Machine Learning and AI

- Methods and Algorithms -

Personnal Notes

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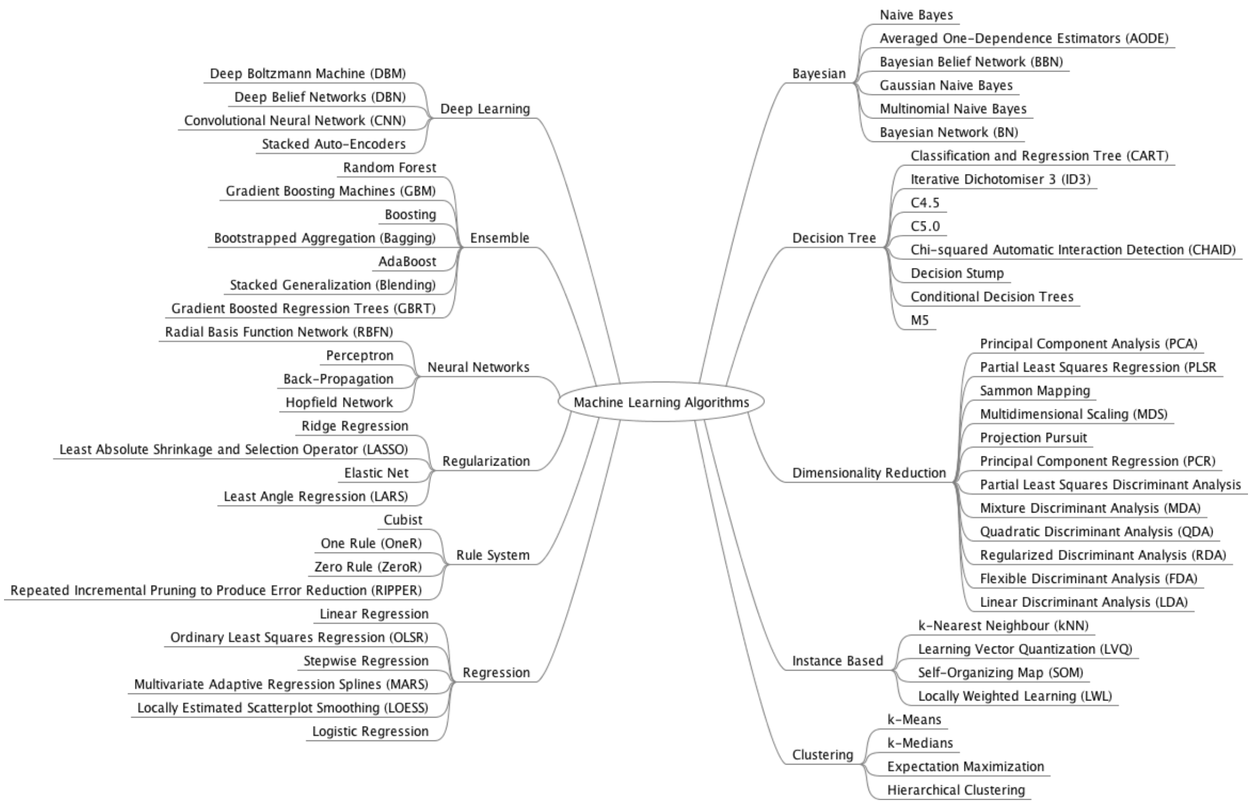
Contents

Todo list

Intro

This document will use the following classification for the machine learning algorithms. However their might be some changes. For exemple, some of them will be part of the commons algorithms and not from their real class.



Machine Learning Algorithms
Deep Learning
Deep Boltzmann Machine (DBM)
Deep Belief Networks (DBN)
Convolutional Neural Network (CNN)
Stacked Auto-Encoders
Ensemble
Random Forest
Gradient Boosting Machines (GBM)
Boosting
Bootstrapped Aggregation (Bagging)
AdaBoost
Stacked Generalization (Blending)
Gradient Boosted Regression Trees (GBRT)
Neural Networks
Radial Basis Function Network (RBFN)
Perceptron
Back-Propagation
Hopfield Network
Regularization
Ridge Regression
Least Absolute Shrinkage and Selection Operator (LASSO)
Elastic Net
Least Angle Regression (LARS)
Rule System
Cubist
One Rule (OneR)
Zero Rule (ZeroR)
Repeated Incremental Pruning to Produce Error Reduction (RIPPER)
Regression
Linear Regression
Ordinary Least Squares Regression (OLSR)
Stepwise Regression
Multivariate Adaptive Regression Splines (MARS)
Locally Estimated Scatterplot Smoothing (LOESS)
Logistic Regression
Bayesian
Naive Bayes
Averaged One-Dependence Estimators (AODE)
Bayesian Belief Network (BBN)
Gaussian Naive Bayes
Multinomial Naive Bayes
Bayesian Network (BN)
Decision Tree
Classification and Regression Tree (CART)
Iterative Dichotomiser 3 (ID3)
C4.5
C5.0
Chi-squared Automatic Interaction Detection (CHAID)
Decision Stump
Conditional Decision Trees
M5
Dimensionality Reduction
Principal Component Analysis (PCA)
Partial Least Squares Regression (PLSR
Sammon Mapping
Multidimensional Scaling (MDS)
Projection Pursuit
Principal Component Regression (PCR)
Partial Least Squares Discriminant Analysis
Mixture Discriminant Analysis (MDA)
Quadratic Discriminant Analysis (QDA)
Regularized Discriminant Analysis (RDA)
Flexible Discriminant Analysis (FDA)
Linear Discriminant Analysis (LDA)
Instance Based
k-Nearest Neighbour (kNN)
Learning Vector Quantization (LVQ)
Self-Organizing Map (SOM)
Locally Weighted Learning (LWL)
Clustering
k-Means
k-Medians
Expectation Maximization
Hierarchical Clustering

Figure 1 – Simple graph for algorithms classification in ML

Chapter 1

Common Machine Learning algorithms

This chapter is dedicated to the most common ML algorithms, a major part of the notes come from the mml-books.com

Add bibtex reference

Find better paragraph layout

1.1 Linear Regression

1.1.1 Maximum Likelihood Estimation (MLE)

Closed-Form Solution

In some cases, a closed-form solution exist, which make computation easy (but not necesseraly cheap)

Maximum A Posteriori Estimation (MAP)

1.2 Gradient Descent

1.2.1 Simple Gradient Descent

1.2.2 Gradient Descent with Momentum

1.2.3 Stochastic Gradient Descent

1.3 Model Selection and Validation

1.3.1 Cross-Validation

1.3.2 Marginal Likelihood

1.4 Bayesian Linear Regression

1.4.1 Mean and Variance

1.4.2 Sample function

Chapter 2

Reinforcement Learning

2.1 Markov Reward and Decision Process

2.1.1 State Value Function Closed-form

For a Markov Reward Process $(\mathcal{S}, \mathcal{P}, \mathcal{R}, \gamma)$, defining the Return R_t and the State Value Function $\mathsf{v}(s) = \mathbb{E}[\mathsf{R}_t S_t = s]$
Then we have, in a vector form :

$$\mathbf{v} = (\mathbb{1} - \gamma\mathcal{P})^{-1}\mathcal{R}$$

Unfortunately, Matrix inversion in costly, so this is only feasible in small Markov Reward Process

2.1.2 Iterative Policy Evaluation Algorithm

2.2 Dynamic Programming in RL

2.2.1 Policy Iteration Algorithm

2.2.2 Value Iteration Algorithm

2.2.3 Assynchronous Backup in RL

Prioritised Sweeping

Real-time Dynamic Programming

2.2.4 Properties and drawbacks of Dynamic Programming

2.3 Model-Free Learning

2.3.1 Monte-Carlo Algorithms

(First Visit) Monte-Carlo Policy Evaluation

Add "you cannot backup death" explanations

Every Visit Monte-Carlo Policy Evaluation

Batch vs Online Monte-Carlo

Incremental Monte-Carlo Update

Runing Mean for Non-Stationnary World

2.3.2 Monte-Carlo Control Algorithms

Monte-Carlo Policy Improvement

Greedy Policy Improvement over State Value Function

Greed Policy Improvement over State-Action Value Function

Don't forget Starting to explore

Exploring Starts Problem

On Policy Soft Control

On-Policy ϵ-greedy first-visit Monte-Carlo control Algorithm

Monte-Carlo Batch Learning to Control

Monte-Carlo Iterative Learning to Control

2.3.3 Temporal Difference Learning

Add Comparison between MC and TD learning

Temporal Difference Value Function Estimation Algorithm

2.3.4 Temporal Difference Learning Control Algorithm

SARSA - On Policy learning Temporal Difference Control: Here is the Sarsa Algorithm :

```
Data: State $\mathcal{S}$, Action $\mathcal{A}$, Reward $\mathcal{R}$ and Discount $\gamma$
Result: The optimal Q(S, A) State-Action Value Function and a greedy
        policy w.r.t Q
Initialise Q(s, a) $\forall a, s$ with Q(terminal state, a) = 0 ;
while Convergence condition (number of epoch, $\Delta \leq$ threshold, ...) do
    Initialise a state S;
    Choose action A from S with $\epsilon$-greedy policy derived from Q;
    while S is not a terminal State do
        Take action A, observe reward R and next state S';
        Choose action A' from S' with $\epsilon$-greedy policy derived from Q;
        Update $Q(S,A) \leftarrow Q(S,A) + \alpha(R + \gamma Q(S',A') - Q(S,A))$;
        $S \leftarrow S', A \leftarrow A'$
    end
end
Return Q and $\pi$ the derived policy
```

Algorithm 1: SARSA algorithm with ϵ-greedy policy

Theorem 1
Convergence of Sarsa
$Q(s,a) \to Q^{\infty}(s,a)$ under :

- *GLIE (Greedy in the Limite with infinite exploration), which mean every state is visited infinitely many times and that the policy converge toward a greedy-policy (ex: ϵ-greedy with $\epsilon \to 0$).*
- *Robbins-Monroe sequence of step-sizes α_t : which imply $\sum \alpha_t$ diverge and $\sum \alpha_t^2$ converge.*

Remark 1
the ϵ-greedy policy can be replaced by any policy derived from Q. (Because Q is the one updated by the algorithm)

SARSA-Lambda

Hindsight Experience Replay

Q-Learning: Off-Policy Temporal Difference Learning

2.4 Reinforcement Learning with Function Approximation

2.4.1 Exemple of features

Coarse Coding

Tile Coding

Radial-Basis Function

Deep Learning

2.4.2 Monte-Carlo with Value Function Approximation

2.4.3 Temporal Difference Learning with Value Function Approximation

2.4.4 Q-Learning with FA

2.4.5 subsection name

2.5 Deep Learning Reinforcement Learning

2.5.1 Experience Replay

2.5.2 Target Network

2.5.3 Clipping of Rewards

2.5.4 Skipping of Frames

Chapter 3

Dimensionality Reduction and Feature Extraction

3.1 Principal Component Analysis

The objective of PCA is to find a set of features, via linear projections, that maximise the variance of the sample data. We need to decide how many dimension d we want to keep.

3.1.1 Simple algorithm

```
Data: Vectors $x_i$ of centered data, number F features and n sample. Dimension
      d of reduction.
Result: $Y$ of size $d \times n$
Compute the product matrix of centered data : $\mathbf{XX}^\top$;
Compute the Eigen Analysis $XX^\top = V\Lambda V^\top$ ;
Order the Eigen Value by descending value, and permute Column of V
  correspondly;
Compute the eigenvectors : $U = XV\Lambda^{-1/2}$;
Keep specific number of first components : $U_d$ the d first d column of U ;
Compute the new features vectors : $Y = U_d^\top X$
```

Algorithm 2: Simple PCA Algorithm

3.1.2 Whitening PCA

The feature given by the PCA algorithm are un-correlated, but the variance in each dimension are not the same (in fact this are the eigen value of $XX^\top$). We can whitening the *features (or "sphering" them) by making the covariance matrix equal to Identity.*

```
Data: Vectors $x_i$ of centered data, number F features and n sample. Dimension
      d of reduction.
Result: $Y$ of size $d \times n$ with Identity Covariance Matrix
Compute the PCA of $X$ and keep $U$ and $\Lambda$;
Compute the Eigen Analysis $XX^\top = V\Lambda V^\top$ ;
Compute the whitened features vectors : $Y = U_d\Lambda^{-1/2}X = (XV\Lambda^{-1})_d X$
```

Algorithm 3: Whitened PCA Algorithm

3.1.3 Kernel PCA

Sometimes we want to compute non-linear features extractions. We use the kernel method to compute this. For a dataset $X = (x_i)_{1...n}$ we know only the kernel matrix $K = [\phi(x_i)\phi(x_j)^\top]_{(1...n)^2} = X^\phi X^{\phi^\top}$ with ϕ the non-linear mapping.

```
Data: Vectors $x_i$ of centered data, number F features and n sample. Dimension
      d of reduction. K the kernel matrix
Result: $Y$ of size $d \times n$
Compute the Eigen Analysis $K = \mathbf{X}^\phi \mathbf{X}^{\phi^\top} = V\Lambda V^\top$ ;
Order the Eigen Value by descending value, and permute Column of V
  correspondly;
Keep specific number of first components : $V_d$ the d first d column of V ;
Compute the vector $g(x_t) = [k(x_i, x_t)]_{1..n}]$ ;
Compute the $E = \frac{1}{n}\mathbf{1}\mathbf{1}^\top$ matrix ;
Compute the new features vectors : $y_t = \Lambda^{-1/2}V^\top(I - E)\left(g(x_t) - \frac{1}{n}K\mathbf{1}\right)$
```

Algorithm 4: Kernel PCA Algorithm

Chapter 4

Useful Computation

4.1 Data Centering using Matrix Multiplication

$$\boldsymbol{X} - \boldsymbol{M} = \boldsymbol{X}\left(\boldsymbol{I}_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^\top\right)$$