THU-70250403, Convex Optimization (Fall 2020) **Homework: 3**

Convex Programming Problems

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Student:

Problem 1

Problems involving l_∞ and l_1-norms. Formulate the following problems as Linear programming problems (LPs). Explain in detail the relation between the optimal solution of each problem and the solution of its equivalent LP.

Problem (a)

$$\min_{\boldsymbol{x}} \quad |A\boldsymbol{x}-\boldsymbol{b}|_1 \tag{1}$$
$$\text{s.t.} \quad |\boldsymbol{x}|_\infty \le 1 \tag{2}$$

Problem (b)

$$\min_{\boldsymbol{x}} \quad |\boldsymbol{x}|_1 \tag{3}$$
$$\text{s.t.} \quad |A\boldsymbol{x}-\boldsymbol{b}|_\infty \le 1 \tag{4}$$

Problem (c)

$$\min_{\boldsymbol{x}} \ |A\boldsymbol{x}-\boldsymbol{b}|_1 + |\boldsymbol{x}|_\infty \tag{5}$$

The matrix $A \in \mathbb{R}^{m\times n}$ and the vector $\boldsymbol{b} \in \mathbb{R}^m$ are given.

Problem 2

l_4-norm approximation via QCQP Formulate the l_4-norm approximation problem

$$\min_{\boldsymbol{x}} \ |A\boldsymbol{x}-\boldsymbol{b}|_4 = \left(\sum_{i=1}^{m}\left(\boldsymbol{a}_i^T\boldsymbol{x}-b_i\right)^4\right)^{1/4} \tag{6}$$

The matrix $A \in \mathbb{R}^{m\times n}$ (with rows $\boldsymbol{a}_i^T$) and the vector $\boldsymbol{b} \in \mathbb{R}^m$ are given.

Problem 3

Suppose real numbers $a \ge b \ge c \ge d > 0$, $a+b+c+d = 1$. Please apply Jensen's Inequality [1] to prove or disprove that

$$a^a b^b c^c d^d \ge a^2+b^2+c^2+d^2 \tag{7}$$

References

[1] http://en.wikipedia.org/wiki/Jensen's_inequality