材料力学

第五章　弯曲应力

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§ 5–2　纯弯曲时的正应力

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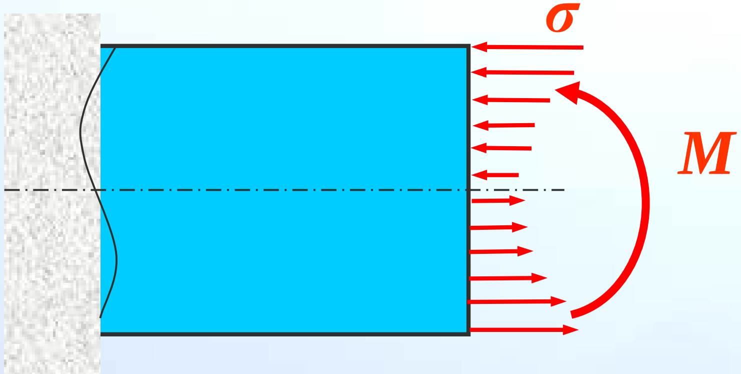
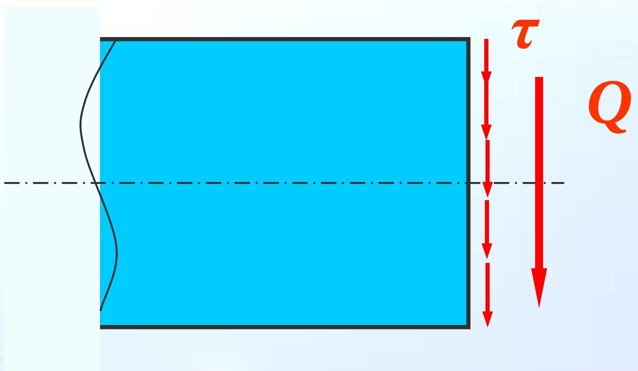
§5–1 纯弯曲

内力

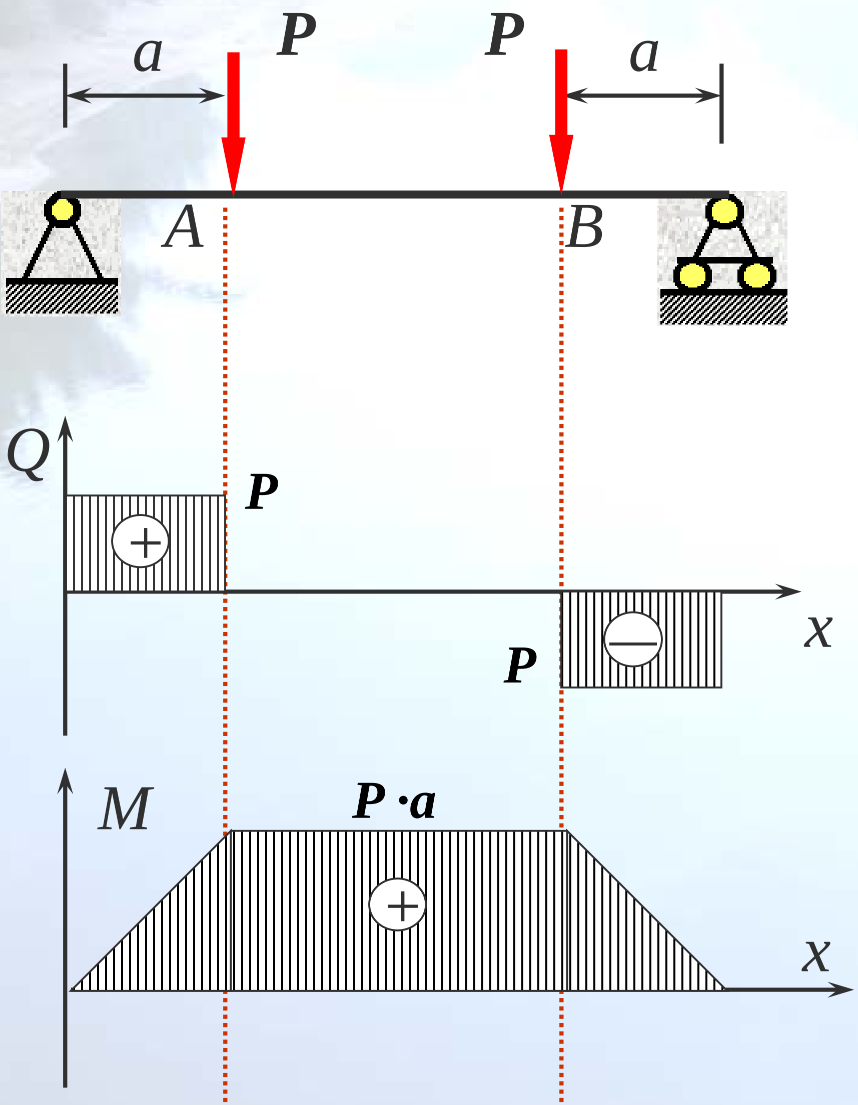
- 弯矩$\boldsymbol{M}$ ⟶ 正应力$\boldsymbol{\sigma}$
- 剪力$\boldsymbol{Q}$ ⟶ 剪应力$\boldsymbol{\tau}$





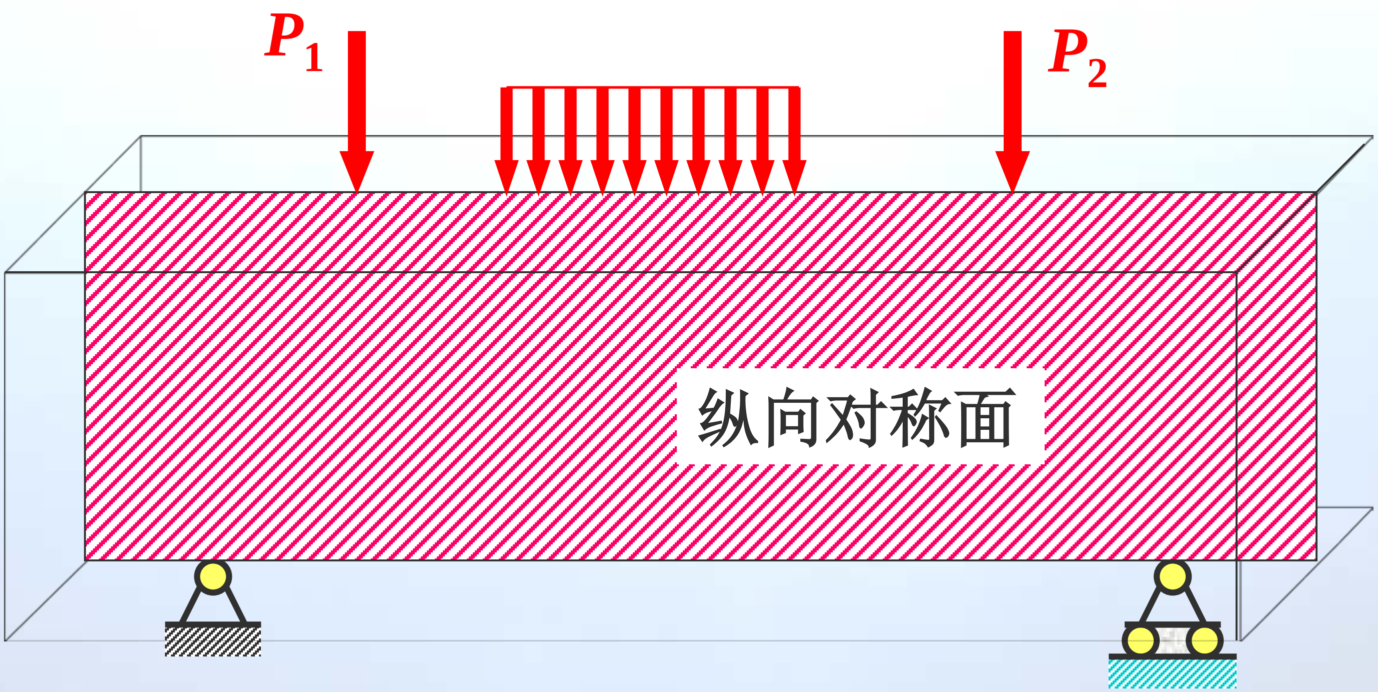
纯弯曲： $Q=0$， $M\neq 0$

横力弯曲：$Q\neq 0$， $M\neq 0$



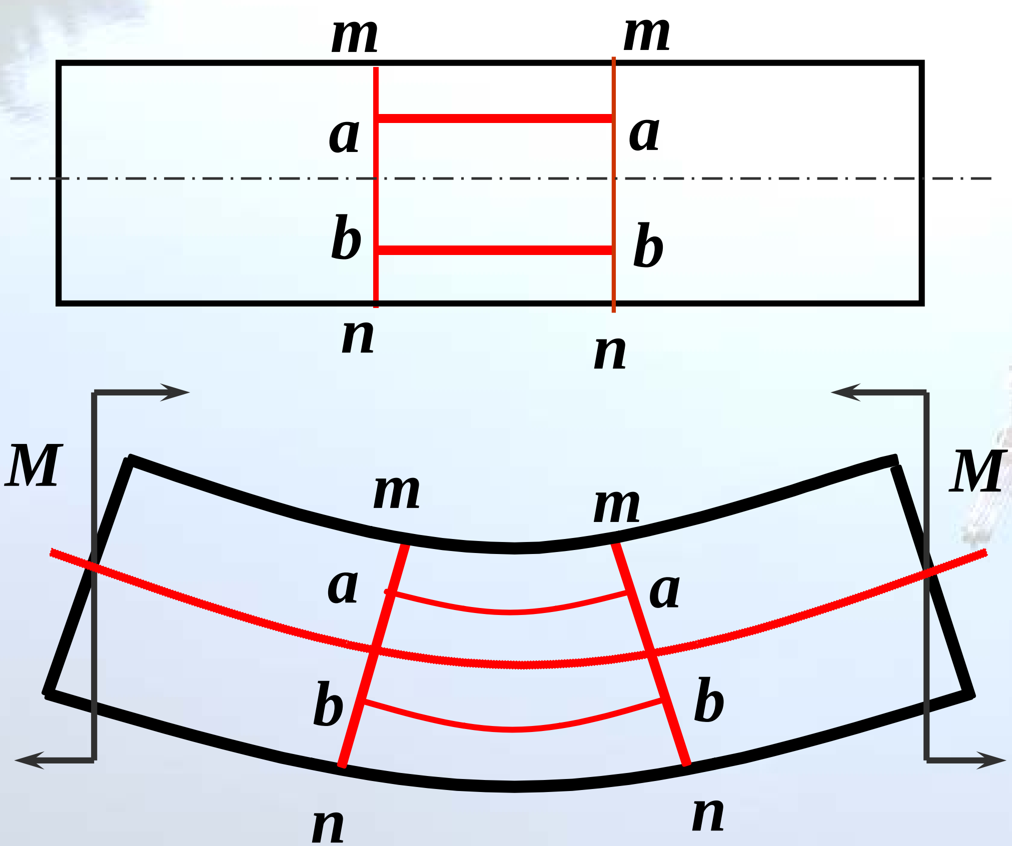
***AB*段纯弯曲**(Pure Bending)**:**

平面弯曲



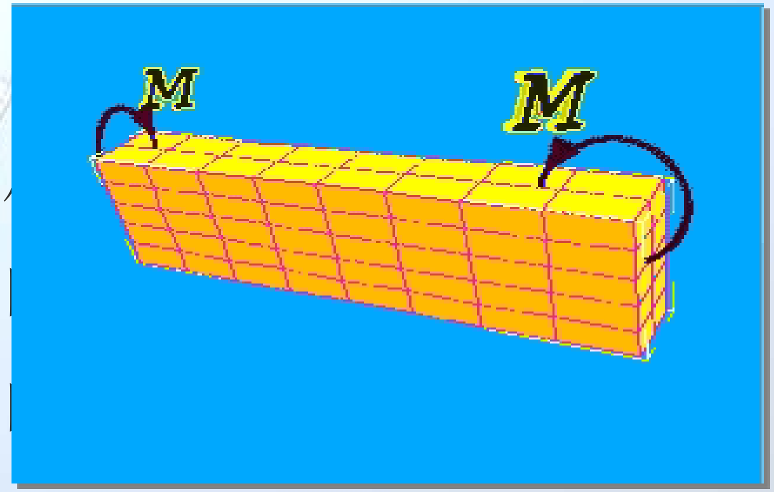
§ 5–2　纯弯曲时的正应力

一、 纯弯曲时梁横截面上的正应力

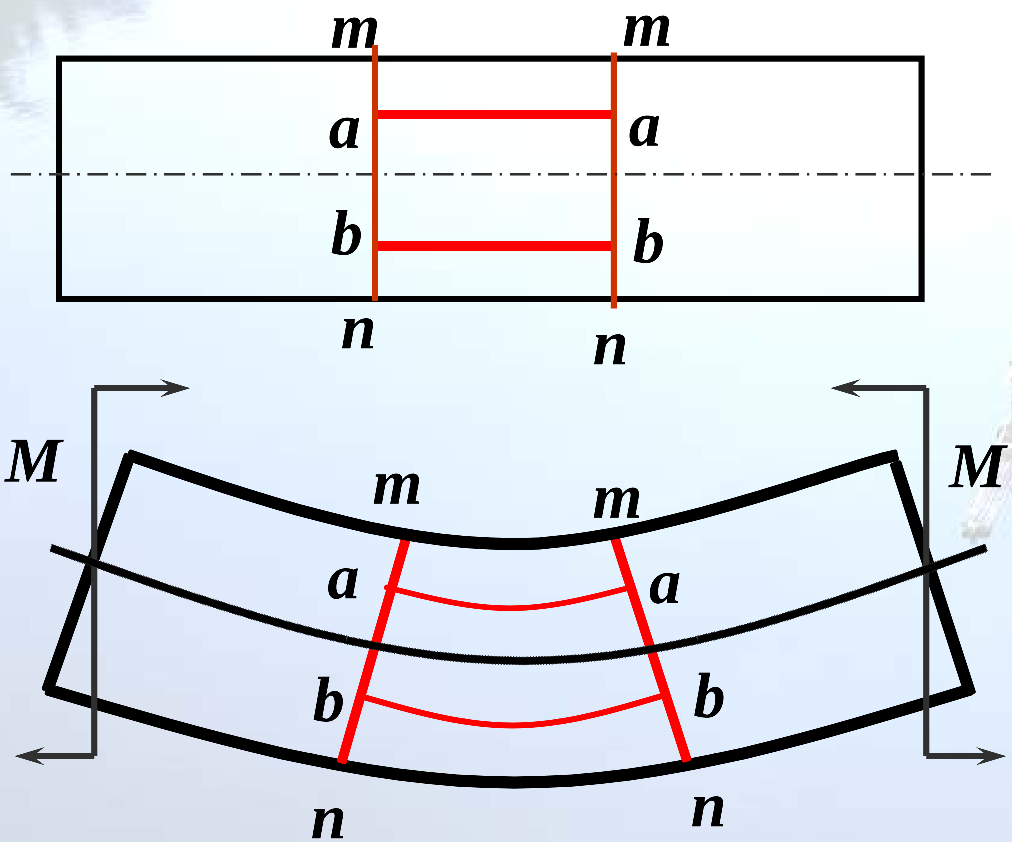


变形几何规律：

1. 梁的纯弯曲实验



2．平面假设： 横截面变形后仍为平面。



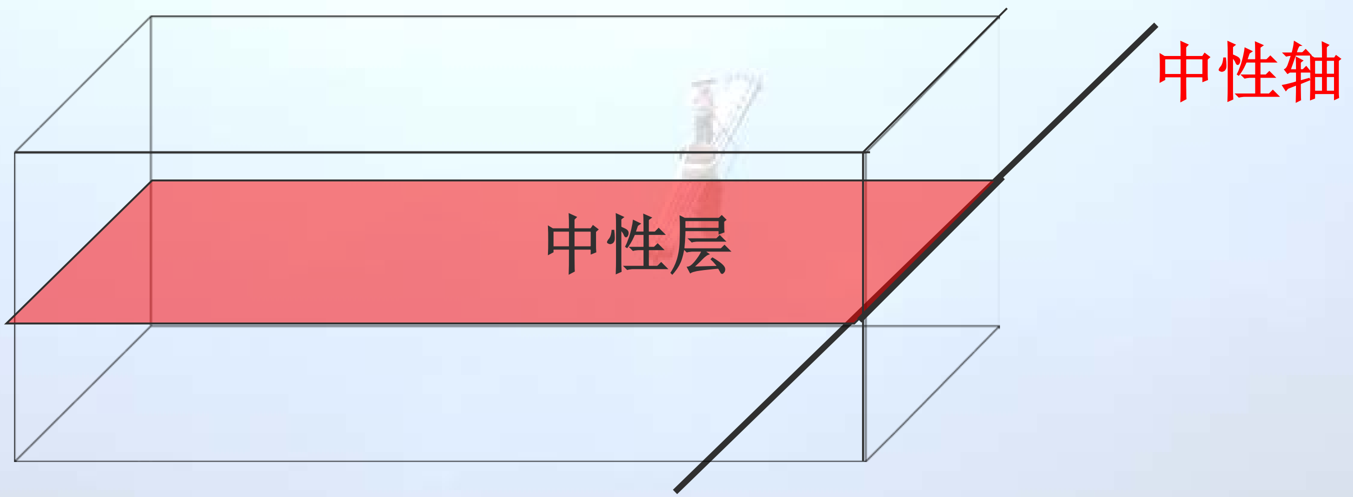
设想梁由无数根平行于轴线的纵向纤维组成，变形后，上部纤维缩短，下部纤维伸长。

有一层纤维变形后不伸长也不缩短。

3. 两个概念

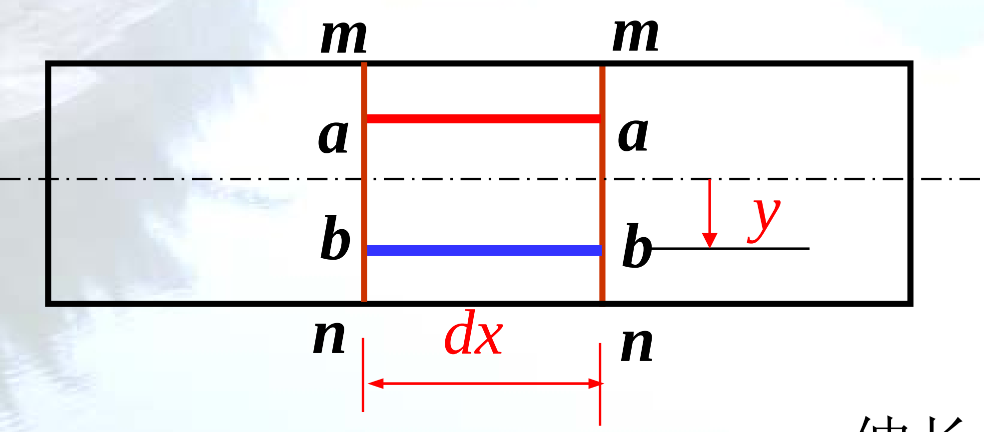
①中性层：梁内既不伸长也不缩短的一层纤维，此层纤维称中性层。

②中性轴：中性层与横截面的交线。



（一）变形几何关系:

建立坐标系

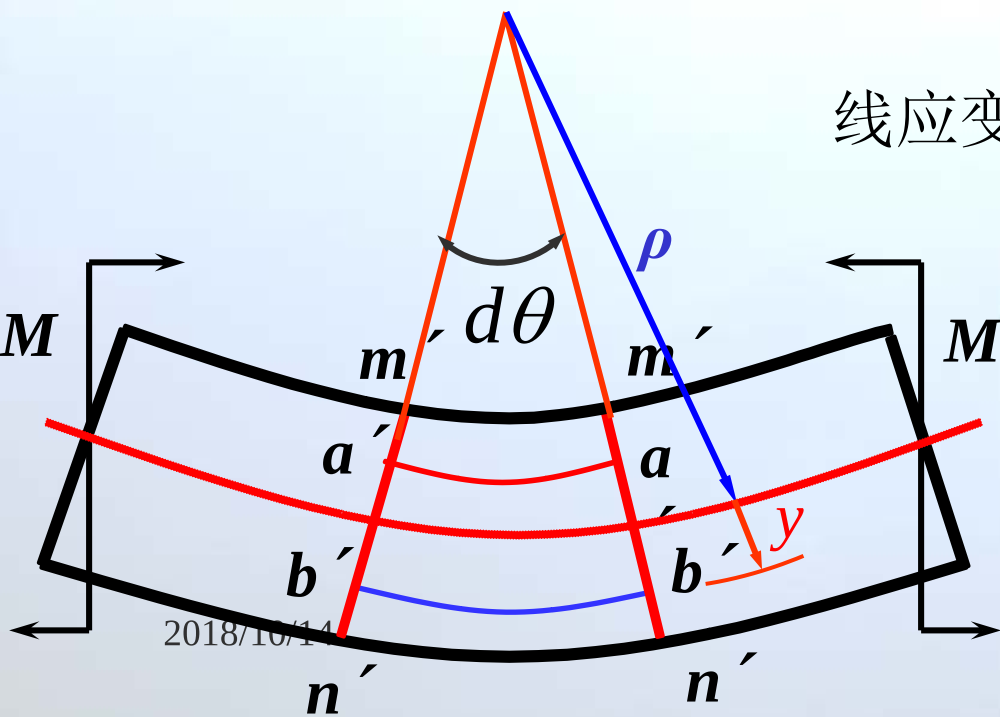


变形前：$l = \overline{bb} = dx = \rho d\theta$

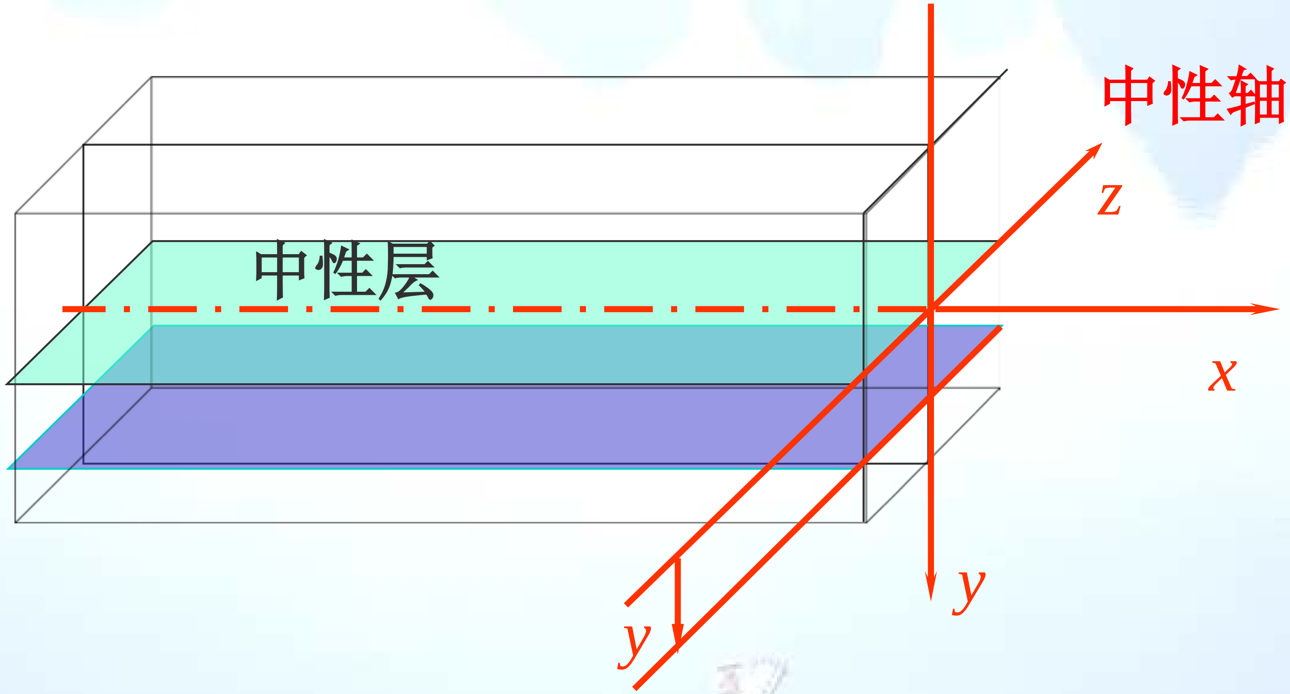
变形后：$l_1 = \overset{\frown}{b'b'} = (\rho + y)d\theta$

伸长量：$\Delta l = l_1 - l = (\rho + y)d\theta - dx$

线应变：$\varepsilon = \dfrac{\Delta l}{l} = \dfrac{(\rho+y)d\theta - dx}{dx} = \dfrac{(\rho+y)\mathrm{d}\theta - \rho\mathrm{d}\theta}{\rho\mathrm{d}\theta} = \dfrac{y}{\rho}$



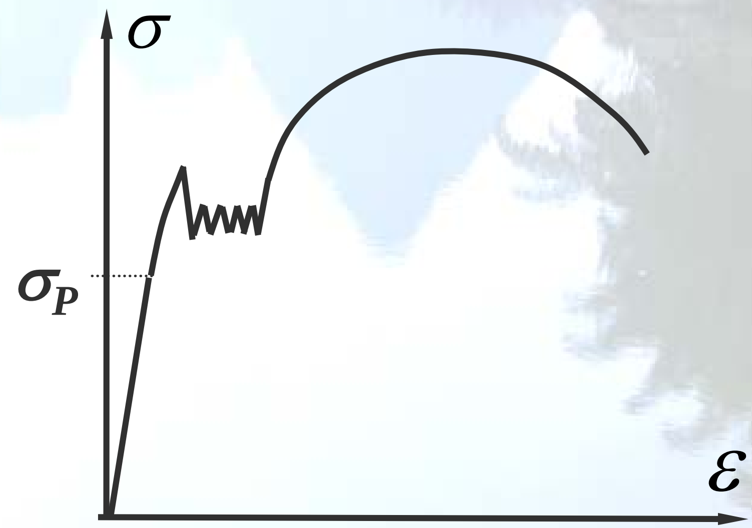
$$\varepsilon = \frac{y}{\rho} \quad ---- \quad （1）$$


中性轴
z
中性层
x
y
y

（二）物理关系：

假设：纵向纤维无挤压。

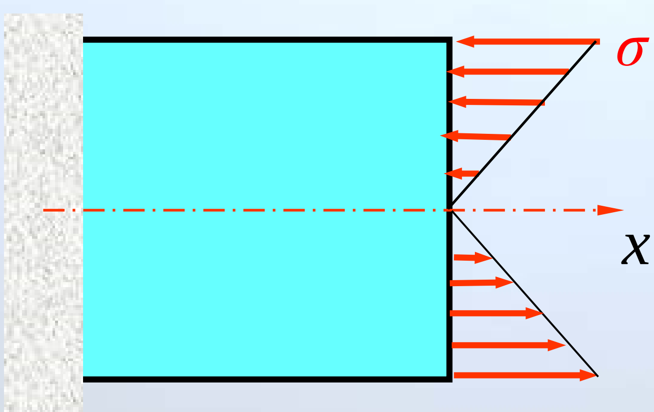
当$\sigma \le \sigma_P$时， $\sigma = E\varepsilon$

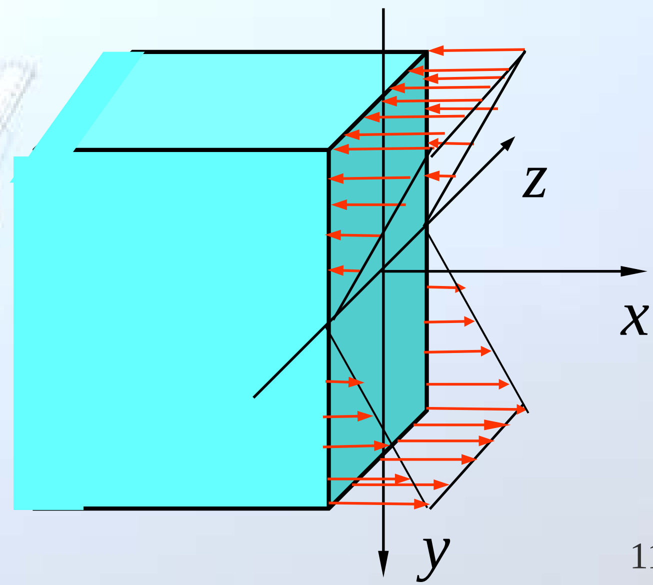


σ—ε曲线

$$\therefore \sigma = E\frac{y}{\rho} \quad \ldots\ldots(2)$$

式中：E和ρ为常数，所以横截面上正应力与 y 成正比。





（三）静力关系：

横截面上的正应力组成一个空间平行力系，可以简化后得到三个内力分量：

$$F_x = 0 \quad \text{----------} \quad (1)$$

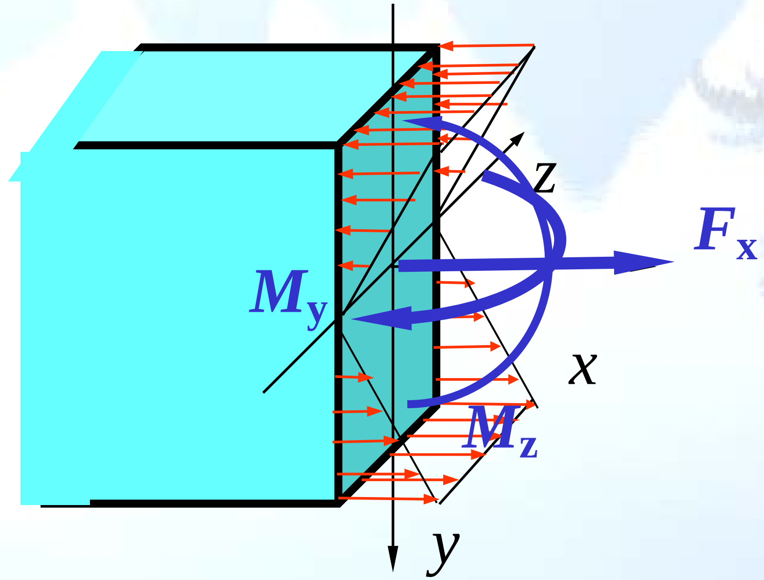
$$F_y = 0$$

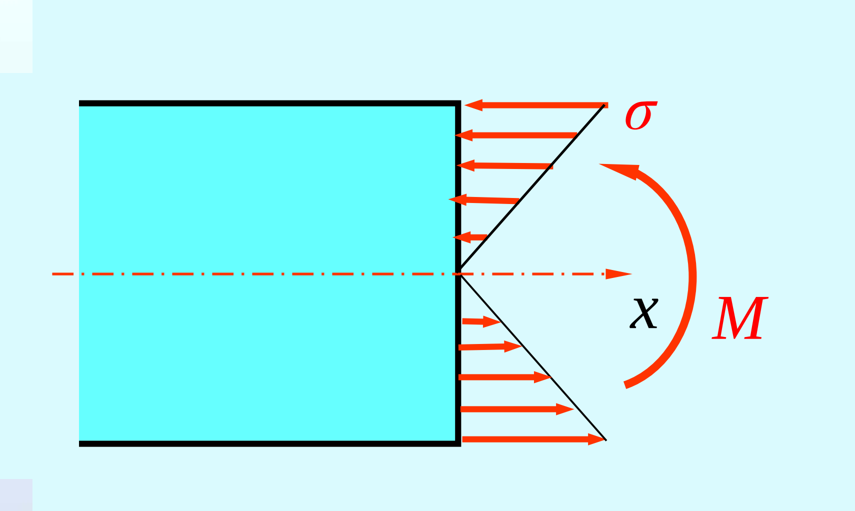
$$F_z = 0$$

$$M_x = 0$$

$$M_y = 0 \quad \text{----------} \quad (2)$$

$$M_z = M \quad \text{---------} (3)$$

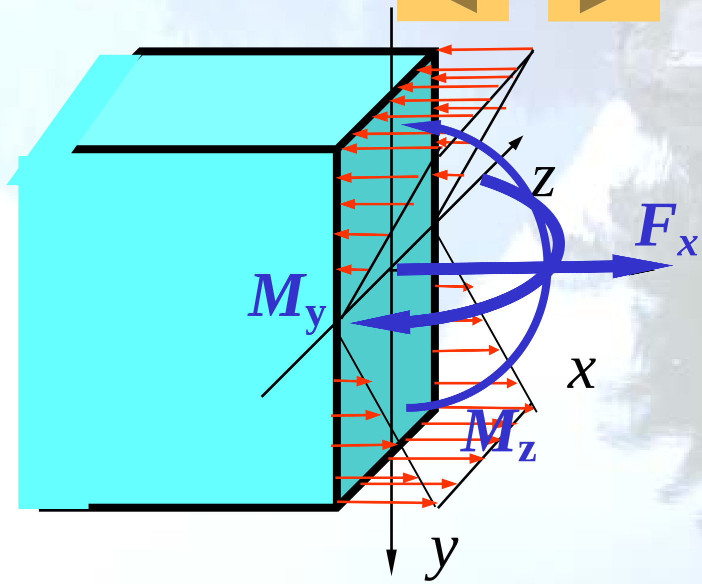




$$F_x = \int_A \sigma \mathrm{d}A = 0 \quad \text{----------} \quad (1)$$

$$M_y = \int_A (\sigma \mathrm{d}A) z = 0 \quad \text{----------} \quad (2)$$

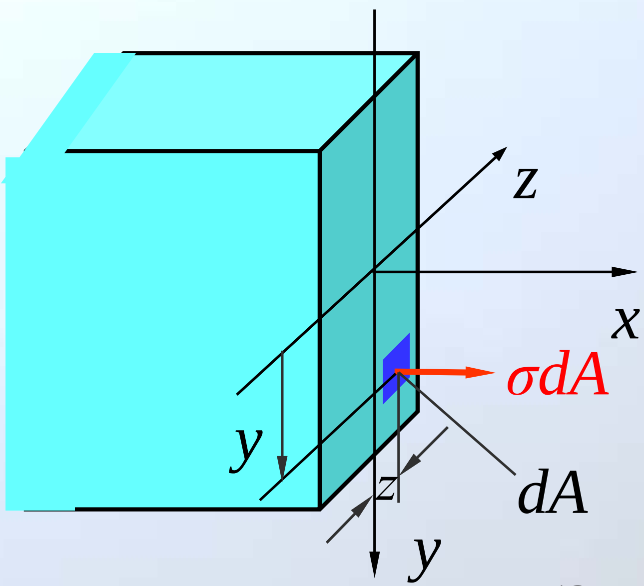
$$M_z = \int_A (\sigma \mathrm{d}A) y = M \quad \text{---------} (3)$$



由（1）式

$$F_x = \int_A \sigma \mathrm{d}A = \int_A E\frac{y}{\rho}\mathrm{d}A = \frac{E}{\rho}\int_A y\mathrm{d}A$$

$$= \frac{E}{\rho}S_Z = 0$$



由于$\frac{E}{\rho}\neq 0$，所以必须 $S_Z = 0$

所以，z (中性)轴必须通过形心

由（2）式

$$M_y=\int_A(\sigma \mathrm{d}A)z=\int_A\frac{Eyz}{\rho}\mathrm{d}A$$

$$=\frac{E}{\rho}\int_A yz\mathrm{d}A=\frac{EI_{yz}}{\rho}=0$$

（平面弯曲，I_{yz}=0）

由（3）式

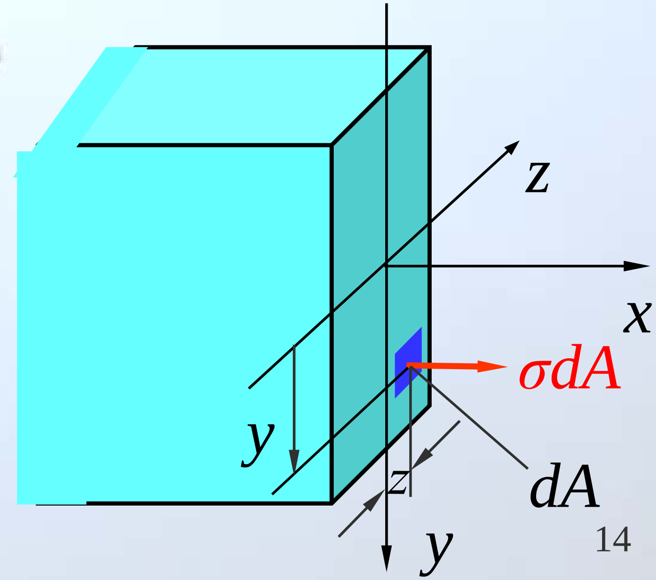
$$M_z=\int_A(\sigma \mathrm{d}A)y=\int_A\frac{Ey^2}{\rho}\mathrm{d}A$$

$$=\frac{E}{\rho}\int_A y^2\mathrm{d}A=\boxed{\frac{EI_z}{\rho}=M}$$

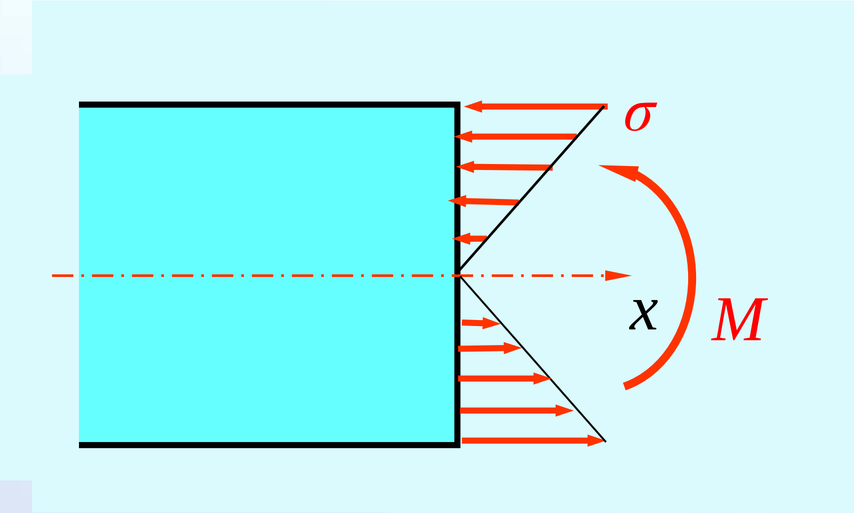
$$\frac{1}{\rho}=\frac{M_z}{EI_z} \quad \text{-----------（5-1）}$$

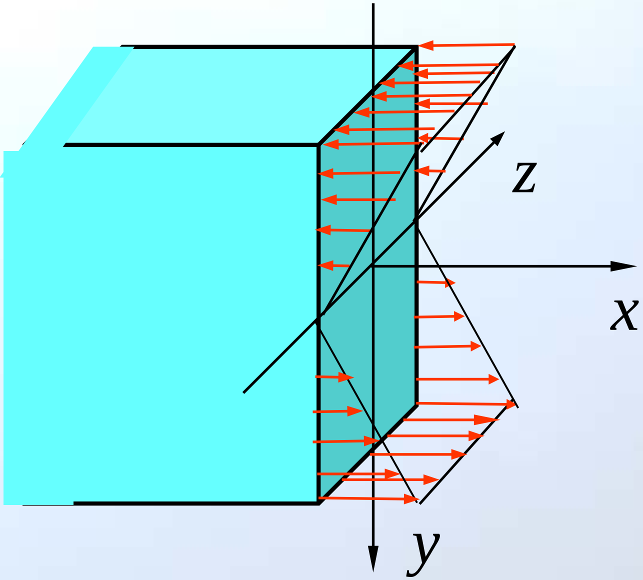
EI_z ⟶ 梁的抗弯刚度。

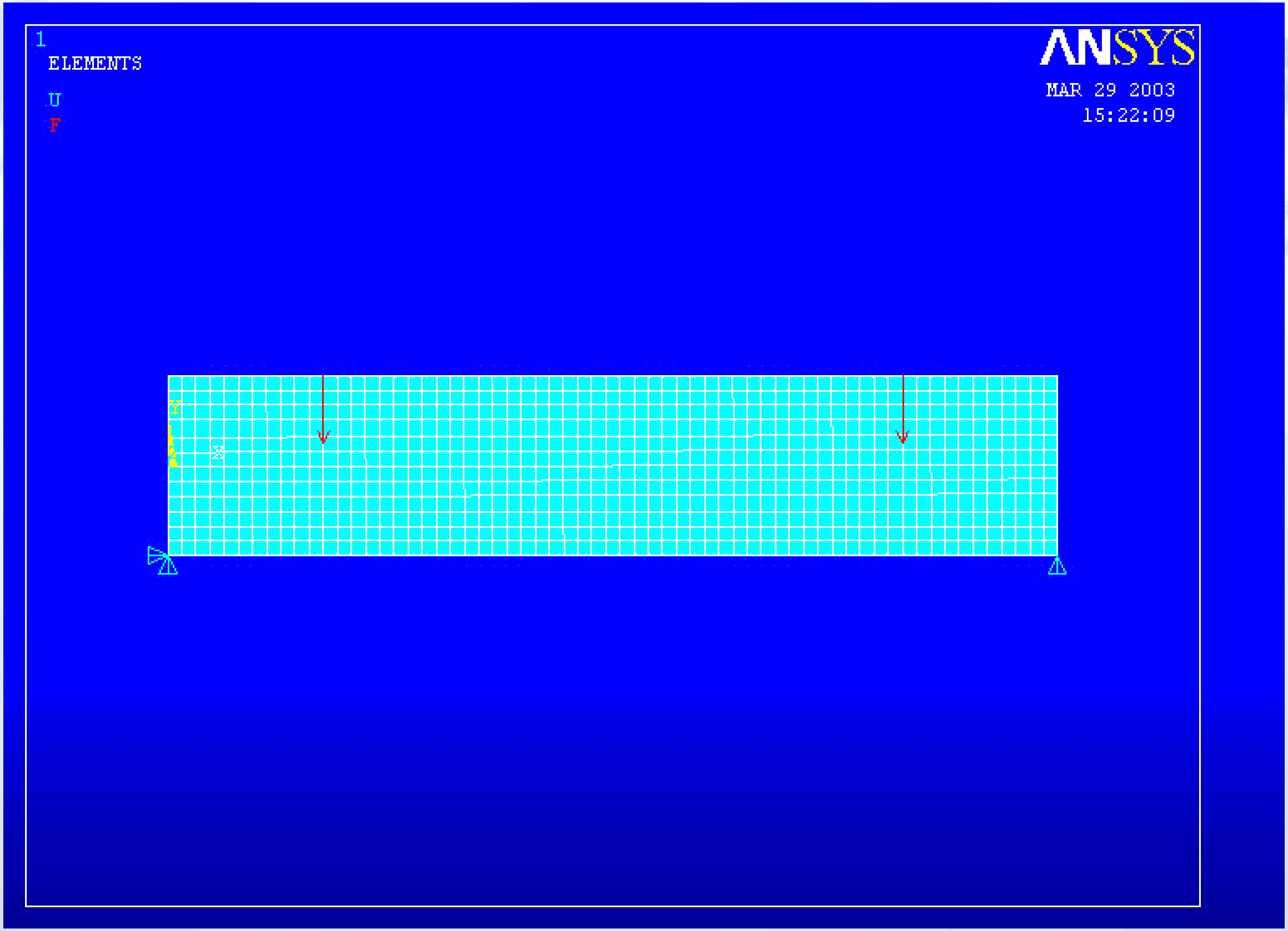
$$\boxed{\sigma=\frac{M\cdot y}{I_z}} \quad \text{-----------（5-2）}$$

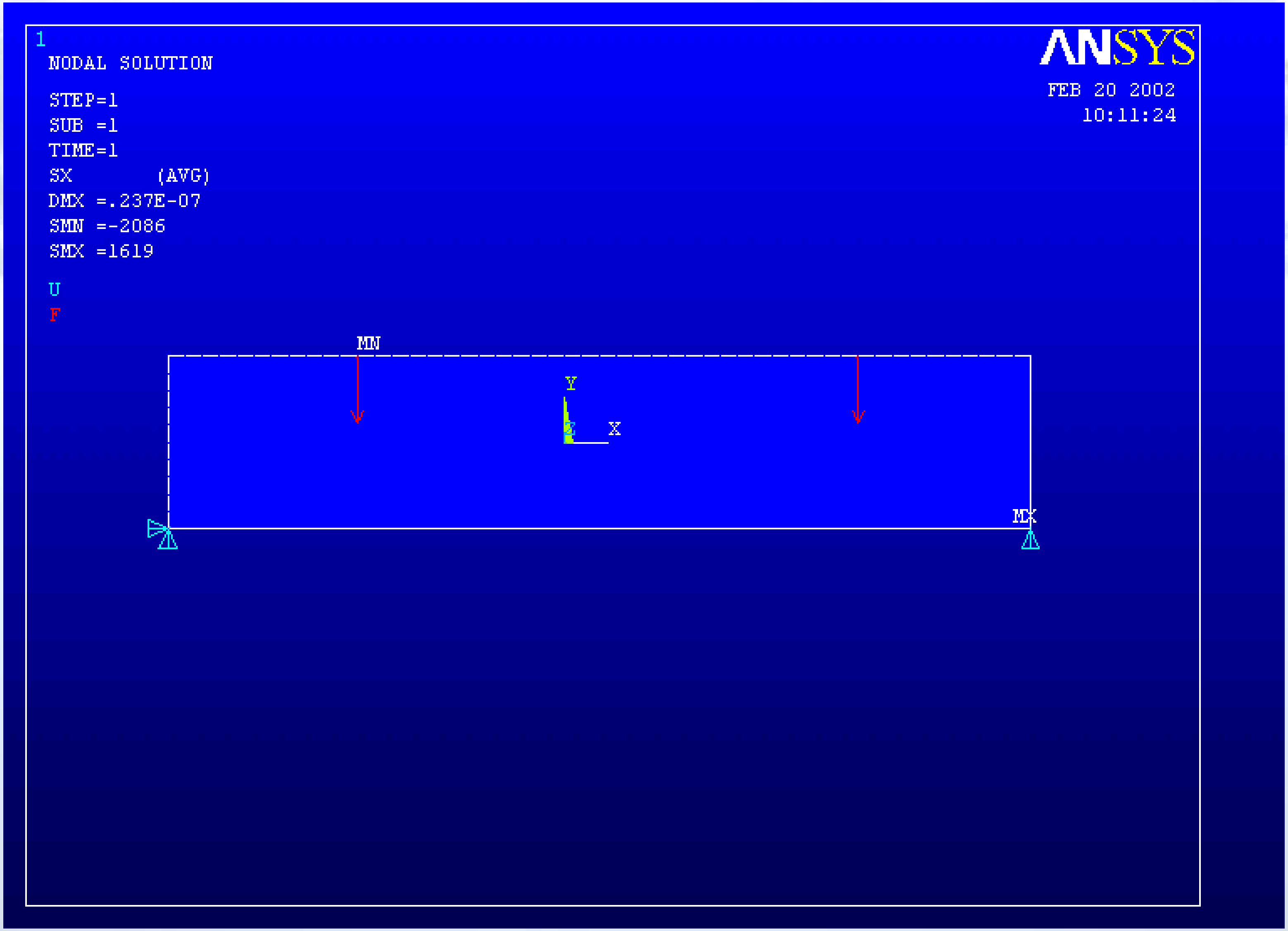


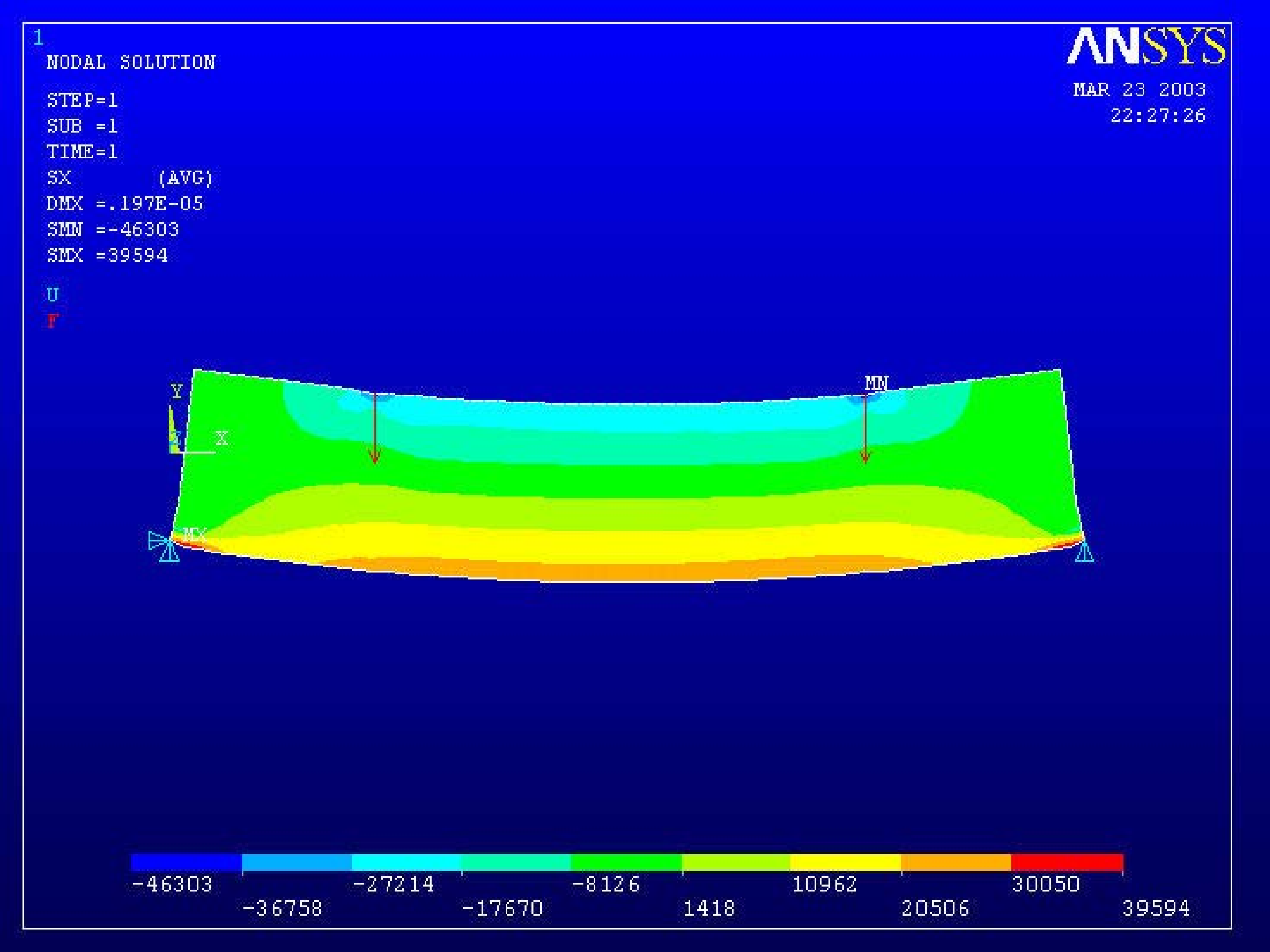
$$\sigma = \frac{M \cdot y}{I_z} \quad \text{（5-2）}$$






1
ELEMENTS
U
F
ANSYS
MAR 29 2003
15:22:09


1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
DMX =.237E-07
SMN =-2086
SMX =1619
U
F
ANSYS
FEB 20 2002
10:11:24
MN
Y
X
MX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NODAL SOLUTION
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TIME=1
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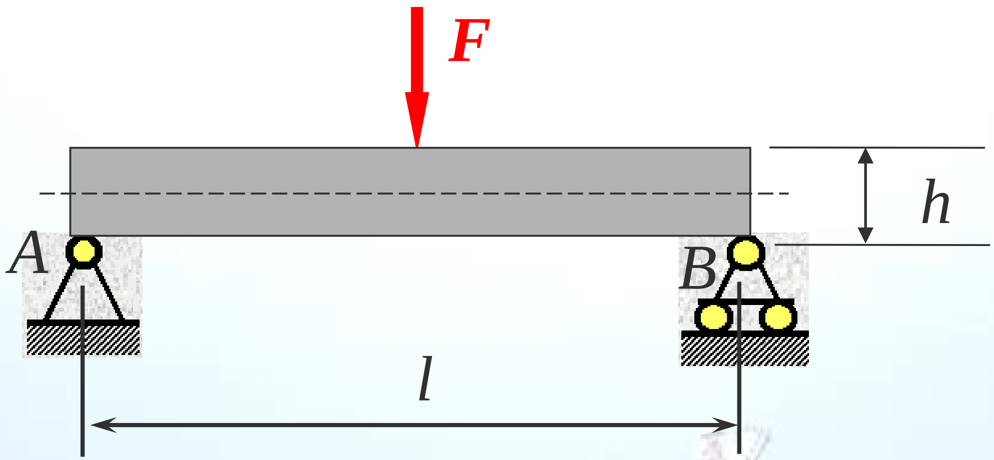
SMN =-46303
SMX =39594
U
F
ANSYS
MAR 23 2003
22:27:26
Y
X
MN
MX
-46303
-36758
-27214
-17670
-8126
1418
10962
20506
30050
39594

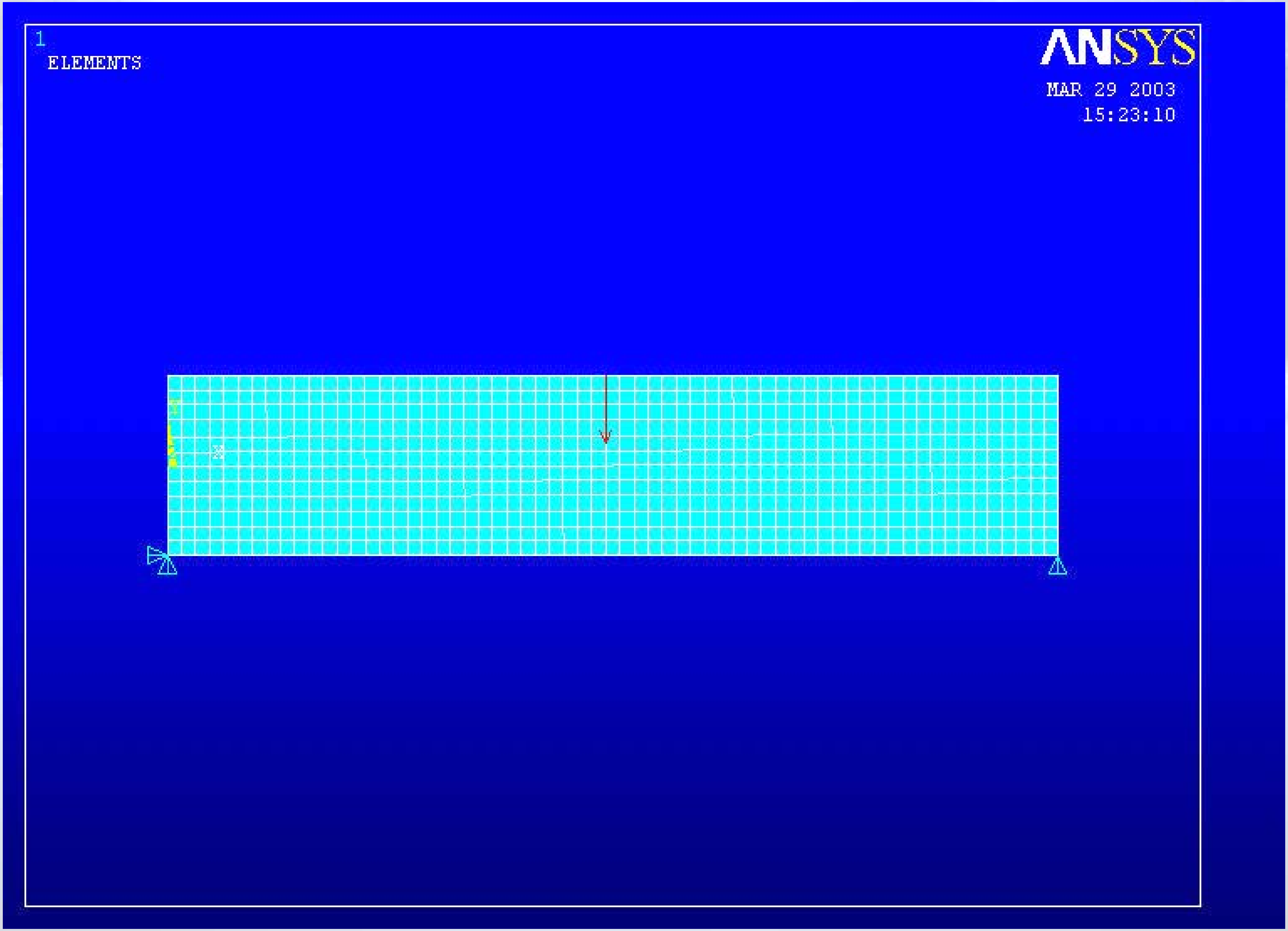
§ 5–3　横力弯曲时的正应力

一、横力弯曲时的正应力



对于横力弯曲，当 $\dfrac{l}{h}>5$ 时，按纯弯曲时的公式计算正应力，误差不超过1%。

$$\sigma=\frac{M\cdot y}{I_z}$$


1
ELEMENTS
ANSYS
MAR 29 2003
15:23:10


1
NODAL SOLUTION
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SUB =1
TIME=1
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SMN =-626.582
SMX =360.233
ANSYS
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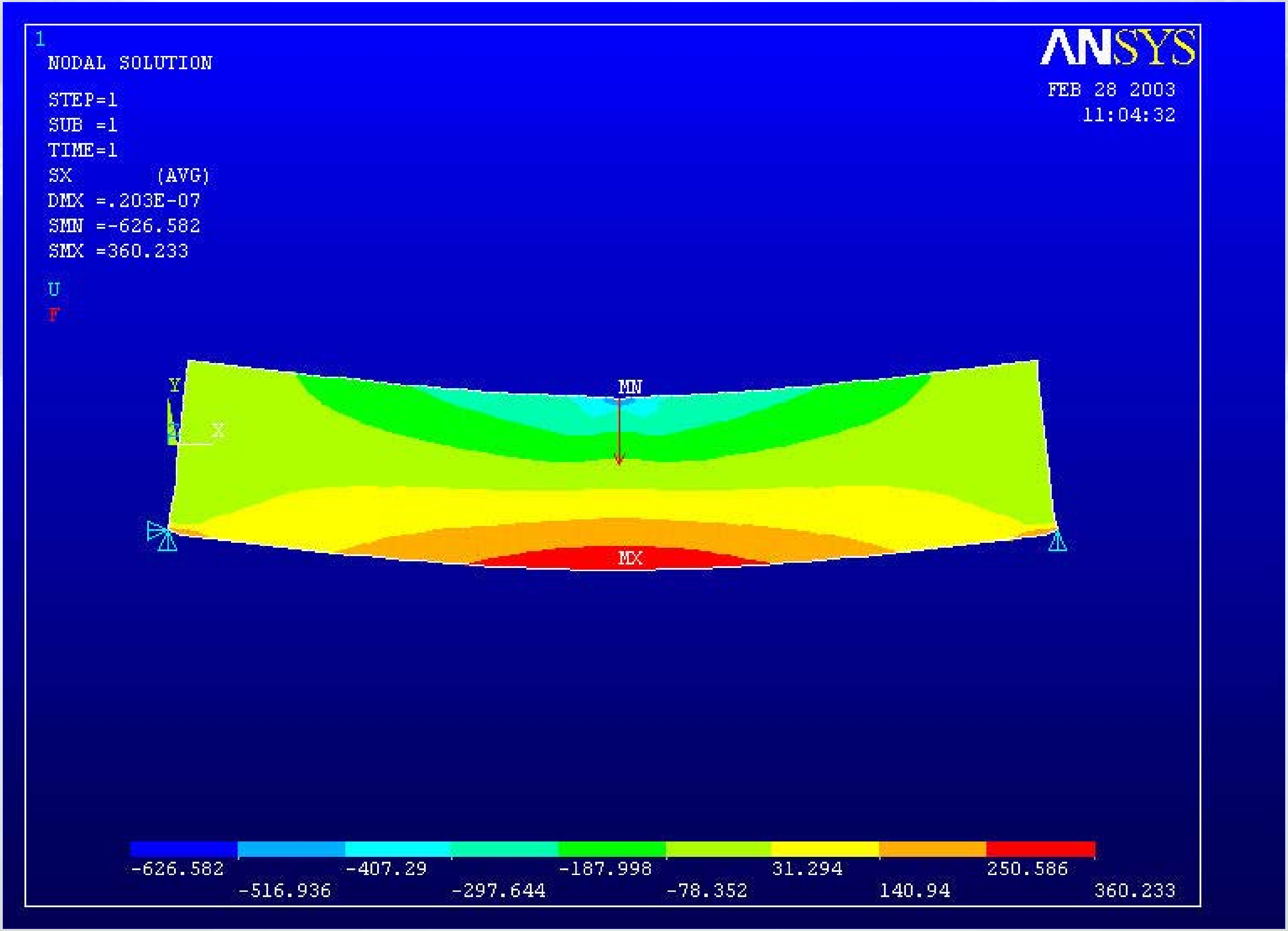
11:16:43
MN
Y
Z
X
MX


1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
DMX =.203E-07
SMN =-626.582
SMX =360.233
U
F
ANSYS
FEB 28 2003
11:04:32
Y
X
MN
MX
-626.582
-516.936
-407.29
-297.644
-187.998
-78.352
31.294
140.94
250.586
360.233

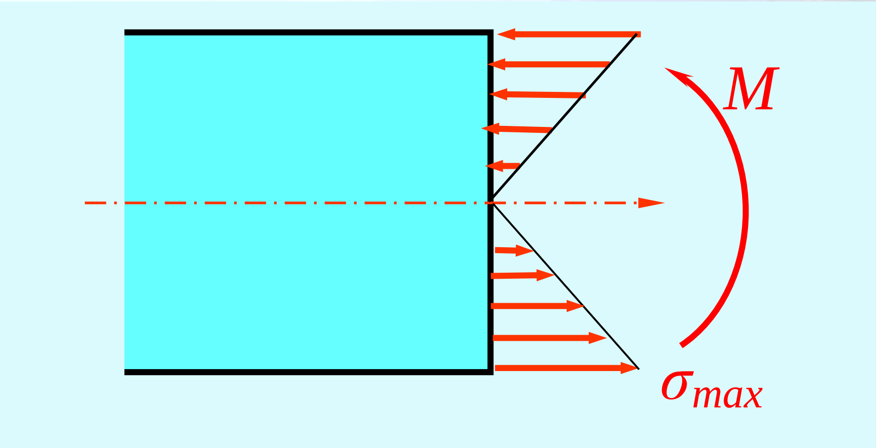
二、最大正应力：

$$\sigma_{\max}=\frac{My_{\max}}{I_z}$$

记：$W_Z=\dfrac{I_z}{y_{\max}}$

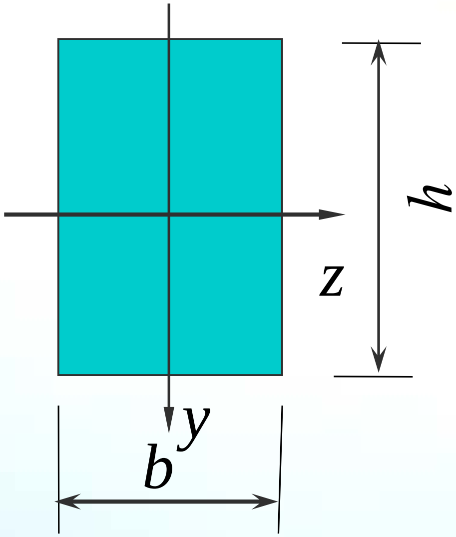
W_z称为抗弯截面系数

$$\boxed{\sigma_{\max}=\frac{M}{W_z}}$$



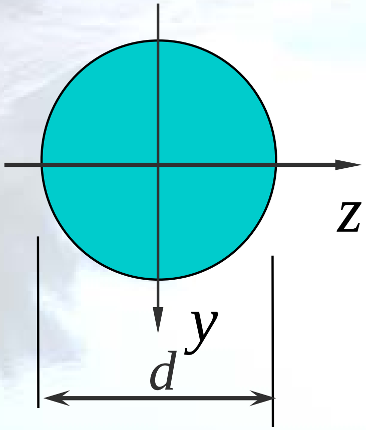
抗弯截面系数：

矩形：



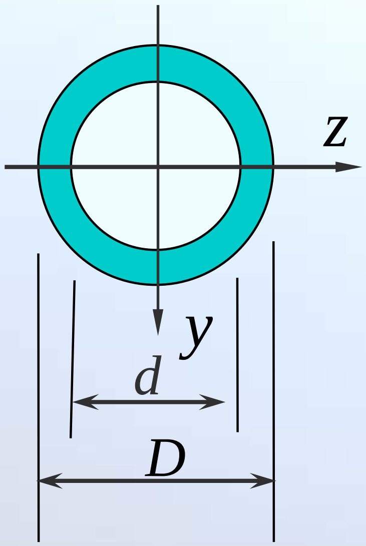
$$I_z = \frac{bh^3}{12}, \quad y_{\max} = \frac{h}{2}$$

$$\boxed{W_z = \frac{bh^2}{6}}$$



实心圆： $I_z = \dfrac{\pi d^4}{64}$， $y_{\max} = \dfrac{d}{2}$

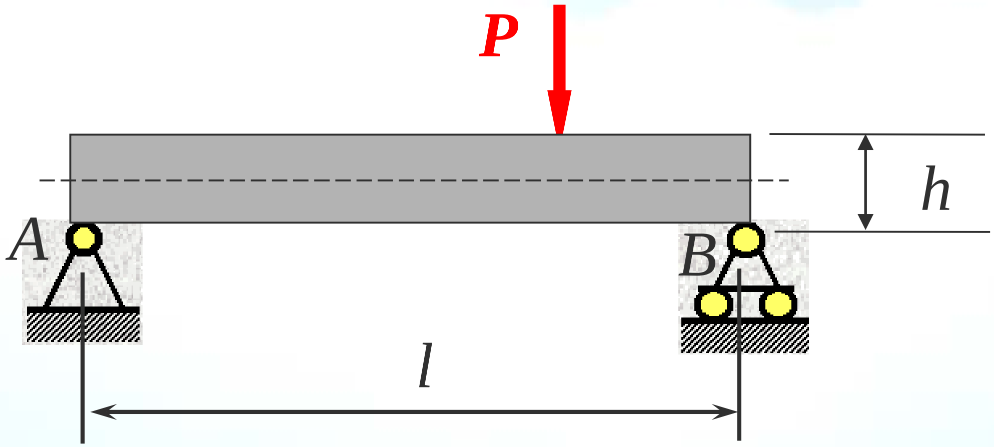
$$\boxed{W_z = \frac{\pi d^3}{32}}$$

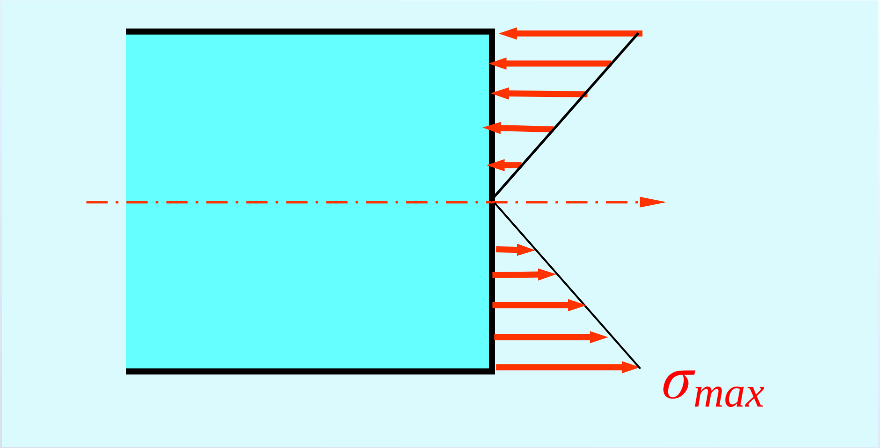


空心圆：$I_z = \dfrac{\pi D^4}{64}(1-\alpha^4)$ ， $y_{\max} = \dfrac{D}{2}$

$$\boxed{W_z = \frac{\pi D^3}{32}(1-\alpha^4)} \qquad \alpha = \frac{d}{D}$$

三、梁的正应力强度条件





$$\sigma_{\max} \leq [\sigma]$$

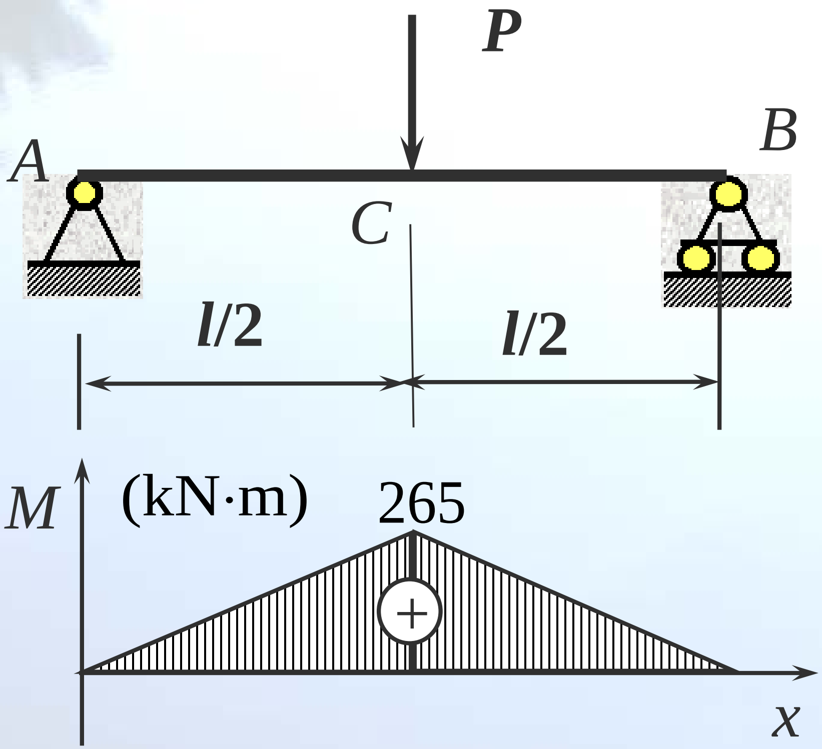
$$\sigma_{\max} = \frac{M_{\max}}{W_z} \leq [\sigma]$$

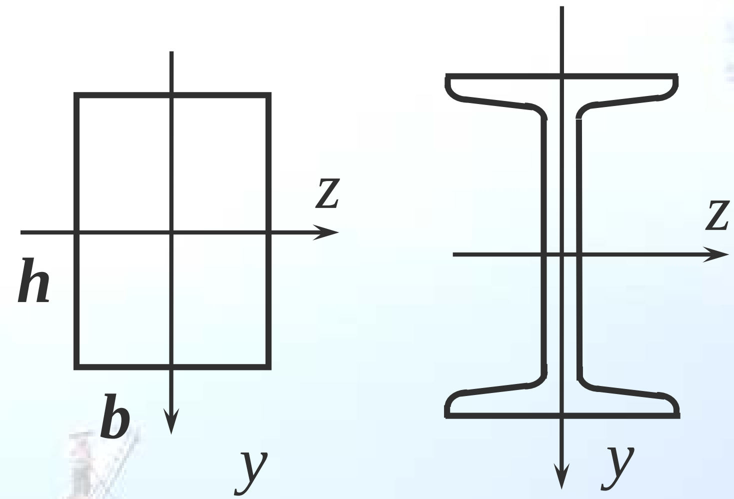
[例1] 图示钢梁，[σ]=170MPa，P=265kN，l=4m，

求：1）设计h/b=1.5的矩形截面梁；

2）选择工字钢型号；

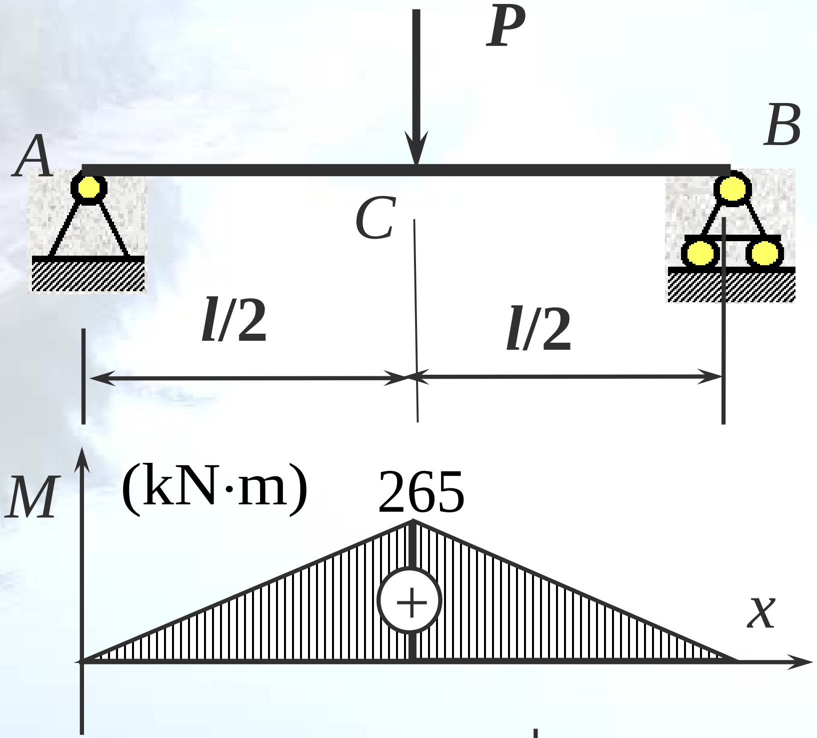
3）比较这两种截面梁的耗材。





解：(1)求支座反力, 画弯矩图

$$M_{\max}=\frac{Pl}{4}=265(\text{kN}\cdot\text{m})$$

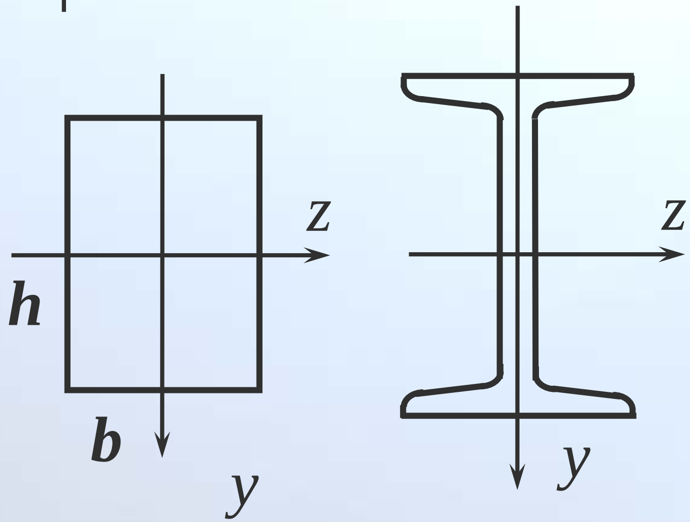


(2) 矩形截面梁

$$\sigma_{\max}=\frac{M_{\max}}{W_z}=\frac{M_{\max}}{\frac{bh^2}{6}}=\frac{6M_{\max}}{1.5^2 b^3}\le[\sigma]$$

$$\therefore\ b\ge\sqrt[3]{\frac{6M_{\max}}{1.5^2[\sigma]}}=160.8(\text{mm})$$

$$\therefore\ h=241.2(\text{mm})$$



(3) 工字形截面梁

$$\sigma_{\max}=\frac{M_{\max}}{W_z}\le[\sigma]$$

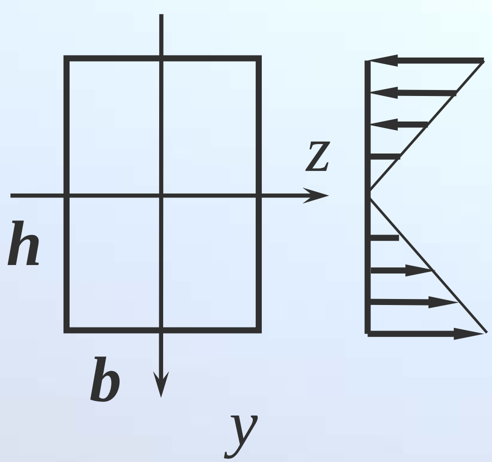
$$\therefore\ W_z\ge\frac{M_{\max}}{[\sigma]}=1558.8(\text{cm}^3)$$

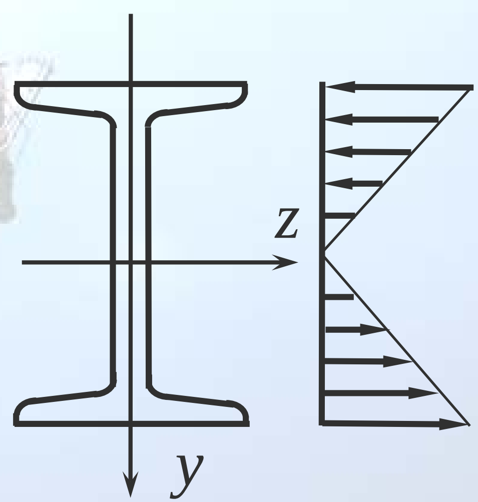
查表，选择No. 45c工字钢　$W_z = 1570\text{cm}^3$

(4) 比较耗材

$$\frac{A_{矩}}{A_{工}} = \frac{160.8 \times 241.2}{12000} = 3.23$$

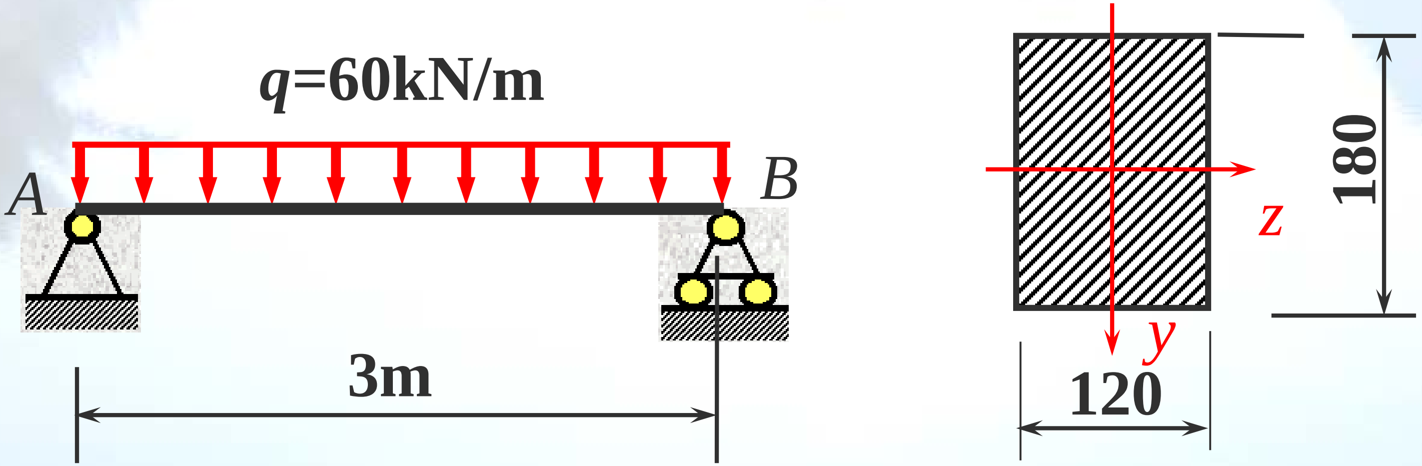
工字钢耗材是矩形截面梁的三分之一。

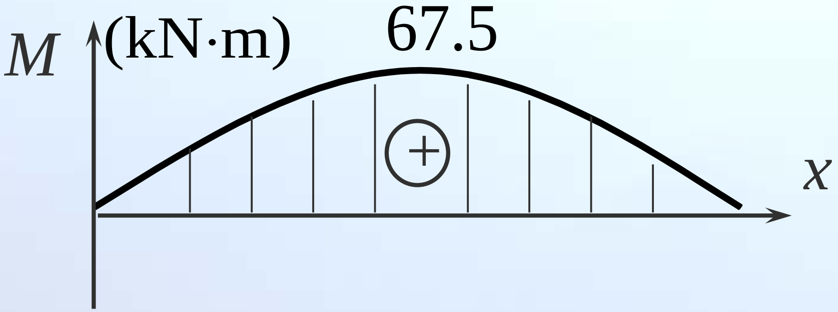




[例2] 受均布载荷作用的简支梁如图所示，

试求：梁内的最大正应力；





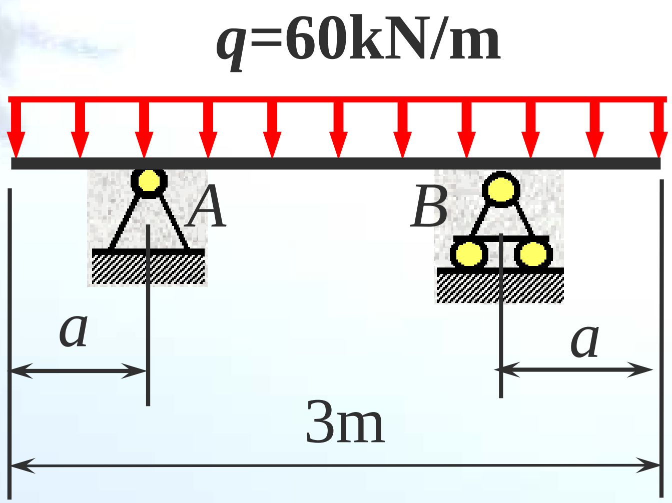
解：$M_{\max}=67.5\text{kNm}$

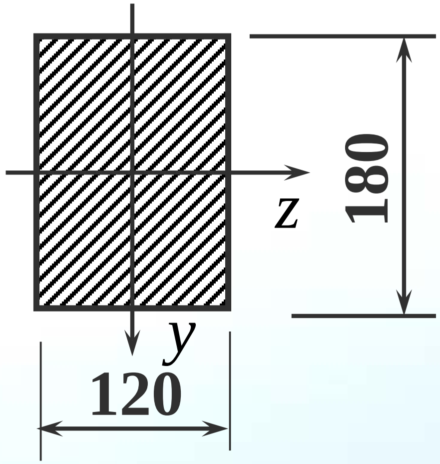
$$W_z=\frac{bh^2}{6}=\frac{120\times180^2}{6}$$

$$=6.48\times10^5\,(\text{mm}^3)$$

$$\sigma_{\max}=\frac{M_{\max}}{W_z}=\frac{67.5\times10^6}{6.48\times10^5}=104.2\text{MPa}$$

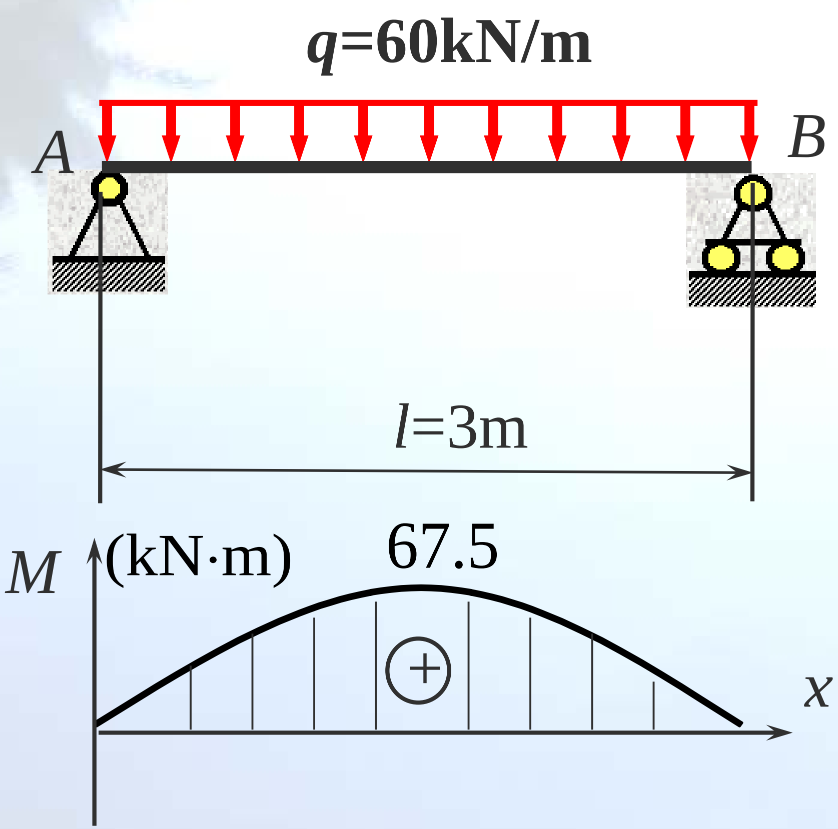
[例3] 支座A和B放在什么位置，梁的受力最合理。

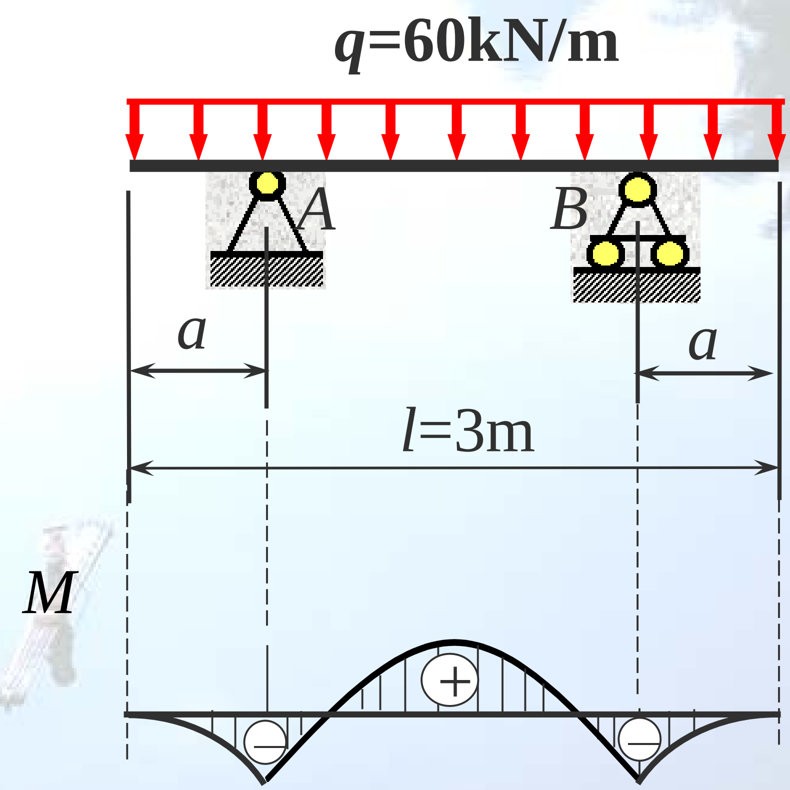


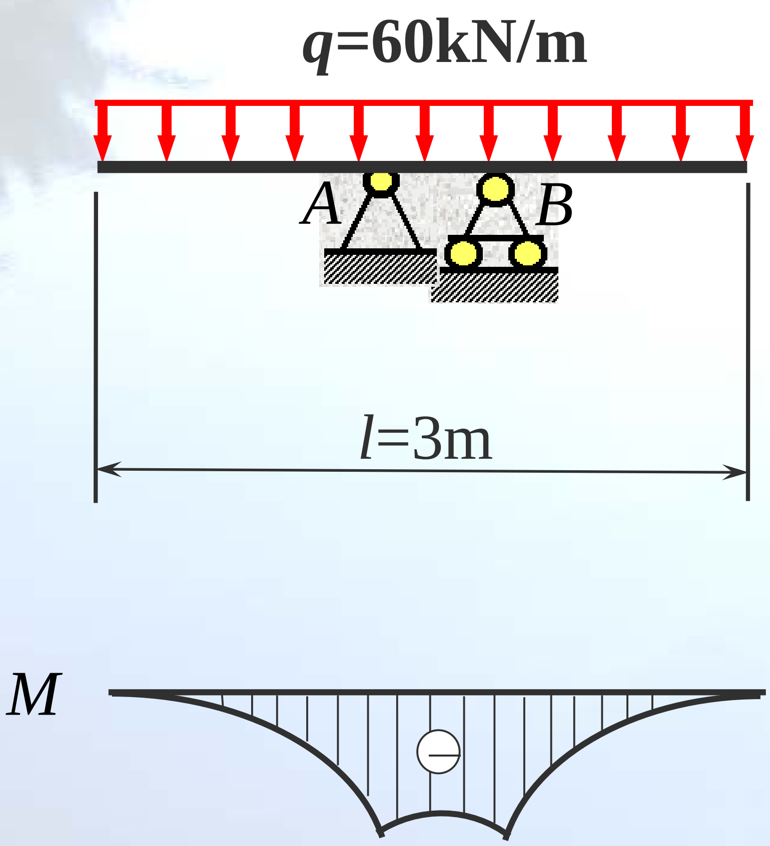


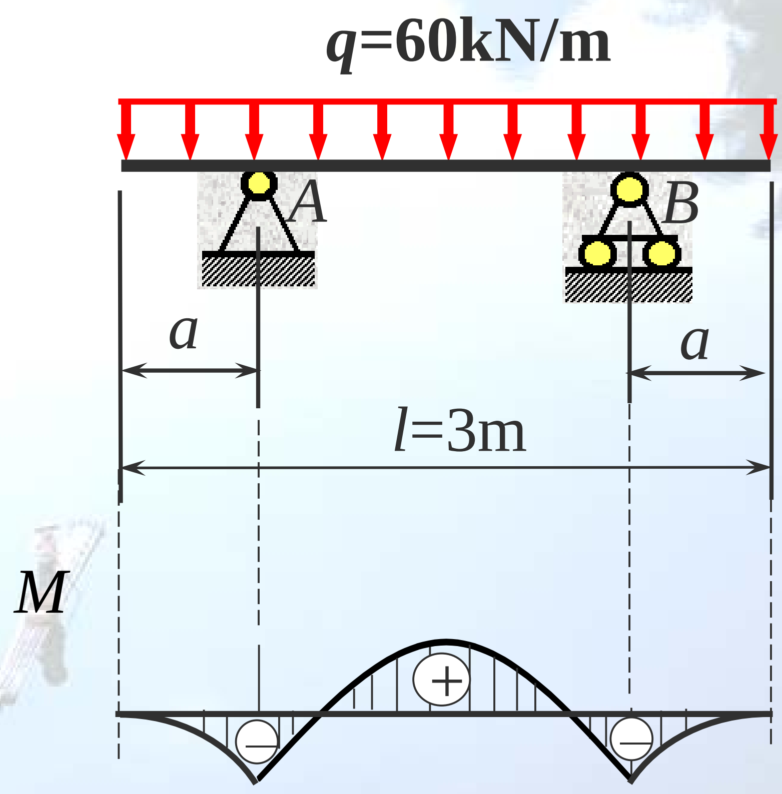
解： 考虑两种极限情况

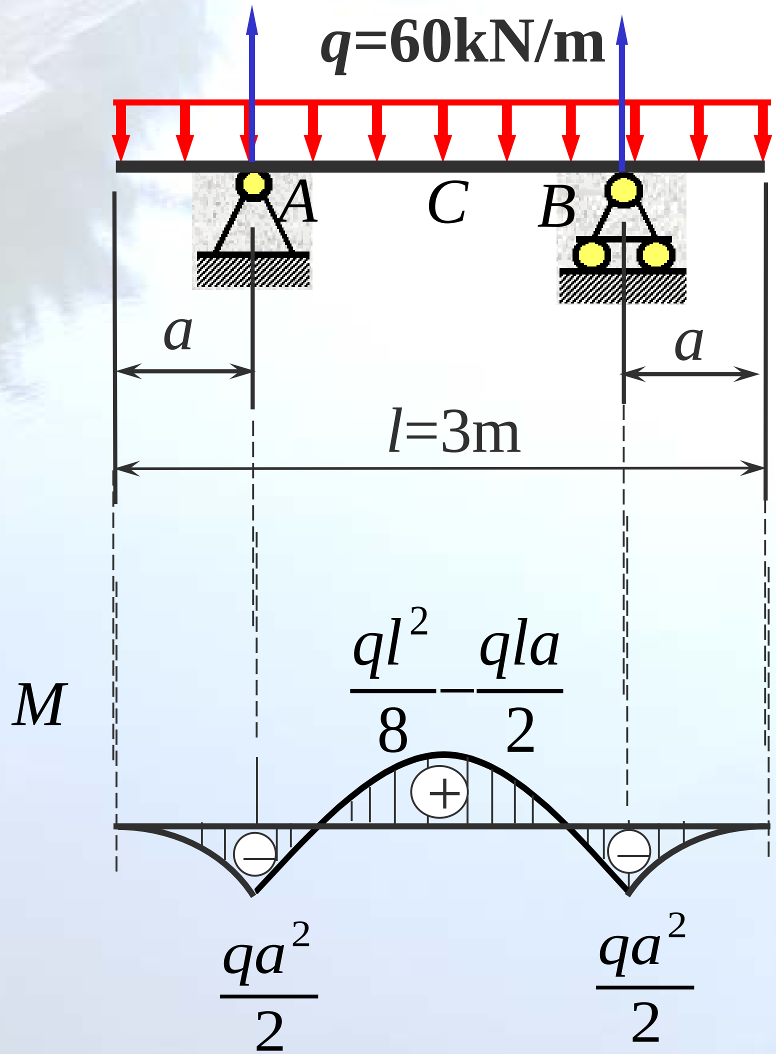
a=0 和 a=1.5m


q=60kN/m
A
B
l=3m
M
(kN·m)
67.5
x


q=60kN/m
A
B
a
a
l=3m
M


q=60kN/m
A
B
l=3m
M


q=60kN/m
A
B
a
a
l=3m
M



当 $|M_C|=|M_A|$ 时，梁受力最合理：

$$M_A=M_B=-\frac{qa^2}{2}$$

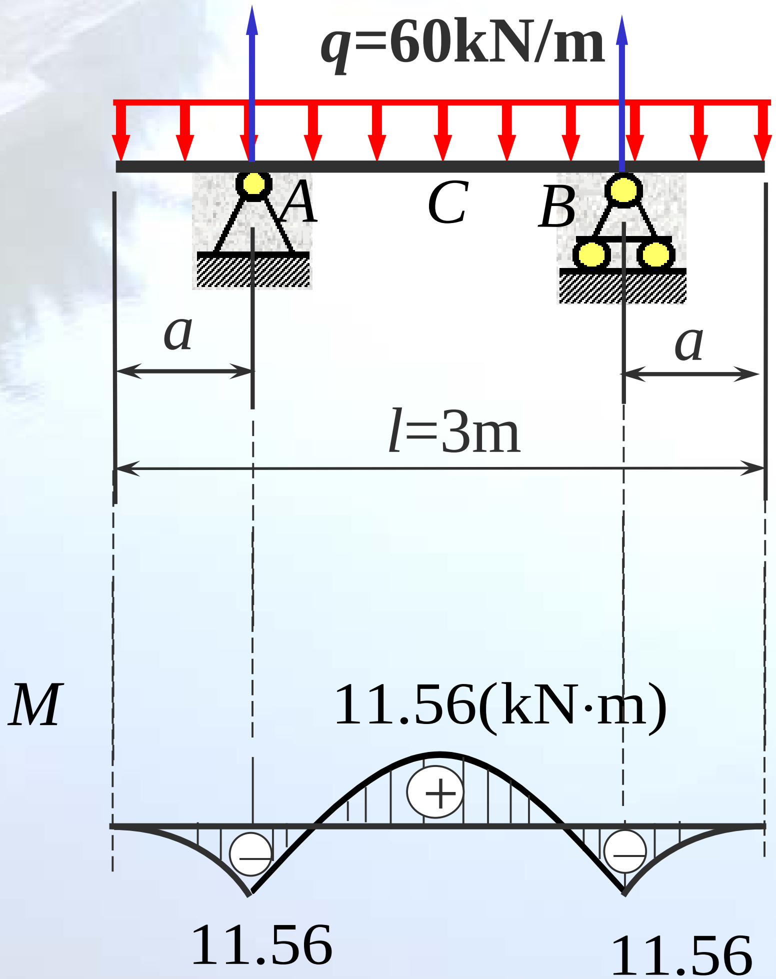
$$M_C=\frac{ql}{2}\times\left(\frac{l}{2}-a\right)-\frac{ql^2}{8}=\frac{ql^2}{8}-\frac{qla}{2}$$

$$\therefore\ \frac{ql^2}{8}-\frac{qal}{2}=\frac{qa^2}{2}$$

$$4a^2+4la-l^2=0$$

$$a=\frac{-l\pm\sqrt{l^2+l^2}}{2}$$ 舍去负值

$$a=\frac{1}{2}(\sqrt{2}-1)l\ =0.207l$$



$$M_A = M_B = -11.56(\mathrm{kN \cdot m})$$

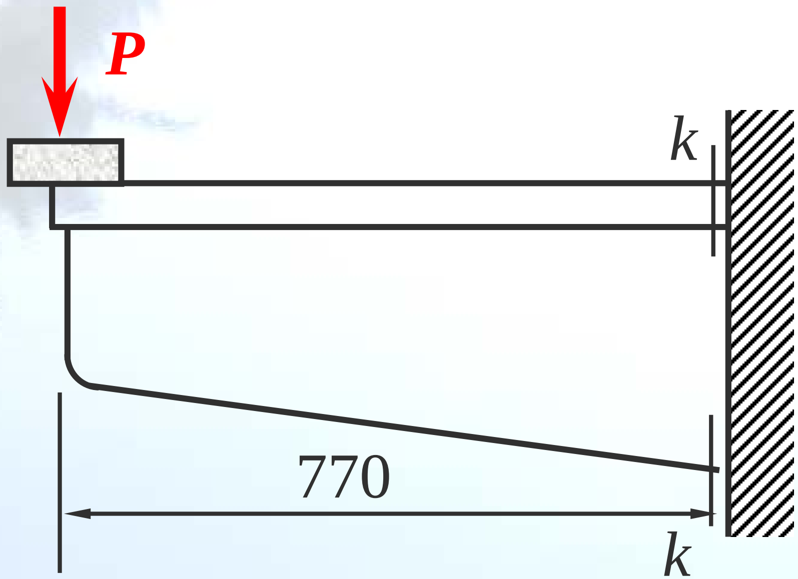
$$M_C = 11.56(\mathrm{kN \cdot m})$$

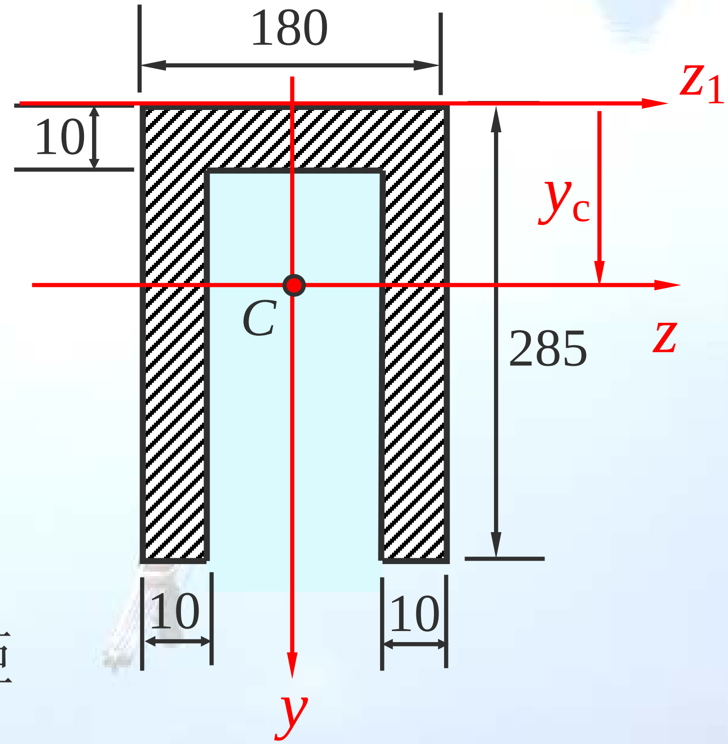
最大弯矩下降了：

$$\frac{67.5-11.56}{67.5} = 0.82 = 82\%$$

梁内最大正应力同样下降了82%。

[例4] 铸铁梁，受力如图，铸铁的$[\sigma_t]$=20MPa，$[\sigma_c]$=60 MPa，试根据危险截面*k-k*的强度，确定最大载荷*P*。





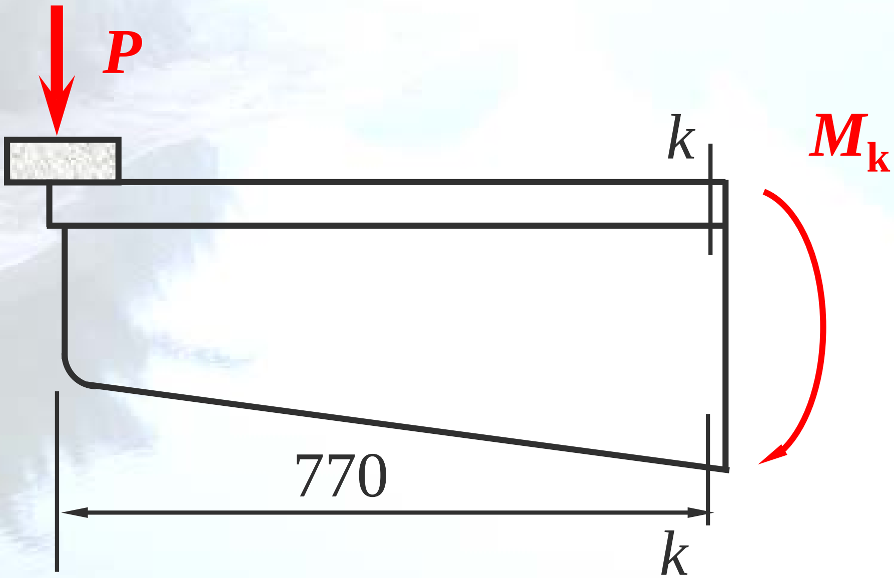
解：(1) 求形心位置和惯性矩

$$y_c = 112.3(\mathrm{mm})$$

$$I_z = 6220 \times 10^4 (\mathrm{mm}^4)$$

(2)求危险截面上的弯矩

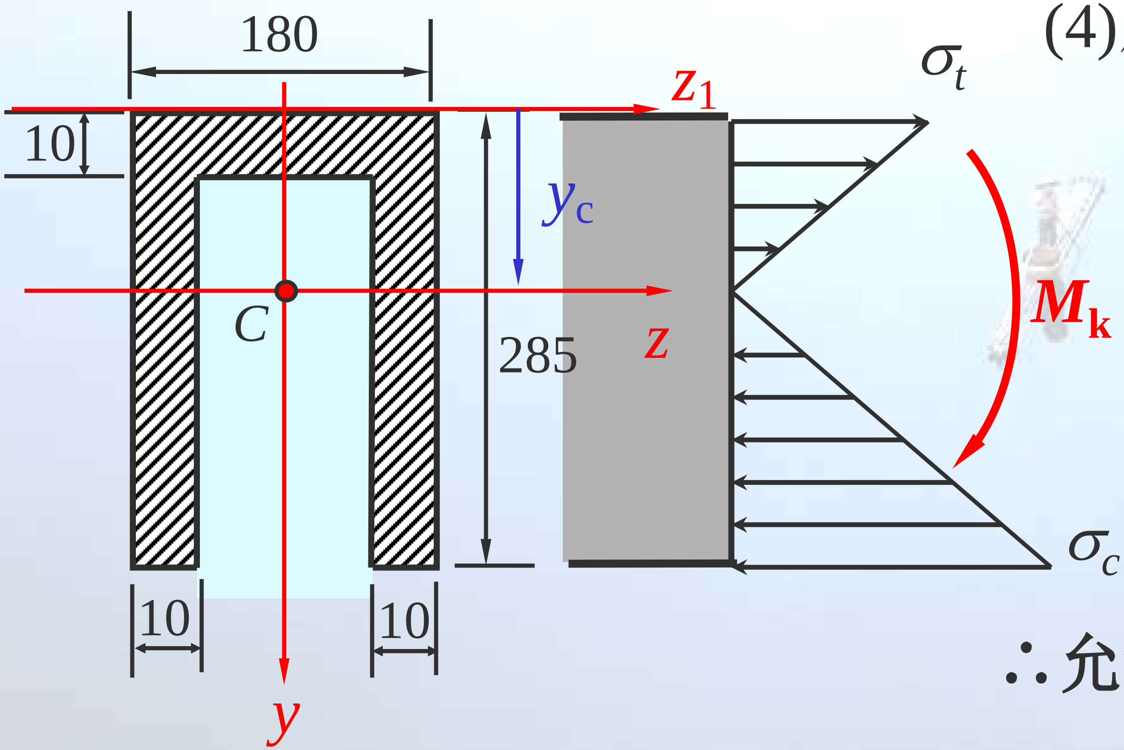
$$M_K = -P \times 770(\mathrm{N \cdot mm})$$



(3) 拉应力强度

$$\sigma_t=\frac{M_k y_c}{I_z}=\frac{P\times770\times112.3}{6220\times10^4}\le[\sigma_t]$$

$$\therefore P\le14.4(\mathrm{kN})$$



(4)压应力强度

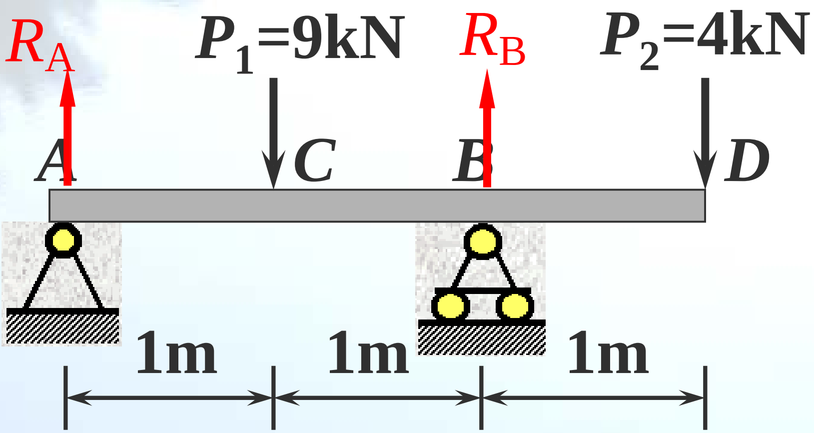
$$\sigma_c=\frac{M_k(285-y_c)}{I_z}$$

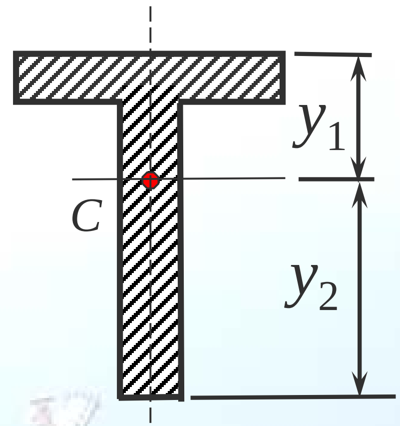
$$=\frac{P\times770\times172.7}{6220\times10^4}\le[\sigma_c]$$

$$\therefore P\le28.1(\mathrm{kN})$$

∴允许的最大载荷P≤14.4kN

[例5] T 字形截面的铸铁梁，受力如图，铸铁的$[\sigma_t]$=30MPa，$[\sigma_c]$=60 MPa，其截面形心位于C点，y_1=52mm， y_2=88mm，I_z=763cm^4 ，试校核此梁的强度。

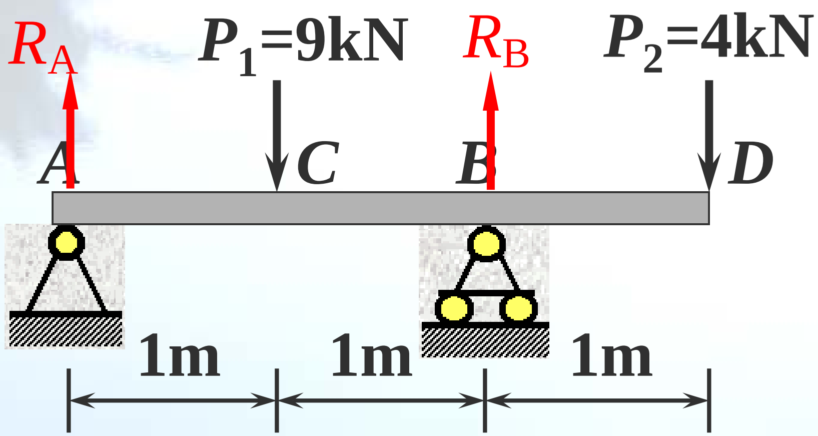


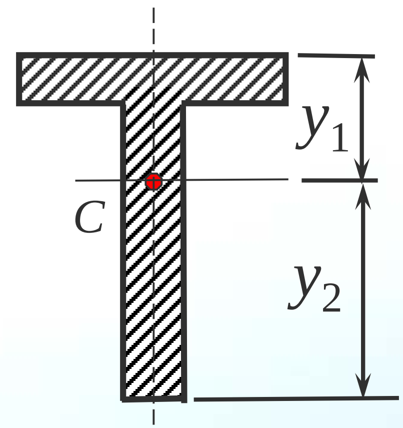


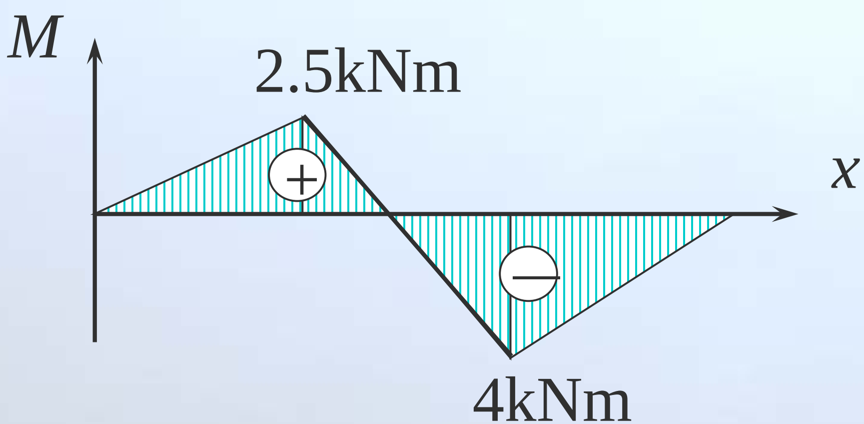
解：(1) 求支座反力

$$R_A=2.5\text{kN}(\uparrow) \qquad R_B=10.5\text{kN}(\uparrow)$$

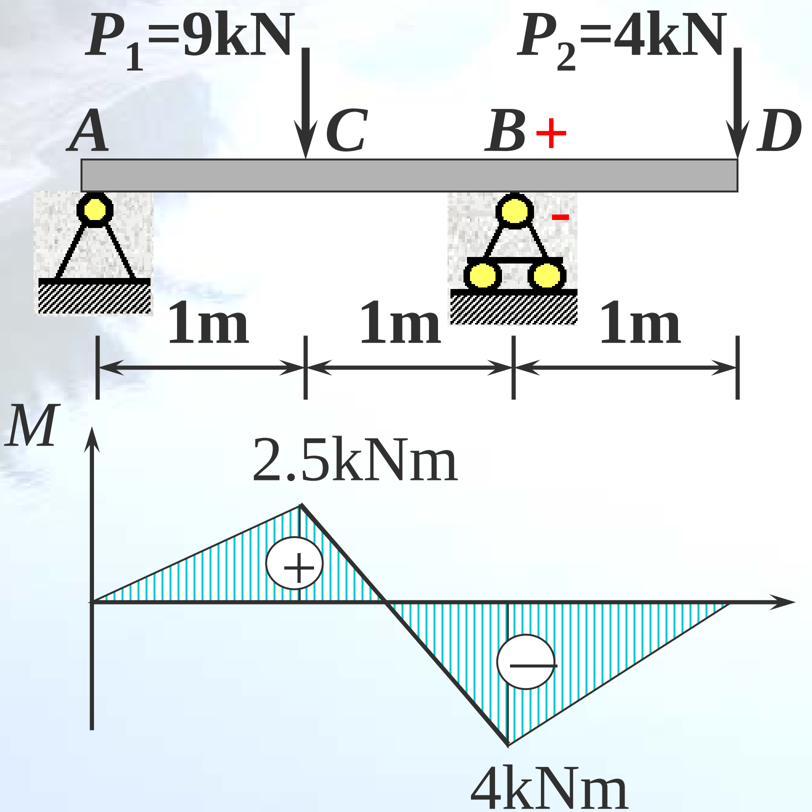
（2）画弯矩图找危险截面

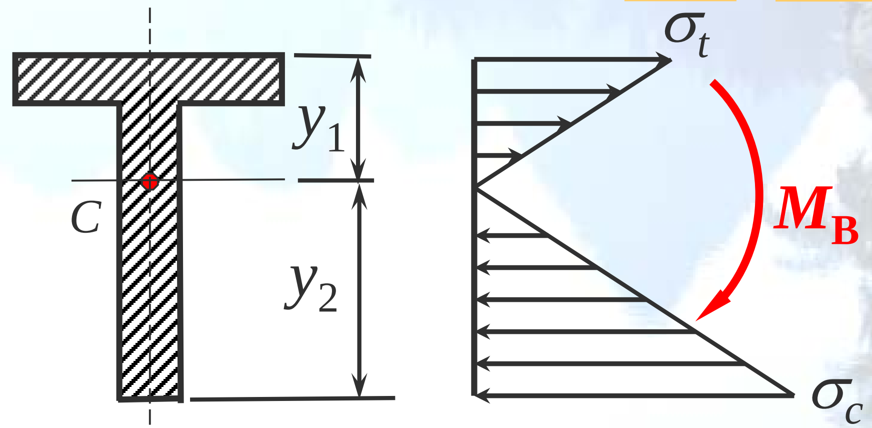






B截面弯矩最大，是危险截面



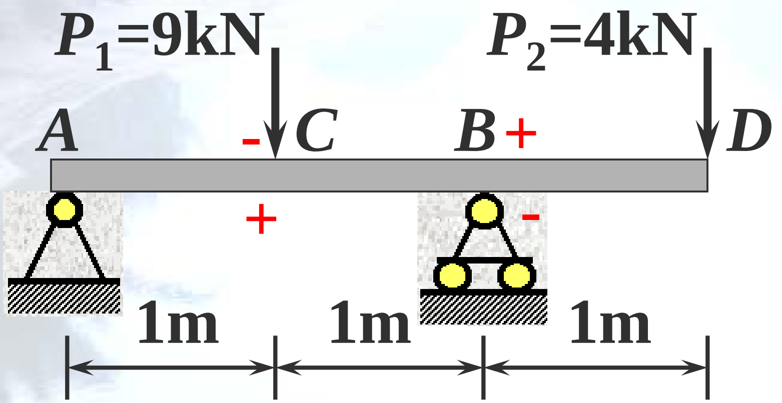


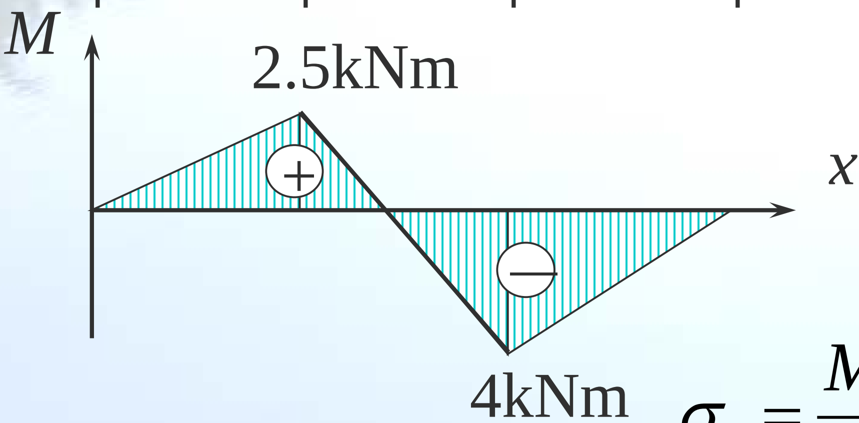
(2) B截面的强度

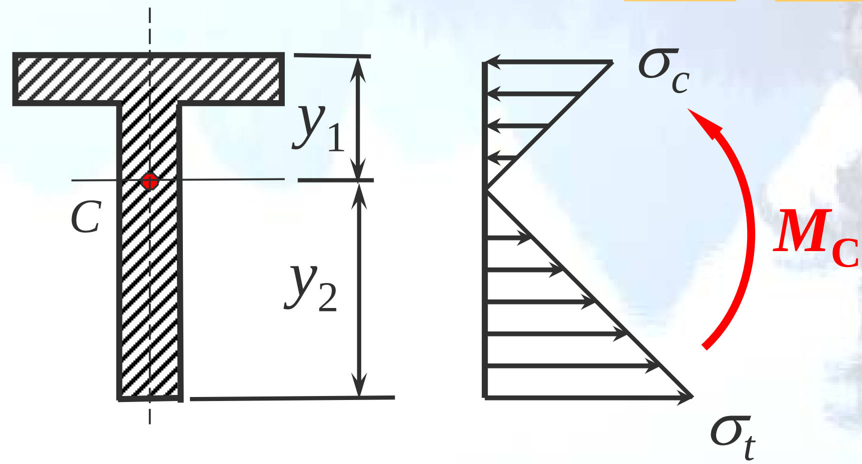
负弯矩，上边缘受拉，下边缘受压

$$\sigma_t = \frac{M_B y_1}{I_z} = \frac{4\times10^6\times52}{763\times10^4} = 27.2\text{MPa} \quad <[\sigma_t]$$

$$\sigma_c = \frac{M_B y_2}{I_z} = \frac{4\times10^6\times88}{763\times10^4} = 46.2\text{MPa} <[\sigma_c]$$







(3) C截面的强度

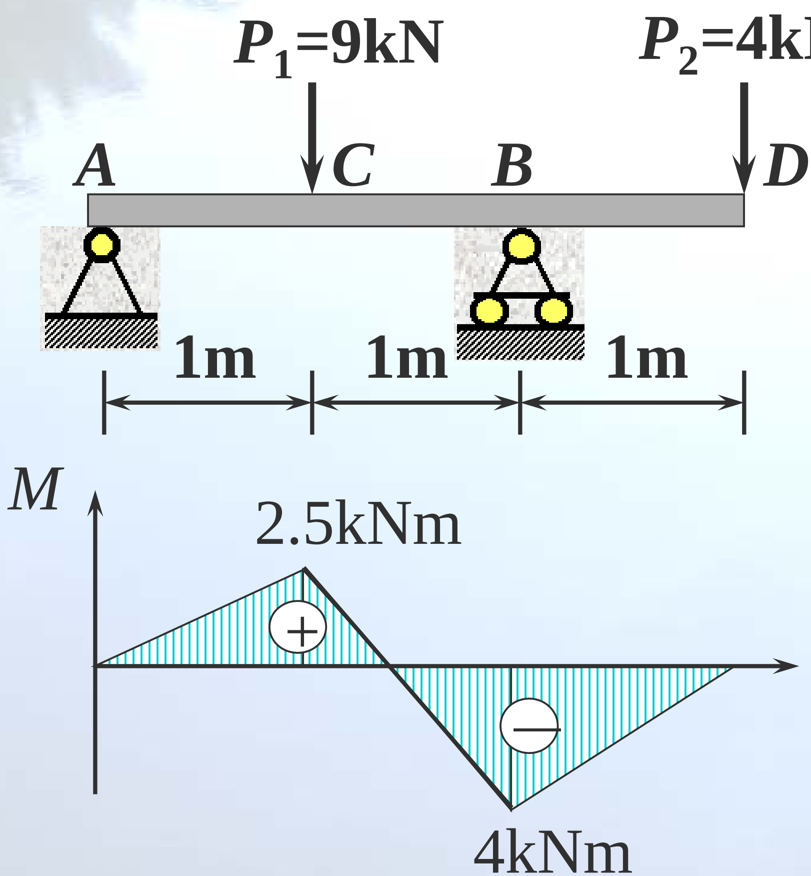
正弯矩，下边缘受拉，上边缘受压

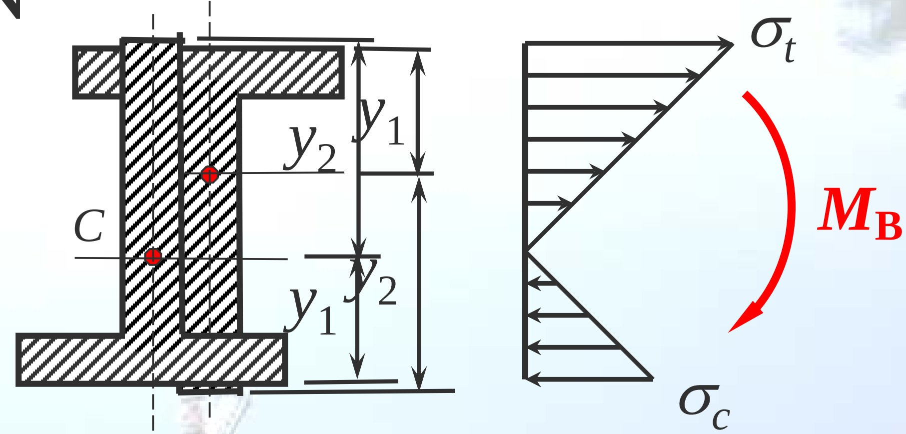
$$\sigma_t = \frac{M_C y_2}{I_z} = \frac{2.5\times10^6\times88}{763\times10^4} = 28.2\text{MPa} < [\sigma_t]$$

$$\sigma_c = \frac{M_C y_1}{I_z} < \frac{M_B y_2}{I_z} \quad < [\sigma_c]$$

∴梁安全

讨论：若将T字形梁倒置，梁是否安全？



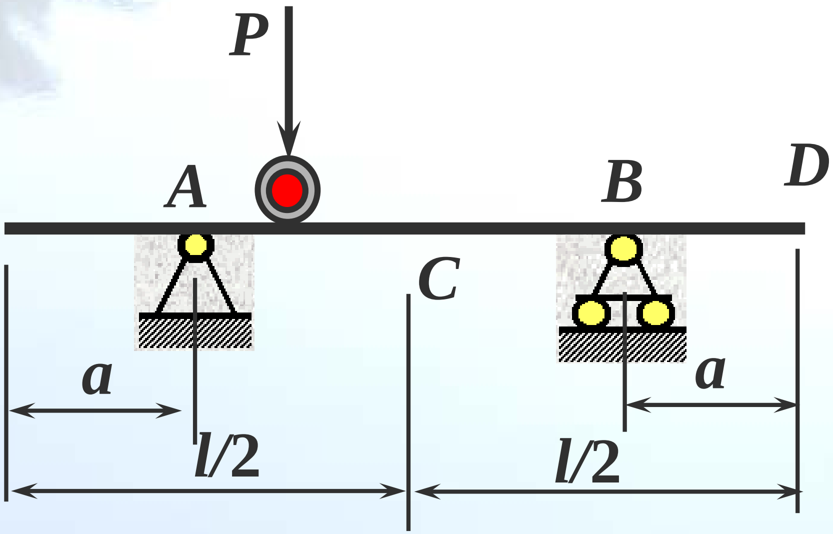


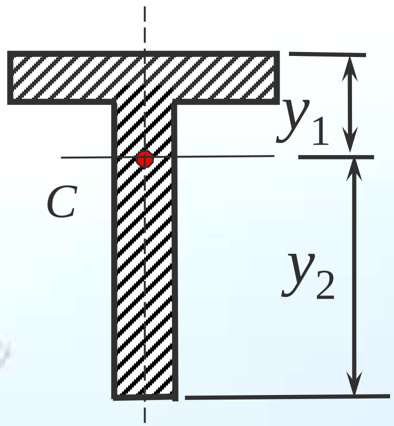
B截面的拉应力：

$$\sigma_t = \frac{M_B y_2}{I_z} = \frac{4\times10^6\times88}{763\times10^4} = 46.2\text{MPa} > [\sigma_t]$$

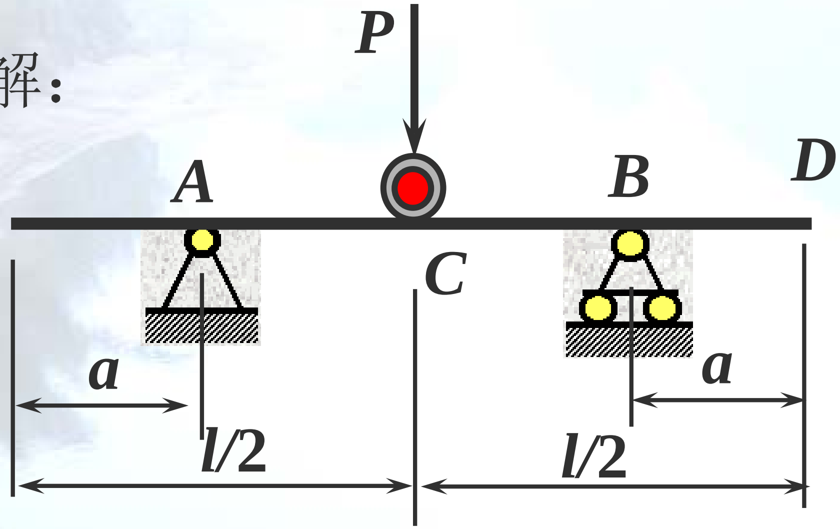
梁的强度不够。

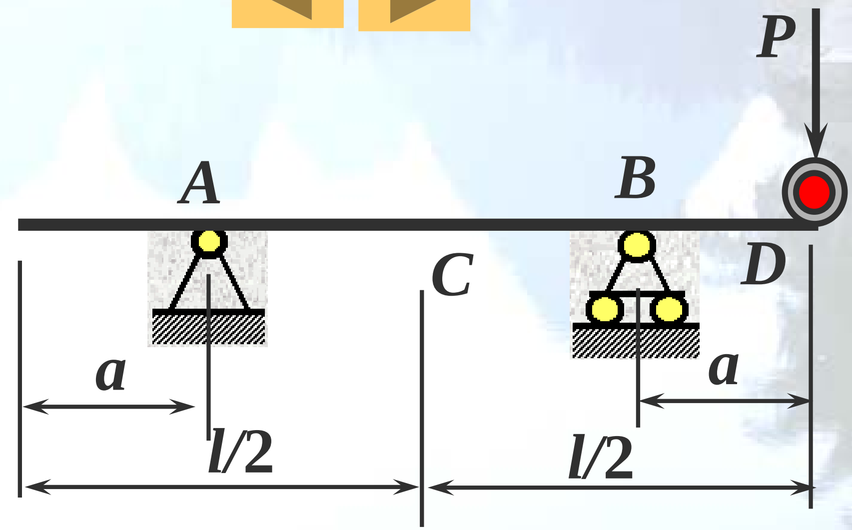
[例6] T 字形截面铸铁梁，梁长为l，受活动载荷，如图，已知许用拉应力与许用压应力之比$[\sigma_t]:[\sigma_c]=1:4$，$y_1:y_2=1:5$，试确定合理的 a 值。

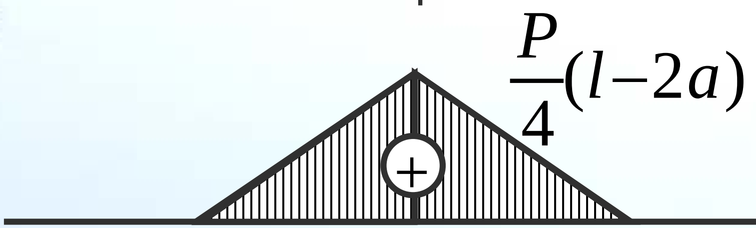


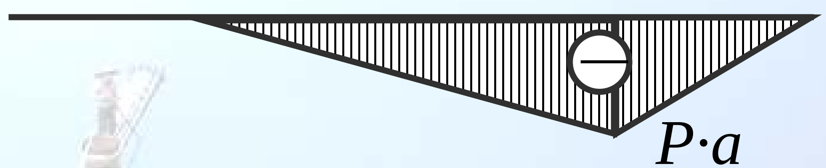


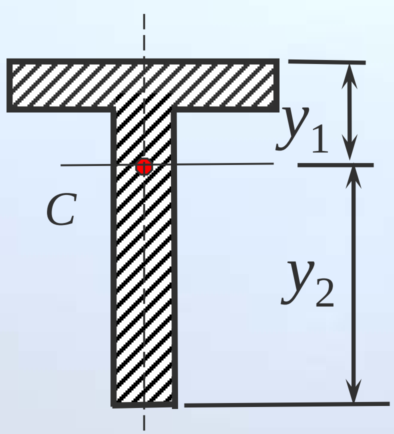
解:







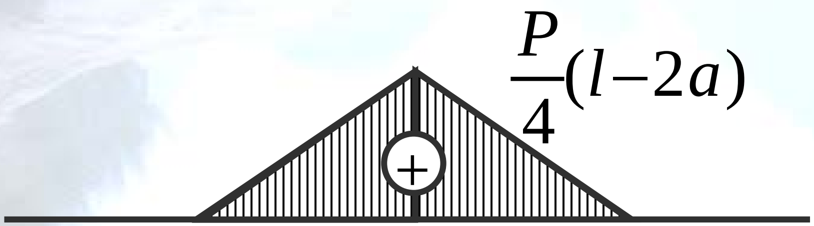


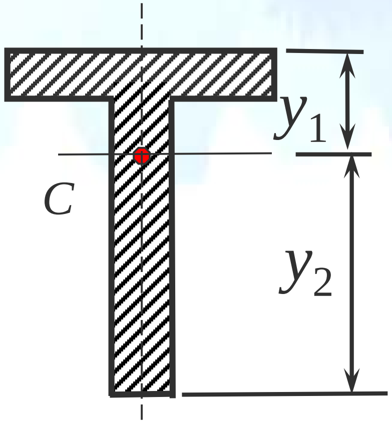


正弯矩：拉应力控制强度

负弯矩：压应力控制强度

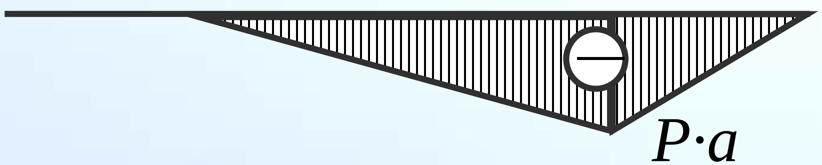
$[\sigma_t]:[\sigma_c]=1:4$，$y_1:y_2=1:5$





正弯矩：拉应力控制强度

$$\sigma_t=\frac{M_C y_2}{I_z}\leq[\sigma_t]$$



负弯矩：压应力控制强度

$$\sigma_c=\frac{M_B y_2}{I_z}\leq[\sigma_c]$$

$$\frac{M_C}{M_B}=\frac{[\sigma_t]}{[\sigma_c]}$$

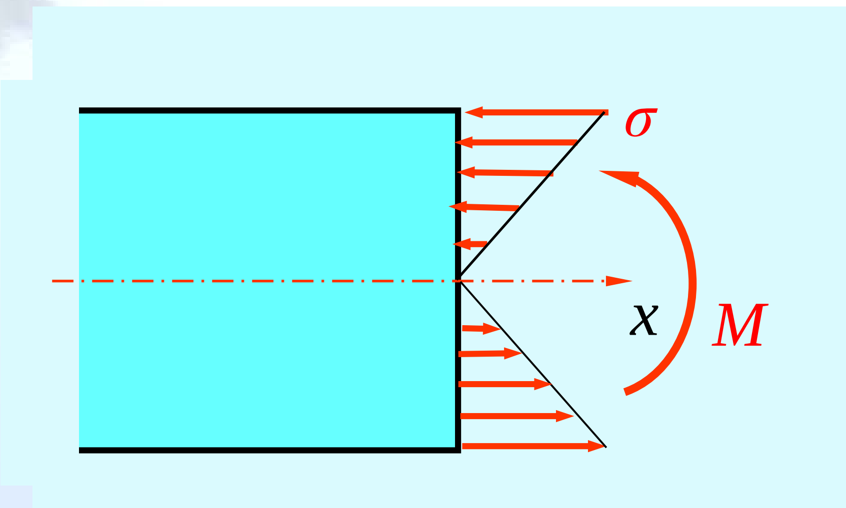
$[\sigma_t]:[\sigma_c]=1:4$，

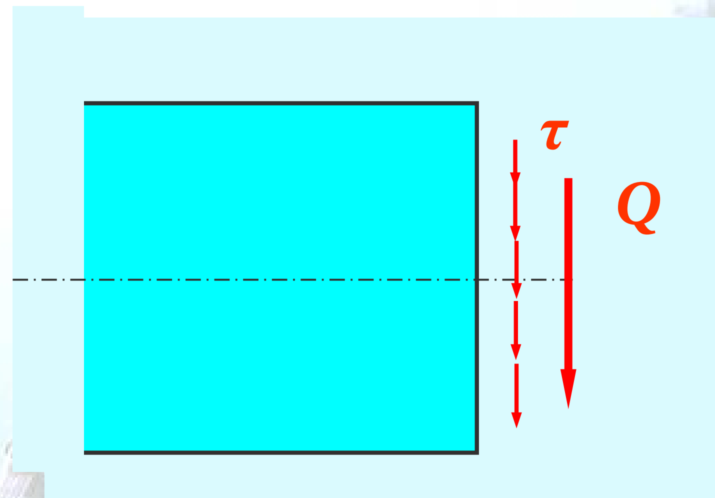
$$M_C=\frac{M_B}{4}$$

$$\frac{P}{4}(l-2a)=\frac{Pa}{4}$$

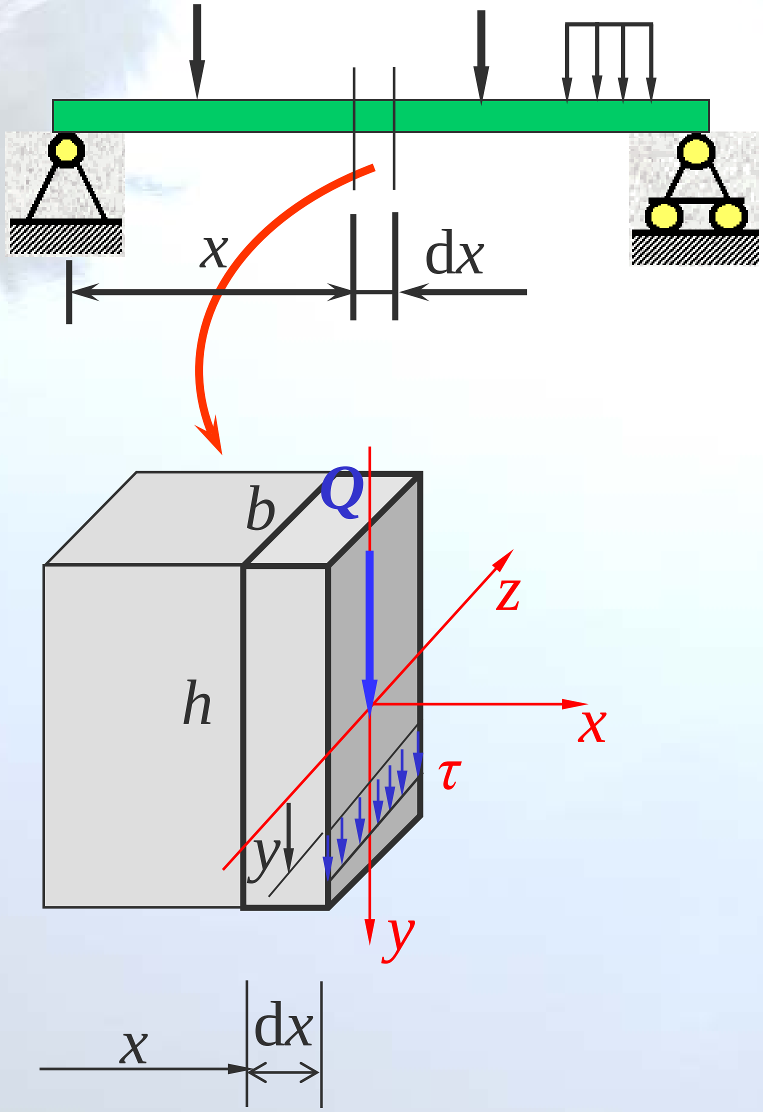
$$a=\frac{l}{3}$$

§ 5–4　弯曲剪应力





§ 5–4　弯曲剪应力

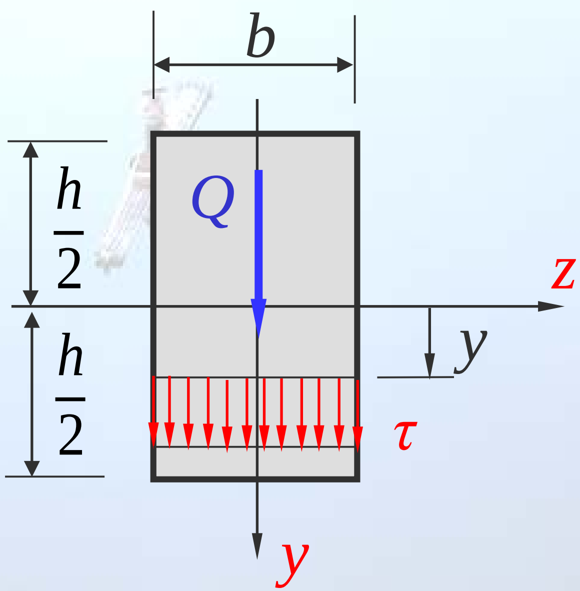


一、　矩形截面梁

1、两点假设：

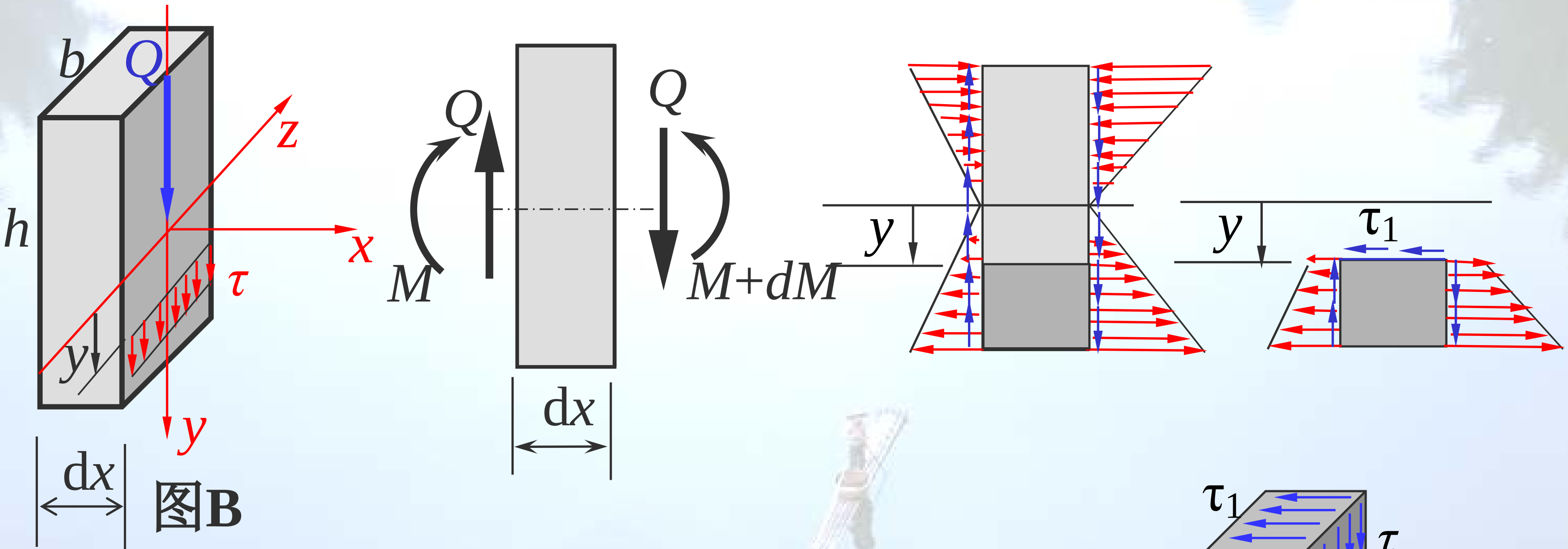
（1）剪应力与剪力平行；

（2）剪应力沿宽度均匀分布。

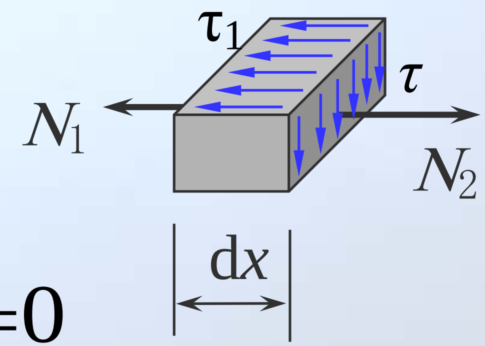


2、研究方法：分离体平衡

（1）在梁上取微段如图**B**；



图**B**

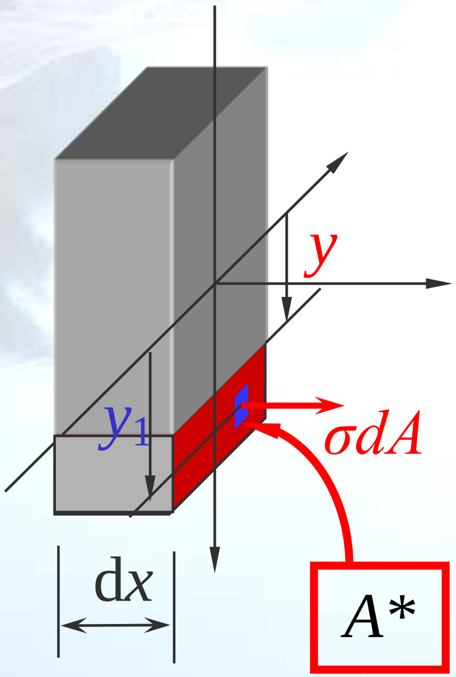


（2）在微段上取一块，求平衡

$$\sum X=0,\quad N_2-N_1-\tau_1 b(dx)=0$$

$$N_2=\int_{A^*}\sigma \mathrm{d}A=\int_{A^*}\frac{(M+dM)}{I_z}\cdot y_1\mathrm{d}A$$

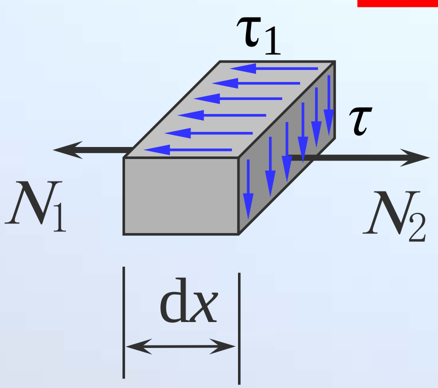
$$=\frac{(M+dM)}{I_z}\int_{A^*}y_1\mathrm{d}A=\frac{(M+\mathrm{d}M)S_z^*}{I_z}$$



同理： $N_1=\frac{MS_z^*}{I_z}$

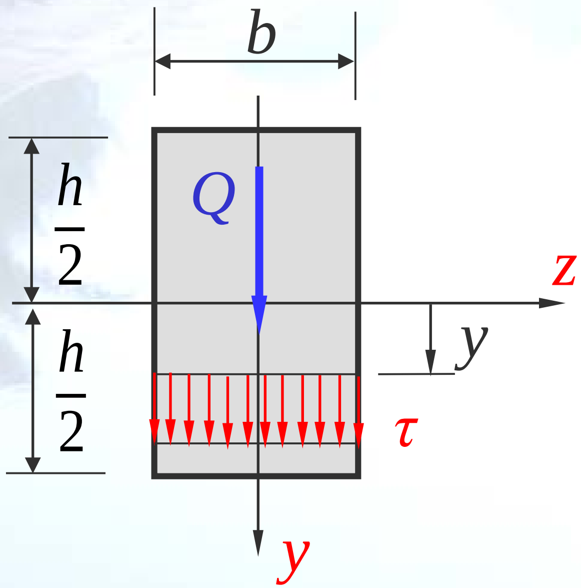
由 $N_2-N_1-\tau_1 b(dx)=0$

$$\therefore \frac{(M+\mathrm{d}M)S_z^*}{I_z}-\frac{MS_z^*}{I_z}-\tau_1 b\mathrm{d}x=0$$



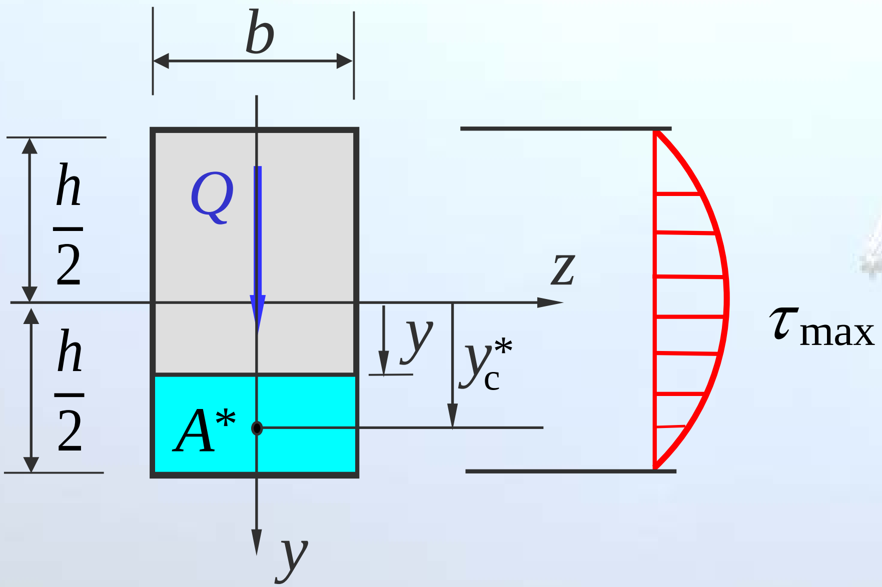
$$\tau_1=\frac{\mathrm{d}M\ S_z^*}{\mathrm{d}x\ bI_z}=\frac{QS_z^*}{I_z b}\quad(\because\ \frac{\mathrm{d}M}{\mathrm{d}x}=Q)$$

由剪应力互等定理 $\boxed{\tau=\frac{QS_z^*}{I_z b}}$



$$\boxed{\tau=\frac{QS_z^*}{I_z b}}$$

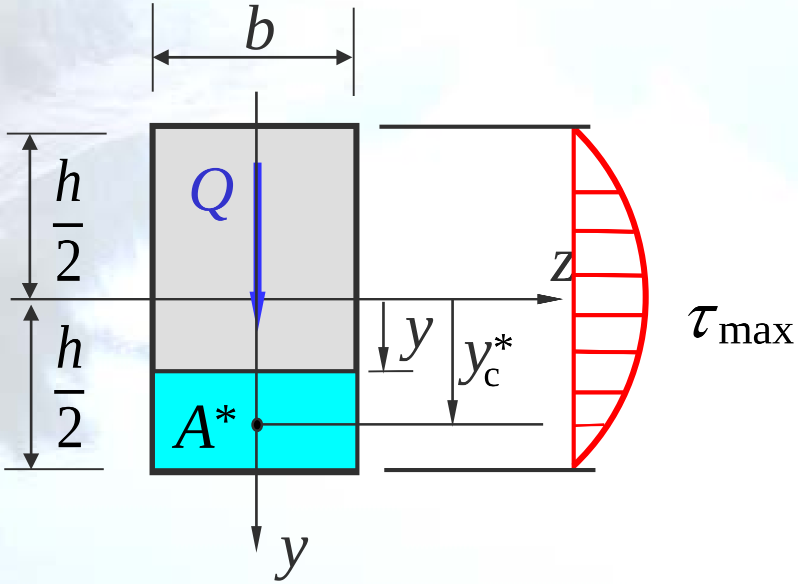
$$S_z^*=A^*\cdot y_c^*$$
$$=b(\frac{h}{2}-y)(y+\frac{h}{4}-\frac{y}{2})$$
$$=b(\frac{h}{2}-y)(\frac{h}{4}+\frac{y}{2})$$
$$=\frac{b}{2}(\frac{h}{2}-y)(\frac{h}{2}+y)$$
$$=\frac{b}{2}(\frac{h^2}{4}-y^2)$$



$$\therefore\quad \tau_{矩}=\frac{Q}{2I_z}(\frac{h^2}{4}-y^2)$$

当 $y=0$ 时， $\tau=\tau_{max}$

$$y=\pm\frac{h}{2}\ ,\quad \tau=0$$



$$\tau_{矩}=\frac{Q}{2I_z}\left(\frac{h^2}{4}-y^2\right)$$

当 $y=0$ 时，

$$\tau_{max}=\frac{Q}{2I_z}\times\frac{h^2}{4}=\frac{Q}{2\times\frac{bh^3}{12}}\times\frac{h^2}{4}$$

$$=\frac{3Q}{2bh}=1.5\frac{Q}{A}=1.5\tau_{平均}$$

$$\boxed{\tau_{max}=1.5\tau_{平均}}$$

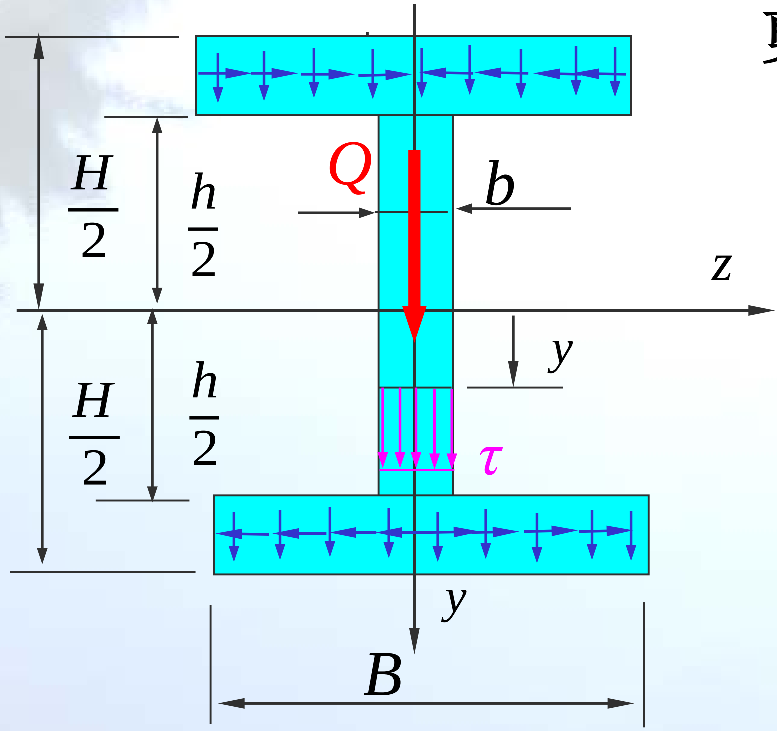
矩形截面弯曲剪应力分布规律：

τ方向：与横截面上剪力方向一致；

τ大小：沿截面宽度均匀分布，沿高度h分布为抛物线。

最大剪应力为平均剪应力的1.5倍。

二、 工字形截面梁

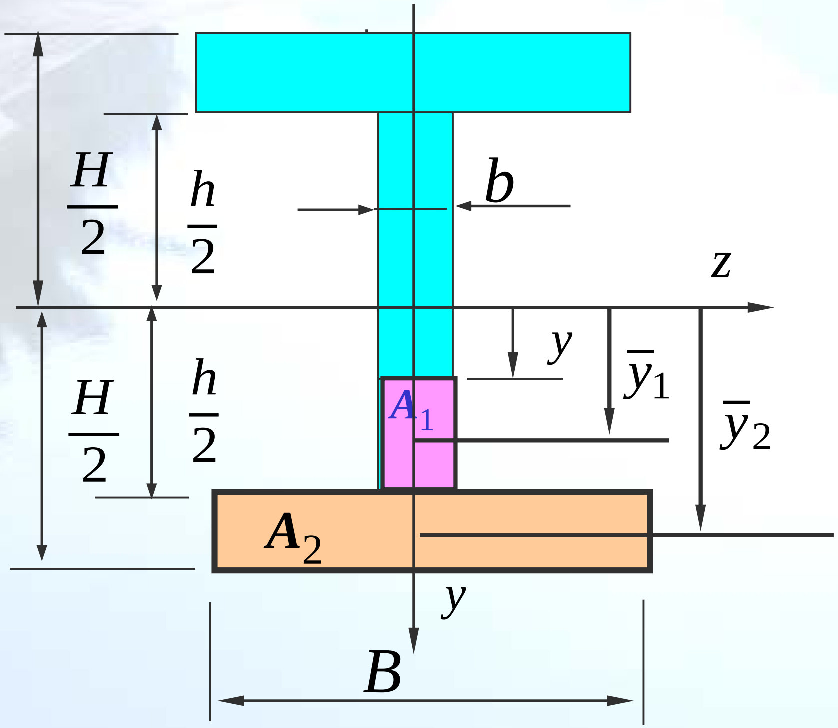


腹板为狭长矩形，可以采用前述的两个假设。采用相同的推导，得到剪应力公式：

$$\tau = \frac{QS_Z^*}{I_Z b}$$

分为腹板和翼板：

翼板上除了有平行于Q的剪应力分量外，还有水平分量。



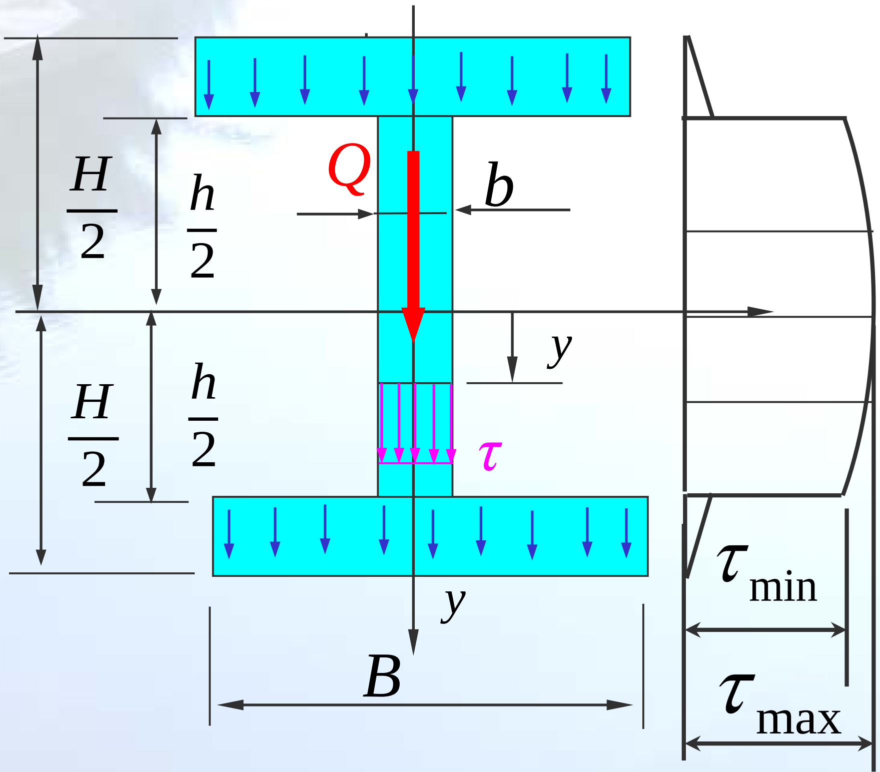
$$S_z^* = A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= b\left(\frac{h}{2} - y\right)\left(y + \frac{\frac{h}{2} - y}{2}\right)$$

$$+ B\left(\frac{H}{2} - \frac{h}{2}\right)\left(\frac{h}{2} + \frac{H}{4} - \frac{h}{4}\right)$$

$$\therefore S_Z^* = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right) + \frac{B}{8}\left(H^2 - h^2\right)$$

$$\therefore \tau = \frac{Q}{I_Z b}\left[\frac{B}{8}(H^2 - h^2) + \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)\right]$$



$y=0$时，

$$\therefore \tau_{\max} = \frac{Q}{I_Z b}\left[\frac{B}{8}(H^2 - h^2) + \frac{bh^2}{8}\right]$$

$$= \frac{Q}{I_Z b}\left[\frac{BH^2}{8} - (B-b)\frac{h^2}{8}\right]$$

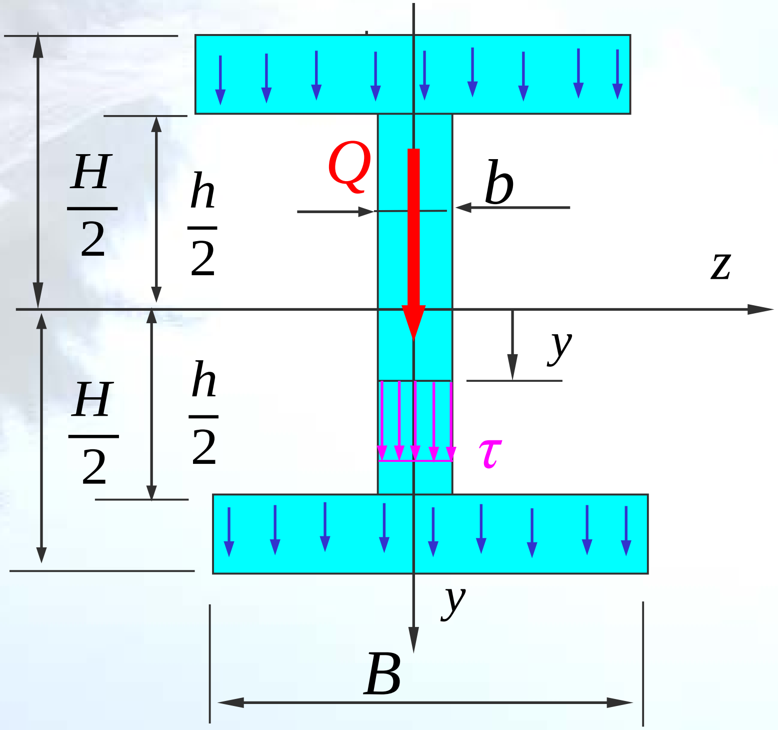
$y=\pm\frac{h}{2}$ 时，

$$\tau_{\min} = \frac{Q}{I_Z b}\left[\frac{BH^2}{8} - \frac{Bh^2}{8}\right]$$

如果 $B>>b$，则$B-b\approx B$

$$\therefore \tau_{\max} \approx \tau_{\min}$$

此时腹板上的剪应力可以看成近似的均匀分布。



计算腹板上剪应力的合力：

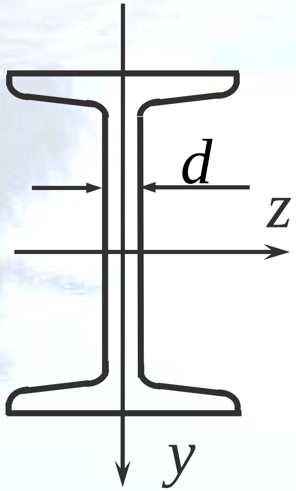
$$\int_{A_{腹}} \tau \, dA = (0.95 \sim 0.97)Q$$

即：腹板承受了95%~97%的剪力

又因为$\tau_{max} \approx \tau_{min}$

$$\therefore \tau_{max} \approx \frac{Q}{A_{腹}}$$

$A_{腹}$ —腹板的面积。

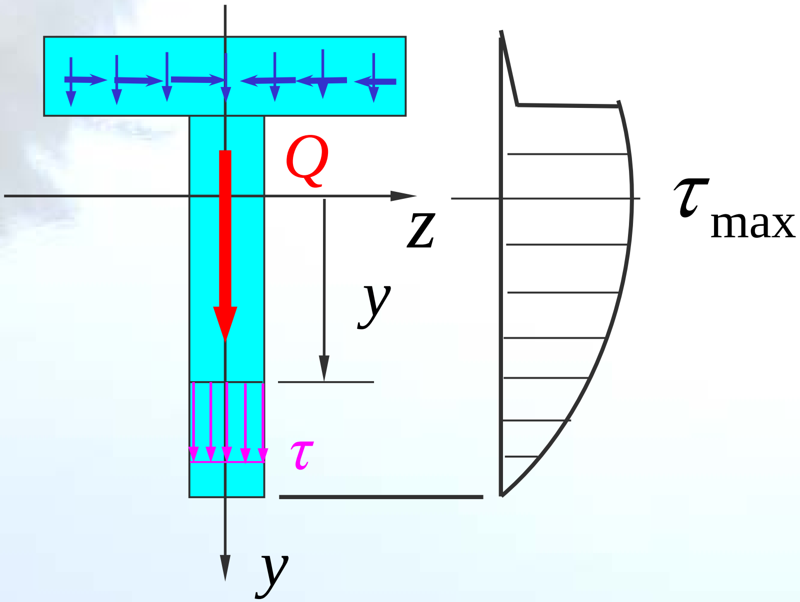


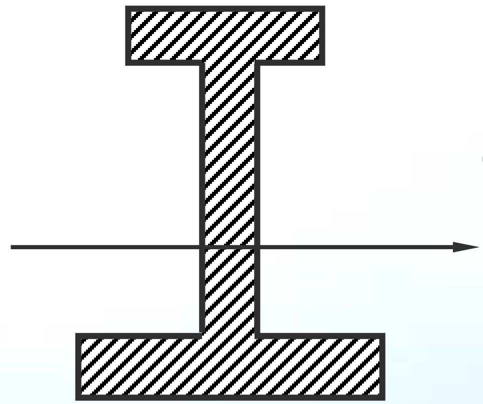
对工字形型钢，剪应力由下式计算：

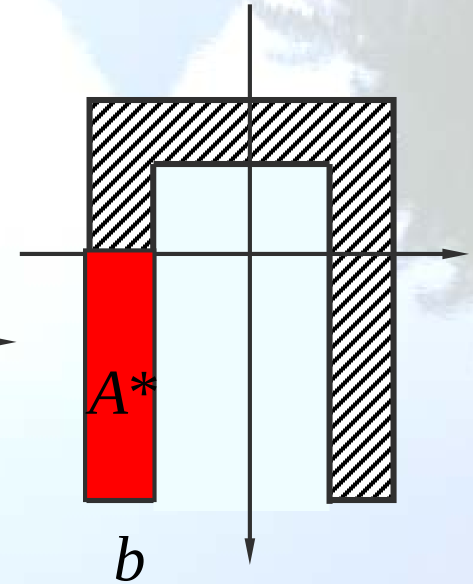
$$\tau_{max}=\frac{Q}{(\frac{I_z}{S_z})\cdot d}$$

式中：$\frac{I_z}{S_z}$由查表得到，d为腹板厚度。

三、 T字形截面梁



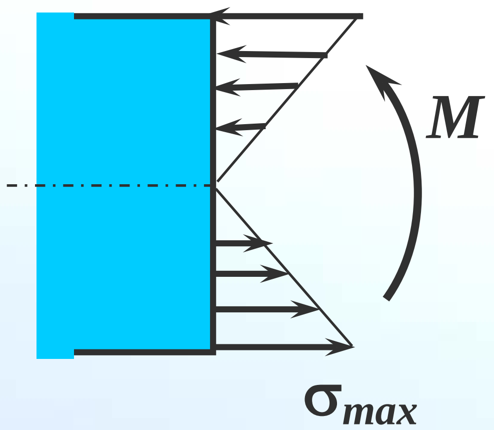


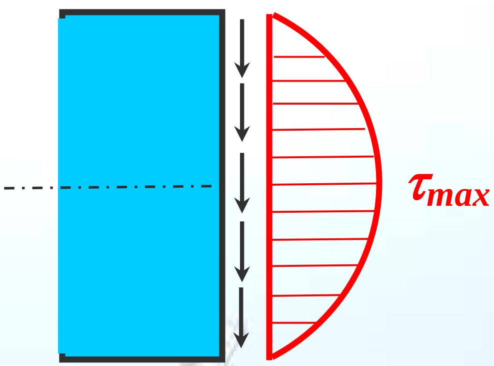


$$\tau = \frac{QS_z^*}{I_z b}$$

三、剪应力强度条件

在梁的横截面上，最大正应力发生梁截面的上下边缘，最大剪应力发生在截面的中性轴处。





剪应力强度条件：

$$\tau_{\max}=\frac{Q_{\max}S_{z\max}^{*}}{I_z b}\leq[\tau]$$

细长梁的控制因素通常是弯曲正应力，只有在下述情况下，需要进行梁的弯曲剪应力强度校核：

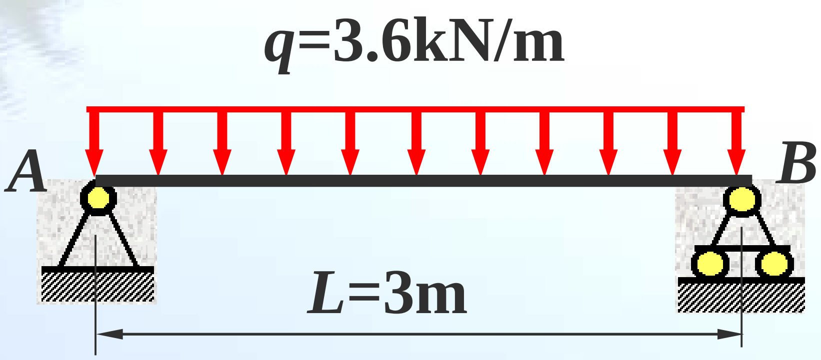
（1）梁的跨度较短，或在支座附近作用较大的载荷，以致梁的M较小，而Q较大时。

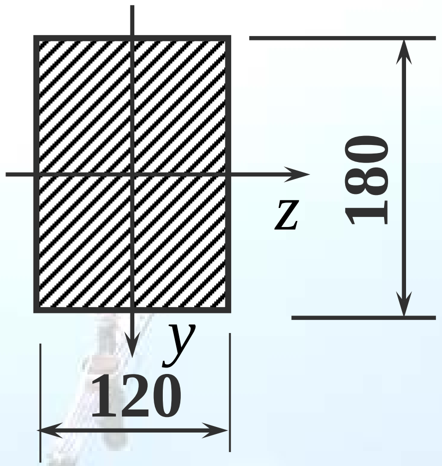
（2）铆接或焊接的组合截面，其腹板的厚度较薄，要校核腹板的剪应力。

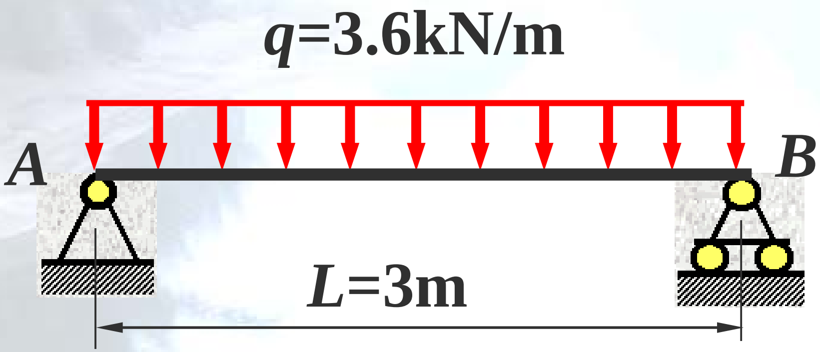
（3）经铆接、焊接或胶合而成的梁，应对焊缝、铆钉、胶合面进行剪应力校核。

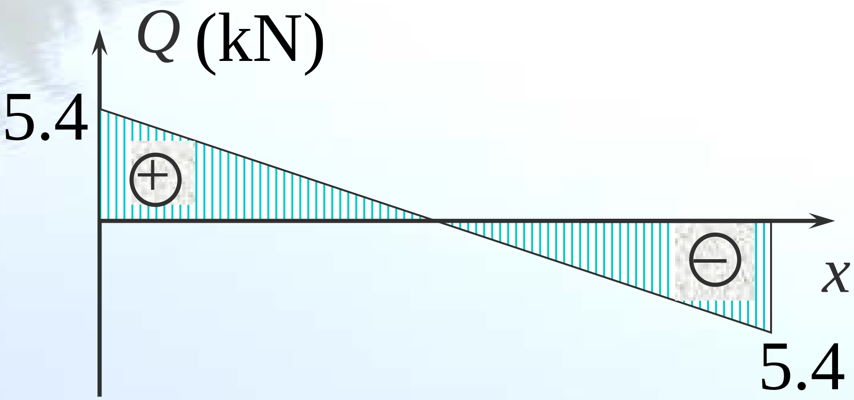
[例7]

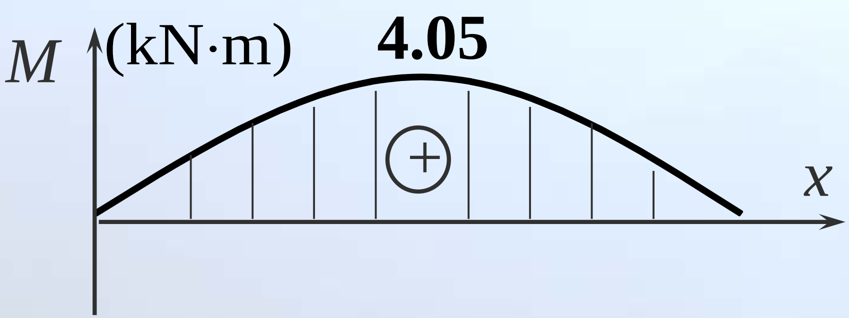
矩形($b\times h$=0.12m×0.18m）截面木梁如图， [σ]=7MPa，[τ]=0. 9 M Pa，校核梁的强度。











解：(1)画内力图找危险截面

（2）校核强度

$$\sigma_{\max}=\frac{M_{\max}}{W_z}=\frac{6M_{\max}}{bh^2}=\frac{6\times4050}{0.12\times0.18^2}$$

$$=6.25\text{MPa}<[\sigma]$$

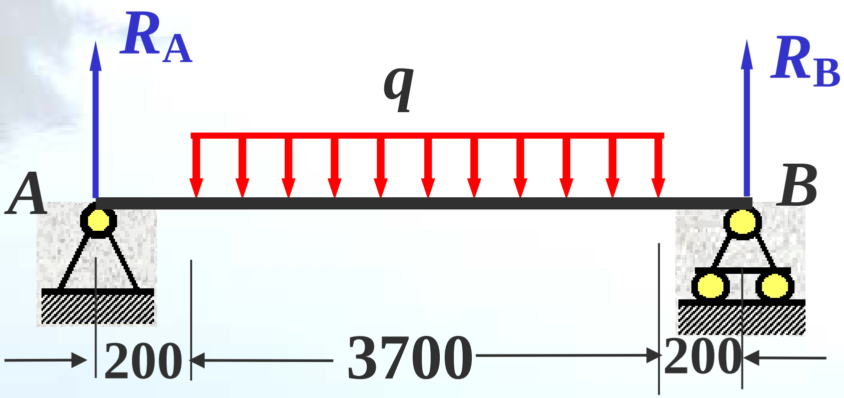
$$\tau_{\max}=1.5\frac{Q_{\max}}{A}$$

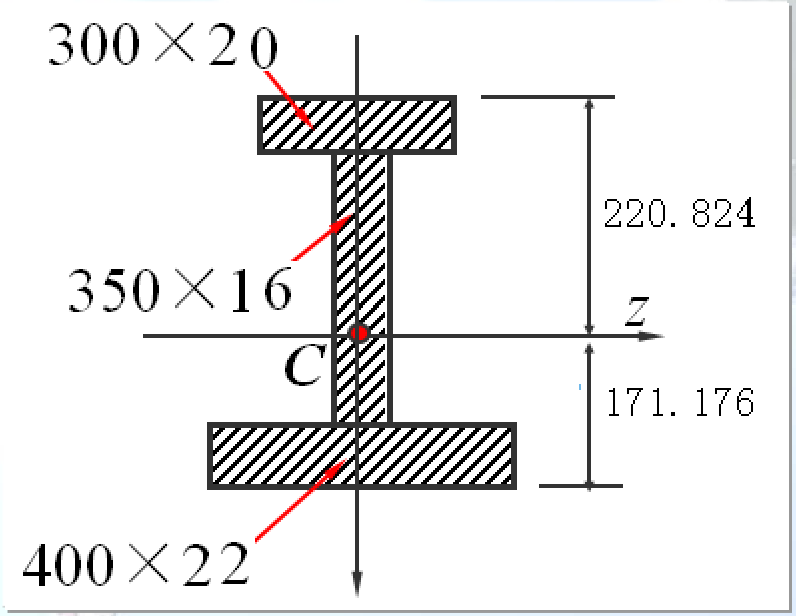
$$=1.5\times\frac{5400}{0.12\times0.18}$$

$$=0.375\text{MPa}\quad<[\tau]$$

∴梁安全！

[例8] 已知：q =407kN/m，[σ]=190MPa，[τ]=130 MPa，校核梁的强度。



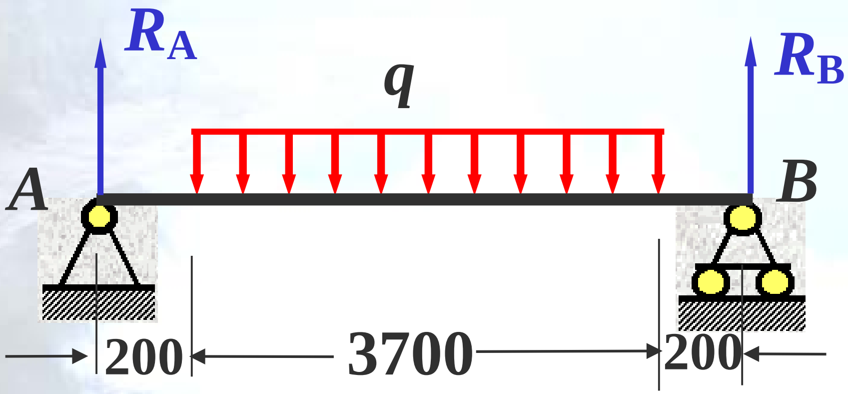


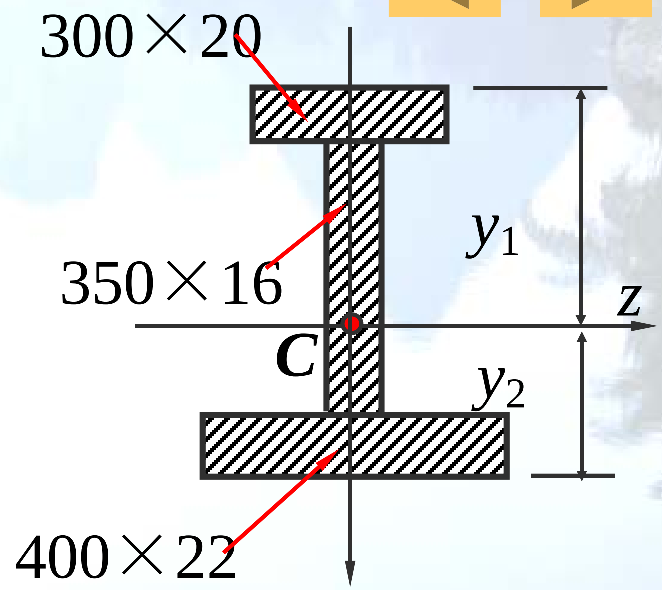
解：（1）求形心位置和惯性矩

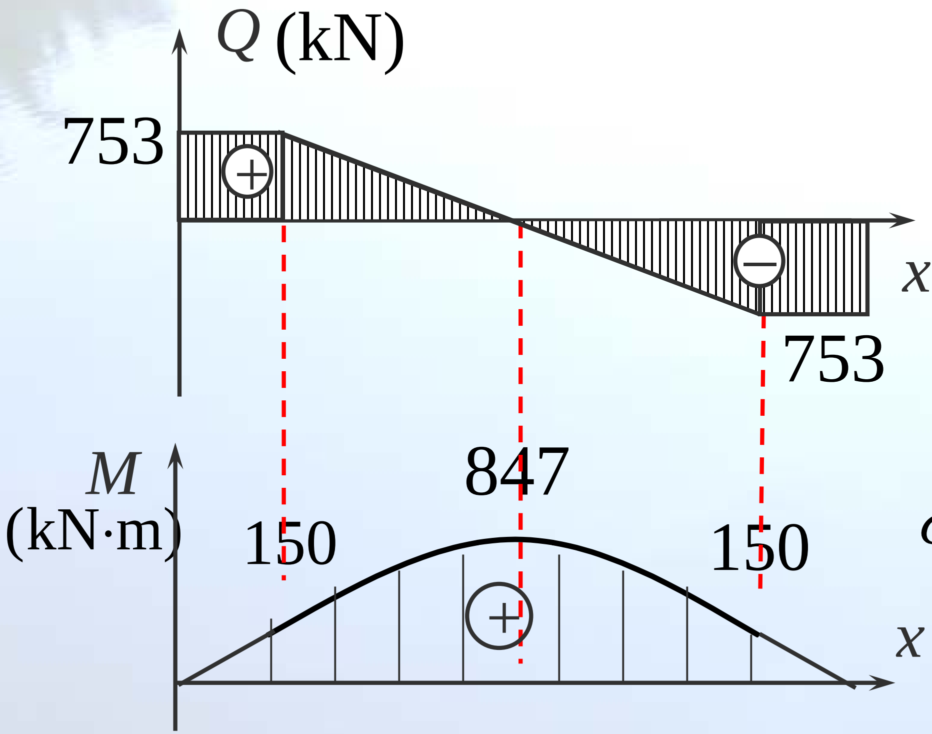
$$y_1 = 220.824(\mathrm{mm})$$

$$y_2 = 171.176(\mathrm{mm})$$

$$I_z = 5.539 \times 10^8 (\mathrm{mm}^4)$$





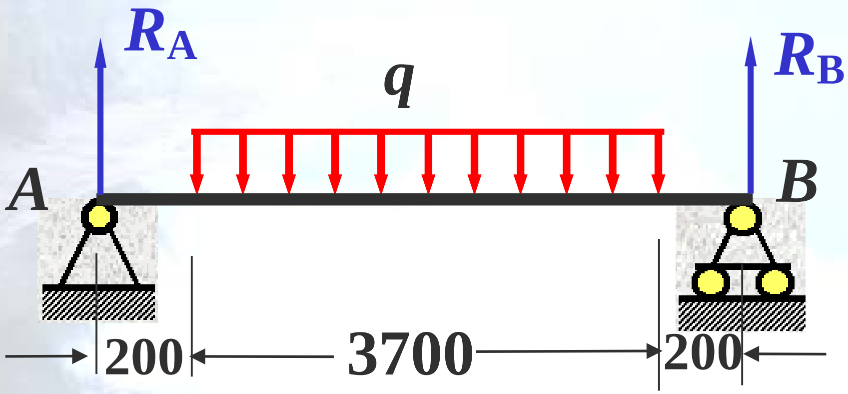


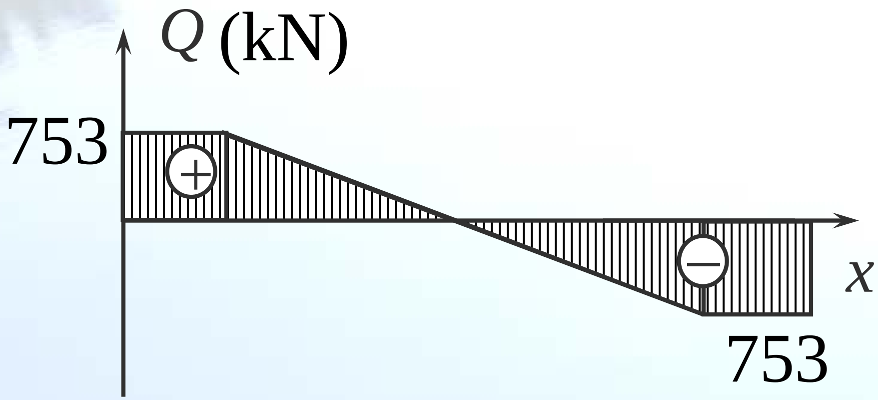
（2）画内力图

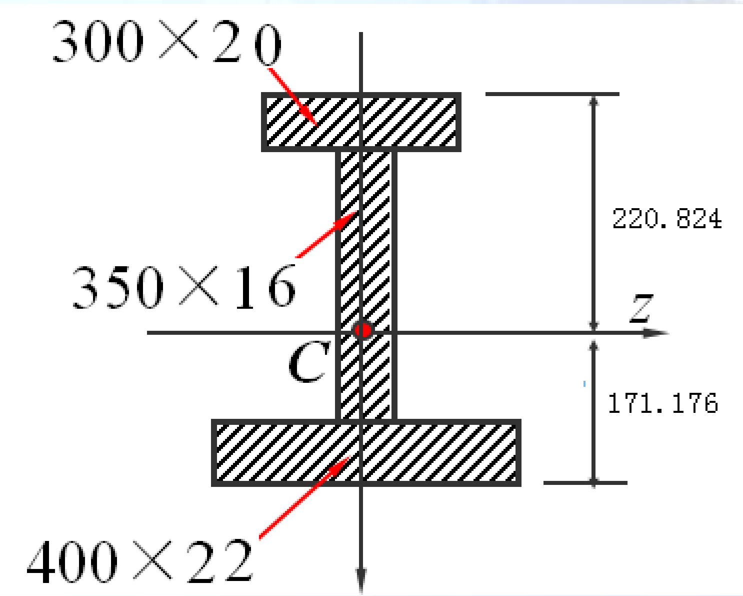
（3）弯曲正应力强度

$$\sigma_{max}=\frac{M_{max}y_1}{I_z}=\frac{847\times10^6\times220.824}{5.539\times10^8}$$

$$=337.62(\text{MPa})\ \rangle[\sigma]$$







（4）弯曲剪应力强度

$$\tau_{max}=\frac{Q_{max}S_z^*}{I_z b}$$

$$S_z^*=400\times22\times298.4+287.4\times16\times\frac{287.4}{2}$$

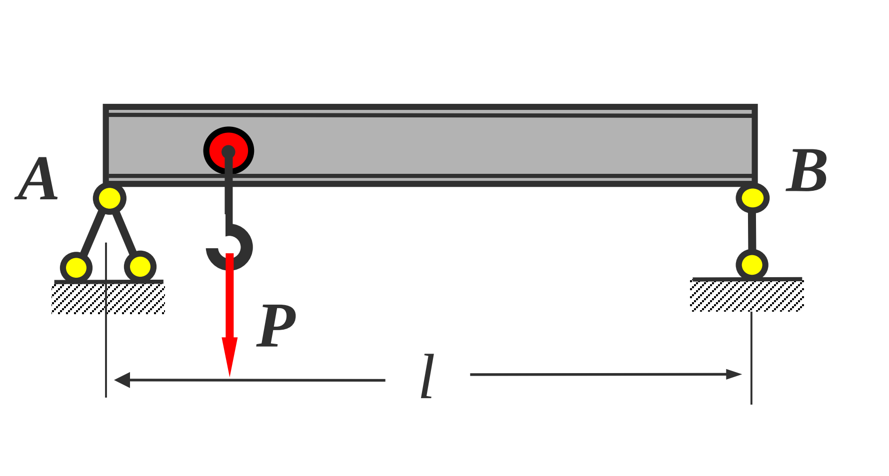
$$=2.819\times10^6(mm^3)$$

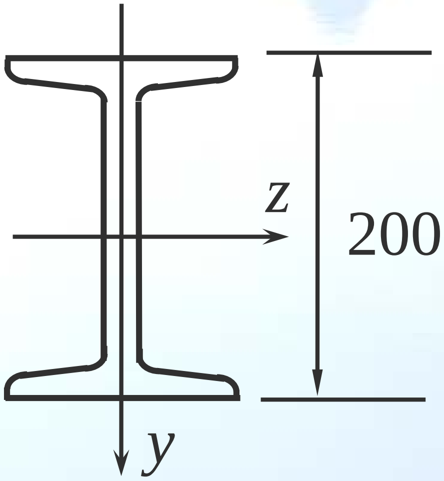
$$\tau_{max}=\frac{Q_{max}S_z^*}{I_z b}=\frac{753\times10^3\times2.819}{5.439\times10^9\times}$$

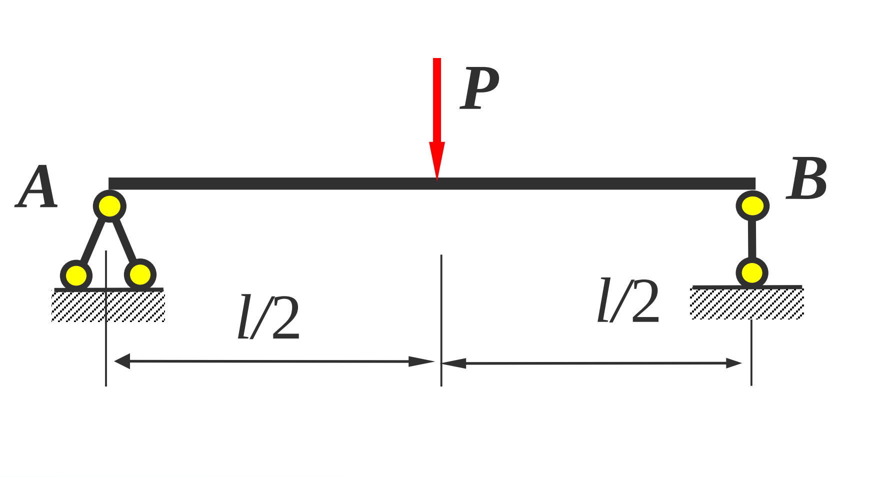
$$=239.47(MPa)\geq[\tau]$$

∴梁不安全

[例9] 已知：P=30kN，l=5m，大梁由20a工字钢制成，[σ]=170MPa，[τ]=100 MPa， 校核梁的强度。



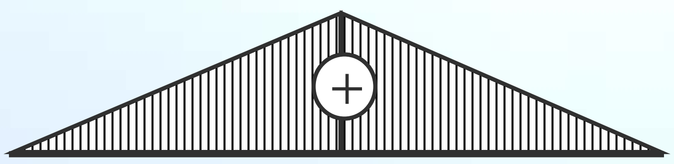




解：（1）弯曲正应力强度

小车在跨中时，梁内弯矩最大，

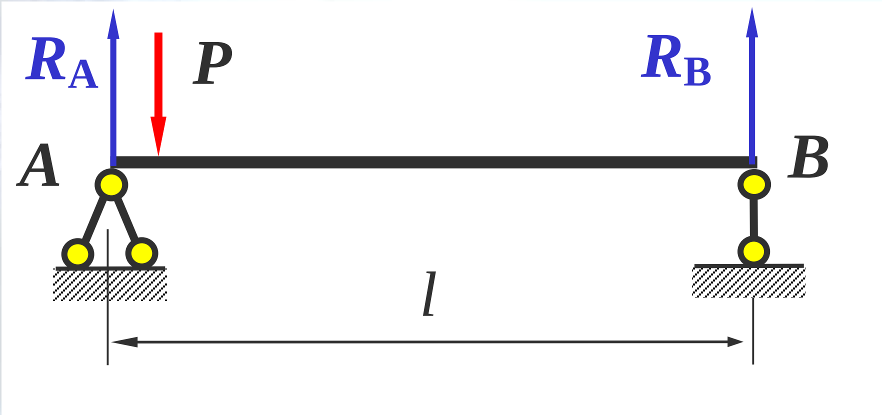
$$M_{max}=37.5(\text{kN}\cdot\text{m})$$

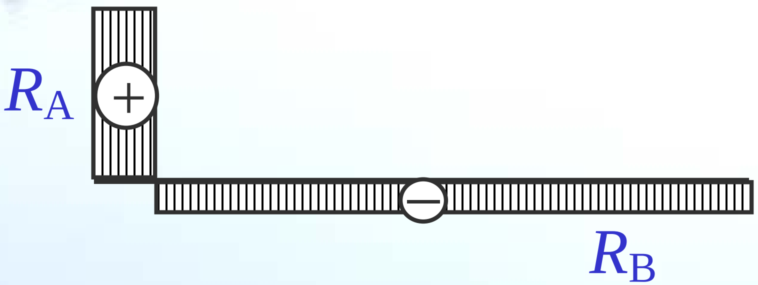


查表得 $W_z=237(\text{cm}^3)$

$$\sigma_{max}=\frac{M_{max}}{W_z}=\frac{37.5\times10^6}{237\times10^3}$$

$$=158(\text{MPa})<[\sigma]$$





（2）弯曲剪应力强度

小车在支座附近时，梁内剪力最大，

$$Q_{max}=P=30(\text{kN})$$

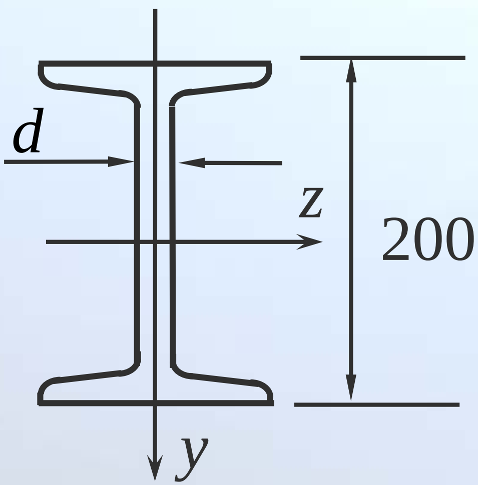
查表得 $\dfrac{I_z}{S_z^*}=17.2(\text{cm})$

$$d=7(\text{mm})$$

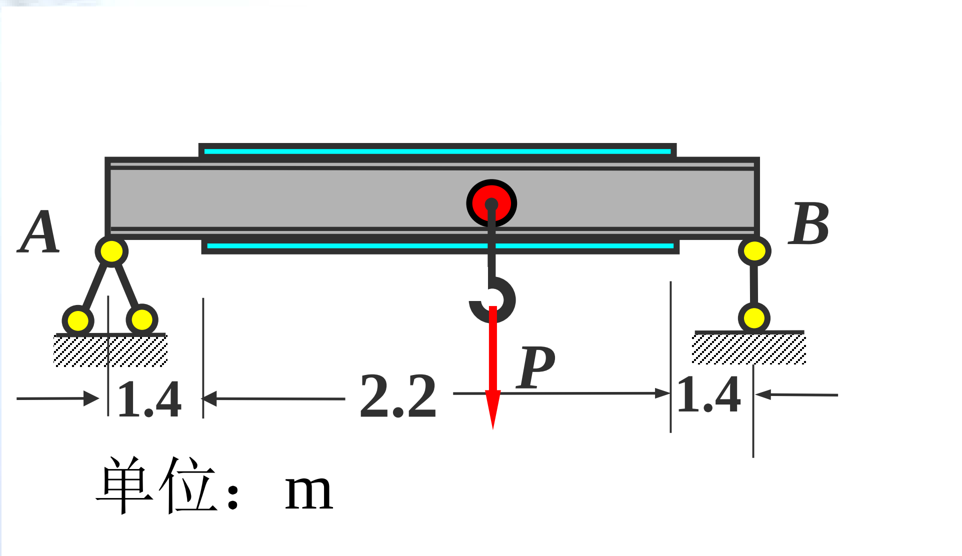
$$\tau_{max}=\frac{Q_{max}}{(I_z/S_z^*)d}=\frac{30\times10^3}{172\times7}$$

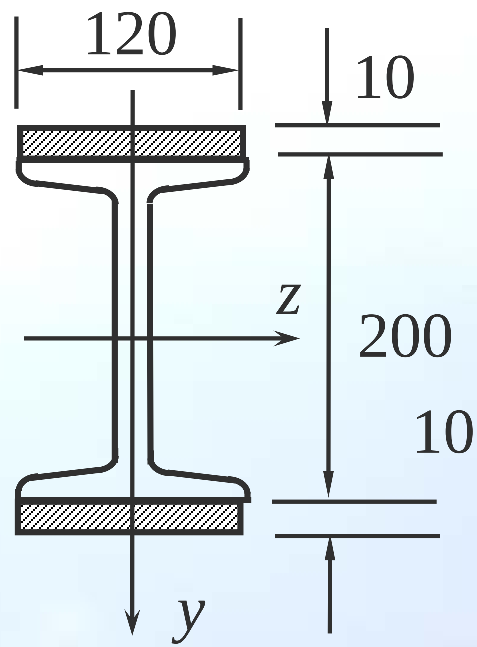
$$=24.9(\text{MPa})<[\tau]$$

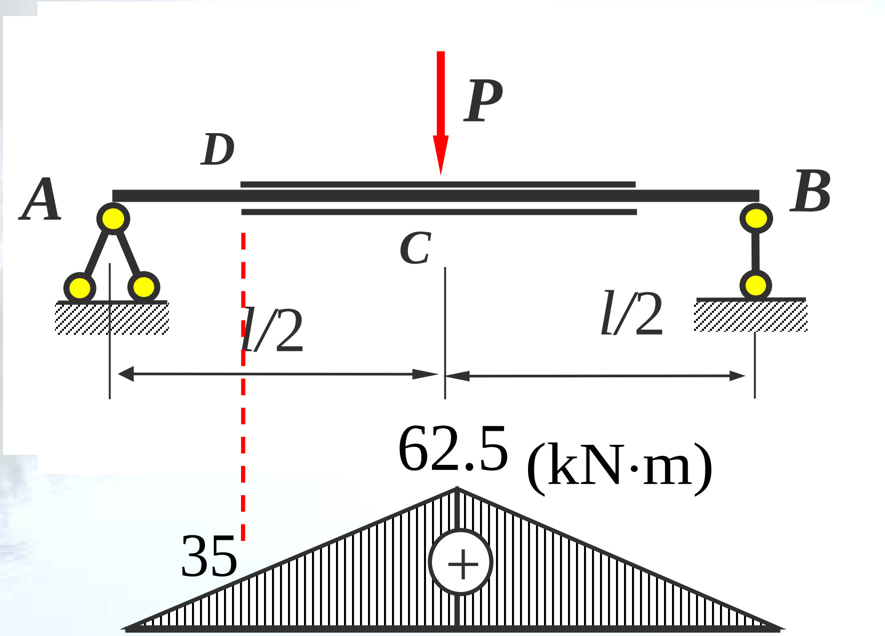
∴梁安全！

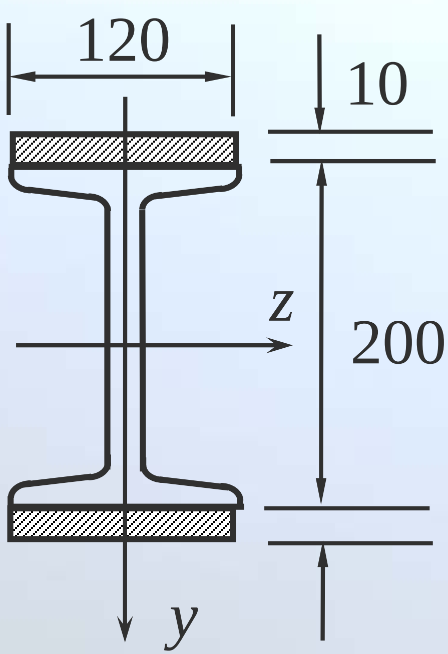


[例10] 已知：P=50kN，大梁由20a工字钢制成，用120×10mm的钢板加强，[σ]=152MPa，[τ]=95 MPa，校核梁的强度。









解：（1）跨中弯曲正应力强度

$$M_{max}=62.5(\mathrm{kN\cdot m})$$

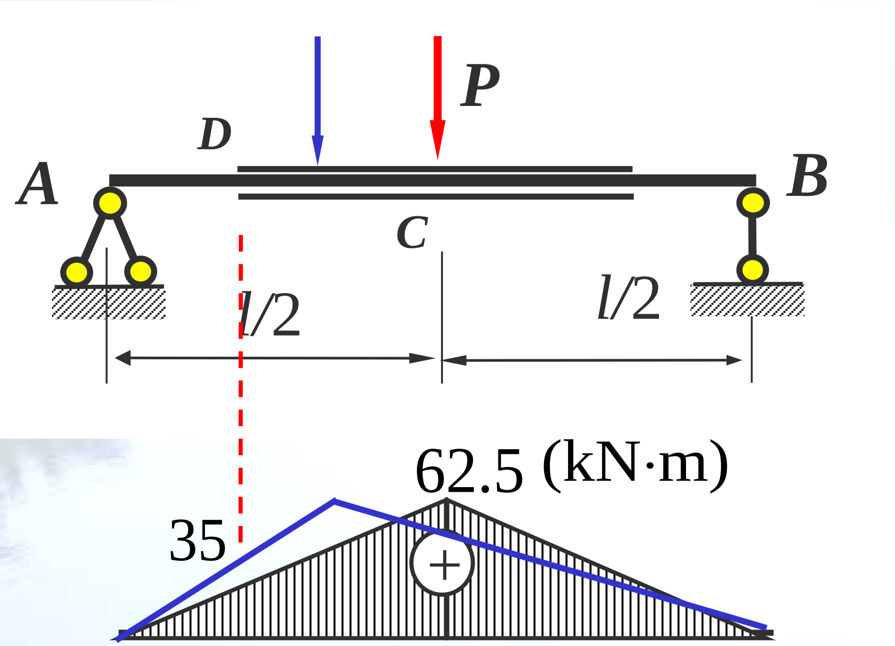
$$I_z=2370+2\times[\frac{12\times1^3}{12}+12\times1\times10.5^2]$$

$$=5018\,(\mathrm{cm}^4)$$

$$\sigma_{max}=\frac{M_{max}y_{max}}{I_z}$$

$$=\frac{62.5\times10^6\times110}{5018\times10^4}$$

$$=137(\mathrm{MPa})<[\sigma]$$



（2）*D*截面弯曲正应力强度

小车在*D*截面时

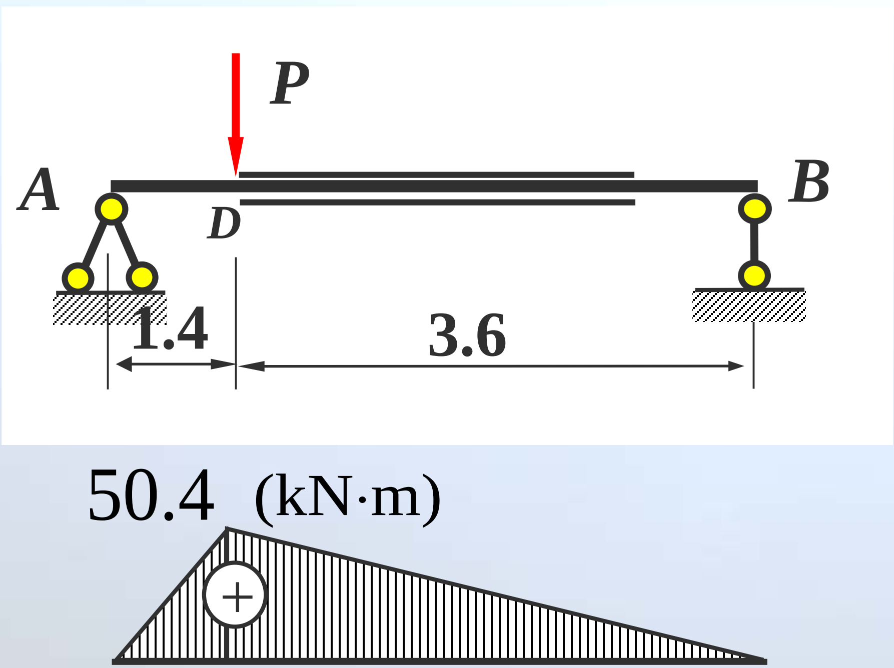
$$M_{D\max}=50.4(\mathrm{kN\cdot m})$$

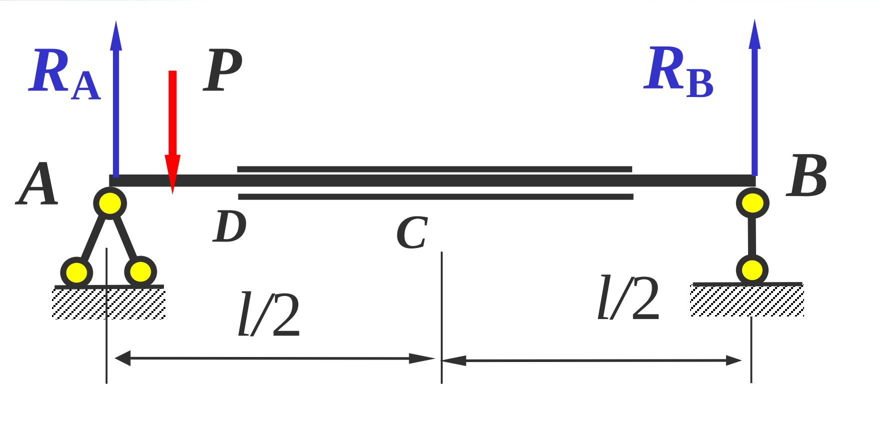
查表得 $W_Z=237(\mathrm{cm}^3)$

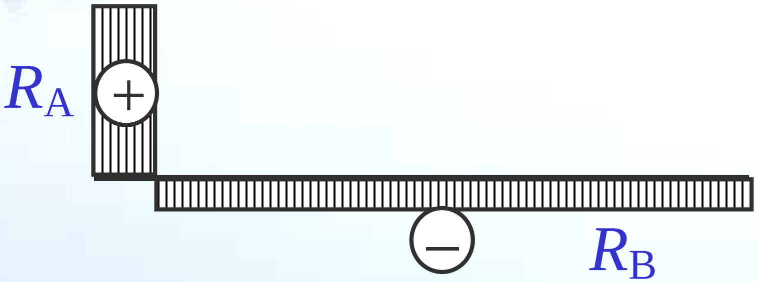
$$\sigma_{\max}=\frac{M_{D\max}}{W_z}=\frac{50.4\times10^6}{237\times10^3}$$

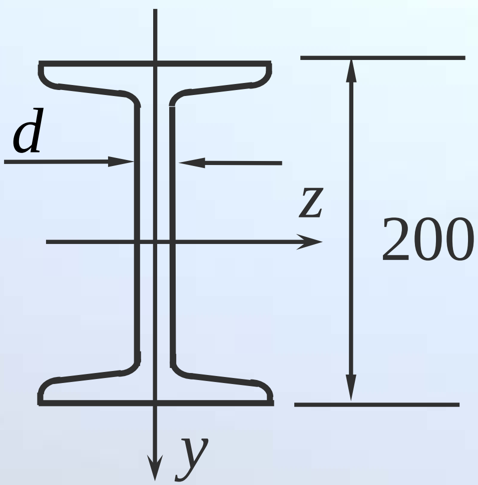
$$=213(\mathrm{MPa})>[\sigma]$$

强度不够，钢板需加长。









（3）弯曲剪应力强度

小车在支座附近时，梁内剪力最大，

$$Q_{max}=P=50(\mathrm{kN})$$

查表得 $$\frac{I_z}{S_z^*}=17.2(\mathrm{cm})$$

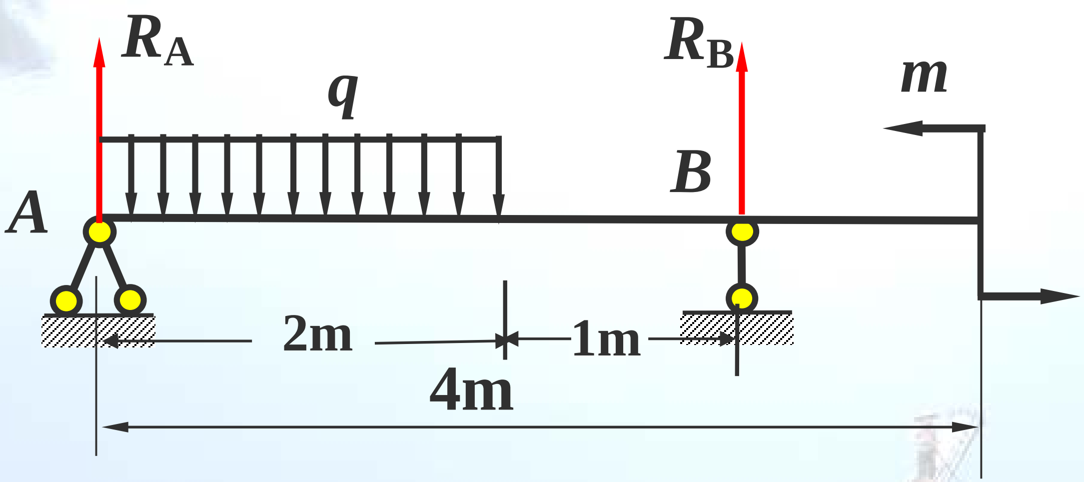
$$d=7(\mathrm{mm})$$

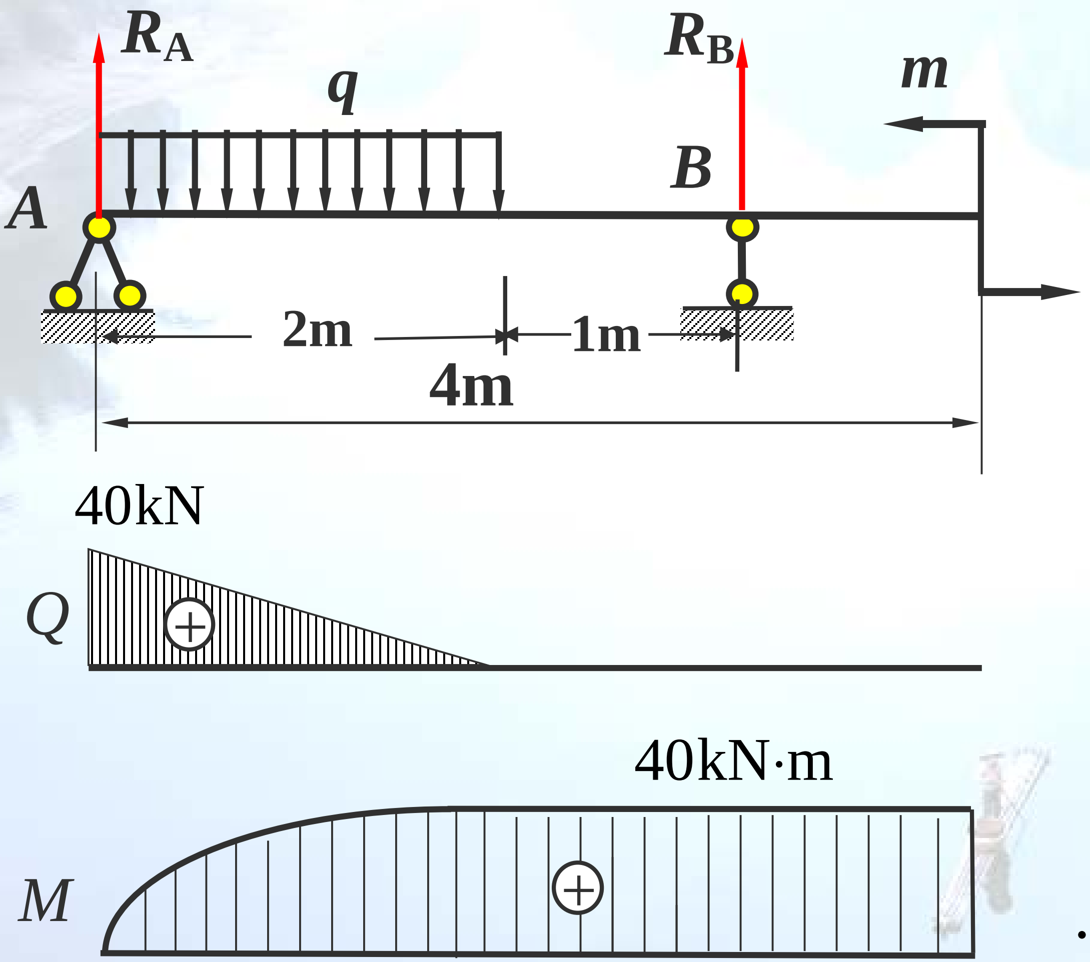
$$\tau_{max}=\frac{Q_{max}}{\left(I_z/S_z^*\right)d}=\frac{50\times10^3}{172\times7}$$

$$=41.5(\mathrm{MPa})<[\tau]$$

∴梁剪应力强度足够！

[例11] 已知：m=40kN·m，q=20kN/m，[σ]=170MPa，[τ]=100 MPa，试选择工字钢型号。





解： $R_A=40\text{kN}$

$R_B=0$

画 Q、M 图

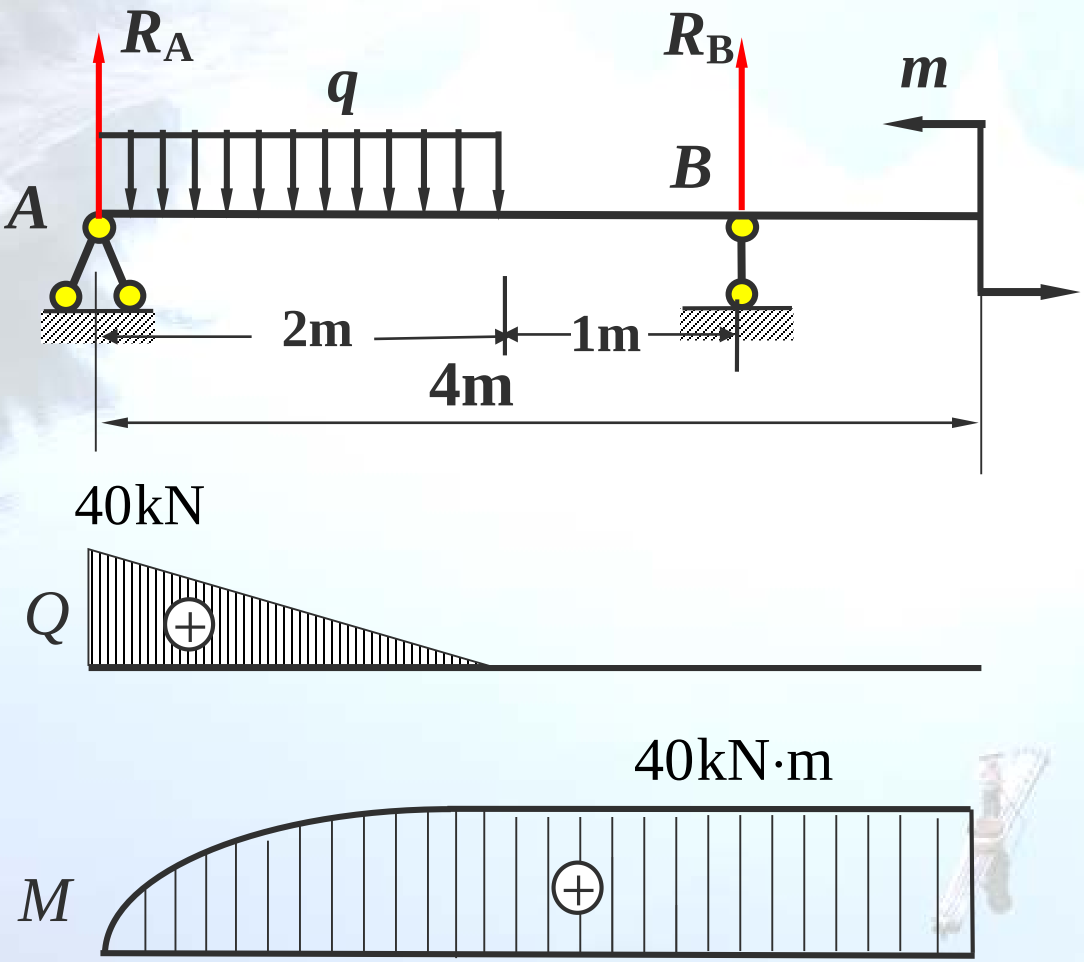
（1）弯曲正应力强度

$$\sigma_{\max}=\frac{M_{\max}}{W_z}\le[\sigma]$$

$$\therefore W_z\ge\frac{M_{\max}}{[\sigma]}=\frac{40\times10^6}{170}$$

$$=235.3\text{cm}^3$$

查表，选择No.20a工字钢，$W_z=237\text{cm}^3$



（2）弯曲剪应力强度

查表得：

$$I_Z : S_Z = 17.2(\text{cm})$$

$$d = 7(\text{mm})$$

$$\tau_{max} = \frac{Q_{max}}{\left(I_z / S_z\right)d}$$

$$= \frac{40 \times 10^3}{172 \times 7}$$

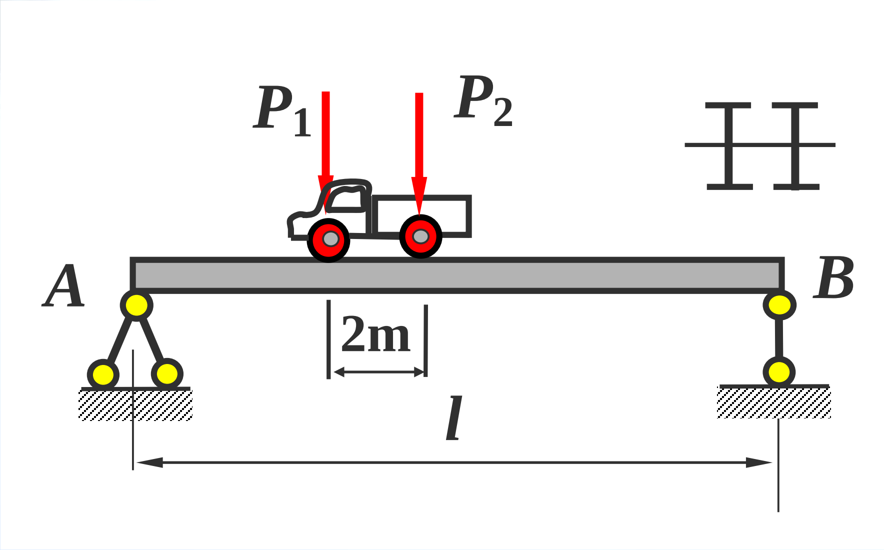
$$= 33.2(\text{MPa}) < [\tau]$$

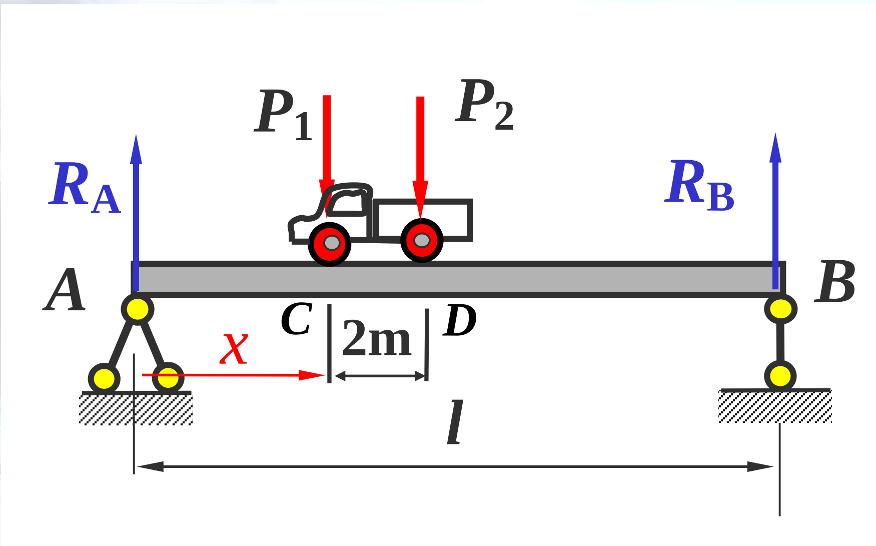
梁剪应力强度足够！

∴选择No.20a工字钢。

注意：先按正应力强度选择型号，再校核剪应力强度。

[例12]已知：前轴重P_1=10kN，后轴重P_2=50kN，l=10m，大梁由两根工字钢制成，[σ]=160MPa，[τ]=100 MPa，试选择工字钢型号。





解：（1）弯曲正应力强度

设小车在距左端 x 距离

$$R_A = 50 - 6x$$

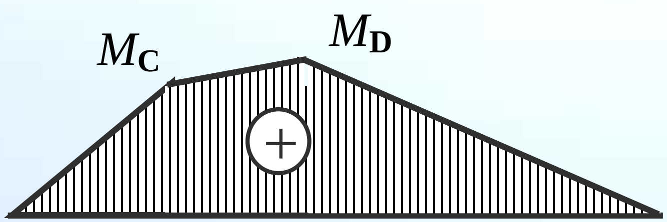
$$R_B = 10 + 6x$$

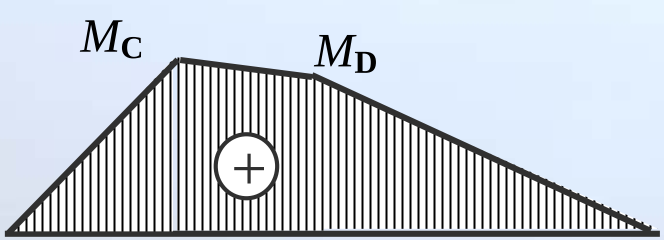
$$M_D = R_A \cdot (x+2) - P_1 \times 2$$

$$= (50-6x)\cdot(x+2) - 2P_2$$

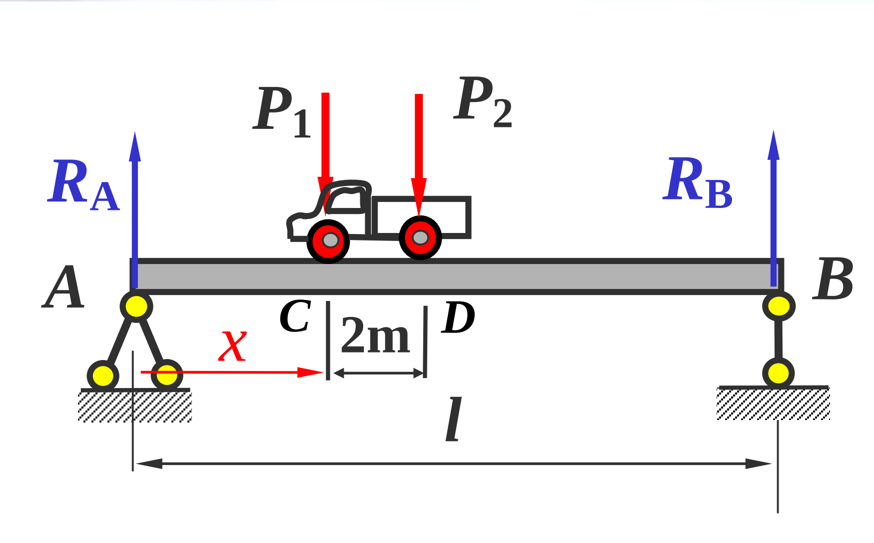
令 $\dfrac{dM_D}{dx} = 0$

$$\therefore\ x_0 = 3.17(\mathrm{m})$$





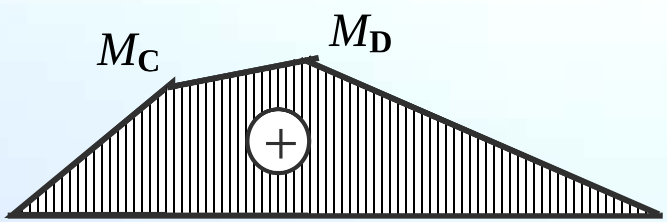
$$M_{D\max} = 140.2(\mathrm{kN\cdot m})$$



$$M_C = R_A \cdot x \quad = (50 - 6x) \cdot x$$

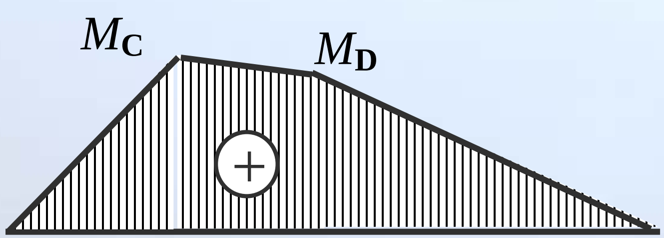
$$令 \ \frac{\mathrm{d}M_C}{\mathrm{d}x} = 0$$

$$\therefore \ x_0 = 4.17(\mathrm{m})$$



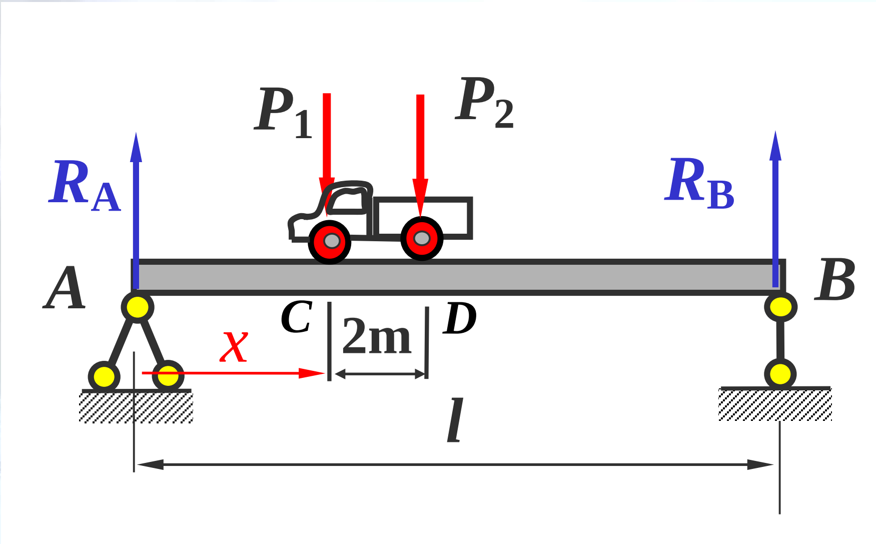
$$M_{C\max} = 104(\mathrm{kN \cdot m})$$

$$\therefore M_{\max} = 140.2\mathrm{kN \cdot m}$$



$$\sigma_{\max} = \frac{M_{\max}}{2W_z} \le [\sigma]$$

$$\therefore W_z \ge \frac{M_{\max}}{2[\sigma]} = \frac{140.2 \times 10^6}{2 \times 160} = 438\mathrm{cm}^3$$



查表，选择No.28a工字钢两根，

$$W_z=508\text{cm}^3$$

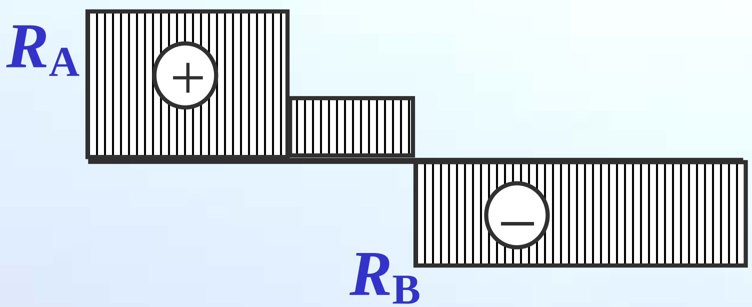
（2）弯曲剪应力强度

小车在距左端 x 距离时

$$R_A=50-6x$$

$$R_B=10+6x$$

当 $x=8$时，$Q_{max}=58\text{kN}$



查表得 $\dfrac{I_z}{S_z^*}=26.42(\text{cm})$

$$d=8.5(\text{mm})$$

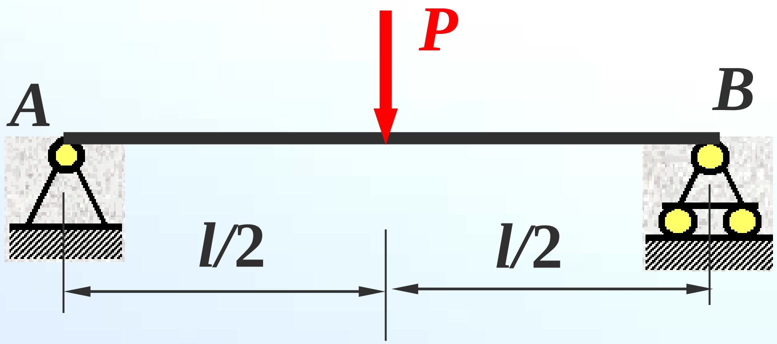
$$\tau_{max}=\frac{\dfrac{Q_{max}}{2}}{\left(I_z/S_z^*\right)d}=13.9(\text{MPa})<[\tau]$$

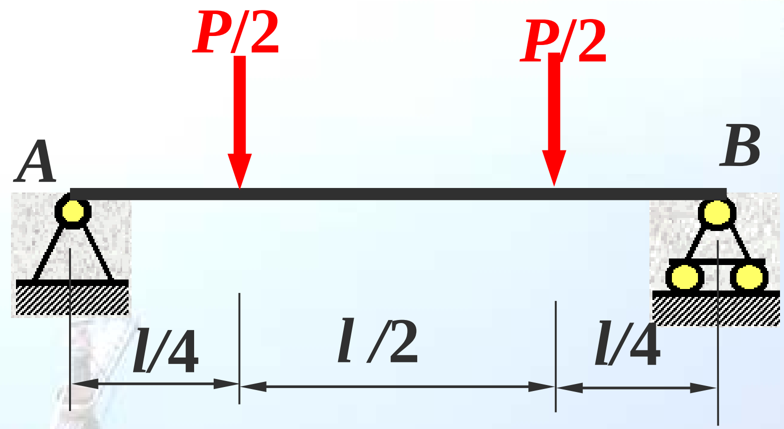
∴梁剪应力强度足够！

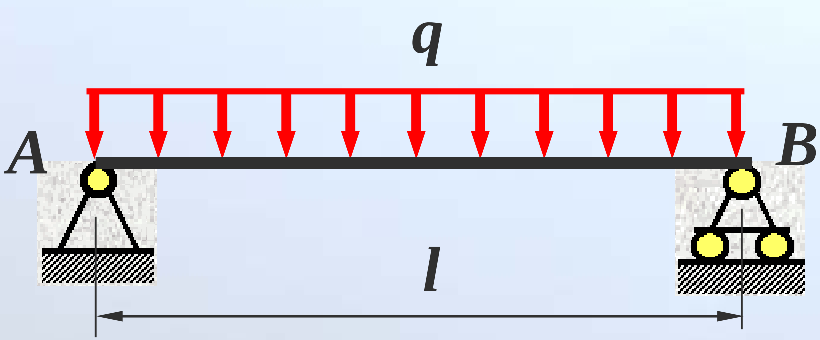
§ 5–6 提高弯曲强度的措施

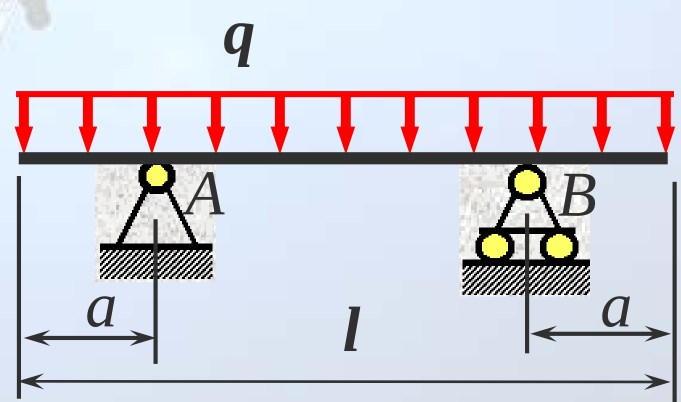
$$\sigma_{\max}=\frac{M_{\max}}{W_z}\le[\sigma]$$

（1）合理安排梁的受力情况



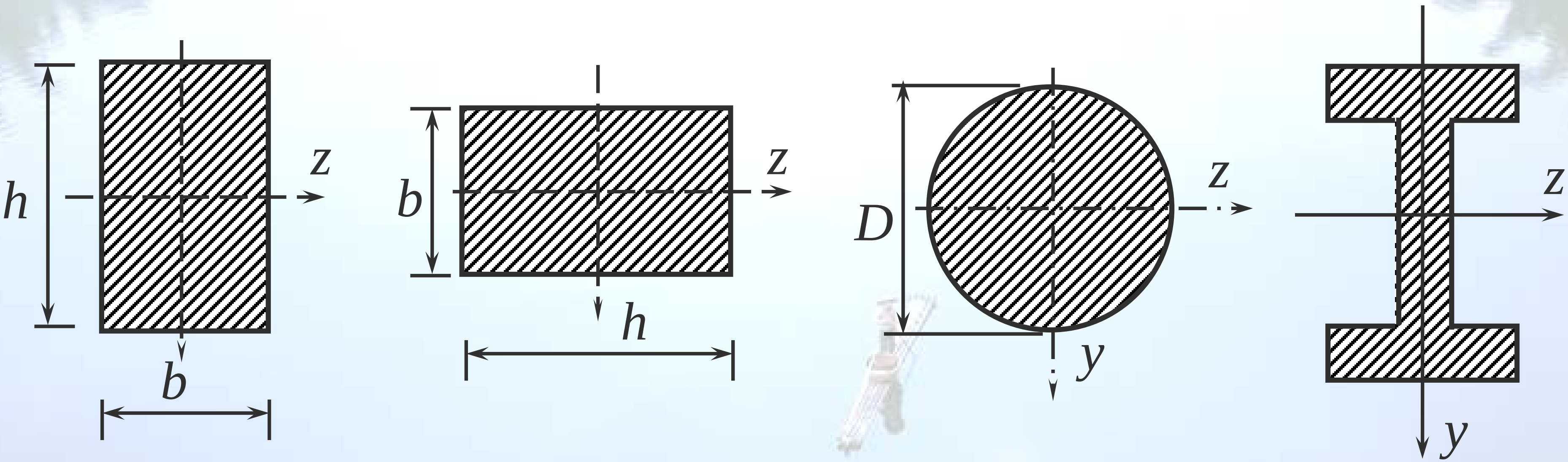






（2）合理选取截面形状

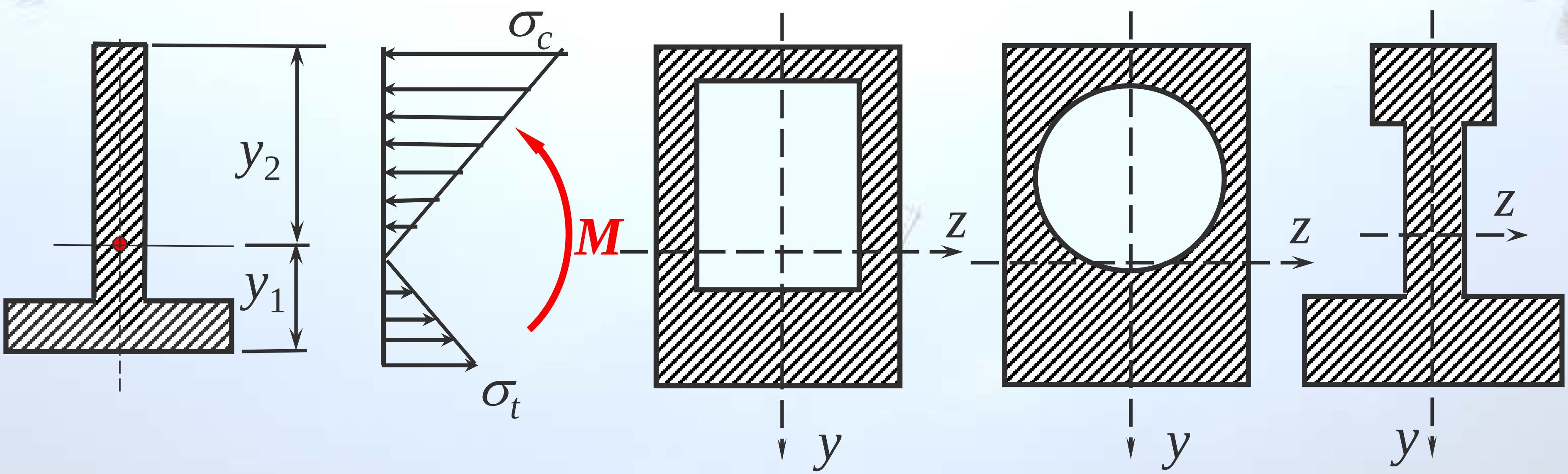
在面积相等的情况下，选择抗弯截面系数大的截面



用比值 $\frac{W_z}{A}$ 来衡量截面形状的合理性，比值越大截面的形状较为合理。

根据材料特性选择截面形状：

对于抗拉、压能力不同的材料(如铸铁、混凝土等脆性材料)，宜采用中性轴偏于受拉一侧的截面形状，充分利用材料抗拉能力差、抗压能力好的特性。



$$\frac{\sigma_{t\max}}{\sigma_{c\max}}=\frac{y_1}{y_2}=\frac{[\sigma_t]}{[\sigma_c]}$$

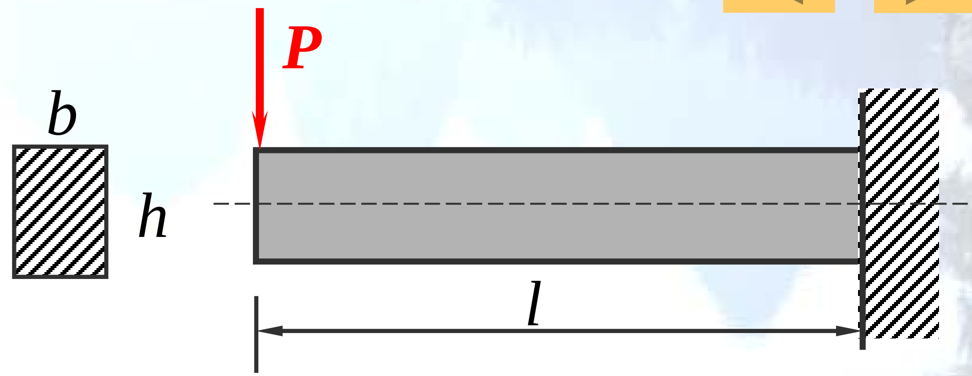
（3）等强度梁

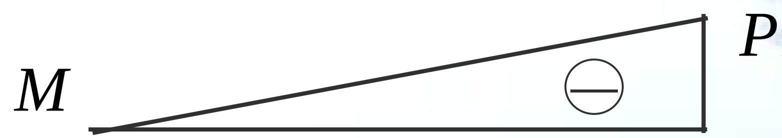
$$\sigma_{\max}(x)=\frac{|M(x)|}{W(x)}=[\sigma]$$

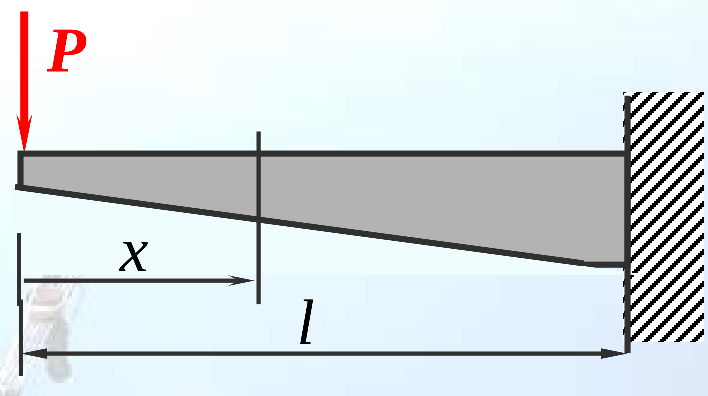
$$\because \quad M(x)=-Px$$

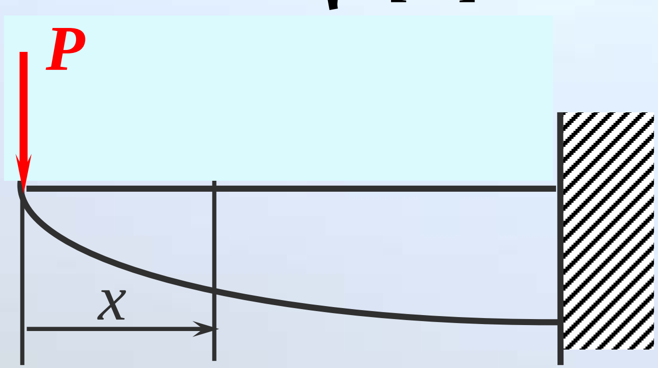
$$又\because \quad W(x)=\frac{bh^2(x)}{6}$$

$$h(x)=\sqrt{\frac{6Px}{b[\sigma]}}$$





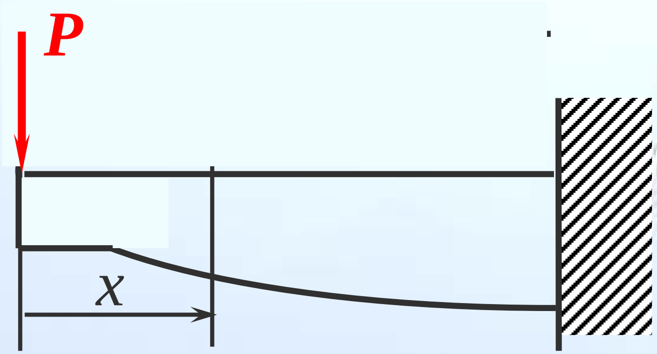




考虑弯曲剪应力强度，则梁高的最小值为：

$$\tau_{\max}=\frac{3Q}{2bh_{\min}}\leq[\tau]$$

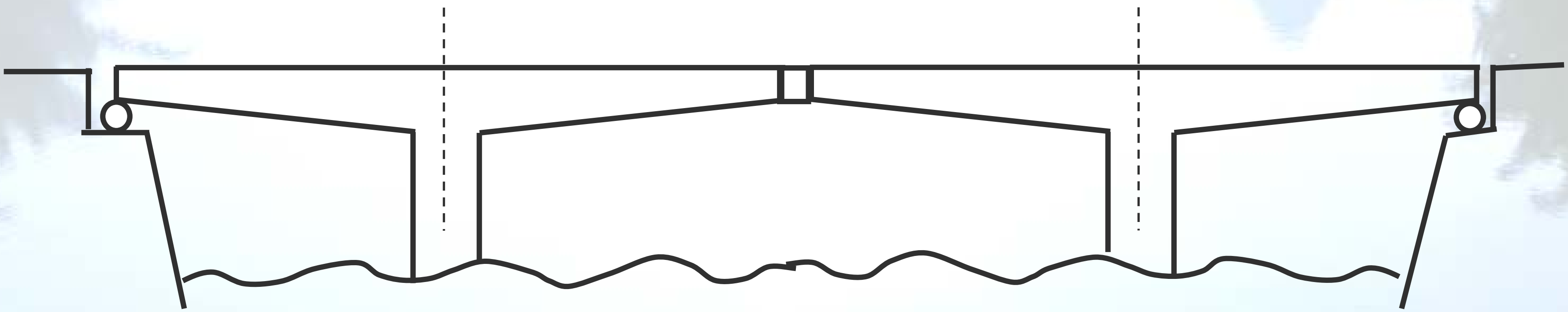
$$\therefore h_{\min}=\frac{3P}{2b[\tau]}$$



蔡甸汉江公路大桥

110m+180m+110m预应力混凝土连续刚构桥

内蒙古磴口黄河大桥



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