Department of Mathematics
IIT Guwahati

Mockquiz	MA101	December 5, 2020	Total marks: NA	Time 10:30–10:40

- The first question is writing your roll number. It is compulsory.
- Each other question carries 2 marks. You get $2, -1$ or 0, for answering a question correctly, wrongly and for not attempting, respectively.
- As it is the mockquiz, it will not be evaluated.
- REMEMBER to press the SUBMIT button by 10:40:59. The form will not accept responses after that.
- ONLY ONE submission per person is allowed. NO REVISION is possible.

1. Write your roll number.
2. Consider the following two statements.

   Statement 1: If $a_n \to 5$ and $b_n = \dfrac{a_1 + 2a_2 + 3a_3 + \cdots + na_n}{\frac{n(n+1)}{2}}$, then $b_n \to 5$.

   Statement 2: If $a_n = \dfrac{n}{2^{\ln n}}$, then (a_n) is convergent.

   Then which of the following options is correct?

   A) Statement 1 is correct but Statement 2 is wrong.

   B) Statement 2 is correct but Statement 1 is wrong.

   C) Both Statement 1 and Statement 2 are wrong.

   D) Both Statement 1 and statement 2 are correct.

   Sol. A)

   Imagine the sequence $(c_n) = (a_1, a_2, a_2, a_3, a_3, a_3, a_4, \cdots)$. As $a_n \to 5$, we see that $c_n \to 5$. Recall the result that (was an exercise), if $x_n \to l$, then $\frac{x_1+x_2+\cdots+x_n}{n} \to l$. Using this result to our (c_n), we see that $\frac{c_1+\cdots+c_n}{n} \to 5$. In particular, being a subsequence, $\frac{c_1+\cdots+c_{n(n+1)/2}}{\frac{n(n+1)}{2}} \to 5$, that is, $b_n \to 5$.

   The other one follows as $a_n = (e/2)^{\ln n}$.
3. Consider the following two statements.

   Statement 1: If $\sum a_n$ converges then $\sum \frac{a_n}{n}$ is convergent.

   Statement 2: $\sum\limits_{n\geq 1} n\sin(\frac{1}{n})$ is convergent.

   Which of the following options is correct?

   A) Statement 1 is correct but Statement 2 is wrong.

   B) Statement 2 is correct but Statement 1 is wrong.

C) Both Statement 1 and Statement 2 are wrong.

D) Both Statement 1 and statement 2 are correct.

Sol. A)

Recall Dirichlet's test: if $\{A_n = \sum_{i=1}^{n} a_i\}$ is bounded and $b_n \downarrow 0$, then $\sum a_n b_n$ converges. Apply with $b_n = \frac{1}{n}$. So statement 1 is correct.

Statement 2 is false as the nth term goes to 1.