

## Math 215 - Homework 2.1

### Gaussian Elimination

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**Instructions:** Solve the following systems using the Gaussian Elimination method. Use Augmented Format

#### Problem 1: A

(a) Solve the system of equations

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 5 \\ 2x_2 + x_3 - 2x_4 + x_5 = 1 \\ 4x_3 + x_4 - 2x_5 = 1 \\ x_4 - 3x_5 = 0 \\ 2x_5 = 2 \end{cases}$$

As the problem is already in row echelon form, we will solve this using back substitution.

(i)

$$2x_5 = 2 \quad (1)$$

$$x_5 = 1 \quad (2)$$

(ii)

$$x_4 - 3x_5 = 0 \quad (3)$$

$$x_4 = 3x_5 \quad (4)$$

$$x_4 = 3 \quad (5)$$

(iii)

$$4x_3 + x_4 - 2x_5 = 1 \quad (6)$$

$$4x_3 + 3 - 2(1) = 1 \quad (7)$$

$$4x_3 + 1 = 1 \quad (8)$$

$$4x_3 = 0 \quad (9)$$

$$x_3 = 0 \quad (10)$$

(iv)

$$2x_2 + x_3 - 2x_4 + x_5 = 1 \quad (11)$$

$$2x_2 + 0 - 2(3) + 1 = 1 \quad (12)$$

$$2x_2 - 6 + 1 = 1 \quad (13)$$

$$2x_2 = 6 \quad (14)$$

$$x_2 = 3 \quad (15)$$

(v)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 5 \quad (16)$$

$$x_1 + 3 + 0 + 3 + 1 = 5 \quad (17)$$

$$x_1 + 6 = 5 - 1 \quad (18)$$

$$x_1 + 6 = 4 \quad (19)$$

$$x_1 = -2 \quad (20)$$

Answer:

$$\begin{pmatrix} -2 \\ 3 \\ 0 \\ 3 \\ 1 \end{pmatrix} \quad (21)$$

Double Checking:

$$\left\{ \begin{array}{rclcl} -2 + 3 & + 0 & + 3 & + 1 & = 5 \\ & 2(3) + 0 & - 2(3) + 1 & & = 1 \\ & & 4(0) + 3 & - 2(1) & = 1 \\ & & & 3 & - 3(1) = 0 \\ & & & & 2(1) = 2 \end{array} \right.$$

### Problem 2: B

Solve using Gaussian Elimination and augmented matrix

$$\left\{ \begin{array}{rcl} 2x_1 & + x_2 - 3x_3 & = -2 \\ x_1 & - x_2 + 2x_3 & = 4 \\ -x_1 & & + 4x_3 = 3 \end{array} \right.$$

Augmented Matrix:

$$\left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 1 & -1 & 2 & 4 \\ -1 & 0 & 4 & 3 \end{array}\right) \xrightarrow{R_2+R_3 \rightarrow R_3} \left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 1 & -1 & 2 & 4 \\ 0 & -1 & 6 & 7 \end{array}\right) \quad (22)$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 1 & -1 & 2 & 4 \\ 0 & -1 & 6 & 7 \end{array}\right) \xrightarrow{\begin{array}{l} -\frac{1}{2}R_1+R_2 \rightarrow R_2 \\ 2R_2 \rightarrow R_2 \end{array}} \left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 0 & -3 & 7 & 10 \\ 0 & -1 & 6 & 7 \end{array}\right) \quad (23)$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 0 & -3 & 7 & 10 \\ 0 & -1 & 6 & 7 \end{array}\right) \xrightarrow{\begin{array}{l} -3R_3+R_2 \rightarrow R_3 \\ -\frac{1}{11}R_3 \rightarrow R_3 \end{array}} \left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 0 & -3 & 7 & 10 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad (24)$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 0 & -3 & 7 & 10 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{-\frac{1}{3}R_2+\frac{7}{3}R_3 \rightarrow R_2} \left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad (25)$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -3 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{\frac{1}{2}R_1-\frac{1}{2}R_2 \rightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad (26)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R_1+\frac{3}{2}R_3 \rightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad (27)$$

Answer:

$$x = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (28)$$

Double Checking:

$$\begin{cases} 2(1) - 1 - 3(1) = -2 \\ 1 - (-1) + 2(1) = 4 \\ -1 + 4(1) = 3 \end{cases}$$

### Problem 3: C

Solve using Gaussian Elimination and augmented matrix

$$\begin{cases} 2x_1 - x_2 + x_3 = 0 \\ 3x_1 + x_2 - x_3 = 5 \\ x_1 + 2x_2 - 3x_3 = 6 \end{cases}$$

Augmented Matrix:

$$\begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 3 & 1 & -1 & | & 5 \\ 1 & 2 & -3 & | & 6 \end{pmatrix} \xrightarrow{-\frac{1}{2}R_1 + R_3 \rightarrow R_3} \begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 3 & 1 & -1 & | & 5 \\ 0 & \frac{5}{2} & -\frac{7}{2} & | & 6 \end{pmatrix} \quad (29)$$

$$\begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 3 & 1 & -1 & | & 5 \\ 0 & \frac{5}{2} & -\frac{7}{2} & | & 6 \end{pmatrix} \xrightarrow{\begin{matrix} -\frac{3}{2}R_1 + R_2 \rightarrow R_2 \\ 2R_3 \rightarrow R_3 \\ \frac{2}{5}R_2 \rightarrow R_2 \end{matrix}} \begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 0 & 1 & -1 & | & 2 \\ 0 & 5 & -7 & | & 12 \end{pmatrix} \quad (30)$$

$$\begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 0 & 1 & -1 & | & 2 \\ 0 & 5 & -7 & | & 12 \end{pmatrix} \xrightarrow{-5R_2 + R_3 \rightarrow R_3} \begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 0 & 1 & -1 & | & 2 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \quad (31)$$

$$\begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 0 & 1 & -1 & | & 2 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \xrightarrow{\begin{matrix} -2R_3 \rightarrow R_3 \\ R_3 + R_2 \rightarrow R_2 \end{matrix}} \begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \quad (32)$$

$$\begin{pmatrix} 2 & -1 & 1 & | & 0 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \xrightarrow{\begin{matrix} R_1 + R_2 + (-1)R_3 \rightarrow R_3 \\ \frac{1}{2}R_1 \rightarrow R_1 \end{matrix}} \begin{pmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \quad (33)$$

Answer:

$$x = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad (34)$$

Double Checking:

$$\begin{cases} 2(1) - 1 - 1 = 0 \\ 3(1) + 1 - (-1) = 5 \\ 1 + 2(1) - 3(-1) = 6 \end{cases}$$