

# Pontificia Universidad Católica del Perú - FCI

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November 9, 2017

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## 1 Centroid Decomposition

```
#define N 100002
```

```
inline ll ma(ll a, ll b){ return ((a-b>0)? a:b);}  
inline ll mi(ll a, ll b){return ((a-b>0)? b:a);}
```

```
struct CD{
```

```
vector< int > graph[N];  
int sub[N],p[N];  
//sub[i]: size del nodo i luego de descomponer el tree  
//p[i]: padre del nodo i luego de descomponer el tree  
//notar que el padre del centroid es -2  
// el tree esó 1 0 base  
//para inicializar addEddge(a,b);  
//para construir el centroid tree, solo llamar init(root); root:  
    root del tree  
void addEdge(int &a, int &b){  
    graph[a].pb(b);  
    graph[b].pb(a);  
}  
inline void dfs(int cur, int parent){  
    sub[cur] = 1;  
    for(int i = 0; i < sz(graph[cur]); ++i){  
        int to = graph[cur][i];  
        if(to != parent && p[to] == -1){  
            dfs(to, cur);  
            sub[cur] += sub[to];  
        }  
    }  
}  
inline void decompose(int cur, int parent, int sb, int prevc){  
    for(int i = 0; i < sz(graph[cur]); ++i){  
        int to = graph[cur][i];  
        if(to != parent && p[to] == -1 && (2 * sub[to] > sb))  
            ){  
            decompose(to, cur, sb, prevc);  
            return;  
        }  
    }  
    p[cur] = prevc;  
    for(int i = 0; i < sz(graph[cur]); ++i){
```

```

        int to = graph[cur][i];
        if(p[to] == -1){
            dfs(to, - 1);
            decompose(to, cur, sub[to], cur);
        }
    }
}

inline void init(int start){
    for(int i = 0; i < N; ++i) p[i] = -1;
    dfs(start, - 1);
    decompose(start, -1, sub[start], -2);
}

};
int cnt=1;
vi adj[N];
int d[N];
inline void make(int &u, int x, int depth){
    d[u]=depth;
    for(auto v : adj[u]) if(v!=x) make(v,u,depth+1);
}
int main() {
    fastio;
    int n; cin>>n;
    CD cd; //cd.n=n;
    REP(i,0,n-1) {
        ll a,b; cin>>a>>b;
        cd.addEdge(a,b);
    }
    cd.init(1);
    int pa, root;
    REP(i,1,n+1) {
        pa=cd.p[i];
        if(pa== -2) root=i;
        if(pa!= -2) {
            adj[i].pb(pa);
            adj[pa].pb(i);
        }
    }
    make(root,0,1);
    char is;map<int, string> m;int k=1,flag=1;
    for(is='A'; is<='Z'; is++) m[k++]=is;
    REP(i,1,n+1) if(d[i]>26) flag=0;
    if(flag==0) cout<<"Impossible!"<<endl;
    if(flag==1) {
        REP(i,1,n+1) cout<<m[d[i]]<<endl;

```

```

    }
    return 0;
}

```

## 2 Closest Pair

//Closest Pair Algorithm with Sweep  
//Complexity:  $O(n \log n)$

```

#define MAX_N 100000
#define px second
#define py first
typedef pair<long long, long long> point;

int N;
point P[MAX_N];
set<point> box;

bool compare_x(point a, point b){ return a.px<b.px; }

inline double dist(point a, point b){
    return sqrt((a.px-b.px)*(a.px-b.px)+(a.py-b.py)*(a.py-b.py));
}

double closest_pair(){
    if(N<=1) return -1;
    sort(P,P+N,compare_x);
    double ret = dist(P[0],P[1]);
    box.insert(P[0]);
    set<point> :: iterator it;
    for(int i = 1,left = 0;i<N;++i){
        while(left<i && P[i].px-P[left].px>ret) box.erase(P[left++]);
        for(it = box.lower_bound(make_pair(P[i].py-ret,P[i].px-ret));
            it!=box.end() && P[i].py+ret>=(*it).py;++it)
            ret = min(ret, dist(P[i],*it));
        box.insert(P[i]);
    }
    return ret;
}

```

### 3 Convex Hull Trick

```
// Simple Hull
struct HullSimple { // Upper envelope for Maximum.
    // Special case: strictly increasing slope in insertions,
    // increasing value in queries.
    deque<pair<ll, ll> > dq;
    ld cross(pair<ll, ll> l1, pair<ll, ll> l2){
        return (ld)(l2.snd - l1.snd) / (ld)(l1.fst - l2.fst);
    }
    void insert_line(ll m, ll b){
        pair<ll, ll> line = mp(m, b);
        while (sz(dq) > 1 && cross(line, dq[sz(dq)-1]) <=
            cross(dq[sz(dq)-1], dq[sz(dq)-2])) dq.pop_back();
        dq.pb(mp(m, b));
    }

    ll eval(pair<ll, ll> line, ll x){
        return line.fst * x + line.snd;
    }

    ll eval(ll x){
        while (sz(dq) > 1 && eval(dq[0], x) < eval(dq[1], x))
            dq.pop_front();
        return eval(dq[0], x);
    }
};

// Dynamic Hull
// Compile with g++ -std=c++11 file.cpp -o file
typedef long double ld;
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

// Upper envelope for Maximum
```

```
struct HullDynamic : public multiset<Line> {
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >=
            (y->b - z->b)*(y->m - x->m);
    }
    void insert_line(ll m, ll b) {
        auto y = insert({ m, b });
        y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
        if (bad(y)) { erase(y); return; }
        while (next(y) != end() && bad(next(y))) erase(next(y));
        while (y != begin() && bad(prev(y))) erase(prev(y));
    }
    ll eval(ll x) {
        auto l = *lower_bound((Line) { x, is_query });
        return l.m * x + l.b;
    }
};
```

### 4 Dates

```
//
// Time - Leap years
//
// A[i] has the accumulated number of days from months previous to i
const int A
    [13] = { 0, 0, 31, 59, 90, 120, 151, 181, 212, 243, 273, 304, 334 };
// same as A, but for a leap year
const int B
    [13] = { 0, 0, 31, 60, 91, 121, 152, 182, 213, 244, 274, 305, 335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y / 400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 == 0 && y % 100 != 0)
    ; }
// number of days in blocks of years
```

```

const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4 = 4*365 + 1;
const int p1 = 365;
int date_to_days(int d, int m, int y)
{
    return (y - 1) * 365 + leap_years(y - 1) + (is_leap(y) ? B[m] : A[m]) +
        d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
    bool top100; // are we in the top 100 years of a 400 block?
    bool top4;   // are we in the top 4 years of a 100 block?
    bool top1;   // are we in the top year of a 4 block?

    y = 1;
    top100 = top4 = top1 = false;

    y += ((days-1) / p400) * 400;
    d = (days-1) % p400 + 1;

    if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
    else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

    if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
    else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

    if (d > p1*3) top1 = true, d -= p1*3, y += 3;
    else y += (d-1) / p1, d = (d-1) % p1 + 1;

    const int *ac = top1 && (!top4 || top100) ? B : A;
    for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
    d -= ac[m];
}

```

## 5 Divide and Conquer Trick

```

// Divide and Conquer DP optimization.
// Problem: dp[i][j] = min{k>j} (func(j,k) + dp[i-1][k]).
// (That is, split n objects into k buckets with cost
// func per bucket). Necessary condition: argmin(dp[i][j]) <=
// argmin(dp[i][j+1]) (this is "opt")

```

```

// Naive complexity: O(kn^2)
// Improved complexity: O(knlog(n))
// Consider checking if opt[i+1][j] <= opt[i][j] <= opt[i][j+1]
// and using a knuth-like O(n^2) loop

```

```

const ll INF = 1e18;
int n, k;

ll c[8100];
ll s[8100];
ll dp[810][8100];

ll func(int i, int j){ return (s[j] - s[i])*(j-i); }

void go(int i, int l, int r, int optl, int optr){
    if (l >= r) return;
    int m = (l+r)/2;
    int opt = n;
    dp[i][m] = INF;
    for(int u = optr; u>= optl; u--){
        ll curr = dp[i-1][u] + func(m,u);
        if(curr < dp[i][m]){
            dp[i][m] = curr;
            opt = u;
        }
    }
    go(i,l,m,optl, opt);
    go(i,m+1,r,opt,optr);
}

int main(){
    fastio;
    cin >> n >> k;
    REP(i,0,n) cin >> c[i];
    s[0] = 0;
    REP(i,0,n+1) s[i] = s[i-1] + c[i-1];
    REP(i,1,k+1) dp[i][n] = INF;
    REP(i,0,n) dp[0][i] = INF;
    dp[0][n] = 0;
    REP(i,1,k+1) go(i,0,n,0,n);
    cout << dp[k][0] << endl;
    return 0;
}

```

```

//Divide and Conquer Trick by Ands

```

```

void compute(int cnt, int l, int r, int optl, int optr){
    if(l > r) return ;
    int mid = ( l + r ) >> 1 ;
    int opt = -1 ;
    ll value = 1e18 ;
    int last = cnt^1 ;
    for(int idx = optl ; idx <= min(mid-1,optr); ++idx){
        ll tmp = dp[last][idx] + C[idx][mid] ;
        if(tmp < value){
            value = tmp ;
            opt = idx ;
        }
    }
    dp[cnt&1][mid] = value ;
    compute(cnt, l, mid-1, optl, opt);
    compute(cnt, mid+1, r, opt, optr);
}

int main(){
    //casos base
    for(int cnt = 2; cnt <= m ; ++cnt) compute(cnt&1, 0, n-1, 0, n-1) ;
}

```

## 6 Fractions

```

struct Frac{
    int num, den;
    Frac(){
        num = 0; den = 1;
    }
    Frac(int a, int b): num(a), den(b){}
    Frac(int a):num(a), den(1){}

    void normalize(){
        if(num == 0){
            den = 1;
        }
        if(den < 0){
            den = -den;
            num = -num;
        }
    }
}

```

```

    }
}

Frac fix(int a, int b){
    if(!a) return Frac(0,1);
    if(!b) return Frac(oo,1);
    int foo = gcd(abs(a),abs(b));
    Frac ret = Frac(a/foo, b/foo);
    ret.normalize();
    return ret;
}

Frac operator + (const Frac& other){
    int num2 = num*other.den + den*other.num, den2 = den*other.den;
    return fix(num2,den2);
}

Frac operator - (const Frac& other){
    int num2 = num*other.den - den*other.num, den2 = den*other.den;
    return fix(num2,den2);
}

Frac operator * (int c){
    int num2 = num*c, den2 = den;
    return fix(num2,den2);
}

Frac operator * (const Frac& other){
    int num2 = num*other.num, den2 = den*other.den;
    return fix(num2,den2);
}

Frac operator / (int c){
    int num2 = num, den2 = den * c;
    return fix(num2,den2);
}

Frac operator / (const Frac& other){
    int num2 = num*other.den, den2 = den*other.num;
    return fix(num2,den2);
}

bool operator < (const Frac& other) const{
    if(num * other.den < other.num*den) return true;
    return false;
}

```

```

    }

    bool operator == (const Frac& other) const{
        if(num == other.num && den == other.den) return true;
        return false;
    }
};

```

## 7 Longest Increasing Subsequence

// Simple  $O(n \log n)$  Longest Increasing Subsequence  
 // Answer is stored in array b[N]

```

int LIS( vi &a ){
    int b[N];
    int sz = 0;
    REP(i,0,a.size()){
        int j = lower_bound( b , b + sz , a[ i ] ) - b;
        // (lower) a < b < c
        // (upper) a <= b <= c
        b[ j ] = a[ i ];
        if( j == sz ) sz++;
    }
    return sz;
}

```

## 8 Matrix Structure

```

const int MN = 111;
const int mod = 10000;

```

```

struct matrix {
    int r, c;
    int m[MN][MN];

    matrix (int _r, int _c) : r (_r), c (_c) {
        memset(m, 0, sizeof m);
    }
}

```

```

void print() {
    for (int i = 0; i < r; ++i) {
        for (int j = 0; j < c; ++j)
            cout << m[i][j] << " ";
        cout << endl;
    }
}

int x[MN][MN];
matrix & operator *=(const matrix &o) {
    memset(x, 0, sizeof x);
    for (int i = 0; i < r; ++i)
        for (int k = 0; k < c; ++k)
            if (m[i][k] != 0)
                for (int j = 0; j < c; ++j) {
                    x[i][j] = (x[i][j] + (m[i][k] * o.m[k][j]) % mod) % mod;
                }
    memcpy(m, x, sizeof(m));
    return *this;
}

};

void matrix_pow(matrix b, long long e, matrix &res) {
    memset(res.m, 0, sizeof res.m);
    for (int i = 0; i < b.r; ++i)
        res.m[i][i] = 1;

    if (e == 0) return;
    while (true) {
        if (e & 1) res *= b;
        if ((e >>= 1) == 0) break;
        b *= b;
    }
}

```

## 9 Ordered Set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef
tree<

```

```

    int,
    null_type,
    less<int>,
    rb_tree_tag,
    tree_order_statistics_node_update
>ordered_set;
// ordered_set
// X.find_by_order(k) returns an iterator to the k-th largest element (
    counting from zero)
// X.order_of_key(v) returns the number of items in a set that are
    strictly smaller than v
int main() {
    int N;
    ordered_set Y;
    Y.insert(5);
    trace (*Y.find_by_order(0));
}

```

## 10 Parallel Binary Search

```

//Cada query esta en (low[i], high[i]]
//Tocheck tiene los valores actuales a verificar
//en el bsearch

```

```

//Solved puede tener 1, -1
//1: el unico valor posible cumple
//-1: no hay respuesta

```

```

int low[MAXN];
int high[MAXN];
char solved[MAXN];
vector< int > tocheck[MAXN];

```

```

int main(){
    // Leer n, m
    // Leer a[i], b[i] (i en [1, m])
    // Leer q: queries
    // Leer x[i], y[i], z[i] (i en [0, q])

    for(int i = 0; i < q; ++i)
        low[i] = 0, high[i] = m;
}

```

```

bool done = 0;
DSU uf(n); // DSU structure
int curvis;
while(!done){
    done = 1;
    for(int i = 0; i < q; ++i){
        int mid = (low[i] + high[i]) >> 1;
        tocheck[mid].pb(i);
    }
    uf.clear(n);
    int last = -1;
    for(int value = 0; value <= m; ++value){
        if(tocheck[value].empty()) continue;
        for(int i = last + 1; i <= value; ++i)
            uf.join(a[i], b[i]);
        last = value;
        while(!tocheck[value].empty()){
            int id = tocheck[value].back();
            tocheck[value].pop_back();
            int u = x[id], v = y[id];
            int visited = z[id];
            if(low[id] + 1 == high[id]) solved[id] = 1;
            if(uf.connected(u, v)) curvis = uf.size(u);
            else curvis = uf.size(u) + uf.size(v);
            if(curvis >= visited) high[id] = value;
            else low[id] = value;
            if(low[id] == high[id]) solved[id] = -1;
        }
    }
    for(int i = 0; i < q; ++i)
        if(solved[i] == 0) done = 0;
}
for(int i = 0; i < q; ++i)
    if(solved[i] == -1) cout << -1 << endl;
    else cout << high[i] << endl;
}

```

## 11 Unordered Map

```

unordered_map<int,int> mp;
mp.reserve(1024); // power of 2 is better
mp.max_load_factor(0.25); // 0.75 used in java

```