Homework 19

Austin Frownfelter Matthew Bialecki

February 26, 2018

1 Problem 29

Show that if $\overline{3SAT} \leq_r 3SAT$ then PH collapses to Σ_3^p

Informal proof:

Assume there is a randomized reduction from $\overline{3SAT}$ to 3SAT. Then there exists a function which maps $\overline{3SAT}$ to 3SAT with some probability of correctness $\geq \frac{2}{3}$.

To make this a certain function (with probability = 1), expand the function to a Σ_3^p function.

This function must take Σ_3^p or harder to decide. Therefore, it is Σ_3^p-complete.

This Σ_3^p function is closed under its complement. Therefore, $\Sigma_3^p = \Pi_3^p$. Therefore, PH collapses to Σ_3^p.

$L \in \overline{3SAT}$

$L \leq_r L'$

$L' \in$ 3SAT

Define a PTM M such that if $x \in L$ most advice A will cause M(x,A) to accept, and if $x \notin L$ most advice A will cause M(x,A) to reject.

There exists a TM N:

$x \in L$: $\forall a \exists w \forall A N(x, a, w, A)$ accepts

$x \notin L$: $\exists a \forall w \exists A N(x, a, w, A)$ rejects

For $\overline{3SAT}$, N(x,a,w,A) runs M(x,A), M(x,a), M(x,a$\oplus w_1$), ..., M(x,a$\oplus w_k$) and accepts if 1 of them accepts and rejects if all reject.

For $3SAT$, N(x,a,w,A) runs M(x,A), M(x,a), M(x,a$\oplus w_1$), ..., M(x,a$\oplus w_k$) and accepts if all reject and rejects if 1 of them accepts.

This shows if there exists a randomized reduction from $\overline{3SAT}$ to 3SAT, then the resulting language is Σ_3^p-complete. Also, it is closed under its complement (the complement is the negation of the quantifiers.) Thus it is also Π_3^p-complete. Therefore, the Polynomial Hierarchy would collapse down to Σ_3^p.