Multiple View Geometry: Solution Exercise Sheet 1

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Part I: Theory

1. **To summarize:**

	B_1	B_2	B_3
(1) Are linearly independent	yes	yes	no
(2) Span $\mathbb{R}^3$	yes	no	yes
(3) Form a basis of $\mathbb{R}^3$	yes	no	no

More details:

B_1 : Can be shown by building a matrix and calculating the determinant: $det\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \neq 0.$

As the determinant is not zero, we know that the vectors are linearly independent. Three linear independent vectors in $\mathbb{R}^3$ span $\mathbb{R}^3$. Furthermore, three spanning vectors build a minimal set, hence, they also form a basis of $\mathbb{R}^3$.

B_2 : To span $\mathbb{R}^3$, there are at least three vectors needed.

B_3 : In $\mathbb{R}^3$, there cannot be more than three independent vectors.

2. **To summarize:**

	G_1	G_2	G_3
Form a group	no	no	yes

More details:

G_1 : Closure not given!

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 5 \end{pmatrix}}_{\in G_1} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{\in G_1} = \underbrace{\begin{pmatrix} 1 & 4 & 9 \\ 2 & 0 & 12 \\ 3 & 8 & 15 \end{pmatrix}}_{\notin G_1}$$

G_2 : Neutral element not included, as $det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 \neq -1$

G_3 : Yes, as can easily be shown using $det(A^{-1}) = \frac{1}{det(A)}$.

3. **To summarize:** No.

   More details (proof): Assuming the existence of four pairwise orthogonal, non-zero vectors $v_1, \ldots, v_4 \in \mathbb{R}^3$, we obtain a contradiction:

   We know that, in $\mathbb{R}^3$, there are at most 3 linearly independent vectors. Hence, we know that $\exists\, a_i : \sum_{i=1}^{4} a_i v_i = 0$, with at least one $a_i \neq 0$. Without loss of generality, we can assume that $a_1 = 1$, giving

$$v_1 = a_2 v_2 + a_3 v_3 + a_4 v_4$$

   As the vectors are pairwise orthogonal, we can derive

$$\begin{aligned} ||v_1||^2 &= \langle v_1, v_1 \rangle \\ &= \langle v_1, a_2 v_2 + a_3 v_3 + a_4 v_4 \rangle \\ &= \langle v_1, v_2 \rangle a_2 + \langle v_1, v_3 \rangle a_3 + \langle v_1, v_4 \rangle a_4 = 0, \end{aligned}$$

   which contradicts $v_1 \neq \mathbf{0}$.

Part I: Matlab

1. Basic image processing

   (a) -

   (b) `I = imread('lena.png');`

   (c)
   ```
   [r,c,ch] = size(I)
   imshow(I)
   ```

   (d)
   ```
   J = rgb2gray(I);
   min_val = min(min(J))
   max_val = max(max(J))
   ```

   (e)
   ```
   h = fspecial('gaussian');
   J2 = im2double(J);
   K = imfilter(J2,h);
   imwrite(K,'smoothed.png','PNG');
   ```

   (f)
   ```
   figure
   subplot(1,3,1), imshow(I), title('original')
   subplot(1,3,2), imshow(J2), title('gray scale')
   subplot(1,3,3), imshow(K), title('smoothed')
   ```

   (g)
   ```
   h = fspecial('gaussian', [9 2], 1);
   K = imfilter(J2,h);
   figure, subplot(1,2,1), imshow(J2), subplot(1,2,2), imshow(K)
   ```

2. Basic operations

   (a) `x = A \ b`

   (b) `B = A`

   (c) `A(1,2) = 4`

   (d)
   ```
   c = 0;
   for i = -4:4:4
       c = c + i*A'*b
   end
   fprintf('c = %0.1f',c)
   ```

   (e) a) element-wise multiplication of matrix-elements
   b) matrix multiplication ($A^\top B$)

3. There is a number of possible solution which do not require a loop, such as:

   - `out = all(all(abs(x-y) <= eps))`
   - `out = sum(sum(abs(x-y) > eps)) == 0`
   - `out = max(abs(x-y)) <= eps`
   - `out = max( (x-y) .* (x-y) ) <= eps*eps`

4. Again there are several possible commands, such as:

   - ```
     A = s:e;
     out = sum(isprime(A) .* A);
     ```
   - ```
     A = s:e;
     out = sum(A(isprime(A)));
     ```

Gaussian Smoothing

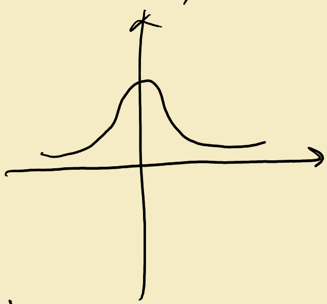
blur images and remove detail and noise

Mean Filter

3x3 filter: $\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$ $\begin{bmatrix} 1 & 7 & 8 \\ 3 & 5 & 1 \\ 4 & 6 & 10 \end{bmatrix}$ $\longrightarrow \frac{1+7+8+3+5+1+4+6+10}{9}$

$= \cdots$

How it works?



1-D: $G(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{x^2}{2\sigma^2}\right\}$

2-D: $G(x,y) = \frac{1}{2\pi\sigma^2} \exp\left\{\frac{-(x^2+y^2)}{2\sigma^2}\right\}$

larger σ means larger kernel size