Project 2

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Github repository: https://github.com/Fslippe/FYS4150/tree/main/project2

We are working with:

- A horizontal beam of length L.
- We let $u(x)$ be the vertical displacement of the beam at horizontal position x, with $x \in [0, L]$.
- A force F is applied at the endpoint ($x = L$), directed into the beam, i.e. towards $x = 0$.
- The beam is fastened with pin endpoints, meaning that $u(0) = 0$ and $u(L) = 0$, but the endpoints are allowed to rotate ($u'(x) \neq 0$).

Second order differential equation describing our buckling beam situation:

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x) \tag{1}$$

Troughout this project we will be working with the scaled equation:

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \tag{2}$$

Where $\hat{x} \equiv x/L$ is a dimensionless variable, $\hat{x} \in [0, 1]$ and $\lambda = \frac{FL^2}{\gamma}$.

PROBLEM 1

Using the defenintion $\hat{x} \equiv x/L$ to show that Eq. 1 can be written as Eq. 2.

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x)$$
$$\frac{d^2 u(x)}{dx^2} = -\frac{F}{\gamma} u(x)$$

Multiplying both sides by L^2.

$$\frac{d^2 u(x) L^2}{dx^2} = -\frac{FL^2}{\gamma} u(x)$$
$$\frac{d^2 u(x)}{d\frac{x^2}{L^2}} = -\frac{FL^2}{\gamma} u(x)$$

Using that $\hat{x} \equiv x/L$ so that $\hat{x}^2 \equiv x^2/L^2$.

$$\frac{d^2 u(x)}{d\hat{x}} = -\frac{FL^2}{\gamma} u(x)$$

We also know that $x = \hat{x}L$, so $u(x) = u(\hat{x}L) = u(\hat{x}) + u(L) = u(\hat{x})$. Since $u(L) = 0$. Finally, using $\lambda = \frac{FL^2}{\gamma}$ gives us the dimensionless Eq. 2:

$$\frac{d^2 u(\hat{x})}{d\hat{x}} = -\lambda u(\hat{x})$$

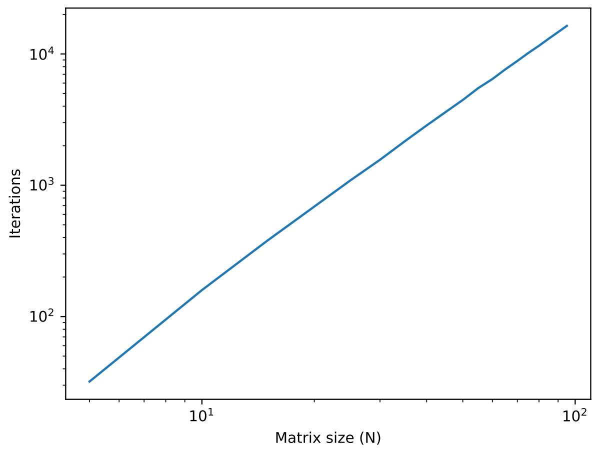
PROBLEM 2

PROBLEM 3

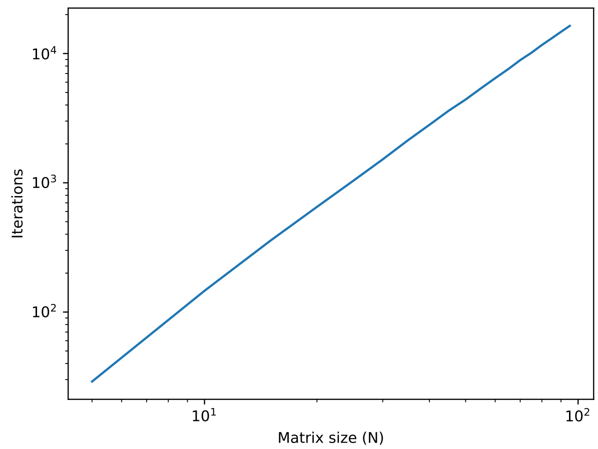
PROBLEM 4

PROBLEM 5

a) By running the jacobi algorithm for different choices of N we can see how the number of jacobi rotation scales with increasing matrix sizes. This is done by reducing all elements to less than $\epsilon = 10^{-9}$.



(a) Iterations for increasing tridiagonal matrix sizes. Both on a logarithmic scale



(b) Iterations for increasing dense matrix sizes. Both on a logarithmic scale

PROBLEM 6