THU-70250403, Convex Optimization (Fall 2020) **Homework: 7**

Norm Approximation

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Student:

Problem 1

Lower bounds in Chebyshev approximation from least-squares. Consider the Chebyshev or ℓ_∞-norm approximation problem

$$\min_{\boldsymbol{x}} \quad |A\boldsymbol{x} - \boldsymbol{b}|_\infty \tag{1}$$

where where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{b} \in \mathbb{R}^m$, $A \in \mathbb{R}^{m\times n}$, and $\operatorname{rank}(A) = n$.

Let $\boldsymbol{x}_{\text{ch}}$ denote an optimal solution (there may be multiple optimal solutions; $\boldsymbol{x}_{\text{ch}}$ denotes one of them).

The Chebyshev problem has no closed-form solution, but the corresponding least-squares problem does. Define

$$x_{\text{ls}} = \operatorname{argmin}|A\boldsymbol{x} - \boldsymbol{b}|_2 = (A^T A)^{-1} A^T b. \tag{2}$$

We address the following question. Suppose that for a particular A and b we have computed the least-squares solution $\boldsymbol{x}_{\text{ls}}$ (but not $\boldsymbol{x}_{\text{ch}}$). How suboptimal is $\boldsymbol{x}_{\text{ls}}$ for the Chebyshev problem? In other words, how much larger is $\|A\boldsymbol{x}_{\text{ls}} - \boldsymbol{b}\|_\infty$ than $|A\boldsymbol{x}_{\text{ch}} - \boldsymbol{b}|_\infty$?

Problem 2

Please read paper and design some numerical tests to study the loss function proposed in [1] for the simplest norm approximation problem

$$\min_{\boldsymbol{x}} \ |A\boldsymbol{x} - \boldsymbol{b}|_{\text{special loss}} \tag{3}$$

Design and use your own testing dataset. Write a brief report.

References

[1] J. T. Barron, "A general adaptive robust loss function," *Proceedings of IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 2019.