Tutorial 3: Series2,3, Limitcontinuity1,2

A) If possible find examples with nonzero terms (wherever necessary).

1. Let l be a fixed real number. Give two series with distinct terms converging to l.
2. If $\sum a_n$ and $\sum b_n$ are divergent, is $\sum(a_n + b_n)$ necessarily divergent?
3. What happens to $\sum(a_n + b_n)$ when $\sum a_n = a$ and $\sum b_n$ diverges?
4. If $\sum a_n$ and $\sum b_n$ are convergent is $\sum(a_n b_n)$ necessarily convergent?
5. Let $a_n, b_n \geq 0$. Suppose that $\sum a_n = l$ and $\sum b_n = t$. Is $\sum a_n b_n$ convergent?
6. Let $a_n \geq 0$. If $\sum a_n^2$ converges, then is $\sum a_n$ convergent?
7. Can $\sum(a_n b_n)$ be convergent, given $\sum a_n, \sum b_n$ are divergent?
8. If $\sum a_n$ is convergent and $\{b_n\}$ is bounded, is $\sum(a_n b_n)$ necessarily convergent?
9. Let $a_n \geq 0$ and $\sum a_n$ be convergent. Is $\sum \frac{\sqrt{a_n}}{n}$ necessarily convergent?
10. If $\sum a_n$ converges and $a_n \geq 0$ then is $\sum \frac{a_n}{n}$ necessarily convergent?
11. Suppose that $a_n > 0$ and $\lim\limits_{n\to\infty} a_n = 0$. Is $\sum\limits_{n=1}^{\infty}(-1)^{n+1}a_n$ necessarily convergent?

B) True or false?

1. Let $f : \mathbb{N} \to \mathbb{R}$ be defined as $f(x) = 5$. Then $\lim\limits_{x\to 4} f(x) = 5$.
2. Let $f : A \to \mathbb{R}$ and a be a cluster point of A. Suppose that for each sequence (a_n) of points from A converging to a, we have $f(a_n) \to l$ (here we are not putting the restriction that $a_n \neq a$). Then $\lim\limits_{x\to a} f(x) = l$.
3. Take $f(x) = x^5 + 7x^4 - 10x^3 + 5$. We want to show that $\lim\limits_{x\to 1} f(x) = 3$. For that we start with 'Let $\epsilon > 0$'. Then
$$\delta = \min\{\sqrt[5]{1+\frac{\epsilon}{3}} - 1,\ \sqrt[4]{1+\frac{\epsilon}{21}} - 1,\ \sqrt[3]{1+\frac{\epsilon}{30}} - 1\}$$
is an appropriate value.

C) Other questions.

1. Define $\lim_{x \to c} f(x) = \infty$ in both ways. Compare with texts.
2. Define $\lim_{x \to \infty} f(x) = l$ in both ways. Compare with the texts. Did you find it similar to that of $\lim_{n \to \infty} a_n = l$, where $a_n = f(n)$?
3. Define f on $\mathbb{R}$ as

$$f(x) = \begin{cases} \frac{1}{n} & \text{if } x = \frac{1}{n},\ n \in \mathbb{N} \\ 0 & \text{else.} \end{cases}$$

   Find the points a at which $\lim_{x \to a} f(x)$ exists.