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### Problem 1

1. Suppose the instructor of the course is convinced that the mean engagement of students who become knowledgeable in the material (i.e., the `engagement_1` population) is 0.75.
  - a. *[5 points]* Formulate null and alternative hypotheses for a statistical test that seeks to challenge this belief. What are the null and alternative hypotheses?

**ANSWER:** The null hypothesis is  $H_0: \mu = 0.75$ . The alternative hypothesis is  $H_1: \mu \neq 0.75$

- b. *[5 points]* What type of test should be used and why?

**ANSWER:** We should use a Z-score test, as we have at least 30 data points.

2. Carry out the statistical test defined in (1b) using the `engagement_1` sample.

- a. *[1 point]* What is the sample size?

**ANSWER:** 937

- b. *[1 point]* What is the sample mean?

**ANSWER:** 0.7430304110448239

- c. *[2 points]* What is the standard error?

**ANSWER:** 0.004153027288269652

- d. *[2 points]* What is the standard score?

**ANSWER:** -1.6781948375012814

- e. *[2 points]* What is the p-value?

**ANSWER:** 0.09330906925243751

- f. [2 points] Are the results statistically significant at a level of 0.05? How about 0.10? What (if anything) can we conclude (i.e., what is the interpretation of the result)?

**ANSWER:** At a level of 0.05, we failed to reject the null hypothesis for the 95% confidence level and the p-value is statistically significant.

At a level of 0.10, we can reject the null hypothesis for the 90% confidence level and the p-value is not statistically significant.

3. [10 points] What is the largest standard error for which the test will be significant at a level of 0.05? What is the corresponding minimum sample size? (You may assume that the population variance and mean does not change.)

**ANSWER:** At a level of 0.05, the largest standard error for which the test will be significant is: 0.004237209220916103  
The minimum sample size is: 900

4. Suppose the instructor is also convinced that the mean engagement is different between students who become knowledgeable (the engagement\_1 population) and those who do not (the engagement\_0 population).
- a. [5 points] Formulate null and alternative hypotheses that seek to validate this belief. What are the null and alternative hypotheses?

**ANSWER:** The null hypothesis is that the mean engagement is 0.75. The alternative hypothesis is that the mean engagement is different between the two.

- b. [5 points] What type of test should be used and why?

**ANSWER:** The z-test should be used, as the sample size is bigger than 30.

5. Carry out the statistical test defined in (4b) using the `engagement_1` and `engagement_2` samples.

- a. [1 point] What are the sample sizes?

**ANSWER:** 1977 and 937

- b. [1 point] What are the sample means?

**ANSWER:** 0.6399545077035914 and 0.7430304110448239

- c. [2 points] What is the standard error?

**ANSWER:** 0.005715989588773277

- d. [2 points] What is the standard score?

**ANSWER:** -19.252220562568542

- e. [2 points] What is the p-value?

**ANSWER:** 1.352401028607762e-82

- f. [2 points] Are the results statistically significant at a level of 0.05? How about 0.10? What (if anything) can we conclude (i.e., what is the interpretation of the result)?

**ANSWER:** We can reject the null hypothesis for the 95% confidence level, and the p-value is not statistically significant.

We can reject the null hypothesis for the 90% confidence level, and the p-value is not statistically significant.

## Problem 2

1. Use the sample to construct a 90% confidence interval for the number of points by which the team wins on average.

- a. *[3 points]* Will you use a t-test or z-test (Hint: Think which distribution should you use here if very few data points are available)? Justify your answer.

**ANSWER:** T-test, as there are fewer than 30 data points.

- b. *[3 points]* What is the sample mean?

**ANSWER:** 7.363636363636363

- c. *[3 points]* What is the standard error?

**ANSWER:** 4.840040856450614

- d. *[3 points]* What is the standard statistic (t or z value)?

**ANSWER:** 1.4308981556180451

- e. *[3 points]* What is the 90% confidence interval?

**ANSWER:** (0.4380308290251964, 14.28924189824753)

2. Repeat Q1 for a 95% confidence interval.

- a. *[2 points]* What is the standard statistic (t or z value)?

**ANSWER:** 1.4407478926091581

- b. [2 points] What is the 95% confidence interval?

**ANSWER:** (0.39035769956291677, 14.33691502770981)

- c. [1 point] Is your interval wider or narrower compared to using the 90% confidence interval in Q1?

**ANSWER:** My interval is wider compared to the 90% interval.

3. Repeat Q2 if you are told that the population standard deviation is 15.836.

- a. [5 points] Will you use a t-test or z-test (Hint: Think which distribution should you use here now that you have the true population standard deviation)? Justify your answer.

**ANSWER:** Since we know the standard deviation, I will use a z-score.

- b. [3 points] What is the standard error?

**ANSWER:** 4.774733652733465

- c. [3 points] What is the standard statistic (t or z value)?

**ANSWER:** 1.6448536269514722

[3 points] What is the 95% confidence interval?

**ANSWER:** (-0.49010160278952775, 15.217374330062254)

- d. [6 points] Is your interval wider or narrower than the interval computed in Q2?

**ANSWER:** My interval is wider than the interval computed in Q2.

4. [10 points] Assume you no longer know the population standard deviation. With what level of confidence can we say that the team is expected to win on average? (Hint: What level of confidence would you get a confidence interval with the lower endpoint being 0?)

**ANSWER:** We can say that the team is expected to win on average with a confidence level of 84%

