LAB #1: OBSERVATIONAL SPECIAL RELATIVITY

In this lab you will analyze astronomical data on an intriguing source called SS 433. This lab is different from the other labs in this class in that you will not acquire any new data. Instead, you will use somebody else's data (some 500+ measurements collected by professional astronomers over a span of ≈ 14 yr), extract the necessary measurements from these data, and analyze these measurements using scientific software to derive some parameters of SS 433. Therefore, the skill you are practicing here is modeling the data when the underlying model is entirely based on special relativistic kinematics.

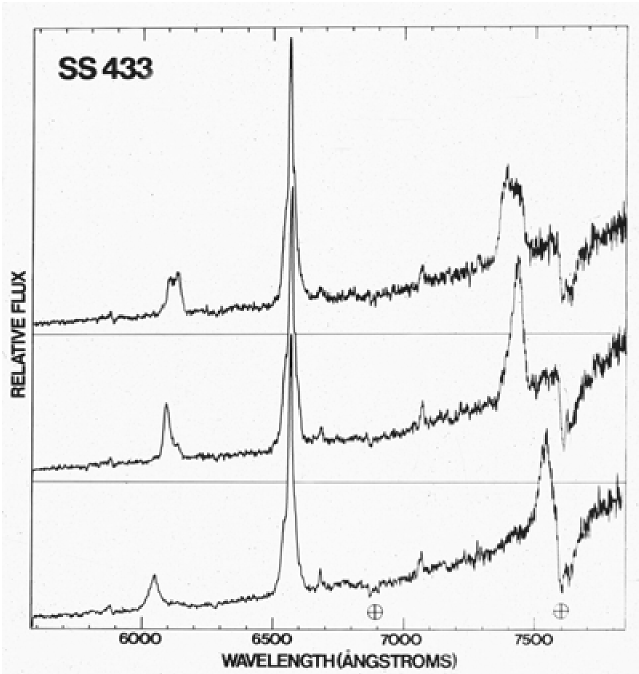
This lab requires you to understand a very complex astrophysical system in the context of a simple kinematic model. This approach—analyzing a very complex physical situation in terms of a simplified picture you hope captures the principal elements of what happened—is usually the first step in any scientific research. Nature rarely presents itself in the form of clean idealized problems!

To prepare for this lab exercise, start by getting a sense of what kind of object SS 433 is—Googling it is fruitful. There is a lot of detailed, interesting and useful information on this famous source. For example, `http://blackholes.stardate.org/resources/article-mystery-of-ss443.html` is an account of how we came to learn about it, written by one of the first astronomers to notice it. There is also a lot of slightly-garbled or out-of-date or even crackpot information that gets turned up by Googling, so, as always when it comes to internet information, be critical and be very aware of the quality of the source. One of the things you will probably learn from your reading is that there is very good reason to think that, at bottom, SS 433 is a binary star system in which one member is a "live" star whose mass is at least several times the mass of the Sun, while the other is a black hole whose mass is rather larger than the live star. In your data analysis, you'll learn what's going on to cause a very special effect exhibited by SS 433, one that, despite a good deal of searching for other examples, remains unique more than 40 years after its discovery.

Given how strangely SS 433 behaves, it's our great good fortune to have lots of observational data from it. It can be detected in many different sorts of EM radiation: radio, optical, UV, and X-rays. It can even be imaged in the radio band.

The data you'll be working with is a series of optical spectra of this system, i.e., measurements of how much light energy is radiated as a function of wavelength. Your analysis of these spectra rests upon an important fact about atoms: when they emit light, they do so at a number of very specific wavelengths/frequencies (the wavelength λ and frequency ν are related by $c = \lambda\nu$). The wavelengths radiated are intrinsic to the specific atom or ion (and are defined, of course, in the atom's rest-frame). Such features are called "lines" because that's how they appear when the spectrum is created by viewing the source through an aperture shaped ike a narrow rectangle and then spreading the light along a line perpendicular to this "slit" (the technical term). Because the sideways displacement of the light-rays is proportional to their wavelength, one can measure how much energy is being delivered as a function of wavelength.

What makes SS 433 so amazing is that although it shows some lines placed essentially where one might expect to see them, it also displays other lines at wavelengths that don't correspond to any line from an astrophysically-abundant element—and most remarkably, move back and forth in wavelength, changing by as much as ~ 1000 Å over timescales of months. In this lab, you'll demonstrate that all of this crazy behavior can be explained by a simple model: the lines actually come from H, the most abundant element in the Universe, and the wandering lines are due to moderately simple—but relativistic—motions within the source.



The specific lines of greatest interest are called "Balmer lines". Named Hα, Hβ, etc. in order of decreasing wavelength, they are created when H atoms de-excite from a more energetic state to the second-lowest energy state in H. Their rest-frame wavelengths are:
Hα 6563 Å
Hβ 4861 Å
Hγ 4341 Å
Hδ 4102 Å.

However, their *observed* wavelengths can be altered by Doppler shifts between their source material and the Earth. As can be seen in the figure above (showing spectra of SS 433 on three successive nights), the precision of the emitted wavelength can be smeared; in addition, there can be a wholesale shift of the entire feature's center. The spread in each line can be attributed to comparatively small random motions in the source region. These random motions might be thermal in origin or perhaps due to fluctuations in the local fluid speed inside the source. The overall shift is due to a large mean relative velocity. Within our own Galaxy, relative velocity is mostly due to differing orbital speeds within the Galaxy, typically ~ 200 km/s; when the source is in an external galaxy, cosmological expansion is usually the dominant effect. Because Galactic orbital speeds are $\sim 10^{-3}c$, the maximum fractional shift in wavelength or frequency they can create is similarly small. Cosmological expansion can cause order-unity shifts from objects sufficiently far away, but—very unlike the shifts in SS 433—they change on Gyr timescales, not weeks or months.

Your lab report will consist of the list of collected measurements and the answers to the questions presented below.

Group 1—to be done during the discussion section

1) (2 points) Sketch a lump of gas containing H moving with arbitrary velocity $\vec{v}$ in space and the line connecting the lump to an observer on Earth. Using Lorentz transformations and the energy-momentum 4-vector of photons, derive the Doppler effect formula:

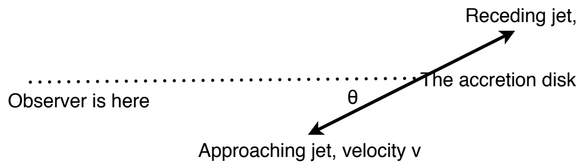
$$\nu_{\rm obs} = \frac{\nu_{\rm em}}{\gamma(1 - \vec{v} \cdot \hat{n}/c)}.$$

where $\nu_{\rm obs}$ and $\nu_{\rm em}$ are the photon frequency in the observer's frame and the photon frequency emitted, respectively. As usual γ is the Lorentz factor of the emitter (in Earth's frame), $\vec{v}$ is its velocity (again in Earth's frame), and $\hat{n}$ is the unit vector in our frame directed from the emitter to the observer.

2) (1 point) Rewrite the formula above for the relationship between wavelengths $\lambda_{\rm em}$ and $\lambda_{\rm obs}$. This is the primary formula we will be using.

3) (3 points) Using any measuring tools you deem necessary, determine the observed wavelengths of the two lines on either side of the central peak in the bottom spectrum of the figure. Describe exactly what measurements you performed and how you determine the wavelength from your measurements. What would you say is the uncertainty in the wavelengths you measure?

4) (3 points) The schematic below shows a simple kinematic model of a double-sided jet inclined at an angle θ to the line of sight to the observer. The model depends on two parameters, θ and v. Assuming that the two flanking lines in the spectrum you analyzed for Question 3 are Hα lines produced by the two sides of the jet, which line is produced by which side? Then use the Doppler shift formula you derived to calculate θ and v. Does your measurement of v agree with what various Internet pages on SS 433 say?

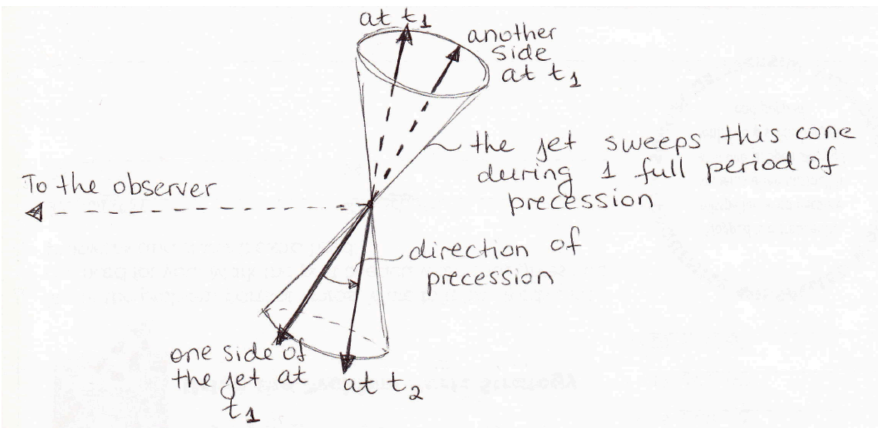


5) (1 point) If the jet were only one-sided, could you find a solution for the two parameters?

Group 2—to be done at home and turned in in class on Tuesday, October 5.

As if having jets moving with relativistic velocities was not exotic enough, it turns out that the jets are not stationary in space, but are precessing. In fact, the jet's precession can be seen directly in radio images: a very nice animation can be seen in `ss433jimovie.gif`. The jet is somewhat clumpy, so you often see individual blobs propagating away from the compact object, rather than smooth lines. And if you look on larger scales, you can see the remnants of old jet material forming a "corkscrew" pattern as a result of the jet precession (`ss433.oct04.jpg`).

It is precisely because of the precession that the flanking emission lines produced by the jet did not stay at the same wavelengths in the three example spectra in Part I. Because the orientation of the jet changes as a function of time, the magnitude of the Doppler effect changes as well, and so does the observed wavelength of Hα. A sketch of what this might look like is shown in the next figure (note that there is a small typo in one place):



In this part of the lab exercise, you'll construct a parameterized model of this tilted, precessing jet whose parameters can be inferred by matching your predicted Doppler shifts to real data. To ease the labor of this task, you can make a number of simplifying assumptions:

- The jet speed is the same everywhere and at all times, even though the orientation of the jet changes.
- The two sides of the jet always move in exactly opposite directions.
- The precession proceeds with a constant angular frequency.

The following questions will help guide your work.

6) (3 points) The stationary problem we discussed in Group 1 had two parameters: the velocity and the angle to the line of sight. How many parameters does the full model have? Assign letters to these parameters and write down an analytic expression for the observed wavelength of Hα as a function of time in the full precession model. (Hint: within our model assumptions, only the $\vec{v} \cdot \hat{n}$ term changes as a function of time in the Doppler formula.)

In order to constrain the parameters of the precession model, the source must be monitored at least over a full precession period (in practice, this object has been monitored over many periods). In every observation, astronomers obtain a spectrum looking more or less like the three you examined in the questions of Group 1.

Astronomers love inventing jargon and special notations. Their favorite way to describe Doppler shifts is through the quantity $z \equiv (\lambda_{\rm obs} - \lambda_0)/\lambda_{\rm obs}$, the ratio between the observed wavelength shfit and the rest-frame wavelength. It's conventionally called a "redshift" because the cosmological expansion biases z to generally be positive. Because SS 433 is in our own Galaxy, the cosmological expansion is not important for it. One of the many remarkable properties of SS 433 is that some emission lines are red-shifted while others seen in the same spectrum are blue-shifted, and the values of these shifts greatly over time.

We'll distribute a file that contains redshift data on SS 433 from a long series of observations taken at few-day intervals. These data were kindly provided by Prof. Bruce Margon, who wrote several of the ground-breaking papers on this system. The three relevant columns are JD, Red and Blue (you can ignore the column headed by Obs). JD is short for "Julian Date". This is a system for counting days since a designated starting point. It's named for the Julian Calendar (introduced by Julius Caesar in Rome ≈ 2070 years ago; it has nothing to do with your professor). Red and Blue are the redshift values for the two sides of the jet; these numbers are dimensionless. Note that some redshift values are shown as 9.999; these indicate bad data.

For the next question, you'll be working on lots of data, so it will definitely pay to automate your work. There are many ways to do this. Here we'll give some examples of how to do it using Mathematica, but if you prefer other software, feel free to use it.

Some basic info on Mathematica:

(1) After youve typed the command (examples given below) you press Shift+Enter to run it. If you made a typo, Mathematica will complain, and you can edit your command and re-run it.

(2) Many functions have their standard names, but with capitalized initial letters; their arguments are placed within square brackets: Exp[x], Sin[2x], Cos[xffl3], Tan[x-15], Log[5,x] (the log of x to base 5), etc.

(3) A reference manual is built into Mathematica ("Help") and is also available at `http:// reference.wolfram.com`. Googling "Mathematica ListPlot would produce the desired result.

7) (2 points) Select about 15–20 entries from the data file (there are more than 500 altogether). These entries should not be completely random: your data should cover approximately a year, roughly uniformly. If you take dates that are too close together, then the orientation of the jet does not change much from one observation to the other and you wont get good constraints on precession. If you take dates that are too far apart, you may miss a large fraction of the precession period altogether and have trouble tying your model to the data points. Therefore, an ideal dataset is one whose data points are approximately

20 days apart (but you are welcome to experiment). Plot the redshift of both sides of the jet as a function of time.

For example, if you use Mathematica, the commands might be

myblue=ListPlot[$\{\{1000., -0.1\}, \{1020., -0.2\}, \{1030., -0.1\}\}$],

which plots three data points for the blue component (each pair is a made-up JD value and a Blue value) and

myred=ListPlot[$\{\{1000., 0.1\}, \{1020., 0.2\}, \{1030., 0.15\}\}$] plots three data points for the red component.

You can use PlotRange to adjust the axes if necessary:

myred=ListPlot[$\{\{1000., 0.1\}, \{1020., 0.2\}, \{1030., 0.15\}\}$, PlotRange-> $\{\{990, 1050\}, \{-0.5, 0.5\}\}$]

To show both datasets together you might use:

Show[myblue,myred,PlotRange-> $\{\{990, 1050\}, \{-0.5, 0.5\}\}$]

However you generate your plots, print them out and enclose them with your lab report.

8) (1 point) You should see that the $z(t)$ curves from the two sides of the jet cross at some moments, which means at these times the two emission lines from the different sides of the jet blend into one. Sketch the line of sight to the observer and the orientation of the jet at the moment when the two curves cross. Do the two curves cross at positive z or negative z? Why? Why not at $z = 0$? Which parameter of the model does the curve-crossing value of z constrain?

9) (2 points) Try to adjust your model parameters from Question 6 to produce an approximate fit to the data in Question 7. (Hint: what is the smallest angle the jet makes with respect to the line of sight? What is the largest angle the jet makes with respect to the line of sight? How are these related to your model parameters? At which orientations of the jet are the peaks of the $z(t)$ curves reached?).

In Mathematica, you can plot your model using the Plot function:

mymodel=Plot[Exp[$-(x - 1000)/10$], $\{x, 1000, 1050\}$]

(for the Plot function you have to specify the range of the argument). Then you can plot your model and your data together:

Show[myblue,myred,mymodel,PlotRange-> $\{\{990, 1050\}, \{-0.5, 0.5\}\}$]

You can now change the parameters of 'mymodel', re-run it, and then re-run the Show function to show the data with the updated model.

At the end of this process, report your best results: show your fit (functions plus data) and the values of the parameters that produced that fit.