Perceptron

1 Perceptron

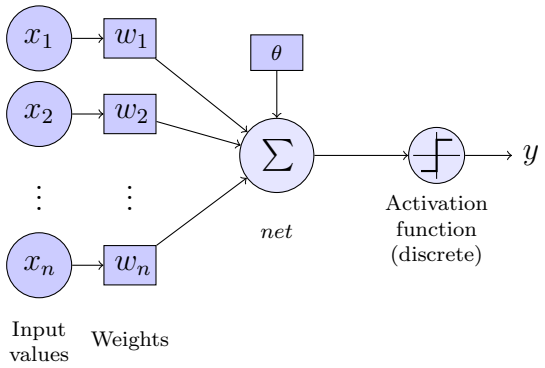


Figure 1: Perceptron with discreta activation function.

A perceptron consists of the following:

- input vector $\mathbf{x} = \begin{pmatrix} x_1 & x_2 & \dots & x_p \end{pmatrix}^{\mathrm{T}}$,
- weight vector $\mathbf{w} = \begin{pmatrix} w_1 & w_2 & \dots & w_p \end{pmatrix}^{\mathrm{T}}$,
- bias θ,
- activation function f.

1.1 Calculating output value

The *net* value is the scalar product of the weight vector and input vector minus the bias:

$$net = \mathbf{w}^{\mathrm{T}}\mathbf{x} - \theta = w_1 \cdot x_1 + w_2 \cdot x_2 + \cdots + w_p \cdot x_p - \theta$$

The output value is:

$$y = f(net) = \begin{cases} 1 & \text{if } net \geq 0 \\ 0 & \text{if } net < 0 \end{cases}$$

2 Geometric interpretation

A perceptron creates a hyperplane, which divides the decision space into two parts. The weight vector is its normal vector. The output value is equal to 1 if the input is on the same side of the hyperplane as the weight vector and 0 otherwise. Below is the hyperplane equation for a perceptron with the weight vector $(w_1, w_2, \ldots, w_p)$ and the bias θ:

$$w_1 x_1 + w_2 x_2 + \cdots + w_p x_p - \theta = 0$$

3 Delta rule

Weights and biases are modified according to the following rule:

$$\begin{aligned} \mathbf{w}' &= \mathbf{w} + \alpha(d - y)\mathbf{x} \\ \theta' &= \theta - \alpha(d - y), \end{aligned}$$

where:

- $\mathbf{w}'$ is the new weight vector,
- $\mathbf{w}$ is the old weight vector,
- θ' is the new bias,
- θ is the old bias,
- α is the learning rate (usually between 0 and 1),
- d is the expected output,
- y is the output,
- $\mathbf{x}$ is the input vector.

4 Perceptron learning algorithm

The input is the training set consisting of input vectors $\mathbf{x}_i$ and expected outputs d_i:

$$D = \{(\mathbf{x}_1, d_1), (\mathbf{x}_2, d_2), \ldots, (\mathbf{x}_n, d_n)\}$$

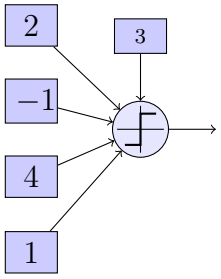
1. Pick learning rate α and pick random initial weights.
2. For each training set vector $\mathbf{x}_i$:
   (a) Calculate output y.
   (b) Update weights and bias according to delta rule.
3. Calculate iteration error:
$$E = \frac{1}{n} \sum_{i=1}^{n} (d_i - y_i)^2$$
4. Go back to step 2, or end training when E is below a given threshold ($E < E_{max}$), or after a given number of iterations.

Questions

Question 1.

Given the following perceptron with the weight vector $\mathbf{w} = (2, -1, 4, 1)$, bias $\theta = 3$, and a unipolar discrete activation function, calculate the output value for the following inputs.

- $\mathbf{x} = (7, -2, -5, 4)$
- $\mathbf{x} = (2, 0, 2, 8)$



Question 2.

What is the hyperplane equation of the perceptron in Question 1.

Question 3.

Design a perceptron to implement the following logical functions: AND, OR, XOR.

Question 4.

How will the weights and bias of the percptron from Q1 be modified by the delta rule given the following inputs ($\alpha = 0.5$).

- $\mathbf{x} = (7, -2, -5, 4)$, $d = 1$
- $\mathbf{x} = (5, 8, -1, -2)$, $d = 1$
- $\mathbf{x} = (0, 1, 2, 1)$, $d = 0$
- $\mathbf{x} = (2, 0, 2, 8)$, $d = 0$
- $\mathbf{x} = (3, 1, 9, -3)$, $d = 0$

Mini-project: Perceptron

The training set in `perceptron.data` contains the *Iris* dataset limited to `Iris-versicolor` and `Iris-virginica`. `perceptron.test.data` contains the test set. Implement the perceptron and train it to classify the two species. Test with the test set and output the accuracy.

The program should have the following capabilities:

- Loading any dataset in csv format, where the last column is the class. The number of weights should be adjusted to the dataset. (!!)
- Picking the learning rate.
- Simple UI to manually input vectors to classify.
- Hints:
  - For best results pick a small learning rage (e.g. 0.01) and repeat the learning procedure for a larger number of iterations.
  - Pick initial weights and bias randomly from the range $[0, 1]$.
  - `Iris-versicolor` and `Iris-virginica` are not linearly separable (the iteration error will never be 0), but the test set can nevertheless be classified with high accuracy.