

## Bending frequencies

Cloudy...



## 1 How finite should it be?

FIR filters have, as the name suggests, an impulse response of finite length. In contrast to IIR filters, where the output is always fed back into the system, FIR filters only have feed-forward elements, such that at one instance in time a pulse travelled through all elements and the impulse response ends. Obviously FIR filter are easier to scale with respect to required resources - if the impulse response is too long, we can simply shorten it. But - if we can simply shorten it, why don't we get rid of most of the length? In this part of the exercise you will investigate what happens when the impulse response of a filter is step wise shortened. Shortening of the impulse responses might be necessary due to limited resources, but of course you always have to make sure that you don't cut the filter down to something that is useless.

### 1.1 Like a hot knife through a Butterworth

Construct a bandpass filter which meets the following specifications (normalized frequency):

- Passband from 0.2 to 0.3
- Stopbands from 0 to 0.1 and 0.4 to 1
- Ripples are OK if they don't exceed 2 dB
- The stopband gain should be at least -100 dB

Compute and plot the frequency response, the impulse response and the pole-zero plot in the z-plane. Also provide in the frequency response plot the details of the filter (approximation, order, corner frequencies, ...).

Now take the impulse response. Let's define for practical reasons an effective length of the impulse response the length (in samples) at which the impulse response decays below a value of 10% of its maximum. Now vary the length of the filter:

- Cut the filter at its effective length
- Cut the filter to 75%/60%/40%/10% of the above length

and compare the frequency response of the resulting filters by plotting them on top of each other. Use a rectangular window and one other window of your choice. Describe what happens to the frequency response when the impulse responses are shortened.

### 1.2 Shorter and shorter

For an application you are to construct a FIR lowpass filter by the frequency domain equivalence, i.e. by sampling the frequency response of a perfect lowpass filter. You got a sheet of paper from you colleague that holds the specs: The filter passes all frequencies between 0 and 5 kHz with a gain of 0 dB. Just over the sampling frequency, there is this huge coffee stain, making it impossible to read, the same for the order of the desired filter. But you see from the sketch that the cutoff of the filter corresponds to one third on a normalized frequency axis, and you can see that the impulse response should be 601 samples long.

- Which sampling rate should you use?
- Construct the filter and plot the frequency and the impulse response

- NOTE: Make sure the IR is real (or has a vanishingly small imaginary part)!

Now you think that the filter might be more efficient by just decreasing the length of the impulse response. Reduce the length of the filter stepwise to only a few samples by windowing. Plot the corresponding frequency responses on top of each other and compare.

## 2 An equalizer - without buttons

Filters are everywhere: Check your amplifier at home. Most likely it has some knobs labelled with “bass” or “treble”. And surely you tried turning them to see what happens. (Certain types of music benefit from a turn of the bass knob, even though the relation to your neighbours might suffer a bit). Possibly you also have an old equalizer at home that allows you to move even more sliders up and down. Behind these knobs are nothing else than filters - a simple lowpass/highpass for the turning knobs and some bandpass filters for each of the sliders on the equalizer. Now that we know how to construct filters, let's do something useful with it: Let's pimp our favourite piece of music!

This part will give you some more experience with handling filters in a real world application. This problem might also be combined with the previous hands-on on block-processing such that the settings can be adjusted and immediately applied!

### 2.1 Bass and treble

Design a function that returns the filter coefficients of a FIR bandpass filter when provided with the following parameters:

- Cut-off frequency separating low and high frequencies (on a normalized frequency scale)
- Gain in dB
- Length of the filter (filter order + 1)

The gain of this filter can be positive or negative (moving the bass/treble knob to the left or right). In order to obtain the filter coefficients, follow the recipe (frequency sampling method):

- Determine the indices in the spectral domain where the corresponding cut-off frequencies of your filter are placed (round to the next integer and remember that the spectrum is periodic/symmetric)
- Create the frequency response of your filter, having values of 1 up to the cutoff frequency and a value of the gain (on a linear scale) as specified by the input variable above the cutoff frequency
- Compute the IR  $h$  by using `ifft`. (Make sure your resulting IR is real-valued - if you have large complex components, your indices in the spectrum do not match up)
- Your resulting IR will have the length as specified by the input argument - check the frequency response of that filter!
- Multiply the resulting IR with a window of your choice (Hann or Hamming might be working nicely) and plot the frequency response of that filter

## 2. *AN EQUALIZER - WITHOUT BUTTONS*

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Use these filter coefficients to filter some piece of music (or any other signal) through your filter. The cut-off frequency you should choose depending on where you want to increase the gain of the frequencies. NOTE: When using positive gains, you might need to rescale your signal into  $] -1, 1[$  to avoid clipping!