Examen Corto #3 - DSP

Pregunta #1 (4 pts)

$$x(n) = \left(-\tfrac{1}{2}\right)^{n+1-1} u(n+1) + \delta(n+3)$$

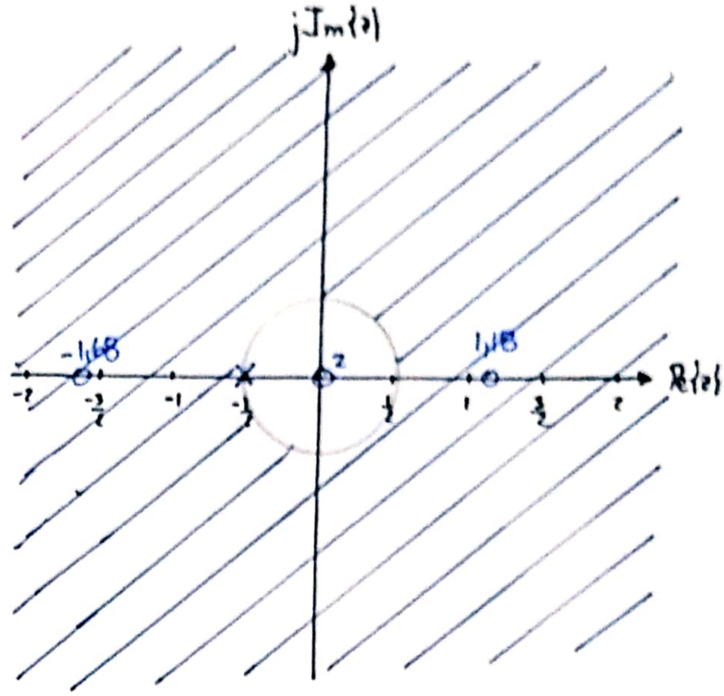
$$x(n) = \left(-\tfrac{1}{2}\right)^{-1} \cdot \left(-\tfrac{1}{2}\right)^{n+1} \cdot u(n+1) + \delta(n+3)$$

$$X(z) = -2 \cdot \frac{1}{1+\frac{1}{2}z^{-1}} \cdot z + z^3$$

$$X(z) = \frac{-2z + z^3\left(1+\frac{1}{2}z^{-1}\right)}{1+\frac{1}{2}z^{-1}}$$

$$X(z) = \frac{z^3\left(-2z^{-2} + \left(1+\frac{1}{2}z^{-1}\right)\right)}{1+\frac{1}{2}z^{-1}}$$

$$\boxed{X(z) = \frac{-2z^{-2}+\frac{1}{2}z^{-1}+1}{z^{-3}\left(1+\frac{1}{2}z^{-1}\right)}}$$ 3pts



2pts

Calculando los polos y los ceros

$$X(z) = \frac{-2z^{-2}+\frac{1}{2}z^{-1}+1}{z^{-3}\cdot\left(1+\frac{1}{2}z^{-1}\right)} \cdot \frac{z^4}{z^4}$$

$$X(z) = \frac{z^2\left(z^2+\frac{1}{2}z-2\right)}{z+\frac{1}{2}}$$

Ceros: 2pts
- $z=0$ → orden 2
- $z = \frac{-1+\sqrt{33}}{4}$ → orden 1
- $z = \frac{-1-\sqrt{33}}{4}$ → orden 1

Polos: 2pts
- $z=-1/2$ orden 1
- $z=\infty$ orden 3

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Pregunta #2 (9pts)

$$x(n) = \{\underset{\uparrow}{0}, 1, 2, 2, 3, 3, 3, \ldots\}$$

$$x(n) = \delta(n-1) + 2\delta(n-2) + 2\delta(n-3) + 3 \cdot u(n-4)$$

$$X(z) = z^{-1} + 2z^{-2} + 2z^{-3} + \frac{3 \cdot z^{-4}}{1 - z^{-1}}$$

$$X(z) = \frac{z^{-1}\cdot(1-z^{-1}) + 2(z^{-2})(1-z^{-1}) + 2z^{-3}(1-z^{-1}) + 3z^{-4}}{1-z^{-1}}$$

$$X(z) = \frac{z^{-1} \cancel{- z^{-2}} + \cancel{2z^{-2}} \cancel{- 2z^{-3}} + \cancel{2z^{-3}} - 2z^{-4} + 3z^{-4}}{1-z^{-1}}$$

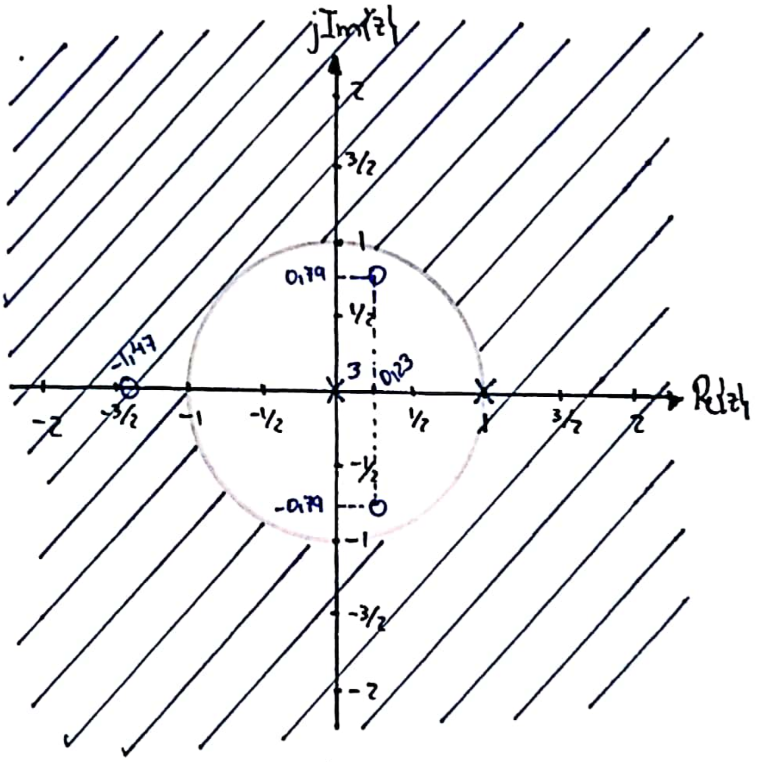
$$\boxed{X(z) = \frac{z^{-1} + z^{-2} + z^{-4}}{1-z^{-1}}}$$ 3pts

$$X(z) = \frac{z^{-1} + z^{-2} + z^{-4}}{1-z^{-1}} \cdot \frac{z^4}{z^4}$$

$$X(z) = \frac{z^3 + z^2 + 1}{z^3(z-1)}$$

polos: 2pts
- $z = 0$ orden 3
- $z = 1$ orden 1

ceros: 2pts
- $z = -1{,}47$ orden 1
- $z = 0{,}23 + 0{,}79j$ orden 1
- $z = 0{,}23 - 0{,}79j$ orden 1
- $z = \infty$ orden 1



2pts

Pregunta #3 (16pts)

$$y(n) = -2y(n-2) + x(n-2) + x(n) \qquad y(-1)=1 \quad y(-2)=-1$$

a) Aplicando la transformada z bilateral (6pts)

$$Y(z) = -2z^{-2}Y(z) + X(z)\cdot z^{-2} + X(z)$$

$$Y(z)\cdot[1+2z^{-2}] = X(z)\cdot[1+z^{-2}]$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1+z^{-2}}{1+2z^{-2}}$$

$$\boxed{H(z) = \frac{1+z^{-2}}{1+2z^{-2}}}$$ 2pts

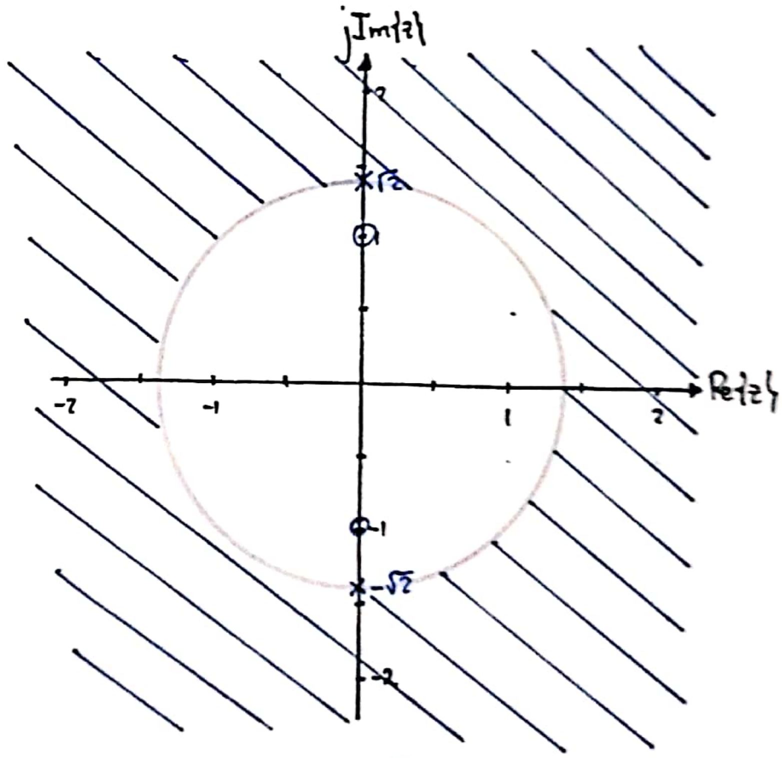
Calculando polos y ceros

$$H(z) = \frac{1+z^{-2}}{1+2z^{-2}}\cdot\frac{z^2}{z^2}$$

$$H(z) = \frac{z^2+1}{z^2+2}$$

ceros: 2pts $\begin{cases} z = j & \text{orden } 1 \\ z = -j & \text{orden } 1 \end{cases}$

polos: 2pts $\begin{cases} j\sqrt{2} & \text{orden } 1 \\ -j\sqrt{2} & \text{orden } 1 \end{cases}$



2pts

b) (8pts) $y(n) + 2y(n-2) = x(n-2) + x(n)$

$$\Big\updownarrow Z_u$$

$$Y(z) + 2\left[z^{-2}Y(z) + y(-1)\cdot z^{-1} + y(-2)\right] = z^{-2}\cancel{X(z)}^{\to 0} + \cancel{x(-1)\cdot z^{-1}}^{\to 0} + \cancel{x(-2)}^{\to 0} + \cancel{X(z)}^{\to 0}$$

$$Y(z)\cdot\left[1 + 2z^{-2}\right] = -2\cancel{y(-1)}^{\to 1}\cdot z^{-1} - 2\cancel{y(-2)}^{\to -1}$$

$$Y(z) = \frac{2 - 2z^{-1}}{1 + 2z^{-2}}\cdot\frac{z^2}{z^2}$$

$$Y(z) = \frac{z(2z-2)}{z^2+2} = \left[\frac{A}{z+j\sqrt{2}} + \frac{B}{z-j\sqrt{2}}\right]\cdot z\cdot 2$$

$$\frac{Y(z)}{2z} = \frac{z-1}{z^2+2} = \frac{A}{z+j\sqrt{2}} + \frac{B}{z-j\sqrt{2}}$$

$$A = \lim_{z\to -j\sqrt{2}} \frac{\cancel{(z+j\sqrt{2})}\cdot(z-1)}{\cancel{(z+j\sqrt{2})}(z-j\sqrt{2})} = \frac{-j\sqrt{2}-1}{-j\sqrt{2}-j\sqrt{2}} = \frac{\sqrt{6}}{4}\measuredangle -35{,}26^\circ$$

$$B = \frac{\sqrt{6}}{4}\measuredangle 35{,}26^\circ$$

$$Y(z) = 2z\cdot\left[\frac{\frac{\sqrt{6}}{4}\measuredangle -35{,}26^\circ}{z+j\sqrt{2}}\cdot\frac{z^{-1}}{z^{-1}} + \frac{\frac{\sqrt{6}}{4}\measuredangle 35{,}26^\circ}{z-j\sqrt{2}}\cdot\frac{z^{-1}}{z^{-1}}\right]$$

$$Y(z) = \frac{\frac{\sqrt{6}}{2}\measuredangle -35{,}26^\circ}{1+j\sqrt{2}\,z^{-1}} + \frac{\frac{\sqrt{6}}{2}\measuredangle 35{,}26^\circ}{1-j\sqrt{2}\,z^{-1}}$$

$$\Big\updownarrow$$

$$y(n) = \frac{\sqrt{6}}{2}e^{-j35{,}26^\circ}\cdot\left(\sqrt{2}e^{-j90^\circ}\right)^n\cdot u(n) + \frac{\sqrt{6}}{2}\cdot e^{j35{,}26^\circ}\cdot\left(\sqrt{2}e^{j90^\circ}\right)^n u(n)$$

$$y(n) = \frac{\sqrt{6}}{2}(\sqrt{2})^n \cdot \left[\frac{e^{-j(90° \cdot n + 35,26°)} + e^{j(90° \cdot n + 35,26°)}}{2} \right] u(n) \cdot 2$$

$$\boxed{\begin{aligned} y(n) &= \sqrt{6}(\sqrt{2})^n \cdot \cos(90° \cdot n + 35,26°)\, u(n) \\ y(n) &= \sqrt{6} \cdot (\sqrt{2})^n \cdot \cos\left(\frac{\pi}{2} \cdot n + 0,62\right) \cdot u(n) \end{aligned}}$$

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Pregunta #4 (16pts)

a) 3pts

$$H(z)=\frac{2\left(1-e^{j\frac{\pi}{4}}\cdot z^{-1}\right)\left(1-e^{-j\frac{\pi}{4}}\cdot z^{-1}\right)}{\left(1-\beta e^{j\frac{\pi}{4}}\cdot z^{-1}\right)\left(1-\beta e^{-j\frac{\pi}{4}}z^{-1}\right)z}$$

$$H(z)=\left[\frac{2\left(1-e^{-j\frac{\pi}{4}}\cdot z^{-1}-e^{j\frac{\pi}{4}}\cdot z^{-1}+e^{j\frac{\pi}{4}}\cdot e^{-j\frac{\pi}{4}}\cdot z^{-2}\right.}{1-\beta e^{-j\frac{\pi}{4}}\cdot z^{-1}-\beta e^{j\frac{\pi}{4}}\cdot z^{-1}+\beta^2 e^{-j\frac{\pi}{4}}\cdot e^{j\frac{\pi}{4}}\cdot z^{-2}}\right]\cdot z^{-1}$$

$$H(z)=\left[\frac{2-2z^{-1}\cdot\cancel{\left(e^{-j\frac{\pi}{4}}+e^{j\frac{\pi}{4}}\right)}+2z^{-2}}{1-\beta z^{-1}\cdot\left(e^{-j\frac{\pi}{4}}+e^{j\frac{\pi}{4}}\right)+\beta^2 z^{-2}}\right]\cdot z^{-1}$$

$$H(z)=\frac{2z^{-1}-4z^{-2}\cdot\cancel{\cos\left(\frac{\pi}{4}\right)}^{\frac{\sqrt{2}}{2}}+2z^{-3}}{1-2\beta z^{-1}\cdot\cancel{\cos\left(\frac{\pi}{4}\right)}^{\frac{\sqrt{2}}{2}}+\beta^2 z^{-2}}$$

$$\boxed{H(z)=\frac{2z^{-1}-2\sqrt{2}z^{-2}+2z^{-3}}{1-\beta\sqrt{2}\cdot z^{-1}+\beta^2 z^{-2}}}$$

ROC: $|z|>\beta$

b) 1pts $\boxed{\text{para}\quad \beta\leq 1}$

$\beta=1$ inclusive, ya que los polos se cancelarían con los ceros.

c) 2pts

$$\frac{Y(z)}{X(z)}=\frac{2z^{-1}-2\sqrt{2}z^{-2}+2z^{-3}}{1-\beta\sqrt{2}z^{-1}+\beta^2 z^{-2}}$$

$$Y(z)\cdot\left[1-\beta\sqrt{2}z^{-1}+\beta^2 z^{-2}\right]=X(z)\left[2z^{-1}-2\sqrt{2}z^{-2}+2z^{-3}\right]$$

$$y(n)-\beta\sqrt{2}y(n-1)+\beta^2 y(n-2)=2x(n-1)-2\sqrt{2}x(n-2)+2x(n-3)$$

$$\boxed{y(n)=\beta\sqrt{2}y(n-1)-\beta^2 y(n-2)+2x(n-1)-2\sqrt{2}x(n-2)+2x(n-3)}$$

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d) 5pts $a(n) = \sqrt{2}\cdot\cos\left(\frac{\pi}{4}n\right)u(n+1)$

$$a(n) = \sqrt{2}\cdot\cos\left(\frac{\pi}{4}\cdot n\right)\cdot\delta(n+1) + \sqrt{2}\cdot\cos\left(\frac{\pi}{4}\cdot n\right)\cdot u(n)$$

$$A(z) = z\cdot\sqrt{2}\cdot\cos\left(\frac{-\pi}{4}\right) + \sqrt{2}\cdot\frac{1-\cos\left(\frac{\pi}{4}\right)\cdot z^{-1}}{1-2z^{-1}\cdot\cos\left(\frac{\pi}{4}\right)+z^{-2}}$$

$$A(z) = z + \frac{\sqrt{2}-z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}}$$

$$A(z) = \frac{z\left(1-\sqrt{2}z^{-1}+z^{-2}\right)+\sqrt{2}-z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}}$$

$$A(z) = \frac{z-\cancel{\sqrt{2}}+\cancel{z^{-1}}+\cancel{\sqrt{2}}-\cancel{z^{-1}}}{1-\sqrt{2}z^{-1}+z^{-2}}$$

$$A(z) = \frac{1}{z^{-1}\left(1-\sqrt{2}z^{-1}+z^{-2}\right)}$$

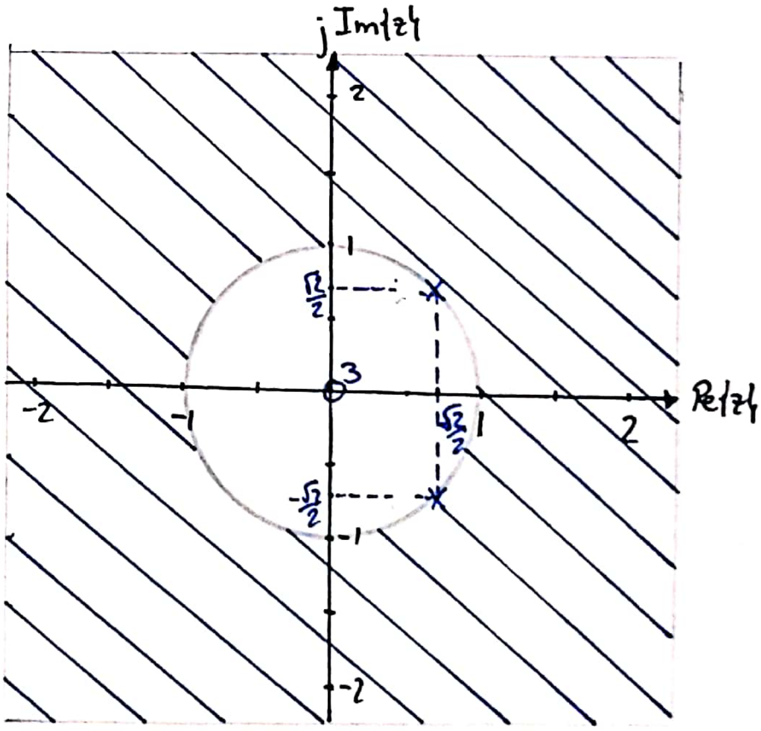
$$\boxed{A(z) = \frac{1}{z^{-1}-\sqrt{2}z^{-2}+z^{-3}}}$$

$$A(z) = \frac{1}{z^{-1}-\sqrt{2}z^{-2}+z^{-3}}\cdot\frac{z^3}{z^3}$$

$$A(z) = \frac{z^3}{z^2-\sqrt{2}z+1}$$

ceros: $\{\ z=0$ orden 3

polos: $\left\{\begin{array}{ll} z=\frac{\sqrt{2}+\sqrt{2}j}{2} & \text{orden } 1 \\ z=\frac{\sqrt{2}-\sqrt{2}j}{2} & \text{orden } 1 \\ z=\infty & \text{orden } 1 \end{array}\right.$



e) 5pts

$$Y(z)=H(z).X(z)$$

$$Y(z)=\frac{2\cancel{(1-\sqrt{2}z^{-1}+z^{-2})}\cancel{z^{-1}}}{1-\beta\sqrt{2}.z^{-1}+\beta^2z^{-2}}\cdot\frac{1}{\cancel{z^{-1}}\cancel{(1-\sqrt{2}z^{-1}+z^{-2})}}\cdot\frac{z^2}{z^2}$$

$$Y(z)=\frac{2z^2}{[1-\beta\sqrt{2}z^{-1}+\beta^2z^{-2}].z^2}$$

$$\frac{Y(z)}{z}=\frac{2z}{z^2-\beta\sqrt{2}z+\beta^2}=\frac{2z}{(z-\beta e^{j\pi/4})(z-\beta e^{-j\pi/4})}=\frac{A}{z-\beta e^{j\frac{\pi}{4}}}+\frac{B}{z-\beta e^{-j\frac{\pi}{4}}}$$

$$A=\lim_{z\to\beta e^{j\pi/4}}\frac{\cancel{(z-\beta e^{j\frac{\pi}{4}})}.2z}{\cancel{(z-\beta e^{j\pi/4})}(z-\beta e^{-j\pi/4})}=\frac{2\cancel{\beta}e^{j\pi/4}}{\cancel{\beta}e^{j\pi/4}-\cancel{\beta}e^{-j\pi/4}}=\frac{\cancel{2}e^{j\frac{\pi}{4}}}{\cancel{2}j\,sen(\ \ )}=e^{-j\frac{\pi}{2}}.e^{j\frac{\pi}{4}}.\sqrt{2}$$

$$A=\sqrt{2}e^{-j\pi/4}$$

$$B=\sqrt{2}e^{j\pi/4}$$

$$Y(z)=z.\left[\frac{\sqrt{2}e^{-j\pi/4}}{z-\beta e^{j\pi/4}}.\frac{z^{-1}}{z^{-1}}+\frac{\sqrt{2}e^{j\pi/4}}{z-\beta e^{-j\pi/4}}.\frac{z^{-1}}{z^{-1}}\right]$$

$$Y(z)=\frac{\sqrt{2}.e^{-j\pi/4}}{1-\beta e^{j\pi/4}.z^{-1}}+\frac{\sqrt{2}e^{j\pi/4}}{1-\beta e^{-j\pi/4}z^{-1}}$$

$$y(n)=\sqrt{2}e^{-j\frac{\pi}{4}}.(\beta e^{j\pi/4})^n u(n)+\sqrt{2}e^{j\pi/4}.(\beta e^{-j\pi/4})^n u(n)$$

$$y(n)=\sqrt{2}\beta^n.\left[\frac{e^{j(\frac{\pi}{4}n-\frac{\pi}{4})}+e^{-j(\frac{\pi}{4}.n-\frac{\pi}{4})}}{2}\right].2.u(n)$$

$$\boxed{y(n)=2\sqrt{2}\beta^n.\cos(\tfrac{\pi}{4}.n-\tfrac{\pi}{4})u(n)}$$