Homework 17

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1 Problem 27

> Describe a real number ρ such that given a random coin that comes up Heads with probability ρ, a Turing machine can decide an undecidable language in polynomial time.

Using the proof for Lemma 7.12:

Assume there exists a polynomial method of calculating the ith digit of an irrational number. Construct a Turing Machine such that every ith step, it flips a coin and compares it to the ith digit in the binary representation of an irrational number ρ, with a 0 being tails and a 1 being heads. If it is equal, it continues. If it is less, it rejects and halts.

The expected running time for this TM = $\Sigma_i i^c \frac{1}{2^i}$, which is bounded by polynomial time for any $c > 0$. Since this expected runtime is polynomial, the runtime for an irrational number $x : 0 < x < 1$ is polynomial time. The probability it accepts is equal to $\Sigma_i p_i \frac{1}{2^i}$, which is equal to ρ, the irrational number in the TM. Thus, there is a polynomial time TM that decides whether a number is irrational.

This TM could be used to decide whether an input is equal to an irrational number ρ. An example is where $\rho = \frac{1}{\sqrt{2}}$. The probability it accepts is equal to ρ. The expected running time is $\Sigma_i i^c \frac{1}{2^i}$, a polynomial time. Since it is impossible to compare equal irrational numbers in polynomial time, the initial assumption must be wrong.

2 Problem 28

(NP $\cup$ CoNP) $\subseteq$ PP

NP $\subseteq$ PP
NP accepts or rejects in polynomial time.
Its probability of being right = 1. Thus, NP $\subseteq$ PP

CoNP $\subseteq$ PP
CoNP accepts or rejects in polynomial time.
Its probability of being right = 1. Thus, CoNP $\subseteq$ PP.