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Markov Random Fields in Image Segmentation

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Overview

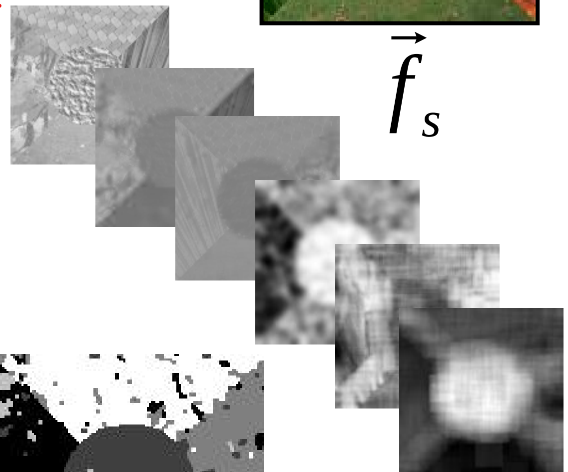
- Segmentation as pixel labeling
- Probabilistic approach
  - Markov Random Field (MRF)
  - Gibbs distribution & Energy function
- Energy minimization
  - Simulated Annealing
  - Markov Chain Monte Carlo (MCMC) sampling
- Example MRF model & Demo
- Parameter estimation (EM)
- More complex models

Segmentation as a Pixel Labelling Task

$\vec{f}_s$

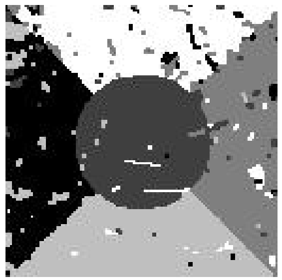
1. Extract features from the input image
   - Each pixel *s* in the image has a feature vector
   - For the whole image, we have

$$f = \{\vec{f}_s : s \in S\}$$



2. Define the set of labels Λ
   - Each pixel *s* is assigned a label $\omega_s \in \Lambda$
   - For the whole image, we have

$$\omega = \{\omega_s, s \in S\}$$



- For an $N \times M$ image, there are $|\Lambda|^{NM}$ possible labelings.
   - **Which one is the right segmentation?**

Probabilistic Approach, MAP

- Define a *probability measure* on the set of all possible labelings and select the most likely one.
- $P(\omega \mid f)$ measures the probability of a labelling, given the observed feature f
- Our goal is to find an optimal labeling $\hat{\omega}$ which *maximizes* $P(\omega \mid f)$
- This is called the *Maximum a Posteriori* (MAP) estimate:

代表一种分类方式

$$\hat{\omega}^{MAP} = \arg\max_{\omega \in \Omega} P(\omega \mid f)$$

Bayesian Framework

- By Bayes Theorem, we have

$$P(\omega \mid f) = \frac{P(f \mid \omega)P(\omega)}{P(f)} \propto \underbrace{P(f \mid \omega)}_{\text{likelihood}}\underbrace{P(\omega)}_{\text{prior}}$$

- $P(f)$ is constant
- We need to define $P(\omega)$ and $P(f \mid \omega)$ in our model

Why MRF Modelization?

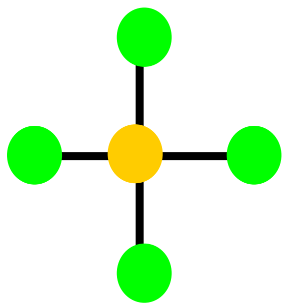
- In real images, regions are often homogenous; neighboring pixels usually have similar properties (intensity, color, texture, …)
- Markov Random Field (MRF) is a probabilistic model which captures such contextual constraints
- Well studied, strong theoretical background
- Allows MCMC sampling of the (hidden) underlying structure ➔ Simulated Annealing

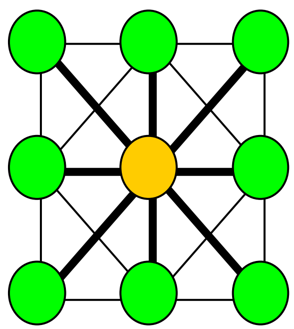
What is MRF?

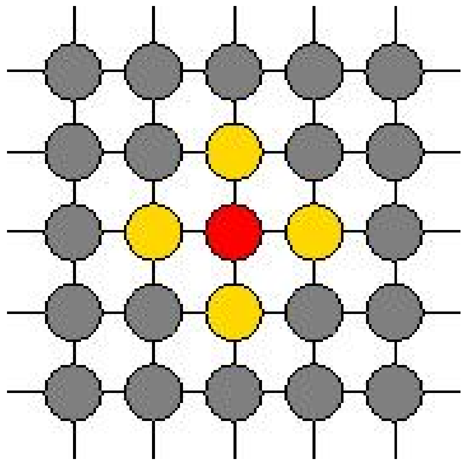
- To give a formal definition for Markov Random Fields, we need some basic building blocks
  - Observation Field and (hidden) Labeling Field
  - Pixels and their Neighbors
  - Cliques and Clique Potentials
  - Energy function
  - Gibbs Distribution

Definition – Neighbors

- For each pixel, we can define some surrounding pixels as its neighbors.
- Example : 1^{st} order neighbors and 2^{nd} order neighbors





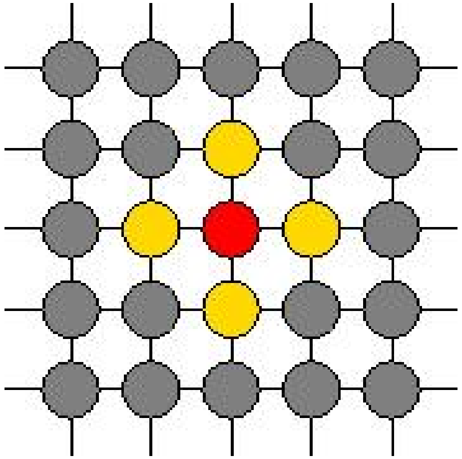


Definition – MRF

- The labeling field *X* can be modeled as a Markov Random Field (MRF) if
  1. For all $\omega \in \Omega : P(X = \omega) > 0$
  2. For every $s \in S$ and $\omega \in \Omega$:

     $$P(\omega_s \mid \omega_r, r \neq s) = P(\omega_s \mid \omega_r, r \in N_s)$$

  N_s denotes the neighbors of pixel *s*

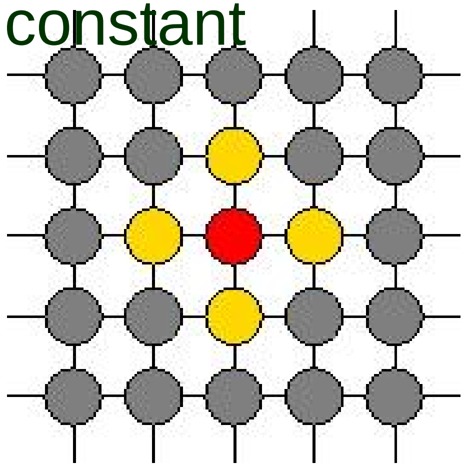


Hammersley-Clifford Theorem

- The Hammersley-Clifford Theorem states that a random field is a MRF if and only if $P(\omega)$ follows a Gibbs distribution.

$$P(\omega) = \frac{1}{Z}\exp(-U(\omega)) = \frac{1}{Z}\exp(-\sum_{c \in C} V_c(\omega))$$

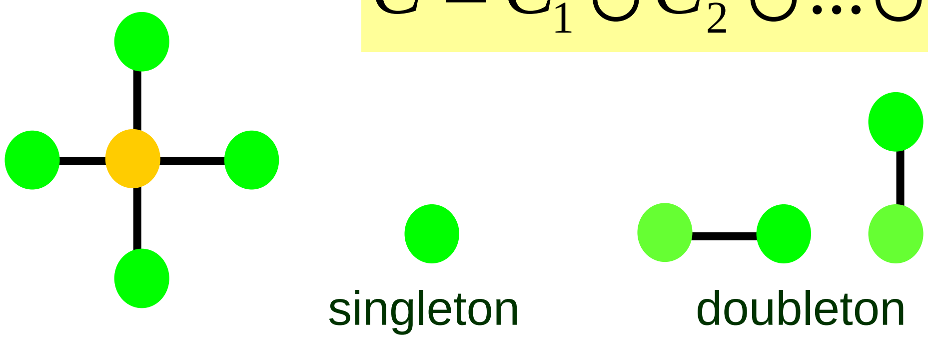
- where $Z = \sum_{\omega \in \Omega} \exp(-U(\omega))$ is a normalization constant
- This theorem provides us an easy way of defining MRF models via *clique potentials*.

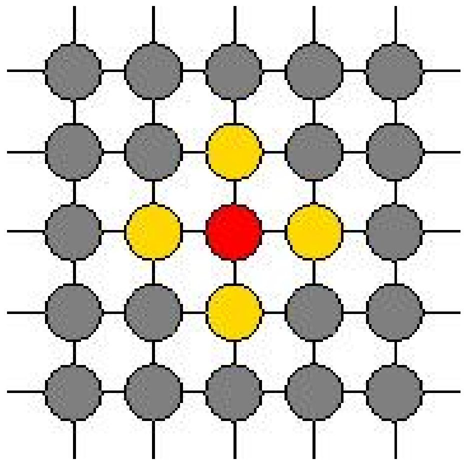


Definition – Clique

- A subset $C \subseteq S$ is called a *clique* if every pair of pixels in this subset are neighbors.
- A clique containing n pixels is called n^{th} *order clique*, *denoted by* C_n.
- The set of cliques in an image is denoted by

$$C = C_1 \cup C_2 \cup \ldots \cup C_k$$

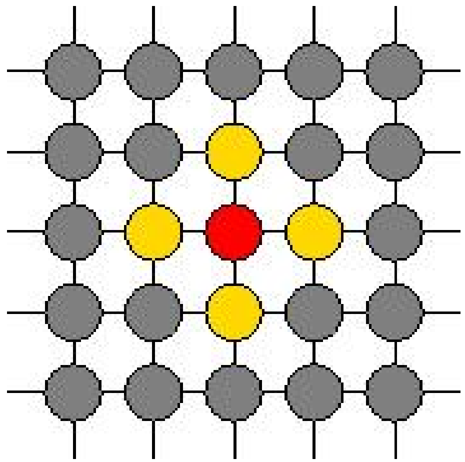




Definition – Clique Potential

- For each clique *c* in the image, we can assign a value $V_c(\omega)$ which is called *clique potential* of *c*, where ω is the configuration of the labeling field
- The sum of potentials of all cliques gives us the energy $U(\omega)$ of the configuration ω

$$U(\omega) = \sum_{c \in C} V_c(\omega) = \sum_{i \in C_1} V_{C_1}(\omega_i) + \sum_{(i,j) \in C_2} V_{C_2}(\omega_i, \omega_j) + \ldots$$



Segmentation of grayscale images: A simple MRF model

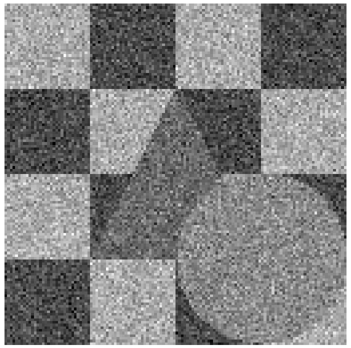
- Construct a segmentation model where regions are formed by spatial clusters of pixels with similar intensity:

Model parameters

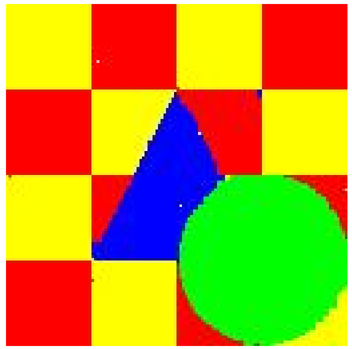
MRF segmentation model

+

find MAP estimate $\hat{\omega}$



Input image



segmentation

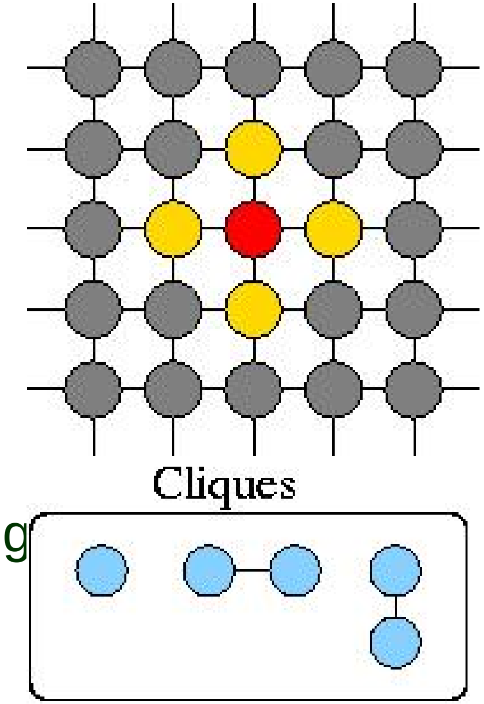
$\hat{\omega}$

MRF segmentation model

- Pixel labels (or classes) are represented by Gaussian distributions:

$$P(f_s \mid \omega_s) = \frac{1}{\sqrt{2\pi}\sigma_{\omega_s}} \exp\left(-\frac{(f_s - \mu_{\omega_s})^2}{2\sigma_{\omega_s}^2}\right)$$

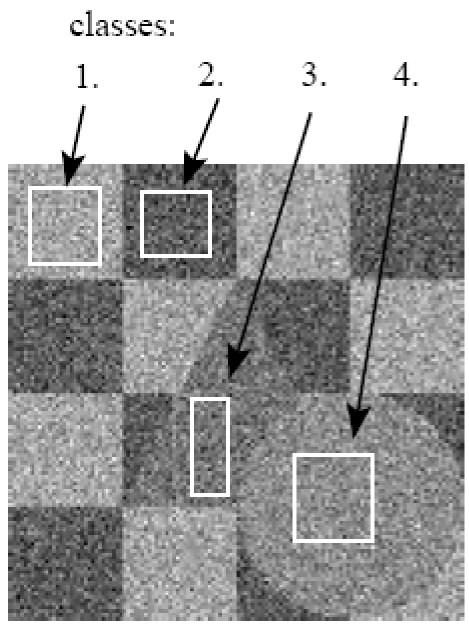
- Clique potentials:
  - ***Singleton***: proportional to the likelihood of features given ω: *log(P(f | ω)).*
  - ***Doubleton***: favours similar labels at neighbouring pixels – ***smoothness prior***



$$V_{c_2}(i,j) = \beta\delta(\omega_i, \omega_j) = \begin{cases} -\beta & \text{if} \quad \omega_i = \omega_j \\ +\beta & \text{if} \quad \omega_i \neq \omega_j \end{cases}$$

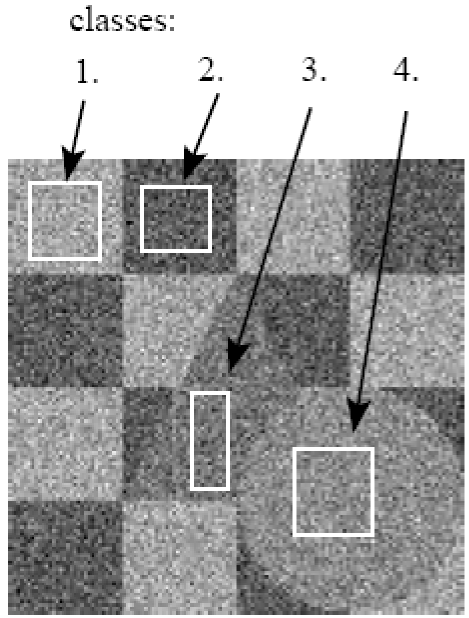
As β increases, regions become more homogenous

Model parameters



- Doubleton potential β
  - less dependent on the input ➔
    - can be fixed a priori
- Number of labels ($|\Lambda|$)
  - Problem dependent➔
    - usually given by the user or
    - inferred from some higher level knowledge
- Each label $\lambda \in \Lambda$ is represented by a Gaussian distribution $N(\mu_\lambda, \sigma_\lambda)$:
  - estimated from the input image

Model parameters



- The class statistics (mean and variance) can be estimated via the *empirical mean and variance*:

$$\forall \lambda \in \Lambda : \quad \mu_\lambda = \frac{1}{|S_\lambda|} \sum_{s \in S_\lambda} f_s,$$

$$\sigma_\lambda^2 = \frac{1}{|S_\lambda|} \sum_{s \in S_\lambda} (f_s - \mu_\lambda)^2$$

  - where *S_λ* denotes the set of pixels in the training set of class λ
  - a training set consists in a representative region selected by the user

Energy function

- Now we can define the energy function of our MRF model:

singleton

doubleton

$$U(\omega)=\sum_s\left(\log(\sqrt{2\pi}\sigma_{\omega_s})+\frac{(f_s-\mu_{\omega_s})^2}{2\sigma_{\omega_s}^2}\right)+\sum_{s,r}\beta\delta(\omega_s,\omega_r)$$

- Recall:

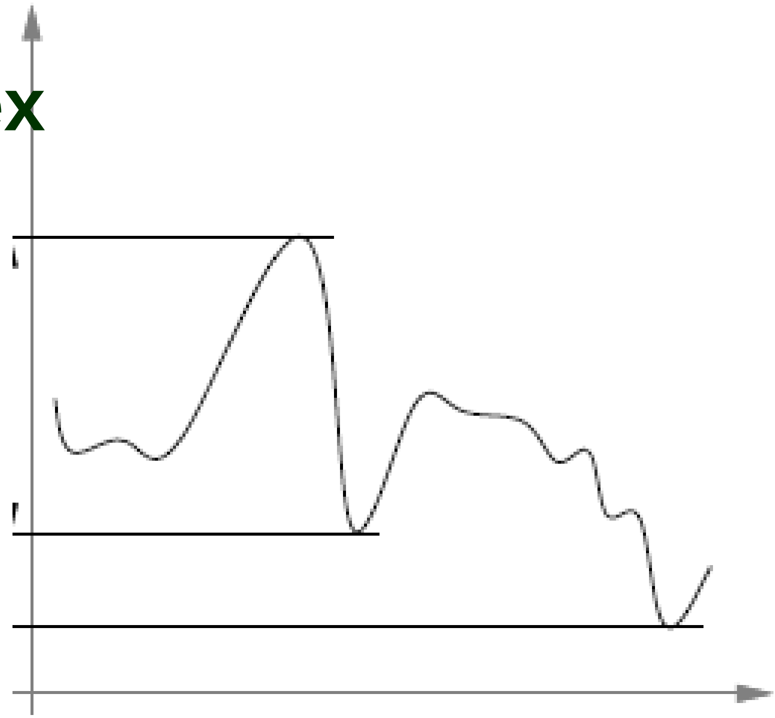
$$P(\omega\mid f)=\frac{1}{Z}\exp(-U(\omega))=\frac{1}{Z}\exp(-\sum_{c\in C}V_c(\omega))$$

- Hence

$$\hat{\omega}^{MAP}=\arg\max_{\omega\in\Omega}P(\omega\mid f)=\arg\min_{\omega\in\Omega}U(\omega)$$

Optimization

- Problem reduced to the minimization of a **non-convex** energy function
  - Many local minima
- Gradient descent?
  - Works only if we have a *good* initial segmentation
- Simulated Annealing
  - Always works (at least in theory)



ICM (~Gradient descent) [Besag86]

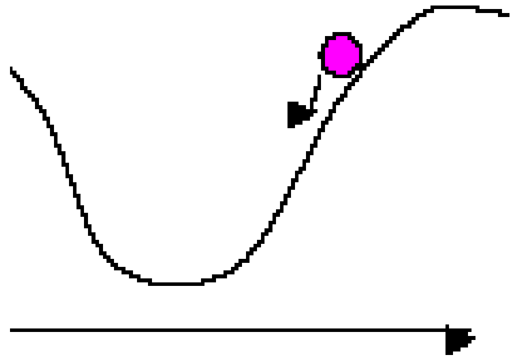
① *Start at a "good" initial configuration* ω^0 *and set* $k = 0$.

② *For each configuration which differs at most in one element from the current configuration* ω^k *(they are denoted by* $\mathcal{N}_{\omega^k}$*), compute the energy* $U(\eta)$ $(\eta \in \mathcal{N}_{\omega^k})$.

③ *From the configurations in* $\mathcal{N}_{\omega^k}$*, select the one which has a minimal energy:*

$$\omega^{k+1} = \arg \min_{\eta \in \mathcal{N}_{\omega^k}} U(\eta). \qquad (6)$$

④ *Goto Step* ② *with* $k = k + 1$ *until convergence is obtained (for example, the energy change is less than a certain threshold).*



Simulated Annealing

① *Set* $k = 0$ *and initialize* ω *randomly. Choose a sufficiently high initial temperature* $T = T_0$.

② *Construct a trial perturbation* η *from the current configuration* ω *such that* η *differs only in one element from* ω.

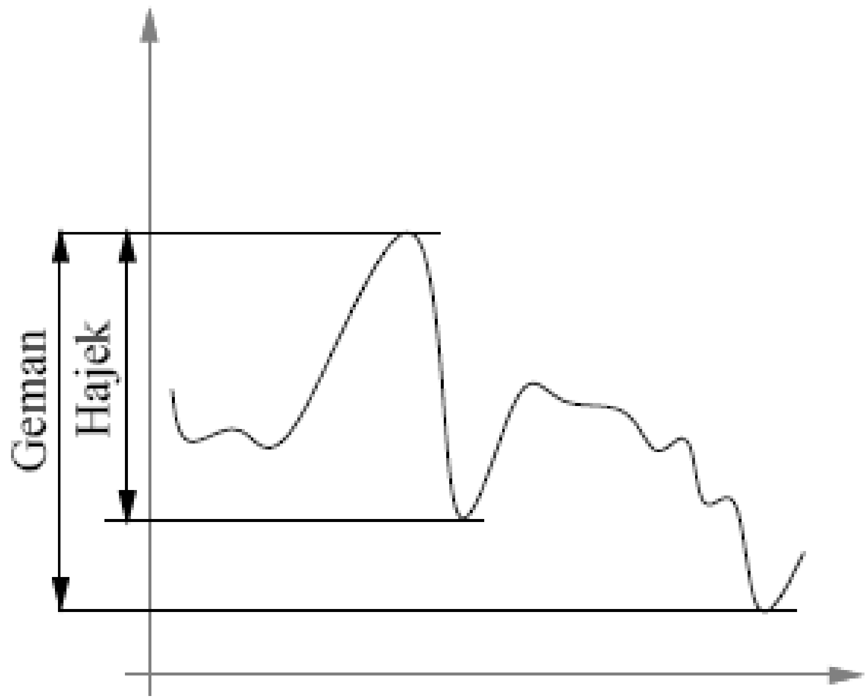
③ **(Metropolis criteria)** *Compute* $\Delta U = U(\eta) - U(\omega)$ *and accept* η *if* $\Delta U < 0$ *else accept with probability* $\exp(-\Delta U/T)$ *(analogy with thermodynamics):*

$$\omega = \begin{cases} \eta & \text{if } \Delta U \leq 0, \\ \eta & \text{if } \Delta U > 0 \text{ and } \xi < \exp(-\Delta U/T), \\ \omega & \text{otherwise} \end{cases} \quad (4)$$

where ξ *is a uniform random number in* $[0, 1)$.

④ *Decrease the temperature:* $T = T_{k+1}$ *and goto Step* ② *with* $k = k + 1$ *until the system is* frozen.

Temperature Schedule

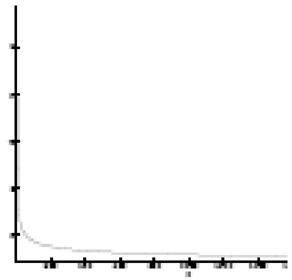


$$T_k \geq \frac{\Gamma}{\ln(k)} \tag{8}$$

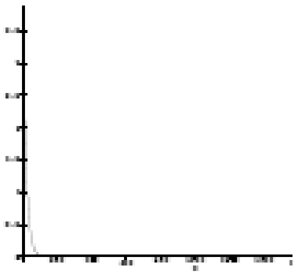
with

$$\Gamma > \max_{\omega \in \Omega} U(\omega) - \min_{\omega \in \Omega} U(\omega) \tag{9}$$

Temperature Schedule



Logarithmic schedule ($4/\ln(k)$).

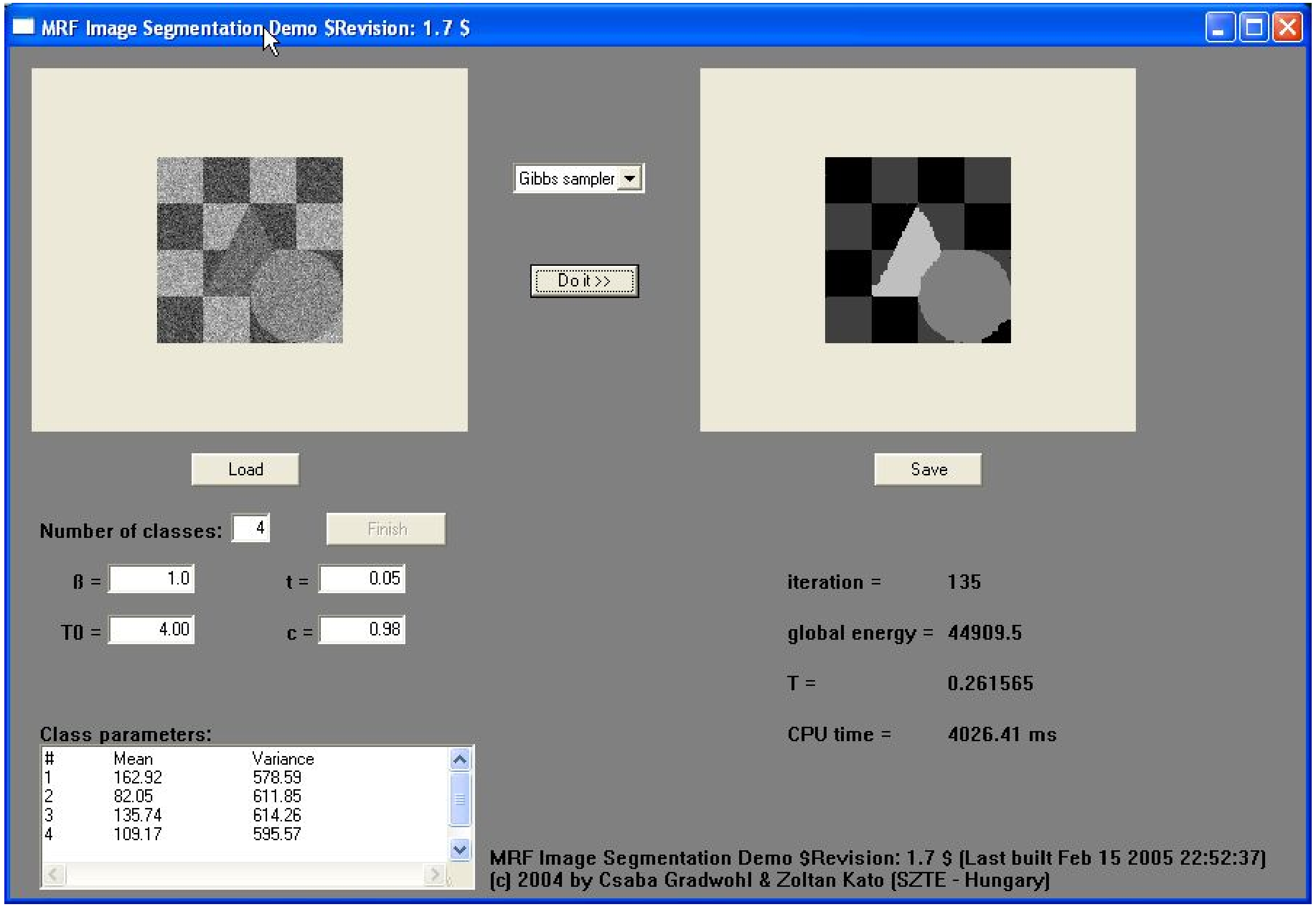


Exponential schedule ($0.95^k \cdot 4$).

- **Initial temperature:** set it to a relatively low value (~4)➔ faster execution
  - must be high enough to allow random jumps at the beginning!
- **Schedule:** $T_{k+1} = c \cdot T_k, \quad k = 0, 1, 2, \ldots$
- **Stopping criteria:**
  - Fixed number of iterations
  - Energy change is less than a threshols

Demo

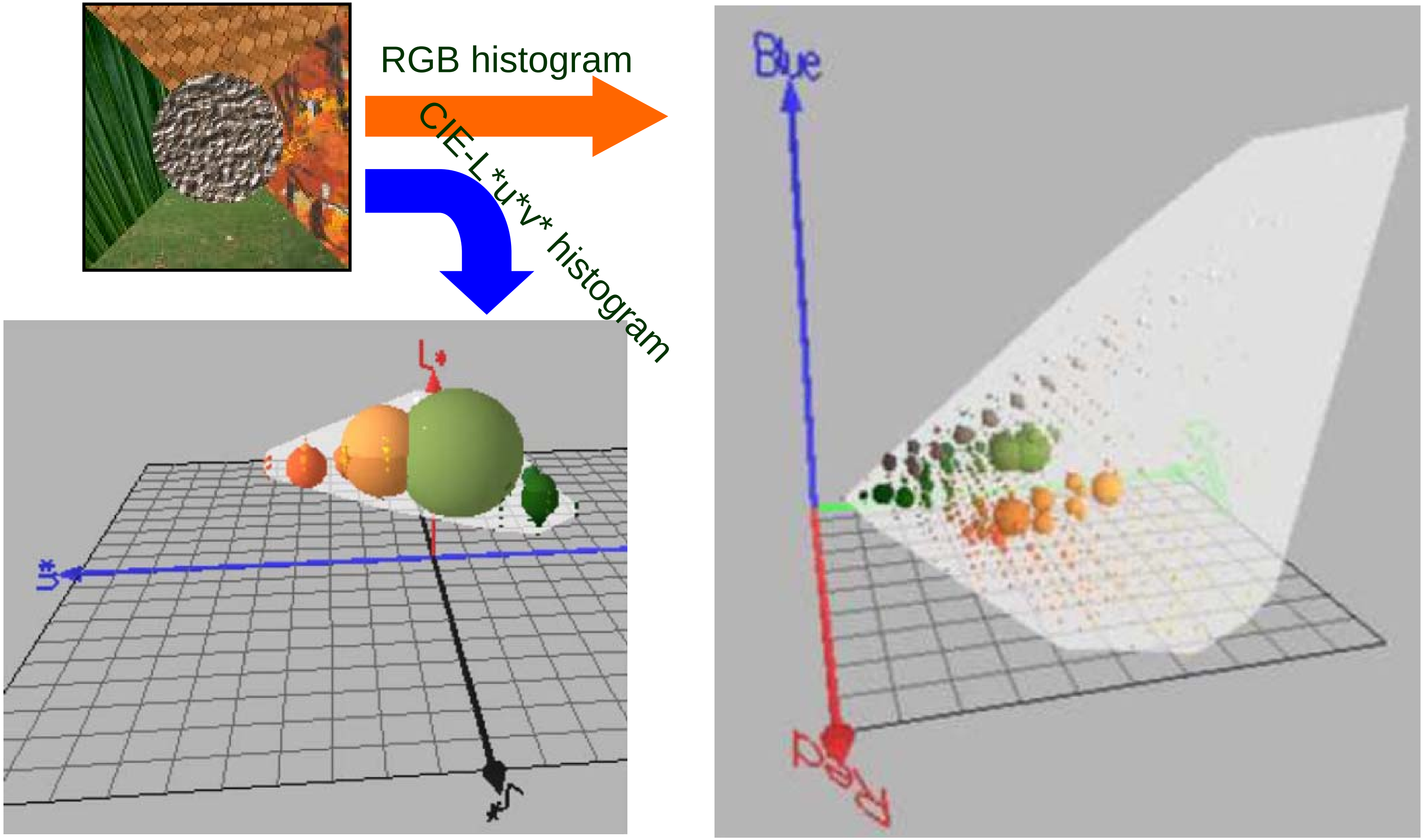
- Download from:
  http://www.inf.u-szeged.hu/~kato/software/



Summary

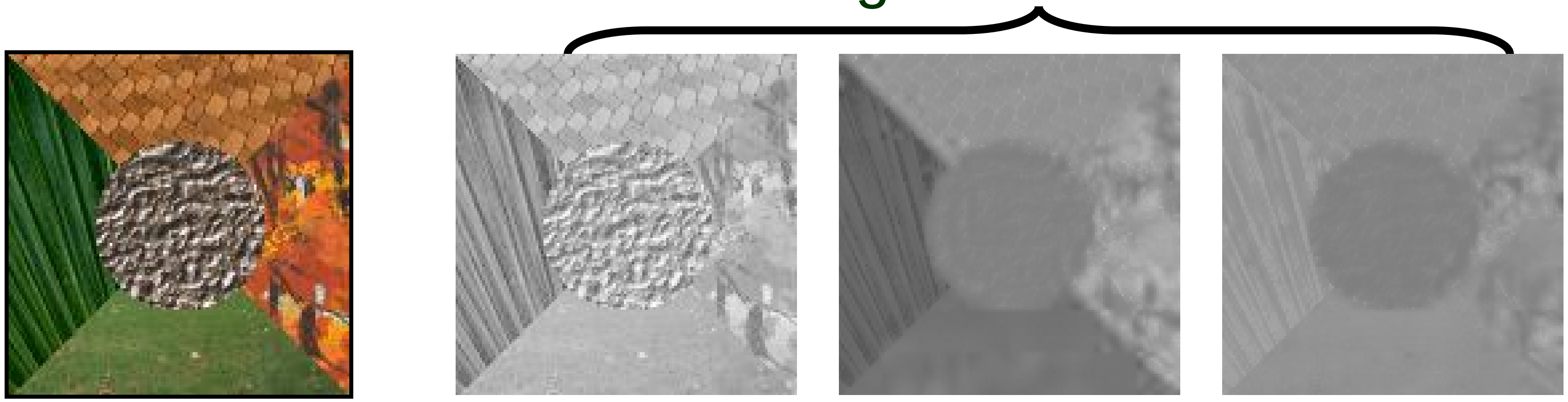
- Design your model carefully
  - Optimization is just a tool, do not expect a good segmentation from a wrong model
- What about other than graylevel features
  - Extension to color is relatively straightforward

What color features?



Extract Color Feature

- We adopt the CIE-L*u*v* color space because it is ***perceptually uniform***.
  - Color difference can be measured by Euclidean distance of two color vectors.
- We convert each pixel from RGB space to CIE-L*u*v* space ➔
  - We have 3 color feature images

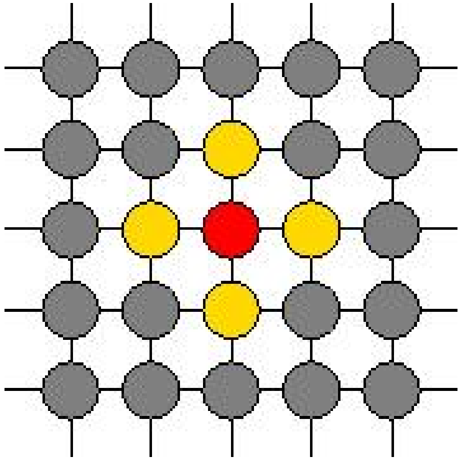


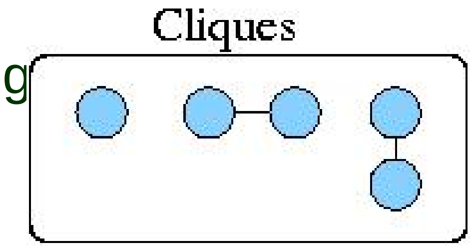
L^* u^* v^*

Color MRF segmentation model

- Pixel labels (or classes) are represented by three-variate Gaussian distributions:

$$P(f_s \mid \omega_s) = \frac{1}{\sqrt{(2\pi)^n \mid \Sigma_{\omega_s} \mid}} \exp(-\frac{1}{2}(\vec{f}_s - \vec{u}_{\omega_s})\Sigma_{\omega_s}^{-1}(\vec{f}_s - \vec{u}_{\omega_s})^T)$$





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As β increases, regions become more homogenous

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- Can we segment images without user interaction?
  - Yes, but you need to estimate model parameters automatically (EM algorithm)

Incomplete data problem

- Supervised parameter estimation
  - we are given a labelled data set to learn from
    - e.g. somebody manually assigned labels to pixels
- How to proceed without labelled data?
  - Learning from incomplete data
  - Standard solution is an iterative procedure called ***E**xpectation-**M**aximization*
    - Assigns labels and estimates parameters simultaneously
    - Chicken-Egg problem

EM principles : The two steps

E Step : For each pixel,
use parameters to compute probability distribution

Parameters :
P(pixel/label)P(label)

Weighted labeling :
P(label/pixel)

M Step : Update the estimates of parameters
based on weighted (or ”soft”) labeling

The basic idea of EM

- Each of the **E** and **M** steps is straightforward assuming the other is solved
  - Knowing the label of each pixel, we can estimate the parameters
    - Similar to supervised learning (hard vs. soft labeling)
  - Knowing the parameters of the distributions, we can assign a label to each pixel
    - by Maximum Likelihood – i.e. using the singleton energies only without pairwise interactions

Parameter estimation via EM

- Basically, we will fit a mixture of Gaussian to the image histogram
  - We know the number of labels $|\Lambda| \equiv$ number of mixture components
- At each pixel, the complete data includes
  - The observed feature $\mathbf{f}_s$
  - Hidden pixel labels $\mathbf{l}_s$ (a vector of size $|\Lambda|$)
    - specifies the contribution of the pixel feature to each of the labels – i.e. a soft labeling

Parameter estimation via EM

- **E** step: recompute $\mathbf{l}_s^i$ at each pixel s:

$$\mathbf{l}_{\mathbf{s}}^{\mathbf{i}} = P(\lambda \,|\, \mathbf{f}_{\mathbf{s}}) = \frac{P(\mathbf{f}_{\mathbf{s}} \,|\, \lambda) P(\lambda)}{\sum_{\lambda \in \Lambda} P(\mathbf{f}_{\mathbf{s}} \,|\, \lambda) P(\lambda)}$$

- **M** step: update Gaussian parameters for each label λ:

$$P(\lambda) = \frac{\sum_{s \in S} P(\lambda \,|\, \mathbf{f}_{\mathbf{s}})}{|S|}, \quad \mu_\lambda = \frac{\sum_{s \in S} P(\lambda \,|\, \mathbf{f}_{\mathbf{s}}) \mathbf{f}_{\mathbf{s}}}{\sum_{s \in S} P(\lambda \,|\, \mathbf{f}_{\mathbf{s}})}, \ldots$$

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- Design your model carefully
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  - Extension to color is relatively
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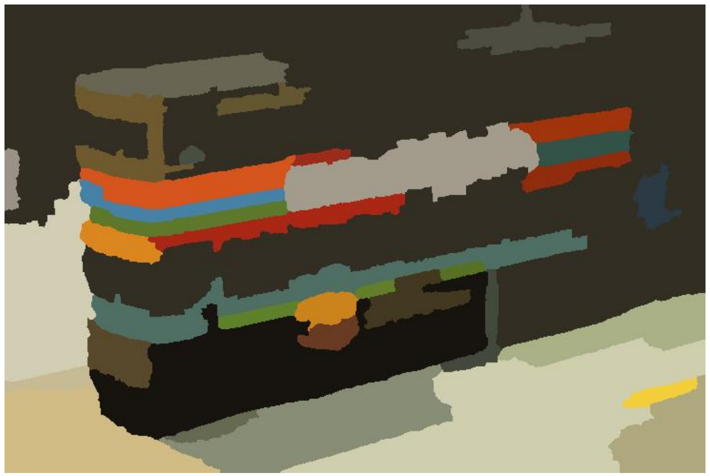
  - Yes, but you need to estimate model parameters automatically (EM algorithm)
- What if we do not know $|\Lambda|$?
  - Fully automatic segmentation requires
    - Modeling of the parameters AND
    - a more sophisticated sampling algorithm (Reversible jump MCMC)

MRF+RJMCMC vs. JSEG

730 X 500

RJMCMC (17 min)



JSEG (1.5 min)

JSEG (Y. Deng, B.S.Manjunath: PAMI'01):

1. **color quantization**: colors are quantized to several representing classes that can be used to differentiate regions in the image.
2. **spatial segmentation**: A region growing method is then used to segment the image.

Benchmark results using the Berkeley Segmentation Dataset

JSEG

RJMCMC

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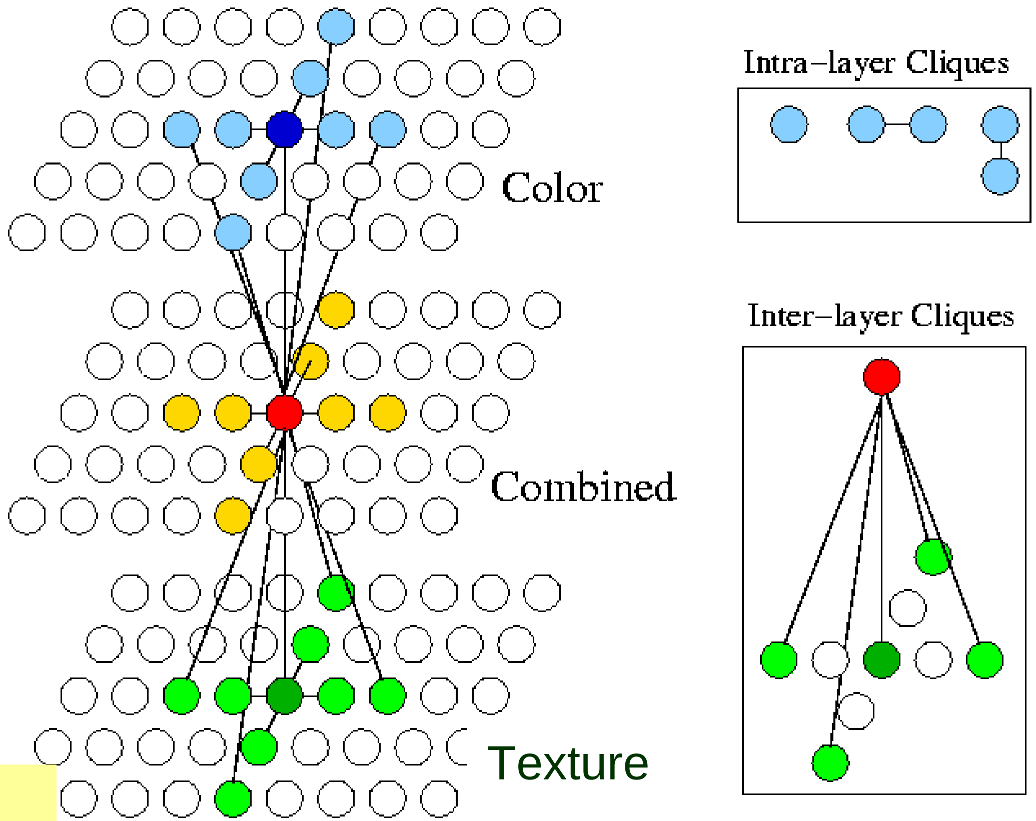
  - Yes, but you need to estimate model parameters automatically (EM algorithm)
- What if we do not know $|\Lambda|$?
  - Fully automatic segmentation requires
    - Modeling of the parameters AND
    - a more sophisticated sampling algorithm (Reversible jump MCMC)
- Can we segment more complex images?
  - Yes but you need a more complex MRF model

Objectives

- Combine different segmentation cues:
  - Color & Texture [ICPR2002,ICIP2003]
  - Color & Motion [ACCV2006,ICIP2007]
  - …?
- How humans do it?
  - Multiple cues are perceived simultaneously and then they are integrated by the human visual system [Kersten *et al. An. Rev. Psych.* 2004]
  - Therefore different image features has to be handled in a parallel fashion.
- We attempt to develop such a model in a Markovian framework

Multi-Layer MRF Model: Neighborhood & Interactions

- ω is modeled as a **MRF**
  - Layered structure
  - "Soft" interaction between features
- ➔ *P*(ω | *f*) follows a **Gibbs distribution**
  - Clique potentials define the local interaction strength
- MAP ⇔ **Energy minimization** (*U*(ω))



Hammersley-Clifford Theorem:

$$P(\omega) = \frac{\exp(-U(\omega))}{Z} = \frac{\exp(-\sum_C V_C(\omega))}{Z}$$

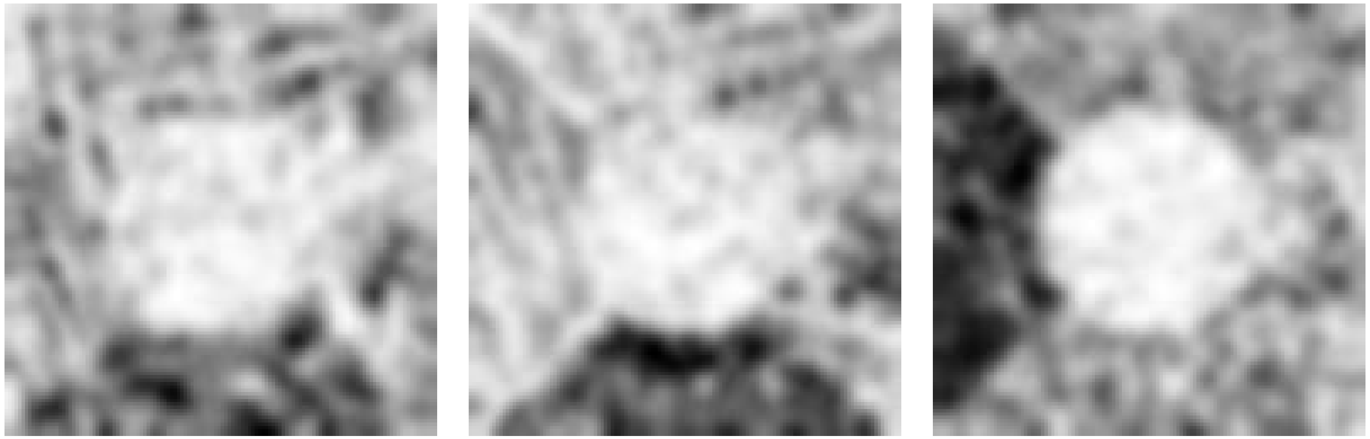
Model ⇔ Definition of clique potentials

Texture Layer: MRF model

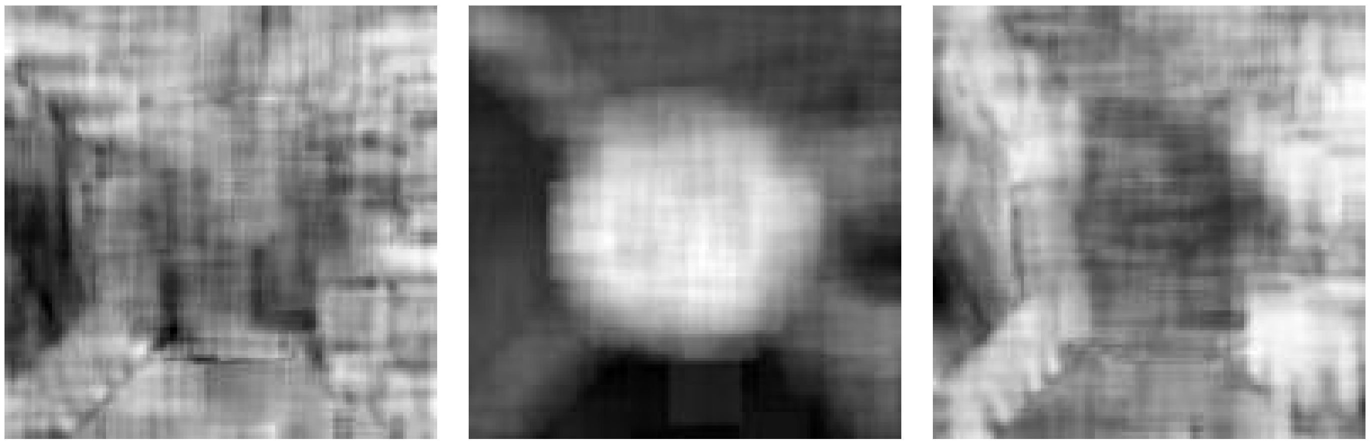
- We extract two type of texture features
  - *Gabor feature* is good at discriminating strong-ordered textures
  - *MRSAR feature* is good at discriminating weak-ordered (or random) textures
  - The number of texture feature images depends on the size of the image and other parameters.
    - Most of these doesn't contain useful information ➔
  - Select feature images with high discriminating power.
- MRF model is similar to the color layer model.

Examples of Texture Features

Gabor features:

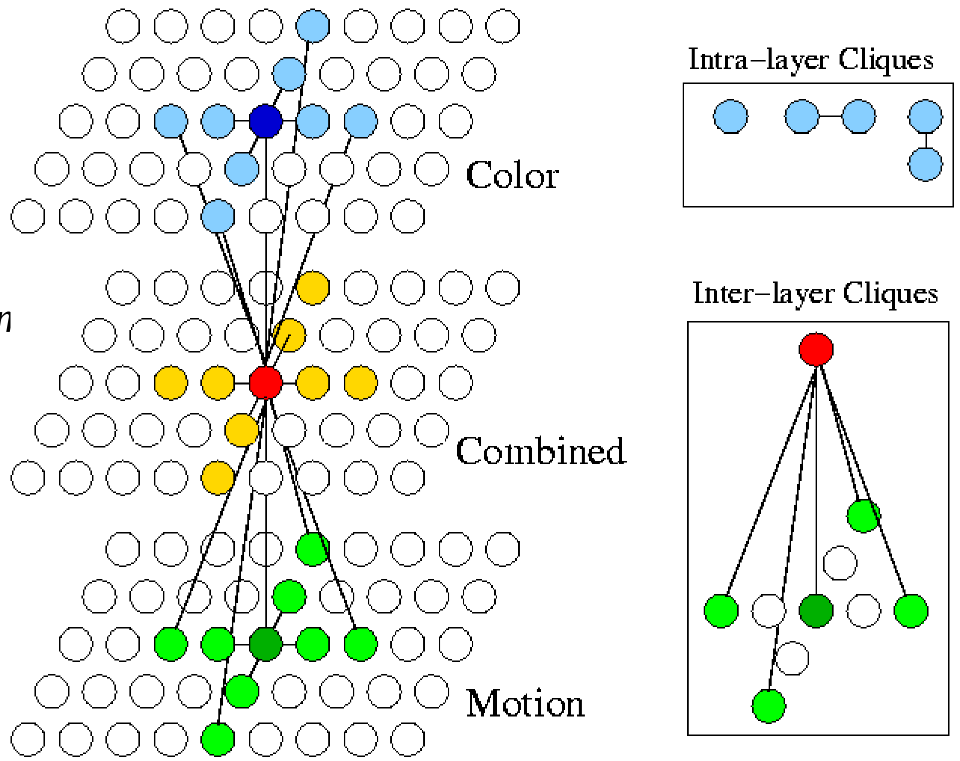


MRSAR features:



Combined Layer: Labels

- A label on the combined layer consists of a pair of color and texture/motion labels such that $\eta_s = <\eta_s^c, \eta_s^m>$ where $\eta_s^c \in \Lambda^c$ and $\eta_s^m \in \Lambda^n$
- The number of possible classes is $L^c \times L^m$
- The combined layer selects the most likely ones.



Combined Layer: ***Singleton*** potential

- Controls the number of classes:

$$V_s(\eta_s) = R\left((10N_{\eta_s})^{-3} + P(L)\right)$$

  - N_{η_s} is the percentage of labels belonging to class η_s
  - *L* is the number of classes present on the combined layer.
- Unlikely classes have a few pixels ➔ they will be penalized and removed to get a lower energy
- $P(L)$ is a log-Gaussian term:
  - Mean value is a guess about the number of classes,
  - Variance is the confidence.

Combined Layer: ***Doubleton*** potential

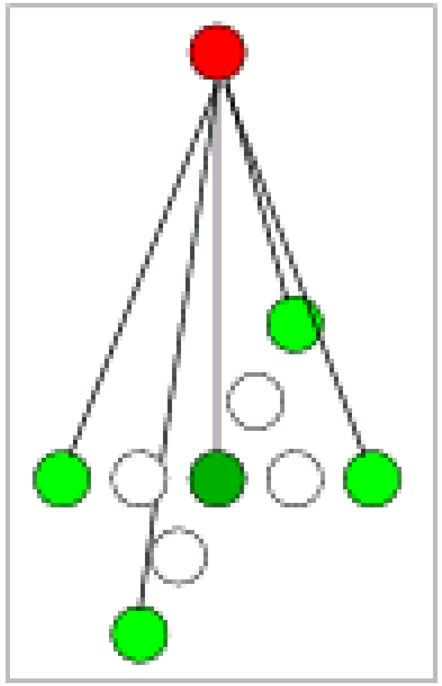
- Preferences are set in this order:
  1. Similar color and motion/texture labels
  2. Different color and motion/texture labels
  3. Similar color (resp. motion/texture) and different motion/texture (resp. color) labels
     - These are contours visible only at one feature layer.

$$\delta(\eta_s,\eta_r)=\begin{cases}-\alpha & \text{if } \eta_s^c=\eta_r^c, \eta_s^m=\eta_r^m \\ 0 & \text{if } \eta_s^c\neq\eta_r^c, \eta_s^m\neq\eta_r^m \\ +\alpha & \text{if } \eta_s^c\neq\eta_r^c, \eta_s^m=\eta_r^m \\ & \text{or } \eta_s^c=\eta_r^c, \eta_s^m\neq\eta_r^m \end{cases}$$

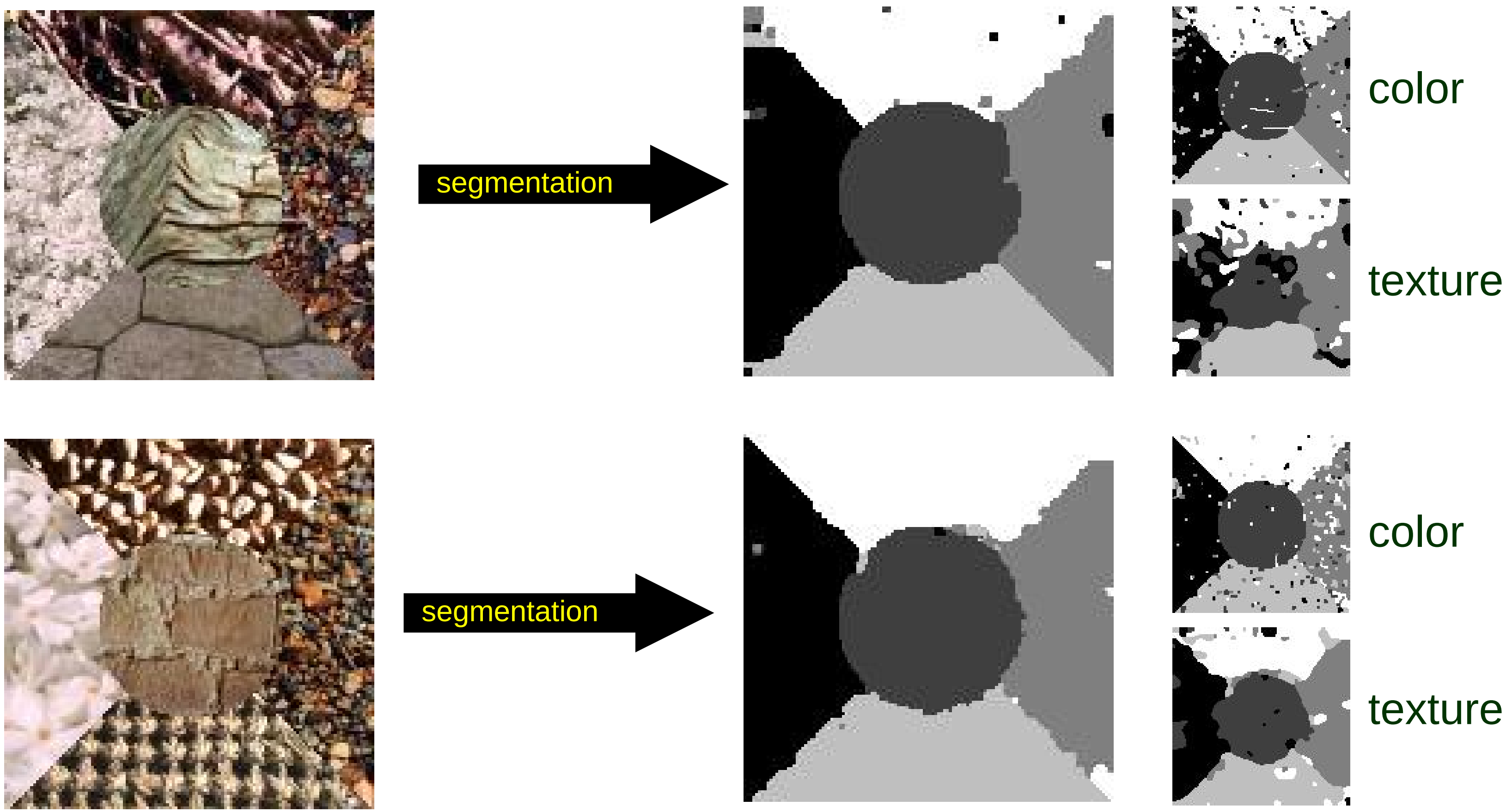
Inter-layer clique potential

- Five pair-wise interactions between a feature and combined layer
- Potential is proportional to the difference of the singleton potentials at the corresponding feature layer.
  - Prefers ω_s and η_s having the same label, since they represent the labeling of the same pixel
  - Prefers ω_s and η_r having the same label, since we expect the combined and feature layers to be homogenous

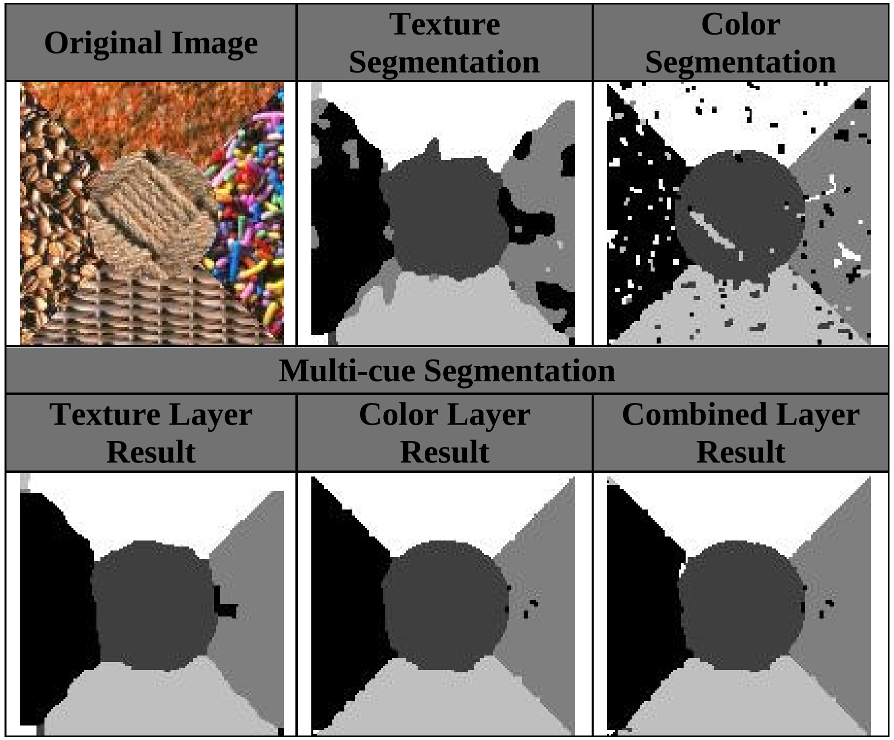
Inter–layer Cliques

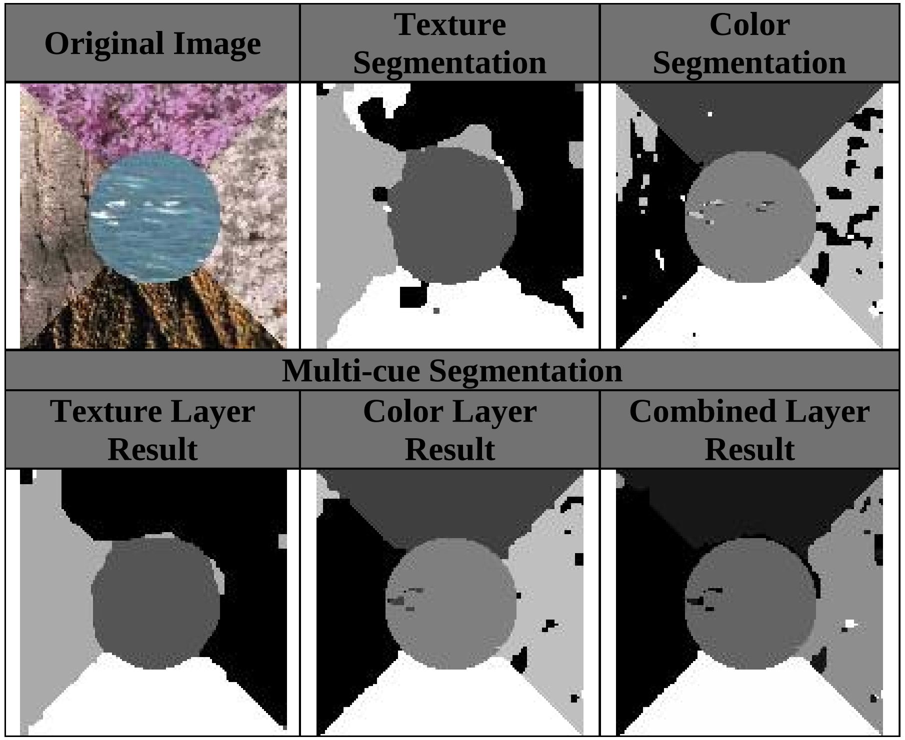


Color Textured Segmentation



Color Textured Segmentation





Color & Motion Segmentation

References

- Visit

http://www.inf.u-szeged.hu/~kato/