

FACULTAD DE INGENIERÍA EN SISTEMAS

Ingeniería en Ciencias de la Computación

Escuela Politécnica Nacional

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Grupo: GR1CC

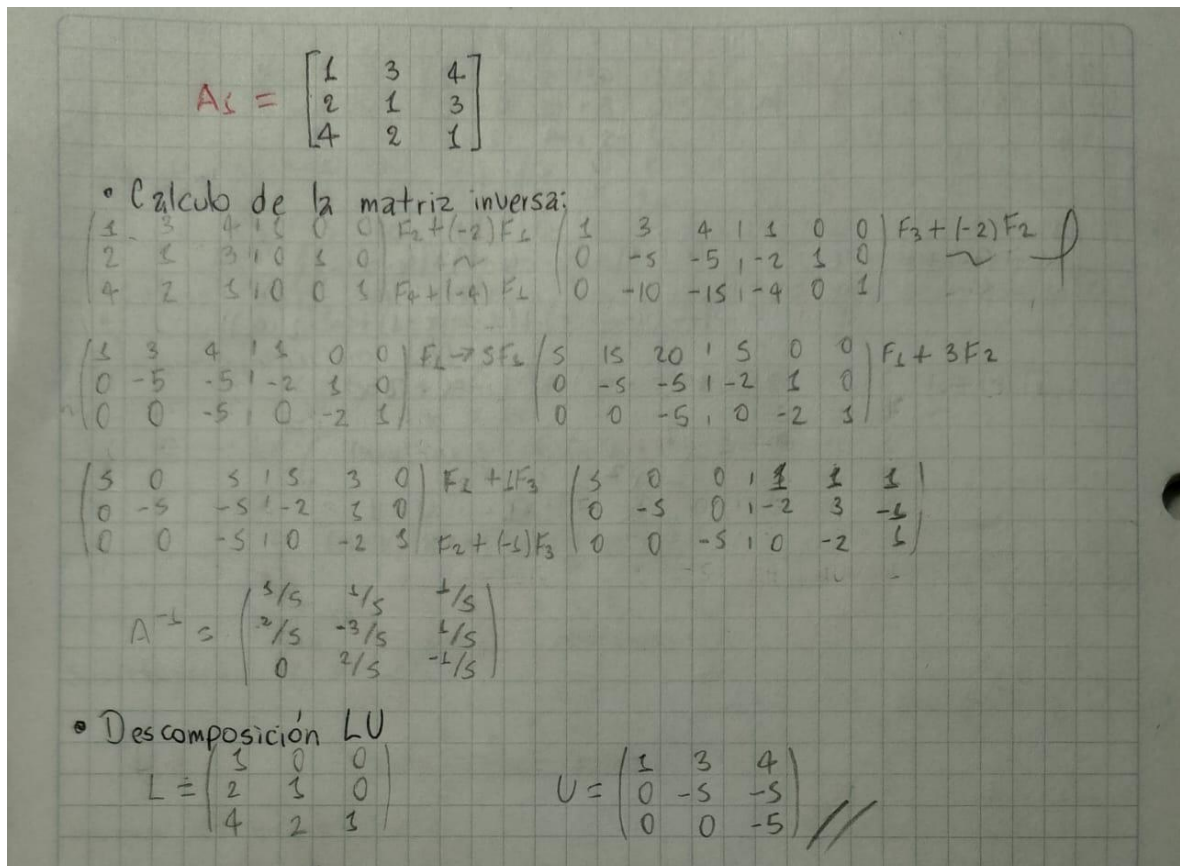
Asignatura: Métodos Numéricos

Tarea: Tarea 10.5

Tema: Factorización LU

- Para las siguientes matrices calcula la matriz inversa con el método de Gauss-Jordan.
- Calcule la descomposición LU.

1.



$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$

• Cálculo de la matriz inversa:

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 4 & 2 & 1 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} F_2 + (-2)F_1 \\ F_3 + (-4)F_1 \end{array} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -5 & -2 & 1 & 0 \\ 0 & -10 & -15 & -4 & 0 & 1 \end{array} \right) \begin{array}{l} F_3 + (-2)F_2 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -5 & -2 & 1 & 0 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array} \right) \begin{array}{l} F_2 \rightarrow 5F_2 \\ F_2 + 3F_3 \end{array} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -5 & -2 & 1 & 0 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -5 & -2 & 1 & 0 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array} \right) \begin{array}{l} F_2 + 1F_3 \\ F_2 + (-1)F_3 \end{array} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & 0 & -2 & 3 & -1 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} 3/5 & 1/5 & 1/5 \\ 2/5 & -3/5 & 1/5 \\ 0 & 2/5 & -1/5 \end{pmatrix}$$

• Descomposición LU

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 3 & 4 \\ 0 & -5 & -5 \\ 0 & 0 & -5 \end{pmatrix}$$

2.

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$A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$

• Cálculo de la matriz inversa.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 5 & 6 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} F_2 \leftrightarrow F_3 \\ F_2 + (-5)F_1 \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -15 & -5 & 0 & 1 \\ 0 & 1 & 4 & 0 & 1 & 0 \end{array} \right) \begin{array}{l} F_3 + \frac{5}{4}F_2 \\ F_2 \rightarrow 4F_2 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -15 & -5 & 0 & 1 \\ 0 & 0 & \frac{1}{4} & \frac{5}{4} & 1 & \frac{5}{4} \end{array} \right) \begin{array}{l} F_2 \rightarrow 4F_2 \\ F_2 + 15F_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & 0 & -80 & 60 & 16 \\ 0 & 0 & \frac{1}{4} & \frac{5}{4} & 1 & \frac{5}{4} \end{array} \right) \begin{array}{l} F_2 \rightarrow \frac{1}{4}F_2 \\ F_2 + 15F_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & \frac{1}{4} & \frac{5}{4} & 1 & \frac{5}{4} \end{array} \right) \begin{array}{l} F_3 + (-3)F_2 \\ F_3 + (-3)F_2 \end{array}$$

- Descomposición LU: Por teorema no aplica.

3.

$A_3 = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

• Cálculo de la matriz inversa.

$$\left(\begin{array}{ccc|ccc} 4 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} F_2 + (-\frac{1}{2})F_1 \\ F_3 + (-\frac{1}{4})F_1 \end{array} \left(\begin{array}{ccc|ccc} 4 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & \frac{5}{2} & \frac{15}{4} & -\frac{1}{4} & 0 & 1 \end{array} \right) \begin{array}{l} F_2 \leftrightarrow F_3 \\ F_2 \rightarrow 4F_2 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 4 & 2 & 1 & 1 & 0 & 0 \\ 0 & 10 & 15 & -1 & 4 & 0 \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{array} \right) \begin{array}{l} F_3 \rightarrow 2F_3 \\ F_2 + (-15)F_3 \end{array} \left(\begin{array}{ccc|ccc} 4 & 2 & 1 & 1 & 0 & 0 \\ 0 & 10 & 15 & -1 & 4 & 0 \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{array} \right) \begin{array}{l} F_2 + (-15)F_3 \\ F_3 + (-1)F_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 4 & 2 & 0 & \frac{16}{5} & -\frac{2}{5} & 0 \\ 0 & 10 & 0 & 2 & -8 & 4 \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{array} \right) \begin{array}{l} F_2 \rightarrow \frac{1}{10}F_2 \\ F_2 + (-2)F_1 \end{array} \left(\begin{array}{ccc|ccc} 4 & 2 & 0 & \frac{16}{5} & -\frac{2}{5} & 0 \\ 0 & 1 & 0 & \frac{1}{5} & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{array} \right) \begin{array}{l} F_1 + (-2)F_2 \\ F_1 + (-2)F_2 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 4 & 0 & 0 & \frac{4}{5} & -\frac{4}{5} & -\frac{4}{5} \\ 0 & 1 & 0 & \frac{1}{5} & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{array} \right) \begin{array}{l} \frac{5}{4}F_1 \rightarrow F_1 \\ F_1 \rightarrow \frac{1}{4}F_1 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & 1 & 0 & \frac{1}{5} & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & \frac{5}{2} & -\frac{1}{2} & 1 & 0 \end{array} \right)$$

- Descomposición LU: Por teorema no aplica.

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4.

$$A_4 = \begin{bmatrix} 2 & 4 & 6 & 1 \\ 4 & 7 & 5 & -6 \\ 2 & 5 & 18 & 10 \\ 6 & 12 & 38 & 16 \end{bmatrix}$$

• Cálculo de la matriz Inversa

$$\left(\begin{array}{cccc|cccc} 2 & 4 & 6 & 1 & 1 & 0 & 0 & 0 \\ 4 & 7 & 5 & -6 & 0 & 1 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 & 1 & 0 \\ 6 & 12 & 38 & 16 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} F_1 \rightarrow \frac{1}{2}F_1 \\ F_2 + (-4)F_1 \\ F_3 + (-2)F_1 \\ F_4 + (-6)F_1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & -1 & -7 & -8 & 0 & 1 & 0 & 0 \\ 0 & 1 & 12 & 9 & 0 & 0 & 1 & 0 \\ 0 & 0 & 20 & 13 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \\ \\ \\ F_4 + (-6)F_1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 1 & 7 & 8 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{3}{5} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} (-1)F_2 \\ F_3 + (-1)F_2 \\ \frac{3}{5}F_3 \\ F_4 + (-20)F_3 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 1 & 7 & 8 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{3}{5} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \\ F_3 + (-\frac{1}{2})F_4 \\ F_2 + (-8)F_4 \\ F_1 + (-\frac{1}{2})F_4 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 16/5 & -77/45 & -167/45 \\ 0 & 1 & 0 & 0 & 0 & -2/5 & 8/15 & 23/15 \\ 0 & 0 & 1 & 0 & 0 & -4/5 & 13/45 & 13/45 \\ 0 & 0 & 0 & 1 & 0 & -4/9 & -4/9 & 1/9 \end{array} \right) \begin{array}{l} \\ F_2 + (-7)F_3 \\ F_1 + (-3)F_3 \\ F_1 + (-2)F_2 \end{array}$$

• Factorización LU:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 3 & 0 & 4 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 4 & 6 & 1 \\ 0 & -1 & -7 & -8 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$