# Package 'rcosmo'

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```
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# R topics documented:

ncestor		4
reCompatibleCMBDFs		4
s.CMBDataFrame		5
ssumedConvex		6
ssumedUniquePix		6
aseNeighbours		7
bind.CMBDataFrame		8
hi2CMB		8
hildren		9
MBDat		10
MBDataFrame		11
MBWindow		12
oords		13
oords.CMBDataFrame		14
oords.CMBWindow		15
oords.data.frame		16
oords.HPDataFrame		17
orrCMB		18
ovCMB		19
ovmodelCMB		21
ovPwSp		22
isplayPixelBoundaries		23
isplayPixels		24
ownloadCMBMap		25
ownloadCMBPS		26
ntropyCMB		27
xprob		28
xtrCMB		29
mf		29
Ren		30
eo2sph		31
eoArea		32
eoArea.CMBDataFrame		
eoArea.CMBWindow		
eoArea.HPDataFrame		
eoDist		
eader		35
IPDataFrame		36
op2p		37
s.CMBDat		38
s.CMBDataFrame		39
s.CMBWindow		39
s.HPDataFrame		40
nesCMB		40
naxDist		41
naxWindowDist		42
ninDist		43
eighbours		44
est2ring		45
estSearch	 •	45

Index

89

nside	 . 46
nside.CMBDataFrame	 . 47
nside.HPDataFrame	 . 47
ordering	 . 48
ordering.CMBDataFrame	 . 48
ordering.HPDataFrame	 . 49
p2bp	 . 50
p2ibp	 . 50
parent	 . 51
pix	 . 51
pix.CMBDataFrame	 . 52
pix.HPDataFrame	 . 52
pix2coords	
pixelArea	 . 54
pixelWindow	
plot.CMBCorrelation	
plot.CMBCovariance	
plot.CMBDataFrame	
plot.CMBWindow	
plot.HPDataFrame	
plot.variogram	
plotAngDis	
plotcovmodelCMB	
plotvariogram	
practicalRangeCMB	
print.CMBDataFrame	
print.HPDataFrame	
pwSpCorr	
qqnormWin	
qqplotWin	
qstat	
rbind.CMBDataFrame	
rcosmo	
resolution	
ring2nest	
sampleCMB	
siblings	 . 72
sphericalHarmonics	
summary.CMBDataFrame	
summary.CMBWindow	
summary.HPDataFrame	
triangulate	
variofitCMB	
variogramCMB	
window	
window.CMBDat	
window.CMBDataFrame	
window.data.frame	
window.HPDataFrame	
winType	
"mijpo	 . 07

ancestor

Return index of kth ancestor pixel

# Description

Gives the pixel at resolution j - k that contains p, where p is specified at resolution j (notice it does not depend on j).

### Usage

```
ancestor(p, k)
```

# Arguments

p A pixel index specified in nested order.k A generation of an ancestor pixel.

# **Examples**

```
ancestor(4,2)
ancestor(17,2)
```

areCompatibleCMBDFs

Check compatibleness of CMBDataFrames

# **Description**

Compare attributes to decide if two CMBDataFrames are compatible

# Usage

```
areCompatibleCMBDFs(cmbdf1, cmbdf2, compare.pix = FALSE)
```

# Arguments

compare.pix A boolean. If TRUE then cmbdf1 and cmbdf2 must share the same pixel indices

to be considered compatible

# **Details**

If the CMBDataFrames do not have compatible attributes then a message is printed indicating the attributes that do not match. To suppress this use the suppressMessages function

as.CMBDataFrame 5

#### **Examples**

```
a <- CMBDataFrame(nside = 2, ordering = "ring", coords = "cartesian")
b <- CMBDataFrame(nside = 1, ordering = "nested", coords = "spherical")
areCompatibleCMBDFs(a,b)
suppressMessages(areCompatibleCMBDFs(a,b))</pre>
```

as.CMBDataFrame

Convert dataframes to CMBDataFrames

#### **Description**

Safely converts a data.frame to a CMBDataFrame. The rows of the data.frame are assumed to be in the HEALPix order given by ordering, and at the HEALPix resolution given by nside. Coordinates, if present, are assumed to correspond to HEALPix pixel centers. The coordinates must be named either x,y,z (cartesian) or theta, phi (spherical colatitude and longitude respectively).

# Usage

```
as.CMBDataFrame(df, ordering, nside, spix)
```

# **Arguments**

df Any data. frame whose rows are in HEALPix order ordering character string that specifies the ordering scheme ("ring" or "nested") nside an integer  $2^k$  that specifies the Nside (resolution) HEALPix parameter an integer vector that specifies the HEALPix pixel index corresponding to each row of df. If spix is left blank and df is a data. frame, then df is assumed to contain data for every pixel at resolution parameter nside (the full sky). In other words, in this case, the number of rows of df must be equal to 12\*nside^2. However, if spix is left blank and df is a CMBDataFrame, then spix is set equal to pix(df)

# Value

A CMBDataFrame

```
## Example 1: Create df with no coords, then create CMBDataFrames cmbdf and
## df2 with spherical coords

df <- data.frame(I=rnorm(12))
df

cmbdf <- as.CMBDataFrame(df,ordering= "ring", nside=1)
summary(cmbdf)
pix(cmbdf)
coords(cmbdf)</pre>
```

6 assumedUniquePix

```
df2 <- coords(cmbdf, new.coords = "spherical")
df2

## Example 2: Create CMBDataFrames for first 10 Healpix centers

df <- data.frame(I=rnorm(10))
df

cmbdf <- as.CMBDataFrame(df,ordering= "ring", nside=2, spix=1:10)
summary(cmbdf)
pix(cmbdf)</pre>
```

assumedConvex

Check if a CMBWindow is assumed convex.

# **Description**

Initially any CMBWindow is not assumed convex. The assumedConvex attribute can be change for any CMBWindow.

# Usage

```
assumedConvex(win, assume.convex)
```

#### **Arguments**

```
win a CMBWindow object
assume.convex optionally change the assumedConvex attribute to TRUE or FALSE
```

#### **Examples**

```
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2)) assumedConvex(win1) win2 <- assumedConvex(win1, assume.convex = TRUE) assumedConvex(win2) assumedConvex(win1) <- TRUE assumedConvex(win1)
```

assumedUniquePix

Check if object was assumed to have unique HEALPix indices

# Description

The function checks object's attribute assumedUniquePix. The attribute is True if the object was assumed to have rows that correspond to unique HEALPix pixel indices.

#### Usage

```
assumedUniquePix(obj)
```

baseNeighbours 7

#### **Arguments**

obj Any object

#### Value

A boolean. This is TRUE if obj is a CMBDataFrame or a HPDataFrame whose rows were assumed to correspond to unique HEALPix pixel indices.

# **Examples**

```
hp1 <- HPDataFrame(I=rnorm(5), nside = 1, spix = c(1,1,2,2,3))
pix(hp1)
coords(hp1, new.coords = "cartesian")
assumedUniquePix(hp1)

sky <- CMBDataFrame(nside = 32, coords = "cartesian", ordering = "nested")
sky.s <- CMBDataFrame(sky, sample.size = 100)
hpdf <- HPDataFrame(sky.s, auto.spix = TRUE)
assumedUniquePix(hpdf)</pre>
```

baseNeighbours

Return neighbours of base pixels

# **Description**

The base-resolution comprises twelve pixels. baseNeighbours returns a map from the base pixel index bp to the vector of base pixels that are neighbours of bp, in counterclockwise order of direction: S,SE,E,NE,N,NW,W,SW. The presence of -1 indicates that the corresponding direction is empty.

### Usage

baseNeighbours(bp)

### **Arguments**

bp

The base pixel index

```
## Return neighbours of base pixel 1
baseNeighbours(1)
## There is no base pixel 14, so baseNeighbours returns NULL
baseNeighbours(14)
```

8 chi2CMB

cbind.CMBDataFrame cbind for CMBDataFrames

#### **Description**

Add a new column or columns (vector, matrix or data.frame) to a CMBDataFrame. Note that method dispatch occurs on the first argument. So, the CMBDataFrame must be the first argument

# Usage

```
## S3 method for class 'CMBDataFrame'
cbind(..., deparse.level = 1)
```

# **Arguments**

```
... (generalized) vectors or matrices. Columns to bind.

deparse.level Integer controlling the construction of labels in the case of non-matrix-like arguments.
```

#### **Details**

See the documentation for cbind

#### **Examples**

```
cmbdf <- CMBDataFrame(nside = 1, ordering = "nested", coords = "spherical")
cmbdf2 <- cbind(cmbdf, myData = rep(1, 12))
cmbdf2</pre>
```

chi2CMB

Chi-squared statistic for two CMBWindows

# **Description**

This function returns the empirical chi-squared statistic for intensities observations from two CMBWindows of the specified CMBDataFrame. The functions employs the function chi2.empirical and uses histogram counts of intensities for computations.

### Usage

```
chi2CMB(cmbdf, win1, win2, intensities = "I")
```

# **Arguments**

intensities A CMBDataFrame column with measured values.

children 9

#### Value

Estimated Chi-squared statistic for observations in two CMBWindows. For small sample sizes and many zero counts Chi-squared statistic is inefficient.

#### References

chi2.empirical

# **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 1000)
#
# win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
# win2 <- CMBWindow(theta = c(pi/2,pi,pi/2), phi = c(0,0,pi/2))
# plot(win1)
# plot(win2,col="green")
#
# chi2CMB(cmbdf, win1, win2)</pre>
```

children

Return children of a pixel

# Description

Gives four pixels at resolution j + 1 that are contained in p, where p is a pixel specified at resolution j (notice it does not depend on j).

# Usage

```
children(p)
```

# **Arguments**

р

A pixel index specified in nested order.

```
children(11)
```

10 CMBDat

|--|

# **Description**

The function CMBDat creates objects of class CMBDat. These are lists containing header information and other metadata as well as an element called data, whose columns may include, for example, the intensity (I), polarisation (Q, U), PMASK and TMASK. It also may contain an mmap object that points to the CMB map data table in the FITS file.

# Usage

```
CMBDat(filename, mmap = FALSE, spix)
```

# **Arguments**

filename The path to the fits file.

mmap A boolean indicating whether to use memory mapping.

spix The sample pixels (rows) to read from the FITS file binary data table (optional)

#### Value

A list containing header information and other metadata as well as an element called data where: If mmap = FALSE then a data. frame is included, named data, whose columns may include, for example, the intensity (I), polarisation (Q, U), PMASK and TMASK. If mmap = TRUE then a mmap object is returned that points to the CMB map data table in the FITS file.

```
## Ensure you have a FITS file with the correct path
## before running the example:
## download a FITS file and use real data
# downloadCMBMap()
# cmbdat <- CMBDat("CMB_map_smica1024.fits", mmap = TRUE)
# class(cmbdat)
# str(cmbdat)

## View metadata
# cmbdat$header1
# cmbdat$header2
# cmbdat$resoln
# cmbdat$method
# cmbdat$coordsys
# cmbdat$nside
# cmbdat$hdr</pre>
```

CMBDataFrame 11

CMBDataFrame	CMBDataFrame class	

# Description

The function CMBDataFrame creates objects of class CMBDataFrame. These are a special type of data.frame that carry metadata about, e.g., the HEALPix ordering scheme, coordinate system, and nside parameter.

# Usage

```
CMBDataFrame(CMBData, coords, win, include.polar = FALSE,
  include.masks = FALSE, spix, sample.size, nside, ordering, I, ...)
```

# Arguments

S	
CMBData	Can be a string location of FITS file, another CMBDataFrame, a CMBDat object, or unspecified.
coords	Can be "spherical," "cartesian", or unspecified (HEALPix only).
win	optional CMBWindow object that specifies a spherical polygon within which to subset the full sky CMB data.
include.polar	TRUE if polarisation data is required, otherwise FALSE.
include.masks	TRUE if TMASK and PMASK are required, otherwise FALSE.
spix	Optional vector of sample pixel indices, or a path to a file containing comma delimited sample pixel indices. The ordering scheme is given by ordering. If ordering is unspecified then CMBData must be either a CMBDataFrame or a FITS file and the ordering scheme is then assumed to match that of CMBData.
sample.size	If a positive integer is given, a simple random sample of size equal to sample.size will be taken from CMBData. If spix is specified then sample.size must be unspecified.
nside	Optionally specify the nside parameter manually nside= $2^k$ (usually 1024 or 2048).
ordering	Specifies the desired HEALPix ordering scheme ("ring" or "nested") for the output CMBDataFrame. If ordering is unspecified then the ordering scheme will be taken from the CMBData object, which must then be either a CMBDataFrame or a path to a FITS file. This parameter also specifies the ordering scheme of spix.
I	A vector of intensities to be included if CMBData is unspecified. Note that length(I) must equal $12*nside^2$ if either spix or sample.size are unspecified.
•••	Optional names data columns of length nrow(CMBData) to add to the CMB-DataFrame.

# Value

A CMBDataFrame whose row.names attribute contains HEALPix indices.

12 CMBWindow

### **Examples**

```
## Method 1: Read the data while constructing the CMBDataFrame
## download a FITS file and use real data
# downloadCMBMap()
# df <- CMBDataFrame("CMB_map_smica1024.fits")</pre>
df <- CMBDataFrame(nside = 16, I = rnorm(12 * 16 ^ 2),</pre>
                   ordering = "nested")
# Specify a sample size for a random sample
df.sample <- CMBDataFrame(df, sample.size = 80)</pre>
plot(df.sample)
# Specify a vector of pixel indices using spix
df.subset <- CMBDataFrame(df, spix = c(2,4,6))
# Take a look at the summary
summary(df)
# Access HEALPix pixel indices using pix function
# (these are stored in the row.names attribute)
pix(df.subset)
```

CMBWindow

CMBWindow class.

# Description

The function CMBWindow creates objects of class CMBWindow. It is either a polygon or a disc type.

# Usage

```
CMBWindow(..., r, set.minus = FALSE, assume.convex = FALSE)
```

# Arguments

•••	these arguments are compulsory and must be labelled either $x$ , $y$ , $z$ (cartesian) or theta, phi (spherical, colatitude and longitude respectively). Alternatively, a single data.frame may be passed in with columns labelled $x$ , $y$ , $z$ or theta, phi.
r	if a disc type window is required then this specifies the radius of the disc
set.minus	when TRUE the window will be the unit sphere minus the window specified
assume.convex	when TRUE the window is assumed to be convex resulting in a faster computation time when the window is used with functions such as subWindow. This argument is irrelevant when the window is not a polygon

coords 13

#### **Details**

If r is unspecified then the rows of . . . correspond to counter-clockwise ordered vertices defining a spherical polygon lying entirely within one open hemisphere on the unit sphere. Counter-clockwise is understood from the perspective outside the sphere, facing the hemisphere that contains the polygon, looking toward the origin. Note that there must be at least 3 rows (vertices) to define a polygon (we exclude bygones). On the other hand, if r is specified then . . . must specify just one row, and this row is taken to be the center of a disc of radius r

#### **Examples**

```
win <- CMBWindow(theta = c(pi/2,pi/2,pi/3, pi/3), phi = c(0,pi/3,pi/3,0))
plot(win)

## Create a disc type window
win1<- CMBWindow(x=0,y=3/5,z=4/5,r=0.8, set.minus =TRUE)
plot(win1)

## Apply a disc type window to CMBDataFrame
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian", ordering = "nested")
window(cmbdf) <- CMBWindow(x=0,y=3/5,z=4/5,r=0.8, set.minus =TRUE)
plot(cmbdf)</pre>
```

coords

Coordinate conversion generic

# Description

Detailed descriptions and and examples can be found in documentation for specific coords functions coords. CMBDataFrame, coords. CMBWindow, coords. HPDataFrame, coords.data.frame

# Usage

```
coords(x, ...)
```

#### Arguments

x An object.

... Unused arguments.

### See Also

coords.CMBDataFrame coords.CMBWindow coords.HPDataFrame coords.data.frame

14 coords.CMBDataFrame

coords.CMBDataFrame

 $Coordinate\ system\ from\ a\ {\tt CMBDataFrame}$ 

# **Description**

If new.coords is unspecified then this function returns the coordinate system used in the CMB-DataFrame cmbdf. The coordinate system is either "cartesian" or "spherical". If a new coordinate system is specified, using e.g. new.coords = "spherical", then this function instead returns a new CMBDataFrame whose coordinates are of the specified type. The original CMBDataFrame, cmbdf, is unaffected. If you would like to change cmbdf without creating a new variable, then use coords<-.CMBDataFrame (see examples below).

# Usage

```
## S3 method for class 'CMBDataFrame'
coords(x, new.coords, ...)
```

### **Arguments**

```
    x A CMBDataFrame, cmbdf.
    new.coords Specifies the new coordinate system ("spherical" or "cartesian") if a change of coordinate system is desired.
    ... Unused arguments.
```

#### Value

If new.coords is unspecified, then the name of the coordinate system of cmbdf is returned. Otherwise a new CMBDataFrame is returned equivalent to cmbdf but having the desired change of coordinates

```
## Create df with no coords, then create df2 with cartesian coords
df <- CMBDataFrame(nside = 16)
df
coords(df)
df2 <- coords(df, new.coords = "cartesian")
coords(df2)

## Change the coords of df directly (to spherical)
coords(df) <- "spherical"
coords(df)</pre>
```

coords.CMBWindow 15

coords.CMBWindow

Coordinate system from a CMBWindow

# **Description**

This function returns the coordinate system used in a CMBWindow. The coordinate system is either "cartesian" or "spherical"

# Usage

```
## S3 method for class 'CMBWindow'
coords(x, new.coords, ...)
```

# **Arguments**

```
    x a CMBWindow, win.
    new.coords specifies the new coordinate system ("spherical" or "cartesian") if a change of coordinate system is desired
    ... Unused arguments.
```

#### **Details**

If a new coordinate system is specified, using e.g. new.coords = "spherical", the coordinate system of the CMBWindow will be converted

# Value

If new.coords is unspecified, then the name of the coordinate system of win is returned. Otherwise a new CMBWindow is returned equivalent to win but having the desired change of coordinates

```
## Create win with sperical coords, then change it to win1 with cartesian coords win <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2)) coords(win) win1 <- coords(win, new.coords = "cartesian") coords(win1) ## Change back to spherical coordinates coords(win1) <- "spherical" coords(win1)
```

16 coords.data.frame

coords.data.frame

Create a new data.frame with a given coordinate system

# **Description**

This does not affect the original object unless new coordinate system is directly assigned.

# Usage

```
## S3 method for class 'data.frame'
coords(x, new.coords, ...)
```

# **Arguments**

```
    a data.frame with columns labelled x, y, z (for cartesian) or theta, phi (for spherical colatitude and longitude respectively)
    specifies the new coordinate system ("spherical" or "cartesian").
    Unused arguments.
```

### Value

A new data.frame whose coordinates are as specified by new.coords

```
## Create df with no coords, then create df2 with spherical coords df <- data.frame(x = c(1,0,0), y = c(0,1,0), z = c(0,0,1)) df df2 <- coords(df, new.coords = "spherical") df2 
## The function coords does not affect the original object. ## To change the coords assign a new value ("spherical or "cartesian") df coords(df, new.coords = "spherical") df coords(df) <- "spherical"
```

coords.HPDataFrame 17

coords.HPDataFrame

Coordinate system from a HPDataFrame

### **Description**

Add or change coordinates in a HPDataFrame. This does not affect the argument object hpdf. Instead it returns a new HPDataFrame with the desired coordinates. To change hpdf directly see coords<-.HPDataFrame.

### Usage

```
## S3 method for class 'HPDataFrame'
coords(x, new.coords, healpixCentered = FALSE, ...)
```

# **Arguments**

```
x a HPDataFrame, hpdf.

new.coords specifies the new coordinate system ("spherical" or "cartesian")

healpixCentered boolean. If TRUE then columns x,y,z or theta, phi will be ignored and removed if present. This forces the coordinates to be found from HEALPix pixel indices only. Then the HEALPixCentered attribute of hpdf will be set to TRUE.

... Unused arguments.
```

#### **Details**

If columns exist labelled x,y,z (cartesian) or theta, phi (colatitude and longitude respectively), then these will be treated as the coordinates of hpdf and converted accordingly. If columns x,y,z or theta,phi are not present then the healpix pixel indices as given by pix(hpdf) are used for assigning coordinates.

# Value

A HPDataFrame with columns x,y,z (cartesian) or theta, phi (colatitude and longitude respectively)

18 corrCMB

```
nside = 1, auto.spix = TRUE)
hp2 <- coords(hp1, new.coords = "spherical", healpixCentered = TRUE)</pre>
```

corrCMB

Sample correlation function

#### **Description**

This function provides an empirical correlation function for data in a CMBDataFrame or data.frame. It assumes that data are from a stationary spherical random field and the correlation depends only on a geodesic distance between locations. Output is a binned correlation.

# Usage

```
corrCMB(cmbdf, num.bins = 10, sample.size, max.dist = pi, breaks,
   equiareal = TRUE, calc.max.dist = FALSE)
```

# **Arguments**

cmbdf	is a CMBDataFrame or data.frame
num.bins	specifies the number of bins
sample.size	optionally specify the size of a simple random sample to take before calculating correlation. This may be useful if the full correlation computation is too slow.
max.dist	an optional number between 0 and pi specifying the maximum geodesic distance to use for calculating correlation. Only used if breaks are unspecified.
breaks	optionally specify the breaks manually using a vector giving the break points between cells. This vector has length num.bins since the last break point is taken as max.dist. If equiareal = TRUE then these breaks should be $cos(r_i)$ where $r_i$ are radii. If equiareal = FALSE then these breaks should be $r_i$ .
equiareal	if TRUE then the bins have equal spherical area. If false then the bins have equal annular widths. Default is TRUE.
calc.max.dist	if TRUE then the max.dist will be calculated from the locations in cmbdf. Otherwise either max.dist must be specified or max.dist will default to pi.

# Value

#' An object of the class CMBCorrelation that is a modification of variog from the package **geoR** with variogram values replaced by correlation.

The attribute "breaks" contains the break points used to create bins. The result has num.bins + 1 values since the first value at distance 0 is not counted as a bin.

- **u** a vector with distances.
- v a vector with estimated correlation values at distances given in u.
- n number of pairs in each bin
- sd standard deviation of the values in each bin

bins.lim limits defining the interval spanned by each bin

covCMB 19

**ind.bin** a logical vector indicating whether the number of pairs in each bin is greater or equal to the value in the argument pairs.min

var.mark variance of the data

beta.ols parameters of the mean part of the model fitted by ordinary least squares

output.type echoes the option argument

max.dist maximum distance between pairs allowed in the correlation calculations

n.data number of data

direction direction for which the correlation was computed

call the function call

#### References

```
geoR package,variog, variogramCMB, covCMB
```

### **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 100000)
# corcmb <- corrCMB(cmbdf, max.dist = 0.03, num.bins = 10, sample.size=1000)
# corcmb</pre>
```

covCMB

Sample covariance function

# Description

This function provides an empirical covariance function for data in a CMBDataFrame or data.frame. It assumes that data are from a stationary spherical random field and the covariance depends only on a geodesic distance between locations. Output is a binned covariance.

# Usage

```
covCMB(cmbdf, num.bins = 10, sample.size, max.dist = pi, breaks,
  equiareal = TRUE, calc.max.dist = FALSE)
```

# Arguments

cmbdf is a CMBDataFrame or data.frame
num.bins specifies the number of bins
sample.size optionally specify the size of a simple random sample to take before calculating covariance. This may be useful if the full covariance computation is too slow.

max.dist an optional number between 0 and pi specifying the maximum geodesic distance to use for calculating covariance. Only used if breaks are unspecified.

20 covCMB

breaks optionally specify the breaks manually using a vector giving the break points between cells. This vector has length num.bins since the last break point is taken as max.dist. If equiareal = TRUE then these breaks should be  $cos(r_i)$  where  $r_i$  are radii. If equiareal = FALSE then these breaks should be  $r_i$ . equiareal if TRUE then the bins have equal spherical area. If false then the bins have equal annular widths. Default is TRUE.

if TRUE then the max.dist will be calculated from the locations in cmbdf. Otherwise either max.dist must be specified or max.dist will default to pi.

#### Value

An object of the class CMBCovariance that is a modification of variog from the package **geoR** with variogram values replaced by covariances.

The attribute "breaks" contains the break points used to create bins. The result has num.bins + 1 values since the first value, the sample variance, is not counted as a bin.

**u** a vector with distances.

calc.max.dist

v a vector with estimated covariance values at distances given in u.

n number of pairs in each bin

sd standard deviation of the values in each bin

bins.lim limits defining the interval spanned by each bin

**ind.bin** a logical vector indicating whether the number of pairs in each bin is greater or equal to the value in the argument pairs.min

var.mark variance of the data

beta.ols parameters of the mean part of the model fitted by ordinary least squares

output.type echoes the option argument

max.dist maximum distance between pairs allowed in the covariance calculations

n.data number of data

direction direction for which the covariance was computed

call the function call

#### References

```
geoR package, variog, variogramCMB, corrCMB
```

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 100000)
# Cov <- covCMB(cmbdf, max.dist = 0.03, num.bins = 10)
# Cov</pre>
```

covmodelCMB 21

covmodelCMB	Computes values of covariance functions
-------------	---

# **Description**

This function computes the covariances given the separation distance of locations. Options for different covariance functions on spheres are available. The function extends the function cov.spatial for additional covariance models on spheres.

### Usage

```
covmodelCMB(obj, cov.model = "matern",
  cov.pars = stop("no cov.pars argument provided"), kappa = 0.5)
```

# **Arguments**

obj	Vector of distances between pairs of spatial locations.
cov.model	A type of the correlation function. Available choices are: "matern", "exponential", "spherical", "powered.exponential", "cauchy", "gencauchy", "pure.nugget", "askey", "c2wendland", "c4wendland", "sinepower", "multiquadric". Default is "matern"
cov.pars	A vector with two covariance parameters. The first parameter corresponds to the variance sigma^2. The second parameter corresponds to the range phi of the correlation function.
kappa	A smoothness parameter of the correlation function.

# **Details**

The function returns the value of the covariance C(h) at the distance h. The covariance function has the form

$$C(h) = sigma^2 * rho(h/phi).$$

The parameters of the covariance are positive numbers sigma^2, phi and kappa.

Expressions for the correlation functions which are not included in the package **geoR**:

# askey

$$rho(h/phi) = (1 - h/phi)^{kappa}, ifh < phi;$$
  $0, otherwise.$ 

# c2wendland

$$rho(h/phi) = (1 + kappa * h/phi) * (1 - h/phi)^{kappa}, ifh < phi;$$
  
 $0, otherwise.$ 

# c4wendland

$$rho(h/phi) = (1 + kappa*h/phi + (kappa^2 - 1)*(h/phi)^2/3)*(1 - h/phi)^{kappa}, if h < phi;$$
 
$$0, otherwise.$$

22 covPwSp

sinepower

$$rho(h/phi) = 1 - (sin(h/(2phi)))^{kappa}$$

multiquadric

$$C(h) = (1 - phi)^{(2*kappa)} / (1 + phi^2 - 2*phi*cos(h))^{kappa}, 0 < phi < 1$$

Additional information can be found in the section Details in cov.spatial.

# Value

Values of a covariance function for the given distances.

#### References

```
geoR package, cov.spatial
```

T. Gneiting. Strictly and non-strictly positive definite functions on spheres. Bernoulli 19 (2013), no. 4, 1327-1349.

# **Examples**

```
## Compute Askey variogram at x = pi/4

1 - covmodelCMB(pi/4, cov.pars = c(1, pi), kappa = 3, cov.model = "askey" )

## Plot of the Askey covariance function

v1.f <- function(x, ...) {covmodelCMB(x, ...)}

curve( v1.f(x, cov.pars = c(1, 0.03), kappa = 3, cov.model = "askey"),

from = 0, to = 0.1, xlab = "distance", ylab = expression(cov(h)), lty = 2,

main = "covariance")</pre>
```

covPwSp

Covariance estimate via power spectra

# **Description**

This function provides a covariance estimate using the values of the estimated power spectra.

# Usage

```
covPwSp(PowerSpectra, Ns)
```

### **Arguments**

Ns

PowerSpectra A data frame which first column lists values of multipole moments and the second column gives the corresponding values of CMB power spectra.

A number of points in which the covariance estimate is computed on the interval

[-1,1]

#### Value

A data frame which first column is 1-d grid starting at -1+1/Ns and finishing at 1 with the step 2/Ns. The second column is the values of estimated covariances on this grid.

#### References

Formula (2.1) in Baran A., Terdik G. Power spectrum estimation of spherical random fields based on covariances. Annales Mathematicae et Informaticae 44 (2015) pp. 15–22.

Power Spectra data are from Planck Legacy Archive http://pla.esac.esa.int/pla/#cosmology

# **Examples**

```
## Download the power spectrum first
# N <- 20000
# COM_PowerSpectra <- downloadCMBPS(link=1)
#
# Cov_est <- covPwSp(COM_PowerSpectra[,1:2], N)
# plot(Cov_est, type="1")

## Plot the covariance estimate as a function of angular distances
# plot(acos(Cov_est[,1]), Cov_est[,2], type ="1",
# xlab ="angular distance", ylab ="Estimated Covariance")</pre>
```

displayPixelBoundaries

Plot HEALPix pixel boundaries

#### **Description**

Plot the HEALPix pixel boundaries at nside

# Usage

```
displayPixelBoundaries(nside, eps = pi/90, col = "gray", lwd = 1,
  ordering, incl.labels = 1:(12 * nside^2), nums.col = col,
  nums.size = 1, font = 2, ...)
```

#### **Arguments**

nside	the HEALPix nside parameter (integer number $2^k$ )
eps	controls the smoothness of the plot, smaller eps implies more samples
col	the colour of plotted boundary lines
lwd	the thickness of the plotted boundary lines
ordering	optionally specify an ordering scheme from which to plot HEALPix pixel numbers. Can be either "ring" or "nested"
incl.labels	If ordering is specified then this parameter sets the pixel indices that will be displayed (default is all indices at nside)
nums.col	specifies the colour of pixel numbers if ordering is specified

24 displayPixels

```
nums.size specifies the size of pixel numbers if ordering is specified

A numeric font number from 1 to 5, used if ordering is specified

arguments passed to rgl::plot3d
```

#### Value

Produces a plot of the HEALPix pixel boundaries.

# **Examples**

```
displayPixelBoundaries(1, eps = pi/90, col = "red")
displayPixelBoundaries(2, eps = pi/90, col = "green")
```

displayPixels Display the pixels and grandchildren

# Description

Display the pixels spix at resolution j by colouring in the grandchildren of spix at resolution plot.j

# Usage

```
displayPixels(boundary.j, j, plot.j = 5, spix, boundary.col = "gray",
  boundary.lwd = 1, incl.labels = 1:(12 * 4^boundary.j),
  col = "blue", size = 3)
```

# **Arguments**

boundary.j	The resolution to display boundaries at. If this is missing then boundaries will not be plotted.
j	The resolution that spix are specified at.
plot.j	The resolution to plot grandchildren at
spix	Integer vector. The pixel indices to display. These must be in nested order.
boundary.col	The boundary colour.
boundary.lwd	The boundary line width.
incl.labels	Integer vector of pixel indices to label at resolution j.
col	The colour to make the grandchildren.
size	The size to make the grandchildren.

```
## Example 1
## Plot base pixels 1,2,3 by colouring their grandchildren at resolution
## 5 (by default). No pixel boundaries.
displayPixels(j=0, spix=c(1,2,3))
## Plot base pixels 1,2,3 display and their boundaries (boundary.j=0)
```

downloadCMBMap 25

```
displayPixels(0,0, spix=c(1,2,3))
## Plot base pixels 1,2,3 by colouring their grandchildren at resolution 2
displayPixels(0,0, plot.j = 2, spix=c(1,2,3))
## Example 2
demoNeighbours <- function(p,j) {</pre>
  neighbours(p, j)
  displayPixels(boundary.j = j, j = j, plot.j = 5,
                spix = neighbours(p, j),
                boundary.col = "gray",
                boundary.lwd = 1,
                incl.labels = neighbours(p, j),
                col = "blue",
                size = 3)
  rcosmo::displayPixelBoundaries(nside = 1, col = "blue", lwd = 3)
}
demoNeighbours(1,2)
```

download CMBMap

Download CMB Maps from Planck Public Data Release.

# **Description**

The function downloadCMBMap downloads CMB maps from http://irsa.ipac.caltech.edu/data/Planck/release\_2/all-sky-maps/matrix\_cmb.html.

#### Usage

```
downloadCMBMap(foreground = "smica", nside = 1024, destfile)
```

#### **Arguments**

foreground A string naming the foreground separation method pipeline. Please choose one

of "COMMANDER", "NILC", "SEVEM" or "SMICA" (not case sensitive).

nside An integer. The nside parameter (resolution) required. The available options are

1024 or 2048.

destfile An optional character string with the path and file name for the downloaded file

to be saved. Defaults to the working directory. Tilde-expansion is performed.

#### **Details**

CMB maps have been produced by the COMMANDER, NILC, SEVEM, and SMICA pipelines, respectively.

For each pipeline, the intensity maps are provided at Nside = 2048, at 5 arcmin resolution, and the polarization maps are provided at Nside = 1024 at 10 arcmin resolution.

26 downloadCMBPS

#### Value

CMB Map FITS File (Flexible Image Transport System). The FITS file can be loaded into a CMBDataFrame using the CMBDataFrame function (see examples).

#### References

```
Planck Public Data Release 2 Maps http://irsa.ipac.caltech.edu/data/Planck/release_ 2/all-sky-maps/matrix_cmb.html
```

Other fits maps can also be downloaded using the general command download.file.

### **Examples**

```
## Download SMICA with \code{nside = 1024}
## and save in working directory
## as "CMB_map_smica1024.fits"
# downloadCMBMap(foreground = "smica", nside = 1024)
## Load the downloaded map into a CMBDataFrame
# sky <- CMBDataFrame("CMB_map_smica1024.fits")

## Download SMICA with Nside=2048 and save in the working directory
## as "CMB_map_smica2048.fits"
# downloadCMBMap(foreground = "smica", nside = 2048)

## Download COMMANDER with Nside=1024 and save in a specified folder,
## for example,
# dest <- "CMB_map_commander1024.fits"
# downloadCMBMap(foreground = "commander", nside = 1024, destfile = dest)</pre>
```

downloadCMBPS

Download CMB Power Spectra from Planck Legacy Archive.

# Description

The function downloadCMBPS downloads CMB power spectra components from http://pla.esac.esa.int/pla/#cosmology.

# Usage

```
downloadCMBPS(link = 1, destfile, save = TRUE)
```

# **Arguments**

link	The link code (an integer from 1 to 6) for the URL to download the file. See code details in this help file.
destfile	A character string with the file name for the downloaded file to be saved. Tilde-expansion is performed.
save	A boolean indicating whether to save or not (since the downloaded data is returned anyway).

entropyCMB 27

#### **Details**

```
link = 1: Best-fit LCDM CMB power spectra from the baseline Planck TT, TE, EE+lowE+lensing (2 <= ell <= 2508).

link = 2: Baseline high-ell Planck TT power spectra (2 <= ell <= 2508).

link = 3: Baseline high-ell Planck EE power spectra (2 <= ell <= 1996).

link = 4: Baseline high-ell Planck TE power spectra (2 <= ell <= 1996).

link = 5: Low-ell Planck EB power spectra (2 <= ell <= 29).

link = 6: Low-ell Planck BB power spectra (2 <= ell <= 29).
```

#### Value

The Data Frame with CMB Power Spectra and, if save = TRUE a txt file is saved in destfile

#### References

```
Planck Legacy Archive http://pla.esac.esa.int/pla/#cosmology
```

### **Examples**

```
## Download the Low-ell Planck BB power spectra (2 <= ell <= 29) and
## save it to C:/PW.txt
# downloadCMBPS(link=6, destfile="C:/PW.txt")

## Download the Best-fit LCDM CMB power spectra
## and plot it
# CMBPS <- downloadCMBPS(link=1, save = FALSE)
# plot(CMBPS$L,CMBPS$TT, type="o",col="red",cex=0.3,
# main="CMB Angular Power Spectra",xlab=expression(1),
# ylab=expression(paste(D[1],"(",mu,K^2,")")))</pre>
```

entropyCMB

CMB Entropy

### **Description**

This function returns an estimated entropy for the specified CMBDataFrame column intensities and CMBWindow region. The functions employs the function entropy and uses histogram counts of intensities for computations. All arguments of the standard entropy can be used.

# Usage

```
entropyCMB(cmbdf, win, intensities = "I", method)
```

#### **Arguments**

 $\begin{array}{ccc} {\sf cmbdf} & {\sf A\;CMBDataFrame.} \\ {\sf win} & {\sf A\;CMBWindow} \end{array}$ 

intensities A CMBDataFrame column with measured values.

method A method to estimate entropy, see entropy

28 exprob

#### Value

Estimated Shannon entropy for observations in CMBWindow

#### References

```
entropy
```

# **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 10000)
# win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
# entropyCMB(cmbdf, win1)</pre>
```

exprob

Threshold exceedance probability

#### **Description**

This function returns an estimated exceedance probability for the specified CMBDataFrame column intensities, threshold level alpha and CMBWindow region.

#### Usage

```
exprob(cmbdf, win, alpha, intensities = "I")
```

#### **Arguments**

cmbdf A CMBDataFrame. win A CMBWindow

alpha A numeric threshold level.

intensities The name of the column in cmbdf that contains the measured values.

# Value

Estimated threshold exceedance probability

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 1000)

# intensities <- "I"
# alpha <- mean(cmbdf[,intensities, drop = TRUE])
# alpha

# win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
# exprob(cmbdf, win1, alpha)</pre>
```

extrCMB 29

extrCMB Extreme values

# **Description**

This function returns n largest extreme values for the specified CMBDataFrame column intensities and CMBWindow region.

# Usage

```
extrCMB(cmbdf, win, n, intensities = "I")
```

### **Arguments**

# Value

A CMBDataFrame with n largest extreme values

#### **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 1000)
#
# win1 <- CMBWindow(theta = c(pi/2,pi,pi/2), phi = c(0,0,pi/2))
# extrCMB(cmbdf, win1,5)
#
## Ploting the window and 5 top extreme values
# plot(win1)
# plot(extrCMB(cmbdf, win1,5), col ="blue", size = 4,add = TRUE)</pre>
```

fmf

First Minkowski functional

# Description

This function returns an area of the spherical region where measured values are above of the specified threshold level alpha.

#### Usage

```
fmf(cmbdf, alpha, intensities = "I")
```

fRen

# **Arguments**

cmbdf A CMBDataFrame.
alpha A numeric threshold level.

intensities A CMBDataFrame column with measured values.

#### Value

The area of the exceedance region

#### References

Leonenko N., Olenko A. (2014) Sojourn measures of Student and Fisher-Snedecor random fields. Bernoulli, 20:1454-1483.

# **Examples**

fRen

Sample Renyi function

### **Description**

This function computes values of the sample Renyi function. Returns the estimated values of T(q) for q taking values on a grid. For large data sets could be rather time consuming.

# Usage

```
fRen(cmbdf, q.min = 1.01, q.max = 10, N = 20,
k.box = log2(nside(cmbdf)) - 3, intensities = "I")
```

# **Arguments**

cmbdf	A CMBDataFrame.
q.min	Left endpoint of the interval to compute the Renyi function. The default value is 1.01,
q.max	Right endpoint of the interval to compute the Renyi function. The default value is 10
N	Number of points to compute the Renyi function. The default value is 20.
k.box	A dyadic decomposition level in computing the Renyi function, see the references in Details. The default value is $log2(nside(cmbdf))-3$
intensities	A CMBDataFrame column with measured values

geo2sph 31

#### Value

Data frame which first column is the sampling grid seq(q.min, q.max, length.out = N) of q values. Another column consists of values of the sample Renyi function T(q) computed on the grid using the k.boxth level dyadic decomposition of the unit ball.

#### References

- (1) Leonenko, N., and Shieh, N. 2013. Rényi function for multifractal random fields. Fractals 21, Article No. 1350009.
- (2) http://mathworld.wolfram.com/RenyiEntropy.html

#### **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
#
# cmbdf <- CMBDataFrame("CMB_map_smica1024.fits")
# win <- CMBWindow(theta = c(pi/4,pi/2,pi/2), phi = c(0,0,pi/2))
# cmbdf<- window(cmbdf, new.window = win)
# Tq <- fRen(cmbdf)
#
# plot(Tq[,1], Tq[,2], ylab =expression(D[q]), xlab = "q",
# main = "Sample Renyi function", pch = 20, col = "blue")</pre>
```

geo2sph

Convert geographic to spherical coordinates

# **Description**

Convert latitude (lat) and longitude (lon) to spherical coordinates (theta, phi) with theta in [0, pi] and phi in [0, 2 \* pi). All values lat, lon, theta, phi are assumed to be in radians.

### Usage

```
geo2sph(...)
```

# **Arguments**

... A data.frame with columns lat and lon, or named vectors of lat and lon.

#### Value

A data.frame with columns theta and phi.

# **Examples**

```
geo <- data.frame( lat = c(0, pi/3, pi/2), lon = c(0, pi/3, pi)) geo sph <- geo2sph(geo) sph
```

geoArea

geoArea generic

# **Description**

Detailed descriptions and and examples can be found in documentation for specific geoArea functions geoArea. CMBDataFrame, geoArea. HPDataFrame, geoArea. CMBWindow

# Usage

```
geoArea(x)
```

# Arguments

Х

An object.

# See Also

geoArea.CMBDataFrame geoArea.HPDataFrame geoArea.CMBWindow

```
geoArea.CMBDataFrame
```

 $Geodesic \ area \ covered \ by \ a \ {\tt CMBDataFrame}$ 

# Description

Gives the surface on the unit sphere that is encompassed by all pixels in cmbdf

# Usage

```
## S3 method for class 'CMBDataFrame'
geoArea(x)
```

# **Arguments**

Х

a CMBDataFrame.

# Value

The sum of the areas of all pixels (rows) in x.

geoArea.CMBWindow 33

# **Examples**

```
## At low resolution, a few data points can
## occupy a large pixel area, e.g.:
cmbdf <- CMBDataFrame(nside = 1, spix = c(1,2,3))
pix(cmbdf)

## The total number of Healpix points at nside=1 equals 12. As cmbdf has 3 Healpix
## it occupies pi = 1/4*(surface area of unit sphere)

geoArea(cmbdf)
plot(cmbdf, size = 5, hp.boundaries = 1)</pre>
```

geoArea.CMBWindow

Geodesic area of a CMBWindow

# **Description**

Geodesic area of a CMBWindow

# Usage

```
## S3 method for class 'CMBWindow'
geoArea(x)
```

# **Arguments**

х

A CMBWindow.

#### Value

The spherical area inside the CMBWindow x.

```
## A window that covers 1/8 of the unit sphere is constructed and its area is ## pi/2 = 1/8*(surface area of unit sphere)
win <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
geoArea(win)
```

34 geoDist

geoArea.HPDataFrame Geodesic area covered by a HPDataFrame

# Description

Gives the surface on the unit sphere that is encompassed by all pixels in x.

# Usage

```
## S3 method for class 'HPDataFrame'
geoArea(x)
```

# Arguments

Х

A HPDataFrame.

#### Value

The sum of the areas of all pixels (rows) in x.

# **Examples**

```
## Generate random I for HPDataFrame
hp1 <- HPDataFrame(I=rnorm(5), nside = 1, spix = c(1,1,2,2,3))
pix(hp1)

## The total number of Healpix points at nside=1 equals 12. As hp1 has five
## I values at 3 Healpix points, then the occupied area is
## pi = 1/4*(surface area of unit sphere)

geoArea(hp1)
plot(hp1, size = 5, hp.boundaries = 1)</pre>
```

geoDist

Geodesic distance on the unit sphere

# Description

Get geodesic distance between points on the unit sphere

# Usage

```
geoDist(p1, p2, include.names = FALSE)
```

header 35

#### **Arguments**

p1 A data. frame with rows specifying numeric points located on the unit sphere.

It should have columns labelled x,y,z for Cartesian or theta, phi for spherical

colatitude and longitude respectively.

p2 Same as p1.

include.names Boolean. If TRUE then the row and column names of the returned matrix will

be taken from the points in p1 and p2 (see examples below).

### Value

Let n denote the number of rows of p1 and let m denote the number of rows of p2. Then the returned object is an n by m matrix whose entry in position ij is the geodesic distance from the ith row of p1 to the jth row of p2.

# **Examples**

```
p1 <- data.frame(diag(3))
colnames(p1) <- c("x", "y", "z")
p1
p2 <- data.frame(x=c(1,0), y=c(0,3/5), z=c(0,4/5))
p2
geoDist(p1, p2, include.names = FALSE)</pre>
```

header

Get the FITS headers from a CMBDataFrame

# Description

Get the FITS headers from a CMBDataFrame

#### Usage

header(cmbdf)

# Arguments

cmbdf

a CMBDataFrame.

#### Value

The FITS headers belonging to the FITS file from which cmbdf data was imported

```
## First download the map
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# df.sample <- CMBDataFrame(df, sample.size = 10000)
# header(df.sample)</pre>
```

36 HPDataFrame

#### **Description**

HPDataFrames are a type of data.frame designed to carry data located on the unit sphere. Each row of a HPDataFrame is associated with a HEALPix pixel index. The HPDataFrame also holds an attribute called nside which stores the HEALPix Nside parameter (i.e., the resolution of the HEALPix grid that is being used). Unlike CMBDataFrame, HPDataFrames may have repeated pixel indices. They are made this way so that multiple data points falling within a given pixel can be stored in different rows of any given HPDataFrame.

# Usage

```
HPDataFrame(..., nside, ordering = "nested", auto.spix = FALSE, spix,
assumedUniquePix = FALSE, delete.duplicates = FALSE)
```

# **Arguments**

	Data. Can be named vectors or a data.frame. May include columns (x,y,z) or (theta, phi) representing Cartesian or spherical coordinates of points on the unit sphere.
nside	Integer number $2^k$ , the nside parameter, i.e, resolution. If nside is unspecified, then the an attempt is made to use columns x,y and z from the provided data, as Cartesian coordinates, to calculate an nside that is sufficient to ensure all points belong to unique pixels.
ordering	The HEALPix ordering scheme ("ring" or "nested").
auto.spix	Boolean. If TRUE then spix will be found from the coordinates provided in the data. That is, each row of data will be assigned the pixel index of its closest HEALPix pixel center. There must be columns x,y,z for cartesian or theta, phi for spherical colatitude and longitude respectively. If auto.spix = FALSE then nside must be specified.

spix A vector of HEALPix pixel indices indicating the pixel locations of the data. Note that spix is ignored if auto.spix = TRUE.

assumedUniquePix

A boolean. Sets the assumedUniquePix attribute of the HPDataFrame. This attribute indicates whether or not the rows of a HPDataFrame can be assumed to belong to unique pixels.

delete.duplicates

Boolean. If TRUE then rows corresponding to duplicate pixel indices will be dropped from the returned HPDataFrame, and assumedUniquePix will be set to TRUE.

### **Details**

HPDataFrame with auto.spix = TRUE can be used to transform any spherical data (not necessarily CMB) to the Healpix representation, see Example 3 below.

*ibp2p* 37

#### **Examples**

```
##Example 1.
hp1 <- HPDataFrame(I=rnorm(5), nside = 1, spix = c(1,1,2,2,3))
pix(hp1)
coords(hp1, new.coords = "cartesian")
class(hp1)
assumedUniquePix(hp1)
##Example 2.
# Where nside is not specified
sky <- CMBDataFrame(nside = 32, coords = "cartesian", ordering = "nested")</pre>
sky.s <- CMBDataFrame(sky, sample.size = 100)</pre>
hpdf <- HPDataFrame(sky.s, auto.spix = TRUE)</pre>
class(hpdf)
assumedUniquePix(hpdf)
##Example 3.
## With earth data.
## Download World Cities Database from
## https://simplemaps.com/static/data/world-cities/basic/simplemaps_worldcities_basicv1.4.zip
## unpack the file worldcities.csv
# worldcities <- read.csv("worldcities.csv")</pre>
# uscities <- worldcities[worldcities$country == "United States",]</pre>
# Prepare a data frame with cities' coordinates
# usdf <- data.frame(phi = pi/180*uscities$lng, theta = pi/2 - pi/180*uscities$lat,
                    I=rep(1,length(uscities$lng)))
# Select k cities with different coordinates
# k <- 1000
# usdf <- usdf[sample(nrow(usdf), k), ]</pre>
# plot(usdf$phi, usdf$theta)
# usdf[duplicated(usdf), ]
# usdf<- usdf[!duplicated(usdf), ]</pre>
# usdf[duplicated(usdf), ]
# usdf <- coords(usdf, new.coords = "cartesian")</pre>
# Create and plot the corresponding HPDataFrame with unique pixels
# ushp <- HPDataFrame(usdf, auto.spix = TRUE)</pre>
# plot(ushp, size = 2)
```

ibp2p

Computes pixel's index using its subindex within base resolution

## **Description**

Find the pixel index p of a given pixel with index ibp in base pixel bp.

is.CMBDat

## Usage

```
ibp2p(ibp, bp, j)
```

# **Arguments**

ibp The pixel index within base pixel bp, at resolution j, in nested order.

bp The base pixel index

j The resolution parameter nside =  $2^{j}$ 

# **Examples**

```
ibp2p(1, 1, 2)
ibp2p(1, 2, 2)
```

is.CMBDat

Check if an object is of class CMBDat

# Description

Check if an object is of class CMBDat

# Usage

```
is.CMBDat(cmbdf)
```

# Arguments

cmbdf

Any R object

# Value

TRUE if cmbdf is a CMBDat object, otherwise FALSE

```
## First download the map
# downloadCMBMap(foreground = "smica", nside = 1024)
# cmbdat <- CMBDat("CMB_map_smica1024.fits", mmap = TRUE)
# class(cmbdat)
# is.CMBDat(cmbdat)</pre>
```

is.CMBDataFrame 39

is.CMBDataFrame

Check if an object is of class CMBDataFrame

# Description

Check if an object is of class CMBDataFrame

# Usage

```
is.CMBDataFrame(cmbdf)
```

# Arguments

 ${\sf cmbdf}$ 

Any R object

## Value

TRUE if cmbdf is a CMBDataFrame, otherwise FALSE

# **Examples**

```
df <- CMBDataFrame(nside = 16)
is.CMBDataFrame(df)
df2 <- coords(df, new.coords = "cartesian")
is.CMBDataFrame(df2)</pre>
```

is.CMBWindow

Check if an object is a CMBWindow

# Description

Check if an object is a CMBWindow

## Usage

```
is.CMBWindow(win)
```

# Arguments

win

any object

# Value

TRUE or FALSE depending if win is a CMBWindow

40 linesCMB

## **Examples**

```
win <- CMBWindow(x=0,y=3/5,z=4/5,r=0.8, set.minus = TRUE) is.CMBWindow(win)
```

is.HPDataFrame

Check if an object is of class HPDataFrame

# Description

Check if an object is of class HPDataFrame

## Usage

```
is.HPDataFrame(hpdf)
```

#### **Arguments**

hpdf

Any R object

#### Value

TRUE if hpdf is a HPDataFrame, otherwise FALSE

# **Examples**

```
df <- CMBDataFrame(nside = 16)
is.HPDataFrame(df)

df <- HPDataFrame(I = rep(0,12), nside = 1)
is.HPDataFrame(df)</pre>
```

linesCMB

Adds lines of fitted variograms to variogram plots

## **Description**

This function adds a line with the variogram model fitted by the function variofitCMB to a current variogram plot. The function modifies lines.variomodel.variofit from the package **geoR** for additional covariance models on spheres.

# Usage

```
linesCMB(x, max.dist, scaled = FALSE, ...)
```

maxDist 41

### **Arguments**

X	An object of the class variofit containing information about the fitted model obtained as an output of the function variofitCMB.
max.dist	A maximum distance to draw the variogram line. The default is x\$max.dist.
scaled	logical. If TRUE the sill in the plot is 1.
	other plotting parameters passed to curve

#### **Details**

The function adds a line with fitted variogram model to a plot. It is used to compare empirical variograms against fitted models returned by variofitCMB.

Available models are: "matern", "exponential", "spherical", "powered.exponential", "cauchy", "gencauchy", "pure.nugget", "askey", "c2wendland", "c4wendland", "sinepower", "multiquadric".

#### Value

A line with a fitted variogram model is added to a plot.

#### References

```
geoR package, lines.variomodel.variofit, covmodelCMB, variofitCMB
```

#### **Examples**

```
## Plot the fitted Matern variogram versus its empirical variogram
#
# df <- CMBDataFrame("../CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 10000)
# varcmb <- variogramCMB(cmbdf, max.dist = 0.1, num.bins = 30)
# varcmb
# ols <- variofitCMB(varcmb, fix.nug=FALSE, wei="equal", cov.model= "matern")
# plot(varcmb)
# lines(ols, lty=2)
#
## Plot the fitted Askey variogram versus its empirical variogram
#
# ols <- variofitCMB(vario1, ini.cov.pars = c(1, 0.03), fix.nug = TRUE,
# kappa = 3, wei = "equal", cov.model = "askey")
# plot(varcmb, main = ols$cov.model)
# linesCMB(ols, lty = 2)</pre>
```

maxDist

Get the maximum geodesic distance between points

# Description

Get the maximum geodesic distance either between all points in a data.frame pairwise, or between all points in a data.frame and one target point.

42 maxWindowDist

#### **Usage**

```
maxDist(df, point)
```

#### **Arguments**

df A data. frame with columns x,y,z for cartesian or theta, phi for spherical colati-

tude and longitude respectively. The rows must correspond to points on the unit sphere. If this is a HPDataFrame or CMBDataFrame and coordinate columns are missing, then coordinates will be assigned based on HEALPix pixel indices.

point An optional target point on the unit sphere in cartesian coordinates, in which

case all distances are calculated between point and the points in df.

#### Value

If point is specified: the longest geodesic distance from point to the points specified by the rows of df. If point is not specified: the longest geodesic distance pairwise between points in df.

#### **Examples**

```
## Using a CMBDataFrame with HEALPix coordinates only
cmbdf \leftarrow CMBDataFrame(nside = 1, spix = c(1,5,12), ordering = "ring")
plot(cmbdf, hp.boundaries = 1, col = "blue", size = 5)
p < -c(0,0,1)
maxDist(cmbdf, p) # no need to have coordinates
## Using a HPDataFrame with HEALPix coordinates only
hp <- HPDataFrame(nside = 1, I = rep(0,3), spix = c(1,5,12))
maxDist(hp, p) # notice no need to have coordinates
## Using a data.frame with cartesian coordinates
coords(hp) <- "cartesian"</pre>
df \leftarrow data.frame(x = hp$x, y = hp$y, z = hp$z)
maxDist(df, p)
## Using a data.frame with spherical coordinates
coords(hp) <- "spherical"</pre>
df <- data.frame(theta = hp$theta, phi = hp$phi)</pre>
maxDist(df, p)
## max distance between points in cmdf
maxDist(cmbdf)
```

maxWindowDist

Get the maximum distance between all points in a CMBWindow

### **Description**

Get the maximum distance between all points in a CMBWindow

#### Usage

```
maxWindowDist(x)
```

minDist 43

### **Arguments**

Χ

A CMBWindow object.

#### Value

The maximum distance between window's points.

#### **Examples**

```
## win is a equilateral spherical triangle which sides are pi/2 win <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2)) maxWindowDist(win)
```

minDist

Get the minimum geodesic distance between points

#### **Description**

Get the minimum geodesic distance either between all points in a data.frame pairwise, or between all points in a data.frame and one target point.

#### Usage

```
minDist(df, point)
```

## **Arguments**

df

A data. frame with columns x,y,z for cartesian or theta, phi for spherical colatitude and longitude respectively. The rows must correspond to points on the unit sphere. If this is a  ${\tt HPDataFrame}$  or  ${\tt CMBDataFrame}$  and coordinate columns are missing, then coordinates will be assigned based on  ${\tt HEALPix}$  pixel indices.

point

An optional target point on the unit sphere in cartesian coordinates, in which case all distances are calculated between point and the points in df.

## Value

If point is specified: the shortest distance from point to the points specified by the rows of df. If point is not specified: the shortest distance pairwise between points in df.

```
## Using a CMBDataFrame with HEALPix coordinates only cmbdf <- CMBDataFrame(nside = 1, spix = c(1,5,12), ordering = "ring") plot(cmbdf, hp.boundaries = 1, col = "blue", size = 5) p <- c(0,0,1) minDist(cmbdf, p) # no need to have coordinates ## Using a HPDataFrame with HEALPix coordinates only hp <- HPDataFrame(nside = 1, I = rep(0,3), spix = c(1,5,12))
```

44 neighbours

```
minDist(hp, p) # notice no need to have coordinates
## Using a data.frame with cartesian coordinates
coords(hp) <- "cartesian"
df <- data.frame(x = hp$x, y = hp$y, z = hp$z)
minDist(df, p)

## Using a data.frame with spherical coordinates
coords(hp) <- "spherical"
df <- data.frame(theta = hp$theta, phi = hp$phi)
minDist(df, p)

## min distance between points in cmdf
minDist(cmbdf)</pre>
```

neighbours

Return neighbouring pixels

### **Description**

Return the neighbouring pixels to a given pixel p that is specified at resolution j, in the nested order.

#### Usage

```
neighbours(p, j)
```

# Arguments

```
p Pixel index p at resolution j.j The resolution parameter with nside = 2^j.
```

```
## Return the neighbouring pixels for base pixel 1
neighbours(1, 0)
## Plot the neighbouring pixels for base pixel 1
demoNeighbours <- function(p,j) {</pre>
  neighbours(p, j)
  displayPixels(boundary.j = j, j = j, plot.j = j+3,
                spix = neighbours(p, j),
                boundary.col = "gray",
                boundary.lwd = 1,
                incl.labels = neighbours(p, j),
                col = "blue",
                size = 3)
  rcosmo::displayPixelBoundaries(nside = 1, col = "blue", lwd = 3)
}
demoNeighbours(1,2)
demoNeighbours(1,0)
```

nest2ring 45

	nest2ring	Convert nest to ring ordering	
--	-----------	-------------------------------	--

## Description

Convert from "nested" to "ring" ordering

nest2ring computes the HEALPix pixel index in the "ring" ordering scheme from the pixel index in the "nested" ordering scheme.

## Usage

```
nest2ring(nside, pix)
```

#### **Arguments**

nside is the HEALPix nside parameter.

pix is the set or subset of pixel indices at nside. If pix is left blank then all pixels are

converted.

#### Value

the output is the corresponding set of pixel in the ring ordering scheme.

## **Examples**

```
# compute HEALPix indices in the ring ordering scheme
nside <- 8
pix <-c(1,2,23)
nest2ring(nside,pix)</pre>
```

nestSearch

Finds the closest pixel center to a point

#### **Description**

Finds the closest HEALPix pixel center to a given target point, specified in Cartesian coordinates, using an efficient nested search algorithm. HEALPix indices are all assumed to be in the "nested" ordering scheme.

#### Usage

```
nestSearch(target, nside, index.only = FALSE)
```

## **Arguments**

target A data.frame, matrix or vector of Cartesian (x,y,z) coordinates for the target

point. If a data frame is used then spherical coordinates can be specified with

row names theta and phi.

nside An integer, the target resolution at which the resulting pixels are returned.

index.only A boolean indicating whether to return only the pixel index (TRUE), or cartesian

coordinates as well (FALSE).

46 nside

#### Value

if index.only = TRUE then the output will be a HEALPix index. If index.only FALSE then the output is the list containing the HEALPix index and Cartesian coordinate vector of the HEALPix point closest to target at resolution nside.

## **Examples**

```
## Find the closest HEALPix pixel center at resolution j=2 for
## the point (0.6,0.8,0)
point <- c(0.6,0.8,0)
j <- 2
cpoint <- nestSearch(point, nside = 2^j)</pre>
## Plot the closest pixel center in blue and the point (0.6,0.8,0) in red
displayPixels(j, j, plot.j=j, spix=c(cpoint$pix),
              size=5, incl.labels =FALSE)
rgl::plot3d(point[1], point[2], point[3],
            col="red", size = 5, add = TRUE)
## Repeat the above for 4 points in a data.frame
points <- data.frame(x = c(1,0,0,0.6),
                     y = c(0,1,0,0.8),
                     z = c(0,0,1,0))
points
j <- 2
cpoints <- nestSearch(points, nside = 2^j)</pre>
## Plot the closest pixel center in blue and the point (0.6,0.8,0) in red
displayPixels(j, j, plot.j=j, spix=c(cpoints$pix),
              size=5, incl.labels =FALSE)
rgl::plot3d(points[,1], points[,2], points[,3],
            col="red", size = 5, add = TRUE)
```

nside

nside generic

## **Description**

Detailed descriptions and and examples can be found in documentation for specific nside functions nside.CMBDataFrame, nside.HPDataFrame

### Usage

```
nside(x)
```

#### **Arguments**

Х

An object.

nside.CMBDataFrame 47

#### See Also

 $nside.CMBDataFrame \ nside.HPDataFrame$ 

nside.CMBDataFrame

HEALPix Nside parameter from a CMBDataFrame

# Description

This function returns the HEALPix Nside parameter of a CMBDataFrame

## Usage

```
## S3 method for class 'CMBDataFrame'
nside(x)
```

## **Arguments**

X

A CMBDataFrame.

#### Value

The HEALPix Nside parameter.

# **Examples**

```
df <- CMBDataFrame(nside = 16)
nside(df)</pre>
```

nside.HPDataFrame

 $HEALPix\ Nside\ parameter\ from\ a\ HPDataFrame$ 

# Description

This function returns the HEALPix Nside parameter of a HPDataFrame

## Usage

```
## S3 method for class 'HPDataFrame'
nside(x)
```

## Arguments

Χ

A HPDataFrame.

# Value

The HEALPix Nside parameter.

#### **Examples**

```
df \leftarrow HPDataFrame(I = rep(0,12), nside = 1)
nside(df)
```

ordering

ordering generic

## **Description**

Detailed descriptions and and examples can be found in documentation for specific ordering functions ordering.CMBDataFrame, ordering.HPDataFrame

#### Usage

```
ordering(x, ...)
```

## **Arguments**

x An object.

... Extra arguments.

#### See Also

ordering.CMBDataFrame ordering.HPDataFrame

ordering.CMBDataFrame HEALPix ordering scheme from a CMBDataFrame

#### **Description**

This function returns the HEALPix ordering scheme from a CMBDataFrame. The ordering scheme is either "ring" or "nested".

## Usage

```
## S3 method for class 'CMBDataFrame'
ordering(x, new.ordering, ...)
```

### **Arguments**

x A CMBDataFrame.

new.ordering Specifies the new ordering ("ring" or "nest") if a change of ordering scheme is

desired.

... Unused arguments.

## **Details**

If a new ordering is specified, using e.g. new.ordering = "ring", the ordering scheme of the CMB-DataFrame will be converted.

ordering.HPDataFrame

49

#### Value

The name of the HEALPix ordering scheme that is used in the CMBDataFrame x.

### **Examples**

```
df <- CMBDataFrame(nside = 1, ordering = "nested")
ordering(df)
df1 <- ordering(df, new.ordering = "ring")
ordering(df1)</pre>
```

ordering.HPDataFrame

HEALPix ordering scheme from a HPDataFrame

## **Description**

This function returns the HEALPix ordering scheme from a HPDataFrame. The ordering scheme is either "ring" or "nested". If a new ordering is specified, using e.g. new.ordering = "ring", the ordering scheme of the HPDataFrame will be converted.

#### Usage

```
## S3 method for class 'HPDataFrame'
ordering(x, new.ordering, ...)
```

# **Arguments**

```
    x a HPDataFrame.
    new.ordering Specifies the new ordering ("ring" or "nest") if a change of ordering scheme is desired.
    ... Unused arguments.
```

## Value

The name of the HEALPix ordering scheme that is used in the HPDataFrame x, or a new HPDataFrame with the desired new.ordering

```
df <- HPDataFrame(I = rep(0,12), nside = 1, ordering = "nested")
ordering(df)
df1 <- ordering(df, new.ordering = "ring")
ordering(df1)</pre>
```

50 p2ibp

p2bp

Return base pixel to which pixel belongs

# Description

The base pixel to which pixel p belongs at resolution j

# Usage

```
p2bp(p, j)
```

# Arguments

p The pixel index at resolution j, in nested order.

j The resolution parameter nside =  $2^j$ 

# **Examples**

```
p2bp(5, 0)
p2bp(5, 1)
```

p2ibp

Return pixel index within its base pixel

# Description

Convert a pixel index p to its index within the base pixel to which p belongs

# Usage

```
p2ibp(p, j)
```

# Arguments

```
p The pixel index at resolution j, in nested order.
```

j The resolution parameter nside =  $2^{j}$ 

```
p2ibp(6, 0)
p2ibp(6, 1)
```

parent 51

parent

Return index of parent pixel

# Description

Gives the pixel at resolution j-1 that contains p, where p is specified at resolution j (notice it does not depend on j).

# Usage

```
parent(p)
```

# Arguments

n

A pixel index specified in nested order.

# **Examples**

```
parent(4)
parent(5)
```

pix

pix generic

# Description

Detailed descriptions and and examples can be found in documentation for specific pix functions pix.CMBDataFrame, pix.HPDataFrame

# Usage

```
pix(x, ...)
```

# Arguments

x An object.

... Extra arguments.

# See Also

```
pix.CMBDataFrame pix.HPDataFrame
```

52 pix.HPDataFrame

pix.CMBDataFrame

HEALPix pixel indices from CMBDataFrame

## **Description**

If new.pix is unspecified then this function returns the vector of HEALPix pixel indices from a CMBDataFrame. If new.pix is specified then this function returns a new CMBDataFrame with the same number of rows as cmbdf, but with pix attribute new.pix. Thus, new.pix must have length equal to nrow(cmbdf).

## Usage

```
## S3 method for class 'CMBDataFrame'
pix(x, new.pix, ...)
```

## Arguments

```
x A CMBDataFrame.new.pix Optional vector of pixel indices with length equal to nrow(x).... Unused arguments.
```

#### Value

The vector of HEALPix pixel indices or, if new.pix is specified, a new CMBDataFrame.

## **Examples**

```
## First download the map
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits", sample.size = 800000)
# pix(df)

df <- CMBDataFrame(nside = 16, sample.size = 10, ordering = "nested")
pix(df)</pre>
```

 $\verb"pix.HPD" ataFrame"$ 

HEALPix pixel indices from HPDataFrame

## **Description**

If new.pix is unspecified then this function returns the vector of HEALPix pixel indices from a HPDataFrame. If new.pix is specified then this function returns a new HPDataFrame with the same number of rows as x, but with pix attribute new.pix. Thus, new.pix must have length equal to nrow(x).

# Usage

```
## S3 method for class 'HPDataFrame'
pix(x, new.pix, ...)
```

pix2coords 53

## **Arguments**

```
x a HPDataFrame.new.pix optional vector of pixel indices with length equal to nrow(x)... Unused arguments.
```

# Value

The vector of HEALPix pixel indices (integers) or, if new.pix is specified, a new HPDataFrame.

# **Examples**

```
df <- HPDataFrame(I = rep(0,12), nside = 1) pix(df)
```

pix2coords

Convert pixel indices to cartesian/spherical coordinates

# Description

Convert HEALPix pixel indices to cartesian or spherical coordinates

## Usage

```
pix2coords(nside, coords = "cartesian", ordering = "nested", spix)
```

# Arguments

nside the nside parameter (integer number  $2^k$ ) coords 'cartesian' or 'spherical' coordinates

ordering 'ring' or 'nested' ordering

spix optional integer or vector of sample pixel indices

## Value

```
a data.frame with columns 'x', 'y', 'z' (cartesian) or 'theta', 'phi' (spherical)
```

```
pix2coords(nside=1, spix=c(2,5))
pix2coords(nside=1, coords = "spherical", spix=c(2,5))
```

54 pixelWindow

pixelArea

Area of a HEALPix pixel

## **Description**

Get the area of a single HEALPix pixel

#### Usage

```
pixelArea(nsideObject)
```

## **Arguments**

```
nsideObject CMBDataFrame, a HPDataFrame, or an integer giving the nside parameter.
```

#### Value

the area of a single HEALPix pixel at the nside resolution of nsideObject

# **Examples**

pixelWindow

Find high resolution pixels falling in a lower resolution window

# Description

Find all pixels in a higher resolution that fall within the specified pixel area at a lower resolution. All pixels are assumed to be in nested ordering.

# Usage

```
pixelWindow(j1, j2, pix.j1)
```

# Arguments

JI	An integer. The lower resolution, with $j1 = < j2$ .
j2	An integer. The upper resolution.
pix.j1	An integer. The pixel index at resolution j1 within which all pixels from resolu-
	tion j2 will be returned. pix. j1 can also be a vector of non-zero pixel indices.

plot.CMBCorrelation 55

#### Value

All pixels in resolution j2 that fall within the pixel pix.j1 specified at resolution j1

#### **Examples**

```
pixelWindow(3, 3, 2)
pixelWindow(3, 4, 2)
pixelWindow(3, 5, 2)
```

plot.CMBCorrelation

Plot sample CMBCorrelation

## **Description**

Plots sample (empirical) correlation function. Uses plot.variogram from geoR package.

## Usage

```
## S3 method for class 'CMBCorrelation' plot(x, ...)
```

## **Arguments**

x An object of class CMBCorrelation.

... Extra arguments as in plot.variogram passed to plot.default.

## Value

Produces a plot with the sample correlation function.

#### References

```
geoR package, corrCMB, variog, plot.variogram
```

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 100000)
# corcmb <- corrCMB(cmbdf, max.dist = 0.03, num.bins = 10, sample.size=1000)
# plot(corcmb)</pre>
```

56 plot.CMBDataFrame

plot.CMBCovariance

Plot sample CMBCovariance

## **Description**

Plots sample (empirical) covariance function. Uses plot.variogram from geoR package.

## Usage

```
## S3 method for class 'CMBCovariance' plot(x, ...)
```

#### **Arguments**

- x An object of class CMBCovariance.
- ... Extra arguments as in plot.variogram passed to plot.default.

#### Value

Produces a plot with the sample covariance function.

```
#'@references geoR package, covCMB, variog, plot.variogram
```

## **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 100000)
# Cov <- covCMB(cmbdf, max.dist = 0.03, num.bins = 10)
# plot(Cov)</pre>
```

plot.CMBDataFrame

Plot CMB Data

# Description

This function produces a plot from a CMBDataFrame.

## Usage

```
## S3 method for class 'CMBDataFrame'
plot(x, intensities = "I", add = FALSE,
   sample.size, type = "p", size = 1, box = FALSE, axes = FALSE,
   aspect = FALSE, col, back.col = "black", labels, hp.boundaries = 0,
   hpb.col = "gray", ...)
```

plot.CMBWindow 57

# Arguments

x	A CMBDataFrame.
intensities	The name of a column that specifies CMB intensities. This is only used if col is unspecified.
add	If TRUE then this plot will be added to any existing plot. Note that if back.col (see below) is specified then a new plot window will be opened and add = TRUE will have no effect.
sample.size	Optionally specifies the size of a simple random sample to take before plotting. This can make the plot less computationally intensive.
type	A single character indicating the type of item to plot. Supported types are: 'p' for points, 's' for spheres, 'l' for lines, 'h' for line segments from $z=0$ , and 'n' for nothing.
size	The size of plotted points.
box	Whether to draw a box.
axes	Whether to draw axes.
aspect	Either a logical indicating whether to adjust the aspect ratio, or a new ratio.
col	Specify the colour(s) of the plotted points.
back.col	Optionally specifies the background colour of the plot. This argument is passed to rgl::bg3d.
labels	Optionally specify a vector of labels to plot, such as words or vertex indices. If this is specified then rgl::text3d is used instead of rgl::plot3d. Then length(labels) must equal nrow(x).
hp.boundaries	Integer. If greater than 0 then HEALPix pixel boundaries at $nside = hp.boundaries$ will be added to the plot.
hpb.col	Colour for the hp. boundaries.
	Arguments passed to rgl::plot3d.

# Value

A plot of the CMB data

# **Examples**

```
## First download the map
# downloadCMBMap(foreground = "smica", nside = 1024)
# sky <- CMBDataFrame("CMB_map_smica1024.fits")
# plot(sky, sample.size = 800000)</pre>
```

plot.CMBWindow

Visualise a CMBWindow

# Description

Visualise a CMBWindow

58 plot.HPDataFrame

#### Usage

```
## S3 method for class 'CMBWindow'
plot(x, add = TRUE, type = "1", col = "red",
    size = 2, box = FALSE, axes = FALSE, aspect = FALSE, back.col,
    ...)
```

#### **Arguments**

X	A CMBWindow.
^	A CIVID WILLIAM.
add	if TRUE then this plot will be added to any existing plot. Note that if back.col (see below) is specified then a new plot window will be opened and add = TRUE will have no effect
type	a single character indicating the type of item to plot. Supported types are: 'p' for points, 's' for spheres, 'l' for lines, 'h' for line segments from $z=0$ , and 'n' for nothing.
col	specify the colour(s) of the plotted points
size	the size of plotted points
box	whether to draw a box
axes	whether to draw axes
aspect	either a logical indicating whether to adjust the aspect ratio, or a new ratio.
back.col	specifies the background colour of the plot. This argument is passed to rgl::bg3d.
	arguments passed to rgl::plot3d

#### **Examples**

```
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
win2 <- CMBWindow(theta = c(2*pi/3,3*pi/4,3*pi/4,2*pi/3), phi = c(pi/4,pi/4,pi/3,pi/3))
plot(win1)
plot(win2)
```

plot.HPDataFrame

Plot HPDataFrame

# Description

This function produces a plot from a HPDataFrame. If columns x,y,z (cartesian) or theta,phi (colatitude and longitude respectively) are present in x, then these will be used as coordinates for plotting. Otherwise, the HEALPix indices as in pix(x) will be used. If HEALPix indices are used and multiple rows correspond to a single pixel index, then beware that values may be obfuscated in the plot, and all locations are pixel centers.

## Usage

```
## S3 method for class 'HPDataFrame'
plot(x, intensities = "I", add = FALSE,
   sample.size, type = "p", size = 1, box = FALSE, axes = FALSE,
   aspect = FALSE, col = "blue", back.col = "black", labels,
   hp.boundaries = 0, hpb.col = "gray", ...)
```

plot.variogram 59

## **Arguments**

x	A HPDataFrame.
intensities	The column name for the data in x that is to be treated as intensities for plotting.
add	if TRUE then this plot will be added to any existing plot. Note that if back.col (see below) is specified then a new plot window will be opened and add = TRUE will have no effect
sample.size	optionally specifies the size of a simple random sample to take before plotting. This can make the plot less computationally intensive
type	a single character indicating the type of item to plot. Supported types are: 'p' for points, 's' for spheres, 'l' for lines, 'h' for line segments from $z=0$ , and 'n' for nothing.
size	the size of plotted points
box	whether to draw a box
axes	whether to draw axes
aspect	either a logical indicating whether to adjust the aspect ratio, or a new ratio.
col	specify the colour(s) of the plotted points
back.col	optionally specifies the background colour of the plot. This argument is passed to rgl::bg3d.
labels	optionally specify a vector of labels to plot, such as words or vertex indices. If this is specified then rgl::text3d is used instead of rgl::plot3d. Then length(labels) must equal nrow(x)
hp.boundaries	integer. If greater than 0 then HEALPix pixel boundaries at $nside = hp.boundaries$ will be added to the plot
hpb.col	colour for the hp.boundaries
	arguments passed to rgl::plot3d

# Value

A plot of the data locations according to coordinate columns or HEALPix index

# **Examples**

plot.variogram Plot sample variogram

# Description

Plots sample (empirical) variogram. Uses  ${\tt plot.variogram}$  from  ${\tt geoR}$  package.

# **Arguments**

x An object of class variogram.

... Extra arguments as in plot.variogram passed to plot.default.

60 plotAngDis

#### Value

Produces a plot with the sample variogram.

#### References

```
geoR package, variogramCMB, variog, plot.variogram
```

#### **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 100000)
# varcmb <- variogramCMB(cmbdf, max.dist = 0.1, num.bins = 30, sample.size=1000)
# plot(varcmb)</pre>
```

plotAngDis

Plot angular scatterplots and means

## **Description**

For specified measurements from CMBDataFrame this function produces scatterplots and binned means versus theta and phi angles.

#### Usage

```
plotAngDis(cmbdf, intensities = "I")
```

## **Arguments**

 ${\sf cmbdf} \qquad \qquad {\sf A} \; {\sf CMBDataFrame} \; object.$ 

intensities The name of a column of cmbdf, containing measured values.

### Value

2x2 plot. The first row shows scatterplots. The second row gives barplots of the corresponding means computed over bins. The first column corresponds to the values of theta and the second one is for psi.

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# df.sample <- CMBDataFrame(df, sample.size = 80000)
# win <- CMBWindow(theta = c(pi/4,pi/2,pi/2,pi/4), phi = c(0,0,pi/2,pi/2))
# cmbdf.win <- window(df.sample, new.window = win)
#
# intensities <- "I"
# plotAngDis(cmbdf.win, intensities)</pre>
```

plotcovmodelCMB 61

|--|

# Description

Plots theoretical covariance functions from the list defined in covmodelCMB

# Usage

```
plotcovmodelCMB(cov.model = "matern", sigmasq = 1, phi = pi,
  kappa = 0.5, from = 0, to = pi, ...)
```

# **Arguments**

cov.model	A type of the correlation function. Available choices are: "matern", "exponential", "spherical", "powered.exponential", "cauchy", "gencauchy", "pure.nugget", "askey", "c2wendland", "c4wendland", "sinepower", "multiquadric". The default is "matern"
sigmasq	The variance parameter as documented in covmodelCMB. The default is 1.
phi	The range parameter as documented in covmode1CMB. The default is pi.
kappa	A smoothness parameter of the correlation function. The default is 0.5.
from	A lower range of the plotting region. The default is 1b =0
to	An upper range of the plotting region. The default is ub= pi.
	optional plotting parameters.

#### Value

Produces a plot with the theoretical covariance function.

#### References

```
covmode1CMB
```

```
plotcovmodelCMB("matern", sigmasq = 5)
plotcovmodelCMB("askey", phi = pi/4, to = pi/2, kappa = 4)
```

62 plotvariogram

plotvariogram	Plot theoretical variogram

# Description

Plots theoretical variogram functions from the list defined in covmodelCMB

# Usage

```
plotvariogram(cov.model = "matern", sigmasq = 1, phi = pi,
  kappa = 0.5, from = 0, to = pi, ...)
```

# **Arguments**

cov.model	A type of the variogram function. Available choices are: "matern", "exponential", "spherical", "powered.exponential", "cauchy", "gencauchy", "pure.nugget", "askey", "c2wendland", "c4wendland", "sinepower", "multiquadric". The default is "matern"
sigmasq	The variance parameter as documented in covmodelCMB. The default is 1.
phi	The range parameter as documented in covmodelCMB. The default is pi.
kappa	A smoothness parameter of the variogram function. The default is 0.5.
from	A lower range of the plotting region. The default is 1b =0
to	An upper range of the plotting region. The default is ub= pi.
	optional plotting parameters.

#### Value

Produces a plot with the theoretical variogram.

#### References

```
covmode1CMB
```

```
plotvariogram("matern", sigmasq = 5)
plotvariogram("askey", phi = pi/4, to = pi/2, kappa = 4)
```

practicalRangeCMB 63

practicalRangeCMB	Practical range for covariance function	

# Description

This function computes the practical range for covariance functions on spheres. The function extends practicalRange from the package **geoR** to additional covariance models on spheres.

#### Usage

```
practicalRangeCMB(cov.model, phi, kappa = 0.5, correlation = 0.05, ...)
```

## **Arguments**

cov.model	A type of the correlation function. Available choices are: "matern", "exponential", "spherical", "powered.exponential", "cauchy", "gencauchy", "pure.nugget", "askey", "c2wendland", "c4wendland", "sinepower", "multiquadric".
phi	The range parameter as documented in covmodelCMB
kappa	A smoothness parameter of the correlation function.
correlation	A correlation threshold (default is 0.05)
	other optimisation parameters

#### **Details**

The practical(effective) range for a covariance function is the distance at which a covariance function first time reaches the specified value correlation. For covariance functions on spheres the practical range does not exceed pi, the distance beyond which a covariance function is not defined. For the covariance functions "spherical", "askey", "c2wendland", "c4wendland" their practical ranges are equal to lengths of their support.

#### Value

Value of the practical range for the covariance function specified in covmodelCMB

## References

```
\textbf{geoR} \text{ package, practicalRange, covmodelCMB}
```

```
practicalRangeCMB(cov.model = "sinepower", phi = 0.1, kappa = 0.5)
practicalRangeCMB(cov.model = "askey", phi = 0.1, kappa = 0.5)
```

64 print.HPDataFrame

print.CMBDataFrame

Print CMBDataFrame

# Description

This function neatly prints the contents of a CMBDataFrame.

## Usage

```
## S3 method for class 'CMBDataFrame'
print(x, ...)
```

# **Arguments**

```
{\sf x} A CMBDataFrame.
```

... arguments passed to print.tbl\_df

## Value

Prints contents of the CMB data frame to the console.

## **Examples**

```
## First download the map
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# print(df)</pre>
```

print.HPDataFrame

Print a HPDataFrame

# Description

This function neatly prints the contents of a HPDataFrame.

## Usage

```
## S3 method for class 'HPDataFrame'
print(x, ...)
```

## **Arguments**

```
x A HPDataFrame.
... arguments passed to print.tbl_df
```

# Value

Prints contents of the HPDataFrame to the console.

pwSpCorr 65

#### **Examples**

```
df <- HPDataFrame(I = rep(0,12), nside = 1, ordering = "nested") print(df) df
```

pwSpCorr

Power spectra estimate via correlation

### **Description**

This function provides an angular power spectra estimate using the values of the sample correlations. The approach is based on Lawson-Hanson algorithm for non-negative least squares.

## Usage

```
pwSpCorr(corcmb, lmax = 20 * length(corcmb$u))
```

#### **Arguments**

corcmb An object of the class CMBCorrelation.

1max A number of angular power spectra components to estimate

#### Value

A data frame which first column is 1-d grid of 1 values from 0 to 1max. The second column is estimated angular power spectra components on this grid.

#### References

Formula (2.1) in Baran A., Terdik G. Power spectrum estimation of spherical random fields based on covariances. Annales Mathematicae et Informaticae 44 (2015) pp. 15–22.

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# Corrf <- corrCMB(df, max.dist = 0.1, num.bins = 30,
# sample.size=10000)
# pw <- pwSpCorr(Corrf)</pre>
```

66 qqnormWin

qqnormWin

Normal QQ plot for CMBWindow

# Description

This function is a modification of standard qqnorm functions to work with CMBWindow regions.

## Usage

```
qqnormWin(cmbdf, win, intensities = "I")
```

# **Arguments**

 $\begin{array}{ccc} {\sf cmbdf} & & {\sf A\;CMBDataFrame.} \\ \\ {\sf win} & & {\sf A\;CMBWindow} \end{array}$ 

intensities A CMBDataFrame column with measured values.

#### **Details**

qqnormWin returns a normal QQ plot of for the specified CMBDataFrame column intensities and CMBWindow region. The function automatically adds a line of a "theoretical" normal quantile-quantile plot.

#### Value

A list with quantile components x and y and a normal QQ plot with QQ line

#### References

```
qqnorm, qqplot, qqplotWin
```

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 1000)
# win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
# qqnormWin(cmbdf, win1)</pre>
```

qqplotWin 67

qqplotWin	Quantile-Quantile plots for CMBWindows

# Description

This function is a modification of standard qqplot functions to work with CMBWindow regions.

# Usage

```
qqplotWin(cmbdf, win1, win2, intensities = "I")
```

## **Arguments**

#### **Details**

qqplotWin produces a QQ plot of quantiles of observations in two CMBWindows against each other for the specified CMBDataFrame column intensities. The function automatically adds a diagonal line.

## Value

A list with quantile components x and y and a QQ plot with a diagonal line

# References

```
qqnormWin, qqnorm, qqplot
```

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 10000)

# win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
# win2 <- CMBWindow(theta = c(2*pi/3,3*pi/4,3*pi/4, 2*pi/3),
# phi = c(pi/4,pi/4,pi/3,pi/3))

# qqplotWin(cmbdf, win1, win2)</pre>
```

68 qstat

#### **Description**

This function returns an estimated q-statistic for the specified column intensities in a CMBDataFrame and the list of CMBWindows.

## Usage

```
qstat(cmbdf, listwin, intensities = "I")
```

## **Arguments**

intensities A CMBDataFrame column with measured values.

#### **Details**

The q-statistics is used to measure spatial stratified heterogeneity and takes values in [0, 1]. It gives the percent of the variance of intensities explained by the stratification. 0 corresponds to no spatial stratified heterogeneity, 1 to perfect spatial stratified heterogeneity.

#### Value

Estimated q-statistics for observations in a list of CMBWindows

## References

Wang, J.F, Zhang, T.L, Fu, B.J. (2016). A measure of spatial stratified heterogeneity. Ecological Indicators. 67: 250–256.

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 1000)
# win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
# win2 <- CMBWindow(theta = c(pi/2,pi,pi/2), phi = c(0,0,pi/2))
# # lw <- list(win1, win2)
# qstat(cmbdf, lw)</pre>
```

rbind.CMBDataFrame 69

rbind.CMBDataFrame

rbind for CMBDataFrames

## **Description**

Add a new row or rows to a CMBDataFrame. All arguments passed to . . . must be CMBDataFrames. If the CMBDataFrame arguments have overlapping pixel indices then all but one of the non-unique rows will be deleted unless unsafe = TRUE. If unsafe = TRUE then a HPDataFrame will be returned instead of a CMBDataFrame.

## Usage

```
## S3 method for class 'CMBDataFrame'
rbind(..., deparse.level = 1, unsafe = FALSE)
```

# **Arguments**

. . . A number of CMBDataFrames

deparse.level See documentation for rbind.data.frame.

unsafe

A boolean. If the CMBDataFrame arguments have overlapping pixel indices then all but one of the non-unique rows will be deleted unless unsafe = TRUE. If unsafe = TRUE then a HPDataFrame will be returned instead of a CMBDataFrame.

#### See Also

See the documentation for rbind

```
df <- CMBDataFrame(nside = 1, I = 1:12)

df.123 <- CMBDataFrame(df, spix = c(1,2,3))
 df.123
 df.234 <- CMBDataFrame(df, spix = c(2,3,4))
 df.234

df.1234 <- rbind(df.123, df.234)
 df.1234
 class(df.1234) # A CMBDataFrame
 pix(df.1234)

df.123234 <- rbind(df.123, df.234, unsafe = TRUE)
 df.123234
 class(df.123234) # A HPDataFrame
 pix(df.123234)</pre>
```

70 resolution

rcosmo

Handling and Analysing CMB data

#### **Description**

Handling and Analysing Spherical, Healpix and Cosmic Microwave Background data on a HEALPix grid.

#### **Details**

The package rcosmo offers various tools for

- Downloading and transforming Cosmic Microwave Background radiation (CMB) and spherical data
- Working with Hierarchical Equal Area isoLatitude Pixelation of a sphere (Healpix)
- · Spherical geometry
- Statistical analysis of CMB and spherical data
- · Visualisation of Healpix data

Most of rcosmo features were developed for CMB, but it can also be used for other spherical data. It contains tools for transforming spherical data in cartesian and geographic coordinates to the Healpix representation.

# **Update**

Current updates are available through URL: https://github.com/VidaliLama/rcosmo

# **BugReports**

https://github.com/VidaliLama/rcosmo/issues

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resolution

 $Get\ the\ arcmin\ resolution\ from\ a\ {\tt CMBDataFrame}$ 

# Description

Get the arcmin resolution from a CMBDataFrame

#### Usage

resolution(cmbdf)

#### **Arguments**

cmbdf

a CMBDataFrame.

ring2nest 71

## Value

The arcmin resolution as specified by the FITS file where the data was sourced

## **Examples**

```
## First download the map
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# resolution(df)</pre>
```

ring2nest

Convert ring to nest ordering.

# Description

ring2nest converts HEALPix pixel indices in the 'ring' ordering scheme to HEALPix pixel indices in the 'nested' ordering scheme.

# Usage

```
ring2nest(nside, pix)
```

## **Arguments**

nside is the HEALPix nside parameter (integer number  $2^k$ )
pix is a vector of HEALPix pixel indices, in the 'ring' ordering scheme.

## Value

the output is a vector of HEALPix pixel indices in the 'nested' ordering scheme.

```
## Convert (1,2,23) from ring to nest at nside = 8
nside <- 8
pix <-c(1,2,23)
ring2nest(nside,pix)</pre>
```

72 siblings

sampleCMB

 $\it Take\ a\ simple\ random\ sample\ from\ a\ {\tt CMBDataFrame}$ 

## **Description**

This function returns a CMBDataFrame which size equals to sample.size, whose rows comprise a simple random sample of the rows from the input CMBDataFrame.

# Usage

```
sampleCMB(cmbdf, sample.size)
```

### **Arguments**

```
cmbdf a CMBDataFrame.
sample.size the desired sample size.
```

#### Value

A CMBDataFrame which size equals to sample.size, whose rows comprise a simple random sample of the rows from the input CMBDataFrame.

## **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# plot(sampleCMB(df, sample.size = 800000))

df <- CMBDataFrame(nside = 16, I = rnorm(12 * 16 ^ 2), ordering = "nested")
df.sample <- sampleCMB(df, sample.size = 100)
df.sample</pre>
```

siblings

Return siblings of pixel

## **Description**

The siblings of pixel p are defined as the children of the parent of p. Note this is resolution independent.

## Usage

```
siblings(p)
```

## **Arguments**

р

Pixel index in nested order.

sphericalHarmonics 73

# **Examples**

```
siblings(11)
siblings(12)
```

sphericalHarmonics

Compute spherical harmonic values at given points on the sphere.

# Description

The function spherical Harmonics computes the spherical harmonic values for the given 3D Cartesian coordinates.

#### Usage

```
sphericalHarmonics(L, m, xyz)
```

## **Arguments**

L The degree of spherical harmonic (L=0,1,2,...)

m The order number of the degree-L spherical harmonic (m=-L,-L+1,...,L-1,L)

xyz Dataframe for given points in 3D cartesian coordinates

# Value

values of spherical harmonics

## References

See https://en.wikipedia.org/wiki/Table\_of\_spherical\_harmonics

It uses equation (7) in Hesse, K., Sloan, I. H., & Womersley, R. S. (2010). Numerical integration on the sphere. In Handbook of Geomathematics (pp. 1185-1219). Springer Berlin Heidelberg,

but instead of the order k=1,...,2L+1 in the book we use m=k-L-1.

```
## Calculate spherical harmonic value at
## the point (0,1,0) with L=5, m=2
point<-data.frame(x=0,y=1,z=0)
sphericalHarmonics(5,2,point)

## Calculate spherical harmonic values at
## the point (1,0,0), (0,1,0), (0,0,1) with L=5, m=2
points<-data.frame(diag(3))
sphericalHarmonics(5,2,points)</pre>
```

```
summary.CMBDataFrame Summarise a CMBDataFrame
```

## **Description**

This function produces a summary from a CMBDataFrame.

#### Usage

```
## S3 method for class 'CMBDataFrame'
summary(object, intensities = "I", ...)
```

## Arguments

```
object A CMBDataFrame.

intensities the name of a column specifying CMB intensities (or potentially another numeric quantity of interest)

... Unused arguments.
```

#### Value

A summary includes window's type and area, total area covered by observations, and main statistics for intensity values

# **Examples**

```
## First download the map
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# df.sample <- CMBDataFrame(df, sample.size = 800000)
# summary(df.sample)

ns <- 16
df <- CMBDataFrame(I = rnorm(12*ns^2), nside = ns, ordering = "nested")

win1 <- CMBWindow(x=0,y=3/5,z=4/5,r=0.8)
df.sample1 <- window(df, new.window = win1)
summary(df)</pre>
```

summary.CMBWindow

Summarise a CMBWindow

# Description

This function produces a summary from a CMBWindow

summary.HPDataFrame 75

# Usage

```
## S3 method for class 'CMBWindow'
summary(object, ...)
```

## **Arguments**

```
object A CMBWindow.
... Unused arguments.
```

# Value

A summary includes window's type and area

## **Examples**

```
win <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2)) summary(win) win1<- CMBWindow(x=0,y=3/5,z=4/5,r=0.8, set.minus = TRUE) summary(win1)
```

 $\verb"summary.HPD" at a Frame"$ 

 $Summarise \ a \ {\tt HPDataFrame}$ 

# Description

This function produces a summary from a HPDataFrame.

## Usage

```
## S3 method for class 'HPDataFrame'
summary(object, intensities = "I", ...)
```

# **Arguments**

object A HPDataFrame.

intensities the name of a column specifying intensities (or potentially another numeric

quantity of interest)

... Unused arguments.

# Value

A summary includes window's type and area, total area covered by observations, and main statistics for intensity values

76 triangulate

#### **Examples**

```
ns <- 2 hpdf <- HPDataFrame(I = rnorm(12*ns^2), nside = 2, ordering = "nested") win <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2)) hpdf.win <- window(hpdf, new.window = win) summary(hpdf.win)
```

triangulate

Triangulate a polygonal CMBWindow

#### **Description**

Triangulate a polygonal CMBWindow

# Usage

```
triangulate(win)
```

#### **Arguments**

win

a CMBWindow object

#### Value

a list of CMBWindow polygons or minus.polygons, each having 3 vertices and representing a triangle. If winType of win does not include "minus" then these triangles have pairwise disjoint interiors and their union is equal to the original polygon, win. Otherwise, if winType of win does include "minus" the triangles are the same as for the non-minus type above, but have "minus" types.

variofitCMB 77

```
win1 <- triangulate(win)
win1
plot(win1[[1]], add= FALSE, col="green")
plot(win1[[2]], col="blue")
plot(win1[[3]], col="yellow")
summary(win1[[1]])
summary(win1[[2]])
summary(win1[[3]])</pre>
```

variofitCMB

Estimates parameters of variograms

# Description

This function estimates variogram parameters by fitting a parametric model from covmodelCMB to a sample variogram. The function extends variofit from the package **geoR** to additional covariance models on spheres.

#### Usage

```
variofitCMB(vario, ini.cov.pars, cov.model, fix.nugget = FALSE,
  nugget = 0, fix.kappa = TRUE, kappa = 0.5, simul.number = NULL,
  max.dist = vario$max.dist, weights, minimisation.function,
  limits = geoR::pars.limits(), messages, ...)
```

## **Arguments**

vario	An object of the class variogram obtained as an output of the function variogramCMB.
ini.cov.pars	A vector with initial values for the variogram parameters. The first parameter corresponds to the variance sigma^2. The second parameter corresponds to the range phi of the correlation function.
cov.model	A type of the variogram function. Available choices are: "matern", "exponential", "spherical", "powered.exponential", "cauchy", "gencauchy", "pure.nugget", "askey", "c2wendland", "c4wendland", "sinepower", "multiquadric". The default is "matern"
fix.nugget	logical. Indicates whether the nugget variance should be regarded as fixed or be estimated. The default is FALSE.
nugget	A value for the nugget parameter. Regarded as a fixed values if fix.nugget = TRUE or as a initial value for the minimization algorithm if fix.nugget = FALSE. The default is zero.
fix.kappa	logical. Indicates whether the parameter kappa should be regarded as fixed or be estimated. The default is TRUE.
kappa	A value for the smoothness parameter. Regarded as a fixed values if fix.kappa = TRUE or as a initial value for the minimization algorithm if fix.kappa = FALSE. Required not in all covariance models, see covmodelCMB. The default is 0.5.
simul.number	number of simulation. Used if vario has empirical variograms for more than one data-set (simulations). The default is NULL
max.dist	A maximum distance to fit a variogram model. The default is x\$max.dist.

78 variofitCMB

weights Weights used in the loss function in the minimization algorithm.

minimisation.function

Minimization function ("optim", "nlm", "nls") to estimate the parameters.

Lower and upper limits for the model parameters used in the numerical minimisation by minimisation.function = "optim".

messages logical. Indicates whether or not status messages are printed on the screen.

other minimisation parameters

#### **Details**

The parameter values of a variogram function from covmodelCMB are found by numerical optimization using one of the functions: optim, nlm and nls.

The function extends variofit from the package **geoR** to additional variogram models on spheres. Available models are: "matern", "exponential", "spherical", "powered.exponential", "cauchy", "gencauchy", "pure.nugget", "askey", "c2wendland", "c4wendland", "sinepower", "multiquadric".

Additionally it rescales an empirical variogram to the range [0,1] before numerical optimisation and then transforms all obtained results to the original scale. If ini.cov.pars are not provided then the 5x5 grid (seq(0,max(vario\$v),1=5), seq(0,vario\$max.dist,1=5)) of initial values of sigma^2 and phi is used.

#### Value

An object of the class variomodel and variofit, see variofit

#### References

```
geoR package, variofit, covmodelCMB
```

```
#
# df <- CMBDataFrame("../CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 10000)
# varcmb <- variogramCMB(cmbdf, max.dist = 0.1, num.bins = 30)
# varcmb
#
# ols <- variofitCMB(varcmb, fix.nug=FALSE, wei="equal", cov.model= "matern")
# plot(varcmb)
# lines(ols, lty=2)
# str(ols)
#
# ols <- variofitCMB(varcmb, fix.nug = TRUE, kappa = 3, wei = "equal",
# cov.model = "askey")
# plot(varcmb, main = ols$cov.model)
# linesCMB(ols, lty = 2)
# str(ols)</pre>
```

variogramCMB 79

## **Description**

This function provides an empirical variogram for data in a CMBDataFrame or data.frame. It assumes that data are from a stationary spherical random field and the covariance depends only on a geodesic distance between locations. Output is a binned variogram.

# Usage

```
variogramCMB(cmbdf, num.bins = 10, sample.size, max.dist = pi, breaks,
   equiareal = TRUE, calc.max.dist = FALSE)
```

## **Arguments**

cmbdf	is a CMBDataFrame or data.frame
num.bins	specifies the number of bins
sample.size	optionally specify the size of a simple random sample to take before calculating variogram. This may be useful if the full covariance computation is too slow.
max.dist	an optional number between 0 and pi specifying the maximum geodesic distance to use for calculating covariance. Only used if breaks are unspecified.
breaks	optionally specify the breaks manually using a vector giving the break points between cells. This vector has length num.bins since the last break point is taken as max.dist. If equiareal = TRUE then these breaks should be $cos(r_i)$ where $r_i$ are radii. If equiareal = FALSE then these breaks should be $r_i$ .
equiareal	if TRUE then the bins have equal spherical area. If false then the bins have equal annular widths. Default is TRUE.
calc.max.dist	if TRUE then the max.dist will be calculated from the locations in cmbdf. Otherwise either max.dist must be specified or max.dist will default to pi.

## Value

An object of class variog specified in the package **geoR**.

The attribute "breaks" contains the break points used to create bins. The result has num.bins + 1 values since the first value at distance 0 is not counted as a bin.

- **u** a vector with distances.
- v a vector with estimated variogram values at distances given in u.
- **n** number of pairs in each bin
- sd standard deviation of the values in each bin

bins.lim limits defining the interval spanned by each bin

**ind.bin** a logical vector indicating whether the number of pairs in each bin is greater or equal to the value in the argument pairs.min

var.mark variance of the data

**beta.ols** parameters of the mean part of the model fitted by ordinary least squares **output.type** echoes the option argument

80 window

```
max.dist maximum distance between pairs allowed in the variogram calculations
n.data number of data
direction direction for which the variogram was computed
call the function call
```

#### References

```
geoR package, variog, covCMB, corrCMB
```

## **Examples**

```
## Download the map first
# downloadCMBMap(foreground = "smica", nside = 1024)
# df <- CMBDataFrame("CMB_map_smica1024.fits")
# cmbdf <- sampleCMB(df, sample.size = 100000)
# varcmb <- variogramCMB(cmbdf, max.dist = 0.1, num.bins = 30, sample.size=100)
# varcmb</pre>
```

window

window generic

## **Description**

Detailed descriptions and and examples can be found in documentation for specific window functions window. CMBDataFrame, window. HPDataFrame, window. data. frame

# Usage

```
window(x, ...)
```

# Arguments

```
x An object.... Extra arguments.
```

#### See Also

```
window. \, CMBDataFrame \,\, window. \, HPDataFrame \,\, window. \, data. \, frame \,\, window. \, data. \, fram
```

window.CMBDat 81

window.CMBDa	et Get a si	ub window from a CMBDat object

#### **Description**

This function returns a data frame containing the data in x restricted to the CMBWindow new.window

## Usage

```
## S3 method for class 'CMBDat'
window(x, new.window, intersect = TRUE, ...)
```

#### **Arguments**

```
    x a CMBDat object.
    new.window A single CMBWindow object or a list of them.
    intersect A boolean that determines the behaviour when new.window is a list containing BOTH regular type and "minus" type windows together (see details).
    ... Unused arguments.
```

#### Details

Windows that are tagged with set.minus (see CMBWindow) are treated differently from other windows.

If the argument is a list of CMBWindows, then interious of all windows whose winType does not include "minus" are united (let A be their union) and exteriors of all windows whose winType does include "minus" are intersected, (let B be their intersection). Then, provided that intersect = TRUE (the default), the returned data.frame will be the points of cmbdat\$data in the the intersection of A and B. Otherwise, if intersect = FALSE, the returned data.frame consists of the points of x\$data in the union of A and B.

Note that if A (resp. B) is empty then the returned data frame will be the points of x in B (resp. A).

#### Value

A CMBDataFrame containing the data in x restricted to the CMBWindow new.window

```
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
## Ensure you have a FITS file with correct path
## before uncommenting and running the rest of the example:
# cmbdat <- CMBDat("CMB_map_smica1024.fits", mmap = TRUE)
# class(cmbdat)
# cmbdat.win <- window(cmbdat, new.window = win1)
# class(cmbdat.win)</pre>
```

82 window.CMBDataFrame

window.CMBDataFrame Get a sub window from CMBDataFrame

Description

When new.window or in.pixels is unspecified this function returns the CMBWindow attribute of a CMBDataFrame. The return value is NULL if the window is full sky. When new.window is specified this function instead returns a new CMBDataFrame whose CMBWindow attribute is new.window

#### Usage

```
## S3 method for class 'CMBDataFrame'
window(x, new.window, intersect = TRUE, in.pixels,
  in.pixels.res = 0, ...)
```

#### **Arguments**

X	A CMBDataFrame.
new.window	Optionally specify a new window in which case a new CMBDataFrame is returned whose CMBWindow is new.window. new.window may also be a list (see details section and examples).
intersect	A boolean that determines the behaviour when new.window is a list containing BOTH regular type and "minus" type windows together (see details).
in.pixels	A vector of pixels at resolution in.pixels.res whose union contains the window(s) new.window entirely, or if new.window is unspecified then this whole pixel is returned.
in.pixels.res	An integer. Resolution (i.e., $j$ such that nside = 2 <sup>j</sup> ) at which the in.pixels parameter is specified
	Unused arguments.

#### **Details**

Windows that are tagged with set.minus (see CMBWindow) are treated differently from other windows.

If the argument new window is a list of CMBWindows, then interious of all windows whose win-Type does not include "minus" are united (let A be their union) and exteriors of all windows whose win-Type does include "minus" are intersected, (let B be their intersection). Then, provided that intersect = TRUE (the default), the returned CMBDataFrame will be the points of cmbdf in the the intersection of A and B. Otherwise, if intersect = FALSE, the returned CMBDataFrame consists of the points of x in the union of A and B.

Note that if A (resp. B) is empty then the returned CMBDataFrame will be the points of x in B (resp. A).

## Value

The window attribute of x or, if new.window/in.pixels is specified, a new CMBDataFrame.

window.CMBDataFrame 83

```
## Example 1: Create a new CMBDataFrame with a window
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian",</pre>
                      ordering = "nested")
win <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
cmbdf.win <- window(cmbdf, new.window = win)</pre>
plot(cmbdf.win)
window(cmbdf.win)
## Example 2: Change the window of an existing CMBDataFrame
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian", ordering = "nested")</pre>
window(cmbdf) <- win2 <- CMBWindow(theta = c(pi/6,pi/3,pi/3, pi/6),
                                   phi = c(0,0,pi/6,pi/6))
plot(cmbdf)
## Example 3: union of windows
## Create 2 windows
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
win2 <- CMBWindow(theta = c(2*pi/3, 3*pi/4, 3*pi/4, 2*pi/3),
                             phi = c(pi/4, pi/4, pi/3, pi/3))
plot(win1)
plot(win2)
## Create CMBDataFrame with points in the union of win1 and win2
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian", ordering = "nested")</pre>
cmbdf.win <- window(cmbdf, new.window = list(win1, win2), intersect = FALSE)</pre>
plot(cmbdf.win)
## Example 4: intersection of windows
## Create 2 windows
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
win2 <- CMBWindow(theta = c(pi/4,pi/3,pi/3, pi/4),
                  phi = c(pi/4,pi/4,pi/3,pi/3))
plot(win1)
plot(win2)
## Create CMBDataFrame with points in the intersection of win1 and win2
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian", ordering = "nested")</pre>
cmbdf.win1 <- window(cmbdf, new.window = win1)</pre>
cmbdf.win12 <- window(cmbdf.win1, new.window = win2)</pre>
plot(cmbdf.win12)
plot(win1)
plot(win2)
## Example 5: intersection of windows with "minus" type
## Create 2 windows with "minus" type
```

84 window.data.frame

```
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2),
                  set.minus =TRUE)
win2 <- CMBWindow(theta = c(pi/4, pi/3, pi/3, pi/4),
                  phi = c(pi/4, pi/4, pi/3, pi/3),
                   set.minus =TRUE)
plot(win1)
plot(win2)
## Create CMBDataFrame with points in the intersection of win1 and win2
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian", ordering = "nested")</pre>
cmbdf.win <- window(cmbdf, new.window = list(win1, win2))</pre>
plot(cmbdf.win)
## Example 6: intersection of windows with different types
##Create 2 windows, one with "minus" type
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
win2 <- CMBWindow(theta = c(pi/4, pi/3, pi/3, pi/4),
                  phi = c(pi/4, pi/4, pi/3, pi/3),
                   set.minus =TRUE)
plot(win1)
plot(win2)
## Create CMBDataFrame with points in the intersection of win1 and win2
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian", ordering = "nested")</pre>
cmbdf.win <- window(cmbdf, new.window = list(win1, win2), intersect = TRUE)</pre>
plot(cmbdf.win)
## Example 7: union of windows with different types
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2), set.minus =TRUE)
win2 <- CMBWindow(theta = c(pi/4,pi/3,pi/3,pi/4), phi = c(pi/4,pi/4,pi/3,pi/3))
plot(win1)
plot(win2)
## Create CMBDataFrame with points in the union of win1 and win2
cmbdf <- CMBDataFrame(nside = 64, coords = "cartesian", ordering = "nested")</pre>
cmbdf.win <- window(cmbdf, new.window = list(win1, win2), intersect = FALSE)</pre>
plot(cmbdf.win)
```

window.data.frame

Get a sub window from a data.frame

## Description

This function returns a data.frame containing the data in x restricted to the CMBWindow new.window

window.HPDataFrame 85

#### Usage

```
## S3 method for class 'data.frame'
window(x, new.window, intersect = TRUE, ...)
```

## **Arguments**

x A data.frame. Must have columns labelled x,y,z specifying cartesian coordi-

nates, or columns labelled theta, phi specifying colatitude and longitude respec-

tively.

new.window A single CMBWindow object or a list of them.

intersect A boolean that determines the behaviour when new. window is a list containing

BOTH regular type and "minus" type windows together (see details).

... Unused arguments.

#### **Details**

Windows that are tagged with set.minus (see CMBWindow) are treated differently from other windows.

If the argument is a list of CMBWindows, then interiors of all windows whose winType does not include "minus" are united (let A be their union) and exteriors of all windows whose winType does include "minus" are intersected, (let B be their intersection). Then, provided that intersect = TRUE (the default), the returned data.frame will be the points of x in the the intersection of A and B. Otherwise, if intersect = FALSE, the returned data.frame consists of the points of x in the union of A and B.

Note that if A (resp. B) is empty then the returned data.frame will be the points of x in B (resp. A).

# Value

A data.frame containing the data in x restricted to the CMBWindow new.window

```
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))

cmbdf <- CMBDataFrame(nside = 4)

df2 <- coords(cmbdf, new.coords = "cartesian")

df <- as.data.frame(df2[,1:3])

df

df.win <- window(df, new.window = win1)

df.win
```

86 window.HPDataFrame

#### **Description**

This function returns a HPDataFrame containing the data in hpdf restricted to the CMBWindow new.window. If the HPDataFrame has columns x,y,z or theta, phi then these will be used to determine locations with priority over the HEALPix indices in pix(hpdf) unless healpixCentered = TRUE is given. Note that if healpixCentered = TRUE then columns x,y,z or theta, phi will be discarded and replaced with pixel center locations.

#### Usage

```
## S3 method for class 'HPDataFrame'
window(x, new.window, intersect = TRUE,
   healpixCentered = FALSE, ...)
```

#### **Arguments**

x A HPDataFrame.

new.window Optional. A single CMBWindow object or a list of them.

intersect A boolean that determines the behaviour when new.window is a list containing

BOTH regular type and "minus" type windows together (see details).

healpixCentered

A boolean. If the HPDataFrame has columns x,y,z or theta, phi then these will be used to determine locations with priority over the HEALPix indices in pix(x) unless healpixCentered = TRUE is given. Note that if healpixCentered = TRUE then columns x,y,z or theta, phi will be discarded and replaced with pixel center

locations.

... Unused arguments.

#### **Details**

Windows that are tagged with set.minus (see CMBWindow) are treated differently from other windows.

If the argument is a list of CMBWindows, then interiors of all windows whose winType does not include "minus" are united (let A be their union) and exteriors of all windows whose winType does include "minus" are intersected, (let B be their intersection). Then, provided that intersect = TRUE (the default), the returned data.frame will be the points of df in the the intersection of A and B. Otherwise, if intersect = FALSE, the returned data.frame consists of the points of df in the union of A and B.

Note that if A (resp. B) is empty then the returned data frame will be the points of df in B (resp. A).

## Value

A HPDataFrame containing the data in x restricted to the CMBWindow new.window. Or, if new.window is unspecified, then the window attribute of x is returned instead (and may be NULL).

```
ns <- 16
hpdf <- HPDataFrame(nside = ns, I = 1:(12*ns^2))
hpdf
win1 <- CMBWindow(theta = c(0,pi/2,pi/2), phi = c(0,0,pi/2))
```

winType 87

```
plot(hpdf); plot(win1)
hpdf.win <- window(hpdf, new.window = win1)
plot(hpdf.win, col = "yellow", size = 4, add = TRUE)
attributes(hpdf.win)
window(hpdf.win)
hpdf.win</pre>
```

winType

Get/change winType

# **Description**

Get/change the winType (polygon or disk) of a CMBWindow. If new.type is missing then the winType of win is returned. Otherwise, a new window is returned with winType equal to new.type. If you want to change the winType of win directly, then use winType<-, see the examples below.

# Usage

```
winType(win, new.type)
```

#### **Arguments**

win a CMBWindow object or a list of such

new.type optionally specify a new type. Use this to change between "polygon" and "mi-

nus.polygon" or to change between "disc" and "minus.disc"

#### Value

If new.type is missing then the winType of win is returned. Otherwise a new window is returned with winType equal to new.type

88 winType

plot(cmbdf.win1)

# Index

ancestor, 4	geo2sph, 31
areCompatibleCMBDFs,4	geoArea, 32
as.CMBDataFrame,5	geoArea.CMBDataFrame, 32,32
assumedConvex, 6	geoArea.CMBWindow, 32, 33
assumedUniquePix, 6	geoArea.HPDataFrame, 32, 34
	geoDist, 34
baseNeighbours, 7	
alada di O	header, 35
cbind, 8	HPDataFrame, 7, 17, 34, 36, 40, 42, 43, 47, 49,
cbind.CMBDataFrame, 8	52–54, 58, 64, 69, 75, 85, 86
chi2.empirical, 8, 9	
chi2CMB, 8	ibp2p, 37
children, 9	is.CMBDat, 38
CMBDat, 10, 81	is.CMBDataFrame, 39
CMBDataFrame, 4, 7, 8, 11, 14, 18, 19, 26–30,	is.CMBWindow, 39
32, 35, 36, 42, 43, 47, 48, 52, 54, 56,	is.HPDataFrame,40
57, 60, 64, 66–70, 72, 74, 79, 82	1
CMBWindow, 6, 8, 9, 11, 12, 15, 27–29, 33, 42,	lines.variomodel.variofit, 40, 41
57, 66–68, 74, 76, 81, 82, 85–87	linesCMB,40
coords, 13	mayDict 41
coords.CMBDataFrame, 13, 14	maxDist, 41
coords. CMBWindow, 13, 15	maxWindowDist, 42
coords.data.frame, 13, 16	minDist, 43
coords.HPDataFrame, 13, 17	mmap, $10$
corrCMB, 18, 20, 55, 80	neighbours, 44
cov.spatial, 21, 22	nest2ring, 45
covCMB, 19, 19, 56, 80	nestSearch, 45
covmodelCMB, 21, 41, 61–63, 77, 78	nlm, 78
covPwSp, 22	nls, 78
curve, <i>41</i>	nside, 46
data Carrie 5 11 10 10 25 70	nside.CMBDataFrame, 46, 47, 47
data.frame, 5, 11, 18, 19, 35, 79	nside.HPDataFrame, 46, 47, 47
displayPixelBoundaries, 23	TISTUE. HE Data Falle, 40, 47, 47
displayPixels, 24	optim, 78
download.file, 26	ordering, 48
downloadCMBMap, 25	ordering.CMBDataFrame, 48, 48
downloadCMBPS, 26	ordering.HPDataFrame, 48, 49
entropy, 27, 28	or der ing. in bacar ame, 70, 19
entropyCMB, 27	p2bp, 50
exprob, 28	p2ibp, 50
extrCMB, 29	parent, 51
	pix, 51
fmf, 29	pix.CMBDataFrame, 51, 52
fRen, 30	pix.HPDataFrame, 51, 52
•	• / /-

90 INDEX

```
pix2coords, 53
pixelArea, 54
pixelWindow, 54
plot.CMBCorrelation, 55
plot.CMBCovariance, 56
plot.CMBDataFrame, 56
plot.CMBWindow, 57
\verb|plot.HPDataFrame|, 58|
plot.variogram, 55, 56, 59, 59, 60
plotAngDis, 60
plotcovmodelCMB, 61
plotvariogram, 62
practicalRange, 63
practicalRangeCMB, 63
print.CMBDataFrame, 64
print. HPDataFrame, 64
print.tbl_df,64
pwSpCorr, 65
qqnorm, 66, 67
\mathsf{qqnormWin}, 66, 66, 67
qqplot, 66, 67
qqplotWin, 66, 67, 67
qstat, 68
rbind, 69
rbind.CMBDataFrame, 69
rbind.data.frame, 69
rcosmo, 70, 70
rcosmo-package (rcosmo), 70
resolution, 70
ring2nest, 71
sampleCMB, 72
siblings, 72
sphericalHarmonics, 73
subWindow, 12
summary.CMBDataFrame, 74
summary.CMBWindow, 74
summary. HPDataFrame, 75
suppressMessages, 4
triangulate, 76
variofit, 77, 78
variofitCMB, 40, 41, 77
variog, 18–20, 55, 56, 60, 79, 80
variogramCMB, 19, 20, 60, 77, 79
window, 80
window.CMBDat, 81
window.CMBDataFrame, 80,82
window.data.frame, 80,84
window. HPDataFrame, 80, 85
winType, 87
```