MA 101 (Mathematics - I)

Quiz - III

Maximum Marks : 25

Date: February 20, 2021 **Time:** 10 am - 11 am

Instructions:

- The answers of this Quiz question paper are to be filled in the Quiz - III response form. You get exactly one hour time (from 10 am to 11 am) for doing this.
- You should submit the response form at 11 am (or before). Although you get extra 5 minutes for submission only (the portal will close at 11:05 am), it is advised not to take any risk of submitting after 11 am. No request will be entertained if you fail to submit your responses through the portal due to any reason.
- The response form permits only one submission. It does not allow revision.

Type of Questions and Marking scheme:

- The first question is writing your Roll number. It is compulsory. It has no marks.
- Q.2 to Q.5 are of single correct option type questions, where exactly one option is correct. Each of these questions carries 2 marks for correct answer, -1 mark for incorrect answer, and 0 mark for no answer. No answer will be considered if you do not mark any option or if you mark option (E) given in the response form.
- Q.6 to Q.12 are of multiple correct option type questions, where one or more of the options is (are) correct. In each of these questions, you get 2 marks if you choose all the correct options and choose no wrong option. In all other cases, you get 0 mark.
- Q.13 is also of multiple correct option type questions, where one or more of the options is (are) correct. For this question, you get 3 marks if you choose all the correct options and choose no wrong option. In all other cases, you get 0 mark.

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1. Write your Roll number.

2. Let C be the curve of intersection of the surfaces $3x^2y + y^2z + 2 = 0$ and $2xz - x^2y = 3$ in $\mathbb{R}^3$. Then the tangent line to the curve C at the point $(1, -1, 1)$ passes through the point
(A) $\big(\frac{3}{2}, \frac{4}{3}, \frac{7}{3}\big)$ (B) $\big(2, \frac{13}{3}, \frac{5}{3}\big)$ (C) $\big(0, -\frac{17}{3}, \frac{2}{3}\big)$ (D) $\big(-\frac{1}{2}, -\frac{11}{3}, -1\big)$

3. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a differentiable function such that $f(x + xy^3, x, y) = 5y - 3x$ for all $x, y \in \mathbb{R}$. If $\mathbf{u} = \big(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\big)$ and $D_{\mathbf{u}}f(4, 2, 1) = 4\sqrt{3}$, then $\|\nabla f(4, 2, 1)\|^2$ is equal to
(A) 48 (B) 96 (C) 100 (D) 102

4. Consider the following two statements **P** and **Q**.
P : There exists a continuous function from $\{(x, y) \in \mathbb{R}^2 : 1 < x^2 + y^2 < 2\}$ onto $\{x \in \mathbb{R} : |x| > 1\}$.
Q : There exists a one-one continuous function from $\{(x, y) \in \mathbb{R}^2 : y^2 = 2x\}$ onto

$\{x \in \mathbb{R} : x > 2\}$. Then

(A) both **P** and **Q** are true (B) **P** is true but **Q** is false

(C) **Q** is true but **P** is false (D) both **P** and **Q** are false

5. For $c \in \mathbb{R}$ with $c > 1$, let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = (cx^2+y^2)e^{-x^2-y^2}$ for all $(x, y) \in \mathbb{R}^2$. Consider the following two statements **P** and **Q**.

P : There exists $c \in \mathbb{R}$ with $c > 1$ such that f has at least six critical points.

Q : There exists $c \in \mathbb{R}$ with $c > 1$ such that f has at least three saddle points.

Then

(A) both **P** and **Q** are true (B) **P** is true but **Q** is false

(C) **Q** is true but **P** is false (D) both **P** and **Q** are false

6. Let $S = [0, 1] \times [0, 1] \subseteq \mathbb{R}^2$ and let $f : S \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} x\left(\cos\left(\frac{1}{x+y}\right) - 1\right) - y & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Then

(A) f is continuous at each point of S

(B) there exists exactly one $\mathbf{x}_0 \in S$ such that $f(\mathbf{x}_0) = \sup\{f(\mathbf{x}) : \mathbf{x} \in S\}$

(C) there exist more than one $\mathbf{x}_0 \in S$ such that $f(\mathbf{x}_0) = \sup\{f(\mathbf{x}) : \mathbf{x} \in S\}$

(D) there exist no $\mathbf{y}_0 \in S$ such that $f(\mathbf{y}_0) = \inf\{f(\mathbf{x}) : \mathbf{x} \in S\}$

7. For $r, s, t \in (0, \infty)$, let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \begin{cases} \frac{\sin(|x|^r|y|^s)}{(x^2+y^2)^t} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

Then

(A) f is differentiable at $(0, 0)$ if $r + s > 2t + 1$

(B) f is differentiable at $(0, 0)$ only if $r + s > 2t + 1$

(C) f is differentiable at $(0, 0)$ if $r > t + 1$ and $s > t + 1$

(D) f is differentiable at $(0, 0)$ only if $r + s > 2t + 2$

8. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \begin{cases} x^2 + y^2 & \text{if } x, y \in \mathbb{Q}, \\ 0 & \text{otherwise.} \end{cases}$

Then

(A) $f_x(x_0, y_0)$ exists (in $\mathbb{R}$) for exactly one point $(x_0, y_0) \in \mathbb{R}^2$

(B) $f_x(x_0, y_0)$ exist (in $\mathbb{R}$) for all points $(x_0, y_0) \in \mathbb{R}^2$ with $x_0,\ y_0 \in \mathbb{Q}$

(C) $f_x(x_0, y_0)$ exist (in $\mathbb{R}$) for all points $(x_0, y_0) \in \mathbb{R}^2$ with $x_0 \in \mathbb{R} \setminus \mathbb{Q},\ y_0 \in \mathbb{Q}$

(D) $f_y(x_0, y_0)$ exist (in $\mathbb{R}$) for infinitely many points $(x_0, y_0) \in \mathbb{R}^2$

9. If $S = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : \text{there exists } x \in \mathbb{R} \text{ such that } a_1x^4+a_2x^3+a_3x^2+a_4x+a_5 = 0\}$, then

(A) $(4, -7, 2, -3, -1) \in S^0$ (B) $(3, 1, -5, -2, 1) \in S^0$

(C) $(-2, 0, 4, -6, 3) \in S^0$ (D) $(-6, -2, 0, -6, -1) \in S^0$

10. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \begin{cases} \frac{xy^2+x^3+xy^4}{x^2+y^4} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

Then which of the following statements is (are) FALSE ?

(A) Starting at $(0,0)$, among all $\mathbf{u} \in \mathbb{R}^2$ with $\|\mathbf{u}\| = 1$, f increases most rapidly in the direction of $\mathbf{u} = (1,0)$

(B) Starting at $(0,0)$, among all $\mathbf{u} \in \mathbb{R}^2$ with $\|\mathbf{u}\| = 1$, f increases most rapidly in the direction of $\mathbf{u} = (0,1)$

(C) Starting at $(0,0)$, among all $\mathbf{u} \in \mathbb{R}^2$ with $\|\mathbf{u}\| = 1$, f increases most rapidly in the direction of $\mathbf{u} = (-1,0)$

(D) Starting at $(0,0)$, among all $\mathbf{u} \in \mathbb{R}^2$ with $\|\mathbf{u}\| = 1$, f decreases most rapidly in the direction of $\mathbf{u} = (0,-1)$

11. Which of the following is (are) closed set(s) in $\mathbb{R}^2$?

(A) $\left\{\left(x, \frac{1}{x}\right) : x \in \mathbb{R},\, x > 0\right\}$

(B) $\left\{\left(x, \frac{1}{x}\right) : x \in \mathbb{R},\, x > 0\right\} \bigcup \left\{(x,y) \in \mathbb{R}^2 : x \geq 0,\, y \geq 0,\, xy = 0\right\}$

(C) $\left\{\left(\frac{1}{4^m}, \frac{1}{5^n}\right) : m, n \in \mathbb{N}\right\}$

(D) $\left\{\left(\frac{1}{4^m}, \frac{1}{5^n}\right) : m, n \in \mathbb{N}\right\} \bigcup \left\{(0,0)\right\}$

12. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be such that $f_x(x,y)$ and $f_y(x,y)$ exist (in $\mathbb{R}$) for all $(x,y) \in \mathbb{R}^2$. Which of the following statements is (are) always true?

(A) If both $f_x : \mathbb{R}^2 \to \mathbb{R}$ and $f_y : \mathbb{R}^2 \to \mathbb{R}$ are bounded, then f is continuous

(B) If f is discontinuous, then at least one of $f_x : \mathbb{R}^2 \to \mathbb{R}$ and $f_y : \mathbb{R}^2 \to \mathbb{R}$ is unbounded

(C) If f is discontinuous, then both $f_x : \mathbb{R}^2 \to \mathbb{R}$ and $f_y : \mathbb{R}^2 \to \mathbb{R}$ are unbounded

(D) If both $f_x : \mathbb{R}^2 \to \mathbb{R}$ and $f_y : \mathbb{R}^2 \to \mathbb{R}$ are unbounded, then f is unbounded

13. For $m,\, n,\, k,\, \ell \in \mathbb{N}$ with $k,\, \ell$ even, let $f : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$ be defined by $f(x,y) = \frac{x^m y^n}{x^k + y^\ell}$ for all $(x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$. Then $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists (in $\mathbb{R}$) if

(A) $m = 5,\, n = 2,\, k = \ell = 6$ (B) $m = 2,\, n = 3,\, k = 6,\, \ell = 2$

(C) $m = 2,\, n = 3,\, k = 6,\, \ell = 4$ (D) $m = n = 3,\, k = 8,\, \ell = 4$

——— **END** ———