

# Pontificia Universidad Católica del Perú - FCI

XieXieLucas Notebook - Froz/Phibrain/Ands

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## 1 Data Structures

### 1.1 Fenwick Tree

---

// Fenwick tree:  $O(\log(n))$  accumulated sum queries.

```

ll bitadd[N] ;
ll bitsub[N] ;
int n ;

void update( int idx, ll val1, ll val2 ){
    while( idx <=n ) {
        bitadd[idx] += val1 ;
        bitsub[idx] += val2 ;
        idx += idx & -idx ;
    }
}

void updaterange( int l , int r , ll val ){
    update( l , val , (l-1)*val ) ;
    update( r+1 , -val , -r*val ) ;
}

ll get( int idx ){
    ll add = 0 , sub = 0, aux = idx ;
    while ( idx > 0 ){
        add += bitadd[idx] ;
        sub += bitsub[idx] ;
        idx -= idx & -idx ;
    }
}

```

```

}
return aux*add - sub ;
}

```

---

### 1.2 Heavy Light Decomposition

---

//Heavy-Light Decomposition Tree for Commutative Operations  
//Phibrain

```

inline ll ma(ll a, ll b){return ((a-b>0)? a:b);}
inline ll mi(ll a, ll b){return ((a-b>0)? b:a);}

struct ST{
    ll n;
    ll t[2*N];
    ll Op(ll &u, ll &v){ return ma(u,v); }
    inline void build(){
        RREP(i,n-1,1) t[i]=Op(t[i<<1], t[i<<1|1]);
    }
    inline void modify(ll p, ll val){
        for(t[p+=n] = val ; p >>= 1;) t[p] = Op(t[p<<1], t[p<<1|1]);
    }
    inline ll que(ll l, ll r){
        ll ans1=min, ansr=min;
        for(l += n, r += n; l < r; l >>= 1, r >>= 1){
            if(l&1) ans1 = Op(ans1, t[l++]);
            if(r&1) ansr = Op(t[--r], ansr);
        }
        return Op(ans1, ansr);
    }
}

};

struct HLDES{
    ll n;
    ST st;
    vi adj[N];
    ll p[N],d[N],tsz[N],id[N],rt[N];
    ll gid;
    inline ll Op(ll val1, ll val2) {return ma(val1,val2);}
    inline ll make1(ll u,ll par,ll depth){
        p[u]=par; d[u]=depth; tsz[u]=1;
        for(auto v:adj[u])if(v!=p[u]) tsz[u]+=make1(v,u,depth+1);
        return tsz[u];
    }
}

```

```

inline void make(){
    ll val=make1(0,-1,0);
}
inline void dfs(ll u, ll root){
    id[u]=gid++; rt[u]=root;
    ll w=0 , wsz=min;
    for(auto v: adj[u]) if(v!=p[u]){
        if(tsz[v]>wsz) {w=v; wsz=tsz[v];}
    }
    if(w) dfs(w,root);
    for(auto v:adj[u]) if(v!=p[u]) if(v!=w) dfs(v,v);
}
inline void upd(ll u, ll val){
    ll a=id[u];
    st.modify(a,val);
}
ll que(ll u, ll v){
    ll ans=0;// neutro?
    while(u!=-1){
        if(rt[u]==rt[v]){
            ll a=id[u], b=id[v];
            if(a>b) swap(a,b);
            ans=Op(ans,st.que(a,b+1));
            u=-1;
        }
        else{
            if(d[rt[u]]>d[rt[v]]) swap(u,v);
            ans=Op(ans,st.que(id[rt[v]],id[v]+1));
            v=p[rt[v]];
        }
    }
    return ans;
}
inline void build(){
    gid=0; st.n=n;
    make(); dfs(0,0);
    REP(i,0,n) st.t[i+n]=0;//val de cada t[i]
    st.build();
}
};

```

//Heavy Light Decomposition General

```
struct HLDES{
```

```

    ll n;
    ST st1,st2;
    vi adj[N];
    vector<ii> ver[N];
    ll p[N],d[N],tsz[N],id[N],rt[N],ar[N],val[N],id1[N];
    ll gid,k;

    inline T Op(T &val1, T &val2){
        T ty;
        //Operacion del Heavy Light
        return ty;
    }

    inline ll make1(ll u,ll par,ll depth){
        p[u]=par; d[u]=depth; tsz[u]=1;
        for(auto v:adj[u])if(v!=p[u]) tsz[u]+=make1(v,u,depth+1);
        return tsz[u];
    }
    inline void make(){
        ll val=make1(0,-1,0);
    }
    inline void dfs(ll u, ll root){
        ar[gid]=val[u];
        id[u]=gid++; rt[u]=root;
        ll w=0LL , wsz=min;
        for(auto v: adj[u]) if(v!=p[u]){
            if(tsz[v]>wsz) {w=v; wsz=tsz[v];}
        }
        if(w) dfs(w,root);
        for(auto v:adj[u]) if(v!=p[u]) if(v!=w) dfs(v,v);
    }
    inline void solve(){
        ll ta;
        REP(i,0,n) ver[rt[i]].pb(mp(id[i],i));
        REP(i,0,n){
            if(ver[i].size()!=0){
                sort(all(ver[i]));
                ta=ver[i].size();
                ta=ver[i][ta-1].fst;
                for(auto j: ver[i]) id1[j.snd]=ta--;
            }
        }
    }
    inline ll LCA(ll u, ll v){
        while(rt[u]!=rt[v]){

```

```

    if(d[rt[u]]<d[rt[v]]) v=p[rt[v]];
    else u=p[rt[u]];
}
return d[u]>d[v]? v:u;
}
inline void upd(ll u, ll v, ll val){
    ll l, r, a, b;
    //Update del Heavy Light
}
inline ll que(ll u, ll v){
    //Query del HLD
}
inline void build(){
    REP(i,0,n) cin>>val[i];
    REP(i,0,n-1) {
        ll a,b; cin>>a>>b;
        a--;b--;
        adj[a].pb(b); adj[b].pb(a);
    }
    gid=0LL; k=0LL; st1.n=n; //st2.n=n; //st.made();
    make();
    dfs(0,0);
    REP(i,0,n) st1.ar[i]=ar[i];
    st1.build();
}
} hld;

```

### 1.3 LCA Tree

```

const int MAX = 1e4;
const int LGMAX = 15;
//LCA construction in O(n*log(n)) with O(log(n)) queries.
struct LCATree{
    int n;
    vector<int> adj[MAX];
    int p[MAX][LGMAX]; // 2^j ancestor of node i
    int L[MAX];        // Depth of node i
    int q[MAX];        // (Queue used internally).

    LCATree(int N):n(N){}

    void dfs(int u, int h){
        L[u] = h;

```

```

        REP(i,0,sz(adj[u])){
            int v = adj[u][i];
            if (v != p[u][0]) {
                p[v][0] = u;
                dfs(v, h+1);
            }
        }
    }
}
void buildlca(int r){
    REP(i,0,n) REP(pw,0,LGMAX) p[i][pw] = -1;
    dfs(r, 0);
    for (int pw = 1; (1<<pw) < n; pw++){
        REP(i,0,n) if (p[i][pw-1] != -1) p[i][pw] = p[p[i][pw-1]][pw-1];
    }
}
int lca(int u, int v){
    if (L[u] < L[v]) swap(u,v);
    for (int pw = LGMAX-1; pw >= 0; pw--){
        if (L[u] - (1<<pw) >= L[v])
            u = p[u][pw];
    }
    if (u == v) return u;
    for (int pw = LGMAX-1; pw >= 0; pw--){
        if (p[u][pw] != p[v][pw]) {
            u = p[u][pw];
            v = p[v][pw];
        }
    }
    return p[u][0];
}

};

int main() {
    int n = 1e3;
    LCATree T(n);
    //Initialize n and the adj[] list
    T.buildlca(0); //Place the root instead of 0
    //Ready to answer queries
    return 0;
}

```

### 1.4 Lazy Propagation Segment Tree

```

// lazy propagation con propagacion y el update
//ejemplo de update en [l,r> la serie de fibonaci con a y b como primeros
//notar la forma de updatepro y proh;
//made preprocess y find el fib de posicion n con a y b como primeros
//numeros

inline ll ss(ll val) {return val%MOD;}

ll dpf[N];

inline void made(){
    dpf[1]=1 ; dpf[2]=1;
    REP(i,3,N) dpf[i]=ss(dpf[i-1]+dpf[i-2]);
}

inline ll find(ll a, ll b, ll n) {
    if(n<3) return n==1? a:b;
    return ss(a*dpf[n-2]+b*dpf[n-1]);
}

struct ST{
    ii lazy[4*N];
    ll tree[4*N], ar[N];
    ll n;
    inline void updatepro(ii laz,ll id, ll l,ll r){
        ll ta=r-l, sum=(find(laz.fst,laz.snd,ta+2)-laz.snd+MOD)%MOD;
        tree[id]=ss(tree[id]+sum);
        lazy[id].fst=ss(lazy[id].fst+laz.fst);
        lazy[id].snd=ss(lazy[id].snd+laz.snd);
    }
    inline void proh(ll id, ll l,ll r){
        ll mid=(l+r)>>1, ta=mid-l;
        updatepro(lazy[id],2*id,l,mid);
        ii laz;
        laz.fst=find(lazy[id].fst,lazy[id].snd,ta+1);
        laz.snd=find(lazy[id].fst,lazy[id].snd,ta+2);
        updatepro(laz,2*id+1,mid,r);
        lazy[id]={0LL,0LL};
    }
    inline void updateRange(ll x, ll y, ll a, ll b, ll id, ll l,ll r){
        if(x>=r || y<=l) return;
        if(x<=l && r<=y){
            ll ta=l-x; ii laz;
            laz.fst=find(a,b,ta+1); laz.snd=find(a,b,ta+2);
            updatepro(laz,id,l,r);
        }
    }
};

```

```

        return;
    }
    proh(id,l,r);ll mid=(l+r)>>1;
    updateRange(x,y,a,b,2*id,l,mid);
    updateRange(x,y,a,b,2*id+1,mid,r);
    tree[id]=ss(tree[2*id]+tree[2*id+1]);
}

inline ll getSum(ll x,ll y,ll id,ll l,ll r){
    if(x>=r || l>=y) return 0;
    if(x<=l && r<=y) return tree[id];
    proh(id,l,r);ll mid=(l+r)>>1;
    ll ez,ez1,ez2;
    ez1=getSum(x,y,2*id,l,mid);
    ez2=getSum(x,y,2*id+1,mid,r);ez=ss(ez1+ez2);
    return ez;
}

inline void build1( ll id, ll l, ll r){
    if (l > r) return ;
    if (r-l<2){tree[id] = ar[l];return;}
    ll mid = (l + r)>>1;
    build1(2*id, l,mid); build1(2*id+1, mid, r);
    tree[id] = ss(tree[id*2 ] + tree[id*2 + 1]);
}

inline void upd(ll x, ll y, ll a, ll b){
    updateRange(x,y,a,b,1,0,n);
}

inline void build(){
    build1(1,0,n);
}

inline ll que(ll x, ll y){
    return getSum(x,y,1,0,n);
}

};

```

## 1.5 Link Cut Tree

```
//Link cut tree
```

```
const int N = 1e5 + 2;
```

```
struct Node {
    Node *left, *right, *parent;
    bool revert;
```

```

Node() : left(0), right(0), parent(0), revert(false) {}
bool isRoot() {
    return parent == NULL ||
        (parent->left != this && parent->right != this);
}
void push() {
    if (revert) {
        revert = false;
        Node *t = left;
        left = right;
        right = t;
        if (left != NULL) left->revert = !left->revert;
        if (right != NULL) right->revert = !right->revert;
    }
}
};

```

```

struct LinkCutTree{
    Node nos[N];

    LinkCutTree(){
        REP(i,0,N) nos[i] = Node();
    }

    void connect(Node *ch, Node *p, bool isLeftChild) {
        if (ch != NULL) ch->parent = p;
        if (isLeftChild) p->left = ch;
        else p->right = ch;
    }

    void rotate(Node *x){
        Node* p = x->parent;
        Node* g = p->parent;
        bool isRoot = p->isRoot();
        bool leftChild = x == p->left;

        connect(leftChild ? x->right : x->left, p, leftChild);
        connect(p, x, !leftChild);
        if (!isRoot) connect(x, g, p == g->left);
        else x->parent = g;
    }

    void splay(Node *x){
        while (!x->isRoot()) {
            Node *p = x->parent;

```

```

        Node *g = p->parent;
        if (!p->isRoot()) g->push();
        p->push();
        x->push();
        if (!p->isRoot()) {
            rotate((x == p->left) == (p == g->left) ? p : x);
        }
        rotate(x);
    }
    x->push();
}

```

```

Node *expose(Node *x) {
    Node *last = NULL, *y;
    for (y = x; y != NULL; y = y->parent) {
        splay(y);
        y->left = last;
        last = y;
    }
    splay(x);
    return last;
}

```

```

void makeRoot(Node *x) {
    expose(x);
    x->revert = !x->revert;
}

```

```

bool connected(Node *x, Node *y) {
    if (x == y) return true;
    expose(x);
    expose(y);
    return x->parent != NULL;
}

```

```

bool link(Node *x, Node *y) {
    if (connected(x, y)) return false;
    makeRoot(x);
    x->parent = y;
    return true;
}

```

```

bool cut(Node *x, Node *y) {
    makeRoot(x);
    expose(y);

```

```

    if (y->right != x || x->left != NULL || x->right != NULL)
        return false;
    y->right->parent = NULL;
    y->right = NULL;
    return true;
}
};

```

## 1.6 Persistent Segment Tree

```

// Persistent segment tree implemented with pointers.
// Consider using a map<int, node*> which represents
// the segment tree at time t.
const int MAX = 1e6;
typedef int T;
T arr[MAX];
struct node {
    T val;
    node *l, *r;
    node(T val) : val(val), l(NULL), r(NULL) {}
    node(T val, node* l, node* r) : val(val), l(l), r(r) {}
};
// Identity element of Op()
const T OpId = 0;
// Associative query operation
T Op(T val1, T val2){
    return val1 + val2;
}
node* build(int a, int b) {
    if (a+1 == b) return new node(arr[a]);
    node* l = build(a, (a+b)/2);
    node* r = build((a+b)/2, b);
    return new node(Op(l->val, r->val), l, r);
}
// Branch and increment position p by val
node* update(node* u, int a, int b, int p, T val) {
    if (a > p || b <= p) return u;
    if (a+1 == b) return new node(Op(u->val, val));
    node* l = update(u->l, a, (a+b)/2, p, val);
    node* r = update(u->r, (a+b)/2, b, p, val);
    return new node(Op(l->val, r->val), l, r);
}
// Query t to get sum of values in range [i, j)

```

```

T query(node* u, int a, int b, int i, int j) {
    if (a >= j || b <= i) return OpId;
    if (a >= i && b <= j) return u->val;
    T q1 = query(u->l, a, (a+b)/2, i, j);
    T q2 = query(u->r, (a+b)/2, b, i, j);
    return Op(q1, q2);
}
map<int, node*> m;
node* st;
T val;
int n, p;
int main() {
    REP(i,0,n) arr[i] = 0; // Any starting values
    m.clear();
    st = build(0,n);
    m[0] = st;
    REP(i,0,n){
        // Modify position p with value val at time t
        st = update(st, 0, n, p, val);
        m[i] = st;
    }
    // Consider for example rectangular queries:
    // Sum of all nodes in [a,b]x[c,d] using one
    // coordinate as time and another as values
}

```

## 1.7 Segment Tree

```

// Iterative, fast, non-commutative segment tree.
typedef int T;
const int MAX = 1e6;

// Identity element of the operation
const T OpId = 0;
// Associative internal operation
T Op(T& val1, T& val2){
    return val1 + val2;
}

// The user should fill t[n, 2*n)
T t[2*MAX];
int n;

```

```

void build(){
    for( int i = n-1 ; i > 0 ; i-- ) t[i] = Op(t[i<<1], t[i<<1|1]);
}

void modify( int p , T val ){
    for( t[p+=n] = val ; p >= 1 ; ) t[p] = Op(t[p<<1], t[p<<1|1]);
}

T get( int l , int r ){ //[l,r]
    T ans1, ansr;
    ans1 = ansr = OpId; //Initialize operation at Identity
    for( l += n, r += n ; l < r ; l >>= 1, r >>= 1 ){
        if(l&1) ans1 = Op(ans1, t[l++]);
        if(r&1) ansr = Op(t[--r], ansr);
    }
    return Op(ans1, ansr);
}

int main(){
    // Read into t[n,2*n]
    build();
    // Answer queries
}

```

## 1.8 Wavelet Tree

```

/*
    Wavelet Tree Implementation
    Construction in  $O(n \log n)$ 
    Queries in  $O(\log(\text{MAX}))$ 

    1 - based array!
*/

typedef vector<int> vi;

struct WT{
    int lo, hi;
    WT *l, *r; vi b;
    WT(int *from, int *to, int x, int y){

```

```

        lo = x, hi = y;
        if(lo == hi or from >= to) return;
        int mid = (lo+hi)/2;
        auto f = [mid](int x){
            return x <= mid;
        };
        b.reserve(to-from+1);
        b.pb(0);
        for(auto it = from; it != to; it++) b.pb(b.back() + f(*it));
        auto pivot = stable_partition(from, to, f);
        l = new WT(from, pivot, lo, mid);
        r = new WT(pivot, to, mid+1, hi);
    }
    //kth en [l,r]
    int kth(int l, int r, int k){
        if(l > r) return 0;
        if(lo == hi) return lo;
        int inLeft = b[r] - b[l-1]; //cantidad en los a primeros b[a]
        int lb = b[l-1];
        int rb = b[r];
        if(k <= inLeft) return this->l->kth(lb+1, rb , k);
        return this->r->kth(l-lb, r-rb, k-inLeft);
    }

    //cantidad de numeros menores a K en [l,r]
    int LTE(int l, int r, int k) {
        if(l > r or k < lo) return 0;
        if(hi <= k) return r - l + 1;
        int lb = b[l-1], rb = b[r];
        return this->l->LTE(lb+1, rb, k) + this->r->LTE(l-lb, r-rb, k);
    }

    //cantidad de numeros en [l,r] iguales a k
    int count(int l, int r, int k) {
        if(l > r or k < lo or k > hi) return 0;
        if(lo == hi) return r - l + 1;
        int lb = b[l-1], rb = b[r], mid = (lo+hi)/2;
        if(k <= mid) return this->l->count(lb+1, rb, k);
        return this->r->count(l-lb, r-rb, k);
    }
    ~WT(){
        delete l;
        delete r;
    }
};

```



## 2 Geometry

### 2.1 Convex Hull

---

```
// INPUT:  a vector of input points, unordered.
// OUTPUT: a vector of points in the convex hull,
//          counterclockwise, starting with bottom left

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
    T x, y;
    PT() {}
    PT(T x, T y) : x(x), y(y) {}
    bool operator<(const PT &rhs) const {
        return mp(y,x) < mp(rhs.y,rhs.x);
    }
    bool operator==(const PT &rhs) const {
        return mp(y,x) == mp(rhs.y,rhs.x);
    }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) {
    return cross(a,b) + cross(b,c) + cross(c,a);
}

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
    return (fabs(area2(a,b,c)) < EPS &&
            (a.x-b.x)*(c.x-b.x) <= 0 &&
            (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end()), pts.end());
    vector<PT> up, dn;
    for (int i = 0; i < pts.size(); i++) {
        while (up.size() > 1 &&
               area2(up[up.size()-2], up.back(), pts[i]) >= 0)
            up.pop_back();
        while (dn.size() > 1 &&
               area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0)
            dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--)
        pts.push_back(up[i]);
}
```

---

```
        up.pop_back();
    while (dn.size() > 1 &&
           area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0)
        dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
}
pts = dn;
for (int i = (int) up.size() - 2; i >= 1; i--)
    pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
if (pts.size() <= 2) return;
dn.clear();
dn.push_back(pts[0]);
dn.push_back(pts[1]);
for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i]))
        dn.pop_back();
    dn.push_back(pts[i]);
}
if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
}
pts = dn;
#endif
}
```

---

### 2.2 Delaunay Triangulation

---

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:  x[] = x-coordinates
//          y[] = y-coordinates
//
// OUTPUT: triples = a vector containing m triples of indices
//           corresponding to triangle vertices

#include<vector>
```

```

using namespace std;

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
    int n = x.size();
    vector<T> z(n);
    vector<triple> ret;

    for (int i = 0; i < n; i++)
        z[i] = x[i] * x[i] + y[i] * y[i];

    for (int i = 0; i < n-2; i++) {
        for (int j = i+1; j < n; j++) {
            for (int k = i+1; k < n; k++) {
                if (j == k) continue;
                double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])*(z[j]-z[i]);
                double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])*(z[k]-z[i]);
                double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])*(y[j]-y[i]);
                bool flag = zn < 0;
                for (int m = 0; flag && m < n; m++)
                    flag = flag && ((x[m]-x[i])*xn +
                                   (y[m]-y[i])*yn +
                                   (z[m]-z[i])*zn <= 0);
                if (flag) ret.push_back(triple(i, j, k));
            }
        }
    }
    return ret;
}

int main(){
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);
}

```

```

//expected: 0 1 3
//          0 3 2

int i;
for(i = 0; i < tri.size(); i++)
    printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
return 0;
}

```

## 2.3 Geometry

// C++ routines for computational geometry.

```

const double INF = 1e100;
const double EPS = 1e-12;
const double PI = acos(-1);

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
};

double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double norm(PT p) { return sqrt(dot(p,p)); }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << ", " << p.y << ")";
}

double angle(PT p){
    double res = acos(p.x / norm(p));
    if (p.y > 0) return res;
    else return 2*PI - res;
}

```

```

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
    double a, double b, double c, double d)
{
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// Whethes lines (a,b), (c,d) are parallel/collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}
bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

```

```

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 &&
            dot(c-b, d-b) > 0) return false;
        return true;
    }
    if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
    if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
    return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
    b=b-a; d=d-c; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon
// (by William Randolph Franklin); returns 1 for strictly
// interior points, 0 for strictly exterior points, and 0 or 1
// for the remaining points. Note that it is possible to
// convert this into an *exact* test using integer arithmetic
// by taking care of the division appropriately (making sure
// to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();

```

```

    if ((p[i].y <= q.y && q.y < p[j].y ||
        p[j].y <= q.y && q.y < p[i].y) &&
        q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) /
            (p[j].y - p[i].y))
        c = !c;
    }
    return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
    for (int i = 0; i < p.size(); i++)
        if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
            return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    double A = dot(b, b);
    double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleInter(PT a, PT b, double r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)

```

```

        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}

// Area or centroid of a (possibly nonconvex) polygon,
// assuming the coordinates are listed in a clockwise or
// counterclockwise order. Note that the centroid is often
// known as the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

// Whether or not a given (CW or CCW) polygon is simple
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == 1 || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

```

```
// Computes the circumcenter of a Triangle PQR
PT circumcenter(PT p, PT q, PT r) {
    PT a = p-r, b = q-r;
    PT c = PT(dot(a, (p + r)) / 2, dot(b, (q + r)) / 2);
    return PT(dot(c, RotateCW90(PT(a.y, b.y))),
              dot(PT(a.x, b.x), RotateCW90(c))) / dot(a, RotateCW90(b));
}

//Check if a polygon is convex
bool isConvex(const vector<point> &P) {
    int sz = (int)P.size();
    if (sz <= 3) return false;
    bool isLeft = ccw(P[0], P[1], P[2]);
    for (int i = 1; i < sz-1; i++)
        if (ccw(P[i], P[i+1], P[(i+2) == sz ? 1 : i+2]) != isLeft)
            return false;
    return true;
}
```

## 2.4 Minkowski Sum

```
//Calcula suma de Minkowski en O(n + m)
//A y B deben estar en sentido antihorario
inline bool compare(PT a, PT b){
    // mas abajo, mas a la izquierda
    if(a.y < b.y) return 1;
    if(a.y == b.y) return a.x < b.x;
    return 0;
}

vector<PT> minkow_sum(const vector<PT>& a, const vector<PT>& b){
    vector< PT > out;
    out.clear();
    int lena = int(a.size());
    int lenb = int(b.size());
    int i = 0, j = 0;
    for(int q = 0; q < lena; ++q) if(compare(a[q], a[i])) i = q;
    for(int q = 0; q < lenb; ++q) if(compare(b[q], b[j])) j = q;
    ll pr;
    int nexti, nextj;
    do{
        out.pb(a[i] + b[j]);
```

```
        nexti = (i + 1) % lena;
        nextj = (j + 1) % lenb;
        pr = cross(a[nexti] - a[i], b[nextj] - b[j]);
        if(pr > 0) i = nexti;
        else if(pr < 0) j = nextj;
        else i = nexti, j = nextj; // paralelas, subo en ambas
    }while((a[i] + b[j]) != out[0]);
    return out;
}
```

## 3 Graphs

### 3.1 2SAT

```
//2-SAT
//Conditions from 0 to 2*number of nodes, i and i~1 are reciprocal
//That means, ~0 is 1, ~1 is 0, ~2 is 3, ~3 is 2, etc
//When adding an edge, make sure to fix values
//For example, node from a to b (a,b >= 1)
//aa = (a-1)*2, bb = (b-1)*2, then a has "aa" as true and aa~1 as false
//Same to b
//To return to the main state, divide by 2 and sum 1

struct TwoSAT{
    int n;
    vector< vi> g, adj;
    vi d, low, scc, ans, lev;
    vector<bool> stacked, ok;
    stack<int> s;
    int ticks, current_scc;

    TwoSAT(int N):
        n(N), ticks(0), current_scc(0), g(N), adj(N), d(N), low(N), scc(N),
        ans(N), lev(N),
        stacked(N), ok(N){}

    void initialize(){
        REP(i,0,n){
            stacked[i] = false;
            d[i] = -1;
            scc[i] = -1;
            ok[i] = false;
```

```

        current_scc = ticks = 0;
    }
}

void addEdge(int a, int b){
    g[a].pb(b);
}

void tarjan(int u){
    d[u] = low[u] = ticks++;
    s.push(u);
    stacked[u] = true;
    const vector<int> &out = g[u];
    for (int k=0, m=out.size(); k<m; ++k){
        const int &v = out[k];
        if (d[v] == -1){
            tarjan(v);
            low[u] = min(low[u], low[v]);
        }else if (stacked[v]){
            low[u] = min(low[u], low[v]);
        }
    }
    if (d[u] == low[u]){
        int v;
        do{
            v = s.top();
            s.pop();
            stacked[v] = false;
            scc[v] = current_scc;
        }while (u != v);
        current_scc++;
    }
}

bool consistent(){
    for(int i = 0; i < n; i+=2){
        if(scc[i] == scc[i^1]){
            return false;
        }
    }
    return true;
}

void build(){
    REP(i,0,n){

```

```

        REP(j,0,sz(g[i])){
            int v = g[i][j];
            if(scc[i] != scc[v]){
                adj[i].pb(v); lev[v]++;
            }
        }
    }
}

void toposort(){
    queue<int> q;
    REP(i,0,current_scc){
        if(lev[i] == 0) q.push(i);
    }
    int x = 1;
    while(!q.empty()){
        int u = q.front(); q.pop();
        ans[u] = x ++;
        REP(i,0,sz(adj[u])){
            int v = adj[u][i];
            lev[v]--;
            if(lev[v] == 0) q.push(v);
        }
    }
}

void solve(){
    for(int i = 0; i<n; i+=2){
        if(ans[scc[i]] < ans[scc[i^1]]){
            ok[i] = false; ok[i^1] = true;
        }
        else{
            ok[i] = true; ok[i^1] = false;
        }
    }
}

bool go(){
    REP(i,0,n){
        if(scc[i] == -1) tarjan(i);
    }
    if(!consistent()) return false;
    else{
        build();
        toposort();

```

```

        solve();
        return true;
    }
}

};

int main(){
    fastio;
    int n,m; cin >> n >> m;
    TwoSAT TS = TwoSAT(2*n);
    TS.initialize();

    //TO DO: ADD EDGES

    bool res = TS.go();
    if(!res) cout << "Impossible" << endl;
    else{
        for(int i = 0; i < 2*n; i+=2){
            int state = i/2 + 1;
            if(TS.ok[i]) //state is true
            else //state is false
        }
        return 0;
    }
}

```

## 3.2 Biconnected Components

//Finds Biconnected Components

```

bool usd[1005];
int low[1005], d[1005], prev[1005], cnt;
vector<int> adj[1005];
stack<ii> S;

void Outcomp( int u , int v ){
    printf("New Component\n");
    ii e;
    do{
        e = S.top(); S.pop();
        cout << e.fst << " " << e.snd << endl;
    } while( e != mp( u , v ) );
}

```

```

void dfs( int u ){
    usd[u] = 1; cnt++;
    low[u] = d[u] = cnt;
    REP(i,0,sz(adj[u])){
        int v = adj[u][i];
        if( !usd[v] ){
            S.push( mp( u , v ) );
            prev[v] = u; dfs( v );
            if( low[v] >= d[u] ) Outcomp( u , v );
            low[u] = min( low[u] , low[v] );
        }
        else if( prev[u] != v and d[v] < d[u] ){
            S.push( mp( u , v ) );
            low[u] = min( low[u] , d[v] );
        }
    }
}

int main(){
    int n, m;
    cin >> n >> m;
    REP(i,0,m){
        int a , b;
        cin >> a >> b;
        adj[a].pb(b);
        adj[b].pb(a);
    }
    cnt = 0;
    memset(usd,0,sizeof(usd));
    memset(prev,-1,sizeof(prev));
    REP(i,0,n){
        if( !usd[i] ) dfs(i);
    }
    return 0;
}

```

## 3.3 Bridges and Articulation Points

//Finding bridges and articulation points

```

int low[N],id[N],parent[N];
bool art[N];

```

```

vi adj[N];
vi bridge[N];
int curr_id = 0;
int root, rootchild;

void dfs(int u) {
    low[u] = id[u] = curr_id++;
    REP(j, 0, sz(adj[u])) {
        int v = adj[u][j];
        if (id[v] == -1) {
            parent[v] = u;
            if (u == root) rootchild++;
            dfs(v);
            if (low[v] >= id[u]) art[u] = true;
            if (low[v] > id[u]) {
                bridge[u].pb(v);
                bridge[v].pb(u); //store bridges in a sub
                                graph
            }
            low[u] = min(low[u], low[v]);
        }
        else if (v != parent[u]) low[u] = min(low[u], id[v]);
    }
}

//inside int main()
REP(i, 0, n) {
    if (id[i] == -1) {
        root = i; rootchild = 0; dfs(i);
        art[root] = (rootchild > 1);
    }
}

```

### 3.4 Eulerian Path

```

// Finds Eulerian Path (visits every edge exactly once)
// CYCLE exists iff all edges even degree, all edges in
// same connected component.
// PATH exists iff cycle exists and once edge removed
// [ Hamiltonian (all vertices) is NP complete ]
struct Edge;
typedef list<Edge>::iterator iter;
struct Edge

```

```

{
    int next_vertex;
    iter reverse_edge;
    Edge(int next_vertex) : next_vertex(next_vertex) { }
};

const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices]; // adjacency list
vector<int> path;

void find_path(int v)
{
    while(adj[v].size() > 0)
    {
        int vn = adj[v].front().next_vertex;
        adj[vn].erase(adj[v].front().reverse_edge);
        adj[v].pop_front();
        find_path(vn);
    }
    path.push_back(v);
}

void add_edge(int a, int b)
{
    adj[a].push_front(Edge(b));
    iter ita = adj[a].begin();
    adj[b].push_front(Edge(a));
    iter itb = adj[b].begin();
    ita->reverse_edge = itb;
    itb->reverse_edge = ita;
}

```

### 3.5 Maximal Cliques

```

// Bron-Kerbosch algorithm for finding all the
// maximal cliques of a graph in  $O(3^{n/3})$ 
//  $3^{13} = 1.6e6$ 

// Call them using clique(0, (1LL << n) - 1, 0)
// n vertices
ll adj[65];
// This algorithm finds all the maximal cliques containing an edge
// The cliques are found explicitly (the vertex of the cliques)
void clique(ll r, ll p, ll x) {

```



```

if (p == 0 && x == 0) {
    /* r is a maximal clique */
    /* Every 1 in r is a vertex of the clique
    Then, __builtin_popcountll(r) is the size of the clique*/
    return;
}
int pivot = -1;
int menor = INF;
for (int i = 0; i < n; i++) {
    if ( ((1LL << i) & p) || ((1LL << i) & x) ) {
        int x = __builtin_popcountll(p & ~(adj[i]));
        if (x < menor) {
            pivot = i;
            menor = x;
        }
    }
}
for (int i = 0; i < n; i++) {
    if ((1LL << i) & p) {
        if (pivot != -1 && adj[pivot] & (1LL << i)) continue;
        clique(r | (1LL << i), p & adj[i], x & adj[i]);
        p = p ^ (1LL << i);
        x = x | (1LL << i);
    }
}

// This one has the same idea, but is faster
// However, it only finds the size of the cliques
void clique2(int r, ll p, ll x){
    if(p == 0 && x == 0){
        // r is the size of the clique
    }
    if(p == 0) return;
    int u = __builtin_ctzll(p | x);
    ll c = p & ~ adj[u];
    while(c){
        int v = __builtin_ctzll(c); //Number of trailing zeros
        clique(r + 1, p & adj[v], x & adj[v]);
        p ^= (1LL << v);
        x |= (1LL << v);
        c ^= (1LL << v);
    }
}

```

### 3.6 Tarjan Strongly Connected Components

```

/* Complexity: O(E + V)
Tarjan's algorithm for finding strongly connected
components.
*d[i] = Discovery time of node i. (Initialize to -1)
*low[i] = Lowest discovery time reachable from node
i. (Doesn't need to be initialized)
*scc[i] = Strongly connected component of node i. (Doesn't
need to be initialized)
*s = Stack used by the algorithm (Initialize to an empty
stack)
*stacked[i] = True if i was pushed into s. (Initialize to
false)
*ticks = Clock used for discovery times (Initialize to 0)
*current_scc = ID of the current_scc being discovered
(Initialize to 0)
*/

//DON'T FORGET TO INITIALIZE d[MAXN] TO -1 !!!!
vector<int> g[MAXN];
int d[MAXN], low[MAXN], scc[MAXN];
bool stacked[MAXN];
stack<int> s;
int ticks, current_scc;
void tarjan(int u){
    d[u] = low[u] = ticks++;
    s.push(u);
    stacked[u] = true;
    const vector<int> &out = g[u];
    for (int k=0, m=out.size(); k<m; ++k){
        const int &v = out[k];
        if (d[v] == -1){
            tarjan(v);
            low[u] = min(low[u], low[v]);
        }else if (stacked[v]){
            low[u] = min(low[u], low[v]);
        }
    }
    if (d[u] == low[u]){
        int v;
        do{
            v = s.top();
            s.pop();
            stacked[v] = false;

```

```

    scc[v] = current_scc;
}while (u != v);
current_scc++;
}
}

```

## 4 Math

### 4.1 Chinese Remainder Theorem

```

/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 */
long long crt(vector<long long> &a, vector<long long> &x) {
    long long z = 0;
    long long n = 1;
    for (int i = 0; i < x.size(); ++i)
        n *= x[i];

    for (int i = 0; i < a.size(); ++i) {
        long long tmp = (a[i] * (n / x[i])) % n;
        tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
        z = (z + tmp) % n;
    }

    return (z + n) % n;
}

```

### 4.2 Cribas

```

// Criba en O(n)
// p[i] indica el valor del primo i-esimo
// A[i] indica que el menor factor primo de i
// es el primo A[i] - esimo
#define MAXN 100000

int A[MAXN + 1], p[MAXN + 1], pc = 0;

void sieve()

```

```

{
    for(int i=2; i<=MAXN; i++){
        if(!A[i]) p[A[i] = ++pc] = i;
        for(int j=1; j<=A[i] && i*p[j]<=MAXN; j++)
            A[i*p[j]] = j;
    }
}

//Criba para phi
int phi[MAXN];

void CribaEuler(){
    REP(i,0, MAX) primo[i] = 1, phi[i] = 1;
    primo[0] = primo[1] = false;
    REP(i,2,MAX){
        if(primo[i]){
            phi[i] = i - 1;
            for(int j = i+i; j < MAX; j += i){
                primo[j] = false;
                int pot = 1, aux = j/i;
                while( aux % i == 0 ){
                    aux /= i, pot *= i;
                }
                phi[j] *= (i-1)*pot ;
            }
        }
    }
}

```

### 4.3 Euler Totient

```

//Euler Totient
//Finds all k <= n where gcd(k,n) == 1

int phi(int n){
    int res = n;
    for(int p = 2; p*p <= n; p++){
        if(n % p == 0){
            while(n % p == 0)
                n/=p;
            res -= res/p;
        }
    }
}

```

```

    if (n > 1) res -= res/n;
    return res;
}

```

## 4.4 Inverso Modular

```

/** Inverso Modular */
#define MAX 100
#define MOD 1000000009

long long inverso[MAX];

void inv(){
    inverso[1] = 1;
    REP(i,2,MAX) inverso[i] = ( (MOD-MOD/i) * inverso[MOD%i] ) % MOD;
}

```

## 4.5 Miller Rabin

```

//Miller-Rabin primality test

ll pow(ll a, ll b, ll c){
    ll ans = 1;
    while(b){
        if(b&1) ans = (1LL*ans*a)%c;
        a = (1LL*a*a)%c;
        b >>= 1;
    }
    return ans;
}

bool miller(ll p, ll it = 10){
    if(p<2) return 0;
    if(p!=2 && (p&1) == 0) return 0;
    ll s=p-1;
    while((s&1) == 0) s>>=1;
    while(it--){
        ll a = rand()%(p-1)+1, temp = s;
        ll mod = pow(a,temp,p);
        while(temp!= p-1 && mod!=1 && mod!=p-1){
            mod = (1LL*mod*mod)%p;

```

```

        temp<<=1;
    }
    if(mod!=p-1 && (temp&1) == 0) return 0;
}
return 1;
}

```

## 4.6 Number Theory

```

// Encuentra el menor positivo de la forma ax+ b , my+ n
// ( x,y enteros no necesariamente positivos)
// Si son positivos, hallar los coeficientes y sumar lo
// que falta para que de positivo

// d: mcd(a,b) ( d > 0 )
// x,y enteros tales que a*x + b*y = d
// las demas soluciones son ( x + (b/d)t , y - (a/d)t )
void gcdextend(ll a, ll b, ll &x, ll &y, ll &d){
    if( b == 0){
        if(a>0) x = 1, y = 0 , d = a ;
        else x = -1 , y = 0 , d = -a ;
        return ;
    }
    gcdextend(b,a%b,x,y,d) ;
    ll x1 = y , y1 = x - (a/b)*y ;
    x = x1 , y = y1 ;
}

// menor positivo que es u modulo modPos
inline ll ADDTOPOSITIVE(ll u, ll modPOS){
    if(modPOS < 0) modPOS = -modPOS ;
    if(u >= 0) return u%modPOS;
    u = -u ;
    if(u%modPOS == 0) return 0;
    return modPOS*((u/modPOS)+1) - u ;
}

// Encuentra el menor positivo que es
// de la forma ax + b , my + n
// los demas son de la forma ans + ((a/d)*m)*t
// retorna -1 si no hay solucion ( mcd(a,m) no divide a n - b )
inline ll FINDmenorLCS(ll a,ll b,ll m,ll n){

```

```

//Cuidado con el caso a = 0 , m = 0 ,
//porque es solo verificar b = n

a = abs(a) ; m = abs(m) ;
if( a == 0 ) {
    swap(a,m);
    swap(b,n) ;
}
if( m == 0 ){
    if( (n-b) % a == 0) return n ;
    else return -1 ;
}
ll x , y , d ;
gcdextend(a,m,x,y,d) ;
if((n-b)%d != 0) return -1 ;
ll temp = a*x*((n-b)/d) + b ;
temp = ADDTOPOSITIVE(temp,a*(m/d)) ;
return temp ;
}

//Finds partition of n (number of ways to obtain n as a sum of positive
numbers)

int partition(int n) {
    int[] dp = new int[n + 1];
    dp[0] = 1;
    for (int i = 1; i <= n; i++) {
        for (int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0; j++, r *= -1) {
            dp[i] += dp[i - (3 * j * j - j) / 2] * r;
            if (i - (3 * j * j + j) / 2 >= 0) {
                dp[i] += dp[i - (3 * j * j + j) / 2] * r;
            }
        }
    }
    return dp[n];
}

```

## 4.7 Pollard Rho

```

#define MAXL (50000>>5)+1
#define GET(x) (mark[x>>5]>>(x&31)&1)
#define SET(x) (mark[x>>5] |= 1<<(x&31))

```

```

int mark[MAXL];
int P[50000], Pt = 0;
void sieve() {
    register int i, j, k;
    SET(1);
    int n = 46340;
    for (i = 2; i <= n; i++) {
        if (!GET(i)) {
            for (k = n/i, j = i*k; k >= i; k--, j -= i)
                SET(j);
            P[Pt++] = i;
        }
    }
}

ll mul(unsigned ll a, unsigned ll b, unsigned ll mod) {
    ll ret = 0;
    for (a %= mod, b %= mod; b != 0; b >>= 1, a <<= 1, a = a >= mod ? a -
        mod : a) {
        if (b&1) {
            ret += a;
            if (ret >= mod) ret -= mod;
        }
    }
    return ret;
}

void exgcd(ll x, ll y, ll &g, ll &a, ll &b) {
    if (y == 0)
        g = x, a = 1, b = 0;
    else
        exgcd(y, x%y, g, b, a), b -= (x/y) * a;
}

ll inverse(ll x, ll p) {
    ll g, b, r;
    exgcd(x, p, g, r, b);
    if (g < 0) r = -r;
    return (r%p + p)%p;
}

ll mpow(ll x, ll y, ll mod) { // mod < 2^32
    ll ret = 1;
    while (y) {
        if (y&1)
            ret = (ret * x)%mod;
        y >>= 1, x = (x * x)%mod;
    }
}

```

```

    }
    return ret % mod;
}

ll mpow2(ll x, ll y, ll mod) {
    ll ret = 1;
    while (y) {
        if (y&1)
            ret = mul(ret, x, mod);
        y >>= 1, x = mul(x, x, mod);
    }
    return ret % mod;
}

int isPrime(ll p) { // implements by miller-babin
    if (p < 2 || !(p&1)) return 0;
    if (p == 2) return 1;
    ll q = p-1, a, t;
    int k = 0, b = 0;
    while (!(q&1)) q >>= 1, k++;
    for (int it = 0; it < 2; it++) {
        a = rand()%(p-4) + 2;
        t = mpow2(a, q, p);
        b = (t == 1) || (t == p-1);
        for (int i = 1; i < k && !b; i++) {
            t = mul(t, t, p);
            if (t == p-1)
                b = 1;
        }
        if (b == 0)
            return 0;
    }
    return 1;
}

ll pollard_rho(ll n, ll c) {
    ll x = 2, y = 2, i = 1, k = 2, d;
    while (true) {
        x = (mul(x, x, n) + c);
        if (x >= n) x -= n;
        d = __gcd(x - y, n);
        if (d > 1) return d;
        if (++i == k) y = x, k <<= 1;
    }
    return n;
}

```

```

}

void factorize(int n, vector<ll> &f) {
    for (int i = 0; i < Pt && P[i]*P[i] <= n; i++) {
        if (n%P[i] == 0) {
            while (n%P[i] == 0)
                f.push_back(P[i]), n /= P[i];
        }
    }
    if (n != 1) f.push_back(n);
}

void llfactorize(ll n, vector<ll> &f) {
    if (n == 1)
        return ;
    if (n < 1e+9) {
        factorize(n, f);
        return ;
    }
    if (isPrime(n)) {
        f.push_back(n);
        return ;
    }

    ll d = n;
    for (int i = 2; d == n; i++)
        d = pollard_rho(n, i);
    llfactorize(d, f);
    llfactorize(n/d, f);
}

vector<ll> f;
map<ll, int> r;

int main() {
    sieve();
    ll n;
    scanf("%lld", &n);
    llfactorize(n, f);
    for (auto &x : f) r[x]++;
    ll last;
    for (auto it = r.begin(); it != r.end(); it++) {
        if (it != r.begin()) printf(" ");
        last = it -> first;
        printf("%lld", last);
    }
}

```

```

    if (it->second > 1){
        for( int i = 0 ; i < it->second - 1 ; ++i ) printf(" %lld",last
            ) ;
    }
}
return 0;
}

```

## 4.8 Simplex Method

```

// Two-phase simplex algorithm for solving linear programs:
//   maximize   c^T x
//   subject to Ax <= b
//               x >= 0
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD &A, const VD &b, const VD &c) :
        m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, VD(n+2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j < n; j++)
            D[i][j] = A[i][j];
        for (int i = 0; i < m; i++)
            B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];
        for (int j = 0; j < n; j++)
            N[j] = j; D[m][j] = -c[j];
    }
}

```

```

        N[n] = -1; D[m+1][n] = 1;
    }

    void Pivot(int r, int s) {
        for (int i = 0; i < m+2; i++) if (i != r)
            for (int j = 0; j < n+2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n+2; j++) if (j != s)
            D[r][j] /= D[r][s];
        for (int i = 0; i < m+2; i++) if (i != r)
            D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }

    bool Simplex(int phase) {
        int x = phase == 1 ? m+1 : m;
        while (true) {
            int s = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                if (s == -1 || D[x][j] < D[x][s] ||
                    D[x][j] == D[x][s] && N[j] < N[s]) s
                    = j;
            }
            if (D[x][s] >= -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] <= 0) continue;
                if (r == -1 ||
                    D[i][n+1] / D[i][s] < D[r][n+1] / D[r][s] ||
                    D[i][n+1] / D[i][s] == D[r][n+1] / D[r][s]
                    && B[i] < B[r]) r = i;
            }
            if (r == -1) return false;
            Pivot(r, s);
        }
    }

    DOUBLE Solve(VD &x) {
        int r = 0;
        for (int i = 1; i < m; i++) if (D[i][n+1] < D[r][n+1]) r =
            i;
    }
}

```

```

    if (D[r][n+1] <= -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m+1][n+1] < -EPS)
            return -numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++) if (B[i] == -1) {
            int s = -1;
            for (int j = 0; j <= n; j++)
                if (s == -1 || D[i][j] < D[i][s] ||
                    D[i][j] == D[i][s] && N[j] < N[s]) s = j;
            Pivot(i, s);
        }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n)
        x[B[i]] = D[i][n+1];
    return D[m][n+1];
}

int main() {

    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        { 6, -1, 0 },
        { -1, -5, 0 },
        { 1, 5, 1 },
        { -1, -5, -1 }
    };
    DOUBLE _b[m] = { 10, -4, 5, -5 };
    DOUBLE _c[n] = { 1, -1, 0 };

    VVD A(m);
    VD b(_b, _b + m);
    VD c(_c, _c + n);
    for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

    LPSolver solver(A, b, c);
    VD x;
    DOUBLE value = solver.Solve(x);

    cerr << "VALUE: " << value << endl;
    cerr << "SOLUTION:";

```

```

    for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
    cerr << endl;
    return 0;
}

```

## 4.9 Teorema de Lucas

```

ll comb[105][105];

//Devuelve la comb(n,k) % m para n,k grandes y m pequeno

ll lucas( ll n , ll k , ll m ){
    //Se puede precalcular la combinatoria afuera
    REP(i,0,52) REP(j,0,52){
        if( j == 0 ) comb[i][0] = 1;
        else if( j > i ) comb[i][j] = 0;
        else comb[i][j] = ( comb[i-1][j] + comb[i-1][j-1] ) % m;
    }

    ll ans = 1, x, y;

    while( n ){
        x = n % m, y = k % m;
        ans = ( ans * comb[x][y] ) % m;
        n /= m, k /= m;
    }
    return ans;
}

```

## 5 Misc

### 5.1 Centroid Decomposition

```

#define N 100002

inline ll ma(ll a, ll b){ return ((a>b)? a:b);}
inline ll mi(ll a, ll b){return ((a>b)? b:a);}

struct CD{
    vector< int > graph[N];

```

```

int sub[N],p[N];
//sub[i]: size del nodo i luego de descomponer el tree
//p[i]: padre del nodo i luego de descomponer el tree
//notar que el padre del centroid es -2
// el tree esó 1 0 base
//para inicializar addEddge(a,b);
//para construir el centroid tree, solo llamar init(root); root:
    root del tree
void addEdge(int &a, int &b){
    graph[a].pb(b);
    graph[b].pb(a);
}
inline void dfs(int cur, int parent){
    sub[cur] = 1;
    for(int i = 0; i < sz(graph[cur]); ++i){
        int to = graph[cur][i];
        if(to != parent && p[to] == -1){
            dfs(to, cur);
            sub[cur] += sub[to];
        }
    }
}
inline void decompose(int cur, int parent, int sb, int prevc){
    for(int i = 0; i < sz(graph[cur]); ++i){
        int to = graph[cur][i];
        if(to != parent && p[to] == -1 && (2 * sub[to] > sb)
        ){
            decompose(to, cur, sb, prevc);
            return;
        }
    }
    p[cur] = prevc;
    for(int i = 0; i < sz(graph[cur]); ++i){
        int to = graph[cur][i];
        if(p[to] == -1){
            dfs(to, -1);
            decompose(to, cur, sub[to], cur);
        }
    }
}
inline void init(int start){
    for(int i = 0; i < N; ++i) p[i] = -1;
    dfs(start, -1);
    decompose(start, -1, sub[start], -2);
}

```

```

};
int cnt=1;
vi adj[N];
int d[N];
inline void make(int &u, int x, int depth){
    d[u]=depth;
    for(auto v :adj[u]) if(v!=x) make(v,u,depth+1);
}
int main() {
    fastio;
    int n; cin>>n;
    CD cd; //cd.n=n;
    REP(i,0,n-1) {
        ll a,b; cin>>a>>b;
        cd.addEdge(a,b);
    }
    cd.init(1);
    int pa, root;
    REP(i,1,n+1) {
        pa=cd.p[i];
        if(pa== -2) root=i;
        if(pa!= -2) {
            adj[i].pb(pa);
            adj[pa].pb(i);
        }
    }
    make(root,0,1);
    char is;map<int, string> m;int k=1,flag=1;
    for(is='A'; is<='Z'; is++) m[k++]=is;
    REP(i,1,n+1) if(d[i]>26) flag=0;
    if(flag==0) cout<<"Impossible!"<<endl;
    if(flag==1) {
        REP(i,1,n+1) cout<<m[d[i]]<<endl;
    }
    return 0;
}

```

## 5.2 Closest Pair

```

//Closest Pair Algorithm with Sweep
//Complexity: O(nlogn)

```

```

#define MAX_N 100000

```



```

#define px second
#define py first
typedef pair<long long, long long> point;

int N;
point P[MAX_N];
set<point> box;

bool compare_x(point a, point b){ return a.px<b.px; }

inline double dist(point a, point b){
    return sqrt((a.px-b.px)*(a.px-b.px)+(a.py-b.py)*(a.py-b.py));
}

double closest_pair(){
    if(N<=1) return -1;
    sort(P,P+N,compare_x);
    double ret = dist(P[0],P[1]);
    box.insert(P[0]);
    set<point> :: iterator it;
    for(int i = 1,left = 0;i<N;++i){
        while(left<i && P[i].px-P[left].px>ret) box.erase(P[left++]);
        for(it = box.lower_bound(make_pair(P[i].py-ret,P[i].px-ret));
            it!=box.end() && P[i].py+ret>=(*it).py;++it)
            ret = min(ret, dist(P[i],*it));
        box.insert(P[i]);
    }
    return ret;
}

```

### 5.3 Convex Hull Trick

```

// Simple Hull
struct HullSimple { // Upper envelope for Maximum.
    // Special case: strictly increasing slope in insertions,
    // increasing value in queries.
    deque<pair<ll, ll> > dq;
    ld cross(pair<ll, ll> l1, pair<ll, ll> l2){
        return (ld)(l2.snd - l1.snd) / (ld)(l1.fst - l2.fst);
    }
    void insert_line(ll m, ll b){
        pair<ll,ll> line = mp(m,b);

```

```

        while (sz(dq) > 1 && cross(line, dq[sz(dq)-1]) <=
            cross(dq[sz(dq)-1], dq[sz(dq)-2])) dq.pop_back();
        dq.pb(mp(m,b));
    }

    ll eval(pair<ll, ll> line, ll x){
        return line.fst * x + line.snd;
    }

    ll eval(ll x){
        while (sz(dq) > 1 && eval(dq[0], x) < eval(dq[1],x))
            dq.pop_front();
        return eval(dq[0],x);
    }
};

// Dynamic Hull
// Compile with g++ -std=c++11 file.cpp -o file
typedef long double ld;
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

// Upper envelope for Maximum
struct HullDynamic : public multiset<Line> {
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >=
            (y->b - z->b)*(y->m - x->m);
    }
    void insert_line(ll m, ll b) {

```

```

    auto y = insert({ m, b });
    y->succ = [=] { return next(y) == end() ? 0: &*next(y); };
    if (bad(y)) { erase(y); return; }
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
}
ll eval(ll x) {
    auto l = *lower_bound((Line) { x, is_query });
    return l.m * x + l.b;
}
};

```

## 5.4 Dates

```

//
// Time - Leap years
//

// A[i] has the accumulated number of days from months previous to i
const int A
[13] = { 0, 0, 31, 59, 90, 120, 151, 181, 212, 243, 273, 304, 334 };
// same as A, but for a leap year
const int B
[13] = { 0, 0, 31, 60, 91, 121, 152, 182, 213, 244, 274, 305, 335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y / 400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 == 0 && y % 100 != 0)
; }

// number of days in blocks of years
const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4 = 4*365 + 1;
const int p1 = 365;
int date_to_days(int d, int m, int y)
{
    return (y - 1) * 365 + leap_years(y - 1) + (is_leap(y) ? B[m] : A[m]) +
        d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
    bool top100; // are we in the top 100 years of a 400 block?
    bool top4;   // are we in the top 4 years of a 100 block?
    bool top1;   // are we in the top year of a 4 block?

```

```

y = 1;
top100 = top4 = top1 = false;

y += ((days-1) / p400) * 400;
d = (days-1) % p400 + 1;

if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

if (d > p1*3) top1 = true, d -= p1*3, y += 3;
else y += (d-1) / p1, d = (d-1) % p1 + 1;

const int *ac = top1 && (!top4 || top100) ? B : A;
for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
d -= ac[m];
}

```

## 5.5 Divide and Conquer Trick

```

// Divide and Conquer DP optimization.
// Problem: dp[i][j] = min{>j} (func(j,k) + dp[i-1][k]).
// (That is, split n objects into k buckets with cost
// func per bucket). Necessary condition: argmin(dp[i][j]) <=
// argmin(dp[i][j+1]) (this is "opt")
// Naive complexity: O(kn^2)
// Improved complexity: O(knlog(n))
// Consider checking if opt[i+1][j] <= opt[i][j] <= opt[i][j+1]
// and using a knuth-like O(n^2) loop

const ll INF = 1e18;
int n, k;

ll c[8100];
ll s[8100];
ll dp[810][8100];

ll func(int i, int j){ return (s[j] - s[i])*(j-i); }

void go(int i, int l, int r, int optl, int optr){

```

```

    if (l >= r) return;
    int m = (l+r)/2;
    int opt = n;
    dp[i][m] = INF;
    for(int u = optr; u >= optl; u--){
        ll curr = dp[i-1][u] + func(m,u);
        if(curr < dp[i][m]){
            dp[i][m] = curr;
            opt = u;
        }
    }
    go(i,l,m,optl, opt);
    go(i,m+1,r,opt,optr);
}

```

```

int main(){
    fastio;
    cin >> n >> k;
    REP(i,0,n) cin >> c[i];
    s[0] = 0;
    REP(i,0,n+1) s[i] = s[i-1] + c[i-1];
    REP(i,1,k+1) dp[i][n] = INF;
    REP(i,0,n) dp[0][i] = INF;
    dp[0][n] = 0;
    REP(i,1,k+1) go(i,0,n,0,n);
    cout << dp[k][0] << endl;
    return 0;
}

```

//Divide and Conquer Trick by Ands

```

void compute(int cnt, int l, int r, int optl, int optr){
    if(l > r) return ;
    int mid = ( l + r ) >> 1 ;
    int opt = -1 ;
    ll value = 1e18 ;
    int last = cnt^1 ;
    for(int idx = optl ; idx <= min(mid-1,optr); ++idx){
        ll tmp = dp[last][idx] + C[idx][mid] ;
        if(tmp < value){
            value = tmp ;
            opt = idx ;
        }
    }
    dp[cnt&1][mid] = value ;
}

```

```

    compute(cnt, l, mid-1, optl, opt);
    compute(cnt, mid+1, r, opt, optr);
}

```

```

int main(){
    //casos base
    for(int cnt = 2; cnt <= m ; ++cnt) compute(cnt&1, 0, n-1, 0, n-1) ;
}

```

## 5.6 Fractions

```

struct Frac{
    int num, den;
    Frac(){
        num = 0; den = 1;
    }
    Frac(int a, int b): num(a), den(b){}
    Frac(int a):num(a), den(1){}

    void normalize(){
        if(num == 0){
            den = 1;
        }
        if(den < 0){
            den = -den;
            num = -num;
        }
    }

    Frac fix(int a, int b){
        if(!a) return Frac(0,1);
        if(!b) return Frac(oo,1);
        int foo = gcd(abs(a),abs(b));
        Frac ret = Frac(a/foo, b/foo);
        ret.normalize();
        return ret;
    }

    Frac operator + (const Frac& other){
        int num2 = num*other.den + den*other.num, den2 = den*other.den;
        return fix(num2,den2);
    }
}

```

```

Frac operator - (const Frac& other){
    int num2 = num*other.den - den*other.num, den2 = den*other.den;
    return fix(num2,den2);
}

Frac operator * (int c){
    int num2 = num*c, den2 = den;
    return fix(num2,den2);
}

Frac operator * (const Frac& other){
    int num2 = num*other.num, den2 = den*other.den;
    return fix(num2,den2);
}

Frac operator / (int c){
    int num2 = num, den2 = den * c;
    return fix(num2,den2);
}

Frac operator / (const Frac& other){
    int num2 = num*other.den, den2 = den*other.num;
    return fix(num2,den2);
}

bool operator < (const Frac& other) const{
    if(num * other.den < other.num*den) return true;
    return false;
}

bool operator == (const Frac& other) const{
    if(num == other.num && den == other.den) return true;
    return false;
}
};

```

## 5.7 Longest Increasing Subsequence

// Simple  $O(n \log n)$  Longest Increasing Subsequence  
 // Answer is stored in array b[N]

```
int LIS( vi &a ){
```

```

    int b[N];
    int sz = 0;
    REP(i,0,a.size()){
        int j = lower_bound( b , b + sz , a[ i ] ) - b;
        // (lower) a < b < c
        // (upper) a <= b <= c
        b[ j ] = a[ i ];
        if( j == sz ) sz++;
    }
    return sz;
}

```

## 5.8 Matrix Structure

```

const int MN = 111;
const int mod = 10000;

```

```

struct matrix {
    int r, c;
    int m[MN][MN];
}

```

```

matrix (int _r, int _c) : r (_r), c (_c) {
    memset(m, 0, sizeof m);
}

```

```

void print() {
    for (int i = 0; i < r; ++i) {
        for (int j = 0; j < c; ++j)
            cout << m[i][j] << " ";
        cout << endl;
    }
}

```

```

int x[MN][MN];
matrix & operator *= (const matrix &o) {
    memset(x, 0, sizeof x);
    for (int i = 0; i < r; ++i)
        for (int k = 0; k < c; ++k)
            if (m[i][k] != 0)
                for (int j = 0; j < c; ++j) {
                    x[i][j] = (x[i][j] + (m[i][k] * o.m[k][j]) % mod) % mod;
                }
    memcpy(m, x, sizeof(m));
}

```

```

    return *this;
}
};

void matrix_pow(matrix b, long long e, matrix &res) {
    memset(res.m, 0, sizeof res.m);
    for (int i = 0; i < b.r; ++i)
        res.m[i][i] = 1;

    if (e == 0) return;
    while (true) {
        if (e & 1) res *= b;
        if ((e >>= 1) == 0) break;
        b *= b;
    }
}

```

## 5.9 Ordered Set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef
tree<
    int,
    null_type,
    less<int>,
    rb_tree_tag,
    tree_order_statistics_node_update
>ordered_set;
// ordered_set
// X.find_by_order(k) returns an iterator to the k-th largest element (
// counting from zero)
// X.order_of_key(v) returns the number of items in a set that are
// strictly smaller than v
int main() {
    int N;
    ordered_set Y;
    Y.insert(5);
    trace (*Y.find_by_order(0));
}

```

## 5.10 Parallel Binary Search

```

//Cada query esta en (low[i], high[i]]
//Tocheck tiene los valores actuales a verificar
//en el bsearch

//Solved puede tener 1, -1
//1: el unico valor posible cumple
//-1: no hay respuesta

int low[MAXN];
int high[MAXN];
char solved[MAXN];
vector< int > tocheck[MAXN];

int main(){
    // Leer n, m
    // Leer a[i], b[i] (i en [1, m])
    // Leer q: queries
    // Leer x[i], y[i], z[i] (i en [0, q])

    for(int i = 0; i < q; ++i)
        low[i] = 0, high[i] = m;

    bool done = 0;
    DSU uf(n); // DSU structure
    int curvis;
    while(!done){
        done = 1;
        for(int i = 0; i < q; ++i){
            int mid = (low[i] + high[i]) >> 1;
            tocheck[mid].pb(i);
        }
        uf.clear(n);
        int last = -1;
        for(int value = 0; value <= m; ++value){
            if(tocheck[value].empty()) continue;
            for(int i = last + 1; i <= value; ++i)
                uf.join(a[i], b[i]);
            last = value;
            while(!tocheck[value].empty()){
                int id = tocheck[value].back();
                tocheck[value].pop_back();
                int u = x[id], v = y[id];
                int visited = z[id];
            }
        }
    }
}

```

```

        if(low[id] + 1 == high[id]) solved[id] = 1;
        if(uf.connected(u, v)) curvis = uf.size(u);
        else curvis = uf.size(u) + uf.size(v);
        if(curvis >= visited) high[id] = value;
        else low[id] = value;
        if(low[id] == high[id]) solved[id] = -1;
    }
}
for(int i = 0; i < q; ++i)
    if(solved[i] == 0) done = 0;
}
for(int i = 0; i < q; ++i)
    if(solved[i] == -1) cout << -1 << endl;
    else cout << high[i] << endl;
}

```

## 5.11 Unordered Map

```

unordered_map<int,int> mp;
mp.reserve(1024); // power of 2 is better
mp.max_load_factor(0.25); // 0.75 used in java

```

## 6 Network Flows

### 6.1 Bipartite Matching

```

// O(V*E) maximum bipartite matching
int p[MAX]; // Parent of right-node v in the matching
int vis[MAX]; // Whether left-node u has been visited
vi adj[MAX]; // Standard adjacency list

int match(int u) {
    if (vis[u]) return 0;
    vis[u] = 1;
    REP(i,0,adj[u].size()){
        int v = adj[u][i];
        if (p[v] == -1 || match(p[v])) {
            p[v] = u;
            return 1;
        }
    }
}

```

```

    }
    return 0;
}

int main(){
    // build adj here with n left nodes
    // and V total nodes
    int n = 1000000;
    int V = 2000000;
    int maxMatch = 0;
    REP(i,0,V) p[i] = -1;
    REP(u,0,n){
        REP(i,0,n) vis[i] = 0;
        maxMatch += match(u);
    }
    printf("Found %d matchings\n", maxMatch)
}

```

### 6.2 Dinic Flow

```

// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
// O(|V|^2 |E|)
//
// INPUT:
// - graph, constructed using AddEdge()
// - source and sink
//
// OUTPUT:
// - maximum flow value
// - To obtain actual flow values, look at edges with capacity > 0
// (zero capacity edges are residual edges).

```

```

typedef long long LL;

struct Edge {
    int u, v;
    LL cap, flow;
    Edge() {}
    Edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};

```

```

struct Dinic {
    int N;
    vector<Edge> E;
    vector<vector<int>> g;
    vector<int> d, pt;

    Dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}

    void AddEdge(int u, int v, LL cap) {
        if (u != v) {
            E.emplace_back(Edge(u, v, cap));
            g[u].emplace_back(E.size() - 1);
            E.emplace_back(Edge(v, u, 0));
            g[v].emplace_back(E.size() - 1);
        }
    }

    bool BFS(int S, int T) {
        queue<int> q({S});
        fill(d.begin(), d.end(), N + 1);
        d[S] = 0;
        while(!q.empty()) {
            int u = q.front(); q.pop();
            if (u == T) break;
            for (int k: g[u]) {
                Edge &e = E[k];
                if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
                    d[e.v] = d[e.u] + 1;
                    q.emplace(e.v);
                }
            }
        }
        return d[T] != N + 1;
    }

    LL DFS(int u, int T, LL flow = -1) {
        if (u == T || flow == 0) return flow;
        for (int &i = pt[u]; i < g[u].size(); ++i) {
            Edge &e = E[g[u][i]];
            Edge &oe = E[g[u][i]^1];
            if (d[e.v] == d[e.u] + 1) {
                LL amt = e.cap - e.flow;
                if (flow != -1 && amt > flow) amt = flow;
                if (LL pushed = DFS(e.v, T, amt)) {

```

```

                    e.flow += pushed;
                    oe.flow -= pushed;
                    return pushed;
                }
            }
        }
        return 0;
    }

    LL MaxFlow(int S, int T) {
        LL total = 0;
        while (BFS(S, T)) {
            fill(pt.begin(), pt.end(), 0);
            while (LL flow = DFS(S, T))
                total += flow;
        }
        return total;
    }
};

```

### 6.3 Edmonds Blossom

---

```

// Maximum general matching (not necessarily bipartite)
// Make sure to set N in main()
// Claimed  $O(N^4)$  running time

```

```

int N; // the number of vertices in the graph
typedef vector<int> vi;
typedef vector< vector<int> > vvi;
vi match;
vi vis;

```

```

void couple(int n, int m) { match[n]=m; match[m]=n; }

```

```

// True if augmenting path or a blossom (if blossom is non-empty).
// the dfs returns true from the moment the stem of the flower is
// reached and thus the base of the blossom is an unmatched node.
// blossom should be empty when dfs is called and
// contains the nodes of the blossom when a blossom is found.
bool dfs(int n, vvi &conn, vi &blossom) {
    vis[n]=0;
    REP(i, 0, N) if(conn[n][i]) {
        if(vis[i]==-1) {

```

```

    vis[i]=1;
    if(match[i]==-1 || dfs(match[i], conn, blossom)) {
        couple(n,i);
        return true;
    }
}
if(vis[i]==0 || SZ(blossom)) { // found flower
    blossom.pb(i); blossom.pb(n);
    if(n==blossom[0]) { match[n]=-1; return true; }
    return false;
}
}
return false;
}

// search for an augmenting path.
// if a blossom is found build a new graph (newconn) where the
// (free) blossom is shrunken to a single node and recurse.
// if a augmenting path is found it has already been augmented
// except if the augmented path ended on the shrunken blossom.
// in this case the matching should be updated along the
// appropriate direction of the blossom.
bool augment(vvi &conn) {
    REP(m, 0, N) if(match[m]==-1) {
        vi blossom;
        vis=vi(N,-1);
        if(!dfs(m, conn, blossom)) continue;
        if(SZ(blossom)==0) return true; // augmenting path found

// blossom is found so build shrunken graph
        int base=blossom[0], S=SZ(blossom);
        vvi newconn=conn;
        REP(i, 1, S-1) REP(j, 0, N)
            newconn[base][j]=newconn[j][base]=conn[blossom[i]][j];
        REP(i, 1, S-1) REP(j, 0, N)
            newconn[blossom[i]][j]=newconn[j][blossom[i]]=0;
        newconn[base][base]=0; // is now the new graph
        if(!augment(newconn)) return false;
        int n=match[base];

// if n!=-1 the augmenting path ended on this blossom
        if(n!=-1) REP(i, 0, S) if(conn[blossom[i]][n]) {
            couple(blossom[i], n);
            if(i&1) for(int j=i+1; j<S; j+=2)

```

```

                couple(blossom[j],blossom[j+1]);
            else for(int j=0; j<i; j+=2)
                couple(blossom[j],blossom[j+1]);
            break;
        }
        return true;
    }
    return false;
}

// conn is the NxN adjacency matrix
// returns the number of edges in a max matching.
int edmonds(vvi &conn) {
    int res=0;
    match=vi(N,-1);
    while(augment(conn)) res++;
    return res;
}

/*****
set<pair<int,int> > used;
int main(){
    int n;
    cin >> n;
    N = n;
    vvi conn;
    vi tmp;
    tmp.assign(n,0);
    REP(i, 0, n) conn.push_back(tmp);
    int u, v;
    while(cin >> u >> v){
        u--; v--;
        if(u > v) swap(u,v);
        if(used.count(make_pair(u,v))) continue;
        used.insert(make_pair(u,v));
        conn[u][v] = conn[v][u] = 1;
    }
    int res = edmonds(conn);
    cout<<res*2<<endl;
    REP(i, 0, n) {
        if(match[i] > i){
            cout<<i+1<<" "<<match[i] + 1<<endl;
        }
    }
    return 0;
}

```



---

}

## 6.4 Min Cost Max Flow

---

```
const int MAXN = 5010;

const ll INF = 1e15;
struct edge { int dest; ll origcap, cap; ll cost; int rev; };

struct MinCostMaxFlow {
    vector<edge> adj[MAXN];
    ll dis[MAXN], cost;
    int source, target, iter;
    ll cap;
    edge* pre[MAXN];
    int queued[MAXN];
    MinCostMaxFlow () {}
    void AddEdge(int from, int to, ll cap, ll cost) {
        adj[from].push_back(edge {to, cap, cap, cost, (int)adj[to].size()});
        adj[to].push_back(edge {from, 0, 0, -cost, (int)adj[from].size() - 1});
    }

    bool spfa() {
        REP(i, 0, MAXN) queued[i] = 0;
        fill(dis, dis + MAXN, INF);
        queue<int> q;
        pre[source] = pre[target] = 0;
        dis[source] = 0;
        q.emplace(source);
        queued[source] = 1;
        while (!q.empty()) {
            int x = q.front();

            ll d = dis[x];
            q.pop();
            queued[x] = 0;
            for (auto& e : adj[x]) {
                int y = e.dest;
                ll w = d + e.cost;
                if (e.cap < 1 || dis[y] <= w) continue;
            }
        }
    }
};
```

```
                dis[y] = w;
                pre[y] = &e;
                if (!queued[y]) {
                    q.push(y);
                    queued[y] = 1;
                }
            }
        }
        edge* e = pre[target];

        if (!e) return 0;
        while (e) {
            edge& rev = adj[e->dest][e->rev];
            e->cap -= cap;
            rev.cap += cap;
            cost += cap * e->cost;
            e = pre[rev.dest];
        }
        return 1;
    }
};

pair<ll, ll> GetMaxFlow(int S, int T) {
    cap = 1, source = S, target = T, cost = 0;
    while(spfa()) {}
    ll totflow = 0;
    for(auto e: adj[source]){
        totflow += (e.origcap - e.cap);
    }
    return make_pair(totflow, cost);
};
```

## 6.5 Push Relabel Max Flow

---

```
// Fast  $O(|V|^3)$  flow, works for  $n \sim 5000$  with no problem
// Actual flow values in edges with cap > 0 (0 cap = residual)

typedef long long LL;

struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};
```

```

};

struct PushRelabel {
    int N;
    vector<vector<Edge> > G;
    vector<LL> excess;
    vector<int> dist, active, count;
    queue<int> Q;

    PushRelabel(int N) :
        N(N), G(N), excess(N), dist(N), active(N), count(2*N) {}

    void AddEdge(int from, int to, int cap) {
        G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
        if (from == to) G[from].back().index++;
        G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
    }

    void Enqueue(int v) {
        if (!active[v] && excess[v] > 0) {
            active[v] = true; Q.push(v);
        }
    }

    void Push(Edge &e) {
        int amt = min(excess[e.from], LL(e.cap - e.flow));
        if (dist[e.from] <= dist[e.to] || amt == 0) return;
        e.flow += amt;
        G[e.to][e.index].flow -= amt;
        excess[e.to] += amt;
        excess[e.from] -= amt;
        Enqueue(e.to);
    }

    void Gap(int k) {
        for (int v = 0; v < N; v++) {
            if (dist[v] < k) continue;
            count[dist[v]]--;
            dist[v] = max(dist[v], N+1);
            count[dist[v]]++;
            Enqueue(v);
        }
    }

    void Relabel(int v) {

```

```

        count[dist[v]]--;
        dist[v] = 2*N;
        for (int i = 0; i < G[v].size(); i++)
            if (G[v][i].cap - G[v][i].flow > 0)
                dist[v] = min(dist[v], dist[G[v][i].to] + 1);
        count[dist[v]]++;
        Enqueue(v);
    }

    void Discharge(int v) {
        for (int i = 0; excess[v] > 0 && i < G[v].size(); i++)
            Push(G[v][i]);
        if (excess[v] > 0) {
            if (count[dist[v]] == 1) Gap(dist[v]);
            else Relabel(v);
        }
    }

    LL GetMaxFlow(int s, int t) {
        count[0] = N-1;
        count[N] = 1;
        dist[s] = N;
        active[s] = active[t] = true;
        for (int i = 0; i < G[s].size(); i++) {
            excess[s] += G[s][i].cap;
            Push(G[s][i]);
        }

        while (!Q.empty()) {
            int v = Q.front();
            Q.pop();
            active[v] = false;
            Discharge(v);
        }

        LL totflow = 0;
        for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;
        return totflow;
    }
};

```

## 7 Strings

### 7.1 Aho Corasick + Compression

---

```
// Aho Corasick automaton. O(n) in size of Trie.
// Allows searching for a dictionary of patterns in a string.
// Consider using DP[u, pos], for instance.

const int MAXN = 500000; // Sum of words*length
const int SZA = 26;      // Alphabet size

map<int,int> adj[MAXN]; // Trie
int isEnd[MAXN];       // Example: How many words end at node u
int gid;               // Id of last node set
int f[MAXN];           // Aho Corasick failure function

void init(int id){
    isEnd[id] = 0;
    adj[id].clear();
}

void add(string s){
    int u = 0; // Current node
    REP(p,0,sz(s)){
        int id = s[p] - 'a';
        if (!adj[u].count(id)){
            adj[u][id] = ++gid; // Lazy initialization
            init(gid);
        }
        u = adj[u][id];
    }
    isEnd[u]++;
}

void build(){
    // BFS-DP Aho Corasick construction
    queue<int> q;
    f[0] = 0;
    REPIT(it, adj[0]){
        int u = it->snd;
        q.push(u);
        f[u] = 0;
    }
    while (!q.empty()){

```

```
        int e = q.front();
        q.pop();
        REPIT(it, adj[e]){
            int i = it->fst;
            int u = it->snd;
            q.push(u);
            int v = f[e];
            while (v && !adj[v].count(i)) v = f[v];
            f[u] = (adj[v].count(i) ? adj[v][i] : 0);
            // Aggregate necessary information here
            // In general, S[u] += S[f[u]]
            isEnd[u] += isEnd[f[u]];
        }
    }

// Search string s for all strings in trie
ll search(string s){
    ll ans = 0;
    int u = 0;
    REP(p,0,sz(s)){
        int id = s[p] - 'a';
        while (u && !adj[u].count(id)) u = f[u];
        if (adj[u].count(id)) u = adj[u][id];
        ans += isEnd[u];
    }
    return ans;
}

int main(){
    gid = 0;
    init(0);
    // Ready for add(s), build(), search(t)
    return 0;
}
```

### 7.2 Aho Corasick

---

```
// -----aho corasick-----
// cantidad de repeticiones de cada string sobre un text en O(M+N)

#define N 100000 // tamaño del text
#define M 1005 // tamaño de cada string a buscar
```

```

ll n;
char text[N]; // string donde buscar
char buf[N]; // string a buscar
ll cnt[M]; // cnt[i]: cantidad de ocurrencias del string i

ll root, nodes;
// nodes: cantidad de nodos en el trie,
//root: que nodo del trie estoy
struct trieNode{
    bool seen;
    ll matchFail, fail;
    vi matches;
    map< char, ll > next;
    trieNode(){}
    trieNode(bool seen, ll &matchFail, ll &fail, vi & matches, map<char,
        ll> & next):
        seen(seen), matchFail(matchFail), fail(fail), matches(matches), next(
            next){}
} trie[N];
// antes de insertar, notar que root=0 y nodes=1
inline void insert(char * s, ll wordId){ //
    //wordId: id del string
    ll x = root, ta=strlen(s);
    REP(i,0,ta){
        ll &nxt = trie[x].next[ s[i] ];
        if (!nxt) nxt = ++nodes;
        x = nxt;
    }
    trie[x].matches.push_back(wordId);
}

inline ll find(ll x, char ch){
    while (x && !trie[x].next.count(ch)) x = trie[x].fail;
    return x ? trie[x].next[ch] : root;
}

inline void bfs(){
    trie[root].fail = 0;
    queue< ll > q;
    q.push(root);
    while(q.empty()){
        ll u = q.front(), v; q.pop();
        char ch;
        for (auto &it: trie[u].next){

```

```

            ch = it.fst, v = it.snd;
            ll f = find(trie[u].fail, ch);
            trie[v].fail = f;
            trie[v].matchFail = trie[f].matches.empty() ? trie[f].matchFail
                : f;
            q.push(v);
        }
    }
}

inline void search(){
    ll x = root;
    ll ta=strlen(text);
    REP(i,0,ta){
        x = find(x, text[i]);
        for (ll t = x; t && !trie[t].seen; t = trie[t].matchFail){
            trie[t].seen = true;
            REP(j,0, sz(trie[t].matches)) cnt[trie[t].matches[j]] ++;
        }
    }
}

int main(){
    root = ++nodes; //inicializacion
    scanf( "%s", &text );
    scanf( "%d", &n );
    REP(i,0, n){
        scanf( "%s", &buf );
        insert(buf, i);
    }
    bfs(); search();
    REP(i,0,n) printf( "%s\n", cnt[i]>0 ? "Y" : "N" );

    return 0;
}

```

## 7.3 Knuth Morris Pratt

// KMP algorithm for finding a pattern in a string in  $O(n+m)$ .

```

const int MAX = 1000000;

int b[MAX]; // Fail function
char p[MAX]; // Pattern string

```

```

char t[MAX]; // Text string

int n; // Text string length
int m; // Pattern string length

void kmpPreprocess(){
    int i=0, j=-1;
    b[i]=j;
    while (i<m){
        while (j>=0 && p[i]!=p[j]) j=b[j];
        i++; j++;
        b[i]=j;
    }
}

void report(int x){
    cout << "Found on: " << x << endl;
}

void kmpSearch(){
    int i=0, j=0;
    while (i<n){
        while (j>=0 && t[i]!=p[j]) j=b[j];
        i++; j++;
        if (j==m){
            report(i-j);
            j=b[j];
        }
    }
}

```

## 7.4 Manacher Algorithm

// Manacher's algorithm for finding all palindromes  
 // in a string in  $O(n)$ .

```

int n;
char s[200200];
char aux[100100];
int p[200200];

int main(){
    scanf("%s", s, &n);

```

```

s[0] = '^';
s[1] = '#';
REP(i,0,n){
    s[2*i+2] = aux[i];
    s[2*i+3] = '#';
}
s[2*n+2] = '\0';
int c = 0, r = 0;
REP(i,0,2*n+2){
    if (i > r) p[i] = 0;
    else p[i] = min(r-i, p[2*c-i]);
    while (s[i+p[i]+1] == s[i-p[i]-1]) p[i]++;
    if (i + p[i] > r){
        c = i;
        r = i + p[i];
    }
}

printf("%s", s);
REP(i,0,2*n+2) {
    printf("%d", p[i]);
}
printf("\n");
return 0;
}

```

## 7.5 Palindromic Tree

// adamant's palindromic tree online  $O(n \log(|E|))$  construction  
 // Tutorial: <http://adilet.org/blog/25-09-14/>  
 // Add/Delete operation can be supported in  $O(\log n)$  by doing  
 // check(link[v]), v = slink[v] in get\_link  
 // (periodicity -> same initial char)  
 const int maxn = 5e5, sigma = 26, INF = 1e9;  
 int s[maxn], len[maxn], link[maxn], to[maxn][sigma];  
 int n, last, sz;  
 // All these optional (palindromic factoring)  
 int d[maxn], slink[maxn], dpe[maxn], dpo[maxn];  
 int anse[maxn], anso[maxn], prve[maxn], prvo[maxn];  
 void init(){ // Call with n=0  
 s[n++] = -1;  
 link[0] = 1;

```

    len[1] = -1;
    sz = 2;
    anse[0] = 0;
    anso[0] = INF;
}

int get_link(int v){
    while(s[n - len[v] - 2] != s[n - 1]) v = link[v];
    return v;
}

ii getmin(int v, int* ans, int* dp, int* prv){
    dp[v] = ans[n - (len[slink[v]] + d[v]) - 1];
    int best = n - (len[slink[v]] + d[v]) - 1;
    if (d[v] == d[link[v]]){
        if (dp[v] > dp[link[v]]){
            dp[v] = dp[link[v]];
            best = prv[n-1-d[v]];
        }
    }
    return mp(dp[v] + 1, best);
}

void add_letter(int c){
    s[n++] = c;
    last = get_link(last);
    if(!to[last][c]) {
        len[sz] = len[last] + 2;
        link[sz] = to[get_link(link[last])][c];
        d[sz] = len[sz] - len[link[sz]];
        if (d[sz] == d[link[sz]]) slink[sz] = slink[link[sz]];
        else slink[sz] = link[sz];
        to[last][c] = sz++;
    }
    last = to[last][c];

    anse[n-1] = INF;
    for (int v = last; len[v] > 0; v = slink[v]){
        ii acte = getmin(v, anso, dpe, prve);
        if (act.fst < anse[n-1]){
            anse[n-1] = act.fst;
            prve[n-1] = act.snd;
        }
    }
}

```

```

    anso[n-1] = INF;
    for (int v = last; len[v] > 0; v = slink[v]){
        ii act = getmin(v, anse, dpo, prvo);
        if (act.fst <= anso[n-1]){
            anso[n-1] = act.fst;
            prvo[n-1] = act.snd;
        }
    }
}

```

## 7.6 Suffix Array

```

// -----Suffix array-----
// construccion en nlog^2(n)
//usa lcp(x,y)=mi[lcp(x,x+1),lcp(x+1,x+2)...lcp(y-1,y)]
//construye el lcp(x,y) con sparce table, notar que los indices son 0 base
//s=ababa
//s1[0]=ababa,s1[1]=baba,s1[2]=aba, s1[3]=ba,s1[4]=a, s1[5]='$'
//s2={$,a,aba,ababa,ba,baba}={5,4,2,0,3,1}=r
//r[i] lista de los sufijos ordenados en 0 base
//indice de s1={ababa,baba,aba,ba,a,$}={3,5,2,4,1,0}=p
//p[i] posicion del i substring en el suffix array (s1) en 0 base

#define N 100010
#define M 20
inline ll ma(ll a, ll b){ return ((a-b>0)? a:b);}
inline ll mi(ll a, ll b){return ((a-b>0)? b:a);}

struct SA{
    //asignar s:string(char), n fitamao del string
    ll n,t;
    ll p[N],r[N],h[N];
    char s[N];
    ll rmq[M][N];
    ll flog2[N];
    inline void fix_index(ll b, ll e){
        ll lastpk, pk, d;
        lastpk = p[r[b]+t];
        d = b;
        REP(i,b,e){
            if (((pk = p[r[i]+t]) != lastpk) && (b > lastpk || pk >= e)){
                lastpk = pk;
                d = i;
            }
        }
    }
}

```

```

    }
    p[r[i]] = d;
}
}
//calcula de r y p
inline void suff_arr(){
    s[n++] = '$';
    ll bc[256];
    REP(i,0,256) bc[i]=0;
    REP(i,0,n) bc[(ll)s[i]]++;
    REP(i,1,256) bc[i] += bc[i-1];
    RREP(i,n-1,0) r[--bc[(ll)s[i]]] = i;
    RREP(i,n-1,0) p[i] = bc[(ll)s[i]];
    for (t = 1; t < n; t<=1){
        for (ll i = 0, j = 1; i < n; i = j++){
            while (j < n && p[r[j]] == p[r[i]]) ++j;
            if (j-i > 1){
                sort(r+i, r+j, [&](const ll &i, const ll &j){return p[i+t] < p[j+t];});
                fix_index(i, j);
            }
        }
    }
}
//calcula h[i] en O(n) usando Kasai algorithm
inline void initlcp(){
    ll tam = 0, j;
    REP(i,0,n-1){
        j = r[p[i]-1];
        while(s[i+tam] == s[j+tam]) ++tam;
        h[p[i]-1] = tam;
        if (tam > 0) --tam;
    }
}
//construccion del RMQ para hallar lcp en un rango
inline void makelcp(){
    initlcp();
    REP(i,0,n-1) rmq[0][i] = h[i];
    ll lg = 0, pw = 1;
    do{
        REP(i,pw,pw*2) flog2[i] = lg;
        lg++; pw*=2;
        REP(i,0,n-1){
            if (i+pw/2 < n-1) rmq[lg][i] = mi(rmq[lg-1][i], rmq[lg-1][i+pw/2]);
            else rmq[lg][i] = rmq[lg-1][i];
        }
    }
}

```

```

    }
    } while(pw < n);
}
//calcula el lcp en [i,j] de s1(suffix array);
inline ll lcp(ll i, ll j){
    if (i == j) return n - r[i] - 1;
    ll lg = flog2[j-i], pw = (1<<lg);
    return mi(rmq[lg][i], rmq[lg][j-pw]);
}
//limpia y construye
inline void build(){
    memset(p,0,sizeof(p));
    memset(r,0,sizeof(r));
    memset(h,0,sizeof(h));
    memset(rmq,0,sizeof(rmq));
    memset(flog2,0,sizeof(flog2));
    suff_arr();
    makelcp();
}
};
int main(){
    //ejemplo, hallar la cantidad de diferentes substrings para t1 strings;
    ll t1; scanf("%lld", &t1);
    REP(ik,0,t1){
        SA sa; scanf("%s", &sa.s);
        ll ta=strlen(sa.s);
        sa.n=ta; sa.build();
        ll ans=0;
        REP(i,1,ta){
            ans+=sa.lcp(i,i+1);
        }
        ll xd=(ta*(ta+1)/2)-ans;
        printf("%lld\n",xd);
    }
    return 0;
}

```

## 7.7 Suffix Automaton

```

// O(n) Online suffix automaton construction
// len[u]: Max length of a string accepted by u
// link[u]: Suffix link of u
// Link edges give the suffix tree of reverse(s)

```

```
// Terminal nodes can be obtained by
// traversing last's links

const int MAX = 1000000;
int len[MAX*2];
int link[MAX*2];
map<char,int> adj[MAX*2];
int sz, last;

// To reuse, clear adj[]
void sa_init() {
    sz = last = 0;
    len[0] = 0;
    link[0] = -1;
    sz++;
}

void sa_extend (char c) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    int p;
    for (p=last; p!=-1 && !adj[p].count(c); p = link[p])
        adj[p][c] = cur;
    if (p == -1)
        link[cur] = 0;
    else {
        int q = adj[p][c];
        if (len[p] + 1 == len[q])
            link[cur] = q;
        else {
            int clone = sz++;
            len[clone] = len[p] + 1;
            adj[clone] = adj[q];
            link[clone] = link[q];
            for (; p != -1 && adj[p][c] == q; p = link[p])
                adj[p][c] = clone;
            link[q] = link[cur] = clone;
        }
    }
    last = cur;
}
```

## 7.8 Z-Algorithm

```
//Zfun(i) devuelve la longitud del maximo prefijo que empieza en i
vi Zfun(string s){
    vi Z(s.sz,0);
    int l = 0, r = 0;
    REP(i,1,sz(s)){
        if ( i<=r ) Z[i] = min(Z[i-l], r-i+1);
        while ( i+Z[i]<s.sz and s[i+Z[i]]==s[Z[i]] ) Z[i]++;
        if ( i+Z[i]-1>r ) l = i, r = i+Z[i]-1;
    }
    return Z;
}
```

## 8 Templates

### 8.1 Header Template

```
#include <bits/stdc++.h>
#include <sstream>
using namespace std;
#define fastio ios_base::sync_with_stdio(0);cin.tie(0);
#define trace(x) cerr << #x << ": " << x << '\n'
#define trace2(x,y) cerr << #x << ": " << x << " | " << #y << ": " << y << '\n';
#define trace3(x,y,z) cerr << #x << ": " << x << " | " << #y << ": " << y << " | " << #z << ": " << z << '\n';
#define all(v) (v).begin(),(v).end()
#define pb push_back
#define sz(v) ((int)v.size())
#define REP(i,x,y) for(int (i)=(x);(i)<(y);(i)++)
#define RREP(i,x,y) for(int (i)=(x);(i)>=(y);(i)--)
#define mp make_pair
#define fst first
#define snd second
typedef long long ll;
typedef pair<ll, ll> ii;

const int MOD = 1e9 + 7;
const int oo = 1e9;
const ll INF = 1e18;
const long double EPS = 1e-11;
```



## 8.2 Makefile

---

```
CXX = g++
CXXFLAGS = -std=c++11 -Wall -Wextra -Wno-sign-compare -O2 -g

all: %
%: %.cpp
    $(CXX) $(CXXFLAGS) -o $@ $@.cpp
```

---

## 8.3 Stack Size

---

```
#include <sys/resource.h>

int main (int argc, char **argv){
    const rlim_t kStackSize = 64L * 1024L * 1024L; // min stack size = 64
        Mb
    struct rlimit rl;
    int result;

    result = getrlimit(RLIMIT_STACK, &rl);
    if (result == 0)
    {
        if (rl.rlim_cur < kStackSize)
        {
            rl.rlim_cur = kStackSize;
            result = setrlimit(RLIMIT_STACK, &rl);
            if (result != 0)
            {
                fprintf(stderr, "setrlimit returned result = %d\n", result)
                    ;
            }
        }
    }

    // ...

    return 0;
}
```

---

## 8.4 Vim Configuration (vimrc)

---

```
set number
set autoindent
set showmode
set backspace=indent,eol,start
set mouse=a
set ts=3
set shiftwidth=3
set pastetoggle=<F10>
colorscheme chroma
syntax on

nmap ,c <Esc>i<Home>//<Esc>
nmap ,d <Esc><Home>i<Del><Del><Esc>
```

---

## 9 Utils

### 9.1 MinXOR

---

```
/*
    Minimum XOR-Pair on an array in O(n)
    Trie-based Implementation
*/

#define INT_SIZE 32

struct TrieNode{
    int value;
    TrieNode * Child[2];
};

TrieNode * getNode(){
    TrieNode * newNode = new TrieNode;
    newNode->value = 0;
    newNode->Child[0] = newNode->Child[1] = NULL;
    return newNode;
}

void insert(TrieNode *root, int key){
    TrieNode *temp = root;

    for (int i = INT_SIZE-1; i >= 0; i--){
```

```

    bool current_bit = (key & (1<<i));

    if (temp->Child[current_bit] == NULL)
        temp->Child[current_bit] = getNode();

    temp = temp->Child[current_bit];
}

temp->value = key ;
}

int minXORUtil(TrieNode * root, int key){
    TrieNode * temp = root;

    for (int i=INT_SIZE-1; i >= 0; i--){

        bool current_bit = ( key & ( 1<<i) );

        if (temp->Child[current_bit] != NULL)
            temp = temp->Child[current_bit];

        else if(temp->Child[1-current_bit] !=NULL)
            temp = temp->Child[1-current_bit];
    }
    return key ^ temp->value;
}

int minXOR(int arr[], int n){
    int min_xor = INT_MAX;

    TrieNode *root = getNode();
    insert(root, arr[0]);

    for (int i = 1 ; i < n; i++){
        min_xor = min(min_xor, minXORUtil(root, arr[i]));
        insert(root, arr[i]);
    }
    return min_xor;
}

int main(){
    int arr[] = {9, 5, 3};
    int n = sizeof(arr)/sizeof(arr[0]);
    cout << minXOR(arr, n) << endl;
    return 0;
}

```

```

}

```

## 9.2 Offline Less K-Counting

```

//-----inversiones en un rango (offline)-----
// ar[]: arreglo, queries=queri.pb(l,r,valor)
//asignar n,q; ez[i] respuesta para la querie i
//hacer read y make;

```

```

struct ST{
    ll n,q;
    vector<tri> querie;
    ll t[2*N],ar[N];
    ll poar[N],pok[N],ark[N],ez[N];
    vii v,v1;
    inline ll Op(ll &a,ll &b){ return a+b;}
    inline void build (){
        RREP(i,n-1,1) t[i]=Op(t[i<<1],t[i<<1|1]);
    }
    inline void modify (ll p, ll val){
        for(t[p+=n]=val;p>1;p>>=1) t[p>>1]=Op(t[p],t[p^1]);
    }
    inline ll que(ll l, ll r){
        ll res=0;
        for(l+=n,r+=n;l<r;l>>=1,r>>=1){
            if(l&1) res+=t[l++];
            if(r&1) res+=t[--r];
        }
        return res;
    }
    ll p1=0, p2=0,po=0;
    inline void read(){
        REP(i,0,n) v.push_back({ar[i],i});
        sort(all(v));
        REP(i,0,n) poar[p1++]=v[i].snd;
        REP(u,0,q){
            ll k=querie[u].itm3;
            ark[u]=k;
            v1.push_back({k,u});
        }
        sort(all(v1));
        REP(i,0,q) pok[p2++]=v1[i].snd;
    }
}

```

```

inline void make(){
    REP(i,0,n) t[i+n]=0; build();
    REP(i,0,q){
        ll x=pok[i];
        // <k, <= k en l,r(despues del &&)
        //inversa , hacer t[i+n]=1;
        while(po<n && ar[poar[po]]<=ark[x]) modify(poar[po++],1);
        ez[x]=que(querie[x].itm1-1,querie[x].itm2);
    }
}
}st;

int main(){fastio;
    ll n; cin>>n;
    st.n=n;
    REP(i,0,n) cin>>st.ar[i];
    ll q; cin>>q;
    st.q=q;
    REP(i,0,q){
        ll l,r,k; cin>>l>>r>>k;
        st.querie.push_back({l,{r,k}});
    }
    st.read(); st.make();
    REP(i,0,q) cout<<st.ez[i]<<endl;
    return 0;
}

```

### 9.3 Online Less K-Counting

```

/*-----inversiones en un rango (online)-----
construccion amortizada a nlog(n);
cada querie en log^2(n);*/

```

```

struct T{
    vi v;
    T () {}
    T (vi v): v(v){}
};
struct ST{
    ll n,ans;
    T t[2*N];
    inline T Op(T &val1, T &val2 ){
        vi v;

```

```

        REP(i,0,val1.v.size()) v.pb(val1.v[i]);
        REP(i,0,val2.v.size()) v.pb(val2.v[i]);
        sort(all(v));
        T ty;
        ty.v=v;
        return ty;
    }
    inline ll Op1( T &val1,ll &k){
        ans=0;
        //usar upper_bound para valores mayores a k
        //usar quitar el val1.v.size() para valores menores o iguales a k
        // usar lower_bound para valores estrictamente menoes a k(sin el val1.
        v.size())
        ans+=val1.v.size()-(upper_bound(all (val1.v),k)-val1.v.begin());
        return ans;
    }
    inline void build(){
        RREP(i,n-1,1) t[i]=Op(t[i<<1],t[i<<1|1]);
    }
    inline ll que(ll l, ll r, ll k){
        ll ans=0;
        for(l+=n,r+=n;l<r;l>=1,r>=1){
            if(l&1) ans+=Op1(t[l++],k);
            if(r&1) ans+=Op1(t[--r],k);
        }
        return ans;
    }
}
}st;

int main(){fastio;
    ll n; cin>>n;
    st.n=n;
    REP(i,0,n) {
        ll x; cin>>x;
        st.t[i+n].v.push_back(x);
    }
    st.build();
    ll q,ans=0,l,r,k; cin>>q;
    REP(i,0,q){
        cin>>l>>r>>k; // queries 1 base
        ans=st.que(l-1,r,k);
        cout<<ans<<endl;
    }
    return 0;
}

```