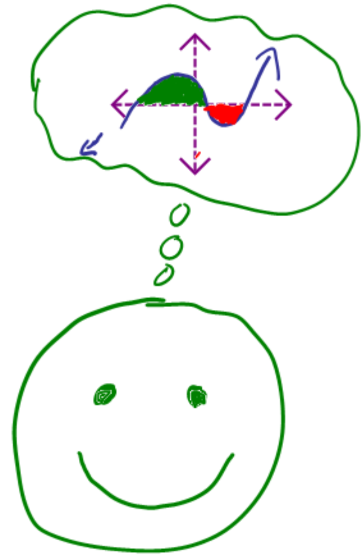
AP Calculus – Day 13



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Welcome to Olympiads!

Today: 1) Take up HW #12

2) Ch. 4 (Integral Calculus)

infinite integrals: e.g., in physics/statistics

Goal: Prepare for AP exams

HW: - Assigned every class,

- Take it up next class,

- Submit it on Classkick.

Submit it AFTER we take it up

Grading: P (pass) vs. I (incomplete) vs. Blank (no submission)

IMPORTANT: you should take down solutions when we take up problems!

This course: about 70 hours. Usual course: 110 hours.

Last day $\rightsquigarrow$ Partial Fractions

Idea: Convert rational functions into different forms which are easier to integrate

Order of Ideas:

1) Integrate polynomials
2) Substitution tricks
   $\int \sin^5(x)\cos(x)\,dx \xrightarrow{\text{u-sub}} \int(\text{polynomial})\,dx$
3) Integrate Rationals

(AP exam up to here)

4) More substitutions
   $\int \frac{\sin(\theta)}{\sin(\theta)+\cos(\theta)}\,d\theta \xrightarrow{\text{u-sub}} \int(\text{rational})\,dx$

(Email me if you a quick file about how to do this technique: "Weierstrass substitution")

D

Partial Fractions

If integrating $\int \frac{P(x)}{Q(x)}dx$ where the degree (largest exponent) of $P(x)$ is smaller than the degree of $Q(x)$ then factor the denominator as completely as possible and find the partial fraction decomposition of the rational expression. Integrate the partial fraction decomposition (P.F.D.). For each factor in the denominator we get term(s) in the decomposition according to the following table.

Factor in $Q(x)$	Term in P.F.D	Factor in $Q(x)$	Term in P.F.D
$ax+b$	$\frac{A}{ax+b}$	$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$	$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$

NOTE: On the actual AP exam, usually only degree 2 shows up.
(maaaaybe degree 3 if it's a simple one)

Big Idea:

# of unknowns to find = degree of the denominator

It's a system of unknowns: n equations in n unknowns. By hand: we can handle n=2

Instructions:

- **Show your full and polished work.**
- Physical submission: Write your work on letter/A4 size papers **stapled at top left corner**.
- All answers should be in **exact form**. For example, write π instead of 3.14. (AP Standard)
- All numerical answers, if needed, are rounded to 3 decimal places. (AP Standard)

Multiple-Choice Questions

Multiple-Choice Questions

Steps to solve:
1) Factor the denominator
2) Write the partial fraction format
3) Solve for the co-efficients (A and B values)
4) Solve your easy integral

1. $\int \frac{1}{x^2 - 6x + 8}\,dx = \int \frac{1}{(x-4)(x-2)}\,dx$

A. $\boxed{\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C}$

B. $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

C. $\frac{1}{2}\ln|(x-2)(x-4)| + C$

D. $\frac{1}{2}\ln|(x-4)(x+2)| + C$

E. $\ln|(x-2)(x-4)| + C$

$$\frac{1}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2} = \frac{A(x-2) + B(x-4)}{\ldots}$$

Numerators: $0x + 1 = (A+B)x + (-2A-4B)$

① $0 = A + B$

② $1 = -2A - 4B$

$\Rightarrow A = \frac{1}{2},\ B = \frac{-1}{2}$

$$\Rightarrow \int \frac{dx}{(x-4)(x-2)} = \int \frac{1/2}{x-4}\,dx + \int \frac{-1/2}{x-2}\,dx$$

$$= \frac{1}{2}\ln|x-4| - \frac{1}{2}\ln|x-2|$$

$$\boxed{= \frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C}$$ A

2. $\displaystyle\int_2^3 \frac{3}{(x-1)(x+2)}\,dx =$

A. $-\frac{33}{20}$ B. $-\frac{9}{20}$ C. $\ln\left(\frac{5}{2}\right)$ $\boxed{\text{D. } \ln\left(\frac{8}{5}\right)}$ E. $\ln\left(\frac{2}{5}\right)$

PFD: $\displaystyle\frac{3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x-2}$

limit method:
Multiply by
(x-1) then take
limit as x-->1

$$\lim_{x\to 1} \frac{3}{x+2} = \lim_{x\to 1}\left[A + \cancel{\frac{(x-1)B}{x-2}}^{\,0}\right]$$

$$\frac{3}{1+2} = A \qquad \therefore A = 1$$

Same idea: $B = -1$

$$\int_2^3 \frac{3}{(x-1)(x+2)}\,dx = \int_2^3 \frac{1}{x-1}\,dx - \int_2^3 \frac{1}{x+2}\,dx$$

$= [\ln(2) - \ln(1)] - [\ln(5) - \ln(4)]$

$$\boxed{= \ln\left(\frac{8}{5}\right)}$$

(D)

3. $\displaystyle\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is $\rightarrow (x+1)(x+2)$

A. $-\ln 8$ B. $\ln\left(\frac{27}{2}\right)$ $\boxed{\text{C. } \ln(18)}$ D. $\ln(288)$ E. divergent

PFD: $\dfrac{5x+8}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$

Limit Method: To get $A \rightsquigarrow$ Mult. by $(x+1)$ & take $\lim\limits_{x \to -1}$

$$A = \lim_{x \to -1} \frac{5x+8}{x+2} = \frac{5(-1)+8}{(-1)+2} = 3$$

$$B = \lim_{x \to -2} \frac{5x+8}{x+1} = \frac{5(-2)+8}{(-2)+1} = 2$$

$\therefore$ Ans $= \displaystyle\int_0^1 \frac{3}{x+1}\,dx + \int_0^1 \frac{2}{x+2}\,dx$

$= 3\ln(2) + 2(\ln(3) - \ln(2))$

$$\boxed{= \ln(18)}$$

(C)

Full-Solution Questions

REMEMBER: Our usual technique to solve a product integral is IBP (integral by parts)

1. Evaluate the following integrals:

(a) $\int x\cos^2 x\,\mathrm{d}x$

(b) $\int \cos^2 x \sin(2x)\,\mathrm{d}x$

(c) $\int \tan^2 x\,\mathrm{d}x$

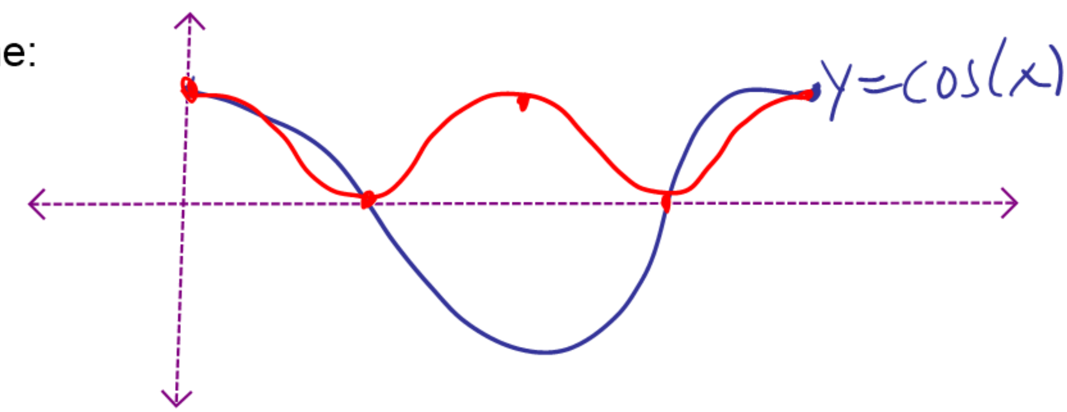
(d) $\int x\sec x\tan x\,\mathrm{d}x$

a)

Clue for sin or cos squared: Use the trig identity that lowers the degree by 1.

$$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$$

After squaring cosine:
-Amplitude of 1/2
-Shifted up by 1/2
-Period is halved.



(Thinking geometrically can really minimize the amount you need to "memorize")

$$\text{Int} = \int \frac{1}{2}x(1+\cos(2x))\,dx$$

$$= \int \frac{1}{2}x\,dx + \underbrace{\int \frac{1}{2}x\cos(2x)\,dx}$$

$f = x \qquad g' = \cos(2x)$

$f' = 1 \qquad g = \frac{1}{2}\sin(2x)$

$$\boxed{I = \frac{1}{4}x^2 + \frac{1}{2}x\sin(2x) + \frac{1}{8}\cos(2x) + C}$$

(b) $\int \cos^2 x \boxed{\sin(2x)}\, dx$ $\sin(2x) = 2\cos(x)\sin(x)$

$$= 2\int \underbrace{\cos^3(x)}_{=u^3} \underbrace{\sin(x)dx}_{=-du}$$

$u = \cos(x)$

$du = -\sin(x)dx$

$$= -2\int u^3 du$$

$$= -2\left[\frac{1}{4}u^4\right]$$

$$\boxed{I = \frac{-1}{2}\cos^4(x) + C}$$

(c) $\int \tan^2 x\, dx = \int [\sec^2(x) - 1]\, dx$ (Pythagorean identity!)

$$\boxed{= \tan(x) - x + C}$$

(d) $\int \underbrace{x}_{=f} \underbrace{\sec x \tan x}_{=g'} \, dx$

$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$

$f = x \qquad g' = \sec(x)\tan(x)$

$f' = 1 \qquad g = \sec(x)$

$I = x\sec(x) - \int \sec(x)\,dx$

This integral is kind of weird, so you may just want to memorize it...

$$\boxed{I = x \cdot \sec(x) - \ln|\sec(x) + \tan(x)| + C}$$

2. A particle moves on a ~~straight line~~ circle with angular velocity function $v(t) = \sin(\omega t)\cos^2(\omega t)$. Find its position function $s = f(t)$ if $f(0) = 0$.

$s(t) = \int v(t)\,dt \quad \& \quad s(0) = 0$

This is called the "initial condition" which helps us solve for the +C term

$s(t) = \int \sin(\omega t)\cos^2(\omega t)\,dt$

$u = \cos(\omega t)$

$\frac{-1}{\omega} du = \sin(\omega t)\,dt$

$= \frac{-1}{\omega} \int u^2\,du$

$= \frac{-1}{\omega} \frac{u^3}{3}$

$$= \frac{-1}{3\omega}\cos^3(\omega t) + C$$

$t=0$: $0 = s(0) = \frac{-1}{3\omega}(1) + C \quad \therefore \quad C = \frac{1}{3\omega}$

$$\Rightarrow \boxed{s(t) = \frac{1-\cos^3(\omega t)}{3\omega}}$$

3. Evaluate the following integrals:

(a) $\displaystyle\int \frac{x-9}{x^2+3x-10}\,dx$

$$= \int\left[\frac{2}{x+5} + \frac{-1}{x-2}\right]dx$$

$$\boxed{= \ln\left|\frac{(x+5)^2}{x-2}\right| + C}$$

(b) $\displaystyle\int \frac{1}{t^2+3t-4}\,dt$

$$= \int\left[\frac{-1/5}{t+4} + \frac{1/5}{t-1}\right]dt$$

$$\boxed{= \frac{1}{5}\ln\left|\frac{t-1}{t+4}\right| + C}$$

(Nothing different from the solution to the multiple choice problems)

Optional Questions

1. Evaluate the following integrals

(a) $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx$

(b) $\displaystyle\int \cos^2 x \tan^3 x\,dx$

(c) $\displaystyle\int_0^{\pi/3} \tan^5 x \sec^4 x\,dx$

(d) $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2}\,dx$

a) Let $u = \sqrt{x} \rightsquigarrow 2du = \frac{dx}{2\sqrt{x}}$

$$\therefore I = 2\int \sin^3(u)\,du$$

$$= 2\int \left[1 - \cos^2(u)\right] \sin(u)\,du$$

Let $t = \cos(u) \rightsquigarrow -dt = \sin(u)$

$$I = -2\int (1 - t^2)\,dt$$

t-integral is solved: $= 2\left(\frac{t^3}{3} - t\right) + C$

turn t back into u: $= 2\left(\frac{\cos^3(u)}{3} - \cos(u)\right) + C$

turn u back into x: $\boxed{= \frac{2}{3}\cos^3(\sqrt{x}) - 2\cos(\sqrt{x}) + C}$

b) $\int \cos^2 x \tan^3 x\,dx$

From our formulas last day: We want (sin and cos) together or (tan and sec) together in order to use our subtitution method.

$$= \int \frac{\sin^3(x)}{\cos(x)}\,dx$$

$$= \int \left[\frac{1 - \cos^2(x)}{\cos(x)}\right] \sin(x)\,dx$$

$u = \cos(x)$

$-du = \sin(x)\,dx$

$$= -\int \frac{1-u^2}{u} du$$

$$\boxed{I = \frac{1}{2}\cos^2(x) - \ln|\cos(x)| + C}$$

c) $\displaystyle\int_0^{\pi/3} \tan^5 x \sec^4 x dx$

$$= \int_0^{\pi/3} \underbrace{\tan^5(x)}_{u^5} \underbrace{\sec^2(x)}_{u^2+1} \underbrace{\sec^2(x) dx}_{du}$$

$u = \tan(\theta)$

$$= \int_0^{\sqrt{3}} (u^7 + u^5) du$$

$$\boxed{= 117/8}$$

d) $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2} dx = \int_0^1 \left[\frac{3}{x+2} - \frac{2}{x+1}\right] dx$

$$\boxed{= \ln\left(\frac{27}{32}\right)}$$

Break until 8:15

For reference:

Find integration of $\sec(x)$**.**

Reframe the integral:

$$\int \sec\ x\ dx = \int \sec\ x\left(\frac{\sec\ x + \tan\ x}{\sec\ x + \tan\ x}\right)\ dx$$

$$\int \sec\ x\ dx = \int \frac{\sec^2 x + \sec\ x\ \tan\ x}{\sec\ x + \tan\ x}\ dx \ldots (1)$$

Take, $\sec\ x + \tan\ x = t$

$$\Rightarrow (\sec\ x\ \tan\ x + \sec^2 x)\ dx = dt.$$

To Find Put the above values in the equation (1):

$$\int \sec\ x\ dx = \int \frac{dt}{t}$$

$$\int \sec\ x\ dx = \ln|\ t| + C\ \left\{\because \int \frac{dt}{t} = \ln\ |t|\right\}$$

$$\int sec\ x\ dx = \ln\ |(sec\ x\ + \tan\ x)| + C$$

$\{\text{Put}\ t = \sec\ x + \tan\ x\}$

Hence, the required integral is

$\int sec\ x\ dx = \ln|(sec\ x\ + \tan\ x)| + C$ **where** C **is a constant of integration .**

§ Improper Integrals

Two conditions for FTC: $\int_a^b f(x)\,dx = F(b) - F(a)$

1) $f(x)$ is conts on $[a,b]$
2) $-\infty < a < b < \infty$ (no infinitely long intervals)

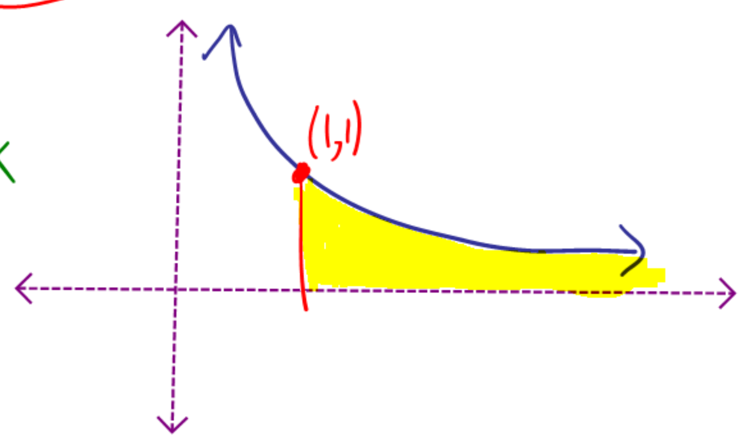
Improper Integrals $\rightsquigarrow$ What do we do if these conditions are NOT satisfied?

Key Idea: Apply FTC using endpoints where it does work... then take a limit of those endpoints.

$\hookrightarrow$ FTC + limits = Improper Integrals

Type I) Infinite Integrals

E.g., $\int_1^\infty \frac{1}{x^2}\,dx$

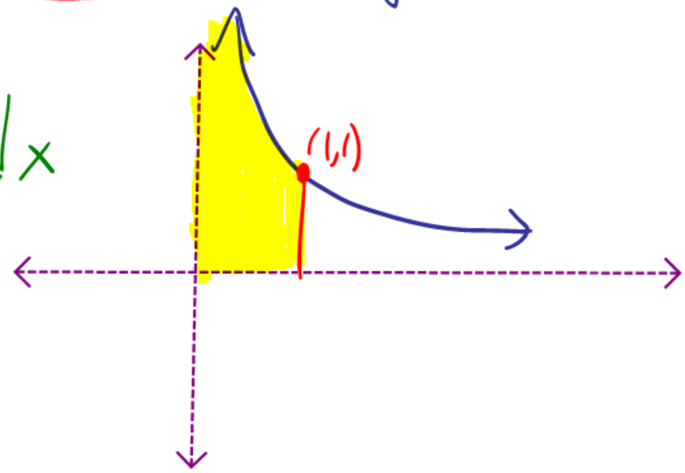


In terms of length: this shape is infintely long...
In terms of area: maybe it could still be finite?

Other Type I: $\int_{-\infty}^{0}$ or $\int_{-\infty}^{\infty}$

Type II) Discontinuous Integrals

E.g., $\int_0^1 \frac{1}{x^2}dx$



Notice: the left endpoint of the interval is a vertical asymptote, so it's a discontinuity.

Try FTC: $\int_0^1 \frac{1}{x^2}dx = \frac{-1}{x}\Big]_0^1 = \frac{-1}{1} - \frac{-1}{0}$

Divide by zero problem :(

Summarize:

Type I $\rightsquigarrow$ Horizontal Asymptotes

Type II $\rightsquigarrow$ Vertical Asymptotes/holes/jumps

Today: Lets focus on Type 1 Improper Integrals (aka infinite integrals)

$\boxed{\text{Def'n}^{(\#1)}}$ Let $f(x)$ be cont's on $[a,\infty)$

Then: $\int_a^\infty f(x)dx = \lim_{b\to\infty}\left[\int_a^b f(x)dx\right]$

This limit 'a' is fixed ahead of time

Here, 'b' is treated as a variable. This integral works with FTC.

Note: If the limit exists: Convergent integral (answer)
If the limit DNE: divergent integral (no answer)

Def'n (#2) Let $f(x)$ be cont's on $(-\infty, b]$

The upper limit is fixed this time

Then: $$\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \left[\int_{a}^{b} f(x)\,dx \right]$$

'a' is treated as a variable here, but FTC works

For these infinite integrals:
1) We first use FTC on a finite, continuous interval
2) THEN take a limit to let your endpoints to go infinity.

Def'n (#3) Let $f(x)$ be cont's on $\mathbb{R} = (-\infty, \infty)$
Fix some $c \in \mathbb{R}$

Then: $$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = \underbrace{\int_{-\infty}^{c} f(x)\,dx}_{\text{(Def'n \#2)}} + \underbrace{\int_{c}^{\infty} f(x)\,dx}_{\text{(Def'n \#1)}}}$$

If BOTH of the integrals on the right are convergent, then our overall integral is convergent. But it requires BOTH to converge.

WARNING: $\int_{-\infty}^{\infty} \neq \lim_{c\to\infty}\int_{-c}^{c}$

You can NOT combine the two limits going in opposite directions together. Both direction limits must exist separately!!

E.g., $I = \int_1^{\infty} \frac{1}{x^2}dx$

First couple steps: Forget about infinity and solve like before.

- $\int \frac{1}{x^2}dx = \int x^{-2}dx = \frac{1}{-1}x^{-1} = \frac{-1}{x}$
- Let $I_b = \int_1^b \frac{1}{x^2}dx = \frac{-1}{x}\Big]_1^b = \frac{-1}{b} - \frac{-1}{1} = 1 - \frac{1}{b}$

Notice: For this integral, we were able to FTC because it was continuous over a finite interval.

$\Rightarrow I = \lim_{b\to\infty} I_b = \lim_{b\to\infty}\left[1 - \cancel{\frac{1}{b}}^{0}\right] = 1$

$\therefore$ $\boxed{\int_1^{\infty} \frac{1}{x^2}dx = 1}$

Same area as a 1x1 square, even though it's an infinitely long and curved region.

In other words: Because this limit exists, the integral is convergent.

E.g., $I = \int_1^\infty \frac{1}{x}\,dx$

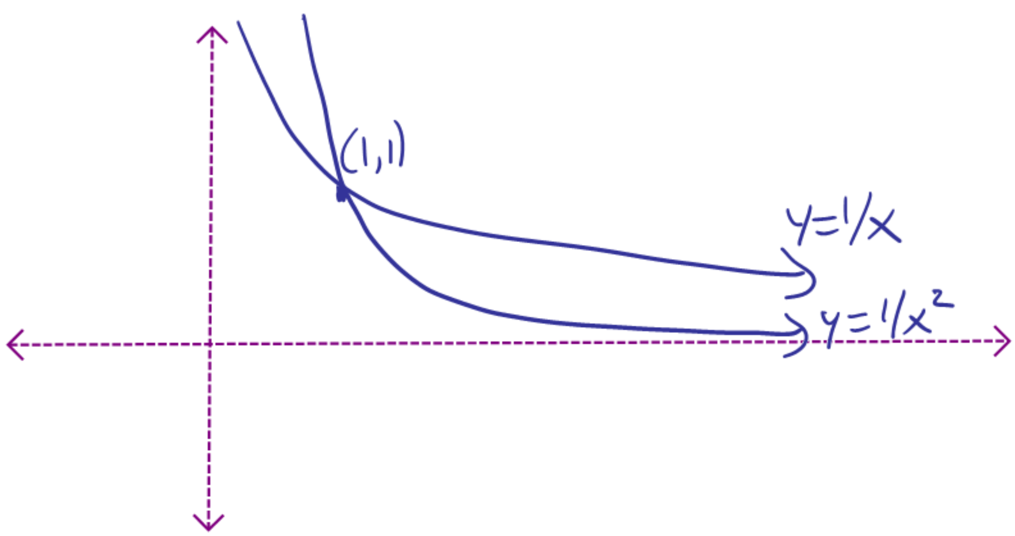
- $\int \frac{1}{x}\,dx = \ln|x|$
- Let $I_b = \int_1^b \frac{1}{x}\,dx = \ln|x|\Big]_1^b = \ln(b) - \ln(1)$

$\therefore\ I_b = \ln(b)$

$$\Rightarrow\ I = \lim_{b\to\infty} I_b = \lim_{b\to\infty} \ln(b) = +\infty$$

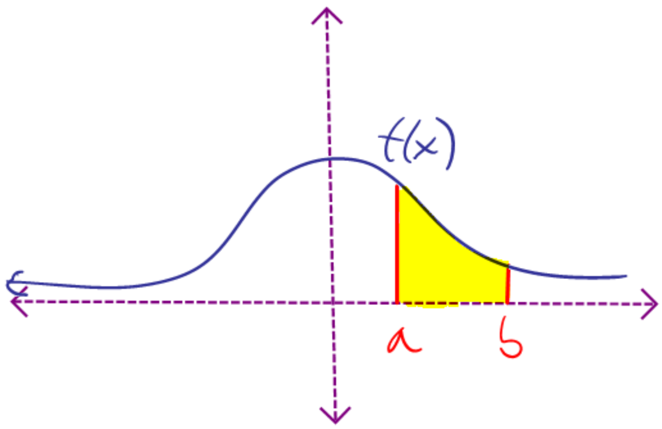
$$\therefore\ \boxed{\int_1^\infty \frac{1}{x}\,dx = +\infty}$$

In other words: This integral is divergent.



1/x^2 decays much faster than 1/x ... Enough that the area under the graph converges rather than diverges.

Application: Continuous Probability



Here: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

This is called a "bell curve"
or "normal distribution"

Random variable: a random real number is outputted (infintely many options)

$$\text{Prob}(a < x < b) = \int_a^b f(x)\,dx$$

Two conditions for $f(x)$ to be a prob. distribution

1. $0 \leq f(x) \leq 1$

2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

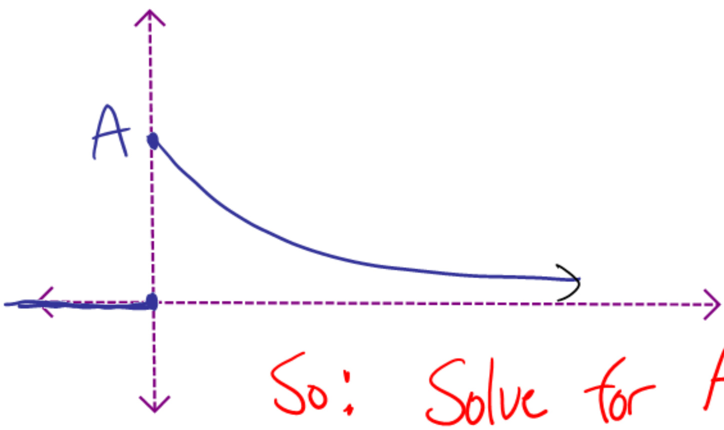
Discrete Prob. $\longleftrightarrow$ Add
Continuous Prob. $\longleftrightarrow$ Integrals

E.g., Exponential Distribution

Fix $k \in \mathbb{R}$ (parameter)

Let $f(x) = A \cdot e^{-kx}$ $\quad (x \geq 0)$

E.g., x = how long an atom lives before decaying



AP level problem: Determine the value of A so this is a probability distribution.

So: Solve for A such that $\int_0^\infty f(x)\,dx = 1$

$$1 = \int_0^\infty Ae^{-kx} = \lim_{b\to\infty}\Big[\underbrace{\int_0^b Ae^{-kx}\,dx}_{\text{FTC}}\Big]$$

$$1 = \lim_{b\to\infty}\left[A \cdot \frac{-1}{k} e^{-kx}\right]_0^b$$

$$1 = \lim_{b\to\infty}\left[A \cdot \frac{-1}{k}\left(\overset{0}{\cancel{e^{-kb}}} - \overset{1}{\cancel{e^{-k(0)}}}\right)\right]$$

$$1 = \left[A\left(\frac{-1}{k}\right)(-1)\right]$$

$$1 = \frac{A}{k} \implies A = k$$

$\int_{0}^{\infty}$

$$\boxed{\text{Exp. Distr.}: \quad f(x) = ke^{-kx}}$$

k = mean of this exponential distribution

AP problem: Given this distribution, what is the median?

Median 'M' means that:

$$\boxed{\int_{-\infty}^{M} f(x) = \frac{1}{2}}$$

Sample problem: For the exponential distribution above, what is the median in terms of the parameter 'k'?

Physics Example

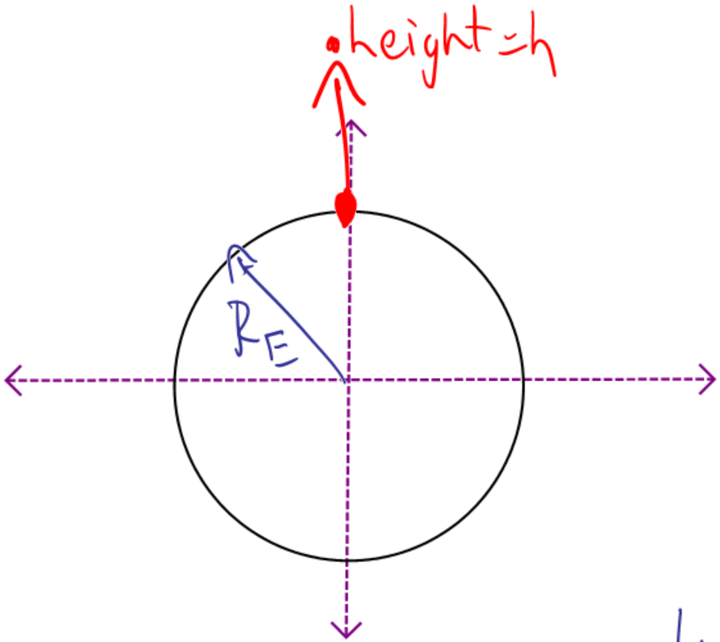
In physics: Work is when you apply a Force on an object over a displacement

(W) (F) (Δx)

Constant Force: $W = \vec{F} \cdot (\vec{\Delta x})$

General Force: $W = \int_a^b F(x)\,dx$

Newton's Gravity Eq'n: $F_G = \frac{G \cdot Mm}{r^2}$



Question: How much work do you have to do to a projectile to launch it up into space?

$$W = \int_{R_E}^{R_E+h} F_G(r)\,dr$$

$$W = \int_{R_E}^{R_E+h} \frac{GMm}{r^2}\,dr$$

$$W = GMm\left[\frac{1}{R_E} - \frac{1}{R_E+h}\right]$$

How much work does it take to escape gravity altogether?

$$\boxed{W_{max} = \int_{R_E}^{\infty} F_G(r)\,dr = \frac{GMm}{R_E}}$$

This is the "Potential Energy" of the Earth-Mass system. If you give an object this much kinetic energy, it will never come back down. (The actual physics formula has a negative sign in it)

Application of this formula: How much speed does the ball have to have in order to never return back down to Earth? This is called the Escape Velocity of the Earth.

"p"-series Integrals (p = power)

Q: Let $I(p) = \int_1^\infty \frac{1}{x^p}\,dx$

For what 'p' does this converge/diverge?

Later on in the course: We'll need these integrals to solve Infinite Series/Taylor Series problems :)

Today's examples: $p=2 \rightsquigarrow I(2)=1$

$p=1 \rightsquigarrow I(1)=+\infty$

Here: Just assume $p \neq 1$

Step 1) Find the anti-derivative, ignoring the infinity and endpoints entirely.

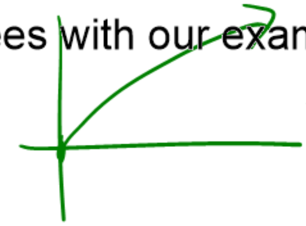
$$\int \frac{1}{x^p}\,dx = \int x^{-p}\,dx = \frac{x^{-p+1}}{-p+1} = \frac{x^{1-p}}{1-p}$$

Step 2) Set a finite upper bound to the interval 'b' and solve the integral using FTC.

For $b \geq 1$, $I_b(p) = \int_1^b \frac{1}{x^p}\,dx = \frac{1}{1-p}\left[\frac{1}{b^{p-1}} - 1\right]$

This agrees with our example above:

$\frac{1}{x^{-1/2}}$:



$\lim_{x\to\infty} = +\infty$

$\frac{1}{x^{1/2}}$:

$\lim_{x\to\infty} = 0$

- $\lim_{b\to\infty} \frac{1}{b^{p-1}} = +\infty$ if $p-1<0$ a.k.a. $p<1$
- $\lim_{b\to\infty} \frac{1}{b^{p-1}} = 0$ if $p-1>0$ aka if $p>1$

$\Rightarrow \lim_{b\to\infty}$: $p-1 \le 0 \Rightarrow$ divergence

$p-1>0 \Rightarrow$ convergence

Step 3) Use that limit to write down the integrals.

If $p>1$: $\lim_{b\to\infty} I_b(p) = \frac{1}{1-p}[0-1]$

$\boxed{\text{Thm}}$ For $I(p) = \int_1^\infty \frac{1}{x^p}dx$, we have:

(this is a dot product equation)

Rule of thumb for physics: Anywhere you see a 'delta' in physics class, change it to a 'd' and you get a calculus formula.

① $\boxed{p \le 1: \int_1^\infty \frac{1}{x^p}dx \text{ diverges}}$

Work done by the force F(x) when you move an object from x=a over to x=b

② $\boxed{p>1: \int_1^\infty \frac{1}{x^p}dx = \frac{1}{p-1}}$

G = some constant
M = mass of Earth
m = your mass
r = radius of separation

$$p=2: \quad \int_1^\infty \frac{1}{x^2}\,dx = \frac{1}{2-1} = \frac{1}{1} = 1 \checkmark$$