Tutorial problems: MA101-Calculus **IIT Guwahati, 2020**

Tutorial 1: Realnumbers1,2,3, Sequence1

1. Let S and T be nonempty and bounded above. Define $S+T = \{s+t \mid s \in S, t \in T\}$. Then show that $\sup(S+T) = \sup S + \sup T$.
2. Give a finite set, a countable set and an uncountable set $S \subseteq \mathbb{R}$ such that $\mathsf{lub}\, S \in S$. Give a finite set, a countable set and an uncountable set $S \subseteq \mathbb{R}$ such that $\mathsf{lub}\, S \notin S$.
3. Let A and B be nonempty and bounded sets such that $A \cap B \neq \emptyset$. Order lub's of $A \cup B$, A and $A \cap B$.
4. Determine the sets $\bigcap_{n=1}^{\infty}(-\frac{1}{n}, \frac{1}{n})$ and $\bigcap_{n=1}^{\infty}(0, \frac{1}{n}]$.
5. Let $S \subseteq [1,2]$ be an infinite set. Show that it has a limit point.
6. Let $a < b$. Supply 3 rationals and 3 irrationals inside (a,b).
7. Consider the sequence $(a_n = \frac{1}{n})$.

   a) Let $a \neq 0$. Then $a_n \not\to a$ as $\exists \epsilon > 0$ such that $B_\epsilon(a)$ misses infinitely many terms of (a_n). Give a value for ϵ.

   b) $a_n \to 0$ as each $B_\epsilon(a)$ contains a tail (which may depend on ϵ) of (a_n). Which tail?
8. Let $s > 0$. Is $\frac{[10^n s]}{10^n} \to s$?