Tutorial 4: continuity

1. T/F? Let (x_n) be a fixed sequence of points in $[a, b]$ such that $x_n \to c$. Let $f : [a, b] \to \mathbb{R}$ be a function such that $f(x_n) \to f(c)$. Then f is continuous at c.
2. T/F? If $f + g$ is continuous at a, then f may be discontinuous at a.
3. T/F? If $f + g$ is continuous at a, then f may be discontinuous at a, even if g is continuous at a.
4. T/F? Let f be continuous at a. Then $|f|$ is also continuous at a.
5. T/F? Let f and g be continuous on $\mathbb{R}$. Then $h(x) := \max\{g(x), f(x)\}$ is continuous.
6. T/F? Let f be continuous. Then the **positive part** $f_+(x)$ which is defined as $\max\{0, f(x)\}$ is continuous.
7. T/F? Let f be continuous on $\mathbb{R}$ with $f(x + y) = f(x) + f(y)$ for each x, y. Then there exists a c such $f(x) = cx$ for all x.
8. Let f, g be continuous at 0. Then $f \circ g$ must be continuous at 0.
9. There is a bijection $f : [0, 1] \to \mathbb{R}$. I claim that such a bijection is discontinuous, how?
10. There is no continuous function f from $(0, 1)$ onto $(0, 1) \cup (2, 3)$. Why?
11. Let $f : [0, 2] \to \mathbb{R}$ be continuous such that $f(0) = 5, f(1) = 4, f(2) = 9$. Then there must be a point $c \in [0, 1]$ such that $f(2c) = 2f(c)$. Why?
12. Let $f : \mathbb{R} \to \mathbb{R}$ be a monotone increasing function. Fix any $a \in \mathbb{R}$. Then $\lim\limits_{x \to a-} f(x)$ must exist. Why?
13. T/F? Let $A = (0, 1) \cup (3, 4)$ and $f : A \to \mathbb{R}$ be defined as $f(x) = 1$ if $x \in A$ and $f(x) = 2$, otherwise. Then f is continuous on A.