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Problem 1 记 $1 = (1, 1, \dots, 1)^T$

$$\begin{aligned} (a) \quad & \text{minimize} \quad 1^T y \\ & \text{subject to} \quad -y \leq Ax - b \leq y \\ & \quad \quad \quad -1 \leq x \leq 1 \end{aligned}$$

$$\begin{aligned} (b) \quad & \text{minimize} \quad 1^T y \\ & \text{subject to} \quad -1 \leq Ax - b \leq 1 \\ & \quad \quad \quad -y \leq x \leq y \end{aligned}$$

$$\begin{aligned} (c) \quad & \text{minimize} \quad 1^T y + m \\ & \text{subject to} \quad -y \leq Ax - b \leq y \\ & \quad \quad \quad -m1 \leq x \leq m1 \end{aligned}$$

Problem 2.

$$\begin{aligned} & \text{minimize} \quad \|Ax - b\|_4 = \left(\sum_{i=1}^m (a_i^T x - b_i)^4 \right)^{\frac{1}{4}} \\ & \text{minimize} \quad \sum_{i=1}^m z_i^2 \\ & \text{subject to} \quad \begin{array}{ll} a_i^T x - b_i = y_i & i=1, 2, \dots, m \\ y_i^2 \leq z_i & i=1, 2, \dots, m \end{array} \end{aligned} \quad \left\{ \begin{array}{l} \text{写成向量的形式} \\ z^T z \\ Ax - b = y \\ y^T y \leq z \end{array} \right.$$

Problem 3.

$a \geq b \geq c \geq d > 0$, $a+b+c+d=1$ 证明 $a^a b^b c^c d^d \leq a^2 + b^2 + c^2 + d^2$

$f(x) = \ln x$ 是凹函数, 有

$$f(aa + \dots + dd) \geq af(a) + \dots + df(d)$$

$$\ln(a^2 + b^2 + c^2 + d^2) \geq \ln a^a b^b c^c d^d$$

两边做指数变换得 $a^2 + b^2 + c^2 + d^2 \geq a^a b^b c^c d^d$ 得证

PS: 题目中不等号符号反了