

Integral

报告主要内容

Write a program to compute the integral

$$I(f) = \int_1^5 \sin(x) dx, h = 0.1$$

with the following methods

- repeated Simpson quadrature
- repeated trapezoid quadrature

and provide the errors

主程序

待积分函数：

```
function f(x)
  implicit none
  real :: f,x
  f=sin(x)
end function f
```

主程序：

```
program main

  interface

    function f(x)
      real :: f,x
    end function f

  end interface
```

```

function repeated_simpson_quadrature(f,a,b,h)
    real :: a,b,h
    real,external :: f
end function repeated_simpson_quadrature

function repeated_trapezoid_quadrature(f,a,b,h)
    real :: a,b,h
    real,external :: f
end function repeated_trapezoid_quadrature

end interface

real :: a,b,h
a=1.0
b=5.0
h=0.1

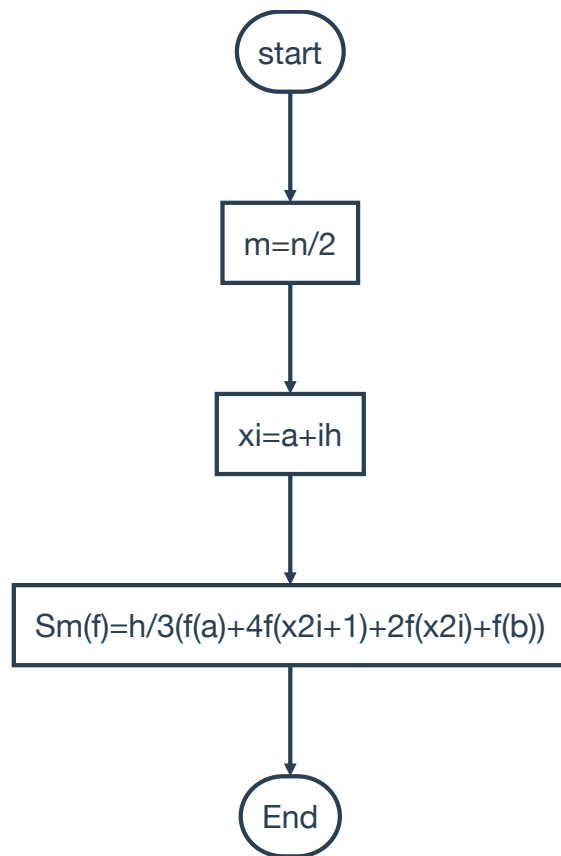
print *
print '(5x,a,5x,f4.1,a,f4.1,5x,a,f4.1)', 'f=sin(x)', a, '<= x'
<=',b,'h=',h
print *
print *, 'Compute the integral...'
print *
print *, 'Repeated Simpson Quadrature'
print
'(5x,a,f9.5)', 'Sm=', repeated_simpson_quadrature(f,a,b,h)
print '(5x,a,f14.10)', 'Error=',
(repeated_simpson_quadrature(f,a,b,h)-
repeated_simpson_quadrature(f,a,b,2*h))/15
print *
print *, 'Repeated Trapezoid Quadrature'
print
'(5x,a,f9.5)', 'Tn=', repeated_trapezoid_quadrature(f,a,b,h)
print '(5x,a,f14.10)', 'Error=',
(repeated_trapezoid_quadrature(f,a,b,h)-
repeated_trapezoid_quadrature(f,a,b,2*h))/3
print *

end program main

```

Repeated Simpson Quadrature

流程图:



原理：

$$h = \frac{b-a}{n}, x_i = a + ih, i = 0, \dots, n; n = 2m$$

$$\int_{x_{2i}}^{x_{2i+2}} f(x) dx = \frac{2h}{6} (f(x_{2i}) + 4f(x_{2i+1}) + f(x_{2i+2}))$$

则：

$$S_m(f) = \frac{h}{3} (f(a) + 4 \sum_{i=1}^{m-1} f(x_{2i+1}) + 2 \sum_{i=1}^{m-1} f(x_{2i}) + f(b))$$

代码：

```

function repeated_simpson_quadrature(f,a,b,h) result(Sm)
  implicit none
  real :: Sm,a,b,h,t1,t2
  real,external :: f
  integer :: n,m,i
  real,allocatable :: x(:)

  n=int((b-a)/h)
  m=n/2
  allocate(x(0:n))
  x=(/ (a+i*h,i=0,n) /)

  t1=sum( (/ (f(x(2*i+1))),i=0,m-1) /) )
  t2=sum( (/ (f(x(2*i))),i=1,m-1) /) )

  Sm=h/3*(f(a)+4*t1+2*t2+f(b))

end function repeated_simpson_quadrature

```

误差分析:

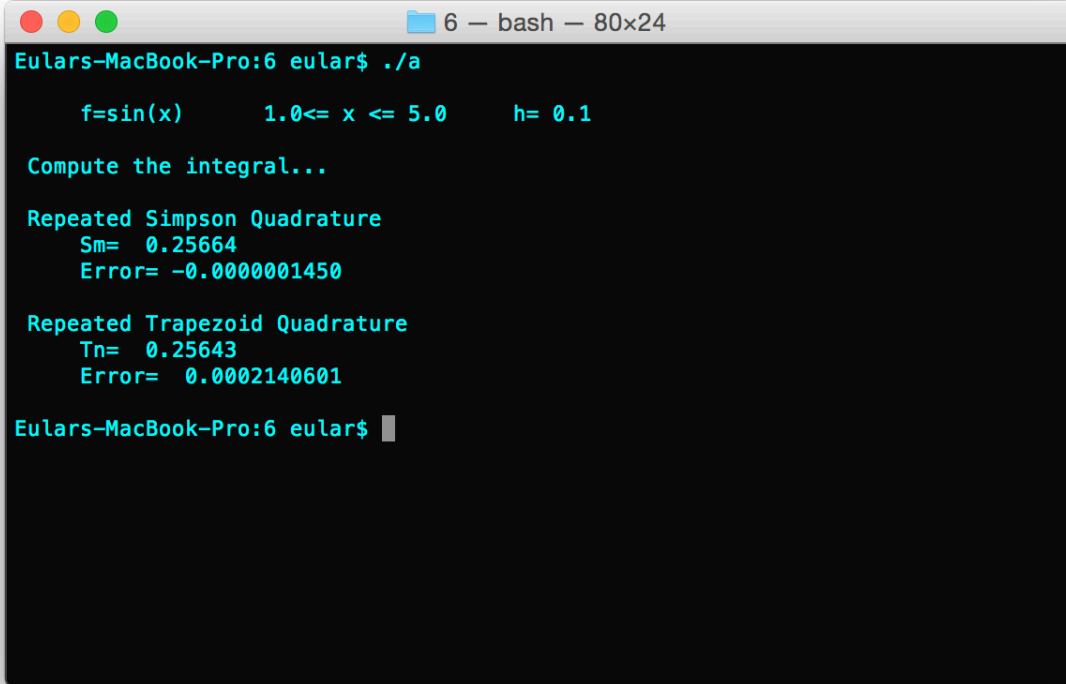
$$E_{2n} = I - S_{2n} = \frac{S_{2n} - S_n}{15}$$

```

(repeated_simpson_quadrature(f,a,b,h)-
repeated_simpson_quadrature(f,a,b,2*h))/15

```

输出结果:

A terminal window titled "6 — bash — 80x24" on a Mac. The prompt is "Eulars-MacBook-Pro:6 eular\$". The user enters "./a". The script outputs: "f=sin(x) 1.0<= x <= 5.0 h= 0.1", "Compute the integral...", "Repeated Simpson Quadrature", "Sm= 0.25664", "Error= -0.000001450", "Repeated Trapezoid Quadrature", "Tn= 0.25643", "Error= 0.0002140601", and returns to the prompt "Eulars-MacBook-Pro:6 eular\$".

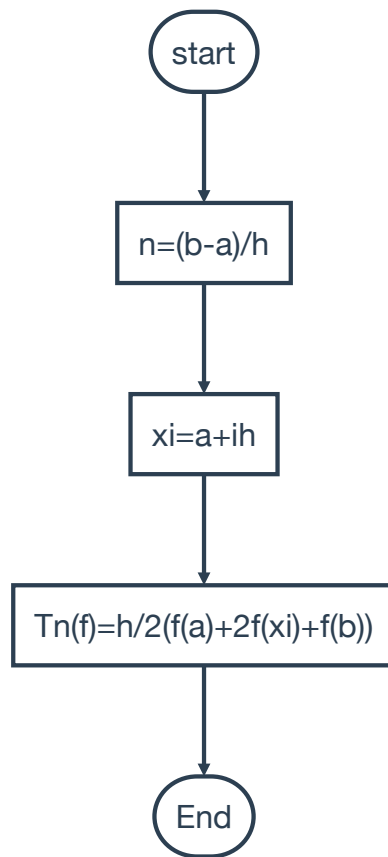
```
Eulars-MacBook-Pro:6 eular$ ./a
f=sin(x) 1.0<= x <= 5.0 h= 0.1
Compute the integral...
Repeated Simpson Quadrature
Sm= 0.25664
Error= -0.000001450
Repeated Trapezoid Quadrature
Tn= 0.25643
Error= 0.0002140601
Eulars-MacBook-Pro:6 eular$
```

分析:

可以看出 **Repeated Simpson Quadrature** 方法求得的结果与真实值十分接近, 误差较小。

Repeated Trapezoid Quadrature

流程图:



原理：

$$h = \frac{b-a}{n}, x_i = a + ih, i = 0, \dots, n$$

$$\int_{x_i}^{x_{i+1}} f(x) dx \approx \frac{h}{2} (f(x_i) + f(x_{i+1}))$$

则：

$$\begin{aligned}
 T_n(f) &= \sum_{i=0}^{n-1} \left(\frac{h}{2} (f(x_i) + f(x_{i+1})) \right) \\
 &= \frac{h}{2} (f(a) + 2 \sum_{i=1}^{n-1} f(x_i) + f(b))
 \end{aligned}$$

代码：

```

function repeated_trapezoid_quadrature(f,a,b,h) result(Tn)
  implicit none
  real :: Tn,a,b,h
  real,external :: f
  integer :: n,i
  real,allocatable :: x(:)

  n=int((b-a)/h)
  allocate(x(0:n))
  x=(/ (a+i*h,i=0,n) /)

  Tn=h/2*(f(a)+2*sum( (/ (f(x(i)),i=1,n-1) /) )+f(b))
end function repeated_trapezoid_quadrature

```

误差分析:

$$E_{2n} = I - T_{2n} = \frac{T_{2n} - T_n}{3}$$

```

(repeated_trapezoid_quadrature(f,a,b,h)-
repeated_trapezoid_quadrature(f,a,b,2*h))/3

```

输出结果:

```

Eulars-MacBook-Pro:6 eular$ ./a

      f=sin(x)      1.0<= x <= 5.0      h= 0.1

Compute the integral...

Repeated Simpson Quadrature
Sm=  0.25664
Error= -0.0000001450

Repeated Trapezoid Quadrature
Tn=  0.25643
Error=  0.0002140601

Eulars-MacBook-Pro:6 eular$

```

分析：

可以看出 Repeated Trapezoid Quadrature 相比 Repeated Simpson Quadrature 方法求得的结果精度较低，误差较大。