

## Problem 1: Matrix Chain Multiplication

Given a sequence of matrices, find the most efficient way to multiply these matrices together. The problem is not actually to perform the multiplications, but merely to decide in which order to perform the multiplications.

We have many options to multiply a chain of matrices because matrix multiplication is associative. In other words, no matter how we parenthesize the product, the result will be the same. For example, if we had four matrices A, B, C, and D, we would have:

$$(ABC)D = (AB)(CD) = A(BCD) = \dots$$

However, the order in which we parenthesize the product affects the number of simple arithmetic operations needed to compute the product, or the efficiency. For example, suppose A is a  $10 \times 30$  matrix, B is a  $30 \times 5$  matrix, and C is a  $5 \times 60$  matrix. Then,

$$(AB)C = (10 \times 30 \times 5) + (10 \times 5 \times 60) = 1500 + 3000 = 4500 \text{ operations}$$

$$A(BC) = (30 \times 5 \times 60) + (10 \times 30 \times 60) = 9000 + 18000 = 27000 \text{ operations.}$$

Clearly the first parenthesization requires less number of operations.

Given an array  $p[]$  which represents the chain of matrices such that the  $i^{\text{th}}$  matrix  $A_i$  is of dimension  $p[i-1] \times p[i]$ . You need to write a program that takes the array  $p$  as input and returns the minimum number of multiplications needed to multiply the chain. The time complexity of your algorithm must be  $O(n^3)$ , where  $n$  is the number of matrices.

Input:  $p[] = \{40, 20, 30, 10, 30\}$

Output: 26000

There are 4 matrices of dimensions  $40 \times 20$ ,  $20 \times 30$ ,  $30 \times 10$  and  $10 \times 30$ .  
Let the input 4 matrices be A, B, C and D. The minimum number of multiplications are obtained by putting parenthesis in following way  
 $(A(BC))D \rightarrow 20 \times 30 \times 10 + 40 \times 20 \times 10 + 40 \times 10 \times 30$

Input:  $p[] = \{10, 20, 30, 40, 30\}$

Output: 30000

There are 4 matrices of dimensions  $10 \times 20$ ,  $20 \times 30$ ,  $30 \times 40$  and  $40 \times 30$ .  
Let the input 4 matrices be A, B, C and D. The minimum number of multiplications are obtained by putting parenthesis in following way  
 $((AB)C)D \rightarrow 10 \times 20 \times 30 + 10 \times 30 \times 40 + 10 \times 40 \times 30$

Input:  $p[] = \{10, 20, 30\}$

Output: 6000

There are only two matrices of dimensions  $10 \times 20$  and  $20 \times 30$ . So there is only one way to multiply the matrices, cost of which is  $10 \times 20 \times 30$

## Problem 2: Longest Common Subsequence

Given two sequences, find the length of longest subsequence present in both of them. A subsequence is a sequence that appears in the same relative order, but not necessarily contiguous. For example, "abc", "abg", "bdf", "aeg", "acefg", .. etc are subsequences of "abcdefg". You need to write a program that takes two strings as input and prints the longest common subsequence. In case of multiple such subsequences, print any one of them. The time complexity of your algorithm must be  $O(mn)$ , where  $m$  and  $n$  are the lengths of the two input strings.

### **Examples:**

LCS for input Sequences "ABCDGH" and "AEDFHR" is "ADH".

LCS for input Sequences "AGGTAB" and "GXTXAYB" is "GTAB".