Homework 3

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1 Problem 4

1.1 Part A

> "Show by reduction from the Halting Problem that there is no Turing machine that takes as input a Turing machine M, and determines whether the language L(M) accepted by M is the empty language."

Assume there is a decider W for the problem described above. Also assume there is a Turing machine Halting that takes as input a Turing machine P and string I. This machine constructs a Turing machine W that takes input x. If x≠I, W rejects. If x=I, W runs P on input I. If P accepts or rejects, W rejects. Halting then runs M on input W. If M accepts, Halting accepts. If M rejects, Halting rejects.

P halts on I iff M accepts no strings. Since Halting is undecidable, it is impossible for M to be a decider. Therefore, the problem above is undecidable.

1.2 Part B

> "Show by reduction from the Halting Problem that there is no Turing machine that takes as input a Turing machine M, and determines whether the language L(M) accepted by M is the language of every string over the input alphabet."

Assume there is a decider W for the problem described above. Also assume there is a Turing machine Halting that takes as input a Turing machine P and string I. This machine constructs a Turing machine W that takes input x. W runs P on I, then accepts. Halting then runs M on input W. If M accepts, Halting accepts. If M rejects, Halting rejects.

P halts on I iff M accepts every string. Since Halting is undecidable, it is impossible for M to be a decider. Therefore, the problem above is undecidable.

1.3 Part C

> "Show by reduction from the Halting Problem that there is no Turing machine that takes as input a Turing machine M, and deter-

> mines whether the language L(M) accepted by M includes the string 11110."

Assume there is a decider W for the problem described above. Also assume there is a Turing machine Halting that takes as input a Turing machine P and string I. This machine constructs a Turing machine W that takes input x. W runs P on I. If P accepts or rejects I, W accepts if x=11110, otherwise it rejects. Halting then runs M on input W. If M accepts, Halting accepts. If M rejects, Halting rejects.

P halts on I iff M accepts every string. Since Halting is undecidable, it is impossible for M to be a decider. Therefore, the problem above is undecidable.

1.4 Part D

> "Let P be some property of languages. Further assume there is a Turing machine M_1 that accepts a language L_1 that has property P, and a Turing machine M_2 that accepts a language L_2 that does not have property P. Show by reduction from the Halting Problem that there is no Turing machine that takes as input a Turing machine M, and determines whether the language L(M) accepted by M satisfies property P."

Assume there is a decider W for the problem described above. Also assume there is a Turing machine Halting that takes as input a Turing machine T and string I. This machine constructs a Turing machine W that takes input x. W runs T on I. If T accepts or rejects I, W accepts if the property P is satisfied, otherwise it rejects. Halting then runs M_1 on input W. If M_1 accepts, Halting accepts. If M_1 rejects, Halting rejects.

T halts on I iff M satisfies the property P. Since Halting is undecidable, it is impossible for M to be a decider. Therefore, the problem above is undecidable.

1.5 Part E

> "Explain why the first three subproblems are consequences of the fourth subproblem."

Part D is a generic form of parts A-C. Each problem is defined by its individual property.