Homework 27

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1 Problem 49

$|\mathbf{v}\rangle = \alpha_{00}|00\rangle + \alpha_{10}|10\rangle + \alpha_{01}|01\rangle + \alpha_{11}|11\rangle$

1.1 Part a

$P(v = 00) = (\alpha_{00})^2$
$P(v = 10) = (\alpha_{10})^2$
$P(v = 01) = (\alpha_{01})^2$
$P(v = 11) = (\alpha_{11})^2$

In the case where the probability of a qubit being 0 or 1 is uniform, all probabilities are $\frac{1}{4}$

1.2 Part b

$$|v_0v_1\rangle = \sqrt{(\alpha_{00})^2 + (\alpha_{01})^2}\,|0v_1\rangle + \sqrt{(\alpha_{10})^2 + (\alpha_{11})^2}\,|1v_1\rangle =$$
$$\sqrt{(\alpha_{00})^2 + (\alpha_{01})^2}\left(\frac{\alpha_{00}}{\sqrt{(\alpha_{00})^2+(\alpha_{01})^2}}|00\rangle + \frac{\alpha_{01}}{\sqrt{(\alpha_{00})^2+(\alpha_{01})^2}}|01\rangle\right)+$$
$$\sqrt{(\alpha_{10})^2 + (\alpha_{11})^2}\left(\frac{\alpha_{10}}{\sqrt{(\alpha_{10})^2+(\alpha_{11})^2}}|10\rangle + \frac{\alpha_{11}}{\sqrt{(\alpha_{10})^2+(\alpha_{11})^2}}|11\rangle\right)$$
$$= \alpha_{00}|00\rangle + \alpha_{10}|10\rangle + \alpha_{01}|01\rangle + \alpha_{11}|11\rangle$$

Thus,

$P(v = 00) = (\alpha_{00})^2$
$P(v = 10) = (\alpha_{10})^2$
$P(v = 01) = (\alpha_{01})^2$
$P(v = 11) = (\alpha_{11})^2$

1.3 Part c

$$|v_0v_1\rangle = \sqrt{(\alpha_{00})^2 + (\alpha_{10})^2}\,|v_00\rangle + \sqrt{(\alpha_{01})^2 + (\alpha_{11})^2}\,|v_01\rangle =$$
$$\sqrt{(\alpha_{00})^2 + (\alpha_{10})^2}\left(\frac{\alpha_{00}}{\sqrt{(\alpha_{00})^2+(\alpha_{10})^2}}|00\rangle + \frac{\alpha_{10}}{\sqrt{(\alpha_{00})^2+(\alpha_{10})^2}}|10\rangle\right)+$$
$$\sqrt{(\alpha_{01})^2 + (\alpha_{11})^2}\left(\frac{\alpha_{01}}{\sqrt{(\alpha_{01})^2+(\alpha_{11})^2}}|01\rangle + \frac{\alpha_{11}}{\sqrt{(\alpha_{01})^2+(\alpha_{11})^2}}|11\rangle\right)$$

$= \alpha_{00} |00\rangle + \alpha_{10} |10\rangle + \alpha_{01} |01\rangle + \alpha_{11} |11\rangle$

Thus,
$P(v = 00) = (\alpha_{00})^2$
$P(v = 10) = (\alpha_{10})^2$
$P(v = 01) = (\alpha_{01})^2$
$P(v = 11) = (\alpha_{11})^2$

2 Problem 50

2.1 Part a

$x = y = 1$

$|ij\rangle = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle$ - This is because the particles are entangled.

Alice rotates by $\frac{\pi}{8}$. This can be represented with a transformation matrix.

After rotating, $|ij\rangle = \frac{1}{\sqrt{2}}(\cos\frac{\pi}{8} |00\rangle + \sin\frac{\pi}{8} |10\rangle) + \frac{1}{\sqrt{2}}(\cos\frac{5\pi}{8} |01\rangle + \sin\frac{5\pi}{8} |11\rangle)$. Each amplitude is the corresponding value in the transformation. Note that the probabilities (square of amplitudes) sum to 1.

Bob rotates by $-\frac{\pi}{8}$. This can also be represented with a transformation matrix.

After rotating, $|ij\rangle = \frac{1}{\sqrt{2}}(\cos\frac{\pi}{8} |00\rangle - \sin\frac{\pi}{8} |10\rangle) + \frac{1}{\sqrt{2}}(\cos\frac{3\pi}{8} |01\rangle + \sin\frac{3\pi}{8} |11\rangle)$. Each amplitude is the corresponding value in the transformation. Note that the probabilities (square of amplitudes) sum to 1.

Alice measures her entangled particle.

$P(0) = (\frac{1}{\sqrt{2}}\cos\frac{\pi}{8})^2 + (\frac{1}{\sqrt{2}}\cos\frac{5\pi}{8})^2 = \frac{1}{2}$. This is the sum of the probabilities Alice's qubit is 0, where Bob's is 0 or 1.
$P(1) = (\frac{1}{\sqrt{2}}\sin\frac{\pi}{8})^2 + (\frac{1}{\sqrt{2}}\sin\frac{5\pi}{8})^2 = \frac{1}{2}$. This is the sum of the probabilities Alice's qubit is 1, where Bob's is 0 or 1.

Bob measures his entangled particle.

When Alice reads a 0, Bob's qubit is in the following state:
$|0b\rangle = \frac{\frac{1}{\sqrt{2}}\cos\frac{\pi}{8}}{\sqrt{(\frac{1}{\sqrt{2}}\cos(\frac{\pi}{8}))^2 + (\frac{1}{\sqrt{2}}\cos(\frac{3\pi}{8}))^2}} |00\rangle + \frac{\frac{1}{\sqrt{2}}\cos\frac{3\pi}{8}}{\sqrt{(\frac{1}{\sqrt{2}}\cos(\frac{\pi}{8}))^2 + (\frac{1}{\sqrt{2}}\cos(\frac{3\pi}{8}))^2}} |01\rangle$

When Alice reads a 1, Bob's qubit is in the following state:
$|0b\rangle = \frac{\frac{1}{\sqrt{2}}\sin -\frac{\pi}{8}}{\sqrt{(\frac{1}{\sqrt{2}}\sin(\frac{\pi}{8}))^2 + (\frac{1}{\sqrt{2}}\sin(\frac{3\pi}{8}))^2}} |00\rangle + \frac{\frac{1}{\sqrt{2}}\sin\frac{3\pi}{8}}{\sqrt{(\frac{1}{\sqrt{2}}\sin(\frac{\pi}{8}))^2 + (\frac{1}{\sqrt{2}}\sin(\frac{3\pi}{8}))^2}} |01\rangle$

Thus,
$P(0|\text{Alice} = 0) = \frac{\frac{1}{2}\cos^2\frac{\pi}{8}}{\frac{1}{2}} = \cos^2\frac{\pi}{8} \approx 0.85355$
$P(0|\text{Alice} = 1) = \frac{\frac{1}{2}\sin^2\frac{\pi}{8}}{\frac{1}{2}} = \sin^2\frac{\pi}{8} \approx 0.14645$

$P(1|\text{Alice} = 0) = \frac{\frac{1}{2}\cos^2\frac{3\pi}{8}}{\frac{1}{2}} = \cos^2\frac{3\pi}{8} \approx 0.14645$
$P(1|\text{Alice} = 1) = \frac{\frac{1}{2}\sin^2\frac{3\pi}{8}}{\frac{1}{2}} = \sin^2\frac{3\pi}{8} \approx 0.85355$

Thus, the resulting probability Alice and Bob win when $x = y = 1 \approx 0.14645$

2.2 Part b

The protocols appear to be the same. Therefore, the probabilities should be equal.

When $x = y = 0$ the probability they win is 1.

When $x \neq y$, the probability they win ≈ 0.85355 (when $a = b$). This is taken from the calculation above, which is equivalent to the book.

When $x = y = 1$, the probability they win is ≈ 0.14645 (when $a \neq b$). This is taken from the calculation above, which should be equivalent to the book.

The resulting probability they win is $\frac{1}{4} * 1 + \frac{1}{2} * 0.8535 + \frac{1}{4} * 0.1465 \approx 0.71$
This is less than the probability in the book, which means the math done in part a for us was incomplete. The 0.1465 should be 0.5. However, the probability they win in the lecture's protocol is equal to the book's protocol, or 0.8.