Department of Mathematics
IIT Guwahati

Quiz1	MA101	12-12-2020	Total marks: 24	Time: 65 min

- The first question is writing your roll number. It is compulsory.
- Each other question carries 2 marks.
- For single-correct-option question, you get 2 for correct answer, -1 for wrong answer and 0 for not attempting.
- For multiple-correct-option question, you get 2 for correct answer and 0 for wrong answer or not attempting.
- You get extra 5 minutes for submission. REMEMBER to press the SUBMIT button by 11:04:59. The form will not accept responses after that.
- The form permits ONLY ONE submission. It does not allow REVISION.

1. Write your roll number.
2. (Multiple correct options) Let A and B be nonempty subsets of $\mathbb{R}$ such that for each $a \in A$ and $b \in B$, we have $a^2 \leq b$. Then which of the following statements are correct?

   A) $\mathsf{lub}\, A$ must exist in $\mathbb{R}$

   B) $\mathsf{lub}\, B$ must exist in $\mathbb{R}$

   C) $\mathsf{glb}\, A$ must exist in $\mathbb{R}$

   D) $\mathsf{glb}\, B$ must exist in $\mathbb{R}$

3. For a natural number n define $val(n) = n(1 - \dfrac{n \ln n}{(n+1)\ln(n+1)})$. Let

   $$(p_n) = (3, 2, 7, 5, 13, 11, 19, 17, \ldots)$$

   be the sequence of prime numbers. We want to find $\lim\limits_{n\to\infty} val(p_n)$. Then which of the following options is correct?

   A) Limit exists and it is less than half.

   B) Limit exists and it is half.

   C) Limit exists and it is more than half.

   D) Limit does not exist.

4. Consider the following two statements.

   Statement 1: Take $a_n = (1 + \frac{1}{\sqrt{n}})^n$. Then the sequence (a_n) is convergent.

   Statement 2: If (c_n) and (d_n) are two Cauchy sequences, then $(c_n d_n)$ must be a Cauchy sequence.

   Which of the following options is correct?

   A) Statement 1 is correct but Statement 2 is wrong.

   B) Statement 2 is correct but Statement 1 is wrong.

   C) Both Statement 1 and Statement 2 are wrong.

   D) Both Statement 1 and statement 2 are correct.

5. Consider the following two statements.

   Statement 1: $\sum(-1)^n \ln\left(1+\frac{1}{n^2}\right)$ is absolutely convergent.

   Statement 2: Let $a_n \geq 0$ and $\sum a_n$ be convergent. Then the sequence (na_n) must be convergent.

   Which of the following options is correct?

   A) Statement 1 is correct but Statement 2 is wrong.

   B) Statement 2 is correct but Statement 1 is wrong.

   C) Both Statement 1 and Statement 2 are wrong.

   D) Both Statement 1 and statement 2 are correct.

6. Consider the following two statements.

   Statement 1: There exists an increasing sequence (a_n) such that $\sum a_n = 2020$.

   Statement 2: $\sum\limits_{n\geq 1} \frac{1}{n^{1+\frac{1}{n}}}$ is divergent.

   Which of the following options is correct?

   A) Statement 1 is correct but Statement 2 is wrong.

   B) Statement 2 is correct but Statement 1 is wrong.

   C) Both Statement 1 and Statement 2 are wrong.

   D) Both Statement 1 and statement 2 are correct.

7. Consider the following two statements.

   Statement 1: Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Then $Z = \{x \mid f(x) = 5\}$ contains all its cluster points.

   Statement 2: Let f be continuous with $\lim\limits_{h\to 0} \frac{f(h)}{h^2} = 2$ and $\lim\limits_{h\to 0} \frac{f(h)}{h} = l$. Then $\lim\limits_{h\to 0} \frac{l}{h} = 2$.

   Which of the following options is correct?

   A) Statement 1 is correct but Statement 2 is wrong.

   B) Statement 2 is correct but Statement 1 is wrong.

   C) Both Statement 1 and Statement 2 are wrong.

   D) Both Statement 1 and statement 2 are correct.

8. (Multiple correct options) I have a polynomial $p(x)$ of degree 2021. Define $f : \mathbb{R} \to \mathbb{R}$ as

$$f(x) = \begin{cases} |p(x)| & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise.} \end{cases}$$

   Then which of the following options are correct?

   A) The function is discontinuous at each point.

   B) The set $\{a \mid f(x) \text{ is continuous at } a\}$ is nonempty and finite.

   C) The function is continuous at every point a where $\lim\limits_{x\to a} f(x)$ exist.

   D) The limit $\lim\limits_{x\to a} f(x)$ exist at infinitely many points $a \in \mathbb{R}$.

9. Consider the following two statements.

   Statement 1: Let $f : \mathbb{R} \to \mathbb{R}$ with $\lim_{h\to 0}\Big(f(x+h) - f(x-h)\Big) = 0$ for all x. Then f is continuous on $\mathbb{R}$.

   Statement 2: If $f : \mathbb{R} \to \mathbb{R}$ is continuous and one-one, then it must be strictly monotone (means strictly increasing or strictly decreasing).

   Which of the following options is correct?

   A) Statement 1 is correct but Statement 2 is wrong.

   B) Statement 2 is correct but Statement 1 is wrong.

   C) Both Statement 1 and Statement 2 are wrong.

   D) Both Statement 1 and statement 2 are correct.

10. Consider the following two statements.

    Statement 1: Let $f, g : \mathbb{R} \to \mathbb{R}$ with $\lim_{x\to 5} f(x) = l$ and $\lim_{x\to 5} g(x) = k$. Then

    $$\lim_{x\to 5} \max\{f(x), g(x)\} = \max\{l, k\}.$$

    Statement 2: Let $f : \mathbb{R} \to \mathbb{R}$ be continuous such at two positive points a, b, we have $f(a) = 5a$ and $f(b) = 7b$. There there must exist a point c for which $f(c) = 6c$.

    Which of the following options is correct?

    A) Statement 1 is correct but Statement 2 is wrong.

    B) Statement 2 is correct but Statement 1 is wrong.

    C) Both Statement 1 and Statement 2 are wrong.

    D) Both Statement 1 and statement 2 are correct.

11. Consider the following two statements.

    Statement 1: The number of continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that the image $f(\mathbb{R}) \subseteq \{0, \pi, e, \sqrt{2}\}$ is at least 2.

    Statement 2: It is given that $\lim_{x\to 5}\Big((f(x))^3 - (f(x))^2 + f(x) - 1\Big)$ exists and it is 0. Then $\lim_{x\to 5} f(x)$ must exist.

    Which of the following options is correct?

    A) Statement 1 is correct but Statement 2 is wrong.

    B) Statement 2 is correct but Statement 1 is wrong.

    C) Both Statement 1 and Statement 2 are wrong.

    D) Both Statement 1 and statement 2 are correct.

12. (Multiple correct options) Let $0 < a < b$. Which of these irrational numbers are necessarily in the interval (a, b)? Here $[x]$ means the greatest integer function.

    A) $[a] + \dfrac{1}{500\sqrt{2}}$

    B) $\dfrac{[na] + \frac{1}{500\sqrt{2}}}{n}$, where $n = [\frac{3}{b-a}]$

C) $\dfrac{[na]+1+\frac{1}{500\sqrt{2}}}{n}$, where $n=[\frac{3}{b-a}]$

D) $\dfrac{[na]+1+\frac{1}{500\sqrt{2}}}{n}$, where $n=[\frac{2}{b-a}]$

13. Let $p(x)=x^4+5x^3-3x^2$ and A be an (arbitrary) infinite bounded set of positive real numbers. Define $B=\{x^4+5y^3-3z^2 \mid x,y,z\in A\}$. Then which of the following statements is correct?

A) We must have $\mathsf{lub}\, B = p(\mathsf{lub}\, A)$.

B) We must have $\mathsf{lub}\, B < p(\mathsf{lub}\, A)$.

C) We must have $\mathsf{lub}\, B > p(\mathsf{lub}\, A)$.

D) No comparisons can be made between $\mathsf{lub}\, B$ and $p(\mathsf{lub}\, A)$ in general.