THU-70250403, Convex Optimization (Fall 2020) **Homework: 2**

Convex Functions

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Student:

(Please choose either two problems from the first three problems and finish them.)

Problem 1

Suppose $f(x) : \mathbb{R} \mapsto \mathbb{R}$ is a convex function. Please prove or disprove that

$$f(x_1) + f(x_2) + f(x_3) + 3f((x_1 + x_2 + x_3)/3) \geq 2f((x_1 + x_2)/2) + 2f((x_1 + x_3)/2) + 2f((x_2 + x_3)/2) \tag{1}$$

Please also discuss whether the above inequality holds for convex function $f(x) : \mathbb{R}^n \mapsto \mathbb{R}$.

Problem 2

Given $a, b > 0$, please solve the following optimization problem.

$$\min_{x \in (0, \frac{\pi}{2})} \frac{a}{\sin x} + \frac{b}{\cos x} \tag{2}$$

Problem 3

Suppose $n \in \mathbb{N}$, $0 \leq x_1 \leq ... \leq x_n \leq \frac{\pi}{2}$ satisfy that

$$\sum_{k=1}^{n} \sin x_k = 1 \tag{3}$$

Please apply Jensen's Inequality [1] to prove or disprove that

$$n \arcsin \frac{1}{n} \leq \sum_{k=1}^{n} x_k \leq \frac{\pi}{2} \tag{4}$$

Problem 4

(This problem is Optional)

Suppose $f(x) : \mathbb{R} \mapsto \mathbb{R}$ is a convex function. Please prove or disprove that

(a) if $f(x)$ is bounded, it must be a constant value function.

(b) if $f(x)$ satisfies

$$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{f(x)}{x} = 0 \tag{5}$$

it must be a constant value function.

References

[1] http://en.wikipedia.org/wiki/Jensen's_inequality