Homework 31

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1 Problem 59

First, we must show that our problem, which we call MAX-LSAT (maximum satisfiable linear equations), can be approximated. We already know that MAXSAT has an approximation algorithm to determine the maximum number of satisfiable clauses. Thus, we can convert the input equations of MAX-LSAT to meet the requirements of the MAXSAT algorithm, then apply the MAXSAT algorithm on these new equations.

We can convert a linear equation into a "boolean" equation by taking each term (normalized to integers and finding the mod 2 of them. For example, the equation $1.5x + 2y + 2.5z = 10.5$ will normalize to $3x + 4y + 5z = 21$, which will convert into $1x + 0y + 1z = 1$, where additions are equivalent to the XOR operator. If this new equation is satisfiable (with a boolean assignment to each variable), then the original equation is satisfiable.

Since there is a set of boolean equations over n variables, there are 2^n possible assignments of these boolean variables. Therefore, MAX-LSAT has an algorithm which decides the maximum number of linear equations which are simultaneously satisfiable.

Now, we must prove approximating MAX-LSAT within a factor of some constant $\rho < 1$ is NP-hard. To do this, we perform a gap reduction for our problem. MAXSAT already has a gap reduction proof to show it is NP-hard for a ρ-approximation. This proof went as such:

Let V be some $(\log n, 1)$-restricted verifier for some language L such that V takes as input I, some random bits R, and advice A. In the case of a yes instance we must accept. In other words, if $\forall R \exists A$ such that $V(I, R, A) = 1$, we must construct a system of linear equations f such that they are all satisfiable. In the case of a no instance, we must reject, or if (for most)$R \forall A$ such that $V(I, R, A) = 0$, we must construct a system of linear equations f such that its maximum number of satisfiable equations is $< \rho n$.