

## Project 2

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(Dated: September 15, 2022)

*Github repository:* <https://github.com/Fslippe/FYS4150/tree/main/project2>

We are working with:

- A horizontal beam of length  $L$ .
- We let  $u(x)$  be the vertical displacement of the beam at horizontal position  $x$ , with  $x \in [0, L]$ .
- A force  $F$  is applied at the endpoint ( $x = L$ ), directed into the beam, i.e. towards  $x = 0$ .
- The beam is fastened with pin endpoints, meaning that  $u(0) = 0$  and  $u(L) = 0$ , but the endpoints are allowed to rotate ( $u'(x) \neq 0$ ).

Second order differential equation describing our buckling beam situation:

$$\gamma \frac{d^2 u(x)}{dx^2} = -Fu(x) \quad (1)$$

Troughout this project we will be working with the scaled equation:

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \quad (2)$$

Where  $\hat{x} \equiv x/L$  is a dimensionless variable,  $\hat{x} \in [0, 1]$  and  $\lambda = \frac{FL^2}{\gamma}$ .

### PROBLEM 1

Using the defenintion  $\hat{x} \equiv x/L$  to show that Eq. 1 can be written as Eq. 2.

$$\begin{aligned} \gamma \frac{d^2 u(x)}{dx^2} &= -Fu(x) \\ \frac{d^2 u(x)}{dx^2} &= -\frac{F}{\gamma} u(x) \end{aligned}$$

Multiplying both sides by  $L^2$ .

$$\begin{aligned} \frac{d^2 u(x)L^2}{dx^2} &= -\frac{FL^2}{\gamma} u(x) \\ \frac{d^2 u(x)}{d\frac{x^2}{L^2}} &= -\frac{FL^2}{\gamma} u(x) \end{aligned}$$

Using that  $\hat{x} \equiv x/L$  so that  $\hat{x}^2 \equiv x^2/L^2$ .

$$\frac{d^2 u(x)}{d\hat{x}} = -\frac{FL^2}{\gamma} u(x)$$

We also know that  $x = \hat{x}L$ , so  $u(x) = u(\hat{x}L) = u(\hat{x}) + u(L) = u(\hat{x})$ . Since  $u(L) = 0$ . Finally, using  $\lambda = \frac{FL^2}{\gamma}$  gives us the dimensionless Eq. 2:

$$\frac{d^2 u(\hat{x})}{d\hat{x}} = -\lambda u(\hat{x})$$

**PROBLEM 2**

**PROBLEM 3**

**PROBLEM 4**

**PROBLEM 5**

**PROBLEM 6**