

## Assignment 1. Linear Regression

**1 (Paper and pencil exercise)** Consider the linear regression problem (with loss function  $J(\theta) = \|Y - X\theta\|^2$ ) for the following data set:

$x_1$	$x_2$	$y$
$\pi/4$	$\pi/4$	2
$-\pi/3$	$\pi/3$	-2
$\pi/8$	$\pi/8$	1

with the model  $y = f_{\theta}(x)$ ,  $f_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 \sin(x_1 + x_2) + \theta_3 \cos(x_1 + x_2)$ .

- Obtain the expression for the gradient  $\nabla J(\theta)$  as a function of the vector of labels  $Y$  and the related matrix of features  $X$ . Obtain explicitly  $X$  and  $Y$ .
- Say if the following statement is true or false, justifying your response. *The previous linear regression problem has a closed-form solution and the minimization of the loss function has led to a unique solution.*

**2 (Python exercise)** Consider the following data for a linear regression problem where  $x_i$  is the feature  $i^{th}$  and  $y$  is the output:

$x_1$	$x_2$	$x_3$	$y$
340	16	356	1.5
665	25	690	2.8
368	15	383	1.7
331	15	346	1.3
954	40	994	5.0

Calculate the solutions for the following regression models. If the solution is not unique, please provide a better approach. Note: do not use the *sklearn library*.

- $\hat{y} = \theta_1 x_1 + \theta_2 x_2$ .
- $\hat{y} = \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$ .