TI Lab 4: Integrals

IN THIS LAB, YOU WILL: CALCULATE DEFINITE INTEGRALS FROM THE HOME SCREEN;
CALCULATE DEFINITE INTEGRALS FROM A GRAPH;
CALCULATE DEFINITE INTEGRALS OF PIECEWISE FUNCTIONS;
GRAPH THE INTEGRAL OF A FUNCTION;
DISCOVER A FUNCTION THAT IS DEFINED BY AN INTEGRAL; AND
USE A PROGRAM TO FIND THE AVERAGE VALUE OF A FUNCTION.

1. Just as you calculate derivatives at a point, you can compute definite integrals on your calculator. The command `fnInt` approximates a definite integral. The format is `fnInt(`*function, variable, lower, upper, tolerance*`)`, where tolerance is the degree of accuracy with which you wish to compute the integral. If you do not specify a value, the calculator assumes that the tolerance is 0.0001.

a) Evaluate $\displaystyle\int_0^1 \frac{4dx}{1+x^2}$.

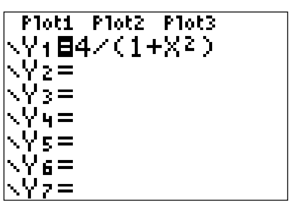
b) Evaluate $\displaystyle\int_{-1}^3 \frac{\sin x}{x}\,dx$. Make sure you are using radians!

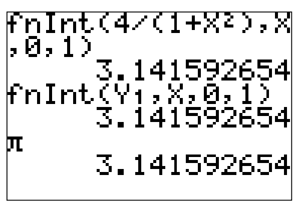
(Notice that the calculator evaluates the integral in part (b) even though the integrand is not defined at $x = 0$!)

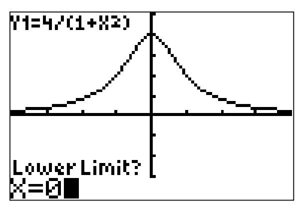
2. Like `nDeriv`, you can store the function you wish to integrate as `Y1` and enter `fnInt(Y1,X,`*lower, upper*`)`. The advantage to entering the function in `Y1` is that there is also an integral function located under the `CALC` menu—`7:∫ f(x)dx`. Not only will the `∫ f(x)dx` command evaluate the definite integral, it will shade the area the integral represents!

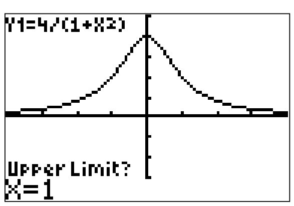
a) Enter `Y1=sin(X)/X` and graph it using `ZTrig`. Use `∫ f(x)dx` to evaluate $\displaystyle\int_{-1}^1 \frac{\sin x}{x}\,dx$. Enter the lower or upper limits (or `TRACE` them) and press `ENTER`.

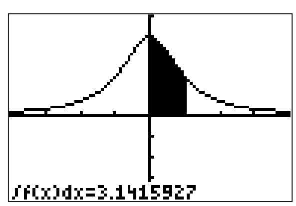
b) Graph the function $f(x) = \begin{cases} x+3 & x \le 3 \\ -(x-5)^2 & x > 3. \end{cases}$ Then evaluate $\displaystyle\int_{-5}^7 f(x)\,dx$ and $\displaystyle\int_{2.5}^{3.5} f(x)\,dx$.







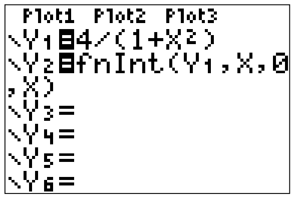


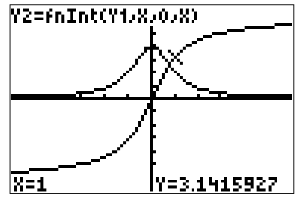


3. You may also graph the integral of a function.

a) Enter `Y1=3X^2-4` and enter `Y2=fnInt(Y1,X,0,X)`. This graphs the integral of `Y1`. Clearly, the integral is equal to $x^3 - 4x$. Enter this expression on `Y3` and compare the accuracy of `Y2` using your table.

You probably noticed that the graph of `Y2` goes very slow. This is because the calculator is evaluating the integral at every pixel. Adjusting `Xres` in the `WINDOW` menu to a higher value (say 2 or 3) will reduce graphing time, but will also reduce accuracy.





b) Find the position of a particle at $x = -\frac{1}{2}$ and $x = 1$ if the particle's velocity is $\dfrac{1}{\sqrt{4-x^2}}$, for $-2 < x < 2$.

c) Clear your `Y=` screen. Enter and graph `nDeriv(fnInt(X^3-4X,X,0,X),X,X)`. Change `Xres` to 2 and use `ZStandard`. (This will take up to 40 seconds to graph.) Does the graph look familiar? Which function is it?

4. Many important functions are defined as integrals. Consider the function

$$L(x) = \int_1^x \frac{1}{t}\,dt,$$

for $x > 0$. The function L is undefined for $x > 0$ since $f(t) = 1/t$ is not continuous at $t = 0$.

a) Graph `Y1=fnInt(1/T,T,1,X)` in the window $0.01 \le x \le 10$ and $-1.5 \le y \le 2.5$.

b) For what values of x is $L(x) = 0$?

c) Use the table to make a list of approximate values for $L(x)$ using 8 equally spaced values starting with $x = 1$.

d) Verify from the table created in part (c) that:

$$L(6) = L(2) + L(3), \qquad L(4) = 2L(2), \qquad L(8) = 3L(2)$$

e) From the home screen, enter {`1,2,3,4,5,6,7,8`}→`L1` then enter `Y1(Ans)`→`L2`. This puts the x-values and y-values from the table in part (c) into lists. Now, from the `STAT CALC` menu, choose `LnReg` to fit a logarithmic regression curve to the data. What is the equation of the curve that best fits the data?

f) What is the derivative of $L(x)$?