Homework 29

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1 Problem 53

1.1 Part a

Alice can do rotations, specifically $\frac{\pi}{2}$ rotations, based on values x, y:
If $x = y = 0$, do not rotate.
If $x = 1, y = 0$, rotate a by $\frac{\pi}{2}$.
If $x = 1, y = 1$, rotate a by π.
If $x = 0, y = 1$, rotate a by $3\frac{\pi}{2}$.

1.2 Part b

The general form for the process described above:
$|a\rangle = \frac{1}{\sqrt{2}}[-2(x - \frac{1}{2})|0\rangle - 2(y - \frac{1}{2})|1\rangle]$
$|b\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

1.3 Part c

In the case of the values (x, y), the qubits are in the following states:
$(0, 0)$: $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$
$(0, 1)$: $\frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$
$(1, 0)$: $\frac{1}{2}(-|00\rangle - |01\rangle + |10\rangle + |11\rangle)$
$(1, 1)$: $\frac{1}{2}(-|00\rangle - |01\rangle - |10\rangle - |11\rangle)$

If an H_2 operation is applied, the following states remain:
$(0, 0)$: $|00\rangle$
$(0, 1)$: $|10\rangle$
$(1, 0)$: $-|10\rangle$
$(1, 1)$: $-|00\rangle$
There is no way to distinguish the values of x, y other than whether $x = y$.

If an H_1 operation is applied to a, the following states remain:
$(0, 0)$: $\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$

$(0,1)$: $\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$

$(1,0)$: $-\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$

$(1,1)$: $-\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$

As before, there is no way to distinguish the values of x, y other than whether $x = y$.

We were unable to come up with a process that would give us a way for Bob to be certain the values x, y. We believe it must involve H_1 operations with measurements, perhaps with something we are missing.