Class 4

Matrix Exponentiation (2) | 18 March / 2020

5. $f(n) = f(n-1) + 2n^2 + 3n + 5$

$= f(n-1) + g(n)$

$= f(n-1) + g(n-1) + 4n + 1$ ... (1)

Here, $g(n) = 2n^2 + 3n + 5$

$g(n-1) = 2(n-1)^2 + 3(n-1) + 5$

or, $g(n-1) = 2(n^2 - 2n + 1) + 3n - 3 + 5$

$= 2n^2 - 4n + 2 + 3n - 3 + 5$

$= 2n^2 + 3n + 5 - 4n - 1$

or, $g(n-1) = g(n) - 4n - 1$

$\therefore g(n) = g(n-1) + 4n + 1$

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From (1),

$$\begin{bmatrix} f(n) \\ g(n) \\ n+1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 4 & 1 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f(n-1) \\ g(n-1) \\ n \\ 1 \end{bmatrix}$$

$f(n) = a\, f(n-1) + b\, f(n-2) + n$

$$\begin{bmatrix} f(n) \\ f(n-1) \\ n+1 \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f(n-1) \\ f(n-2) \\ n \\ 1 \end{bmatrix}$$

Given,

\# $a+b=p$

$ab=q$

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$a^n+b^n=?$

$a^1+a^1=a+b$

$a^2+b^2=(a+b)^2-2ab$

$a^3+b^3=(a^2+b^2)(a+b)-ab\,(a+b)$

$a^4+b^4=(a^3+b^3)(a+b)-ab\,(a^2+b^2)$

$\vdots$

$a^n+b^n=(a^{n-1}+b^{n-1})(a+b)-ab\,(a^{n-2}+b^{n-2})$

$\therefore\ f(n)=f(n-1)\cdot p-f(n-2)\cdot q \quad \cdots (1)$

$$\begin{vmatrix} f(n) \\ f(n-1) \end{vmatrix} = \begin{vmatrix} p & -q \\ 1 & 0 \end{vmatrix}^n \begin{vmatrix} p \\ 2 \end{vmatrix}$$

from (1),

$$\begin{vmatrix} f(n) \\ f(n-1) \end{vmatrix} = \begin{vmatrix} p & -q \\ 1 & 0 \end{vmatrix} \begin{vmatrix} f(n-1) \\ f(n-2) \end{vmatrix}$$

if, $n=2$ $\quad f(n-1)=f(2-1)=f(1)$

if, $n=1$ $\quad f(n)=a^1+b^1=a+b=p$

$f(1)=p$

$f(0)=a^0+b^0=2$

Given,

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$(a^1+a^2+a^3+\cdots+a^n)\%M$

$$f(n) = a^1+a^2+a^3+\cdots+a^n$$

$$f(2n) = a^1+a^2+a^3+\cdots+a^{2n}$$

$$= (a^1+a^2+a^3+\cdots+a^n)+(a^{n+1}+a^{n+2}+a^{n+3}+\cdots+a^{2n})$$

$$= (a^1+a^2+a^3+\cdots+a^n)+a^n(a^1+a^2+a^3+\cdots+a^n)$$

$$= f(n)+a^n f(n)$$

$$f(2n) = f(n)\,[1+a^n] \quad \cdots (1)$$

if n is even, then we can write from (1)

$$f(n) = f\left(\frac{n}{2}\right)\left[1+a^{\frac{n}{2}}\right]$$

if n is odd, then

$$f(n) = f(n-1)+a^n$$

if $n=3$,

$$f(3) = f(2)+a^3$$

$$= a^1+a^2+\underline{a^3}$$

$$\therefore f(n) = \overset{\text{even}}{\overset{\uparrow}{E}}\left[f\left(\frac{n}{2}\right)\left[1+a^{\frac{n}{2}}\right]\right] + \overset{\text{odd}}{\overset{\uparrow}{O}}\left[f(n-1)+a^n\right]$$

$$= (!(n\&1))\left[f\left(\frac{n}{2}\right)\left[1+a^{\frac{n}{2}}\right]\right] + (n\&1)\left[f(n-1)+a^n\right]$$