Calculus Study Sheet

Kylie Pollack - AP Calculus

Limits & Continuity

$\lim_{x\to a} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(x)$] limit is how a function behaves near a point

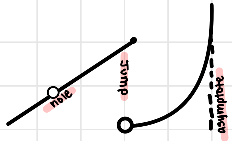
limits from right & left

$\lim_{x\to\infty}$ = horizontal asymptote

$\lim_{x\to a} = \infty$ = vertical asymptote

↳ horizontal asymptote limits $\frac{a}{b}$ → $a>b$ = DNE → $b>a$ = $\lim = 0$ → $a=b$ = coefficient ratio

·continuous when: $f(a)$ exists, $\lim_{x\to a} f(x)$ exists, $\lim_{x\to a} f(x) = f(a)$



Trig Differentiation

$f(x)$	$f'(x)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$

Differentiation

Power rule: $\frac{dy}{dx}x^n = nx^{n-1}$

Quotient rule: $\frac{dy}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Product rule: $\frac{dy}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$

Chain rule: $\frac{dy}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$

Implicit differentiation: $\frac{dy}{dx}$ = in terms of x $(x' = 1,\ y' = y')$

Contextual Applications

Particle motion:

Speeding up: v & a same sign
Slowing down: v & a different signs

$f(x)$ = position

$f'(x)$ = velocity $(v(t))$
- $v(t) > 0$ moving right
- $v(t) = 0$ at rest
- $v(t) < 0$ moving left

$f''(x)$ = acceleration $(a(t))$

Related rates: derivative of shape formula
- ○ πr^2, $2\pi r$
- ⊖ $\pi r^2 h$
- ● $\frac{4}{3}\pi r^3$

Function Behavior

$f'(x) > 0$ increasing
$f'(x) < 0$ decreasing
$f'(x)$ + to − relative max
$f'(x)$ − to + relative min
$f''(x) > 0$ CCU
$f''(x) < 0$ CCD
$f''(x) = 0$ inflection point

Analytical Applications

mean value theorem: continuous, differentiable AROC = IROC

extreme value theorem: → continuous local minimum & maximum

critical points: $f'(x) = 0$, und

global extrema: CPs & endpoints into $f(x)$

Integration & Accumulation

reimann sums → L: $\Sigma y_1 \Delta x$, R: $\Sigma y_2 \Delta x$, M: Σ midpoint $\cdot \Delta x$, T: $\frac{y_1 + y_2}{2} \cdot \Delta x$

Integrals: $\int_a^b f(x)dx = F(b) - F(a)$ | $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$ | $\int f(x)dx = F(x) + C$ | $\int_b^a f(x)dx = -\int_a^b f(x)dx$

antiderivatives

$\int f'(x)$	$f(x)$
$\int \sin(x)$	$-\cos(x) + C$
$\int \cos(x)$	$\sin(x) + C$
$\int \sec^2(x)$	$\tan(x) + C$
$\int e^x$	$e^x + C$

antiderivatives: $\int x^n dx = \frac{x^{n+1}}{n+1}$

Fundamental Theorem: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$

average value: $\frac{1}{b-a}\int_a^b f(x)dx$

U-Substitution

1. find "u" ex. $\int (x^2+2)^3$ u
2. $\frac{du}{dx}$ u ex. $2x$ (u')
3. $\frac{du}{u'} = dx$ ex. $\int (u)^3 \frac{du}{2x}$
4. take antiderivative of 'du'
5. Substitute u-value back in

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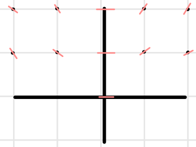
Differential Equations

Separation of variables:

1. multiply both sides by dx
2. move 'y' terms to dy side
3. take integral of both sides
4. plug in initial conditions (x,y)
5. solve for +C value
6. solve for y alone



Slope Fields

Applications of Integrals

area = top f(x) minus bottom → $\int_a^b f(x) - g(x)\,dx$

volume (disk) = $\pi \int_a^b \text{radius}^2\,dx$

volume (washer) = $\pi \int_a^b (R)^2 - (r)^2\,dx$

volume (cross section) = $\int_a^b (\underbrace{\text{area of cross sect.}}_{\text{base of shape}})\,dx$