Project 2

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Github repository: *https://github.com/Fslippe/FYS4150/tree/main/project2*

We are working with:

- A horizontal beam of length L.
- We let $u(x)$ be the vertical displacement of the beam at horizontal position x, with $x \in [0, L]$.
- A force F is applied at the endpoint ($x = L$), directed into the beam, i.e. towards $x = 0$.
- The beam is fastened with pin endpoints, meaning that $u(0) = 0$ and $u(L) = 0$, but the endpoints are allowed to rotate ($u'(x) \neq 0$).

Second order differential equation describing our buckling beam situation:

$$\gamma \frac{d^2u(x)}{dx^2} = -Fu(x) \tag{1}$$

Troughout this project we will be working with the scaled equation:

$$\frac{d^2u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \tag{2}$$

Where $\hat{x} \equiv x/L$ is a dimensionless variable, $\hat{x} \in [0, 1]$ and $\lambda = \frac{FL^2}{\gamma}$.

PROBLEM 1

Using the defenintion $\hat{x} \equiv x/L$ to show that Eq. 1 can be written as Eq. 2.

$$\begin{aligned}
\gamma \frac{d^2u(x)}{dx^2} &= -Fu(x) \\
\frac{d^2u(x)}{dx^2} &= -\frac{F}{\gamma}u(x)
\end{aligned}$$

Multiplying both sides by L^2.

$$\begin{aligned}
\frac{d^2u(x)L^2}{dx^2} &= -\frac{FL^2}{\gamma}u(x) \\
\frac{d^2u(x)}{d\frac{x^2}{L^2}} &= -\frac{FL^2}{\gamma}u(x)
\end{aligned}$$

Using that $\hat{x} \equiv x/L$ so that $\hat{x}^2 \equiv x^2/L^2$.

$$\frac{d^2u(x)}{d\hat{x}} = -\frac{FL^2}{\gamma}u(x)$$

We also know that $x = \hat{x}L$, so $u(x) = u(\hat{x}L) = u(\hat{x}) + u(L) = u(\hat{x})$. Since $u(L) = 0$. Finally, using $\lambda = \frac{FL^2}{\gamma}$ gives us the dimensionless Eq. 2:

$$\frac{d^2u(\hat{x})}{d\hat{x}} = -\lambda u(\hat{x})$$

PROBLEM 5

We will now look at how many similarity transformations we need before we reach a result where all non-diagonal matrix elements are close to zero.

PROBLEM 6