Homework 11

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1 Problem 16

> "Show that if there is a log space Turing machine S that accepts B, then there is a log space Turing Machine U that accepts A."

There is a TM T which its output is in B iff $x \in A$. If there is a log space TM S that accepts B, then we can construct a TM U that accepts A:

U inputs x, and it must run T on x to get its output, which is the input to S to decide if it is in B. If it is in B, then x is in A.

We can simulate T on x until it outputs its first symbol, which is then fed into S. T maintains its work tape, along with its current output, on the work tape of U. Once T has output its first symbol, S is simulated until it requires another output symbol of T. This required symbol's index i is stored on the work tape of U, along with the work of S. U then simulates T until it reaches its ith output, which is stored on the work tape. S is then simulated from where it left off until it requires another output of T, where the process is then repeated. U continues this process until S either accepts or rejects, which U will then accept or reject, respectively.

U has a work tape which stores the individual work tapes of S and T. It also has to store the state of S when it runs T such that it can continue S later, along with the ith output of T. In total, the work tape of U uses $2 * log(x) + c$ where c is the constants which are the state of S and the ith output of T.

Therefore, if there is a log space TM S that accepts B, there exists a log space TM U that accepts A.

2 Problem 17

2.1 Part a

> "Define a language C, and show that C is complete for EXPSPACE under polynomial time reductions."

$C = \{(M, I, 2^{n^k}) \mid M \text{ accepts } I \text{ in space } 2^{n^k}\}$

C is complete for EXPSPACE under polynomial time reductions.

1) $C \in$ EXPSPACE

There exists a TM N(TM M, input I, 2^{n^k}) that decides C:

Run M on I for 2^{n^k} steps

If M accepted or rejected, accept or reject respectively

Otherwise, reject

N decides C in EXPSPACE, therefore $C \in$ EXPSPACE.

2) $\forall L \in$ EXPSPACE, $L \leq_p C$

Let $L \in$ EXPSPACE

$\exists$ TM M, integer $k \mid M$ accepts x iff $x \in L$, M on x uses space $\leq 2^{|x|^k}$.

$\exists$ TM A which constructs input $(M, I, 2^{|x|^k})$ and runs a decider for C on it. If the decider accepts, A accepts; otherwise it rejects.

M and I can be constructed in polynomial time. 2^{n^k} can be encoded using polynomial space in polynomial time.

2.2 Part b

"Define a language C, and show that C is complete for EXPSPACE under polynomial time reductions."

$C = \{(M, I, 1^{c^{n^k}}) \mid M \text{ accepts } I \text{ in space } c^{n^k}\}$

C is complete for EXPSPACE under polynomial time reductions.

1) $C \in$ EXPSPACE

There exists a TM N(TM M, input I, c^{n^k}) that decides C:

Run M on I for c^{n^k} steps

If M accepted or rejected, accept or reject respectively

Otherwise, reject

N decides C in EXPSPACE, therefore $C \in$ EXPSPACE.

2) $\forall L \in$ EXPSPACE, $L \leq_p C$

Let $L \in$ EXPSPACE

$\exists$ TM M, integer $k \mid M$ accepts x iff $x \in L$, M on x uses space $\leq c^{|x|^k}$.

$\exists$ TM A which constructs input $(M, I, c^{|x|^k})$ and runs a decider for C on it. If the decider accepts, A accepts; otherwise it rejects.

M and I can be constructed in polynomial time. c^{n^k} can be encoded using polynomial space in polynomial time.