Importance sampling in Thunderloom

Vidar Nelson

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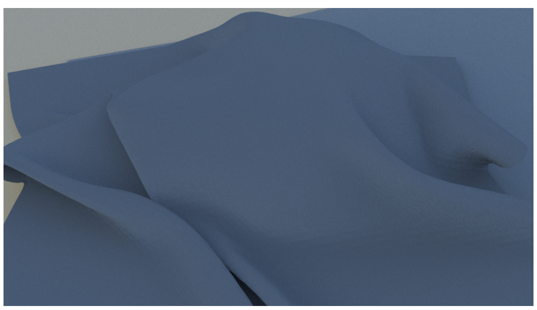
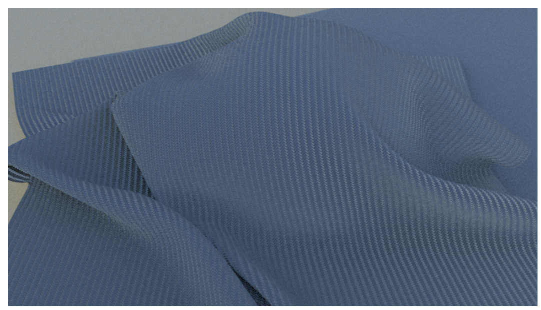
1 Introduction

In order for the Thunderloom shader to be useful in modern rendering applications, it is important that it correctly responds to indirect illumination. At the moment, no consideration has been given to importance sampling and this document is an initial step towards handling this properly. Fig. 1 shows a comparison between uniform sampling of incident directions, which is the current approach used in Thunderloom, and the sampling method described here. It is clear from the images that the level of detail can be improved considerably by handling indirect illumination.



(a) Indirect illumination using diffuse sampling.



(b) Indirect illumination using the method described here.

Figure 1: A comparison between the indirect illumination component of a piece of fabric using diffuse sampling and the sampling method described in this document. The surface of (a) is very uniform while the structure the treads can be seen clearly in (b).

2 Sampling the cone of specular directions

In the BRDF that Thunderloom uses [2], specular reflection occurs when

$$\mathbf{h} \cdot \mathbf{t} = 0, \tag{1}$$

where $\mathbf{h} = (\omega_i + \omega_o)/||\omega_i + \omega_o||$ is the half angle vector and $\mathbf{t}$ is the fiber tangent direction, determined by the geometry of the cloth. Note that Eq. (1) defines a cone of possible incident directions since it implies that any normalized ω_i that fulfills

$$\omega_i \cdot \mathbf{t} = -\omega_o \cdot \mathbf{t} \tag{2}$$

is valid. Disregarding the phase function of the BRDF for now, we can sample from this cone of directions by first constructing a basis consisting of the vectors $\mathbf{t}, \mathbf{r}$ and $\mathbf{s}$, where $\mathbf{s}$ and $\mathbf{r}$ are orthogonal to $\mathbf{t}$ and to each other. For later convenience, we will choose

$$\mathbf{r} = \frac{\mathbf{t} \times \mathbf{n}}{||\mathbf{t} \times \mathbf{n}||}, \ \mathbf{s} = \frac{\mathbf{t} \times \mathbf{r}}{||\mathbf{t} \times \mathbf{r}||}. \tag{3}$$

Let us now sample a random vector on the cone of specular directions. Observe that the **t**-component of such a vector should be

$$t = -(\omega_o \cdot \mathbf{t}). \tag{4}$$

What remains then is to pick a random vector with length $\sqrt{1-t^2}$ in the **s**, **r** plane. Let $\theta \sim U[0, 2\pi]$ be a uniformly distributed random variable and

$$\begin{aligned} r &= \sqrt{1-t^2}\cos\theta \\ s &= \sqrt{1-t^2}\sin\theta. \end{aligned} \tag{5}$$

Then

$$\tilde{\omega_i} = t\ \mathbf{t} + r\ \mathbf{r} + s\ \mathbf{s} \tag{6}$$

is a random vector uniformly distributed on the cone of specular directions. If the phase function had been completely uniform, this would sample the BRDF.

3 Sampling the phase function

In this section we limit our discussion to the circle parameterized by θ in the 2 dimensional plane spanned by **r** and **s**. The goal is to sample a position on the circle according to the phase function of the BRDF. The phase function currently used in Thunderloom is the von Mises distribution, which has a *concentration* parameter β and a *location* parameter μ. This is mixed with a uniform distribution over the circle, with the parameter α determining the proportion of the two distributions. In the shader, it is assumed that the distribution of incident light directions on the circle is concentrated around the location of the outgoing light direction, i.e. that

$$\mu = \tan^{-1}\frac{\omega_o \cdot \mathbf{r}}{\omega_o \cdot \mathbf{s}}. \tag{7}$$

The concentration parameter β determines the concentration around this mean, with $\beta = 0$ giving a completely uniform distribution on the circle and the samples getting more and more tightly concentrated around μ as β increases. Unfortunately, there is no straight forward way of sampling the von Mises distribution. To get around this, we can change to a similar distribution, the wrapped Cauchy distribution [3, p. 51]

$$f_{\mathrm{wc}}(\theta; \mu, \rho) = \frac{1}{2\pi}\frac{1-\rho^2}{1-2\rho\cos(\theta-\mu)+\rho^2} \tag{8}$$

where μ is equivalent to the location parameter of the von Mises distribution and ρ controls the concentration. The benefit of using the wrapped Cauchy distribution as our phase function, instead of the von Mises distribution, is that it is easy to sample a value of θ from the former using inverse transform sampling [1]. The CDF of the wrapped Cauchy distribution is

$$F_{\mathrm{wc}}(\theta; \mu, \rho) = \frac{1}{2\pi}\cos^{-1}\left(\frac{(1+\rho^2)\cos(\theta-\mu)-2\rho}{1+\rho^2-2\rho\cos(\theta-\mu)}\right) \tag{9}$$

If $\xi \sim \mathrm{U}[0, 1]$ is a uniformly distributed random variable, then we can express θ in terms of it by the transformation

$$\begin{aligned} \cos(2\pi\xi) &= \frac{(1+\rho^2)\cos(\theta-\mu)-2\rho}{1+\rho^2-2\rho\cos(\theta-\mu)} \\ (1+\rho^2)\cos(2\pi\xi) - 2\rho\cos(\theta-\mu)\cos(2\pi\xi) &= (1+\rho^2)\cos(\theta-\mu) - 2\rho \\ (1+\rho^2)\cos(2\pi\xi) + 2\rho &= \big((1+\rho^2) + 2\rho\cos(2\pi\xi)\big)\cos(\theta-\mu) \\ \mu + \cos^{-1}\left(\frac{(1+\rho^2)\cos(2\pi\xi)+2\rho}{(1+\rho^2)+2\rho\cos(2\pi\xi)}\right) &= \theta, \end{aligned} \tag{10}$$

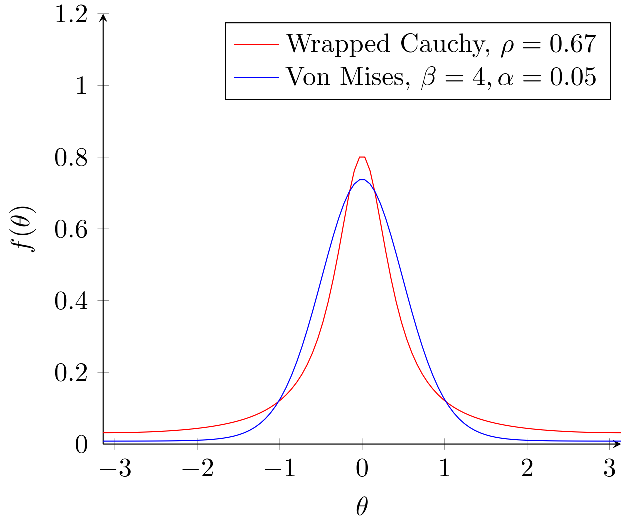


Figure 2: The wrapped Cauchy distribution, where ρ has been chosen to approximately match the von Mises distribution with $\beta = 4$.

which gives us a way of sampling the wrapped Cauchy distribution.

The remaining question is then how to set the concentration parameter, ρ. Most importantly, the default value should not differ too much from the current default. Fig. 2 shows a comparison between the Wrapped Cauchy distribution with $\rho = 0.67$ and von Mises with the default $\beta = 4, \alpha = 0.05$. Although the distributions differ a bit, this is a reasonable match and the suggestion is to make $\rho = 0.67$ the new default. Having ρ as a parameter will hopefully make a bit more sense than β and α did, since ρ is a single parameter in the range $[0, 1)$. The only issue it that when $\rho \to 1$, then $f_{\mathrm{wc}}(\theta; \mu, \rho)$ goes to the Dirac delta function. By keeping this in mind, the implementation can be made to handle values of ρ in the full range of $[0, 1]$. The formula in Eq. (10) does not need modification to allow $\rho = 1$, in that case it will simplify to $\theta = \mu$, as one would expect. However, when evaluating the phase function in Eq. (8) a special case is needed when $\rho = 1$. For this value of ρ, the phase function should simply return 0. This is analogous with the implementation of a mirror BRDF.

4 Remaining issues

The scheme for importance sampling described here improves indirect illumination in Thunderloom. However, several questions still remain.

Sampled directions below the mesh surface

With the sampling scheme described here, some directions will be located below the surface (i.e with $\omega_i \cdot \mathbf{n} < 0$), which is an unneccessary waste of samples. It should be possible to modify the formula for θ in Eq. (10) to only produce directions above the surface, but this remains to be done.

The attenuation factor

In the BRDF used by Thunderloom, there is a factor not considered in the sampling described here, the *attenuation factor*. It would be worthwhile to investigate if this could also be included in the sampling procedure.

The sampling PDF

To be correct, the color found by the sampled rays should be scaled by the ratio between the BRDF and the PDF of the sampled direction. So far, the PDF of the sampled direction has not been analyzed and this factor has just been set to the attenuation factor in the V-Ray implementation of Thunderloom.

References

[1] Luc Devroye. Non-uniform random variate generation. `http://luc.devroye.org/handbooksimulation1.pdf`.

[2] Piti Irawan and Steve Marschner. Specular reflection from woven cloth. *ACM Trans. Graph.*, 31(1), February 2012.

[3] Kanti V Mardia and Peter E Jupp. *Directional statistics.* John Wiley & Sons, 2000.