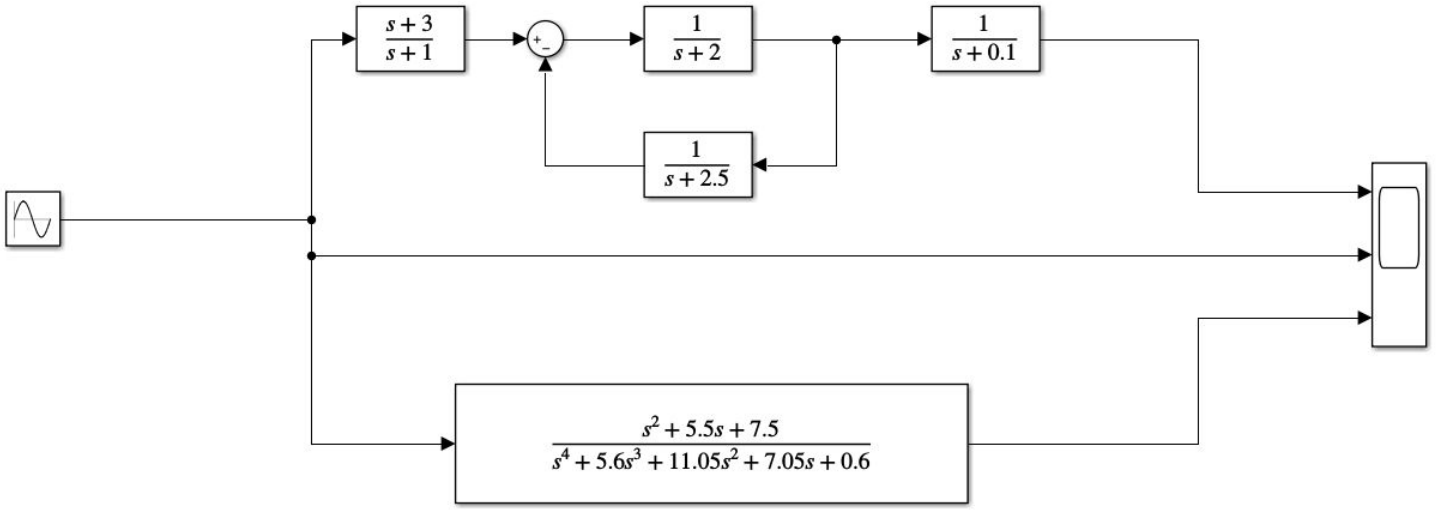


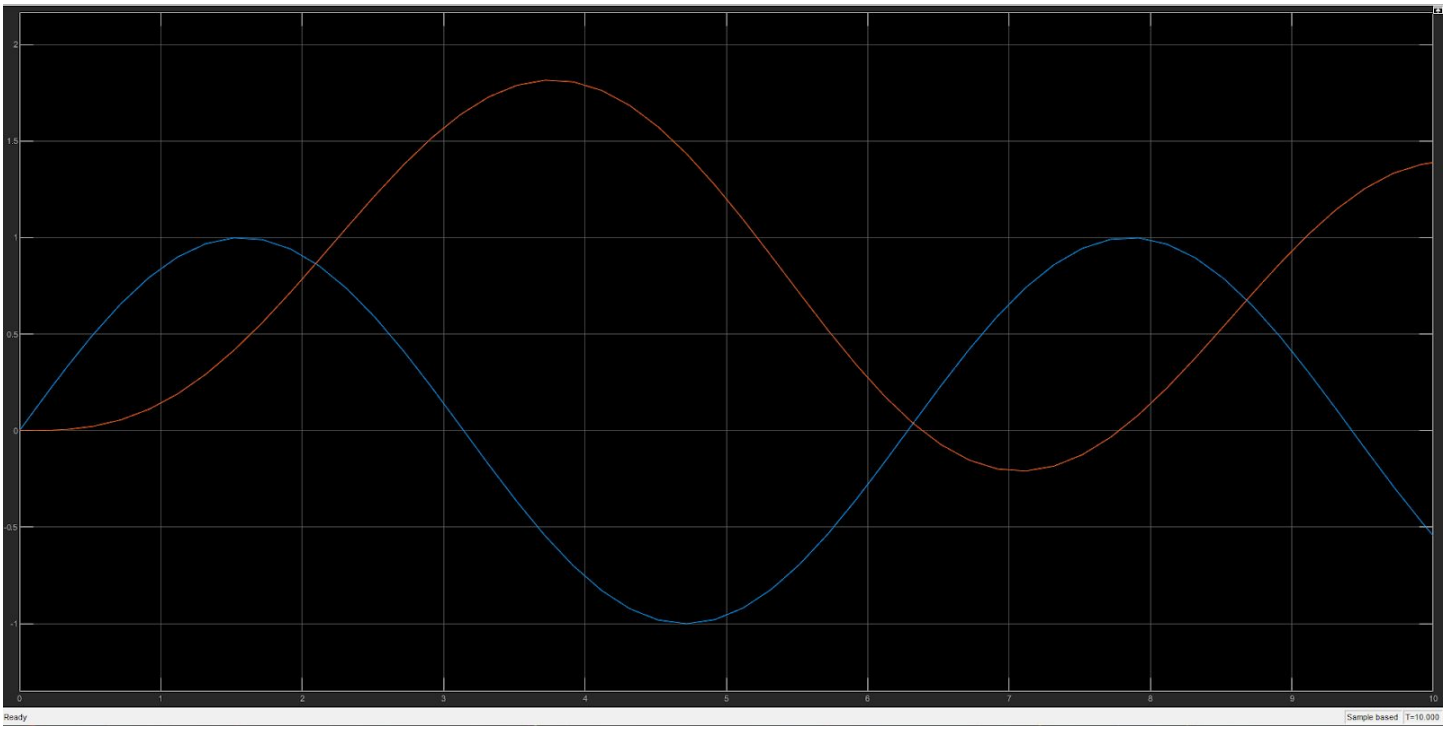
Step:





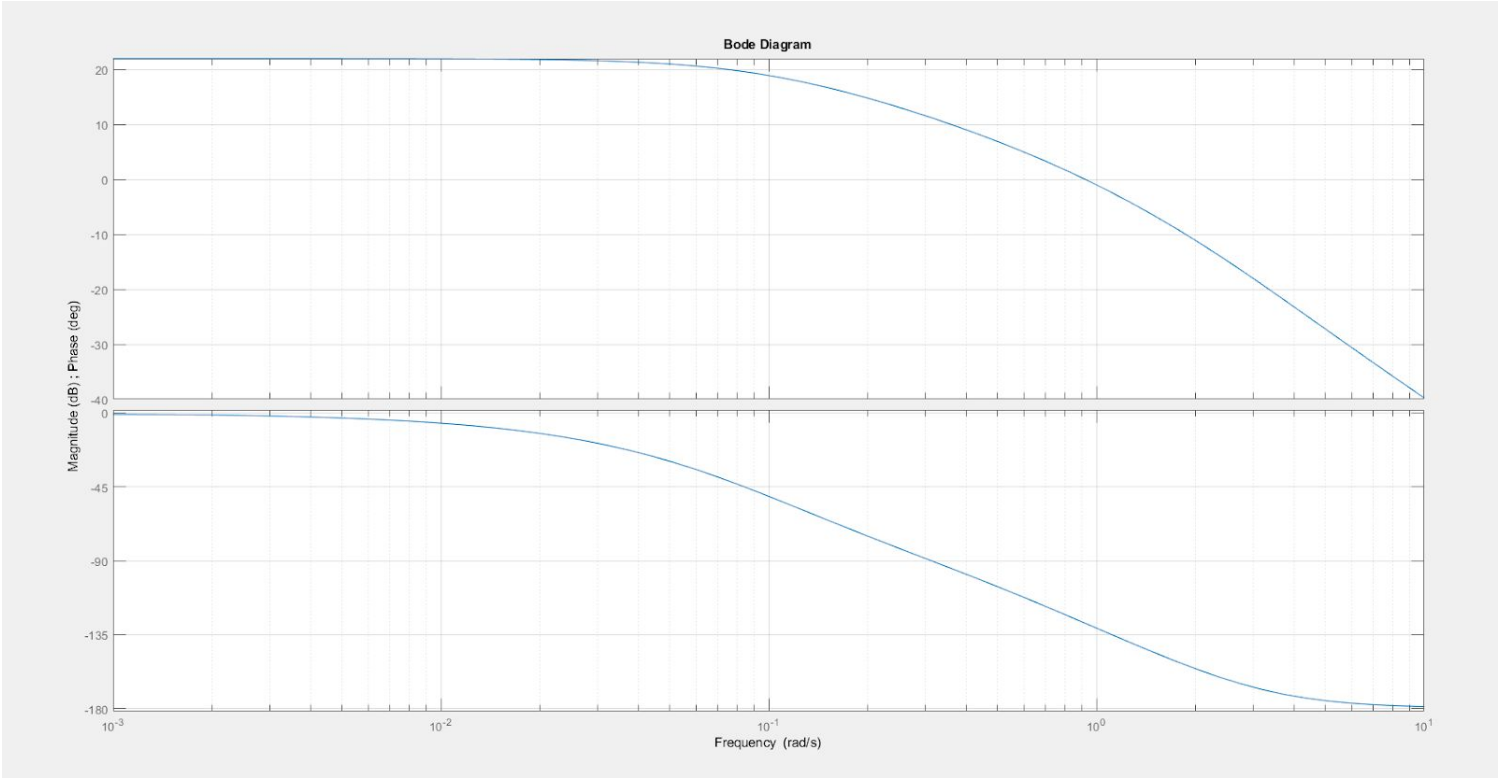
Frequency:

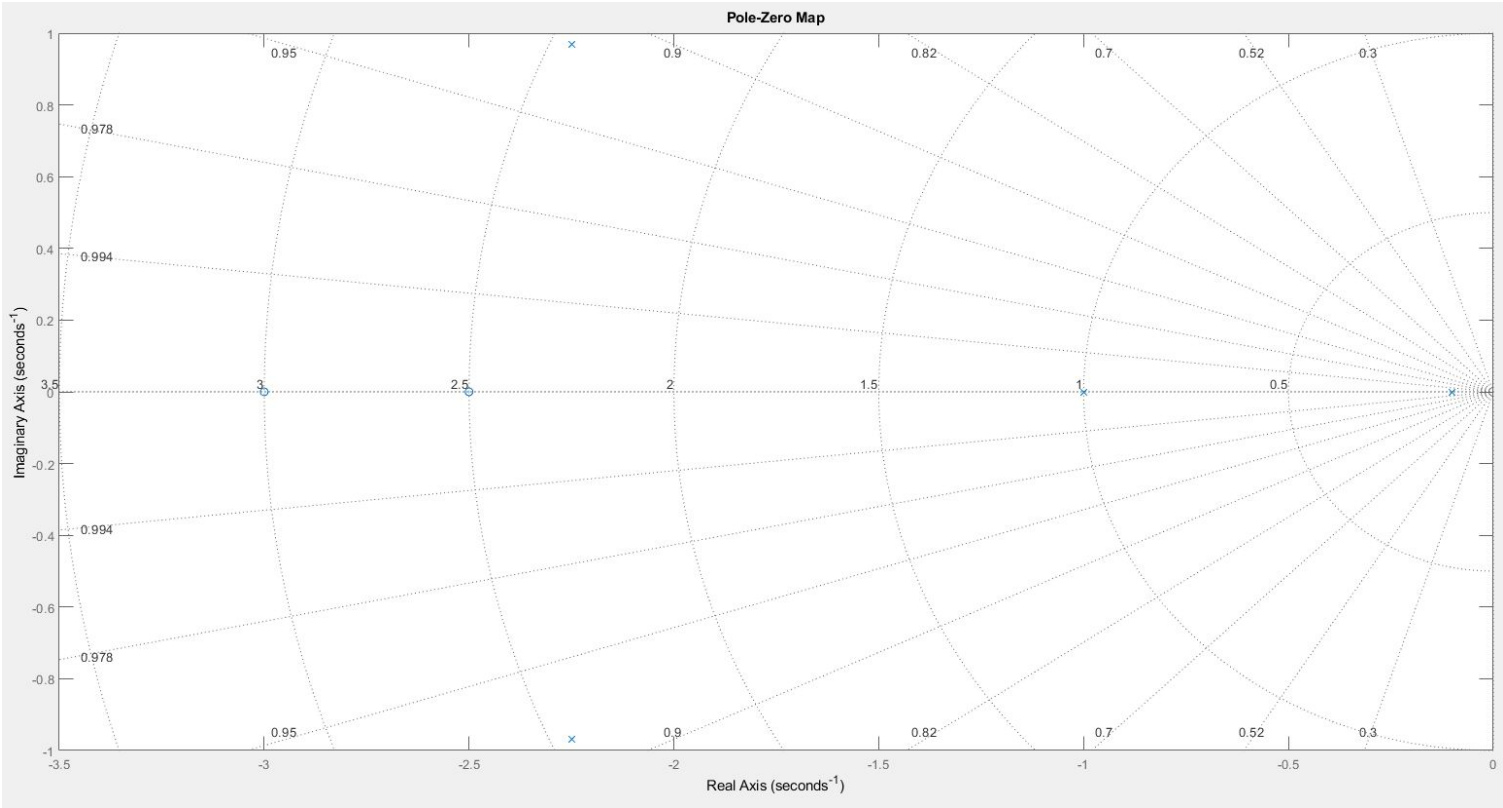




(C) For one of the inputs (write down what you choose) generate a Bode and Pole-Zero map plots. Put plots and result - stable or unstable is system and why - in the report.

I choose Impulse.

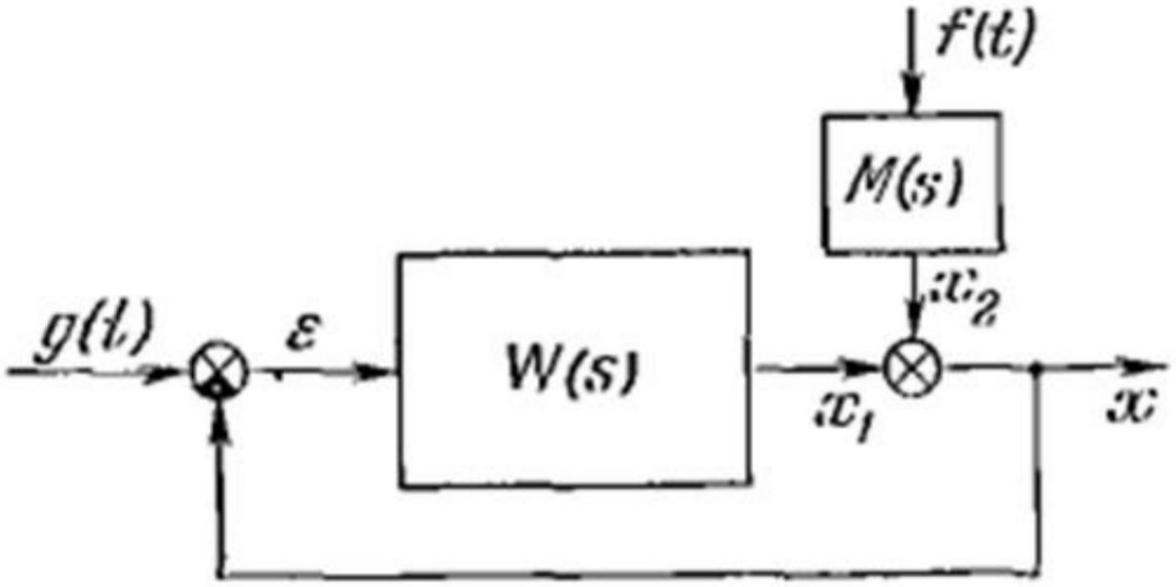




Conclusion: The system is stable because gain margin and phase margin is greater than 0.

№3.

Find total transfer function for a closed-loop system.



$W(s) = \frac{s+1}{s^2+3s+2}$

$M(s) = \frac{1}{s+3}$

$\Phi(s) = \frac{X}{G} = \frac{W(s)}{1+W(s)}$

$\Phi_f(s) = \frac{X}{F} = \frac{M(s)}{1+W(s)}$

$X = \Phi(s)G + \Phi_f(s)F = \frac{W(s)}{1+W(s)}G + \frac{M(s)}{1+W(s)}F$

$W(s) + 1 = \frac{s+1+s^2+3s+2}{s^2+3s+2} = \frac{s^2+4s+3}{s^2+3s+2}$

$\Phi(s) = \frac{W(s)}{1+W(s)} = \frac{s+1}{s^2+3s+2} * \frac{s^2+3s+2}{s^2+4s+3} = \frac{s+1}{s^2+4s+3} = \frac{(s+1)}{(s-1)(s+3)}$

$\Phi_f(s) = \frac{M(s)}{1+W(s)} = \frac{1}{s+3} * \frac{s^2+3s+2}{s^2+4s+3} = \frac{s^2+3s+2}{(s+3)(s^2+4s+2)} = \frac{(s-1)(s-2)}{(s+3)^2(s-1)} = \frac{(s-2)}{(s+3)^2}$

$X = \frac{(s+1)}{(s-1)(s+3)}G + \frac{s-2}{(s+3)^2}F$

$Answer: \frac{(s+1)}{(s-1)(s+3)}G + \frac{s-2}{(s+3)^2}F$

№4.

Find transfer function of the system.

$$\text{f) } A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, C = \begin{bmatrix} -1 & 4 \end{bmatrix}, D = \begin{bmatrix} 2 \end{bmatrix}$$

$$Y(s) = (C(sI - A)^{-1}B + D)U(s)$$

$$H(s) - transferfunction$$

$$H(s) = \frac{Y(s)}{U(s)} = (C(sI - A)^{-1}B + D)$$

```
1 -    syms s
2
3 -    A=[1 0; 2 1];
4 -    B=[2; 2];
5 -    C=[-1  4];
6 -    D=2;
7
8 -    Phi=inv(s*eye(2)-A);
9
10 -   H=C*Phi*B+D;
11
12 -   disp(H);
```

Output:

```
6/(s - 1) + 16/(s - 1)^2 + 2
```

№5

$$\text{(f) } A = \begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 5 \\ 3 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} -2 & 0 \end{pmatrix}, D = \begin{pmatrix} 2 & 4 \end{pmatrix}$$

Same formula as in №4.

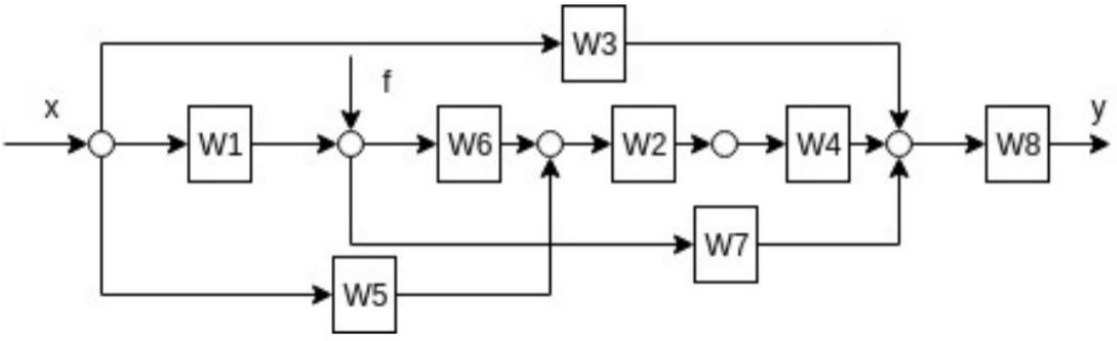
```
1 -    syms s
2
3 -    A=[2 1; -3 1];
4 -    B=[-1 5; 3 1];
5 -    C=[-2  0];
6 -    D=[2 4];
7
8 -    Phi=inv(s*eye(2)-A);
9 -    H=C*Phi*B+D;
10
11 -   disp(H);
```

Output:

```
[ (2*(s - 1))/(s^2 - 3*s + 5) - 6/(s^2 - 3*s + 5) + 2, 4 - (10*(s - 1))/(s^2 - 3*s + 5) - 2/(s^2 - 3*s + 5)]
```

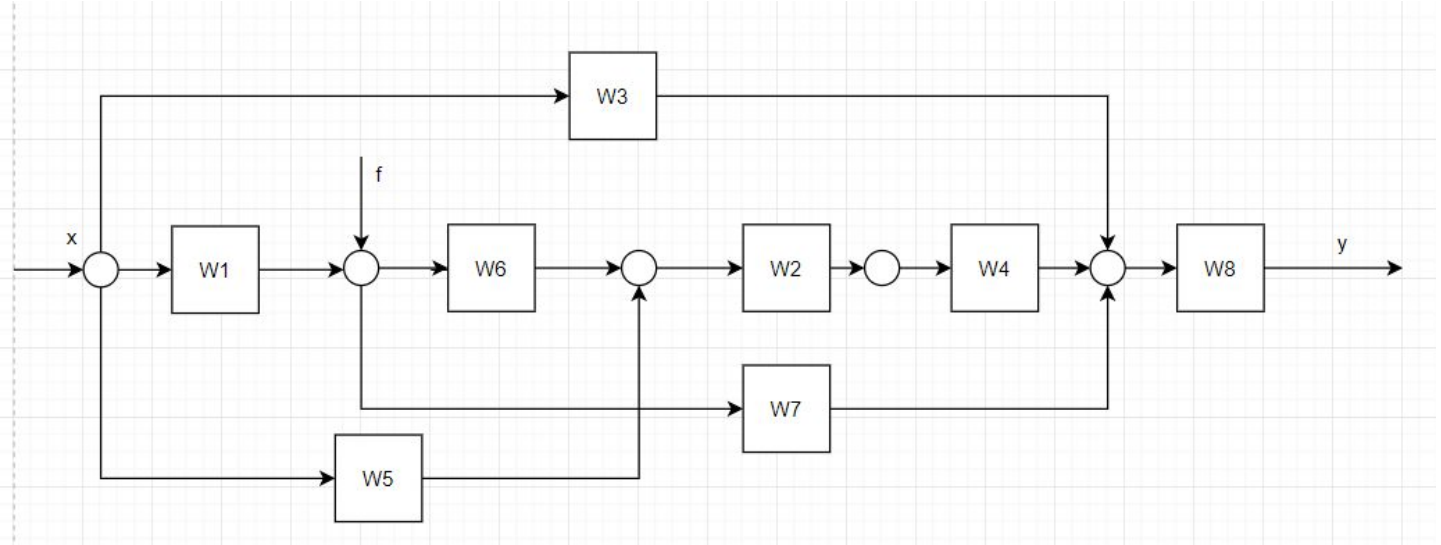
№6.

Simplify the system step by step and calculate total transfer function for both inputs x and f.

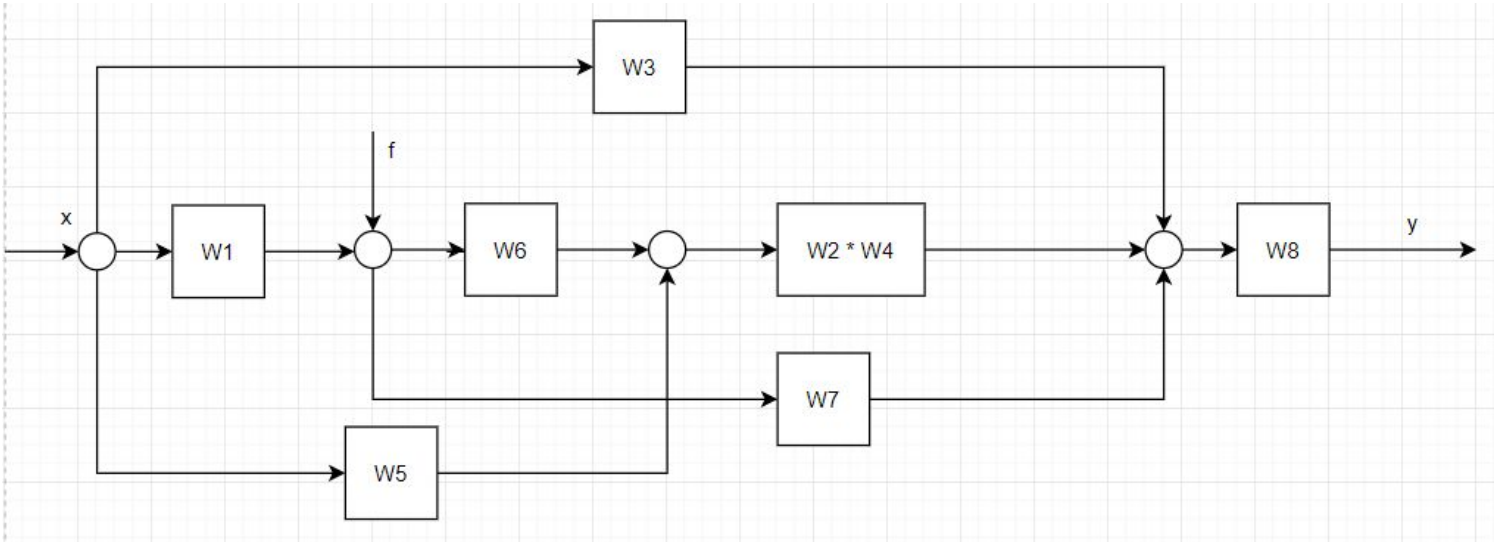


(f)

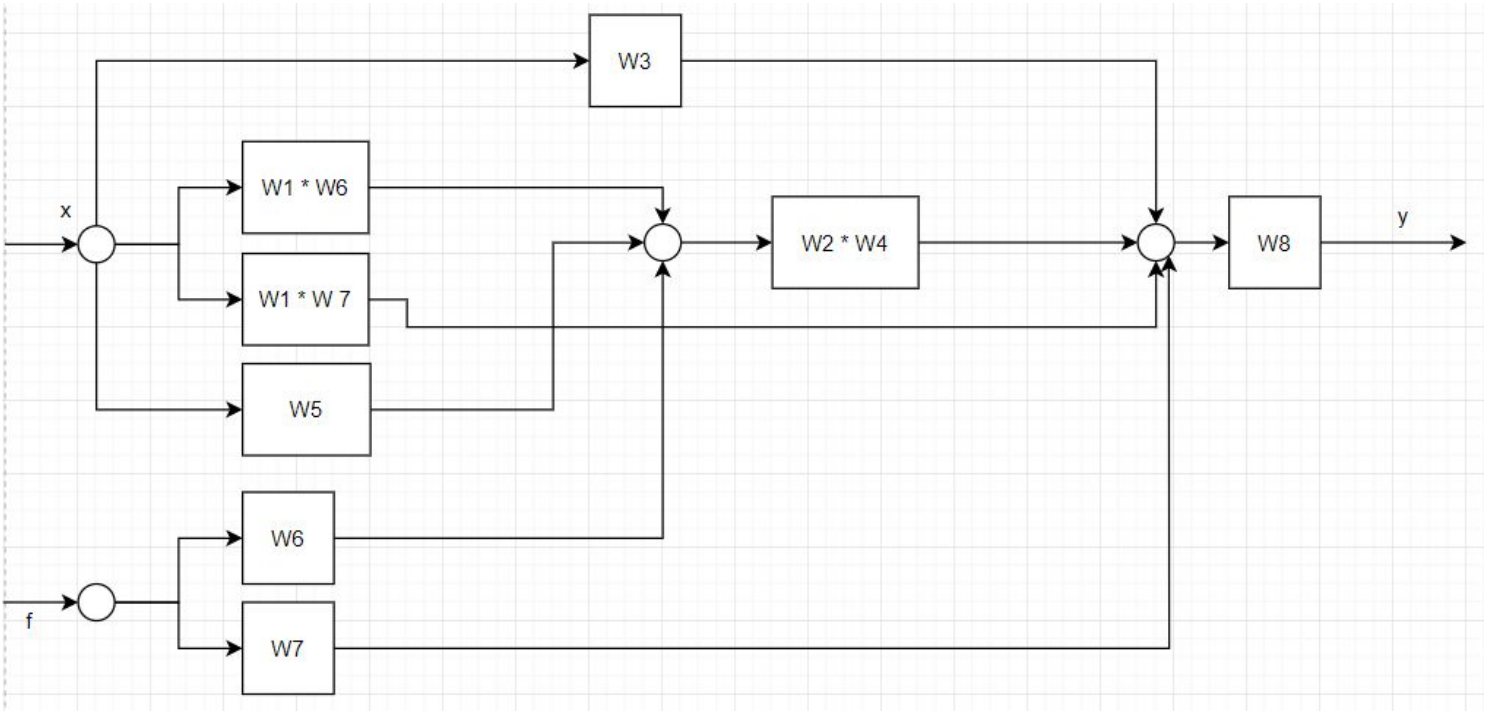
Step 0:



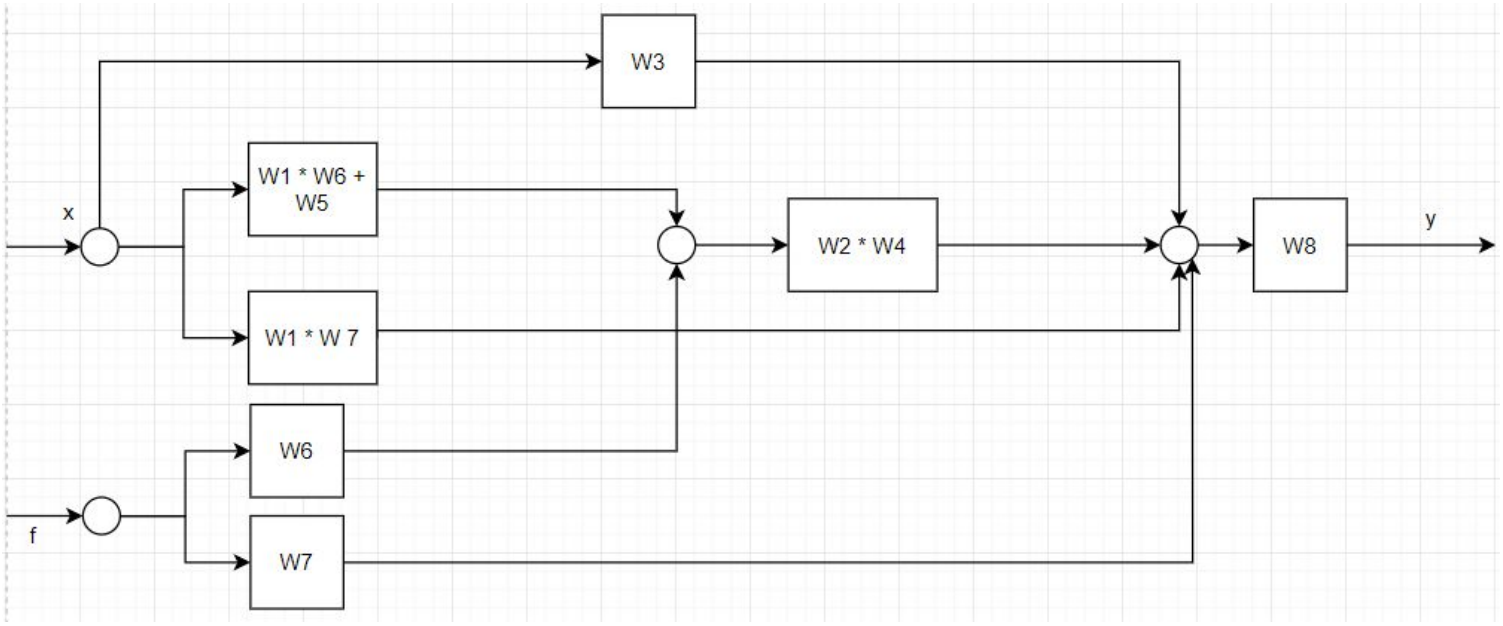
Step 1:



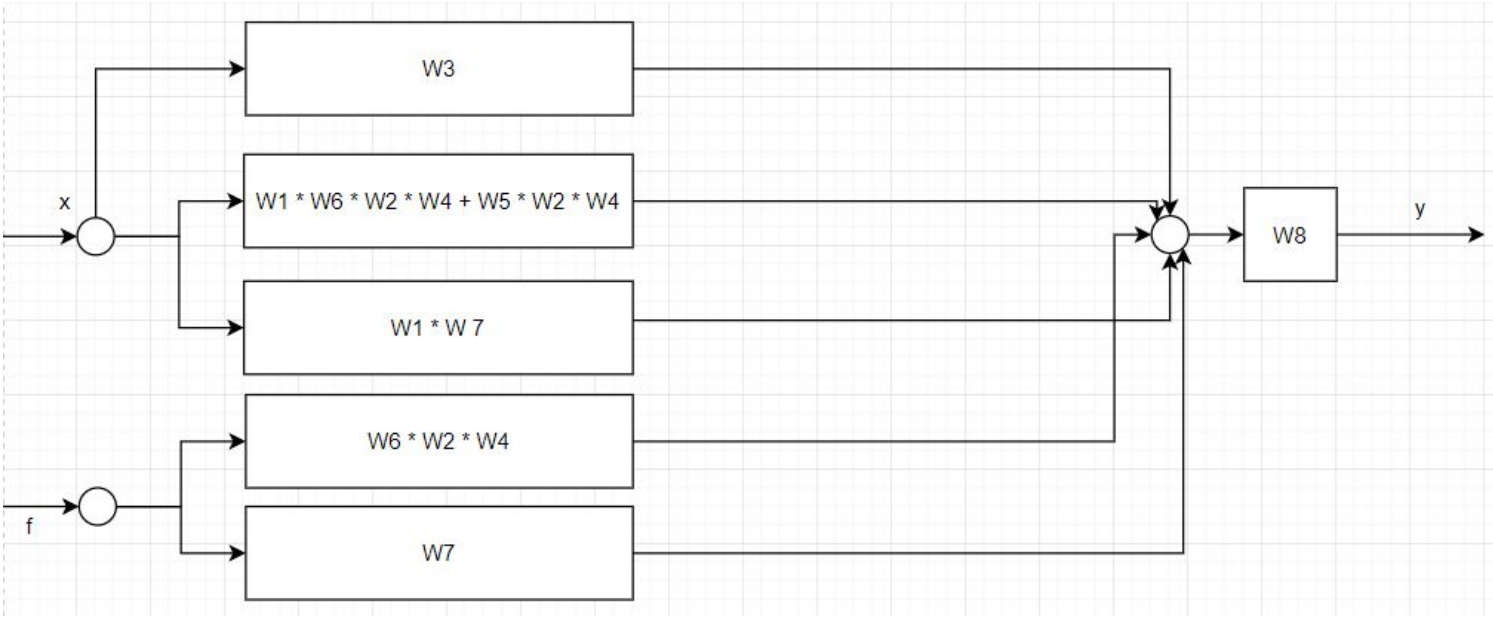
Step 2:



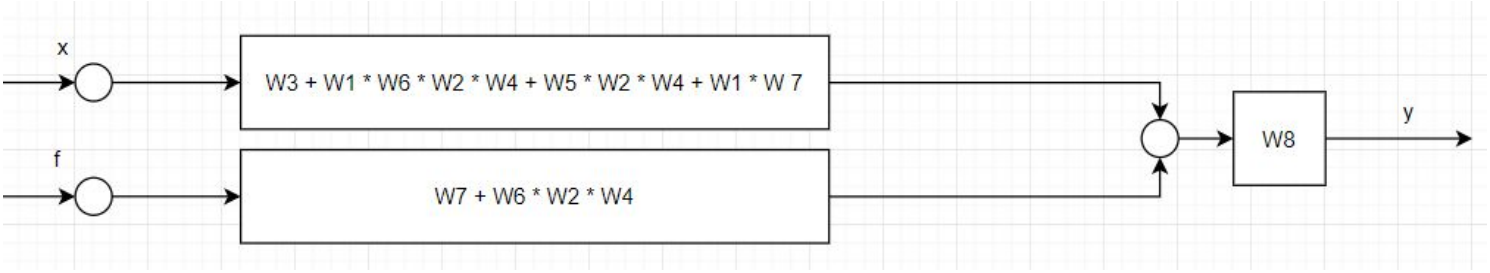
Step 3:



Step 4:



Step 5:



6 Step: