Homework 5

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January 24, 2018

1 Problem 7

1.1 Part A

Show one valid computation history H for the input $I = 101$ for Turing machine M.

$$\# > q_0 101\_\# > 1q_1 01\_\# > 11q_1 1\_\# > 110q_0\_\# > 110q_0\_\#...$$

1.2 Part B

Give the Godel sentence S for the M and I in the previous subproblem.
$\exists H|$ H is a computation history that proves M halts on I.

H is a valid computation history that proves M halts on I iff

1. First configuration is the starting configuration
2. Each configuration follows the previous one
3. Final configuration is the halting configuration

where H can be represented as a number of base B where B is the size of the tape alphabet plus the number of final states in set Q + the pound sign.
where

1) $\exists l\ l = log_B H$ where

$$PLACE(l) = \# \wedge$$
$$PLACE(l-1) => \wedge$$
$$PLACE(l-2) = q_0 \wedge$$
$$\forall i PLACE(i) \in I \mid l-2 > i > NEXT(l-2, \#)$$

2) $\forall q\ STATE(q)\ q < log_B H - 3$
$TABLE(PREVS(q) - 1 - PREV(PREVS(q), \#) + PREV(q, \#),$
$PREVS(q) - 1)$

3) $\exists p\ PLACE(p) = q_h\ \forall q\ STATE(q) \mid q \geq p$

PLACE(l) $\iff$ $H \ mod \ B^{P+1} \ div \ B^P$

STATE(S) $\iff$ $PLACE(S) = q_0 \vee PLACE(S) = q_1 \vee PLACE(S) = q_h$

NEXT(i,s) $\iff$ $\exists j \ PLACE(j) = s \mid j < i \mid \neg \exists k \ PLACE(k) = s \ k < j < i$

PREV(i,s) $\iff$ $\exists j \ PLACE(j) = s \mid j > i \mid \neg \exists k \ PLACE(k) = s \ i < j < k$

NEXTS(i) $\iff$ $\exists s \ STATE(s) \wedge NEXT(i,s)$

PREVS(i) $\iff$ $\exists s \ STATE(s) \wedge PREV(i,s)$

TABLE(i,j) $\iff$ $PLACE(i) = PLACE(j) \wedge$

$PLACE(i+1) = q_0 \wedge PLACE(i+2) = 0 \wedge PLACE(j+1) = 0 \wedge PLACE(j+2) = q_0$

$PLACE(i+1) = q_0 \wedge PLACE(i+2) = 1 \wedge PLACE(j+1) = 1 \wedge PLACE(j+2) = q_1$

$PLACE(i+1) = q_0 \wedge PLACE(i+2) = \_ \wedge PLACE(j+1) = q_0 \wedge PLACE(j+2) = \_$

$PLACE(i+1) = q_1 \wedge PLACE(i+2) = 0 \wedge PLACE(j+1) = 1 \wedge PLACE(j+2) = q_1$

$PLACE(i+1) = q_1 \wedge PLACE(i+2) = 1 \wedge PLACE(j+1) = 0 \wedge PLACE(j+2) = q_0$

$PLACE(i+1) = q_1 \wedge PLACE(i+2) = \_ \wedge PLACE(j+1) = q_h \wedge PLACE(j+2) = \_$

By these three statements the Godel sentence is true iff H is a valid computation history.