Project 2

Alessio Canclini, Filip von der Lippe
(Dated: September 26, 2022)

Github repository: $https://github.com/Fslippe/FYS4150/tree/main/project2$

We are working with:

- A horizontal beam of length L.

- We let $u(x)$ be the vertical displacement of the beam at horizontal position x, with $x \in [0, L]$.

- A force F is applied at the endpoint $(x = L)$, directed into the beam, i.e. towards $x = 0$.

- The beam is fastened with pin endpoints, meaning that $u(0) = 0$ and $u(L) = 0$, but the endpoints are allowed to rotate $(u'(x) \neq 0)$.

Second order differential equation describing our buckling beam situation:

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x) \tag{1}$$

Troughout this project we will be working with the scaled equation:

$$\frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}) \tag{2}$$

Where $\hat{x} \equiv x/L$ is a dimensionless variable, $\hat{x} \in [0, 1]$ and $\lambda = \frac{FL^2}{\gamma}$.

PROBLEM 1

Using the definition $\hat{x} \equiv x/L$ to show that Eq. 1 can be written as Eq. 2.

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x)$$
$$\frac{d^2 u(x)}{dx^2} = -\frac{F}{\gamma} u(x)$$

Multiplying both sides by L^2.

$$\frac{d^2 u(x) L^2}{dx^2} = -\frac{FL^2}{\gamma} u(x)$$
$$\frac{d^2 u(x)}{d\frac{x^2}{L^2}} = -\frac{FL^2}{\gamma} u(x)$$

Using that $\hat{x} \equiv x/L$ so that $\hat{x}^2 \equiv x^2/L^2$.

$$\frac{d^2 u(x)}{d\hat{x}} = -\frac{FL^2}{\gamma} u(x)$$

We also know that $x = \hat{x}L$, so $u(x) = u(\hat{x}L) = u(\hat{x}) + u(L) = u(\hat{x})$. Since $u(L) = 0$. Finally, using $\lambda = \frac{FL^2}{\gamma}$ gives us the dimensionless Eq. 2:

$$\frac{d^2 u(\hat{x})}{d\hat{x}} = -\lambda u(\hat{x})$$

PROBLEM 5

We will now look at how many similarity transformations we need before we reach a result where all non-diagonal matrix elements are close to zero.

Running our program for different choices of N gives us the following scaling data in figure 1. We obeserve that the number of required transformations T are square proportional to the matrix size N. So $T = N^2$.

In figure 1 we see that our program shows the close to the same scaling behavior for both tridiagonal and dense matrixes. This is as expected, since our tridiagonal matrix is is no longer tridiagonal after the first jacobi rotation of our algorithm. Therfore the tridiagonal matrix is treated just like a dens matrix after a few rotations in this case.
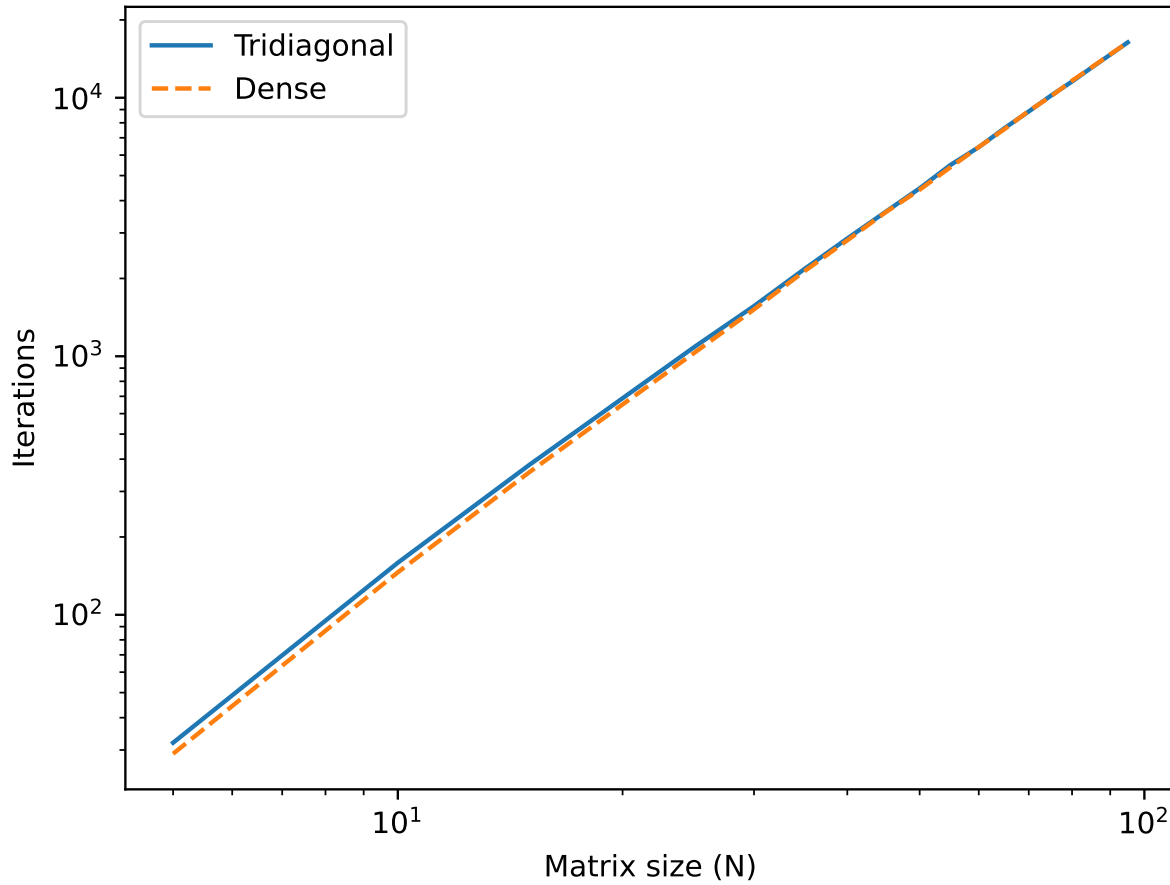

Figure 1: Transformation scaling data for a matrixes on a Log scale.

PROBLEM 6

a) Solving the equation $\mathbf{A}\tilde{\mathbf{v}} = \lambda\tilde{\mathbf{v}}$ for a discretization of $\hat{x}$ with steps $n = 10$ steps gives us the three eigenvectors in figure 2
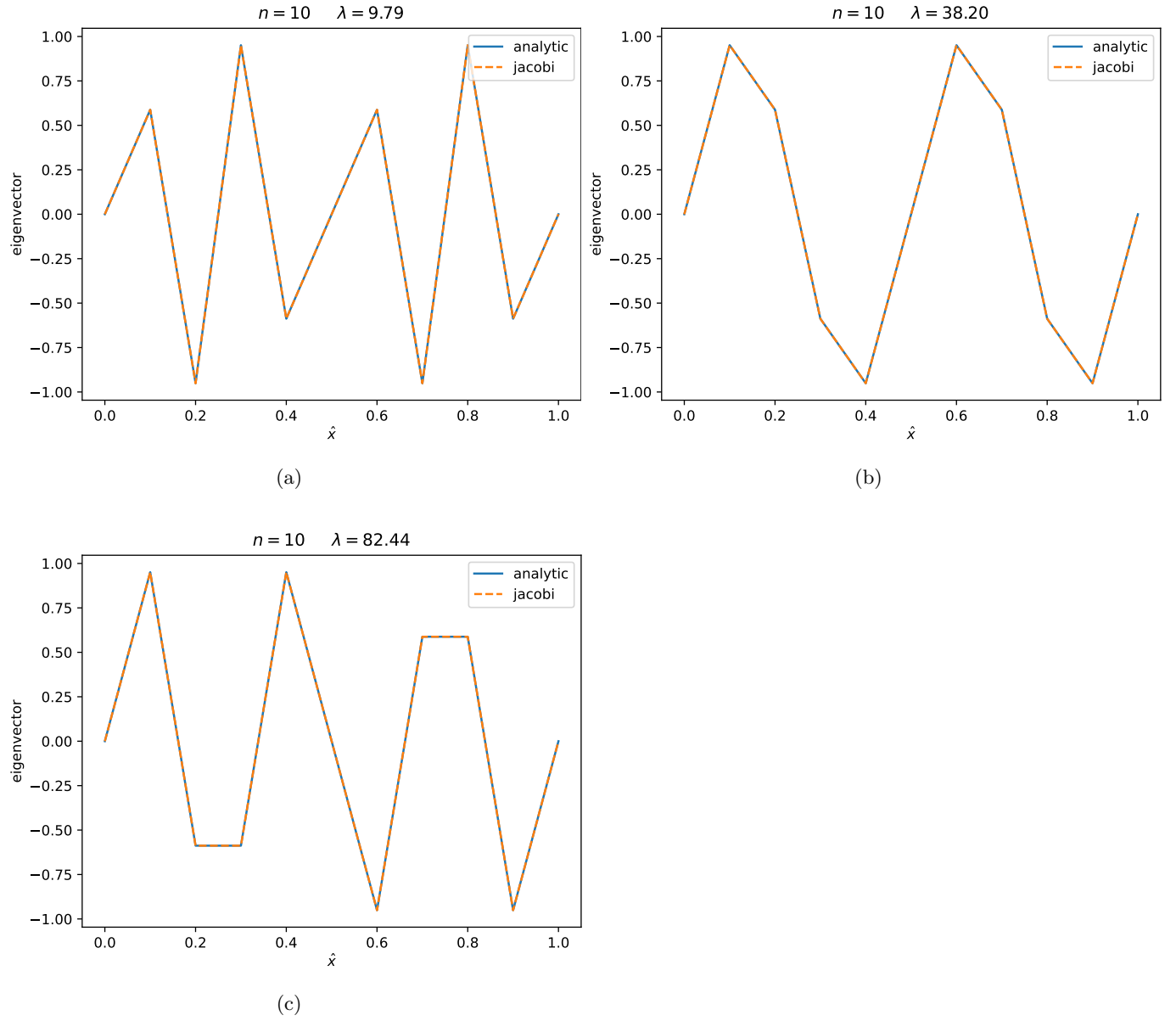

Figure 2: Eigenvectors corresponding to the three lowest eigenvalues for $n = 10$. Shows the vector v_i elements against the corresponding positions

b) Figure 3 shows the same plots for discretization of with $n = 100$ steps.
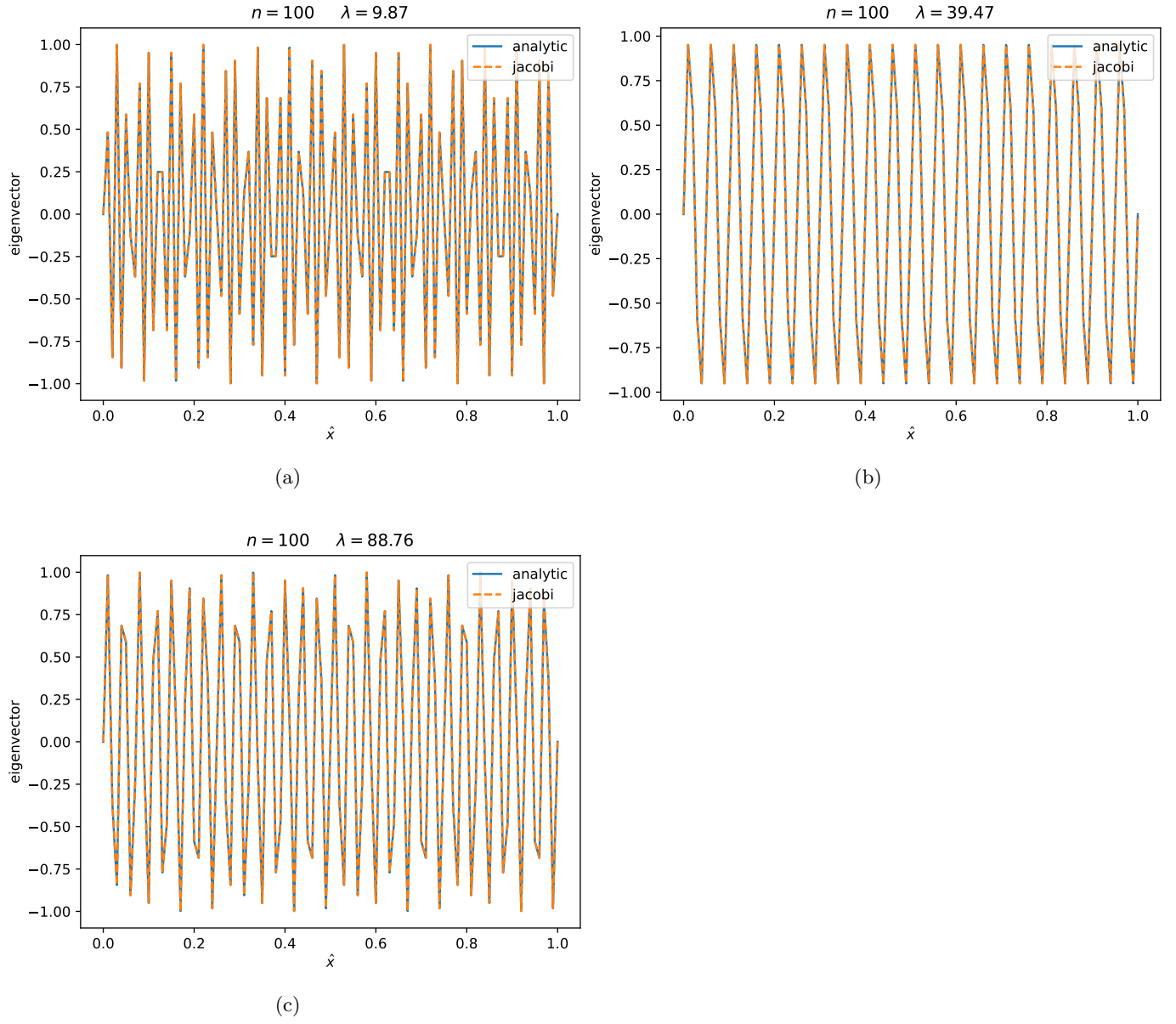

(a)

(b)

(c)

Figure 3: Eigenvectors corresponding to the three lowest eigenvalues for $n = 100$.