Related Rates

Useful Knowledge

Geometric Quantity	Formula	Derivative
Area of a Circle	$A = \pi r^2$	$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
Circumference of a Circle	$C = 2\pi r$	$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$
Volume of a Sphere	$V = \frac{4}{3}\pi r^3$	$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$
Surface Area of a Sphere	$SA = 4\pi r^2$	$\frac{dSA}{dt} = 8\pi r \frac{dr}{dt}$
Volume of a Cube	$V = s^3$	$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$
Surface Area of a Cube	$SA = 6s^2$	$\frac{dSA}{dt} = 12s \frac{ds}{dt}$
Pythagorean Theorem	$a^2 + b^2 = c^2$	$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$
Volume of a Cone	$V = \frac{1}{3}\pi r^2 h$	$\frac{dv}{dt} = \frac{1}{3}\pi[r^2 \frac{dh}{dt} + h(2r\frac{dr}{dt})]$

*In related rates, everything changes with respect to time.

*Nothing is a variable, everything is a function of time with an inside piece (chain rule)

$\rightarrow y' = \frac{dy}{dt}, v' = \frac{dv}{dt}$.

Solve

1. Write relevant geometric formula
2. Take the derivative
3. Plug in known values
4. Solve

*Plug in ratios if needed (to avoid product rule)

Example Question

Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down; it has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 feet deep?

Theorems

Intermediate Value Theorem

The intermediate value theorem states that if f is a continuous function whose domain contains the interval [a, b], then it takes on any given value between f(a) and f(b) at some point within the interval.

What to say: *Since ?(x) is continuous, the ivt states that the value (x) will be at some point in the interval [a,b]*

Example:

x	0	3	4	8	9
f(x)	1	-5	3	7	-1

1. On the interval $0<x<9$, must there be a value of x for which f(x)=2? explain?

F(0)=1 F(4)=3 Since f(x) is continuous, the IVT states that there is a value c such that f(c)= 2 and $0<x<4$

Mean Value Theorem

The mean value theorem states, roughly, that for a given planar arc between two endpoints, there is at least one point at which the tangent to the arc is parallel to the secant through its endpoints. (X2-X1/Y2-Y1)

What to say? *Since f(x) is continuous and differentiable, there is such a value c in the interval [a.b]*

Example:

x	0	2	4	6
f(x)	0	-2	3	7

Is there a value x of f'(x)=5? If so make it justifiable

$$\frac{f(6)-f(4)}{6-4} = 2$$

Since the number does not equal five the MVT does not apply.

Extreme Value Theorem

The extreme value theorem states that if a real-valued function f is continuous on the closed interval [a, b], then f must attain a maximum and a minimum, each at least once. For the EVT

Topic/Question	Steps to Solve	Explanation/Reasoning given f(x)
Relative Max	1. f'(x)=0 or UND 2. Sign chart for f'(x)	f'(x) changes from + to -
Relative Min	1. f'(x)=0 or UND 2. Sign chart for f'(x)	f'(x) changes from - to +
Increasing	1. f'(x)=0 or UND 2. Sign chart for f'(x)	f'(x)>0 (F is increasing if f' is positive)
Decreasing	1. f'(x)=0 or UND 2. Sign chart for f'(x)	f'(x)<0 (F is decreasing f' is negative)
Point of Inflection	1. f''(x)=0 of UND 2. Sign chart for F''(x)	f''(x) changes signs (F has an inflection point when it changes from increasing to decreasing, f' slope is 0)
Concave Up	1. f''(x)=0 of UND 2. Sign chart for F''(x)	f''(x)>0 (f is positive f' is concave up) (slope of f' is positive f'' is concave up)
Concave Down	1. f''(x)=0 of UND 2. Sign chart for F''(x)	f''(x)<0 (f is negative f' is concave down) (slope of f' is negative f'' is concave down)
Absolute max/min	1. f''(x)=0 of UND 2. Sign chart for F''(x)	Make a table with CP and ENDPOINTS. Plug in values into f(x)

Washer Method/Disc Method

Normal Area: $\int (top) - (botton)$

Volume (rotated around an axis): $\boldsymbol{\pi} \int (R)^2 - (r)^2$

Volume with known cross section: ∫ (area of the cross section)

*all the integrals go from points on the x-axis which will be the restrictions/end points of the area you are finding. B is on the top point a is on the bottom

Riemann Sums

To approximate the area on [a,b] with a equal subdivision, use (b-a/n) as the width of each rectangle

LRAM: Evaluate the function at the left end point (Increasing- Underestimate Decreasing- Over estimate)

RRAM: Evaluate the function at the right end point(Increasing- Overestimate Decreasing- Underestimate)

MRAM Evaluate the function at the midpoint

TRAPIZOIDAL SUMS: Use a trapezoid equation (b1+b2/2) x the h

*add all the parts together in the interval to get the area under the curve that you want. When doing it by hand if the area is under the x-axis then it will be negative