Session 1 Challenge

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```
In [1]: # For data handling and visualization
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
```

1 Linear Regression example

https://scikit-learn.org/stable/datasets/index.html#diabetes-dataset

```
In [2]: # Load the dataset
        from sklearn import datasets
        diabetes = datasets.load_diabetes()
```

```
In [3]: # Get input, output, and their names
        X = diabetes.data
        y = diabetes.target
        names = diabetes.feature_names
```

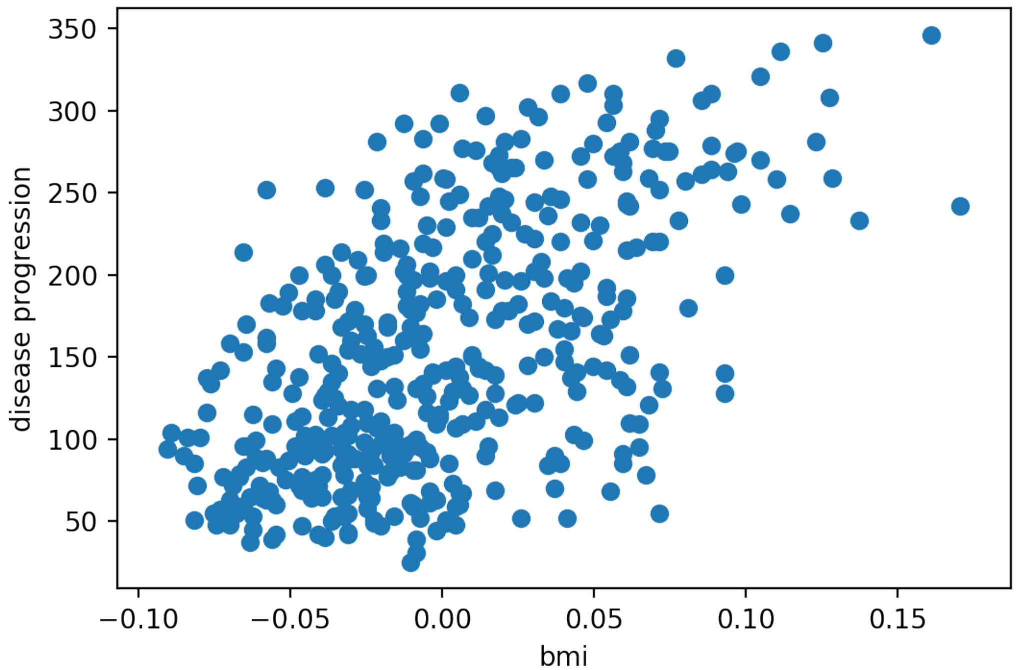
```
In [4]: # First 5 data points
        df = pd.DataFrame(np.concatenate((X, y[:, np.newaxis]), axis=1), columns=names + ['dise
        df.head()
```

```
Out[4]:         age       sex       bmi        bp        s1        s2        s3  \
        0  0.038076  0.050680  0.061696  0.021872 -0.044223 -0.034821 -0.043401
        1 -0.001882 -0.044642 -0.051474 -0.026328 -0.008449 -0.019163  0.074412
        2  0.085299  0.050680  0.044451 -0.005671 -0.045599 -0.034194 -0.032356
        3 -0.089063 -0.044642 -0.011595 -0.036656  0.012191  0.024991 -0.036038
        4  0.005383 -0.044642 -0.036385  0.021872  0.003935  0.015596  0.008142

                 s4        s5        s6  disease progression
        0 -0.002592  0.019908 -0.017646                151.0
        1 -0.039493 -0.068330 -0.092204                 75.0
        2 -0.002592  0.002864 -0.025930                141.0
        3  0.034309  0.022692 -0.009362                206.0
        4 -0.002592 -0.031991 -0.046641                135.0
```

```
In [5]: # Only one feature
        bmi = X[:, np.newaxis, 2]
```

In [6]:
```
# Plot bmi vs. target variable
plt.figure(dpi=200)
plt.scatter(bmi, y)
plt.xlabel('bmi')
plt.ylabel('disease progression')
```

Out[6]:
```
Text(0, 0.5, 'disease progression')
```



In [7]:
```
# Split the dataset into train and test

X_train = bmi[:300]
y_train = y[:300]

X_test = bmi[300:]
y_test = y[300:]
```

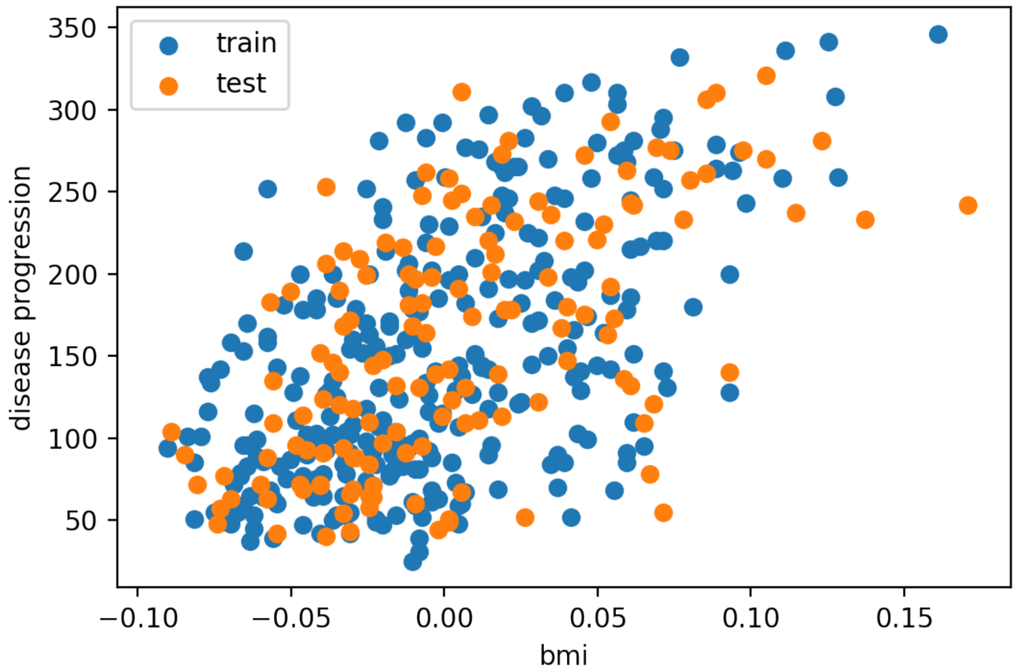
In [8]:
```
# Sanity check the shapes
X_train.shape, y_train.shape, X_test.shape, y_test.shape
```

Out[8]:
```
((300, 1), (300,), (142, 1), (142,))
```

In [9]:
```
# Plot train and test
plt.figure(dpi=200)
plt.scatter(X_train, y_train, label='train')
plt.scatter(X_test, y_test, label='test')
```

```
plt.xlabel('bmi')
plt.ylabel('disease progression')
plt.legend()
```

Out[9]: <matplotlib.legend.Legend at 0x11e1bd9e8>



In [10]:
```
# Call the linear regression object
from sklearn import linear_model
model1 = linear_model.LinearRegression()
```

In [11]:
```
# LEARN = .fit(input, target)
model1.fit(X_train, y_train)
```

Out[11]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)
```

In [12]:
```
# Get predictions for the test set
preds = model1.predict(X_test)
```

In [13]:
```
# Check predictions vs. labels for the first 10 data points
preds[:10], y_test[:10]
```

Out[13]:
```
(array([222.78610979, 127.10097299, 183.88116406, 184.93264908,
        113.43166773, 112.38018271, 149.1821584 , 120.79206287,
        106.07127259, 152.33661346]),
 array([275.,  65., 198., 236., 253., 124.,  44., 172., 114., 142.]))
```

In [14]:
```python
# Quantify the error
preds[:10] - y_test[:10]
```

Out[14]:
```
array([ -52.21389021,   62.10097299,  -14.11883594,  -51.06735092,
       -139.56833227,  -11.61981729,  105.1821584 ,  -51.20793713,
         -7.92872741,   10.33661346])
```

In [15]:
```python
# Use absolute values to avoid positive/negative errors canceling each other
abs(preds[:10] - y_test[:10])
```

Out[15]:
```
array([ 52.21389021,  62.10097299,  14.11883594,  51.06735092,
       139.56833227,  11.61981729, 105.1821584 ,  51.20793713,
         7.92872741,  10.33661346])
```

In [16]:
```python
# Find the mean abs error
np.mean(abs(preds[:10] - y_test[:10]))
```

Out[16]:
```
50.53446360349993
```

In [17]:
```python
# Find the mean abs error, for all test data
np.mean(abs(preds - y_test))
```

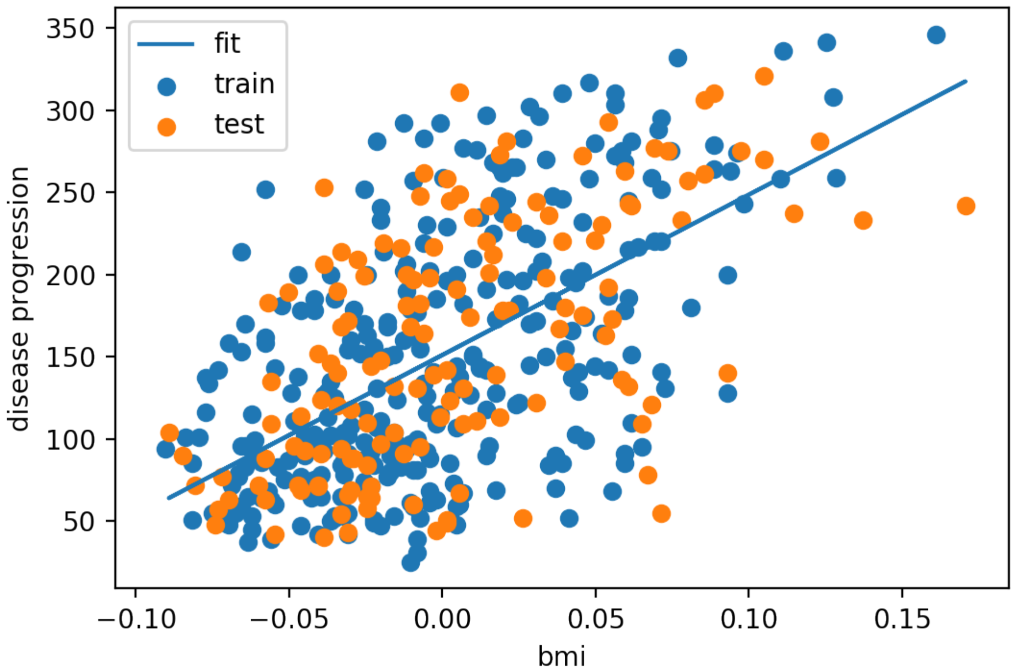
Out[17]:
```
50.59683538302396
```

In [18]:
```python
# This and many other metrics are available in sklearn
from sklearn import metrics
metrics.mean_absolute_error(y_test, preds)
```

Out[18]:
```
50.59683538302396
```

In [19]:
```python
# Plot train/test data with fit
plt.figure(dpi=200)
plt.scatter(X_train, y_train, label='train')
plt.plot(X_test, model1.predict(X_test), label='fit')
plt.scatter(X_test, y_test, label='test')
plt.xlabel('bmi')
plt.ylabel('disease progression')
plt.legend()
```

Out[19]:
```
<matplotlib.legend.Legend at 0x11ea96f28>
```



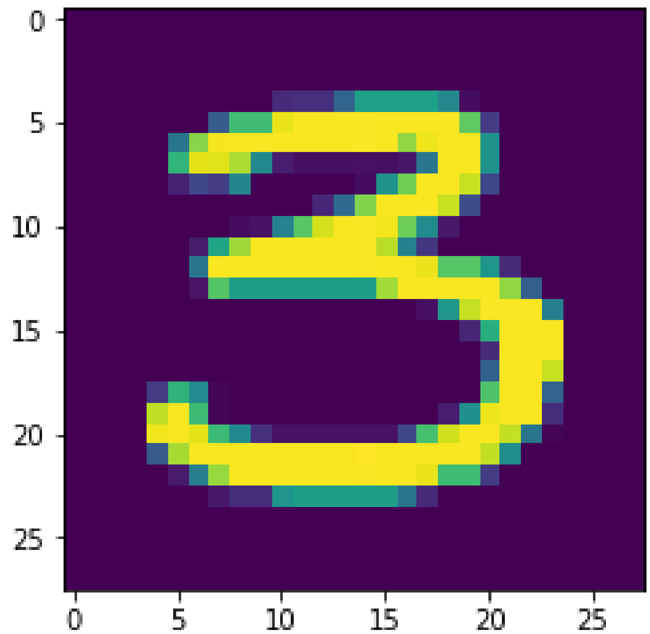
2 Deep learning example

```
In [20]: # Download the MNIST ataset
         from keras.datasets import mnist
         (x_train, y_train), (x_test, y_test) = mnist.load_data()
```

```
Using TensorFlow backend.
```

```
In [21]: # Random digit from the training set.
         index = np.random.randint(0, 60000)
         plt.imshow(x_train[index])
         print('Label: ', y_train[index])
```

```
Label:  3
```



In [22]:
```
# Build the architecture
from keras.models import Sequential
from keras.layers import Conv2D, Flatten, Activation, Dense

model2 = Sequential()
model2.add(Conv2D(32, (3, 3), padding='same', input_shape=(28, 28, 1)))
model2.add(Activation('relu'))
model2.add(Conv2D(32, (3, 3), strides=2, padding='same'))
model2.add(Activation('relu'))
model2.add(Flatten())
model2.add(Dense(10))
model2.add(Activation('softmax'))
model2.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_1 (Conv2D)            (None, 28, 28, 32)        320
_________________________________________________________________
activation_1 (Activation)    (None, 28, 28, 32)        0
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 14, 14, 32)        9248
_________________________________________________________________
activation_2 (Activation)    (None, 14, 14, 32)        0
_________________________________________________________________
flatten_1 (Flatten)          (None, 6272)              0
```

```
_________________________________________________________________
dense_1 (Dense)              (None, 10)                62730
_________________________________________________________________
activation_3 (Activation)    (None, 10)                0
=================================================================
Total params: 72,298
Trainable params: 72,298
Non-trainable params: 0
_________________________________________________________________
```

In [23]:
```python
# Set the optimizer and the loss
from keras.optimizers import Adam, SGD
opt = SGD(lr=0.001)
model2.compile(optimizer=opt, loss='binary_crossentropy', metrics=['accuracy'])
```

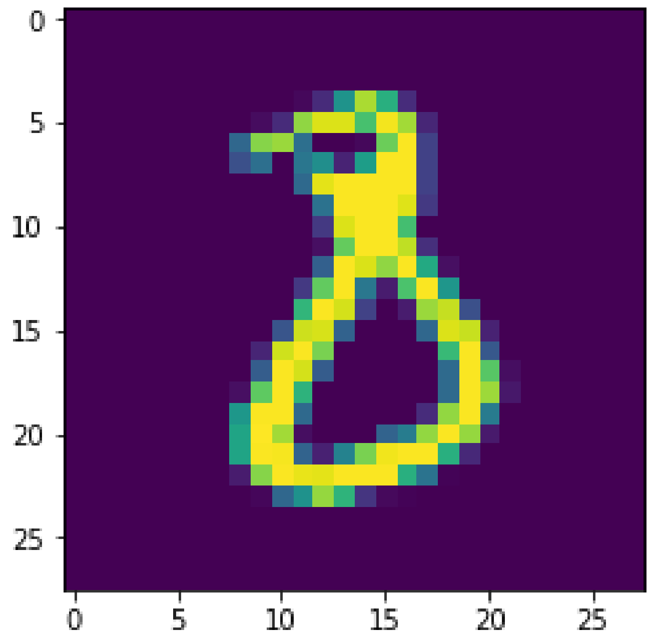
In [24]:
```python
# Put data into suitable shape
from keras.utils import to_categorical
y_test = to_categorical(y_test)
y_train = to_categorical(y_train)

x_train = x_train.reshape(-1, 28, 28, 1)
x_test = x_test.reshape(-1, 28, 28, 1)
```

In [25]:
```python
# Train and test
H = model2.fit(x_train, y_train, batch_size=32, epochs=1, validation_data=(x_test, y_
```

```
Train on 60000 samples, validate on 10000 samples
Epoch 1/1
60000/60000 [==============================] - 53s 889us/step - loss: 0.1489 - acc: 0.9710 - va
```

In [26]:
```python
# Random image from the test set
index = np.random.randint(0, 10000)
random_image = x_test[np.newaxis, index]
plt.imshow(random_image.reshape(28, 28))
print('Label:', np.argmax(y_test[index]))
print('Prediction: ', np.argmax(model2.predict(random_image)))  # model.predict(image
```

```
Label: 8
Prediction:  3
```



3 Challenge 1

```
In [27]: from sklearn import datasets
         diabetes = datasets.load_diabetes()
```

- Repeat the example: predict disease progression from bmi
- Check errors with different metrics e.g. mean squared error (mse) https://scikit-learn.org/stable/modules/classes.html#module-sklearn.metrics
- Compare mse with mae
- Find the intercept and slope using sklearn.linear_model.LinearRegression attributes https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html
- Instead of bmi only, use more features and fit again, check if the performance improves
- Use other algorithms e.g. Random Forests

4 Data = Signal + Noise

- Data generation
  - Signal
  - Noise
  - Signal + Noise

In [28]:
```python
# Newton's second law as signal

m = 2  # kg
F = 10  # N

a = F/m  # m/s2

# a = intercept + slope * F

a
```

Out[28]: 5.0

In [29]:
```python
# 10 random F's between 0 and 1
F = np.random.rand(10, 1)
F
```

Out[29]:
```
array([[0.27265566],
       [0.45639664],
       [0.59645395],
       [0.93229163],
       [0.42039381],
       [0.79091579],
       [0.92921021],
       [0.89619034],
       [0.93065236],
       [0.95336649]])
```

In [30]:
```python
# 10 random F's between 0 and 100
F = np.random.rand(10, 1)*100
F
```

Out[30]:
```
array([[17.48382141],
       [76.99769087],
       [ 9.11378162],
       [89.1453672 ],
       [69.00352034],
       [18.27328023],
       [93.63434462],
       [41.13464976],
       [56.54294344],
       [20.48335529]])
```

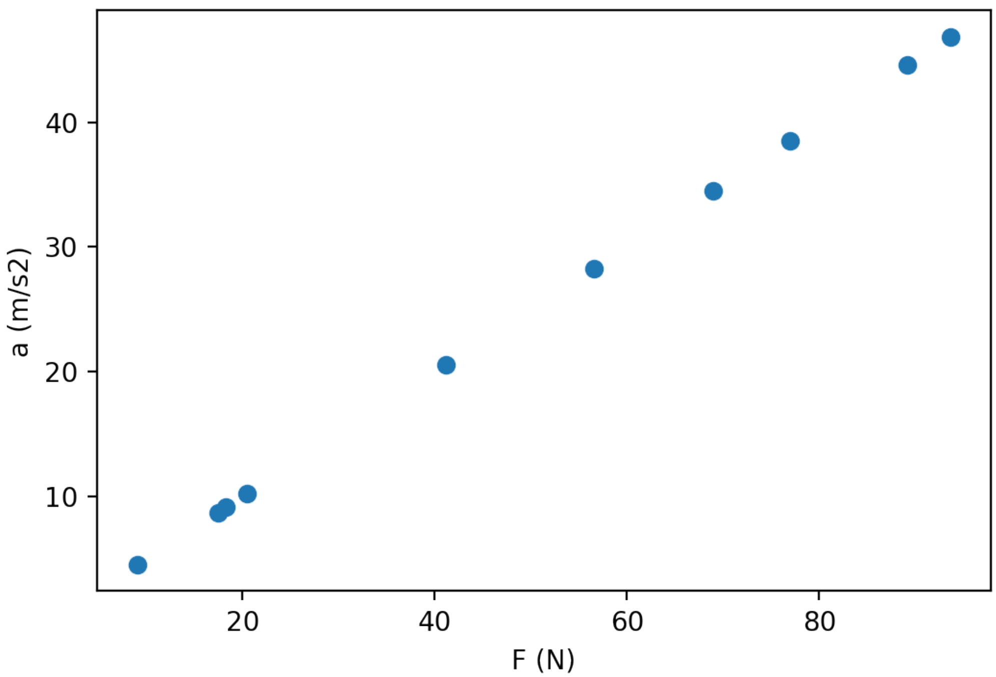
In [31]:
```python
# Calculate a using Newton's second law
a = F/m
a
```

Out[31]:
```
array([[ 8.7419107 ],
       [38.49884543],
```

```
       [ 4.55689081],
       [44.5726836 ],
       [34.50176017],
       [ 9.13664012],
       [46.81717231],
       [20.56732488],
       [28.27147172],
       [10.24167764]])
```

```
In [32]: # Plot F vs. a
         plt.figure(dpi=200)
         plt.scatter(F, a)
         plt.xlabel('F (N)')
         plt.ylabel('a (m/s2)')
```

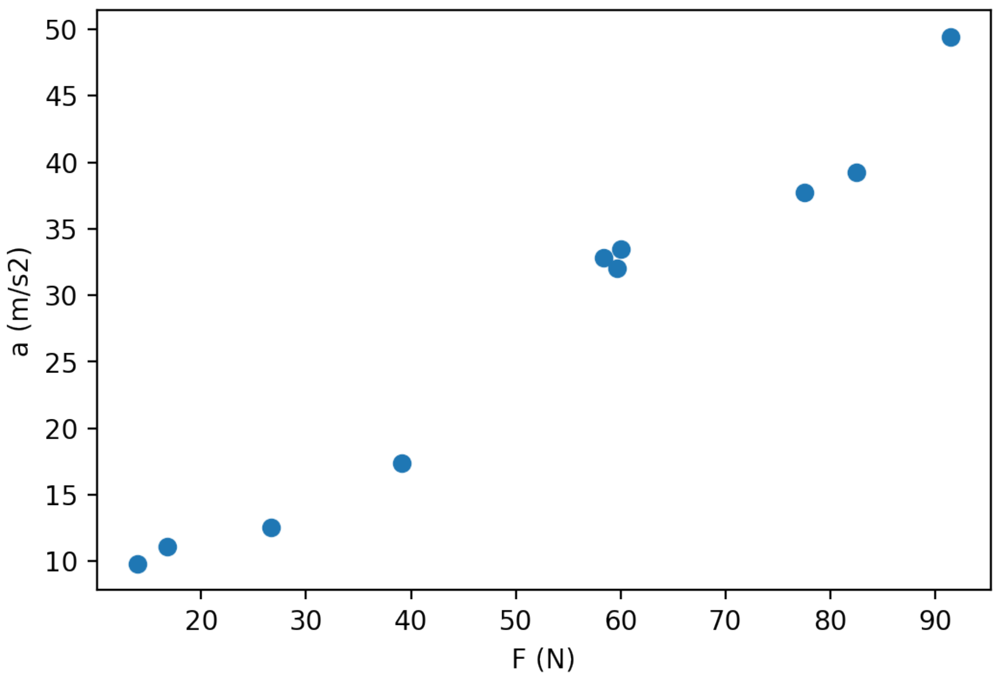
```
Out[32]: Text(0, 0.5, 'a (m/s2)')
```




```
In [33]: # Let's add noise to the signal
         m = 2  # kg
         F = np.random.rand(10, 1) * 100  # N
         noise = np.random.rand(10, 1) * 10 - 5
         a = F/m + noise  # m/s2

         # Plot F vs. a
         plt.figure(dpi=200)
```

```
plt.scatter(F, a)
plt.xlabel('F (N)')
plt.ylabel('a (m/s2)')
```

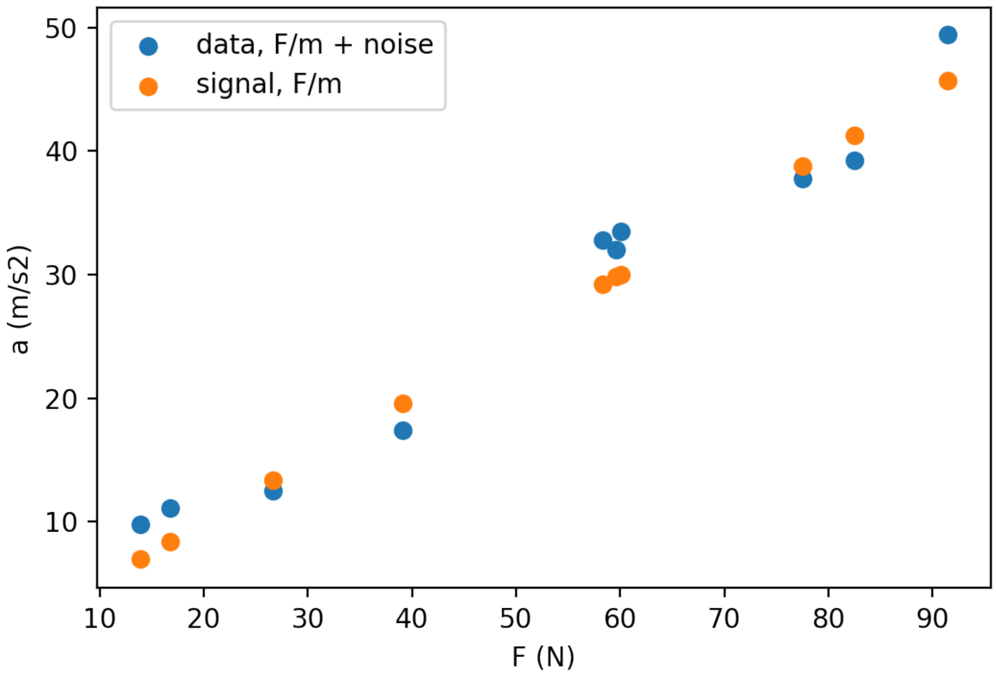
Out[33]: Text(0, 0.5, 'a (m/s2)')



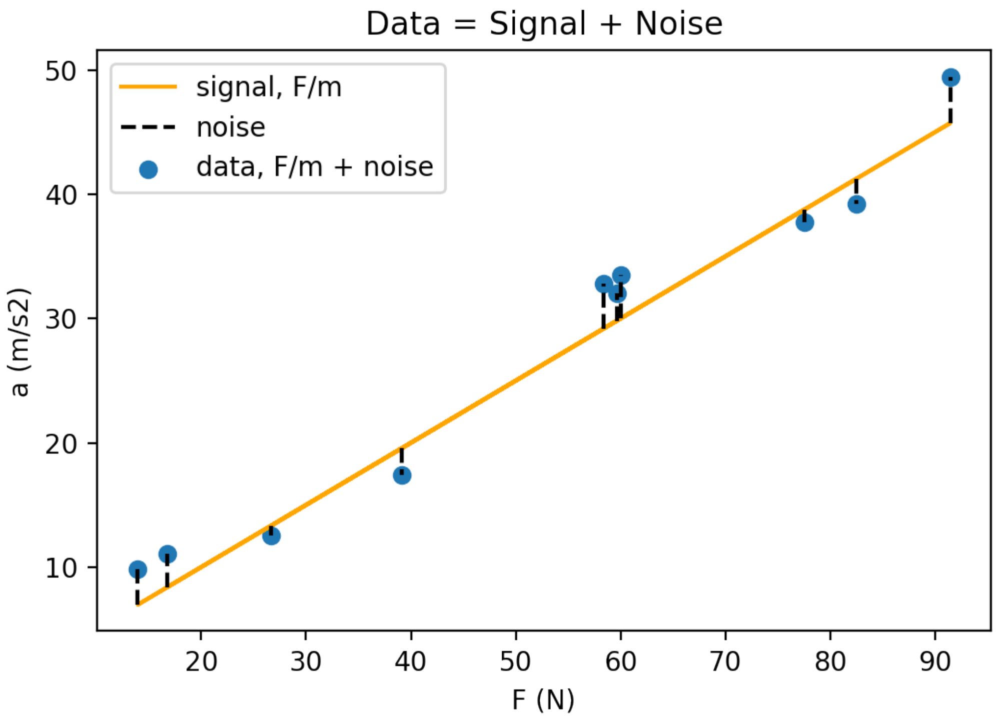
Data = Signal + Noise
a = F/m + noise

In [34]:
```
# Signal vs. Data
plt.figure(dpi=200)
plt.scatter(F, a, label='data, F/m + noise')
plt.scatter(F, F/m, label='signal, F/m')
plt.xlabel('F (N)')
plt.ylabel('a (m/s2)')
plt.legend()
```

Out[34]: <matplotlib.legend.Legend at 0x135919eb8>

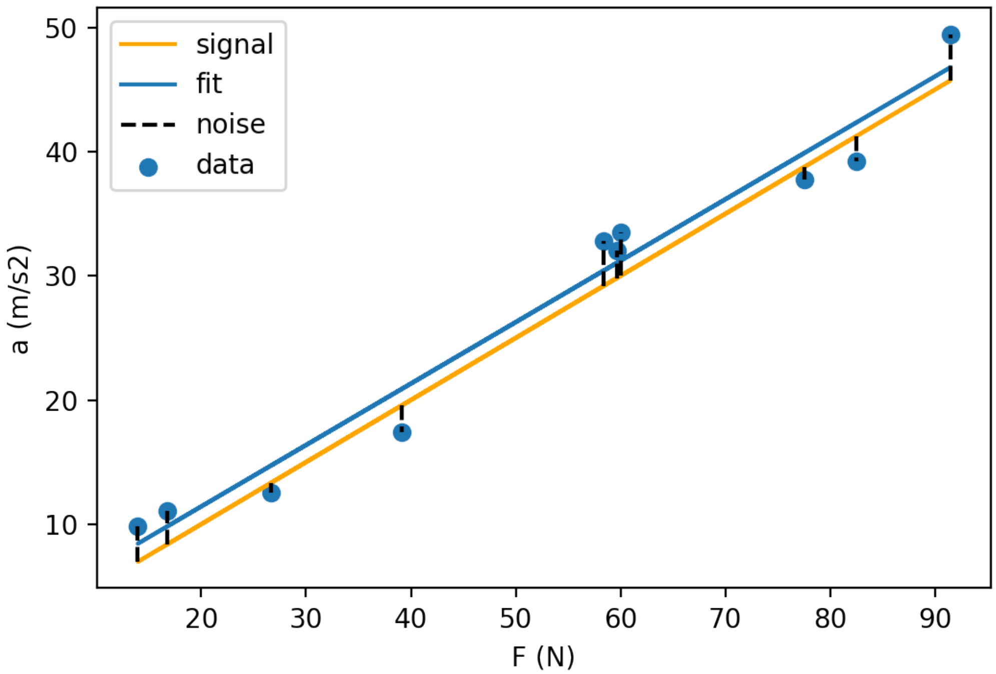



```
In [35]: # Data = Signal + Noise
         plt.figure(dpi=200)
         plt.scatter(F, a, label='data, F/m + noise')
         plt.plot(F, F/m, label='signal, F/m', color='orange')

         for i in range(len(F)):
             plt.plot([F[i], F[i]], [F[i]/m, a[i]], 'k--', label='noise' if i == 0 else None)

         plt.xlabel('F (N)')
         plt.ylabel('a (m/s2)')
         plt.legend()
         plt.title('Data = Signal + Noise')

Out[35]: Text(0.5, 1.0, 'Data = Signal + Noise')
```



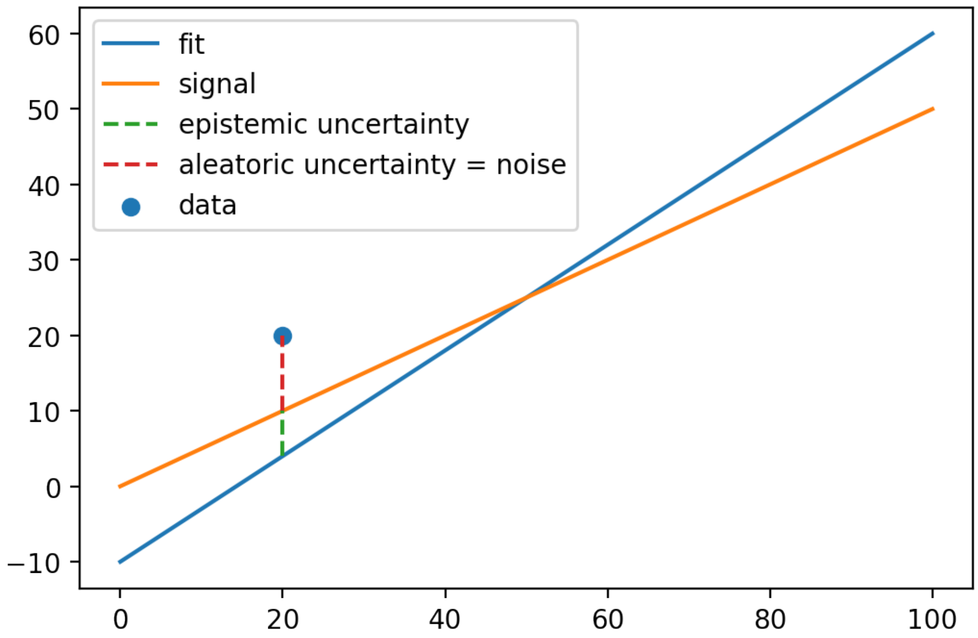
The aim of machine learning is to find the signal

```
In [36]: # We know the signal, let's see if ML can find it
         from sklearn.linear_model import LinearRegression
         linear_model = LinearRegression()
         linear_model.fit(F, a)

Out[36]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
                  normalize=False)

In [37]: # Signal vs. Fit
         plt.figure(dpi=200)
         plt.scatter(F, a, label='data')
         plt.plot(F, F/m, label='signal', color='orange')
         plt.plot(F, linear_model.predict(F), label='fit')

         for i in range(len(F)):
             plt.plot([F[i], F[i]], [F[i]/m, a[i]], 'k--', label='noise' if i == 0 else None)

         plt.xlabel('F (N)')
         plt.ylabel('a (m/s2)')
         plt.legend()

Out[37]: <matplotlib.legend.Legend at 0x136332320>
```




```
In [38]: # Aleatoric vs. Epistemic uncertainty
         plt.figure(dpi=200)
         plt.plot([0, 100], [-10, 60], label='fit')
         plt.plot([0, 100], [0, 50], label='signal')
         plt.scatter([20], [20], label='data')
         plt.plot([20, 20], [4, 10], '--', label='epistemic uncertainty')
         plt.plot([20, 20], [10, 20], '--', label='aleatoric uncertainty = noise')
         plt.legend()
```

```
Out[38]: <matplotlib.legend.Legend at 0x136370198>
```



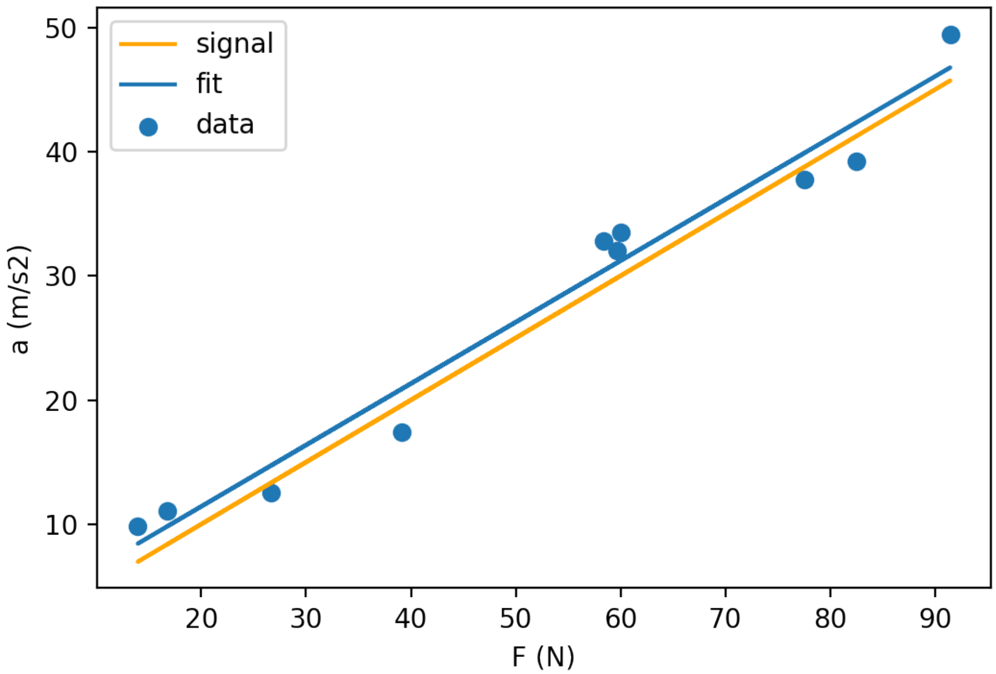
5 Challenge 2

- Plot how epistemic and aleatoric uncertainity will change wrt the number of data. (fixed noise)
- Plot how epistemic and aleatoric uncertainity will change wrt the noise. (fixed number of data)
- Make an educated guess before plotting.

6 Overfitting

In [39]:
```
# Plot again the prev. data
plt.figure(dpi=200)
plt.scatter(F, a, label='data')
plt.plot(F, F/m, label='signal', color='orange')
plt.plot(F, linear_model.predict(F), label='fit')

plt.xlabel('F (N)')
plt.ylabel('a (m/s2)')
plt.legend()
```

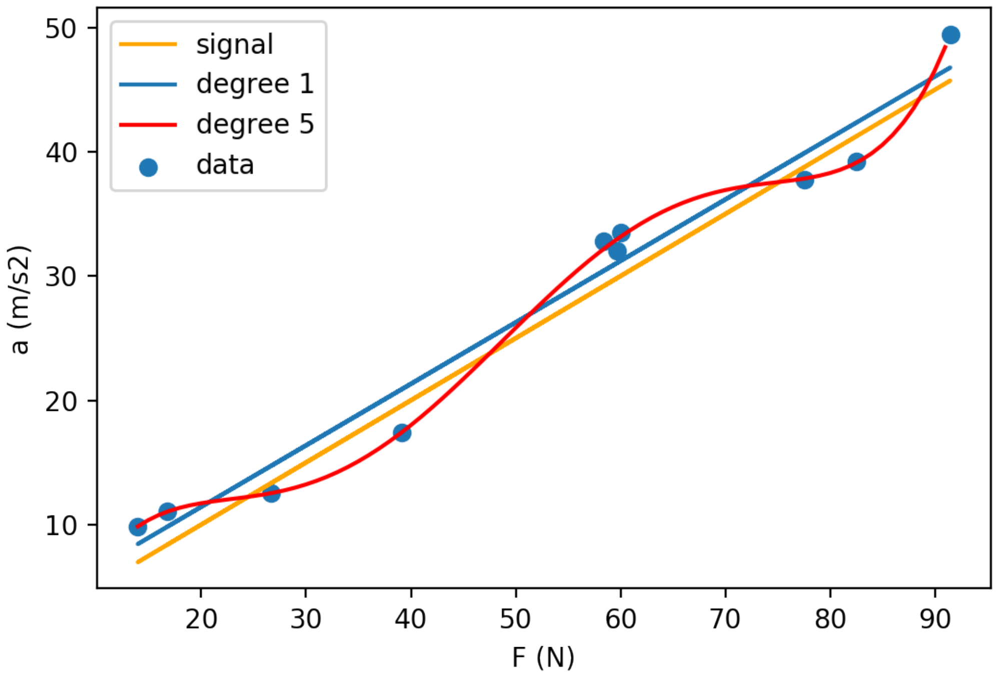
Out[39]:
```
<matplotlib.legend.Legend at 0x10c472a90>
```




```
In [40]: # Fit a degree 5 polynomial to F and a
         z = np.poly1d(np.polyfit(F.reshape(-1), a.reshape(-1), 5))
```

```
In [41]: # Plot both fits
         plt.figure(dpi=200)
         plt.scatter(F, a, label='data')
         plt.plot(F, F/m, label='signal', color='orange')
         plt.plot(F, linear_model.predict(F), label='degree 1')

         plt.plot(np.arange(min(F), max(F)), z(np.arange(min(F), max(F))), label='degree 5', co

         plt.xlabel('F (N)')
         plt.ylabel('a (m/s2)')
         plt.legend()
```

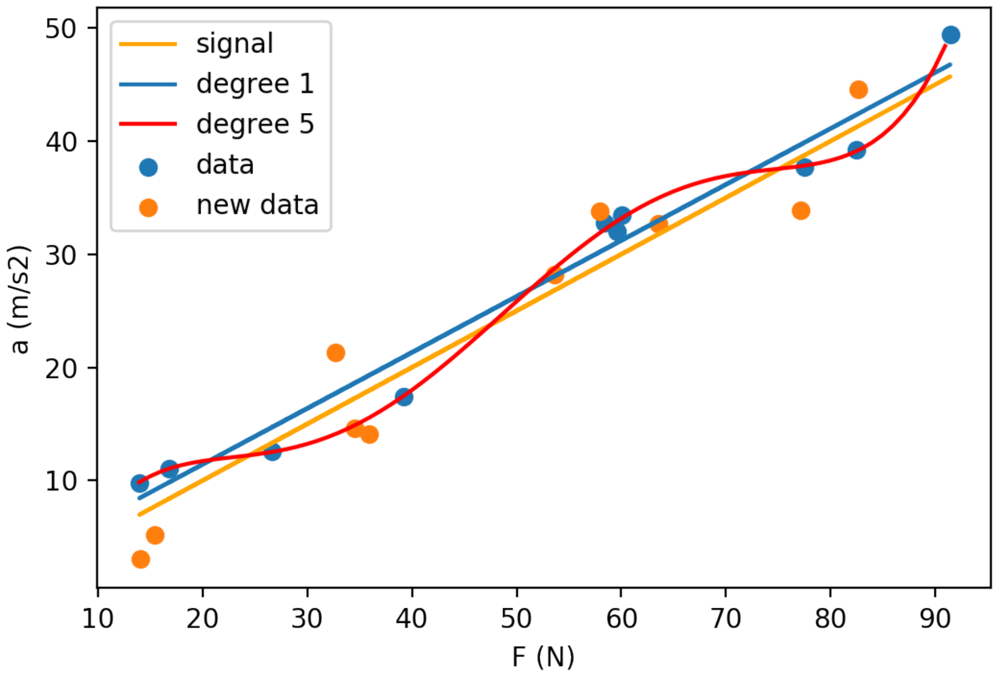
```
Out[41]: <matplotlib.legend.Legend at 0x136353ba8>
```




```
In [42]: # Generate new data
         m = 2  # kg
         F_new = np.random.rand(10, 1) * (max(F)-min(F)) + min(F)  # N
         noise = np.random.rand(10, 1) * 10 - 5
         a_new = F_new/m + noise  # m/s2
```

```
In [43]: # Both fits on train and test data
         plt.figure(dpi=200)
         plt.scatter(F, a, label='data')
         plt.scatter(F_new, a_new, label='new data')
         plt.plot(F, F/m, label='signal', color='orange')
         plt.plot(F, linear_model.predict(F), label='degree 1')

         plt.plot(np.arange(min(F), max(F)), z(np.arange(min(F), max(F))), label='degree 5', co

         plt.xlabel('F (N)')
         plt.ylabel('a (m/s2)')
         plt.legend()
```

```
Out[43]: <matplotlib.legend.Legend at 0x136cd4710>
```



7 Challenge 3

- Calculate training error for both fits
- Calculate test error for both fits
- Fit different degree polynomials and plot training and test errors wrt degree of the polynomial fit
- Generate data using a quartic signal, repeat previous step, and discover yourself what "underfitting" is.

8 Challenge 4

Implement linear regression using gradient descent

In [44]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [45]:
```
# Generate data
# 1. Define m and n
m = 10  # number of examples
n = 1  # number of features
```

In [46]:
```
# 2. Generate X
X = np.random.uniform(size=(m, n))
X
```

```
Out[46]: array([[0.64985039],
                [0.62500976],
                [0.69915575],
                [0.33744647],
                [0.31746079],
                [0.50201737],
                [0.77397666],
                [0.22890078],
                [0.193001  ],
                [0.29300021]])
```

```
In [47]: # 3. Add 1's to X for the bias term for the dot product
         X = np.concatenate((np.ones((m, 1)), X), axis=1)
         X
```

```
Out[47]: array([[1.        , 0.64985039],
                [1.        , 0.62500976],
                [1.        , 0.69915575],
                [1.        , 0.33744647],
                [1.        , 0.31746079],
                [1.        , 0.50201737],
                [1.        , 0.77397666],
                [1.        , 0.22890078],
                [1.        , 0.193001  ],
                [1.        , 0.29300021]])
```

```
In [48]: # 4. Randomly generate parameters
         theta_true = np.random.uniform(size=(n+1, 1))
         theta_true
```

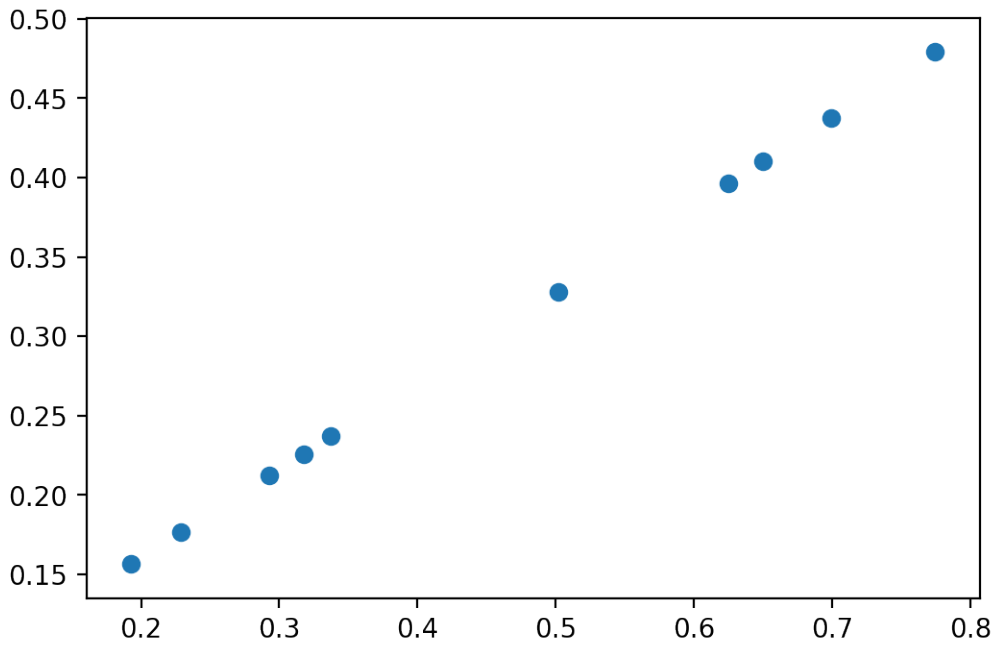
```
Out[48]: array([[0.04970101],
                [0.55483883]])
```

```
In [49]: # 5. Calculate y
         y = np.dot(X, theta_true)
         y
```

```
Out[49]: array([[0.41026324],
                [0.39648069],
                [0.43761977],
                [0.23692942],
                [0.22584059],
                [0.32823974],
                [0.47913332],
                [0.17670406],
                [0.15678546],
                [0.21226891]])
```

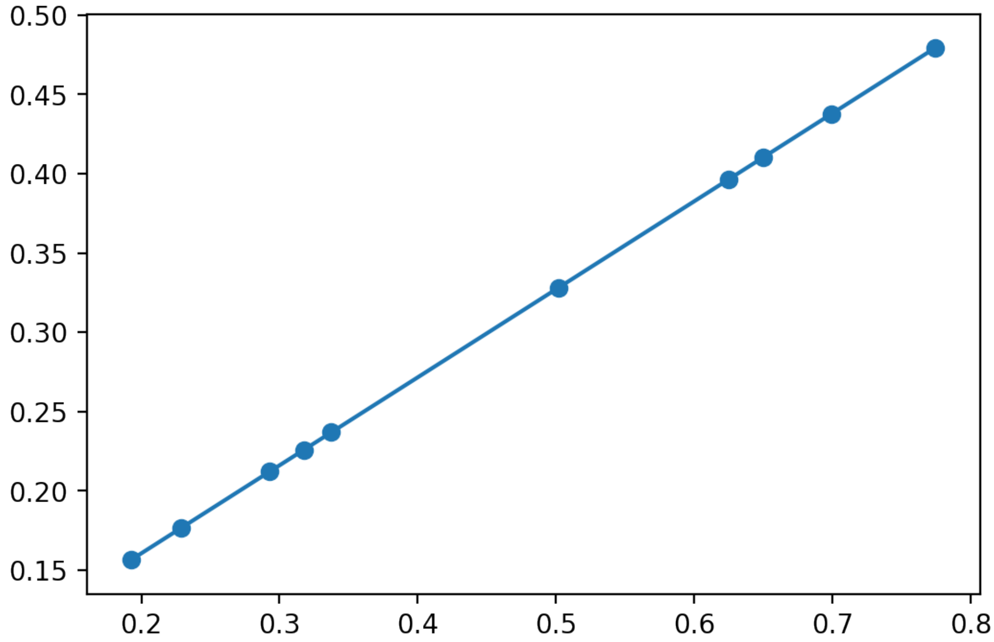
```
In [50]: # 6. Plot the generated X and y
         plt.figure(dpi=200)
         plt.scatter(X[:, 1], y)
```

Out[50]: <matplotlib.collections.PathCollection at 0x1376178d0>



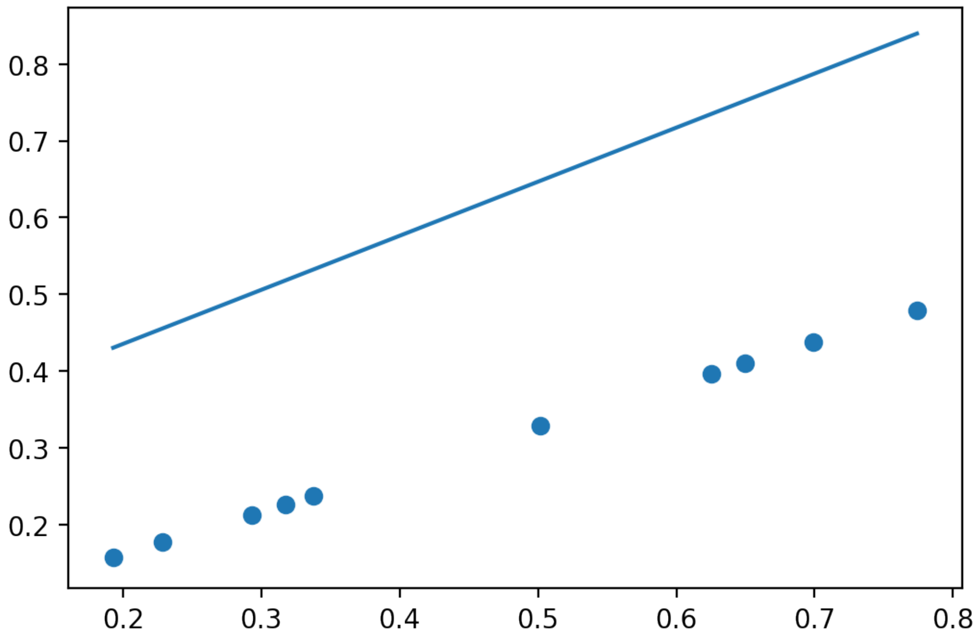
In [51]:
```
def plotter(X, y, theta):
    '''
    Arguments:
        n = 1
        X: np array with shape (m, 1+n)
        y: np array with shape (m, 1)
        theta: np array with shape (1+n, 1)
    Returns:
        None
        Plots X, y and the fit defined by theta
    '''
    plt.figure(dpi=200)
    plt.scatter(X[:, 1], y)
    linex = np.linspace(X[:, 1].min(), X[:, 1].max())
    liney = theta[0] + theta[1]*linex
    plt.plot(linex, liney)
```

In [52]:
```
# 7. Plot the generated X and y, together with the signal used to generate them
plotter(X, y, theta_true)
```



8.1 Gradient Descent

```
In [53]: # Random weight initialization
         theta = np.random.uniform(size=(n+1, 1))
         theta

Out[53]: array([[0.29412786],
                [0.70486621]])

In [54]: # X, y together with random fit
         plotter(X, y, theta)
```




```
In [55]: # Sub-challenge 1
         def cost_function(X, y, theta):
             '''
             Arguments:
                 X: np array with shape (m, 1+n)
                 y: np array with shape (m, 1)
                 theta: np array with shape (1+n, 1)
             Returns:
                 cost, a float.
             '''

             return cost
```

```
In [56]: # It should be a positive number
         cost_function(X, y, theta)
```

```
Out[56]: 4.9675336216256785
```

```
In [57]: # It should be zero
         cost_function(X, y, theta_true)
```

```
Out[57]: 0.0
```

```
In [58]: # Sub-challenge 2
         # Take the theta, update it using the GD rule
         def GD_one_step(X, y, theta, lr):
```

```
        '''
        Updates theta using one gradient descent step.
        Arguments:
            X: np array with shape (m, 1+n)
            y: np array with shape (m, 1)
            theta: np array with shape (1+n, 1)
            lr: learning rate, a float
        Returns:
            theta_new: np array with shape (1+n, 1), updated after one GD step
        '''

        return theta_new
```

```
In [59]: print('theta: ', theta.T)  # Theta before update
         print('theta_true: ', theta_true.T)  # True theta
```

```
theta:  [[0.29412786 0.70486621]]
theta_true:  [[0.04970101 0.55483883]]
```

```
In [60]: theta = GD_one_step(X, y, theta, lr=0.01)
```

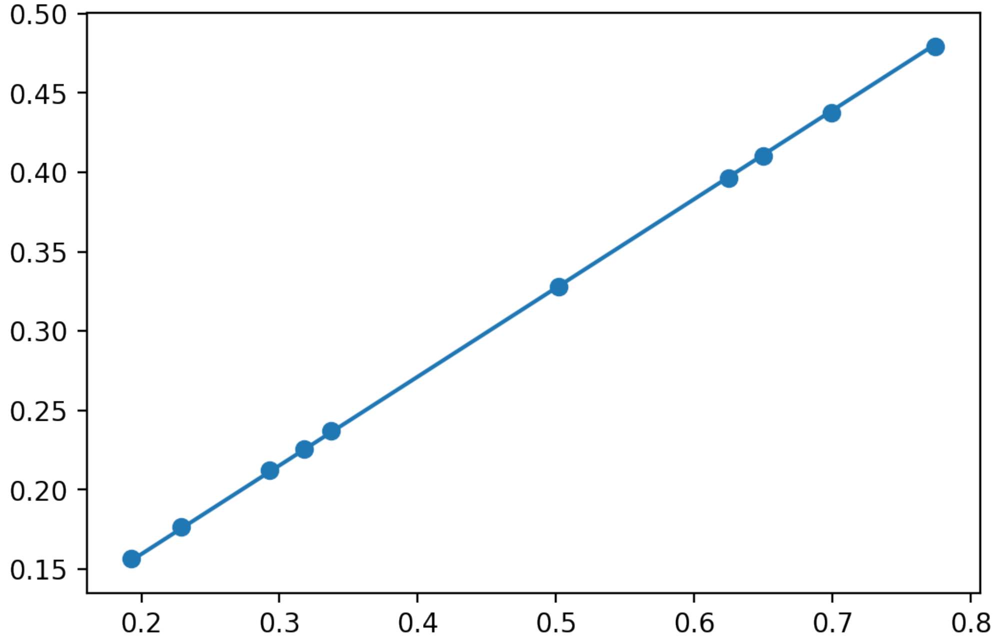
```
In [61]: print('theta: ', theta.T) # Theta after the first update
         print('theta_true: ', theta_true.T)  # True theta
```

```
theta:  [[0.2909905  0.70335549]]
theta_true:  [[0.04970101 0.55483883]]
```

```
In [62]: # Sub-challenge 3
         # Repeat update
         def GD(X, y, lr, epoch):
             '''
             Finds theta from X and y using gradient descent.
             Starts by random init. of theta
             Repeats GD_one_step for predefined number of epochs
             Arguments:
                 X: np array with shape (m, 1+n)
                 y: np array with shape (m, 1)
                 lr: learning rate, a float
                 epoch: an integer
             Returns:
                 theta: np array with shape (1+n, 1), final theta after training
             '''

             return theta
```

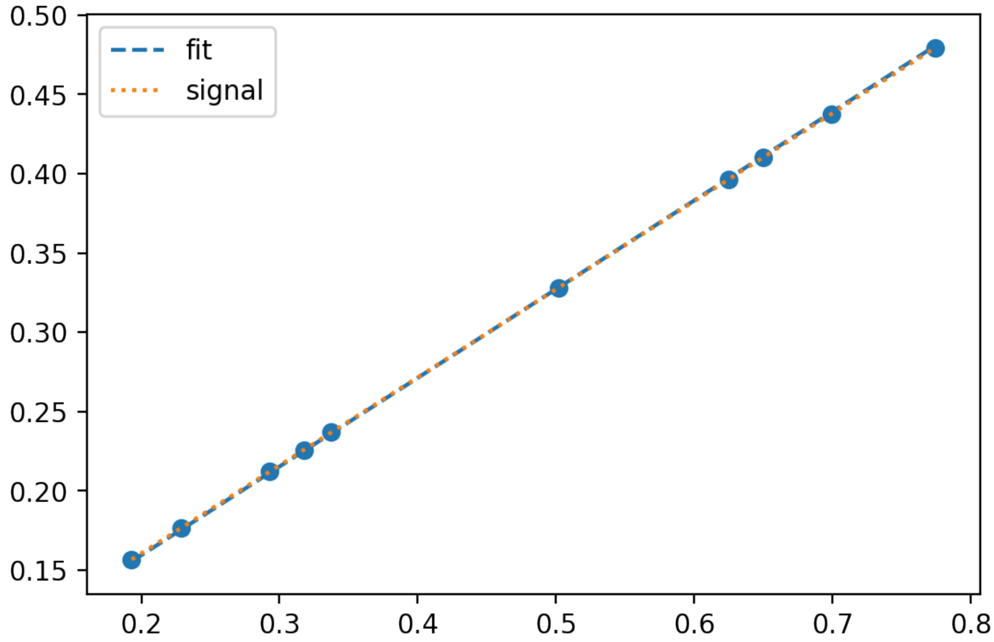
```
In [63]: # Linear regression with Gradient Descent
         theta = GD(X, y, lr=0.01, epoch=10000)
```

In [64]:
```python
# Plot the learned theta
plotter(X, y, theta)
```



In [65]:
```python
def plotter_multiple(X, y, theta, theta_true):
    '''
    For comparing the learned theta with the true theta
    Arguments:
        n = 1
        X: np array with shape (m, 1+n)
        y: np array with shape (m, 1)
        theta: np array with shape (1+n, 1)
        theta_true: np array with shape (1+n, 1)
    Returns:
        None
        Plots X, y, fit(theta), signal(theta_true)
    '''
    plt.figure(dpi=200)
    plt.scatter(X[:, 1], y)
    line_x = np.array([X[:, 1].min(), X[:, 1].max()])
    plt.plot(line_x, theta[0] + theta[1]*line_x, '--', label='fit')
    plt.plot(line_x, theta_true[0] + theta_true[1]*line_x, ':', label='signal')
    plt.legend()
```

In [66]:
```python
# Fit (learned theta) vs. Signal (true theta)
plotter_multiple(X, y, theta, theta_true)
```



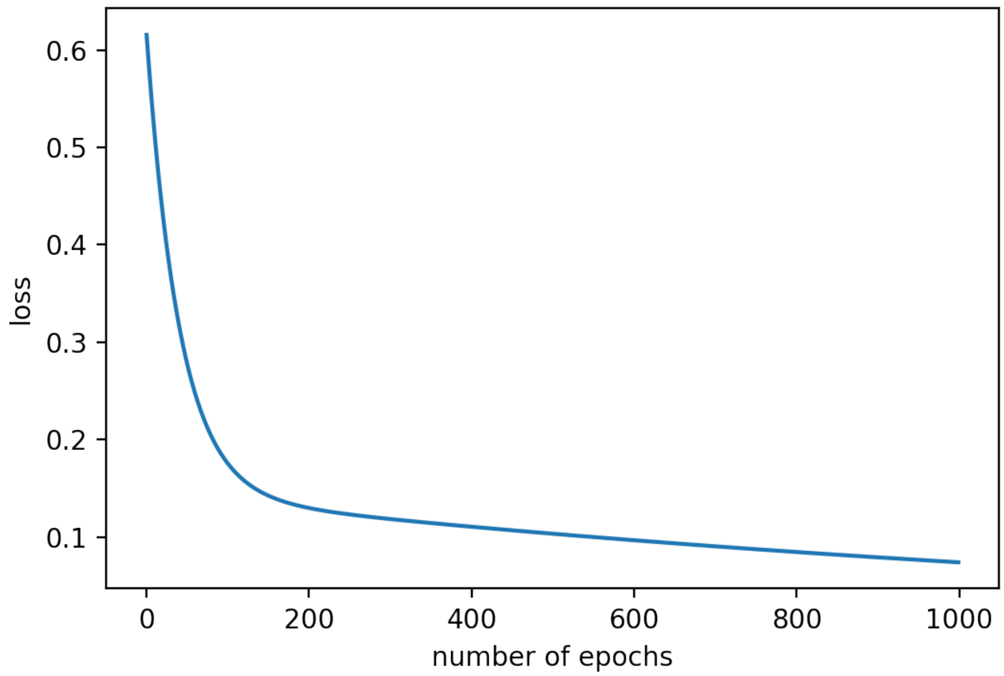
In [67]:
```
# Sub-challenge 3 cont.
# Save loss at each epoch for plotting
def GD_memory(X, y, lr, epoch):
    '''
    Finds theta from X and y using gradient descent.
    Starts by random init. of theta
    Repeats GD_one_step for predefined number of epochs
    At each epoch calculates the loss and saves it into a list
    Arguments:
        X: np array with shape (m, 1+n)
        y: np array with shape (m, 1)
        lr: learning rate, a float
        epoch: an integer
    Returns:
        theta: np array with shape (1+n, 1), final theta after training
        memory: a list of floats, each item is loss at each epoch
    '''
    memory = []

    return theta, memory
```

In [68]:
```
theta, memory = GD_memory(X, y, lr=0.01, epoch=1000)
```

In [69]:
```
def loss_plotter(memory):
    '''
    Plots the list of losses
```

```
    '''
    plt.figure(dpi=200)
    plt.plot(memory)
    plt.ylabel('loss')
    plt.xlabel('number of epochs')
```

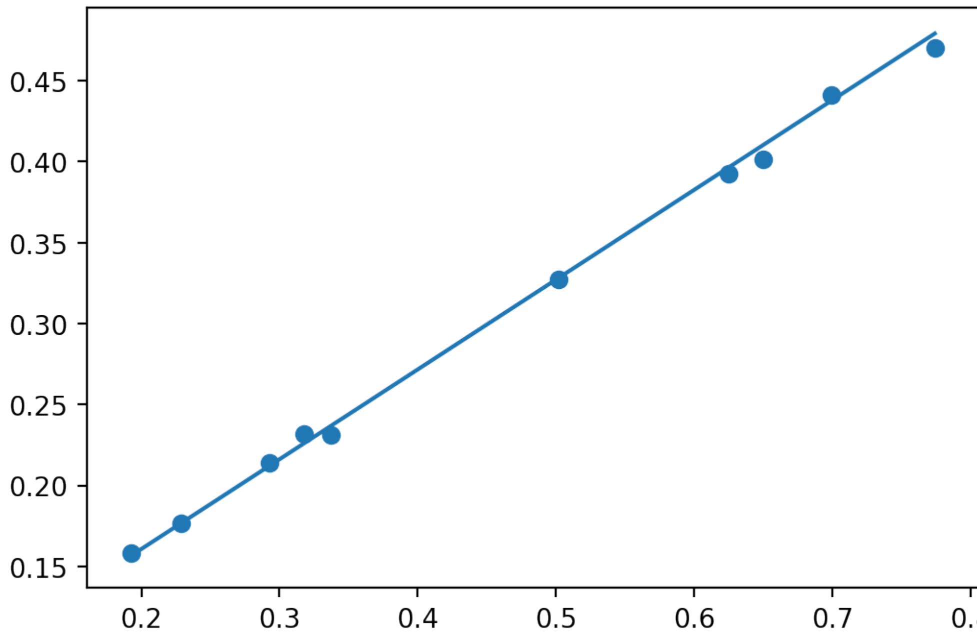
In [70]:
```
loss_plotter(memory)
```



8.2 It should work with noisy data as well

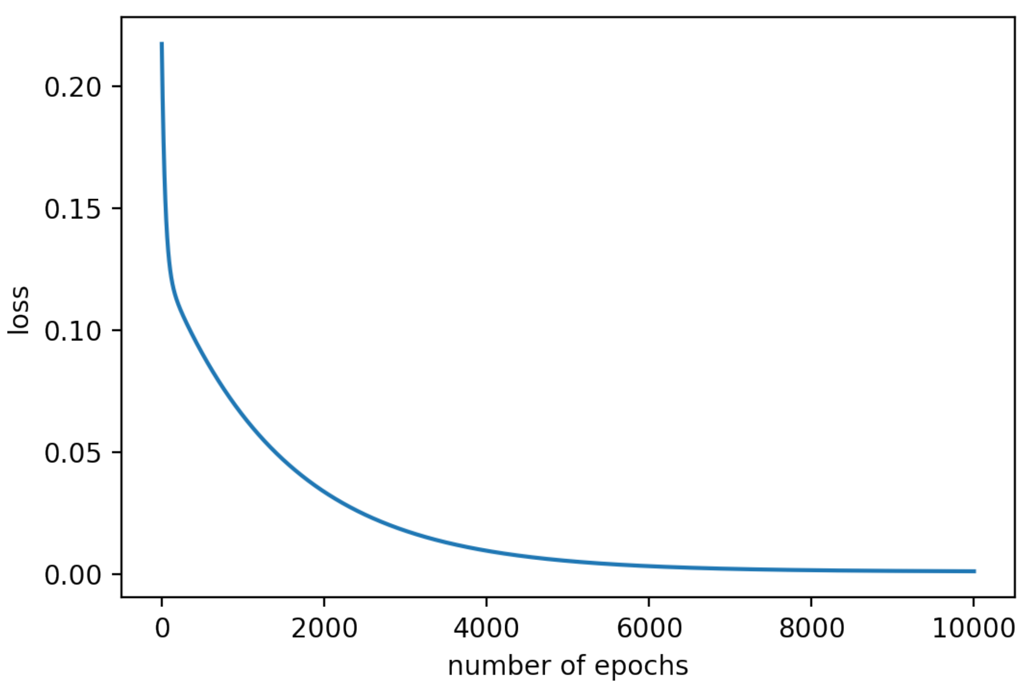
In [71]:
```
# let's introduce noise
y_real = y * np.random.uniform(0.97, 1.03, m).reshape(-1, 1)
```

In [72]:
```
# Plot noisy X and y, with signal
plotter(X, y_real, theta_true)
```

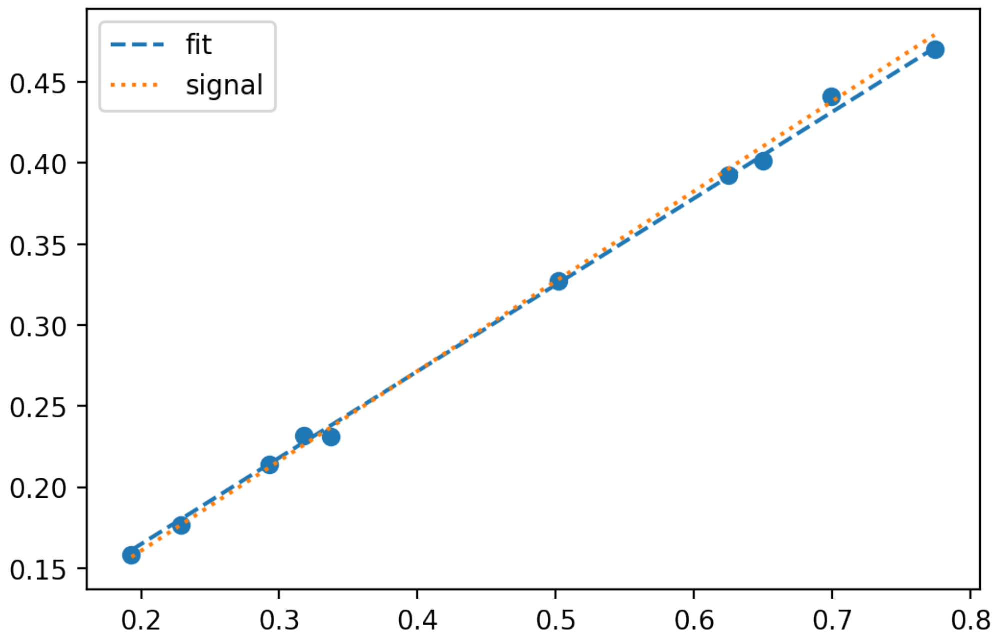



```
In [73]: theta, memory = GD_memory(X, y_real, lr=0.01, epoch=10000)
```

```
In [74]: loss_plotter(memory)
```



```
In [75]: # Fit vs. Signal for noisy data
         plotter_multiple(X, y_real, theta, theta_true)
```



8.3 How about more features and noise

```
In [76]: # Repeat data generation with more features + noise
         m = 10  # number of examples
         n = 2  # number of features

In [77]: X = np.random.uniform(size=(m, n))
         X

Out[77]: array([[0.20011324, 0.06261688],
                [0.953954  , 0.24166609],
                [0.07787241, 0.92590823],
                [0.96146176, 0.19724506],
                [0.17035948, 0.99479614],
                [0.00209128, 0.02980649],
                [0.05672694, 0.07903966],
                [0.87311992, 0.01305432],
                [0.12750353, 0.7170062 ],
                [0.41163831, 0.59416031]])

In [78]: X = np.concatenate((np.ones((m, 1)), X), axis=1)
         X
```

Out[78]: array([[1.   , 0.20011324, 0.06261688],
       [1.   , 0.953954 , 0.24166609],
       [1.   , 0.07787241, 0.92590823],
       [1.   , 0.96146176, 0.19724506],
       [1.   , 0.17035948, 0.99479614],
       [1.   , 0.00209128, 0.02980649],
       [1.   , 0.05672694, 0.07903966],
       [1.   , 0.87311992, 0.01305432],
       [1.   , 0.12750353, 0.7170062],
       [1.   , 0.41163831, 0.59416031]])

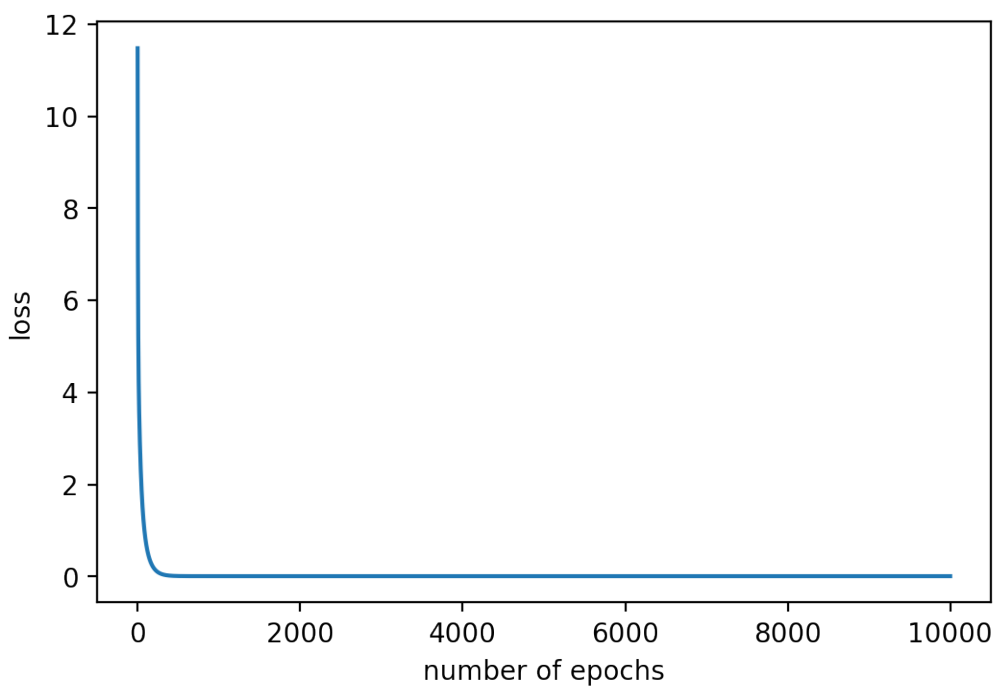
In [79]:
```
theta_true = np.random.uniform(size=(n+1, 1))
theta_true
```

Out[79]: array([[0.10835929],
       [0.88547444],
       [0.82740538]])

In [80]:
```
y = np.dot(X, theta_true)
```

In [81]:
```
# Add noise
y = y * np.random.uniform(0.97, 1.03, m).reshape(-1, 1)
```

In [82]:
```
theta, memory = GD_memory(X, y, lr=0.1, epoch=10000)
loss_plotter(memory)
```



```
In [83]: # Learned params vs. True params
         print('theta: ', theta.T)
         print('theta_true: ', theta_true.T)
```

```
theta:  [[0.11139771 0.86932284 0.82249641]]
theta_true:  [[0.10835929 0.88547444 0.82740538]]
```

```
In [84]: # Prediction vs. Label
         print('Prediction:', np.dot(X, theta)[0])
         print('Ground truth:', y[0])
```

```
Prediction: [0.33686288]
Ground truth: [0.33565044]
```

```
In [ ]:
```