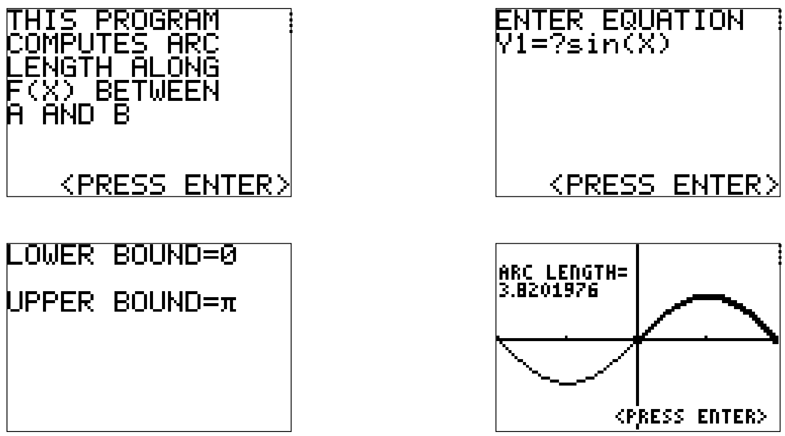
TI Lab 7: Applications of Integrals

In this lab, you will: use a program to approximate the arc length of a curve; and
use a program to approximate the volume of a solid of revolution.

1. The program `ARC` uses the the calculator's definite integral function to determine the arc length of a function. In the figures below, the arc length of $\sin x$ from 0 to π is calculated. Notice that the program shaded the part of the curve whose length you found.



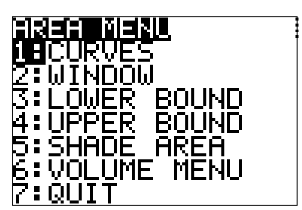
a) Set up the integral that represents the length of the curve $\arctan x$ from 0 to π, then use the program to find the length.

b) Set up the integral that represents the length of the curve $e^x + e^{-x}$ from -1 to 1, then use the program to find the length.

2. The prorgam `VOLUME` determines the volume of a solid of revolution. This program does a lot to accurately compute the volume and draw the solid—therefore, it is quite invovled!

a) In this example, we compute the volume of the solid formed by revolving the region between x^2 and the x-axis from 1 to 3 around the line $y = -1$.

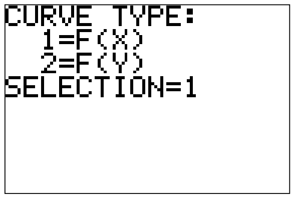
We begin with the Area Menu. This is the first screen you see upon running the program. You must choose options 1 through 4 (in that order) before you choose option 6 to compute volume!

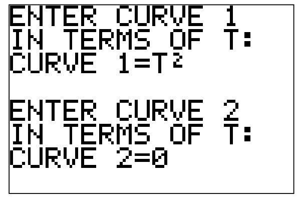


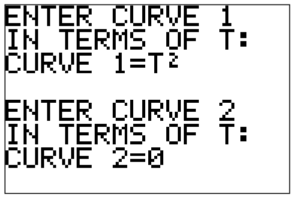
After selecting option 1, you are asked to choose which type of curve you have. If the solid you wish to form is revolved around a horizontal axis, choose `F(X)`; if it is revolved around a vertical axis, choose `F(Y)`. In this case, we have a horizontal axis of rotation.

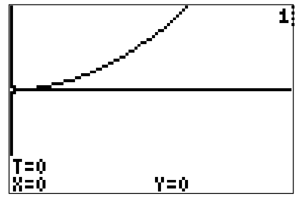
Then you are asked to enter the curves. (Notice that it says "in terms of t." This is becuase the calculator will graph the curve parametrically in order to then graph the solid formed.) Here, we enter t^2 as the first curve and 0 as the second, since the second curve is the x-axis.

The next screen asks you to set the graph window, then the graph is shown. Notice that you want to set the window appropriately so that you will be able to see the entire region once it is rotated.

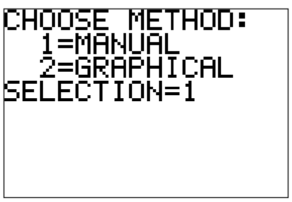


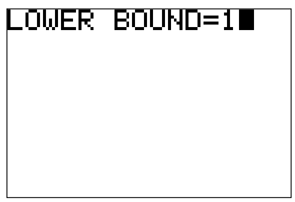


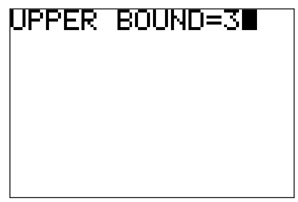


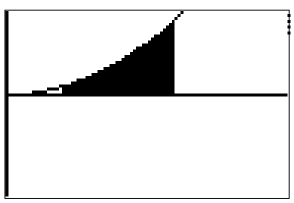


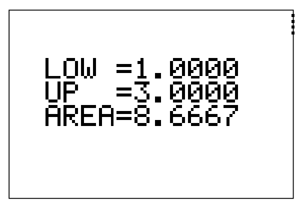
Next, you must set the upper and lower bounds for the region. For both bounds, you are given the option of entering the bound (manual) or tracing the graph to select the bound. Since we know exactly what are bounds are, we choose Manual for both. Finally, you may choose option 5 to shade the region you will be rotating. After the shading is complete, pressing `ENTER` gives you the area of the region.



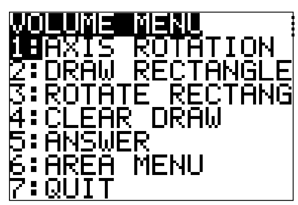




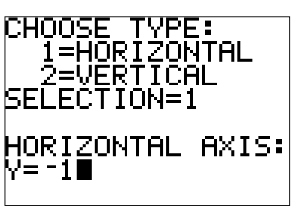


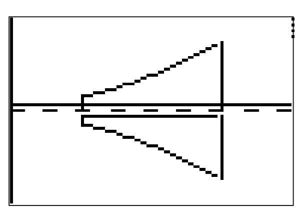


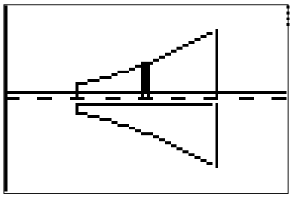
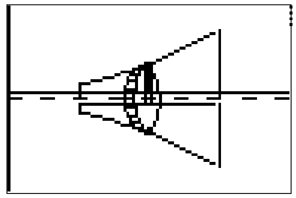
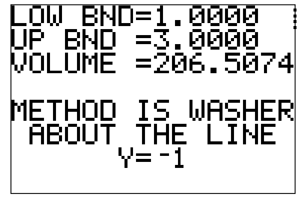
Finally, we are ready for the Volume Menu. This is where you choose the rotation axis. You may also choose tohave the calculator graph a representative rectangular region in the solid to revolve, although this is not necessary.



In our example, we want to revolve the region around $y = -1$. We choose option 1 and we are asked to choose whether the line is horizontal or vertical and then enter the line. Then the calculator graphs an "outline" of the solid with the axis of rotation as a dotted line. To draw a representative rectangular region, we choose option 2, and we are shown the graph of the region. Move the cursor up to any location on the graph of the function and press `ENTER`. Then a shaded rectangle is drawn. Next, choose 3 to rotate the rectangle. Finally, choose 5 for the answer to the volume question.



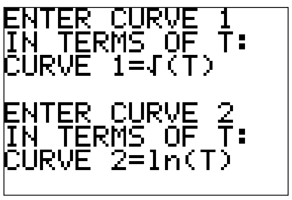


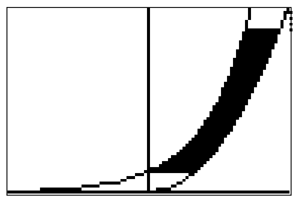
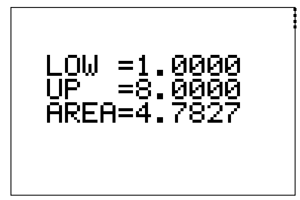




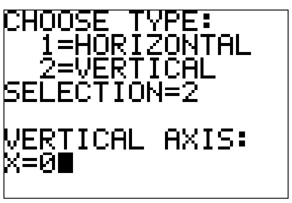
Drawing the figure is not necessary to compute the answer. Once the axis of rotation is selected, you can go immediately to the answer. Also notice that if you mess up, you have the option of going back to the Area Menu and re-entering your functions and bounds.

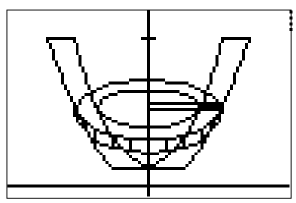
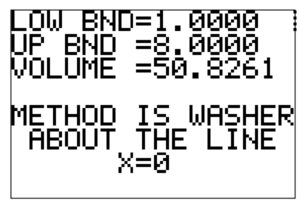
b) Find the volume of the solid formed by revolving the region between $y = x^2$ and $y = e^x$ about the y-axis from $y = 1$ to $y = 8$.

Since the rotation axis is vertical, we must select curve type `F(Y)`. This requires that both funtions and both limits must be entered in terms of y. The limits are already given in terms of y; simply solve both functions for y to get $x = \sqrt{y}$ and $x = \ln y$ and you are ready to find the volume.





c) The region R is bounded by the curves $y = \dfrac{-5(x+2)}{x} + 10$ and $y = \ln x$ from $x = 3$ to $x = 7$. Set up an integral that represents the volume of the solid generated by revolving the region around the line $y = 6$, then find the volume.

d) The region R is bounded by the curves $y = x^3$ and $y = e^{x/2}$ from $y = 3$ to $y = 7$. Set up an integral that represents the volume of the solid generated by revolving the region around the line $x = 1$, then find the volume.