

MA 101 (Mathematics I)

Multivariable Calculus : Tutorial Problem Set - 3

1. Let S be a nonempty open subset of \mathbb{R}^2 and let $f : S \rightarrow \mathbb{R}$ be such that the partial derivatives f_x and f_y exist at each point of S . If $f_x : S \rightarrow \mathbb{R}$ and $f_y : S \rightarrow \mathbb{R}$ are bounded, then show that f is continuous.
2. Find all $\mathbf{u} \in \mathbb{R}^2$ with $\|\mathbf{u}\| = 1$ for which the directional derivative $D_{\mathbf{u}}f(0,0)$ exists (in \mathbb{R}), if for all $(x,y) \in \mathbb{R}^2$, $f(x,y) = \begin{cases} 1 & \text{if } y < x^2 < 2y, \\ 0 & \text{otherwise.} \end{cases}$
3. State TRUE or FALSE with justification: If $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous such that all the directional derivatives of f at $(0,0)$ exist (in \mathbb{R}), then f must be differentiable at $(0,0)$.
4. Determine all the points of \mathbb{R}^2 where $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is differentiable, if for all $(x,y) \in \mathbb{R}^2$,
$$f(x,y) = \begin{cases} x^{4/3} \sin\left(\frac{y}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$
5. Let $f : S \subseteq \mathbb{R}^m \rightarrow \mathbb{R}$ be differentiable at $\mathbf{x}_0 \in S^0$ and let $f(\mathbf{x}_0) = 0$. If $g : S \rightarrow \mathbb{R}$ is continuous at \mathbf{x}_0 , then show that $fg : S \rightarrow \mathbb{R}$, defined by $(fg)(\mathbf{x}) = f(\mathbf{x})g(\mathbf{x})$ for all $\mathbf{x} \in S$, is differentiable at \mathbf{x}_0 .
6. Show that $f : S \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$ is differentiable at $(x_0, y_0) \in S^0$ iff there exist functions $\varphi, \psi : S \rightarrow \mathbb{R}$ such that φ, ψ are continuous at (x_0, y_0) and $f(x,y) - f(x_0, y_0) = (x - x_0)\varphi(x,y) + (y - y_0)\psi(x,y)$ for all $(x,y) \in S$.
7. Let the temperature $T(x,y)$ at any point $(x,y) \in \mathbb{R}^2$ be given by $T(x,y) = 2x^2 + xy + y^2$. An insect is at the point $(1,1)$.
 - (a) What is the best direction for the insect to move to feel cooler?
 - (b) In which direction should the insect move to feel no change in temperature?