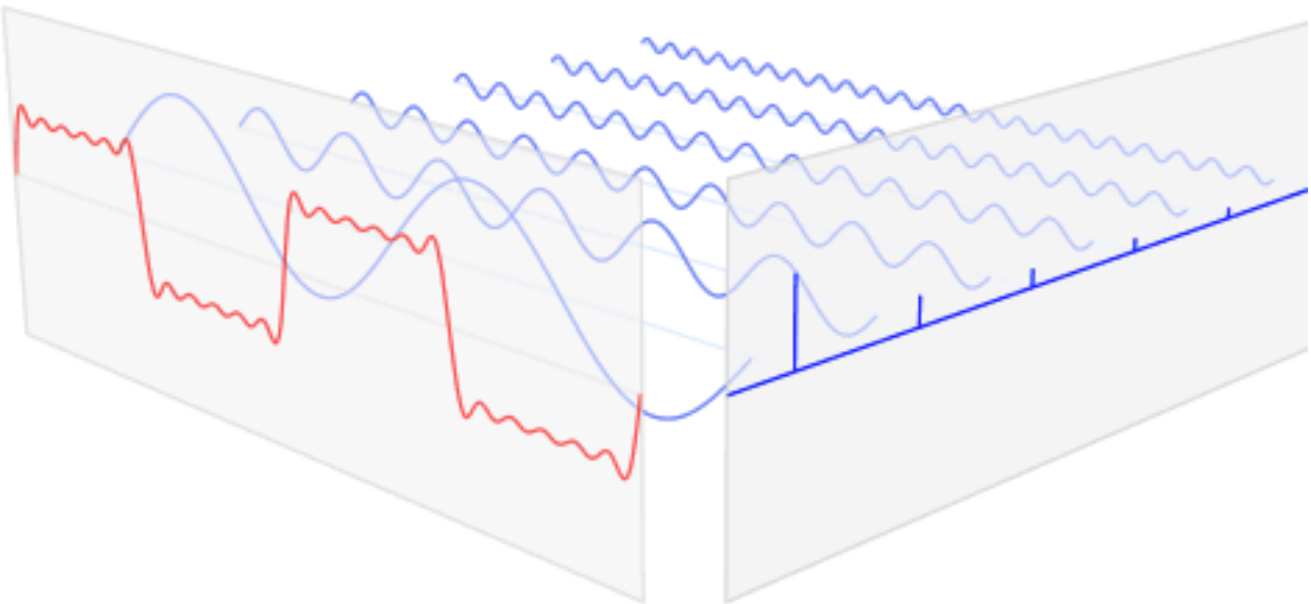
DTU Elektro, Hearing Systems Group
Building 352, DTU, 2800 Lyngby
Phone: 45 25 39 30, Fax: 45 88 05 77 www.hea.elektro.dtu.dk

Course 31606: Signals and Linear Systems in Discrete Time

Cloudy...

Bastian Epp: 45 25 39 53 (office), e-mail: bepp@elektro.dtu.dk



Fourier analysis of digital signals

Submission deadline: Thursday 27.09.2018 before the start of the lecture

Assignment

These exercises are part of the final evaluation, and must be handed in as hard-copy and as .pdf named 31606_e18_assignment_<number>_<your_group_number>.pdf (via DTU Learn). Please work on the assignment in groups of (at most) three, and produce a short report about your results written in English. Please address and answer all questions in the report. If you are asked to plot something, include the plot in the report. **For each problem, outline the problem in your own words, your approach, what you did and why you did it.** Make sure the figures are readable (see the general comments handed out before hands-on 1), and use the figure caption to describe the figure. Provide all the code in a .zip file and upload it together with your assignment. Make sure the code is **runnable, well commented and follows the general guidelines** uploaded on CANVAS. Organize the code in a folder structure as shown below:

```
31606_e18_grp_<your_group_number>
└─ assignment_<assignment_number>
   ├─ <multiple files if you feel like it>
   ├─ <add a cooking recipe if you have a good one>
   └─ ...
```

For statistical (non-commercial) purposes, please indicate the number of hours spend for each of the group members on the report.

1 Time to frequency and frequency to time

In the first part you will analyze signals generated by yourself and some recorded signals. For each of the signals, provide a plot of the time signal and the different spectra (real part, imaginary part, magnitude). If it makes sense, also plot the phase spectrum. For acoustical signals, it makes sense to plot the spectra on a decibel (dB) scale. To do so, convert your vector holding the spectrum from linear amplitude to 'dB re 1' via:

```
signal_dB = 20*log10(abs(vector_with_spectrum));
```

1.1 Toolbox: Spectrum

To make coding easier, you can pack operations that you use frequently into a function, such that you don't have to write the whole code, but just call the function with the appropriate input parameters. Since there will be quite some spectra involved, we start with a function that takes a signal and its sampling frequency, and returns the corresponding complex spectrum along with the correct frequency vector.

- Write a function that returns the complex spectrum of a signal:

```
function [Y, freq] = make_spectrum(signal, fs)
% Here goes the help message
% ...

% compute spectrum (note: it will be complex-valued).
Y = fft(signal);

% The FFT needs to be scaled in order to give a physically plausible scaling.
Y = Y/(length(Y));
  % NOTE: If you do an IFFT, this scaling must NOT be done.
  % We'll get to this in the lecture. If you are only interested
  % in the positive frequencies, you need to scale by <length(Y)/2>.

% frequency vector
delta_f = ...
freq = 0:delta_f: ...
  % NOTE: The first element that comes out of the FFT is the DC offset
  % (i.e., frequency 0). Each subsequent
  % bin is spaced by the frequency resolution <delta_f> which you can
  % calculate from the properties of the inut signal. Remember the highest
  % frequency you can get in a digitized signal....

% ...
% convert into column vector (if required)
Y = Y(:);
freq = freq(:);

% eof
end
```

- Generate a sinusoid of frequency 500 Hz, duration of 0.5 seconds, and amplitude of 1. Pass the signal to your new function. Plot the result over the frequency and make sure the peak is on the correct frequency.

- Change the frequency of the signal to 499 Hz and plot the result in the same graphs as above. Please comment on the result.

1.2 Fourier transform of a synthetic signal

Synthesize the following signal of duration `T = 4` seconds with a suitable sampling frequency:

$$s(n) = \sum_{k=0}^{4} \cos(2\pi \cdot 2^k f_0 \cdot t + k \cdot \frac{\pi}{3}) \quad f_0 = 25; \tag{1.1}$$

- Plot the time signal in the time window from [0.8 s, 0.9 s].

- Compute and plot the spectrum of the whole signal with Hz on the x-axis and linear amplitude on the y-axis. Plot 1) Magnitude and phase for the positive frequencies in two separate panels (magnitude on top of phase) and 2) the real part and imaginary part in two panels (imaginary part on top of real part), including the DC component (0 Hz). Use Euler's formula to explain why the spectrum looks as it does. What would it look like if the signal was composed of sin() rather than cos()?

  `useful command:  subplot`

- Plot the magnitude spectrum with Hz on a log scale on the x-axis and dB on the y-axis. Mark the first peak in the spectrum with a circle.

  `useful command:  semilogx`

- Save the signal as a .wav file with a bit depth of 16 bits and load it again. Make sure the signal is not distorted by saving into a .wav file. Plot the loaded signal on top of the synthetic time signal using a different color in the time interval [0.85 s,0.925 s]. Provide comments on your coding.

2 Some analytical work

Show that the Fourier transform of a *rect*-function in the time domain corresponds to a *sinc* function in the frequency domain and vice versa. Provide an expression of the zero-crossings of the sinc function as a function of the width W:

$$\mathrm{rect}_W(x) = \begin{cases} \frac{1}{W} & \text{if} \quad 0 \leq x < W \\ 0 & \text{if} \quad x \geqq W \end{cases} \tag{2.1}$$

3 Some stuff from the previous hands-on

The remaining part of the assignment is based on the hands-on. The focus in this part is to document that you understood how the exercises have been solved, why they were solved in the way you did it and what the theoretical background behind them is. You may use snipplets of your code to explain how it works and how you solved the task. The code needs to be complete, a solution without explanation is not a solution!

4 Hands-on 2

Please provide a solution and the corresponding explanation for 1.3.