

## Examen Corto #2 - DSP

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## Pregunta #1 (2pts)

$$y(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$h(n) = u(n)$$

$$x(n) = \{1, -2, 1\}$$

$$\begin{array}{rcl}
 \{1, 1, 1, 1, \dots\} & \rightarrow & h(k) \\
 \uparrow & & \\
 \{1, -2, 1\} & \rightarrow & x(0-k) \\
 \uparrow & & \\
 \{1, -2, 1\} & \rightarrow & x(1-k) \\
 \uparrow & & \\
 \{1, -2, 1\} & \rightarrow & x(2-k) \\
 \uparrow & & 
 \end{array}$$

$$y(n) = \{1, -1\}$$

## Pregunta #2 (10pts)

a)  
(1pts)

$$h(n) = h_1(n) * [h_2(n) + h_3(n) * h_4(n)]$$

b)  
(5pts)Primero: Calcular  $h_0(n) = h_3(n) * h_4(n)$   
2pts

$$h_0(n) = u(n) * [-\delta(n-2)]$$

$$h_0(n) = -u(n-2)$$

Segundo: Calcular  $h_x(n) = h_2(n) + h_0(n)$   
1pts

$$h_x(n) = u(n) - u(n-2) = \{1, 1\}$$

Tercero: Calcular  $h(n) = h_1(n) * h_2(n) = \sum_{k=-\infty}^{\infty} h_1(k) \cdot h_2(n-k)$   
(2pts)

$$\begin{array}{c} \{ \frac{1}{2}, \frac{1}{4}, \frac{1}{2} \} \\ \uparrow \\ \{ 1, 1 \} \\ \uparrow \\ \{ 1, 1 \} \\ \uparrow \\ \{ 0, 1, 1 \} \\ \uparrow \\ \{ 0, 0, 1, 1 \} \end{array}$$

$$y(n) = h(n) = \{ \frac{1}{2}, \frac{3}{4}, \frac{3}{4}, \frac{1}{2} \}$$

c) (4pts)

$$y(n) = h(n) * x(n)$$

$$h(n) = \{ \frac{1}{2}, \frac{3}{4}, \frac{3}{4}, \frac{1}{2} \}$$

$$x(n) = \{ 1, 0, 0, 3, 0, -4 \}$$

|        |               |               |               |               |               |               |               |               |               |               |                |
|--------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|----------------|
|        |               |               | 1             | 0             | 0             | 3             | 0             | -4            |               |               |                |
| $n=-2$ | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ |               |               |               |               |               |               | $\frac{1}{2}$  |
| $n=-1$ |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ |               |               |               |               |               | $\frac{3}{4}$  |
| $n=0$  |               |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ |               |               |               |               | $\frac{3}{4}$  |
| $n=1$  |               |               |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ |               |               |               | 2              |
| $n=2$  |               |               |               |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ |               |               | $\frac{9}{4}$  |
| $n=3$  |               |               |               |               |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ |               | $\frac{1}{4}$  |
| $n=4$  |               |               |               |               |               |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ | $-\frac{3}{2}$ |
| $n=5$  |               |               |               |               |               |               |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $-\frac{1}{2}$ |
| $n=6$  |               |               |               |               |               |               |               |               | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$  |
|        |               |               |               |               |               |               |               |               |               | $\frac{1}{2}$ | $-\frac{1}{2}$ |

$$y(n) = \{ \frac{1}{2}, \frac{3}{4}, \frac{3}{4}, 2, \frac{9}{4}, \frac{1}{4}, -\frac{3}{2}, -3, -2 \}$$

## Pregunta #3 (3 pts)

$$x(n) = \{1, 3, 3, 1\} \rightarrow N_x = 4$$

$$y(n) = \{1, 4, 6, 4, 1\} \rightarrow N_y = 5$$

$$h(n) = \{a, b\}$$

$$N_h + N_x - 1 = N_y$$

$$N_h = 5 + 1 - 4 = 2$$

$$N_h = 2$$

|     |   |   |   |   |   |   |
|-----|---|---|---|---|---|---|
|     |   | 1 | 3 | 3 | 1 |   |
| n=0 | b | a |   |   |   |   |
| n=1 |   | b | a |   |   |   |
| n=2 |   |   | b | a |   |   |
| n=3 |   |   |   | b | a |   |
| n=4 |   |   |   |   | b | a |

$$h(n) = \{1, 1\}$$

## Pregunta #4 (4 pts)



$$y(n) = -\frac{1}{2}y(n-1) + x(n) + 2x(n-2)$$

$$x(n) = \{1, 2, 3, 4, 2, 1\}$$

|    |           | A        | B    | C      |
|----|-----------|----------|------|--------|
| n  | y(n)      | y(n-1)   | x(n) | x(n-2) |
| -2 | 1         | 0        | 1    | 0      |
| -1 | 3/2       | 1        | 2    | 0      |
| 0  | 17/4      | 3/2      | 3    | 1      |
| 1  | 47/8      | 17/4     | 4    | 2      |
| 2  | 81/16     | 47/8     | 2    | 3      |
| 3  | 207/32    | 81/16    | 1    | 4      |
| 4  | 49/64     | 207/32   | 0    | 2      |
| 5  | 207/128   | 49/64    | 0    | 1      |
| 6  | -207/256  | 207/128  | 0    | 0      |
| 7  | 207/512   | -207/256 | 0    | 0      |
| 8  | -207/1024 | 207/512  | 0    | 0      |

$$y(n) = \left\{ 1, \frac{3}{2}, \frac{17}{4}, \frac{47}{8}, \frac{81}{16}, \frac{207}{32}, \frac{49}{64}, \frac{207}{128}, -\frac{207}{256}, \frac{207}{512}, \dots \right\}$$

Pregunta #5 (6pts)A. Respuesta homogénea (natural) (2pts)

$$y(n) - 4y(n-1) + 4y(n-2) = 0$$

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}[\lambda^2 - 4\lambda + 4] = 0$$

$$\lambda_1 = 2 \text{ (doble)}$$

$$y_h(n) = A2^n + B \cdot n2^n$$

C. Respuesta total (3pts)

$$y(n) = A2^n + B \cdot n2^n + C \cdot \left(\frac{1}{2}\right)^n$$

$$y(0) = A + C$$

$$y(1) = 2A + 2B + \frac{1}{2}C$$

$$y(2) = 4A + 8B + \frac{1}{4}C$$

Por lo tanto:

$$A + C = 1$$

$$2A + 2B + \frac{1}{2}C = -\frac{1}{2}$$

$$4A + 8B + \frac{1}{4}C = -\frac{25}{4}$$

resolviendo

$$A = 10/9$$

$$B = -4/3$$

$$C = -1/9$$

B. Respuesta Particular (1pts)

$$y_p(n) = C \cdot \left(\frac{1}{2}\right)^n$$

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$y(n) = 4y(n-1) - 4y(n-2) + x(n) - x(n-1)$$

$$y(0) = 1$$

$$y(1) = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$y(2) = 4 \cdot \frac{1}{2} - 4 \cdot 1 + \frac{1}{4} - \frac{1}{2}$$

$$y(2) = -\frac{25}{4}$$

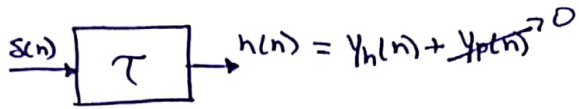
$$y(n) = \frac{10}{9} \cdot (2)^n - \frac{4}{3} \cdot n(2)^n - \frac{1}{9} \left(\frac{1}{2}\right)^n$$

$$y(0) = 1 \quad \checkmark$$

$$y(1) = -1/2 \quad \checkmark$$

$$y(2) = -\frac{25}{4} \quad \checkmark$$

## Pregunta #6 (5pts)



$$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$$

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2} [\lambda^2 - 0.6\lambda + 0.08] = 0$$

$$\lambda_1 = 2/5$$

$$\lambda_2 = 1/5$$

$$y_h(n) = A\left(\frac{2}{5}\right)^n + B\left(\frac{1}{5}\right)^n$$

$$y(0) = A + B$$

$$y(1) = \frac{2}{5}A + \frac{1}{5}B$$

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

$$y(0) = 1$$

$$y(1) = 0.6$$

$$\begin{cases} A + B = 1 \\ \frac{2}{5}A + \frac{1}{5}B = 0.6 \end{cases} \quad \begin{cases} \text{Resolviendo} \\ A = 2 \\ B = -1 \end{cases}$$

$$h(n) = 2\left(\frac{2}{5}\right)^n - \left(\frac{1}{5}\right)^n$$

## Pregunta #7 (2pts)

$$r_{yx}(n) = r_{xy}(-n) = \{2, 0, -1, 2, 3, -1\}$$

↑

$$r_{yx}(-n) = \{-1, 3, 2, -1, 0, 2\}$$

↑

$$r_{yx}(-n+1) = \{-1, 3, 2, -1, 0, 2\}$$

↑