CHAPTER I

MATHEMATICS IN OUR WORLD

In the 21st century, problems of science and technology have been the main focus for scientists and mathematicians. They are using the concepts of mathematics in different fields of specializations as a necessary tool to obtain the significant results of their experiments and programs such as forecasts the behaviour of one variable given the other known variables, develop vaccines for any unprecedented bacterial and viral infections and diseases, locate the epicentre of any earthquakes and estimates the possible damages, create programs and algorithms, and other undertakings that made mathematics as inevitable.

With these tangible evidences, would you consider mathematics as a science or as an art? What is the nature of the world without mathematics? Is mathematics considered the queen of science given that Number Theory is the queen of mathematics, and sciences and other discipline use numbers as a tool to express their ideas and concepts?

Though problems of science have been the chief interest for mathematicians there are still other motivations for mathematical works. One of these is the search for beauty. Certainly, there many theorems, axioms and algorithms that have no bearing on science but that appeal to many mathematicians as beautiful such as the Fibonacci sequence, Pascal's triangle and other numerical patterns and their applications to our daily lives, nature, and in the world in general.

Learning outcomes:

At the end of this chapter, the students are expected to:

1. Specify mathematical patterns in nature and regularities in the world;
2. Recognize mathematics as a science and as an art;
3. Make and test conjectures based on their own exploration especially when the material presented is carefully chosen;
4. Appreciate the beauty of mathematics and its application in our daily lives.

Nature of Mathematics

The existence of mathematics in our environment is undeniable. The shapes and heights of the buildings, the speed of the moving vehicles, the formations of mountains and terrains, the growth of the plants and pistil formations, and the human daily activities such as the way we buy in the market, asking weights and prices of any commodities, and even in dancing; they all involve numbers, and needless to say, mathematics.

As rational creatures, we tend to identify and follow patterns, whether consciously or subconsciously. Recognizing patterns feels natural like our brain is hardwired to recognized them. Early humans recognized the repeating interval of day and night, the cycle of the moon, the rising and falling of tides, and the changing of the seasons. Awareness of these allowed humans to survive. In a similar fashion, many flora and fauna also follow certain patterns such as the arrangement of leaves and stems in a plant, the shape of a snowflake, the flower's petals, or even the shape of snail's shell.

Patterns

In the general sense of the word, **patterns** are regular, repeated, or recurring forms or designs.

Patterns indicate a sense of structure and organization that it seems only humans are capable of producing these intricate, creative, and amazing formations. It is from this perspective that some people see an "intelligent design" in the way that nature forms.

- What number comes next in 1, 3, 5, 7, 9, ___?
- What comes next in A, C, E, G, I,__?

Mathematics as a Science

Basically, mathematics is a science of numbers and magnitudes. Everything that involves quantity and its equivalent and those activities and experiments that show direction are mathematics. Generally, mathematics is a science because it follows the scientific procedure from problem to conclusion.

Mathematics as an Art

Math is an art because it can deduce any data or set of information into forms without losing any informative details through graphs, diagrams, statements and figures. The following illustrations features the beauty of mathematics.

Illustration 1: Graphs

Classroom population **50**

- Male 20
- Female 30

Pie graph

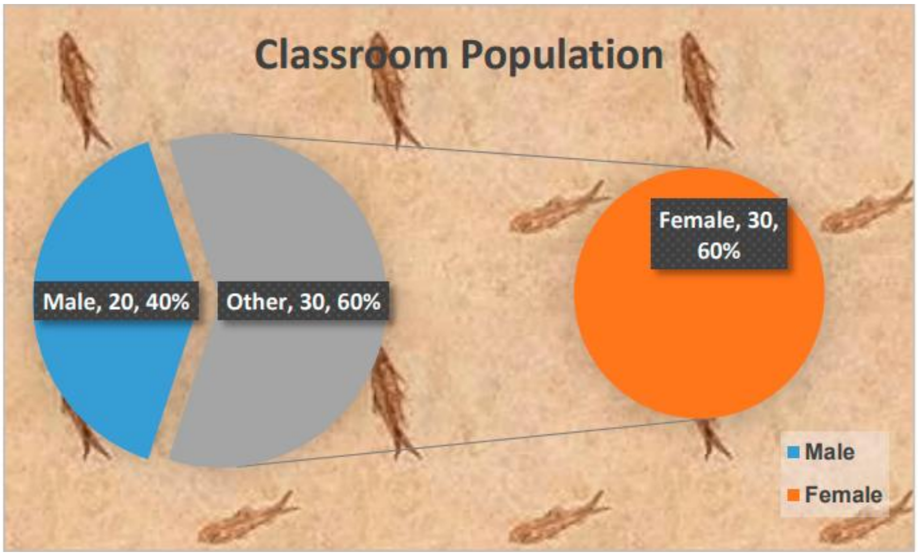


Illustration 2 : VEDIC MATH

23 x 32 = ?

Direction:

- Draw 2 horizontal lines and not in the far distance below draw another 3 horizontal lines for the multiplicand 23.

- Also draw 3 vertical lines across the horizontal lines and another 2 vertical lines not far distance to the right for multiplier 32.

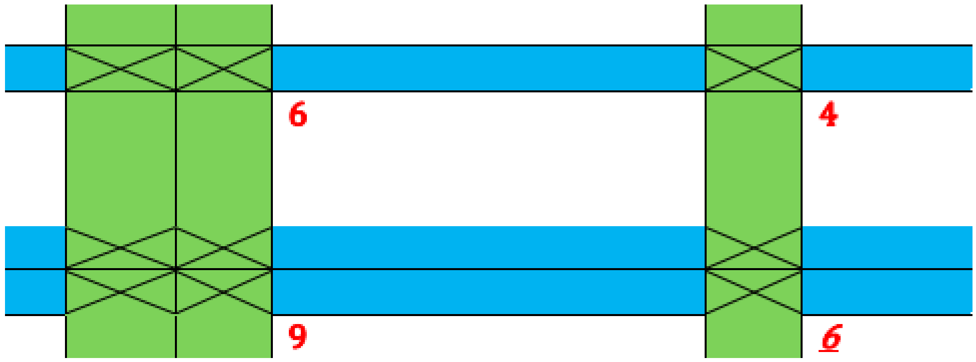
- Then count all the dots or vertices for every group.

- Group 1 the unit digit or the lower right (in case of 10 or more retain the unit digit and add the ten digit to the next group) in this case only 6.

- Group 2 is the sum of the diagonal group counts " say" 4 + 9 = 13, since 13 exceeds 10 retained 3 and carry over 1 to the next group.

- Group3 the hundreds or possibly the thousands digits, in this case 6 + 1 = 7.

$1 + 6 = \underline{7}$



$9 + 4 = 13$

$\underline{3}$ Carry 1

Therefore,

$23 \times 32 = \underline{736}$

ILLUSTRATION 3. THE PASCAL'S TRIANGLE

Diagram	Binomial equivalent
1	$(x+y)^0$
1 1	$(x+y)^1$
1 2 1	$(x+y)^2$
1 3 3 1	$(x+y)^3$
1 4 6 4 1	$(x+y)^4$
1 5 10 10 5 1	$(x+y)^5$
1 6 15 20 15 6 1	$(x+y)^6$
....	.
....	.
....	$(x+y)^n$

Example: Simplify $(x+y)^5$

$$(x+y)^5 = \mathbf{x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5}$$

Explanation:

The digits coloured red above serve as the numerical coefficients of our answer. Note that our answer might be in the forms of

$$(x+y)^5 = \mathbf{1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5}$$

ILLUSTRATION 4: STATEMENTS

Dr. OLRUGSTAVE, a mathematician and a psychiatrist, promotes a new approach of diagnosing patients. Every patient who comes to visit his clinic will just select 3 numbers that correspond to every letter of his name.

0 = O, 1 = L, 2 = R, 3 = U, 4 = G, 5 = S, 6 = T, 7 = A, 8 = V, 9 = E.

One day, Nikka and Algasher visit the clinic for consultation regarding the BSMATH course they are going to take looking if it fits their field of interest. Without much ado Dr. Olrugstave told them to choose 3 numbers from 0 to 9 to determine how interested they are in mathematics.

Nikka chose 143 while Algasher select 761. To determine their interest towards Mathematics these are the following steps.

Step 1. Be sure that the 3 numbers are not repeated.
Step 2. Scrutinize if the unit digit of their chosen number is greater than the hundred digits to avoid negative difference. Otherwise, retain the number.
Step 3. Subtract the number from the reverse of the chosen number or the Chosen number will be subtracted by the reverse of it.
Step 4. Add the difference from step 3 and its reverse.
Step 5. The corresponding sum will be substituted directly by their corresponding equivalent letter of the doctor's name for the female patient.
Step 6. The sum reveals the result or her interest in mathematics.
Step 7. For male patients multiply 40 to the sum of step 5 (40 is the international mortality age average for men being the head of the family).
Step 8. Substitute the product per digit by their corresponding equivalent letter of the doctor's name.
Step 9. The product divulge his interest in mathematics.

Example 1: Nikka -------------143

341 – 143 = 198
198 + 891 = 1089

by substitution

1 = L, 0 = O, 8 = V, 9 = E

Thus, **LOVE**

Example 2: Algasher----------761

761 – 167 = 594
594 + 495 = 1089
1089 x 40 = 43560

By substitution

4 = G, 3 = U, 5 = S, 6 = T, 0 = O

Thus, **GUSTO**

As per result the doctor's diagnosed that both Nikka and Algasher were both interested in mathematics with an adjectival rating of LOVE and GUSTO respectively.

FIBONACCI SEQUENCE

A **sequence** is an ordered list of numbers, called terms, that may have repeated values. The arrangement of these terms is set by a definite rule.

Fibonacci sequence is an array of numbers 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 and so on and so forth. These numbers were found by adding the two preceding numbers i.e,

- the 1 is found by adding 0 and 1
- the 3 is found by adding 1 and 2
- the 8 is found by adding 3 and 5
- the 55 is found by adding 21 and 34
- the 144 is found by adding 55 and 89

Mathematically, it is represented by this rule

$$F_n = f_{n-1} + f_{n-2}$$

Where f_{n-1} and f_{n-2} are the two preceding numbers. Actually, the sequence was known in India hundred years ago but it was an Italian mathematician, Leonardo Pisano Bogollo, who published it and called it Fibonacci sequence. For better understanding and appreciation on how this sequence thrives in the environment. You may watch the short movie by Cristobal Vila and Prof. John Adam at www.youtube.com entitled Nature by Numbers.

Example:

Let F_n be the nth term of the Fibonacci sequence, with $f_1 = 1, f_2 = 1, f_3 = 2,$ and so on.
Find the F_9.

Solution:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,…

$$F_n = f_{n-1} + f_{n-2}$$

$$F_9 = f_{9-1} + f_{9-2}$$

$$F_9 = f_8 + f_7$$

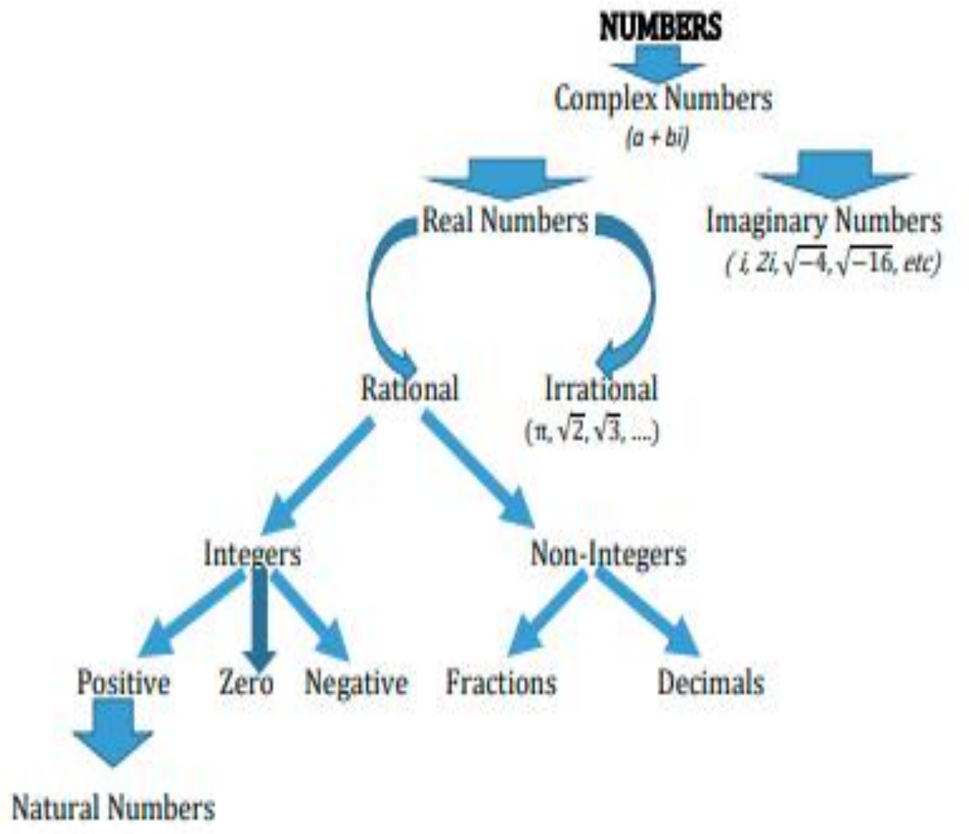
$$F_9 = 21 + 13$$

$$F_9 = 34$$

Numbers

What are numbers? Someone says, one, two, three, four, five or thousands, millions, billions, trillions. Nowadays, the most sophisticated computers could accommodate only 36-digit number or undecillion but there are still numbers beyond this. Thus, numbers are abstract ideas because without losing our generality (WOLOG) numbers are unbounded and unsaturated.

Hierarchy of Numbers



Definition of Terms

- Numbers - are abstract ideas. We count things using numbers. There are many ways in which we can classify numbers.
- Complex Numbers – Any number that can be written in the form of a + bi where a and b are real numbers, a is called the real part while bi is the imaginary part.
- Real Numbers – The union of all rational and irrational numbers.
- Imaginary Numbers – A non-real number that is usually a multiple of i which is the square root of -1.

- Rational Numbers – Any number that can be expressed as a fraction a/b where a and b are both integers but b could never be zero.
- Irrational Numbers – Any number that cannot be represented as a fraction.
- Integers – Consists of the whole numbers including its negative and Zero.
- Zero – Neither positive nor negative.
- Natural Numbers – Sometimes called counting Numbers. An integer greater than zero.

Numbers' Name based on the number of zeroes

The digit zero plays a gigantic role as we count very large number. It helps to track these multiples of 10 because the larger the number is, the more zeroes are needed.

Name	Number of zeroes	Scientific Notations
Ten	1	1×10^{1}
Hundred	2	1×10^{2}
Thousand	3	1×10^{3}
Ten Thousand	4	1×10^{4}
Hundred Thousand	5	1×10^{5}
Million	6	1×10^{6}
Billion	9	1×10^{9}
Trillion	12	1×10^{12}
Quadrillion	15	1×10^{15}
Quintillion	18	1×10^{18}
Sextillion	21	1×10^{21}
Septillion	24	1×10^{24}
Octillion	27	1×10^{27}
Nonillion	30	1×10^{30}
Decillion	33	1×10^{33}
Undecillion	36	1×10^{36}
Duodecillion	39	1×10^{39}
Tredecillion	42	1×10^{42}
Quattuordecillion	45	1×10^{45}
Quindecillion	48	1×10^{48}
Sexdecillion	51	1×10^{51}
Septen-decillion	54	1×10^{54}
Octodecillion	57	1×10^{57}
Novemdecillion	60	1×10^{60}
Vigintillion	63	1×10^{63}
Centillion	303	1×10^{303}
Googol	100	1×10^{100}
Googolplex		1×10^{googol}

Remarks: ***Scientific Notation*** *is a way of expressing numbers that are too big or too small to be conveniently written in decimal. It is commonly used by scientists, mathematicians and engineers in particular because it can simplify certain arithmetic operations.*

Mathematical Symbols and Notations

Mathematicians are fond of using mathematical jargons to express its ideas and concepts. These notations plays a gigantic role as we go along with our course. The following are the mathematical symbols that you might be encountered in the succeeding topics.

Symbols	Unicode Names	Representations
$\forall$	For all or for any	Proof jargon
$\exists$	There exists	Proof jargon
$\ni$	Contain as number or such that	Proof jargon
$\Rightarrow$	Implies to	Implication
$\Leftrightarrow$	Bi-conditional	If and only if or iff
$\therefore$	Therefore	Concluding statement
$\mid$	Divides	a **$\mid$ b , a can divides b**
$\nmid$	Does not divide	a **$\nmid$ b, a cannot divides b**
$\sum$	Capital letter Sigma	Summation notation
$\in$	Element of a set	Proof jargon
μ	Small Mu	Population Mean
π	pi	Ratio (Circumference & diameter of a circle)
ρ	Small letter Rho	Rank Correlation
σ	Small letter Sigma	Standard Deviation
$\vee$	Logical OR	Disjunction
$\wedge$	Logical AND	Conjunction
$\sim$	Tilde Operator	Negation
$\neq$	Not equal	Not equal
$\cong$	Congruent	Linear Congruence
$\ncong$	Not Congruent	Linear Congruence
Φ	Greek letter Phi	Euler's Phi Function
$\bullet$	Dot	Multiplication Sign
$\leq$	Less than or equal to	Inclusion of least integer