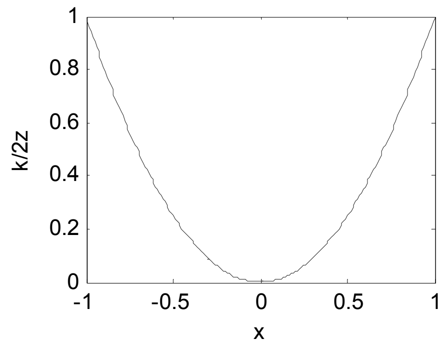

Figure 4.1 *x*-axis phase profile of the chirp function in Eq. (4.11).

variety of situations to model a contracting or expanding optical field. An example 1D profile of the phase of Eq. (4.11) is plotted in Fig. 4.1.

An important concept is *leading* and *lagging* phase. The temporal phasor $\exp(-j2\pi\nu t)$ defined in Eq. (4.6) indicates that the phase of the optical field becomes more *negative* as time progresses. Therefore, we say the phase in the center of the profile in Fig. 4.1 *leads* the rest of the function since it has the "most negative" value. The further away from the center, the more the phase *lags*. Interpreting the phase as a representation of an optical wavefront, the center of the wave crest in Fig. 4.1 leads the edges, and the wave can be imagined to be "propagating downward" in Fig. 4.1. Further physical interpretation of the optical phase is discussed in Section 5.3.

4.4 Analytic Diffraction Solutions

4.4.1 Rayleigh–Sommerfeld solution I

Consider the propagation of monochromatic light from a 2D plane (source plane) indicated by the coordinate variables ξ and η (Fig. 4.2). At the source plane, an area Σ defines the extent of a source or an illuminated aperture. The field distribution in the source plane is given by $U_1(\xi, \eta)$, and the field $U_2(x, y)$ in a distant observation plane can be predicted using the first *Rayleigh–Sommerfeld diffraction solution*

$$U_2(x,y) = \frac{z}{j\lambda} \iint_{\Sigma} U_1(\xi,\eta) \frac{\exp(jkr_{12})}{r_{12}^2} d\xi d\eta. \tag{4.12}$$

Here, λ is the optical wavelength; k is the wavenumber, which is equal to $2\pi/\lambda$ for free space; z is the distance between the centers of the source and observation coordinate systems; and r_{12} is the distance between a position on the source plane and a position in the observation plane. ξ and η are variables of integration, and the integral limits correspond to the area of the source Σ. With the source and observation positions defined on parallel planes, the distance r_{12} is

$$r_{12} = \sqrt{z^2 + (x - \xi)^2 + (y - \eta)^2} \ . \tag{4.13}$$

Expression (4.12) is a statement of the Huygens–Fresnel principle. This principle supposes the source acts as an infinite collection of fictitious point sources, each producing a spherical wave associated with the actual source field at any position (ξ, η). The contributions of these spherical waves are summed at the observation position (x, y), allowing for interference. The extension of Eqs. (4.12) and (4.13) to nonplanar geometries is straightforward; for example, involving a more complicated function for r, but the planar geometry is more commonly encountered, and this is our focus here.

Expression (4.12) is, in general, a superposition integral, but with the source and observation areas defined on parallel planes, it becomes a convolution integral, which can be written as

$$U_2(x,y) = \iint U_1(\xi,\eta) h(x - \xi, y - \eta) d\xi d\eta \ , \tag{4.14}$$

where the general form of the Rayleigh–Sommerfeld impulse response is

$$h(x,y) = \frac{z}{j\lambda} \frac{\exp(jkr)}{r^2} , \tag{4.15}$$

and

$$r = \sqrt{z^2 + x^2 + y^2} \ . \tag{4.16}$$

The Fourier convolution theorem is applied to write Eq. (4.14) as

$$U_2(x,y) = \Im^{-1}\{\Im\{U_1(x,y)\}\Im\{h(x,y)\}\}. \tag{4.17}$$

For this convolution interpretation the source and observation plane variables are

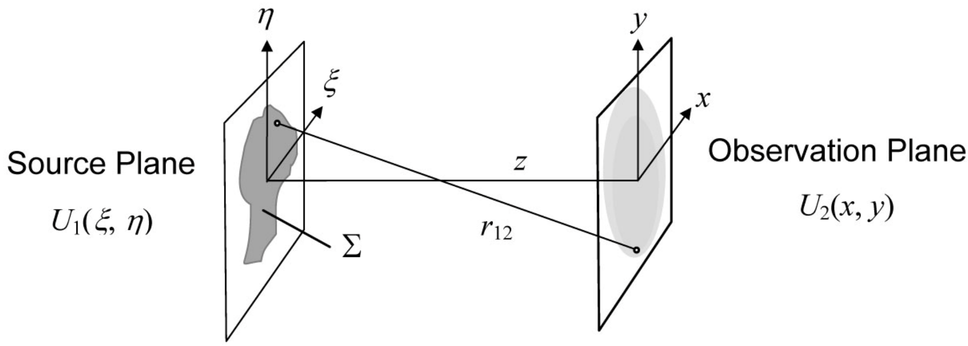

Figure 4.2 Propagation geometry for parallel source and observation planes.

simply re-labeled as x and y. An equivalent expression for Eq. (4.17) is

$$U_2(x,y) = \Im^{-1}\{\Im\{U_1(x,y)\}H(f_X, f_Y)\}, \tag{4.18}$$

where H is the Rayleigh–Sommerfeld transfer function given by

$$H(f_X, f_Y) = \exp\left(jkz\sqrt{1-(\lambda f_X)^2 - (\lambda f_Y)^2}\right). \tag{4.19}$$

Strictly speaking, $\sqrt{f_X^2 + f_Y^2} < 1/\lambda$ must be satisfied for propagating field components. An angular spectrum analysis is often used to derive Eq. (4.19).

The Rayleigh–Sommerfeld expression is the most accurate diffraction solution considered in this book. Other than the assumption of scalar diffraction, this solution only requires that $r >> \lambda$, the distance between the source and the observation position, be much greater than a wavelength.

4.4.2 Fresnel approximation

The square root in the distance terms of Eq. (4.13) or (4.16) can make analytic manipulations of the Rayleigh–Sommerfeld solution difficult and add execution time to a computational simulation. By introducing approximations for these terms, a more convenient scalar diffraction form is developed. Consider the binomial expansion

$$\sqrt{1+b} = 1 + \frac{1}{2}b - \frac{1}{8}b^2 + \ldots, \tag{4.20}$$

where b is a number less than 1, then expand Eq. (4.13) and keep the first two terms to yield

$$r_{12} \approx z\left[1 + \frac{1}{2}\left(\frac{x-\xi}{z}\right)^2 + \frac{1}{2}\left(\frac{y-\eta}{z}\right)^2\right]. \tag{4.21}$$

This approximation is applied to the distance term in the phase of the exponential in Eq. (4.12), which amounts to assuming a parabolic radiation wave rather than a spherical wave for the fictitious point sources. Furthermore, use the approximation $r_{12} \approx z$ in the denominator of Eq. (4.12) to arrive at the *Fresnel diffraction* expression:[1]

$$U_2(x,y) = \frac{e^{jkz}}{j\lambda z}\iint U_1(\xi,\eta)\exp\left\{j\frac{k}{2z}\left[(x-\xi)^2 + (y-\eta)^2\right]\right\}d\xi d\eta\,. \tag{4.22}$$