# Integral

## 报告主要内容

> **Write a program to compute the integral**
>
> $$I(f) = \int_1^5 sin(x)\mathrm{d}x, h = 0.1$$
>
> **with the following methods**
>
> - repeated Simpson quadrature
> - repeated trapezoid quadrature
>
> **and provide the errors**

## 主程序

待积分函数:

```
function f(x)
    implicit none
    real :: f,x
    f=sin(x)
end function f
```

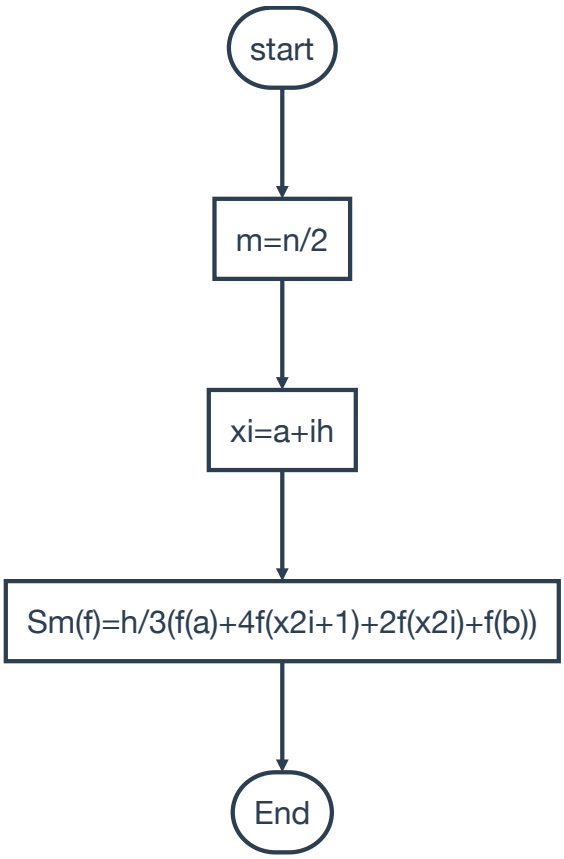
主程序:

```
program main

    interface

        function f(x)
            real :: f,x
        end function f
```

```
        function repeated_simpson_quadrature(f,a,b,h)
            real :: a,b,h
            real,external :: f
        end function repeated_simpson_quadrature

        function repeated_trapezoid_quadrature(f,a,b,h)
            real :: a,b,h
            real,external :: f
        end function repeated_trapezoid_quadrature

    end interface

    real :: a,b,h
    a=1.0
    b=5.0
    h=0.1

    print *
    print '(5x,a,5x,f4.1,a,f4.1,5x,a,f4.1)','f=sin(x)',a,'<= x
<=',b,'h=',h
    print *
    print *,'Compute the integral...'
    print *
    print *,'Repeated Simpson Quadrature'
    print
'(5x,a,f9.5)','Sm=',repeated_simpson_quadrature(f,a,b,h)
    print '(5x,a,f14.10)','Error=',
(repeated_simpson_quadrature(f,a,b,h)-
repeated_simpson_quadrature(f,a,b,2*h))/15
    print *
    print *,'Repeated Trapezoid Quadrature'
    print
'(5x,a,f9.5)','Tn=',repeated_trapezoid_quadrature(f,a,b,h)
    print '(5x,a,f14.10)','Error=',
(repeated_trapezoid_quadrature(f,a,b,h)-
repeated_trapezoid_quadrature(f,a,b,2*h))/3
    print *

end program main
```

# Repeated Simpson Quadrature

流程图:




原理:

$$h = \frac{b-a}{n}, x_i = +ih, i = 0, \ldots, n; n = 2m$$

$$\int_{x_{2i}}^{x_{2i+2}} f(x) = \frac{2h}{6}\left(f(x_{2i}) + 4f(x_{2i+1}) + f(x_{2i+2})\right)$$

则:

$$S_m(f) = \frac{h}{3}\left(f(a) + 4\sum_{i=1}^{m-1} f(x_{2i+1}) + 2\sum_{i=1}^{m-1} f(x_{2i}) + f(b)\right)$$

代码:

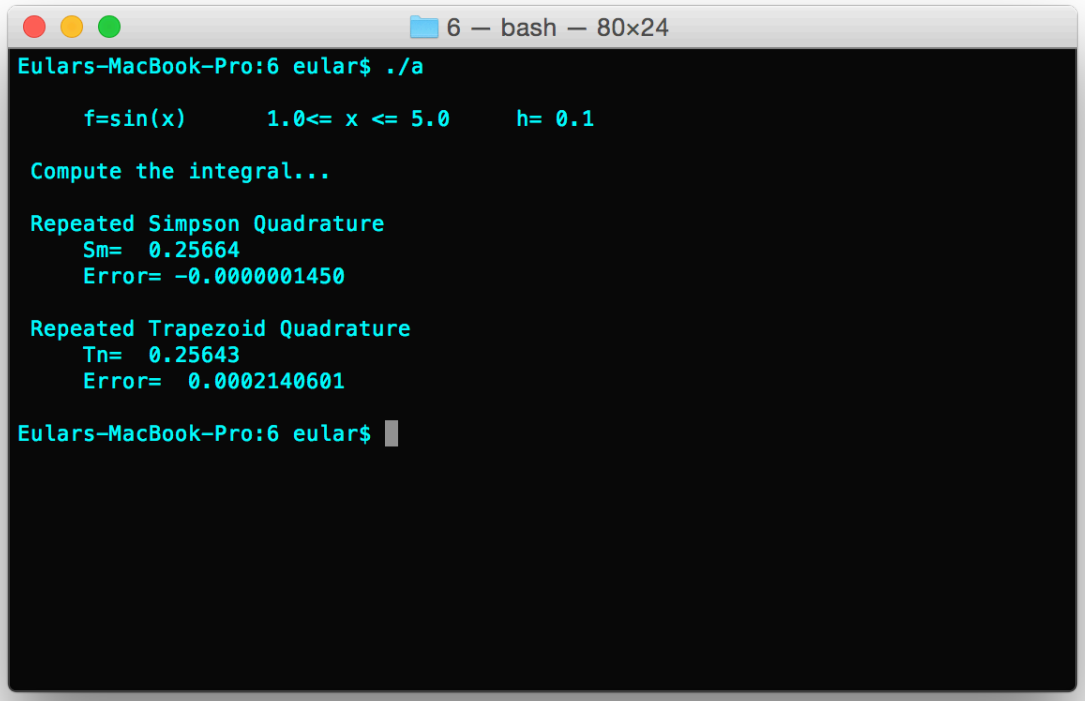
```
function repeated_simpson_quadrature(f,a,b,h) result(Sm)
    implicit none
    real :: Sm,a,b,h,t1,t2
    real,external :: f
    integer :: n,m,i
    real,allocatable :: x(:)

    n=int((b-a)/h)
    m=n/2
    allocate(x(0:n))
    x=(/ (a+i*h,i=0,n) /)

    t1=sum( (/ (f(x(2*i+1)),i=0,m-1) /) )
    t2=sum( (/ (f(x(2*i)),i=1,m-1) /) )

    Sm=h/3*(f(a)+4*t1+2*t2+f(b))

end function repeated_simpson_quadrature
```

误差分析:

$$E_{2n} = I - S_{2n} = \frac{S_{2n} - S_n}{15}$$

```
(repeated_simpson_quadrature(f,a,b,h)-
repeated_simpson_quadrature(f,a,b,2*h))/15
```

输出结果：

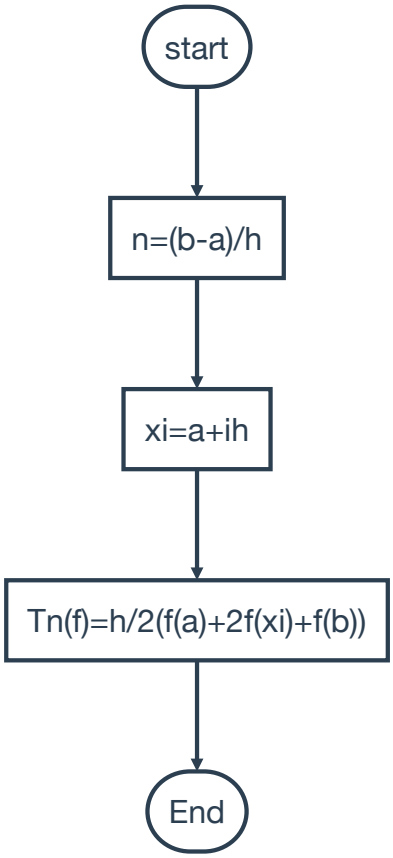



分析：

> 可以看出`Repeated Simpson Quadrature`方法求得的结果与真实值十分接近，误差较小。

# Repeated Trapezoid Quadrature

流程图：




原理:

$$h = \frac{b-a}{n}, x_i = +ih, i = 0, \ldots, n$$

$$\int_{x_i}^{x_{i+1}} f(x) = \frac{h}{2}\left(f(x_i) + f(x_{i+1})\right)$$

则:

$$T_n(f) = \sum_{i=0}^{n-1}\left(\frac{h}{2}\left(f(x_i) + f(x_{i+1})\right)\right)$$

$$= \frac{h}{3}\left(f(a) + 4\sum_{i=1}^{m-1} f(x_{2i+1}) + 2\sum_{i=1}^{m-1} f(x_{2i}) + f(b)\right)$$

代码:

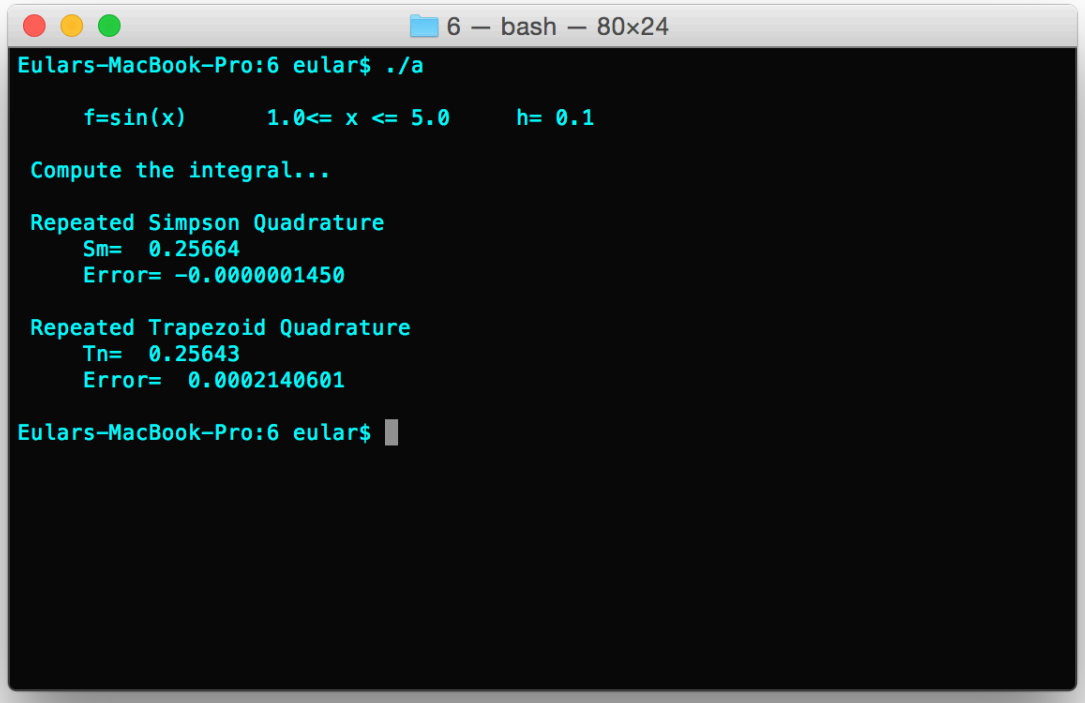
```
function repeated_trapezoid_quadrature(f,a,b,h) result(Tn)
    implicit none
    real :: Tn,a,b,h
    real,external :: f
    integer :: n,i
    real,allocatable :: x(:)

    n=int((b-a)/h)
    allocate(x(0:n))
    x=(/ (a+i*h,i=0,n) /)

    Tn=h/2*(f(a)+2*sum( (/ (f(x(i)),i=1,n-1) /) )+f(b))
end function repeated_trapezoid_quadrature
```

误差分析:

$$E_{2n} = I - T_{2n} = \frac{T_{2n} - T_n}{3}$$

```
(repeated_trapezoid_quadrature(f,a,b,h)-
repeated_trapezoid_quadrature(f,a,b,2*h))/3
```

输出结果:



分析：

> 可以看出`Repeated Trapezoid Quadrature`相比`Repeated Simpson Quadrature`方法求得的结果精度较低，误差较大。