Master de sciences et technologie 1 | MU4MA029
Mention : mathématiques et applications | 2022-2023

TP 3 : Boundary and connected components

Exercise 1

We consider the mesh format introduced in the previous tutorial, and we assume that the point cloud consists in a table `vtx` of pairs of `float`. According to this format, in a triangular mesh, each triangle is represented by a triplet of `int`. In the same manner, we can model the mesh of a 1D contour as a collection of edges that, with `Python`, is represented by a table of pairs of `int`.

Coming back to the example of `maillage1.msh` in appendix of tutorial sheet no. 2, the boundary of the triangular mesh is then a contour represented by the following table.

```
eltb =
[[0,  1  ]
 [1,  2  ]
 [2,  3  ]
 [3,  7  ]
 [7,  11 ]
 [10, 11 ]
 [9,  10 ]
 [8,  9  ]
 [4,  8  ]
 [0,  4  ]]
```

Question 1.1 Write a function `Boundary` taking as input the table of `int` named `elt` of size `nbr_elt`$\times 3$ modelling a triangular mesh, and returning as output a table `int` named `eltb` of size `nbr_eltb`$\times 2$ representing the boundary of the mesh. You may use the types `set` and `dict` of the `Python` standard library.

Question 1.2 Modify the function `PlotMesh` that you wrote for tutorial sheet no.2 so that it can take as input a table of size `N`$\times 2$, in which case `PlotMesh` shall perform a graphic display of the contour. You shall test your work by plotting the boundary of the domain from the file `maillage3.msh`.

Question 1.3 Modify the function `Boundary` so that it returns a tuple `(eltb,be2e)` where `eltb` refers to the same table as in question 1.1, and `be2e` is a table of `int` of size `nbr_eltb` representing the connectivity table relating the triangles of `elt` and the edges of `eltb`. This connectivity table shall comply with the following format :

```
be2e[j] = 3*p+k
for  j = 0,..., nbr_eltb-1
     p = 0,..., nbr_elt-1
     k = 0,1,2
```

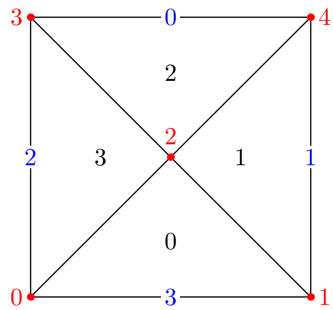
means that the `j`-th edge of the boundary is the `k`-th edge the `p`-th triangle in `elt`. We shall adopt the convention that, in a triangle, the `k`-th edge is the edge opposite to the `k`-th vertex. Below we give an example of such a connectivity table for a very simple mesh with 4 triangles.

```
vtx =
[[0. ,   0. ]
 [1. ,   0. ]
 [0.5 ,  0.5]
 [0. ,   1. ]
 [1. ,   1. ]]
```

```
elt =
[[0 ,  1 ,  2 ]
 [1 ,  2 ,  4 ]
 [2 ,  3 ,  4 ]
 [0 ,  2 ,  3 ]]
```

```
eltb =
[[3 ,  4 ]
 [1 ,  4 ]
 [0 ,  3 ]
 [0 ,  1 ]]

be2e =
[6 ,  4 ,  10 ,  2]
```



Exercice 2

Question 2.1 Write a function `CCmpt` taking as input argument a table `elt` modelling a triangular mesh of size `nbr_elt`×3, and returning as output a table of `int` named `cc` of size `nbr_elt` such that `cc[j] = k` if the `j`-th triangle of the mesh belongs to the `k`-th connected component of the mesh. You can choose the way you number the connected component of the mesh. You may use the function `scipy.sparse.csgraph.connected_components`.

Question 2.2 Using the function of the previous question, provide a graphical display of the computational domain from file `maillage4.msh` where you shall have each connected component appear in a different colour.

Question 2.3 Modify `CCmpt` so that it can take as input parameter a table representing the mesh of a contour (the input argument `elt` shall then be of size `nbr_elt`×2) and that returns as output argument a table `cc` indicating its decomposition in connected components.

Question 2.4 Using the function of the previous question, perform a graphical display of the boundary of the computational domain from file `maillage5.msh` where each connected component of the boundary appears in a different colour.

Exercice 3

Write a function `Refine` taking as input argumentthe tables `vtx` and `elt` representing a mesh, and returning as output two other tables `refined_vtx` and `refined_elt` corresponding to the same mesh after a barycentric refinement operation. A barycentric refinement consists in sub-dividing each triangle in 4 sub-triangles according to the picture below where we have introduced the mid-pints of each edge. Of course, you shall test and verify the correctness of your work by means of a graphical display.