Project 1

Alessio Canclini, Filip von der Lippe

(Dated: September 2, 2022)

List a link to your github repository here!

PROBLEM 1

$$-\frac{d^2u}{dx^2} = f(x) \tag{1}$$

- source term: $f(x) = 100e^{-10x}$
- x range $x \in [0, 1]$
- boundary conditions: $u(0) = 0$ and $u(1) = 0$

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \tag{2}$$

Checking analytically that an exact solution to Eq. 1 is given by Eq. 2.

$$\begin{aligned}
\frac{du}{dx} &= 1 - e^{-10} + 10e^{-10x} \\
\frac{d^2u}{dx^2} &= -100e^{-10x} \\
-\frac{d^2u}{dx^2} &= 100e^{-10x} \\
-\frac{d^2u}{dx^2} &= f(x)
\end{aligned}$$

PROBLEM 2

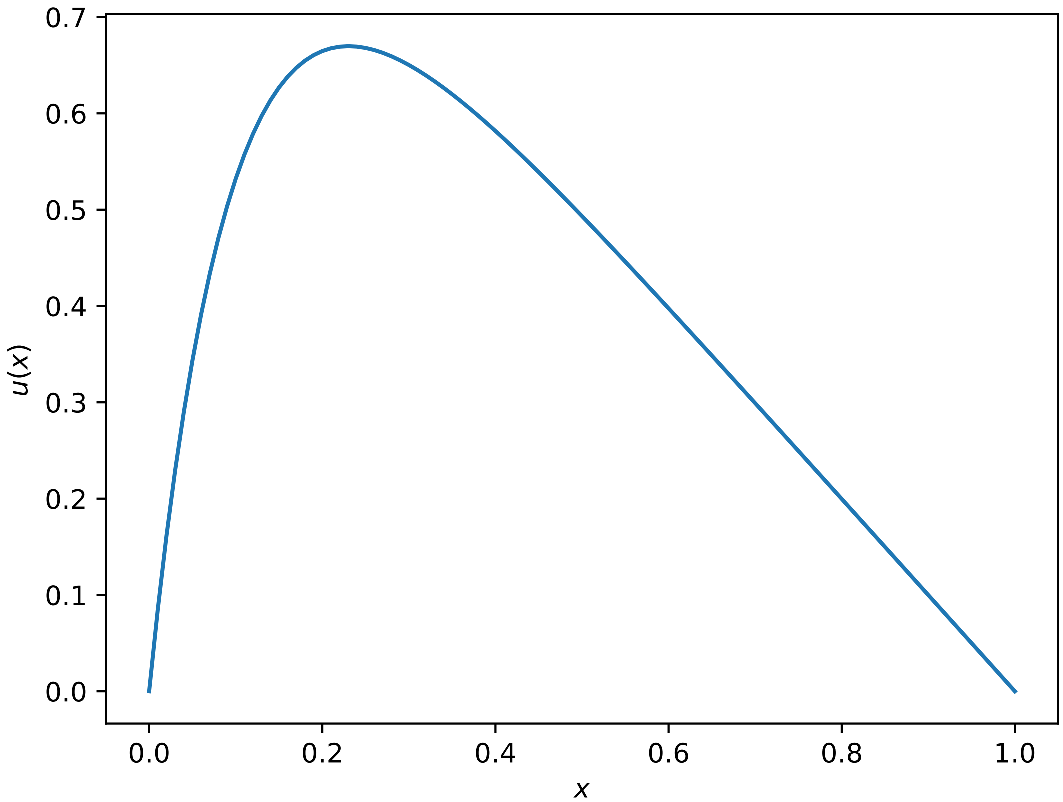

FIG. 1. Plot of $u(x)$.

PROBLEM 3

By using the taylor approximation of the second derivative we can discretize the second derivative in the Poission equation:

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + O(h^2)$$

Here we have the stepsize h and the truncation error O. The one-dimensional Poisson equation can then be written for the approximated version of u as v like:

$$-\frac{v_{i+1} - 2v_i + v_{i-1}}{h^2} = f_i \tag{3}$$

PROBLEM 4

We can rewrite the discretized equation as a matrix equation for $n+2$ number of points with the marix $n \times 2$ matrix A. We rewrite the discretized Poisson funcion:

$$\begin{aligned} 2v_1 - v_2 &= f_1 h^2 \\ -v_1 + 2v_2 - v_3 &= f_2 h^2 \\ &\vdots \\ -v_{n-2} + 2v_{n-1} - v_n &= f_{n-1} h^2 \\ -v_n + 2v_{n+1} &= f_n h^2 \end{aligned}$$

This can be written in terms of the following matrix equation where we rewrite $f_i h^2$ for $i = 1, 2, \ldots n-2$ as g_i

$$\begin{bmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 2 & -1 \\ 0 & 0 & \dots & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{n-1} \\ v_n \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{n-1} \\ g_n \end{bmatrix}$$

PROBLEM 5

¡¡¡¡¡¡¡ HEAD======= Since the vector $\vec{v*}$ of length m represents a complete solution of the descretized Poisson equation it contains all values in $\vec{v}$ in adition to the boundary conditions. The relation bethween n and m is therfore $m = n + 2$. Solving $\mathbf{A}\vec{v} = \vec{g}$ for $\vec{v}$ gives us all but the first an last value in $\vec{v*}$. So all but the boundary values. ¿¿¿¿¿¿¿ b4a1574ad2808bfd35dc1c1351eb43d36ca950de

PROBLEM 6

PROBLEM 7

PROBLEM 8

PROBLEM 9

PROBLEM 10

We write equations using the LaTeX `equation` (or `align`) environments. Here is an equation with numbering

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}, \tag{4}$$

and here is one without numbering:

$$\oint_C \mathbf{F} \cdot \mathrm{d}\mathbf{r} = 0.$$

Sometimes it is useful to refer back to a previous equation, like we're demonstrating here for equation 4.

We can include figures using the `figure` environment. Whenever we include a figure or table, we *must* make sure to actually refer to it in the main text, e.g. something like this: "In figure 2 we show ...". Also, note the LaTeX code we used to get correct quotation marks in the previous sentence. (Simply using the `"` key on your keyboard will give the wrong result.) Figures should preferably be vector graphics (e.g. a `.pdf` file) rather than raster graphics (e.g. a `.png` file).

By the way, don't worry too much about where LaTeX decides to place your figures and tables — LaTeX knows more than we do about proper document layout. As long as you label all your figures and tables and refer to them

FIG. 2. Write a descriptive caption here that explains the content of the figure. Note the font size for the axis labels and ticks — the size should approximately match the document font size.

Number of points	Output
10	0.3086
100	0.2550

TABLE I. Write a descriptive caption here, explaining the content of your table.

in the text, it's all good. Of course, in some cases it can be worth trying to force a specific placement, to avoid the figure/table appearing many pages away from the main text discussing it, but this isn't something you should spend time on until the very end of the writing process.

Next up is a table, created using the `table` and `tabular` environments. We refer to it by table I.

Finally, we can list algorithms by using the `algorithm` environment, as demonstrated here for algorithm 1.

Algorithm 1 Some algorithm

Some maths, e.g $f(x) = x^2$. ▷ Here's a comment
for $i = 0, 1, ..., n-1$ **do**
  Do something here
while Some condition **do**
  Do something more here
Maybe even some more math here, e.g $\int_0^1 f(x)\mathrm{d}x$