Homework 28

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1 Problem 51

In the final observation in Simon's algorithm, we observe y in the first n bits iff $a \odot y \mod 2 = 0$. When $a = 0$, the inner product of a and y will always be zero. Thus, we will always observe y in the first n bits when $a = 0$.

In the case where $x = 0^n$ and $a = 0^n$, $y = 0^n$ and we will learn no bits of a $(0 = 0)$. In all other cases, we discover the sum of a_i bits where $y_i = 1$ is equal to zero. If the algorithm is run $2n$ times, there is an extremely high likelihood that we get n independent equations, allowing us to solve for all bits in a.

Therefore, if $a = 0^n$, Simon's algorithm still works.

2 Problem 52

2.1 Part a

We applied the Kroenecker product ($\otimes$) on H_1 and the basis (denoted as I):

$H_1 \otimes I = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right) \otimes \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right) =$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

2.2 Part b

In Part A we did $H_1 \otimes I$ because the H_1 is applied to the first qubit, and the second qubit remains the same. Thus, we took the Kroenecker product ($\otimes$) on the basis I and H_1:

$I \otimes H_1 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right) \otimes \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right) =$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

2.3 Part c

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix}$$

2.4 Part d

$H_2(a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle) =$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix}a & 0 & 0 & 0\\ 0 & b & 0 & 0\\ 0 & 0 & c & 0\\ 0 & 0 & 0 & d\end{bmatrix}=\frac{1}{2}\begin{bmatrix}a & b & c & d\\ a & -b & c & -d\\ a & b & -c & -d\\ a & -b & -c & d\end{bmatrix}=$$

$a\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)+$
$b\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)+$
$c\frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)+$
$d\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle) =$
$\frac{1}{2}[(a+b+c+d)\,|00\rangle+(a-b+c-d)\,|01\rangle+(a+b-c-d)\,|10\rangle+(a-b-c+d)\,|11\rangle)]$

2.5 Part e

$(H_1 \otimes I)\cdot(a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle) =$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{bmatrix}\cdot\frac{1}{2}\begin{bmatrix}a & 0 & 0 & 0\\ 0 & b & 0 & 0\\ 0 & 0 & c & 0\\ 0 & 0 & 0 & d\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}a & 0 & c & 0\\ 0 & b & 0 & d\\ a & 0 & -c & 0\\ 0 & b & 0 & -d\end{bmatrix}=$$

$a\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)+$
$b\frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)+$
$c\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)+$
$d\frac{1}{\sqrt{2}}(|01\rangle - |11\rangle) =$
$\frac{1}{\sqrt{2}}((a+c)\,|00\rangle + (b+d)\,|01\rangle + (a-c)\,|10\rangle + (b-d)\,|11\rangle)$

2.6 Part f

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix}a & 0 & 0 & 0\\ 0 & b & 0 & 0\\ 0 & 0 & c & 0\\ 0 & 0 & 0 & d\end{bmatrix}=\frac{1}{2}\begin{bmatrix}(a+c) & (b+d) & 0 & 0\\ (a+c) & -(b+d) & 0 & 0\\ 0 & 0 & (a-c) & (b-d)\\ 0 & 0 & (a-c) & -(b-d)\end{bmatrix}=$$

$\frac{1}{2}[(a+c)(|00\rangle + |01\rangle)+$
$(b+d)(|00\rangle - |01\rangle)+$

$(a-c)(|10\rangle + |11\rangle)+$
$(b-d)(|10\rangle - |11\rangle)] =$
$\frac{1}{2}[(a{+}b{+}c{+}d)\,|00\rangle{+}(a{-}b{+}c{-}d)\,|01\rangle{+}(a{+}b{-}c{-}d)\,|10\rangle{+}(a{-}b{-}c{+}d)\,|11\rangle)]$